PRACTICAL - 1

1] $$\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$$

Sol^n

$$\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right] \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \times \frac{\sqrt{3a+x}+}{\sqrt{3a+x+}}$$

$$\lim_{x \to a} \frac{(a+2x-3x)}{(3a+x-4x)} \frac{(\sqrt{3a+x} + 2\sqrt{x})}{(\sqrt{a+2x} + \sqrt{3x})}$$

$$\lim_{x \to a} \frac{(a-x)}{(3a-3x)} \frac{(\sqrt{3a+x} + 2\sqrt{x})}{(\sqrt{a+2x} + \sqrt{3x})}$$

$$\frac{1}{3} \lim_{x \to a} \frac{\cancel{(a-x)}(\sqrt{3a+x} + 2\sqrt{x})}{\cancel{(a-x)}(\sqrt{a+2x} + \sqrt{3x})}$$

$$\lim_{x \to a} \frac{\sqrt{3a+x}}{\sqrt{a+2a}} + \frac{2\sqrt{a}}{\sqrt{3a}}$$

$$\lim_{x \to a} \frac{\sqrt{4a}}{\sqrt{3a}} + \frac{2\sqrt{a}}{\sqrt{3a}}$$

$$\frac{1}{3} \times \frac{4\sqrt{a}}{2\sqrt{3a}}$$

$$= \frac{2}{3\sqrt{3}}$$

2) $$\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$$

$$\lim_{y \to 0} \left(\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y} + \sqrt{a}}{\sqrt{a+y} + \sqrt{a}} \right)$$

$$\lim_{y \to 0} \frac{a+y-a}{y\sqrt{a+y}(\sqrt{a+y}+\sqrt{a})}.$$

$$\frac{y}{y\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}$$

$$\frac{1}{\sqrt{a+0}\,(\sqrt{a+0}+\sqrt{a})}$$

$$\frac{1}{\sqrt{a}\,(\sqrt{a}+\sqrt{a})}$$

$$\frac{1}{\sqrt{a}\,(2\sqrt{a})}$$

$$= \frac{1}{2a}$$

$$\lim_{x\to\frac{\pi}{6}}\left[\frac{\cos x-\sqrt{3}\sin x}{\pi-6x}\right]$$

Sol^n

$$\frac{\left(\cos\frac{\sqrt{3}}{2}h-\sin\frac{h}{2}\right)-\left(\sin\frac{3h}{2}+\cos\frac{\sqrt{3}}{2}h\right)}{6h}$$

$$\therefore \frac{+\sin 4h/2}{+6h} \qquad \because \left(\cos\frac{\sqrt{3}}{2}h-\cos\frac{\sqrt{3}}{2}h\right) = 0$$

$$\therefore \frac{\sin 4h/2}{6} \qquad \therefore \frac{\sin 4h}{12h} \quad \therefore \frac{1}{3}\times\frac{\sin h}{h}$$

$$\therefore \frac{1}{3}$$

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4] $\lim\limits_{x \to \infty} \left[\dfrac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$

$$\lim_{x \to \infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \times \frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+3} + \sqrt{x^2+1}} \times \frac{\sqrt{x^2+5} + \sqrt{x^2-3}}{\sqrt{x^2+5} + \sqrt{x^2-3}} \right]$$

$$\lim_{x \to \infty} \frac{(x^2+5-x^2+3)}{(x^2+3-x^2-1)} \times \frac{(\sqrt{x^2+3} + \sqrt{x^2+1})}{(\sqrt{x^2+5} + \sqrt{x^2-3})}$$

$$\cancel{\frac{8}{2}}^{\,4} \left(\frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+5} + \sqrt{x^2-3}} \right)$$

$$\frac{4\left(\sqrt{x^2\left(1+\frac{3}{x^2}\right)} + \sqrt{x^2\left(1+\frac{1}{x^2}\right)} \right)}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)} + \sqrt{x^2\left(1-\frac{3}{x^2}\right)}}$$

$$4 \lim_{x \to \infty} \frac{\sqrt{x^2(1+3/x^2)} + \sqrt{x^2(1+1/x^2)}}{\sqrt{x^2(1+5/x^2)} + \sqrt{x^2(1-3/x^2)}}$$

$= 4$

5) i) $f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$ for $0 < x \le \dfrac{\pi}{2}$

$= \dfrac{\cos x}{\pi - 2x}$ for $\dfrac{\pi}{2} < x < \pi$ } at $x = \pi/2$

Soln $f\left(\dfrac{\pi}{2}\right) = \dfrac{\sin x\left(\frac{\pi}{2}\right)}{\sqrt{1-\cos 2(\pi/2)}}$

$f(\pi/2) = 0$

f at $x = \pi/2$ define

ii) $\lim\limits_{x\to\pi/2} f(x) = \lim\limits_{x\to\pi/2} \dfrac{\cos x}{\pi - 2x}$

$x - \dfrac{\pi}{2} = h$

$x = h + \pi/2$

when $h \to 0$

$\lim\limits_{h\to 0} \dfrac{\cos \left(h + \frac{\pi}{2}\right)}{\pi - 2\left(h + \frac{\pi}{2}\right)}$

$\lim\limits_{h\to 0} \dfrac{\cos\left(h + \frac{\pi}{2}\right)}{\pi - \left(\frac{2h}{x} + \pi\right)}$ $\cancel{\pi} - 2h - \cancel{\pi}$

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$$\lim_{h\to 0} \frac{\cos\left(h + \frac{\pi}{2}\right)}{-2h}$$

$$\lim_{h\to 0} \frac{\cos h \cdot \cos\frac{\pi}{2} - \sin h \sin\frac{\pi}{2}}{2h}$$

$$\lim_{h\to 0} \cos \cdot 0 - \frac{\sin h}{-2h}$$

$$\lim_{h\to 0} \cancel{\cos h} \cdot \pounds - \frac{\sin h}{-2h} = \frac{\sin h}{2h}$$

$$= \frac{1}{2}$$

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b) $\lim_{x \to \pi/2} f(x) = \lim_{x \to \pi/2} = \frac{\sin 2x}{\sqrt{1 - \cos 2x}}$

$\lim_{x \to \pi/2} = \frac{2 \sin x \cos x}{\sqrt{2 \sin^2 x}}$

$\lim_{x \to \pi/2} = \frac{2 \cancel{\sin x} \cdot \cos x}{\sqrt{2} \cancel{\sin x}}$

$\lim_{x \to \pi/2} = \frac{2 \cos x}{\sqrt{2}}$

$\frac{2}{\sqrt{2}} \lim_{x \to \pi/2^2} \cos x$

L·H·L = R·H·L

f is not continous at $x = \pi/2$

J. ii) $f(x) = \frac{x^2-0}{x-3}$ $\quad 0 < x < 3$

$= x+3 \quad 3 \leq x < 6$

$= \frac{x^2-a}{x+3} \quad 6 \leq x < a$

at $x=3$ & $x=6$

at $x = 3$

ii) ① $f(3) = \frac{x^2-a}{x-3} = 0$

f at $x=3$ define

② $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+}$

1) $f(3) = x+3 = 3+3 = 6$

f is define at $x=3$

2) $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (x+3) = 6$

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3} \frac{x^2-9}{x-3} = \frac{(x-3)(x+3)}{(x-3)}$

$= L \cdot H \cdot L = R \cdot H \cdot L$

f is countinous at $x = 3$

for $x = 6$

i) $f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{27}{9} = 3$

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2 $\lim_{x \to 6^+} = \frac{x^2-9}{x+3}$

$\lim_{x \to 6^+} = \frac{(x-3)(x+3)}{(x+3)}$

$\lim_{x \to 6^+} (x-3) = 6-3$

$= 3$

$\lim_{x \to 6^+} (x+3) = 3+6 = 9$

$L.H.L \neq R.H.L$

function is not continous.

i) $f(x) = \dfrac{1-\cos 4x}{x^2}$ $\quad x<0$
$\quad\quad = k$ $\quad\quad\quad\quad x=0$ $\Big\}$ at $x=0$

Sol^n - f is ct at $x=0$

$$\lim_{x\to 0} f(x) = f(0)$$

$$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = k$$

$$2\lim_{x\to 0} \frac{2\sin^2 x}{x^2} = k$$

$$2\lim_{x\to 0} \left(\frac{\sin^2 2x}{x}\right)^2 = k$$

$$2(2)^2 = k$$

$$k = 8$$

iii) $f(x) = (\sec^2 x)^{\cot^2 x}$ $\quad x \neq 0$
$\quad = k$ $\quad\quad x = 0$ $\Big\}$ at $x = 0$

$$\frac{\dfrac{\sqrt{3} - 3\tan h - \sqrt{3} - \tan h}{1 - \sqrt{3}\tan h}}{-3h}$$

$$= \frac{4\tan h}{(-3h)(1 - \sqrt{3}\tan h)}$$

$$= \frac{4}{3}\left(\frac{1}{1-0}\right)$$

$$= 4$$

iii) $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$ $\left.\begin{array}{l} x = \frac{\pi}{3} \\ x = \frac{\pi}{2} \end{array}\right\}$ at $x = \frac{\pi}{3}$

$x - \frac{\pi}{3} = 4$

$h + \frac{\pi}{3}$ $= \tan(A+B)$

$h \to 0$

$$f\left(\frac{\pi}{3} + h\right) = \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

$$\frac{\sqrt{3} - \dfrac{\tan\frac{\pi}{3} + \tan h}{1 - \tan\frac{\pi}{3} \cdot \tan h}}{\pi - \pi - 3h}$$

$$\frac{\dfrac{\sqrt{3} - \sqrt{3}\times\sqrt{3}\tan h - \sqrt{3} - \tan h}{1 - \sqrt{3} \cdot \tan h}}{-3h}$$

$$\frac{\dfrac{\cancel{\sqrt{3}} - \cancel{\sqrt{3}}\ 3\tan h - \cancel{\sqrt{3}} \cdot \tan h}{1 - \sqrt{3}\tan h}}{-3h}$$

$$\frac{\frac{-4\tan h}{1-\sqrt{3}\tan h}}{-3h}$$

$$= \frac{\cancel{-}4\tan h}{\cancel{-}3h(1-\sqrt{3}\tan h)}$$

$$\frac{4}{3}\lim_{h\to 0}\frac{\tan h}{h}\lim_{h\to 0}\frac{1}{1-\sqrt{3}\tan h}$$

$$= \frac{4}{3}\times\frac{1}{1-3(0)}$$

$$= \frac{4}{3}\times\frac{1}{1} = \frac{4}{3}$$

7,8,0

viii) i) $f(x) = \frac{1-\cos(3x)}{x\tan x}$ $\left.\begin{array}{l} x \neq 0 \\ x = 0 \end{array}\right\}$ at x_0

$$f(x) = \frac{1-\cos 3x}{x\tan x}$$

$$= \frac{2\sin^2 3/2\, x}{x\tan x}$$

$$= \frac{\frac{2\sin^2 \frac{3x}{2}}{x^2} \times x^2}{x \cdot \frac{\tan x}{x^2} \times x^2}$$

$$= 2 \lim_{x\to 0} \frac{\left(\frac{3}{2}\right)^2}{1}$$

$$2 \times \frac{9}{4} = \frac{9}{2}$$

$\lim_{x\to 0} f(x) = \frac{9}{2}$ $9 = f(0)$

f is not cts at $x = 0$
Redefine function

$$f(x) = \begin{cases} \dfrac{1-\cos 3x}{x\tan \cdot x} & x = 0 \\ \dfrac{9}{2} & x = 0 \end{cases}$$

Now $\lim\limits_{x\to 0} f(x) = f(0)$

f has removable discts at $x = 0$

ii) $f(x) = \dfrac{[e^{3x}-1]\sin x^2}{x^2}$ $x \neq 0$ } at $x=0$

$x = 0$

$= \dfrac{\pi}{6}$ $x = 0$

at $x = 0$

$\lim\limits_{x\to 4} (e^{3x}-1)\sin\left(\dfrac{\pi x}{180}\right)$

$3\lim\limits_{x\to \cancel{0}} \dfrac{e^{3x}-1}{3x} \cdot \lim\limits_{x\to 0} \dfrac{\sin\left(\frac{\pi x}{100}\right)}{x}$

$3\log e \dfrac{\pi}{180} = \dfrac{\pi}{60} = f(0)$

f is ~~cts~~ at $x = 0$
continous

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8) $f(x) = \dfrac{e^{x^2} - \cos x}{x^2}$, $x = 0$

is continous at $x = 0$

$$\lim_{x \to 0} f(x) = f(0)$$

$$= \frac{e^{x^2} - \cos x}{x^2} = f(0)$$

$$= \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$$

$$\frac{(e^{x^2} - 1) + (1 - \cos x)}{x^2}$$

$$\frac{e^{x^2} - 1}{x^2} + \lim_{x \to 0} \frac{2 \sin^2 x/2}{x^2}$$

$$= \log e + \left\{ 2 \left(\frac{\sin x/2}{x} \right)^2 \right.$$

$$= 1 + \cancel{2}\, 2 \times \frac{1}{4} = \frac{3}{2} = f(0)$$

9) $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ $\quad x \neq \pi/2$

$\therefore f(0)$ is cots at $x = \pi/2$

$$\frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$$

$$\therefore \frac{2-1+\sin x}{1-\sin^2 x(\sqrt{2}+\sqrt{1+\sin x})}$$

$$\therefore \frac{1+\sin x}{1-\sin^2 x(\sqrt{2}+\sqrt{1+\sin x}}$$

$$\therefore \frac{1}{(1-\sin x)(\sqrt{2}+\sqrt{1+\sin x})}$$

$$\therefore \frac{1}{(1-\sin x)(\sqrt{2}+\sqrt{1+\sin x})}$$

$$\therefore \frac{1}{2(\sqrt{2}+\sqrt{2})} \quad \therefore \frac{1}{4\sqrt{2}} \quad \therefore f(\pi/2) = \frac{1}{4\sqrt{2}}$$

14/02/2020

PRACTICAL NO: 2

Topic - Derivation

Q.1] Show that the following function defined from IR to IR differentiable

i] $\cot x$ ii] $\operatorname{cosec} x$ iii] $\sec x$

Q.2] If $f(x) = 4x + 1$, $x \leq 2$
$= x^2 + 5$, $x > 0$ at $x = 2$, then find f is differentiable or not.

Q.3] If $f(x) = 4x + 7$, $x < 3$
$= x^2 + 3x + 1$ at $x = 3$, then find f is differentiable or not?

Q.4] If $f(x) = 8x - 5$, $x \leq 2$
$= 3x^2 - 4x + 7$ $x > 2$ at $x = 2$
then f is differentiable or not?

P.T.O.

> Topic : Derivation

Q.1> Show that the following functi[on] defined from $\mathbb{R}$ to $\mathbb{R}$ are differ-ble

i> $\cot x$

$f(x) = \cot x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x - a}$$

$$= \lim_{x \to a} \frac{1/\tan x - 1/\tan a}{x - a}$$

$$= \lim_{x \to 0} \frac{\tan a - \tan x}{(x-a)\tan x \tan a}$$

put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a + h - a)\tan(a+h)\tan a}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h)\tan a}$$

$\tan A - \tan B = \tan(A - B)(1 + \tan A \tan B)$

$$= \lim_{h \to 0} \frac{(a - a - h) - (1 + \tan a \tan(a+h)}{h \times \tan(a+h)\tan a}$$

$$= \lim_{h\to 0} -\frac{\tan h}{h} \times \frac{1+\tan a \tan(a+h)}{\tan(a+h)\tan a}$$

$$= -1 \times \frac{1+\tan^2 a}{\tan^2 a}$$

$$= -\frac{\sec^2 a}{\tan^2 a} = \frac{-1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$$

$$= -\operatorname{cosec}^2 a$$

$\therefore Df(a) = -\cos^2 a$

$\therefore$ F is differentiable $\forall a \in R$

ii) cosec x

$f(x) = \operatorname{cosec} x$

$$Df(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$$

$$= \lim_{x\to a} \frac{\sin a - \sin x}{(x-a)\sin a \sin x}$$

put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h\to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \sin(a+h)}$$

formula:

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \sin(a+h)}$$

$$= \lim_{h\to 0} \frac{-\sin h/2}{h/2} \times \frac{1}{2} \times \frac{2\cos(2a+h/2)}{\sin a \sin(a+h)}$$

$$= -\frac{1}{2} \times \frac{2\cos(2a+0/2)}{\sin(a+0)}$$

$$= \frac{-\cos a}{\sin^2 a} = -\cot a \operatorname{cosec} a$$

iii) $\sec x$

$f(x) = \sec x$

$$Df(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\sec x - \sec a}{x-a}$$

$$= \lim_{x\to a} \frac{\cos a - \cos x}{(x-a)\cos a \cos x}$$

Put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h\to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cos(a+h)}$$

Formula :- $2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \cos a \cos(a+h)}$$

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$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right)\sin\frac{-h}{2}}{\cos a \cos(a+h) \times \frac{-h}{2}} \times \frac{-1}{2}$$

$$= \frac{-1}{2} \times 2 \frac{\sin a}{\cos a \times \cos a}$$

$$= \tan a \sec a$$

Q.2) If $f(x) = 4x+1, \quad x \le 2$
$= x^2+5 \quad x > 0$, at $x=2$, then
Find function is differiantable or not

LHD:

$$Df(2^-) = \lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x-8}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4(x-2)}{(x-2)} = 4$$

$$Df(2^-) = 4$$

RHD:

$$Df(2^+) = \lim_{x\to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x\to 2^+} \frac{x^2-4}{x-4}$$

$$= \lim_{x\to 2^+} \frac{(x+2)(x-2)}{x-2}$$

$$= 2+2 = 4$$

RHD = LHD $\therefore$ F is differentiable at $x=2$

Q·3) If $f(x) = 4x + 7$, $x < 3$

$= x^2 + 3x + 1$, $x \geq 3$ at $x = 3$

find f is differentiable or not

Soln

RHD:

$$D(f)(3^+) = \lim_{x \to 2^+} \frac{f(x) - f(3)}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 3x + 1 - (3^2 + 3 + 3+}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 3x + 1 - (3^2 + 3 + 3 + 1)}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 3x + 1 - 19}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x(x+6) - 3(x+6)}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{(x+6)(x-3)}{(x-3)} = 3 + 6 = 9$$

LHD = $Df(3^-)$

$$= \lim_{x \to 3^-} \frac{f(x) - f(3)}{x - 3}$$

$$= \lim_{x \to 3^-} \frac{4x + 7 - 19}{x - 3}$$

$$= \lim_{x \to 3^-} \frac{4x - 12}{x - 3}$$

$$= \lim_{x \to 3} \frac{4(\cancel{x-3})}{(\cancel{x-3})}$$

$DLf(3^+) = 4$

RHD $\neq$ LHD

F is not differentiable at $x = 3$

Q.4) If $f(x) = 8x - 5 \quad , x \le 2$
$= 3x^2 - 4x + 7 \quad , x > 2$ at $x = 2$

Then

Soln:

$f(2) = 8 \times 2 - 5 = 16 - 5 = 11$

RHD

$$Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 6x + 2x - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{(3x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= 3 \times 2 + 2 = 8$$

$DF(2^+) = 8$

AV
20/1/23

PRACTICAL: 3

Topic: Application of Derivative

Q.1) Find

i) $f(x) = x^3 - 5x - 11$

$\rightarrow f(x) = 3x^2 - 5$

f is increasing iff $f'(x) > 0$

$\therefore 3x^2 - 5 > 0$

$3x^2 > 5$

$x^2 > 5/3$

$x > \pm \sqrt{5/3}$

$x \in \left(-\infty, -\sqrt{\frac{5}{3}}\right) \cup \left(\sqrt{\frac{5}{3}}, \infty\right)$

$\therefore$ f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$3x^2 < 5$

$x^2 < 5/3$

$x < \pm \sqrt{5/3}$

$\therefore x \in \left(-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right)$

ii) $f(x) = x^2 - 4x$

$f(x) = 2x - 4$

$\therefore$ f is increasing iff $f(x) > 0$

$\therefore 2x - 4 > 0$

$x - 2 > 0$

$x > 2$

$\therefore x \in (2, \infty)$

iii) f

$\therefore f$ is decreasing iff $f'(x) < 0$

$2x - 4 < 0$

$x - 2 < 0$

$x < 2$

$x \in (-\infty, 2)$

ii) $f(x) = 2x^2 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$\therefore f$ is increasing iff $f'(x) > 0$

$6x^2 + 2x - 20 > 0$

$6x^2 + 12x - 10x - 20 > 0$

$6x(x+2) - 10(x+2) > 0$

$(6x - 10)(x+2) > 0$

$+ \quad\quad - \quad\quad +$

$-2 \quad\quad\quad 10/6$

$\therefore x \in (-\infty, 2) \cup (10/6, \infty)$

f is decreasing iff $f'(x) < 0$

$6x^2 + 2x - 20 < 0$

$6x^2 + 12x - 10x - 20 < 0$

$6x(x+2) - 10(x-2) < 0$

$(6x - 10)(x+2) < 0$

$\therefore x \in \left(-2, \frac{10}{6}\right)$

iv) $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$= 3(x^2 - 9)$

f is increasing iff $f'(x) > 0$

$\therefore 3(x^2 - 9) > 0$

$x^2 - 9 > 0$

$(x-3)(x+3) > 0$

$+ \quad - \quad +$

$-3 \quad 3$

$x \in (-\infty, -3) \cup (3, \infty)$

is decreasing iff $f'(x) < 0$

$3(x^2 - a) < 0$

$x^2 - a < 0$

$(x-3)(x+3) < 0$

$+ \quad - \quad +$

$-3 \quad 3$

$\therefore x \in (-3, 3)$

$f(x) = 69 - 24x - 9x^2 + 2x^2$

$f'(x) = -24 - 18x + 6x^2$

i.e $6x^2 - 18x - 24$

$6(x^2 - 3x - 4)$

$\therefore f$ is increasing iff $f'(x) > 0$

$\therefore 6(x^2 - 3x - 4) > 0$

$x^2 - 3x - 4 > 0$

$x^2 - 4x + x - 4 > 0$

$x(x-4) + 1(x-4) > 0$

$(x+1)(x-4) > 0$

$+ \quad - \quad +$

$-1 \quad 4$

$x \in (-\infty, -1) \cup (4, \infty)$

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f is decreasing iff $f'(x) < 0$

$$6(x^2 - 3x - 4) < 0$$

$$x^2 - 3x - 4 < 0$$

$$x^2 - 4x + x - 4 < 0$$

$$x(x-4) + 1(x-4) < 0$$

$$(x+1)(x-4) < 0$$

$+$ $\quad -1 \quad$ $-$ $\quad 4 \quad$ $+$

$\therefore x \in (1, 4)$

Q.2) i) $y = 3x^2 - 2x^3$

Let

$$f(x) = y = 3x^2 - 2x^3$$

$$\therefore f'(x) = 6x - 6x^2$$

$$= 6x(1-x)$$

$$f''(x) = 6 - 12x$$

$$= 6(1-2x)$$

$f''(x)$ is concave upwards iff,

$$f''(x) > 0$$

$$6(1-2x) > 0$$

$$1 - 2x > 0$$

$$-2x > -1$$

$$x < \frac{1}{2}$$

$$\therefore x \in \left(-\infty, \frac{1}{2}\right)$$

$f''(x)$ is concave downwards iff

$f''(x) < 0$

$6(1-2x) < 0$

$1-2x < 0$

$-2x < -1$

$2x > 1$

$x = \frac{1}{2}$

$\therefore x \in \left(\frac{1}{2}, \infty\right)$

) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

Let,

$f(x) = y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$\therefore f'(x) = 4x^3 - 18x^2 + 24x + 5$

$= 12(x^2 - 3x + 2)$

$f''(x)$ is concave upward iff

$f''(x) > 0$

$12(x^2 - 3x + 2) > 0$

$x^2 - 3x + 2 > 0$

$x^2 - x - 2x + 2 > 0$

$x(x-1) - 2(x-1) > 0$

$(x-2)(x-1) > 0$

$\therefore x \in (-\infty, 1) \cup (2, \infty)$

$f''(x)$ is concave downwards iff

$f''(x) < 0$

$12(x^2 - 3x + 2) < 0$

$x^2 - 3x + 2 < 0$

$x(x-1) - 2(x-1) < 0$

$(x-2)(x-1) < 0$

$+ \quad - \quad +$

$1 \quad 2$

$\therefore x \in (1, 2)$

ii) $y = x^3 - 27x + 5$

Let

$f(x) = y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

$f''(x)$ is concave iff upwards iff.

$f''(x) > 0$

$6x > 0$

$x = 0$

$\therefore x \in (0, \infty)$

$f''(x)$ is concave downwards iff.

$f''(x) < 0$

$6x < 0$

$x < 0$

$\therefore x \in (\infty, 0)$

iv) $y = 69 - 24x - 9x^2 + 2x^3$

Let,

$f(x) = y = 69 - 24x - 9x^2 + 2x^3$

$\therefore f'(x) = -24 - 18x + 6x^2$

$\therefore f''(x) = -18 + 12x$

$f''(x)$ is concave upward iff

$f''(x) > 0$

$-18 + 12x > 0$

$12x > 18$

$x = \frac{18}{12}$

$\therefore x \in (\frac{3}{2}, \infty)$

$f''(x)$ is concave downward iff

$f''(x) < 0$

$-18 + 12x < 0$

$12x < 18$

$x < \frac{18}{12}$

$\therefore x \in (-\infty, \frac{3}{2})$

v) $y = 2x^3 + x^2 - 20x + 4$

Let

$f(x) = y = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

$= 2(6x + 1)$

$\therefore$ $f''(x)$ is concave upwards iff

$f''(x) > 0$

$2(6x+1) > 0$

$6x+1 > 0$

$x > \frac{-1}{6}$

$\therefore x \in \left(\frac{-1}{6}, \infty\right)$

$f''(x)$ is concave downward iff

$f''(x) < 0$

$2(6x+1) < 0$

$6x < -1$

$x < \frac{-1}{6}$

$\therefore x \in \left(-\infty, \frac{-1}{6}\right)$.

20/12/19

Date

PRACTICAL : 4

Q:1) i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - 32/x^3$

Now consider, $f'(x) = 0$

$\therefore 2x - 32/x^3 = 0$

$2x = 32/x^3$

$x^4 = 32/2$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + 96/x^4$

$f''(2) = 2 + \frac{96}{2^4}$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum value at $x = 2$

$f(2) = 2^2 + 16/2^2$

$= 4 + 16/4$

$= 4 + 4$

$= 8$

$f''(x-2) = 2 + 96/(-2)^4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum value at $x = -2$

$\therefore$ function reaches min value at $x = 2$, and $x = -2$

ii) $f(x) = 3 - 5x^3 + 3x^5$

$\therefore f'(x) = -15x^2 + 15x^4$

consider, $f'(x) = 0$ 058

$\therefore -15x^2 + 15x^4 = 0$

$x^2 = 1$

$x = \pm 1$

$\therefore f''(x) = -30x + 60x^3$

$f(1) = -30 + 60$

$= 30 > 0 \therefore f$ has minimum value at $x=1$

$f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3 = 5$

$\therefore f$ has maximum value at 5 at $x = -1$ & has the minimum value 1 at $x = 1$

iii) $f(x) = x^3 - 3x^2 + 1$

$\therefore f'(x) = 3x^2 - 6x$

consider, $f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$3x(x-2) = 0$

$\therefore 3x = 0$ or $x - 2 = 0$

$x = 0$ $x = 2$

$f''(x) = 6x - 6$

$f''(0) = 6(0) - 6$

$= -6 < 0 \therefore f$ has maximum value at $x = 0$

$\therefore f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

$\therefore f$ has minimum value at $x = 2$

$f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12 = -3$

$\therefore f$ has maximum value 1 at $x = 0$ & f has minimum value -3 at $x = 2$

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$\therefore f''(x) = 6x^2 - 6x - 12$

consider, if $f''(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$\therefore 6(x^2 - x - 2) = 0$

$\therefore x^2 - x - 2 = 0$

$\therefore x^2 + x - 2x - 2 = 0$

$\therefore x(x+1) - 2(x+1) = 0$

$(x-2)(x+1) = 0$

$\therefore x = 2$ or $x = -1$

$f''(x) = 12x - 6$

$f''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$

$\therefore$ f has minimum value at $x = 2$

$\therefore f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1$

$= -19$

$\therefore f''(-1) = 12(-1) - 6$

$= -12 - 6$

$= -18 < 0$

f has maximum value a $x = -1$

$\therefore f(-1) = 2(-1)^3 - 3(-1)^2 - 12(1) + 1$

$= -2 - 3 + 12 + 1$

$= 8$

$\therefore$ f has maximum value 8 at $x = -1$ and f has ~~mc~~ minimum value -19 at $x = 2$

1) $f(x) = x^3 - 3x^2 - 55x + 9.5$ $x_0 = 0 \rightarrow$ given

$f'(x) = 3x^2 - 6x - 55$

By Newton's method

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_0)/f'(x_0)$

$x_1 = 0 + 9.5/55$

$x_1 = 0.1727$

$f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829$

$\therefore f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 0.0895 - 1.0362 - 55$

$= -55.9467$

$\therefore x_2 = x_1 - f(x_1)/f'(x_1)$

$= 0.1727 - 0.0829/55.9467$

$= 0.1712$

$f(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= 0.0879 - 1.0272 - 55$

$= -55.9393$

$\therefore x_3 = x_2 - f(x_2)/f'(x_2)$

$= 0.1712 + 0.0011/55.9393$

$= 0.1712$

$\therefore$ The root of equation is 0.1712.

ii) $f(x) = x^3 - 4x - 9$

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9$

$f(3) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

Let $x_0 = 3$ be the initial approximation

$\therefore$ By Newton's method

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = x_0 - f(x_0) / f'(x_0)$

$= 3 - 6/23$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 22.5096 - 4$

$= 18.5096$

$x_2 = x_1 - f(x_1) / f'(x_1)$

$= 2.7392 - 6.596 / 18.5096$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071)$

$= 19.8386 - 10.8284$

$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$

$= 21.9851 - 4$

$= 17.98$

$= 2.7071 - \frac{0.0102}{17.9851}$

$= 2.7071 - 0.0056 = 2.7015$ 060:

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$
$= 19.7158 - 10.806 - 9 = -0.0901$

$f'(x) = 3(2.7015)^2 - 4 = 21.8943 = 17.8493$

$x_4 = 2.7015 + 0.0901/17.8493 = 2.7015 + 0.0050$
$= 2.7065$

$f(x) = x^3 - 1.8x^2 - 10x + 17$ $[1,2]$
$f'(x) = 3x^2 - 3.6x - 10$
$f(1) = (1)^3 - (1.8)(1)^2 - 10(1) + 17$
$= 6.2$
$f(2) = (2)^3 - 18(2)^2 - 10(2) + 17$
$= -2.2$

let $x_0 = 2$ be initial approximation By Newton's method

$x_{n+1} = x_n - f(x_n) / f'(x_n)$
$x_1 = x - f(x_2) / f'(x_2)$
$= 2 - 2.2/5.2$
$= 2 - 0.4230 = 1.577$

$f(x_1) = (1.577)^3 - 1.8(1.57)^2 - 10(1.57) + 17$
$= 3.9219 - 4.4764 - 15.77 + 17$
$= 6.6755$

$f'(x) = 3(1.577)^2 - 3.6(1.577) - 10$
$= 7.4068 - 5.6772 - 10$
$= -8.2164$

$x_2 = x_1 - f(x_1) / f'(x_1)$
$= 1.577 + 0.6755/8.2164$
$= 1.6592$

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$f(x_2) = 1.6592)^3 \; 1.8(1.6592)^2 - 10(1.6592) + 17$

$= 4.5677 - 4.9553 - 16.592 + 17$

$= 0.0204$

$f'(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$

$= 4.5677 - 4.9553 - 16.592 + 17$

$= 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$

$= 8.2588 - 5.97312 - 10$

$= -7.7143$

$x_3 = x_2 - f(x_2) / f(x_2)$

$= 1.6592 + 0.0204 / 7.7143$

$= 1.6188$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$

$= 4.5892 - 4.9708 - 16.618 + 17$

$= 0.0004$

$f'(x_3) = 3(1.6618)^2 - 3.6(2.6618) - 10$

$= -7.6977$

$x_4 = x_3 - f(x_3) / f'(x_3)$

$= 1.618 + \frac{0.0004}{7.697}$

$= 1.6618$

PRACTICAL : 5

Q.1) $\int \frac{1}{x^2+2x-3} \, dx$

$$= \int \frac{1}{\sqrt{x^2+2x-3}} \, dx$$

$$= \int \frac{1}{\sqrt{x^2+2x+1-4}}$$

$a^2+2ab+b^2 = (a+b)^2$

$$= \int \frac{1}{\sqrt{(x+1)^2-4}} \, dx$$

substitute

put $x+1 = t$

$dx = \frac{1}{t} \times dt$ where $t = 1$ $t = x+1$

$$\int \frac{1}{\sqrt{t^2-4}} \, dt$$

using

$$\int \frac{1}{\sqrt{x^2-a^2}} \, dx = \ln(|x+\sqrt{x^2-a^2})$$

$$= \ln(|t+\sqrt{t^2-4})$$

$t = x+1$

$$= \ln(|x+1+\sqrt{(x+1)^2-4})$$

$$= \ln(|x+1+\sqrt{x^2+2x-3})$$

$$= \ln(|x+1|\sqrt{x^2+2x-3}) + C$$

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2) $\int (4e^{3x} + 1) dx$
$= \int 4e^{3x} dx + \int 1 dx$
$= 4\int e^{3x} dx + \int 1 dx$ # $\int e^{ax} dx = \frac{1}{a} e^{ax}$
$= \frac{4e^{3x}}{3} + x$
$= \frac{4e^{3x}}{3} + x + C$

3) $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\, dx$
$= \int 2x^2 - 3\sin(x) + 5x^{1/2} dx$
$= \int 2x^2 dx - \int 3\sin(x) dx + \int 5x^{1/2} dx$
$= \frac{2x^3}{3} + 3\cos x + \frac{10 \times \sqrt{x}}{3} + C$
$= \frac{2x^3 + 10x\sqrt{x}}{3} + 3\cos x + C$

4) $\int \frac{x^3 + 3x + 4}{\sqrt{x}} dx$

$= \int \frac{x^3 + 3x + 4}{x^{1/2}} dx$

split the denominator
$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}} dx$
$= \int x^{5/2} + 3x^{1/2} + \frac{4}{x^{1/2}} dx$
$= \int x^{5/2} dx + \int 3x^{1/2} dx + \int \frac{4}{x^{1/2}} dx$
$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + C$

5) $\int t^{7x} s$
put
du
$= \int t$
$= \int$
$= \int t$
s
$= \int$
$=$
$= \frac{1}{16}$
#

Put $u = 2t^4$
$du = 2 \times 4t^3$

$= \int t^7 \times \sin(2t^4) \times \frac{1}{2 \times 4t^3} du$

$= \int t^4 \sin(2t^4) \times \frac{1}{2 \times 4} du$

$= \int t^4 \sin(2t^4) \times \frac{1}{8} du = \frac{t^4 \times \sin(2t^4) du}{8}$

Substitute t^4 with $\frac{u}{2}$

$= \int \frac{u/2 \times \sin u}{8} du$

$= \int \frac{u \times \sin(u)}{16} du$

$= \frac{1}{16} \int u \times \sin(u) du$

\# $\int u dv = uv - \int v du$
where $u = u$
$dv = \sin(u) \times du$
$du = 1 du \quad v = -\cos(u)$

$= \frac{1}{16} \times (u \times (-\cos u)) + \int \cos(u) du$

\# $\int \cos x \, dx = \sin(x)$.

$= \frac{1}{16} \times (u \times (-\cos(u)) + \sin(u))$

Return the substitution $u = 2t^4$

$= 1/16 \times (2t^4 \times (-\cos(2t^4)) + \sin(2t^4))$

$= -\frac{t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + C$

vi) $\int \sqrt{x}.(x^2-1)\,dx$

$= \int \sqrt{x}.x^2 - \sqrt{x}.dx$

$= \int x^{1/2} \times x^2 - x^{1/2} dx$

$= \int x^{5/2} - x^{1/2} dx$

$= \int x^{5/2} dx - \int x^{1/2} dx$

$= I_1 \; \frac{x^{5/2+1}}{5/2+1} = \frac{x^{7/2}}{7/2} = \frac{2x^{7/2}}{7} = \frac{2\sqrt{x}^7}{7} = \frac{2x^3\sqrt{x}}{7}$

$= I_2 = \frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3/2} = \frac{2\sqrt{x}^3}{3}$

$= \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x}^3}{3} + C$

iii) $\int \frac{\cos x}{\sqrt[3]{\sin(x)^2}}\,dx$

$= \int \frac{\cos x}{\sin x^{2/3}}\,dx$

put $t = \sin(x)$

$t = \cos x$

$= \int \frac{\cos(x)}{\sin(x)^{3/2}} \times \frac{1}{\cos(x)dt}$

$= \frac{1}{\sin x^{3/2}}\,dt$

$= \frac{1}{t^{2/3}}\,dt$

$I = \int \frac{1}{t^{2/3}}\,dt = \frac{-1}{(2/3-1)t^{2/3-1}} = \frac{-1}{-1/3 + 2/3 - 1} = \frac{t^{1/3}}{1/3}$

$= 3t^{1/3}$

063

$= 3\sqrt[3]{\sin(x)} + C$

(x) $\int \frac{x^2-2x}{x^3-3x^2+1}\,dx$

put $x^3-3x^2+1 = dt$

$I = \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-3\times 2x}\,dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-6x}\,dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3(x^2-2x)}\,dt$

$= \int \frac{1}{x^3-3x^2+1} \times \frac{1}{3}\,dt$

$= \int \frac{1}{3(x^3-3x+1)\,dt} = \int \frac{1}{3t}\,dt$

$= \frac{1}{3}\int \frac{1}{t}\,dt \quad \int \frac{1}{x}\,dx = \ln|x|$

$= \frac{1}{3} \times \ln|t| + C$

$= \frac{1}{3} \times \ln(|x^3-3x^2+1|) + C$

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PRACTICAL - 6

1] $y = \sqrt{4-x^2}$

$\frac{dy}{dx}$ $-2\int_{0}^{2} \frac{1}{2\sqrt{4-x^2}} dx$

$y = \sqrt{\cancel{4-}x^2}$ $2\int_{0}^{2} \sqrt{1 + \frac{x^2}{4-x^2}} dx$

$= 4\int_{0}^{4} \frac{1}{\sqrt{4-x^2}} dx$

$= 4\left(\sin^{-1}(x/2)\right)_{0}^{2}$

$= 2\pi$

i) $\int_{a}^{b} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2} dt$

$x = t - \sin t$ $\therefore \frac{dx}{dt} = 1 - \cos t$

$\therefore y = 1 - \cos t$ $\therefore \frac{dy}{dt} = 6 - (-\sin t) = \sin$

$L = \int_{0}^{2\pi} \sqrt{1 - 2\cos t + 1} \, dt$

$= \int_{0}^{2\pi} \sqrt{2 - 2\cos t} \, dt$ $= \sqrt{4\sin t/2}$

$= \int_{0}^{2\pi} 2\sin\frac{t}{2}\,dt$ 061

$= \left(-4\cos\left(\frac{t}{2}\right)\right)_0^{2\pi} = (-4\cos\pi) - (-4\cos 0)$

$= 4+4$

$= 8$

) $y = x^{3/2}$ in $(0,4)$

$F(x) = \frac{3}{2}x^{1/2}$

$L = \int_a^b \sqrt{1+(f'(x))^2}\,dx$

$= \int_0^4 \frac{\sqrt{4+9x}}{4}\,dx$

$= \frac{1}{2}\int_0^4 \sqrt{4+9}\,dx$

$= \frac{1}{2}\left[\frac{(4+9x)^{1/2+1}}{1/2+1}\right]_0^4 dx$

$\frac{1}{27}\left[(4+9x)^{3/2}\right]_0^4$

$\frac{1}{27}\left[(4+0)^{3/2} - (4+36^{3/2})\right]$

$= \frac{1}{27}(4)^{3/2} - (40)^{3/2}]$

iv) $x = 3\sin t \quad y = 3\cos t$

$$\frac{dx}{dt} = 3\cos t \quad \frac{dy}{dt} = -3\sin t$$

$$I = \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t)}\, dt$$

$$= \int_0^{2\pi} \sqrt{9(\sin^2 t + \cos^2 t)}\, dt$$

$$= \int_0^{2\pi} \sqrt{9(1)}\, dt$$

$$= \int_0^{2\pi} 3\, dt = 3\int_0^{2\pi} dt = 3[x]^{2\pi}$$

$$= 3(2\pi - 0)$$

$$= 6\pi$$

v) $x = \frac{1}{t} y^3 + \frac{1}{2y} \quad y = [1,2]$

$$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2} = \frac{y^4 - 1}{2y^2}$$

$$= \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y)^2}}\, dy$$

$$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$$

$$= \frac{1}{2}\int_1^2 y^2\, dy + \frac{1}{2}\int_1^2 y^{-2}\, dy$$

$$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-2}}{1}\right]_1^2$$

$$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$$

$$= \frac{1}{2}\left[\frac{7}{3} - \frac{1}{2}\right] = \frac{17}{12}$$

Q.2 $\int_0^2 e^{x^2}\, dx$ with $n = 4$

$a = 0$, $b = 2$, $n = 4$

$$h = \frac{b-a}{n} = \frac{2-0}{4} = \frac{1}{2}$$

x	0	0.5	~~1.5~~ 1	1.5	2
y	1	1.2540	2.7182	9.4877	54.5981

By simpson's Rule

$$\int_0^2 e^{x^2}\, dx = \frac{0.5}{3}\left[(1 + 54.5981) + 4(1.2840 + 9.4877) + 2(2.7182)\right]$$

$$= 17.3535$$

2) $\int_6^4 x^2 \, dx$ with $n = a$

$a = 0$, $b = a$, $n = a$

$h = \frac{b-a}{n} = \frac{4-0}{4} = 1$

x	0	1	2	3	4
y	0	1	4	9	16

By Simpson's Rule

$$\int_0^4 x^2 dx = \frac{1}{3}\left[(0+16) + 4(1+9) + 2(4)\right]$$

$$= \frac{1}{3}(16 + 40 + 8)$$

$$= \frac{1}{3} 64$$

$$= \frac{64}{3}$$

3) $\int_0^{\pi/3} \sqrt{\sin x}\, dx$ with $n=6$

C66

$a=0$, $b=\frac{\pi}{3}$, $n=6$

$h = \frac{\frac{\pi}{3} - 0}{6} = \pi/18$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$5\pi/18$	$6\pi/18$
y	0	0.4167	0.5848	0.7071	0.8017	0.8752	0.9306

By Simpson's Rule,

$$\int_0^{\pi/3} \sqrt{\sin x} = \frac{h}{3}\left[(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)\right]$$

$$= \frac{\pi}{54}\left[(0.0.9306)+4(0.4167+0.7071+0.8752) + 2(0.5848+0.8017)\right]$$

$$= \frac{\pi}{54} \times 11.6996$$

$$= 0.6806$$

$$\int_0^{\pi/3} \sqrt{\sin x} = 0.6806$$

PRACTICAL - 07

Topic - Differential equation.

i] $x\frac{dy}{dx} + y = e^x$

$\therefore \frac{dy}{dx} + \frac{1}{x} y = \frac{e^x}{x}$

comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

$P(x) = \frac{1}{x}$; $Q(x) = \frac{e^x}{x}$

$I.F = e^{\int 1/x \, dx}$

$= e^{\log x}$

$I.F = x$

$\therefore y(I.F) = \int Q(\cancel{Q}\, x)(I.F)dx$

$y(x) = \int \frac{e^x}{x} \cdot x dx$

$yx = \int e^x dx$

$xy = e^x + c$

ii) $e^x \frac{dy}{dx} + 2e^x y = 1$

$e^x\left(\frac{dy}{dx} + 2y\right) = 1$

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

comparing with $\frac{dy}{dx} + P(x) y = Q(x)$

$\therefore P(x) = 2$; $Q(x) = \frac{1}{e^x}$

$$I.F = e^{\int 2 dx}$$
$$= e^{2x}$$
$$y(I.F) = \int \varphi(x)(I.F)\,dx$$
$$y\,e^{2x} = \int \frac{1}{e^x} e^{2x}\,dx$$
$$ye^{2x} = \int e^{-x} \cdot e^{e^x}\,dx$$
$$ye^{2x} = \int e^{x}\,dx$$
$$ye^{2x} = e^x + c$$

iii) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$$x\frac{dy}{dx} + 2y = \frac{\cos x}{x}$$
$$\frac{dy}{dx} + \left(\frac{2}{x}\right) y = \frac{\cos x}{x^2}$$

comparing with $\frac{dy}{dx} + P(x)y = \varphi(x)$

$\therefore P(x) = \frac{2}{x}$; $\varphi(x) = \frac{\cos x}{x^2}$

$$I.F = e^{\int \frac{2}{x} dx}$$
$$= e^{2\log x}$$
$$= e^{\log x^2}$$
$$I.F = x^2$$

$$y(I.F) = \int \varphi(x)(I.F)\,dx$$
$$x^2 y = \sin x + C$$

iv) $\frac{\ldots}{dx} \ldots 3y = \frac{\ldots}{x^2}$

$$\frac{dy}{dx} + \left(\frac{3}{x}\right)y = \frac{\sin x}{x^2}$$

Comparing with $\frac{dy}{dx} + P(x)\,y = \varphi(x)$

$$P(x) = 3x^{-1} \qquad \varphi(x) = \frac{\sin x}{x^3}$$

$$I.F = e^{\int \frac{3}{x} dx}$$
$$= e^{3\log x}$$
$$= e^{\log x^3}$$
$$= x^3$$

$$y \cdot (I.F) = \int \varphi(x)(I.F)\,dx$$
$$y(x^3) = \int \frac{\sin x}{x^3}(x^3)\,dx$$
$$x^3 y = -\cos x + C$$

v) $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

$$e^{2x}\left(\frac{dy}{dx} + 2y\right) = 2x$$

$$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$$

Comparing with $\frac{dy}{dx} + P(x)\,y = \varphi(x)$

$$\therefore P(x) = 2 \qquad \varphi(x) = \frac{2x}{e^{2x}}$$

$$\therefore I.F. = e^{\int 2dx}$$
$$= e^{2x}$$

$$y\,(I.F) = \int \varphi(x)(I.F)\,dx$$

$$y(e^{2x}) = \int \frac{2x}{e^{2x}} (e^{2x})\, dx$$

$$ye^{2x} = \frac{\cancel{2}x^2}{\cancel{2}} + c$$

$$ye^{2x} = x^2 + c$$

vi) $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$

$$\sec^2 x \tan y\, dx = -\sec^2 y \tan x\, dy$$

$$\frac{\sec^2 x}{\tan x} dx = \frac{-\sec^2 y}{\tan y} dy$$

$$\therefore \int \frac{\sec^2 x}{\tan x} dx = -\int \frac{\sec^2 y}{\tan y} dy$$

$$\log|\tan x| = -\log|\tan y| + c$$

$$\log|\tan x| + \log|\tan y| = C$$

$$\log|\tan x \cdot \tan y| = C \Rightarrow \tan x \cdot \tan y = c$$

vii) $\frac{dy}{dx} = \sin^2(x-y+1)$

put $x-y+1 = v$

$$1 - \frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = 1 - \frac{dv}{dx}$$

$$\therefore 1 + \frac{dv}{dx} = \sin^2 v$$

$$1 - \sin^2 v = \frac{dv}{dx}$$

$$dx = \frac{dv}{1-\sin^2 v}$$

$$\int dx = \int \sec^2 v \, dv$$

$x = \tan v + c$

But $v = x+y-1$

$\therefore x = \tan(x+y-1)+c$

vii) $$\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$$

$$\frac{dy}{dx} = \frac{2x+3y-1}{3(2x+3y+2)}$$

Put $2x+3y = v$

$$2+3\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx}-2\right)$$

$$\therefore \frac{1}{3}\left(\frac{dv}{dx}-2\right) = \frac{v-1}{3(v+2)}$$

$$\frac{dv}{dx} = \frac{v-1}{v+2}+2$$

$$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$$

eac

$$\frac{dv}{dx} = \frac{3v+3}{v+2}$$

$$\frac{v+2\,dv}{3(v+1)} = dx$$

$$\frac{1}{3}\int 1 + \frac{1}{v+1}\,dv = \int dx$$

$$\frac{1}{3}\left(v + \log(v+1)\right) = x + c$$

But $v = 2x + 3y$

$$\therefore 2x + 3y + \log|2x+3y+1| = 3x + c$$

$$3y = x - \log|2x+3y+1| + c$$

10/01/2020

PRACTICAL : 8

1) $\frac{dy}{dx} = y + e^{x} - 2$

$f(x, y) = y + e^{x} - 2$, $y_0 = 2$, $x_0 = 0$, $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.87	3.57345
2	1	3.5743	4.2925	5.3615

$y_{n+1} = y_n + h\, f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
3	1.5	5.3615	7.8431	9.28305
4	2	9.2831		

$\therefore$ By Euler's Formula

$y(2) = 9.2831$

$$\frac{dy}{dx} = 1 + y^2$$

$f(x, y) = 1 + y^2$, $y_0 = 0$, $x_0 = 0$, $h = 0.2$

using Euler's iteration formula,
$y_{n+1} = y_n + h\, f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	~~0~~ 1.04	0.408
2	0.4	0.408	1.1665	0.6413
3	0.6	0.6413	1.4113	0.9236
4	0.8	0.9236	1.8530	1.2942
5	1	1.2442		

By Eulers Formula,

$y(1) = 1.2942$

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3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0)=1$ $x_0 = 0$, $h = 0.2$

Using Euler's Formula,

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
		1	0	0
0	0	0		
1	0.2			
2	0.4			
3	0.6			
4	0.8			
5	1			

4) $\frac{dy}{dx} = 3x^2+1$ $y_0 = 2$, $x_0 = 1$, $h = 0$

for $h = 0.5$

Using Euler's Formula

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	49	28.5
2	2	28.5		

For

n
0
1
2
3
4

5) $\frac{d}{d}$

n
0
1

AL 17/01/2

By Euler's Formula.
$y(2) = 28.5$

For $h = 0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4219
2	1.5	4.4219	7.75	6.3594
3	1.75	6.3594	10.181	8.9048
4	2	8.9048		

By Euler's Formula
$y(2) = 8.9048$

5) $\frac{dy}{dx} = \sqrt{xy} + 2 \quad y_0 = 1, \; x_0 = 1, \; h = 0.2$

Using Euler's Iteration Formula.

$$y_{n+1} = y_n + h f(x_n, y_n)$$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	1.6
1	1.2	1.6		

By Euler's Formula.

$y(1.2) = 1.6$

AV
17/01/2020

PRACTICAL - 9

Q.1)
Aim → Limits & partial order derivative.

1) $\lim_{(x,y)\to(-4,-1)} \dfrac{x^3 - 3y + y^2 - 1}{xy + 5}$

At $(-4,-1)$, Den $\neq 0$
$\therefore$ By Applying limit

$= \dfrac{(-4)^3 - 3(-1) + (-1)^2 - 1}{-4(-1) + 5}$

$= -\dfrac{61}{9}$

ii) $\lim_{(x,y)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}$

At $(2,0)$, Denominator $\neq 0$
$\therefore$ By applying lim,

$= \dfrac{(0+1)((2)^2+0-(4\times 2))}{2+0}$

$= -4/2$

$= -2$

iii) $\lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2z^2}{x^3-x^2yz}$

At $(1,1,1)$, Denominator $= 0$

$\therefore \lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2z^2}{x^3-x^2yz}$

$= \lim_{(x,y,z)\to(1,1,1)} \dfrac{\cancel{(x-yz)}(x+yz)}{x^2\cancel{(x-yz)}}$

$= \lim_{(x,y,z)\to(1,1,1)} \dfrac{x+yz}{x^2}$

on Applyin limit $= \dfrac{1+1(1)}{(1)^2} = 2$

Q.2)

2) $f(x,y) = xye^{x^2} + y^2$

$\therefore f_x = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(xye^{x^2} + y^2)$

$= ye^{x^2} + y^2(2x)$

$\therefore f_x = 2xye^{x^2} + y^2$

$f_y = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(xye^{x^2} + y^2)$

$= xe^{x^2+y^2}(2y)$

$\therefore f_y = 2yxe^{x^2+y^2}$

ii) $f(x,y) = e^x \cos y$

$f_x = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(e^x \cos y)$

$\therefore f_x = e^x \cos y$

$f_y = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(e^x \cos y)$

$\therefore f_y = -e^x \sin y$

3) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$$f_x = \frac{\partial}{\partial x}(f(x,y))$$

$$= \frac{\partial}{\partial x}(x^3y^2 - 3x^2y + y^3 + 1)$$

$$f_x = 3x^2y^2 - 6xy$$

$$f_y = \frac{\partial}{\partial y}(f(x,y))$$

$$= \frac{\partial}{\partial y}(x^3y^2 - 3x^2y + y^3 + 1)$$

$$\therefore f_y = 2x^3y - 3x^2 + 3y^2$$

Q-3) i) $f(x,y) = \frac{2x}{1+y^2}$

$$f_x = \frac{\partial}{\partial x}\left(\frac{2x}{1+y^2}\right)$$

$$= \frac{1+y^2\frac{\partial}{\partial x}(2x) - 2x\frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$$

$$= \frac{2+2y^2-0}{(1+y^2)^2}$$

$$= \frac{2(1+y^2)}{(1+y^2)(1+y)^2}$$

$$= \frac{2}{1+y^2}$$

At $(0,0)$

$$= \frac{2}{1+0} = 2$$

$$f_y = \frac{\partial}{\partial y}\left(\frac{2x}{1+y^2}\right)$$

$$= \frac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x \frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$$ 071

$$= \frac{1+y^2(0) - 2x(2y)}{(1+y^2)^2}$$

$$= -\frac{4xy}{(1+y^2)^2}$$

$$\text{At } (0,0) = -\frac{4(0)(0)}{1+0)^2} = 0$$

Q.4) i) $f(x,y) = \frac{y^2 - xy}{x^2}$

$$f_x = \frac{x^2 \frac{\partial}{\partial x}(y^2 - xy) - (y^2 - xy)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$$= \frac{-x^2 y - 2x(y^2 - xy)}{x^4}$$

$$f_y = \frac{2y - x}{x^2}$$

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{-x^2 y - 2x(y^2 - xy)}{x^4}\right)$$

$$= \frac{x^4\left(\frac{\partial}{\partial x}(-x^2 y - 2xy^2 + 2x^2 y)\right) - (-x^2 y - 2xy + 2x^2 y)\frac{\partial}{\partial x}(x^4)}{(x^4)^2}$$

2

$$f_{yy} = \frac{\partial}{\partial y}\left(\frac{2y-x}{x^2}\right)$$

$$= \frac{2-0}{x^2} = \frac{2}{x^2} \quad -(2)$$

$$f_{xy} = \frac{\partial}{\partial y}\left(\frac{-x^2y - 2xy^2 + 2x^2y}{x^4}\right)$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4} \quad -(3)$$

$$f_{yx} = \frac{\partial}{\partial x}\left(\frac{2y-x}{x^2}\right)$$

$$= \frac{x^2\frac{\partial}{\partial x}(2y-x) - (2y-x)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4} \quad -(4)$$

from (3) & (4)

$$f_{xy} = f_{yx}$$

ii) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$$f_x = \frac{\partial}{\partial x}(x^3 + 3x^2y^2 - \log(x^2+1)) = 3x^2 + 6xy^2 - \frac{2x}{x^2}$$

$$f_y = \frac{\partial}{\partial y}(x^3 + 3x^2y^2 - \log(x^2+1))$$

$= \cancel{3x}$

$$= 0 + 6x^2y - 0$$

$$= 6x^2y$$

$$f_{xx} = 6x + 6y^2 - \left(\frac{x^2+1 \frac{\partial(2x)}{\partial x} - 2x \frac{\partial(x^2+1)}{\partial x}}{(x^2+1)^2} \right)$$

$$= 6x + 6y^2 - \left(\frac{2(x^2+1) - 4x^2}{(x^2+1)^2} \right) \quad — ①$$

$$f_{yy} = \frac{\partial}{\partial y}(6x^2y) \quad — ②$$

$$= 6x^2$$

$$f_{xy} = \frac{\partial}{\partial y}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$$

$$= 0 + 12xy - 0$$

$$= 12xy \quad — ④$$

$$f_{yx} = \frac{\partial}{\partial x}(6x^2y) \quad — \cancel{③}$$

$$= 12xy \quad — ③$$

from ③ & ~~④~~ ~~③~~ ④

$$f_{xy} = f_{yx}$$

iii) $f(x,y) = \sin(xy) + e^{x+y}$

$\rightarrow f_x = y\cos(xy) + e^{x+y}(1)$

$= y\cos(xy) + e^{x+y}$

$f_y = x\cos(xy) + e^{x+y}(1)$

$= x\cos(xy) + e^{x+y}$

$f_{xx} = \frac{d}{dx}(y\cos(xy) + e^{x+y})$

$= -y\sin(xy)\cdot(y) + e^{x+y}(1)$

$= -y^2\sin(xy) + e^{x+y}$

$f_{yy} = \frac{\partial}{\partial y}(x\cos(xy) + e^{x+y})$

$= -x^2\sin(xy) + e^{x+y}$ — (2)

$f_{xy} = \frac{\partial}{\partial y}(y\cos(xy) + e^{x+y})$

$= -y^2\sin(xy) + \cos(xy) + e^{x+y}$ — (

$f_{yx} = \frac{\partial}{\partial x}(x\cos(xy) + e^{x+y})$

$= -x^2\sin(xy) + \cos(xy) + e^{x}$

from (3) & (4)

$f_{xy} \neq f_{yx}$

Scanned with CamScanner

Q.5) i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$f(1,1) = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

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$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x)$ $\qquad$ $f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$

$= \frac{x}{\sqrt{x^2+y^2}}$ $\qquad$ $= \frac{y}{\sqrt{x^2+y^2}}$

f_x at $(1,1) = \frac{1}{\sqrt{2}}$ $\qquad$ f_y at $(1,1) = \frac{1}{\sqrt{2}}$

$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x+y-2)$

$= \cancel{\sqrt{2}} \; \frac{x}{\sqrt{2}} + \frac{1}{\sqrt{2}}y - \frac{2}{\sqrt{2}}$

$= \frac{x+y}{\sqrt{2}}$

ii) $f(x,y) = 1 - x + y\sin x$ at $(\frac{\pi}{2}, 0)$

$f(\pi/2, 0) = 1 - \pi/2 + 0 = 1 - \pi/2$

$f_x = 0 - 1 + y\cos x$ $\qquad$ $f_y = 0 - 0 + \sin x$

f_x at $(\pi/2, 0) = -1 + 0 = -1$ $\qquad$ f_y at $(\pi/2, 0) = \sin \pi/2 = 1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 1 - \pi/2 + (-1)(x - \pi/2) + 1(y-0)$

$= 1 - \pi/2 - x + \pi/2 + y$

$= 1 - x + y$

iii) $f(x,y) = \log x + \log y$ at $(1,1)$

$f(1,1) = \log(1) + \log(1) = 0$

$f_x = \frac{1}{x} + 0$

$f_y = 0 + \frac{1}{y}$

f_x at $(1,1) = 1$

f_y at $(1,1) = 1$

$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

$= x - 1 + y - 1$

$= x + y - 2$

AK 24/01/2020

PRACTICAL - 10

i) $f(x, y) = x + 2y - 3$ $\quad a = (1, -1)$ $\quad u = 3i - j$

Here, $u = 3i - j$ is not a unit vector

$|\bar{u}| = \sqrt{(3)^2 - (-1)^2} = \sqrt{9+1} = \sqrt{10}$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3, -1)$

$$\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$$

$f(a+hu) = f(1, -1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1, -1) = (1) + 2(1) - 3 = 1 - 2 - 3 = -4$

$f(a+hu) = f(1, -1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$= f\left[\left(1 + \frac{3}{\sqrt{10}}\right), \left(-1 - \frac{h}{\sqrt{10}}\right)\right]$

$f(a+hu) = \left(1 + \frac{3}{\sqrt{10}}\right) + 2\left(-1 - \frac{h}{\sqrt{10}}\right) - 3$

$= \left(1 + \frac{3}{\sqrt{10}}\right) + 2\left(-1 - \frac{h}{\sqrt{10}}\right) - 3$

$= 1 + \cancel{\frac{3}{\sqrt{10}}} - 2 - \frac{2h}{\sqrt{20}} - 3$

$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$

$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h \to 0} \frac{-4 + h/\sqrt{10} + 4}{h}$

$D_u f(a) = \frac{1}{\sqrt{10}}$ //

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ii) $f(x) = y^2 - 4x + 1$ $a = (3, 4)$ $u = i + 5j$

Here $u = i + 5j$ is not a unit vector

$|\bar{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1, 5)$

$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$f(a) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{26}\right)$

$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$

$f_{x,y}(a+hu) = \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$

$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$

$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$

$= \frac{25h^2}{26} + \frac{40h - 4h}{\sqrt{26}} + 5$

$f(a+hu) = \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$ ~~$+ \frac{-4+h}{\sqrt{10}}$~~

$D_u f(a) = \lim_{h \to a} \frac{f(a+hu) - f(a)}{h}$

$= \cancel{\lim_{h\to 0} -4 + h/\sqrt{10} +}$

$$= \lim_{h\to 0} \frac{\frac{25h}{26} + \frac{36h}{\sqrt{26}} + 5 - 5}{h}$$

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$$h\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)$$

$$\therefore D_u f(a) = \frac{25h}{26} + \frac{36}{\sqrt{26}}$$

ii) $2x+3y \quad a=(1,2)\ ,\ u=(3i+4j)$

Here $u = 3i+4j$ is not a unit vector

$|\bar{u}| = \sqrt{3^2+4^2} = \sqrt{25} = 5$

unit vector along u is $\frac{u}{|u|} = \frac{1}{5}(3,4)$

$= \left(\frac{3}{5}, \frac{4}{5}\right)$

$f(a) = f(1,2) = 2(1)+3(2) = 8$

$f(a+hu) = f(1,2) + h\left(3/5, 4/5\right)$

$= f\left(1+3h/5, 2+4/5\right)$

$f(a+hu) = 2\left(1+3h/5 + 3\left(3+4h/5\right)\right)$

$= 2+6h/5 + 6 + 12h/5$

$= 18h/5 + 8$

$$D_u f(a) = \lim_{h\to 0} \frac{\frac{18h}{5}+8-8}{h} = 18/5$$

2) $f(x,y) = x^y + y^x$ & $a=(1,1)$

$f_x = y \cdot x^{y-1} + y^x \log y$

$f_y = x^y \log x + x y^{x-1}$

$f(x,y) = (f_x, f_y)$

$= (y x^{y-1} + y^2 \log y, x^y \log x + x y^{x-1})$

$f(1,1) = (1+0, 1+0)$

$= (1,1)$

ii) $f(x,y) = (\tan^{-1}x)\cdot y^2 \quad a = (1,-1)$

$f_x = \frac{1}{1+x^2} \cdot y^2$

$f(y) = 2y \tan^{-1} x$

$\nabla f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2}, 2y\tan^{-1}x \right)$

$f(1,-1) = \left(\frac{1}{2}, \tan^{-1}(1)(-2) \right) = \left(\frac{1}{2}, \frac{\pi}{4}(-2) \right)$

$= \left(\frac{1}{2}, \frac{-\pi}{2} \right)$

iii) $f(x,y,z) = xyz - e^{x+y+z}, \quad a = (1,-1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = f_x, f_y, f_z$

$= yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z}$

$f(1,-1,0) = ((-1)(0) - e^{1+(-1)+0}, (1)(0) - e^{1+(-1)+0}, (1)(-1) - e^{1+(-1)+0}$

$= (0 - e^0, 0 - e^0, -1 - e^0)$

$= (-1, -1, -2)$

Q-3) Find the eqn.

i) $x^2 \cos y + e^{xy} = 2 \quad at (1,0)$

$f_x = \cos y\, 2x + e^{xy}\, y$

$f_y = x^2(-\sin y) + e^{xy}\cdot x$

$(x_0, y_0) = (1,0) = \therefore x_0 = 1, \; y_0 = 0$

eqn of tangent

$f_x(x, x_0) + f_y(y - y_0) = 0$

$f_x(x_0, y_0) = \cos 0\, 2(1) + e^0\, 0$

$= 1(2) + 0$

$= 2$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$ It is the required eqn of tangent

eqn of Normal

$= ax + by + c = 0$

$= bx + ay + d = 0$

$1(1) + 2(y) + d = 0$

$1 + 2y + d = 0$

$1 + 2(0) + d = 0$

$d + 1 = 0$

$\therefore d = -1$

ii) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2, -2)$

$fx = 2x + 0 - 2 + 0 + 0$

$= 2x - 2$

$fy = 0 + 2y - 0 + 3 + 0$

$= 2y + 3$

$fx(x_0, y_0) = 2(2) + (-2) = 2$

$fy(x_0, y_0) = 2(-2) + 3 = 1$

eqn of tangent

$fx(x - x_0) + fy(y - y_0) = 0$

$2(x-2) + (-1(y+2)) = 0$

$2(x-2) - y - 2 = 0$

$2x - y - 4 = 0$ → It is required eqn

eqn of normal $= ax + by + c = 0$

$bx + ay - 1d = 0$

$= -1(x) + 2(y) + d = 0$

$-x + 2y + d = 0$ at $(2, -2)$

$$-6 + d = 0$$
$$\therefore d = 6$$

Q.4) Find.

i) $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$

$$f_x = 2x - 0 + 0 + z$$
$$f_x = 2x + z$$
$$f_y = 0 - 2z + 3 + 0$$
$$= 2z + 3$$
$$f_z = 0 - 2y + 0 + x$$
$$= -2y + x$$

$(x_0, y_0, z_0) = (2,1,0) \therefore x_0 = 2,\ y_0 = 1,\ z_0 = 0$

$$f_x(x_0, y_0, z_0) = 2(2) + 0 = 4$$
$$f_y(x_0, y_0, z_0) = 2(0) + 3 = 0$$
$$f_z(x_0, y_0, z_0) = -2(1) + 2 = 0$$

eq^n of tangent

$$f_x(x_0 - x_0) + f_y(y_0 - y_0) + f_z(z - z_0) = 0$$
$$= 4(x-2) + 3(y-1) + 0(z-0) = 0$$
$$= 4x - 8 + 3y - 3 = 0$$
$$4x + 3y - 11 = 0 \text{ - reqard eq}^n \text{ of tangent}$$

eq^n of normal at $(4, 3, -11)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_x} = \frac{z - z_0}{f_z}$$
$$= \frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{0}$$

ii) $3xy^2 - x - y + z = -4$ at $(1,-1,2)$

$3xyz - x - y + z + 4 = 0$ at $(1,-1,2)$

$f_x = 3yz - 1 - 0 + 0 + 0$
$= 3yz - 1$

$f_y = 3xz - 0 - 1 + 0 + 0$
$= 3xz - 1$

$f_z = 3xy - 0 - 0 + 1 + 0$
$= 3xz - 1$

$f(x_0, y_0, z_0) = (1,-1,2)$ $\therefore x_0 = 1, y_0 = -1, z_0 = 2$

$f_x(x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$

$f_y(x_0, y_0, z_0) = 3(1)(2) - 1 = 5$

$f_z(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

eq^n of tangent

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$-7x + 7 + 5y - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0$ → This is required equation

Eq^n of normal at $(-7, 5, -2)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$= \frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$

Q.5)

i) $f(x, y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 - 0$
$= 6x - 3y + 6$

$f_y = 0 + 2y - 3x + 0 - 4$
$= 2y - 3x - 4$

$f_x = 0$

$6x - 3y + 6 = 0$

$2x - y + 2 = 0$ — (1)

$2x - y = -2$

$2x - y = -2$

$f_y = 0$

$2y - 3x - 4 = 0$

or $2y - 3x = 4$ — (2)

multiply eqn (1) with 2

~~$4x =$~~ $4x - 2y = -4$

$2y - 3y = 4$

$x = 0$

Substitute value of x in eqn (1)

$2(0) - y = -2$

$-y = -2$ $\therefore y = 2$

$\therefore$ Critical points are $(0, 2)$

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

Here $r > 0$

$= rt - s^2$

$= 6(2) - (-3)^2$

$= 12 - 9$

$= 3 > 0$

$\therefore f$ has maximum at $(0, 2)$

$3x^2 + y^2 - 3y + 6x - 4y$ at $(0, 2)$

$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$0 + 4 - 0 + 0 - 8$

$= -4$

iii) $f(x, y) = 2x^4 + 3x^2y - y^2$

~~f_y~~ $f_x = 8x^3 + 6x$

$f_y = 3x^2 - 2y$

$f_x = 0$

$\therefore 8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0$

$4x^2 + 3y = 0$ — (1)

$f_y = 0$

$3x^2 - 2y = 0$ — (2)

Multiply eq^n ① with 3
② with 4

$12x^2 + 9y = 0$
$-12x^2 + -8y = 0$
$17y = 0$
$\therefore y = 0$ //

Substitute value of y in eq^n ①
$4x^2 + 3(0) = 0$
$4x^2 = 0$
$x = 0$

critical point is $(0, 0)$
$r = f_{xx} = 24x^2 + 6x$
$t = f_{yy} = 0 - 2 = 2$
$s = f_{xy} = 6x - 0 = 6x = 6(0) = 0$

$r = (0, 0)$

$= 24(0) + 6(0) = 0$
$\therefore r = (0)$
$rt - s^2 = 0(-2) - (s)^2$
$= 0 - 0 = 0$
$r = 0$ & $rt = s^2 = 0$
(noting to say)

iii) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$$f_x = 2x + 2$$

$$f_y = -2y + 8$$

$$f_x = 0 \qquad \therefore 2x + 2 = 0$$

$$x = -1$$

$$f_y = 0 \qquad -2y + 8 = 0$$

$$y = \frac{-8}{-2} 4$$

$$\therefore y = 4$$

$\therefore$ critical point is $(-1, 4)$

$$r = f_{xx} = 2$$

$$t = f_{yy} = -2$$

$$s = f_{xy} = 0$$

$$r > 0$$

$$rt - s^2 = 2(-2) - (0)^2$$

$$= -4 - 0$$

$$= -4 < 0$$

$f(x, y)$ at $(-1, 4)$

$$(-1)^2 - (4)^2 + 2(1) + 8(4) - 70$$

$$= 1 + 16 - 2 + 32 - 70$$

$$= 17 + 30 - 70$$

$$= 37 - 70 \qquad = -33$$

All
14/02/2020