Ex 1.1

Relations Ex 1.1 Q1(i)

A be the set of human beings.
$R = \{(x, y) :$ x and y work at the same place$\}$

Reflexive:

$\therefore$ x and x works together
$\therefore$ $(x,x) \in R$

$\Rightarrow$ R is reflexive

Symmetric: If x and y work at the same place, which implies,
y and x work at the same place

$\therefore$ $(y,x) \in R$

$\Rightarrow$ R is symmetric

Transitive: If x and y work at the same place
then x and y work at the same place and y and z work at the same place

$\Rightarrow$ $(x,z) \in R$ and

Hence,
$\Rightarrow$ R is transitive

Relations Ex 1.1 Q1(ii)

A be the set of human beings.

$R = \{(x, y): \text{ x and y lives in the same locality }\}$

Reflexive: since x and x lives in the same locality

$\Rightarrow \quad (x,x) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $(x,y) \in R$

$\Rightarrow \quad x$ and y lives in the same locality

$\Rightarrow \quad y$ and x lives in the same locality

$\Rightarrow \quad (y,x) \in R$

Transitive: Let $(x, y) \in R$ and $(y, z) \in R$

$(x, y) \in R$

$\Rightarrow$ x and y lives in the same locality

and $(y, z) \in R$

$\Rightarrow$ y and z lives in the same locality

$\Rightarrow$ x and z lives in the same locality

$\Rightarrow \quad (x, z) \in R$

$\Rightarrow$ R is transitive

Relations Ex 1.1 Q1(iii)

$R = \{(x, y): x \text{ is wife of } y\}$

Reflexive: since x can not be wife of x

$\therefore \quad (x,x) \notin R$

$\Rightarrow \quad R$ is not reflexive

Symmetric: Let $(x,y) \in R$

$\Rightarrow \quad x$ is wife of y

$\Rightarrow \quad y$ is husband of x

$\Rightarrow \quad (y,x) \notin R$

$\Rightarrow \quad R$ is not symmetric

Transitive: Let $(x,y) \in R$ and $(y,z) \in R$

$\Rightarrow \quad x$ is wife of y and y is husband of z

which is a contradiction

$\Rightarrow \quad (x,z) \notin R$

$\Rightarrow \quad R$ is not transitive

Relations Ex 1.1 Q1(iv)

A be the set of human beings
R = {(x, y) : x is father of y}
Reflexive: since x can not be father of x
$\therefore$ $(x, x) \notin R$

$\Rightarrow$ R is not reflexive

Symmetric: Let $(x, y) \in R$

$\Rightarrow$ x is father of y
$\Rightarrow$ y can not be father of x

$\Rightarrow$ $(y, x) \notin R$

$\Rightarrow$ R is not symmetric

Transitive: Let $(x, y) \in R$ and $(y, z) \in R$

$\Rightarrow$ x is father of y and y is father of z
$\Rightarrow$ x is grandfather of z

$\Rightarrow$ $(x, z) \notin R$

$\Rightarrow$ R is not transitive

Relations Ex 1.1 Q2

We have, $A = \{a, b, c\}$
$R_1 = \{(a,a)(a,b)(a,c)(b,b)(b,c)(c,a)(c,b)(c,c)\}$

R_1 is reflexive as $(a,a) \in R_1, (b,b) \in R_1$ & $(c,c) \in R_1$
R_1 is not symmetric as $(a,b) \in R_1$ but $(b,a) \in R_1$
R_1 is not transitive as $(b,c) \in R_1$ and $(c,a) \in R_1$ but $(b,a) \notin R_1$

$R_2 = \{(a,a)\}$
R_2 is not reflexive as $(b,b) \notin R_2$
R_2 is symmetric and transitive.

$R_3 = \{(b,c)\}$
R_3 is not reflexive as $(b,b) \notin R_3$
R_3 is not symmetric
R_3 is not transitive.

$R_4 = \{(a,b)(b,c)(c,a)\}$
R_4 is not reflexive on set A as $(a,a) \notin R_4$
R_4 is not symmetric as $(a,b) \in R_4$ but $(b,a) \notin R_4$
R_4 is not transitive as $(a,b) \in R_4$ and $(b,c) \in R_4$ but $(a,c) \notin R_4$

Relations Ex 1.1 Q3

$R_1 = \left\{(x, y), x, y \in Q_0, x = \frac{1}{y}\right\}$

Reflexivity: Let, $x \in Q_0$

$\Rightarrow \quad x \neq \frac{1}{x}$

$\Rightarrow \quad (x, x) \in R_1$

$\therefore \quad R_1$ is not reflexive

Symmetric: Let, $(x, y) \in R_1$

$\Rightarrow \quad x = \frac{1}{y}$

$\Rightarrow \quad y = \frac{1}{x}$

$\Rightarrow \quad (y, x) \in R_1$

$\therefore \quad R_1$ is symmetric symmetric

Transitive: Let, $(x, y) \in R_1$ and $(y, z) \in R_1$

$\Rightarrow \quad x = \frac{1}{y}$ and $y = \frac{1}{z}$

$\Rightarrow \quad x = z$

$\Rightarrow \quad (x, z) \notin R_1$

$\therefore \quad R_1$ is not trasitive

Relations Ex 1.1 Q3(ii)

Reflexivity: Let, $a \in z$

$\Rightarrow \quad |a - a| = 0 \leq 5$

$\therefore \quad (a, a) \in R_2 \Rightarrow \quad R_2$ is reflexive

Symmetricity:Let, $(a, b) \in R_2$

$\Rightarrow \quad |a - a| \leq 5$

$\Rightarrow \quad |b - a| \leq 5$

$\Rightarrow \quad |b, a| \in R_2 \quad \Rightarrow \quad R_2$ is symmetric

Transitivity: Let, $(a,b) \in R_2$ and $(b,c) \in R_2$

$\Rightarrow \quad |a - b| \leq 5$ and $|b - c| \leq 5$

$\not\Rightarrow \quad |a - c| \leq 5$

$\Rightarrow \quad R_2$ is not transitive

$$\left[\begin{array}{l} \therefore \text{ if } a = 15, b = 11, c = 7 \\ \qquad \Rightarrow \quad |15 - 11| \leq 5 \text{ and } |11 - 7| \leq 5 \\ \qquad\qquad \text{but } |15 - 7| \geq 5 \end{array}\right]$$

Relations Ex 1.1 Q4

(i) We have, $A = \{1, 2, 3\}$ and

$R_1 = \{(1,1)(1,3)(3,1)(2,2)(2,1)(3,3)\}$

$\therefore \quad (1,1), (2,2)$ and $(3,3) \in R_1$

$\therefore \quad R_1$ is not Reflexive

Now,

$\therefore \quad (2,1) \in R_1$ but $(1,2) \notin R_1$

$\therefore \quad R_1$ is not Symmetric

Again,

$\therefore \quad (2,1) \in R_1$ and $(1,3) \in R_1$ but $(2,3) \notin R_1$

$\therefore \quad R_1$ is not Transitive

(ii) $R_2 = \{(2,2), (3,1), (1,3)\}$

$\therefore \quad (1,1) \notin R_2$

$\Rightarrow \quad R_2$ is not reflexive

Now, $(1,3) \in R_2$

$\Rightarrow \quad (3,1) \in R_2$

$\Rightarrow \quad R_2$ is symmetric

Again, $(3,1) \in R_2$ and $(1,3) \in R_2$ but $(3,3) \notin R_1$

$\therefore \quad R_2$ is not transitive

(iii) $R_3 = \{(1,3)(3,3)\}$

$\therefore \quad (1,1) \notin R_3$

$\Rightarrow \quad R_3$ is not reflexive

Now, $(1,3) \in R_3$ but $(3,1) \in R_3$

$\Rightarrow \quad R_3$ is not symmetric

Again, It is clear that R_3 is transitive

Relations Ex 1.1 Q5.

(i) aRb if a-b $\rangle$ 0
Let R be the set of real numbers.

Reflexivity: Let a $\in R$

$\Rightarrow$ $a - a = 0$
$\Rightarrow$ $(a,a) \notin R$

$\therefore$ R is not reflexive

Symmetric: Let aRb

$\Rightarrow$ $a - a > 0$
$\Rightarrow$ $b - a < 0$

$\therefore$ $b \not R a$

$\therefore$ R is not Symmetric

Transitive: Let aRb and bRc

$\Rightarrow$ $a - a >$ and b $- c > 0$
$\Rightarrow$ $a - c > 0$
$\Rightarrow$ aRc

$\therefore$ R is Transitive

Relations Ex 1.1 Q5(ii)

We have, aRb iff $1 + ab > 0$
Let R be the set of real numbers

Reflexive: Let a $\in R$

$\Rightarrow$ $1 + a^2 > 0$
$\Rightarrow$ aRa

$\Rightarrow$ R is reflexive

Symmetric: Let aRb

$\Rightarrow$ $1 + ab > 0$
$\Rightarrow$ $1 + ba > 0$
$\Rightarrow$ bRa

$\Rightarrow$ R is symmetric

Transitive: Let aRb and bRc

$\Rightarrow$ $1 + ab > 0$ and $1 + bc > 0$
$\not\Rightarrow$ $1 + ac > 0$

$\Rightarrow$ R is not transitive

Relations Ex 1.1 Q5(iii)

We have, aRb if $|a| \le b$

Reflexivity: Let $a \in R$

$\Rightarrow \quad |a| \not\le a \quad [\because \quad |-2| = 2 > -2]$

$\Rightarrow \quad R$ is not reflexive

Symmetric: Let aRb

$\Rightarrow \quad |a| \le b$

$\not\Rightarrow \quad |b| \le a \quad \left[\because \quad \begin{matrix} \text{Let } a = 4,\ b = 6 \\ |4| \le 8 \text{ but } |8| > 4 \end{matrix}\right]$

$\Rightarrow \quad R$ is not symmetric

Transitive: Let aRb and bRc

$\Rightarrow \quad |a| \le b$ and $|b| \le c$
$\Rightarrow \quad |a| \le |b| \le c$
$\Rightarrow \quad |a| \le c$
$\Rightarrow \quad a\ R\ c$

$\Rightarrow \quad R$ is transitive

Relations Ex 1.1 Q6.

Let $A = \{1, 2, 3, 4, 5, 6\}$.
A relation R is defined on set A as:
$R = \{(a, b): b = a + 1\}$
Therefore, $R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$
We find $(a, a) \notin R$, where $a \in A$.
For instance, $(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) \notin R$
Therefore, R is not reflexive.
It can be observed that $(1, 2) \in R$, but $(2, 1) \notin R$.
Therefore, R is not symmetric.
Now, $(1, 2), (2, 3) \in \mathbf{R}$
But, $(1, 3) \notin R$
Therefore, R is not transitive
Hence, R is neither reflexive, nor symmetric, nor transitive.

Relations Ex 1.1 Q7.

$R = \{(a, b): a \le b^3\}$
It is observed that $\left(\frac{1}{2}, \frac{1}{2}\right) \notin R$ as $\frac{1}{2} > \left(\frac{1}{2}\right)^3 = \frac{1}{8}$.
Therefore, R is not reflexive.
Now, $(1, 2) \in R$ (as $1 < 2^3 = 8$)
But, $(2, 1) \notin R$ (as $2^3 > 1$)
Therefore, R is not symmetric.
We have
$\left(3, \frac{3}{2}\right), \left(\frac{3}{2}, \frac{6}{5}\right) \in R$ as $3 < \left(\frac{3}{2}\right)^3$ and $\frac{3}{2} < \left(\frac{6}{5}\right)^3$.
But $\left(3, \frac{6}{5}\right) \notin R$ as $3 > \left(\frac{6}{5}\right)^3$.
Therefore, R is not transitive.
Hence, R is neither reflexive, nor symmetric, nor transitive.

Relations Ex 1.1 Q8

Let A be a set.
Then $I_A = \{(a,a)\ ;\ a \in A\}$ is the identity relation on A.

Hence, every identity relation on a set is reflexive by definition.

Converse:

Let $A = \{(a,b,c)\}$ be a set.
Let $R = \{(a,a)(b,b)(c,c)(a,b)\}$ be a relation defined on A.

Clearly R is reflexive on set A, but it is not identity relation on set A as $(a,b) \in R$

Hence, a reflexive relation need not be identity relation.

Relations Ex 1.1 Q9

We have, $A = \{1,2,3,4\}$

(i) $R = \{(1,1)(2,2)(3,3)(4,4)(1,2)\}$ is a relation on set A which is reflexive,transitive but not symmetric

(ii) $R = \{(2,3)(3,2)\}$ is a relation on set A which is symmetric but neither reflexive nor transitive

(iii) $R = \{(1,1)(2,2)(3,3)(4,4)(1,2)(2,1)\}$ is a relation on set A which is reflexive, symmetric and transitive

Relations Ex 1.1 Q10

We have, $R - \{(x,y); x, y \in N, 2x + y = 41\}$

Then Domain of R is $x \in N$, such that

$$2x + y = 41$$
$$\Rightarrow \quad x = \frac{41 - y}{2}$$

Since $y \in N$, largest value that x can take corresponds to the smallest value that y can take.

$$\therefore \quad x = \{1,2,3,..........20\}$$

Range of R is $y \in N$ such that

$$2x + y = 41$$
$$\Rightarrow \quad y = 41 - 2x$$

Since, $x = \{1,2,3,..........20\}$

$$\therefore \quad y = \{39,37,35,33,............7,5,3,1\}$$

Since, $(2,2) \notin R, R$ is not reflexive.

Also, since $(1,39) \in R$ but $(39,1) \notin R$, R is not symmetric.

Finally, since, $(15,11) \in R$ and $(11,19) \in R$ but $(15,19) \notin R$

$\therefore$ R is not trasitive.

Relations Ex 1.1 Q11

No, it is not necessary that a relation which is symmetric and transitive is reflexive as well.
For Example,
Let $A = \{a,b,c\}$ be a set and
$R_2 = \{(a,a)\}$ is a relation defined on A.
Clearly,
R_2 is symmetric and transitive but not reflexive.

Relations Ex 1.1 Q12

It is given that an integer m is said to be relative to another integer n if m is a multiple of n.
In other words

$$R = \{(m,n); \quad m = kn, k \in z\}$$

Reflexivity: Let, $m \in z$

$\Rightarrow \quad m = 1.m$
$\Rightarrow \quad (m,m) \in R$

$\therefore \quad R$ is reflexive

Transitive: Let $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow \quad a = kb$ and $b = k'c$
$\Rightarrow \quad a = kk'c \qquad [\therefore \quad kk' \in z]$
$\Rightarrow \quad a = lc \qquad [\therefore \quad l = kk' \in z]$
$\Rightarrow \quad (a,c) \in R$

$\therefore \quad R$ is transitive

Symmetric: Let $(a,b) \in R$

$\Rightarrow \quad a = kb$
$\Rightarrow \quad b = \frac{1}{k}a \qquad$ but $\frac{1}{k} \notin z$ if $k \in z$
$\therefore \quad (b,a) \notin R$

$\therefore \quad R$ is not symmetric

Relations Ex 1.1 Q13

We have,
relation $R = " \geq "$ on the set R of all real numbers

Reflexivity: Let $a \in R$

$\Rightarrow \quad a \geq a$
$\Rightarrow \quad " \geq "$ is reflexive

Symmetric: Let $a,b \in R$
such that $a \geq b \not\Rightarrow b \geq a$

$\therefore \quad " \geq "$ not symmetric

Transitivity: Let $a,b,c \in R$
and $a \geq b$ & $b \geq c$

$\Rightarrow \quad a \geq c$

$\Rightarrow \quad " \geq "$ is transitive

Relations Ex 1.1 Q14

(i)Let $A = \{4, 6, 8\}$.

Define a relation R on A as:

$A = \{(4, 4), (6, 6), (8, 8), (4, 6), (6, 4), (6, 8), (8, 6)\}$

Relation R is reflexive since for every $a \in A$, $(a, a) \in$R i.e., $(4, 4), (6, 6), (8, 8)\} \in$ R.

Relation R is symmetric since $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in R$.

Relation R is not transitive since $(4, 6), (6, 8) \in R$, but $(4, 8) \notin R$.

Hence, relation R is reflexive and symmetric but not transitive.

(ii) Define a relation R in **R** as:

$R = \{a, b): a^3 \geq b^3\}$

Clearly $(a, a) \in R$ as $a^3 = a^3$. $a = a$.

Therefore, R is reflexive.

Now, $(2, 1) \in R$ (as $2^3 \geq 1^3$)

But, $(1, 2) \notin R$ (as $1^3 < 2^3$)

Therefore, R is not symmetric.

Now, Let $(a, b), (b, c) \in R$.

$\Rightarrow a^3 \geq b^3$ and $b^3 \geq c^3$

$\Rightarrow a^3 \geq c^3$

$\Rightarrow (a, c) \in R$

Therefore, R is transitive.

Hence, relation R is reflexive and transitive but not symmetric.

Hence, relation R is transitive but not reflexive and symmetric.

(iv)Let $A = \{5, 6, 7\}$.

Define a relation R on A as $R = \{(5, 6), (6, 5)\}$.

Relation R is not reflexive as $(5, 5), (6, 6), (7, 7) \notin R$.

Now, as $(5, 6) \in R$ and also $(6, 5) \in R$, R is symmetric.

$\Rightarrow (5, 6), (6, 5) \in R$, but $(5, 5) \notin R$

Therefore, R is not transitive.

Hence, relation R is symmetric but not reflexive or transitive.

(v)Consider a relation R in **R** defined as:

$R = \{(a, b): a < b\}$

For any $a \in R$, we have $(a, a) \notin R$ since a cannot be strictly less than a itself. In fact, $a = a$.

Therefore, R is not reflexive.

Now, $(1, 2) \in R$ (as $1 < 2$)

But, 2 is not less than 1.

Therefore, $(2, 1) \notin R$

Therefore, R is not symmetric.

Now, let $(a, b), (b, c) \in R$.

$\Rightarrow a < b$ and $b < c$

$\Rightarrow a < c$

$\Rightarrow (a, c) \in R$

Therefore, R is transitive.

Hence, relation R is transitive but not reflexive and symmetric.

Relations Ex 1.1 Q15

We have,

$A = \{1,2,3\}$ and $R\{(1,2)(2,3)\}$

Now,

To make R reflexive, we will add $(1,1)(2,2)$ and $(3,3)$ to get

$\therefore \quad R' = \{(1,2)(2,3)(1,1,)(2,2)(3,3)\}$ is reflexive

Again to make R' symmetric we shall add $(3,2)$ and $(2,1)$

$\therefore \quad R'' = \{(1,2)(2,3)(1,1)(2,2)(3,3)(3,2)(2,1)\}$ is reflexive and symmetric

Now,

To make R'' transitive we shall add $(1,3)$ and $(3,1)$

$\therefore \quad R''' = \{(1,2)(2,3)(1,1)(2,2)(3,3)(3,2)(2,1)(1,3)(3,1)\}$

$\therefore \quad R'''$ is reflexive, symmetric and transitive

Relations Ex 1.1 Q16

We have, $A = \{1, 2, 3\}$ and $R = \{(1,2)(1,1)(2,3)\}$

To make R transitive we shall add $(1,3)$ only.

$\therefore \quad R' = \{(1,2)(1,1)(2,3)(1,3)\}$

Relations Ex 1.1 Q17

A relation R in A is said to be reflexive if aRa for all a∈A

R is said to be transitive if aRb and bRc ⇒ aRc
for all a, b, c ∈ A.
Hence for R to be reflexive (b, b) and (c, c) must be there in the set R.
Also for R to be transitive (a, c) must be in R because (a, b) ∈ R and (b, c) ∈ R so (a, c) must be in R.
So at least 3 ordered pairs must be added for R to be reflexive and transitive.

Relations Ex 1.1 Q18

A relation R in A is said to be reflexive if aRa for all a∈A, R is symmetric if aRb ⇒ bRa, for all a, b ∈ A and it is said to be transitive if aRb and bRc ⇒ aRc for all a, b, c ∈ A.

• x > y, x, y ϵ N
(x, y) ϵ {(2, 1), (3, 1).......(3, 2), (4, 2)....}
This is not reflexive as (1, 1), (2, 2)....are absent.
This is not symmetric as (2,1) is present but (1,2) is absent.
This is transitive as (3, 2) ϵ R and (2,1) ϵ R also (3,1) ϵ R ,similarly this property satisfies all cases.

• x + y = 10, x, y ϵ N
(x, y)ϵ {(1, 9), (9, 1), (2, 8), (8, 2), (3, 7), (7, 3), (4, 6), (6, 4), (5, 5)}
This is not reflexive as (1, 1),(2, 2)..... are absent.
This only follows the condition of symmetric set as (1, 9)ϵR also (9, 1)ϵR similarly other cases are also satisfy the condition.
This is not transitive because {(1, 9),(9, 1)}ϵR but (1, 1) is absent.

• xy is square of an integer, x, y ϵ N
(x, y) ϵ {(1, 1), (2, 2), (4, 1), (1, 4), (3, 3), (9, 1), (1, 9), (4, 4), (2, 8), (8, 2), (16, 1), (1, 16)..........}
This is reflexive as (1,1),(2,2)..... are present.
This is also symmetric because if aRb ⇒ bRa, for all a,bϵN.
This is transitive also because if aRb and bRc ⇒ aRc for all a, b, c ϵ N.

• x + 4y = 10, x, y ϵ N
(x, y) ϵ {(6, 1), (2, 2)}
This is not reflexive as (1, 1), (2, 2).....are absent.
This is not symmetric because (6,1) ϵ R but (1,6) is absent.
This is not transitive as there are only two elements in the set having no element common.

Ex 1.2

Relations Ex 1.2 Q1

We have,

$R = \{(a,b): \ a-b \text{ is divisible by } 3; \ a,b, \in Z\}$

To prove: R is an equivalence relation

Proff:

Reflexivity: Let $a \in Z$

$\Rightarrow \quad a - a = 0$

$\Rightarrow \quad a - a$ is divisible by 3

$\Rightarrow \quad (a,a) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $a,b \in Z$ and $(a,b) \in R$

$\Rightarrow \quad a - b$ is divisible by 3

$\Rightarrow \quad a - b = 3p \qquad$ For some $p \in Z$

$\Rightarrow \quad b - a = 3 \times (-p)$

$\Rightarrow \quad b - a \in R$

$\Rightarrow \quad R$ is symmetric

Transitive: Let $a,b,c \in Z$ and such that $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow \quad a - b = 3p$ and $b - c = 3q$ For some $p,q \in Z$

$\Rightarrow \quad a - c = 3(p+q)$

$\Rightarrow \quad a - c$ is divisible by 3

$\Rightarrow \quad (a,c) \in R$

$\Rightarrow \quad R$ is transitive

Since, R is reflexive, symmetric and transitive, so R is equivalence relation.

Relations Ex 1.2 Q2

We have,
$R = \{(a,b) : a-b \text{ is divisible by } 2;\ a,b, \in Z\}$
To prove: R is an equivalence relation

Proff:
Reflexivity: Let $a \in Z$

$\Rightarrow \quad a-a=0$
$\Rightarrow \quad a-a$ is divisible by 2
$\Rightarrow \quad (a,a) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $a,b \in Z$ and $(a,b) \in R$

$\Rightarrow \quad a-b$ is divisible by 2
$\Rightarrow \quad a-b = 2p \quad$ For some $p \in Z$
$\Rightarrow \quad b-a = 2 \times (-p)$
$\Rightarrow \quad b-a \in R$

$\Rightarrow \quad R$ is symmetric

Transitive: Let $a,b,c \in Z$ and such that $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow \quad a-b=2p$ and $b-c = q$ For some $p,q \in Z$
$\Rightarrow \quad a-c = 2(p+q)$
$\Rightarrow \quad a-c$ is divisible by 2
$\Rightarrow \quad (a,c) \in R$
$\Rightarrow \quad R$ is transitive

Relations Ex 1.2 Q3

We have,
$R = \{(a,b) : (a-b) \text{ is divisible by } 5\}$ on Z.
We want to prove that R is an equivalence relation on Z.

Now,
Reflexivity: Let $a \in Z$

$\Rightarrow \quad a-a=0$
$\Rightarrow \quad a-a$ is divisible by 5.

$\therefore \quad (a,a) \in R$, so R is reflexive

Symmetric: Let $(a,b) \in R$

$\Rightarrow \quad a-b=5P \quad$ For some $P \in Z$
$\Rightarrow \quad b-a = 5 \times (-P)$
$\Rightarrow \quad b-a$ is divisible by 5

$\Rightarrow \quad (b,a) \in R$, so R is symmetric

Transitive: Let $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow \quad a-b=5p$ and $b-c=5q$ For some $p,q \in Z$
$\Rightarrow \quad a-c = 5(p+q)$
$\Rightarrow \quad a-c$ is divisible by 5.

$\Rightarrow \quad R$ is transitive.

Thus, R being reflexive,symmetric and transitive on Z.

Hence, R is equivalence relation on Z

Relations Ex 1.2 Q4

$R = \{(a,b):$ a-b is divisible by n$\}$ on Z.

Now,

Reflexivity: Let $a \in Z$

$\Rightarrow$ $a - a = 0 \times n$

$\Rightarrow$ $a - a$ is divisible by n

$\Rightarrow$ $(a,a) \in R$

$\Rightarrow$ R is reflexive

Symmetric: Let $(a,b) \in R$

$\Rightarrow$ $a - b = np$ For some p $\in$ Z

$\Rightarrow$ $b - a = n(-p)$

$\Rightarrow$ $b - a$ is divisible by n

$\Rightarrow$ $(b,a) \in R$

$\Rightarrow$ R is symmetric

Transitive: Let $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow$ $a - b = xp$ and $b - c = xq$ For some p,q $\in$ Z

$\Rightarrow$ $a - c = n(p + q)$

$\Rightarrow$ $a - c$ is divisible by n

$\Rightarrow$ $(a,c) \in R$

$\Rightarrow$ R is transitive

Thus, R being reflexive, symmetric and transitive on Z.

Hence, R is an equivalence relation on Z

Relations Chapter 1 Ex 1.2 Q5

We have, Z be set of integers and
$R = \{(a,b) : \text{a,b} \in \text{Z} \text{ and } a+b \text{ is even }\}$ be a relation on Z.

We want to prove that R is an equivalence relation on Z.

Now,
Reflexivity: Let a $\in Z$

$\Rightarrow \quad a+a$ is even $\quad \begin{bmatrix} \text{if a is even} \Rightarrow a+a \text{ is even} \\ \text{if a is odd} \Rightarrow a+a \text{ is even} \end{bmatrix}$

$\Rightarrow \quad (a,a) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let a,b $\in Z$ and $(a,b) \in R$

$\Rightarrow \quad a+b$ is even
$\Rightarrow \quad b+a$ is even
$\Rightarrow \quad (b,a) \in R,$

$\Rightarrow \quad R$ is symmetric

Transitivity: Let $(a,b) \in R$ and $(b,c) \in R$ For some a,b,c $\in$ Z

$\Rightarrow \quad a+b$ is even and $b+c$ is even

$\Rightarrow \quad a+c$ is even $\quad \begin{bmatrix} \text{if b is odd, then a and c must be odd} \Rightarrow a+c \text{ is even,} \\ \text{If b is even, then a and c must be even} \Rightarrow a+c \text{ is even} \end{bmatrix}$

$\Rightarrow \quad (a,c) \in R$

$\Rightarrow \quad R$ is transitive

Hence, R is an equivalence relation on Z

Relations Ex 1.2 Q6

Let Z be set of integers

$R = \{(m,n) : m - n \text{ is divisible by } 13\}$ be a relation on Z.

Now,

Reflexivity: Let $m \in Z$

$\Rightarrow \quad m - m = 0$

$\Rightarrow \quad m - m$ is divisible by 13

$\Rightarrow \quad (m,m) \in R,$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $m, n \in Z$ and $(m,n) \in R$

$\Rightarrow \quad m - n = 13.p$ For some $p \in Z$

$\Rightarrow \quad n - m = 13 \times (-p)$

$\Rightarrow \quad n - m$ is divisible by 13

$\Rightarrow \quad (n - m) \in R,$

so

$\Rightarrow \quad R$ is symmetric

Transitivity: Let $(m,n) \in R$ and $(n,q) \in R$ For some $m,n,q \in Z$

$\Rightarrow \quad m - n = 13p$ and $n - q = 13s$ For some $p,s \in Z$

$\Rightarrow \quad m - q = 13(p + s)$

$\Rightarrow \quad m - q$ is divisible by 13

$\Rightarrow \quad (m,q) \in R$

$\Rightarrow \quad R$ is transitive

Hence, R is an equivalence relation on Z

Relations Ex 1.2 Q7

(x, y) R (u, v) $\Leftrightarrow$ xv = yu

TPT Reflexive $\because$ xy = yx

$\therefore$ (x, y) R (x, y)

TPT Symmetric Let (x, y) R (u, v)

TPT (u, v) R (x, y)

Given xv = yu

$\Rightarrow$ yu = xv

$\Rightarrow$ uy = vx

$\therefore$ (u, v) R (x, y)

Transitive Let (x, y) R (u, v) and (u, v) R (p, q) ....(i)

TPT (x, y) R (p, q)

TPT xq = yp

from (1) xv = yu & uq = vp

xvuq = yuvp

xq = yp

$\therefore$ R is transitive

since R is reflexive symmetric & transitive all means it is an equivalence relation.]

Relations Ex 1.2 Q8

We have, $A = \{x \in z : \ 0 \le x \le 12\}$ be a set and
$R = \{(a,b) : \ a = b\}$ be a relation on A
Now,
Reflexivity: Let $a \in A$

$\Rightarrow \quad a = a$
$\Rightarrow \quad (a,a) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $a,b \in A$ and $(a,b) \in R$

$\Rightarrow \quad a = b$
$\Rightarrow \quad b = a$
$\Rightarrow \quad (b,a) \in R$

$\Rightarrow \quad R$ is symmetric

Transitive: Let a,b & $c \in A$
and Let $(a,b) \in R$ and $(b,c) \in R$
$\Rightarrow \quad a = b$ and $b = c$
$\Rightarrow \quad a = c$
$\Rightarrow \quad (a,c) \in R$

$\Rightarrow \quad R$ is transitive

Since R is being relfexive,symmetric and transitive, so
R is an equivalance relation.

Also, we need to find the set of all elements related to 1.
Since the relation is given by, R={(a,b):a=b}, and 1 is an element of A,
R={(1,1):1=1}
Thus, the set of all elements related to 1 is 1.

Relations Ex 1.2 Q9

(i) We have, L is the set of lines.

$R = \{(L_1, L_2): \text{ L}_1 \text{ is parallel to L}_2\}$ be a relation on L

Now,

Reflexivity: Let $L_1 \in L$

Since a line is always parallel to itself.

$\therefore \quad (L_1, L_2) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $L_1, L_2 \in L$ and $(L_1, L_2) \in R$

$\Rightarrow \quad L_1$ is parallel to L_2

$\Rightarrow \quad L_2$ is parallel to L_1

$\Rightarrow \quad (L_1, L_2) \in R$

$\Rightarrow \quad R$ is symmetric

Transitive: Let $L_1, L_2 and L_3 \in L$ such that $(L_1, L_2) \in R$ and $(L_2, L_3) \in R$

$\Rightarrow \quad L_1$ is parallel to L_2 and L_2 is parallel to L_3

$\Rightarrow \quad L_1$ is parallel to L_3

$\Rightarrow \quad (L_1, L_3) \in R$

$\Rightarrow \quad R$ is transitive

Since, R is reflexive, symmetric and transitive, so R is an equivalence relation.

(ii) The set of lines parallel to the line $y = 2x + 4$ is

$y = 2x + c$ For all $c \in R$

Where R is the set of real numbers.

Relations Ex 1.2 Q10

R = {(P_1, P_2): P_1 and P2 have same the number of sides}

R is reflexive since (P_1, P_1) ∈ R as the same polygon has the same number of sides with itself.

Let (P_1, P_2) ∈ R.

⇒ P_1 and P_2 have the same number of sides.

⇒ P_2 and P_1 have the same number of sides.

⇒ (P_2, P_1) ∈ R

∴R is symmetric.

Now,

Let (P_1, P_2), (P_2, P3) ∈ R.

⇒ P_1 and P_2 have the same number of sides. Also, P_2 and P3 have the same number of sides.

⇒ P_1 and P3 have the same number of sides.

⇒ (P_1, P3) ∈ R

∴R is transitive.

Hence, R is an equivalence relation.

The elements in A related to the right-angled triangle (T) with sides 3, 4, and 5 are those polygons which have 3 sides (since T is a polygon with 3 sides).

Hence, the set of all elements in A related to triangle T is the set of all triangles.

Relations Ex 1.2 Q11

Let A be set of points on plane.
Let $R = \{(P,Q): OP = OQ\}$ be a relation on A where O is the origin.

To prove R is an equivalence relation, we need to show that R is reflexive, symmetric and transitive on A.

Now,
Reflexivity: Let $p \in A$

Since $OP = OP \Rightarrow (P,P) \in R$

$\Rightarrow$ R is reflexive

Symmetric: Let $(P,Q) \in R$ for $P,Q \in A$

Then $OP = OQ$
$\Rightarrow$ $OQ = OP$
$\Rightarrow$ $(Q,P) \in R$
$\Rightarrow$ R is symmetric

Transitive: Let $(P,Q) \in R$ and $(Q,S) \in R$

$\Rightarrow$ $OP = OQ$ and $OQ = OS$
$\Rightarrow$ $OP = OS$
$\Rightarrow$ $(P,S) \in R$

$\Rightarrow$ R is transitive

Thus, R is an equivalence relation on A

Relations Ex 1.2 Q12

Given A={1,2,3,4,5,6,7} and R={(a,b):both a and b are either odd or even number}
Therefore,
R={(1,1),(1,3),(1,5),(1,6),(3,3),(3,5),(3,7),(5,5),(5,7),(7,7),(7,5),(7,3),(5,3),(6,1),(5,1),(3,1),
(2,2),(2,4),(2,6),(4,4),(4,6),(6,6),(6,4),(6,2),(4,2)}
Form the relation R it is seen that R is symmetric, reflecive and transitive also. Therefore R is an equivalent relation.
From the relation R it is seen that {1,3,5,7} are related with each other only and {2,4,6} are related with each other

Relations Ex 1.2 Q13

$S = \{(a,b): a^2 + b^2 = 1\}$

Now,
Reflexivity: Let $a = \frac{1}{2} \in \mathrm{R}$
Then, $a^2 + a^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \neq 1$
$\Rightarrow$ $(a,a) \notin S$

$\Rightarrow$ S is not reflexive

Hence, S in not an equivalenve relation on R

Relations Ex 1.2 Q14

We have, Z be set of integers and Z_0 be the set of non-zero integers.
$R = \{(a,b)(c,d) : ad = bc\}$ be a relation on $z \times z_0$

Now,
Reflexivity: $(a,b) \in Z \times Z_0$

$\Rightarrow \quad ab = ba$
$\Rightarrow \quad ((a,b),(a,b)) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $((a,b),(c,d)) \in R$

$\Rightarrow \quad ad = bc$
$\Rightarrow \quad cd = da$
$\Rightarrow \quad ((c,d),(a,b)) \in R$

$\Rightarrow \quad R$ is symmetric

Transitive: Let $(a,b),(c,d) \in R$ and $(c,d),(e,f) \in R$

$\Rightarrow \quad ad = bc$ and $cf = de$
$\Rightarrow \quad \frac{a}{b} = \frac{c}{d}$ and $\frac{c}{d} = \frac{e}{f}$
$\Rightarrow \quad \frac{a}{b} = \frac{e}{f}$
$\Rightarrow \quad af = be$

We have, Z be set of integers and Z_0 be the set of non-zero integers.
$R = \{(a,b)(c,d) : ad = bc\}$ be a relation on Z and Z_0.

Now,
Reflexivity: $(a,b) \in Z \times Z_0$

$\Rightarrow \quad ab = ba$
$\Rightarrow \quad ((a,b),(a,b)) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $((a,b),(c,d)) \in R$

$\Rightarrow \quad ad = bc$
$\Rightarrow \quad cd = da$
$\Rightarrow \quad ((c,d),(a,b)) \in R$

$\Rightarrow \quad R$ is symmetric

Transitive: Let $(a,b),(c,d) \in R$ and $(c,d),(e,f) \in R$

$\Rightarrow \quad ad = bc$ and $cf = de$
$\Rightarrow \quad \frac{a}{b} = \frac{c}{d}$ and $\frac{c}{d} = \frac{e}{f}$
$\Rightarrow \quad \frac{a}{b} = \frac{e}{f}$
$\Rightarrow \quad af = be$
$\Rightarrow \quad (a,b)(e,f) \in R$

$\Rightarrow \quad R$ is transitive

Hence, R is an equivalence relation on $Z \times Z_0$

Relations Ex 1.2 Q15.

R and S are two symmetric relations on set A

(i) To prove: $R \cap S$ is symmetric
Let $(a,b) \in R \cap S$

$\Rightarrow \quad (a,b) \in R$ and $(a,b) \in S$
$\Rightarrow \quad (b,a) \in R$ and $(b,a) \in S \quad [\because R$ and S are symmetric$]$
$\Rightarrow \quad (b,a) \in R \cap S$
$\Rightarrow \quad R \cap S$ is symmetric

To prove: $R \cup S$ is symmetric.
Let $(a,b) \in R \cup S$

$\Rightarrow \quad (a,b) \in R$ or $(a,b) \in S$
$\Rightarrow \quad (b,a) \in R$ or $(b,a) \in S \quad [\because R$ and S are symmetric$]$
$\Rightarrow \quad (b,a) \in R \cup S$
$\Rightarrow \quad R \cup S$ is symmetric

(ii) R and S are two relations on A such that R is reflexive.

To prove: $R \cup S$ is reflexive

Suppose $R \cup S$ is not reflexive.
This means that there is an $a \in R \cup S$ such that $(a,a) \notin R \cup S$

Since $a \in R \cup S$,
$\therefore \quad a \in R$ or $a \in S$

If $a \in R$, then $(a,a) \in R \quad [\because R$ is reflexive$]$
$\Rightarrow \quad (a,a) \in R \cup S$
Hence, $R \cup S$ is reflexive

Relations Ex 1.2 Q16.

We will prove this by means of an example.
Let $A = \{a,b,c\}$ be a set and
$R = \{(a,a)(b,b)(c,c)(a,b)(b,a)\}$ and
$S = \{(a,a)(b,b)(c,c)(b,c)(c,b)\}$ are two relations on A

Clearly R and S are transitive relation on A

Now, $R \cup S = \{(a,a)(b,b)(c,c)(a,b)(b,a)(b,c)(c,b)\}$
Here, $(a,b) \in R \cup S$ and $(b,c) \in R \cup S$
but $(a,c) \notin R \cup S$

$\therefore \quad R \cup S$ is not transitive

Ex 2.1

Functions Ex 2.1 Q1(i)

Example of a function which is one-one but not only.

let $f : N \to N$ given by $f(x) = x^2$

Check for injectivity:

let $x, y \in N$ such that

$f(x) = f(y)$

$\Rightarrow \quad x^2 = y^2$

$\Rightarrow \quad (x - y)(x + y) = 0 \qquad [\because x, y \in N \Rightarrow x + y > 0]$

$\Rightarrow \quad x - y = 0$

$\Rightarrow \quad x = y$

$\therefore \quad f$ is one-one

Surjectivity: let $y \in N$ be arbitrary, then

$f(x) = y$

$\Rightarrow \quad x^2 = y$

$\Rightarrow \quad x = \sqrt{y} \notin N$ for non-perfect square value of y.

$\therefore$ No non-perfect square value of y has a pre image in domain N.

$\therefore$ $f : N \to N$ given by $f(x) = x^2$ is one-one but not onto.

Functions Ex 2.1 Q1(ii)

Example of a function which is onto but not one-one.

let $f : R \to R$ defined by $f(x) = x^3 - x$

Check for injectivity:

let $x, y \in R$ such that

$f(x) = f(y)$

$\Rightarrow \quad x^3 - x = y^3 - y$

$\Rightarrow \quad x^3 - y^3 - (x - y) = 0$

$\Rightarrow \quad (x - y)(x^2 + xy + y^2 - 1) = 0$

$\because \quad x^2 + xy + y^2 \geq 0 \quad \Rightarrow \quad x^2 + xy + y^2 - 1 \geq -1$

$\therefore \quad x \neq y$ for some $x, y \in R$

$\therefore \quad f$ is not one-one.

Surjectivity: let $y \in R$ be arbitrary

then, $f(x) = y$

$\Rightarrow \quad x^3 - x = y$

$\Rightarrow \quad x^3 - x - y = 0$

we know that a degree 3 equation has a real root.

let $x = \alpha$ be that root

$\therefore \quad \alpha^3 - \alpha = y$

$\Rightarrow \quad f(\alpha) = y$

Thus for clearly $y \in R$. there exist $\alpha \in R$ such that $f(x) = y$

$\therefore \quad f$ is onto

$\therefore$ Hence $f : R \to R$ defined by $f(x) = x^3 - x$ is not one-one but onto.

Functions Ex 2.1 Q1(iii)

Example of a function which is neither one-one nor onto.

let $f : R \to R$ defined by $f(x) = 2$

We know that a constant function in neither one-one nor onto

Here $f(x) = 2$ is a constant function

$\therefore \quad f : R \to R$ defined by $f(x) = 2$ is neither one-one nor onto.

Functions Ex 2.1 Q2

i) $f_1 = \{(1,3),(2,5),(3,7)\}$

$A = \{1,2,3\}$, $B = \{3,5,7\}$

We can earily observe that in f_1 every element of A has different image from B.

$\therefore$ f_1 in one-one

also, each element of B is the image of some element of A.

$\therefore$ f_1 in onto.

ii)

$f_2 = \{(2,a),(3,b),(4,c)\}$

$A = \{2,3,4\}$ $B = \{a,b,c\}$

It in clear that different elements of A have different images in B

$\therefore$ f_2 in one-one

Again, each element of B is the image of some element of A.

$\therefore$ f_2 in onto

iii) $f_3 = \{(a,x),(b,x),(c,z)(d,z)\}$

$A = \{a,b,c,d\}$ $B = \{x,y,z\}$

Since, $f_3(a) = x = f_3(b)$ and $f_3(c) = z = f_3(d)$

$\therefore$ f_3 in not one-one

Again, $y \in B$ in not the image of any of the element of A

$\therefore$ f_3 in not onto

Functions Ex 2.1 Q3

We have, $f : N \to N$ defined by $f(x) = x^2 + x + 1$

Check for injectivity:

Let $x, y \in N$ such that

$$
\begin{aligned}
& f(x) = f(y) \\
\Rightarrow \quad & x^2 + x + 1 = y^2 + y + 1 \\
\Rightarrow \quad & x^2 - y^2 + x - y = 0 \\
\Rightarrow \quad & (x-y)(x+y+1) = 0 \\
\Rightarrow \quad & x - y = 0 \qquad [\because x, y \in N \Rightarrow x + y + 1 > 0] \\
\Rightarrow \quad & x = y
\end{aligned}
$$

$\therefore$ f is one-one.

Surjectivity:

Let $y \in N$, then

$$
\begin{aligned}
& f(x) = y \\
\Rightarrow \quad & x^2 + x + 1 - y = 0 \\
\Rightarrow \quad & x = \frac{-1 \pm \sqrt{1 - 4(1-y)}}{2} \notin N \quad \text{for } y > 1
\end{aligned}
$$

$\therefore$ for $y > 1$, we do not have any pre-image in domain N.

$\therefore$ f is not onto.

Functions Ex 2.1 Q4.

We have, $A = \{-1, 0, 1\}$ and $f : A \to A$

defined by $f = \{(x, x^2) : x \in A\}$

clearly $f(1) = 1$ and $f(-1) = 1$

$\therefore \quad f(1) = f(-1)$

$\therefore \quad f$ is not one-one

Again $y = -1 \in A$ in the co-domain does not have any pre image in domain A.

$\therefore \quad f$ is not onto.

Functions Ex 2.1 Q5(i)

$f : N \to N$ given by $f(x) = x^2$

let $\quad x_1 = x_2 \quad$ for $\quad x_1, x_2 \in N$

$\Rightarrow \quad x_1^2 = x_2^2 \quad \Rightarrow \quad f(x_1) = f(x_2)$

$\therefore \quad f$ in one-one.

Surjectivity: Since f takes only square value like 1,4,9,16......

so, non-perfect square values in N (co-domain) do not have pre image in domain N.

Thus, f is not onto.

Functions Ex 2.1 Q5(ii)

$f : Z \to Z$ given by $f(x) = x^2$

Injectivity: let $x_1 \& -x_1 \in Z$

$\Rightarrow \quad x_1 \neq -x_1$

$\Rightarrow \quad x_1^2 = (-x_1)^2 \quad \Rightarrow \quad f(x_1) = f(-x_1)$

$\Rightarrow \quad f$ is not one-one.

Surjective: Again, f takes only square values 1,4,9,16,...

So, no non-perfect square values in Z have a pre image in domain Z.

$\therefore \quad f$ is not onto.

Functions Ex 2.1 Q5(iii)

$f : N \to N$, given by $f(x) = x^3$

Injectivity: let $y, x \in N$ such that

$x = y$

$\Rightarrow \quad x^3 = y^3$

$\Rightarrow \quad f(x) = f(y)$

$\therefore \quad f$ is one-one

Surjective:

$\because \quad f$ attain only cubic number like 1,8,27,64,...

So, no non-cubic values of N (co-domain) have pre image in N (Domain)

$\therefore \quad f$ is not onto.

Functions Ex 2.1 Q5(iv)

$f : Z \to Z \quad$ given by $f(x) = x^3$

Injectivity: let $x, y \in Z$ such that

$x = y$

$\Rightarrow \quad x^3 = y^3$

$\Rightarrow \quad f(x) = f(y)$

$\Rightarrow \quad f(x) = f(y)$

$\Rightarrow \quad f$ is one-one.

Surjective: Since f attains only cubic values like $\pm 1, \pm 8, \pm 27, \ldots\ldots\ldots\ldots$

so, no non-cubic values of Z (co-domain) have pre image in Z (domain)

$\therefore \quad f$ is not onto.

Functions Ex 2.1 Q5(v)

$f : R \to R$ given by $f(x) = |x|$

Injectivity: let $x, y \in R$ such that

$x = y$ but if $y = -x$

$\Rightarrow \quad |x| = |y| \quad \Rightarrow \quad |y| = |-x| = x$

$\therefore \quad f$ is not one-one.

Surjective: Since f attains only positive values, for negative real numbers in R, there is no pre-image in domain R.

$\therefore \quad f$ is not onto.

Functions Ex 2.1 Q5(vi)

$f : Z \to Z$ given by $f(x) = x^2 + x$

Injective: let $x, y \in Z$ such that

$f(x) = f(y)$

$\Rightarrow \quad x^2 + x = y^2 + y$

$\Rightarrow \quad x^2 - y^2 + x - y = 0$

$\Rightarrow \quad (x - y)(x + y + 1) = 0$

$\Rightarrow$ either $x - y = 0$ or $x + y + 1 = 0$

Case I: if $x - y = 0$

$\Rightarrow \quad x = y$

$\therefore \quad f$ is injective

Case II if $x + y + 1 = 0$

$\Rightarrow \quad x + y = -1$

$\Rightarrow \quad x \neq y$

$\therefore \quad f$ is not one to one

Thus, in general, f is not one-one

Surjective:

Since $1 \in Z$ (co-domain)

Now, we wish to find if there is any pre-image in domain Z.

let $x \in Z$ such that $f(x) = 1$

$\Rightarrow \quad x^2 + x = 1 \quad \Rightarrow \quad x^2 + x - 1 = 0$

$\Rightarrow \quad x = \dfrac{-1 \pm \sqrt{1+4}}{2} \notin Z.$

So, f is not onto.

Functions Ex 2.1 Q5(vil)

$f : Z \to Z$ given by $f(x) = x - 5$

Injective: let $x, y \in Z$ such that

$f(x) = f(y)$

$\Rightarrow \quad x - 5 = y - 5$

$\Rightarrow \quad x = y$

$\therefore \quad f$ is one-one.

Surjective: let $y \in Z$ be an arbitrary element

then $f(x) = y$

$\Rightarrow \quad x - 5 = y$

$\Rightarrow \quad x = y + 5 \quad \in Z$ (domain)

Thus, for each element in co-domain Z there exists an element in domain Z such that $f(x) = y$

$\therefore \quad f$ in onto.

Since, f in one-one and onto,

$\therefore \quad f$ in bijective.

Functions Ex 2.1 Q5(viii)

$f: R \to R$ given by $f(x) = \sin x$

Injective: let $x, y \in R$ such that

$f(x) = f(y)$

$\Rightarrow \quad \sin x = \sin y$

$\Rightarrow \quad x = n\pi + (-1)^n y$

$\Rightarrow \quad x \neq y$

$\therefore \quad f$ is not one-one.

Surjective: let $y \in R$ be arbitrary such that

$f(x) = y$

$\Rightarrow \quad \sin x = y$

$\Rightarrow \quad x = \sin^{-1} y$

Now, for $y > 1$ $x \notin R$ (domain)

$\therefore \quad f$ is not onto.

Functions Ex 2.1 Q5(ix)

$f: R \to R$ difined by $f(x) : x^3 + 1$

Injective: let $x, y \in R$ such that

$f(x) = f(y)$

$\Rightarrow \quad x^3 + 1 = y^3 + 1$

$\Rightarrow \quad x^3 = y^3$

$\Rightarrow \quad x = y$

$\therefore \quad f$ is one-one.

Surjective:
let $y \in R$, then

$f(x) = y$

$\Rightarrow \quad x^3 + 1 = y \quad \Rightarrow x^3 + 1 - y = 0$

We know that degree 3 equation has atleast one real root.

$\therefore \quad$ let $x = \alpha$ be the real root.

$\therefore \quad \alpha^3 + 1 = y$

$\Rightarrow \quad f(\alpha) = y$

Thus, for each $y \in R$, there exist $\alpha \in R$ such that $f(\alpha) = y$

$\therefore \quad f$ is onto.

Since f is one-one and onto, f is bijective.

Functions Ex 2.1 Q5(x)

$f : R \to R$ defined by $f(x) = x^3 - x$

Injective: let $x, y \in R$ such that

$f(x) = f(y)$

$\Rightarrow \quad x^3 - x = y^3 - y$

$\Rightarrow \quad x^3 - y^3 - (x - y) = 0$

$\Rightarrow \quad (x - y)(x^2 + xy + y^2 - 1) = 0$

$\because \quad x^2 + xy + y^2 \geq 0 \Rightarrow x^2 + xy + y^2 - 1 \geq -1$

$\therefore \quad x^2 + xy + y^2 - 1 \neq 0$

$\Rightarrow \quad x - y = 0 \quad \Rightarrow \quad x = y$

$\therefore \quad f$ is one-one.

Surjective:

let $y \in R$, then

$f(x) = y$

$\Rightarrow \quad x^3 - x - y = 0$

We know that a degree 3 equation has atleast one real solution.

let $x = \alpha$ be that real solution

$\therefore \quad \alpha^3 - \alpha = y$

$\Rightarrow \quad f(\alpha) = y$

$\therefore$ For each $y \in R$, there exist $x = \alpha \in R$ such that $f(\alpha) = y$

$\therefore \quad f$ is onto.

Functions Ex 2.1 Q5(xi)

$f : R \to R$ defined by $f(x) = sin^2 x + cos^2 x$.

Injective: since $f(x) = sin^2 x + cos^2 x = 1$

$\Rightarrow \quad f(x) = 1$ which is a constant function we know that a constant function in neither injective nor surjective

$\therefore \quad f$ is not one-one and not onto.

Functions Ex 2.1 Q5(xii)

$f: Q-[3] \to Q$ defined by $f(x) = \frac{2x+3}{x-3}$

Injective: let $x, y \in Q-[3]$ such that

$f(x) = f(y)$

$\Rightarrow \quad \frac{2x+3}{x-3} = \frac{2y+3}{y-3}$

$\Rightarrow \quad 2xy - 6x + 3y - 9 = 2xy + 3x - 6y - 9$

$\Rightarrow \quad -6x + 3y - 3x + 6y = 0$

$\Rightarrow \quad -9(x-y) = 0$

$\Rightarrow \quad x = y$

$\Rightarrow \quad f$ is one-one.

Surjective:

let $y \in Q$ be arbitrary. then

$f(x) = y$

$\Rightarrow \quad \frac{2x+3}{x-3} = y$

$\Rightarrow \quad 2x + 3 = xy - 3y$

$\Rightarrow \quad x(2-y) = -3(y+1)$

$\therefore \quad x = \frac{-3(y+1)}{2-y} \notin Q-[3]$ for $y = 2$

$\therefore \quad f$ is not onto

Functions Ex 2.1 Q5(xiii)

$f: Q \to Q$ defined by $f(x) = x^3 + 1$

Injective: let $x, y \in Q$ such that

$f(x) = f(y)$

$\Rightarrow \quad x^3 + 1 = y^3 + 1$

$\Rightarrow \quad (x^3 - y^3) = 0$

$\Rightarrow \quad (x-y)(x^2 + xy + y^2) = 0$

but $x^2 + xy + y^2 \geq 0$

$\therefore \quad x - y = 0$

$\Rightarrow \quad x = y$

$\therefore \quad f$ is injective.

Surjective: let $y \in Q$ be arbitrary, then

$f(x) = y$

$\Rightarrow \quad x^3 + 1 - y = 0$

we know that a degree 3 equation has alteast one real solution.

let $x = \alpha$ be that solution

$\therefore \quad \alpha^3 + 1 = y$

$\therefore \quad f(\alpha) = y$

$\therefore \quad f$ is onto.

Functions Ex 2.1 Q5(xiv)

$f : R \to R$ defined by $f(x) = 5x^3 + 4$

Injective: let $x, y \in R$ such that

$f(x) = f(y)$

$\Rightarrow$ $5x^3 + 4 = 5y^3 + 4$

$\Rightarrow$ $5\left(x^3 - y^3\right) = 0$

$\Rightarrow$ $5(x - y)\left(x^2 + xy + y^2\right) = 0$

but $5\left(x^2 + xy + y^2\right) \geq 0$

$\Rightarrow$ $x - y = 0$ $\Rightarrow$ $x = y$

$\therefore$ f is one-one

Surjective: let $y \in R$ be arbitrary, then

$f(x) = y$

$\Rightarrow$ $5x^3 + 4 = y$

$\Rightarrow$ $5x^3 + 4 - y = 0$

we know that a degree 3 equation has alteast one real solution.

let $x = \alpha$ be that real solution

$\therefore$ $5\alpha^3 + 4 = y$

$\therefore$ $f(\alpha) = y$

$\therefore$ For each $y \in Q$, there $\alpha \in R$ such that $f(\alpha) = y$

$\therefore$ f is onto

Since f in one-one and onto

$\therefore$ f in bijective.

Functions Ex 2.1 Q5(xv)

$f : R \to R$ defined by $f(x) = 3 - 4x$

Injective: let $x, y \in R$ such that

$f(x) = f(y)$

$\Rightarrow$ $3 - 4x = 3 - 4y$

$\Rightarrow$ $-4(x - y) = 0$

$\Rightarrow$ $x = y$

$\therefore$ f is one-one.

Surjective: let $y \in R$ be arbitrary, such that

$f(x) = y$

$\Rightarrow$ $3 - 4x = y$

$\Rightarrow$ $x = \dfrac{3 - y}{4} \in R$

Thus for each $y \in R$, there exist $x \in R$ such that

$f(x) = y$

$\therefore$ f is onto.

Hence, f is one-one and onto and therefore bijective.

Functions Ex 2.1 Q5(xvi)

$f : R \to R$ defined by $f(x) = 1 + x^2$

Injective: let $x, y \in R$ such that

$f(x) = f(y)$

$\Rightarrow \quad 1 + x^2 = 1 + y^2$

$\Rightarrow \quad x^2 - y^2 = 0$

$\Rightarrow \quad (x - y)(x + y) = 0$

either $x = y$ or $x = -y$ or $x \neq y$

$\therefore \quad f$ is not one-one.

Surjective: let $y \in R$ be arbitrary, then

$f(x) = y$

$\Rightarrow \quad 1 + x^2 = y$

$\Rightarrow \quad x^2 + 1 - y = 0$

$\therefore \quad x = \pm\sqrt{y - 1} \notin R$ for $y < 1$

$\therefore \quad f$ is not onto.

Functions Ex 2.1 Q6

Given, $f : A \to B$ is injective such that range$(f) = \{a\}$

We know that in injective map different elements have different images.

$\therefore$ A has only one element.

Functions Ex 2.1 Q7

A = **R** − {3}, B = **R** − {1}

f: A → B is defined as $f(x) = \left(\frac{x-2}{x-3}\right)$.

Let $x,\ y \in \mathrm{A}$ such that $f(x) = f(y)$.

$$\Rightarrow \frac{x-2}{x-3} = \frac{y-2}{y-3}$$
$$\Rightarrow (x-2)(y-3) = (y-2)(x-3)$$
$$\Rightarrow xy - 3x - 2y + 6 = xy - 3y - 2x + 6$$
$$\Rightarrow -3x - 2y = -3y - 2x$$
$$\Rightarrow 3x - 2x = 3y - 2y$$
$$\Rightarrow x = y$$

Therefore, f is one-one.

Let $y \in$B = **R** − {1}.

Then, $y \neq 1$.

The function f is onto if there exists $x \in$A such that $f(x) = y$.

Now,

$$f(x) = y$$
$$\Rightarrow \frac{x-2}{x-3} = y$$
$$\Rightarrow x - 2 = xy - 3y$$
$$\Rightarrow x(1-y) = -3y + 2$$
$$\Rightarrow x = \frac{2-3y}{1-y} \in \mathrm{A} \qquad [y \neq 1]$$

Thus, for any $y \in$ B, there exists $\frac{2-3y}{1-y} \in \mathrm{A}$ such that

$$f\left(\frac{2-3y}{1-y}\right) = \frac{\left(\frac{2-3y}{1-y}\right) - 2}{\left(\frac{2-3y}{1-y}\right) - 3} = \frac{2-3y-2+2y}{2-3y-3+3y} = \frac{-y}{-1} = y.$$

$\therefore f$ is onto.

Hence, function f is one-one and onto.

Functions Ex 2.1 Q8

We have $f : R \to R$ given by $f(x) = x - [x]$

Now,

check for injectivity:

$\because\ f(x) = x - [x] \Rightarrow f(x) = 0$ for $x \in Z$

$\therefore$ Range of $f = [0, 1] \neq R$

$\therefore$ f is not one-one, where as many-one

Again, Range of $f = [0, 1] \neq R$

$\therefore$ f is an into function

Functions Ex 2.1 Q9

Suppose $f(n_1) = f(n_2)$

If n_1 is odd and n_2 is even, then we have

$n_1 + 1 = n_2 - 1 \Rightarrow n_2 - n_1 = 2$, not possible

If n_1 is even and n_2 is odd, then we have

$n_1 - 1 = n_2 + 1 \Rightarrow n_1 - n_2 = 2$, not possible

Therefore, both n_1 and n_2 must be either odd or even.

Suppose both n_1 and n_2 are odd.

Then, $f(n_1) = f(n_2) \Rightarrow n_1 + 1 = n_2 + 1 \Rightarrow n_1 = n_2$

Suppose both n_1 and n_2 are even.

Then, $f(n_1) = f(n_2) \Rightarrow n_1 - 1 = n_2 - 1 \Rightarrow n_1 = n_2$

Thus, f is one – one.

Also, any odd number $2r + 1$ in the co – domain $\mathbf{N}$ will have an even number as image in domain $\mathbf{N}$ which is

$f(n) = 2r + 1 \Rightarrow n - 1 = 2r + 1 \Rightarrow n = 2r + 2$

any even number $2r$ in the co – domain $\mathbf{N}$ will have an odd number as image in domain $\mathbf{N}$ which is

$f(n) = 2r \Rightarrow n + 1 = 2r \Rightarrow n = 2r - 1$

Thus, f is onto.

Functions Ex 2.1 Q10

We have $A = \{1, 2, 3\}$

All one-one functions from $A = \{1, 2, 3\}$ to itself are obtained by re-arranging elements of A.

Thus all possible one-one functions are:

i) $f(1) = 1,\ \ f(2) = 2,\ \ f(3) = 3$

ii) $f(1) = 2,\ \ f(2) = 3,\ \ f(3) = 1$

iii) $f(1) = 3,\ \ f(2) = 1,\ \ f(3) = 2$

iv) $f(1) = 1,\ \ f(2) = 3,\ \ f(3) = 2$

v) $f(1) = 3,\ \ f(2) = 2,\ \ f(3) = 1$

vi) $f(1) = 2,\ \ f(2) = 1,\ \ f(3) = 3$

Functions Ex 2.1 Q11

We have $f : R \to R$ given by $f(x) = 4x^3 + 7$

Let $x, y \in R$ such that

$$f(a) = f(b)$$
$$4a^3 + 7 = 4b^3 + 7$$
$$a = b$$

f is one-one.

Now let $y \in R$ be arbitrary, then

$$f(x) = y$$
$$4x^3 + 7 = y$$
$$x = (y-7)^{\frac{1}{3}} \in R$$

f is onto.

Hence the function is a bijection

Functions Ex 2.1 Q12

We have $f : R \to R$ given by $f(x) = e^x$

let $x, y \in R$, such that

$f(x) = f(y)$

$\Rightarrow \quad e^x = e^y$

$\Rightarrow \quad e^{x-y} = 1 = e^0$

$\Rightarrow \quad x - y = 0$

$\Rightarrow \quad x = y$

$\therefore$ f is one-one

clearly range of $f = (0, \infty) \neq R$

$\therefore$ f is not onto

When co-domain in replaced by R_0^+ *i.e*, $(0, \infty)$ then f becomes an onto function.

Functions Ex 2.1 Q13

We have $f : R_0^+ \to R$ given by $f(x) = log_a x : a > 0$

let $x, y \in R_0^+$, such that

$f(x) = f(y)$

$\Rightarrow \quad log_a x = log_a y$

$\Rightarrow \quad log_a^x \left(x/y\right) = 0$

$\Rightarrow \quad x/y = 1$

$\Rightarrow \quad x = y$

$\therefore$ f is one-one

Now, let $y \in R$ be arbitrany, then

$f(x) = y$

$\Rightarrow \quad log_a x = y \quad \Rightarrow \quad x = a^y \in R_0^+ \qquad \left[\because \ a > 0 \Rightarrow a^y > 0\right]$

Thus, for all $y \in R$, there exist $x = a^y$ such that $f(x) = y$

$\therefore$ f is onto

$\because$ f is one-one and onto $\quad \therefore$ f is bijective

Functions Ex 2.1 Q14

Since f is one-one, three elements of {1, 2, 3} must be taken to 3 different elements of the co-domain {1, 2, 3} under f.

Hence, f has to be onto.

Functions Ex 2.1 Q15

Suppose f is not one-one.

Then, there exists two elements, say 1 and 2 in the domain whose image in the co-domain is same.

Also, the image of 3 under f can be only one element.

Therefore, the range set can have at most two elements of the co-domain {1, 2, 3}

i.e f is not an onto function, a contradiction.

Hence, f must be one-one.

Functions Ex 2.1 Q16

Onto functions from the set $\{1, 2, 3, \ldots, n\}$ to itself is simply a permutation on n symbols 1, 2, ..., n.
Thus, the total number of onto maps from $\{1, 2, \ldots, n\}$ to itself is the same as the total number of permutations on n symbols 1, 2, ..., n, which is $n!$.

Functions Ex 2.1 Q17

Let $f_1 : R \to R$ and $f_2 : R \to R$ be two functions given by:

$$f_1(x) = x$$
$$f_2(x) = -x$$

We can earily verify that f_1 and f_2 are one-one functions.

Now,

$$(f_1 + f_2)(x) = f_1(x) + f_2(x) = x - x = 0$$

$\therefore$ $f_1 + f_2 : R \to R$ is a function given by

$$(f_1 + f_2)(x) = 0$$

Since $f_1 + f_2$ is a constant function, it is not one-one.

Functions Ex 2.1 Q18

Let $f_1 : Z \to Z$ defined by $f_1(x) = x$ and
$f_2 : Z \to Z$ defined by $f_2(x) = -x$

Then f_1 *and* f_2 are surjective functions.

Now,

$f_1 + f_2 : Z \to Z$ is given by

$$(f_1 + f_2)(x) = f_1(x) + f_2(x) = x - x = 0$$

Since $f_1 + f_2$ is a constant function, it is not surjective.

Functions Ex 2.1 Q19

Let $f_1 : R \to R$ be defined by $f_1(x) = x$
and $f_2 : R \to R$ be defined by $f_2(x) = x$

clearly f_1 *and* f_2 are one-one functions.

Now,

$F = f_1 \times f_2 : R \to R$ *is* defined by

$$F(X) = (f_1 \times f_2)(x) = f_1(x) \times f_2(x) = x^2 \quad \ldots\ldots\ldots \text{(i)}$$

Clearly, $F(-1) = 1 = F(1)$

$\therefore$ F is not one-one

Hence, $f_1 \times f_2 : R \to R$ is not one-one.

Functions Ex 2.1 Q20

Let $f_1 : R \to R$ and $f_2 : R \to R$ are two functions defined by $f_1(x) = x^3$ and

$f_2(x) = x$

clearly f_1 & f_2 are one-one functions.

Now,

$\frac{f_1}{f_2} : R \to R$ given by

$$\left(\frac{f_1}{f_2}\right)(x) = \frac{f_1(x)}{f_2(x)} = x^2 \text{ for all } x \in R.$$

let $\frac{f_1}{f_2} = f$

$\therefore$ $F = R \to R$ defined by $f(x) = x^2$

now, $F(1) = 1 = F(-1)$

$\therefore$ F is not one-one

$\therefore$ $\frac{f_1}{f_2} = R \to R$ is not one-one.

Functions Ex 2.1 Q22

We have $f : R \to R$ given by $f(x) = x - [x]$

Now,

check for injectivity:

$\because$ $f(x) = x - [x] \Rightarrow f(x) = 0$ for $x \in Z$

$\therefore$ Range of $f = [0,1] \neq R$

$\therefore$ f is not one-one, where as many-one

Again, Range of $f = [0,1] \neq R$

$\therefore$ f is an into function

Functions Ex 2.1 23

Suppose $f(n_1) = f(n_2)$

If n_1 is odd and n_2 is even, then we have

$n_1 + 1 = n_2 - 1 \Rightarrow n_2 - n_1 = 2$, not possible

If n_1 is even and n_2 is odd, then we have

$n_1 - 1 = n_2 + 1 \Rightarrow n_1 - n_2 = 2$, not possible

Therefore, both n_1 and n_2 must be either odd or even.

Suppose both n_1 and n_2 are odd.

Then, $f(n_1) = f(n_2) \Rightarrow n_1 + 1 = n_2 + 1 \Rightarrow n_1 = n_2$

Suppose both n_1 and n_2 are even.

Then, $f(n_1) = f(n_2) \Rightarrow n_1 - 1 = n_2 - 1 \Rightarrow n_1 = n_2$

Thus, f is one – one.

Also, any odd number $2r + 1$ in the co – domain $\mathbf{N}$ will have an even number as image in domain $\mathbf{N}$ which is

$f(n) = 2r + 1 \Rightarrow n - 1 = 2r + 1 \Rightarrow n = 2r + 2$

any even number $2r$ in the co – domain $\mathbf{N}$ will have an odd number as image in domain $\mathbf{N}$ which is

$f(n) = 2r \Rightarrow n + 1 = 2r \Rightarrow n = 2r - 1$

Thus, f is onto.

Ex 2.2

Functions Ex2.2 Q1(i)

Since, $f : R \to R$ and $g : R \to R$

$\therefore \quad f \circ g : R \to R$ and $gof : R \to R$

Now, $f(x) = 2x + 3$ and $g(x) = x^2 + 5$

$g \circ f(x) = g(2x+3) = (2x+3)^2 + 5$

$\Rightarrow g \circ f(x) = 4x^2 + 12x + 14$

$f \circ g(x) = f(g(x)) = f(x^2+5) = 2(x^2+5) + 3$

$\Rightarrow f \circ g(x) = 2x^2 + 13$

Functions Ex2.2 Q1(ii)

$$f(x) = 2x + x^2 \quad \text{and} \quad g(x) = x^3$$

$$g \circ f(x) = g(f(x)) = g(2x + x^2)$$

$$g \circ f(x) = (2x + x^2)^3$$

$$f \circ g(x) = f(g(x)) = f(x^3)$$

$$\therefore \quad f \circ g(x) = 2x^3 + x^6$$

Functions Ex2.2 Q1(iii)

$f(x)=x^2+8$ and $g(x)=3x^3+1$

Thus, $g\circ f(x)=g[f(x)]$

$\Rightarrow\ g\circ f(x)=g\left[x^2+8\right]$

$\Rightarrow\ g\circ f(x)=3\left[x^2+8\right]^3+1$

Similarly, $f\circ g(x)=f[g(x)]$

$\Rightarrow\ f\circ g(x)=f\left[3x^3+1\right]$

$\Rightarrow\ f\circ g(x)=\left[3x^3+1\right]^2+8$

$\Rightarrow\ f\circ g(x)=\left[9x^6+1+6x^3\right]+8$

$\Rightarrow\ f\circ g(x)=9x^6+6x^3+9$

Functions Ex2.2 Q1(iv)

$f(x)=x$ and $g(x)=|x|$

Now, $g\circ f(x)=g(f(x))=g(x)$

$\therefore\ g\circ f(x)=|x|$

and, $f\circ g(x)=f(g(x))=f(|x|)$

$\therefore\ f\circ g(x)=|x|$

Functions Ex2.2 Q1(v)

$f(x)=x^2+2x-3$ and $g(x)=3x-4$

Now, $g\circ f(x)=g(f(x))=g\left(x^2+2x-3\right)$

$\therefore\ g\circ f(x)=3\left(x^2+2x-3\right)-4$

$\Rightarrow\ g\circ f(x)=3x^2+6x-13$

and, $f\circ g(x)=f(g(x))=f(3x-4)$

$\therefore\ f\circ g(x)=(3x-4)^2+2(3x-4)-3$

$=gx^2+16-24x+6x-8-3$

$\therefore\ f\circ g(x)=9x^2-18x+5$

Functions Ex2.2 Q1(vi)

$f(x)=8x^3$ and $g(x)=x^{1/3}$

Now, $g\circ f(x)=g(f(x))=g\left(8x^3\right)$

$=\left(8x^3\right)^{1/3}$

$\therefore\ g\circ f(x)=2x$

and, $f\circ g(x)=f(g(x))=f\left(x^{1/3}\right)$

$=8\left(x^{1/3}\right)^3$

$\therefore\ f\circ g(x)=8x$

Functions Ex2.2 Q2

Let $f = \{(3,1), (9,3), (12,4)\}$ and

$g = \{(1,3), (3,3), (4,9), (5,9)\}$

Now,

range of $f = \{1, 3, 4\}$

domain of $f = \{3, 9, 12\}$

range of $g = \{3, 9\}$

domain of $g = \{1, 3, 4, 5\}$

since, range of $f \subset$ domain of g

$\therefore \quad g \circ f$ in well defined.

Again, range of $g \subseteq$ domain of f

$\therefore \quad f \circ g$ in well defined.

Now $g \circ f = \{(3,3), (9,3), (12,9)\}$

$f \circ g = \{(1,1), (3,1)(4,3), (5,3)\}$

Functions Ex2.2 Q3

We have,

$f = \{(1,-1), (4,-2), (9,-3), (16,4)\}$ and

$g = \{(-1,-2), (-2,-4), (-3,-6), (4,8)\}$

Now,

Domain of $f = \{1, 4, 9, 16\}$

Range of $f = \{-1, -2, -3, 4\}$

Domain of $g = \{-1, -2, -3, 4\}$

Range of $g = \{-2, -4, -6, 8\}$

Clearly range of f = domain of g

$\therefore$ $g \circ f$ is defined.

but, range of $g \neq$ domain of f

$\therefore \quad f \circ g$ in not defined.

Now,

$g \circ f(1) = g(-1) = -2$

$g \circ f(4) = g(-2) = -4$

$g \circ f(9) = g(-3) = -6$

$g \circ f(16) = g(4) = 8$

$\therefore \quad g \circ f = \{(1,-2), (4,-4), (9,-6), (16,8)\}$

Functions Ex2.2 Q4

$A = \{a, b, c\}$, $B = \{u, v, w\}$ and
$f = A \to B$ and $g : B \to A$ defined by
$f = \{(a, v), (b, u), (c, w)\}$ and
$g = \{(u, b), (v, a), (w, c)\}$

For both f and g, different elements of domain have different images
$\therefore$ f and g are one-one

Again for each element in co-domain of f and g, there in a pre image in domain
$\therefore$ f and g are onto

Thus, f and g are bijectives.

Now,
$$g \circ f = \{(a, a), (b, b), (c, c)\} \text{ and}$$
$$f \circ g = \{(u, u), (v, v), (w, w)\}$$

Functions Ex2.2 Q5

We have, $f : R \to R$ given by $f(x) = x^2 + 8$ and
$g : R \to R$ given by $g(x) = 3x^3 + 1$

$$\therefore\ f \circ g(x) = f(g(x)) = f(3x^3 + 1)$$
$$= (3x^3 + 1)^2 + 8$$

$$\therefore\ f \circ g(2) = (3 \times 8 + 1)^2 + 8 = 625 + 8 = 633$$

Again
$$g \circ f(x) = g(f(x)) = g(x^2 + 8)$$
$$= 3(x^2 + 8)^3 + 1$$

$$\therefore\ g \circ f(1) = 3(1 + 8)^3 + 1 = 2188$$

Functions Ex2.2 Q6

We have, $f : R^+ \to R^+$ given by
$f(x) = x^2$
$g : R^+ \to R^+$ given by
$g(x) = \sqrt{x}$

$$\therefore\ f \circ g(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x$$

Also,
$$g \circ f(x) = g(f(x)) = g(x^2) = \sqrt{x^2} = x$$

Thus,
$$f \circ g(x) = g \circ f(x)$$

Functions Ex2.2 Q7

We have, $f : R \to R$ and $g : R \to R$ are two functions defined by
$f(x) = x^2$ and $g(x) = x + 1$

Now,
$$f \circ g(x) = f(g(x)) = f(x + 1) = (x + 1)^2$$
$$\therefore f \circ g(x) = x^2 + 2x + 1 \ldots\ldots\ldots\ldots\ldots \text{(i)}$$
$$g \circ f(x) = g(f(x)) = g(x^2) = x^2 + 1 \ldots\ldots\ldots\text{(ii)}$$
from (i) & (ii)
$$f \circ g \neq g \circ f$$

Functions Ex2.2 Q8

Let $f : R \to R$ and $g : R \to R$ are defined as
$f(x) = x + 1$ and $g(x) = x - 1$

Now,
$$f \circ g(x) = f(g(x)) = f(x-1) = x - 1 + 1$$
$$= x = I_R \,\ldots\ldots\ldots (i)$$

Again,
$$f \circ g(x) = f(g(x)) = g(x+1) = x + 1 - 1$$
$$= x = I_R \,\ldots\ldots\ldots (ii)$$

from (i) & (ii)
$$f \circ g = g \circ f = I_R$$

Functions Ex2.2 Q9

We have, $f : N \to Z_0$, $g : Z_0 \to Q$ and
$h : Q \to R$

Also, $f(x) = 2x$, $g(x) = \frac{1}{x}$ and $h(x) = e^x$

Now, $f : N \to Z_0$ and $h \circ g : Z_0 \to R$
$\therefore$ $(h \circ g) \circ f : N \to R$

also, $g \circ f : N \to Q$ and $h : Q \to R$
$\therefore$ $h \circ (g \circ f) : N \to R$

Thus, $(h \circ g) \circ f$ and $h \circ (g \circ f)$ exist and are function from N to *set* R.

Finally. $(h \circ g) \circ f\ (x) = (h \circ g)(f(x)) = (h \circ g)(2x)$
$$= h\left(1/2x\right)$$
$$= e^{1/2x}$$

now, $h \circ (g \circ f)(x) = h \circ (g(2x)) = h\left(1/2x\right)$
$$= e^{1/2x}$$
Hence, associativity verified.

Functions Ex2.2 Q10

We have,
$$h \circ (g \circ f)(x) = h(g \circ f(x)) = h(g(f(x)))$$
$$= h(g(2x)) = h(3(2x) + 4)$$
$$= h(6x + 4) = \sin(6x + 4) \quad \forall x \in \mathbf{N}$$
$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = (h \circ g)(2x)$$
$$= h(g(2x)) = = h(3(2x) + 4)$$
$$= h(6x + 4) = \sin(6x + 4) \quad \forall x \in \mathbf{N}$$
This shows, $h \circ (g \circ f) = (h \circ g) \circ f$

Functions Ex2.2 Q11

Define f: $\mathbf{N} \to \mathbf{N}$ by, $f(x) = x + 1$
And, g: $\mathbf{N} \to \mathbf{N}$ by,
$$g(x) = \begin{cases} x - 1 & \text{if } x > 1 \\ 1 & \text{if } x = 1 \end{cases}$$
We first show that f *is* not onto.
For this, consider element 1 in co-domain $\mathbf{N}$. It is clear that this element is not an image of any of the elements in domain $\mathbf{N}$.
Therefore, f is not onto.
Now, *gof*: $\mathbf{N} \to \mathbf{N}$ is defined by,

Functions Ex2.2 Q12

Define f: $\mathbf{N} \to \mathbf{Z}$ as $f(x) = x$ and g: $\mathbf{Z} \to \mathbf{Z}$ as $g(x) = |x|$.

We first show that g is not injective.

It can be observed that:

$g(-1) = |-1| = 1$

$g(1) = |1| = 1$

Therefore, $g(-1) = g(1)$, but $-1 \neq 1$.

Therefore, g is not injective.

Now, gof: $\mathbf{N} \to \mathbf{Z}$ is defined as $gof(x) = g(f(x)) = g(x) = |x|$.

Let $x, y \in \mathbf{N}$ such that $gof(x) = gof(y)$.

$\Rightarrow |x| = |y|$

Since x and $y \in \mathbf{N}$, both are positive.

$\therefore |x| = |y| \Rightarrow x = y$

Hence, gof is injective

Functions Ex2.2 Q13

We have, $f: A \to B$ and $g: B \to C$ are one-one functions

Now we have to prove : $g \circ f: A \to C$ in one-one

let $x, y \in A$ such that

$$g \circ f(x) = g \circ f(y)$$

$$\Rightarrow \quad g(f(x)) = g(f(y))$$

$$\Rightarrow \quad f(x) = f(y) \qquad [\because g \text{ in one-one}]$$

$$\Rightarrow \quad x = y \qquad [\because f \text{ in one-one}]$$

$\therefore$ $g \circ f$ is one-one function

Functions Ex2.2 Q14

We have, $f: A \to B$ and $g: B \to C$ are onto functions.

Now, we need to prove: $g \circ f: A \to C$ in onto.

let $y \in C$, then

$$g \circ f(x) = y$$

$$\Rightarrow \quad g(f(x)) = y \quad \ldots\ldots\ldots\ldots\text{(i)}$$

Since g is onto, for each element in C, then exists a preimage in B.

$$\therefore \quad g(x) = y \quad \ldots\ldots\ldots\ldots\text{(ii)}$$

From (i) & (ii)

$$f(x) = \alpha.$$

Since f is onto, for each element in B there exists a preimage in A

$$\therefore f(x) = \alpha \quad \ldots\ldots\ldots\ldots\text{(iii)}$$

From (ii) and (iii) we can conclude that for each $y \in C$, there exists a pre image in A such that $g \circ f(x) = y$

$\therefore$ $g \circ f$ is onto

Ex 2.3

Functions Ex 2.3 Q 1(i)

$f(x) = e^x$ and $g(x) = log_e x$

Now, $f \circ g(x) = f(g(x)) = f(log_e x) = e^{log_e x} = x$

$f \circ g(x) = x$

$g \circ f(x) = g(f(x)) = g(e^x) = log_e e^x = x$

$\Rightarrow \quad g \circ f(x) = x$

Functions Ex 2.3 Q 1(ii)

$f(x) = x^2, \quad g(x) = \cos x$

Domain of f and Domain of $g = R$

Range of $f = (0, \infty)$

Range of $g = (-1, 1)$

$\therefore$ Range of $f \subset$ domain of $g \Rightarrow g \circ f$ exist

Range of $g \subset$ domain of $f \Rightarrow f \circ g$ exist

Now,

$g \circ f(x) = g(f(x)) = g(x^2) = \cos x^2$

And

$f \circ g(x) = f(f(x)) = f(\cos x) = \cos^2 x$

Functions Ex 2.3 Q1(iii)

$f(x) = |x|$ and $g(x) = \sin x$

Range of $f = (0, \infty) \subset$ Domain $g(R) \Rightarrow g \circ f$ exist

Range of $g = [-1, 1] \subset$ Domain of $(R) \Rightarrow f \circ g$ exist

Now,

$$f \circ g(x) = f(g(x)) = f(\sin x) = |\sin x|$$

And

$$g \circ f(x) = g(f(x)) = g(|x|) = \sin |x|$$

Functions Ex 2.3 Q1(iv)

$f(x) = x + 1$ and $g(x) = e^x$

Range of $f = R \subset$ Domain of $g = R \Rightarrow g \circ f$ exist

Range of $g = (0, \infty) \subset$ Domain of $f = R \Rightarrow f \circ g$ exist

Now,

$$g \circ f(x) = g(f(x)) = g(x+1) = e^{x+1}$$

And

$$f \circ g(x) = f(g(x)) = f(e^x) = e^x + 1$$

Functions Ex 2.3 Q1(v)

$f(x) = \sin^{-1} x$ and $g(x) = x^2$

Range of $f = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \subset$ Domain of $g = R \Rightarrow g \circ f$ exist

Range of $g = (0, \infty) \subseteq$ Domain of $f = R \Rightarrow f \circ g$ exist

Now,

$$f \circ g(x) = f(g(x)) = f(x^2) = \sin^{-1} x^2$$

And

$$g \circ f(x) = g(f(x)) = g(\sin^{-1} x) = (\sin^{-1} x)^2$$

Functions Ex 2.3 Q 1(vi)

$f(x) = x + 1$ and $g(x) = \sin x$

Range of $f = R \subset$ Domain of $g = R \Rightarrow g \circ f$ exists

Range of $g = [-1, 1] \subset$ Domain of $f = R \Rightarrow f \circ g$ exists

Now,

$$f \circ g(x) = f(g(x)) = f(\sin x) = \sin x + 1$$

And

$$g \circ f(x) = g(f(x)) = g(x+1) = \sin(x+1)$$

Functions Ex 2.3 Q1(vii)

$f(x) = x + 1$ and $g(x) = 2x + 3$

Range of $f = R \subseteq$ Domain of $g = R \Rightarrow g \circ f$ exist

Range of $g = R \subseteq$ Domain of $R = R \Rightarrow f \circ g$ exist

Now,

$$f \circ g(x) = f(g(x)) = f(2x+3) = (2x+3) + 1 = 2x + 4$$

And

$$g \circ f(x) = g(f(x)) = g(x+1) = 2(x+1) + 3$$

$$\Rightarrow \quad g \circ f(x) = 2x + 5$$

Functions Ex 2.3 Q1(viii)

$f(x) = c, \quad c \in R$ and
$g(x) = \sin x^2$

Range of $f = R \subset$ Domain of $g = R \Rightarrow g \circ f$ exist
Range of $g = [-1, 1] \subset$ Domain of $f = R \Rightarrow f \circ g$ exist

Now,
$$g \circ f(x) = g(f(x)) = g(c) = \sin c^2$$
And
$$f \circ g(x) = f(g(x)) = f(\sin x^2) = c$$

Functions Ex 2.3 Q1(ix)

$f(x) = x^2 + 2$ and $g(x) = 1 - \frac{1}{1-x}$

Range of $f = (2, \infty) \subset$ Domain of $g = R \Rightarrow g \circ f$ exist
Range of $g = R - [1] \subset$ Domain of $f = R \Rightarrow f \circ g$ exist

Now,
$$f \circ g(x) = f(g(x)) = f\left(\frac{-x}{1-x}\right) = \frac{x^2}{(1-x)^2} + 2$$
And
$$g \circ f(x) = g(f(x)) = g(x^2 + 2) = \frac{-(x^2+2)}{1-(x^2+2)}$$

$$\Rightarrow \quad g \circ f(x) = \frac{x^2+2}{x^2+1}$$

Functions Ex 2.3 Q2

We have, $f(x) = x^2 + x + 1$ and $g(x) = \sin x$

Now,
$$f \circ g(x) = f(g(x)) = f(\sin x)$$
$$\Rightarrow \quad f \circ g(x) = \sin^2 x + \sin x + 1$$

Again, $g \circ f(x) = g(f(x)) = g(x^2 + x + 1)$
$$\Rightarrow \quad g \circ f(x) = \sin(x^2 + x + 1)$$
Clearly
$$f \circ g \neq g \circ f$$

Functions Ex 2.3 Q3

We have $\quad f(x) = |x|$

We assume the domain of $f = R$
Range of $f = (0, \infty)$
$\therefore$ Range of f $\subset$ domain of f
$\therefore$ $f \circ f$ exists.

Now,
$$f \circ f(x) = f(f(x)) = f(|x|) = ||x|| = f(x)$$

$\therefore \quad f \circ f = f$

Functions Ex 2.3 Q4

$f(x) = 2x + 5$ and $g(x) = x^2 + 1$

$\therefore$ Range of $f = R$ and range of $g = [1, \infty]$

$\therefore$ Range of $f \subseteq$ Domain of $g(R)$ and range of $g \subseteq$ domain of $f(R)$

$\therefore$ both fog and gof exist.

i) $f \circ g(x) = f(g(x)) = f(x^2 + 1)$

$= 2(x^2 + 1) + 5$

$\Rightarrow$ $f \circ g(x) = 2x^2 + 7$

ii) $g \circ f(x) = g(f(x)) = g(2x + 5)$

$= (2x + 5)^2 + 1$

$\Rightarrow$ $g \circ f(x) = 4x^2 + 20x + 26$

iii) $f \circ f(x) = f(f(x)) = f(2x + 5)$

$= 2(2x + 5) + 5$

$f \circ f(x) = 4x + 15$

iv) $f^2(x) = [f(x)]^2 = (2x + 5)^2$

$= 4x^2 + 20x + 25$

$\therefore$ from (iii)& (iv)

$f \circ f \neq f^2$

Functions Ex 2.3 Q5

We have, $f(x) = \sin x$ and $g(x) = 2x$.

Domain of f and g is R

Range of $f = [-1, 1]$

Range of $g = R$

$\therefore$ Range of $f \subset$ Domain g and

Range of $g \subseteq$ Domain f

$\therefore$ fog and gof both exist.

i) $g \circ f(x) = g(f(x)) = g(\sin x) = g \circ f(x) = 2 \sin x$

ii) $f \circ g(x) = f(g(x)) = f(2x) = \sin 2x$

$\therefore$ $g \circ f \neq f \circ g$

Functions Ex 2.3 Q6

$f, g,$ and h are real functions given by $f(x) = \sin x$, $g(x) = 2x$ and $h(x) = \cos x$

To prove: $f \circ g = g \circ (fh)$

L.H.S

$f \circ g(x) = f(g(x))$

$= f(2x) = \sin 2x$

$\Rightarrow f \circ g(x) = 2 \sin x \cos x$ ....(A)

R.H.S

$g \circ (fh)(x) = go(f(x).h(x))$

$= g(\sin x \cos x)$

$g \circ (fh)(x) = 2 \sin x \cos x$ ....(B)

from A & B

$f \circ g(x) = g \circ (fh)(x)$

Functions Ex 2.3 Q7

We are given that f is a real function and g is a function given by $g(x) = 2x$

To prove; $g \circ f = f + f$.

L.H.S

$$g \circ f(x) = g(f(x)) = 2f(x)$$
$$= f(x) + f(x) = \text{R.H.S}$$
$$\Rightarrow \quad g \circ f = f + f$$

Functions Ex 2.3 Q8

$$f(x) = \sqrt{1-x}\ , \quad g(x) = log_e^x$$

Domain of f and g are R.

Range of $f = (-\infty, 1)$

Range of $g = (0, e)$

Clearly Range $f \subset$ Domain $g \Rightarrow g \circ f$ exists

Range $g \subset$ Domain $f \Rightarrow$ f∘g exists

$$\therefore\ g \circ f(x) = g(f(x)) = g\left(\sqrt{1-x}\right)$$
$$g \circ f(x) = log_e^{\sqrt{1-x}}$$

Again

$$f \circ g(x) = f(g(x)) = f\left(log_e^x\right)$$
$$f \circ g(x) = \sqrt{1 - log_e^x}$$

Functions Ex 2.3 Q9

$f : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R$ and $g : [-1, 1] \to R$ defined as $f(x) = tan\,x$ and $g(x) = \sqrt{1-x^2}$

Range of f : let $y = f(x) \Rightarrow y = tan\,x$

$$\Rightarrow x = tan^{-1} y$$

Since $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), y \in (-\infty, \infty)$

$\therefore$ Range of f $\subset$ domain of $g = [-1, 1]$

$\therefore\ g \circ f$ exists.

By similar argument $f \circ g$ exists.

Now,

$$f \circ g(x) = f(g(x)) = f\left(\sqrt{1-x^2}\right)$$

$$f \circ g(x) = tan\sqrt{1-x^2}$$

Again

$$g \circ f(x) = g(f(x))$$
$$= g(tan\,x)$$
$$g \circ f(x) = \sqrt{1 - tan^2 x}$$

Functions Ex 2.3 Q10

$f(x) = \sqrt{x+3}$ and $g(x) = x^2 + 1$

Now,

Range of $f = [-3, \infty]$ and

Range of $g = (1, \infty)$

Then, Range of $f \subset$ Domain g and

Range of $g \subset$ Domain f

$\therefore$ $f \circ g$ and $g \circ f$ exist.

Now,

$$f \circ g(x) = f(g(x)) = f(x^2 + 1)$$

$$f \circ g(x) = \sqrt{x^2 + 4}$$

Again,

$$g \circ f(x) = g(f(x)) = g(\sqrt{x+3})$$

$$= (\sqrt{x+3})^2 + 1$$

$$g \circ f(x) = x + 4$$

Functions Ex 2.3 Q11(i)

We have, $f(x) = \sqrt{x-2}$

Clearly, Domain$(f) = [2, \infty)$ and Range$(f) = [0, \infty)$.

We observe that range(f) is not a subset of domain of f.

$$\therefore \text{Domain of (fof)} = \{x : x \in \text{Domain (f) and } f(x) \in \text{Domain (f)}\}$$

$$= \{x : x \in [2, \infty) \text{ and } \sqrt{x-2} \in [2, \infty)\}$$

$$= \{x : x \in [2, \infty) \text{ and } \sqrt{x-2} \geq 2\}$$

$$= \{x : x \in [2, \infty) \text{ and } x - 2 \geq 4\}$$

$$= \{x : x \in [2, \infty) \text{ and } x \geq 6\}$$

$$= [6, \infty)$$

Now,

$$(fof)(x) = f(f(x)) = f(\sqrt{x-2}) = \sqrt{\sqrt{x-2} - 2}$$

$\therefore$ fof : $[6, \infty) \to R$ defined as

$$(fof)(x) = \sqrt{\sqrt{x-2} - 2}$$

Functions Ex 2.3 Q11(ii)

We have, $f(x)=\sqrt{x-2}$

Clearly, $\text{Domain}(f)=[2,\infty)$ and $\text{Range}(f)=[0,\infty)$.

We observe that range(f) is not a subset of domain of f.

$$
\begin{aligned}
\therefore \text{ Domain of } (fof) &= \{x : x \in \text{Domain } (f) \text{ and } f(x) \in \text{Domain } (f)\} \\
&= \{x : x \in [2,\infty) \text{ and } \sqrt{x-2} \in [2,\infty)\} \\
&= \{x : x \in [2,\infty) \text{ and } \sqrt{x-2} \ge 2\} \\
&= \{x : x \in [2,\infty) \text{ and } x-2 \ge 4\} \\
&= \{x : x \in [2,\infty) \text{ and } x \ge 6\} \\
&= [6,\infty)
\end{aligned}
$$

Clearly, range of $f=[0,\infty) \not\subset$ Domain of (fof).

$$
\begin{aligned}
\therefore \text{ Domain of } ((fof)of) &= \{x : x \in \text{Domain } (f) \text{ and } f(x) \in \text{Domain } (fof)\} \\
&= \{x : x \in [2,\infty) \text{ and } \sqrt{x-2} \in [6,\infty)\} \\
&= \{x : x \in [2,\infty) \text{ and } \sqrt{x-2} \ge 6\} \\
&= \{x : x \in [2,\infty) \text{ and } x-2 \ge 36\} \\
&= \{x : x \in [2,\infty) \text{ and } x \ge 38\} \\
&= [38,\infty)
\end{aligned}
$$

Now,

$$(fof)(x)=f(f(x))=f(\sqrt{x-2})=\sqrt{\sqrt{x-2}-2}$$

$$(fofof)(x)=(fof)(f(x))=(fof)(\sqrt{x-2})=\sqrt{\sqrt{\sqrt{x-2}-2}-2}$$

$\therefore$ $fofof:[38,\infty)\to R$ defined as

$$(fofof)(x)=\sqrt{\sqrt{\sqrt{x-2}-2}-2}$$

Functions Ex 2.3 Q11(iii)

We have, $f(x)=\sqrt{x-2}$

Clearly, $\text{Domain}(f)=[2,\infty)$ and $\text{Range}(f)=[0,\infty)$.

We observe that range(f) is not a subset of domain of f.

$\therefore$ Domain of (fof) $=\{x : x\in \text{Domain }(f) \text{ and } f(x)\in \text{Domain }(f)\}$

$$=\{x : x\in[2,\infty) \text{ and } \sqrt{x-2}\in[2,\infty)\}$$

$$=\{x : x\in[2,\infty) \text{ and } \sqrt{x-2}\geq 2\}$$

$$=\{x : x\in[2,\infty) \text{ and } x-2\geq 4\}$$

$$=\{x : x\in[2,\infty) \text{ and } x\geq 6\}$$

$$=[6,\infty)$$

Clearly, range of $f=[0,\infty)\not\subset$ Domain of (fof).

$\therefore$ Domain of $((fof)of)$ $=\{x : x\in \text{Domain }(f) \text{ and } f(x)\in \text{Domain }(fof)\}$

$$=\{x : x\in[2,\infty) \text{ and } \sqrt{x-2}\in[6,\infty)\}$$

$$=\{x : x\in[2,\infty) \text{ and } \sqrt{x-2}\geq 6\}$$

$$=\{x : x\in[2,\infty) \text{ and } x-2\geq 36\}$$

$$=\{x : x\in[2,\infty) \text{ and } x\geq 38\}$$

$$=[38,\infty)$$

Now,

$$(fof)(x)=f(f(x))=f\left(\sqrt{x-2}\right)=\sqrt{\sqrt{x-2}-2}$$

$$(fofof)(x)=(fof)(f(x))=(fof)\left(\sqrt{x-2}\right)=\sqrt{\sqrt{\sqrt{x-2}-2}-2}$$

$\therefore$ $fofof : [38,\infty)\to R$ defined as

$$(fofof)(x)=\sqrt{\sqrt{\sqrt{x-2}-2}-2}$$

$$(fofof)(38)=\sqrt{\sqrt{\sqrt{38-2}-2}-2}=\sqrt{\sqrt{\sqrt{36}-2}-2}=\sqrt{\sqrt{6-2}-2}=\sqrt{\sqrt{4}-2}=\sqrt{2-2}=0$$

Functions Ex 2.3 Q11(iv)

We have, $f(x)=\sqrt{x-2}$

Clearly, $\text{Domain}(f)=[2,\infty)$ and $\text{Range}(f)=[0,\infty)$.

We observe that range(f) is not a subset of domain of f.

$\therefore$ Domain of (fof) $=\{x : x\in \text{Domain }(f) \text{ and } f(x)\in \text{Domain }(f)\}$

$$=\{x : x\in[2,\infty) \text{ and } \sqrt{x-2}\in[2,\infty)\}$$

$$=\{x : x\in[2,\infty) \text{ and } \sqrt{x-2}\geq 2\}$$

$$=\{x : x\in[2,\infty) \text{ and } x-2\geq 4\}$$

$$=\{x : x\in[2,\infty) \text{ and } x\geq 6\}$$

$$=[6,\infty)$$

Now,

$$(fof)(x)=f(f(x))=f\left(\sqrt{x-2}\right)=\sqrt{\sqrt{x-2}-2}$$

$\therefore$ $fof : [6,\infty)\to R$ defined as

$$(fof)(x)=\sqrt{\sqrt{x-2}-2}$$

$$f^2(x)=[f(x)]^2=\left[\sqrt{x-2}\right]^2=x-2$$

$\therefore$ $f^2 : [2,\infty)\to R$ defined as

$$f^2(x)=x-2$$

$\therefore$ $fof\neq f^2$

Functions Ex 2.3 Q12

$$f(x)\begin{cases}1+x & 0 \le x \le 2\\ 3-x & 2 \le x \le 3\end{cases}$$

$\therefore$ Range of $f = [0,3] \subseteq$ Domain of f.

$$\therefore\ f \circ f(x) = f(f(x)) = f\begin{cases}1+x & 0 \le x \le 2\\ 3-x & 2 < x \le 3\end{cases}$$

$$f \circ f(x) = \begin{cases}2+x & 0 \le x \le 1\\ 2-x & 1 < x \le 2\\ 4-x & 2 < x \le 3\end{cases}$$

Ex 2.5

Functions Ex 2.5 Q 1.

i) $f : \{1,2,3,4\} \to \{10\}$ given by

$f\{(1,10),(2,10),(3,10),(4,10)\}$

clearly f is many-one function

$\Rightarrow$ f is not bijective

$\Rightarrow$ f is not invertible

ii) $g : \{5,6,7,8\} \to \{1,2,3,4\}$ given by

$g\{(5,4),(6,3),(7,4),(8,2)\}$

Since, 5 and 7 have same mage 4

$\therefore$ g is not bijectible

$\Rightarrow$ g is not bijective

$\Rightarrow$ g is not invertible

iii) $h : \{2,3,4,5\} \to \{7,9,11,13\}$ given by

$h\{(2,7),(3,9),(4,11),(5,13)\}$

We can observe that different element of domain have defferent image is co-domain.

Functions Ex 2.5 Q2

$A = \{0, -1, -3, 2\}, \quad B = \{-9, -3, 0, 6\}$

$f : A \to B$ is defined by $f(x) = 3x$

Since different elements of A have different images in B.

$\therefore \quad f$ is one-one

Again, each element in B has a preimage in A.

$\therefore \quad f$ is onto

$\because f$ in one-one bijective

$\Rightarrow \quad f^{-1} : B \to A$ exists and is given by

$$f^{-1}(x) = \frac{x}{3}$$

$A = \{1, 3, 5, 7, 9\}, \quad B = \{0, 1, 9, 25, 49, 81\}$

$f : A \to B$ be a function defined by $f(x) = x^2$

Since different elements of A have different images in B.

$\therefore \quad f$ is one-one

Again, $0 \in B$ does not have a preimage in A.

$\therefore \quad f$ is not onto

Hence, f^{-1} does not exist.

Functions Ex 2.5 Q3

Given that $f:\{1,2,3\} \to \{a,b,c\}$ *and* $g:\{a,b,c\} \to \{apple, ball, cat\}$ *such that*

$f(1) = a$, $f(2) = b$, $f(3) = c$, $g(a) = apple$, $g(b) = ball$ *and* $g(c) = cat$

We need to prove that f, g *and* $g \circ f$ *are invertible.*

In order to prove that f is invertiblem is is sufficient to show that

$f:\{1,2,3\} \to \{a,b,c\}$ *is a bijection.*

f is one – one:

Each and every element of the set $\{1,2,3\}$ *is having an image in the set* $\{a,b,c\}$

Thus, f is one – one.

Obviously, the number of element of the sets $\{1,2,3\}$ *and* $\{a,b,c\}$ *are equal and hence*

f is onto.

Thus, the function f is invertible.

Similarly, let us observe for the function g:

g is one – one:

Each and every element of the set $\{a,b,c\}$ *is having an image in the set* $\{apple, ball, cat\}$

Thus, g is one – one.

Obviously, the number of element of the sets $\{a,b,c\}$ *and* $\{apple, ball, cat\}$ *are equal and hence*

g is onto.

Thus, the function g is invertible.

$Now\ let\ us\ consider\ the\ function\ g \circ f = g[f(x)]$
$Each\ and\ every\ element\ of\ of\ the\ set\ \{1,2,3\}\ is\ having\ an\ image\ in\ the\ set$
$\{apple,\ ball,\ cat\}.$
$Therefore,\ g \circ f = \{(1,apple),\ (2,ball),\ (3,cat)\ \}$
$Thus,\ g \circ f\ is\ one-one.$
$Since\ the\ number\ of\ elemenets\ in\ the\ sets\ \{1,2,3\}\ and\ \{apple,\ ball,\ cat\}\ are\ equal.$
$Hence\ \ g \circ f\ is\ onto.$
$Therefore,\ function\ \ g \circ f\ is\ invertible.$
$Let\ us\ now\ find\ f^{-1}:$
$We\ have\ f:\{1,2,3\} \rightarrow \{a,b,c\}$
$Thus,\ f^{-1}:\{a,b,c\} \rightarrow \{1,2,3\}$
$\Rightarrow f^{-1} = \{(a,1),(b,2),(c,3)\}$
$Let\ us\ now\ find\ g^{-1}:$
$We\ have\ g:\{a,b,c\} \rightarrow \{apple,ball,cat\}$
$Thus,\ g^{-1}:\{apple,ball,cat\} \rightarrow \{a,b,c\}$
$\Rightarrow g^{-1} = \{(apple,a),(ball,b),(cat,c)\}$
$Let\ us\ now\ find\ f^{-1} \circ g^{-1}:$
$\Rightarrow\ f^{-1} \circ g^{-1} = \{(apple,1),\ (ball,\ 2),\ (cat,3)\}.....(1)$
$Also,\ let\ us\ find,\ (g \circ f)^{-1}:$
$\Rightarrow\ (g \circ f)^{-1} = \{(apple,\ 1),\ (ball,2),\ (cat,3)\ \}....(2)$
$From\ (1)\ and\ (2),\ we\ have,$
$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

Functions Ex 2.5 Q4

Given that

$$A = \{1, 2, 3, 4\},\quad B = \{3, 5, 7, 9\},\quad C = \{7, 23, 47, 79\}$$

$f : A \to B$ and $g : B \to C$ are two functions defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 2$

Now,

$$g \circ f(x) = g(f(x)) = g(2x + 1) = (2x + 1)^2 - 2$$

$$\Rightarrow \quad g \circ f(x) = 4x^2 + 4x - 1$$

Now,

$$f : A \to B \text{ given by } f(x) = 2x + 1$$

Clearly f in one-one and onto, $\therefore f$ in bijective

$\Rightarrow \quad f^{-1}$ exist

$\therefore \quad f^{-1} = \{(3, 1), (5, 2), (7, 3), (9, 7_1)\}$

Again, $g : B \to C$ given by $g(x) = x^2 - 2$

Clearly g in one-one and onto $\Rightarrow g^{-1}$ exists

$g^{-1} = \{(7, 3), (23, 5), (47, 7), (79, 9)\}$

$f \circ^{-1} g^{-1} = \{(7, 1), (23, 2), (47, 3), (79, 4)\}$ .... (A)

Now, $g \circ f(x) = 4x^2 + 4x - 1$

Clearly gof in one-one and onto $\Rightarrow (g \circ f)^{-1}$ exists.

Hence,

$$(g \circ f)^{-1} = \{(7, 1), (23, 2), (47, 3), (79, 4)\} \ .... (B)$$

From (A) & (B) we have $\quad g \circ f^{-1} = f \circ^{-1} g^{-1}$

Functions Ex 2.5 Q5

Given that $f: Q \to Q$ *defined by* $f(x) = 3x + 5$.

To prove that f *is invertible, we need to prove that* f *is one – one and onto.*

Let $(x, y) \in Q$ *be such that,* $f(x) = f(y)$

$\Rightarrow 3x + 5 = 3y + 5$

$\Rightarrow x = y$

So, f *is an injection.*

Let y *be an arbitrary element of* Q *such that* $f(x) = y$.

$\Rightarrow 3x + 5 = y$

$\Rightarrow 3x = y - 5$

$\Rightarrow x = \dfrac{y-5}{3}$

Thus, for any $y \in Q$ *there exists* $x = \dfrac{y-5}{3} \in Q$ *such that*

$f(x) = f\left(\dfrac{y-5}{3}\right) = 3\dfrac{y-5}{3} + 5 = y$

Thus, $f: Q \to Q$ *is a bijection and hence invertible.*

Let f^{-1} *denotes the inverse of* f.

Thus, $f \circ f^{-1}(x) = x$ *for all* $x \in Q$

$\Rightarrow f\left[f^{-1}(x)\right] = x$ *for all* $x \in Q$.

$\Rightarrow 3f^{-1}(x) + 5 = x$ *for all* $x \in Q$.

$\Rightarrow f^{-1}(x) = \dfrac{x-5}{3}$ *for all* $x \in Q$

Functions Ex 2.5 Q6

f: **R** → **R** is given by, $f(x) = 4x + 3$

One-one:

Let $f(x) = f(y)$.

$\Rightarrow 4x + 3 = 4y + 3$

$\Rightarrow 4x = 4y$

$\Rightarrow x = y$

Therefore f is a one-one function.

Onto:

For $y \in \mathbf{R}$, let $y = 4x + 3$.

$\Rightarrow x = \dfrac{y-3}{4} \in \mathbf{R}$

Therefore, for any $y \in \mathbf{R}$, there exists $x = \dfrac{y-3}{4} \in \mathbf{R}$ such that

$f(x) = f\left(\dfrac{y-3}{4}\right) = 4\left(\dfrac{y-3}{4}\right) + 3 = y.$

Therefore, f is onto.

Thus, f is one-one and onto and therefore, f^{-1} exists.

Let us define g: **R**→ **R** by $g(x) = \dfrac{x-3}{4}$

$$\text{Now,} (gof)(x) = g(f(x)) = g(4x+3) = \frac{(4x+3)-3}{4} = x$$

$$(fo\,g)(y) = f(g(y)) = f\left(\frac{y-3}{4}\right) = 4\left(\frac{y-3}{4}\right) + 3 = y - 3 + 3 = y$$

Therefore, gof = fog = I_R

Hence, f is invertible and the inverse of f is given by

$f^{-1}(y) = g(y) = \dfrac{y-3}{4}.$

Functions Ex 2.5 Q7

f: $\mathbf{R}_+ \to [4, \infty)$ is given as $f(x) = x^2 + 4$.
One-one:
Let $f(x) = f(y)$.

$\Rightarrow x^2+4 = y^2+4$

$\Rightarrow x^2 = y^2$

$\Rightarrow x = y \qquad [\text{as } x = y \in \mathbf{R}_+]$

Therefore, f is a one-one function.
Onto:
For $y \in [4, \infty)$, let $y = x^2 + 4$.

$\Rightarrow x^2 = y-4 \geq 0 \qquad [\text{as } y \geq 4]$

$\Rightarrow x = \sqrt{y-4} \geq 0$

Therefore, for any $y \in \mathbf{R}$, there exists $x = \sqrt{y-4} \in \mathbf{R}$ such that

$f(x) = f\left(\sqrt{y-4}\right) = \left(\sqrt{y-4}\right)^2 + 4 = y-4+4 = y$.

Therefore, f is onto.
Thus, f is one-one and onto and therefore, f^{-1} exists.
Let us define g: $[4, \infty) \to \mathbf{R}_+$ by,

$g(y) = \sqrt{y-4}$

$\text{Now, } gof(x) = g(f(x)) = g(x^2+4) = \sqrt{(x^2+4)-4} = \sqrt{x^2} = x$

$\text{And, } fog(y) = f(g(y)) = f\left(\sqrt{y-4}\right) = \left(\sqrt{y-4}\right)^2 + 4 = (y-4)+4 = y$

Therefore, gof = fog = I_R
Hence, f is invertible and the inverse of f is given by

$f^{-1}(y) = g(y) = \sqrt{y-4}$.

Functions Ex 2.5 Q8

It is given that $f(x) = \dfrac{(4x+3)}{(6x-4)}$, $x \neq \dfrac{2}{3}$.

$$(fof)(x) = f(f(x)) = f\left(\frac{4x+3}{6x-4}\right)$$

$$= \frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4} = \frac{16x+12+18x-12}{24x+18-24x+16} = \frac{34x}{34} = x$$

Therefore, $fof(x) = x$, for all $x \neq \dfrac{2}{3}$.

$\Rightarrow fof = I$

Hence, the given function f is invertible and the inverse of f is f itself.

Functions Ex 2.5 Q9

f: $\mathbf{R}_+ \to [-5, \infty)$ is given as $f(x) = 9x^2 + 6x - 5$.
Let y be an arbitrary element of $[-5, \infty)$.
Let $y = 9x^2 + 6x - 5$.

$\Rightarrow y = (3x+1)^2 - 1 - 5 = (3x+1)^2 - 6$

$\Rightarrow (3x+1)^2 = y+6$

$\Rightarrow 3x+1 = \sqrt{y+6} \qquad [\text{as } y \geq -5 \Rightarrow y+6 > 0]$

$\Rightarrow x = \dfrac{\sqrt{y+6}-1}{3}$

Therefore, f is onto, thereby range $f = [-5, \infty)$.

Let us define g: $[-5, \infty) \to \mathbf{R}_+$ as $g(y) = \dfrac{\sqrt{y+6}-1}{3}$.

We now have:

$$\begin{aligned}(gof)(x) &= g(f(x)) = g(9x^2+6x-5) \\ &= g\left((3x+1)^2-6\right) \\ &= \frac{\sqrt{(3x+1)^2-6+6}-1}{3} \\ &= \frac{3x+1-1}{3} = x\end{aligned}$$

$$\text{And,}\ (fog)(y) = f(g(y)) = f\left(\frac{\sqrt{y+6}-1}{3}\right)$$

$$= \left[3\left(\frac{\sqrt{y+6}-1}{3}\right)+1\right]^2 - 6$$

$$= \left(\sqrt{y+6}\right)^2 - 6 = y + 6 - 6 = y$$

Therefore, $gof = I_R$ and $fog = I_{(-5, \infty)}$

Hence, f is invertible and the inverse of f is given by

$$f^{-1}(y) = g(y) = \frac{\sqrt{y+6}-1}{3}.$$

Functions Ex 2.5 Q10

$f : R \to R$ be a function defined by

$f(x) = x^3 - 3$

Injectivity:

let $f(x_1) = f(x_2)$

$\Rightarrow$ $x_1^3 - 3 = x_2^3 - 3$

$\Rightarrow$ $x_1^3 = x_2^3$

$\Rightarrow$ $x_1 = x_2$

$\Rightarrow$ f in one-one

Surjectivity: let $y \in R$ be arbitrary such that

$f(x) = y$

$\Rightarrow$ $x^3 - 3 - y = 0$

We know that an equation of odd degree must have atleast one real solution.

let $x = \alpha$ be that solution

$\therefore$ $\alpha^3 - 3 = y$

$\Rightarrow$ $f(\alpha) = y$

so, for each $y \in R$ in co-domain there exist $\alpha \in R$ in domain

$\Rightarrow$ f in onto

Thus, f in one-one and onto, so

f^{-1} exists

Now,

$\because$ $f(x) = x^3 - 3 = y$

$\Rightarrow$ $x^3 = 3 + y$

$\Rightarrow$ $x = \sqrt[3]{3+y}$

$\Rightarrow$ $f^{-1}(x) = \sqrt[3]{3+x}$

Thus, $f^{-1} : R \to R$ be the inverse function defined by $f^{-1}(x) = (x+3)^{1/3}$

finally,

$f^{-1}(24) = (24+3)^{1/3} = 3$

$f^{-1}(5) = (5+3)^{1/3} = 2$

Functions Ex 2.5 Q11

We have,

$f : R \to R$ in a function defined by

$f(x) = x^3 + 4$

Injectivity: let $f(x_1) = f(x_2)$ for $x_1 x_2 \in R$

$\Rightarrow \quad x_1^3 + 4 = x_2^3 + 4$

$\Rightarrow \quad x_1^3 = x_2^3$

$\Rightarrow \quad x_1 = x_2$

$\Rightarrow \quad f$ in one-one

Surjectivity: let $y \in R$ be artritrary such that

$f(x) = y$

$\Rightarrow \quad x^3 + 4 = y$

$\Rightarrow \quad x^3 + 4 - y = 0$

We know that an odd degree equation must have a real root.

$\Rightarrow \quad \alpha^3 + 4 = y \Rightarrow f(\alpha) = y$

$\Rightarrow \quad f$ in onto

Since f in one-one and onto

$\Rightarrow \quad f$ in bijective

finally.

$f(x) = y$

$\Rightarrow \quad x^3 + 4 = y$

$\Rightarrow \quad x^3 = y - 4$

$\Rightarrow \quad x = (y - 4)^{1/3}$

$\therefore \quad f^{-1}(x) = (x - 4)^{1/3}$

$\therefore \quad f^{-1}(3) = (3 - 4)^{1/3} = -1$

Functions Ex 2.5 Q12

$Given\ that\ f(x)=2x\ and\ g(x)=x+2.$
$We\ need\ to\ prove\ that\ f\ and\ g\ are\ bijective\ maps.$
$Let\ x,y\in Q.$
$Consider\ f(x)=f(y)$
$\Rightarrow 2x=2y$
$\Rightarrow x=y$
$\Rightarrow f\ is\ one-one.$
$Let\ y\ be\ an\ arbitrary\ element\ of\ Q\ such\ that\ f(x)=y$
$Then\ f(x)=y=2x\Rightarrow x=\frac{y}{2}$
$Thus,\ for\ any\ y\in Q,\ there\ exists\ x=\frac{y}{2}\in Q\ such\ that,$
$f(x)=f\left(\frac{y}{2}\right)=2\frac{y}{2}=y$
$So\ f:Q\rightarrow Q\ is\ a\ bijection\ and\ hence\ invertible.$
$Let\ f^{-1}\ denote\ the\ inverse\ of\ f.$
$Thus,\ f^{-1}(x)=\frac{x}{2}...(1)$
$Let\ x,y\in Q.$
$Consider\ g(x)=g(y)$
$\Rightarrow x+2=y+2$
$\Rightarrow x=y$
$\Rightarrow g\ is\ one-one.$
$Let\ y\ be\ an\ arbitrary\ element\ of\ Q\ such\ that\ g(x)=y$
$Then\ g(x)=y=x+2\Rightarrow x=y-2$
$Thus,\ for\ any\ y\in Q,\ there\ exists\ x=y-2,\ y\in Q\ such\ that,$
$g(x)=g(y-2)=y-2+2=y$
$So\ g:Q\rightarrow Q\ is\ a\ bijection\ and\ hence\ invertible.$
$Let\ g^{-1}\ denote\ the\ inverse\ of\ g.$
$Thus,\ g^{-1}(x)=x-2...(2)$
$Now\ consider\ g\circ f=g[f(x)]=g(2x)=2x+2$
$Thus,\ (g\circ f)^{-1}=\frac{x-2}{2}...(3)$
$From\ (1)\ and\ (2),\ we\ have$
$f^{-1}\circ g^{-1}=f^{-1}[g^{-1}(x)]=f^{-1}[x-2]=\frac{x-2}{2}...(4)$
$From\ (3)\ and\ (4),\ it\ is\ clear\ that$
$(g\circ f)^{-1}=f^{-1}\circ g^{-1}$

Functions Ex 2.5 Q13

$Given\ that\ f(x) = \dfrac{x-2}{x-3};$

$Let\ f(x) = y;$

$\Rightarrow y = \dfrac{x-2}{x-3}$

$Interchange\ x\ and\ y;$

$\Rightarrow x = \dfrac{y-2}{y-3}$

$\Rightarrow (y-3)x = y-2$

$\Rightarrow xy - 3x = y - 2$

$\Rightarrow xy - y = 3x - 2$

$\Rightarrow y(x-1) = 3x - 2$

$\Rightarrow y = \dfrac{3x-2}{x-1}$

$\Rightarrow f^{-1}(x) = \dfrac{3x-2}{x-1}$

Functions Ex 2.5 Q14

$f : R^{+} \to [-9, \infty)$ given by $f(x) = 5x^2 + 6x - 9$

For any $x,\ y \in R^{+}$

$f(x) = f(y)$

$\Rightarrow 5x^2 + 6x - 9 = 5y^2 + 6y - 9$

$\Rightarrow 5(x^2 - y^2) + 6(x - y) = 0$

$\Rightarrow (x-y)[5(x+y)+6] = 0$

$\Rightarrow x - y = 0 \qquad [\because 5(x+y)+6 \neq 0 \text{ as } x, y \in R^{+}]$

$\Rightarrow x = y$

So, f is an injection.

Let y be an arbitrary element of $[-9, \infty)$.

$f(x) = y$

$\Rightarrow 5x^2 + 6x - 9 = y$

$\Rightarrow 25x^2 + 30x - 45 = 5y$

$\Rightarrow 25x^2 + 30x + 9 - 54 = 5y$

$\Rightarrow (5x+3)^2 = 5y + 54$

$\Rightarrow (5x+3) = \sqrt{5y+54}$

$\Rightarrow x = \dfrac{\sqrt{5y+54} - 3}{5}$

Now, $y \in [-9, \infty)$

$\Rightarrow y \geq -9$

$\Rightarrow 5y + 54 \geq 9$

$\Rightarrow \sqrt{5y+54} \geq 3$

$\Rightarrow \sqrt{5y+54} - 3 \geq 0$

$\Rightarrow \dfrac{\sqrt{5y+54} - 3}{5} \geq 0$

$\Rightarrow x \geq 0 \Rightarrow x \in R^+$

Thus, for every $y \in [-9, \infty)$ there exist $x = \dfrac{\sqrt{5y+54} - 3}{5} \in R^+$ such that $f(x) = y$.

So, $f : R^+ \to [-9, \infty)$ is onto.

Thus, $f : R^+ \to [-9, \infty)$ is a bijection and hence invertible.

Let f^{-1} denote the inverse of f.

Then,

$(fof^{-1})(y) = y$ for all $y \in [-9, \infty)$

$f(f^{-1}(y)) = y$ for all $y \in [-9, \infty)$

$\Rightarrow 5\{f^{-1}(y)\}^2 + 6\{f^{-1}(y)\} - 9 = y$ for all $y \in [-9, \infty)$

$\Rightarrow 25\{f^{-1}(y)\}^2 + 30\{f^{-1}(y)\} - 45 = 5y$ for all $y \in [-9, \infty)$

$\Rightarrow 25\{f^{-1}(y)\}^2 + 30\{f^{-1}(y)\} + 9 = 5y + 54$ for all $y \in [-9, \infty)$

$\Rightarrow \{5f^{-1}(y) + 3\}^2 = 5y + 54$ for all $y \in [-9, \infty)$

$\Rightarrow 5f^{-1}(y) + 3 = \sqrt{5y + 54}$ for all $y \in [-9, \infty)$

$\Rightarrow f^{-1}(y) \dfrac{\sqrt{5y+54} - 3}{5}$

Functions Ex 2.5 Q15

We have given that

$f : R \to (-1, 1)$ defined by

$f(x) = \dfrac{10^x - 10^{-x}}{10^x + 10^{-x}}$ is invertible

let $f(x) = y$

$\Rightarrow \quad \dfrac{10^x - 10^{-x}}{10^x + 10^{-x}} = y$

$\Rightarrow \quad \dfrac{10^{2x} - 1}{10^{2x} - 1} = y$

$\Rightarrow \quad 10^{2x} - 1 = y(10^{2x} + 1)$

$\Rightarrow \quad 10^{2x} - 10^{2x}y = y + 1$

$\Rightarrow \quad 10^{2x}(1 - y) = y + 1$

$\Rightarrow \quad 10^{2x} = \dfrac{y+1}{1-y}$

$\Rightarrow \quad 2x = log_{10}\left(\dfrac{1+y}{1-y}\right)$

$x = \dfrac{1}{2} log_{10}\left(\dfrac{1+y}{1-y}\right)$

$\therefore \quad f^{-1}(x) = \dfrac{1}{2} log_{10}\left(\dfrac{1+x}{1-x}\right)$

Functions Ex 2.5 Q16

We have given that

$f : R \to (0, 2)$ defined by

$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} + 1$ is invertible.

let $f(x) = y$

$$\Rightarrow \quad \frac{e^x - e^{-x}}{e^x + e^{-x}} + 1 = y$$

$$\Rightarrow \quad \frac{2e^x}{e^x + e^{-x}} = y$$

$$\Rightarrow \quad \frac{2e^{2x}}{e^{2x} + 1} = y$$

$$\Rightarrow \quad 2e^{2x} = y\left(e^{2x} + 1\right)$$

$$\Rightarrow \quad e^{2x}\left(2 - y\right) = y$$

$$\Rightarrow \quad e^{2x} = \frac{y}{2 - y} \Rightarrow x = \frac{1}{2} log_e \left(\frac{y}{2 - y}\right)$$

$$\Rightarrow \quad f^{-1}(x) = \frac{1}{2} log_e \left(\frac{x}{2 - x}\right)$$

Functions Ex 2.5 Q17

Given: that

$f:[-1,\infty]\to[-1,\infty]$ is a function

given by $f(x)=(x+1)^2-1$

In order to show that f in invertible, we need to prove that f in bijective.

Injective: let $x,y\in[-1,\infty]$, Such that

$f(x)=f(y)$

$\Rightarrow \quad (x+1)^2-1=(y+1)^2-1$

$\Rightarrow \quad (x+1)^2=(y+1)^2$

$\Rightarrow \quad x+1=y+1 \qquad [x,y\in[-1,\infty]]$

$\Rightarrow \quad x=y$

$\Rightarrow \quad f$ is one-one

Surjectivity: let $y\in[-1,\infty]$ be arbitrary

such that $f(x)=y$

$\Rightarrow \quad (x+1)^2-1=y$

$= \quad (x+1)^2=y+1$

$\Rightarrow \quad x+1=\sqrt{y+1}$

$\Rightarrow \quad x=\sqrt{y+1}-1\in[-1,\infty]$

So, for each $y\in[-1,\infty]$ (co-domain) there exist $x=\sqrt{y+1}-1\in[-1,\infty]$ (domain)

$\therefore \quad f$ is onto

Thus, f is bijective $\Rightarrow$ f is invertible.

Now,

$f(x)=f^{-1}(x)$

$\Rightarrow \quad (x+1)^2-1=\sqrt{x+1}-1$

$\Rightarrow \quad (x+1)^2-\sqrt{x+1}=0$

$\Rightarrow \quad \sqrt{x+1}\left((x+1)^{3/2}-1\right)=0$

$\Rightarrow \quad \sqrt{x+1}=0$ or $(x+1)^{3/2}-1=0$

$\Rightarrow \quad x=-1$ or $x=0$

$\therefore \quad x=0,-1$

Hence, $S=\{0,-1\}$

Functions Ex 2.5 Q18

$A = \{x \in R : -1 \le x \le 1\}$ and $f : A \to A$, $g : A \to A$ are two functions defined by $f(x) = x^2$ and $g(x) = \sin\left(\frac{\pi x}{2}\right)$

Here, $f : A \to A$ is defined by

$$f(x) = x^2$$

Clearly f in not injective, $\because f(1) = f(-1) = 1$

So, f is not bijective and hence not invertible.

Hence, f^{-1} does not exist

Now, $g : A \to A$ defined by

$$g(x) = \sin\left(\frac{\pi x}{2}\right)$$

Injectivity: Let $x_1 = x_2$

$$\Rightarrow \quad \frac{\pi x_1}{2} = \frac{\pi x_2}{2}$$

$$\Rightarrow \quad \sin\left(\frac{\pi x_1}{2}\right) = \sin\left(\frac{\pi x_2}{2}\right) \qquad [\because -1 \le x \le 1]$$

$$\Rightarrow \quad g(x_1) = g(x_2)$$

$\Rightarrow \quad g$ is one-one ....(i)

Surjectivity: let y be aribitrary such that

$$g(x) = y$$

$$\Rightarrow \quad \sin\left(\frac{\pi x}{2}\right) = y$$

$$\Rightarrow \quad \frac{\pi x}{2} = \sin^{-1} y$$

$$\Rightarrow \quad x = \frac{2}{\pi}\sin^{-1} y = [-1, 1]$$

Thus, for each y in codomain, there exists x in domain, such that

$$g(x) = y$$

$\Rightarrow \quad g$ is surjective ....(ii)

From (i) & (ii)

Functions Ex 2.5 Q19

Given: $f : R \to R$ is a function defined by

$$f(x) = \cos(x + 2)$$

Injectivity: let $x, y \in R$ such that

$$f(x) = f(y)$$

$$\Rightarrow \quad \cos(x + 2) = \cos(y + 2)$$

$$\Rightarrow \quad x + 2 = 2n\pi \pm y + 2$$

$$\Rightarrow \quad x = 2n\pi \pm y$$

$$\Rightarrow \quad x \ne y$$

$\Rightarrow \quad f$ is not one-one

Hence, f is not bijective

$\Rightarrow \quad f$ is not invertible

Functions Ex 2.5 Q20

We have, $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c, d\}$

We know that a function from A to B is said to be bijection if it is one-one and onto. This means different elements of A has different image in B. Also each element of B has preimage in A.

Let f_1, f_2, f_3 and f_4 are the functions from A to B.

$$f_1 = \{(1, a), (2, b), (3, c), (4, d)\}$$
$$f_2 = \{(1, b), (2, c), (3, d), (4, a)\}$$
$$f_3 = \{(1, c), (2, d), (3, a), (4, b)\}$$
$$f_4 = \{(1, d), (2, a), (3, b), (4, c)\}$$

we can verify that f_1, f_2, f_3 and f_4 are bijective from A to B.

Now,

$$f_1^{-1} = \{(a, 1), (b, 2), (c, 3), (d, 4)\}$$
$$f_2^{-1} = \{(b, 1), (c, 2), (d, 3), (a, 4)\}$$
$$f_3^{-1} = \{(c, 1), (d, 2), (a, 3), (b, 4)\}$$
$$f_4^{-1} = \{(d, 1), (a, 2), (b, 3), (c, 4)\}$$

Functions Ex 2.5 Q21

Given: A and B are two sets with finite elements.

$f : A \to B$ and $g : B \to A$ are injective map.

To prove: f in bijective

Proof: *Since*, $f : A \to B$ in injective we need to show f in surjective only.

Now,

$g : B \to A$ in injective

$\Rightarrow$ each element of B has image in A.

Functions Ex 2.5 Q22

We have,

$f : Q \to Q$ and $g : Q \to Q$ are two function defined by

$f(x) = 2x$ and $g(x) = x + 2$

Now, $f : Q \to Q$ defined by $f(x) = 2x$

Injectivity: let $x, y \in Q$ such that

$f(x) = f(y) \Rightarrow 2x = 2y \quad \Rightarrow x = y$

$\Rightarrow$ f in one-one

Surjectivity: let $y \in Q$ such that

$f(x) = y \quad \Rightarrow \quad 2x = y \quad \Rightarrow x = y/2 \in Q$

$\therefore$ For each $y \in Q$ (co-domain) there exist $x = y/2 \in Q$ (domain) such that $f(x) = y$

$\Rightarrow$ f is onto

$\therefore$ f in bijective

Again for $g : Q \to Q$ defined by

$g(x) = x + 2$

Injectivity: let $x, y \in Q$ such that

$g(y) = g(x) \Rightarrow \quad y + 2 = x + 2 \Rightarrow \quad y = x$

$\Rightarrow$ g is one-one

Surjectivity: let $y \in Q$ be arbitrary such that

$g(x) = y \Rightarrow \quad x + 2 = y \Rightarrow \quad x = y - 2 \in Q$

Thus, for each $y \in Q$ (co-domain), there exist $x = y - 2 \in Q$ such that $g(x) = y$

$\therefore$ g in onto

Hence, g is bijective.

$g \circ f(x) = g(f(x)) = g(2x) = 2x + 2$

$\Rightarrow$ $gof(x) = 2x + 2$

f and g are bijective $\Rightarrow g \circ f$ is bijective

$\Rightarrow$ $(g \circ f)^{-1}$ exist

Now, $(g \circ f)(x) = 2x + 2$

$\Rightarrow (g \circ f)^{-1}(2x + 2) = x$

$\Rightarrow (g \circ f)^{-1}(2x) = x - 2$

$(g \circ f)^{-1}(x) = \frac{1}{2}(x - 2) \qquad \ldots A$

Again,

f is bijective $\Rightarrow f^{-1}$ exist

$\therefore$ $f^{-1} : Q \to Q$ defined by

$f^{-1}(x) = x/2$

Also, g is bijective $\Rightarrow g^{-1}$ exist.

$\therefore$ $g^{-1} : Q \to Q$ defined by

$g^{-1}(x) = x - 2$

$\therefore$ $f^{-1} \circ g^{-1}(x) = f^{-1}(g^{-1}(x))$

$= f^{-1}(x - 2)$

$(f^{-1} \circ g^{-1})(x) = \frac{1}{2}(x - 2) \ldots\ldots\ldots\ldots\ldots (B)$

From (A) & (B)

$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

Ex 3.1

Binary Operations Ex 3.1 Q1(i)

We have,

$$a * b = a^b \text{ for all } a, b \in N$$

Let $a \in N$ and $b \in N$

$\Rightarrow \quad a^b \in N$

$\Rightarrow \quad a * b \in N$

The operation $*$ defines a binary operation on N

Binary Operations Ex 3.1 Q1(ii)

We have,

$$a \circ b = a^b \text{ for all } a, b \in Z$$

Let $a \in Z$ and $b \in Z$

$\Rightarrow \quad a^b \notin Z \qquad \Rightarrow a \circ b \notin Z$

For example, if $\quad a = 2, \;\; b = -2$

$\Rightarrow \quad a^b = 2^{-2} = \frac{1}{4} \notin Z$

$\therefore \quad$ The operation '$\circ$' does not define a binary operation on Z.

Binary Operations Ex 3.1 Q1(iii)

We have,

$$a * b = a + b - 2 \text{ for all } a, b \in N$$

Let $a \in N$ and $b \in N$

Then, $a + b - 2 \notin N$ for all $a, b \in N$

$\Rightarrow \quad a * b \notin N$

For example $a = 1$, $b = 1$

$\Rightarrow \quad a + b - 2 = 0 \notin N$

$\therefore$ The operation $*$ does not define a binary operation on N

Binary Operations Ex 3.1 Q1(iv)

We have,

$$S = \{1, 2, 3, 4, 5\}$$

and, $a \times_6 b =$ Remainder when ab is divided by 6

Let $a \in S$ and $b \in S$

$\Rightarrow \quad a \times_6 b \notin S$ for all $a, b \in S$

For example, $a = 2$, $b = 3$

$\Rightarrow \quad 2 \times_6 3 =$ Remainder when 6 is divided by $6 = 0 \notin S$

$\therefore \quad \times_6$ does not define a binary oparation on S

Binary Operations Ex 3.1 Q1(v)

We have,

$$S = \{0, 1, 2, 3, 4, 5\}$$

and, $a +_6 b = \begin{cases} a + b; \text{ if } a + b < 6 \\ a + b - 6; \text{ if } a + b \geq 6 \end{cases}$

Let $a \in S$ and $b \in S$ such that $a + b < 6$

Then $a +_6 b = a + b \in S \quad [\because a + b < 6 = 0, 1, 2, 3, 4, 5]$

Let $a \in S$ and $b \in S$ such that $a + b > 6$

Then $a +_6 b = a + b - 6 \in S \quad [\because \text{if } a + b \geq 6 \text{ then } a + b - 6 \geq 0 = 0, 1, 2, 3, 4, 5]$

$\therefore \quad a +_6 b \in S$ for $a, b \in S$

$\therefore \quad +_6$ defines a binary oparation on S

Binary Operations Ex 3.1 Q1(vi)

We have,

$$a \circ b = a^b + b^a \text{ for all } a, b \in N$$

Let $a \in N$ and $b \in N$

$\Rightarrow \quad a^b \in N$ and $b^a \in N$

$\Rightarrow \quad a^b + b^a \in N$

$\Rightarrow \quad a \circ b \in N$

Thus, the operation '$\circ$' defines a binary relation on N

Binary Operations Ex 3.1 Q1(vii)

We have,

$$a*b=\frac{a-1}{b+1} \text{ for all } a,b\in Q$$

Let $a\in Q$ and $b\in Q$

Then $\frac{a-1}{b+1}\notin Q$ for $b=-1$

$\Rightarrow$ $a*b\notin Q$ for all $a,b\in Q$

Thus, the operation $*$ does not define a binary operation on Q

Binary Operations Ex 3.1 Q2

(i) On $\mathbf{Z}^+$, * is defined by $a * b = a - b$.
It is not a binary operation as the image of (1, 2) under * is $1 * 2 = 1 - 2 = -1 \notin \mathbf{Z}^+$.

(ii) On $\mathbf{Z}^+$, * is defined by $a * b = ab$.
It is seen that for each $a, b \in \mathbf{Z}^+$, there is a unique element ab in $\mathbf{Z}^+$.
This means that * carries each pair (a, b) to a unique element $a * b = ab$ in $\mathbf{Z}^+$.
Therefore, * is a binary operation.

(iii) On $\mathbf{R}$, * is defined by $a * b = ab^2$.
It is seen that for each $a, b \in \mathbf{R}$, there is a unique element ab^2 in $\mathbf{R}$.
This means that * carries each pair (a, b) to a unique element $a * b = ab^2$ in $\mathbf{R}$.
Therefore, * is a binary operation.

(iv) On $\mathbf{Z}^+$, * is defined by $a * b = |a - b|$.
It is seen that for each $a, b \in \mathbf{Z}^+$, there is a unique element $|a - b|$ in $\mathbf{Z}^+$.
This means that * carries each pair (a, b) to a unique element $a * b = |a - b|$ in $\mathbf{Z}^+$.
Therefore, * is a binary operation.

(v) On $\mathbf{Z}^+$, * is defined by $a * b = a$.
* carries each pair (a, b) to a unique element $a * b = a$ in $\mathbf{Z}^+$.
Therefore, * is a binary operation.

(vi) on R, * is defined by $a * b = a + 4b^2$
it is seen that for each element $a, b \in R$, there is unique element $a + 4b^2$ in R
This means that * carries each pair (a, b) to a unique element $a * b = a + 4b^2$ in R.
Therefore, * is a binary operation.

Binary Operations Ex 3.1 Q3

It is given that, $a*b=2a+b-3$
Now

$$3*4=2\times3+4-3$$
$$=10-3$$
$$=7$$

Binary Operations Ex 3.1 Q4

The operation * on the set A = {1, 2, 3, 4, 5} is defined as
$a * b$ = L.C.M. of a and b.
2*3 = L.C.M of 2 and 3 = 6. But 6 does not belong to the given set.
Hence, the given operation * is not a binary operation.

Binary Operations Ex 3.1 Q5

We have,

$$S=\{a,b,c\}$$

We know that the total number of binary operation on a set S with n element is n^{n^2}

$\Rightarrow$ Total number of binary operation on $S=\{a,b,c\} = 3^{3^2}=3^9$

Binary Operations Ex 3.1 Q6

We have,

$$S=\{a,b\}$$

The total number of binary operation on $S=\{a,b\}$ in $2^{2^2}=2^4=16$

Binary Operations Ex 3.1 Q7

We have,

$$M = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : \ a, b \in R - \{0\} \right\} \text{ and}$$

$$A * B = AB \text{ for all } A, B \in M$$

Let $A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \in M$ and $B = \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} \in M$

Now, $AB = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} = \begin{bmatrix} ac & 0 \\ 0 & bd \end{bmatrix}$

$\therefore \quad a \in R,\ b \in R,\ c \in R,\ \&\ d \in R$

$\Rightarrow \quad ac \in R$ and $bd \in R$

$\Rightarrow \quad \begin{bmatrix} ac & 0 \\ 0 & bd \end{bmatrix} \in M$

$\Rightarrow \quad A * B \in M$

Thus, the operater $*$ difines a binary operation on M

Binary Operations Ex 3.1 Q8

S = set of rational numbers of the form $\frac{m}{n}$ where $m \in Z$ and $n = 1, 2, 3$

Also, $a * b = ab$

Let $a \in S$ and $b \in S$

$\Rightarrow \quad ab \notin S$

For example $a = \frac{7}{3}$ and $b = \frac{5}{2}$

$\Rightarrow \quad ab = \frac{35}{6} \notin S$

$\therefore \quad a * b \notin S$

Hence, the operater $*$ does not define a binary operation on S

Binary Operations Ex 3.1 Q9

It is given that, a*b = 2a + b

Now

$$(2*3) = 2 \times 2 + 3 = 4 + 3 = 7$$

$$(2*3)*4 = 7*4 = 2 \times 7 + 4 = 14 + 4 = 18$$

Binary Operations Ex 3.1 Q10

It is given that, a*b = LCM (a, b)

Now

$$5*7 = \text{LCM}(5, 7) = 35$$

Ex 3.2

Binary Operations Ex 3.2 Q1

We have,

$$a * b = l.c.m.(a, b) \text{ for all } a, b \in N$$

(1)
Now,

$$2 * 4 = l.c.m\ (2,4) = 4$$
$$3 * 5 = l.c.m\ (3,5) = 15$$
$$1 * 6 = l.c.m\ (1,6) = 6$$

(ii)
Commutativity:
Let $a, b \in N$ then,

$$\begin{aligned} a * b &= l.c.m(a, b) \\ &= l.c.m(b, a) \\ &= b * a \end{aligned}$$

$\Rightarrow \quad a * b = b * a$

$\therefore \quad *$ is commutative on N.

Associativity:
Let $a, b, c \in N$ then,

$$\begin{aligned} (a * b) * c &= l.c.m(a, b) * c \\ &= l.c.m(a, b, c) \qquad ---(i) \end{aligned}$$

and, $$\begin{aligned} a * (b * c) &= a * l.c.m(b, c) \\ &= l.c.m(a, b, c) \qquad ---(ii) \end{aligned}$$

From (i) and (ii)

$$(a * b) * c = a * (b * c)$$

$\therefore \quad *$ is associative on N.

Binary Operations Ex 3.2 Q2

(i) Clearly, by definition $a * b = 1 = b * a$, $\forall a, b \in N$
Also, $(a * b) * c = (1 * c) = 1$
and $a * (b * c) = (a * 1) = 1 \qquad \forall a, b, c \in N$
Hence, N is both associative and commutative.

(ii) $a * b = \frac{a+b}{2} = \frac{b+a}{2} = b * a$,
which shows $*$ is commutative.

Further, $(a * b) * c = \left(\frac{a+b}{2}\right) * c = \frac{\left(\frac{a+b}{2}\right) + c}{2} = \frac{a+b+2c}{4}$

$$a * (b * c) = a * \left(\frac{b+c}{2}\right) = \frac{a + \left(\frac{b+c}{2}\right)}{2} = \frac{2a+b+c}{2} \neq \frac{a+b+2c}{4}$$

Hence, $*$ is not associative.

Binary Operations Ex 3.2 Q3

We have, binary operator $*$ defined on A and is given by

$$a*b=b \text{ for all } a,b\in A$$

Commutativity: Let $a,b\in A$, then

$$a*b=b\neq a=b*a$$

$\Rightarrow$ $a*b\neq b*a$

$\therefore$ '$*$' is not commutative on A.

Associativity: Let $a,b,c\in A$, then

$$(a*b)*c=b*c=c \qquad ---(i)$$

and, $a*(b*c)=a*c=c \qquad ---(ii)$

From (i) and (ii)

$$(a*b)*c=a*(b*c)$$

$\Rightarrow$ '$*$' is associative on A.

Binary Operations Ex 3.2 Q4(i)

'$*$' is a binary operator on Z defined by $a*b=a+b+ab$ for all $a,b\in Z$.

Commutativity of '$*$':

Let $a,b\in Z$, then

$$a*b=a+b+ab=b+a+ba=b*a$$

$\therefore$ $a*b=b*a$

Associative of '$*$':

Let $a,b\in Z$, then

$$(a*b)*c=(a+b+ab)*c=a+b+ab+c+ac+bc+abc$$
$$=a+b+c+ab+bc+ac+abc \qquad ---(i)$$

Again, $a*(b*c)=a*(b+c+bc)$

$$=a+b+c+bc+ab+ac+abc \qquad ---(ii)$$

From (i) & (ii), we get

$$(a*b)*c=a*(b*c)$$

$\therefore$ $*$ is commutative and associative on Z

Binary Operations Ex 3.2 Q4(ii)

Commutative:

Let $a, b \in N$, then

$$a * b = 2^{ab} = 2^{ba} = b * a$$

$\therefore \quad a * b = b * a$

$\therefore \quad *$ is commutative on N

Associative:

Let $a, b, c \in N$, then

$$(a * b) * c = 2^{ab} * c = 2^{2^{ab} \cdot c} \qquad ---(i)$$

and, $\quad a * (b * c) = a * 2^{bc} = 2^{a \cdot 2^{bc}} \qquad ---(ii)$

From (i) & (ii), we get

$$(a * b) * c \neq a * (b * c)$$

$\therefore \quad *$ is not associative on N

Binary Operations Ex 3.2 Q4(iii)

Commutativity:

Let $a, b \in Q$, then

$$a * b = a - b \neq b - a = b * a$$

$\therefore \quad a * b \neq b * a$

$\Rightarrow \quad *$ is not commutative on Q

Associative:

Let $a, b, c \in Q$, then

$$(a * b) * c = (a - b) * c = a - b - c \qquad ---(i)$$

and, $\quad a * (b * c) = a * (b - c) = a - b + c \qquad ---(ii)$

From (i) & (ii), we get

$$(a * b) * c \neq a * (b * c)$$

$\therefore \quad *$ is not associative on Q

Binary Operations Ex 3.2 Q4(iv)

Commutative:

Let $a, b \in Q$, then

$$a \,e\, b = a^2 + b^2 = b^2 + a^2 = b \,e\, a$$

$\Rightarrow \quad a \,e\, b = b \,e\, a$

$\therefore \quad e$ is commutative on Q.

Associative:

Let $a, b, c \in Q$, then

$$(a \,e\, b) \,e\, c = (a^2 + b^2) \,e\, c = (a^2 + b^2)^2 + c^2 \qquad ---(i)$$

and, $\quad a \,e\, (b \,e\, c) = a \,e\, (a^2 + b^2) = a^2 + (b^2 + c^2)^2 \qquad ---(ii)$

From (i) & (ii),

$$(a \,e\, b) \,e\, c \neq a \,e\, (b \,e\, c)$$

$\therefore \quad e$ is not associative on Q.

Binary Operations Ex 3.2 Q4(v)

Binary operation '$\circ$' defined on Q, given by $aob = \frac{ab}{2}$ for all $a, b \in Q$

Commutative:

Let $a, b \in Q$, then

$$a \circ b = \frac{ab}{2} = \frac{ba}{2} = b \circ a$$

$\Rightarrow \quad a \circ b = b \circ a$

$\therefore \quad \circ$ is commutative on Q.

Associativity:

Let $a, b, c \in Q$, then

$$(a \circ b) \circ c = \left(\frac{ab}{2}\right) \circ c = \frac{abc}{4} \qquad ---(i)$$

$$a \circ (b \circ c) = a \circ \left(\frac{bc}{2}\right) = \frac{abc}{4} \qquad ---(ii)$$

From (i) & (ii) we get

$$(a \circ b) \circ c = a \circ (b \circ c)$$

$\therefore \quad$ '$\circ$' is associative on Q.

Binary Operations Ex 3.2 Q4(vi)

Commutative:

Let $a, b \in Q$, then

$$a * b = ab^2 \neq ba^2 = b * a$$

$\Rightarrow \quad a * b \neq b * a$

$\therefore \quad *$ is not commutative on Q

Associativity:

Let $a, b, c \in Q$, then

$$(a * b) * c = ab^2 * c = ab^2c^2 \qquad ---(i)$$

& $\quad a * (b * c) = a * bc^2 = a\left(bc^2\right)^2 \qquad ---(ii)$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

$\therefore \quad *$ is not associative on Q

Binary Operations Ex 3.2 Q4(vii)

Commutativity:

Let $a, b \in Q$, then

$$a * b = a + ab \qquad ---(i)$$
$$b * a = b + ab \qquad ---(ii)$$

From (i) & (ii)

$$a * b \neq b * a$$

$\Rightarrow$ $*$ is not commutative on Q

Associativity:

Let $a, b, c \in Q$, then

$$(a * b) * c = (a + ab) * c = a + ab + ac + abc \qquad ---(i)$$
$$a * (b * c) = a * (b + bc)$$
$$= a + ab + abc \qquad ---(ii)$$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

$\Rightarrow$ $*$ is not associative on Q

Binary Operations Ex 3.2 Q4(viii)

Commutativity: Let $a, b \in R$, then

$$a * b = a + b - 7$$
$$= b + a - 7$$
$$= b * a$$

$\Rightarrow$ $a * b = b * a$

$\Rightarrow$ $*$ is commutative on R

Associativity: Let $a, b, c \in Q$, then

$$(a * b) * c = (a + b - 7) * c$$
$$= a + b - 7 + c - 7$$
$$= a + b + c - 17 \qquad ---(i)$$

and, $a * (b * c) = a * (b + c - 7)$

$$= a + b + c - 7 - 7$$
$$= a + b + c - 17 \qquad ---(ii)$$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

$\Rightarrow$ $*$ is associative on R

Binary Operations Ex 3.2 Q4(ix)

Commutativity:

Let $a, b \in R - \{-1\}$, then

$$a * b = \frac{a}{b+1} \neq \frac{b}{a+1} = b * a$$

$\Rightarrow \quad a * b \neq b * a$

$\Rightarrow \quad *$ is not commutative on $R - \{-1\}$

Associativity:

Let $a, b, c \in R - \{-1\}$, then

$$(a * b) * c = \left(\frac{a}{b+1}\right) * c$$

$$= \frac{\frac{a}{b+1}}{c+1} = \frac{a}{(b+1)(c+1)} \quad --- (i)$$

& $\quad a * (b * c) = a * \left(\frac{b}{c+1}\right)$

$$= \frac{a}{\frac{b}{c+1} + 1} = \frac{a(c+1)}{b+c+1} \quad --- (ii)$$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

$\Rightarrow \quad *$ is not associative on $R - \{-1\}$

Binary Operations Ex 3.2 Q4(x)

Commutativity:

Let $a, b \in Q$, then

$$a * b = ab + 1 = ba + 1 = b * a$$

$\Rightarrow \quad a * b = b * a$

$\Rightarrow \quad *$ is commutative on Q

Associativity:

Let $a, b, c \in Q$, then

$$(a * b) * c = (ab + 1) * c$$

$$= abc + c + 1 \quad --- (i)$$

$$a * (b * c) = a * (bc + 1)$$

$$= abc + a + 1 \quad --- (ii)$$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

$\Rightarrow \quad *$ is not associative on Q.

Binary Operations Ex 3.2 Q4(xi)

Commutativity:
Let $a, b \in N$, then
$$a * b = a^b \neq b^a = b * a$$
$\Rightarrow \quad a * b \neq b * a$

$\Rightarrow \quad$ '$*$' is not commutative on N

Associativity:
Let $a, b, c \in N$, then
$$(a * b) * c = a^b * c = \left(a^b\right)^c = a^{bc} \qquad ---(i)$$
$$a * (b * c) = a * b^c = (a)^{b^c} \qquad ---(ii)$$
From (i) and (ii)
$$a^{bc} \neq (a)^{b^c}$$
$\Rightarrow (a * b) * c \neq a * (b * c)$

$\Rightarrow \quad$ '$*$' is not associative on N.

Binary Operations Ex 3.2 Q4(xii)

Commutativity:
Let $a, b \in N$, then
$$a * b = a^b \neq b^a = b * a$$
$\Rightarrow \quad a * b \neq b * a$

$\Rightarrow \quad$ '$*$' is not commutative on N

Associativity:
Let $a, b, c \in N$, then
$$(a * b) * c = a^b * c = \left(a^b\right)^c = a^{bc} \qquad ---(i)$$
$$a * (b * c) = a * b^c = (a)^{b^c} \qquad ---(ii)$$
From (i) and (ii)
$$a^{bc} \neq (a)^{b^c}$$
$\Rightarrow (a * b) * c \neq a * (b * c)$

$\Rightarrow \quad$ '$*$' is not associative on N.

Binary Operations Ex 3.2 Q4(xiii)

Commutativity:
Let $a, b \in Z$ then,

$$a * b = a - b \neq b - a = b * a$$

$\Rightarrow$ $a * b \neq b * a$

$\Rightarrow$ $*$ is not commutative on Z

Associativity:
Let $a, b, c \in Z$, then

$$(a * b) * c = (a - b) * c = (a - b - c) \quad ---(i)$$

& $a * (b * c) = a * (b - c) = (a - b + c) \quad ---(ii)$

From (i) & (ii)

$$(a * b) * c \neq a * (b * c)$$

$\Rightarrow$ '$*$' is not associative on Z.

Binary Operations Ex 3.2 Q4(xiv)

Commutativity:
Let $a, b \in Q$ then,

$$a * b = \frac{ab}{4} = \frac{ba}{4} = b * a$$

$\Rightarrow$ $a * b = b * a$

$\therefore$ $*$ is commutative on Q

Associativity:
Let $a, b, c \in Q$ then,

$$(a * b) * c = \frac{ab}{4} * c = \frac{abc}{16} \quad ---(i)$$

and, $a * (b * c) = a * \frac{bc}{4} = \frac{abc}{16} \quad ---(ii)$

From (i) and (ii)

$$(a * b) * c = a * (b * c)$$

$\therefore$ '$*$' is associative on Q.

Binary Operations Ex 3.2 Q4(xv)

Commutativity:
Let $a, b \in Q$ then,

$$a * b = (a - b)^2 = (b - a)^2 = b * a$$

$\Rightarrow$ $a * b = b * a$

$\therefore$ '$*$' is commutative on Q.

Associativity:
Let $a, b, c \in Q$ then,

$$(a * b) * c = (a - b)^2 * c = \left[(a - b)^2 - c\right]^2 \quad ---(i)$$

and, $a * (b * c) = a * (b - c)^2 = \left[a - (b - c)^2\right]^2 \quad ---(ii)$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

$\therefore$ $*$ is not associative on Q.

Binary Operations Ex 3.2 Q5

The binary operator o defined on $Q - \{-1\}$ is given by

$$a \circ b = a + b - ab \text{ for all } a, b \in Q - \{-1\}$$

Commutativity:

Let $a, b \in Q - \{-1\}$, then

$$a \circ b = a + b - ab = b + a - ba = b \circ a$$

$\Rightarrow \quad a \circ b = b \circ a$

$\Rightarrow \quad$ '$\circ$' is commutative on $Q - \{-1\}$.

Binary Operations Ex 3.2 Q6

The binary operator $*$ defined on Z and is given by

$$a * b = 3a + 7b$$

Commutativity: Let $a, b \in Z$, then

$$a * b = 1a + 7b \text{ and}$$
$$b * a = 3b + 7a$$

$\therefore \quad a * b \neq b * a$

Hence, '$*$' is not commutative on Z.

Binary Operations Ex 3.2 Q7

We have, $*$ is a binary operator defined on Z is given by

$$a * b = ab + 1 \text{ for all } a, b \in Z$$

Associativity: Let $a, b, c \in Z$, then

$$(a * b) * c = (ab + 1) * c$$
$$= abc + c + 1 \qquad ---(i)$$

and, $\quad a * (b * c) = a * (bc + 1)$
$$= abc + a + 1 \qquad ---(ii)$$

From (i) & (ii)

$\therefore \quad (a * b) * c \neq a * (b * c)$

Hence, '$*$' is not associative on Z.

Binary Operations Ex 3.2 Q8

We have, set of real numbers except -1 and $*$ is an operator given by

$$a * b = a + b + ab \text{ for all } a, b \in S = R - \{-1\}$$

Now, $\forall\ a, b \in S$

$$a * b = a + b + ab \in S$$

$\because$ if $\quad a + b + ab = -1$

$\Rightarrow \quad a + b(1 + a) + 1 = 0$

$\Rightarrow \quad (a + 1)(b + 1) = 0$

$\Rightarrow \quad a = -1$ or $b = -1$

but $a \neq -1$ and $b \neq -1$ (given)

$\therefore \quad a + b + ab \neq -1$

$\Rightarrow \quad a * b \in S$ for $ab \in S$

$\Rightarrow \quad$ '$*$' is a binary operator on S

Commutativity: Let $a, b \in S$

$\Rightarrow \quad a * b = a + b + ab = b + a + ba = b * a$

$\Rightarrow \quad a * b = b * a$

and, $a*(b*c) = a*(b+c+bc)$
$$= a+b+c+bc+ab+ac+abc \quad ---(ii)$$

From (i) and (ii)
$$(a*b)*c = a*(b*c)$$

$\therefore$ '*' is associative on S.

Now, $(2*x)*3 = 7$
$\Rightarrow (2+x+2x)*3 = 7$
$\Rightarrow 2+x+2x+3+6+3x+6x = 7$
$\Rightarrow 11+12x = 7$
$\Rightarrow 12x = -4$
$\Rightarrow x = \frac{-4}{12} \quad \Rightarrow x = \frac{-1}{3}$

Binary Operations Ex 3.2 Q9

The binary operator * defined as
$$a*b = \frac{a-b}{2} \text{ for all } a,b \in Q.$$

Now,
Associativity: Let $a,b,c \in Q$, then
$$(a*b)*c = \frac{a-b}{2}*c = \frac{\frac{a-b}{2}-c}{2}$$
$$= \frac{a-b-2c}{4} \quad ---(i)$$

and, $a*(b*c) = a*\frac{b-c}{2} = \frac{a-\frac{b-c}{2}}{2}$
$$= \frac{2a-b+c}{4} = \quad ---(ii)$$

From (i) & (ii)
$$(a*b)*c \neq a*(b*c)$$

Hence, '*' is not associative on Q.

Binary Operations Ex 3.2 Q10

The binary operator * defined as
$$a*b = a+3b-4 \text{ for all } a,b \in Z$$

Now,
Commutativity: Let $a,b \in Z$, then
$$a*b = a+3b-4 \neq b+3a-4 = b*a$$

$\Rightarrow a*b \neq b*a$

$\Rightarrow$ '*' is not commutative on Z.

Associativity: Let $a,b,c \in Z$, then
$$(a*b)*c = (a+3b-4)*c = a+3b-4+3c-4$$
$$= a+3b+3c-8 \quad ---(i)$$

and, $a*(b*c) = a*(b+3c-4) = a+3(b+3c-4)-4$
$$= a+3b+9c-16 \quad ---(ii)$$

From (i) & (ii)
$$(a*b)*c \neq a*(b*c)$$

Hence, '*' is not associative on Z.

Binary Operations Ex 3.2 Q11

Q be the set of rational numbers and $*$ be a binary operation defined as

$$a*b=\frac{ab}{5} \text{ for all } a,b\in Q$$

Now,

Associativity: Let $a,b,c\in Q$, then

$$(a*b)*c=\frac{ab}{5}*c=\frac{abc}{25} \qquad ---(i)$$

and, $$a*(b*c)=a*\frac{bc}{5}=\frac{abc}{25} \qquad ---(ii)$$

From (i) & (ii)

$\therefore$ $$(a*b)*c=a*(b*c)$$

$\Rightarrow$ $*$ is associative on Q.

Binary Operations Ex 3.2 Q12

The binary operator $*$ is defined as

$$a*b=\frac{ab}{7} \text{ for all } a,b\in Q$$

Now,

Associativity: Let $a,b,c\in Q$, then

$$(a*b)*c=\frac{ab}{7}*c=\frac{abc}{49} \qquad ---(i)$$

and, $$a*(b*c)=a*\frac{bc}{7}=\frac{abc}{49} \qquad ---(ii)$$

From (i) & (ii)

$$(a*b)*c=a*(b*c)$$

$\Rightarrow$ '$*$' is associative on Q.

Binary Operations Ex 3.2 Q13

The binary operator $*$ defined as

$$a*b=\frac{a+b}{2} \text{ for all } a,b\in Q.$$

Now,

Associativity: Let $a,b,c\in Q$, then

$$(a*b)*c=\frac{a+b}{2}*c=\frac{\frac{a+b}{2}+c}{2}$$

$$=\frac{a+b+2c}{4} \qquad ---(i)$$

and, $$a*(b*c)=a*\frac{b+c}{2}$$

$$=\frac{a+\frac{b+c}{2}}{2}$$

$$=\frac{2a+b+c}{4}= \qquad ---(ii)$$

From (i) & (ii)

$$(a*b)*c\neq a*(b*c)$$

Hence, '$*$' is not associative on Q.

Ex 3.3

Binary Operations Ex 3.3 Q1

The binary operator $*$ is defined on I^+
and is given by,

$a * b = a + b$ for all $a, b \in I^+$

Let $a \in I^+$ and $e \in I^+$ be the identity element with respect to $*$.
by identity property, we have,

$a * e = e * a = a$

$\Rightarrow \quad a + e = a$
$\Rightarrow \quad e = 0$

Thus the required identity element is 0.

Binary Operations Ex 3.3 Q2

Let $R - \{-1\}$ be the set and $*$ be a binary operator, given by

$a * b = a + b + ab$ for all $a, b \in R - \{-1\}$

Now,
Let $a \in R - \{-1\}$ and $e \in R - \{-1\}$ be the identity element with respect to $*$.
by identity property, we have,

$a * e = e * a = a$

$\Rightarrow \quad a + e + ae = a$
$\Rightarrow \quad e(1 + a) = 0$
$\Rightarrow \quad e = 0 \qquad [\because 1 + a \neq 0 \text{ as } a \neq -1]$

$\therefore$ The required identity element is 0.

Binary Operations Ex 3.3 Q3

We are given the binary operator $*$ defined on Z as

$$a * b = a + b - 5 \text{ for all } a, b \in Q.$$

Let e be the identity element with respect to $*$

Then, $a * e = e * a = a$ [By identity property]

$\Rightarrow$ $a + e - 5 = a$

$\Rightarrow$ $e = 5$

Hence, the required identity element with respect to $*$ is 5.

Binary Operations Ex 3.3 Q4

The binary operator $*$ is defined on Z, and is given by

$$a * b = a + b + 2 \text{ for all } a, b \in Z.$$

Let $a \in Z$ and $e \in Z$ be the identity element with respect to $*$, then

$a * e = e * a = a$ [By identity property]

$\Rightarrow$ $a + e + 2 = a$

$\Rightarrow$ $e = -2 \in Z$

Hence, the identity element with respect to $*$ is -2.

Ex 3.4

Binary Operations Ex 3.4 Q1

Given,

$$a * b = a + b - 4 \text{ for all } a, b \in Z$$

(i)

Commutative: Let $a, b \in Z$, then

$\Rightarrow \quad a * b = a + b - 4 = b + a - 4 = b * a$

$\Rightarrow \quad a * b = b * a$

So, '$*$' is commutative on Z.

Associativity: Let $a, b, c \in Z$, then

$$(a * b) * c = (a + b - 4) * c = a + b - 4 + c - 4$$

$$= a + b + c - 8 \qquad ---(i)$$

and, $\quad a * (b * c) = a * (b + c - 4) = a + b + c - 8 \qquad ---(ii)$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

So, '$*$' is associative on Z.

(ii)

Let $e \in Z$ be the identity element with respect to $*$.

By identity property, we have

$$a * e = e * a = a \quad \text{for all } a \in Z$$

$\Rightarrow \quad a + e - 4 = a$

$\Rightarrow \quad e = 4$

So, $e = 4$ will be the identity element with respect to $*$

(iii)

Let $b \in Z$ be the inverse element of $a \in Z$

Then, $\quad a * b = b * a = e$

$\Rightarrow \quad a + b - 4 = e$

$\Rightarrow \quad a + b - 4 = 4 \qquad [\because e = 4]$

$\Rightarrow \quad b = 8 - a$

Thus, $\quad b = 8 - a$ will be the inverse element of $a \in Z$.

Binary Operations Ex 3.4 Q2

We have,

$$a * b = \frac{3ab}{5} \text{ for all } a, b \in Q_0$$

(i)

Commutative: Let $a, b \in Q_0$, then

$$a * b = \frac{3ab}{5} = \frac{3ba}{5} = b * a$$

$\Rightarrow \quad a * b = b * a$

So, '$*$' is commutative on Q_0

Associativity: Let $a, b, c \in Q_0$, then

$$(a * b) * c = \frac{3ab}{5} * c$$

$$= \frac{9abc}{25} \qquad ---(i)$$

and, $\quad a * (b * c) = a * \frac{3bc}{5}$

$$= \frac{9abc}{25} \qquad ---(ii)$$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

So, '$*$' is associative on Q_0

(ii)

Let $e \in Q_0$ be the identity element with respect to $*$, then

$$a * e = e * a = a \text{ for all } a \in Q_0$$

$\Rightarrow \quad \frac{3ae}{5} = a$

$\Rightarrow \quad e = \frac{5}{3}$

will be the identity element with respect to $*$.

(iii)

Let $b \in Q_0$ be the inverse element of $a \in Q_0$, then

$$a * b = b * a = e$$

$\Rightarrow \quad \frac{3}{5}ab = e$

$\Rightarrow \quad \frac{3}{5}ab = \frac{5}{3} \qquad \left[\because e = \frac{5}{3}\right]$

$\Rightarrow \quad b = \frac{25}{9a}$

$\therefore \quad b = \frac{25}{9a}$ is the inverse of $a \in Q_0$.

Binary Operations Ex 3.4 Q3

We have,

$$a * b = a + b + ab \text{ for all } a, b \in Q - \{-1\}$$

(i)

Commutativity: Let $a, b \in Q - \{-1\}$

$\Rightarrow \quad a * b = a + b + ab = b + a + ba = b * a$

$\Rightarrow \quad a * b = b * a$

$\Rightarrow \quad$ '$*$' is commutative on $Q - \{-1\}$

Associativity: Let $a, b, c \in Q - \{-1\}$, then

$\Rightarrow \quad (a * b) * c = (a + b + ab) * c$

$\quad = a + b + ab + c + ac + bc + abc \qquad ---(i)$

and, $\quad a * (b * c) = a * (b + c + bc)$

$\quad = a + b + c + bc + ab + ac + abc \qquad ---(ii)$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

$\Rightarrow \quad *$ is associative on $Q - \{-1\}$

(ii)

Let e be identity element with respect to $*$.

By identity property,

$$a * e = a = e * a \text{ for all } a \in Q - \{-1\}$$

$\Rightarrow \quad a + e + ae = a$

$\Rightarrow \quad e(1 + a) = 0 \quad \Rightarrow e = 0 \qquad [\because 1 + a \neq 0 \text{ as } a \neq -1]$

$\therefore \quad e = 0$ is the identity element with respect to $*$

(iii)

Let b be the inverse of $a \in Q - \{-1\}$

Then, $\quad a * b = b * a = e \qquad$ [e is the identity element]

$\Rightarrow \quad a + b + ab = e$

$\Rightarrow \quad a + b + ab = 0$

$\Rightarrow \quad b(1 + a) = -a$

$\Rightarrow \quad b = \dfrac{-a}{1 + a} \qquad \left[\begin{array}{l}\because \dfrac{-a}{1+a} \neq -1, \text{ because if } \dfrac{-a}{1+a} = -1 \\ \Rightarrow a = 1 + a \Rightarrow 1 = 0 \text{ Not possible}\end{array}\right]$

$\therefore \quad b = \dfrac{-a}{1 + a}$ is the inverse of a with respect to $*$

Binary Operations Ex 3.4 Q4

We have,

$$(a,b) \odot (c,d) = (ac, bc+d) \text{ for all } (a,b),(c,d) \in R_0 \times R$$

(i)

Commutativity: Let $(a,b),(c,d) \in R_0 \times R$, then

$\Rightarrow \quad (a,b) \odot (c,d) = (ac, bc+d) \qquad \text{---}(i)$

and, $\quad (c,d) \odot (a,b) = (ca, da+b) \qquad \text{---}(ii)$

From (i) & (ii)

$$(a,b) \odot (c,d) \neq (c,d) \odot (a,b)$$

$\Rightarrow \quad$ '$\odot$' is not commutative on $R_0 \times R$.

Associativity: Let $(a,b),(c,d),(e,f) \in R_0 \times R$, then

$$\begin{aligned} \Rightarrow \quad ((a,b) \odot (c,d)) \odot (e,f) &= (ac, bc+d) \odot (e,f) \\ &= (ace, bce, de+f) \qquad \text{---}(i) \end{aligned}$$

$$\begin{aligned} \text{and,} \quad (a,b) \odot (c,d \odot (e,f)) &= (a,b) \odot (ce, de+f) \\ &= (ace, bce+de+f) \qquad \text{---}(ii) \end{aligned}$$

$\Rightarrow \quad ((a,b) \odot (c,d)) \odot (e,f) = (a,b) \odot ((c,d) \odot (e,f))$

$\Rightarrow \quad$ '$\odot$' is associative on $R_0 \times R$.

(ii)

Let $(x,y) \in R_0 \times R$ be the identity element with respect to $\odot$, then

$$(a,b) \odot (x,y) = (x,y) \odot (a,b) = (a,b) \text{ for all } (a,b) \in R_0 \times R$$

$\Rightarrow \quad (ax, bx+y) = (a,b)$

$\Rightarrow \quad ax = a$ and $bx + y = b$

$\Rightarrow \quad x = 1$, and $y = 0$

$\therefore \quad (1,0)$ will be the identity element with respect to $\odot$.

(iii)

Let $(c,d) \in R_0 \times R$ be the inverse of $(a,b) \in R_0 \times R$, then

$$(a,b) \odot (c,d) = (c,d) \odot (a,b) = e$$

$\Rightarrow \quad (ac, bc+d) = (1,0) \qquad [\because e = (1,0)]$

$\Rightarrow \quad ac = 1$ and $bc + d = 0$

$\Rightarrow \quad c = \dfrac{1}{a}$ and $d = -\dfrac{b}{a}$

$\therefore \quad \left(\dfrac{1}{a}, -\dfrac{b}{a}\right)$ will be the inverse of (a,b).

Binary Operations Ex 3.4 Q5

We have,

$$a * b = \frac{ab}{2} \text{ for all } a, b \in Q_0$$

(i)

Commutativity: Let $a, b \in Q_0$, then

$$\Rightarrow \quad a * b = \frac{ab}{2} = \frac{ba}{2} = b * a$$

$$\Rightarrow \quad a * b = b * a$$

Hence, '$*$' is commutative on Q_0.

Associativity: Let $a, b, c \in Q_0$, then

$$\Rightarrow \quad (a * b) * c = \frac{ab}{2} * c = \frac{abc}{4} \qquad ---(i)$$

and, $$a * (b * c) = a * \frac{bc}{2} = \frac{abc}{4} \qquad ---(ii)$$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

$\Rightarrow \quad *$ is associative on Q_0.

(ii)

Let $e \in Q_0$ be the identity element with respect to $*$.

By identity property, we have,

$$a * e = e * a = a \text{ for all } a \in Q_0$$

$$\Rightarrow \quad \frac{ae}{2} = a \qquad \Rightarrow e = 2$$

Thus, the required identity element is 2.

(iii)

Let $b \in Q_0$ be the inverse of $a \in Q_0$ with respect to $*$, then,

$$a * b = b * a = e \text{ for all } a \in Q_0$$

$$\Rightarrow \quad \frac{ab}{2} = e \qquad \Rightarrow \frac{ab}{2} = 2$$

$$\Rightarrow b = \frac{4}{a}$$

Thus, $b = \frac{4}{a}$ is the inverse of a with respect to $*$.

Binary Operations Ex 3.4 Q6

We have,

$$a * b = a + b - ab \text{ for all } a, b \in R - \{+1\}$$

(i)

Commutative: Let $a, b \in R - \{+1\}$, then,

$\Rightarrow \quad a * b = a + b - ab = b + a - ba = b * a$

$\Rightarrow \quad a * b = b * a$

So, '$*$' is commutative on $R - \{+1\}$.

Associativity: Let $a, b, c \in R - \{+1\}$, then

$$\begin{aligned}(a * b) * c &= (a + b - ab) * c \\ &= a + b - ab + c - ac - bc + abc \\ &= a + b + c - ab - ac - bc + abc \qquad ---(i)\end{aligned}$$

and, $$\begin{aligned}a * (b * c) &= a * (b + c - bc) \\ &= a + b + c - bc - ab - ac + abc \qquad ---(ii)\end{aligned}$$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

So, '$*$' is associative on $R - \{+1\}$.

(ii)

Let $e \in R - \{+1\}$ be the identity element with respect to $*$, then

$$a * e = e * a = a \text{ for all } a \in R - \{+1\}$$

$\Rightarrow \quad a + e - ae = a$

$\Rightarrow \quad e(1 - a) = 0$

$\Rightarrow \quad e = 0 \qquad [\because a \neq 1 \Rightarrow 1 - a \neq 0]$

$\therefore \quad e = 0$ will be the identity element with respect to $*$.

(iii)

Let $b \in R - \{1\}$ be the inverse element of $a \in R - \{1\}$, then

$$a * b = b * a = e$$

$\Rightarrow \quad a + b - ab = 0 \qquad [\because e = 0]$

$\Rightarrow \quad b(1 - a) = -a$

$\Rightarrow \quad b = \dfrac{-a}{1 - a} \neq 1 \qquad \left[\begin{array}{l}\because \text{ if } \dfrac{-a}{1-a} = 1 \\ \Rightarrow -a = 1 - a \quad \Rightarrow 1 = 0 \\ \quad \text{Not possible}\end{array}\right]$

$\therefore \quad b = \dfrac{-a}{1 - a}$ is the inverse of $a \in R - \{1\}$ with respect to $*$.

Binary Operations Ex 3.4 Q7

We have,

$$(a,b)*(c,d)=(ac,bd) \text{ for all } (a,b),(c,d)\in A$$

(i)

Let $(a,b),(c,d)\in A$, then

$$\begin{aligned}(a,b)*(c,d) &= (ac,bd)\\ &= (ca,db) && [\because ac=ca \text{ and } bd=db]\\ &= (c,d)*(a,b)\end{aligned}$$

$\Rightarrow \quad (a,b)*(c,d)=(c,d)*(a,b)$

So, '$*$' is commutative on A

Associativity: Let $(a,b),(c,d),(e,f)\in A$, then

$$\begin{aligned}\Rightarrow \quad ((a,b)*(c,d))*(e,f) &= (ac,bd)*(e,f)\\ &= (ace,bdf) && ---(i)\end{aligned}$$

$$\begin{aligned}\text{and,} \quad (a,b)*((c,d)*(e,f)) &= (a,b)*(ce,df)\\ &= (ace,bdf) && ---(ii)\end{aligned}$$

From (i) & (ii)

$\Rightarrow \quad ((a,b)*(c,d))*(e,f)=(a,b)*((c,d)*(e,f))$

So, '$*$' is associative on A.

(ii)

Let $(x,y)\in A$ be the identity element with respect to $*$.

$$(a,b)*(x,y)=(x,y)*(a,b)=(a,b) \text{ for all } (a,b)\in A$$

$\Rightarrow \quad (ax,by)=(a,b)$

$\Rightarrow \quad ax=a$ and $by=b$

$\Rightarrow \quad x=1$, and $y=1$

$\therefore \quad (1,1)$ will be the identity element

(iii)

Let $(c,d)\in A$ be the inverse of $(a,b)\in A$, then

$$(a,b)*(c,d)=(c,d)*(a,b)=e$$

$\Rightarrow \quad (ac,bd)=(1,1) \qquad [\because e=(1,1)]$

$\Rightarrow \quad ac=1$ and $bd=1$

$\Rightarrow \quad c=\frac{1}{a}$ and $d=\frac{1}{b}$

$\therefore \quad \left(\frac{1}{a},\frac{1}{b}\right)$ will be the inverse of (a,b) with respect to $*$.

Binary Operations Ex 3.4 Q8

The binary operation * on **N** is defined as:

$a * b$ = H.C.F. of a and b

It is known that:

H.C.F. of a and b = H.C.F. of b and a, $a, b \in$ **N**.

Therefore, $a * b = b * a$

Thus, the operation * is commutative.

For $a, b, c \in$ **N**, we have:

$(a * b)* c$ = (H.C.F. of a and b) * c = H.C.F. of a, b, and c

$a *(b * c)= a *$(H.C.F. of b and c) = H.C.F. of a, b, and c

Therefore, $(a * b) * c = a * (b * c)$

Thus, the operation * is associative.

Now, an element $e \in$ **N** will be the identity for the operation * if $a * e = a = e* a$, $\forall$ $a \in$ **N**.

But this relation is not true for any $a \in$ **N**.

Thus, the operation * does not have any identity in **N**.

Ex 3.5

Binary Operations Ex 3.5 Q1

$a \times_4 b$ = the remainder when ab is divided by 4.

eg. (i) $2 \times 3 = 6 \Rightarrow 2 \times_4 3 = 2$

[When 6 is divided by 4 we get 2 as remainder]

(ii) $2 \times 3 = 4 \Rightarrow 2 \times_4 2 = 0$

[When 4 is divided by 4 we get 0 as remainder]

The composition table for $\times_4$ on set $S = \{0, 1, 2, 3\}$ *is* :

$\times_4$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

Binary Operations Ex 3.5 Q 2

$a +_5 b =$ the remainder when $a + b$ is divided by 5.

eg. $2 + 4 = 6 \Rightarrow 2 +_5 4 = 1$ $\because$ [we get 1 as remainder when 6 is divided by 5]

$2 + 4 = 7 \Rightarrow 3 +_5 4 = 2$ $\because$ [we get 2 as remainder when 7 is divided by 5]

The composition table for $+_5$ on set $S = \{0, 1, 2, 3, 4\}$.

$+_5$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

Binary Operations Ex 3.5 Q3

$a \times_6 b$ = the remainder when the product of ab is divided by 6.

The composition table for $\times_6$ on set $S = \{0, 1, 2, 3, 4, 5\}$.

$\times_6$	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

Binary Operations Ex 3.5 Q4

$a \times_5 b$ = the remainder when the product of ab is divided by 5.

The composition table for $\times_5$ on $Z_5 = \{0, 1, 2, 3, 4\}$.

$\times_5$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

Binary Operations Ex 3.5 Q5

$a \times_{10} b$ = the remainder when the product of ab is divided by 10.

The composition table for $\times_{10}$ on set $S = \{1, 3, 7, 9\}$

$\times_{10}$	1	3	7	9
1	1	3	7	9
3	3	9	1	7
7	7	1	9	3
9	9	7	3	1

We know that an element $b \in S$ will be the inverse of $a \in S$

if $a \times_{10} b = 1$ $\left[\begin{array}{l}\because 1 \text{ is the identity element with} \\ \text{respect to multiplication}\end{array}\right]$

$\Rightarrow$ $3 \times_{10} b = 1$

From the above table $b = 7$

$\therefore$ Inverse of 3 is 7.

Binary Operations Ex 3.5 Q6

$a \times_7 b$ = the remainder when the product of ab is divided by 7.

The composition table for $\times_7$ on $S = \{1, 2, 3, 4, 5, 6\}$

$\times_7$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

We know that 1 is the identity element with respect to multiplication

Also, b will be the inverse of a
if, $a \times_7 b = e = 1$

$\Rightarrow$ $3 \times_7 b = 1$

From the above table $3 \times_7 5 = 1$

$\therefore$ $b = 3^{-1} = 5$

Now, $3^{-1} \times_7 4 = 5 \times_7 4 = 6$

Binary Operations Ex 3.5 Q7

$a \times_{11} b$ = the remainder when the product of ab is divided by 11.

The composition table for $\times_{11}$ on Z_{11}

$\times_{11}$	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	1	3	5	7	9
3	3	6	9	1	4	7	10	3	5	8
4	4	8	1	5	9	2	6	10	3	7
5	5	10	4	9	3	8	2	7	1	6
6	6	1	7	2	8	3	9	4	10	5
7	7	3	10	6	2	9	5	1	8	4
8	8	5	3	10	7	4	1	9	6	3
9	9	7	5	3	1	10	8	6	4	2
10	10	9	8	7	6	5	4	3	2	1

From the above table

$5 \times_{11} 9 = 1$ $\quad\quad [\because 1 \text{ is the identity element}]$

$\therefore$ Inverse of 5 is 9.

Binary Operations Ex 3.5 Q8

$Z_5 = \{0, 1, 2, 3, 4\}$

$a \times_5 b$ = the remainder when the product of ab is divided by 5.

The composition table for $\times_5$ on $Z_5 = \{0, 1, 2, 3, 4\}$

$\times_5$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

Binary Operations Ex 3.5 Q9

(i)

From the above table we can say that

$a*b=b*a=b$

$a*c=c*a=c$

$a*d=d*a=d$

$b*c=c*b=d$

$b*d=d*b=c$

$c*d=d*c=b$

$\therefore$ '$*$' is commutative

Again, $a,b,c\in S$

$\Rightarrow$ $(a*b)*c=b*c=d$ and

$a*(b*c)=a*d=d$

$\therefore$ $(a*b)*c=a*(b*c)$

$\therefore$ $*$ is associative

We know that e will be identity element with respect to $*$ if

$a*e=e*a=a$ for all $a\in S$

$\Rightarrow$ $a*a=a,\ a*b=b,\ a*c=c,\ a*d=d$

$\therefore$ a will be the identity element

Again,

b will be the inverse of a if

$b*a=a*b=e$

From the above table

$a*a=a,$ $b*b=b, c*c=c$ and $d*d=d$

$\therefore$ Inverse of $a=a$

$b=b$

$c=c$

$d=d$

(ii)

From the above table, we can observe

$a \circ b = b \circ a, \quad b \circ c = c \circ b$

$a \circ c = c \circ a, \quad b \circ d = d \circ b$

$a \circ d = d \circ a, \quad c \circ d = d \circ c$

$\therefore$ "o" is commutative on S

Again, for $a, b, c \in S$

$(a \circ b) \circ c = a \circ c = a \qquad ---(i)$

$a \circ (b \circ c) = a \circ c = a \qquad ---(ii)$

From (i) & (ii)

$(a \circ b) \circ c = a \circ (b \circ c)$

So, "o" is associative on S

Now, we have,

$a \circ b = a$

$b \circ b = b$

$c \circ b = c$

$d \circ b = d$

$\Rightarrow$ b is the identity element with respect to "o"

We know that x will be inverse of y

If $x \circ y = y \circ x = e$

$\Rightarrow \quad x \circ y = y \circ x = b \qquad [\because e = b]$

Now, from the above table we find that

$b \circ b = b$

$c \circ d = b$

$d \circ c = b$

$\therefore \quad b^{-1} = b,\ c^{-1} = d$, and $d^{-1} = c$

Not: a^{-1} does ont exist.

Binary Operations Ex 3.5 Q10

Let $X = \{0, 1, 2, 3, 4, 5\}$.

The operation * on X is defined as:

$$a * b = \begin{cases} a+b & \text{if } a+b<6 \\ a+b-6 & \text{if } a+b \geq 6 \end{cases}$$

An element $e \in X$ is the identity element for the operation *, if

$a * e = a = e * a \ \forall a \in X$.

For $a \in X$, we observed that:

$a * 0 = a + 0 = a \qquad [a \in X \Rightarrow a + 0 < 6]$

$0 * a = 0 + a = a \qquad [a \in X \Rightarrow 0 + a < 6]$

$\therefore a * 0 = a = 0 * a \ \forall a \in X$

Thus, 0 is the identity element for the given operation *.

An element $a \in X$ is invertible if there exists $b \in X$ such that $a * b = 0 = b * a$.

$$\text{i.e.,} \begin{cases} a+b=0=b+a, & \text{if } a+b<6 \\ a+b-6=0=b+a-6, & \text{if } a+b \geq 6 \end{cases}$$

i.e.,

$a = -b$ or $b = 6 - a$

But, $X = \{0, 1, 2, 3, 4, 5\}$ and $a, b \in X$. Then, $a \neq -b$.

Therefore, $b = 6 - a$ is the inverse of a $a \in X$.

Hence, the inverse of an element $a \in X$, $a \neq 0$ is $6 - a$ i.e., $a^{-1} = 6 - a$.

Ex 4.1

Inverse Trigonometric Functions Ex 4.1 Q1.

Let $\tan^{-1}\left(-\sqrt{3}\right) = y$. Then, $\tan y = -\sqrt{3} = -\tan\frac{\pi}{3} = \tan\left(-\frac{\pi}{3}\right)$.

We know that the range of the principal value branch of $\tan^{-1}$ is

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\tan\left(-\frac{\pi}{3}\right)$ is $-\sqrt{3}$.

Therefore, the principal value of $\tan^{-1}\left(\sqrt{3}\right)$ is $-\frac{\pi}{3}$.

Concept Insight:

The range for $\tan^{-1}$ is same as $\sin^{-1}$ except that it is an open interval, as $\tan(-\pi/2)$ and $\tan(\pi/2)$ are not defined. So the method of finding principal value is same as $\sin^{-1}$ given in the first problem. Also note that $\tan(-x) = -\tan x$.

Let $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = y$. Then, $\cos y = -\frac{1}{\sqrt{2}} = -\cos\left(\frac{\pi}{4}\right) = \cos\left(\pi - \frac{\pi}{4}\right) = \cos\left(\frac{3\pi}{4}\right)$.

We know that the range of the principal value branch of $\cos^{-1}$ is

$[0,\pi]$ and $\cos\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is $\frac{3\pi}{4}$.

Let $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right) = y$. Then, $\operatorname{cosec} y = -\sqrt{2} = -\operatorname{cosec}\left(\frac{\pi}{4}\right) = \operatorname{cosec}\left(-\frac{\pi}{4}\right)$.

We know that the range of the principal value branch of

$\operatorname{cosec}^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ and $\operatorname{cosec}\left(-\frac{\pi}{4}\right) = -\sqrt{2}$.

Therefore, the principal value of $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$ is $-\frac{\pi}{4}$.

We know that for any $x \in [-1, 1]$, $\cos^{-1} x$ represents angle in $[0, \pi]$

$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ = an angle in $[0, \pi]$ whose cosine is $\left(-\frac{\sqrt{3}}{2}\right)$

$$= \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$\therefore \quad \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$

We know that, for any $x \in R$, $\tan^{-1} x$ represents an angle in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is x.

So,

$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ = An angle in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ whose tangest is $\frac{1}{\sqrt{3}}$

$$= \frac{\pi}{6}$$

$\therefore \ \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}.$

We know that, for $x \in R$, $sec^{-1}x$ represents an angle in $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.

$sec^{-1}\left(-\sqrt{2}\right)$ = An angle in $[0, \pi] - \left\{\frac{\pi}{2}\right\}$ whose secant is $\left(-\sqrt{2}\right)$

$$= \pi - \frac{\pi}{4}$$

$$= \frac{3\pi}{4}$$

$sec^{-1}\left(-\sqrt{2}\right) = \frac{3\pi}{4}.$

We know that, for any $x \in R$, $cot^{-1}x$ represents an angle in $(0, \pi)$

$cot^{-1}\left(-\sqrt{3}\right)$ = An angle in $(0, \pi)$ whose contangent is $\left(-\sqrt{3}\right)$

$$= \pi - \frac{\pi}{6}$$

$$= \frac{5\pi}{6}$$

$\therefore \ cot^{-1}\left(-\sqrt{3}\right) = \frac{5\pi}{6}.$

We know that, for any $x \in R$, $\sec^{-1} x$ represents an angle in $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.

$\sec^{-1}(2)$ = An angle is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$ whose secant is 2

$$= \frac{\pi}{3}$$

$\therefore \ \sec^{-1}(2) = \frac{\pi}{3}.$

We know that, for any $x \in R$. $\text{cosec}^{-1}x$ is an angle in $\left[\frac{-\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

$\cos ec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ = An angle is $\left[\frac{-\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$ whose cosecant is $\left(\frac{2}{\sqrt{3}}\right)$

$$= \frac{\pi}{3}$$

$\therefore \ \cos ec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{3}.$

Inverse Trigonometric Functions Ex 4.1 Q2.

Let $\cos^{-1}\left(\frac{1}{2}\right)=x$. Then, $\cos x=\frac{1}{2}=\cos\left(\frac{\pi}{3}\right)$.

$\therefore \cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}$

Let $\sin^{-1}\left(\frac{1}{2}\right)=y$. Then, $\sin y=\frac{1}{2}=\sin\left(\frac{\pi}{6}\right)$.

$\therefore \sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$

$\therefore \cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}+\frac{2\pi}{6}=\frac{\pi}{3}+\frac{\pi}{3}=\frac{2\pi}{3}$

$\therefore \sin^{-1}\frac{1}{2}=\frac{\pi}{6}$

$\therefore \tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]=\tan^{-1}\left[2\cos\left(2\times\frac{\pi}{6}\right)\right]$

$=\tan^{-1}\left[2\cos\frac{\pi}{3}\right]=\tan^{-1}\left[2\times\frac{1}{2}\right]$

$=\tan^{-1}1=\frac{\pi}{4}$

Concept Insight:
Solve the innermost bracket first, so first find the principal value of $\sin^{-1}(1/2)$

Let $\tan^{-1}(1)=x$. Then, $\tan x=1=\tan\frac{\pi}{4}$.

$\therefore \tan^{-1}(1)=\frac{\pi}{4}$

Let $\cos^{-1}\left(-\frac{1}{2}\right)=y$. Then, $\cos y=-\frac{1}{2}=-\cos\left(\frac{\pi}{3}\right)=\cos\left(\pi-\frac{\pi}{3}\right)=\cos\left(\frac{2\pi}{3}\right)$.

$\therefore \cos^{-1}\left(-\frac{1}{2}\right)=\frac{2\pi}{3}$

Let $\sin^{-1}\left(-\frac{1}{2}\right)=z$. Then, $\sin z=-\frac{1}{2}=-\sin\left(\frac{\pi}{6}\right)=\sin\left(-\frac{\pi}{6}\right)$.

$\therefore \sin^{-1}\left(-\frac{1}{2}\right)=-\frac{\pi}{6}$

$\therefore \tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$

$=\frac{\pi}{4}+\frac{2\pi}{3}-\frac{\pi}{6}$

$=\frac{3\pi+8\pi-2\pi}{12}=\frac{9\pi}{12}=\frac{3\pi}{4}$

$\tan^{-1}\left(\sqrt{3}\right)$ = Angle in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is $\sqrt{3}$

$= \frac{\pi}{3}$

$\sec^{-1}(-2)$ = An angle in $[0, \pi] - \left\{\frac{\pi}{2}\right\}$ whose secant is (-2)

$= \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

$cosec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ = An angle in $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ whose cosecant is $\left(\frac{2}{\sqrt{3}}\right)$

$= \frac{\pi}{3}$

Hence,

$$\tan^{-1}\sqrt{3} - \sec^{-1}(-2) + cosec^{-1}\frac{2}{\sqrt{3}}$$
$$= \frac{\pi}{3} - \frac{2\pi}{3} + \frac{\pi}{3}$$
$$= 0$$

$$\therefore \tan^{-1}\sqrt{3} - \sec^{-1}\left(-\sqrt{2}\right) + cosec^{-1}\left(\frac{2}{\sqrt{3}}\right) = 0$$

$$\text{Let } \cos^{-1}\left(\frac{1}{2}\right) = x. \text{ Then, } \cos x = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$$
$$\therefore \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$
$$\text{Let } \sin^{-1}\left(-\frac{1}{2}\right) = y. \text{ Then, } \sin y = -\frac{1}{2} = -\sin\left(\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right)$$
$$\therefore \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$
$$\therefore \cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{3} - \left(-\frac{2\pi}{6}\right) \qquad = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$$

Inverse Trigonometric Functions Ex 4.1 Q3.

$$\text{Let } \sin^{-1}\left(\frac{1}{2}\right) = x. \text{ Then, } \sin x = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$$
$$\therefore \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$
$$\text{Let } \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = y. \text{ Then, } \sin y = \frac{1}{\sqrt{2}} = \sin\left(\frac{\pi}{4}\right)$$
$$\therefore \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$
$$\therefore \sin^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{6} - \frac{2\pi}{4} = \frac{\pi}{6} - \frac{\pi}{2} = \frac{\pi - 3\pi}{6} = -\frac{\pi}{3}$$

$$\text{Let } \sin^{-1}\left(-\frac{1}{2}\right) = x. \text{ Then, } \sin x = -\frac{1}{2} = \sin\left(-\frac{\pi}{6}\right)$$
$$\therefore \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$
$$\text{Let } \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = y. \text{ Then, } \cos y = \frac{-\sqrt{3}}{2} = \cos\left(\pi - \frac{\pi}{6}\right)$$
$$\therefore \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \frac{5\pi}{6}$$
$$\therefore \sin^{-1}\left(-\frac{1}{2}\right) + 2\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = -\frac{\pi}{6} + \frac{10\pi}{6} = \frac{-\pi + 10\pi}{6} = \frac{9\pi}{6} = \frac{3\pi}{2}$$

Let $\tan^{-1}(-1) = x$. Then, $\tan x = -1 = -\tan\left(\frac{\pi}{4}\right) = \tan\left(\pi - \frac{\pi}{4}\right)$

$\therefore \tan^{-1}(-1) = \frac{3\pi}{4}$

Let $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = y$. Then, $\cos y = \frac{-1}{\sqrt{2}} = -\cos\left(\frac{\pi}{4}\right) = \cos\left(\pi - \frac{\pi}{4}\right)$

$\therefore \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = \frac{3\pi}{4}$

$\therefore \tan^{-1}(-1) + \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = \frac{3\pi}{4} + \frac{3\pi}{4} = \frac{6\pi}{4} = \frac{3\pi}{2}$

Let $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = x$. Then, $\sin x = -\frac{\sqrt{3}}{2} = -\sin\left(\frac{\pi}{3}\right) = \sin\left(\pi - \frac{\pi}{3}\right)$

$\therefore \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{2\pi}{3}$

Let $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = y$. Then, $\cos y = \frac{\sqrt{3}}{2} = \cos\left(\frac{\pi}{6}\right)$

$\therefore \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

$\therefore \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{2\pi}{3} + \frac{\pi}{6} = \frac{4\pi + \pi}{6} = \frac{5\pi}{6}$

Let $\tan^{-1}\left(\sqrt{3}\right) = x$. Then, $\tan x = \sqrt{3} = \tan\left(\frac{\pi}{3}\right)$

$\therefore \tan^{-1}\left(\sqrt{3}\right) = \frac{\pi}{3}$

Let $\sec^{-1}(-2) = y$. Then, $\sec y = -2 = \sec\left(\pi - \frac{\pi}{3}\right)$

$\therefore \sec^{-1}(-2) = \frac{2\pi}{3}$

$\therefore \tan^{-1}\left(\sqrt{3}\right) - \sec^{-1}(-2) = \frac{\pi}{3} - \frac{2\pi}{3} = \frac{\pi - 2\pi}{3} = -\frac{\pi}{3}$

Ex 4.2

Inverse Trigonometric Functions Ex 4.2 Q1

8.i

$$= \cos^{-1}\left(\frac{a\cos\theta + b}{a + b\cos\theta}\right) \qquad \left\{\text{Since } \cos\theta = \frac{1 - \tan^2\frac{\theta}{2}}{1 + \tan^2\frac{\theta}{2}}\right\}$$

$= \text{RHS}$

Hence,

$$2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\,\tan\frac{\theta}{2}\right) = \cos^{-1}\left(\frac{a\cos\theta + b}{a + b\cos\theta}\right)$$

Ex 5.1

Algebra of Matrices Ex 5.1 Q1

We know that if a matrix is of the order $m \times n$, it has mn elements. Thus, to find all the possible orders of a matrix having 8 elements, we have to find all the ordered pairs of natural numbers whose products is 8.

The ordered pairs are: $(1 \times 8), (8 \times 1), (2 \times 4), (4 \times 2)$

$(1,5)$ *and* $(5,1)$ are the ordered pairs of natural numbers whose product is 5.

Hence, the possible orders of a matrix having 5 elements are 1×5 *and* 5×1

Algebra of Matrices Ex 5.1 Q2

$$\text{If } A = \left[a_{ij}\right] = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix} \text{ and } \quad B = \left[b_{ij}\right] = \begin{bmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 2 \end{bmatrix}$$

(i) $a_{22} + b_{21} = 4 + (-3) = 1$

Hence, $a_{22} + b_{21} = 1$

(ii) $a_{11}\, b_{11} + a_{22} b_{22} = (2)(2) + (4)(4) = 4 + 16 = 20$

Hence,

$$a_{11}\, b_{11} + a_{22} b_{22} = 20$$

Algebra of Matrices Ex 5.1 Q3

Here, $A = [a_{ij}]_{3\times 4}$

R_1 = first row of $A = [a_{11} a_{12} a_{13} a_{14}]_{1\times 4}$

So, order of $R_1 = 1 \times 4$

C_2 = Second column of A

$$= \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix}_{3\times 1}$$

Order of $C_2 = 3 \times 1$

Algebra of Matrices Ex 5.1 Q4

Let $A = (a_{ij})_{2\times 3}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \qquad ---(i)$$

$(i) \qquad a_{ij} = i.j$

$a_{11} = 1.1 = 1,\ a_{12} = 1.2 = 2,\ a_{13} = 1.3 = 3$

$a_{21} = 2.1 = 2,\ a_{22} = 2.2 = 4,\ a_{23} = 2.3 = 6$

So, using equation (i)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$$

(ii) $a_{ij} = 2i - j$

$a_{11} = 2(1) - 1 = 1,\ a_{12} = 2(1) - 2 = 0,\ a_{13} = 2(1) - 3 = -1$

$a_{21} = 2(2) - 1 = 3,\ a_{22} = 2(2) - 2 = 2,\ a_{23} = 2(2) - 3 = 1$

Using equation (i)

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \end{bmatrix}$$

(iii) $a_{ij} = i + j$

$a_{11} = 1+1 = 2,\ a_{12} = 1+2 = 3,\ a_{13} = 1+3 = 4$

$a_{21} = 2+1 = 3,\ a_{22} = 2+2 = 4,\ a_{23} = 2+3 = 5$

Using equation (i)

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

(iv) $a_{ij} = \dfrac{(i+j)^2}{2}$

$a_{11} = \dfrac{(1+1)^2}{2} = 2,\ a_{12} = \dfrac{(1+2)^2}{2} = \dfrac{9}{2},\ a_{13} = \dfrac{(1+3)^2}{2} = 8$

$a_{21} = \dfrac{(2+1)^2}{2} = \dfrac{9}{2},\ a_{22} = \dfrac{(2+2)^2}{2} = 8,\ a_{23} = \dfrac{(2+3)^2}{2} = \dfrac{25}{2}$

Using equation(i),

$$A = \begin{bmatrix} 2 & \frac{9}{2} & 8 \\ \frac{9}{2} & 8 & \frac{25}{2} \end{bmatrix}$$

Algebra of Matrices Ex 5.1 Q5

Here,

$$A = \left[a_{ij}\right]_{2\times2} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad \text{---(i)}$$

(i) $\quad a_{ij} = \dfrac{(i+j)^2}{2}$

$$a_{11} = \frac{(1+1)^2}{2} = 2,\ a_{12} = \frac{(1+2)^2}{2} = \frac{9}{2},$$

$$a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2},\ a_{22} = \frac{(2+2)^2}{2} = 8,$$

Using equation (i)

$$A = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$$

(ii) $\quad a_{ij} = \dfrac{(i-j)^2}{2}$

$$a_{11} = \frac{(1-1)^2}{2} = 0,\ a_{12} = \frac{(1+2)^2}{2} = \frac{1}{2},$$

$$a_{21} = \frac{(2-1)^2}{2} = \frac{1}{2},\ a_{22} = \frac{(2-2)^2}{2} = 0,$$

Using equation (i)

$$A = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$$

(iii) $a_{ij} = \dfrac{(i-2j)^2}{2}$

$a_{11} = \dfrac{(1-2(1))^2}{2} = \dfrac{1}{2}$, $a_{12} = \dfrac{(1-2(2))^2}{2} = \dfrac{9}{2}$,

$a_{21} = \dfrac{(2-2(1))^2}{2} = 0$, $a_{22} = \dfrac{(2-2(2))^2}{2} = 2$,

Using equation (i)

$$A = \begin{bmatrix} \dfrac{1}{2} & \dfrac{9}{2} \\ 0 & 2 \end{bmatrix}$$

(iv) $a_{ij} = \dfrac{(2i+j)^2}{2}$

$a_{11} = \dfrac{(2(1)+1)^2}{2} = \dfrac{9}{2}$, $a_{12} = \dfrac{(1(1)+2)^2}{2} = 8$,

$a_{21} = \dfrac{(2(2)+2)^2}{2} = \dfrac{25}{2}$, $a_{22} = \dfrac{(2(2)+2)^2}{2} = 18$

Using equation (i)

$$A = \begin{bmatrix} \dfrac{9}{2} & 8 \\ \dfrac{25}{2} & 18 \end{bmatrix}$$

(v) $$a_{ij} = \frac{\left(\left|2i - 3j\right|\right)^2}{2}$$

$$a_{11} = \frac{\left|2(1) - 3(1)\right|}{2} = \frac{1}{2},\ a_{12} = \frac{\left|2(1) - 3(2)\right|}{2} = 2$$

$$a_{21} = \frac{\left|2(2) - 3(1)\right|}{2} = \frac{1}{2},\ a_{22} = \frac{\left|2(2) - 3(2)\right|}{2} = 1$$

Using equation (i)

$$A = \begin{bmatrix} \frac{1}{2} & 2 \\ \frac{1}{2} & 1 \end{bmatrix}$$

Algebra of Matrices Ex 5.1 Q6

Here, $$A = \left(a_{ij}\right)_{3\times 4} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \qquad ---(i)$$

(i) $a_{ij} = i + j$

$a_{11} = 1 + 1 = 2,\ a_{12} = 1 + 2 = 3,\ a_{13} = 1 + 3 = 4,\ a_{14} = 1 + 4 = 5$
$a_{21} = 2 + 1 = 3,\ a_{22} = 2 + 2 = 4,\ a_{23} = 2 + 3 = 5,\ a_{24} = 2 + 4 = 6$
$a_{31} = 3 + 1 = 4,\ a_{32} = 3 + 2 = 5,\ a_{33} = 3 + 3 = 6,\ a_{34} = 3 + 4 = 7$

Using equation (i)

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix}$$

(ii) $a_{ij} = i - j$

$a_{11} = 1 - 1 = 0,\ a_{12} = 1 - 2 = -1,\ a_{13} = 1 - 3 = -2,\ a_{14} = 1 - 4 = -3$
$a_{21} = 2 - 1 = 1,\ a_{22} = 2 - 2 = 0,\ a_{23} = 2 - 3 = -1,\ a_{24} = 2 - 4 = -2$
$a_{31} = 3 - 1 = 2,\ a_{32} = 3 - 2 = 1,\ a_{33} = 3 - 3 = 0,\ a_{34} = 3 - 4 = -1$

Using equation (i)

$$A = \begin{bmatrix} 0 & -1 & -2 & -3 \\ 1 & 0 & -1 & -2 \\ 2 & 1 & 0 & -1 \end{bmatrix}$$

(iii) $a_{ij} = 2i$

$a_{11} = 2(1) = 2,\ a_{12} = 2(1) = 2,\ a_{13} = 2(1) = 2,\ a_{14} = 2(1) = 2$

$a_{21} = 2(2) = 4,\ a_{22} = 2(2) = 4,\ a_{23} = 2(2) = 4,\ a_{24} = 2(2) = 4$

$a_{31} = 2(3) = 6,\ a_{32} = 2(3) = 6,\ a_{33} = 2(3) = 6,\ a_{34} = 2(3) = 6$

Using Equation (i),

$$A = \begin{bmatrix} 2 & 2 & 2 & 2 \\ 4 & 4 & 4 & 4 \\ 6 & 6 & 6 & 6 \end{bmatrix}$$

(iv) $a_{ij} = j$

$a_{11} = 1,\ a_{12} = 2,\ a_{13} = 3,\ a_{14} = 4$

$a_{21} = 1,\ a_{22} = 2,\ a_{13} = 3,\ a_{14} = 4$

$a_{31} = 1,\ a_{32} = 2,\ a_{33} = 3,\ a_{34} = 4$

Using Equation (i),

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

Algebra of Matrices Ex 5.1 Q7

Here,

$$A = \left[a_{ij}\right]_{4\times 3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}$$

(a) $\quad a_{ij} = 2i + \dfrac{i}{j}$

$a_{11} = 2(1) + \dfrac{1}{1} = 3,\ a_{12} = 2(1) + \dfrac{1}{2} = \dfrac{5}{2},\ a_{13} = 2(1) + \dfrac{1}{3} = \dfrac{7}{3}$

$a_{21} = 2(2) + \dfrac{2}{1} = 6,\ a_{22} = 2(2) + \dfrac{2}{2} = 5,\ a_{23} = 2(2) + \dfrac{2}{3} = \dfrac{14}{3}$

$a_{31} = 2(3) + \dfrac{3}{1} = 9,\ a_{32} = 2(3) + \dfrac{3}{2} = \dfrac{15}{2},\ a_{33} = 2(3) + \dfrac{3}{3} = 7$

$a_{41} = 2(4) + \dfrac{4}{1} = 12,\ a_{42} = 2(4) + \dfrac{4}{2} = 10,\ a_{43} = 2(4) + \dfrac{4}{3} = \dfrac{28}{3}$

Using equation (i),

$$A = \begin{bmatrix} 3 & \dfrac{5}{2} & \dfrac{7}{3} \\ 6 & 5 & \dfrac{14}{3} \\ 9 & \dfrac{15}{2} & 7 \\ 12 & 10 & \dfrac{28}{3} \end{bmatrix}$$

(c) $a_{ij} = i$

$a_{11} = 1,\ a_{12} = 1,\ a_{13} = 1,$

$a_{21} = 2,\ a_{22} = 2,\ a_{23} = 2$

$a_{31} = 3,\ a_{32} = 3,\ a_{33} = 3$

$a_{41} = 4,\ a_{42} = 4,\ a_{43} = 4$

Using equation (i)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \\ 4 & 4 & 4 \end{bmatrix}$$

Algebra of Matrices Ex 5.1 Q8

Given,

$$\begin{bmatrix} 3x+4y & 2 & x-2y \\ a+b & 2a-b & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 4 \\ 5 & -5 & -1 \end{bmatrix}$$

Since corresponding entries of equal matrix are equal.
So,

$$3x + 4y = 2 \qquad \text{---(i)}$$
$$x - 2y = 4 \qquad \text{---(ii)}$$
$$a + b = 5 \qquad \text{---(iii)}$$
$$2a - b = -5 \qquad \text{---(iv)}$$

Solving equation (i) and (iii)

$$\begin{array}{l} 3x - 4y = 2 \\ 3x - 6y = 12 \\ (-)\quad (+)\quad (-) \\ \hline \qquad 10y = -10 \end{array}$$

$$y = \frac{-10}{10} = -1$$

Put $y = 1$ in equation (ii)

$$x - 2y = 4$$
$$x - 2(-1) = 4$$
$$x = 4 - 2$$
$$x = 2$$

Now, solving equation $(\text{iii})\,and\,(\text{iv})$,

$$\begin{array}{l} 2a + 2b = 10 \\ 2a - b = -5 \\ (-)\quad (+)\quad (+) \\ \hline \qquad 3b = 15 \end{array}$$

$$b = \frac{15}{3}$$
$$b = 5$$

Put the value of b in equation of (iii)

$$a + b = 5$$
$$a + 5 = 5$$
$$a = 5 - 5$$
$$a = 0$$

Hence,

$$x = 2,\ y = -1,\ a = 0,\ b = 5$$

Algebra of Matrices Ex 5.1 Q9

Given,

$$\begin{bmatrix} 2x-3y & a-b & 3 \\ 1 & x+4y & 3a+4b \end{bmatrix} = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 6 & 29 \end{bmatrix}$$

Since corresponding entries of equal matrix are equal.

So,

$$2x - 3y = 1 \qquad \text{---(i)}$$

$$x - b = -2 \qquad \text{---(ii)}$$

$$x - 4y = 6 \qquad \text{---(iii)}$$

$$3a + 4b = 29 \qquad \text{---(iv)}$$

Solving equation (i) and (iii)

$$\begin{array}{l} 2x - 3y = 1 \\ \underline{\overset{}{2x} - 8y = 12} \\ (-) \quad (-) \quad (-) \end{array}$$

$$-11y = -11$$

$$y = \frac{-11}{-11}$$

$$y = 1$$

Put the value of y in equation (i),

$$2x - 3y = 1$$

$$2x - 3(i) = 1$$

$$2x - 3 = 1$$

$$2x = 1 + 3$$

$$2x = 4$$

$$x = 2$$

Solving equation (ii) and (iv)

$$4a - 4b = -8$$

$$\underline{3a - 4b = 29}$$

$$7a \quad = 21$$

$$a = \frac{21}{7}$$

$$a = 3$$

Put $a = 3$ in equation (ii),

$$3 - b = -2$$
$$b = 3 + 2$$
$$b = 5$$

Hence,

$$x = 2,\ y = 1,\ a = 3,\ b = 5$$

Algebra of Matrices Ex 5.1 Q10

As the given matrices are equal, therefore their corresponding elements must be equal.

Comparing the corresponding elements, we get

$2a + b = 4$ ----(i)

$a - 2b = -3$ ----(ii)

$5c - d = 11$ ----(iii)

$4c + 3d = 24$ ----(iv)

Multiplying (i) by 2 and adding to (ii)

$5a = 5 \Rightarrow a = 1$

(i) $\Rightarrow b = 4 - 2.1 = 2$

Multiplying (iii) by 3 and adding to (iv)

$19c = 57 \Rightarrow c = 3$

(iii) $\Rightarrow d = 5.3 - 11 = 4$

Hence, $a = 1$, $b = 2$, $c = 3$, $d = 4$

Algebra of Matrices Ex 5.1 Q11

Given,

$$A = B$$

$$\begin{bmatrix} x-2 & 3 & 2z \\ 18z & y+2 & 6z \end{bmatrix} = \begin{bmatrix} y & z & 6 \\ 6y & x & 2y \end{bmatrix}$$

Since corresponding entries of equal matrices are equal, So

$x - 2 = y$ ---(i)

$3 = z$ ---(ii)

$2z = 6$ ---(iii)

$18z = 6$ ---(iv)

$y + 2 = x$ ---(v)

$6z = 2y$ ---(vi)

Equation (ii) gives, $z = 3$

Put the value of z in equal (iv),

$$18z = 6y$$
$$18(3) = 6y$$
$$54 = 6y$$
$$y = \frac{54}{6}$$
$$y = 9$$

Put $y = 9$ in equation (v)

$$y + 2 = x$$
$$9 + 2 = x$$
$$11 = x$$

Hence,

$$x = 11, y = 9, z = 3$$

Algebra of Matrices Ex 5.1 Q12

Given,

$$\begin{bmatrix} x & 3x-y \\ 2x+z & 3y-w \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 7 \end{bmatrix}$$

Since corresponding entries of equal matrices are equal, So

$$x = 3 \qquad \text{---(i)}$$

$$3x - y = 2 \qquad \text{---(ii)}$$

$$2x + z = 4 \qquad \text{---(iii)}$$

$$3y - w = 7 \qquad \text{---(iv)}$$

Put the value of $x = 3$ from equation on (i) in eqation (ii),

$$3x - y = 2$$

$$3(3) - y = 2$$

$$9 - y = 2$$

$$y = 9 - 2$$

$$y = 7$$

Put the value of $y = 7$ in equation (iv),

$$3y - w = 7$$

$$3(7) - w = 7$$

$$w = 21 - 7$$

$$w = 14$$

Put the value of $x = 3$ in equation (iii),

$$2x + z = 4$$

$$2(3) + z = 4$$

$$6 + z = 4$$

$$z = 4 - 6$$

$$z = -2$$

Hence,

$$x = 3, y = 7, z = -2, w = 14$$

Algebra of Matrices Ex 5.1 Q13

Given,

$$\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$$

Since corresponding entries of equal matrices are equal, So

$$x - y = -1 \qquad \text{---(i)}$$
$$z = 4 \qquad \text{---(ii)}$$
$$2x - y = 0 \qquad \text{---(iii)}$$
$$w = 5 \qquad \text{---(iv)}$$

Solving equation (i) and (iii)

$$\begin{array}{l} x - y = -1 \\ \underset{(-)}{2x} \underset{(+)}{- y} = \underset{(-)}{0} \\ \hline -x \quad = -1 \\ \quad x = 1 \end{array}$$

Put $x = 1$ in equation (i),

$$x - y = -1$$
$$1 - y = -1$$
$$-y = -1 - 1$$
$$-y = -2$$
$$y = 2$$

equation (ii) and (iv) give the values of z and w respectively, so

$$z = 4, \quad w = 5$$

Hence,

$$x = 1, y = 2, z = 4, w = 5$$

Algebra of Matrices Ex 5.1 Q14

By the definition of equality of matrices we know that if two matrices

$A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{m\times n}$

are equal then $a_{ij} = b_{ij}$ for i = 1,2,3.......m and j = 1,2,3.......n.

Given that $\begin{bmatrix} x+3 & z+4 & 2y-7 \\ 4x+6 & a-1 & 0 \\ b-3 & 3b & z+2c \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y-2 \\ 2x & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{bmatrix}$

$\therefore$ Equating the entries gives:

x + 3 = 0, z + 4 = 6 and 2y - 7 = 3y - 2

$\Rightarrow$ x = - 3 , z = 2 and 2y - 3y = - 2 + 7

$\Rightarrow$ x = - 3 , z = 2 and - y = 5

$\Rightarrow$ x = - 3 , z = 2 and y = - 5

Similarly, $a - 1 = -3$ and 2c + 2 =0

$\Rightarrow$ a = - 3 + 1 and 2c = - 2

$\Rightarrow$ a = - 2 and c = - 1

Lastly, b - 3 = 2b + 4

$\Rightarrow$ b - 2b = 4 + 3

$\Rightarrow$ -b = 7

$\Rightarrow$ b = - 7

The values of x, y, z, a, b, c are - 3, - 5 , 2 , -2 , - 7 , - 1 respectively.

Algebra of Matrices Ex 5.1 Q15

Given that $\begin{bmatrix} 2x+1 & 5x \\ 0 & y^2+1 \end{bmatrix} = \begin{bmatrix} x+3 & 10 \\ 0 & 26 \end{bmatrix}$

The corresponding entries of the equal matrices are equal.

$\Rightarrow$ 2x + 1 = x + 3 , $y^2 + 1 = 26$,

$\Rightarrow$ 2x - x = 2 , $y^2 = 25$

$\Rightarrow$ x = 2 , $y = \pm 5$

$\Rightarrow$ x = 2 ,y = 5 or x = 2 , y = -5

$\therefore$ x + y = 7 or - 3

Algebra of Matrices Ex 5.1 Q16

$\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$

The corresponding entries of the two equal matrices are equal.

$\Rightarrow$ xy = 8 ......(1),

w = 4 ....(2),

z + 6 = 0 ....(3),

and x + y = 6 ....(4)

from equation (2) and equation(3) we get z = - 6 and w = 4.

from equation(4) we have,

x + y = 6,

$\Rightarrow$ x = 6 - y,

subsituting value of x in equation (1) we get,

$\Rightarrow$ (6 - y)y = 8,

$\Rightarrow$ y^2- 6y + 8 = 0,

$\Rightarrow$ (y - 2) (y - 4)= 0,

$\Rightarrow$ y = 2, 4

subsituting the value of y in equation(1) we get ,

$\Rightarrow$ x = 4, 2

Therefore, value of x , y , z ,w are 2 , 4 , - 6, 4 or 4, 2 , -6 , 4 .

Algebra of Matrices Ex 5.1 Q17

(i) We know that,

Order of a row matrix $= 1 \times n$

order of a column matrix $= m \times 1$

So, order of a row as well as column matrix $= 1 \times 1$

Therefore,

Required matrix $= [a]_{1\times 1}$

(ii) A diagonal matrix has only a_{11}, a_{22}, a_{33} for a 3×3 matrix such that a_{11}, a_{22}, a_{33} are equal or different and all other entries zero while scalor matrix has $a_{11} = a_{22} = a_{33} = m$ (say) So, A diagonal matrix which is not scalar must have, $a_{11} \neq a_{22} \neq a_{33}$ and $aij = 0$ for $i \neq j$, So

$$\text{Required Matrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

(iii) A triangular matrix is a square matrix $A = [aij]$ such that $aij = 0$ for all $i > j$, so

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 0 & 4 & 3 \\ 0 & 0 & -6 \end{bmatrix}$$

Algebra of Matrices Ex 5.1 Q18

Given data is,

For January 2013:

	Deluxe	Premium	Standard Cars
Dealer A	5	3	4
Dealer B	7	2	3

For January-February :

	Deluxe	Premium	Standard Cars
Dealer A	8	7	6
Dealer B	10	5	7

Hence,

$$A = \begin{array}{c} \\ \text{Dealer A} \\ \text{Dealer B} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Deluxe} & \text{Premum} & \text{Standard} \end{array} \\ \begin{bmatrix} 5 & 3 & 4 \\ 7 & 2 & 3 \end{bmatrix} \end{array}$$

$$B = \begin{array}{c} \\ \text{Dealer A} \\ \text{Dealer B} \end{array} \begin{array}{c} \begin{array}{ccc} Deluxe & Premum & \text{Standard} \end{array} \\ \begin{bmatrix} 8 & 7 & 6 \\ 10 & 5 & 7 \end{bmatrix} \end{array}$$

Algebra of Matrices Ex 5.1 Q19

Given,

$$A = B$$

$$\begin{bmatrix} 2x+1 & 2y \\ 0 & y^2-5y \end{bmatrix} = \begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$$

Since equal matrics has all corresponing entries equal,
So,

$$2x+1 = x+3 \qquad ---(i)$$
$$2y = y^2+2 \qquad ---(ii)$$
$$y^2-5y = -6 \qquad ---(iii)$$

Solving equation (i)

$$2x+1 = x+3$$
$$2x-x = 3-1$$
$$x = 2$$

Solving equation (ii)

$$2y = y^2+2$$
$$y^2-2y+2 = 0$$
$$D = b^2-4ac$$
$$= (-2)^2-4(i)(ii)$$
$$= 4-8$$
$$= -2$$

So, There is no real value of y from equation (ii).

Solving equation (iii)

$$y^2-5y = -6$$
$$y^2-5y+6 = 0$$
$$y^2-3y-2y+6 = 0$$
$$y(y-3)-2(y-3) = 0$$
$$(y-3)(y-2) = 0$$
$$y = 3 \qquad or \qquad y = 2$$

From solution of equation (i), (ii) and (iii), We can say that
A and B can not be equal for any value of y.

Algebra of Matrices Ex 5.1 Q20

Given,

$$\begin{bmatrix} x+10 & y^2+2y \\ 0 & -4 \end{bmatrix} = \begin{bmatrix} 3x+4 & 3 \\ 0 & y^2-5y \end{bmatrix}$$

Since corresponding entries of equal matrices are equal, So

$$x+10 = 3x+4 \qquad ---(i)$$

$$y^2+2y = 3 \qquad ---(ii)$$

$$-4 = y^2-5y \qquad ---(iii)$$

Solving equation (i),

$$x+10 = 3x+4$$

$$x-3x = 4-10$$

$$-2x = -6$$

$$x = \frac{6}{2}$$

Solving equation (ii),

$$y^2+2y = 3$$

$$y^2+2y-3 = 0$$

$$y^2+3y-y-3 = 0$$

$$y(y+3)(y-1) = 0$$

$\Rightarrow \qquad y = -3$ and $\quad y = 1$

Solving equation (iii)

$$-4 = y^2-5y$$

$$y^2-5y+4 = 0$$

$$y^2-4y-y+(y-4) = 0$$

$$y(y-4)-1(y-4) = 0$$

$$(y-4)(y-1) = 0$$

$\Rightarrow \qquad y = 4$ and $y = 1$

From equation (ii) *and* (iii),

The common value of y = 1

So, $\qquad x = 3, y = 1$

Algebra of Matrices Ex 5.1 Q21

$$A = \begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix},\ B = \begin{bmatrix} 2a+2 & b^2+2 \\ 8 & b^2-5b \end{bmatrix}$$

Given that A = B

Corresponding element of two equal matrices are equal

$\Rightarrow \qquad a + 4 = 2a + 2,\ 3b = b^2 + 2 \quad$ and $\ -6 = b^2 - 5b$

$\Rightarrow \qquad a - 2a = 2 - 4,\ b^2 - 3b + 2 = 0$ and $b^2 - 5b + 6 = 0$

$\Rightarrow \qquad -a = -2,\ (b-1)(b-2) = 0$ and $(b-2)(b-3) = 0$

$\Rightarrow \qquad a = 2,\ b = 1,2 \qquad$ and $b = 2,3$

So value of a = 2, b=2 respectively.

Ex 5.2

Algebra of Matrices Ex 5.2 Q1

(i) $$\begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3-2 & -2+4 \\ 1+1 & 4+3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 2 & 7 \end{bmatrix}$$

Hence,

$$\begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 7 \end{bmatrix}$$

(ii) $$\begin{bmatrix} 2 & 1 & 3 \\ 0 & 3 & 5 \\ -1 & 2 & 5 \end{bmatrix} + \begin{bmatrix} 1 & -2 & 3 \\ 2 & 6 & 1 \\ 0 & -3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+1 & 1-2 & 3+3 \\ 0+2 & 3+6 & 5+1 \\ -1+0 & 2-3 & 5+1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 & 6 \\ 2 & 9 & 6 \\ -1 & -1 & 6 \end{bmatrix}$$

Hence,

$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & 3 & 5 \\ -1 & 2 & 5 \end{bmatrix} + \begin{bmatrix} 1 & -2 & 3 \\ 2 & 6 & 1 \\ 0 & -3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 6 \\ 2 & 9 & 6 \\ -1 & -1 & 6 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q2

Given, $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

$2A - 3B$

$$= 2\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - 3\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 8 \\ 6 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 9 \\ -6 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3 & 8-9 \\ 6+6 & 4-15 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \\ 12 & -11 \end{bmatrix}$$

Hence,

$$2A - 3B = \begin{bmatrix} 1 & -1 \\ 12 & -11 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q2(ii)

Given, $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

$B - 4c$

$$= \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} - 4\begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} - \begin{bmatrix} -8 & 20 \\ 12 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} 1+8 & 3-20 \\ -2-12 & 5-16 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -17 \\ -14 & -11 \end{bmatrix}$$

Hence,

$$B - 4c = \begin{bmatrix} 9 & -17 \\ -14 & -11 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q2(iii)

Given, $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

$3A - C$

$$= 3\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6+2 & 12-5 \\ 9-3 & 6-4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$$

Hence,

$$3A - C = \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q2(iv)

Given, $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

$3A - 2B + 3C$

$$= 3\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - 2\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} + 3\begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} 2 & 6 \\ -4 & 10 \end{bmatrix} + \begin{bmatrix} -6 & 15 \\ 9 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 6-2-6 & 12-6+15 \\ 9+4+9 & 6-10+12 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 21 \\ 22 & 8 \end{bmatrix}$$

Hence,

$$3A - 2B + 3C = \begin{bmatrix} -2 & 21 \\ 22 & 8 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q3

Given, $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 4 & 1 \end{bmatrix}, C = \begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \end{bmatrix}$

(i) $A + B$

$A + B$ is not possible as order of A is 2×2 and order of B is 2×3.
And we know that sum of matrix is possible only when their order is same.

Hence,

$A + B$ is not possible

$B + C$

$$= \begin{bmatrix} -1 & 0 & 2 \\ 3 & 4 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1-1 & 0+2 & 2+3 \\ 3+2 & 4+1 & 1+0 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 2 & 5 \\ 5 & 5 & 1 \end{bmatrix}$$

So,

$$B + C = \begin{bmatrix} -2 & 2 & 5 \\ 5 & 5 & 1 \end{bmatrix}$$

We need to find $2B+3A$ *and* $3C-4B$

Thuss, $2B+3A$ *does not exist as the order of* A *and* B *are different.*

Let us find $3C-4B = 3\begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \end{bmatrix} - 4\begin{bmatrix} -1 & 0 & 2 \\ 3 & 4 & 1 \end{bmatrix}$

$$= \begin{bmatrix} -3 & 6 & 9 \\ 6 & 3 & 0 \end{bmatrix} - \begin{bmatrix} -4 & 0 & 8 \\ 12 & 16 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 6 & 9 \\ 6 & 3 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 & -8 \\ -12 & -16 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 6 & 1 \\ -6 & -13 & -4 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q4

Given, $A = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix}, B = \begin{bmatrix} 0 & -2 & 5 \\ 1 & -3 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & -5 & 2 \\ 6 & 0 & -4 \end{bmatrix}$

$2A - 3B + 4C$

$$= 2\begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix} - 3\begin{bmatrix} 0 & -2 & 5 \\ 1 & -3 & 1 \end{bmatrix} + 4\begin{bmatrix} 1 & -5 & 2 \\ 6 & 0 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 0 & 4 \\ 6 & 2 & 8 \end{bmatrix} - \begin{bmatrix} 0 & -6 & 15 \\ 3 & -9 & 3 \end{bmatrix} + \begin{bmatrix} 4 & -20 & 8 \\ 24 & 0 & -16 \end{bmatrix}$$

$$= \begin{bmatrix} -2-0+4 & 0+6-20 & 4-15+8 \\ 6-3+24 & 2+9+0 & 8-3-16 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -14 & -3 \\ 27 & 11 & -11 \end{bmatrix}$$

Hence,

$$2A - 3B + 4C = \begin{bmatrix} 2 & -14 & -3 \\ 27 & 11 & -11 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q5

Given, $A = diag\begin{pmatrix}2 & -5 & 9\end{pmatrix}, B = diag\begin{pmatrix}1 & 1 & -4\end{pmatrix}$

and $C = diag\ \begin{pmatrix}-b & 3 & 4\end{pmatrix}$

(i) $A - 2B$

$= diag\begin{pmatrix}2 & -5 & 9\end{pmatrix} - 2diag\begin{pmatrix}1 & 1 & 9-4\end{pmatrix}$

$= diag\begin{pmatrix}2 & -5 & 9\end{pmatrix} - diag\begin{pmatrix}2 & 2 & -8\end{pmatrix}$

$= diag\begin{pmatrix}2-2 & -5-2 & 9+8\end{pmatrix}$

$= diag\begin{pmatrix}0 & -7 & -17\end{pmatrix}$

$So,$ $A - 2B = diag\begin{pmatrix}0 & -7 & 17\end{pmatrix}$

(ii) $B + C - 2A$

$= diag\begin{pmatrix}1 & 1 & -4\end{pmatrix} + diag\begin{pmatrix}-6 & 3 & 4\end{pmatrix} - 2diag\begin{pmatrix}2 & -5 & 9\end{pmatrix}$

$= diag\begin{pmatrix}1 & 1 & -4\end{pmatrix} + diag\begin{pmatrix}-6 & 3 & 4\end{pmatrix} - diag\begin{pmatrix}4 & -10 & 18\end{pmatrix}$

$= diag\begin{pmatrix}1-6-4 & 1+3+10 & -4+4-18\end{pmatrix}$

$= diag\begin{pmatrix}-9 & 14 & -18\end{pmatrix}$

So, $\mathrm{B} + C - 2A = diag\begin{pmatrix}-9 & 14 & -18\end{pmatrix}$

(iii) $2A + 3B - 5C$

$= 2\ diag\begin{pmatrix}2 & -5 & 9\end{pmatrix} + 3\ diag\begin{pmatrix}1 & 1 & -4\end{pmatrix} - 5\ diag\begin{pmatrix}-6 & 3 & 4\end{pmatrix}$

$= diag\begin{pmatrix}4 & -10 & 18\end{pmatrix} + diag\begin{pmatrix}3 & 3 & -12\end{pmatrix} - diag\begin{pmatrix}-30 & 15 & 20\end{pmatrix}$

$= diag\begin{pmatrix}4+3+30 & -10+3-15 & 18-12-20\end{pmatrix}$

$= diag\begin{pmatrix}37 & -22 & -14\end{pmatrix}$

So,

$2A + 3B - 5C = diag\begin{pmatrix}37 & -22 & -14\end{pmatrix}$

Algebra of Matrices Ex 5.2 Q6

Given,

$$A=\begin{bmatrix}2&1&1\\3&-1&0\\0&2&4\end{bmatrix}, B=\begin{bmatrix}9&7&-1\\3&5&4\\2&1&6\end{bmatrix}, C=\begin{bmatrix}2&-4&3\\1&-1&0\\9&4&5\end{bmatrix}$$

$$\text{LHS} = (A+B)+C$$

$$=\left\{\begin{bmatrix}2&1&1\\3&-1&0\\0&2&4\end{bmatrix}+\begin{bmatrix}9&7&-1\\3&5&4\\2&1&6\end{bmatrix}\right\}+\begin{bmatrix}2&-4&3\\1&-1&0\\9&4&5\end{bmatrix}$$

$$=\begin{bmatrix}2+9&1+7&1-1\\3+2&-1+5&0+4\\0+2&2+1&4+6\end{bmatrix}+\begin{bmatrix}2&-4&3\\1&-1&0\\9&4&5\end{bmatrix}$$

$$=\begin{bmatrix}11&8&0\\6&4&4\\2&3&10\end{bmatrix}+\begin{bmatrix}2&-4&3\\1&-1&0\\9&4&5\end{bmatrix}$$

$$=\begin{bmatrix}11+2&8-4&0+3\\6+1&4-1&4+0\\2+9&3+4&10+5\end{bmatrix}$$

$$\text{LHS} = \begin{bmatrix}13&4&3\\7&3&4\\11&7&15\end{bmatrix} \qquad ---(i)$$

$$\text{RHS} = A+(B+C)$$

$$=\begin{bmatrix}2&1&1\\3&-1&0\\0&2&4\end{bmatrix}+\left\{\begin{bmatrix}9&7&-1\\3&5&4\\2&1&6\end{bmatrix}+\begin{bmatrix}2&-4&3\\1&-1&0\\9&4&5\end{bmatrix}\right\}$$

$$=\begin{bmatrix}2&1&1\\3&-1&0\\0&2&4\end{bmatrix}+\begin{bmatrix}9+2&7-4&-1+3\\3+1&5-1&4+0\\2+9&1+4&6+5\end{bmatrix}$$

$$=\begin{bmatrix}2&1&1\\3&-1&0\\0&2&4\end{bmatrix}+\begin{bmatrix}11&3&2\\4&4&4\\11&5&11\end{bmatrix}$$

$$=\begin{bmatrix}2+11&1+3&1+2\\3+4&-1+4&0+4\\0+11&2+5&4+11\end{bmatrix}$$

$$RHS = \begin{bmatrix} 13 & 4 & 3 \\ 7 & 3 & 4 \\ 11 & 7 & 15 \end{bmatrix} \qquad ---(ii)$$

From equation (i) and , we get

$$(A+B)+C = A+(B+C)$$

Algebra of Matrices Ex 5.2 Q7

We have

$$(X+Y)+(X-Y) = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow \ 2X = \begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix}$$

$$\Rightarrow \ X = \frac{1}{2}\begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$$

$$\text{Also, } (X+Y)-(X-Y) = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow \ 2Y = \begin{bmatrix} 5-3 & 2-6 \\ 0-0 & 9+1 \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ 0 & 10 \end{bmatrix}$$

$$\Rightarrow \ Y = \frac{1}{2}\begin{bmatrix} 2 & -4 \\ 0 & 10 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q8

$$2X+Y=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}$$

$$\Rightarrow 2X+\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}$$

$$\Rightarrow 2X=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}-\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}=\begin{bmatrix}1-3 & 0-2\\ -3-1 & 2-4\end{bmatrix}$$

$$\Rightarrow 2X=\begin{bmatrix}-2 & -2\\ -4 & -2\end{bmatrix}$$

$$\therefore X=\frac{1}{2}\begin{bmatrix}-2 & -2\\ -4 & -2\end{bmatrix}=\begin{bmatrix}-1 & -1\\ -2 & -1\end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q9

Given,

$$2x-y=\begin{bmatrix}6 & -6 & 0\\ -4 & 2 & 1\end{bmatrix} \quad ---(i)$$

$$x+2y=\begin{bmatrix}3 & 2 & 5\\ -2 & 1 & -7\end{bmatrix} \quad ---(ii)$$

Now find

$$2(2x-y)+(x+2y)=2\begin{bmatrix}6 & -6 & 0\\ -4 & 2 & 1\end{bmatrix}+\begin{bmatrix}3 & 2 & 5\\ -2 & 1 & -7\end{bmatrix} \quad \{\text{using equation (i) and (ii)}\}$$

$$\Rightarrow \quad 4x-2y+x+2y=\begin{bmatrix}12 & -12 & 0\\ -8 & 4 & 2\end{bmatrix}+\begin{bmatrix}3 & 2 & 5\\ -2 & 1 & -7\end{bmatrix}$$

$$\Rightarrow \quad 5x=\begin{bmatrix}12+3 & -12+2 & 0+5\\ -8-2 & 4+1 & 2-7\end{bmatrix}$$

$$\Rightarrow \quad 5x=\begin{bmatrix}15 & -10 & 5\\ -10 & 5 & -5\end{bmatrix}$$

$$\Rightarrow \quad 5x=5\begin{bmatrix}3 & -2 & 1\\ -2 & 1 & -1\end{bmatrix}$$

$$\Rightarrow \quad x=\begin{bmatrix}3 & -2 & 1\\ -2 & 1 & -1\end{bmatrix}$$

Now find,

$$(2x-y)-2(x+2y)=\begin{bmatrix}6 & -6 & 0\\ -4 & 2 & 1\end{bmatrix}-2\begin{bmatrix}3 & 2 & 5\\ -2 & 1 & -7\end{bmatrix} \quad \{\text{using equation (i) and (ii)}\}$$

$$\Rightarrow \quad 2x=y-2x-4y=\begin{bmatrix}6 & -6 & 0\\ -4 & 2 & 1\end{bmatrix}-\begin{bmatrix}6 & 4 & 10\\ -4 & 2 & -14\end{bmatrix}$$

$$\Rightarrow \quad -y-4y=\begin{bmatrix}6-6 & -6-4 & 0-10\\ -4+4 & 2-2 & 1+14\end{bmatrix}$$

$$\Rightarrow \quad -5y=\begin{bmatrix}0 & -10 & -10\\ 0 & 0 & 15\end{bmatrix}$$

$$\Rightarrow \quad -5y=-5\begin{bmatrix}0 & 2 & 2\\ 0 & 0 & -3\end{bmatrix}$$

$$\Rightarrow \quad y=\begin{bmatrix}0 & 2 & 2\\ 0 & 0 & -3\end{bmatrix}$$

Hence,

$$x=\begin{bmatrix}3 & -2 & 1\\ -2 & 1 & -1\end{bmatrix}, y=\begin{bmatrix}0 & 2 & 2\\ 0 & 0 & -3\end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q10

Given,

$$x - y = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$x + y = \begin{bmatrix} 3 & 5 & 1 \\ -1 & 1 & 4 \\ 11 & 8 & 0 \end{bmatrix}$$

Now find,

$$(x - y) + (x + y) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 5 & 1 \\ -1 & 1 & 4 \\ 11 & 8 & 0 \end{bmatrix}$$

$$\Rightarrow \quad 2x = \begin{bmatrix} 1+3 & 1+5 & 1+1 \\ 1-1 & 1+1 & 0+4 \\ 1+11 & 0+8 & 0+0 \end{bmatrix}$$

$$\Rightarrow \quad 2x = \begin{bmatrix} 4 & 6 & 2 \\ 0 & 2 & 4 \\ 12 & 8 & 0 \end{bmatrix}$$

$$\Rightarrow \quad 2x = 2\begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 2 \\ 6 & 4 & 0 \end{bmatrix}$$

$$\Rightarrow \quad x = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 2 \\ 6 & 4 & 0 \end{bmatrix}$$

Now find,

$$(x - y) - (x + y) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 3 & 5 & 1 \\ -1 & 1 & 4 \\ 11 & 8 & 0 \end{bmatrix}$$

$$\Rightarrow \quad x - y - x - y = \begin{bmatrix} 1-3 & 1-5 & 1-1 \\ 1+1 & 1-1 & 0-4 \\ 1-11 & 0-8 & 0-0 \end{bmatrix}$$

$$\Rightarrow \quad -2y = \begin{bmatrix} -2 & -4 & 0 \\ 2 & 0 & -4 \end{bmatrix}$$

$$\begin{bmatrix} -10 & -8 & 0 \end{bmatrix}$$

$$\Rightarrow \qquad -2y = -2\begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 2 \\ 5 & 4 & 0 \end{bmatrix}$$

$$\Rightarrow \qquad y = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 2 \\ 5 & 4 & 0 \end{bmatrix}$$

Hence,

$$x = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 2 \\ 6 & 4 & 0 \end{bmatrix}, y = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 2 \\ 5 & 4 & 0 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q11

Given,

$$\begin{bmatrix} 1 & 2 & 4 \\ -1 & 0 & 9 \end{bmatrix} + A = \begin{bmatrix} 9 & -1 & 1 \\ 4 & -2 & 3 \end{bmatrix}$$

$$\Rightarrow \qquad A = \begin{bmatrix} 9 & -1 & 1 \\ 4 & -2 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 4 \\ -1 & 0 & 9 \end{bmatrix}$$

$$\Rightarrow \qquad A = \begin{bmatrix} 9-1 & -1-2 & 1-4 \\ 4+1 & -2-0 & 3-9 \end{bmatrix}$$

$$\Rightarrow \qquad A = \begin{bmatrix} 8 & -3 & -3 \\ 5 & -2 & -6 \end{bmatrix}$$

Hence,

$$A = \begin{bmatrix} 8 & -3 & -3 \\ 5 & -2 & -6 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q12

Given, $A = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$

Let, $C = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$

Since, $5A + 3B + 2C$ is a null matrix, so

$$5A + 3B + 2C = 0$$

$$\Rightarrow \quad 5\begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix} + 3\begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix} + 2\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 45 & 5 \\ 35 & 40 \end{bmatrix} + \begin{bmatrix} 3 & 15 \\ 21 & 36 \end{bmatrix} + \begin{bmatrix} 2x & 2y \\ 2z & 2w \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 45 + 3 + 2x & 5 + 15 + 2y \\ 35 + 21 + 2z & 40 + 36 + 2w \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 48 + 2x & 20 + 2y \\ 56 + 2z & 76 + 2w \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Since, corresponding entries of equalmatrices are equal.

$48 + 2x = 0$

$x = -\dfrac{48}{2}$

$x = -24$

$20 + 2y = 0$

$y = -\dfrac{20}{2}$

$y = -10$

$56 + 2z = 0$

$z = -\dfrac{56}{2}$

$z = -28$

$76 + 2w = 0$

$w = -\dfrac{76}{2}$

$w = -38$

Hence, $C = \begin{bmatrix} -24 & -10 \\ -28 & -38 \end{bmatrix}$

Given,

$$A = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}, B = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$$

And

$$2A + 3X = 5B$$

$$\Rightarrow \quad 3X = 5B - 2A$$

$$\Rightarrow \quad 3X = 5\begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix} - 2\begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$$

$$\Rightarrow \quad 3X = \begin{bmatrix} 40 & 0 \\ 20 & -10 \\ 15 & 30 \end{bmatrix} - \begin{bmatrix} 4 & -4 \\ 8 & 4 \\ -10 & 2 \end{bmatrix}$$

$$\Rightarrow \quad 3X = \begin{bmatrix} 40-4 & 0+4 \\ 20-8 & -10-4 \\ 15+10 & 30-2 \end{bmatrix}$$

$$\Rightarrow \quad 3X = \begin{bmatrix} 36 & 4 \\ 12 & -14 \\ 25 & 28 \end{bmatrix}$$

$$\Rightarrow \quad X = \begin{bmatrix} \frac{36}{3} & \frac{4}{3} \\ \frac{12}{3} & \frac{-14}{3} \\ \frac{25}{3} & \frac{28}{3} \end{bmatrix}$$

$$\Rightarrow \quad X = \begin{bmatrix} 12 & \frac{4}{3} \\ 4 & \frac{-14}{3} \\ \frac{25}{3} & \frac{28}{3} \end{bmatrix}$$

Hence,

$$\begin{bmatrix} 12 & \frac{4}{3} \\ & -14 \end{bmatrix}$$

$$
X = \begin{bmatrix} 4 & \frac{\text{[illegible]}}{3} \\ \frac{25}{3} & \frac{28}{3} \end{bmatrix}
$$

Algebra of Matrices Ex 5.2 Q14

Given,

$$
A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}
$$

And

$$
A + B + C = 0
$$

$$
\Rightarrow \quad C = -A - B + 0
$$

$$
\Rightarrow \quad C = -A - B
$$

$$
\Rightarrow \quad C = -\begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}
$$

$$
\Rightarrow \quad C = \begin{bmatrix} -1 & 3 & -2 \\ -2 & 0 & -2 \end{bmatrix} - \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}
$$

$$
\Rightarrow \quad C = \begin{bmatrix} -1-2 & 3+1 & -2+1 \\ -2-1 & 0-0 & -2+1 \end{bmatrix}
$$

$$
\Rightarrow \quad C = \begin{bmatrix} -3 & 4 & -1 \\ -3 & 0 & -1 \end{bmatrix}
$$

Hence,

$$
C = \begin{bmatrix} -3 & 4 & -1 \\ -3 & 0 & -1 \end{bmatrix}
$$

Algebra of Matrices Ex 5.2 Q15(i)

$$\begin{bmatrix} x-y & 2 & -2 \\ 4 & x & 6 \end{bmatrix} + \begin{bmatrix} 3 & -2 & 2 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 5 & 2x+y & 5 \end{bmatrix}$$

$$\begin{bmatrix} x-y+3 & 2-2 & -2+2 \\ 4+1 & x+0 & 6-1 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 5 & 2x+y & 5 \end{bmatrix}$$

$$\begin{bmatrix} x-y+3 & 0 & 0 \\ 5 & x & 5 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 5 & 2x+y & 5 \end{bmatrix}$$

Use know that, corresponding entries of equal matrices are equal. So,

$$x - y + 3 = 6$$

$$\Rightarrow \quad x - y = 3 \qquad \text{---(i)}$$

and $\quad x = 2x + y$

$$\Rightarrow \quad 2x - x + y = 0$$

$$\Rightarrow \quad x + y = 0 \qquad \text{---(ii)}$$

Adding equation (i), (ii),

$$x - y + x + y = 3 + 0$$

$$\Rightarrow \quad 2x = 3$$

$$\Rightarrow \quad x = \frac{3}{2}$$

Put in equation (i),

$$x - y = 3$$

$$\Rightarrow \quad \frac{3}{2} - y = 3$$

$$\Rightarrow \quad -y = \frac{3-3}{2}$$

$$\Rightarrow \quad y = \frac{-3}{2}$$

Hence,

$$x = \frac{3}{2}, y = \frac{-3}{2}$$

Algebra of Matrices Ex 5.2 Q15(ii)

$$[x \quad y+2 \quad z-3]+[y \quad 4 \quad 5]=[4 \quad 9 \quad 12]$$
$$\Rightarrow \quad [x+y \quad y+2+4 \quad z-3+5]=[4 \quad 9 \quad 12]$$
$$\Rightarrow \quad [x+y \quad y+6 \quad z+2]=[4 \quad 9 \quad 12]$$

We know that, corresponding entries, of equal matrices are equal, So

$$x+y=4 \qquad ---(i)$$
$$y+6=9 \qquad ---(ii)$$
$$z+2=12 \qquad ---(iii)$$

From equation (ii), We get

$$y=9-6$$
$$y=3$$

Put the value of y in equation (i),

$$x+y=4$$
$$\Rightarrow \quad x+3=4$$
$$\Rightarrow \quad x=4-3$$
$$\Rightarrow \quad x=1$$

From equation (iii)

$$z=2=12$$
$$z=12-2$$
$$z=10$$

Hence,

$$x=1, y=3, z=10$$

Algebra of Matrices Ex 5.2 Q16

Given,

$$2\begin{bmatrix}3 & 4\\5 & x\end{bmatrix}+\begin{bmatrix}1 & y\\0 & 1\end{bmatrix}=\begin{bmatrix}7 & 0\\10 & 5\end{bmatrix}$$
$$\Rightarrow \quad \begin{bmatrix}6 & 8\\10 & 2x\end{bmatrix}+\begin{bmatrix}1 & y\\0 & 1\end{bmatrix}=\begin{bmatrix}7 & 0\\10 & 5\end{bmatrix}$$
$$\Rightarrow \quad \begin{bmatrix}6+1 & 8+y\\10+0 & 2x+1\end{bmatrix}=\begin{bmatrix}7 & 0\\10 & 5\end{bmatrix}$$
$$\Rightarrow \quad \begin{bmatrix}7 & 8+y\\10 & 2x+1\end{bmatrix}=\begin{bmatrix}7 & 0\\10 & 5\end{bmatrix}$$

Since corresponding entries of equal matrices are equal, So

$$8+y=0$$
$$y=-8$$

And

$$2x+1=5$$
$$2x=5-1$$
$$x=\frac{4}{2}$$
$$x=2$$

Hence,

$$x=2, y=-8$$

Algebra of Matrices Ex 5.2 Q17

Given,

$$\lambda\begin{bmatrix}1 & 0 & 2\\ 3 & 4 & 5\end{bmatrix}+2\begin{bmatrix}1 & 2 & 3\\ -1 & -3 & 2\end{bmatrix}=\begin{bmatrix}4 & 4 & 10\\ 4 & 2 & 14\end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix}\lambda & 0 & 2\lambda\\ 3\lambda & 4\lambda & 5\lambda\end{bmatrix}+\begin{bmatrix}2 & 4 & 6\\ -2 & -6 & 4\end{bmatrix}=\begin{bmatrix}4 & 4 & 10\\ 4 & 2 & 14\end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix}\lambda+2 & 4 & 2\lambda+6\\ 3\lambda-2 & 4\lambda-6 & 5v+4\end{bmatrix}=\begin{bmatrix}4 & 4 & 10\\ 4 & 2 & 14\end{bmatrix}$$

Since corresponding entries of equal matrices are equal, So

$$\lambda = 2 = 4$$

$$\Rightarrow \quad \lambda = 2$$

and

$$3\lambda - 2 = 4$$

$$3\lambda = 6$$

$$\Rightarrow \quad \lambda = 2$$

Hence,

$$\lambda = 2$$

Algebra of Matrices Ex 5.2 Q18(i)

Given,

$$A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$$

and

$$2A + B + X = 0$$

$$\Rightarrow \quad X = -2A - B$$

$$\Rightarrow \quad X = -2\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$$

$$\Rightarrow \quad X = \begin{bmatrix} 2 & -4 \\ -6 & -8 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$$

$$\Rightarrow \quad X = \begin{bmatrix} 2-3 & -4+2 \\ -6-1 & -8-5 \end{bmatrix}$$

$$\Rightarrow \quad X = \begin{bmatrix} -1 & -2 \\ -7 & -13 \end{bmatrix}$$

Hence,

$$X = \begin{bmatrix} -1 & -2 \\ -7 & -13 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q18(ii)

$$Given, A = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix} and \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$$

Also we have $2A + 3X = 5B$

Thus, we have, $3X = 5B - 2A$

$$\Rightarrow 3x = 5\begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix} - 2\begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$$

$$\Rightarrow 3x = \begin{bmatrix} 10 & -10 \\ 20 & 10 \\ -25 & 5 \end{bmatrix} - \begin{bmatrix} 16 & 0 \\ 8 & -4 \\ 6 & 12 \end{bmatrix}$$

$$\Rightarrow 3x = \begin{bmatrix} 10-16 & -10-0 \\ 20-8 & 10-(-4) \\ -25-6 & 5-12 \end{bmatrix}$$

$$\Rightarrow 3x = \begin{bmatrix} -6 & -10 \\ 12 & 14 \\ -31 & -7 \end{bmatrix}$$

$$\Rightarrow x = \begin{bmatrix} -2 & \frac{-10}{3} \\ 4 & \frac{14}{3} \\ \frac{-31}{3} & \frac{-7}{3} \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q19(i)

$$3\begin{bmatrix} x & y \\ z & t \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2t \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+t & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3x & 3y \\ 3z & 3t \end{bmatrix} = \begin{bmatrix} x+4 & 6+x+y \\ -1+z+t & 2t+3 \end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we get:

$3x = x+4$
$\Rightarrow 2x = 4$
$\Rightarrow x = 2$

$3y = 6+x+y$
$\Rightarrow 2y = 6+x = 6+2 = 8$
$\Rightarrow y = 4$

$3t = 2t+3$
$\Rightarrow t = 3$

$3z = -1+z+t$
$\Rightarrow 2z = -1+t = -1+3 = 2$
$\Rightarrow z = 1$

$\therefore x = 2,\ y = 4,\ z = 1,$ and $t = 3$

Algebra of Matrices Ex 5.2 Q19(ii)

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 14 \\ 15 & 14 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x & 10 \\ 14 & 2y-6 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 14 \\ 15 & 14 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+3 & 14 \\ 15 & 2y-4 \end{bmatrix} = \begin{bmatrix} 7 & 14 \\ 15 & 14 \end{bmatrix}$$

Comparing the corresponding elements from both sides,
$2x+3 = 7 \Rightarrow 2x = 4 \Rightarrow x = 2$
$2y-4 = 14 \Rightarrow 2y = 18 \Rightarrow y = 9$
Hence, $x = 2,\ y = 9$

Algebra of Matrices Ex 5.2 Q20

Let us solve this problem using simultaneous linear
equation and algebra of matrices.

$$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \quad .... (1)$$

$$3X + 2Y = \begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix} \quad ....(2)$$

multiplying the first equation by 3 and second equation by 2 we get,

$$6X + 9Y = 3\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}........(3),$$

$$6X + 4Y = 2\begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix}............(4)$$

Subtracting equation (4) from equation (3) we have,

$$5Y = 3\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - 2\begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix}$$

$$\Rightarrow 5Y = \begin{bmatrix} 6 & 9 \\ 12 & 0 \end{bmatrix} - \begin{bmatrix} -4 & 4 \\ 2 & -10 \end{bmatrix}$$

$$\Rightarrow 5Y = \begin{bmatrix} 10 & 5 \\ 10 & 10 \end{bmatrix}$$

$$\Rightarrow Y = \frac{1}{5}\begin{bmatrix} 10 & 5 \\ 10 & 10 \end{bmatrix}$$

$$\Rightarrow Y = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}$$

Similarly, multiplying the equation (1) by 2 and equation (2) by 3 we get,

$$4X + 6Y = 2\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}........(5),$$

$$9X + 6Y = 3\begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix}............(6)$$

Subtracting equation (6) from equation (5) we have,

$$-5X = 2\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - 3\begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix}$$

$$\Rightarrow -5X = \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix} - \begin{bmatrix} -6 & 6 \\ 3 & -15 \end{bmatrix}$$

$$\Rightarrow -5X = \begin{bmatrix} 10 & 0 \\ 5 & 15 \end{bmatrix}$$

$$\Rightarrow \quad X = -\frac{1}{5}\begin{bmatrix} 10 & 0 \\ 5 & 15 \end{bmatrix}$$

$$\Rightarrow \quad X = \begin{bmatrix} -2 & 0 \\ -1 & -3 \end{bmatrix}$$

Hence the value of $X = \begin{bmatrix} -2 & 0 \\ -1 & -3 \end{bmatrix}$ and $Y = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}$.

Algebra of Matrices Ex 5.2 Q21

Let A represent the post allocation matrix for a college, So

$$A = \begin{bmatrix} 15 \\ 6 \\ 1 \\ 1 \end{bmatrix} \begin{array}{c} \text{Peons} \\ \text{Clerks} \\ \text{Typist} \\ \text{Section officer} \end{array}$$

The total number of posts of each kind in 30 colleges in given by:

$$= 30A$$

$$= 30\begin{bmatrix} 15 \\ 6 \\ 1 \\ 1 \end{bmatrix}$$

$$30A = \begin{bmatrix} 450 \\ 90 \\ 30 \\ 30 \end{bmatrix} \begin{array}{c} \text{Peons} \\ \text{Clerks} \\ \text{Typists} \\ \text{Section Officers} \end{array}$$

Ex 5.3

Algebra of Matrices Ex 5.3 Q1

(i) $$\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$= \begin{bmatrix} (a)(a)+(b)(b) & (a)(-b)+(b)(a) \\ (-b)(a)+(a)(b) & (-b)(-b)+(a)(a) \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2 & -ab+ab \\ -ab+ab & b^2+a^2 \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix}$$

Hence,

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix}$$

(ii) $$\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ -3 & 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} (1)(1)+(-2)(-3) & (1)(2)+(-2)(2) & (1)(3)+(-2)(-1) \\ (2)(1)+(3)(-3) & (2)(2)+(3)(2) & (2)(3)+(3)(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 1+6 & 2-4 & 3+2 \\ 2-9 & 4+6 & 6-3 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -2 & 5 \\ -7 & 10 & 3 \end{bmatrix}$$

Hence,

$$\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ -3 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 7 & -2 & 5 \\ -7 & 10 & 3 \end{bmatrix}$$

(iii) $$\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} (2)(1)+(3)(0)+(4)(3) & (2)(-3)+(3)(2)+(4)(0) & (2)(5)+(3)(4)+(4)(5) \\ (3)(1)+(4)(0)+(5)(3) & (3)(-3)+(4)(2)+(5)(0) & (3)(5)+(4)(4)+(5)(5) \\ (4)(1)+(5)(0)+(6)(3) & (4)(-3)+(5)(2)+(6)(0) & (4)(5)+(5)(4)+(6)(5) \end{bmatrix}$$

$$= \begin{bmatrix} 2+0+12 & -6+6+0 & 10+12+20 \\ 3+0+15 & -9+8+0 & 15+16+25 \\ 4+0+18 & -12+10+0 & 20+20+30 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q2(i)

Given, $A = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$

$$AB = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 10-3 & 5-4 \\ 12+21 & 6+28 \end{bmatrix}$$

$$AB = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix} \qquad \text{---(i)}$$

$$BA = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 10+6 & -2+7 \\ 15+24 & -3+28 \end{bmatrix}$$

$$BA = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix} \qquad \text{---(ii)}$$

From equation (i) and (ii), we get

$$AB \neq BA$$

Algebra of Matrices Ex 5.3 Q2(ii)

$$\text{Given, } A = \begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$BA = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1+0+0 & -2+1+0 & -3+0+0 \\ +0+01 & 0-1+1 & 0+0+0 \\ 2+0+4 & 4+3+4 & 6+0+0 \end{bmatrix}$$

$$AB = \begin{bmatrix} -1 & -1 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \qquad ---(i)$$

$$BA = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1+0+6 & 1-2+9 & 0+2+12 \\ 0+0+0 & 0-1+0 & 0+1+0 \\ -1+0+0 & 1-1+0 & 0+1+0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix} \qquad ---(ii)$$

From (i) and (ii), $\quad AB \neq BC$

Algebra of Matrices Ex 5.3 Q2(iii)

Given, $A = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix}$

$$BA = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+3+0 & 1+0+0 & 0+0+0 \\ 0+1+0 & 1+0+0 & 0+0+0 \\ 0+1+0 & 4+0+0 & 0+0+0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 4 & 0 \end{bmatrix} \qquad \text{---(i)}$$

$$BA = \begin{bmatrix} 0 & 10 & 1 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1+0 & 0+1+0 & 0+0+0 \\ 1+0+0 & 3+0+0 & 0+0+0 \\ 0+5+4 & 0+5+1 & 0+0+0 \end{bmatrix} \qquad ---(ii)$$

$$BA = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 3 & 0 \\ 9 & 6 & 0 \end{bmatrix} \qquad \text{---(ii)}$$

From equation (i) and (ii), we get

$$AB \neq BA$$

Algebra of Matrices Ex 5.3 Q3(i)

$A = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

Since order of A is 2×2 and order of B is 2×3,
So AB is possible but BA is not possible order of AB is 2×3.

$$AB = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1)(1)+(-2)(2) & (1)(2)+(-2)(3) & (1)(3)+(-2)(1) \\ (2)(1)+(3)(2) & (2)(2)+(3)(3) & (2)(3)+(3)(1) \end{bmatrix}$$

$$= \begin{bmatrix} 1-4 & 2-6 & 3-2 \\ 2+6 & 4+9 & 6+3 \end{bmatrix}$$

$$AB = \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$

Hence,

$$AB = \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$

BA does not exits

Algebra of Matrices Ex 5.3 Q3(ii)

Here, $A=\begin{bmatrix}3 & 2\\ -1 & 0\\ -1 & 1\end{bmatrix}, B=\begin{bmatrix}4 & 5 & 6\\ 0 & 1 & 2\end{bmatrix}$

Order of $A = 3\times 2$ and order of $B = 2\times 3$ So,

AB and BA Both exits and order of $AB = 3\times 3$ and order of $BA = 2\times 2$

$$AB=\begin{bmatrix}3 & 2\\ -1 & 0\\ -1 & 1\end{bmatrix}\begin{bmatrix}4 & 5 & 6\\ 0 & 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}(3)(4)+(2)(0) & (3)(5)+(2)(1) & (3)(6)+(2)(2)\\ (-1)(4)+(0)(0) & (-1)(5)+(0)(0) & (-1)(6)+(0)(2)\\ (-1)(4)+(1)(0) & (-1)(5)+(1)(1) & (-1)(6)+(1)(2)\end{bmatrix}$$

$$=\begin{bmatrix}12+0 & 15+2 & 18+4\\ -4+0 & -5+0 & -6+0\\ -4+0 & -5+0 & -6+2\end{bmatrix}$$

$$=\begin{bmatrix}12 & 17 & 22\\ -4 & -5 & -6\\ -4 & -4 & -4\end{bmatrix}$$

$$BA=\begin{bmatrix}4 & 5 & 6\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}3 & 2\\ -1 & 0\\ -1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}(4)(3)+(5)(-1)+(6)(-1) & (4)(2)+(5)(0)+(6)(1)\\ (0)(3)+(1)(-1)+(2)(-1) & (0)(2)+(1)(0)+(2)(1)\end{bmatrix}$$

$$=\begin{bmatrix}12-5-6 & 8+0+6\\ 0-1-2 & 0+0+2\end{bmatrix}$$

$$=\begin{bmatrix}1 & 14\\ -3 & 2\end{bmatrix}$$

Hence,

$$AB=\begin{bmatrix}12 & 17 & 22\\ -4 & -5 & -6\\ -4 & -4 & -4\end{bmatrix}, BA=\begin{bmatrix}1 & 14\\ -3 & 2\end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q3(iii)

Here,

$$A = \begin{bmatrix} 1 & -1 & 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 2 \end{bmatrix}$$

Order of A = 1×4 and order of B=4×1 So,

AB and BA both exist and order of AB = 1×1 and order of BA=4×4, So

$$AB = \begin{bmatrix} 1 & -1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 3 \\ 2 \end{bmatrix}$$

$$= \left[(1)(0) + (-1)(1) + (2)(3) + (3)(2) \right]$$

$$= \left[0 - 1 + 6 + 6 \right]$$

$$AB = \left[11 \right]$$

$$BA = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 & 3 \end{bmatrix}$$

$$BA = \begin{bmatrix} (0)(1) & (0)(-1) & (0)(2) & (0)(3) \\ (1)(1) & (1)(-1) & (1)(2) & (1)(3) \\ (3)(1) & (3)(-1) & (3)(2) & (3)(3) \\ (2)(1) & (2)(-1) & (3)(2) & (2)(3) \end{bmatrix}$$

$$BA = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & -1 & 2 & 3 \\ 3 & -3 & 6 & 9 \\ 2 & -2 & 4 & 6 \end{bmatrix}$$

Hence,

$$AB = \left[11 \right]$$

$$BA = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & -1 & 2 & 3 \\ 3 & -3 & 6 & 9 \\ 2 & -2 & 4 & 6 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q3(iv)

$$\begin{bmatrix} a & b \end{bmatrix}\begin{bmatrix} c \\ d \end{bmatrix} + \begin{bmatrix} a & b & c & d \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$= \left[ac + bd \right] + \left[a^2 + b^2 + c^2 + d^2 \right]$$

$$= \left[ac + bd = a^2 + b^2 + c^2 + d^2 \right]$$

Hence,

$$\begin{bmatrix} a & b \end{bmatrix}\begin{bmatrix} c \\ d \end{bmatrix} + \begin{bmatrix} a & b & c & d \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$= \left[ac + bd + a^2 + b^2 + c^2 + d^2 \right]$$

Algebra of Matrices Ex 5.3 Q4(i)

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix}, B = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 2 & -4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -2-3+6 & 3+6-9 & -1-3+4 \\ -4+1+6 & 6-2-9 & -2+1+4 \\ -6+0+6 & 9+0-9 & -3+0+4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0 & 0 \\ 3 & -5 & 3 \\ 0 & 0 & 1 \end{bmatrix} \qquad ---(i)$$

$$BA = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -2+6-3 & -6-3+0 & 2-3+1 \\ -1+4-3 & -3-2+0 & 1-2+1 \\ -6+18-12 & -18-9+0 & 6-9+4 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & -9 & 0 \\ 0 & -5 & 0 \\ 0 & -27 & 1 \end{bmatrix} \qquad ---(ii)$$

From equation (i) and (ii),

$$AB \neq BA$$

Algebra of Matrices Ex 5.3 Q4(ii)

$$A=\begin{bmatrix}10 & -4 & -1\\ -11 & 5 & 0\\ 9 & -5 & 1\end{bmatrix}, B=\begin{bmatrix}1 & 2 & 1\\ 3 & 4 & 2\\ 1 & 3 & 2\end{bmatrix}$$

$$AB=\begin{bmatrix}10 & -4 & -1\\ -11 & 5 & 0\\ 9 & -5 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 1\\ 3 & 4 & 2\\ 1 & 3 & 2\end{bmatrix}$$

$$=\begin{bmatrix}10-12-1 & 20-16-3 & 10-8-2\\ -11+15+0 & -22+20+0 & -11+10+0\\ 9-15+1 & 18-20+3 & 9-10+2\end{bmatrix}$$

$$AB=\begin{bmatrix}-3 & 1 & 0\\ 4 & -2 & -1\\ -5 & 1 & 1\end{bmatrix} \quad ---(i)$$

$$BA=\begin{bmatrix}1 & 2 & 1\\ 3 & 4 & 2\\ 1 & 3 & 2\end{bmatrix}\begin{bmatrix}10 & -4 & -1\\ -11 & 5 & 0\\ 9 & -5 & 1\end{bmatrix}$$

$$=\begin{bmatrix}10-22+9 & -4+10-5 & -9+0+1\\ 30-44+10 & -12+20-10 & -3+0+2\\ 10-33+18 & -4+15-10 & -1+0+2\end{bmatrix}$$

$$BA=\begin{bmatrix}-3 & 1 & 0\\ 4 & -2 & -1\\ -5 & 1 & 1\end{bmatrix} \quad ---(ii)$$

From equation (i) and (ii)

$$AB \neq BA$$

Algebra of Matrices Ex 5.3 Q5(i)

$$\left(\begin{bmatrix}1 & 3\\ -1 & -4\end{bmatrix}+\begin{bmatrix}3 & -2\\ -1 & 1\end{bmatrix}\right)\begin{bmatrix}1 & 3 & 5\\ 2 & 4 & 6\end{bmatrix}$$

$$=\left(\begin{bmatrix}1+3 & 3-2\\ -1-1 & -4+1\end{bmatrix}\right)\begin{bmatrix}1 & 3 & 5\\ 2 & 4 & 6\end{bmatrix}$$

$$=\begin{bmatrix}4 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}1 & 3 & 5\\ 2 & 4 & 6\end{bmatrix}$$

$$=\begin{bmatrix}4+2 & 12+4 & 20+6\\ -2-6 & -6-12 & -10-18\end{bmatrix}$$

$$=\begin{bmatrix}6 & 16 & 26\\ -8 & -18 & -28\end{bmatrix}$$

Hence,

$$\left(\begin{bmatrix}1 & 3\\ -1 & -4\end{bmatrix}+\begin{bmatrix}3 & -2\\ -1 & 1\end{bmatrix}\right)\begin{bmatrix}1 & 3 & 5\\ 2 & 4 & 6\end{bmatrix}=\begin{bmatrix}6 & 16 & 26\\ -8 & -18 & -28\end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q5(ii)

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+0 & 0+0+3 & 2+0+6 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 3 & 10 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} 10+12+60 \end{bmatrix}$$

$$= \begin{bmatrix} 82 \end{bmatrix}$$

Hence,

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} = \begin{bmatrix} 82 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q5(iii)

$$\begin{bmatrix}1 & -1\\0 & 2\\2 & 3\end{bmatrix}\left(\begin{bmatrix}1 & 0 & 2\\2 & 0 & 1\end{bmatrix}-\begin{bmatrix}0 & 1 & 2\\1 & 0 & 2\end{bmatrix}\right)$$

$$=\begin{bmatrix}1 & -1\\0 & 2\\2 & 3\end{bmatrix}\left(\begin{bmatrix}1-0 & 0-1 & 2-2\\2-1 & 0-0 & 1-2\end{bmatrix}\right)$$

$$=\begin{bmatrix}1 & -1\\0 & 2\\2 & 3\end{bmatrix}\begin{bmatrix}1 & -1 & 0\\1 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1-1 & -1+0 & 0+1\\0+2 & 0+0 & 0-2\\2+3 & -2+0 & 0-3\end{bmatrix}$$

$$=\begin{bmatrix}0 & -1 & 1\\2 & 0 & -2\\5 & -2 & -3\end{bmatrix}$$

Hence,

$$\begin{bmatrix}1 & -1\\0 & 2\\2 & 3\end{bmatrix}\left(\begin{bmatrix}1 & 0 & 2\\2 & 0 & 1\end{bmatrix}-\begin{bmatrix}0 & 1 & 2\\1 & 0 & 2\end{bmatrix}\right)=\begin{bmatrix}0 & -1 & 1\\2 & 0 & -2\\5 & -2 & -3\end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q6

Given, $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, C \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^2 = I_2 \qquad \text{---(i)}$$

$$B^2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B^2 = I_2 \qquad \text{---(ii)}$$

$$C^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1 & 0+0 \\ 0+0 & 1+0 \end{bmatrix}$$

$$C^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C^2 = I_2 \qquad \text{---(iii)}$$

Hence,

From equation (i), (ii) and (iii),

$$A^2 = B^2 = C^2 = I_2$$

Algebra of Matrices Ex 5.3 Q7

Given, $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$

$$3A^2 - 2B + I$$

$$= 3\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} - 2\begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= 3\begin{bmatrix} 4-3 & -2-2 \\ 6+6 & -3+4 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= 3\begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -12 \\ 36 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3-0+1 & -12+8+0 \\ 36+2+0 & 3-14+1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -20 \\ 38 & -10 \end{bmatrix}$$

Hence,

$$3A^2 - 2B + I = \begin{bmatrix} 4 & -20 \\ 38 & -10 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q8

Given, $A=\begin{bmatrix}4 & 2\\ -1 & -1\end{bmatrix}$

$$(A-2I)(A-3I)$$

$$=\left(\begin{bmatrix}4 & 2\\ -1 & -1\end{bmatrix}-2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right)\left(\begin{bmatrix}4 & 2\\ -1 & -1\end{bmatrix}-3\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right)$$

$$=\left(\begin{bmatrix}4 & 2\\ -1 & -1\end{bmatrix}-\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}\right)\left(\begin{bmatrix}4 & 2\\ -1 & -1\end{bmatrix}-\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}\right)$$

$$=\left(\begin{bmatrix}4-2 & 2-0\\ -1-0 & 1-2\end{bmatrix}\right)\left(\begin{bmatrix}4-3 & 2-0\\ -1-0 & 1-3\end{bmatrix}\right)$$

$$=\begin{bmatrix}2 & 2\\ -1 & -1\end{bmatrix}\begin{bmatrix}1 & 2\\ -1 & -2\end{bmatrix}$$

$$=\begin{bmatrix}2-2 & 4-4\\ -1+1 & -2+2\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$=0$$

Hence,

$$(A-2I)(A-3I)=0$$

Algebra of Matrices Ex 5.3 Q9

Given, $A=\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$

$$A^2=\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1+0 & 1+1\\0+0 & 0+1\end{bmatrix}$$

$$A^2=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}$$

$$A^3=A^2\cdot A$$

$$=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1+0 & 1+2\\0+0 & 0+1\end{bmatrix}$$

$$A^3=\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}$$

Hence,

$$A^2=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}, A^3=\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q10

$$\text{Given, } A=\begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$$

$$A^2 = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}\begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$$

$$= \begin{bmatrix} a^2b^2 - a^2b^2 & ab^3 - ab^3 \\ -a^3b + a^3b & -a^2b^2 + a^2b^2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

Hence,

$$A^2 = 0$$

Algebra of Matrices Ex 5.3 Q11

$$\text{Given,} \qquad A = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$A^2 = A.A$$

$$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 2\theta - \sin^2 2\theta & \cos 2\theta \sin^2 + \cos 2\theta \sin^2 \theta \\ -\cos 2\theta \sin^2 \theta - \sin^2 \theta \cos^2 \theta & -\sin^2 2\theta + \cos^2 2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 4\theta & 2 \sin^2 \theta \cos^2 \theta \\ -2 \sin^2 \cos 2\theta & \cos 4\theta \end{bmatrix}$$

$$\{\text{since } \cos^2 \theta - \sin^2 \theta = \cos 2\theta\}$$

$$= \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$$

$$\{\text{since } \sin^2 \theta = 2 \sin \theta \cos \theta\}$$

Hence,

$$A^2 = \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q12

Given, $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}, B = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$

$$AB = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}\begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -2-3+5 & 6+9-15 & 10+15+25 \\ 1+4-5 & -3-12+15 & -5-20+25 \\ -1-3+4 & 3+9-12 & 5+15-20 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$AB = O_{3\times 3} \qquad ---(i)$$

$$BA = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -2-3+5 & 3+12-15 & 5+15-20 \\ 2+3-5 & -3-12+15 & -5-15+20 \\ -2-3+5 & 3+12-15 & 5+15-20 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$BA = O_{3\times 3} \qquad ---(ii)$$

From equation (i) and (ii),

$$AB = BA = O_{3\times 3}$$

Algebra of Matrices Ex 5.3 Q13

Given, $A = \begin{bmatrix} 0 & c & -b \\ -c & o & a \\ b & -a & 0 \end{bmatrix}, B = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$

$$AB = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix} \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 + abc - abc & 0 + b^2c - b^2c & 0 + bc^2 - bc^2 \\ -a^2c + 0 + a^2c & -abc + 0 + abc & -ac^2 + 0 + ac^2 \\ a^2b - a^2b + 0 & ab^2 - ab^2 + 0 & abc - abc + 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$AB = O_{3\times 3} \qquad \text{---(ii)}$$

From equation (i) and (ii),

$$AB = BA = O_{3\times 3}$$

Algebra of Matrices Ex 5.3 Q14

$$\text{Given, } A=\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}, B=\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$AB=\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$=\begin{bmatrix} 4+3-5 & -4-9+10 & -8-12+15 \\ -2-4+5 & 2+12-10 & 4+16-15 \\ 2+3-4 & -2-9+18 & -4-12+12 \end{bmatrix}$$

$$=\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

$$AB=A$$

$$BA=\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

$$=\begin{bmatrix} 4+2-4 & -6-8+12 & -10-10+16 \\ -2-3+4 & 3+12-12 & 5+15-16 \\ 2+2-3 & -3-8+9 & -5-10+12 \end{bmatrix}$$

$$=\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$BA=B$$

Algebra of Matrices Ex 5.3 Q15

Given, $A = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$

$$A^2 = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix}\begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1+3-5 & -1-3-5 & 1+3-5 \\ -3-9+15 & 3+9+15 & -3-9+15 \\ -5+15+25 & 5-15+25 & -5+15+25 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & -9 & -1 \\ 3 & 27 & 3 \\ 35 & 15 & 35 \end{bmatrix} \quad ---(\text{i})$$

$$B^2 = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}\begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0+4-3 & 0-12+12 & 0-12+12 \\ 0-3+3 & 4+9-12 & 3+9-12 \\ 0+4-4 & -4-12+16 & -3-12+16 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ---(\text{ii})$$

Subtracting equation (ii) from equation (i),

$$A^2 - B^2 = \begin{bmatrix} -1 & -9 & -1 \\ 3 & 27 & 3 \\ 35 & 15 & 35 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1-1 & -9-0 & -1-0 \\ 3-0 & 27-1 & 3-0 \\ 35-0 & 15-0 & 35-1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -9 & -1 \\ 3 & 26 & 3 \\ 35 & 15 & 34 \end{bmatrix}$$

Hence,

$$A^2 - B^2 = \begin{bmatrix} -2 & -9 & -1 \\ 3 & 26 & 3 \\ 35 & 15 & 34 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q16(i)

Given, $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix}$ and

$$C = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$(AB)C = \left(\begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix} \right) \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2+0 & 0+4+0 \\ -1+0+0 & +0+0+3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 4 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -4 \\ -1 & -3 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -5 \\ -4 \end{bmatrix} \qquad ---(i)$$

$$A(BC) = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1+0 \\ -1-2 \\ 0-3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-6+0 \\ -1+0-3 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} -5 \end{bmatrix} \qquad ---(ii)$$

$[-4]$

From equation (i) and (ii) we get,

$$(AB)C = A(BC)$$

(ii) Given,

$$A=\begin{bmatrix}4&2&3\\1&1&2\\3&0&1\end{bmatrix}, B=\begin{bmatrix}1&-1&1\\0&1&2\\2&-1&1\end{bmatrix}, C=\begin{bmatrix}1&2&-1\\3&0&1\\0&0&1\end{bmatrix}$$

$$(AB)C=\left(\begin{bmatrix}4&2&3\\1&1&2\\3&0&1\end{bmatrix}\begin{bmatrix}1&-1&1\\0&1&2\\2&-1&1\end{bmatrix}\right)\begin{bmatrix}1&2&-1\\3&0&1\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}4+0+6&-4+2-3&4+4+3\\1+0+4&-1+1-2&1+2+2\\3+0+2&-3+0-1&3+0+1\end{bmatrix}\begin{bmatrix}1&2&-1\\3&0&1\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}10&-5&11\\5&-2&5\\5&-4&4\end{bmatrix}\begin{bmatrix}1&2&-1\\3&0&1\\8&0&1\end{bmatrix}$$

$$=\begin{bmatrix}10-15+0&20+0+0&-10+5+11\\5-6+0&10+0+0&-5-2+5\\5-12+0&10+0+0&-5-4+4\end{bmatrix}$$

$$(AB)C=\begin{bmatrix}-5&20&-4\\-1&10&-2\\-7&10&-5\end{bmatrix} \qquad ---(i)$$

$$A(BC)=\begin{bmatrix}4&2&3\\1&1&2\\3&0&1\end{bmatrix}\left(\begin{bmatrix}1&-1&1\\0&1&2\\2&-1&1\end{bmatrix}\begin{bmatrix}1&2&-1\\3&0&1\\0&0&1\end{bmatrix}\right)$$

$$=\begin{bmatrix}4&2&3\\1&1&2\\3&0&1\end{bmatrix}\begin{bmatrix}1-3+0&2+0+0&-1-1+1\\0+3+0&0+0+0&0+1+2\\2-3+0&4+0+0&-2-1+1\end{bmatrix}$$

$$=\begin{bmatrix}4&2&3\\1&1&2\\3&0&1\end{bmatrix}\begin{bmatrix}-2&2&-1\\3&0&3\\-1&4&-2\end{bmatrix}$$

$$=\begin{bmatrix}-8+6-3&8+0+12&-4+6-6\\-2+3-2&2+0+8&-1+3-4\\-6+0-1&6+0+4&-3+0-2\end{bmatrix}$$

$$A(BC)=\begin{bmatrix}-5&20&-4\\-1&10&-2\\-7&10&-5\end{bmatrix} \qquad --(ii)$$

From equation (i) and (ii),

$$(AB)C=A(BC)$$

Algebra of Matrices Ex 5.3 Q17(i)

Given, $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$

$$A(B+C) = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\left(\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -1+0 & 0+1 \\ 2+1 & 1-1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1-3 & 1+0 \\ 0+6 & 0+0 \end{bmatrix}$$

$$A(B+C) = \begin{bmatrix} -4 & 1 \\ 6 & 0 \end{bmatrix} \qquad ---(i)$$

$$AB = AC = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1-2 & 0-1 \\ 0+4 & 0+2 \end{bmatrix} + \begin{bmatrix} 0+-1 & 1+1 \\ 0+2 & 0-2 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -3-1 & -1+2 \\ 4+2 & 2-2 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} -4 & 1 \\ 6 & 0 \end{bmatrix} \qquad ---(ii)$$

Using equation (i) and (ii),

$$A(B+C) = AB + AC$$

Algebra of Matrices Ex 5.3 Q17(ii)

$$\text{Given,} \qquad A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$A\left(B + C\right) = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}\left(\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 0+1 & 1-1 \\ 1+0 & 1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 0+2 \\ 1+1 & 0+2 \\ -1+2 & 0+4 \end{bmatrix}$$

$$A\left(B + C\right) = \begin{bmatrix} 1 & -2 \\ 2 & 2 \\ 1 & 4 \end{bmatrix} \qquad \text{---(i)}$$

$$AB + AC = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1 & 2-1 \\ 0+1 & 1+1 \\ 0+2 & -1+2 \end{bmatrix} + \begin{bmatrix} 2+0 & -2-1 \\ 1+0 & -1+1 \\ -1+0 & 1+2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 \\ 1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1+2 & 1-3 \\ 1+1 & 2+0 \\ 2-1 & 1+3 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} 1 & -2 \\ 2 & 2 \\ 1 & 4 \end{bmatrix} \qquad \text{---(ii)}$$

From equation (i) and (ii),

$$A\left(B + C\right) = AB + AC$$

Algebra of Matrices Ex 5.3 Q18

Given,

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$A(B-C) = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}\left[\begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\right]$$

$$= \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0-1 & 5-5 & -4-2 \\ -2+1 & 1-1 & 3-0 \\ -1-0 & 0+1 & 2-1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 & -6 \\ -1 & 0 & 3 \\ -1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1+0+2 & 0+0-2 & -6+0-2 \\ -3+1+0 & 0+0+0 & -18-3+0 \\ 2-1-1 & 0+0+1 & 12+3+1 \end{bmatrix}$$

$$A(B-C) = \begin{bmatrix} 1 & -2 & -8 \\ -2 & 0 & -21 \\ 0 & 1 & 16 \end{bmatrix} \qquad ---(i)$$

$$AB - AC = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+2 & 5+0+0 & -4+0-4 \\ 0+2+0 & 15-1+0 & -12-3+0 \\ 0-2-1 & -10+1+0 & 8+3+2 \end{bmatrix} - \begin{bmatrix} 1+0+0 & 5+0+2 & 2+0-2 \\ 3+1+0 & 15-1+0 & 6+0+0 \\ 0-2-1 & -10+1+1 & -4+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 5 & -8 \\ 2 & 14 & -15 \\ -3 & -9 & 13 \end{bmatrix} - \begin{bmatrix} 1 & 7 & 0 \\ 4 & 14 & 6 \\ -3 & -10 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 5-7 & -8-0 \\ 2-4 & 14-14 & -14-6 \\ -3+3 & -9+10 & 13+3 \end{bmatrix}$$

$$AB - AC = \begin{bmatrix} 1 & -2 & -8 \\ -2 & 0 & -21 \\ 0 & 1 & 16 \end{bmatrix} \qquad ---(ii)$$

From equation (i) and (ii),

$$A(B-C) = AB - AC$$

Algebra of Matrices Ex 5.3 Q19

Given,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 2 \\ 0 & 3 & 2 \\ 4 & 0 & 4 \end{bmatrix} 0 \begin{bmatrix} 2 & -1 \\ -3 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 & 2 & -2 \\ 3 & -3 & 4 & -4 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0-3+0 & 0+2+0 \\ 4+0+8 & -2+0+6 \\ 0-9+8 & 0+6+6 \\ 8+0+16 & -4+0+12 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 & 2 & -2 \\ 3 & -3 & 4 & -4 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} -3 & 2 \\ 12 & 4 \\ -1 & 12 \\ 24 & 8 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 & 2 & -2 \\ 3 & -3 & 4 & -4 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0+6 & -3-6 & 3+8 & -6-8 & 6+0 \\ 0+12 & 12-12 & -12+16 & 24-16 & -24+0 \\ 0+36 & -1-36 & 1+48 & -2-48 & 2+0 \\ 0+24 & 24-24 & -24+34 & 48-32 & -48+0 \end{bmatrix}$$

$$A = \begin{bmatrix} 6 & -9 & 11 & -14 & 6 \\ 12 & 0 & 4 & 8 & -24 \\ 36 & -37 & 49 & -50 & 2 \\ 24 & 0 & 8 & 16 & -48 \end{bmatrix}$$

Here, $a_{43} = 8, a_{22} = 0$

Algebra of Matrices Ex 5.3 Q20

Given,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$$

$$A^2 = A \times A$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+0 & 0+0+0 & 0+1+0 \\ 0+0+p & 0+0+q & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \end{bmatrix}$$

$$A^3 = A^2 \times A$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ p & q & r \\ pr & p+qr & q+r^2 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+p & 0+0+q & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \\ 0+0+pq+pr^2 & pr+0+q^2+qr^2 & 0+p+qr+qr+r^2 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} p & q & r \\ pr & p+qr & q+r^2 \\ pq+pr^2 & pr+q^2+qr^2 & p+2qr+r^2 \end{bmatrix} \qquad \text{---(i)}$$

$$pI + qA + rA^2$$

$$= p\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + q\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix} + r\begin{bmatrix} 0 & 0 & 1 \\ p & q & r \\ pr & p+qr & q+r^2 \end{bmatrix}$$

$$= \begin{bmatrix} p+0+0 & 0+q+0 & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \\ 0+pq+pr^2 & 0+q^2+pr+qr^2 & p+qr+qr+r^2 \end{bmatrix}$$

$$pI + qA + rA^2$$

$$= \begin{bmatrix} p & q & r \\ pr & p+qr & q+r^2 \\ pq+pr^2 & pr+q^2+qr^2 & p+2qr+r^2 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q21

Given, $\quad W$ is a complex cube root of unity,

$$\left(\begin{bmatrix}1 & w & w^2\\ w & w^2 & 1\\ w^2 & 1 & w\end{bmatrix}+\begin{bmatrix}w & w^2 & 1\\ w^2 & 1 & w\\ w & w^2 & 1\end{bmatrix}\right)\begin{bmatrix}1\\ w\\ w^2\end{bmatrix}$$

$$=\begin{bmatrix}1+w & w+w^2 & w^2+1\\ w+w^2 & w^2+1 & 1+w\\ w^2+w & 1+w^2 & w+1\end{bmatrix}\begin{bmatrix}1\\ w\\ w^2\end{bmatrix}$$

$$=\begin{bmatrix}-w^2 & -1 & -w\\ -1 & -w & -w^2\\ -1 & -w & -w^2\end{bmatrix}\begin{bmatrix}1\\ w\\ w^2\end{bmatrix} \qquad \left\{\begin{array}{l}Since\ 1+w+w^2=0\\ and\ w^3\ =1\end{array}\right\} \qquad ---(i)$$

$$=\begin{bmatrix}-w^2-w-w^3\\ -1-w^2-w^4\\ -1-w^2-w^2\end{bmatrix}$$

$$=\begin{bmatrix}-w\left(1+w+w^2\right)\\ -1-w^2-w^3w\\ -1-w^2-w^3w\end{bmatrix}$$

$$=\begin{bmatrix}-w.0\\ -1-w^2-w\\ -1-w^2-w\end{bmatrix} \qquad \{\text{using reason (i)}\}$$

$$=\begin{bmatrix}0\\ -\left(1+w+w^2\right)\\ -\left(1+w+w^2\right)\end{bmatrix}$$

$$=\begin{bmatrix}0\\ -(0)\\ -(0)\end{bmatrix}$$

$$=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q22

Given, $\quad A=\begin{bmatrix}2 & -3 & -5\\ -1 & 4 & 5\\ 1 & -3 & -4\end{bmatrix}$

$$A^2 = A, A$$

$$=\begin{bmatrix}2 & -3 & -5\\ -1 & 4 & 5\\ 1 & -3 & -4\end{bmatrix}\begin{bmatrix}2 & -3 & -5\\ -1 & 4 & 5\\ 1 & -3 & -4\end{bmatrix}$$

$$=\begin{bmatrix}4+3-5 & -6-12+15 & -10-15+20\\ -2-4+5 & 3+16-15 & 5+20-20\\ 2+3-4 & -3-12+12 & -5-15+16\end{bmatrix}$$

$$=\begin{bmatrix}2 & -3 & -5\\ -1 & 4 & 5\\ 1 & -3 & -4\end{bmatrix}$$

$$= A$$

Hence,

$$A^2 = A$$

Algebra of Matrices Ex 5.3 Q23

Given, $A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$

$$A^2 = A.A$$

$$= \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix} \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 16-3-12 & -4+0+4 & -16+4+12 \\ 12+0-12 & -3+0+4 & -12+0+12 \\ 12-3-9 & -3+0+3 & -12+4+9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= I_3$$

Hence,

$$A^2 = I_3$$

Algebra of Matrices Ex 5.3 Q24(i)

Given,

$$\begin{bmatrix}1 & 1 & x\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\0 & 2 & 1\\2 & 1 & 0\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix}1+0+2x & 0+2+x & 2+1+0\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix}2x+1 & 2+x & 3\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix}2x+1+2+x+3\end{bmatrix} = 0$$

$$\Rightarrow \quad 3x + 6 = 0$$

$$\Rightarrow \quad x = -\frac{6}{3}$$

$$\Rightarrow \quad x = -2$$

Algebra of Matrices Ex 5.3 Q24(ii)

Given that $\begin{bmatrix}2 & 3\\5 & 7\end{bmatrix}\begin{bmatrix}1 & -3\\-2 & 4\end{bmatrix} = \begin{bmatrix}-4 & 6\\-9 & x\end{bmatrix}$

By multiplication of matrices, we have,

$$\begin{bmatrix}2\times1+3\times(-2) & 2\times(-3)+3\times4\\5\times1+7\times(-2) & 5\times(-3)+7\times4\end{bmatrix} = \begin{bmatrix}-4 & 6\\-9 & x\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}-4 & 6\\-9 & 13\end{bmatrix} = \begin{bmatrix}-4 & 6\\-9 & x\end{bmatrix}$$

$$\Rightarrow x = 13$$

Algebra of Matrices Ex 5.3 Q25

Given,

$$\begin{bmatrix} x & 4 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix} 2x+4+0 & x+0+2 & 2x+8-4 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix} 2x+4 & x+2 & 2x+4 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

$$\Rightarrow \quad \left[(2x+4)x + 4(x+2) - 1(2x+4)\right] = 0$$

$$\Rightarrow \quad 2x^2 + 4x + 4x + 8 - 2x - 4 = 0$$

$$\Rightarrow \quad 2x + 6x + 4 = 0$$

$$\Rightarrow \quad 2x^2 + 2x + 4x + 4 = 0$$

$$\Rightarrow \quad 2x(x+1) + 4(x+1) = 0$$

$$\Rightarrow \quad (x+1)(2x+4) = 0$$

$$\Rightarrow \quad x+1 = 0 \text{ or } 2x+4 = 0$$

$$\Rightarrow \quad x = -1 \text{ or } x = -2$$

Hence, $x = -1$ or -2

Algebra of Matrices Ex 5.3 Q26

Given,

$$\begin{bmatrix}1 & -1 & x\end{bmatrix}\begin{bmatrix}0 & 1 & -1\\2 & 1 & 3\\1 & 1 & 1\end{bmatrix}\begin{bmatrix}0\\1\\1\end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix}0-2+x & 1-1+x & -1-3+x\end{bmatrix}\begin{bmatrix}0\\1\\1\end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix}x-2 & x & x-4\end{bmatrix}\begin{bmatrix}0\\1\\1\end{bmatrix} = 0$$

$$\Rightarrow \quad \left[0(x-2)+x.1+1.(x-4)\right] = 0$$

$$\Rightarrow \quad 0+x+x-4=0$$

$$\Rightarrow \quad 2x-4=0$$

$$\Rightarrow \quad x=2$$

Hence,

$$x = 2$$

Algebra of Matrices Ex 5.3 Q27

Given, $A = \begin{bmatrix}3 & -2\\4 & -2\end{bmatrix}, I = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$$A^2 - A + 2I$$

$$= \begin{bmatrix}3 & -2\\4 & -2\end{bmatrix}\begin{bmatrix}3 & -2\\4 & -2\end{bmatrix} - \begin{bmatrix}3 & -2\\4 & -2\end{bmatrix} + 2\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}9-8 & -6+4\\12-8 & -8+4\end{bmatrix} - \begin{bmatrix}3 & -2\\4 & -2\end{bmatrix} + \begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}$$

$$= \begin{bmatrix}1 & -2\\4 & -4\end{bmatrix} - \begin{bmatrix}3 & -2\\4 & -2\end{bmatrix} + \begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}$$

$$= \begin{bmatrix}1-3+2 & -2+2+0\\4-4+0 & -4+2+2\end{bmatrix}$$

$$= \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

$$= 0$$

Hence,

$$A^2 - A + 2I = 0$$

Algebra of Matrices Ex 5.3 Q28

Given, $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

And

$A^2 = 5A + \lambda I$

$$\Rightarrow \quad \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + \lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 9-1 & 3+2 \\ -3+2 & -1+4 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 15+\lambda & 5 \\ -5 & 10+\lambda \end{bmatrix}$$

Since, Corresponding entries of equal matrices are equal, So

$8 = 15 + \lambda$

$\lambda = 8 - 15$

$\lambda = -7$

Algebra of Matrices Ex 5.3 Q29

Given, $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$A^2 - 5A + 7I_2$

$$= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$= 0$

Hence, $A^2 - 5A + 7I_2 = 0$

Algebra of Matrices Ex 5.3 Q30

Given, $A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$

$$A^2 - 2A + 3I_2$$

$$= \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} - 2\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3 & 6+0 \\ -2+0 & -3+0 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ -2 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ -2 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-4+3 & 6-6+0 \\ -2+2+0 & -3+0+3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

Hence,

$$A^2 - 2A + 3I_2 = 0$$

Algebra of Matrices Ex 5.3 Q31

Given, $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$$

$$A^3 = A^2.A$$

$$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 14+12 & 21+24 \\ 8+7 & 12+14 \end{bmatrix}$$

$$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix}$$

Hence, $A^3 - 4A^2 + A$

$$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - 4\begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 26-28+2 & 45-48+3 \\ 15-16+1 & 26-28+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

So, $\quad A^3 - 4A^2 + A = 0$

Algebra of Matrices Ex 5.3 Q32

Given, $A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$

$$A^2 - 12A - I$$

$$= \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix} - 12\begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 25+36 & 15+21 \\ 60+84 & 36+49 \end{bmatrix} - \begin{bmatrix} 60 & 36 \\ 144 & 84 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 61 & 36 \\ 144 & 85 \end{bmatrix} - \begin{bmatrix} 60 & 36 \\ 144 & 84 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 61-60-1 & 36-36-0 \\ 144-144-0 & 85-84-1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

Since $A^2 - 12A - I = 0$

So,

A is a root of the equation $A^2 - 12A - I = 0$

Algebra of Matrices Ex 5.3 Q33

Given, $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$

$$A^2 = 5A - 14I$$

$$= \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} - 5\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} - 14\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9+20 & -15-10 \\ -12-8 & 20+4 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 29-15-14 & -25+25-0 \\ -20+20-0 & 24-10-14 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

So,

$$A^2 - 5A - 14I = 0$$

Algebra of Matrices Ex 5.3 Q34

Given, $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$$

$$A^3 = A^2.A$$

$$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 14+12 & 21+24 \\ 8+7 & 12+14 \end{bmatrix}$$

$$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix}$$

Hence, $A^3 - 4A^2 + A$

$$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - 4\begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 26-28+2 & 45-48+3 \\ 15-16+1 & 26-28+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

So, $\quad A^3 - 4A^2 + A = 0$

It is given that $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$

$$\therefore A^2 = A\cdot A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}3(3)+1(-1) & 3(1)+1(2)\\ -1(3)+2(-1) & -1(1)+2(2)\end{bmatrix}$$

$$=\begin{bmatrix}9-1 & 3+2\\ -3-2 & -1+4\end{bmatrix}=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}$$

$\therefore$ L.H.S. $= A^2-5A+7I$

$$=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}-5\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}+7\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}-\begin{bmatrix}15 & 5\\ -5 & 10\end{bmatrix}+\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$$

$$=\begin{bmatrix}-7 & 0\\ 0 & -7\end{bmatrix}+\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$= O =$ R.H.S.

$$\therefore A^2-5A+7I=O$$

$Since\ A^2 - 5A + 7I = O,\ we\ have$

$A^2 = 5A - 7I$

$Therefore,\ A^4 = A^2 \times A^2 = (5A - 7I)(5A - 7I)$

$\Rightarrow A^4 = 25A^2 - 35AI - 35IA + 49I$

$\Rightarrow A^4 = 25A^2 - 70A + 49I$

$\Rightarrow A^4 = 25(5A - 7I) - 70A + 49I$

$\Rightarrow A^4 = 125A - 175I - 70A + 49I$

$\Rightarrow A^4 = 55A - 126I$

$$\Rightarrow A^4 = 55\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 126\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 165 & 55 \\ -55 & 110 \end{bmatrix} - \begin{bmatrix} 126 & 0 \\ 0 & 126 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 165 - 126 & 55 - 0 \\ -55 - 0 & 110 - 126 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 39 & 55 \\ -55 & -16 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q35

$$A^2 = A \cdot A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 3(3)+(-2)(4) & 3(-2)+(-2)(-2) \\ 4(3)+(-2)(4) & 4(-2)+(-2)(-2) \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

Now $A^2 = kA - 2I$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$$

Comparing the corresponding elements, we have:

$3k - 2 = 1$

$\Rightarrow 3k = 3$

$\Rightarrow k = 1$

Thus, the value of k is 1.

Algebra of Matrices Ex 5.3 Q36

Here,

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$$

And

$$A^2 - 8A + kI = 0$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} - 8\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} + k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 1+0 & 0+0 \\ -1-7 & 0+49 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1-8+k & 0+0+0 \\ -8+8+0 & 49-56+k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -7+k & 0 \\ 0 & -7+k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Since,

corresponding entries of equal matrices are equal, so

$-7 + k = 0$

$k = 7$

Algebra of Matrices Ex 5.3 Q37

Given,

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \text{ and } f(x) = x^2 - 2x - 3$$

$f(A) = A^2 - 2A - 3I$

$$= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4 & 2+2 \\ 2+2 & 4+1 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5-2-3 & 4-4-0 \\ 4-4-0 & 5-2-3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

So,

$$f(A) = 0$$

Algebra of Matrices Ex 5.3 Q38

Given, $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Given,

$$A^2 = \lambda A + \mu I$$

$$\Rightarrow \quad \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \lambda \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \mu \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix} = \begin{bmatrix} 2\lambda & 3\lambda \\ \lambda & 2\lambda \end{bmatrix} + \begin{bmatrix} \mu & 0 \\ 0 & \mu \end{bmatrix}$$

$$\begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 2\lambda + \mu & 3\lambda \\ \lambda & 2\lambda + \mu \end{bmatrix}$$

Since corresponding entries of equal matrices are equal, so

$2\lambda + \mu = 7$ ---(i)

$\lambda = 4$ ---(ii)

Put λ from equation (ii) in equation (i),

$2(4) + \mu = 7$

$\mu = 7 - 8$

$\mu = -1$

Hence. $\lambda = 4, \mu = -1$

Given, $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^3 - 4A^2 + A$

$$= \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} - 4\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 14+12 & 21+24 \\ 8+7 & 12+14 \end{bmatrix} - \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 26-7+2 & 45-12+3 \\ 15-4+1 & 26-7+2 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 36 \\ 12 & 21 \end{bmatrix}$$

Hence, $A^3 - 4A^2 + A = \begin{bmatrix} 21 & 36 \\ 12 & 21 \end{bmatrix}$

Algebra of Matrices Ex 5.3 Q39

Given,

$$\begin{bmatrix} 2 & 0 & 7 \\ 0 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} -x & 14x & 7x \\ 0 & 1 & 0 \\ x & -4x & -2x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -2x+0+7x & 28x+0-28x & 14x+0-14x \\ 0+0+0 & 0+1+0 & 0+0+0 \\ -x+0+x & 14x-2-4x & 7x+0-2x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5x & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 10x-2 & 5x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$5x = 1$ and $10x - 2 = 0$

$\Rightarrow$ $x = \frac{1}{5}$ and $x = \frac{1}{5}$

Hence, $x = \frac{1}{5}$

Algebra of Matrices Ex 5.3 Q40(i)

Here,

$$\begin{bmatrix} x & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} x \\ 5 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} x-2 & 0-3 \end{bmatrix}\begin{bmatrix} x \\ 5 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} (x-2)x - 15 \end{bmatrix} = 0$$

$$\Rightarrow x^2 - 2x - 15 = 0$$

$$\Rightarrow x^2 - 5x + 3x - 15 = 0$$

$$\Rightarrow x(x-5) + 3(x-5) = 0$$

$$\Rightarrow (x-5)(x+3) = 0$$

$$\Rightarrow x - 5 = 0 \quad \text{or} \quad x + 3 = 0$$

$$\Rightarrow x = 5 \quad \text{or} \quad x = -3$$

So,

$$x = 5 \text{ or } -3$$

Algebra of Matrices Ex 5.3 Q40(ii)

We have:

$$\begin{bmatrix} x & -5 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} x+0-2 & 0-10+0 & 2x-5-3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} x-2 & -10 & 2x-8 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow \left[x(x-2)-40+2x-8\right] = O$$

$$\Rightarrow \left[x^2-2x-40+2x-8\right] = [0]$$

$$\Rightarrow \left[x^2-48\right] = [0]$$

$$\therefore x^2-48=0$$

$$\Rightarrow x^2 = 48$$

$$\Rightarrow x = \pm 4\sqrt{3}$$

Algebra of Matrices Ex 5.3 Q41

Given, $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix}$

$$A^2 - 4A + 3I_3$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix} - 4\begin{bmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+6+0 & 2-8+0 & 0+10+0 \\ 3-12+0 & 6+16-5 & 0-20+15 \\ 0-3+0 & 0+4-3 & 0-5+9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 0 \\ 12 & -16 & 20 \\ 0 & -4 & 12 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -6 & 10 \\ -9 & 17 & -5 \\ -3 & 1 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 0 \\ 12 & -16 & 20 \\ 0 & -4 & 12 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 7-4+3 & -6-8+0 & 10-0+0 \\ -9-12+0 & 17+16+3 & -5-20+0 \\ -3-0+0 & 1+4+0 & 4-12+3 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -14 & 10 \\ -21 & 36 & -25 \\ -3 & 5 & -5 \end{bmatrix}$$

Hence,

$$A^2 - 4A + 3I_3 = \begin{bmatrix} 6 & -14 & 10 \\ -21 & 36 & -25 \\ -3 & 5 & -5 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q42

Given, $A = \begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 0 \\ 0 & 2 & 3 \end{bmatrix}$

And $f(x) = x^2 - 2x$

$\Rightarrow \quad f(A) = A^2 - 2A$

$$\Rightarrow \quad f(A) = \begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 0 \\ 0 & 2 & 3 \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 0 \\ 0 & 2 & 3 \end{bmatrix} - 2\begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 0 \\ 0 & 2 & 3 \end{bmatrix}$$

$$\Rightarrow \quad f(A) = \begin{bmatrix} 0+4+0 & 0+5+4 & 0+0+6 \\ 0+20+0 & 4+25+0 & 8+0+0 \\ 0+8+0 & 0+10+6 & 0+0+9 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 4 \\ 8 & 10 & 0 \\ 0 & 4 & 6 \end{bmatrix}$$

$$\Rightarrow \quad f(A) = \begin{bmatrix} 4 & 9 & 6 \\ 20 & 29 & 8 \\ 8 & 16 & 9 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 4 \\ 8 & 10 & 0 \\ 0 & 4 & 6 \end{bmatrix}$$

$$\Rightarrow \quad f(A) = \begin{bmatrix} 4-0 & 9-2 & 6-4 \\ 20-8 & 29-10 & 8-0 \\ 8-0 & 16-4 & 9-6 \end{bmatrix}$$

$$\Rightarrow \quad f(A) = \begin{bmatrix} 4 & 7 & 2 \\ 12 & 19 & 8 \\ 8 & 12 & 3 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q43

Given,

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$$

And $\quad f(x) = x^3 + 4x^2 - x$

$\Rightarrow \quad f(x) = A^3 + 4A^2 - A \qquad$ ---(i)

$$A^2 = A \times A$$

$$= \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+2+2 & 0-3-2 & 0+0+0 \\ 0-6+0 & 2+9+0 & 4+0+0 \\ 0-2+0 & 1+3+0 & 0+0+0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4 & -5 & 0 \\ -6 & 11 & 4 \\ -2 & 4 & 2 \end{bmatrix}$$

$$A^3 = A^2 \times A$$

$$= \begin{bmatrix} 4 & -5 & 0 \\ -6 & 11 & 4 \\ -2 & 4 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0-10+0 & 4+15+0 & 8+0+0 \\ 0+22+4 & -6-33-4 & -12+0+0 \\ 0+8+2 & -2-12-2 & -4+0+0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} -10 & 19 & 8 \\ 26 & -43 & -12 \\ 10 & -16 & -4 \end{bmatrix}$$

Put the value of A, A^2, A^3 in equation (i)

$$f(A) = A^3 + 4A^2 - A$$

$$= \begin{bmatrix} -10 & 19 & 8 \\ 26 & -43 & -12 \\ 10 & -16 & -4 \end{bmatrix} + 4\begin{bmatrix} 4 & -15 & 0 \\ -6 & 11 & 4 \\ -2 & 4 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -10+16-0 & 19-20-1 & 8+0-2 \\ 26-24-2 & -43+44+3 & -12+16+0 \\ 10-8-1 & -16+16+1 & -4+8-0 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -2 & 6 \\ 0 & 4 & 4 \\ 1 & 1 & 4 \end{bmatrix}$$

Hence,

$$f(A) = \begin{bmatrix} 6 & -2 & 6 \\ 0 & 4 & 4 \\ 1 & 1 & 4 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q44

$$Given\ that,\ A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} and\ f(x) = x^3 - 6x^2 + 7x + 2$$

$$Therefore,\ f(A) = A^3 - 6A^2 + 7A + 2I_3$$

$$First\ find\ A^2:$$

$$A^2 = A \times A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

$$Now,\ Let\ us\ find\ A^3:$$

$$A^3 = A^2 \times A = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

$$Thus,$$

$$f(A) = A^3 - 6A^2 + 7A + 2I_3$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6\begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 21 - 30 + 7 + 2 & 0 & 34 - 48 + 14 + 0 \\ 12 - 12 + 0 & 8 - 24 + 14 + 2 & 23 - 30 + 7 + 0 \\ 34 - 48 + 14 + 0 & 0 & 55 - 78 + 21 + 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

$$Thus,\ A\ is\ a\ root\ of\ the\ polynomial.$$

Algebra of Matrices Ex 5.3 Q45

Given,

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$A^2 - 4A - 5I$

$$= \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 9-4-5 & 8-8-0 & 8-8-0 \\ 8-8-0 & 9-4-5 & 8-8-0 \\ 8-8-0 & 8-8-0 & 9-4-5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= 0$$

Hence,

$$A^2 - 4A - 5I = 0$$

Algebra of Matrices Ex 5.3 Q46

Given,

$$A = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$A^2 - 7A + 10I_3$

$$= \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} - 7\begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} + 10\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9+2+0 & 6+8+0 & 0+0+0 \\ 3+4+0 & 2+16+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+25 \end{bmatrix} - \begin{bmatrix} 21 & 14 & 0 \\ 7 & 28 & 0 \\ 0 & 0 & 35 \end{bmatrix} + \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 14 & 0 \\ 7 & 18 & 0 \\ 0 & 0 & 25 \end{bmatrix} - \begin{bmatrix} 21 & 14 & 0 \\ 7 & 28 & 0 \\ 0 & 0 & 35 \end{bmatrix} + \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 11-21+10 & 14-14+0 & 0-0+0 \\ 7-7+0 & 18-28+10 & 0-0+0 \\ 0-0+0 & 0-0+0 & 25-35+10 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$= 0$

Hence,

$$A^2 - 7A + 10I_3 = 0$$

Algebra of Matrices Ex 5.3 Q47

Given,

$$\begin{bmatrix}5 & -7\\ -2 & 3\end{bmatrix}\begin{bmatrix}x & y\\ z & u\end{bmatrix}=\begin{bmatrix}-16 & -6\\ 7 & 2\end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix}5x-7z & 5y-7u\\ -2x+3z & -2y+3u\end{bmatrix}=\begin{bmatrix}-16 & -6\\ 7 & 2\end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$$5x-7z=-16 \qquad ---(i)$$

$$-2x+3z=7 \qquad ---(ii)$$

$$5y-7u=-6 \qquad ---(iii)$$

$$-2y+3u=2 \qquad ---(iv)$$

Solving equation (i) and (ii)

$$10x-14z=-32$$

$$\underline{-10x+15z=35}$$

$$z=3$$

Put the value of z in equation (i)

$$5x-7(3)=-16$$

$$\Rightarrow \quad 5x=16+21$$

$$\Rightarrow \quad 5x=5$$

$$\Rightarrow \quad x=1$$

Solving equation (iii) and (iv)

$$10y-14u=-12$$

$$\underline{-10y+15u=10}$$

$$u=-2$$

Put the value of u in equation (iii)

$$5y-7u=-6$$

$$\Rightarrow \quad 5y-7(-2)=-6$$

$$\Rightarrow \quad 5y+14=-6$$

$$\Rightarrow \quad 5y=-20$$

$$\Rightarrow \quad y=-4$$

Algebra of Matrices Ex 5.3 Q48

Given,

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

Since, $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}_{2\times 2} \qquad A = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}_{2\times 3}$

$\Rightarrow \qquad A$ is a matrix of order 2×3

So,

$$\text{Let } A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$$\Rightarrow \qquad \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \qquad \begin{bmatrix} a+d & b+e & c+c \\ 0+d & 0+e & 0+f \end{bmatrix} = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$d = 1,\ e = 0,\ f = 1$

And $\quad a + d = 3$

$a + 1 = 3$

$a = 3 - 1$

$a = 2$

$b + e = 3$

$b + 0 = 3$

$b = 3$

And $\quad c + f = 5$

$c + 1 = 5$

$c = 4$

Hence,

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 1 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q48(ii)

It is given that:

$$A\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$$

The matrix given on the R.H.S. of the equation is a 2×3 matrix and the one given on the L.H.S. of the equation is a 2×3 matrix. Therefore, X has to be a 2×2 matrix.

Now, let $X=\begin{bmatrix}a & c\\ b & d\end{bmatrix}$

Therefore, we have:

$$\begin{bmatrix}a & c\\ b & d\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}a+4c & 2a+5c & 3a+6c\\ b+4d & 2b+5d & 3b+6d\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$$

Equating the corresponding elements of the two matrices, we have:

$a+4c=-7, \quad 2a+5c=-8, \quad 3a+6c=-9$

$b+4d=2, \quad 2b+5d=4, \quad 3b+6d=6$

Now, $a+4c=-7 \Rightarrow a=-7-4c$

$\therefore 2a+5c=-8 \Rightarrow -14-8c+5c=-8$

$\Rightarrow -3c=6$

$\Rightarrow c=-2$

$\therefore a=-7-4(-2)=-7+8=1$

Now, $b+4d=2 \Rightarrow b=2-4d$

$\therefore 2b+5d=4 \Rightarrow 4-8d+5d=4$

$\Rightarrow -3d=0$

$\Rightarrow d=0$

$\therefore b=2-4(0)=2$

Thus, $a=1,\ b=2,\ c=-2,\ d=0$

Hence, the required matrix X is $\begin{bmatrix}1 & -2\\ 2 & 0\end{bmatrix}$.

Algebra of Matrices Ex 5.3 Q48(iii)

We know that two matrices B and C are eligible for the product BC only when number of columns of B is equal to number of rows in C. So, from the given definition we can conclude that the order of matrix A is 1×3 i.e. we can assume $A=\begin{bmatrix}x_1 & x_2 & x_3\end{bmatrix}$.

Therefore,

$$\begin{bmatrix}4\\1\\3\end{bmatrix}_{3\times1}\begin{bmatrix}x_1 & x_2 & x_3\end{bmatrix}_{1\times3}=\begin{bmatrix}-4 & 8 & 4\\ -1 & 2 & 1\\ -3 & 6 & 3\end{bmatrix}_{3\times3},$$

$$\Rightarrow \begin{bmatrix}4\times(x_1) & 4\times(x_2) & 4\times(x_3)\\ 1\times(x_1) & 1\times(x_2) & 1\times(x_3)\\ 3\times(x_1) & 3\times(x_2) & 3\times(x_3)\end{bmatrix}_{3\times3}=\begin{bmatrix}-4 & 8 & 4\\ -1 & 2 & 1\\ -3 & 6 & 3\end{bmatrix}_{3\times3}$$

$$\Rightarrow \begin{bmatrix}4x_1 & 4x_2 & 4x_3\\ x_1 & x_2 & x_3\\ 3x_1 & 3x_2 & 3x_3\end{bmatrix}_{3\times3}=\begin{bmatrix}-4 & 8 & 4\\ -1 & 2 & 1\\ -3 & 6 & 3\end{bmatrix}_{3\times3}$$

$\Rightarrow \quad 4x_1=-4, \quad 4x_2=8, \quad 4x_3=4$

Solving $\quad x_1=-1,\ x_2=2,\ x_3=1$

So, matrix $A=\begin{bmatrix}-1 & 2 & 1\end{bmatrix}$.

Algebra of Matrices Ex 5.3 Q48(iv)

Using matrix multiplication,

Let, $A_1 = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}$, $A_2 = \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ and $A_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

Now, $A_1.A_2 = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} =$

$$= \begin{bmatrix} (2\times -1)+(1\times -1)+(3\times 0) & (2\times 0)+(1\times 1)+(3\times 1) & (2\times -1)+(1\times 0)+(3\times 1) \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 4 & 1 \end{bmatrix}$$

and $(A_1.A_2)A_3 = \begin{bmatrix} -3 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$$= \begin{bmatrix} (-3\times 1)+(4\times 0)+(1\times -1) \end{bmatrix}$$

$(A_1.A_2)A_3 = \begin{bmatrix} -4 \end{bmatrix} = A$

Therefore matrix $A = \begin{bmatrix} -4 \end{bmatrix}$

Note : The problem can also be solved by calculating (A_2A_3) first then pre multiplying it with A_1 as matrix multiplication is associative but one must not change the order of multiplication.

Algebra of Matrices Ex 5.3 Q49

Let, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Given,

$$A \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = 6 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} a+b & -2a+4b \\ c+d & -2c+4d \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$a + b = 6$ ---(i)

$-2a + 4b = 0$ ---(ii)

$c + d = 0$ ---(iii)

$-2c + 4d = 6$ ---(iv)

Solving equation (i) and (ii)

$$\begin{array}{l} 4a + 4b = 24 \\ -2a + 4b = 0 \\ \;(+)\quad\; (-) \\ \hline 6a \qquad = 24 \end{array}$$

$$\Rightarrow \quad a = \frac{24}{6}$$

$$a = 4$$

Put $a = 4$ in equation (i)

$$a + b = 6$$

$$4 + b = 6$$

$$b = 6 - 4$$

$$b = 2$$

Solving equation (iii) and (iv)

$$\begin{array}{r} 2c + 2d = 0 \\ \underline{-2c + 4d = 6} \\ 6d = 6 \end{array}$$

$$d = \frac{6}{6}$$

$$d = 1$$

Put $d = 1$ in equation (iii)

$$c + d = 0$$

$$c = -1$$

Hence,

$$A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q50

Given,

$$A = \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$$

$$A^2 = A \times A$$

$$= \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

$$A^4 = A^2 \times A^2$$

$$= 0 \times 0$$

$$= 0$$

$$A^{16} = A^4 \times A^4$$

$$= 0 \times 0$$

$$= 0$$

So,

A^{16} is a nill matrix

Algebra of Matrices Ex 5.3 Q51

Solving the LHS of the given equation we have ,

$$\Rightarrow \quad A + B = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 0 & -x + 1 \\ x + 1 & 0 \end{bmatrix}$$

$$(A + B)^2 = \begin{bmatrix} 0 & -x + 1 \\ x + 1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & -x + 1 \\ x + 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -x + 1 \\ x + 1 & 0 \end{bmatrix}$$

$$(A + B)^2 = \begin{bmatrix} (0 \times 0) + ((-x + 1) \times (x + 1)) & (0 \times (-x + 1)) + ((-x + 1) \times 0) \\ ((x + 1) \times 0) + (0 \times (x + 1)) & ((x + 1) \times (-x + 1)) + (0 \times 0) \end{bmatrix}$$

$$(A + B)^2 = \begin{bmatrix} 1 - x^2 & 0 \\ 0 & 1 - x^2 \end{bmatrix}.$$

Solving the RHS we get,

$$\Rightarrow \quad A^2 + B^2 = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}\begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 + B^2 = \begin{bmatrix} (0 \times 0) + ((-x) \times (x)) & (0 \times (-x)) + ((-x) \times 0) \\ ((x) \times 0) + (0 \times (x)) & ((x) \times (-x)) + (0 \times 0) \end{bmatrix} + \begin{bmatrix} (0 \times 0) + (1 \times 1) & (0 \times 1) + (1 \times 0) \\ (1 \times 0) + (0 \times 1) & (1 \times 1) + (0 \times 0) \end{bmatrix}$$

$$A^2 + B^2 = \begin{bmatrix} -x^2 & 0 \\ 0 & -x^2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^2 + B^2 = \begin{bmatrix} 1 - x^2 & 0 \\ 0 & 1 - x^2 \end{bmatrix}$$

Subsituting the value of $x^2 = -1$ in the LHS and RHS above,

$$\Rightarrow \quad (A + B)^2 = \begin{bmatrix} 1 - x^2 & 0 \\ 0 & 1 - x^2 \end{bmatrix} = \begin{bmatrix} 1+1 & 0 \\ 0 & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \text{ and}$$

$$A^2 + B^2 = \begin{bmatrix} 1 - x^2 & 0 \\ 0 & 1 - x^2 \end{bmatrix} = \begin{bmatrix} 1+1 & 0 \\ 0 & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \quad (A + B)^2 = A^2 + B^2.$$

Algebra of Matrices Ex 5.3 Q52

Solving the LHS i.e.

$$A^2 + A = \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}^2 + \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 & -6 \\ 4 & 4 & 0 \\ 2 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 & -9 \\ 6 & 5 & 3 \\ 2 & 3 & 5 \end{bmatrix}$$

Solving the RHS i.e.

$$A(A + I) = \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}\left(\begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 & -3 \\ 2 & 2 & 3 \\ 2 & 3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 & -9 \\ 6 & 5 & 3 \\ 2 & 3 & 5 \end{bmatrix}$$

So, LHS = RHS verified.

Algebra of Matrices Ex 5.3 Q53

We have,

$$A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

Now, $A^2 = AA = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} (3\times3)+(-5\times-4) & (3\times-5)+(-5\times2) \\ (-4\times3)+(2\times-4) & (-4\times-5)+(2\times2) \end{bmatrix}$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix},$$

$$-5A = \begin{bmatrix} -15 & 25 \\ 20 & -10 \end{bmatrix} \text{ and } -14I = \begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix}$$

$$\therefore \quad A^2-5A-14I = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} + \begin{bmatrix} -15 & 25 \\ 20 & -10 \end{bmatrix} + \begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix}$$

$$= \begin{bmatrix} 29-15-14 & -25+25+0 \\ -20+20+0 & 24-10+-14 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Now,

$$A^2-5A-14I = 0$$

$\Rightarrow \quad A^2 = 5A + 14I$

$\Rightarrow \quad A^3 = A^2.A = (5A + 14I)A$

$\Rightarrow \quad A^3 = A^2.A = 5A^2 + 14A \quad$ [By using dist. of matrices over matrix addition]

$\Rightarrow \quad A^3 = 5\begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} + 14\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$

$\Rightarrow \quad A^3 = \begin{bmatrix} 145 & -125 \\ -100 & 120 \end{bmatrix} + \begin{bmatrix} 42 & -70 \\ -56 & 28 \end{bmatrix}$

$\Rightarrow \quad A^3 = \begin{bmatrix} 187 & -195 \\ -156 & 148 \end{bmatrix}$

Algebra of Matrices Ex 5.3 Q54

We have,

$$P(x).\ P(y) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}\begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}$$

$\Rightarrow P(x).\ P(y) = \begin{bmatrix} \cos x \cos y - \sin x \sin y & \sin y \cos x + \sin x \cos y \\ -\sin x \cos y - \cos x \sin y & -\sin x \sin y + \cos x \cos y \end{bmatrix}$

$\Rightarrow P(x).\ P(y) = \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix} = P(x+y)$

Now,

$$P(y).\ P(x) = \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

$\Rightarrow \quad P(y).\ P(x) = \begin{bmatrix} \cos y \cos x - \sin y \sin x & \sin x \cos y + \sin y \cos x \\ -\sin y \cos x - \cos y \sin x & -\sin y \sin x + \cos y \cos x \end{bmatrix}$

$\Rightarrow \quad P(y).\ P(x) = \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix} = P(x+y)$

$\therefore \quad P(x).\ P(y) = P(x+y) = P(y).\ P(x)$

We have,

$$P = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}, \; Q = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

So, $PQ = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$

$$= \begin{bmatrix} x \times a & 0 & 0 \\ 0 & y \times b & 0 \\ 0 & 0 & z \times c \end{bmatrix}$$

$$= \begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix}$$

and $QP = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$

$$= \begin{bmatrix} a \times x & 0 & 0 \\ 0 & b \times y & 0 \\ 0 & 0 & c \times z \end{bmatrix}$$

$$= \begin{bmatrix} ax & 0 & 0 \\ 0 & by & 0 \\ 0 & 0 & zc \end{bmatrix}$$

as, $xa = ax$, $yb = by$, $zc = cz$

$$\therefore \; PQ = \begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix} = QP.$$

Algebra of Matrices Ex 5.3 Q55

We have,

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

Then ,

$$A^2 = A.A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 2\times2+0\times2+1\times1 & 2\times0+0\times1+1\times-1 & 2\times1+0\times3+1\times0 \\ 2\times2+1\times2+3\times1 & 2\times0+0\times1+1\times-1 & 2\times1+1\times3+3\times0 \\ 1\times2+-1\times2+0\times1 & 1\times0+-1\times1+0\times-1 & 1\times1+-1\times3+0\times0 \end{bmatrix}.$$

$$-5A = \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix}, \; 4I = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

Hence, $A^2\text{-}5A+4I = \begin{bmatrix} 5 & -1 & 5 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} + \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

$$A^2\text{-}5A+4I = \begin{bmatrix} 5\text{-}10+4 & -1+0+0 & 5\text{-}5+0 \\ 9\text{-}10+0 & -2\text{-}5+4 & 5\text{-}15+0 \\ 0\text{-}5\text{-}0 & -1+5+0 & -2+0+4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$$

Now, given is $A^2\text{-}5A+4I+X=O$

$\Rightarrow X = -(A^2\text{-}5A+4I)$

$$X = -\begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & 4 & 2 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q56

Given,

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

To prove $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ we will use the principle of mathematical induction.

Step 1: Put $n = 1$

$$A^1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

So,

A^n is true for $n = 1$

Step 2: Let, A^n be true for $n = k$, then

$$A^k = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} \qquad \text{---(i)}$$

Step 3: We have to show that $A^{k+1} = \begin{bmatrix} 1 & k+1 \\ 0 & 1 \end{bmatrix}$

So,

$$A^{k+1} = A^k \times A$$

$$= \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \qquad \{\text{using equation (i) and given}\}$$

$$= \begin{bmatrix} 1+0 & 1+k \\ 0+0 & 0+1 \end{bmatrix}$$

$$A^{k+1} = \begin{bmatrix} 1 & 1+k \\ 0 & 1 \end{bmatrix}$$

This shows that A^n is true for $n = k + 1$ whenever it is true for $n = k$

Hence, by the principle of mathematical induction A^n is true for all positive integer.

Algebra of Matrices Ex 5.3 Q52

Given,

$$A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$$

To prove $A^n = \begin{bmatrix} a^n & \dfrac{b\left(a^n - 1\right)}{a - 1} \\ 0 & 1 \end{bmatrix}$ we will use the principle of mathematical induction.

Step 1: Put $n = 1$

$$A^1 = \begin{bmatrix} a^1 & \dfrac{b\left(a^1 - 1\right)}{a - 1} \\ 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$$

So,

A^n is true for $n = 1$

Step 2: Let, A^n is true for $n = k$, so,

$$A^k = \begin{bmatrix} a^k & \dfrac{b\left(ak - 1\right)}{a - 1} \\ 0 & 1 \end{bmatrix} \qquad \text{---(i)}$$

Step 3: We have to show that

$$A^{k+1} = \begin{bmatrix} a^{k+1} & \dfrac{b\left(a^k - 1\right)}{a - 1} \\ 0 & 1 \end{bmatrix}$$

Now,

$$A^{k+1} = A^k \times A$$

$$= \begin{bmatrix} a^k & \dfrac{b\left(a^k - 1\right)}{a - 1} \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix} \qquad \{\text{using equation (i) and given}\}$$

$$= \begin{bmatrix} a^{k+1} + 0 & a^k b + \dfrac{b\left(a^k - 1\right)}{a - 1} \\ 0 + 0 & 0 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} a^{k+1} & \dfrac{a^{k+1}b - a^k b + a^k b - b}{a - 1} \\ 0 & 1 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q57 pending

$$A^{k+1} = \begin{bmatrix} a^{k+1} & \dfrac{b\left(a^{k+1} - 1\right)}{a - 1} \\ 0 & 1 \end{bmatrix}$$

So,

A^n is true for $n = k + 1$ whenever it is true $n = k$.

Hence, by principle of mathematical induction A^n is true for all positive integer n.

Algebra of Matrices Ex 5.3 Q58

Given,

$$A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$$

To show that,

$$A^n = \begin{bmatrix} \cos n\theta & i\sin n\theta \\ i\sin n\theta & \cos n\theta \end{bmatrix} \text{ for all } n \in N.$$

Put $n = 1$

$$A^1 = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$$

So,

A^n is true for $n = 1$

Let, A^n is true for $n = k$, so

$$A^k = \begin{bmatrix} \cos k\theta & i\sin k\theta \\ i\sin k\theta & \cos k\theta \end{bmatrix} \qquad \text{---(i)}$$

Now, we have to show that,

$$A^{k+1} = \begin{bmatrix} \cos(k+1)\theta & i\sin(k+1)\theta \\ i\sin(k+1)\theta & \cos(k+1)\theta \end{bmatrix}$$

Now, $A^{k+1} = A^k \times A$

$$= \begin{bmatrix} \cos k\theta & i\sin k\theta \\ i\sin k\theta & \cos k\theta \end{bmatrix} \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos k\theta\cos\theta + i^2\sin k\theta\sin\theta & i^2\cos k\theta\sin\theta + i\sin k\theta\cos\theta \\ i\sin k\theta\cos\theta + i\cos k\theta\sin\theta & i^2\sin k\theta\sin\theta + \cos\theta\cos k\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos k\theta\cos\theta - \sin k\theta\sin\theta & i(\cos k\theta\sin\theta + \sin k\theta\cos\theta) \\ i(\sin k\theta\cos k\theta\sin\theta) & \cos k\theta\cos\theta - \sin k\theta\sin\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(k+1)\theta & i\sin(k+1)\theta \\ i\sin(k+1)\theta & \cos(k+1)\theta \end{bmatrix}$$

So, A^n is true for $n = k+1$ whenever it is true for $n = k$.

Hence, By principle of mathematical induction A^n is true for all positive integer.

Algebra of Matrices Ex 5.3 Q59

Given,

$$A = \begin{bmatrix} \cos\alpha + \sin\alpha & \sqrt{2}\sin\alpha \\ -\sqrt{2}\sin\alpha & \cos\alpha - \sin\alpha \end{bmatrix}$$

To prove P(n): $A^n = \begin{bmatrix} \cos n\alpha + \sin n\alpha & \sqrt{2}\sin n\alpha \\ -\sqrt{2}\sin n\alpha & \cos n\alpha - \sin n\alpha \end{bmatrix}$ we use mathematical induction.

Step 1: To show P(1) is true.

A^n is true for $n = 1$

Step 2: Let, $P(k)$ be true, so

$$A^k = \begin{bmatrix} \cos k\alpha + \sin k\alpha & \sqrt{2}\sin k\alpha \\ -\sqrt{2}\sin k\alpha & \cos k\alpha - \sin k\alpha \end{bmatrix} \quad \text{---(i)}$$

Step 3: Let, $P(k)$ is true.

Now, we have to show that

$$A^{k+1} = \begin{bmatrix} \cos(k+1)\alpha + \sin(k+1)\alpha & \sqrt{2}\sin(k+1)\alpha \\ -\sqrt{2}\sin(k1)\alpha & \cos(k+1)\alpha - \sin(k+1)\alpha \end{bmatrix}$$

Now,

$$A^{k+1} = A^k \times A$$

$$= \begin{bmatrix} \cos k\alpha + \sin k\alpha & \sqrt{2}\sin k\alpha \\ -\sqrt{2}\sin k\alpha & \cos k\alpha - \sin k\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha + \sin\alpha & \sqrt{2}\sin\alpha \\ -\sqrt{2}\sin\alpha & \cos\alpha - \sin\alpha \end{bmatrix}$$

$$= \begin{bmatrix} (\cos k\alpha + \sin k\alpha)(\cos\alpha + \sin\alpha) - 2\sin\alpha\sin k\alpha & \begin{matrix}(\cos k\alpha + \sin k\alpha)\sqrt{2}\sin\alpha \\ +\sqrt{2}\sin k\alpha\,(\cos\alpha - \sin\alpha)\end{matrix} \\ (\cos\alpha + \sin\alpha)\left(-\sqrt{2}\sin k\alpha\right) - \sqrt{2}\sin\alpha\,(\cos k\alpha - \sin k\alpha) & \begin{matrix}-2\sin k\alpha\sin\alpha + (\cos k\alpha - \sin k\alpha) \\ (\cos\alpha - \sin\alpha)\end{matrix} \end{bmatrix}$$

$$= \begin{bmatrix} \begin{matrix}\cos k\alpha\cos\alpha + \sin k\alpha\cos\alpha + \cos k\alpha\sin\alpha \\ +\sin\alpha\sin k\alpha - 2\sin\alpha\sin k\alpha\end{matrix} & \begin{matrix}\sqrt{2}\cos k\alpha\sin\alpha + \sqrt{2}\sin\alpha\sin k\alpha + \\ \sqrt{2}\sin k\alpha\cos\alpha - \sqrt{2}\sin k\alpha\sin\alpha\end{matrix} \\ \begin{matrix}-\sqrt{2}\cos\alpha\sin\alpha - \sqrt{2}\sin\alpha\sin k\alpha - \sqrt{2}\sin\alpha \\ \cos k\alpha + \sqrt{2}\sin\alpha\sin k\alpha\end{matrix} & \begin{matrix}-2\sin k\alpha\sin\alpha + \cos k\alpha\cos\alpha - \cos\alpha \\ \sin k\alpha - \sin\alpha\cos k\alpha\sin\alpha\sin k\alpha\end{matrix} \end{bmatrix}$$

$$= \begin{bmatrix} \begin{matrix}\cos\alpha\cos k\alpha + \sin\alpha\sin k\alpha \\ \sin\alpha\cos k\alpha + \sin k\alpha\cos\alpha\end{matrix} & \sqrt{2}(\sin k\alpha\cos\alpha + \cos k\alpha\sin\alpha) \\ -\sqrt{2}(\sin k\alpha\cos\alpha + \cos k\alpha\sin\alpha) & \begin{matrix}\cos k\alpha\cos\alpha - \sin k\alpha\sin\alpha - \\ (\sin k\alpha\cos\alpha + \sin\alpha\cos k\alpha)\end{matrix} \end{bmatrix}$$

$$= \begin{bmatrix} \cos(k+1)\alpha + \sin(k+1)\alpha & \sqrt{2}\sin(k+1)\alpha \\ -\sqrt{2}\sin(k+1)\alpha & \cos(k+1)\alpha - \sin(k+1)\alpha \end{bmatrix}$$

So, $P(k+1)$ is true whenever $P(k)$ is true.

Hence, by principle of mathematical induction $P(n)$ is true for all positive integer.

Algebra of Matrices Ex 5.3 Q60

Given,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

To prove, $A^n = \begin{bmatrix} 1 & n & \frac{n(n+1)}{2} \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$, we will use the principle of mathematical induction.

Step 1: Put $n = 1$

$$A^1 = \begin{bmatrix} 1 & 1 & \frac{1(1+1)}{2} \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

So, A^n is true for $n = 1$

Step 2: Let, A^n be true for $n = k$, so,

$$A^k = \begin{bmatrix} 1 & k & \frac{k(k+1)}{2} \\ 0 & 1 & (k+1) \\ 0 & 0 & 1 \end{bmatrix}$$

Step 3: We will prove that A^n be true for $n = k + 1$

Now,

$$A^{k+1} = A^k \times A$$

$$= \begin{bmatrix} 1 & k & \frac{k(k+1)}{2} \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad \{\text{using equation (i) and given}\}$$

$$= \begin{bmatrix} 1+0+0 & 1+k+0 & 1+k+\frac{k(k+1)}{2} \\ 0+0+0 & 0+1+0 & 0+1+k \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & (k+1) & \frac{(k+1)(k+2)}{2} \\ 0 & 1 & (k+1) \\ 0 & 0 & 1 \end{bmatrix}$$

Hence, A^n is true for $n = k + 1$ whenever it is true for $n = k$.

So, by principle of mathematical induction A^n is true for all positive integer n.

Algebra of Matrices Ex 5.3 Q61

We will prove P(n): $A^{n+1} = B^n\left[B+(n+1)C\right]$ is true for all natural numbers using mathematical induction.

Given,

$$A = B + C,\ BC = CB,\ C^2 = 0$$
$$A = B + C$$

Squaring both the sides, so

$$A^2 = (B+C)^2$$
$$\Rightarrow \quad A^2 = (B+C)(B+C)$$
$$\Rightarrow \quad A^2 = B\times B + BC + CB + C\times C \qquad \{\text{using distributive property}\}$$
$$\Rightarrow \quad A^2 = B^2 + BC + BC + C^2 \qquad \{\text{using } BC = CB \text{ given}\}$$
$$\Rightarrow \quad A^2 = B^2 + 2BC + 0 \qquad \left\{\text{since, given } C^2 = 0\right\}$$
$$\Rightarrow \quad A^2 = B^2 + 2BC \qquad \text{---(1)}$$
$$A^2 = B(B+2C)$$

Now, consider

$$P(n):\ A^{n+1} = B^n\left[B+(n+1)C\right]$$

Step 1: Tp prove P(1) is true, put $n = 1$

$$A^{1+1} = B^1\left[B+(1+1)C\right]$$
$$A^2 = B\left[B+2C\right]$$
$$A^2 = B^2 + 2BC$$

From equation (i), P(1) is true.

Step 2: Suppose P(k) is true.

$$\therefore A^{k+1} = B^k\left[B+(k+1)C\right] \qquad \text{---(2)}$$

Step 3: Now, we have to show that P(k+1) is true.

That is we need to prove that,

$$A^{k+2} = B^{k+1}\left[B+(k+2)C\right]$$

Now,

$$A^{k+2} = A^k \times A^2$$
$$= B^{(k-1)}\left[B+kC\right]\times\left[B(B+2C)\right]$$
$$= B^k\left[B+kC\right]\times\left[B+2C\right]$$
$$= B^k\left[B\times B + B\times 2C + kC\times B + 2kC^2\right]$$
$$= B^k\left[B^2 + 2BC + kBC + 2k\times 0\right] \qquad \left\{\text{since } BC = CB,\ C^2 = 0\right\}$$
$$= B^k\left[B^2 + BC(2+k)\right]$$
$$= B^k \times B\left[B+(k+2)C\right]$$
$$= B^{k+1}\left[B+(k+2)C\right]$$

So, $P(n)$ is true for $n = k+1$ whenever $P(n)$ is true for $n = k$

Therefore by principle of mathematical induction $P(n)$ is true for all natural number.

Algebra of Matrices Ex 5.3 Q62

Given,

$$A = \text{diag}(a, b, c)$$

Show that,

$$A^n = \text{diag}\left(a^n, b^n, c^n\right)$$

Step 1: Put $n = 1$

$$A^1 = \text{diag}\left(a^1, b^1, c^1\right)$$

$$A = \text{diag}(a, b, c)$$

So,

A^n is true for $n = 1$

Step 2: Let, A^n be true for $n = k$, so,

$$A^k = \text{diag}\left(a^k, b^k, c^k\right) \qquad \text{---(i)}$$

Step 3: Now, we have to show that,

$$A^{k+1} = \text{diag}\left(a^{k+1}, b^{k+1}, c^{k+1}\right)$$

Now,

$$A^{k+1} = A^k \times k3$$

$$= \text{diag}\left(a^k, b^k, c^k\right) \times \text{diag}(a, b, c) \qquad \{\text{using equation (i) and given}\}$$

$$A^{k+1} = \begin{bmatrix} a^k & 0 & 0 \\ 0 & b^k & 0 \\ 0 & 0 & c^k \end{bmatrix}\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$= \begin{bmatrix} a^k \times a + 0 + 0 & 0 + 0 + 0 & 0 + 0 + 0 \\ 0 + 0 + 0 & 0 + b^k \times b + 0 & 0 + 0 + 0 \\ 0 + 0 + 0 & 0 + 0 + 0 & 0 + 0 + c^k \times c \end{bmatrix}$$

$$= \begin{bmatrix} a^{k+1} & 0 & 0 \\ 0 & b^{k+1} & 0 \\ 0 & 0 & c^{k+1} \end{bmatrix}$$

$$A^{k+1} = \text{diag}\left(a^{k+1}, b^{k+1}, c^{k+1}\right)$$

So, $P(n)$ is true for $n = k + 1$ whenever $P(n)$ is true for $n = k$.

Hence, by principle of mathematical induction A^n is true for all positive integer.

Algebra of Matrices Ex 5.3 Q64

Given,

order of matrix $X = (a+b)\times(a+2)$

order of matrix $Y = (b+1)\times(a+3)$

Given, $X_{(a+b)\times(a+2)} \cdot Y_{(b+1)\times(a+3)}$ exist.

$\Rightarrow \quad a+2 = b+1$

$\Rightarrow \quad a-b = -1 \qquad ---(i)$

And

$Y_{(b+1)\times(a+3)} \cdot X_{(a+b)\times(a+2)}$ exists.

$\Rightarrow \quad a+3 = a+b$

$\Rightarrow \quad b = 3$

Put $b = 3$ in equation (i),

$a-b = -1$

$a-3 = -1$

$a = 3-1$

$a = 2$

So, $\quad a = 2, b = 3$

So,

$Order\ of\ X = (a+b)\times(a+2)$

$= (2+3)\times(2+2)$

$= 5\times 4$

$Order\ of\ Y = (b+1)\times(a+3)$

$= (3+1)\times(2+3)$

$= 4\times 5$

$Order\ of\ X_{5\times 4} \cdot Y_{4\times 5} = 5\times 5$

$Order\ of\ X_{4\times 5} \cdot Y_{5\times 4} = 4\times 4$

So, $\quad$ *order* of XY and YX are not same and they are not equal but both are square matrices.

Algebra of Matrices Ex 5.3 Q65(i)

Let, $A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}$, $B \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$

$$AB = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & ab+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & ab \\ 0 & 0 \end{bmatrix} \quad ---(i)$$

$$BA = \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

From equation (i) and (ii)

$$AB \neq BA$$

when $A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$

Algebra of Matrices Ex 5.3 Q65(ii)

Let, $A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} \neq 0$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix} \neq 0$$

$$AB = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence,

$$AB = 0$$

When,

$$A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} \neq 0$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix} \neq 0$$

Let, $A = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} b & 0 \\ 0 & 0 \end{bmatrix}$

$$AB = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}\begin{bmatrix} b & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$AB = 0$$

$$BA = \begin{bmatrix} b & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & ab+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & ab \\ 0 & 0 \end{bmatrix}$$

$$BA \neq 0$$

Hence,

for $AB = 0$ and $BA \neq 0$ we have,

$$A = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}, \ B = \begin{bmatrix} b & 0 \\ 0 & 0 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q65(iv)

Let, $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

Here,

$$A \neq 0,\ B \neq C$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

LHS = RHS

So,

for $A \neq 0,\ BC \neq 0$ but $AB = AC$

We have,

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix},\ C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q66

Given,

A and B are square matices of same order

$(A+B)^2 = (A+B)(A+B)$

$= A(A+B) + B(A+B)$ {using distributive property}

$= A \times A + AB + BA + B^2$

$= A^2 + AB + BA + B^2$

But,

$(A+B)^2 = A^2 + 2AB + B^2$ is possible only when $AB = BA$

Here, we can not say that $AB = BA$

So,

$(A+B)^2 = A^2 + 2AB + B^2$ does not hold.

Algebra of Matrices Ex 5.3 Q67

Given, A and B are square matrices of same order.

(i) $(A+B)^2 = (A+B)(A+B)$

$= A(A+B)+B(A+B)$ {using distributive property}

$= A\times A + AB + BA + B\times B$

$= A^2 + AB + BA + B^2$

$\neq A^2 + 2AB + B^2$

Since, in general matix multiplication is not commutative $(AB \neq BA)$

So, $(A+B)^2 \neq A^2 + 2AB + B^2$

(ii) $(A-B)^2 = (A-B)(A-B)$

$= A(A-B) - B(A-B)$ {using distributive property} .

$= A\times A - AB - BA + B\times B$

$= A^2 - AB - BA + B^2$

$\neq A^2 - 2AB + B^2$

Since, in general matrix multiplication is not commutative $(AB \neq BA)$, so

So, $(A-B)^2 \neq A^2 - 2AB + B^2$

(iii) $(A+B)(A-B) = A(A-B) + B(A-B)$ {using distributive property}

$= A\times A - AB + BA - B\times B$

$= A^2 - AB + BA - B^2$

$= A^2 - B^2$

Since, in general matix multiplication is not commutative $(AB \neq BA)$,

So, $(A+B)(A-B) \neq A^2 - B^2$

Algebra of Matrices Ex 5.3 Q68

The given equality is true only when we choose A and B to be a square matrix in such a way that AB = BA else the result is not true in general.

Example: Let A = $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Here AB = $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1\times 0+0\times 2+0\times 0 & 1\times 1+0\times 1+0\times 0 & 1\times 0+0\times 0+0\times 1 \\ 1\times 0+1\times 2+0\times 0 & 1\times 1+1\times 1+0\times 0 & 1\times 0+1\times 0+0\times 1 \\ 0\times 0+0\times 2+1\times 0 & 0\times 1+0\times 1+1\times 0 & 0\times 0+0\times 0+1\times 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

and BA = $\begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0\times 1+1\times 1+0\times 0 & 0\times 0+1\times 1+0\times 0 & 0\times 0+1\times 0+0\times 1 \\ 2\times 1+1\times 1+0\times 0 & 2\times 0+1\times 1+0\times 0 & 2\times 0+1\times 0+0\times 1 \\ 0\times 1+0\times 1+1\times 0 & 0\times 0+0\times 1+1\times 0 & 0\times 0+0\times 0+1\times 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

AB $\neq$ BA

Now, $(AB)^2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 0\times0+1\times1+0\times0 & 0\times1+1\times2+0\times0 & 0\times0+1\times0+0\times1 \\ 1\times0+2\times1+0\times0 & 1\times1+1\times2+0\times0 & 1\times0+2\times0+0\times1 \\ 0\times0+0\times1+1\times0 & 0\times1+0\times2+1\times0 & 0\times0+0\times0+1\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1+0\times1+0\times0 & 1\times0+0\times1+0\times0 & 1\times0+0\times0+0\times1 \\ 1\times1+1\times1+0\times0 & 1\times0+1\times1+0\times0 & 1\times0+1\times0+0\times1 \\ 0\times1+0\times1+1\times0 & 0\times0+0\times1+1\times0 & 0\times0+0\times0+1\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0\times0+1\times2+0\times0 & 0\times1+1\times1+0\times0 & 0\times0+1\times0+0\times1 \\ 2\times0+1\times2+0\times0 & 2\times1+1\times1+0\times0 & 2\times0+1\times0+0\times1 \\ 0\times0+0\times2+1\times0 & 0\times1+0\times1+1\times0 & 0\times0+0\times0+1\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^2B^2 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1+0\times1+0\times0 & 1\times1+0\times2+0\times0 & 1\times0+0\times0+0\times1 \\ 2\times1+1\times1+0\times0 & 2\times1+1\times2+0\times0 & 2\times0+1\times0+0\times1 \\ 0\times1+0\times1+1\times0 & 0\times1+0\times2+1\times0 & 0\times0+0\times0+1\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We can see that if we have A and B two square matrices with $AB \neq BA$ then $(AB)^2 \neq A^2B^2$

Algebra of Matrices Ex 5.3 Q69

Given,

A and B two square matrices of same order such that $AB = BA$.

To prove : $(A+B)^2 = A^2 + 2AB + B^2$

Now, solving LHS gives,

$$(A+B)^2 = (A+B)(A+B)$$

$$= A(A+B) + B(A+B) \quad \text{[by dist. of matrix multiplication over addition]}$$

$$= A^2 + AB + BA + B^2 \quad \text{[by dist. of matrix multiplication over addition]}$$

$$= A^2 + 2AB + B^2 \quad [\text{As, } AB = BA]$$

$$= RHS$$

Hence proved.

Algebra of Matrices Ex 5.3 Q70

Given, $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix}, B = \begin{bmatrix} 3 & 1 \\ 5 & 2 \\ -2 & 4 \end{bmatrix}, C = \begin{bmatrix} 4 & 2 \\ -3 & 5 \\ 5 & 0 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 5 & 2 \\ -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3+5-2 & 1+2+4 \\ 9+15-6 & 3+6+12 \end{bmatrix}$$

$$AB = \begin{bmatrix} 6 & 7 \\ 18 & 21 \end{bmatrix} \qquad \text{---(i)}$$

$$AC = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ -3 & 5 \\ 5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3+5 & 2+5+0 \\ 12-9+15 & 6+15+0 \end{bmatrix}$$

$$AC = \begin{bmatrix} 6 & 7 \\ 18 & 21 \end{bmatrix} \qquad \text{---(ii)}$$

From equation (i) and (ii)

$$AB = AC$$

Algebra of Matrices Ex 5.3 Q71

The number of items purchased by A, B and C are represented in matrix form as,

$$X = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} \text{Notebook} & \text{Pens} & \text{Pencils} \end{array} \\ \begin{bmatrix} 144 & 60 & 72 \\ 120 & 72 & 84 \\ 132 & 156 & 96 \end{bmatrix} \end{array}$$

Now, matrix formed by the cost of each items is given by,

$$Y = \begin{bmatrix} 0.40 \\ 1.25 \\ 0.35 \end{bmatrix} \begin{array}{c} \text{Note book} \\ \text{Pen} \\ \text{Pencil} \end{array}$$

Individual bill can be calculated by

$$XY = \begin{bmatrix} 144 & 60 & 72 \\ 120 & 72 & 84 \\ 132 & 156 & 96 \end{bmatrix} \begin{bmatrix} 0.40 \\ 1.25 \\ 0.35 \end{bmatrix}$$

$$XY = \begin{bmatrix} 57.60 + 75.00 + 25.20 \\ 48.00 + 90.00 + 29.40 \\ 52.80 + 195.00 + 33.60 \end{bmatrix}$$

$$XY = \begin{bmatrix} 157.80 \\ 167.40 \\ 281.40 \end{bmatrix}$$

So,

Bill of A = Rs 157.80

Bill of B = Rs 167.40

Bill of C = Rs 281.40

Algebra of Matrices Ex 5.3 Q72

Matrix representation of stock of various types of book in the store is given by,

$$X = \begin{array}{c} \begin{array}{ccc} \text{Physics} & \text{Chemistry} & \text{Mathematics} \end{array} \\ \begin{bmatrix} 120 & 96 & 60 \end{bmatrix} \end{array}$$

Matrix representation of sellin price (Rs.) of each book is given by

$$Y = \begin{bmatrix} 8.30 \\ 3.45 \\ 4.50 \end{bmatrix} \begin{array}{c} \text{Physics} \\ \text{Chemistry} \\ \text{Mathematics} \end{array}$$

So, totaol amount recieved by the store from sellin all the items is given by,

$$XY = \begin{bmatrix} 120 & 96 & 60 \end{bmatrix} \begin{bmatrix} 8.30 \\ 3.45 \\ 4.50 \end{bmatrix}$$

$$= \begin{bmatrix} (120)(8.30) + (96)(3.45) + (60)(4.50) \end{bmatrix}$$

$$= \begin{bmatrix} 996 + 331.20 + 270 \end{bmatrix}$$

$$= \begin{bmatrix} 1597.20 \end{bmatrix}$$

Required amount = Rs 1597.20

Algebra of Matrices Ex 5.3 Q73

Given,

The cost per contact (in paise) is given by

$$A = \begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix} \begin{bmatrix} \text{Telephone} \\ \text{Housecall} \\ \text{Letter} \end{bmatrix}$$

The number of contact of each type made in two cities X and y is given by.

$$\begin{array}{cc} & \begin{array}{ccc} \text{Telephone} & \text{Housecall} & \text{Letter} \end{array} \\ B = & \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix} \end{array}$$

Total amount spent by the group in the two cities X and y can be given by

$$BA = \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix} \begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix}$$

$$= \begin{bmatrix} 40000 + 50000 + 250000 \\ 120000 + 100000 + 500000 \end{bmatrix}$$

$$= \begin{bmatrix} 340000 \\ 720000 \end{bmatrix} \begin{matrix} X \\ Y \end{matrix}$$

Hence,

Amount spend on X = Rs 3400

Amount spend on Y = Rs 7200

Algebra of Matrices Ex 5.3 Q74

(a) Let Rs x be invested in the first bond. Then, the sum of money invested in the second bond will be Rs $(30000 - x)$.

It is given that the first bond pays 5% interest per year and the second bond pays 7% interest per year.

Therefore, in order to obtain an annual total interest of Rs 1800, we have:

$$\begin{bmatrix} x & (30000 - x) \end{bmatrix} \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 1800 \qquad \left[\text{S.I. for 1 year} = \frac{\text{Principal} \times \text{Rate}}{100} \right]$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000 - x)}{100} = 1800$$

$$\Rightarrow 5x + 210000 - 7x = 180000$$

$$\Rightarrow 210000 - 2x = 180000$$

$$\Rightarrow 2x = 210000 - 180000$$

$$\Rightarrow 2x = 30000$$

$$\Rightarrow x = 15000$$

Thus, in order to obtain an annual total interest of Rs 1800, the trust fund should invest Rs 15000 in the first bond and the remaining Rs 15000 in the second bond.

(b) Let Rs x be invested in the first bond. Then, the sum of money invested in the second bond will be Rs $(30000-x)$.

Therefore, in order to obtain an annual total interest of Rs 2000, we have:

$$\begin{bmatrix} x & (30000-x) \end{bmatrix}\begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 2000$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000-x)}{100} = 2000$$

$$\Rightarrow 5x + 210000 - 7x = 200000$$

$$\Rightarrow 210000 - 2x = 200000$$

$$\Rightarrow 2x = 210000 - 200000$$

$$\Rightarrow 2x = 10000$$

$$\Rightarrow x = 5000$$

Thus, in order to obtain an annual total interest of Rs 2000, the trust fund should invest Rs 5000 in the first bond and the remaining Rs 25000 in the second bond

Algebra of Matrices Ex 5.3 Q75

The cost for each mode per attempt is represented by 3 x 1 matrix:

$$A = \begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix}$$

The number of attempts made in the three villages X, Y, and Z are represented by a 3 x 3 matrix:

$$B = \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix}$$

The total cost incurred by the prganization for the three villages seperately is given by matrix multiplication

$$BA = \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix}\begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix}$$

$$BA = \begin{bmatrix} 400 \times 50 + 300 \times 20 + & 100 \times 40 \\ 300 \times 50 + 250 \times 20 + & 75 \times 40 \\ 500 \times 50 + 400 \times 20 + & 150 \times 40 \end{bmatrix}$$

$$= \begin{bmatrix} 30{,}000 \\ 23{,}000 \\ 39{,}000 \end{bmatrix}$$

Note: The answer given in the book is incorrect.

Algebra of Matrices Ex 5.3 Q76

Let F be the family matrix and R be the requirement matrix. Then,

$$F = \begin{array}{c} \\ \text{Family A} \\ \text{Family B} \end{array}\begin{array}{c} \begin{array}{ccc} \text{Men} & \text{Women} & \text{Children} \end{array} \\ \begin{bmatrix} 4 & 6 & 2 \\ 2 & 2 & 4 \end{bmatrix} \end{array}$$

$$R = \begin{array}{c} \\ \text{Men} \\ \text{Women} \\ \text{Children} \end{array}\begin{array}{c} \begin{array}{cc} \text{Calories} & \text{Portein} \end{array} \\ \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix} \end{array}$$

The requirement of calories and protein of each of the two families is given by the product matrix FR, as matrix F has number of columns equal to number of rows of R thus ,

$$FR = \begin{bmatrix} 4 & 6 & 2 \\ 2 & 2 & 4 \end{bmatrix}\begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix}$$

$$FR = \begin{bmatrix} 4 \times 2400 + 6 \times 1900 + 2 \times 1800 & 4 \times 45 + 6 \times 55 + 2 \times 33 \\ 2 \times 2400 + 2 \times 1900 + 4 \times 1800 & 2 \times 45 + 2 \times 55 + 4 \times 33 \end{bmatrix}$$

$$FR = \begin{array}{c} \\ \text{Family A} \\ \text{Family B} \end{array}\begin{array}{c} \begin{array}{cc} \text{Calories} & \text{Protein} \end{array} \\ \begin{bmatrix} 24600 & 576 \\ 15800 & 332 \end{bmatrix} \end{array}$$

we can say that balanced diet having the required amount of calories and protein must be taken by each of the family.

Algebra of Matrices Ex 5.3 Q77

The cost per contact (in paisa) is given in matrix A as

$$A = \begin{bmatrix} 140 \\ 200 \\ 150 \end{bmatrix}\begin{array}{l} \text{Telephone} \\ \text{House calls} \\ \text{Letters} \end{array}$$

The number of contacts of each type made in two cities X and Y is given in the matrix B as

$$B = \begin{array}{c} \\ \text{City X} \\ \text{City Y} \end{array}\begin{array}{c} \begin{array}{ccc} \text{Telephone} & \text{House calls} & \text{Letters} \end{array} \\ \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix} \end{array}$$

The total amout of money spent by party in each of the city for the election is given by the matrix multiplication :

$$BA = \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix}\begin{bmatrix} 140 \\ 200 \\ 150 \end{bmatrix}$$

$$= \begin{bmatrix} 1000 \times 140 + 500 \times 200 + 5000 \times 150 \\ 3000 \times 140 + 1000 \times 200 + 10000 \times 150 \end{bmatrix}$$

$$= \begin{array}{c} \text{City X} \\ \text{City Y} \end{array}\begin{bmatrix} 990000 \\ 2120000 \end{bmatrix}$$

The total amout of money spent by party in each of the city for the election in rupees is given by

$$= \left(\frac{1}{100}\right)\begin{array}{c} \text{City X} \\ \text{City Y} \end{array}\begin{bmatrix} 990000 \\ 2120000 \end{bmatrix}$$

$$= \begin{array}{c} \text{City X} \\ \text{City Y} \end{array}\begin{bmatrix} 9900 \\ 21200 \end{bmatrix}$$

One sould consider social activities before casting his/her vote to the party.

Ex 5.4

Algebra of Matrices Ex 5.4 Q1(i)

Given,

$$A = \begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

$$(2A)^T = 2 \times A^T$$

$$\Rightarrow \quad \left(2\begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix}\right)^T = 2\begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix}^T$$

$$\Rightarrow \quad \begin{bmatrix} 4 & -6 \\ -14 & 10 \end{bmatrix}^T = 2\begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 4 & -14 \\ -6 & 10 \end{bmatrix} = \begin{bmatrix} 4 & -14 \\ -6 & 10 \end{bmatrix}$$

LHS = RHS

So,

$$(2A)^T = 2A^T$$

Algebra of Matrices Ex 5.4 Q1(ii)

Given,

$$A = \begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

$$(A+B)^T = A^T + B^T$$

$$\left(\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}\right)^T = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T + \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}^T$$

$$\Rightarrow \quad \begin{bmatrix} 2+1 & -3+0 \\ -7+2 & 5-4 \end{bmatrix}^T = \begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 3 & -3 \\ -5 & 1 \end{bmatrix}^T = \begin{bmatrix} 2+1 & -7+2 \\ -3+0 & 5-4 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 3 & -5 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -3 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \text{LHS} = \text{RHS}$$

So,

$$(A+B)^T = A^T + B^T$$

Algebra of Matrices Ex 5.4 Q1(iii)

Given,

$$A = \begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

$(A - B)^T = A^T - B^T$

$$\Rightarrow \quad \left(\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}\right)^T = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T - \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}^T$$

$$\Rightarrow \quad \begin{bmatrix} 2-1 & -3-0 \\ -7-2 & 5+4 \end{bmatrix}^T = \begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 1 & -3 \\ -9 & 9 \end{bmatrix}^T = \begin{bmatrix} 2-1 & -7-2 \\ -3-0 & 5+4 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 1 & -9 \\ -3 & 9 \end{bmatrix} = \begin{bmatrix} 1 & -9 \\ -3 & 9 \end{bmatrix}$$

$\Rightarrow \quad$ LHS = RHS

So,

$$(A - B)^T = A^T - B^T$$

Algebra of Matrices Ex 5.4 Q1(iv)

Given,

$$A = \begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

$$(AB)^T = B^T A^T$$

$$\Rightarrow \quad \left(\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}\right)^T = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}^T \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T$$

$$\Rightarrow \quad \begin{bmatrix} 2-6 & 0+12 \\ -7+10 & 0-20 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix}\begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} -4 & 12 \\ 3 & -20 \end{bmatrix}^T = \begin{bmatrix} 2-6 & -7+10 \\ 0+12 & 0-20 \end{bmatrix}.$$

$$\Rightarrow \quad \begin{bmatrix} -4 & 3 \\ 12 & -20 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 12 & -20 \end{bmatrix}$$

$$\Rightarrow \quad \text{HS} = \text{RHS}$$

So,

$$(AB)^T = B^T A^T$$

Algebra of Matrices Ex 5.4 Q2

Given,

$$A = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix},\ B = \begin{bmatrix} 1 & 0 & 4 \end{bmatrix}$$

$$(AB)^T = B^T A^T$$

$$\Rightarrow \quad \left(\begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 4 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & 0 & 4 \end{bmatrix}^T \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}^T$$

$$\Rightarrow \quad \begin{bmatrix} 3 & 0 & 12 \\ 5 & 0 & 20 \\ 2 & 0 & 8 \end{bmatrix}^T = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix} \begin{bmatrix} 3 & 5 & 2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 3 & 5 & 2 \\ 0 & 0 & 0 \\ 12 & 20 & 8 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 2 \\ 0 & 0 & 0 \\ 12 & 20 & 8 \end{bmatrix}$$

$$\Rightarrow \quad \text{LHS} = \text{RHS}$$

So,

$$(AB)^T = B^T A^T$$

Algebra of Matrices Ex 5.4 Q3(i)

Given,

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}, \ B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \qquad A^T = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix}, \ B^T = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$$(A+B)^T = A^T + B^T$$

$$\Rightarrow \qquad \left(\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}^T + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}^T$$

$$\Rightarrow \qquad \begin{bmatrix} 1+1 & -1+2 & 0+3 \\ 2+2 & 1+1 & 3+3 \\ 1+0 & 2+1 & 1+1 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$$\Rightarrow \qquad \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 6 \\ 1 & 3 & 2 \end{bmatrix}^T = \begin{bmatrix} 1+1 & 2+2 & 1+0 \\ -1+2 & 1+1 & 2+1 \\ 0+3 & 3+3 & 1+1 \end{bmatrix}$$

$$\Rightarrow \qquad \begin{bmatrix} 2 & 4 & 1 \\ 1 & 2 & 3 \\ 3 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 1 \\ 1 & 2 & 3 \\ 3 & 6 & 2 \end{bmatrix}$$

$$\Rightarrow \qquad \text{LHS} = \text{RHS}$$

So,

$$(A+B)^T = A^T + B^T$$

Algebra of Matrices Ex 5.4 Q3(ii)

Given,

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix},\ B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \qquad A^T = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix},\ B^T = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$$(AB)^T = B^T A^T$$

$$\Rightarrow \qquad \left(\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}\right)^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}^T \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}^T$$

$$\Rightarrow \qquad \begin{bmatrix} 1-2+0 & 2-1+0 & 3-3+0 \\ 2+2+0 & 4+1+3 & 6+3+3 \\ 1+4+0 & 2+2+1 & 3+6+1 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix}$$

$$\Rightarrow \qquad \begin{bmatrix} -1 & 1 & 0 \\ 4 & 8 & 12 \\ 5 & 5 & 10 \end{bmatrix}^T = \begin{bmatrix} 1-2+0 & 2+2+0 & 1+4+0 \\ 2-1+0 & 4+1+3 & 2+2+1 \\ 3-3+0 & 6+3+3 & 3+6+1 \end{bmatrix}$$

$$\Rightarrow \qquad \begin{bmatrix} -1 & 4 & 5 \\ 1 & 8 & 5 \\ 0 & 12 & 10 \end{bmatrix} = \begin{bmatrix} -1 & 4 & 5 \\ 1 & 8 & 5 \\ 0 & 12 & 10 \end{bmatrix}$$

$$\Rightarrow \qquad \text{LHS} = \text{RHS}$$

So,

$$(AB)^T = B^T A^T$$

Algebra of Matrices Ex 5.4 Q3(iii)

Given,

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \qquad A^T = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix}, \quad B^T = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$$(2A)^T = 2A^T$$

$$\Rightarrow \qquad \left(2\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}\right)^T = 2\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}^T$$

$$\Rightarrow \qquad \begin{bmatrix} 2 & -2 & 0 \\ 4 & 2 & 6 \\ 2 & 4 & 2 \end{bmatrix}^T = 2\begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix}$$

$$\Rightarrow \qquad \begin{bmatrix} 2 & 4 & 2 \\ -2 & 2 & 4 \\ 0 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 2 \\ -2 & 2 & 4 \\ 0 & 6 & 2 \end{bmatrix}$$

$$\Rightarrow \qquad \text{LHS} = \text{RHS}$$

So,

$$(2A)^T = 2 \times A^T$$

Algebra of Matrices Ex 5.4 Q4

Given,

$$A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}, \ B = \begin{bmatrix} 1 & 3 & -6 \end{bmatrix}$$

$$(AB)^T = B^T A^T$$

$$\Rightarrow \quad \left(\begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix} \begin{bmatrix} 1 & 3 & -6 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & 3 & -6 \end{bmatrix}^T \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}^T$$

$$\Rightarrow \quad \begin{bmatrix} -2 & -6 & 12 \\ 4 & 12 & -24 \\ 5 & 15 & -30 \end{bmatrix}^T = \begin{bmatrix} 1 \\ 3 \\ -6 \end{bmatrix} \begin{bmatrix} -2 & 4 & 5 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -24 & -30 \end{bmatrix} = \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -24 & -30 \end{bmatrix}$$

$$\Rightarrow \quad \text{LHS} = \text{RHS}$$

So,

$$(2A)^T = 2 \times AT$$

Algebra of Matrices Ex 5.4 Q5

Given,

$$A = \begin{bmatrix} 2 & 4 & -1 \\ -1 & 0 & 2 \end{bmatrix}, \; B = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 2 & 1 \end{bmatrix}$$

$(AB)^T$

$$= \left(\begin{bmatrix} 2 & 4 & -1 \\ -1 & 0 & 2 \end{bmatrix}\right)\begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 2 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 6-4-2 & 8+8-1 \\ -3+0+4 & -4+0+2 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & 15 \\ 1 & -2 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & 1 \\ 15 & -2 \end{bmatrix}$$

So,

$$(AB)^T = \begin{bmatrix} 0 & 1 \\ 15 & -2 \end{bmatrix}$$

Algebra of Matrices Ex 5.4 Q6

Given,

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix},\ B = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix}$$

$(AB)^T = B^T A^T$

$$\Rightarrow \quad \left(\begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix}\right)^T = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix}^T \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix}^T$$

$$\Rightarrow \quad \begin{bmatrix} 2+0+15 & -2+20 \\ 4+0+0 & -4+2+0 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & 5 \\ -1 & 2 & 0 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 1 & 1 \\ 3 & 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 17 & 0 \\ 4 & -2 \end{bmatrix}^T = \begin{bmatrix} 2+0+15 & 4+0+0 \\ -2+2+0 & -4+2+0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 17 & 4 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 17 & 4 \\ 0 & -2 \end{bmatrix}$$

$\Rightarrow \quad$ LHS = RHS

So,

$$(AB)^T = B^T A^T$$

Algebra of Matrices Ex 5.4 Q6(ii)

Given,

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix},\ B = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$$

$$(AB)^T = B^T A^T$$

$$\Rightarrow \quad \left(\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}\right)^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}^T \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}^T$$

$$\Rightarrow \quad \begin{bmatrix} 1+6 & 4+15 \\ 2+8 & 8+20 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 7 & 19 \\ 10 & 28 \end{bmatrix}^T = \begin{bmatrix} 1+6 & 2+8 \\ 4+15 & 8+20 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 7 & 10 \\ 19 & 28 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 19 & 28 \end{bmatrix}$$

$$\Rightarrow \quad \text{LHS} = \text{RHS}$$

So,

$$(AB)^T = B^T A^T$$

Algebra of Matrices Ex 5.4 Q7

$$Given\ that\ A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} and\ B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}.$$

$$We\ need\ to\ find\ A^T - B^T.$$

$$Given\ that\ ,\ B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$\Rightarrow B^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}^T = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

$$Let\ us\ find\ A^T - B^T:$$

$$A^T - B^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

$$\Rightarrow A^T - B^T = \begin{bmatrix} 3+1 & 4-1 \\ -1-2 & 2-2 \\ 0-1 & 1-3 \end{bmatrix}$$

$$\Rightarrow A^T - B^T = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

Algebra of Matrices Ex 5.4 Q8

(i)

$$A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$$

$$\therefore A'=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$$

$$A'A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$$
$$=\begin{bmatrix}(\cos\alpha)(\cos\alpha)+(-\sin\alpha)(-\sin\alpha) & (\cos\alpha)(\sin\alpha)+(-\sin\alpha)(\cos\alpha)\\ (\sin\alpha)(\cos\alpha)+(\cos\alpha)(-\sin\alpha) & (\sin\alpha)(\sin\alpha)+(\cos\alpha)(\cos\alpha)\end{bmatrix}$$
$$=\begin{bmatrix}\cos^2\alpha+\sin^2\alpha & \sin\alpha\cos\alpha-\sin\alpha\cos\alpha\\ \sin\alpha\cos\alpha-\sin\alpha\cos\alpha & \sin^2\alpha+\cos^2\alpha\end{bmatrix}$$
$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

Hence, we have verified that $A'A=I$.

Algebra of Matrices Ex 5.4 Q9

(ii)

$$A=\begin{bmatrix}\sin\alpha & \cos\alpha\\ -\cos\alpha & \sin\alpha\end{bmatrix}$$

$$\therefore A'=\begin{bmatrix}\sin\alpha & -\cos\alpha\\ \cos\alpha & \sin\alpha\end{bmatrix}$$

$$A'A=\begin{bmatrix}\sin\alpha & -\cos\alpha\\ \cos\alpha & \sin\alpha\end{bmatrix}\begin{bmatrix}\sin\alpha & \cos\alpha\\ -\cos\alpha & \sin\alpha\end{bmatrix}$$

$$\begin{bmatrix}\sin\alpha & -\cos\alpha\\ \cos\alpha & \sin\alpha\end{bmatrix}\begin{bmatrix}\sin\alpha & \cos\alpha\\ -\cos\alpha & \sin\alpha\end{bmatrix}$$
$$=\begin{bmatrix}(\sin\alpha)(\sin\alpha)+(-\cos\alpha)(-\cos\alpha) & (\sin\alpha)(\cos\alpha)+(-\cos\alpha)(\sin\alpha)\\ (\cos\alpha)(\sin\alpha)+(\sin\alpha)(-\cos\alpha) & (\cos\alpha)(\cos\alpha)+(\sin\alpha)(\sin\alpha)\end{bmatrix}$$
$$=\begin{bmatrix}\sin^2\alpha+\cos^2\alpha & \sin\alpha\cos\alpha-\sin\alpha\cos\alpha\\ \sin\alpha\cos\alpha-\sin\alpha\cos\alpha & \cos^2\alpha+\sin^2\alpha\end{bmatrix}$$
$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

Hence, we have verified that $A'A=I$.

Algebra of Matrices Ex 5.4 Q10

Given,

l_i, m_i, n_i are direction cosines of three mutually perpendicular vectors

$$\Rightarrow \left.\begin{matrix} l_1 l_2 + m_1 m_2 + n_1 n_2 = 0 \\ l_2 l_3 + m_2 m_3 + n_2 n_3 = 0 \\ l_1 l_3 + m_1 m_3 + n_1 n_3 = 0 \end{matrix}\right\} \quad ---(A)$$

And,

$$\left.\begin{matrix} {l_1}^2 + {m_1}^2 + {n_1}^2 = 1 \\ {l_2}^2 + {m_2}^2 + {n_2}^2 = 1 \\ {l_3}^2 + {m_3}^2 + {n_3}^2 = 1 \end{matrix}\right\} \quad ---(B)$$

Given,

$$A = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}^T$$

$$= \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \begin{bmatrix} l_1 & l_1 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix}$$

$$= \begin{bmatrix} {l_1}^2 + {m_1}^2 + {n_1}^2 & l_1 l_2 + m_1 m_2 + n_1 n_2 & l_1 l_3 + m_1 m_3 + n_1 n_3 \\ l_1 l_2 + m_1 m_2 + n_1 n_2 & {l_2}^2 + {m_2}^2 + {n_2}^2 & l_2 l_3 + m_2 m_3 + n_2 n_3 \\ l_1 l_3 + m_1 m_3 + n_1 n_3 & l_3 l_2 + m_3 m_2 + n_3 n_2 & {l_3}^2 + {m_3}^2 + {n_3}^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \{\text{Using (A) and (B)}\}$$

$$= I$$

Hence,

$$AA^T = I$$

Ex 5.5

Algebra of Matrices Ex 5.5 Q1

Given,

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

$$\left(A - A^T\right) = \left(\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^T\right)$$

$$= \left(\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 2-2 & 3-4 \\ 4-3 & 5-5 \end{bmatrix}$$

$$\left(A - A^T\right) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad ---(i)$$

$$-\left(A - A^T\right)^T = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^T$$

$$= -\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$-\left(A - A^T\right)^T = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad ---(ii)$$

From (i) and (ii),

$$\left(A - A^T\right) = -\left(A - A^T\right)^T$$

We know that, x is a skew symmetric matrix if $x = -x^T$

So, $\left(A - A^T\right)$ is skew symmetric.

Algebra of Matrices Ex 5.5 Q2

Given,

$$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

$$A - A^T = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3-3 & -4-1 \\ 1+4 & -1+1 \end{bmatrix}$$

$$A - A^T = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} \qquad \text{---(i)}$$

$$-\left(A - A^T\right)^T = -\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}^T$$

$$= -\begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}$$

$$-\left(A - A^T\right)^T = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} \qquad \text{---(ii)}$$

From equation (i) and (ii),

$$\left(A - A^T\right) = -\left(A - A^T\right)^T$$

We know that, x is skewsymmetric matrx if $x = -x^T$

So, $\left(A - A^T\right)$ is skewsymmetric matrix.

Algebra of Matrices Ex 5.5 Q3

Given,

$$A = \begin{bmatrix} 5 & 2 & x \\ y & z & -3 \\ 4 & t & -7 \end{bmatrix} \text{ is a symmetric matrix.}$$

We know that $A = \left[aij\right]_{m \times n}$ is a symmetric matrix if $aij = aji$

So, $x = a_{13} = a_{31} = 4$

$y = a_{21} = a_{12} = 2$

$z = a_{22} = a_{22} = z$

$t = a_{32} = a_{32} = -3$

Hence,

$x = 4, y = 2, t = -3$ and z can have any value.

Algebra of Matrices Ex 5.5 Q4

Given,

$$A = \begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix} \qquad \Rightarrow \qquad A^T = \begin{bmatrix} 3 & 1 & -2 \\ 2 & 4 & 5 \\ 7 & 3 & 8 \end{bmatrix}$$

$$\therefore X = \frac{1}{2}\left(A + A^T\right)$$

$$= \frac{1}{2}\left(\begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix} + \begin{bmatrix} 3 & 1 & -2 \\ 2 & 4 & 5 \\ 7 & 3 & 8 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 3+3 & 2+1 & 7-2 \\ 1+2 & 4+4 & 3+5 \\ -2+7 & 5+3 & 8+8 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 6 & 3 & 5 \\ 3 & 8 & 8 \\ 5 & 8 & 16 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix}$$

Now, $Y = \frac{1}{2}\left(A - A^T\right)$

$$= \frac{1}{2}\left(\begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix} - \begin{bmatrix} 3 & 1 & -2 \\ 2 & 4 & 5 \\ 7 & 3 & 8 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 3-3 & 2-1 & 7+2 \\ 1-2 & 4-4 & 3-5 \\ -2-7 & 5-3 & 8-8 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & 1 & 9 \\ -1 & 0 & -2 \\ -9 & 2 & 0 \end{bmatrix}$$

$$Y = \begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ \frac{-1}{2} & 0 & -1 \\ \frac{-9}{2} & 1 & 0 \end{bmatrix}$$

Now,

$$X^T = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix}^T = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix} = X$$

$\Rightarrow$ x is a symmetric matrix

Now,

$$-Y^T = -\begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ \frac{-1}{2} & 0 & -1 \\ \frac{-9}{2} & 1 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & \frac{-1}{2} & \frac{-9}{2} \\ \frac{1}{2} & 0 & 1 \\ \frac{9}{2} & -1 & 0 \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ \frac{-1}{2} & \frac{1}{2} & -1 \\ \frac{-9}{2} & 1 & 0 \end{bmatrix}$$

$$\Rightarrow \quad -Y' = Y$$

$\therefore \quad Y$ is skew symmetric matrix.

$$X+Y = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ \frac{-1}{2} & 0 & -1 \\ \frac{-9}{2} & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3+0 & \frac{3}{2}+\frac{1}{2} & \frac{5}{2}+\frac{9}{2} \\ \frac{3}{2}-\frac{1}{2} & 4+0 & 4-1 \\ \frac{5}{2}-\frac{9}{2} & 4+1 & 8-0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix}$$

$$= A$$

Hence,

$$X = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{4} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix}, \quad Y = \begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ \frac{-1}{2} & 0 & -1 \\ \frac{-9}{2} & 1 & 0 \end{bmatrix}$$

Algebra of Matrices Ex 5.5 Q5

Given,

$$A = \begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix} \qquad \Rightarrow \qquad A^T = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 5 & -2 \\ -1 & 7 & 1 \end{bmatrix}$$

Let $\quad X = \frac{1}{2}\left(A + A^T\right)$

$$= \frac{1}{2}\left(\begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 3 & 1 \\ 2 & 5 & -2 \\ -1 & 7 & 1 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 4+4 & 2+3 & -1+1 \\ 3+2 & 5+5 & 7-2 \\ 1-1 & -2+7 & 1+1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 8 & 5 & 0 \\ 5 & 10 & 5 \\ 0 & 5 & 2 \end{bmatrix} = \begin{bmatrix} 4 & \frac{5}{2} & 0 \\ \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{bmatrix}$$

$$X^T = \begin{bmatrix} 4 & \frac{5}{2} & 0 \\ \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{bmatrix}^T = \begin{bmatrix} 4 & \frac{5}{2} & 0 \\ \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{bmatrix}^T = X$$

$\therefore \qquad X$ is symmetric matrix

Now,

$$Y = \frac{1}{2}\left(A - A^T\right)$$

$$= \frac{1}{2}\left(\begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 3 & 1 \\ 2 & 5 & -2 \\ -1 & 7 & 1 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 4-4 & 2-3 & -1-1 \\ 3-2 & 5-5 & 7+2 \\ 1+1 & -2-7 & 1-1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & 9 \\ 2 & -9 & 0 \end{bmatrix}$$

$$\therefore Y = \begin{bmatrix} 0 & \frac{-1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & \frac{-9}{2} & 0 \end{bmatrix}$$

$$\Rightarrow -Y^T = -\begin{bmatrix} 0 & \frac{-1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & \frac{-9}{2} & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & \frac{-1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & \frac{-9}{2} & 0 \end{bmatrix} = Y$$

$\Rightarrow \qquad Y$ is a skew symmetric maatrix.

Now,

$$X+Y=\begin{bmatrix}4 & \frac{5}{2} & 0\\ \frac{5}{2} & 5 & \frac{5}{2}\\ 0 & \frac{5}{2} & 1\end{bmatrix}+\begin{bmatrix}0 & \frac{-1}{2} & -1\\ \frac{1}{2} & 0 & \frac{9}{2}\\ 1 & \frac{-9}{2} & 0\end{bmatrix}$$

$$=\begin{bmatrix}4+0 & \frac{5}{2}-\frac{1}{2} & 0-1\\ \frac{5}{2}+\frac{1}{2} & 5+0 & \frac{5}{2}+\frac{9}{2}\\ 0+1 & \frac{5}{2}-\frac{9}{2} & 1+0\end{bmatrix}$$

$$=\begin{bmatrix}4 & 2 & -1\\ 3 & 5 & 7\\ 1 & -2 & 1\end{bmatrix}=A$$

Algebra of Matrices Ex 5.5 Q6

A square matrix A is called a symmetric matrix, if $A^T = A$

Here,

$$A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$$

$$A + A^T = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}^T$$

$$= \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & 4+5 \\ 5+4 & 6+6 \end{bmatrix}$$

$$A + A^T = \begin{bmatrix} 4 & 9 \\ 9 & 12 \end{bmatrix} \qquad ---(i)$$

$$\left(A + A^T\right)^T = \begin{bmatrix} 4 & 9 \\ 9 & 12 \end{bmatrix}^T$$

$$\left(A + A^T\right)^T = \begin{bmatrix} 4 & 9 \\ 9 & 12 \end{bmatrix} \qquad ---(ii)$$

From equation (i) and (ii),

$$\left(A + A^T\right)^T = \left(A + A^T\right)$$

So,

$\left(A + A^T\right)$ is a symmetric matrix.

Algebra of Matrices Ex 5.5 Q7

Here,

$$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \quad \Rightarrow A^T = \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$$

$$Let, \quad X = \frac{1}{2}\left(A + A^T\right) = \frac{1}{2}\left(\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 3+3 & -4+1 \\ 1-4 & -1-1 \end{bmatrix} = \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix}$$

$$Now, \quad X^T = \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix}^T = \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix}^T = X$$

$\Rightarrow \quad X$ is symmetric matrix

$$Let \quad Y = \frac{1}{2}\left(A - A^T\right) = \frac{1}{2}\left(\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 3-3 & -4-1 \\ 1+4 & -1+1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$$

$$Now, \quad -Y^T = -\begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix} = Y$$

$\Rightarrow \quad Y$ is skew symmetric

$$Now, \quad X + Y = \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix} = \begin{bmatrix} 3+0 & \frac{-3}{2} - \frac{5}{2} \\ \frac{-3}{2} + \frac{5}{2} & -1+0 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = A$$

Algebra of Matrices Ex 5.5 Q8

Let,

$$A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} \quad \Rightarrow \quad A^T = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

$$\text{Let, } X = \frac{1}{2}\left(A + A^T\right) = \frac{1}{2}\left(\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 3+3 & -2+3 & -4-1 \\ 3-2 & -2-2 & -5+1 \\ -1-4 & 1-5 & 2+2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix}$$

$$\text{Now, } \quad X^T = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix}^T = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix} = X$$

$\Rightarrow$ X is a symmetric matrix

$$\text{Let, } Y = \frac{1}{2}\left(A - A^T\right) = \frac{1}{2}\left(\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 3-3 & -2-3 & -4+1 \\ 3+2 & -2+2 & -5-1 \\ -1+4 & 1+5 & 2-2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{-5}{2} & \frac{-3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix}$$

$$-Y^T = -\begin{bmatrix} 0 & \frac{-5}{2} & \frac{-3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & \frac{-5}{2} & \frac{-3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix} = Y$$

$\Rightarrow$ Y is a skew symmetric matrix

$$X + Y = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-5}{2} & \frac{-3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix} = \begin{bmatrix} 3+0 & \frac{1}{2}-\frac{5}{2} & \frac{-5}{2}-\frac{3}{2} \\ \frac{1}{2}+\frac{5}{2} & -2+0 & -2-3 \\ \frac{-5}{2}+\frac{3}{2} & -2+3 & 2+0 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} = A$$

$$\text{Hence, } \quad \text{Symmetric matrix } X = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix}$$

Ex 6.1

Determinants Ex 6.1 Q1(i)

Let M_{ij} and C_{ij} represents the minor and co-factor respectively of an element which is placed at the i^{th} row and j^{th} column.

Now,

$M_{11} = -1$ $\left[\begin{array}{l}\text{In a } 2\times 2 \text{ matrix, the minor is obtained for a particular element, by} \\ \text{deleting that row and column where the element is present.}\end{array}\right]$

$M_{21} = 20$

$C_{11} = (-1)^{1+1} \times M_{11}$ $\left[\because C_{ij} = (-1)^{i+j} \times M_{ij}\right]$

$= (+1)(-1)$

$= -1$

$C_{21} = (-1)^{2+1} M_{21}$

$= (-1)^3 \times 20$

$= -20$

Also,

$|A| = 5(-1) - (0) \times (20)$ $\left[\text{If } A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{then } |A| = a_{11}a_{22} - a_{21}a_{12}\right]$

$= -5$

Determinants Ex 6.1 Q1(ii)

Let M_{ij} and C_{ij} represents the minor and co-factor respectively of an element which is present at the i^{th} row and j^{th} column.

Now,

$M_{11} = 3$ $\qquad \left[\begin{array}{l}\text{In a } 2\times 2 \text{ matrix, the minor of an element is obtained by}\\ \text{deleting that row and that column in which it is present.}\end{array}\right]$

$M_{21} = 4$

$C_{11} = (-1)^{1+1} \times M_{11}$ $\qquad \left[C_{ij} = (-1)^{i+j} \times M_{ij}\right]$

$C_{21} = (-1)^{2+1} \times M_{21}$

$= (-1)^3 \times 4$

$= -4$

Also,

$|A| = (-1)\times(3) - (2)\times(4)$

$= -3 - 8$

$= -11$

Determinants Ex 6.1 Q1(iii)

Let M_{ij} and C_{ij} represents the minor and co-factor respectively of an element which is placed at the i^{th} row and j^{th} column.

Now,

$M_{11} = \begin{bmatrix}-1 & 2\\ 5 & 2\end{bmatrix}$ $\qquad \left[\begin{array}{l}\text{In a } 3\times 3 \text{ matrix, } M_{ij} \text{ equals to the determinant of the } 2\times 2\\ \text{sub-matrix obtained by leaving the } i^{th} \text{ row and } j^{th} \text{ column of } A.\end{array}\right]$

$= (-1)\times(2) - (5)\times(2)$

$= -2 - 10$

$= -12$

$M_{21} = \begin{bmatrix}-3 & 2\\ 5 & 2\end{bmatrix} = (-3)\times(2) - (5)\times(2) = -6 - 10 = -16$

$M_{31} = \begin{bmatrix}-3 & 2\\ -1 & 2\end{bmatrix} = (-3)(2) - (-1)(2) = -6 + 2 = -4$

$C_{11} = (-1)^{1+1} M_{11}$ $\qquad \left(C_{ij} = (-1)^{i+j} \times M_{ij}\right)$

$= (+)(-12) = -12$

$C_{21} = (-1)^{2+1} M_{21} = (-1)^3(-16) = 16$

$C_{31} = (-1)^{3+1} M_{31} = (-1)^4(-4) = -4$

Also, expanding the determinant along the first column.

$|A| = a_{11} \times \left((-1)^{1+1} \times M_{11}\right) + a_{21} \times \left((-1)^{2+1} \times M_{21}\right) + a_{31} \times \left((-1)^{3+1} \times M_{31}\right)$

$= a_{11} \times C_{11} + a_{21} \times C_{21} + a_{31} \times C_{31}$

$= 1 \times (-12) + 4 \times 16 + 3 \times (-4)$

$= -12 + 48 - 12 = 24$

Determinants Ex 6.1 Q1(iv)

Let M_{ij} and C_{ij} are respectively the minor and co-factor of the element a_{ij}.

Now,

$$M_{11} = \begin{bmatrix} b & ca \\ c & ab \end{bmatrix}$$
$$= ab^2 - ac^2$$

$$M_{21} = \begin{bmatrix} a & bc \\ c & ab \end{bmatrix}$$
$$= a^2b - c^2b$$

$$M_{31} = \begin{bmatrix} a & bc \\ b & ca \end{bmatrix}$$
$$= a^2c - b^2c$$

$$C_{11} = (-1)^{1+1} \times M_{11} = +\left(ab^2 - ac^2\right)$$
$$C_{21} = (-1)^{2+1} \times M_{21} = -\left(a^2b - c^2b\right)$$
$$C_{31} = (-1)^{3+1} \times M_{31} = +\left(a^2c - b^2c\right)$$

Also, expanding the determinant, along the first column.

$$\begin{aligned}|A| &= a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31} \\ &= 1\left(ab^2 - ac^2\right) + 1\left(c^2b - a^2b\right) + 1 \times \left(a^2c - b^2c\right) \\ &= ab^2 - ac^2 + c^2b - a^2b + a^2c - b^2c\end{aligned}$$

Determinants Ex 6.1 Q1(v)

Let M_{ij} and C_{ij} are respectively the minor and co-factor of the element a_{ij}.

Now,

$$M_{11} = \begin{bmatrix} 5 & 0 \\ 7 & 1 \end{bmatrix} = 5 - 0 = 5$$
$$M_{21} = \begin{bmatrix} 2 & 6 \\ 7 & 1 \end{bmatrix} = 2 - 42 = -40$$
$$M_{31} = \begin{bmatrix} 2 & 6 \\ 5 & 0 \end{bmatrix} = 0 - 30 = -30$$

$$C_{11} = (-1)^{1+1} \times M_{11} = +5$$
$$C_{21} = (-1)^{2+1} \times M_{21} = (-)(-40) = 40$$
$$C_{31} = (-1)^{3+1} \times M_{31} = +(-30) = -30$$

Now, expanding the determinant along the first column.

$$\begin{aligned}|A| &= a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31} \\ &= 0 \times 5 + 1 \times (40) + 3 \times (-30) \\ &= 40 - 90 \\ &= -50\end{aligned}$$

Determinants Ex 6.1 Q1(vi)

Let M_{ij} and C_{ij} are respectively the minor and co-factor of the element a_{ij}.

Now,

$$M_{11} = \begin{bmatrix} b & f \\ f & c \end{bmatrix} = bc - f^2$$

$$M_{21} = \begin{bmatrix} h & g \\ f & c \end{bmatrix} = hc - gf$$

$$M_{31} = \begin{bmatrix} h & g \\ b & f \end{bmatrix} = hf - bg$$

$$Also\ C_{11} = (-1)^{1+1} M_{11} = bc - f^2$$

$$C_{21} = (-1)^{2+1} M_{21} = -(hc - gf)$$

$$C_{31} = (-1)^{3+1} M_{31} = hf - bg$$

Also, expanding along the first column.

$$\begin{aligned} |A| &= a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31} \\ &= a(bc - f^2) + h(-)(hc - gf) + g(hf - bg) \\ &= abc - af^2 + hgf - h^2c + ghf - bg^2 \end{aligned}$$

Determinants Ex 6.1 Q1(vii)

We have,

$$\begin{bmatrix} 2 & -1 & 0 & 1 \\ -3 & 0 & 1 & -2 \\ 1 & 1 & -1 & 1 \\ 2 & -1 & 5 & 0 \end{bmatrix}$$

Here, $M_{11} = \begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & 1 \\ -1 & 5 & 0 \end{bmatrix} = -1(0+10) - 1(1-2) = -9$

$$M_{21} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 1 \\ -1 & 5 & 0 \end{bmatrix} = 9$$

$$M_{31} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & -2 \\ -1 & 5 & 0 \end{bmatrix} = -9$$

$$M_{41} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & - & 1 \end{bmatrix} = 0$$

$\therefore$

$$C_{11} = (-1)^{1+1} M_{11} = -9$$

$$C_{21} = (-1)^{3} M_{21} = -9$$

$$C_{31} = (-4)^{4} M_{31} = -9$$

$$C_{41} = (-1)^{5} M_{41} = 0$$

Hence,

$$\begin{bmatrix} 2 & -1 & 0 & 1 \\ -3 & 0 & 1 & -2 \\ 1 & 1 & -1 & 1 \\ 2 & -1 & 5 & 0 \end{bmatrix} = 2 \times C_{11} + (-3)C_{21} + 1 \times C_{31} + 2 \times C_{41} = -9[2 - 3 + 1] = 0$$

Determinants Ex 6.1 Q2(i)

Let $A = \begin{vmatrix} x & -7 \\ x & 5x+1 \end{vmatrix}$

$$\begin{aligned} |A| &= x(5x+1) + 7 \times x \\ &= 5x^2 + x + 7x \\ &= 5x^2 + 8x \end{aligned}$$

Hence $|A| = 5x^2 + 8x$

Determinants Ex 6.1 Q2(ii)

Let $A = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$

$$\begin{aligned} |A| &= \cos\theta \times \cos\theta + \sin\theta \times \sin\theta \\ &= \cos^2\theta + \sin^2\theta \\ &= 1 \end{aligned}$$

Hence $|A| = 1$

Determinants Ex 6.1 Q2(iii)

Let $A = \begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$

$$\begin{aligned} |A| &= \cos 15^\circ \cos 75^\circ - \sin 15^\circ \sin 75^\circ \\ &= \cos(75+15) \qquad (\because \cos A \cos B - \sin A \sin B = \cos(A+B)) \\ &= \cos 90^\circ \\ &= 0 \end{aligned}$$

Hence $|A| = 0$

Determinants Ex 6.1 Q2(iv)

Let $A = \begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

$$\begin{aligned} |A| &= (a+ib)(a-ib) - (c+id)(-c+id) \\ &= (a^2+b^2) + (c+id)(c-id) \qquad (\text{Taking } (-) \text{ sign common from } -c+id) \end{aligned}$$

$$\left(Also\ (a+ib)(a-ib) = a^2+b^2\right)$$

$= a^2+b^2+c^2+d^2$

Hence $|A| = a^2+b^2+c^2+d^2$

Determinants Ex 6.1 Q3

Since $|AB| = |A| \times |B|$

Hence $|A|^2 = |A| \times |A| \qquad ---(1)$

Now let $A = \begin{vmatrix} 2 & 3 & 7 \\ 13 & 17 & 5 \\ 15 & 20 & 12 \end{vmatrix}$

Expanding along the first column, we get

$$\begin{aligned} |A| &= 2\begin{vmatrix} 17 & 5 \\ 20 & 12 \end{vmatrix} - 3\begin{vmatrix} 13 & 5 \\ 15 & 12 \end{vmatrix} + 7\begin{vmatrix} 13 & 17 \\ 15 & 20 \end{vmatrix} \\ &= 2(204-100) - 3(156-75) + 7(260-255) \\ &= 2(104) - 3(81) + 7(5) \\ &= 208 - 243 + 35 \\ &= 243 - 243 \\ &= 0 \end{aligned}$$

Hence from eq.(1)

$|A|^2 = |A| \times |A| = 0 \times 0 = 0$

Determinants Ex 6.1 Q4

Evaluating the given determinant

$sin\,10^\circ \times cos\,80^\circ + cos\,10^\circ\ sin\,80^\circ$

$= sin\left(10^\circ + 80^\circ\right)$ $\left[\because sin\,A\,cos\,B + cos\,A\,sin\,B = sin\left(A+B\right)\right]$

$= sin\,90^\circ$

$= 1$

Henceproved

Determinants Ex 6.1 Q5

We will evaluate the given determinant

(i) Along the first row

(ii) Along the first column

(i) Along the first row

$$|A| = 2\begin{vmatrix}1 & -2\\4 & 1\end{vmatrix} - 3\begin{vmatrix}7 & -2\\-3 & 1\end{vmatrix} - 5\begin{vmatrix}7 & 1\\-3 & 4\end{vmatrix}$$
$$= 2\left(1+8\right) - 3\left(7-6\right) - 5\left(28+3\right)$$
$$= 2\left(9\right) - 3\left(1\right) - 5\left(31\right)$$
$$= 18 - 3 - 155 = -140$$

(ii)Along the first column

$$|A| = 2\begin{vmatrix}1 & -2\\4 & 1\end{vmatrix} - 7\begin{vmatrix}3 & -5\\4 & 1\end{vmatrix} - 3\begin{vmatrix}3 & -5\\1 & -2\end{vmatrix}$$
$$= 2\left(1+8\right) - 7\left(3+20\right) - 3\left(-6+5\right)$$
$$= 18 - 7\left(23\right) - 3\left(-1\right)$$
$$= 18 - 161 + 3$$
$$= 21 - 161$$
$$= -140$$

We can see, the answer is same with both the methods.

Determinants Ex 6.1 Q6

$$\Delta = \begin{vmatrix}0 & \sin\alpha & -\cos\alpha\\-\sin\alpha & 0 & \sin\beta\\\cos\alpha & -\sin\beta & 0\end{vmatrix}$$
$$= -\sin\alpha(-\sin\beta\cos\alpha) - \cos\alpha(\sin\alpha\sin\beta)$$
$$= 0$$

Determinants Ex 6.1 Q7

$$\Delta = \begin{vmatrix}\cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha\\-\sin\beta & \cos\beta & 0\\\sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha\end{vmatrix}$$

Expanding along C_3, we have:

$$\Delta = -\sin\alpha\left(-\sin\alpha\sin^2\beta - \cos^2\beta\sin\alpha\right) + \cos\alpha\left(\cos\alpha\cos^2\beta + \cos\alpha\sin^2\beta\right)$$
$$= \sin^2\alpha\left(\sin^2\beta + \cos^2\beta\right) + \cos^2\alpha\left(\cos^2\beta + \sin^2\beta\right)$$
$$= \sin^2\alpha\left(1\right) + \cos^2\alpha\left(1\right)$$
$$= 1$$

Determinants Ex 6.1 Q8

Let $A = \begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}$

$\Rightarrow |A| = 2 - 10 = -8$

$B = \begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix}$

$\Rightarrow |B| = 20 + 6 = 26$

$$\text{Now } AB = \begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix}$$
$$= \begin{bmatrix} 2\times 4 + 5\times 2 & 2\times(-3) + 5\times 5 \\ 2\times 4 + 1\times 2 & 2\times(-3) + 1\times 5 \end{bmatrix}$$
$$= \begin{bmatrix} 8+10 & -6+25 \\ 8+2 & -6+5 \end{bmatrix}$$
$$= \begin{bmatrix} 18 & 19 \\ 10 & -1 \end{bmatrix}$$

$$\Rightarrow \quad |AB| = 18\times(-1) - (10)(19)$$
$$= -18 - 190 = -208$$

Now $|AB| = |A| \times |B|$

$-208 = (-8)\times(26)$

$-208 = -208$

Hence verified.

Determinants Ex 6.1 Q9

Let $A = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix}$

Evaluating the determinant along the first column

$$|A| = 1\begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix} - 0\begin{vmatrix} 0 & 1 \\ 0 & 4 \end{vmatrix} + 0\begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix}$$
$$= 1\times(4-0) - 0 + 0$$
$$= 4$$

Again $3A = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{bmatrix}$ (every element of A will be multiplied by 3)

Now, evaluating this determinant

$$|3A| = 3\begin{vmatrix} 3 & 6 \\ 0 & 12 \end{vmatrix} - 0\begin{vmatrix} 0 & 3 \\ 0 & 12 \end{vmatrix} + 0\begin{vmatrix} 0 & 3 \\ 3 & 6 \end{vmatrix}$$
$$= 3(36 - 0) - 0 + 0$$
$$= 108$$

Now, according to the question

$|3A| = 27|A|$

$108 = 27(4)$ (Substituting values)

$108 = 108$

Hence proved

Determinants Ex 6.1 Q10

(i) $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

$\Rightarrow 2\times1-5\times4=2x\times x-6\times4$
$\Rightarrow 2-20=2x^2-24$
$\Rightarrow 2x^2=6$
$\Rightarrow x^2=3$
$\Rightarrow x=\pm\sqrt{3}$

(ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

$\Rightarrow 2\times5-3\times4=x\times5-3\times2x$
$\Rightarrow 10-12=5x-6x$
$\Rightarrow -2=-x$
$\Rightarrow x=2$

(iii)

$\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$
$3-x^2=3-8$
$x^2=8$
$x=\pm2\sqrt{2}$

(iv)

$\begin{vmatrix} 3x & 7 \\ 2 & 4 \end{vmatrix} = 10$
$12x-14=10$
$12x=24$
$x=2$

Determinants Ex 6.1 Q11

Let $A = \begin{vmatrix} x^2 & x & 1 \\ 0 & 2 & 1 \\ 3 & 1 & 4 \end{vmatrix}$

Expanding the given determinant along the first column

$|A| = x^2\begin{vmatrix} 2 & 1 \\ 1 & 4 \end{vmatrix} - 0\begin{vmatrix} x & 1 \\ 1 & 4 \end{vmatrix} + 3\begin{vmatrix} x & 1 \\ 2 & 1 \end{vmatrix}$
$28 = x^2(8-1)-0(4x-1)+3(x-2)$
$28 = 7x^2+3x-6$
or
$7x^2+3x-6=28$
$7x^2+3x-34=0$
Solving using quadratic formula, we get x = 2.

Determinants Ex 6.1 Q12(i)

A matrix A is called singular if $|A| = 0$

Now expanding along the first row $|A|$

$$= (x-1)\begin{vmatrix} x-1 & 1 \\ 1 & x-1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & x-1 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ x-1 & 1 \end{vmatrix}$$
$$= (x-1)\left[(x-1)^2 - 1\right] - 1\left[x-1-1\right] + 1\left[1-x+1\right]$$
$$= (x-1)\left(x^2+1-2x-1\right) - 1(x-2) + 1(2-x)$$
$$= (x-1)\left(x^2-2x\right) - x + 2 + 2 - x$$
$$= (x-1) \times x \times (x-2) + (4-2x)$$
$$= (x-1) \times x \times (x-2) + 2(2-x)$$
$$= (x-1) \times x \times (x-2) - 2(x-2)$$
$$= (x-2)\left[x(x-1) - 2\right] \qquad \text{(Taking } (x-2) \text{ common)}$$

Since A is a singular matrix, so $|A| = 0$

$i.e\ (x-2)\left(x^2 - x - 2\right) = 0$

either $(x-2) = 0$ $\qquad$ $or\ x^2 - x - 2 = 0$

$x = 2$ $\qquad$ $or\ x^2 - 2x + x - 2 = 0$

$x(x-2) + 1(x-2) = 0$

$(x-2)(x+1) = 0$

$x = 2, -1$

$x = 2 \text{ or } -1$

Determinants Ex 6.1 Q12(ii)

A matrix A is said to be singular if $|A| = 0$

Now

$$\begin{vmatrix} 1+x & 7 \\ 3-x & 8 \end{vmatrix} = 0$$
$$8 + 8x - 21 + 7x = 0$$
$$15x = 13$$
$$x = \frac{13}{15}$$

Ex 6.2

Chapter 6 Determinants Ex 6.2 Q1-i

$$\begin{vmatrix} 1 & 3 & 5 \\ 2 & 6 & 10 \\ 31 & 11 & 38 \end{vmatrix} = 2\begin{vmatrix} 1 & 3 & 5 \\ 1 & 3 & 5 \\ 31 & 11 & 38 \end{vmatrix} = 0$$

Chapter 6 Determinants Ex 6.2 Q1-ii

Consider the determinant

$$\Delta = \begin{vmatrix} 67 & 19 & 21 \\ 39 & 13 & 14 \\ 81 & 24 & 26 \end{vmatrix}$$

Applying $C_1 \to C_1 - 4C_3$, *we get*,

$$\Delta = \begin{vmatrix} 4 & 19 & 21 \\ -3 & 13 & 14 \\ -3 & 24 & 26 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 4 & 19 & 21 \\ -3 & 13 & 14 \\ -3 & 24 & 26 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 32 & 35 \\ -3 & 13 & 14 \\ 0 & 11 & 12 \end{vmatrix} \quad [\text{Applying } R_3 \to R_3 - R_2 \text{ and } R_1 \to R_1 + R_2]$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 32 & 35 \\ 0 & 109 & 119 \\ 0 & 11 & 12 \end{vmatrix} \quad [\text{Applying } R_2 \to 3R_1 + R_2\ ]$$

$$\Rightarrow \Delta = 1(109 \times 12 - 119 \times 11)$$

$$\Rightarrow \Delta = -1$$

Chapter 6 Determinants Ex 6.2 Q1-iii

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

$= a(bc - f^2) - h(hc - fg) + g(hf - bg)$

$= abc - af^2 - h^2c + hfg + ghf - bg^2$

Chapter 6 Determinants Ex 6.2 Q1-iv

$$\begin{vmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{vmatrix} = 2\begin{vmatrix} 1 & -3 & 1 \\ 4 & -1 & 1 \\ 3 & 5 & 1 \end{vmatrix} = 2\begin{vmatrix} 1 & -3 & 1 \\ 3 & 2 & 0 \\ 2 & 8 & 0 \end{vmatrix} = 2(24 - 4) = 40$$

Chapter 6 Determinants Ex 6.2 Q1-v

Let Δ be the determinant.

$$\Delta = \begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix}$$

Applying $R_3 \to R_3 - R_2$, we get,

$$\Delta = \begin{vmatrix} 1 & 4 & 9-4 \\ 4 & 9 & 16-9 \\ 9 & 16 & 25-16 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 4 & 5 \\ 4 & 9 & 7 \\ 9 & 16 & 9 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 5 & 5 \\ 4 & 13 & 7 \\ 9 & 25 & 9 \end{vmatrix} \quad [Applying\ C_2 \to C_1 + C_2]$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 0 & 0 \\ 4 & -7 & -13 \\ 9 & -20 & -36 \end{vmatrix} \quad [Applying\ C_2 \to 5C_1 - C_2\ and\ C_3 \to 5C_1 - C_3]$$

$$\Rightarrow \Delta = 1(7 \times 36 - 13 \times 20) = 252 - 260 = -8$$

Chapter 6 Determinants Ex 6.2 Q1-vi

$$\begin{vmatrix} 6 & -3 & 2 \\ 2 & -1 & 2 \\ -10 & 5 & 2 \end{vmatrix}$$

Apply: $R_1 \to R_1 + (-3)R_2$ and $R_3 \to R_3 + 5R_2$

$$= \begin{vmatrix} 0 & 0 & -4 \\ 2 & -1 & 2 \\ 0 & 0 & 12 \end{vmatrix} = 0$$

Chapter 6 Determinants Ex 6.2 Q1-vii

$$\begin{vmatrix} 1 & 3 & 9 & 27 \\ 3 & 9 & 27 & 1 \\ 9 & 27 & 1 & 3 \\ 27 & 1 & 3 & 9 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 3^2 & 3^3 \\ 3 & 3^2 & 3^3 & 1 \\ 3^2 & 3^3 & 1 & 3 \\ 3^3 & 1 & 3 & 3^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1+3+3^2+3^3 & 3 & 3^2 & 3^3 \\ 1+3+3^2+3^3 & 3^2 & 3^3 & 1 \\ 1+3+3^2+3^3 & 3^3 & 1 & 3 \\ 1+3+3^2+3^3 & 1 & 3 & 3^2 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right) \begin{vmatrix} 1 & 3 & 3^2 & 3^3 \\ 1 & 3^2 & 3^3 & 1 \\ 1 & 3^3 & 1 & 3 \\ 1 & 1 & 3 & 3^2 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right) \begin{vmatrix} 1 & 3 & 3^2 & 3^3 \\ 0 & 3^2-3 & 3^3-3^2 & 1-3^3 \\ 0 & 3^3-3 & 1-3^2 & 3-3^3 \\ 0 & 1-3 & 3-3^2 & 3^2-3^3 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right) \begin{vmatrix} 6 & 18 & -26 \\ 24 & -8 & -24 \\ -2 & -6 & -18 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right) 2^3 \begin{vmatrix} 3 & -9 & 13 \\ 12 & 4 & 12 \\ -1 & 3 & 9 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right) 2^3 \begin{vmatrix} 0 & 0 & 40 \\ 12 & 4 & 12 \\ -1 & 3 & 9 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right) 2^3 \times 40(36+4) = 512000$$

Chapter 6 Determinants Ex 6.2 Q1-viii

$$Let\ \Delta = \begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$

Applying $R_3 \to 17R_2 - R_3$, *we get,*

$$\Delta = \begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 0 & 48 & 62 \end{vmatrix}$$

Applying $R_2 \to 102R_2 - R_1$, *we get,*

$$\Delta = \begin{vmatrix} 102 & 18 & 36 \\ 0 & 288 & 372 \\ 0 & 48 & 62 \end{vmatrix}$$

Thus,

$\Delta = 102(288 \times 62 - 372 \times 48)$

$\Rightarrow \Delta = 0$

Chapter 6 Determinants Ex 6.2 Q2(i)

$$\begin{vmatrix} 8 & 2 & 7 \\ 12 & 3 & 5 \\ 16 & 4 & 3 \end{vmatrix}$$

Apply: $R_3 \to R_3 - R_2$

$$= \begin{vmatrix} 8 & 2 & 7 \\ 12 & 3 & 5 \\ 4 & 1 & -2 \end{vmatrix}$$

Apply: $R_2 \to R_2 - R_1$

$$= \begin{vmatrix} 8 & 2 & 7 \\ 4 & 1 & -2 \\ 4 & 1 & -2 \end{vmatrix}$$

Since, $R_3 = R_2$, the value of the determinant is zero.

Chapter 6 Determinants Ex 6.2 Q2(ii)

$$\begin{vmatrix} 6 & -3 & 2 \\ 2 & -1 & 2 \\ -10 & 5 & 2 \end{vmatrix}$$

Taking (-2) common from C_1, we get

$$= (-2)\begin{vmatrix} -3 & -3 & 2 \\ -1 & -1 & 2 \\ 5 & 5 & 2 \end{vmatrix}$$

$= 0$

$\because C_1$ and C_2 are identical.

Chapter 6 Determinants Ex 6.2 Q2(iii)

$$\begin{vmatrix} 2 & 3 & 7 \\ 13 & 17 & 5 \\ 15 & 20 & 12 \end{vmatrix}$$

Use: $R_3 \to R_3 - R_2$

$$= \begin{vmatrix} 2 & 3 & 7 \\ 13 & 17 & 5 \\ 2 & 3 & 7 \end{vmatrix}$$

$= 0$

$\because R_3 = R_1$

Chapter 6 Determinants Ex 6.2 Q2(iv)

$$\begin{vmatrix} \frac{1}{a} & a^2 & bc \\ \frac{1}{b} & b^2 & ca \\ \frac{1}{c} & c^2 & ab \end{vmatrix}$$

Multiply: R_1, R_2 and R_3 by a, b and c respectively, we get

$$= \frac{1}{abc}\begin{vmatrix} 1 & a^3 & abc \\ 1 & b^3 & bca \\ 1 & c^3 & cab \end{vmatrix}$$

Take abc common from C_3, we get,

$$= \begin{vmatrix} 1 & a^3 & 1 \\ 1 & b^3 & 1 \\ 1 & c^3 & 1 \end{vmatrix}$$

$= 0$

$\because \quad C_1 = C_3$

Chapter 6 Determinants Ex 6.2 Q2(v)

$$\begin{vmatrix} a+b & 2a+b & 3a+b \\ 2a+b & 3a+b & 4a+b \\ 4a+b & 5a+b & 6a+b \end{vmatrix}$$

Apply: $C_3 \to C_3 - C_2$

$$= \begin{vmatrix} a+b & 2a+b & a \\ 2a+b & 3a+b & a \\ 4a+b & 5a+b & a \end{vmatrix}$$

Apply: $C_2 \to C_2 - C_1$

$$= \begin{vmatrix} a+b & a & a \\ 2a+b & a & a \\ 4a+b & a & a \end{vmatrix}$$

$= 0$

$\because \quad C_3 = C_2$

Chapter 6 Determinants Ex 6.2 Q2(vi)

$$\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ac \\ 1 & c & c^2-ab \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & bc \\ 1 & b & ac \\ 1 & c & ab \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix} - \begin{vmatrix} 1 & a & bc \\ 0 & b-a & (a-b)c \\ 0 & c-a & (a-c)b \end{vmatrix}$$

$$= (b-a)(c-a)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 1 & c+a \end{vmatrix} - (b-a)(c-a)\begin{vmatrix} 1 & a & bc \\ 0 & 1 & -c \\ 0 & 1 & -b \end{vmatrix}$$

$= (b-a)(c-a)(c+a-b-a) - (b-a)(c-a)(-b+c)$

$= (b-a)(c-a)(c-b) - (b-a)(c-a)(-b+c)$

$= 0$

Chapter 6 Determinants Ex 6.2 Q2(vii)

$$\begin{vmatrix} 49 & 1 & 6 \\ 39 & 7 & 4 \\ 26 & 2 & 3 \end{vmatrix}$$

Apply: $C_1 \to C_1 + (-8)C_3$

$$= \begin{vmatrix} 1 & 1 & 6 \\ 7 & 7 & 4 \\ 2 & 2 & 3 \end{vmatrix} = 0$$

$\because \quad C_1 = C_2$

Chapter 6 Determinants Ex 6.2 Q2(viii)

$$\begin{vmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{vmatrix}$$

Multiply C_1, C_2 and C_3 by z, y, and x respectively

$$= \frac{1}{xyz}\begin{vmatrix} 0 & xy & yx \\ -xz & 0 & zx \\ -yz & -zy & 0 \end{vmatrix}$$

Take y, x, and z common from R_1, R_2 and R_3 respectively

$$= \begin{vmatrix} 0 & x & x \\ -z & 0 & z \\ -y & -y & 0 \end{vmatrix}$$

Apply: $C_2 \to C_2 - C_3$

$$= \begin{vmatrix} 0 & 0 & x \\ -z & -z & z \\ -y & -y & 0 \end{vmatrix}$$

$= 0$

$\because C_1 = C_2$

Chapter 6 Determinants Ex 6.2 Q2(ix)

$$\begin{vmatrix} 1 & 43 & 6 \\ 7 & 35 & 4 \\ 3 & 17 & 2 \end{vmatrix}$$

Apply: $C_2 \to C_2 + (-7)C_3$

$$= \begin{vmatrix} 1 & 1 & 6 \\ 7 & 7 & 4 \\ 3 & 3 & 2 \end{vmatrix}$$

$= 0$

$\because \quad C_1 = C_2$

Chapter 6 Determinants Ex 6.2 Q2(x)

$$\begin{vmatrix} 1^2 & 2^2 & 3^2 & 4^2 \\ 2^2 & 3^2 & 4^2 & 5^2 \\ 3^2 & 4^2 & 5^2 & 6^2 \\ 4^2 & 5^2 & 6^2 & 7^2 \end{vmatrix}$$

$Apply: C3 \to C3 - C2, C4 \to C4 - C1$

$$= \begin{vmatrix} 1^2 & 2^2 & 3^2 - 2^2 & 4^2 - 1^2 \\ 2^2 & 3^2 & 4^2 - 3^2 & 5^2 - 2^2 \\ 3^2 & 4^2 & 5^2 - 4^2 & 6^2 - 3^2 \\ 4^2 & 5^2 & 6^2 - 5^2 & 7^2 - 4^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1^2 & 2^2 & 5 & 15 \\ 2^2 & 3^2 & 7 & 21 \\ 3^2 & 4^2 & 9 & 27 \\ 4^2 & 5^2 & 11 & 33 \end{vmatrix}$$

Take 3 common from C_4

$$= 3\begin{vmatrix} 1^2 & 2^2 & 5 & 5 \\ 2^2 & 3^2 & 7 & 7 \\ 3^2 & 4^2 & 9 & 9 \\ 4^2 & 5^2 & 11 & 11 \end{vmatrix}$$

$= 0$

$\because \quad C_3 = C_4$

Chapter 6 Determinants Ex 6.2 Q2(xi)

$$\begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c \\ 2a+2x & 2b+2y & 2c+2z \\ x+a & y+b & z+c \end{vmatrix}$$

$$= 2\begin{vmatrix} a & b & c \\ a+x & b+y & c+z \\ x+a & y+b & z+c \end{vmatrix}$$

$= 0$

Chapter 6 Determinants Ex 6.2 Q3

$$\begin{vmatrix} a & b+c & a^2 \\ b & c+a & b^2 \\ c & a+b & c^2 \end{vmatrix}$$

Apply: $C_2 \to C_2 + C_1$.

$$= \begin{vmatrix} a & b+c+a & a^2 \\ b & c+a+b & b^2 \\ c & a+b+c & c^2 \end{vmatrix}$$

Take $(a+b+c)$ common from C_2

$$= (b+c+a)\begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & c^2 \end{vmatrix}$$

Apply: $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$= (b+c+a)\begin{vmatrix} a & 1 & a^2 \\ b-a & 0 & b^2-a^2 \\ c-a & 0 & c^2-a^2 \end{vmatrix}$$

$$= (b+c+a)(b-a)(c-a)\begin{vmatrix} a & 1 & a^2 \\ 1 & 0 & b+a \\ 1 & 0 & c+a \end{vmatrix}$$

$$= (b+c+a)(b-a)(c-a)(b-c)$$

Chapter 6 Determinants Ex 6.2 Q4

$$Let\ \Delta = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ *and* $R_3 \to R_3 - R_1$ *we get,*

$$\Delta = \begin{vmatrix} 1 & a & bc \\ 0 & b-a & ca-bc \\ 0 & c-a & ab-ba \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & a & bc \\ 0 & b-a & c(a-b) \\ 0 & c-a & b(a-c) \end{vmatrix}$$

Taking $(a-b)$ *and* $(a-c)$ *common, we have*

$$\Delta = (a-b)(a-c)\begin{vmatrix} 1 & a & bc \\ 0 & -1 & c \\ 0 & -1 & b \end{vmatrix}$$

$$\Rightarrow \Delta = (a-b)(c-a)(b-c)$$

Chapter 6 Determinants Ex 6.2 Q5

$$Let\ \Delta = \begin{vmatrix} x+\lambda & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$ *we get,*

$$\Delta = \begin{vmatrix} 3x+\lambda & x & x \\ 3x+\lambda & x+\lambda & x \\ 3x+\lambda & x & x+\lambda \end{vmatrix}$$

Taking $(3x+\lambda)$ *common, we have*

$$\Delta = (3x+\lambda)\begin{vmatrix} 1 & x & x \\ 1 & x+\lambda & x \\ 1 & x & x+\lambda \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$, *we get,*

$$\Delta = (3x+\lambda)\begin{vmatrix} 1 & x & x \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix}$$

$$\Rightarrow \Delta = \lambda^2(3x+\lambda)$$

Chapter 6 Determinants Ex 6.2 Q6

$$Let\ \Delta = \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

$Applying\ C_1 \to C_1 + C_2 + C_3\ we\ get,$

$$\Delta = \begin{vmatrix} a+b+c & b & c \\ a+b+c & a & b \\ a+b+c & c & a \end{vmatrix}$$

$Taking\ (a+b+c)\ common,\ we\ have$

$$\Delta = (a+b+c) \begin{vmatrix} 1 & b & c \\ 1 & a & b \\ 1 & c & a \end{vmatrix}$$

$Applying\ R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1,\ we\ get,$

$$\Delta = (a+b+c) \begin{vmatrix} 1 & b & c \\ 0 & a-b & b-c \\ 0 & c-b & a-c \end{vmatrix}$$

$$\Rightarrow \Delta = (a+b+c)[(a-b)(a-c)-(b-c)(c-b)]$$

$$\Rightarrow \Delta = (a+b+c)\left[a^2 - ac - ab + bc + b^2 + c^2 - 2bc\right]$$

$$\Rightarrow \Delta = (a+b+c)\left[a^2 + b^2 + c^2 - ac - ab - bc\right]$$

Chapter 6 Determinants Ex 6.2 Q7

$$\begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix} = \begin{vmatrix} 2+x & 1 & 1 \\ 2+x & x & 1 \\ 2+x & 1 & x \end{vmatrix} = (2+x) \begin{vmatrix} 1 & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix}$$

$$= (2+x) \begin{vmatrix} 1 & 1 & 1 \\ 0 & x-1 & 0 \\ 0 & 0 & x-1 \end{vmatrix}$$

$$= (2+x)(x-1)^2$$

Chapter 6 Determinants Ex 6.2 Q8

$$\begin{vmatrix} 0 & xy^2 & xz^2 \\ x^2y & 0 & yz^2 \\ x^2z & zy^2 & 0 \end{vmatrix}$$

$$= 0\left(0 - y^3z^3\right) - xy^2\left(0 - x^2yz^3\right) + xz^2\left(x^2y^3z - 0\right)$$

$$= 0 + x^3y^3z^3 + x^3y^3z^3$$

$$= 2x^3y^3z^3$$

Chapter 6 Determinants Ex 6.2 Q9

$$\text{Let } \Delta = \begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$ and $R_3 \to R_3 - R_2$

$$\Delta = \begin{vmatrix} a & -a & 0 \\ x & a+y & z \\ 0 & -a & a \end{vmatrix}$$

Applying $C_2 \to C_2 + C_1$

$$\Delta = \begin{vmatrix} a & 0 & 0 \\ x & a+x+y & z \\ 0 & -a & a \end{vmatrix}$$

$$\Delta = a\left[a(a+x+y) + az\right] + 0 + 0$$

$$\Delta = a^2(a+x+y+z)$$

Chapter 6 Determinants Ex 6.2 Q10

$$\Delta + \Delta_1 = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ yz & zx & xy \\ x & y & z \end{vmatrix}$$

$$= \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + \begin{vmatrix} 1 & yz & x \\ 1 & zx & y \\ 1 & xy & z \end{vmatrix} \ldots\ldots\ldots \left[\because |A| = |A^T|\right]$$

$$= \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} - \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$$

$\ldots\ldots$ [If any two rows (columns) of the determinant are interchanged then value of the determinant changes in sign.]

$$= \begin{vmatrix} 0 & 0 & x^2 - yz \\ 0 & 0 & y^2 - zx \\ 0 & 0 & z^2 - xy \end{vmatrix}$$

$= 0 \ldots\ldots\ldots [\because C_1$ and C_2 are identical$]$

Chapter 6 Determinants Ex 6.2 Q11

$$\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$$

$$\text{LHS} = \begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$$

Apply: $C_1 \to C_1 + C_2 + C_3$.

$$= \begin{vmatrix} a+b+c & b & c \\ 0 & b-c & c-a \\ 2(a+b+c) & c+a & a+b \end{vmatrix}$$

Take $(a+b+c)$ common from C_1

$$= (a+b+c) \begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 2 & c+a & a+b \end{vmatrix}$$

Apply: $R_3 \to R_3 - 2R_1$

$$= (a+b+c) \begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 0 & c+a-2b & a+b-2c \end{vmatrix}$$

$$= (a+b+c)\left[(b-c)(a+b-2c) - (c-a)(c+a-2b)\right]$$

$$= a^3 + b^3 + c^3 - 3abc$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q12

$$\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3abc - a^3 - b^3 - c^3$$

$$\text{LHS} = \begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix}$$

$$= \begin{vmatrix} b+c+a & -b & a \\ c+a+b & -c & b \\ a+b+c & -a & c \end{vmatrix}$$

$$= -(b+c+a)\begin{vmatrix} 1 & b & a \\ 1 & c & b \\ 1 & a & c \end{vmatrix}$$

$$= -(b+c+a)\begin{vmatrix} 1 & b & a \\ 0 & c-b & b-a \\ 0 & a-b & c-a \end{vmatrix}$$

$$= -(b+c+a)\left[(c-b)(c-a) - (b-a)(a-b)\right]$$

$$= 3abc - a^3 - b^3 - c^3$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q13

$$\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$\text{LHS} = \begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix}$$

Apply: $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 2(a+b+c) & b+c & c+a \\ 2(a+b+c) & c+a & a+b \\ 2(a+b+c) & a+b & b+c \end{vmatrix}$$

$$= 2\begin{vmatrix} a+b+c & b+c & c+a \\ a+b+c & c+a & a+b \\ a+b+c & a+b & b+c \end{vmatrix}$$

Apply: $C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$

$$= 2\begin{vmatrix} a+b+c & -a & -b \\ a+b+c & -b & -c \\ a+b+c & -c & -a \end{vmatrix}$$

$$= 2\begin{vmatrix} a+b+c & a & b \\ a+b+c & b & c \\ a+b+c & c & a \end{vmatrix}$$

$$= 2\left(\begin{vmatrix} c & a & b \\ a & b & c \\ b & c & a \end{vmatrix} + \begin{vmatrix} a & a & b \\ b & b & c \\ c & c & a \end{vmatrix} + \begin{vmatrix} b & a & b \\ c & b & c \\ a & c & a \end{vmatrix}\right)$$

$$= 2\begin{vmatrix} c & a & b \\ a & b & c \\ b & c & a \end{vmatrix}$$

$$= 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q14

We need to prove the following identity:

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

Applying $C_1 \to C_1 + C_2 + C_3$, we have,

$$L.H.S = \begin{vmatrix} 2a+2b+2c & a & b \\ 2a+2b+2c & b+c+2a & b \\ 2a+2b+2c & a & c+a+2b \end{vmatrix}$$

Taking the term $2a+2b+2$ as common, we have

$$L.H.S = (2a+2b+2c)\begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}$$

$$\Rightarrow L.H.S = 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$ we have

$$L.H.S = 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 0 & a+b+c & 0 \\ 0 & 0 & a+b+c \end{vmatrix}$$

Thus, we have,

$$\begin{aligned} L.H.S &= 2(a+b+c)\left[1 \times (a+b+c)^2\right] \\ &= 2(a+b+c)(a+b+c)^2 \\ &= 2(a+b+c)^3 \end{aligned}$$

Chapter 6 Determinants Ex 6.2 Q15

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

$$\text{LHS} = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Apply: $R_1 \to R_1 + R_2 + R_3$.

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Take $(a+b+c)$ common from R_1

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Apply: $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$$= (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -b-c-a & 0 \\ 2c & 0 & -c-a-b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & b+c+a & 0 \\ 2c & 0 & b+c+a \end{vmatrix}$$

$$= (a+b+c)^3$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q16

$$LHS = \begin{vmatrix} 1 & b+c & b^2+c^2 \\ 1 & c+a & c^2+a^2 \\ 1 & a+b & a^2+b^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & b+c & b^2+c^2 \\ 0 & a-b & a^2-b^2 \\ 0 & a-c & a^2-c^2 \end{vmatrix}$$

$$= (a-b)(a-c)\begin{vmatrix} 1 & b+c & b^2+c^2 \\ 0 & 1 & a+b \\ 0 & 1 & a+c \end{vmatrix}$$

$$= (a-b)(a-c)\begin{vmatrix} 1 & b+c & b^2+c^2 \\ 0 & 1 & a+b \\ 0 & 0 & c-b \end{vmatrix}$$

$$= (a-b)(b-c)(c-a)$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q17

$$LHS = \begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$$

$$= \begin{vmatrix} 3a+3b & 3a+3b & 3a+3b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$$

$$= (3a+3b)\begin{vmatrix} 1 & 1 & 1 \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$$

$$= 3(a+b)\begin{vmatrix} 0 & 1 & 0 \\ 2b & a & b \\ -b & a+2b & -2b \end{vmatrix}$$

$$= 3(a+b)b^2\begin{vmatrix} 0 & 1 & 0 \\ 2 & a & 1 \\ -1 & a+2b & -2 \end{vmatrix}$$

$$= 9(a+b)b^2$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q18

$$\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

Apply $R_1 \to R_1a,\ R_2 \to R_2b,\ R_3 \to R_3c$

$$= \frac{1}{abc}\begin{vmatrix} a & a^2 & abc \\ b & b^2 & cab \\ c & c^2 & abc \end{vmatrix}$$

$$= \frac{abc}{abc}\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$$

$$= -\begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & c^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Chapter 6 Determinants Ex 6.2 Q19

$$\begin{vmatrix} z & x & y \\ z^2 & x^2 & y^2 \\ z^4 & x^4 & y^4 \end{vmatrix} = \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^4 & y^4 & z^4 \end{vmatrix} = \begin{vmatrix} x^2 & y^2 & z^2 \\ x^4 & y^4 & z^4 \\ x & y & z \end{vmatrix} = xyz\,(x-y)(y-z)(z-x)(x+y+z)$$

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^4 & y^4 & z^4 \end{vmatrix}$$

$$= xyz\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix}$$

$$= xyz\begin{vmatrix} 0 & 1 & 0 \\ x-y & y & z-y \\ x^3-y^3 & y^3 & z^3-y^3 \end{vmatrix}$$

$$= xyz(x-y)(z-y)\begin{vmatrix} 0 & 1 & 0 \\ 1 & y & 1 \\ x^2+y^2+xy & y^3 & z^2+y^2+zy \end{vmatrix}$$

$$= -xyz(x-y)(z-y)\left[z^2+y^2+zy-x^2-y^2-xy\right]$$

$$= -xyz(x-y)(z-y)\left[(z-x)(z+x)+y(z-x)\right]$$

$$= -xyz(x-y)(z-y)(z-x)\left[z+x+y\right]$$

$$= xyz(x-y)(y-z)(z-x)(x+y+z)$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q20

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)\left(a^2+b^2+c^2\right)$$

$$\text{LHS} = \begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix}$$

Apply: $C_1 \to C_1 + C_2 - 2C_3$

$$= \begin{vmatrix} (b+c)^2+a^2-2bc & a^2 & bc \\ (c+a)^2+b^2-2ca & b^2 & ca \\ (a+b)^2+c^2-2ab & c^2 & ab \end{vmatrix}$$

$$= \begin{vmatrix} a^2+b^2+c^2 & a^2 & bc \\ a^2+b^2+c^2 & b^2 & ca \\ a^2+b^2+c^2 & c^2 & ab \end{vmatrix}$$

Take $\left(a^2+b^2+c^2\right)$ common from C_1

$$= \left(a^2+b^2+c^2\right)\begin{vmatrix} 1 & a^2 & bc \\ 1 & b^2 & ca \\ 1 & c^2 & ab \end{vmatrix}$$

$$= \left(a^2+b^2+c^2\right)\begin{vmatrix} 1 & a^2 & bc \\ 0 & b^2-a^2 & ca-bc \\ 0 & c^2-a^2 & ab-bc \end{vmatrix}$$

$$= \left(a^2+b^2+c^2\right)(b-a)(c-a)\begin{vmatrix} 1 & a^2 & bc \\ 0 & b+a & -c \\ 0 & c+a & -b \end{vmatrix}$$

$$= \left(a^2+b^2+c^2\right)(b-a)(c-a)\left[(b+a)(-b)-(-c)(c+a)\right]$$

$$= (a-b)(b-c)(c-a)(a+b+c)\left(a^2+b^2+c^2\right)$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q21

$$\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix} = -2$$

$$\text{LHS} = \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$$

Apply $R_3 \to R_3 - R_2$

$$= \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)2 & 1 & 0 \end{vmatrix}$$

Apply $R_2 \to R_2 - R_1$

$$= \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)2 & 1 & 0 \\ (a+3)2 & 1 & 0 \end{vmatrix}$$

$= \left[(2a+4)(1) - (1)(2a+6)\right]$

$= -2$

$= \text{RHS}$

Chapter 6 Determinants Ex 6.2 Q22

$$\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)\left(a^2+b^2+c^2\right)$$

$$\text{LHS} = \begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix}$$

Apply: $C_2 \to C_2 - 2C_1 - 2C_3$

$$= \begin{vmatrix} a^2 & a^2-(b-c)^2-2a^2-2bc & bc \\ b^2 & b^2-(c-a)^2-2b^2-2ca & ca \\ c^2 & c^2-(a-b)^2-2c^2-2ab & ab \end{vmatrix}$$

$$= \begin{vmatrix} a^2 & -\left(b^2+c^2+a^2\right) & bc \\ b^2 & -\left(b^2+c^2+a^2\right) & ca \\ c^2 & -\left(b^2+c^2+a^2\right) & ab \end{vmatrix}$$

Take $-\left(a^2+b^2+c^2\right)$ common from C_2

$$= -\left(b^2+c^2+a^2\right)\begin{vmatrix} a^2 & 1 & bc \\ b^2 & 1 & ca \\ c^2 & 1 & ab \end{vmatrix}$$

$$= -\left(b^2+c^2+a^2\right)\begin{vmatrix} a^2 & 1 & bc \\ b^2-a^2 & 0 & ca-bc \\ c^2-a^2 & 0 & ab-bc \end{vmatrix}$$

$$= -\left(b^2+c^2+a^2\right)(a-b)(c-a)\begin{vmatrix} a^2 & 1 & bc \\ -(b+a) & 0 & c \\ c+a & 0 & -b \end{vmatrix}$$

$= -\left(b^2+c^2+a^2\right)(a-b)(c-a)\left[(-(b+a))(-b) - (c)(c+a)\right]$

$= (a-b)(b-c)(c-a)(a+b+c)\left(a^2+b^2+c^2\right)$

$= RHS$

Chapter 6 Determinants Ex 6.2 Q23

$$\begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ca & b^3 \\ 1 & c^2+ab & c^3 \end{vmatrix} = -(a-b)(b-c)(c-a)\left(a^2+b^2+c^2\right)$$

$$\text{LHS} = \begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ca & b^3 \\ 1 & c^2+ab & c^3 \end{vmatrix}$$

Apply: $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= \begin{vmatrix} 1 & a^2+bc & a^3 \\ 0 & b^2+ca-a^2-bc & b^3-a^3 \\ 0 & c^2+abb^2+ca-a^2-bc & c^3-a^3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a^2+bc & a^3 \\ 0 & \left(b^2-a^2\right)-c(b-a) & b^3-a^3 \\ 0 & \left(c^2-a^2\right)-b(c-a) & c^3-a^3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a^2+bc & a^3 \\ 0 & (b-a)(b+a-c) & b^3-a^3 \\ 0 & (c-a)(c+a-b) & c^3-a^3 \end{vmatrix}$$

$$= (b-a)(c-a)\begin{vmatrix} 1 & a^2+bc & a^3 \\ 0 & (b+a-c) & b^2+a^2+ab \\ 0 & (c+a-b) & c^2+a^2+ac \end{vmatrix}$$

$$= (b-a)(c-a)\left[((b+a-c))\left(c^2+a^2+ac\right)-\left(b^2+a^2+ab\right)\left(c^2+a^2+ac\right)\right]$$

$$= -(a-b)(b-c)(c-a)\left(a^2+b^2+c^2\right)$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q24

We need to prove the following identity:

$$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$

Taking the term a,b,c *common from* C_1, C_2 *and* C_3, *respectively, we have,*

$$L.H.S = abc\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$, *we have,*

$$L.H.S = abc\begin{vmatrix} 2a+2c & c & a+c \\ 2a+2b & b & a \\ 2b+2c & b+c & c \end{vmatrix}$$

$$\Rightarrow L.H.S = 2abc\begin{vmatrix} a+c & c & a+c \\ a+b & b & a \\ b+c & b+c & c \end{vmatrix}$$

$Applying\ C_2 \to C_2 - C_1\ and\ C_3 \to C_3 - C_1,\ we\ have,$

$$L.H.S = 2abc\begin{vmatrix} a+c & -a & 0 \\ a+b & -a & -b \\ b+c & 0 & -b \end{vmatrix}$$

$Applying\ C_1 \to C_1 + C_2 + C_3,\ we\ have,$

$$\Rightarrow L.H.S = 2abc\begin{vmatrix} c & -a & 0 \\ 0 & -a & -b \\ c & 0 & -b \end{vmatrix}$$

$Taking\ c,\ a,\ and\ b\ \ from\ C_1,\ C_2\ and\ C_3\ respectively,\ we\ have,$

$$L.H.S = 2a^2b^2c^2\begin{vmatrix} 1 & -1 & 0 \\ 0 & -1 & -1 \\ 1 & 0 & -1 \end{vmatrix}$$

$Applying\ R_3 \to R_3 - R_1,\ we\ have$

$$L.H.S = 2a^2b^2c^2\begin{vmatrix} 1 & -1 & 0 \\ 0 & -1 & -1 \\ 0 & 1 & -1 \end{vmatrix}$$

$$= 4a^2b^2c^2$$

Chapter 6 Determinants Ex 6.2 Q25

$We\ need\ to\ prove\ the\ following\ identity:$

$$\begin{vmatrix} x+4 & x & x \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix} = 16(3x+4)$$

$Let\ us\ consider\ the\ L.H.S\ of\ the\ above\ equation.$

$$\Delta = \begin{vmatrix} x+4 & x & x \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix}$$

$Applying\ C_1 \to C_1 + C_2 + C_3,\ we\ get,$

$$\Delta = \begin{vmatrix} 3x+4 & x & x \\ 3x+4 & x+4 & x \\ 3x+4 & x & x+4 \end{vmatrix}$$

$Taking\ the\ common\ term\ 3x+4,\ we\ get,$

$$\Delta = (3x+4)\begin{vmatrix} 1 & x & x \\ 1 & x+4 & x \\ 1 & x & x+4 \end{vmatrix}$$

$Applying\ R_2 \to R_2 - R_1\ and\ R_3 \to R_3 - R_1,\ we\ get,$

$$\Delta = (3x+4)\begin{vmatrix} 1 & x & x \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{vmatrix}$$

$$\Rightarrow \Delta = 16(3x+4)$$

Chapter 6 Determinants Ex 6.2 Q26

We need to prove the following identity:

$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} = 1$$

Let us consider the L.H.S of the above equation.

Applying $C_2 \to C_2 - pC_1$ *and* $C_3 \to C_3 - qC_1$, *we get*

$$\Delta = \begin{vmatrix} 1 & 1 & 1+p \\ 2 & 3 & 4+3p \\ 3 & 6 & 10+6p \end{vmatrix}$$

Applying $C_3 \to C_3 - pC_2$, *we get*

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 6 & 10 \end{vmatrix}$$

Applying $C_2 \to C_2 - pC_1$ *and* $C_3 \to C_3 - qC_1$, *we get*

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 2 \\ 3 & 3 & 7 \end{vmatrix}$$

$$\Rightarrow \Delta = 1[7-6] = 1$$

Chapter 6 Determinants Ex 6.2 Q27

$$\begin{vmatrix} a & b-c & c-b \\ a-c & b & c-a \\ a-b & b-a & c \end{vmatrix}$$

$R_1 \to R_1 - R_2 - R_3$

$$= \begin{vmatrix} -a+c+b & -b-c+a & -c-b+a \\ a-c & b & c-a \\ a-b & b-a & c \end{vmatrix}$$

$$= (b+c-a)\begin{vmatrix} 1 & -1 & -1 \\ a-c & b & c-a \\ a-b & b-a & c \end{vmatrix}$$

$$= (b+c-a)\begin{vmatrix} 1 & 0 & 0 \\ a-c & b+a-c & 0 \\ a-b & 0 & c+a-b \end{vmatrix}$$

$$= (a+b-c)(b+c-a)(c+a-b)$$

$= RHS$

Chapter 6 Determinants Ex 6.2 Q28

$$LHS = \begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix}$$

$$= \begin{vmatrix} a^2+b^2+2ab & 2ab & b^2 \\ a^2+b^2+2ab & a^2 & 2ab \\ a^2+b^2+2ab & b^2 & a^2 \end{vmatrix}$$

$$= \left(a^2+b^2+2ab\right)\begin{vmatrix} 1 & 2ab & b^2 \\ 1 & a^2 & 2ab \\ 1 & b^2 & a^2 \end{vmatrix}$$

$$= \left(a^2+b^2+2ab\right)\begin{vmatrix} 1 & 2ab & b^2 \\ 0 & a^2-2ab & 2ab-b^2 \\ 0 & b^2-2ab & a^2-b^2 \end{vmatrix}$$

$$= \left(a^2+b^2+2ab\right)\begin{vmatrix} 1 & 2ab & b^2 \\ 0 & a^2-b^2 & 2ab-a^2 \\ 0 & b^2-2ab & a^2-b^2 \end{vmatrix}$$

$$= (a+b)^2\left[\left(a^2-b^2\right)\left(a^2-b^2\right)-\left(2ab-a^2\right)\left(b^2-2ab\right)\right]$$

$$= (a+b)^2\left(a^2+b^2-ab\right)^2$$

$$= \left(a^3+b^3\right)^2$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q29

We need to prove the following identity:

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$$

Let us consider the L.H.S of the above equation.

$$\Delta = \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$$

Applying $R_1 \to R_1(a)$, $R_2 \to R_2(b)$ *and* $R_3 \to R_3(c)$, *we get*

$$\Delta = \frac{1}{abc}\begin{vmatrix} a\left(a^2+1\right) & a^2b & a^2c \\ ab^2 & b\left(b^2+1\right) & b^2c \\ c^2a & c^2b & c\left(c^2+1\right) \end{vmatrix}$$

Taking a,b, and c common from C_1, C_2 *and* C_3, *respectively, we get,*

$$\Delta = \frac{abc}{abc}\begin{vmatrix} \left(a^2+1\right) & a^2 & a^2 \\ b^2 & \left(b^2+1\right) & b^2 \\ c^2 & c^2 & \left(c^2+1\right) \end{vmatrix}$$

Applying $R_1 \to R_1+R_2+R_3$, *we get,*

$$\Delta = \frac{abc}{abc}\begin{vmatrix} \left(a^2+b^2+c^2+1\right) & \left(a^2+b^2+c^2+1\right) & \left(a^2+b^2+c^2+1\right) \\ b^2 & \left(b^2+1\right) & b^2 \\ c^2 & c^2 & \left(c^2+1\right) \end{vmatrix}$$

Taking the term, $(a^2+b^2+c^2+1)$ *common from the above equation, we have,*

$$\Delta=(a^2+b^2+c^2+1)\begin{vmatrix}1 & 1 & 1\\ b^2 & (b^2+1) & b^2\\ c^2 & c^2 & (c^2+1)\end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$, *we get,*

$$\Delta=(a^2+b^2+c^2+1)\begin{vmatrix}1 & 0 & 1\\ b^2 & 1 & 0\\ c^2 & 0 & 1\end{vmatrix}$$

$$\Rightarrow \Delta=(a^2+b^2+c^2+1)$$

Chapter 6 Determinants Ex 6.2 Q30

Let us consider the L.H.S of the given equation.

$$Let\ \Delta=\begin{vmatrix}1 & a & a^2\\ a^2 & 1 & a\\ a & a^2 & 1\end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$, *we have,*

$$\Delta=\begin{vmatrix}1+a+a^2 & a & a^2\\ 1+a+a^2 & 1 & a\\ 1+a+a^2 & a^2 & 1\end{vmatrix}$$

Taking the term $(1+a+a^2)$ *common, we have,*

$$\Delta=(1+a+a^2)\begin{vmatrix}1 & a & a^2\\ 1 & 1 & a\\ 1 & a^2 & 1\end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ *and* $R_3 \to R_3 - R_1$, *we have*

$$\Delta=(1+a+a^2)\begin{vmatrix}1 & a & a^2\\ 0 & 1-a & a(1-a)\\ 0 & -a(1-a) & (1-a)(1+a)\end{vmatrix}$$

Taking the term $(1-a)$ *common from* R_2 *and* R_3, *we have*

$$\Rightarrow \Delta=(1+a+a^2)(1-a)^2\begin{vmatrix}1 & a & a^2\\ 0 & 1 & a\\ 0 & -a & (1+a)\end{vmatrix}$$

$$\Rightarrow \Delta=(1+a+a^2)(1-a)^2(1+a+a^2)$$

$$\Rightarrow \Delta=(1+a+a^2)^2(1-a)^2$$

$$\Rightarrow \Delta=\left[(1+a+a^2)(1-a)\right]^2$$

$$\Rightarrow \Delta=\left[(a^3-1)\right]^2$$

Chapter 6 Determinants Ex 6.2 Q31

$$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a)$$

$$\text{LHS} = \begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$$

Apply: $C_1 \to C_1 + C_3$ and $C_2 \to C_2 + C_3$

$$= \begin{vmatrix} a+c & -(c+b) & -b \\ -(c+a) & b+c & -a \\ a+c & b+c & a+b+c \end{vmatrix}$$

$$=(c+a)(c+b)\begin{vmatrix} 1 & -1 & -b \\ -1 & 1 & -a \\ 1 & 1 & a+b+c \end{vmatrix}$$

$$=(c+a)(c+b)\begin{vmatrix} 1 & -1 & -b \\ 0 & 0 & -a-b \\ 0 & 2 & a+c \end{vmatrix}$$

$$= 2(a+b)(b+c)(c+a)$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q32

We need to prove the following identity:

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$$

Let us consider the L.H.S of the above equation.

$$\Delta = \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$, *we have,*

$$\Delta = \begin{vmatrix} 2(b+c) & 2(a+c) & 2(a+b) \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Taking 2 common from the above equation, we have,

$$\Delta = 2\begin{vmatrix} (b+c) & (a+c) & (a+b) \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ *and* $R_3 \to R_3 - R_1$, *we have,*

$$\Delta = 2\begin{vmatrix} (b+c) & (a+c) & (a+b) \\ -c & 0 & -a \\ -b & -a & 0 \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$, *we have,*

$$\Delta = 2\begin{vmatrix} 0 & c & b \\ -c & 0 & -a \\ -b & -a & 0 \end{vmatrix}$$

$$\Rightarrow \Delta = 2(0 + 2abc + abc)$$

$$\Rightarrow \Delta = 4abc$$

Chapter 6 Determinants Ex 6.2 Q33

$$\begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$$

$$LHS = \begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix}$$

Multiply R_1, R_2 and R_3 by a, b and c respectively.

$$= \frac{1}{abc}\begin{vmatrix} ab^2+ac^2 & a^2b & a^2c \\ b^2a & bc^2+ba^2 & b^2c \\ c^2a & c^2b & ca^2+cb^2 \end{vmatrix}$$

Take a, b and c common from C_1, C_2 and C_3 respectively.

$$= \frac{abc}{abc}\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

Now apply $R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} 2\left(b^2+c^2\right) & 2\left(c^2+a^2\right) & 2\left(a^2+b^2\right) \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

$$= 2\begin{vmatrix} \left(b^2+c^2\right) & \left(c^2+a^2\right) & \left(a^2+b^2\right) \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

$$= 2\begin{vmatrix} c^2 & 0 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

$$= 2\left[c^2\left\{\left(c^2+a^2\right)\left(a^2+b^2\right) - b^2c^2\right\} + a^2\left\{b^2c^2 - \left(c^2+a^2\right)c^2\right\}\right]$$

$$= 4a^2b^2c^2$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q34

$$\begin{vmatrix} 0 & b^2a & c^2a \\ a^2b & 0 & c^2b \\ a^2c & b^2c & 0 \end{vmatrix} = 2a^3b^3c^3$$

$$\text{LHS} = \begin{vmatrix} 0 & b^2a & c^2a \\ a^2b & 0 & c^2b \\ a^2c & b^2c & 0 \end{vmatrix}$$

$$= a^2b^2c^2\begin{vmatrix} 0 & a & a \\ b & 0 & b \\ c & c & 0 \end{vmatrix}$$

$$= a^3b^3c^3\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= a^3b^3c^3\begin{vmatrix} 0 & 0 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= 2a^3b^3c^3$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q35

$$\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{a} & a \\ b & b & \frac{c^2+a^2}{b} \end{vmatrix}$$

$$= \frac{1}{abc}\begin{vmatrix} a^2+b^2 & c^2 & c^2 \\ a^2 & c^2+b^2 & a^2 \\ b^2 & b^2 & c^2+a^2 \end{vmatrix}$$

$$= \frac{1}{abc}\begin{vmatrix} 0 & -2b^2 & -2a^2 \\ a^2 & c^2+b^2 & a^2 \\ b^2 & b^2 & c^2+a^2 \end{vmatrix}$$

$$= \frac{-2}{abc}\begin{vmatrix} 0 & b^2 & a^2 \\ a^2 & c^2+b^2 & a^2 \\ b^2 & b^2 & c^2+a^2 \end{vmatrix}$$

$$= \frac{-2}{abc}\begin{vmatrix} 0 & b^2 & a^2 \\ a^2 & c^2+b^2 & a^2 \\ b^2 & 0 & c^2 \end{vmatrix}$$

$$= \frac{-2}{abc}\begin{vmatrix} 0 & b^2 & a^2 \\ a^2 & c^2 & 0 \\ b^2 & 0 & c^2 \end{vmatrix}$$

$$= \frac{-2}{abc}\left[\left(-a^2\right)\left(b^2c^2\right)+\left(b^2\right)\left(-a^2c^2\right)\right]$$

$$= \frac{-2}{abc}\left(-2a^2b^2c^2\right)$$

$= 4abc$

$= RHS$

Chapter 6 Determinants Ex 6.2 Q36

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix}$$

Multiply R_1, R_2 and R_3 by a, b and c respectively

$$= \frac{1}{abc}\begin{vmatrix} -abc & ab^2+abc & ac^2+abc \\ a^2b+abc & -abc & bc^2+abc \\ a^2c+abc & b^2c+abc & -abc \end{vmatrix}$$

Take a, b and c common from C_1, C_2 and C_3 respectively.

$$= \frac{abc}{abc}\begin{vmatrix} -bc & ab+ac & ac+ab \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

Apply: $R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} ab+bc+ca & ab+bc+ca & ab+bc+ca \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

$$= (ab+bc+ca)\begin{vmatrix} 1 & 1 & 1 \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

$$= (ab+bc+ca)\begin{vmatrix} 0 & 1 & 0 \\ ab+bc+ac & -ac & bc+ab+ac \\ 0 & bc+ac & -ab-bc-ac \end{vmatrix}$$

$$= (ab+bc+ca)^3\begin{vmatrix} 0 & 1 & 0 \\ 1 & -ac & 1 \\ 0 & bc+ac & -1 \end{vmatrix}$$

$= (ab+bc+ca)^3$

$= RHS$

Chapter 6 Determinants Ex 6.2 Q37

L.H.S.,

$$\begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix}$$

$$= \begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix} [C_1 \to C_1 - C_3, C_2 \to C_2 - C_3]$$

$$= \begin{vmatrix} \lambda - x & 0 & 2x \\ 0 & \lambda - x & 2x \\ x-\lambda & x-\lambda & x+\lambda \end{vmatrix}$$

$$= (\lambda - x)(\lambda - x)\begin{vmatrix} 1 & 0 & 2x \\ 0 & 1 & 2x \\ -1 & -1 & x+\lambda \end{vmatrix}$$

$$= (\lambda - x)^2 \begin{vmatrix} 1 & 0 & 2x \\ 0 & 1 & 2x \\ -1 & -1 & x+\lambda \end{vmatrix}$$

$= (\lambda - x)^2[1(x+\lambda) + 2x + 2x(0+1)]$

$= (\lambda - x)^2[x + \lambda + 2x + 2x]$

$= (\lambda - x)^2[5x + \lambda]$

= R.H.S

Hence Proved

Chapter 6 Determinants Ex 6.2 Q38

$$LHS = \begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

Apply $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 5x+4 & 2x & 2x \\ 5x+4 & x+4 & 2x \\ 5x+4 & 2x & x+4 \end{vmatrix}$$

$$= (5x+4)\begin{vmatrix} 1 & 2x & 2x \\ 1 & x+4 & 2x \\ 1 & 2x & x+4 \end{vmatrix}$$

$$= (5x+4)\begin{vmatrix} 1 & 2x & 2x \\ 0 & -x+4 & 0 \\ 0 & 0 & -x+4 \end{vmatrix}$$

$$= (5x+4)(4-x)^2\begin{vmatrix} 1 & 2x & 2x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$= (5x+4)(4-x)^2$

$= RHS$

Chapter 6 Determinants Ex 6.2 Q39

$$\text{Let } \Delta = \begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$

$$\Delta = \begin{vmatrix} y & -x & y-x \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$$

Applying $R_1 \to R_1 - R_3$

$$\Delta = \begin{vmatrix} 0 & -2x & -2x \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$$

$$\Delta = 2x\left[z(x+y) - xy\right] - 2x\left[zx - y(z+x)\right]$$

$$\Delta = 2x\left[zx + zy - xy - zx + yz + yx\right]$$

$$\Delta = 4xyz$$

Chapter 6 Determinants Ex 6.2 Q40

$$\begin{vmatrix} -a\left(b^2+c^2-a^2\right) & 2b^3 & 2c^3 \\ 2a^3 & -b\left(c^2+a^2-b^2\right) & 2c^3 \\ 2a^3 & 2b^3 & -c\left(a^2+b^2-c^2\right) \end{vmatrix} = abc\left(a^2+b^2+c^2\right)^3$$

$$\text{LHS} = \begin{vmatrix} -a\left(b^2+c^2-a^2\right) & 2b^3 & 2c^3 \\ 2a^3 & -b\left(c^2+a^2-b^2\right) & 2c^3 \\ 2a^3 & 2b^3 & -c\left(a^2+b^2-c^2\right) \end{vmatrix}$$

Take a, b and c common from C_1, C_2 and C_3 respectively.

$$=\text{abc}\begin{vmatrix} -\left(b^2+c^2-a^2\right) & 2b^2 & 2c^2 \\ 2a^2 & -\left(c^2+a^2-b^2\right) & 2c^2 \\ 2a^2 & 2b^2 & -\left(a^2+b^2-c^2\right) \end{vmatrix}$$

Apply: $R_1 \to R_1 - R_3, R_2 \to R_2 - R_3$

$$=\text{abc}\begin{vmatrix} -\left(b^2+c^2-a^2\right)-2a^2 & 0 & 2c^2+\left(a^2+b^2-c^2\right) \\ 0 & -\left(c^2+a^2-b^2\right)-2b^2 & 2c^2+\left(a^2+b^2-c^2\right) \\ 2a^2 & 2b^2 & -\left(a^2+b^2-c^2\right) \end{vmatrix}$$

$$=\text{abc}\begin{vmatrix} -\left(b^2+c^2+a^2\right) & 0 & \left(a^2+b^2+c^2\right) \\ 0 & -\left(c^2+a^2+b^2\right) & \left(a^2+b^2+c^2\right) \\ 2a^2 & 2b^2 & -\left(a^2+b^2-c^2\right) \end{vmatrix}$$

$$=\text{abc}\left(b^2+c^2+a^2\right)^2\begin{vmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2a^2 & 2b^2 & -\left(a^2+b^2-c^2\right) \end{vmatrix}$$

$$=\text{abc}\left(b^2+c^2+a^2\right)^2\begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 1 \\ 2a^2 & 2b^2 & -\left(a^2+b^2-c^2\right)+2a^2 \end{vmatrix}$$

$$=\text{abc}\left(b^2+c^2+a^2\right)^2\begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 1 \\ 2a^2 & 2b^2 & -b^2+c^2+a^2 \end{vmatrix}$$

$$=\text{-abc}\left(b^2+c^2+a^2\right)^2\left[(-1)\left(-b^2+c^2+a^2\right)-(1)\left(2b^2\right)\right]$$

$$abc\left(a^2+b^2+c^2\right)^3$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q41

$$LHS = \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix}$$

$$= \begin{vmatrix} 3+a & 3+a & 3+a \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix}$$

$$= (3+a)\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix}$$

$$= (3+a)\begin{vmatrix} 1 & 1 & 1 \\ 0 & a & 0 \\ 0 & 1 & a \end{vmatrix}$$

$$= (3+a)a^2$$
$$= a^3 + 3a^2$$
$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q42

L.H.S.,

$$\begin{vmatrix} 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \\ x-y-z & 2x & 2x \end{vmatrix}$$

$$= \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ 2z & 2z & z-x-y \\ x-y-z & 2x & 2x \end{vmatrix} [R_1 = R_1 + R_2 + R_3]$$

$$= (x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ 2z & 2z & z-x-y \\ x-y-z & 2x & 2x \end{vmatrix}$$

$$= (x+y+z)\begin{vmatrix} 1 & 0 & 0 \\ 2z & 0 & -x-y-z \\ x-y-z & x+y+z & x+y+z \end{vmatrix} [C_2 = C_2 - C_1, C_3 = C_3 - C_1]$$

$$= (x+y+z)[1\{0+(x+y+z)(x+y+z)\}]$$
$$= (x+y+z)^3$$
$$= R.H.S.$$

Hence Proved

Chapter 6 Determinants Ex 6.2 Q43

$$\begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix}$$

$$= \begin{vmatrix} 2(y+z+x) & y+z+x & y+z+x \\ z+x & z & x \\ x+y & y & z \end{vmatrix}$$

$$= (x+y+z)\begin{vmatrix} 2 & 1 & 1 \\ z+x & z & x \\ x+y & y & z \end{vmatrix}$$

$$= (x+y+z)\begin{vmatrix} 0 & 1 & 1 \\ z+x-z-x & z & x \\ x+y-y-z & y & z \end{vmatrix}$$

$$= (x+y+z)\begin{vmatrix} 0 & 1 & 1 \\ 0 & z & x \\ x-z & y & z \end{vmatrix}$$

$$= (x+y+z)(x-z)^2$$
$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q44

L.H.S. =

$$\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix}$$

$$= \begin{vmatrix} a+x+y+z & y & z \\ a+x+y+z & a+y & z \\ a+x+y+z & x & a+z \end{vmatrix} [C_1 = C_1 + C_2 + C_3]$$

$$= (a+x+y+z)\begin{vmatrix} 1 & y & z \\ 1 & a+y & z \\ 1 & x & a+z \end{vmatrix}$$

$$= (a+x+y+z)\begin{vmatrix} 1 & y & z \\ 0 & a & 0 \\ 0 & x-y & a \end{vmatrix} [R_2 \to R_2 - R_1, R_3 \to R_3 - R_1]$$

$= (a+x+y+z)[1(a^2 - 0)]$

$= a^2(a+x+y+z)$

= R.H.S.

Hence Pr oved.

Chapter 6 Determinants Ex 6.2 Q45

$$\text{Let } \Delta = \begin{vmatrix} a^3 & 2 & a \\ b^3 & 2 & b \\ c^3 & 2 & c \end{vmatrix}$$

$$\Delta = 2\begin{vmatrix} a^3 & 1 & a \\ b^3 & 1 & b \\ c^3 & 1 & c \end{vmatrix}$$

$$\Delta = 2\left\{a^3(c-b) - 1\left(b^3c - bc^3\right) + a\left(b^3 - c^3\right)\right\}$$

$$\Delta = 2\left\{a^3(c-b) - bc(b-c)(b+c) + a(b-c)\left(b^2 + bc + c^2\right)\right\}$$

$$\Delta = 2(b\text{-}c)\left\{-a^3 - bc(b+c) + a\left(b^2 + bc + c^2\right)\right\}$$

$$\Delta = 2(a\text{-}b)(b\text{-}c)(c-a)(a+b+c)$$

Chapter 6 Determinants Ex 6.2 Q46

$$\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} = -\begin{vmatrix} x & y & z \\ a & b & c \\ p & q & r \end{vmatrix} = (-1)^2\begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix} = \begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix}$$

$$= (-1)\begin{vmatrix} y & x & z \\ q & p & r \\ b & a & c \end{vmatrix}$$

$$= (-1)^2\begin{vmatrix} y & x & z \\ b & a & c \\ q & p & r \end{vmatrix}$$

Taking transpose, we get

$$= \begin{vmatrix} y & b & p \\ x & a & q \\ z & c & r \end{vmatrix}$$

Chapter 6 Determinants Ex 6.2 Q47

Consider the determinant $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$, *where* a, b, c *are in A.P.*

Let $\Delta = \begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$, *we have,*

$$\Delta = \begin{vmatrix} 3x+1+2+a & x+2 & x+a \\ 3x+2+3+b & x+3 & x+b \\ 3x+3+4+c & x+4 & x+c \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 3x+3+a & x+2 & x+a \\ 3x+5+b & x+3 & x+b \\ 3x+7+c & x+4 & x+c \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ *and* $R_3 \to R_3 - R_2$, *we have,*

$$\Rightarrow \Delta = \begin{vmatrix} 3x+3+a & x+2 & x+a \\ 2+b-a & 1 & b-a \\ 2+c-b & 1 & c-b \end{vmatrix}$$

Since a, b *and* c *are in arithmetic progression, we have*
$b - a = c - b = k(say)$.
Thus,

$$\Delta = \begin{vmatrix} 3x+3+a & x+2 & x+a \\ 2+k & 1 & k \\ 2+k & 1 & k \end{vmatrix}$$

Since the second row and the third row are identical, we have
$\Delta = 0$

Chapter 6 Determinants Ex 6.2 Q48

Since, α, β, γ are in A.P, $2\beta = \alpha + \gamma$

$$LHS = \begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix}$$

$R_2 \to R_2 - \frac{R_1}{2} - \frac{R_3}{2}$

$$= \begin{vmatrix} x-3 & x-4 & x-\alpha \\ (x-2) - \frac{x-3}{2} - \frac{x-1}{2} & (x-3) - \frac{x-4}{2} - \frac{x-2}{2} & (x-\beta) - \frac{x-\alpha}{2} - \frac{x-\gamma}{2} \\ x-1 & x-2 & x-\gamma \end{vmatrix}$$

$$= \begin{vmatrix} x-3 & x-4 & x-\alpha \\ 0 & 0 & 0 \\ x-1 & x-2 & x-\gamma \end{vmatrix} \qquad [\because 2\beta = \alpha + \gamma]$$

$= 0$

Chapter 6 Determinants Ex 6.2 Q49

$$\Delta=\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

Applying $R_1 \to R_1+R_2+R_3$, we have:

$$\Delta=\begin{vmatrix} 2(a+b+c) & 2(a+b+c) & 2(a+b+c) \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

$$=2(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

Applying $C_2 \to C_2-C_1$ and $C_3 \to C_3-C_1$, we have:

$$\Delta=2(a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ c+a & b-c & b-a \\ a+b & c-a & c-b \end{vmatrix}$$

Expanding along R_1, we have:

$$\Delta=2(a+b+c)(1)\left[(b-c)(c-b)-(b-a)(c-a)\right]$$
$$=2(a+b+c)\left[-b^2-c^2+2bc-bc+ba+ac-a^2\right]$$
$$=2(a+b+c)\left[ab+bc+ca-a^2-b^2-c^2\right]$$

It is given that $\Delta=0$.

$$(a+b+c)\left[ab+bc+ca-a^2-b^2-c^2\right]=0$$

$\Rightarrow$ Either $a+b+c=0$, or $ab+bc+ca-a^2-b^2-c^2=0$.

Now,

$$ab+bc+ca-a^2-b^2-c^2=0$$
$$\Rightarrow -2ab-2bc-2ca+2a^2+2b^2+2c^2=0$$
$$\Rightarrow (a-b)^2+(b-c)^2+(c-a)^2=0$$
$$\Rightarrow (a-b)^2=(b-c)^2=(c-a)^2=0 \qquad \left[(a-b)^2,(b-c)^2,(c-a)^2 \text{ are non-negative}\right]$$
$$\Rightarrow (a-b)=(b-c)=(c-a)=0$$
$$\Rightarrow a=b=c$$

Hence, if $\Delta=0$, then either $a+b+c=0$ or $a=b=c$.

Chapter 6 Determinants Ex 6.2 Q50

$$\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} p-a & 0 & c-r \\ 0 & q-b & c-r \\ a & b & r \end{vmatrix} = 0 [R_1 = R_1 - R_3, R_2 = R_2 - R_3]$$

$$\Rightarrow (p-a)[r(q-b) - b(c-r)] + (c-r)[0 - a(q-b)] = 0$$

$$\Rightarrow (p-a)r(q-b) - (p-a)b(c-r) - (c-r)a(q-b) = 0$$

$$\Rightarrow \frac{(p-a)r(q-b)}{(p-a)(q-b)(r-c)} - \frac{(p-a)b(c-r)}{(p-a)(q-b)(r-c)} - \frac{(c-r)a(q-b)}{(p-a)(q-b)(r-c)} = 0$$

$$\Rightarrow \frac{r}{(r-c)} + \frac{b}{(q-b)} + \frac{a}{(p-a)} = 0$$

$$\Rightarrow \frac{r}{(r-c)} + \frac{b-q+q}{(q-b)} + \frac{a+p-p}{(p-a)} = 0$$

$$\Rightarrow \frac{r}{(r-c)} + \frac{q}{(q-b)} + \frac{(b-q)}{(q-b)} + \frac{(a-p)}{(p-a)} + \frac{p}{(p-a)} = 0$$

$$\Rightarrow \frac{r}{(r-c)} + \frac{q}{(q-b)} - 1 - 1 + \frac{p}{(p-a)} = 0$$

$$\Rightarrow \frac{r}{(r-c)} + \frac{q}{(q-b)} + \frac{p}{(p-a)} = 2$$

$$\therefore \frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$$

Chapter 6 Determinants Ex 6.2 Q51

Let us show that $x = 2$ is a root of the given equation:
Putting $x = 2$ in the LHS, we get

$$\begin{vmatrix} 2 & -6 & -1 \\ 2 & -6 & -1 \\ -3 & 4 & 4 \end{vmatrix} = 0$$

$\because R_1 = R_2$

Hence, $x = 2$ is a root of the given equation.

Now, we see if there are any other roots. For this we need to solve the equation:

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-1 & -6 & -1 \\ x-1 & -3x & x-3 \\ x-1 & 2x & x+2 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)\begin{vmatrix} 1 & -6 & -1 \\ 1 & -3x & x-3 \\ 1 & 2x & x+2 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)\begin{vmatrix} 1 & -6 & -1 \\ 1 & -3x & x-3 \\ 1 & 2x & x+2 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)\begin{vmatrix} 1 & -6 & -1 \\ 0 & -3x+6 & x-3+1 \\ 0 & 2x+6 & x+2+1 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)\begin{vmatrix} 1 & -6 & -1 \\ 0 & -3(x-2) & x-2 \\ 0 & 2(x+3) & x+3 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)(x-2)(x+3)\begin{vmatrix} 1 & -6 & -1 \\ 0 & -3 & 1 \\ 0 & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)(x-2)(x+3) = 0$$

$$\Rightarrow (x-1) = 0 \qquad (x-2) = 0 \qquad (x+3) = 0$$

$$\Rightarrow x = 1 \qquad x = 2 \qquad x = -3$$

Chapter 6 Determinants Ex 6.2 Q52-i

$$\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = 0$$

Apply $C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+b & c \\ x+a+b+c & b & x+c \end{vmatrix} = 0$$

$$\Rightarrow (x+a+b+c) \begin{vmatrix} 1 & b & c \\ 1 & x+b & c \\ 1 & b & x+c \end{vmatrix} = 0$$

$$\Rightarrow (x+a+b+c) \begin{vmatrix} 1 & b & c \\ 0 & x & 0 \\ 0 & 0 & x \end{vmatrix} = 0$$

$$\Rightarrow (x+a+b+c) x^2 = 0$$

$$\Rightarrow x = -(a+b+c) \quad or \quad x = 0$$

Chapter 6 Determinants Ex 6.2 Q52-ii

$$\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

Applying $R_1 \to R_1 + R_2 + R_3$, we get:

$$\begin{vmatrix} 3x+a & 3x+a & 3x+a \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

$$\Rightarrow (3x+a) \begin{vmatrix} 1 & 1 & 1 \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$, we have:

$$(3x+a) \begin{vmatrix} 1 & 0 & 0 \\ x & a & 0 \\ x & 0 & a \end{vmatrix} = 0$$

Expanding along R_1, we have:

$$(3x+a)\left[1 \times a^2\right] = 0$$

$$\Rightarrow a^2(3x+a) = 0$$

But $a \neq 0$.

Therefore, we have:

$$3x + a = 0$$

$$\Rightarrow x = -\frac{a}{3}$$

Chapter 6 Determinants Ex 6.2 Q52-iii

$$\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$$

Apply $C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \begin{vmatrix} 3x-2 & 3 & 3 \\ 3x-2 & 3x-8 & 3 \\ 3x-2 & 3 & 3x-8 \end{vmatrix} = 0$$

$$\Rightarrow (3x-2)\begin{vmatrix} 1 & 3 & 3 \\ 1 & 3x-8 & 3 \\ 1 & 3 & 3x-8 \end{vmatrix} = 0$$

$$\Rightarrow (3x-2)\begin{vmatrix} 1 & 3 & 3 \\ 0 & 3x-11 & 0 \\ 0 & 0 & 3x-11 \end{vmatrix} = 0$$

$$\Rightarrow (3x-2)(3x-11)^2 = 0$$

$$\Rightarrow (3x-2) = 0 \quad or \quad (3x-11)^2 = 0$$

$$\Rightarrow x = \frac{2}{3} \text{ or } \quad x = \pm\frac{11}{3}$$

Chapter 6 Determinants Ex 6.2 Q52-iv

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & a & a^2 \\ 1 & b & b^2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & x & x^2 \\ 0 & a-x & a^2-x^2 \\ 0 & b-x & b^2-x^2 \end{vmatrix} = 0$$

$$\Rightarrow (a-x)(b-x)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & a+x \\ 0 & 1 & b+x \end{vmatrix} = 0$$

$$\Rightarrow (a-x)(b-x)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & a+x \\ 0 & 0 & b-a \end{vmatrix} = 0$$

$$\Rightarrow (a-x)(b-x)(b-a) = 0$$

$$\Rightarrow (a-x) = 0 \quad or \quad (b-x) = 0$$

$$\Rightarrow a = x \quad or \quad b = x$$

Chapter 6 Determinants Ex 6.2 Q52-v

$$\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = 0$$

Apply $C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \begin{vmatrix} x+9 & 3 & 5 \\ x+9 & x+2 & 5 \\ x+9 & 3 & x+4 \end{vmatrix} = 0$$

$$\Rightarrow (x+9)\begin{vmatrix} 1 & 3 & 5 \\ 1 & x+2 & 5 \\ 1 & 3 & x+4 \end{vmatrix} = 0$$

$$\Rightarrow (x+9)\begin{vmatrix} 1 & 3 & 5 \\ 0 & x-1 & 0 \\ 0 & 0 & x-1 \end{vmatrix} = 0$$

$$\Rightarrow (x+9)(x-1)^2 = 0$$

$$\Rightarrow (x+9) = 0 \quad or \quad (x-1)^2 = 0$$

$$\Rightarrow x = -9 \quad or \quad x = 1$$

Chapter 6 Determinants Ex 6.2 Q52-vi

$$\Rightarrow \begin{vmatrix} 1 & x & x^3 \\ 0 & b-x & b^3-x^3 \\ 0 & c-x & c^3-x^3 \end{vmatrix} = 0$$

$$\Rightarrow (b-x)(c-x)\begin{vmatrix} 1 & x & x^3 \\ 0 & 1 & b^2+x^2+bx \\ 0 & 1 & c^2+x^2+cx \end{vmatrix} = 0$$

$$\Rightarrow (b-x)(c-x)\begin{vmatrix} 1 & x & x^3 \\ 0 & 1 & b^2+x^2+bx \\ 0 & 0 & c^2+x^2+cx-\left(b^2+x^2+bx\right) \end{vmatrix} = 0$$

$$\Rightarrow (b-x)(c-x)\begin{vmatrix} 1 & x & x^3 \\ 0 & 1 & b^2+x^2+bx \\ 0 & 0 & c^2-b^2+cx-bx \end{vmatrix} = 0$$

$$\Rightarrow (b-x)(c-x)(c-b)\begin{vmatrix} 1 & x & x^3 \\ 0 & 1 & b^2+x^2+bx \\ 0 & 0 & b+c+x \end{vmatrix} = 0$$

$$\Rightarrow (b-x)(c-x)(c-b)(b+c+x) = 0$$

$$\Rightarrow (b-x) = 0 \quad (c-x) = 0 \quad (b+c+x) = 0$$

$$\Rightarrow x = b \quad x = c \quad x = -(b+c)$$

Chapter 6 Determinants Ex 6.2 Q52-vii

$$\begin{vmatrix} 15-2x & 11-3x & 7-x \\ 11 & 17 & 14 \\ 10 & 16 & 13 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 15-2x & 11-3x & 7-x \\ 1 & 1 & 1 \\ 10 & 16 & 13 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 15-2x & -x-4 & 7-x \\ 1 & 0 & 1 \\ 10 & 6 & 13 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 8-x & -x-4 & 7-x \\ 0 & 0 & 1 \\ -3 & 6 & 13 \end{vmatrix} = 0$$

$$\Rightarrow -\left[(8-x)(6)-(-x-4)(-3)\right] = 0$$

$$\Rightarrow -\left[36-9x\right] = 0$$

$$\Rightarrow x = 4$$

Chapter 6 Determinants Ex 6.2 Q52-viii

$$\begin{vmatrix} 1 & 1 & x \\ p+1 & p+1 & p+x \\ 3 & x+1 & x+2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & x \\ p & p & p \\ 2 & x & 2 \end{vmatrix} = 0$$

$$\Rightarrow p\begin{vmatrix} 1 & 1 & x \\ 1 & 1 & 1 \\ 2 & x & 2 \end{vmatrix} = 0$$

$$\Rightarrow p\begin{vmatrix} 1 & 1 & x \\ 0 & 0 & 1-x \\ 2 & x & 2 \end{vmatrix} = 0$$

$$\Rightarrow p(x-1)(x-2) = 0$$

$$\Rightarrow (x-1) = 0 \qquad (x-2) = 0$$

$$\Rightarrow x = 1 \qquad x = 2$$

Chapter 6 Determinants Ex 6.2 Q52-ix

$$\begin{vmatrix} 3 & -2 & \sin 3\theta \\ -7 & 8 & \cos 2\theta \\ -11 & 14 & 2 \end{vmatrix} = 0$$

$\Rightarrow 3(16 - 14\cos 2\theta) + 2(-14 + 11\cos 2\theta) + \sin 3\theta(-98 + 88) = 0$

$\Rightarrow 20(1 - \cos 2\theta) + 10\sin 3\theta = 0$

$\Rightarrow 20(2\sin^2\theta) + 10(3\sin\theta - 4\sin^3\theta) = 0$

$\Rightarrow 4\sin^2\theta + 3\sin\theta - 4\sin^3\theta = 0$

$\Rightarrow 4\sin\theta + 3 - 4\sin^2\theta = 0$

$\Rightarrow 4\sin^2\theta - 4\sin\theta - 3 = 0$

$\Rightarrow (2\sin\theta + 1)(2\sin\theta - 3) = 0$

$\Rightarrow \sin\theta = -\frac{1}{2}$ or $\sin\theta = \frac{3}{2} = 1.5$

As $\sin\theta \in [-1, 1]$

$\therefore \sin\theta = -\frac{1}{2}$

$\Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{6}, n \in Z$

Ex 6.3

Chapter Determinants Ex 6.3 Q1(i)

If the vertices of a triangle are $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) then the area of the triangle is given by :

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Substituting the values

$$\Delta = \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & -1 & 1 \end{vmatrix}$$

expanding the determinant along R_1

$$= \frac{1}{2}\left[3\begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} - 8\begin{vmatrix} -4 & 1 \\ 5 & 1 \end{vmatrix} + 1\begin{vmatrix} -4 & 2 \\ 5 & -1 \end{vmatrix}\right]$$

$$= \frac{1}{2}\left[3(3) - 8(-9) + 1(-6)\right]$$

$$= \frac{1}{2}\left[9 + 72 - 6\right] = \frac{75}{2} \text{ sq. units}$$

The area of the Δ is $\frac{75}{2}$ sq. units

Chapter Determinants Ex 6.3 Q1(ii)

The area is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

expanding along R_1

$= \frac{1}{2}\left[2(-7) - 7(-9) + 1(-2)\right]$

$= \frac{1}{2}\left[-14 + 63 - 2\right]$

$= \frac{47}{2}$ sq. units

The area of the Δ is $\frac{47}{2}$ sq. units

Chapter Determinants Ex 6.3 Q1(iii)

The area is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} -1 & -8 & 1 \\ -2 & -3 & 1 \\ 3 & 2 & 1 \end{vmatrix}$$

$= \frac{1}{2}\left[-1(-5) + 8(-5) + 1(5)\right]$

$= \frac{1}{2}\left[5 - 40 + 5\right] = \frac{-30}{2} = 15$ sq. units

$\because$ Area can not be negative, so answer will be 15 sq. units.

The area of the Δ is 15 sq. units.

Chapter Determinants Ex 6.3 Q1(iv)

The area is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$$

Expanding along R_1

$= \frac{1}{2}\left[0 - 0 + 1(18)\right] = 9$ sq. units

The area is 9 sq. units

Chapter Determinants Ex 6.3 Q2(i)

If 3 points are collinear, then the area of the triangle then form will be zero.

Hence

$$\frac{1}{2}\begin{vmatrix} 5 & 5 & 1 \\ -5 & 1 & 1 \\ 10 & 7 & 1 \end{vmatrix} = 0$$

Expanding along R_1

$= \frac{1}{2}\left[5(-6) - 5(-15) + 1(-35 - 10)\right]$

$= \frac{1}{2}\left[-35 + 75 - 45\right]$

$= \frac{1}{2}\left[0\right]$

$= 0$

Since the area of the triangle is zero, hence the points are collinear.

Chapter Determinants Ex 6.3 Q2(ii)

If 3 points are collinear, then the area of the triangle then form will be zero.

Hence

$$\frac{1}{2}\begin{vmatrix}1 & -1 & 1\\ 2 & 1 & 1\\ 4 & 5 & 1\end{vmatrix} = 0$$

Expanding along R_1

$= \frac{1}{2}\left[1(-4)+1(-2)+1(6)\right]$

$= 0$

Since the area of the triangle is zero, hence the points are collinear.

Chapter Determinants Ex 6.3 Q2(iii)

If the points are collinear, then the area of the triangle will be zero.

So

$$\frac{1}{2}\begin{vmatrix}3 & -2 & 1\\ 8 & 8 & 1\\ 5 & 2 & 1\end{vmatrix} = 0$$

L.H.S

Expanding along R_1

$= \frac{1}{2}\left[3(6)+2(3)+1(-24)\right]$

$= \frac{1}{2}\left[18+6-24\right]$

$= \frac{1}{2}\left[0\right]$

$= 0$

Since the area of the triangle is zero, hence given points are collinear.

Chapter Determinants Ex 6.3 Q2(iv)

If given points are collinear, then the area of the triangle must be zero.

Hence

$$= \frac{1}{2}\begin{vmatrix}2 & 3 & 1\\ -1 & -2 & 1\\ 5 & 8 & 1\end{vmatrix}$$

$= \frac{1}{2}\left[2(-10)-3(-6)+1(2)\right]$

$= \frac{1}{2}\left[-20+18+2\right]$

$= \frac{1}{2}\left[0\right]$

$= 0$

Hence the given points are collinear.

Chapter Determinants Ex 6.3 Q3

If the given points are collinear, the area of the triangle must be zero.

Hence

$$\frac{1}{2}\begin{vmatrix}a & 0 & 1\\ 0 & b & 1\\ 1 & 1 & 1\end{vmatrix}$$

Expanding along R_1

$= \frac{1}{2}\left[a(b-1)-0(0-1)+1(-b)\right] = 0$

$or\ ab - a - 0 - b = 0$

$or\ ab = a + b$

Hence proved

Chapter Determinants Ex 6.3 Q4

If the given points are collinear, then the area of the triangle must be zero.

Hence

$$\frac{1}{2}\begin{vmatrix} a & b & 1 \\ a' & b' & 1 \\ a-a' & b-b' & 1 \end{vmatrix} = 0$$

or

$$\frac{1}{2}\left[a\left(b' - b + b'\right) - b\left(a' - a + a'\right) + 1\left(a'b - a'b' - ab' + a'b'\right)\right] = 0$$

or $\frac{1}{2}\left[ab' - ab + ab' - a'b + ab - a'b + a'b - ab'\right] = 0$

or $ab' - a'b = 0$

$ab' = a'b$

Hence proved

Chapter Determinants Ex 6.3 Q5

If the points are collinear, then the area of the triangle must be zero.

Hence

$$\begin{vmatrix} 1 & -5 & 1 \\ -4 & 5 & 1 \\ \lambda & 7 & 1 \end{vmatrix} = 0$$

Expanding along R_1

$$1(-2) + 5(-4 - \lambda) + 1(-28 - 5\lambda) = 0$$

$$-2 - 20 - 5\lambda - 28 - 5\lambda = 0$$

$$-50 - 10\lambda = 0$$

$$\lambda = -5$$

Hence $\lambda = -5$

Chapter Determinants Ex 6.3 Q6

$$\text{Area} = \left|\frac{1}{2}\begin{vmatrix} x & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix}\right|$$

$$\pm 2 \times 35 = \begin{vmatrix} x & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix}$$

$$\pm 70 = x(-10) - 4(-3) + 1(38)$$

$$\pm 70 = -10x + 12 + 38$$

$$\pm 70 = -10x + 50 \qquad ---(1)$$

Taking $(+)$ sign

$+70 = -10x + 50$

$10x = -20$ or $x = -2$

Again taking $(-)$ sign

$-70 = -10x + 50$

$10x = 120$ or $x = 12$

Hence $x = -2, 12$

Chapter Determinants Ex 6.3 Q7

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 1 & 4 & 1 \\ 2 & 3 & 1 \\ -5 & -3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[1(6) - 4(7) + 1(-6+15)\right]$$

$$= \frac{1}{2}\left[6 - 28 + 9\right]$$

$$= \frac{1}{2}\left[-13\right]$$

$$= \frac{13}{2}\text{ sq. units} \qquad [\because \text{Area can not be negative}]$$

Also, since the area of the triangle is non-zero.

Hence these points are non-collinear.

Chapter Determinants Ex 6.3 Q8

$$\text{Area} = \frac{1}{2}\begin{vmatrix} -3 & 5 & 1 \\ 3 & -6 & 1 \\ 7 & 2 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[-3(-8) - 5(-4) + 1(48)\right]$$

$$= \frac{1}{2}\left[24 + 20 + 48\right]$$

$$= 46 \text{ sq. units}$$

Hence the area is 46 sq. units.

Chapter Determinants Ex 6.3 Q9

If the given points are collinear, then the area of the triangle must be zero.

$$\text{so } \frac{1}{2}\begin{vmatrix} k & 2-2k & 1 \\ -k+1 & 2k & 1 \\ -4-k & 6-2k & 1 \end{vmatrix} = 0$$

expanding along R_1

$k(2k - 6 + 2k) - (2-2k)(-k+1+4+k) + 1(1-k)\times(6-2k) - 2k(-4-k) = 0$

$k(4k-6) - (2-2k)(5) + 1\left[6 - 2k - 6k + 2k^2 + 8k + 2k^2\right] = 0$

$4k^2 - 6k - 10 + 10k + 6 + 4k^2 = 0$

$8k^2 + 4k - 4 = 0$

$8k^2 + 8k - 4k - 4 = 0$ (Middle term splitting)

$8k(k+1) - 4(k+1) = 0$

$(8k-4)(k+1) = 0$

If $8k - 4 = 0$ or if $k + 1 = 0$

$k = \frac{1}{2}$ $\qquad$ $k = -1$

Hence $k = -1, \frac{1}{2}$

Chapter Determinants Ex 6.3 Q10

Since the points are collinear, hence the area of the triangle must be zero.

$$\text{so } \frac{1}{2}\begin{vmatrix} x & -2 & 1 \\ 5 & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix} = 0$$

or $x(-6) + 2(-3) + 1(24) = 0$

or $-6x - 6 + 24 = 0$

$-6x + 18 = 0$

$x = 3$

Hence $x = 3$

Chapter Determinants Ex 6.3 Q11

Since the points are collinear, hence the area of the triangle must be zero.

$$\frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ x & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix} = 0$$

$$3(-6) + 2(x - 8) + 1(8x - 16) = 0$$
$$-18 + 2x - 16 + 8x - 16 = 0$$
$$10x = 50$$
$$x = 5$$

Hence $x = 5$

Chapter Determinants Ex 6.3 Q12(i)

Let $A(x, y)$, $B(1, 2)$ and $C(3, 6)$ are 3 points in a line.

Since these points are collinear, hence area of the triangle must be zero.

$$\frac{1}{2}\begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix} = 0$$

Expanding along R_1

$$x(-4) - y(-2) + 1(0) = 0$$
$$-4x + 2y = 0$$
$$\text{or } 2x - y = 0$$
$$\text{or } y = 2x$$

Hence the equation is $y = 2x$

Chapter Determinants Ex 6.3 Q12(ii)

Let $A(x, y)$, $B(3, 1)$ and $C(9, 3)$ are 3 points in a line.

Since these points are collinear, hence the area of the triangle ABC must be zero.

$$\frac{1}{2}\begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0$$

Expanding along R_1

$$x(-2) - y(-6) + 1(0) = 0$$
$$-2x + 6y = 0$$
$$x - 3y = 0$$

Hence the equation of the line is $x - 3y = 0$

Chapter Determinants Ex 6.3 Q13(i)

$$\text{Area} = \frac{1}{2}\left\|\begin{matrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{matrix}\right\|$$

$$\pm 4 = \frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix}$$

Expanding along R_1

$$\pm 8 = k(-2) - 0(4 - 0) + 1(8)$$
$$\pm 8 = -2k + 8$$

Taking positive $(+)$ sign

$+8 = -2k + 8$ or $k = 0$

Taking negative $(-)$ sign

$-8 = -2k + 8$ or $k = 8$

Hence $k = 0, 8$

Chapter Determinants Ex 6.3 Q13(ii)

$$4 = \frac{1}{2}\left\|\begin{matrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{matrix}\right\|$$

$$\pm 8 = \begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix}$$

Expanding along R_1

$\pm 8 = -2(4-k) - 0(0-0) + 1(0)$

$\pm 8 = -8 + 2k$

Taking positive $(+)$ sign

$+8 = -8 + 2k$ or $k = 8$

Taking negative $(-)$ sign

$-8 = -8 + 2k$ or $k = 0$

Hence $k = 0, 8$

Ex 6.4

Chapter 6 Determinants Ex 6.4 Q1

Let $D = \begin{vmatrix} 1 & -2 \\ -3 & 5 \end{vmatrix} = 5 - 6 = -1$

$D_1 = \begin{vmatrix} 4 & -2 \\ -7 & 5 \end{vmatrix} = 20 - 14 = 6$

$D_2 = \begin{vmatrix} 1 & 4 \\ -3 & -7 \end{vmatrix} = -7 + 12 = 5$

by definition $x = \dfrac{D_1}{D} = \dfrac{6}{-1} = -6$

$y = \dfrac{D_2}{D} = \dfrac{5}{-1} = -5$

Hence $x = -6$

$y = -5$

Chapter 6 Determinants Ex 6.4 Q2

Let $D = \begin{vmatrix} 2 & -1 \\ 7 & -2 \end{vmatrix} = -4 + 7 = 3$

$D_1 = \begin{vmatrix} 1 & -1 \\ -7 & -2 \end{vmatrix} = -9$

$D_2 = \begin{vmatrix} 2 & 1 \\ 7 & -7 \end{vmatrix} = -21$

Now, $x = \dfrac{D_1}{D} = \dfrac{-9}{3} = -3$

$y = \dfrac{+D_2}{D} = \dfrac{-21}{3} = -7$

Hence $x = -3$

$y = -7$

Chapter 6 Determinants Ex 6.4 Q3

Let $D = \begin{vmatrix} 2 & -1 \\ 3 & 5 \end{vmatrix} = 13$

$D_1 = \begin{vmatrix} 17 & -1 \\ 6 & 5 \end{vmatrix} = 91$

$D_2 = \begin{vmatrix} 2 & 17 \\ 3 & 6 \end{vmatrix} = -39$

$x = \frac{D_1}{D} = \frac{91}{13} = 7$

$y = \frac{D_2}{D} = \frac{-39}{13} = -3$

Hence $x = 7$
$y = -3$

Chapter 6 Determinants Ex 6.4 Q4

Let $D = \begin{vmatrix} 3 & 1 \\ 3 & -1 \end{vmatrix} = -6$

$D_1 = \begin{vmatrix} 19 & 1 \\ 23 & -1 \end{vmatrix} = -42$

$D_2 = \begin{vmatrix} 3 & 19 \\ 3 & 23 \end{vmatrix} = 12$

$x = \frac{D_1}{D} = \frac{-42}{-6} = 7$

$y = \frac{D_2}{D} = \frac{12}{-6} = -2$

Hence $x = 7$
$y = -2$

Chapter 6 Determinants Ex 6.4 Q5

Let $D = \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix} = 11$

$D_1 = \begin{vmatrix} -2 & -1 \\ 3 & 4 \end{vmatrix} = -5$

$D_2 = \begin{vmatrix} 2 & -2 \\ 3 & 3 \end{vmatrix} = 12$

$x = \frac{D_1}{D} = \frac{-5}{11}$

$y = \frac{D_2}{D} = \frac{12}{11}$

Chapter 6 Determinants Ex 6.4 Q6

Let $D = \begin{vmatrix} 3 & a \\ 2 & a \end{vmatrix} = a$

$D_1 = \begin{vmatrix} 4 & a \\ 2 & a \end{vmatrix} = 2a$

$D_2 = \begin{vmatrix} 3 & 4 \\ 4 & 2 \end{vmatrix} = -2$

$x = \frac{D_1}{D} = \frac{2a}{a} = 2$

$y = \frac{D_2}{D} = \frac{-2}{a}$

Chapter 6 Determinants Ex 6.4 Q7

$$\text{Let } D = \begin{vmatrix} 2 & 3 \\ 1 & 6 \end{vmatrix} = 9$$

$$D_1 = \begin{vmatrix} 10 & 3 \\ 4 & 6 \end{vmatrix} = 48$$

$$D_2 = \begin{vmatrix} 2 & 10 \\ 1 & 4 \end{vmatrix} = -2$$

$$x = \frac{D_1}{D} = \frac{48}{9} = \frac{16}{3}$$

$$y = \frac{D_2}{D} = \frac{-2}{9}$$

Chapter 6 Determinants Ex 6.4 Q8

$$\text{Let } D = \begin{vmatrix} 5 & 7 \\ 4 & 6 \end{vmatrix} = 2$$

$$D_1 = \begin{vmatrix} -2 & 7 \\ -3 & 6 \end{vmatrix} = 9$$

$$D_2 = \begin{vmatrix} 5 & -2 \\ 4 & -3 \end{vmatrix} = -7$$

$$x = \frac{D_1}{D} = \frac{9}{2}$$

$$y = \frac{D_2}{D} = \frac{-7}{2}$$

Chapter 6 Determinants Ex 6.4 Q9

$$\text{Let } D = \begin{vmatrix} 9 & 5 \\ -2 & 3 \end{vmatrix} = 37$$

$$D_1 = \begin{vmatrix} 10 & 5 \\ 8 & 3 \end{vmatrix} = -10$$

$$D_2 = \begin{vmatrix} 9 & 10 \\ -2 & 8 \end{vmatrix} = 92$$

$$x = \frac{D_1}{D} = \frac{-10}{37}$$

$$y = \frac{D_2}{D} = \frac{92}{37}$$

Chapter 6 Determinants Ex 6.4 Q10

$$\text{Let } D = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = -5$$

$$D_1 = \begin{vmatrix} 1 & 2 \\ 4 & 1 \end{vmatrix} = -7$$

$$D_2 = \begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} = 1$$

$$x = \frac{D_1}{D} = \frac{7}{5}$$

$$y = \frac{D_2}{D} = \frac{-1}{5}$$

Chapter 6 Determinants Ex 6.4 Q11

$$\text{Let } D = \begin{vmatrix} 3 & 1 & 1 \\ 2 & -4 & 3 \\ 4 & 1 & -3 \end{vmatrix}$$

Expanding along R_1

$= 3(9) + (-1)(-18) + 1(18)$

$= 27 + 18 + 18 = 63$

$$\text{Again } D_1 = \begin{vmatrix} 2 & 1 & 1 \\ -1 & -4 & 3 \\ -11 & 1 & -3 \end{vmatrix}$$

Expanding along R_1

$= 2(9) + (-1)(36) + 1(-45)$

$= 18 - 36 - 45 = -63$

$$\text{Again } D_2 = \begin{vmatrix} 3 & 2 & 1 \\ 2 & -1 & 3 \\ 4 & -11 & -3 \end{vmatrix}$$

Expanding along R_1 $= 3(3 + 33) - 2(-18) + 1(-22 + 4)$

$= 108 + 36 - 18 = 126$

$$\text{Also } D_3 = \begin{vmatrix} 3 & 1 & 2 \\ 2 & -4 & -1 \\ 4 & 1 & -11 \end{vmatrix}$$

Expanding along R_1

$= 3(45) - 1(-18) + 2(18) = 135 + 18 + 36 = 189$

Now $x = \dfrac{D_1}{D} = \dfrac{-63}{63} = -1$

$y = \dfrac{D_2}{D} = \dfrac{126}{63} = 2$

$z = \dfrac{D_3}{D} = \dfrac{189}{63} = 3$

Chapter 6 Determinants Ex 6.4 Q12

$$\text{Let } D = \begin{vmatrix} 1 & -4 & -1 \\ 2 & -5 & 2 \\ -3 & 2 & 1 \end{vmatrix}$$

Expanding along R_1

$$= 1(-9) + 4(8) - 1(-11) = -9 + 32 + 11 = 34$$

$$\text{Again } D_1 = \begin{vmatrix} 11 & -4 & -1 \\ 39 & -5 & 2 \\ 1 & 2 & 1 \end{vmatrix}$$

Expanding along R_1

$$= 11(-9) + 4(37) - 1(83) \quad = -99 + 148 - 83$$
$$= 148 - 182$$
$$= -34$$

$$\text{Also } D_2 = \begin{vmatrix} 1 & 11 & -1 \\ 2 & 39 & 2 \\ -3 & 1 & 1 \end{vmatrix}$$

Expanding along R_1

$$= 1(37) - 11(8) - 1(119)$$
$$= 37 - 88 - 119 = -170$$

$$\text{Also } D_3 = \begin{vmatrix} 1 & -4 & 11 \\ 2 & -5 & -39 \\ -3 & 2 & 1 \end{vmatrix}$$

Expanding along R_1

$$= 1(-5 - 78) + 4(2 + 117) + 11(4 - 15)$$
$$= -83 + 476 - 121 = 272$$

Now $\quad x = \dfrac{D_1}{D} = \dfrac{-34}{34} = -1$

$$y = \frac{D_2}{D} = \frac{-170}{34} = -5$$

$$z = \frac{D_3}{D} = \frac{272}{34} = 8$$

Hence $x = -1, y = -5, z = 8$

Chapter 6 Determinants Ex 6.4 Q13

$$\text{Let } D = \begin{vmatrix} 6 & 1 & -3 \\ 1 & 3 & -2 \\ 2 & 1 & 4 \end{vmatrix}$$

Expanding along R_1

$$= 6(14) - 1(8) - 3(-5)$$
$$= 84 - 8 + 15 = 91$$

$$\text{Also } D_1 = \begin{vmatrix} 5 & 1 & -3 \\ 5 & -3 & -2 \\ 8 & 1 & 4 \end{vmatrix}$$

Expanding along R_1

$$= 5(14) - 1(36) - 3(-19) = 70 - 36 + 57 = 91$$

$$\text{Again } D_2 = \begin{vmatrix} 6 & 5 & -3 \\ 1 & 5 & -2 \\ 2 & 8 & 4 \end{vmatrix}$$

Expanding along R_1

$$= 6(36) - 5(8) - 3(-2) = 216 - 40 + 6 = 182$$

$$\text{Also } D_3 = \begin{vmatrix} 6 & 1 & 5 \\ 1 & 3 & 5 \\ 2 & 1 & 8 \end{vmatrix}$$

Expanding along R_1

$$= 6(19) - 1(-2) + 5(-5) = 114 + 2 - 25 = 91$$

Now $\quad x = \frac{D_1}{D} = \frac{91}{91} = 1$

$$y = \frac{D_2}{D} = \frac{182}{91} = 2$$

Also $\quad z = \frac{D_3}{D} = \frac{91}{91} = 1$

Hence $x = 1, y = 2, z = 1$

Chapter 6 Determinants Ex 6.4 Q14

$$\text{Let } D = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

Expanding along R_1

$$= 1(1) - 1(-1) + 0(-1) = 1 + 1 + 0 = 2$$

$$\text{Also } D_1 = \begin{vmatrix} 5 & 1 & 0 \\ 3 & 1 & 1 \\ 4 & 0 & 1 \end{vmatrix}$$

Expanding along R_1

$$= 5(1) - 1(-1) + 0(-4) = 5 + 1 + 0 = 6$$

$$\text{Again } D_2 = \begin{vmatrix} 1 & 5 & 0 \\ 0 & 3 & 1 \\ 1 & 4 & 1 \end{vmatrix}$$

Expanding along R_1

$$= 1(-1) - 5(-1) + 0(-3) = -1 + 5 + 0 = 4$$

$$\text{Also } D_3 = \begin{vmatrix} 1 & 1 & 5 \\ 0 & 1 & 3 \\ 1 & 0 & 4 \end{vmatrix}$$

$$= 1(4) - 1(-3) + 5(-1) = 4 + 3 - 5 = 2$$

Now $x = \frac{D_1}{D} = \frac{6}{2} = 3$

$$y = \frac{D_2}{D} = \frac{4}{2} = 2$$

$$z = \frac{D_3}{D} = \frac{2}{2} = 1$$

Hence $x = 3, y = 2, z = 1$

Chapter 6 Determinants Ex 6.4 Q15

$$\text{Let } D = \begin{vmatrix} 0 & 2 & -3 \\ 1 & 3 & 0 \\ 3 & 4 & 0 \end{vmatrix}$$

Expanding along R_1

$$= 0(0) - 2(0) - 3(-5) = 15$$

$$\text{Also } D_1 = \begin{vmatrix} 0 & 2 & -3 \\ -4 & 3 & 0 \\ 3 & 4 & 0 \end{vmatrix}$$

Expanding along R_1

$$= 0(0) - 2(0) - 3(-25) = 75$$

$$\text{Again } D_2 = \begin{vmatrix} 0 & 0 & -3 \\ 1 & -4 & 0 \\ 3 & 3 & 0 \end{vmatrix}$$

Expanding along R_1

$$= 0(0) - 0(0) - 3(15) = -45$$

$$\text{Also } D_3 = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 3 & -4 \\ 3 & 4 & 3 \end{vmatrix}$$

$$= 0(25) - 2(15) + 0(1) = -30$$

Now $x = \dfrac{D_1}{D} = \dfrac{75}{15} = 5$

$y = \dfrac{D_2}{D} = \dfrac{-45}{15} = -3$

$z = \dfrac{D_3}{D} = \dfrac{-30}{15} = -2$

Hence $x = 5, y = -3, z = -2$

Chapter 6 Determinants Ex 6.4 Q16

Here $D = \begin{vmatrix} 5 & -7 & 1 \\ 6 & -8 & -1 \\ 3 & 2 & -6 \end{vmatrix} = 5(48+2) + 7(-33) + 1(36)$

$= 250 - 231 + 36 = 55$

$D_1 = \begin{vmatrix} 11 & -7 & 1 \\ 15 & -8 & -1 \\ 7 & 2 & -6 \end{vmatrix} = 11(50) + 7(-83) + 1(86)$

$= 550 - 581 + 86 = 55$

$D_2 = \begin{vmatrix} 5 & 11 & 1 \\ 6 & 15 & -1 \\ 3 & 7 & -6 \end{vmatrix} = 5(-83) - 11(-33) + 1(-3)$

$= -415 + 363 - 3 = -55$

$D_3 = \begin{vmatrix} 5 & -7 & 11 \\ 6 & -8 & 15 \\ 3 & 2 & 7 \end{vmatrix} = 5(-86) + 7(-3) + 11(36)$

$= -430 - 21 + 396$

$= -55$

Now $x = \dfrac{D_1}{D} = \dfrac{55}{55} = 1$

$y = \dfrac{D_2}{D} = \dfrac{-55}{55} = -1$

$z = \dfrac{D_3}{D} = \dfrac{-55}{55} = -1$

Hence $x = 1, y = -1, z = -1$

Chapter 6 Determinants Ex 6.4 Q17

2x - 3y - 4z = 29

-2x + 5y - z = -15

3x - y + 5z = - 11

From the given system of equation we have

$$D = \begin{vmatrix} 2 & -3 & 4 \\ -2 & 5 & -1 \\ 3 & -1 & 5 \end{vmatrix} = 2(25-1)+3(-10+3)+4(2-15) = 48-21-52 = -25$$

$$D_1 = \begin{vmatrix} 29 & -3 & 4 \\ -15 & 5 & -1 \\ 11 & -1 & 5 \end{vmatrix} = 29(25-1)+3(-75+11)+4(15-55) = 696-192-160 = 344$$

$$D_2 = \begin{vmatrix} 2 & 29 & 4 \\ -2 & -15 & -1 \\ 3 & 11 & 5 \end{vmatrix} = 2(-75+11)-29(-10+3)+4(-22+45) = -128+203+92 = 167$$

$$D_3 = \begin{vmatrix} 2 & -3 & 29 \\ -2 & 5 & -15 \\ 3 & -1 & 11 \end{vmatrix} = 2(55-15)+3(-22+45)+29(2-15) = 80+69-377 = -228$$

So, by Cramer's Rule, we obtain

$$x = \frac{D_1}{D} = -\frac{344}{25}$$

$$y = \frac{D_2}{D} = -\frac{167}{25}$$

$$z = \frac{D_3}{D} = \frac{228}{25}$$

Note: Answer given in the book is incorrect.

Chapter 6 Determinants Ex 6.4 Q18

Here $D = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & -2 \end{vmatrix} \quad = 1(1) - 1(-3) = 1 + 3 = 4$

$$D_1 = \begin{vmatrix} 1 & 1 & 0 \\ -6 & 0 & 1 \\ 3 & -1 & -2 \end{vmatrix} \quad = 1(1) - 1(9) = -8$$

$$D_2 = \begin{vmatrix} 1 & 1 & 0 \\ 1 & -6 & 1 \\ 1 & 3 & -2 \end{vmatrix} \quad = 1(9) - 1(-3) = 12$$

$$D_3 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & -6 \\ 1 & -1 & 3 \end{vmatrix} \quad = 1(-6) - 1(9) + 1(-1) = -6 - 9 - 1 = -16$$

Now $x = \frac{D_1}{D} = \frac{-8}{4} = -2$

$y = \frac{D_2}{D} = \frac{12}{4} = 3$

$z = \frac{D_3}{D} = \frac{-16}{4} = -4$

Hence $x = -2, y = 3, z = -4$

Chapter 6 Determinants Ex 6.4 Q19

$$D=\begin{vmatrix}1&1&1\\a&b&c\\a^2&b^2&c^2\end{vmatrix}$$

$$c_2\to c_2-c_1, c_3\to c_3-c_1$$

$$=\begin{vmatrix}1&0&0\\a&b-a&c-a\\a^2&b^2-a^2&c^2-a^2\end{vmatrix}$$

Now taking $(b-a)$ from c_2, and $(c-a)$ c_3 common

$$=(b-a)(c-a)\begin{vmatrix}1&0&0\\a&1&1\\a^2&b+a&c+a\end{vmatrix}$$

Expanding along R_1

$=(b-a)(c-a)[c+a-b-a]$

$=(b-a)(c-a)(c-b)$

$=(a-b)(b-c)(c-a)$

Again $D_1=-\begin{vmatrix}1&1&1\\d&b&c\\d^2&b^2&c^2\end{vmatrix}$

$$c_2\to c_2-c_1, c_3\to c_3-c_1$$

$$D_1=-\begin{vmatrix}1&0&0\\d&b-d&c-d\\d^2&b^2-d^2&c^2-d^2\end{vmatrix}$$

Taking $(b-d)$ common from c_2 and $(c-d)$ from c_3

$$-(b-d)(c-d)\begin{vmatrix}1&0&0\\d&1&1\\d^2&b+d&c+d\end{vmatrix}$$

Expanding along R_1

$=-(b-d)(c-d)[1(c+d-b-d)]$

$=-(b-d)(c-d)(c-b)$

$=-(b-c)(c-d)(d-b)$

Again $D_2=-\begin{vmatrix}1&1&1\\a&d&c\\a^2&b^2&c^2\end{vmatrix}$

$$c_2\to c_2-c_1, c_3\to c_3-c_1$$

$$=-\begin{vmatrix}1&0&0\\a&d-a&c-a\\a^2&d^2-a^2&c^2-a^2\end{vmatrix}$$

Taking $(d-a)$ *common from* c_2 *and* $(c-a)$ *from* c_3

$$=-(d-a)(c-a)\begin{vmatrix}1&0&0\\a&1&1\\a^2&d+a&c+a\end{vmatrix}$$

Expanding along R_1

$=-(d-a)(c-a)\times 1[c+a-d-a]$

$=-(d-a)(c-a)(c-d)$

$=-(a-d)(d-c)(c-a)$

Also $\quad D_3 = -\begin{vmatrix} 1 & 1 & 1 \\ a & b & d \\ a^2 & b^2 & d^2 \end{vmatrix}$

$c_2 \to c_2 - c_1, c_3 \to c_3 - c_1$

$$= -\begin{vmatrix} 1 & 0 & 0 \\ a & b-a & d-a \\ a^2 & b^2-a^2 & d^2-a^2 \end{vmatrix}$$

Now, taking $(b-a)$ *common from* c_2 *and* $(d-a)$ *from* c_3

$$= -(b-a)(d-a)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & b+a & d+a \end{vmatrix}$$

Expanding along R_1

$= -(b-a)(d-a) \times 1[d+a-b-a]$

$= -(b-a)(d-a)(d-b)$

$= -(a-b)(b-d)(d-a)$

Now $\quad x = \dfrac{D_1}{D} = -\dfrac{(b-c)(c-d)(d-b)}{(a-b)(b-c)(c-a)}$

$y = \dfrac{D_2}{D} = -\dfrac{(a-d)(d-c)(c-a)}{(a-b)(b-c)(c-a)}$

$z = \dfrac{D_3}{D} = -\dfrac{(a-b)(b-d)(d-a)}{(a-b)(b-c)(c-a)}$

Chapter 6 Determinants Ex 6.4 Q20

Here $D = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -2 & 2 & 2 \\ 2 & 1 & -2 & 2 \\ 3 & -1 & 3 & -3 \end{vmatrix}$

$$\therefore D = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & -3 & 1 & 1 \\ 2 & -1 & -4 & 0 \\ 3 & -4 & 0 & -6 \end{vmatrix} = 1\begin{vmatrix} -3 & 1 & 1 \\ -1 & -4 & 0 \\ -4 & 0 & -6 \end{vmatrix} \qquad \begin{bmatrix} C2 \to C2 - C1 \\ C3 \to C3 - C1 \\ C4 \to C4 - C1 \end{bmatrix}$$

$$\therefore \quad D = \begin{vmatrix} 0 & 0 & 1 \\ -1 & -4 & 0 \\ -22 & 6 & -6 \end{vmatrix} \qquad \begin{bmatrix} C1 \to C1 + 3C3 \\ C2 \to C2 - C3 \end{bmatrix}$$

$$= 1(-6 - 88) = -94$$

$$D_1 = \begin{vmatrix} 2 & 1 & 1 & 1 \\ -6 & -2 & 2 & 2 \\ -5 & 1 & -2 & 2 \\ -3 & -1 & 3 & -3 \end{vmatrix} = 188$$

$$D_2 = \begin{vmatrix} 1 & 2 & 1 & 1 \\ 1 & -6 & 2 & 2 \\ 2 & -5 & -2 & 2 \\ 3 & -3 & 3 & -3 \end{vmatrix} = -282$$

$$D_3 = \begin{vmatrix} 1 & 1 & 2 & 1 \\ 1 & -2 & -6 & 2 \\ 2 & 1 & -5 & 2 \\ 3 & -1 & -3 & -3 \end{vmatrix} = -141$$

$$D_4 = \begin{vmatrix} 1 & 1 & 1 & 2 \\ 1 & -2 & 2 & -6 \\ 2 & 1 & -2 & -5 \\ 3 & -1 & 3 & -3 \end{vmatrix} = 47$$

Now $x = \frac{D_1}{D} = \frac{188}{-94} = -2$

$$y = \frac{D_2}{D} = \frac{-282}{-94} = 3$$

$$z = \frac{D_3}{D} = \frac{-141}{-94} = \frac{3}{2}$$

$$w = \frac{D_4}{D} = \frac{47}{-94} = -\frac{1}{2}$$

Hence $x = -2, y = 3, z = \frac{3}{2}, w = -\frac{1}{2}$

Chapter 6 Determinants Ex 6.4 Q21

Here $D = \begin{vmatrix} 2 & 0 & -3 & 1 \\ 1 & -1 & 0 & 2 \\ 0 & -3 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}$

$$\therefore D = \begin{vmatrix} 2 & -2 & -5 & 1 \\ 1 & -2 & -1 & 2 \\ 0 & -3 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{vmatrix} = -1 \begin{vmatrix} -2 & -5 & 1 \\ -2 & -1 & 2 \\ -3 & 1 & 1 \end{vmatrix} \qquad \begin{bmatrix} C2 \to C2 - C1 \\ C3 \to C3 - C1 \end{bmatrix}$$

$$\therefore \quad D = -1 \begin{vmatrix} 1 & -6 & 1 \\ 4 & -3 & 2 \\ 0 & 0 & 1 \end{vmatrix} \qquad \begin{bmatrix} C1 \to C1 + 3C3 \\ C2 \to C2 - C3 \end{bmatrix}$$

$$= -1(-3 + 24) = -21$$

$$D_1 = \begin{vmatrix} 1 & 0 & -3 & 1 \\ 1 & -1 & 0 & 2 \\ 1 & -3 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = -21$$

$$D_2 = \begin{vmatrix} 2 & 1 & -3 & 1 \\ 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = -6$$

$$D_3 = \begin{vmatrix} 2 & 0 & 1 & 1 \\ 1 & -1 & 1 & 2 \\ 0 & -3 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = -6$$

$$D_4 = \begin{vmatrix} 2 & 0 & -3 & 1 \\ 1 & -1 & 0 & 1 \\ 0 & -3 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{vmatrix} = 3$$

Now $x = \frac{D_1}{D} = \frac{-21}{-21} = 1$

$$y = \frac{D_2}{D} = \frac{-6}{-21} = \frac{2}{7}$$

$$z = \frac{D_3}{D} = \frac{-6}{-21} = \frac{2}{7}$$

$$w = \frac{D_4}{D} = \frac{3}{-21} = -\frac{1}{7}$$

Chapter 6 Determinants Ex 6.4 Q22

Let $D = \begin{vmatrix} 2 & -1 \\ 4 & -2 \end{vmatrix}$

Expanding along R_1

$= -4 + 4 = 0$

Also $D_1 = \begin{vmatrix} 5 & -1 \\ 7 & -2 \end{vmatrix} = -3$

Also $D_2 = \begin{vmatrix} 2 & 5 \\ 4 & 7 \end{vmatrix} = -6$

And since $D = 0$ and D_1 and D_2 are non-zero, hence the given system of equations is inconsistent.

Hence proved.

Chapter 6 Determinants Ex 6.4 Q23

$$D = \begin{vmatrix} 3 & 1 \\ -6 & -2 \end{vmatrix} = -6+6 = 0$$

$$D_1 = \begin{vmatrix} 5 & 1 \\ 9 & -2 \end{vmatrix} = -10-9 = -19 \neq 0$$

Since $D = 0$ but $D_1 \neq 0$

Hence the given system of equations is inconsistent.

Chapter 6 Determinants Ex 6.4 Q24

Here $D = \begin{vmatrix} 3 & -1 & 2 \\ 2 & 1 & 3 \\ 1 & -2 & -1 \end{vmatrix}$

Expanding along R_1

$$= 3(5) + 1(-5) + 2(-5)$$
$$= 15 - 5 - 10 = 15 - 15 = 0$$

Also $D_1 = \begin{vmatrix} 3 & -1 & 2 \\ 5 & 1 & 3 \\ 1 & -2 & -1 \end{vmatrix}$

Expanding along R_1

$$= 3(5) + 1(-8) + 2(-11)$$
$$= 15 - 8 - 22$$
$$= -15 \neq 0$$

Since $D = 0$ and $D_1 \neq 0$

Hence the given system of equations is inconsistent.

Chapter 6 Determinants Ex 6.4 Q25

Here $D = \begin{vmatrix} 3 & -1 & 2 \\ 2 & -1 & 1 \\ 3 & 6 & 5 \end{vmatrix} = 3(-11) + 1(7) + 2(15) = -33 + 7 + 30 = 4$

$$D_1 = \begin{vmatrix} 6 & -1 & 2 \\ 2 & -1 & 1 \\ 20 & 6 & 5 \end{vmatrix} = 12$$

$$D_2 = \begin{vmatrix} 3 & 6 & 2 \\ 2 & 2 & 1 \\ 3 & 20 & 5 \end{vmatrix} = -4$$

$$D_3 = \begin{vmatrix} 3 & -1 & 6 \\ 2 & -1 & 2 \\ 3 & 6 & 20 \end{vmatrix} = 28$$

Now $x = \frac{D_1}{D} = \frac{12}{4} = -3$

$$y = \frac{D_2}{D} = \frac{-4}{4} = -1$$

$$z = \frac{D_3}{D} = \frac{28}{4} = 7$$

Chapter 6 Determinants Ex 6.4 Q26

We have,

$$D = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ -1 & -2 & 2 \end{vmatrix} = \begin{vmatrix} 0 & -1 & 0 \\ 3 & 1 & 0 \\ -3 & -2 & 0 \end{vmatrix} = 0$$

$$D_1 = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & -2 & 2 \end{vmatrix} = \begin{vmatrix} 3 & -1 & 0 \\ 2 & 1 & 0 \\ 1 & -2 & 0 \end{vmatrix} = 0$$

$$D_2 = \begin{vmatrix} 1 & 3 & 1 \\ 2 & 2 & -1 \\ -1 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & -4 & -3 \\ -1 & 4 & 3 \end{vmatrix} = 1(-12+12) = 0$$

$$D_3 = \begin{vmatrix} 1 & -1 & 3 \\ 2 & 1 & 2 \\ -1 & -2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 3 & -4 \\ -1 & -3 & 4 \end{vmatrix} = 1(12-12) = 0$$

$\therefore \quad D = D_1 = D_2 = D_3 = 0$

So, either the system is consistent with infinitely many solutions or it is inconsistent.

Consider the first two equations, written as

$$x - y = 3 - z$$
$$2x + y = 2 + z$$

To solve these equations we use Cramer's rule.

Here,

$$D = \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 1 + 2 = 3$$

$$D_1 = \begin{vmatrix} 3-z & -1 \\ 2+z & 1 \end{vmatrix} = (3-z) + (2+z) = 5$$

$$D_2 = \begin{vmatrix} 1 & 3-z \\ 2 & 2+z \end{vmatrix} = (2+z) - (6-2z) = -4 + 3z$$

$\therefore \quad x = \frac{D_1}{D} = \frac{5}{3}$

$$y = \frac{D_2}{D} = \frac{-4+3z}{3}$$

Let $z = k$, then the equations have the solution.

$$x = \frac{5}{3},\ y = \frac{-4+3k}{3},\ z = k$$

Chapter 6 Determinants Ex 6.4 Q27

Here,

$$D = \begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix} = 6 - 6 = 0$$

$$D_1 = \begin{vmatrix} 5 & 2 \\ 15 & 6 \end{vmatrix} = 30 - 30 = 0$$

$$D_2 = \begin{vmatrix} 1 & 5 \\ 3 & 15 \end{vmatrix} = 15 - 15 = 0$$

So, $D = D_1 = D_2 = 0$

Let $y = k$, then we have,

$$x + 2y = 5$$

$\Rightarrow \quad x = 5 - 2y = 5 - 2k$

$\therefore \quad x = 5 - 2k,\ y = k$ are the infinite solutions of the given system.

Chapter 6 Determinants Ex 6.4 Q28

Here,

$$D = \begin{vmatrix} 1 & 1 & -1 \\ 1 & -2 & 1 \\ 3 & 6 & -5 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & -3 & 2 \\ 3 & 3 & -2 \end{vmatrix} = 1(6-6) = 0$$

$$D_1 = \begin{vmatrix} 0 & 1 & -1 \\ 0 & -2 & 1 \\ 0 & 6 & -5 \end{vmatrix} = 0$$

$$D_2 = \begin{vmatrix} 1 & 0 & -1 \\ 1 & 0 & 1 \\ 3 & 0 & -5 \end{vmatrix} = 0$$

$$D_3 = \begin{vmatrix} 1 & 1 & 0 \\ 1 & -2 & 0 \\ 3 & 6 & 0 \end{vmatrix} = 0$$

So, $D = D_1 = D_2 = D_3 = 0$

The given system either has infinite solutions or it is inconsistent.

Consider the first two equations, written as

$$x + y = z$$
$$x - 2y = -z$$

To solve this we will use Cramer's rule

Here,

$$D = \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = -2 - 1 = -3$$

$$D_1 = \begin{vmatrix} z & 1 \\ -z & -2 \end{vmatrix} = -2z - z = -z$$

$$D_2 = \begin{vmatrix} 1 & z \\ 1 & -z \end{vmatrix} = -z - z = -2z$$

$$\therefore \quad x = \frac{D_1}{D} = \frac{-z}{-3} = \frac{z}{3}$$

$$y = \frac{D_2}{D} = \frac{-2z}{-3} = \frac{2z}{3}$$

Let $z = k$, then the solutions of the given system are

$$x = \frac{k}{3},\ y = \frac{2k}{3},\ z = k$$

Chapter 6 Determinants Ex 6.4 Q29

Here,

$$D = \begin{vmatrix} 2 & 1 & -2 \\ 1 & -2 & 1 \\ 5 & -5 & 1 \end{vmatrix} = \begin{vmatrix} 12 & 9 & -2 \\ -4 & -3 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 1(-36+36) = 0$$

$$D_1 = \begin{vmatrix} 4 & 1 & -2 \\ -2 & -2 & 1 \\ -2 & -5 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 1 & -2 \\ 0 & -2 & 1 \\ 0 & -5 & 1 \end{vmatrix} = 0$$

$$D_2 = \begin{vmatrix} 2 & 4 & -2 \\ 1 & -2 & 1 \\ 5 & -2 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 0 & -2 \\ 1 & 0 & 1 \\ 5 & 0 & 1 \end{vmatrix} = 0$$

$$D_3 = \begin{vmatrix} 2 & 1 & 4 \\ 1 & -2 & -2 \\ 5 & -5 & -2 \end{vmatrix} = \begin{vmatrix} 4 & -3 & 0 \\ 1 & -2 & -2 \\ 4 & -3 & 0 \end{vmatrix} = 2(-12+12) = 0$$

So, $D = D_1 = D_2 = D_3 = 0$

So, the given system is either inconsistent or has infinite solutions.

Consider the 2nd and 3rd equation, written as

$$x - 2y = -2 - z$$
$$5x - 5y = -2 - z$$

Then,

$$D = \begin{vmatrix} 1 & -2 \\ 5 & -5 \end{vmatrix} = -5 - (-10) = 5$$

$$D_1 = \begin{vmatrix} -2-z & -2 \\ -2-z & -5 \end{vmatrix} = (2+z)(5) - 2(2+z) = 3(2+z) = 6 + 3z$$

$$D_2 = \begin{vmatrix} 1 & -(2+z) \\ 5 & -(2+z) \end{vmatrix} = -(2+z) + 5(2+z) = 4(2+z) = 8 + 4z$$

$$\therefore \quad x = \frac{D_1}{D} = \frac{6+3z}{5}$$

$$y = \frac{D_2}{D} = \frac{8+4z}{5}$$

Let $z = k$, then

$x = \dfrac{6+3k}{5},\ y = \dfrac{8+4k}{5},\ z = k$ are the infinite solution of the given system of equations.

Chapter 6 Determinants Ex 6.4 Q30

Here,

$$D = \begin{vmatrix} 1 & -1 & 3 \\ 1 & 3 & -3 \\ 5 & 3 & 3 \end{vmatrix} = \begin{vmatrix} 2 & 2 & 0 \\ 1 & 3 & -3 \\ 6 & 6 & 0 \end{vmatrix} = 3(12-12) = 0$$

$$D_1 = \begin{vmatrix} 6 & -1 & 3 \\ -4 & 3 & -3 \\ 10 & 3 & 3 \end{vmatrix} = \begin{vmatrix} 2 & 2 & 0 \\ -4 & 3 & -3 \\ 6 & 6 & 0 \end{vmatrix} = 0$$

$$D_2 = \begin{vmatrix} 1 & 6 & 3 \\ 1 & -4 & -3 \\ 5 & 10 & 3 \end{vmatrix} = \begin{vmatrix} 2 & 2 & 0 \\ 1 & -4 & -3 \\ 6 & 6 & 0 \end{vmatrix} = 0$$

$$D_3 = \begin{vmatrix} 1 & -1 & 6 \\ 1 & 3 & -4 \\ 5 & 3 & 10 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 6 \\ 0 & 4 & -10 \\ 0 & 8 & -20 \end{vmatrix} = 1(-80+80) = 0$$

So, $D = D_1 = D_2 = D_3 = 0$

So, the given system is either inconsistent or has infinite solutions.

Consider the first to equations, written as

$$x - y = 6 - 3z$$
$$x + 3y = -4 + 3z$$

Here,

$$D = \begin{vmatrix} 1 & -1 \\ 1 & 3 \end{vmatrix} = 3 + 1 = 4$$

$$D_1 = \begin{vmatrix} 6-3z & -1 \\ -4+3z & 3 \end{vmatrix} = 3(6-3z) + (-4+3z) = 14 - 6z$$

$$D_2 = \begin{vmatrix} 1 & 6-3z \\ 1 & -4+3z \end{vmatrix} = (-4+3z) - (6-3z) = -10 + 6z$$

$$\therefore \quad x = \frac{D_1}{D} = \frac{14-6z}{4} = \frac{7-3z}{2}$$

$$y = \frac{D_2}{D} = \frac{6z-10}{4} = \frac{3z-5}{2}$$

Let $z = k$, then

$x = \frac{7-3k}{2}$, $y = \frac{3k-5}{2}$, $z = k$ are the infinite solution of the given system of equations.

Chapter 6 Determinants Ex 6.4 Q31

Let the rates of commissions on items A, B and C be x, y and z respectively.

Then we can express the given model as a system of linear equations

$$90x + 100y + 20z = 800$$
$$130x + 50y + 40z = 900$$
$$60x + 100y + 30z = 850$$

We will solve this using the Cramer's rule.

Here,

$$D = \begin{vmatrix} 90 & 100 & 20 \\ 130 & 50 & 40 \\ 60 & 100 & 30 \end{vmatrix} = \begin{vmatrix} -170 & 0 & -60 \\ 130 & 50 & 40 \\ -200 & 0 & -50 \end{vmatrix} = 50(8500 - 12000) = -175000$$

$$D_1 = \begin{vmatrix} 800 & 100 & 20 \\ 900 & 50 & 40 \\ 850 & 100 & 30 \end{vmatrix} = \begin{vmatrix} -1000 & 0 & -60 \\ 900 & 50 & 40 \\ -950 & 0 & -50 \end{vmatrix} = 50(50000 - 57000) = -350000$$

$$D_2 = \begin{vmatrix} 90 & 800 & 20 \\ 130 & 900 & 40 \\ 60 & 850 & 30 \end{vmatrix} = \begin{vmatrix} 90 & 800 & 20 \\ -50 & -700 & 0 \\ -75 & -350 & 0 \end{vmatrix} = 20(17500 - 52500) = -700000$$

$$D_3 = \begin{vmatrix} 90 & 100 & 800 \\ 130 & 50 & 900 \\ 60 & 100 & 850 \end{vmatrix} = \begin{vmatrix} -170 & 0 & -1000 \\ 130 & 50 & 900 \\ -200 & 0 & -950 \end{vmatrix} = 50(161500 - 200000) = -1925000$$

$$\therefore \quad x = \frac{D_1}{D} = \frac{-350000}{-175000} = 2$$

$$y = \frac{D_2}{D} = \frac{-700000}{-175000} = 4$$

$$z = \frac{D_3}{D} = \frac{-1925000}{-175000} = 11$$

$\therefore$ The rates of commission of items A, B and C are 2%, 4% and 11% respectively.

Chapter 6 Determinants Ex 6.4 Q32

Expressing the given information as a system of linear equations we get

$$2x + 3y + 4z = 29$$
$$x + y + 2z = 13$$
$$3x + 2y + z = 16$$

Where $x,\ y,\ z$ is the number of cars $C_1,\ C_2$ and C_3 produced.

We use Cramer's rule to solve this system.

Here,

$$D = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 1 & 2 \\ 3 & 2 & 1 \end{vmatrix} = \begin{vmatrix} -10 & -5 & 0 \\ -5 & -3 & 0 \\ 3 & 2 & 1 \end{vmatrix} = 1(30 - 25) = 5$$

$$D_1 = \begin{vmatrix} 29 & 3 & 4 \\ 13 & 1 & 2 \\ 16 & 2 & 1 \end{vmatrix} = \begin{vmatrix} -35 & -5 & 0 \\ -19 & -3 & 0 \\ 16 & 2 & 1 \end{vmatrix} = 1(105 - 95) = 10$$

$$D_2 = \begin{vmatrix} 0 & 29 & 4 \\ 1 & 13 & 2 \\ 3 & 16 & 1 \end{vmatrix} = \begin{vmatrix} -10 & -35 & 0 \\ -5 & -19 & 0 \\ 3 & 16 & 1 \end{vmatrix} = 1(190 - 175) = 15$$

$$D_3 = \begin{vmatrix} 2 & 3 & 29 \\ 1 & 1 & 13 \\ 3 & 2 & 16 \end{vmatrix} = \begin{vmatrix} -2 & 0 & 0 \\ 1 & 1 & 13 \\ 3 & 2 & 16 \end{vmatrix} = -2(16 - 26) = 20$$

$$\therefore \quad x = \frac{D_1}{D} = \frac{10}{5} = 2$$

$$y = \frac{D_2}{D} = \frac{15}{5} = 3$$

and $$z = \frac{D_3}{D} = \frac{20}{5} = 4$$

Hence, the number of cars produced of type $C_1,\ C_2$ and C_3 are 2, 3 and 4 respectively.

Ex 6.5

Chapter 6 Determinants Ex 6.5 Q1

Here $D = \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix}$

$= 1(3) - 1(-3) - 2(3)$

$= 3 + 3 - 6$

$= 0$

Since $D = 0$, so the system has infinite solutions:

Now let $z = k$,

$x + y = 2k$

$2x + y = 3k$

Solving there equations by cramer's Rule

$$x = \frac{D_1}{D} = \frac{\begin{vmatrix} 2k & 1 \\ 3k & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}} = \frac{-k}{-1} = k$$

$$y = \frac{D_2}{D} = \frac{\begin{vmatrix} 1 & 2k \\ 2 & 3k \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}} = \frac{-k}{-1} = k$$

thus, we have $x = k, y = k, z = k$

and there values satisfy eq. (3)

Hence $x = k, y = k, z = k$

Chapter 6 Determinants Ex 6.5 Q2

Here $D = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{vmatrix}$

$= 2(4) - 3(1) + 4(-3)$
$= 8 - 3 - 7$
$= -2$
$\neq 0$

So, the given system of equations has only the trivial solutions i.e $x = 0 = y = z$:

Hence $x = 0$
$y = 0$
$z = 0$

Chapter 6 Determinants Ex 6.5 Q3

Here $D = \begin{vmatrix} 3 & 1 & 1 \\ 1 & -4 & 3 \\ 2 & 5 & -2 \end{vmatrix}$

$= 3(8 - 15) - 1(-2 - 6) + 1(13)$
$= -21 + 8 + 13$
$= 0$

So, the system has infinite solutions:
Let $z = k$,
so, $3x + y = -k$
$x - 4y = -3k$
Now,

$$x = \frac{D_1}{D} = \frac{\begin{vmatrix} -k & 1 \\ -3k & -4 \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ 1 & -4 \end{vmatrix}} = \frac{7k}{-13}$$

$$y = \frac{D_2}{D} = \frac{\begin{vmatrix} 3 & -k \\ 1 & -3k \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ 1 & -4 \end{vmatrix}} = \frac{-8k}{-13}$$

$$x = \frac{-7k}{13}, y = \frac{8k}{13}, z = k$$

and there values satisfy eq.(3)
Hence $x = -7k, y = 8k, z = 13k$

Chapter 6 Determinants Ex 6.5 Q4

$D = \begin{vmatrix} 2\lambda & -2 & 3 \\ 1 & \lambda & 2 \\ 2 & 0 & \lambda \end{vmatrix}$

$= 3\lambda^3 + 2\lambda - 8 - 6\lambda$
$= 2\lambda^3 - 4\lambda - 8$

which is satisfied by $\lambda = 2$ $[\because$ for non-trivial solutions $\lambda = 2]$
Now Let $z = k$,
$4x - 2y = -3k$
$x + 2y = -3k$

$$x = \frac{D_1}{D} = \frac{\begin{vmatrix} -3k & -2 \\ -2k & 2 \end{vmatrix}}{\begin{vmatrix} 4 & -2 \\ 1 & 2 \end{vmatrix}} = \frac{-10k}{10} = -k$$

$$y = \frac{D_2}{D} = \frac{\begin{vmatrix} 4 & -3k \\ 1 & -2k \end{vmatrix}}{\begin{vmatrix} 4 & -2 \\ 1 & 2 \end{vmatrix}} = \frac{-5k}{10} = \frac{-k}{2}$$

Hence solution is $x = -k, y = \frac{-k}{2}, z = k$

Chapter 6 Determinants Ex 6.5 Q5

$$D = \begin{vmatrix} (a-1) & -1 & -1 \\ -1 & (b-1) & -1 \\ -1 & -1 & (c-1) \end{vmatrix}$$

Now for non-trivial solution, $D = 0$

$$0 = (a-1)\left[(b-1)(c-1) - 1\right] + 1\left[-c + \cancel{1} - \cancel{1}\right] - \left[\cancel{1} + b - \cancel{1}\right]$$

$$0 = (a-1)\left[bc - b - c + \cancel{1} - \cancel{1}\right] - c - b$$

$$0 = abc - ab - ac + \cancel{b} + \cancel{c} - \cancel{c} - \cancel{b}$$

$$ab + bc + ac = abc$$

Hence proved

Ex 7.1

Adjoint and Inverse of Matrix Ex 7.1 Q1(i)

Here, $A = \begin{bmatrix} -3 & 5 \\ 2 & 4 \end{bmatrix}$

Cofactors of A are:

$C_{11} = 4$
$C_{12} = -2$
$C_{21} = -5$
$C_{22} = -3$

$\therefore \quad \text{adj}\, A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

$$\therefore \quad (\text{adj}\, A) = \begin{bmatrix} 4 & -2 \\ -5 & -3 \end{bmatrix}^T = \begin{bmatrix} 4 & -5 \\ -2 & -3 \end{bmatrix}$$

Now, $(\text{adj}\, A) A = \begin{bmatrix} 4 & -5 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} -3 & 5 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -22 & 0 \\ 0 & -22 \end{bmatrix}$

And, $|A|.I = \begin{vmatrix} -3 & 5 \\ 2 & 4 \end{vmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = (-22)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -22 & 0 \\ 0 & -22 \end{bmatrix}$

Also,

$$A.(\text{adj}\, A) = \begin{bmatrix} -3 & 5 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 4 & -5 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} -22 & 0 \\ 0 & -22 \end{bmatrix}$$

Therefore, $(\text{adj}\, A) A = |A|.I = A.(\text{adj}\, A)$

Adjoint and Inverse of Matrix Ex 7.1 Q1(ii)

Here, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Cofactors of A are:

$C_{11} = d$
$C_{12} = -c$
$C_{21} = -b$
$C_{22} = a$

$\therefore \quad \text{adj}A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

$\therefore \quad \text{adj}A = \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}^T$

$= \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Now, $(\text{adj}A)(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ad-bc & bd-bd \\ -ac+ac & ad-bc \end{bmatrix} = \begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$

And, $|A|.I = \begin{vmatrix} a & b \\ c & d \end{vmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = (ad-bc)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$

Also,

$A(\text{adj}A) = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$

$\therefore \quad (\text{adj}A)(A) = |A|.I = A(\text{adj}A)$

Adjoint and Inverse of Matrix Ex 7.1 Q1(iii)

Here, $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

Cofactors of A are:

$C_{11}=\cos\alpha$

$C_{12}=-\sin\alpha$

$C_{21}=-\sin\alpha$

$C_{22}=\cos\alpha$

$\therefore \quad \text{Adj}\,A=\begin{bmatrix}C_{11} & C_{12}\\ C_{21} & C_{22}\end{bmatrix}^T$

$$\text{Adj}\,A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}^T$$
$$=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$$

Now, $(\text{adj}A).(A)=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha & \sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$
$$=\begin{bmatrix}\cos^2\alpha-\sin^2\alpha & \cos\alpha\sin\alpha-\sin\alpha\cos\alpha\\ -\cos\alpha\sin\alpha+\sin\alpha\cos\alpha & -\sin^2\alpha+\cos^2\alpha\end{bmatrix}$$
$$=\begin{bmatrix}\cos2\alpha & 0\\ 0 & \cos2\alpha\end{bmatrix}$$

And, $A(\text{adj}A)=\begin{bmatrix}\cos\alpha & \sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha & -\sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$
$$=\begin{bmatrix}\cos^2\alpha-\sin^2\alpha & -\cos\alpha\sin\alpha+\sin\alpha\cos\alpha\\ \sin\alpha\cos\alpha-\cos\alpha\sin\alpha & -\sin^2\alpha+\cos^2\alpha\end{bmatrix}$$
$$=\begin{bmatrix}\cos2\alpha & 0\\ 0 & \cos2\alpha\end{bmatrix}$$

Also,

$$|A|.I=\begin{vmatrix}\cos\alpha & \sin\alpha\\ \sin\alpha & \cos\alpha\end{vmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\left(\cos^2\alpha-\sin^2\alpha\right)\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$
$$=\begin{bmatrix}\cos^2\alpha-\sin^2\alpha & 0\\ 0 & \cos^2\alpha-\sin^2\alpha\end{bmatrix}$$
$$=\begin{bmatrix}\cos2\alpha & 0\\ 0 & \cos2\alpha\end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q1(iv)

We have,

$$A = \begin{bmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{bmatrix}$$

Cofactors of A are:

$c_{11} = 1, \quad c_{12} = -\left(-\tan \alpha/2\right) = \tan \alpha/2$

$c_{21} = -\tan \alpha/2, \quad c_{22} = 1$

$$\therefore \text{ adj } A = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & -\tan \alpha/2 \\ \tan \alpha/2 & 1 \end{bmatrix}$$

Now,

$$|A| = \begin{vmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{vmatrix}$$

$$= 1 + \tan^2 \alpha/2$$

$$= \sec^2 \alpha/2$$

We have,

$$A = \begin{bmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{bmatrix}$$

Cofactors of A are:

$c_{11} = 1, \quad c_{12} = -\left(-\tan \alpha/2\right) = \tan \alpha/2$

$c_{21} = -\tan \alpha/2, \quad c_{22} = 1$

$$\therefore \text{ adj } A = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & -\tan \alpha/2 \\ \tan \alpha/2 & 1 \end{bmatrix}$$

Now,

$$|A| = \begin{vmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{vmatrix}$$

$$= 1 + \tan^2 \alpha/2$$

$$= \sec^2 \alpha/2$$

Adjoint and Inverse of Matrix Ex 7.1 Q2(i)

Here $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

Cofactors of A are:

$C_{11} = -3$ $\quad C_{21} = +2$ $\quad C_{31} = 2$

$C_{12} = +2$ $\quad C_{22} = -3$ $\quad C_{32} = 2$

$C_{13} = 2$ $\quad C_{23} = 2$ $\quad C_{33} = -3$

$$\therefore \quad \text{adj}\,A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}^T$$

Therefore,

$$\text{adj}\,A = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

Now,

$$(\text{adj}\,A).A = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$|A|.I = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= (-3+4+4)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$A.(\text{adj}\,A) = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\therefore \quad (\text{adj}\,A).A = |A|.I = A.(\text{adj}\,A)$$

Adjoint and Inverse of Matrix Ex 7.1 Q2(ii)

Here, $A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

Cofactors of A are:

$C_{11} = 2 \qquad C_{21} = 3 \qquad C_{31} = -13$

$C_{12} = -3 \qquad C_{22} = 6 \qquad C_{32} = 9$

$C_{13} = 5 \qquad C_{23} = -3 \qquad C_{33} = -1$

$$\therefore \quad \text{adj}\, A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 2 & -3 & 5 \\ 3 & 6 & -3 \\ -13 & 9 & -1 \end{bmatrix}^T$$

Therefore,

$$\text{adj}\, A = \begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix}$$

Now,

$$(\text{adj}\, A).A = \begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{bmatrix}$$

$$|A|.I = \begin{vmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{vmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{bmatrix}$$

$$A.(\text{adj}\, A) = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix} = \begin{bmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{bmatrix}$$

$$\therefore \quad (\text{adj}\, A).A = |A|.I = A.(\text{adj}\, A)$$

Adjoint and Inverse of Matrix Ex 7.1 Q2(iii)

Here, $A=\begin{bmatrix}2 & -1 & 3\\ 4 & 2 & 5\\ 0 & 4 & -1\end{bmatrix}$

Cofactors of A are:

$C_{11}=-22$ $\quad C_{21}=11$ $\quad C_{31}=-11$

$C_{12}=4$ $\quad C_{22}=-2$ $\quad C_{32}=2$

$C_{13}=16$ $\quad C_{23}=-8$ $\quad C_{33}=8$

$$\therefore \quad \text{adj}A=\begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}^T$$

$$\therefore \quad =\begin{bmatrix}-22 & 4 & 16\\ 11 & -2 & -8\\ -11 & 2 & 8\end{bmatrix}^T$$

Therefore,

$$\text{adj}A=\begin{bmatrix}-22 & 11 & -11\\ 4 & -2 & 2\\ 16 & -8 & 8\end{bmatrix}$$

Now,

$$(\text{adj}A).A=\begin{bmatrix}-22 & 11 & -11\\ 4 & -2 & 2\\ 16 & -8 & 8\end{bmatrix}\begin{bmatrix}2 & -1 & 3\\ 4 & 2 & 5\\ 0 & 4 & -1\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$|A|.I=\begin{vmatrix}2 & -1 & 3\\ 4 & 2 & 5\\ 0 & 4 & -1\end{vmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=(44-4+48)\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=0$$

$$A(\text{adj}A)=\begin{bmatrix}2 & -1 & 3\\ 4 & 2 & 5\\ 0 & 4 & -1\end{bmatrix}\begin{bmatrix}-22 & 11 & -11\\ 4 & -2 & 2\\ 16 & -8 & 8\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$\therefore \quad (\text{adj}A).A=|A|I=A(\text{adj}A)$$

Adjoint and Inverse of Matrix Ex 7.1 Q2(iv)

Here, $A=\begin{bmatrix}2&0&-1\\5&1&0\\1&1&3\end{bmatrix}$

Cofactors of A are:

$C_{11}=3 \qquad C_{21}=-1 \qquad C_{31}=1$

$C_{12}=-15 \qquad C_{22}=7 \qquad C_{32}=-5$

$C_{13}=4 \qquad C_{23}=-2 \qquad C_{33}=2$

$$\therefore \quad \operatorname{adj}A=\begin{bmatrix}C_{11}&C_{12}&C_{13}\\C_{21}&C_{22}&C_{23}\\C_{31}&C_{32}&C_{33}\end{bmatrix}^T$$

$$=\begin{bmatrix}3&-15&4\\-1&7&-2\\1&-5&2\end{bmatrix}^T$$

Therefore,

$$\operatorname{adj}A=\begin{bmatrix}3&-1&1\\-15&7&-5\\4&-2&2\end{bmatrix}$$

Now,

$$(\operatorname{adj}A)A=\begin{bmatrix}3&-1&1\\-15&7&-5\\4&-2&2\end{bmatrix}\begin{bmatrix}2&0&-1\\5&1&0\\1&1&3\end{bmatrix}=\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}$$

$$|A|.I=\begin{vmatrix}3&-1&1\\-15&7&-5\\4&-2&2\end{vmatrix}I_3$$

$$=(6-4)I_3=2\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$A.(\operatorname{adj}A)=\begin{bmatrix}2&0&-1\\5&1&0\\1&1&3\end{bmatrix}\begin{bmatrix}3&-1&1\\-15&7&-5\\4&-2&2\end{bmatrix}=\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}$$

$$\therefore \quad (\operatorname{adj}A)A=|A|.I=A.(\operatorname{adj}A)$$

Here, $A=\begin{bmatrix}1 & 2 & 3\\ 0 & 5 & 0\\ 2 & 4 & 3\end{bmatrix}$

Cofactors of A are:

$C_{11} = 15 \qquad C_{21} = 6 \qquad C_{31} = -15$

$C_{12} = 0 \qquad C_{22} = -3 \qquad C_{32} = 0$

$C_{13} = -10 \qquad C_{23} = 0 \qquad C_{33} = 5$

$$\therefore \quad \text{adj}\,A = \begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}^T$$

$$= \begin{bmatrix}15 & 0 & -10\\ 6 & -3 & 0\\ -15 & 0 & 5\end{bmatrix}^T$$

Therefore,

$$\text{adj}\,A = \begin{bmatrix}15 & 6 & -15\\ 0 & -3 & 0\\ -10 & 0 & 5\end{bmatrix}$$

Now,

$$(\text{adj}\,A)\,A = \begin{bmatrix}15 & 6 & -15\\ 0 & -3 & 0\\ -10 & 0 & 5\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 0 & 5 & 0\\ 2 & 4 & 3\end{bmatrix} = \begin{bmatrix}-15 & 0 & 0\\ 0 & -15 & 0\\ 0 & 0 & -15\end{bmatrix}$$

$$|A|.I = \begin{vmatrix}1 & 2 & 3\\ 0 & 5 & 0\\ 2 & 4 & 3\end{vmatrix} \; I_3$$

$$= (\;-15)\,I_3 = \begin{bmatrix}-15 & 0 & 0\\ 0 & -15 & 0\\ 0 & 0 & -15\end{bmatrix}$$

$$A.(\text{adj}\,A) = \begin{bmatrix}1 & 2 & 3\\ 0 & 5 & 0\\ 2 & 4 & 3\end{bmatrix}\begin{bmatrix}15 & 6 & -15\\ 0 & -3 & 0\\ -10 & 0 & 5\end{bmatrix} = \begin{bmatrix}-15 & 0 & 0\\ 0 & -15 & 0\\ 0 & 0 & -15\end{bmatrix}$$

$$\therefore \quad (\text{adj}\,A)\,A = |A|.I = A.(\text{adj}\,A)$$

Adjoint and Inverse of Matrix Ex 7.1 Q3

Here

$$A=\begin{bmatrix}1 & -1 & 1\\ 2 & 3 & 0\\ 18 & 2 & 10\end{bmatrix}$$

Cofactors of A are

$$\begin{array}{lll} C_{11}=30 & C_{21}=12 & C_{31}=-3\\ C_{12}=-20 & C_{22}=-8 & C_{32}=2\\ C_{13}=-50 & C_{23}=-20 & C_{33}=5\end{array}$$

Therefore,

$$\begin{aligned} adj\,A &= \begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}^T\\ &= \begin{bmatrix}30 & -20 & -50\\ 12 & -8 & -20\\ -3 & 2 & 5\end{bmatrix}^T \end{aligned}$$

So,

$$adjA=\begin{bmatrix}30 & 12 & -3\\ -20 & -8 & 2\\ -50 & -20 & 5\end{bmatrix}$$

Now,

$$\begin{aligned} A(adjA) &= \begin{bmatrix}1 & -1 & 1\\ 2 & 3 & 0\\ 18 & 2 & 10\end{bmatrix}\begin{bmatrix}30 & 12 & -3\\ -20 & -8 & 2\\ -50 & -20 & 5\end{bmatrix}\\ &= \begin{bmatrix}1 & -1 & 1\\ 2 & 3 & 0\\ 18 & 2 & 10\end{bmatrix}(0)\\ &= 0 \end{aligned}$$

Hence proved.

Adjoint and Inverse of Matrix Ex 7.1 Q4

Here, $A=\begin{bmatrix}-4 & -3 & -3\\ 1 & 0 & 1\\ 4 & 4 & 3\end{bmatrix}$

Cofactors of A are:

$$\begin{array}{lll} C_{11}=-4 & C_{21}=-3 & C_{31}=-3\\ C_{12}=1 & C_{22}=0 & C_{32}=1\\ C_{13}=4 & C_{23}=4 & C_{33}=3\end{array}$$

$$\begin{aligned} \therefore \quad \text{adj}A &= \begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}^T\\ &= \begin{bmatrix}-4 & 1 & 4\\ -3 & 0 & 4\\ -3 & 1 & 3\end{bmatrix}^T \end{aligned}$$

Therefore, $\text{adj}A=\begin{bmatrix}-4 & -3 & -3\\ 1 & 0 & 1\\ 4 & 4 & 3\end{bmatrix}$

So, $\text{adj}A = A$

Adjoint and Inverse of Matrix Ex 7.1 Q5

Here $A=\begin{bmatrix}-1 & -2 & -2\\ 2 & 1 & -2\\ 2 & -2 & 1\end{bmatrix}$

Cofactors of A are:

$C_{11}=-3$ $\quad C_{21}=6$ $\quad C_{31}=6$

$C_{12}=-6$ $\quad C_{22}=3$ $\quad C_{32}=-6$

$C_{13}=-6$ $\quad C_{23}=-6$ $\quad C_{33}=3$

$$\therefore \quad \operatorname{adj}A=\begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}^T$$

$$=\begin{bmatrix}-3 & -6 & -6\\ 6 & 3 & -6\\ 6 & -6 & 3\end{bmatrix}^T$$

$$\text{Therefore, } \operatorname{adj}A=\begin{bmatrix}-3 & 6 & 6\\ -6 & 3 & -6\\ -6 & -6 & 3\end{bmatrix} \qquad ---(i)$$

$$\text{Now, } 3.A^T=3\begin{bmatrix}-1 & 2 & 2\\ -2 & 1 & -2\\ -2 & -2 & 1\end{bmatrix}=\begin{bmatrix}-3 & 6 & 6\\ -6 & 3 & -6\\ -6 & -6 & 3\end{bmatrix} \qquad ---(ii)$$

$$\therefore \quad \operatorname{adj}A=3.A^T$$

Adjoint and Inverse of Matrix Ex 7.1 Q6

Here, $A=\begin{bmatrix}1 & -2 & 3\\ 0 & 2 & -1\\ -4 & 5 & 2\end{bmatrix}$

Cofactors of A are:

$C_{11}=9$ $\quad C_{21}=19$ $\quad C_{31}=-4$

$C_{12}=4$ $\quad C_{22}=14$ $\quad C_{32}=1$

$C_{13}=8$ $\quad C_{23}=3$ $\quad C_{33}=2$

$$\therefore \quad \operatorname{adj}A=\begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}^T$$

$$=\begin{bmatrix}9 & 4 & 8\\ 19 & 14 & 3\\ -4 & 1 & 2\end{bmatrix}^T$$

Therefore,

$$\operatorname{adj}A=\begin{bmatrix}9 & 19 & -4\\ 4 & 14 & 1\\ 8 & 3 & 2\end{bmatrix}$$

$$\text{Now, } A\operatorname{adj}A=\begin{bmatrix}1 & -2 & 3\\ 0 & 2 & -1\\ -4 & 5 & 2\end{bmatrix}\begin{bmatrix}9 & 19 & -4\\ 4 & 14 & 1\\ 8 & 3 & 2\end{bmatrix}$$

$$=\begin{bmatrix}25 & 0 & 0\\ 0 & 25 & 0\\ 0 & 0 & 25\end{bmatrix}$$

$$=25\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=25I_3$$

Adjoint and Inverse of Matrix Ex 7.1 Q7 (i)

$$A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$$

Now, $|A| = 1 \neq 0$

Hence A^{-1} exists.

Cofactors of A are:

$$C_{11}=\cos\theta \qquad C_{21}=-\sin\theta$$
$$C_{12}=\sin\theta \qquad C_{22}=\cos\theta$$

$$\therefore \qquad \operatorname{adj}A=\begin{bmatrix}C_{11} & C_{12}\\ C_{21} & C_{22}\end{bmatrix}^T$$
$$=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$$

$$\therefore \qquad \operatorname{adj}A=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$$

Now, $A^{-1}=\dfrac{1}{|A|}.\operatorname{adj}A$

$$A^{-1}=\frac{1}{1}\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$$

$$A^{-1}=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q7 (ii)

$$A=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$$

Now, $|A| = -1 \neq 0$

Hence A^{-1} exists.

Cofactors of A are:

$$C_{11}=0 \qquad C_{12}=-1$$
$$C_{21}=-1 \qquad C_{22}=0$$

$$\therefore \qquad \operatorname{adj}A=\begin{bmatrix}C_{11} & C_{12}\\ C_{21} & C_{22}\end{bmatrix}^T$$
$$=\begin{bmatrix}0 & -1\\ -1 & 0\end{bmatrix}$$

Also, $A^{-1}=\dfrac{1}{|A|}(\operatorname{adj}A)$

$$A^{-1}=\frac{1}{(-1)}\begin{bmatrix}0 & -1\\ -1 & 0\end{bmatrix}$$

$$A^{-1}=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q7 (iii)

$$A=\begin{bmatrix} a & b \\ c & \frac{1+bc}{a} \end{bmatrix}$$

Now, $|A|=\frac{a+abc}{a}-bc=\frac{a+abc-abc}{a}=1\neq 0$

Hence A^{-1} exists.

Now, cofactors of A are:

$$C_{11}=\frac{1+bc}{a} \qquad C_{12}=-c$$
$$C_{21}=-b \qquad C_{22}=a$$

$\therefore$ $\operatorname{adj}A=\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

$$=\begin{bmatrix} \frac{1+bc}{a} & -c \\ -b & a \end{bmatrix}^T$$

$\therefore$ $\operatorname{adj}A=\begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$

Also, $A^{-1}=\frac{1}{|A|}.\operatorname{adj}A$

$$A^{-1}=\frac{1}{1}\begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$$

or $A^{-1}=\begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q7 (iv)

$$A=\begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$$

Now, $|A|=2+15=17\neq 0$

Hence A^{-1} exists.

Now, cofactors of A are:

$$C_{11}=1 \qquad C_{12}=3$$
$$C_{21}=-5 \qquad C_{22}=2$$

$\therefore$ $\operatorname{adj}A=\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

$$=\begin{bmatrix} 1 & 3 \\ -5 & 2 \end{bmatrix}$$

$\therefore$ $\operatorname{adj}A=\begin{bmatrix} 1 & -5 \\ 3 & 2 \end{bmatrix}$

$$A^{-1}=\frac{1}{|A|}.(\operatorname{adj}A)$$

$$A^{-1}=\frac{1}{17}\begin{bmatrix} 1 & -5 \\ 3 & 2 \end{bmatrix}$$

Hence, $A^{-1}=\frac{1}{17}\begin{bmatrix} 1 & -5 \\ 3 & 2 \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q8(i)

Here, $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$

Expanding using 1^{st} row, we get

$$\begin{aligned} |A| &= 1\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} + 3\begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} \\ &= 1(6-1) - 2(4-3) + 3(2-9) \\ &= 5 - 2 \times 1 + 3 \times (-7) \\ &= 5 - 2 - 21 = -18 \neq 0 \end{aligned}$$

Therefore, A^{-1} exists.

Cofactors of A are:

$$\begin{array}{lll} C_{11} = 5 & C_{21} = -1 & C_{31} = -7 \\ C_{12} = -1 & C_{22} = -7 & C_{32} = 5 \\ C_{13} = -7 & C_{23} = 5 & C_{33} = -1 \end{array}$$

$$\therefore \quad \operatorname{adj} A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & -1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & -1 \end{bmatrix}$$

Now,

$$A^{-1} = \frac{1}{|A|} . \operatorname{adj} A$$

Hence, $A^{-1} = \dfrac{1}{(-18)}\begin{bmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & -1 \end{bmatrix} = \begin{bmatrix} \frac{-5}{18} & \frac{1}{18} & \frac{7}{18} \\ \frac{1}{18} & \frac{7}{18} & \frac{-5}{18} \\ \frac{7}{18} & \frac{-5}{18} & \frac{1}{18} \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q8(ii)

Here, $A=\begin{bmatrix}1 & 2 & 5\\ 1 & -1 & -1\\ 2 & 3 & -1\end{bmatrix}$

Expanding using first column, we get

$$|A| = 1\begin{vmatrix}-1 & -1\\ 3 & -1\end{vmatrix} - 2\begin{vmatrix}1 & -1\\ 2 & -1\end{vmatrix} + 5\begin{vmatrix}1 & -1\\ 2 & 3\end{vmatrix}$$
$$= 1\times(1+3) - 2(-1+2) + 5(3+2)$$
$$= 4 - 2(1) + 5(5) = 27 \neq 0$$

Therefore, A^{-1} exists.

Cofactors of A are:

$C_{11} = 4 \qquad C_{21} = +17 \qquad C_{31} = 3$

$C_{12} = -1 \qquad C_{22} = -11 \qquad C_{32} = +6$

$C_{13} = 5 \qquad C_{23} = +1 \qquad C_{33} = -3$

$$\therefore \quad \text{adj} A = \begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}^T$$
$$= \begin{bmatrix}4 & -1 & 5\\ 17 & -11 & 1\\ 3 & 6 & -3\end{bmatrix}^T$$
$$= \begin{bmatrix}4 & 17 & 3\\ -1 & -11 & 6\\ 5 & 1 & -3\end{bmatrix}$$

Now, $A^{-1} = \frac{1}{|A|}.\text{adj} A$

$$\therefore \quad A^{-1} = \frac{1}{27}\begin{bmatrix}4 & 17 & 3\\ -1 & -11 & 6\\ 5 & 1 & -3\end{bmatrix} = \begin{bmatrix}\frac{4}{27} & \frac{17}{27} & \frac{3}{27}\\ \frac{-1}{27} & \frac{-11}{27} & \frac{6}{27}\\ \frac{5}{27} & \frac{1}{27} & \frac{-3}{27}\end{bmatrix} = \begin{bmatrix}\frac{4}{27} & \frac{17}{27} & \frac{1}{9}\\ \frac{-1}{27} & \frac{-11}{27} & \frac{2}{9}\\ \frac{5}{27} & \frac{1}{27} & \frac{-1}{9}\end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q8(iii)

Here, $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$

Expanding using first column, we get

$$|A| = 2\begin{vmatrix}2 & -1\\ -1 & 2\end{vmatrix} + 1\begin{vmatrix}-1 & -1\\ 1 & 2\end{vmatrix} + 1\begin{vmatrix}-1 & 2\\ 1 & -1\end{vmatrix}$$
$$= 2(4-1)+1(-2+1)+1(1-2)$$
$$= 2(3)+1(-1)+1(-1) = 6-2 = 4 \neq 0$$

Therefore, A^{-1} exists

Cofactors of A are:

$C_{11}=3$ $\quad C_{21}=+1$ $\quad C_{31}=-1$

$C_{12}=+1$ $\quad C_{22}=3$ $\quad C_{32}=+1$

$C_{13}=-1$ $\quad C_{23}=+1$ $\quad C_{33}=3$

$$\therefore \quad \text{adj}A = \begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}^T$$
$$= \begin{bmatrix}3 & 1 & -1\\ 1 & 3 & 1\\ -1 & 1 & 3\end{bmatrix}^T$$
$$= \begin{bmatrix}3 & 1 & -1\\ 1 & 3 & 1\\ -1 & 1 & 3\end{bmatrix}$$

Now, $A^{-1} = \frac{1}{|A|}.\text{adj}A$

$$\therefore \quad A^{-1} = \frac{1}{4}\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & 1\\ -1 & 1 & 3\end{bmatrix} = \begin{bmatrix}\frac{3}{4} & \frac{1}{4} & \frac{-1}{4}\\ \frac{1}{4} & \frac{3}{4} & \frac{1}{4}\\ \frac{-1}{4} & \frac{1}{4} & \frac{3}{4}\end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q8(iv)

Here, $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

Expanding using first column, we get

$$|A| = 2\begin{vmatrix} 1 & 0 \\ 1 & 3 \end{vmatrix} - 0\begin{vmatrix} 5 & 0 \\ 0 & 3 \end{vmatrix} - 1\begin{vmatrix} 5 & 1 \\ 0 & 1 \end{vmatrix}$$
$$= 2(3-0) - 0 - 1(5)$$
$$= 2(3) - 1(5) = 1 \neq 0$$

Therefore, A^{-1} exists

Cofactors of A are:

$C_{11} = 3$ $\quad C_{21} = -1$ $\quad C_{31} = 1$

$C_{12} = -15$ $\quad C_{22} = 6$ $\quad C_{32} = -5$

$C_{13} = 5$ $\quad C_{23} = -2$ $\quad C_{33} = 2$

$$\therefore \quad \text{adj}A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$
$$= \begin{bmatrix} 3 & -15 & 5 \\ -1 & 6 & -2 \\ 1 & -5 & 2 \end{bmatrix}^T$$
$$= \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}.\text{adj}\,A$$
$$= \frac{1}{1}\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

Hence, $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q8(v)

Here, $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$

Expanding using first column, we get

$$|A| = 0\begin{vmatrix} -3 & 4 \\ -3 & 4 \end{vmatrix} - 1\begin{vmatrix} 4 & 4 \\ 3 & 4 \end{vmatrix} - 1\begin{vmatrix} 4 & -3 \\ 3 & -3 \end{vmatrix}$$

$$= 0 - 1(6 - 12) - 1(-12 + 9)$$

$$= -1(4) - 1(-3) = -1 \neq 0$$

Therefore, A^{-1} exists

Cofactors of A are:

$C_{11} = 0 \qquad C_{21} = -1 \qquad C_{31} = 1$

$C_{12} = -4 \qquad C_{22} = 3 \qquad C_{32} = -4$

$C_{13} = -3 \qquad C_{23} = +3 \qquad C_{33} = -4$

$$\therefore \quad \operatorname{adj} A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & -4 & -3 \\ -1 & 3 & 3 \\ 1 & -4 & -4 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & -1 & 1 \\ -4 & 3 & -4 \\ -3 & 3 & -4 \end{bmatrix}$$

Now, $A^{-1} = \frac{1}{|A|}.\operatorname{adj} A$

$$A^{-1} = \frac{1}{(-1)}\begin{bmatrix} 0 & -1 & 1 \\ -4 & 3 & -4 \\ -3 & 3 & -4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q8(vi)

Here, $A=\begin{bmatrix}0 & 0 & -1\\ 3 & 4 & 5\\ -2 & -4 & -7\end{bmatrix}$

Expanding using first column, we get

$$|A| = 0\begin{vmatrix}4 & 5\\ -4 & -7\end{vmatrix} - 0\begin{vmatrix}3 & 5\\ -2 & -7\end{vmatrix} - 1\begin{vmatrix}3 & 4\\ -2 & -4\end{vmatrix}$$

$$= 0 - 0 - 1(-12+8)$$

$$= -1(-4) = 4 \neq 0$$

Therefore, A^{-1} exists

Cofactors of A are:

$C_{11} = -8 \qquad C_{21} = +4 \qquad C_{31} = 4$

$C_{12} = +11 \qquad C_{22} = -2 \qquad C_{32} = -3$

$C_{13} = -4 \qquad C_{23} = +0 \qquad C_{33} = 0$

$$\therefore \quad \text{adj}\,A = \begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}^T$$

$$= \begin{bmatrix}-8 & 11 & -4\\ 4 & -2 & 0\\ 4 & -3 & 0\end{bmatrix}^T$$

$$= \begin{bmatrix}-8 & 4 & 4\\ 11 & -2 & -3\\ -4 & 0 & 0\end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}.\text{adj}\,A$$

$$A^{-1} = \frac{1}{4}\begin{bmatrix}-8 & 4 & 4\\ 11 & -2 & -3\\ -4 & 0 & 0\end{bmatrix}$$

Hence, $A^{-1} = \frac{1}{4}\begin{bmatrix}-8 & 4 & 4\\ 11 & -2 & -3\\ -4 & 0 & 0\end{bmatrix} = \begin{bmatrix}-2 & 1 & 1\\ \frac{11}{4} & \frac{-1}{2} & \frac{-3}{4}\\ -1 & 0 & 0\end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q8(vii)

Here, $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$

Expanding using first column, we get

$$|A| = 1\begin{vmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{vmatrix} - 0 + 0$$

$$= -\cos^2\alpha + \sin^2\alpha$$

$$= -\left(\cos^2\alpha + \sin^2\alpha\right)$$

$$|A| = -1 \neq 0$$

Therefore, A^{-1} exists

Cofactors of A are:

$$\begin{array}{lll} C_{11} = -1 & C_{21} = 0 & C_{31} = 0 \\ C_{12} = 0 & C_{22} = -\cos\alpha & C_{32} = -\sin\alpha \\ C_{13} = 0 & C_{23} = -\sin\alpha & C_{33} = \cos\alpha \end{array}$$

$$\therefore \quad \operatorname{adj} A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}^T$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

Now, $A^{-1} = \frac{1}{|A|}.\operatorname{adj} A$

$$\therefore \quad A^{-1} = \frac{1}{(-1)}\begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

Hence, $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q9(i)

$$A=\begin{bmatrix}1&3&3\\1&4&3\\1&3&4\end{bmatrix}$$

Expanding using 1^{st} row, we get

$$\begin{aligned}|A| &= 1\begin{vmatrix}4&3\\3&4\end{vmatrix}-3\begin{vmatrix}1&3\\1&4\end{vmatrix}+3\begin{vmatrix}1&4\\1&3\end{vmatrix}\\
&=1(16-9)-3(4-3)+3(3-4)\\
&=7-3(1)+3(-1)\\
&=7-3-3=+1=1\neq 0\end{aligned}$$

Therefore, A^{-1} exists

Cofactors of A are:

$$\begin{array}{lll}C_{11}=7 & C_{21}=-3 & C_{31}=-3\\ C_{12}=-1 & C_{22}=1 & C_{32}=-0\\ C_{13}=-1 & C_{23}=-0 & C_{33}=1\end{array}$$

$$\begin{aligned}\therefore\quad \operatorname{adj}A &= \begin{bmatrix}C_{11}&C_{12}&C_{13}\\C_{21}&C_{22}&C_{23}\\C_{31}&C_{32}&C_{33}\end{bmatrix}^T\\
&=\begin{bmatrix}7&-1&-1\\-3&1&0\\-3&0&1\end{bmatrix}^T\\
&=\begin{bmatrix}7&-3&-3\\-1&1&0\\-1&0&1\end{bmatrix}\end{aligned}$$

Now, $\quad A^{-1}=\dfrac{1}{|A|}.\operatorname{adj}A$

$$\therefore\quad A^{-1}=\frac{1}{1}\begin{bmatrix}7&-3&-3\\-1&1&0\\-1&0&1\end{bmatrix}$$

Also, $\quad A^{-1}.A=\begin{bmatrix}7&-3&-3\\-1&1&0\\-1&0&1\end{bmatrix}\begin{bmatrix}1&3&3\\1&4&3\\1&3&4\end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q9(ii)

$$A=\begin{bmatrix}2&3&1\\3&4&1\\3&7&2\end{bmatrix}$$

Expanding 1st row, we get

$$|A|=2\begin{vmatrix}4&1\\7&2\end{vmatrix}-3\begin{vmatrix}3&1\\3&2\end{vmatrix}+1\begin{vmatrix}3&4\\3&7\end{vmatrix}$$
$$=2(8-7)-3(6-3)+1(21-12)$$
$$=2-3(3)+1(9)=2\neq 0$$

Therefore, A^{-1} exists

Cofactors of A are:

$$\begin{array}{lll} C_{11}=1 & C_{21}=+1 & C_{31}=-1\\ C_{12}=-3 & C_{22}=1 & C_{32}=+1\\ C_{13}=9 & C_{23}=-5 & C_{33}=-1\end{array}$$

$$\therefore\quad \operatorname{adj}A=\begin{bmatrix}C_{11}&C_{12}&C_{13}\\C_{21}&C_{22}&C_{23}\\C_{31}&C_{32}&C_{33}\end{bmatrix}^T$$
$$=\begin{bmatrix}1&-3&9\\1&1&-5\\-1&1&-1\end{bmatrix}^T=\begin{bmatrix}1&1&-1\\-3&1&1\\9&-5&-1\end{bmatrix}$$

Now, $A^{-1}=\dfrac{1}{|A|}.\operatorname{adj}A\Rightarrow A^{-1}=\dfrac{1}{2}\begin{bmatrix}1&1&-1\\-3&1&1\\9&-5&-1\end{bmatrix}$

Also, $A^{-1}.A=\dfrac{1}{2}\begin{bmatrix}1&1&-1\\-3&1&1\\9&-5&-1\end{bmatrix}\begin{bmatrix}2&3&1\\3&4&1\\3&7&2\end{bmatrix}$

$$=\frac{1}{2}\begin{bmatrix}2+3-3&3+4-7&1+1-2\\-6+3+3&-9+4+7&-3+1+2\\18-15-3&27-20-7&9-5-2\end{bmatrix}$$
$$=\frac{1}{2}\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$\therefore\ A^{-1}.A=I_3$

Adjoint and Inverse of Matrix Ex 7.1 Q10(i)

$$A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} \qquad \therefore |A| = 1 \neq 0$$

$$\text{adj}\, A = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix} \Rightarrow A^{-1} = \frac{\text{adj}\, A}{|A|} = \frac{1}{1}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 4 & 6 \\ 3 & 2 \end{bmatrix} \qquad \therefore |B| = -10 \neq 0$$

$$\text{adj} B = \begin{bmatrix} 2 & -6 \\ -3 & 4 \end{bmatrix} \Rightarrow B^{-1} = \frac{\text{adj} B}{|B|} = \frac{1}{-10}\begin{bmatrix} 2 & -6 \\ -3 & 4 \end{bmatrix}$$

Also, $AB = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}\begin{bmatrix} 4 & 6 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 18 & 22 \\ 43 & 52 \end{bmatrix}$

$$|AB| = 936 - 946 = -10 \neq 0$$

$$\text{adj}(AB) = \begin{bmatrix} 52 & -22 \\ -43 & 18 \end{bmatrix}$$

$$(AB)^{-1} = \frac{\text{adj}(AB)}{|AB|} = \frac{1}{|AB|}\begin{bmatrix} 52 & -22 \\ -43 & 18 \end{bmatrix} = \frac{1}{-10}\begin{bmatrix} +52 & -22 \\ -43 & +18 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} -52 & 22 \\ 43 & -18 \end{bmatrix}$$

$$B^{-1}.A^{-1} = \frac{-1}{10}\begin{bmatrix} 2 & -6 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix} = \frac{-1}{10}\begin{bmatrix} 52 & -22 \\ -43 & 18 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} -52 & 22 \\ 43 & -18 \end{bmatrix}$$

Hence, $(AB)^{-1} = B^{-1}.A^{-1}$

Adjoint and Inverse of Matrix Ex 7.1 Q10(ii)

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \qquad \therefore |A| = 1 \neq 0 \text{ and adj}\, A = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

$$\therefore \ A^{-1} = \frac{\text{adj}\, A}{|A|} = \frac{1}{1}\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} \qquad \therefore |A| = -1 \ \neq 0 \text{ and adj} B = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$$

$$\therefore \ B^{-1} = \frac{\text{adj} B}{|B|} = \frac{1}{1}\begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$$

Also, $A.B = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 11 & 14 \\ 29 & 27 \end{bmatrix}$

$$|AB| = 407 - 406 = 1 \neq 0$$

and, $\text{adj}(AB) = \begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix}$

$$\therefore \quad (AB)^{-1} = \frac{1}{|AB|}.\text{adj}(AB)$$

$$= \frac{1}{1}\begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix} = \begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix}$$

Again, $B^{-1}.A^{-1} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix}$$

Hence, $(AB)^{-1} = B^{-1}.A^{-1}$

Adjoint and Inverse of Matrix Ex 7.1 Q11

$$A=\begin{bmatrix}3 & 2\\ 7 & 5\end{bmatrix} \quad \therefore |A| = 1 \neq 0 \text{ and } \operatorname{adj} A = \begin{bmatrix}5 & -2\\ -7 & 3\end{bmatrix}$$

$$\therefore \ A^{-1} = \frac{\operatorname{adj} A}{|A|} = \frac{1}{1}\begin{bmatrix}5 & -2\\ -7 & 3\end{bmatrix}$$

$$B=\begin{bmatrix}6 & 7\\ 7 & 9\end{bmatrix} \quad \therefore |B| = -2 \neq 0 \text{ and } \operatorname{adj} B = \begin{bmatrix}9 & -7\\ -8 & 6\end{bmatrix}$$

$$\therefore \ B^{-1} = \frac{\operatorname{adj} B}{|B|} = \frac{1}{(-2)}\begin{bmatrix}9 & -7\\ -8 & 6\end{bmatrix}$$

Now, $(AB)^{-1} = B^{-1}.A^{-1}$

$$(AB)^{-1} = \frac{1}{(-2)}\begin{bmatrix}9 & -7\\ -8 & 6\end{bmatrix}\begin{bmatrix}5 & -2\\ -7 & 3\end{bmatrix}$$

$$(AB)^{-1} = -\frac{1}{2}\begin{bmatrix}94 & -39\\ -82 & 34\end{bmatrix}$$

$$(AB)^{-1} = \begin{bmatrix}-47 & \frac{39}{2}\\ 41 & -17\end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q12

$$A=\begin{bmatrix}2 & -3\\ -4 & 7\end{bmatrix} \quad \therefore |A| = 2 \neq 0$$

$$\operatorname{adj} A = \begin{bmatrix}7 & 3\\ 4 & 2\end{bmatrix}$$

$$\therefore \ A^{-1} = \frac{1}{2}\begin{bmatrix}7 & 3\\ 4 & 2\end{bmatrix}$$

To show: $2A^{-1} = 9I - A$

LHS: $2A^{-1} = 2.\frac{1}{2}\begin{bmatrix}7 & 3\\ 4 & 2\end{bmatrix} = \begin{bmatrix}7 & 3\\ 4 & 2\end{bmatrix}$

RHS: $9I - A = \begin{bmatrix}9 & 0\\ 0 & 9\end{bmatrix} - \begin{bmatrix}2 & -3\\ -4 & 7\end{bmatrix} = \begin{bmatrix}7 & 3\\ 4 & 2\end{bmatrix}$

Therefore, $2A^{-1} = 9I - A$

Adjoint and Inverse of Matrix Ex 7.1 Q13

$$A=\begin{bmatrix}4 & 5\\ 2 & 1\end{bmatrix} \quad \therefore |A| = -6 \text{ and } \operatorname{adj} A = \begin{bmatrix}1 & -5\\ -2 & 4\end{bmatrix} \quad \therefore A^{-1} = \frac{1}{-6}\begin{bmatrix}1 & -5\\ -2 & 4\end{bmatrix}$$

To show: $A - 3I = 2\left(I + 3A^{-1}\right)$

$\therefore$ LHS$=A - 3I = \begin{bmatrix}4 & 5\\ 2 & 1\end{bmatrix} - \begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix} = \begin{bmatrix}1 & 5\\ 2 & -2\end{bmatrix}$

$$\text{RHS:} \quad 2\left(I + 3A^{-1}\right) = 2I + 2.3.A^{-1} = \begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix} + 2.3.\frac{1}{6}\begin{bmatrix}-1 & 5\\ 2 & -4\end{bmatrix}$$

$$= \begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix} + \begin{bmatrix}-1 & 5\\ 2 & -4\end{bmatrix}$$

$$= \begin{bmatrix}1 & 5\\ 2 & -2\end{bmatrix}$$

$\therefore$ $A - 3I = 2\left(I + 3A^{-1}\right)$

Adjoint and Inverse of Matrix Ex 7.1 Q14

$$A = \begin{bmatrix} a & b \\ c & \frac{1+bc}{a} \end{bmatrix}$$

$$\Rightarrow |A| = (1+bc) - bc = 1 \neq 0$$

$$A^{-1} = \frac{1}{|A|}.adjA = \frac{1}{1}\begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$$

Now $aA^{-1} = (a^2+bc+1)I - aA$

$$LHS: aA^{-1} = a\begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} 1+bc & -ab \\ -ac & a^2 \end{bmatrix}$$

$$RHS: (a^2+bc+1)I - aA = \begin{bmatrix} a^2+bc+1 & 0 \\ 0 & a^2+bc+1 \end{bmatrix} - \begin{bmatrix} a^2 & ab \\ ac & 1+bc \end{bmatrix} = \begin{bmatrix} 1+bc & -ab \\ -ac & a^2 \end{bmatrix}$$

Since, $L.H.S = RHS$

Hence, proved

Adjoint and Inverse of Matrix Ex 7.1 Q15

Here

$$(AB)^{-1} = B^{-1}A^{-1}$$

Now we need to find A^{-1}.

We have

$$A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

So,

$$|A| = -5 + 4 = -1$$

Co-factors of A are

$$C_{11} = -1 \qquad C_{21} = 8 \qquad C_{31} = -12$$
$$C_{12} = 0 \qquad C_{22} = 1 \qquad C_{32} = -2$$
$$C_{13} = 1 \qquad C_{23} = -10 \qquad C_{33} = 15$$

Therefore,

$$adjA = \begin{bmatrix} -1 & 8 & -12 \\ 0 & 1 & -2 \\ 1 & -10 & 15 \end{bmatrix}$$

So,

$$A^{-1} = \frac{1}{|A|}adjA = \begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$$

Hence,

$$(AB)^{-1} = B^{-1}A^{-1}$$
$$= \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$$
$$= \begin{bmatrix} -2 & 19 & -27 \\ -2 & 18 & -25 \\ -3 & 29 & -42 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q16(i)

$$F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow |F(\alpha)| = \cos^2\alpha + \sin^2\alpha = 1$$

$C_{11} = \cos\alpha \quad C_{21} = +\sin\alpha \quad C_{31} = 0$
$C_{12} = -\sin\alpha \quad C_{22} = \cos\alpha \quad C_{32} = 0$
$C_{13} = 0 \quad C_{23} = 0 \quad C_{33} = 1$

$$[F(\alpha)]^{-1} = \frac{adj(F(\alpha))}{|F(\alpha)|} = \frac{1}{1}\begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad ---(1)$$

Now

$$F(-\alpha) = \begin{bmatrix} \cos(-\alpha) & -\sin(-\alpha) & 0 \\ \sin(-\alpha) & \cos(-\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad ---(2)$$

From (1) & (2) $F(-\alpha) = [F(\alpha)]^{-1}$

Hence, proved

Adjoint and Inverse of Matrix Ex 7.1 Q16(ii)

$$G(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \Rightarrow |G(\beta)| = \cos^2\beta + \sin^2$$

$C_{11} = \cos\beta \quad C_{21} = 0 \quad C_{31} = \sin\beta$
$C_{12} = +0 \quad C_{22} = 1 \quad C_{32} = 0$
$C_{13} = \sin\beta \quad C_{23} = 0 \quad C_{33} = \cos\beta$

$$[G(\beta)]^{1} = \frac{adj(G(\beta))}{|G(\beta)|} = \frac{1}{1}\begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \qquad ---(1)$$

Now

$$G(-\beta) = \begin{bmatrix} \cos(-\beta) & 0 & \sin(-\beta) \\ 0 & 1 & 0 \\ -\sin(-\beta) & 0 & \cos(-\beta) \end{bmatrix}$$

$$= \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \qquad ---(2)$$

From (1) & (2)

$[G(\beta)]^{1} = G(-\beta)$

Adjoint and Inverse of Matrix Ex 7.1 Q16(iii)

We have to show that

$$[F(\alpha)G(\beta)]^{-1} = G(-\beta)F(-\alpha)$$

We have already shown that

$$G(-\beta) = [G(\beta)]^{-1}$$

and $F(-\beta) = [F(\beta)]^{-1}$

$\therefore$ $\text{LHS} = [F(\alpha)G(\beta)]^{1}$

$= [G(\beta)]^{-1}[F(\alpha)]^{-1} \qquad \left[\because (AB)^{-1} = B^{-1}A^{-1}\right]$

$= G(-\beta) \times F(-\alpha)$

$= \text{RHS}$

Adjoint and Inverse of Matrix Ex 7.1 Q17

We have $A^2 = A.A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$

Hence $A^2 - 4A + I = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} - 4\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 7-8+1 & 12-12+0 \\ 4-4+0 & 7-8+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

Now, $A^2 - 4A + I = O$

$\Rightarrow A.A - 4A = -I$

Post multiplying both sides by A^{-1}, since $|A| \neq 0$

$AA\left(A^{-1}\right) - 4AA^{-1} = -IA^{-1}$

$\Rightarrow A\left(AA^{-1}\right) - 4I = -A^{-1}$

$\Rightarrow AI - 4I = -A^{-1}$

or $A^{-1} = 4I - A = 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4-2 & 0-3 \\ 0-1 & 4-2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q18

$A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$

Now $A^2 + 4A - 42I = 0$

For this $A^2 = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 74 & -20 \\ -8 & 26 \end{bmatrix}$

Hence,

$A^2 + 4A - 42I = \begin{bmatrix} 74 & -20 \\ -8 & 26 \end{bmatrix} + \begin{bmatrix} -32 & 20 \\ 8 & 16 \end{bmatrix} - \begin{bmatrix} 42 & 0 \\ 0 & 42 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Hence proved.

Now, $A^2 + 4A - 42I = 0$

$\Rightarrow \quad A^{-1}.A.A + 4A^{-1}.A - 42A^{-1}.I = 0$

$\Rightarrow \quad IA + 4I - 42A^{-1} = 0$

$\Rightarrow \quad 42A^{-1} = A + 4I$

$\Rightarrow \quad A^{-1} = \frac{1}{42}\left[A + 4I\right] = \frac{1}{42}\left\{\begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}\right\} = \frac{1}{42}\begin{bmatrix} -4 & 5 \\ 2 & 8 \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q19

Here

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

Now,

$$A^2 - 5A + 7I = 0$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

So,

$$A^2 - 5A + 7I = 0$$

Px-multipling with A^{-1}

$$A^{-1}A^2 - 5A^{-1}A + 7IA^{-1} = 0$$

$$A - 5I + 7A^{-1} = 0$$

$$A^{-1} = \frac{1}{7}[5I - A]$$

$$= \frac{1}{7}\left\{\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\right\}$$

$$= \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q20

$$A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$$

$$\therefore\ A^2 = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 22 & 27 \\ 18 & 31 \end{bmatrix}$$

Now $A^2 - xA + yI = 0$

$$\Rightarrow \quad \begin{bmatrix} 22 & 27 \\ 18 & 31 \end{bmatrix} - \begin{bmatrix} 4x & 3x \\ 2x & 5x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow \quad 22 - 4x + y = 0 \quad$ or $\quad 4x - y = 22$

$\Rightarrow \quad 18 - 2x = 0 \quad$ or $\quad x = 9$

$\therefore \quad y = 14$

Again,

$A^2 - 9A + 14I = 0$

$\Rightarrow \quad 9A = A^2 + 14I = 0$

$\Rightarrow \quad 9A^{-1}A = A^{-1}.A.A + 14A^{-1}$

$\Rightarrow \quad 9I = IA + 14A^{-1}$

$$\Rightarrow \quad A^{-1} = \frac{1}{14}\{9I - A\} = \frac{1}{14}\left\{\begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}\right\} = \frac{1}{14}\left\{\begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}\right\}$$

Adjoint and Inverse of Matrix Ex 7.1 Q21

$$A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

If $A^2 = \lambda A - 2I$

$$\lambda A = A^2 + 2I$$
$$= \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\lambda \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$
$$\begin{bmatrix} 3\lambda & -2\lambda \\ 4\lambda & -2\lambda \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$
$$3\lambda = 3$$
$$\lambda = 1 \qquad \text{Ans } \lambda = 1$$

$$A^2 = A - 2I$$

Px multiplying by A^{-1}

$$A^{-1}.AA = A^{-1}.A - 2A^{-1}.I$$
$$A = I - 2A^{-1}$$
$$2A^{-1} = I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ -4 & 3 \end{bmatrix}$$

$$\therefore \qquad A^{-1} = \frac{1}{2}\begin{bmatrix} -2 & 2 \\ -4 & 3 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q22

We have $\quad A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$

To prove: $\quad A^2 - 3A - 7 = 0$

Now, $\quad A^2 = AA = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix}$

So, $\quad A^2 - 3A - 7 = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 15 & 9 \\ -3 & -6 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Now, $\quad A^2 - 3A - 7 = 0$

$\Rightarrow \quad AA^{-1}.A - 3A^{-1}.A - 7A^{-1} = 0$

$\Rightarrow \quad A - 3I - 7A^{-1} = 0$

$\Rightarrow \quad A - 3I - 7A^{-1} = 0$

$\Rightarrow \quad 7A^{-1} = A - 3I$

$\Rightarrow \quad A^{-1} = \frac{1}{7}\left(\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right) = \frac{1}{7}\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix} = \begin{bmatrix} \frac{2}{7} & \frac{3}{7} \\ \frac{-1}{7} & \frac{-5}{7} \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q23

Show that $A = \begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix}$ satisfies the equation $x^2 - 12x + I = 0$. Thus, find A^{-1}.

$A = \begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix}$

$\therefore\ A^2 = \begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix}\begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix} = \begin{bmatrix} 71 & 60 \\ 84 & 71 \end{bmatrix}$

Now $A^2 - 12A + I = \begin{bmatrix} 71 & 60 \\ 84 & 71 \end{bmatrix} - 12\begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$A^2 - 12A + I = 0$

$\Rightarrow \quad A - 12I + A^{-1} = 0$

$\Rightarrow \quad A^{-1} = 12I - A = \left\{\begin{bmatrix} 12 & 0 \\ 0 & 12 \end{bmatrix} - \begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix}\right\} = \left\{\begin{bmatrix} 6 & -5 \\ -7 & 6 \end{bmatrix}\right\}$

Adjoint and Inverse of Matrix Ex 7.1 Q24

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+2 & 1+2-1 & 1-3+3 \\ 1+2-6 & 1+4+3 & 1-6-9 \\ 2-1+6 & 2-2-3 & 2+3+9 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2+2 & 4+4-1 & 4-6+3 \\ -3+8-28 & -3+16+14 & -3-24-42 \\ 7-3+28 & 7-6-14 & 7+9+42 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$$

$\therefore A^3-6A^2+5A+11I$

$$=\begin{bmatrix}8&7&1\\-23&27&-69\\32&-13&58\end{bmatrix}-6\begin{bmatrix}4&2&1\\-3&8&-14\\7&-3&14\end{bmatrix}+5\begin{bmatrix}1&1&1\\1&2&-3\\2&-1&3\end{bmatrix}+11\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}8&7&1\\-23&27&-69\\32&-13&58\end{bmatrix}-\begin{bmatrix}24&12&6\\-18&48&-84\\42&-18&84\end{bmatrix}+\begin{bmatrix}5&5&5\\5&10&-15\\10&-5&15\end{bmatrix}+\begin{bmatrix}11&0&0\\0&11&0\\0&0&11\end{bmatrix}$$

$$=\begin{bmatrix}24&12&6\\-18&48&-84\\42&-18&84\end{bmatrix}-\begin{bmatrix}24&12&6\\-18&48&-84\\42&-18&84\end{bmatrix}$$

$$=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}=O$$

Thus, $A^3-6A^2+5A+11I=O.$

Now,

$A^3-6A^2+5A+11I=O$

$\Rightarrow (AAA)A^{-1}-6(AA)A^{-1}+5AA^{-1}+11IA^{-1}=0 \qquad$ [Post-multiplying by A^{-1} as $|A|\neq 0$]

$\Rightarrow AA(AA^{-1})-6A(AA^{-1})+5(AA^{-1})=-11(IA^{-1})$

$\Rightarrow A^2-6A+5I=-11A^{-1}$

$\Rightarrow A^{-1}=-\frac{1}{11}(A^2-6A+5I) \qquad \ldots(1)$

Now,

$A^2-6A+5I$

$$=\begin{bmatrix}4&2&1\\-3&8&-14\\7&-3&14\end{bmatrix}-6\begin{bmatrix}1&1&1\\1&2&-3\\2&-1&3\end{bmatrix}+5\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}4&2&1\\-3&8&-14\\7&-3&14\end{bmatrix}-\begin{bmatrix}6&6&6\\6&12&-18\\12&-6&18\end{bmatrix}+\begin{bmatrix}5&0&0\\0&5&0\\0&0&5\end{bmatrix}$$

$$=\begin{bmatrix}9&2&1\\-3&13&-14\\7&-3&19\end{bmatrix}-\begin{bmatrix}6&6&6\\6&12&-18\\12&-6&18\end{bmatrix}$$

$$=\begin{bmatrix}3&-4&-5\\-9&1&4\\-5&3&1\end{bmatrix}$$

From equation (1), we have:

$$A^{-1}=-\frac{1}{11}\begin{bmatrix}3&-4&-5\\-9&1&4\\-5&3&1\end{bmatrix}=\frac{1}{11}\begin{bmatrix}-3&4&5\\9&-1&-4\\5&-3&-1\end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q25

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} = \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix}$$

$$A^3 = A^2.A = \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 0 & 7 & 10 \\ 7 & 12 & 7 \end{bmatrix}$$

$$\text{Now } A^3 - A^2 - 3A - I_3 = \begin{bmatrix} -1 & -8 & -10 \\ 0 & 7 & 10 \\ 7 & 12 & 7 \end{bmatrix} - \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 & -6 \\ -6 & -3 & 6 \\ 9 & 12 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore \quad A^3 - A^2 - 3A - I_3 = 0$

$\Rightarrow \quad A^2 - A - 3I - A^{-1} = 0$

$$\Rightarrow \quad A^{-1} = A^2 - A - 3I = \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q26

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+1 & -2-2-1 & 2+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$A^3 = A^2 A = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 12+5+5 & -6-10-5 & 6+5+10 \\ -10-6-5 & 5+12+5 & -5-6-10 \\ 10+5+6 & -5-10-6 & 5+5+12 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

Now,

$A^3-6A^2+9A-4I$

$$=\begin{bmatrix}22 & -21 & 21\\ -21 & 22 & -21\\ 21 & -21 & 22\end{bmatrix}-6\begin{bmatrix}6 & -5 & 5\\ -5 & 6 & -5\\ 5 & -5 & 6\end{bmatrix}+9\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}-4\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}22 & -21 & 21\\ -21 & 22 & -21\\ 21 & -21 & 22\end{bmatrix}-\begin{bmatrix}36 & -30 & 30\\ -30 & 36 & -30\\ 30 & -30 & 36\end{bmatrix}+\begin{bmatrix}18 & -9 & 9\\ -9 & 18 & -9\\ 9 & -9 & 18\end{bmatrix}-\begin{bmatrix}4 & 0 & 0\\ 0 & 4 & 0\\ 0 & 0 & 4\end{bmatrix}$$

$$=\begin{bmatrix}40 & -30 & 30\\ -30 & 40 & -30\\ 30 & -30 & 40\end{bmatrix}-\begin{bmatrix}40 & -30 & 30\\ -30 & 40 & -30\\ 30 & -30 & 40\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$\therefore A^3-6A^2+9A-4I=O$

Now,

$A^3-6A^2+9A-4I=O$

$\Rightarrow (AAA)A^{-1}-6(AA)A^{-1}+9AA^{-1}-4IA^{-1}=O$ $\quad\left[\text{Post-multiplying by } A^{-1} \text{ as } |A|\neq 0\right]$

$\Rightarrow AA(AA^{-1})-6A(AA^{-1})+9(AA^{-1})=4(IA^{-1})$

$\Rightarrow AAI-6AI+9I=4A^{-1}$

$\Rightarrow A^2-6A+9I=4A^{-1}$

$\Rightarrow A^{-1}=\frac{1}{4}(A^2-6A+9I)$ $\quad\ldots(1)$

$A^2-6A+9I$

$$=\begin{bmatrix}6 & -5 & 5\\ -5 & 6 & -5\\ 5 & -5 & 6\end{bmatrix}-6\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}+9\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$=\begin{bmatrix}6 & -5 & 5\\ -5 & 6 & -5\\ 5 & -5 & 6\end{bmatrix}-\begin{bmatrix}12 & -6 & 6\\ -6 & 12 & -6\\ 6 & -6 & 12\end{bmatrix}+\begin{bmatrix}9 & 0 & 0\\ 0 & 9 & 0\\ 0 & 0 & 9\end{bmatrix}$$

$$=\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & 1\\ -1 & 1 & 3\end{bmatrix}$$

From equation (1), we have:

$$A^{-1}=\frac{1}{4}\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & 1\\ -1 & 1 & 3\end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q27

$A=\frac{1}{9}\begin{bmatrix}-8 & 1 & 4\\ 4 & 4 & 7\\ 1 & -8 & 4\end{bmatrix}$ and $A^T=\frac{1}{9}\begin{bmatrix}-8 & 4 & 1\\ 1 & 4 & -8\\ 4 & 7 & 4\end{bmatrix}$

$|A|=\frac{1}{9}\left[-8(16+56)-1(9)+4(-36)\right]=-81$

$C_{11}=72 \qquad C_{21}=-36 \qquad C_{31}=-9$

$C_{12}=-9 \qquad C_{22}=-36 \qquad C_{32}=+72$

$C_{13}=-36 \qquad C_{23}=-63 \qquad C_{33}=-36$

$$A^{-1}=\frac{1}{-81}\begin{bmatrix}72 & -36 & -9\\ -9 & -36 & 72\\ -36 & -63 & -36\end{bmatrix}=\frac{1}{9}\begin{bmatrix}-8 & 4 & 1\\ 1 & 4 & -8\\ 4 & 7 & 4\end{bmatrix}=A^T$$

Hence proved.

Adjoint and Inverse of Matrix Ex 7.1 Q28

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} \text{ and } |A| = 3 + 6 - 8 = 1$$

$C_{11} = 1$ $\quad C_{21} = -1$ $\quad C_{31} = 0$
$C_{12} = -2$ $\quad C_{22} = 3$ $\quad C_{32} = -4$
$C_{13} = -2$ $\quad C_{23} = +3$ $\quad C_{33} = -3$

$$A^{-1} = \frac{1}{|A|}.adjA = \frac{1}{1}\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} \qquad ---(1)$$

Now

$$A^2 = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$A^3 = A^2.A = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} \qquad ---(2)$$

From (1) and (2)

$A^{-1} = A^3$

Hence proved.

Adjoint and Inverse of Matrix Ex 7.1 Q29

$$A = \begin{bmatrix} -1 & 2 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Expanding using 1^{st} row, we get

$$|A| = -1\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} - 2\begin{vmatrix} -1 & 1 \\ 0 & 0 \end{vmatrix} + 0$$
$$= -1(0-1) - 2(0) + 0$$
$$= 1 - 0 + 0$$
$$|A| = 1$$

$$A^2 = AA = \begin{bmatrix} -1 & 2 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 2 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 1 \\ -1 & 1 & 2 \end{bmatrix}$$

Cofactors of A are:

$C_{11} = -1$ $\quad C_{21} = 0$ $\quad C_{31} = 2$
$C_{12} = 0$ $\quad C_{22} = 0$ $\quad C_{32} = 1$
$C_{13} = -1$ $\quad C_{23} = +1$ $\quad C_{33} = 1$

$$\therefore \quad \text{adj}A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T = \begin{bmatrix} -1 & 0 & -1 \\ 0 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}^T = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

Now, $A^{-1} = \frac{1}{|A|}.\text{adj}A$

$$A^{-1} = \frac{1}{1}\begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix} = A^2$$

Hence, $A^2 = A^{-1}$

Adjoint and Inverse of Matrix Ex 7.1 Q30

$$\begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$$

Let $A = \begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$

So, $\quad AX = B$

or $\quad X = A^{-1}.B \qquad ---(i)$

$$|A| = 1 \neq 0$$

Cofactors of A are:

$C_{11} = 1 \qquad C_{12} = -1$

$C_{21} = -4 \qquad C_{22} = 5$

$$\therefore \quad \text{adj } A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & -1 \\ -4 & 5 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix}$$

Now, $\quad A^{-1} = \frac{1}{|A|}.\text{adj} A$

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix}$$

So from (i)

$$X = \begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$$

$$X = \begin{bmatrix} -3 & -14 \\ 4 & 17 \end{bmatrix}$$

Ans.

Adjoint and Inverse of Matrix Ex 7.1 Q31

$$X\begin{bmatrix}5 & 3\\-1 & -2\end{bmatrix}=\begin{bmatrix}14 & 7\\7 & 7\end{bmatrix}$$

Let $B=\begin{bmatrix}5 & 3\\-1 & -2\end{bmatrix}$ and $C=\begin{bmatrix}14 & 7\\7 & 7\end{bmatrix}$

So, $XB=C$

$XBB^{-1}=CB^{-1}$

$XI=C.B^{-1}$

$X=C.B^{-1}$ $\quad ---(i)$

Now, $|B|=-7\neq 0$

Cofactors of B are:

$C_{11}=-2 \qquad C_{12}=1$

$C_{21}=-3 \qquad C_{22}=5$

$$\therefore \quad \text{adj}A=\begin{bmatrix}C_{11} & C_{12}\\C_{21} & C_{22}\end{bmatrix}^T$$

$$=\begin{bmatrix}-2 & 1\\-3 & 5\end{bmatrix}^T$$

$$=\begin{bmatrix}-2 & -3\\1 & 5\end{bmatrix}$$

$$B^{-1}=\frac{1}{|B|}.\text{adj}(B)$$

$$=\frac{1}{(-7)}\begin{bmatrix}-2 & -3\\1 & 5\end{bmatrix} \; \frac{-1}{7}\begin{bmatrix}2 & 3\\-1 & -5\end{bmatrix}$$

Now from (i)

$$X=\begin{bmatrix}14 & 7\\7 & 7\end{bmatrix}.\frac{1}{7}.\begin{bmatrix}2 & 3\\-1 & -5\end{bmatrix}$$

$$=\frac{7}{7}\begin{bmatrix}2 & 1\\1 & 1\end{bmatrix}\begin{bmatrix}2 & 3\\-1 & -5\end{bmatrix}$$

$$=\begin{bmatrix}3 & 1\\1 & -2\end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q32

Let $A=\begin{bmatrix}3 & 2\\7 & 5\end{bmatrix}$

$B=\begin{bmatrix}-1 & 1\\-2 & 1\end{bmatrix}$

$C=\begin{bmatrix}2 & -1\\0 & 4\end{bmatrix}$

Then the given equation becomes

$A\times B=C$

$\Rightarrow \quad X=A^{-1}CB^{-1}$

Now $|A|=35-14=21$

$|B|=-1+2=1$

$$A^{-1}=\frac{adj\,(A)}{|A|}=\frac{1}{21}\begin{bmatrix}5 & -2\\-7 & 3\end{bmatrix}$$

$$B^{-1}=\frac{adj\,(B)}{|B|}=\begin{bmatrix}1 & -1\\2 & -1\end{bmatrix}$$

$$\therefore \quad X=A^{-1}CB^{-1}=\frac{1}{21}\begin{bmatrix}5 & -2\\-7 & 3\end{bmatrix}\begin{bmatrix}2 & -1\\0 & 4\end{bmatrix}\begin{bmatrix}1 & -1\\2 & -1\end{bmatrix}$$

$$=\begin{bmatrix}-16 & 3\\24 & -5\end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q33

Let $\quad A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$

$B = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$

Then the given equation can be written as

$A \times B = I$

$\Rightarrow \quad X = A^{-1}B^{-1}$

Now $\quad |A| = 6 - 5 = 1$

$|B| = 10 - 9 = 1$

$$A^{-1} = \frac{adj\,(A)}{|A|} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

$$B^{-1} = \frac{adj\,(B)}{|B|} = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$$

$$\therefore \quad X = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} 9 & -14 \\ -16 & 25 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q34

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

Now, $\quad A^2 + 4A - 5I$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Also, $\quad A^2 - 4A + 5I = O$

$A^{-1}.AA - 4A^{-1}.A - 5A^{-1}.I = 0$

$A - 4I - 5A^{-1} = 0$

$$A^{-1} = \frac{1}{5}[A - 4I]$$

$$= \frac{1}{5}\left\{\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}\right\}$$

$$= \frac{1}{5}\left(\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}\right) = \frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q35

$|A \text{ adj} A| = |A|^n$

LHS $\quad = |A \text{ Adj} A|$

$= |A| \cdot |\text{Adj} A|$

$= |A| \cdot |A|^{n-1}$

$= |A|^{n-1+1}$

$= |A|^n$

$= \text{RHS}$

Adjoint and Inverse of Matrix Ex 7.1 Q36

Here

$$B^{-1} = \frac{1}{|B|} adj\,|B|$$

Co-factors of B are

$$\begin{array}{lll} C_{11} = 3 & C_{21} = 2 & C_{31} = 6 \\ C_{12} = 1 & C_{22} = 1 & C_{32} = 2 \\ C_{13} = 2 & C_{33} = 2 & C_{33} = 5 \end{array}$$

Therefore,

$$adj\,B = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix}^T$$

$$= \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

Therefore,

$$B^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

Now,

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$= \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 61 & -24 & 22 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q37

$$\text{Let } B = A^T = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$$

$$|B| = \begin{vmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{vmatrix} = (-1-8) - 0 - 2(-8+3) = -9 + 10 = 1 \neq 0$$

So, B is invertible matrix.

$$B_{11} = (-1)^{1+1}(-9) = -9;\ B_{12} = (-1)^{1+2}(-8) = 8;\ B_{13} = (-1)^{1+3}(-5) = -5$$
$$B_{21} = (-1)^{2+1}(8) = -8;\ B_{22} = (-1)^{2+2}(7) = 7;\ B_{23} = (-1)^{2+3}(4) = -4$$
$$B_{31} = (-1)^{3+1}(-2) = -2;\ B_{32} = (-1)^{3+2}(-2) = 2;\ B_{33} = (-1)^{3+3}(-1) = -1$$

$$\text{adj } B = \begin{bmatrix} -9 & 8 & -5 \\ -8 & 7 & -4 \\ -2 & 2 & -1 \end{bmatrix}^T = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} \text{adj} B$$

$$\Rightarrow B^{-1} = \frac{1}{1} \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

$$\Rightarrow \left(A^T\right)^{-1} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q38

$$|A| = \begin{vmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{vmatrix} = -1(1-4)+2(2+4)-2(-4-2) = 3+12+12 = 27$$

$$A_{11} = (-1)^{1+1}(-3) = -3;\ A_{12} = (-1)^{1+2}(6) = -6;\ A_{13} = (-1)^{1+3}(-6) = -6$$
$$A_{21} = (-1)^{2+1}(-6) = 6;\ A_{22} = (-1)^{2+2}(3) = 3;\ A_{23} = (-1)^{2+3}(6) = -6$$
$$A_{31} = (-1)^{3+1}(6) = 6;\ A_{32} = (-1)^{3+2}(6) = -6;\ A_{33} = (-1)^{3+3}(3) = 3$$

$$\text{adj } A = \begin{bmatrix} -3 & -6 & -6 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{bmatrix}^T = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$$A(\text{adj}A) = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}\begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$
$$\Rightarrow A(\text{adj}A) = \begin{bmatrix} 27 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 27 \end{bmatrix}$$
$$\Rightarrow A(\text{adj}A) = 27\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$\Rightarrow A(\text{adj}A) = |A| I_3$$

Adjoint and Inverse of Matrix Ex 7.1 Q39

$$|A| = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = 0-1(0-1)+1(1-0) = 0+1+1 = 2 \neq 0$$

So, A is invertible matrix.

$$A_{11} = (-1)^{1+1}(-1) = -1;\ A_{12} = (-1)^{1+2}(-1) = 1;\ A_{13} = (-1)^{1+3}(1) = 1$$
$$A_{21} = (-1)^{2+1}(-1) = 1;\ A_{22} = (-1)^{2+2}(-1) = -1;\ A_{23} = (-1)^{2+3}(-1) = 1$$
$$A_{31} = (-1)^{3+1}(1) = 1;\ A_{32} = (-1)^{3+2}(-1) = 1;\ A_{33} = (-1)^{3+3}(-1) = -1$$

$$\text{adj } A = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}^T = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \text{..........(i)}$$

$$A^2 - 3I = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$A^2 - 3I = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$
$$A^2 - 3I = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \text{.................(ii)}$$

From (i) and (ii) we can see that,

$$A^{-1} = \frac{1}{2}\left(A^2 - 3I\right)$$

Ex 7.2

Adjoint and Inverse of Matrix Ex 7.2 Q1

$$A = \begin{bmatrix} 7 & 1 \\ 4 & -3 \end{bmatrix}$$

For row transformations $A = IA$

$$\begin{bmatrix} 7 & 1 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to \frac{1}{7} R_1$

$$\begin{bmatrix} 1 & \frac{1}{7} \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} \frac{1}{7} & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 4R_1$

$$\begin{bmatrix} 1 & \frac{1}{7} \\ 0 & \frac{-25}{7} \end{bmatrix} = \begin{bmatrix} \frac{1}{7} & 0 \\ \frac{-4}{7} & 1 \end{bmatrix} A$$

Applying $R_2 \to \left(-\frac{7}{25}\right) R_2$

$$\begin{bmatrix} 1 & \frac{1}{7} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{7} & 0 \\ \frac{4}{25} & \frac{-7}{25} \end{bmatrix} A$$

Applying $R_1 \to R_1 - \frac{1}{7} R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{21}{175} & \frac{1}{25} \\ \frac{4}{25} & \frac{-7}{25} \end{bmatrix} A$$

Hence, $I = B\,A$

So, $B = \begin{bmatrix} \frac{3}{25} & \frac{1}{25} \\ \frac{4}{25} & -\frac{7}{25} \end{bmatrix}$ is the inverse of A.

Adjoint and Inverse of Matrix Ex 7.2 Q2

$$A=\begin{bmatrix}5 & 2\\ 2 & 1\end{bmatrix}$$

For row- transformation $A = IA$

$$\begin{bmatrix}5 & 2\\ 2 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}A$$

Applying $R_1 \to \frac{1}{5}R_1$

$$\begin{bmatrix}1 & \frac{2}{5}\\ 2 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{5} & 0\\ 0 & 1\end{bmatrix}A$$

Applying $R_1 \to R_2 - 2R_1$

$$\begin{bmatrix}1 & \frac{2}{5}\\ 0 & \frac{1}{5}\end{bmatrix}=\begin{bmatrix}\frac{1}{5} & 0\\ -\frac{2}{5} & 1\end{bmatrix}A$$

Applying $R_2 \to 5.R_2$

$$\begin{bmatrix}1 & \frac{2}{5}\\ 0 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{5} & 0\\ -2 & 5\end{bmatrix}A$$

Applying $R_1 \to R_1 - \frac{2}{5}R_2$

$$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & -2\\ -2 & 5\end{bmatrix}A$$

$$I = B.A$$

Hence, $B=\begin{bmatrix}1 & -2\\ -2 & 5\end{bmatrix}$ is the inverse of A.

Adjoint and Inverse of Matrix Ex 7.2 Q3

Let $A=\begin{bmatrix}1 & 6\\ -3 & 5\end{bmatrix}$

For row transformations $A = IA$

$$\begin{bmatrix}1 & 6\\ -3 & 5\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}A$$

Applying $R_2 \to R_2 + 3R_1$

$$\begin{bmatrix}1 & 6\\ 0 & 23\end{bmatrix}=\begin{bmatrix}1 & 0\\ 3 & 1\end{bmatrix}A$$

Applying $R_2 \to \frac{1}{23}.R_2$

$$\begin{bmatrix}1 & 6\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\ \frac{3}{23} & \frac{1}{23}\end{bmatrix}A$$

Applying $R_1 \to R_1 - 6R_2$

$$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}\frac{5}{23} & \frac{-6}{23}\\ \frac{3}{23} & \frac{1}{23}\end{bmatrix}A$$

$$I = B.A$$

Hence, $B=\frac{1}{23}\begin{bmatrix}5 & -6\\ 3 & 1\end{bmatrix}$ is the inverse of A.

Adjoint and Inverse of Matrix Ex 7.2 Q4

$$A=\begin{bmatrix}2 & 5\\ 1 & 3\end{bmatrix}$$

Now, $A = I\,A$

$$\begin{bmatrix}2 & 5\\ 1 & 3\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}A$$

Applying $R_1 \to \frac{1}{2}R_1$

$$\begin{bmatrix}1 & \frac{5}{2}\\ 1 & 3\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & 0\\ 0 & 1\end{bmatrix}A$$

Applying $R_2 \to R_2 - R_1$

$$\begin{bmatrix}1 & \frac{5}{2}\\ 0 & \frac{1}{2}\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & 0\\ \frac{-1}{2} & 1\end{bmatrix}A$$

Applying $R_2 \to 2.R_2$

$$\begin{bmatrix}1 & \frac{5}{2}\\ 0 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & 0\\ -1 & 2\end{bmatrix}A$$

Applying $R_1 \to R_1 - \frac{5}{2}R_2$

$$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}3 & -5\\ -1 & 2\end{bmatrix}.A$$

or $I = B.A$

Hence, B is the inv. of A.

Adjoint and Inverse of Matrix Ex 7.2 Q5

$$A=\begin{bmatrix}3 & 10\\ 2 & 7\end{bmatrix}$$

Now, $A = I\,A$

$$\begin{bmatrix}3 & 10\\ 2 & 7\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}A$$

Applying $R_1 \to \frac{1}{3}R_1$

$$\begin{bmatrix}1 & \frac{10}{3}\\ 2 & 7\end{bmatrix}=\begin{bmatrix}\frac{1}{3} & 0\\ 0 & 1\end{bmatrix}A$$

Applying $R_2 \to R_2 - 2R_1$

$$\begin{bmatrix}1 & \frac{10}{3}\\ 0 & \frac{1}{3}\end{bmatrix}=\begin{bmatrix}\frac{1}{3} & 0\\ \frac{-2}{3} & 1\end{bmatrix}A$$

Applying $R_2 \to 3R_2$

$$\begin{bmatrix}1 & \frac{10}{3}\\ 0 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{3} & 0\\ -2 & 3\end{bmatrix}A$$

Applying $R_1 \to R_1 - \frac{10}{3}R_2$

$$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}7 & -10\\ -2 & 3\end{bmatrix}.A$$

$I = B.A$

Hence, B is the inv. of A.

Adjoint and Inverse of Matrix Ex 7.2 Q6

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

$$A = I\,A$$

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} .A$$

Applying $R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} .A$$

Applying $R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} .A$$

Applying $R_1 \to R_1 - 2R_2,\ R_3 \to R_3 + 5R_2$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} .A$$

Applying $R_3 \to \frac{R_3}{2}$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix} .A$$

Applying $R_1 \to R_1 + R_3,\ R_2 \to R_2 - 2R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ -4 & 3 & 1 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix} .A$$

$$I = B.A$$

Hence, $A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ -4 & 3 & 1 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.2 Q7

$$A = I\,A$$

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to \frac{R_1}{2}$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{2} \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 5R_1$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ \frac{-5}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ \frac{-5}{2} & 1 & 0 \\ \frac{5}{2} & -1 & 1 \end{bmatrix} A$$

Applying $R_3 \to 2R_3$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ \frac{-5}{2} & 1 & 0 \\ 5 & -2 & 2 \end{bmatrix} A$$

Applying $R_1 \to R_1 + \frac{1}{2}R_3,\ R_2 \to R_2 - \frac{5}{2}R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$$I = B.A$$

Adjoint and Inverse of Matrix Ex 7.2 Q8

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$$

Here, $A = I\,A$

$$\begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to \frac{1}{2}R_1$

$$\begin{bmatrix} 1 & \frac{3}{2} & \frac{1}{2} \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 2R_1,\ R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & \frac{3}{2} & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & \frac{5}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -1 & 1 & 0 \\ \frac{-3}{2} & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 - \frac{3}{2}R_2,\ R_3 \to R_3 - \frac{5}{2}R_2$

$$\begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 2 & \frac{-3}{2} & 0 \\ -1 & 1 & 0 \\ 1 & \frac{-5}{2} & 1 \end{bmatrix} A$$

Applying $R_3 \to 2R_3$

$$\begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & \frac{-3}{2} & 0 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix} A$$

Applying $R_1 \to R_1 - \frac{1}{2}R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix} A$$

Hence, $A^{-1} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.2 Q9

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

Now, $A = I\,A$

$$\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.A$$

Applying $R_1 \to \frac{1}{3}R_1$

$$\begin{bmatrix} 1 & -1 & \frac{4}{3} \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.A$$

Applying $R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & -1 & \frac{4}{3} \\ 0 & -1 & \frac{4}{3} \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ \frac{-2}{3} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.A$$

Applying $R_2 \to (-1)R_2$

$$\begin{bmatrix} 1 & -1 & \frac{4}{3} \\ 0 & 1 & \frac{-4}{3} \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ \frac{2}{3} & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.A$$

Applying $R_1 \to R_1 + R_2,\ R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{-4}{3} \\ 0 & 0 & -\frac{1}{3} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ \frac{2}{3} & -1 & 0 \\ \frac{2}{3} & -1 & 1 \end{bmatrix}.A$$

Applying $R_3 \to (-3)R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{-4}{3} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ \frac{2}{3} & -1 & 0 \\ -2 & 3 & -3 \end{bmatrix}.A$$

$R_2 \to R_2 + \frac{4}{3}R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} . A$$

$$I = B.A$$

Hence, B is the inv. of A.

Adjoint and Inverse of Matrix Ex 7.2 Q10

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

$$A = I\ A$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 2R_1,\ R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & -1 \\ 0 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} . A$$

Applying $R_2 \to (-1)R_2$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix} . A$$

Applying $R_1 \to R_1 - 2R_2,\ R_3 \to R_3 + 3R_2$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 6 \end{bmatrix} = \begin{bmatrix} -3 & 2 & 0 \\ 2 & -1 & 0 \\ 5 & -3 & 1 \end{bmatrix} . A$$

Applying $R_3 \to \frac{R_3}{6}$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 2 & 0 \\ 2 & -1 & 0 \\ \frac{5}{6} & \frac{-1}{2} & \frac{1}{6} \end{bmatrix} . A$$

Applying $R_1 \to R_1 + 2R_3,\ R_2 \to R_2 - R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-4}{3} & 1 & \frac{1}{3} \\ \frac{7}{6} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{5}{6} & \frac{-1}{2} & \frac{1}{6} \end{bmatrix} . A$$

Hence, $A^{-1} = \begin{bmatrix} \frac{-4}{3} & 1 & \frac{1}{3} \\ \frac{7}{6} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{5}{6} & \frac{-1}{2} & \frac{1}{6} \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.2 Q11

$$A=\begin{bmatrix}2 & -1 & 3\\ 1 & 2 & 4\\ 3 & 1 & 1\end{bmatrix}$$

$$A = I\,A$$

$$\begin{bmatrix}2 & -1 & 3\\ 1 & 2 & 4\\ 3 & 1 & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}.A$$

Applying $R_1 \to \frac{R_1}{2}$

$$\begin{bmatrix}1 & \frac{-1}{2} & \frac{3}{2}\\ 1 & 2 & 4\\ 3 & 1 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}.A$$

Applying $R_2 \to R_2 - R_1,\ R_3 \to R_3 - 3R_1$

$$\begin{bmatrix}1 & \frac{-1}{2} & \frac{3}{2}\\ 0 & \frac{5}{2} & \frac{5}{2}\\ 0 & \frac{5}{2} & \frac{-7}{2}\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & 0 & 0\\ \frac{-1}{2} & 1 & 0\\ \frac{-3}{2} & 0 & 1\end{bmatrix}.A$$

Applying $R_2 \to \left(\frac{2}{5}\right)R_2$

$$\begin{bmatrix}1 & \frac{-1}{2} & \frac{3}{2}\\ 0 & 1 & 1\\ 0 & \frac{5}{2} & \frac{-7}{2}\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & 0 & 0\\ \frac{-1}{5} & \frac{2}{5} & 0\\ \frac{-3}{2} & -1 & 1\end{bmatrix}.A$$

Applying $R_1 \to R_1 + \frac{1}{2}R_2,\ R_3 \to R_3 - \frac{5}{2}R_2$

$$\begin{bmatrix}1 & 0 & 2\\ 0 & 1 & 1\\ 0 & 0 & -6\end{bmatrix}=\begin{bmatrix}\frac{2}{5} & \frac{1}{5} & 0\\ \frac{-1}{5} & \frac{2}{5} & 0\\ -1 & -1 & 1\end{bmatrix}.A$$

Applying $R_3 \to \frac{R_3}{-6}$

$$\begin{bmatrix}1 & 0 & 2\\ 0 & 1 & 1\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}\frac{2}{5} & \frac{1}{5} & 0\\ \frac{-1}{5} & \frac{2}{5} & 0\\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6}\end{bmatrix}.A$$

Applying $R_2 \to R_2 - R_3,\ R_1 \to R_1 - 2R_3$

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{15} & \frac{-2}{15} & \frac{-1}{3}\\ \frac{-11}{30} & \frac{7}{30} & \frac{1}{6}\\ \frac{1}{6} & \frac{1}{6} & \frac{-1}{6}\end{bmatrix}.A \qquad \left[\because I = A^{-1}.A\right]$$

Ans.

Adjoint and Inverse of Matrix Ex 7.2 Q12

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$$

$$A = I\,A$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} .A$$

Applying $R_2 \to R_2 - 3R_1,\ R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -5 \\ 0 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} .A$$

Applying $R_2 \to \dfrac{R_2}{(-2)}$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & \frac{5}{2} \\ 0 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{3}{2} & \frac{-1}{2} & 0 \\ -2 & 0 & 1 \end{bmatrix} .A$$

Applying $R_1 \to R_1 - R_2,\ R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & \frac{-11}{2} \end{bmatrix} = \begin{bmatrix} \frac{-1}{2} & \frac{1}{2} & 0 \\ \frac{3}{2} & \frac{-1}{2} & 0 \\ \frac{-7}{2} & \frac{1}{2} & \frac{-2}{11} \end{bmatrix} .A$$

Applying $R_3 \to R_3.\left(\dfrac{-2}{11}\right)$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-1}{2} & \frac{1}{2} & 0 \\ \frac{3}{2} & \frac{-1}{2} & 0 \\ \frac{7}{11} & \frac{-1}{11} & \frac{-2}{11} \end{bmatrix} .A$$

Applying $R_1 \to R_1 + \dfrac{1}{2}R_3,\ R_2 \to R_2 - \dfrac{5}{2}R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-2}{11} & \frac{5}{11} & \frac{-1}{11} \\ \frac{-1}{11} & \frac{-3}{11} & \frac{5}{11} \\ \frac{7}{11} & \frac{-1}{11} & \frac{-2}{11} \end{bmatrix} .A$$

Adjoint and Inverse of Matrix Ex 7.2 Q13

$$A=\begin{bmatrix}2 & -1 & 4\\ 4 & 0 & 2\\ 3 & -2 & 7\end{bmatrix}$$

Now, $A = I\,A$

$$\begin{bmatrix}2 & -1 & 4\\ 4 & 0 & 2\\ 3 & -2 & 7\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}.A$$

Applying $R_1 \to \frac{1}{2}R_1$

$$\begin{bmatrix}1 & \frac{-1}{2} & 2\\ 4 & 0 & 2\\ 3 & -2 & 7\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}.A$$

Applying $R_2 \to R_2 - 4R_1,\ R_3 \to R_3 - 3R_1$

$$\begin{bmatrix}1 & \frac{-1}{2} & 2\\ 0 & 2 & -6\\ 0 & \frac{-1}{2} & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & 0 & 0\\ -2 & 1 & 0\\ \frac{-3}{2} & 0 & 1\end{bmatrix}A$$

Applying $R_2 \to \frac{1}{2}.R_2$

$$\begin{bmatrix}1 & \frac{-1}{2} & 2\\ 0 & 1 & -3\\ 0 & \frac{-1}{2} & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & 0 & 0\\ -1 & \frac{1}{2} & 0\\ \frac{-3}{2} & 0 & 1\end{bmatrix}A$$

Applying $R_1 \to R_1 + \frac{1}{2}R_2,\ R_3 \to R_3 + \frac{1}{2}R_2$

$$\begin{bmatrix}1 & 0 & \frac{1}{2}\\ 0 & 1 & -3\\ 0 & 0 & \frac{-1}{2}\end{bmatrix}=\begin{bmatrix}0 & \frac{1}{4} & 0\\ -1 & \frac{1}{2} & 0\\ -2 & \frac{1}{4} & 1\end{bmatrix}.A$$

Applying $R_3 \to (-2).R_3$

$$\begin{bmatrix}1 & 0 & \frac{1}{2}\\ 0 & 1 & -3\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}0 & \frac{1}{4} & 0\\ -1 & \frac{1}{2} & 0\\ 4 & \frac{-1}{2} & -2\end{bmatrix}.A$$

Applying $R_1 \to R_1 - \frac{1}{2}R_3,\ R_2 \to R_2 + 3R_3$

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}-2 & \frac{1}{2} & 1\\ 11 & -1 & -6\\ 4 & \frac{-1}{2} & -2\end{bmatrix}.A$$

$I = B.A$

Hence, B is the inv. of A.

Adjoint and Inverse of Matrix Ex 7.2 Q14

$$A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$$

Now, $A = I\,A$

$$\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.A$$

Applying $R_1 \to \frac{1}{3}R_1$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{3} \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.A$$

Applying $R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{3} \\ 0 & 3 & \frac{2}{3} \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ \frac{-2}{3} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}A$$

Applying $R_2 \to \frac{1}{3}R_2$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{3} \\ 0 & 1 & \frac{2}{9} \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ \frac{-2}{9} & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix}A$$

Applying $R_3 \to R_3 - 4R_2$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{3} \\ 0 & 1 & \frac{2}{9} \\ 0 & 0 & \frac{1}{9} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ \frac{-2}{9} & \frac{1}{3} & 0 \\ \frac{8}{9} & \frac{-4}{3} & 1 \end{bmatrix}A$$

Applying $R_3 \to 9.R_3$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{3} \\ 0 & 1 & \frac{2}{9} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ \frac{-2}{9} & \frac{1}{3} & 0 \\ 8 & -12 & 9 \end{bmatrix}A$$

Applying $R_1 \to R_1 + \frac{1}{3}R_3,\ R_2 \to R_2 - \frac{2}{9}R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix}.A$$

or $\quad I = B.A$

Hence, B is the inv. of A.

Adjoint and Inverse of Matrix Ex 7.2 Q15

Consider the given matrix:

$$\text{Let } A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}$$

We know that $A = IA$

Thus, we have,

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to 3R_1 + R_2$ *and* $R_3 \to R_3 - 2R_1$, *we have,*

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -5 \\ 0 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 - 3R_2$ *and* $R_3 \to R_3 + 5R_2$, *we have,*

$$\begin{bmatrix} 1 & 0 & \frac{-1}{3} \\ 0 & 1 & \frac{-5}{9} \\ 0 & 0 & \frac{11}{9} \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{3} & 0 \\ \frac{1}{3} & \frac{1}{9} & 0 \\ \frac{-1}{3} & \frac{5}{9} & 1 \end{bmatrix} A$$

Applying $R_1 \to \dfrac{R_1}{\frac{11}{9}}$ *we have,*

$$\begin{bmatrix} 1 & 0 & \frac{-1}{3} \\ 0 & 1 & \frac{-5}{9} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{3} & 0 \\ \frac{1}{3} & \frac{1}{9} & 0 \\ \frac{-3}{11} & \frac{5}{11} & \frac{9}{11} \end{bmatrix} A$$

Applying $R_2 \to R_2 + \frac{5}{9}R_3$ *and* $R_1 \to R_1 + \frac{1}{3}R_3$, *we have,*

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-1}{11} & \frac{-2}{11} & \frac{3}{11} \\ \frac{2}{11} & \frac{4}{11} & \frac{5}{11} \\ \frac{-3}{11} & \frac{5}{11} & \frac{9}{11} \end{bmatrix} A$$

$\Rightarrow$ *Inverse of the given matrix is* $\begin{bmatrix} \frac{-1}{11} & \frac{-2}{11} & \frac{3}{11} \\ \frac{2}{11} & \frac{4}{11} & \frac{5}{11} \\ \frac{-3}{11} & \frac{5}{11} & \frac{9}{11} \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.2 Q16

Consider the given matrix $\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

Let $A = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

We know that $A = IA$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to (-1)R_1$, *we have*

$$\begin{bmatrix} 1 & -1 & -2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - 3R_1$, *we have*

$$\begin{bmatrix} 1 & -1 & -2 \\ 0 & 3 & 5 \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to \frac{R_2}{3}$, *we have,*

$$\begin{bmatrix} 1 & -1 & -2 \\ 0 & 1 & \frac{5}{3} \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ 3 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 + R_2$ *and* $R_3 \to R_3 - 4R_2$, *we have*

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{5}{3} & \frac{-4}{3} & 1 \end{bmatrix} A$$

Applying $R_3 \to \frac{R_3}{3}$, *we have*

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ 5 & -4 & 3 \end{bmatrix} A$$

Applying $R_1 \to R_1 + \frac{1}{3}R_3$, $R_2 \to R_2 - \frac{5}{3}R_3$, *we have,*

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} A$$

Thus, the inverse of the given matrix is $\begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix}$.

Ex 8.1

Solution of Simultaneous Linear Equations Ex 8.1 Q1(i)

We have,

$$5x + 7y = -2$$
$$4x + 6y = -3$$

The above system of equations can be written in the matrix form as

$$\begin{bmatrix} 5 & 7 \\ 4 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

or $\quad AX = B$

where $A = \begin{bmatrix} 5 & 7 \\ 4 & 6 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$

Now, $\quad |A| = 30 - 28 = +2 \neq 0$

So the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A, then

$$C_{11} = 6$$
$$C_{12} = -4$$
$$C_{21} = -7$$
$$C_{22} = 5$$

Also,

$$\text{adj } A = \begin{bmatrix} 6 & -4 \\ -7 & 5 \end{bmatrix}^T = \begin{bmatrix} 6 & -7 \\ -4 & 5 \end{bmatrix}$$

$$\therefore\ A^{-1} = \frac{1}{|A|}\text{adj} A = \frac{1}{+2}\begin{bmatrix} 6 & -7 \\ -4 & 5 \end{bmatrix}$$

$\therefore$

$$X = A^{-1}B$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{+1}{2}\begin{bmatrix} 6 & -7 \\ -4 & 5 \end{bmatrix}\begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{+1}{2}\begin{bmatrix} -12 & +21 \\ 8 & -15 \end{bmatrix} = \begin{bmatrix} \frac{9}{2} \\ \frac{-7}{2} \end{bmatrix}$$

Hence, $x = \frac{9}{2}, y = \frac{-7}{2}$

Solution of Simultaneous Linear Equations Ex 8.1 Q1(ii)

The above system can be written in matrix form as

$$\begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

or $\quad A\,X = B$

Where,

$$A = \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

Now, $\quad |A| = 10 - 6 = 4 \neq 0$

So the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A, then

$$C_{11} = 2$$
$$C_{12} = -3$$
$$C_{21} = -2$$
$$C_{22} = 5$$

Also,

$$\text{Adj}\,A = \begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix}^T = \begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$$

$$\therefore\ A^{-1} = \frac{1}{|A|}\,\text{adj}\,A = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$$

Now, $\quad X = A^{-1}B$

$$= \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} -4 \\ 16 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

Hence, $x = -1$

$y = 4$

Solution of Simultaneous Linear Equations Ex 8.1 Q1(iii)

The above system can be written in matrix form as:

$$\begin{bmatrix} 3 & 4 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

or $\quad A X = B$

Where,

$$A = \begin{bmatrix} 3 & 4 \\ 1 & -1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

Now, $\quad |A| = -7 \neq 0$

So the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A, then

$$C_{11} = -1$$
$$C_{12} = -1$$
$$C_{21} = -4$$
$$C_{22} = 3$$

Now,

$$\text{Adj}\, A = \begin{bmatrix} -1 & -1 \\ -4 & 3 \end{bmatrix}^T = \begin{bmatrix} -1 & -4 \\ -1 & 3 \end{bmatrix}$$

$$\therefore\ A^{-1} = \frac{1}{|A|}\text{adj}\, A = \frac{1}{-7}\begin{bmatrix} -1 & -4 \\ -1 & 3 \end{bmatrix}$$

Now, $\quad X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{-1}{7}\begin{bmatrix} -1 & -4 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

$$= \frac{-1}{7}\begin{bmatrix} 7 \\ -14 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

Hence, $x = -1$

$y = 2$

Solution of Simultaneous Linear Equations Ex 8.1 Q1(iv)

The above system can be written in matrix form as:

$$\begin{bmatrix} 3 & 1 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19 \\ 23 \end{bmatrix}$$

or $\quad AX = B$

Where,

$$A = \begin{bmatrix} 3 & 1 \\ 3 & -1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 19 \\ 23 \end{bmatrix}$$

Now, $\quad |A| = -6 \neq 0$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A, then

$$C_{11} = -1$$
$$C_{12} = -3$$
$$C_{21} = -1$$
$$C_{22} = 3$$

Now,

$$\operatorname{Adj} A = \begin{bmatrix} -1 & -3 \\ -1 & 3 \end{bmatrix}^T = \begin{bmatrix} -1 & -1 \\ -3 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\operatorname{adj} A = \frac{1}{-6}\begin{bmatrix} -1 & -1 \\ -3 & 3 \end{bmatrix}$$

Now, $\quad X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{-1}{6}\begin{bmatrix} -1 & -1 \\ -3 & 3 \end{bmatrix}\begin{bmatrix} 19 \\ 23 \end{bmatrix}$$

$$= \frac{-1}{6}\begin{bmatrix} -19 & -23 \\ -57 & +69 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -2 \end{bmatrix}$$

Hence, $x = 7$

$y = -2$

Solution of Simultaneous Linear Equations Ex 8.1 Q1(v)

The above system can be written in matrix form as:

$$\begin{bmatrix} 3 & 7 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

or $\quad AX = B$

where $A = \begin{bmatrix} 3 & 7 \\ 1 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$

Now,

$$|A| = -1 \neq 0$$

So the above system has a unique solution, given by

$$X = A^{-1}B$$

Now, let C_{ij} be the co-factor of a_{ij} in A

$C_{11} = 2$

$C_{12} = -1$

$C_{21} = -7$

$C_{22} = 3$

$$\text{Adj } A = \begin{bmatrix} 2 & -1 \\ -7 & 3 \end{bmatrix}^T = \begin{bmatrix} 2 & -7 \\ -1 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}.\text{adj } A = \frac{1}{(-1)}\begin{bmatrix} 2 & -7 \\ -1 & 3 \end{bmatrix}$$

Now, $\quad X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-1}\begin{bmatrix} 2 & -7 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-1}\begin{bmatrix} 15 \\ -7 \end{bmatrix} = \begin{bmatrix} -15 \\ 7 \end{bmatrix}$$

Hence, $x = -15$

$y = 7$

Solution of Simultaneous Linear Equations Ex 8.1 Q1(vi)

The given system can be written in matrix form as:

$$\begin{bmatrix} 3 & 1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 12 \end{bmatrix}$$

or $\quad A X = B$

Where,

$$A = \begin{bmatrix} 3 & 1 \\ 5 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 7 \\ 12 \end{bmatrix}$$

Since, $|A| = 4 \neq 0$, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$C_{11} = 3$
$C_{12} = -5$
$C_{21} = -1$
$C_{22} = 3$

$$\text{adj } A = \begin{bmatrix} 3 & -5 \\ -1 & 3 \end{bmatrix}^T = \begin{bmatrix} 3 & -1 \\ -5 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{4}\begin{bmatrix} 3 & -1 \\ -5 & 3 \end{bmatrix}$$

Now, $\quad X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 3 & -1 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 7 \\ 12 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 9 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{9}{4} \\ \frac{1}{4} \end{bmatrix}$$

Hence, $x = \frac{9}{4}$

$y = \frac{1}{4}$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(i)

The given system can be written in matrix form as:

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$$

or $\quad A X = B$

Where,

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$$

Now, $|A| = 1\begin{bmatrix} 3 & 1 \\ -1 & -7 \end{bmatrix} - 1\begin{bmatrix} 2 & 1 \\ 3 & -7 \end{bmatrix} - 1\begin{bmatrix} 2 & 3 \\ 3 & -1 \end{bmatrix}$

$$= (-20) - 1(-17) - 1(-11)$$

$$= -20 + 17 + 11 = 8 \neq 0$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$C_{11} = -20 \qquad C_{21} = 8 \qquad C_{31} = 4$

$C_{12} = -(-17) = 17 \qquad C_{22} = -4 \qquad C_{32} = -3$

$C_{13} = -11 \qquad C_{23} = -(-4) = 4 \qquad C_{33} = 1$

$$\text{adj } A = \begin{bmatrix} -20 & 17 & -11 \\ 8 & -4 & 4 \\ 4 & -3 & 1 \end{bmatrix}^T = \begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix}$$

Now, $X = A^{-1}B = \dfrac{1}{8}\begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ 8 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

Hence, $x = 3$

$y = 1$

$z = 1$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(ii)

The above system can be written in matrix form as:

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 2 & 1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ -9 \end{bmatrix}$$

or $\quad A\,X = B$

Where,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 2 & 1 & -3 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ -1 \\ -9 \end{bmatrix}$$

Since, $|A| = 14 \neq 0$, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$$\begin{array}{lll} C_{11} = 2 & C_{21} = 4 & C_{31} = 2 \\ C_{12} = 8 & C_{22} = -5 & C_{32} = 1 \\ C_{13} = 4 & C_{23} = 1 & C_{33} = -3 \end{array}$$

$$\text{Adj}\,A = \begin{bmatrix} 2 & 8 & 4 \\ 4 & -5 & 1 \\ 2 & 1 & -3 \end{bmatrix}^T = \begin{bmatrix} 2 & 4 & 2 \\ 8 & -5 & 1 \\ 4 & 1 & -3 \end{bmatrix}$$

Now, $\quad X = A^{-1}B = \dfrac{1}{|A|} \times \text{Adj}\,A \times B$

$$= \frac{1}{14}\begin{bmatrix} 2 & 4 & 2 \\ 8 & -5 & 1 \\ 4 & 1 & -3 \end{bmatrix}\begin{bmatrix} 3 \\ -1 \\ -9 \end{bmatrix}$$

$$= \frac{1}{14}\begin{bmatrix} -16 \\ 20 \\ 38 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{-8}{7} \\ \frac{10}{7} \\ \frac{19}{7} \end{bmatrix}$$

Hence, $x = \dfrac{-8}{7}$, $y = \dfrac{10}{7}$, $z = \dfrac{19}{7}$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(iii)

The above system can be written in matrix form as:

$$\begin{bmatrix} 6 & -12 & 25 \\ 4 & 15 & -20 \\ 2 & 18 & 15 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 10 \end{bmatrix}$$

or $\quad A X = B$

Where,

$$A = \begin{bmatrix} 6 & -12 & 25 \\ 4 & 15 & -20 \\ 2 & 18 & 15 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 4 \\ 3 \\ 10 \end{bmatrix}$$

Now,

$$\begin{aligned} |A| &= 6(225 + 360) + 12(60 + 40) + 25(72 - 30) \\ &= 6(585) + 1200 + 25(42) \\ &= 3510 + 1200 + 1050 \\ &= 5760 \neq 0 \end{aligned}$$

So, the above system will have a unique solution, given by

$$X = A^{-1}B$$

$$\begin{array}{lll} C_{11} = 585 & C_{21} = -(-180 - 450) = 630 & C_{31} = -135 \\ C_{12} = -100 & C_{22} = 40 & C_{32} = 220 \\ C_{13} = 42 & C_{23} = -132 & C_{33} = 138 \end{array}$$

$$X = A^{-1}B = \frac{1}{|A|}(\text{Adj}\,A) \times B = \frac{1}{5760}\begin{bmatrix} 585 & 630 & -135 \\ -100 & 40 & 220 \\ 42 & -132 & 138 \end{bmatrix}\begin{bmatrix} 2880 \\ 1920 \\ 1152 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5760}\begin{bmatrix} 2880 \\ 1920 \\ 1152 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

Hence, $x = \frac{1}{2}$

$y = \frac{1}{3}$

$z = \frac{1}{5}$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(iv)

The above system can be written as

$$\begin{bmatrix} 3 & 4 & 7 \\ 2 & -1 & 3 \\ 1 & 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$$

or $\quad A\,X = B$

$$\begin{aligned} |A| &= 3(-3) - 4(-9) + 7(5) \\ &= -9 + 36 + 35 \\ &= 62 \neq 0 \end{aligned}$$

So, the above system will have a unique solution, given by

$$X = A^{-1}B$$

Now,
$C_{11} = -3 \qquad C_{21} = 26 \qquad C_{31} = 19$
$C_{12} = 9 \qquad C_{22} = -16 \qquad C_{32} = 5$
$C_{13} = 5 \qquad C_{23} = -2 \qquad C_{33} = -11$

$$\text{adj } A = \begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}$$

$$\begin{aligned} X = A^{-1}B &= \frac{1}{|A|}(\text{Adj } A)B \\ &= \frac{1}{62}\begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}\begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix} \\ &= \frac{1}{62}\begin{bmatrix} -42 + 104 + 0 \\ 126 - 64 + 0 \\ 70 - 8 + 0 \end{bmatrix} = \frac{1}{62}\begin{bmatrix} 62 \\ 62 \\ 62 \end{bmatrix} \end{aligned}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Hence, $x = 1, y = 1, z = 1$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(v)

The above system can be written as

$$\begin{bmatrix} 2 & 6 & 0 \\ 3 & 0 & -1 \\ 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \\ -3 \end{bmatrix}$$

Or $\quad AX = B$

$$|A| = 2(-1) - 6(5) + 0(-3) = -32 \neq 0$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$$\begin{array}{lll} C_{11} = -1 & C_{21} = -6 & C_{31} = -6 \\ C_{12} = -5 & C_{22} = 2 & C_{32} = 2 \\ C_{13} = -3 & C_{23} = 14 & C_{33} = -18 \end{array}$$

$$\text{adj}A = \begin{bmatrix} -1 & -5 & -3 \\ -6 & 2 & 14 \\ -6 & 2 & -18 \end{bmatrix}^T = \begin{bmatrix} -1 & -6 & -6 \\ -5 & 2 & 2 \\ -3 & 14 & -18 \end{bmatrix}$$

Now,

$$X = A^{-1}B = \frac{1}{|A|}(\text{Adj}A) \times B$$

$$= \frac{1}{-32}\begin{bmatrix} -1 & -6 & -6 \\ -5 & 2 & 2 \\ -3 & 14 & -18 \end{bmatrix}\begin{bmatrix} 2 \\ -8 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-32}\begin{bmatrix} 64 \\ -32 \\ -64 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

Hence, $x = -2, y = 1, z = 2$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(v)

Let $\frac{1}{x} = u, \frac{1}{y} = v, \frac{1}{z} = w$

$$2u - 3v + 3w = 10$$
$$u + v + w = 10$$
$$3u - v + 2w = 13$$

Which can be written as

$$\begin{bmatrix} 2 & -3 & 3 \\ 1 & 1 & 1 \\ 3 & -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ 13 \end{bmatrix}$$

$$|A| = 2(3) + 3(-1) + 3(-4)$$
$$= 6 - 3 - 12 = -9 \neq 0$$

Hence, the system has a unique solution, given by

$$X = A^{-1} \times B$$

$$\begin{array}{lll} C_{11} = 3 & C_{21} = 3 & C_{31} = -6 \\ C_{12} = 1 & C_{22} = -5 & C_{32} = 1 \\ C_{13} = -4 & C_{23} = -7 & C_{33} = 5 \end{array}$$

$$X = \frac{1}{|A|}(\text{Adj}\,A) \times (B)$$
$$= \frac{1}{-9}\begin{bmatrix} 3 & 3 & -6 \\ 1 & -5 & 1 \\ -4 & -7 & 5 \end{bmatrix}\begin{bmatrix} 10 \\ 10 \\ 13 \end{bmatrix}$$
$$= \frac{-1}{9}\begin{bmatrix} 30 + 30 - 78 \\ 10 - 50 + 13 \\ -40 - 70 + 65 \end{bmatrix}$$
$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{-1}{9}\begin{bmatrix} -18 \\ -27 \\ -45 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

Hence, $x = \frac{1}{2}, y = \frac{1}{3}, z = \frac{1}{5}$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(vi)

$$\begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ 19 \\ 25 \end{bmatrix}$$

or $\quad AX = B$

$$|A| = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}$$
$$= 5(-2) - 3(5) + 1(3)$$
$$= -10 - 15 + 3 = -22 \neq 0$$

Hence, it has a unique solution, given by

$$X = A^{-1} \times B$$

$$\begin{array}{lll} C_{11} = -2 & C_{21} = -10 & C_{31} = 8 \\ C_{12} = -5 & C_{22} = 19 & C_{32} = -13 \\ C_{13} = 3 & C_{23} = -7 & C_{33} = -1 \end{array}$$

$$X = A^{-1} \times B = \frac{1}{|A|}(\text{Adj}\,A) \times B$$
$$= \frac{1}{-22}\begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}\begin{bmatrix} 16 \\ 19 \\ 25 \end{bmatrix}$$
$$= \frac{-1}{22}\begin{bmatrix} -32 - 190 + 200 \\ -80 + 361 - 325 \\ 48 - 133 - 25 \end{bmatrix}$$
$$= \frac{-1}{22}\begin{bmatrix} -22 \\ -44 \\ -110 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

Hence, $x = 1, y = 2, z = 5$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(vii)

$$\begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix}$$

or $\quad AX = B$

$$\begin{aligned} |A| &= 3(6) - 4(3) + 2(-2) \\ &= 18 - 12 - 4 \\ &= 2 \neq 0 \end{aligned}$$

Hence, the system has a unique solution, given by

$$X = A^{-1}B$$

$$\begin{array}{lll} C_{11} = 6 & C_{21} = -28 & C_{31} = -16 \\ C_{12} = -3 & C_{22} = 16 & C_{32} = 9 \\ C_{13} = -2 & C_{23} = 10 & C_{33} = 6 \end{array}$$

Next, $X = A^{-1}B = \frac{1}{|A|}(\text{Adj}\, A) \times B$

$$= \frac{1}{2}\begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & 9 \\ 2 & 10 & 6 \end{bmatrix}\begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 48 - 84 + 32 \\ -24 + 48 - 18 \\ -16 + 30 - 12 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} -4 \\ 6 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$$

Hence, $x = -2, y = 3, z = 1$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(viii)

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & -1 \\ 3 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$$

$$\begin{aligned} |A| &= 2(-5) - 1(1) + 1(-8) \\ &= -10 - 1 - 8 = -19 \neq 0 \end{aligned}$$

Hence, the unique solution, given by

$$X = A^{-1} \times B$$

$$\begin{array}{lll} C_{11} = -5 & C_{21} = 3 & C_{31} = -4 \\ C_{12} = -1 & C_{22} = -7 & C_{32} = 3 \\ C_{13} = -8 & C_{23} = 1 & C_{33} = 5 \end{array}$$

Next, $X = A^{-1} \times B \quad = \frac{1}{|A|}\begin{bmatrix} -5 & 3 & -4 \\ -1 & -7 & 3 \\ -8 & 1 & 5 \end{bmatrix}\begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$

$$= \frac{1}{-19}\begin{bmatrix} -10 + 15 - 24 \\ -2 - 35 + 18 \\ -16 + 5 + 30 \end{bmatrix}$$

$$= \frac{-1}{19}\begin{bmatrix} -19 \\ -19 \\ 19 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

Hence, $x = 1, y = 1, z = -1$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(x)

The above system of equations can be written as

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 0 & 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

or $\quad AX = B$

$$|A| = 1(1) + 1(-2) + 1(4) = 1 - 2 + 4 = 3 \neq 0$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$C_{11} = 1 \qquad C_{21} = 1 \qquad C_{31} = +1$

$C_{12} = 2 \qquad C_{22} = -1 \qquad C_{32} = 2$

$C_{13} = 4 \qquad C_{23} = -2 \qquad C_{33} = 1$

$$\text{adj } A = \begin{bmatrix} 1 & 2 & 4 \\ 1 & -1 & -2 \\ +1 & 2 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 1 & +1 \\ 2 & -1 & 2 \\ 4 & -2 & 1 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{|A|}(\text{Adj } A) \times B$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & +1 \\ 2 & -1 & 2 \\ 4 & -2 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence, $x = 1, y = 2, z = 3$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(xi)

The above system can be written as

$$\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 18 \\ 5 \\ 5 \end{bmatrix}$$

or $\quad AX = B$

$$|A| = 8(-1) - 4(1) + 3(3) = -8 - 4 + 9 = -3 \neq 0$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$C_{11} = -1 \qquad C_{21} = 2 \qquad C_{31} = 1$

$C_{12} = -1 \qquad C_{22} = 5 \qquad C_{32} = -2$

$C_{13} = 3 \qquad C_{23} = -12 \qquad C_{33} = 0$

$$\text{adj } A = \begin{bmatrix} -1 & -1 & 3 \\ 2 & 5 & -12 \\ 1 & -2 & 0 \end{bmatrix}^T = \begin{bmatrix} -1 & 2 & 1 \\ -1 & 5 & -2 \\ 3 & -12 & 0 \end{bmatrix}$$

Now, $\quad X = A^{-1}B = \frac{1}{|A|}(\text{Adj } A) \times B$

$$= \frac{-1}{3}\begin{bmatrix} -1 & 2 & 1 \\ -1 & 5 & -2 \\ 3 & -12 & 0 \end{bmatrix}\begin{bmatrix} 18 \\ 5 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{3}\begin{bmatrix} -3 \\ -3 \\ -6 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

Hence, $x = 1, y = 1, z = 2$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(xii)

This system can be written as

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

or $\quad AX = B$

$$|A| = 1(-2) - 1(-5) + 1(1) = -2 + 5 + 1 = 4 \neq 0$$

So, $AX = B$ has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$$C_{11} = -2 \qquad C_{21} = 0 \qquad C_{31} = 2$$
$$C_{12} = +5 \qquad C_{22} = -2 \qquad C_{32} = -1$$
$$C_{13} = 1 \qquad C_{23} = 2 \qquad C_{33} = -1$$

$$\text{adj } A = \begin{bmatrix} -2 & 5 & 1 \\ 0 & -2 & 2 \\ 2 & -1 & -1 \end{bmatrix}^T = \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

$$X = A^{-1} \times B = \frac{1}{|A|}(\text{Adj } A) \times B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -12 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

Hence, $x = -3, y = 1, z = 2$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(xiii)

Let

$$\frac{1}{x}=u, \frac{1}{y}=v, \frac{1}{z}=w$$

The above system can be written as

$$\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix}=\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

Or $\quad AX = B$

$$|A|=2(75)-3(-110)+10(72)=1200 \neq 0$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$$\begin{array}{lll} C_{11} = 75 & C_{21} = 150 & C_{31} = 75 \\ C_{12} = 110 & C_{22} = -100 & C_{32} = 30 \\ C_{13} = 72 & C_{23} = 0 & C_{33} = -24 \end{array}$$

$$\text{adj}A = \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}^T = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

Now,

$$X = A^{-1}B = \frac{1}{|A|}(\text{Adj}A)\times B$$

$$=\frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix}=\frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}=\begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

Hence, $x = 2, y = 3, z = 5$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(xiv)

The above system can be written as

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

Or $AX = B$

$$|A| = 1(7)+1(19)+2(-11) = 4 \neq 0$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$$\begin{array}{lll} C_{11} = 7 & C_{21} = 1 & C_{31} = -3 \\ C_{12} = -19 & C_{22} = -1 & C_{32} = 11 \\ C_{13} = -11 & C_{23} = -1 & C_{33} = 7 \end{array}$$

$$\text{adj}A = \begin{bmatrix} 7 & -19 & -11 \\ 1 & -1 & -1 \\ -3 & 11 & 7 \end{bmatrix}^T = \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$

Now,

$$X = A^{-1}B = \frac{1}{|A|}(\text{Adj}A)\times B$$

$$= \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Hence, $x = 2, y = 1, z = 3$

Solution of Simultaneous Linear Equations Ex 8.1 Q3(i)

The above system can be written as

$$\begin{bmatrix} 6 & 4 \\ 9 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

or $AX = B$

$$|A| = 36 - 36 = 0$$

So, A is singular. Now, X will be consistent if $(\text{adj}A)\times B = 0$

$$C_{11} = 6$$
$$C_{12} = -9$$
$$C_{21} = -4$$
$$C_{22} = 6$$

$$\text{adj }A = \begin{bmatrix} 6 & -9 \\ -4 & 6 \end{bmatrix}^T = \begin{bmatrix} 6 & -4 \\ -9 & 6 \end{bmatrix}$$

$$(\text{Adj }A)\times B = \begin{bmatrix} 6 & -4 \\ -9 & 6 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 12-12 \\ -18+18 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Thus, $AX = B$ will have infinite solutions.

Let $y = k$

Hence, $6x = 2 - 4k$ or $9x = 3 - 6k$

$x = \frac{1-2k}{3}$ or $x = \frac{1-2k}{3}$

Hence, $x = \frac{1-2k}{3}, y = k$

Solution of Simultaneous Linear Equations Ex 8.1 Q3(ii)

The system can be written as

$$\begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 15 \end{bmatrix}$$

or $\quad A\,X = B$

$$|A| = 18 - 18 = 0$$

So, A is singular. Now the system will be inconsistent if $(\text{adj } A) \times B \neq 0$

$C_{11} = 9 \qquad C_{21} = -3$

$C_{12} = -6 \qquad C_{22} = 2$

$$\text{adj } A = \begin{bmatrix} 9 & -6 \\ -3 & 2 \end{bmatrix}^T = \begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix}$$

$$(\text{Adj } A) \times B = \begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 15 \end{bmatrix}$$

$$= \begin{bmatrix} 45 - 45 \\ -30 + 30 \end{bmatrix} = [0]$$

Since, $(\text{Adj } A \times B) = 0$, the system will have infinite solutions.

Now,

Let $y = k$

$$2x = 5 - 3k \quad \text{or} \quad x = \frac{5 - 3k}{2}$$

$$x = 15 - 9k \quad \text{or} \quad x = \frac{5 - 3k}{2}$$

Hence, $x = \dfrac{5 - 3k}{2}, y = k$

Solution of Simultaneous Linear Equations Ex 8.1 Q3(iii)

This can be written as

$$\begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix}$$

or $\quad A\,X = B$

$$|A| = 5(256) - 3(16) + 7(6 - 182)$$
$$= 0$$

So, A is singular. Thus, the given system is either inconsistent or it is consistent with infinitely many solutions according as

$$(\text{Adj}\,A) \times B \neq 0 \quad \text{or} \quad (\text{Adj}\,A) \times B = 0$$

Let C_{ij} be the co-factor of a_{ij} in A

$$C_{11} = 256 \qquad C_{21} = -16 \qquad C_{31} = -176$$
$$C_{12} = -16 \qquad C_{22} = 1 \qquad C_{32} = 11$$
$$C_{13} = -176 \qquad C_{23} = 11 \qquad C_{33} = 121$$

$$\text{adj}\,A = \begin{bmatrix} 256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121 \end{bmatrix}^T = \begin{bmatrix} 256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121 \end{bmatrix}$$

$$\text{adj}\,A \times B = \begin{bmatrix} 256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Thus, $AX = B$ has infinite many solutions.

Now, $\quad$ let $z = k$

then, $\quad 5x + 3y = 4 - 7k$

$\quad 3x + 26y = 9 - 2k$

Which can be written as

$$\begin{bmatrix} 5 & 3 \\ 3 & 26 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 - 7k \\ 9 - 2k \end{bmatrix}$$

or $\quad A\,X = B$

$$|A| = 2$$

$$\text{adj}\,A = \begin{bmatrix} 26 & -3 \\ -3 & 5 \end{bmatrix}$$

Now, $\quad X = A^{-1}B = \dfrac{1}{|A|} \times \text{adj}\,A \times B$

$$= \frac{1}{121}\begin{bmatrix} 26 & -3 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 4 - 7k \\ 9 - 2k \end{bmatrix}$$

$$= \frac{1}{121}\begin{bmatrix} 77 - 176k \\ 11k + 33 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \dfrac{7 - 16k}{11} \\ \dfrac{k + 3}{11} \end{bmatrix}$$

There values of x, y, z satisfies the third eq.

Hence, $x = \dfrac{7 - 16k}{11}, y = \dfrac{k + 3}{11}, z = k$

Solution of Simultaneous Linear Equations Ex 8.1 Q3(iv)

This above system can be written as

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ -1 & -2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

or $\quad A\,X = B$

$$|A| = 1(2-2) + 1(4-1) + 1(-3)$$
$$= 0 + 3 - 3$$
$$= 0$$

So, A is singular. Thus, the given system is either inconsistent or consistent with infinitely many solutions according as

$$(\text{Adj } A) \times (B) \neq 0 \quad \text{or} \quad (\text{Adj } A) \times B = 0$$

Let C_{ij} be the co-factor of a_{ij} in A

$$C_{11} = 0 \qquad C_{21} = 0 \qquad C_{31} = 0$$
$$C_{12} = -3 \qquad C_{22} = 3 \qquad C_{32} = 3$$
$$C_{13} = -3 \qquad C_{23} = -3 \qquad C_{33} = 3$$

$$\text{adj } A = \begin{bmatrix} 0 & -3 & -3 \\ 0 & 3 & 3 \\ 0 & 3 & 3 \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & 0 \\ -3 & 3 & 3 \\ -3 & 3 & 3 \end{bmatrix}$$

$$(\text{adj } A) \times B = \begin{bmatrix} 0 & 0 & 0 \\ -3 & 3 & 3 \\ -3 & 3 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Thus, $AX = B$ has infinite many solutions.

Now, $\quad$ let $z = k$

So, $\quad x - y = 3 - k$

$$2x + y = 2 + k$$

Which can be written as

$$\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3-k \\ 2+k \end{bmatrix}$$

or $\quad A\,X = B$

$$|A| = 1 + 2 = 3 \neq 0$$

$$\text{adj } A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$$

and, $\quad X = A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|A|}\begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 3-5 \\ 2+k \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 3-k+2+k \\ -6+2k+2+k \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{5}{3} \\ \frac{3k-4}{3} \end{bmatrix}$$

Hence, $x = \frac{5}{3}, y = k - \frac{4}{3}, z = k$

Solution of Simultaneous Linear Equations Ex 8.1 Q3(v)

This system can be written as

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix}$$

or $\quad A\,X = B$

$$\begin{aligned} |A| &= 1(2) - 1(4) + 1(2) \\ &= 2 - 4 + 2 \\ &= 0 \end{aligned}$$

So, A is singular. Thus, the given system has either no solution or infinite solutions depending on as

$$(\text{Adj } A) \times (B) \neq 0 \quad \text{or} \quad (\text{Adj } A) \times (B) = 0$$

Let C_{ij} be the co-factor of a_{ij} in A

$$\begin{array}{lll} C_{11} = 2 & c_{21} = -3 & C_{31} = 1 \\ c_{12} = -4 & c_{22} = 6 & C_{32} = -2 \\ c_{13} = 2 & c_{23} = -3 & C_{33} = 1 \end{array}$$

$$\text{adj } A = \begin{bmatrix} 2 & -4 & 2 \\ -3 & 6 & -3 \\ 1 & -2 & 1 \end{bmatrix}^T = \begin{bmatrix} 2 & -3 & 1 \\ -4 & 6 & -2 \\ 2 & -3 & 1 \end{bmatrix}$$

$$(\text{adj } A) \times B = \begin{bmatrix} 2 & -3 & 1 \\ -4 & 6 & -2 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix} = \begin{bmatrix} 12 - 42 + 30 \\ -24 + 84 - 60 \\ 12 - 42 + 30 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

So, $AX = B$ has infinite solutions.

Now, $\quad$ let $z = k$

So, $\quad x + y = 6 - k$

$\qquad x + 2y = 14 - 3k$

Which can be written as

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 - k \\ 14 - 3k \end{bmatrix}$$

or $\quad A\,X = B$

$$|A| = 1 \neq 0$$

$$\text{adj } A = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}^T = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{|A|} \text{adj } A \times B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{1} \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 6 - k \\ 14 - 3k \end{bmatrix}$$

$$= \begin{bmatrix} 12 - 2k - 14 + 3k \\ -6 + k + 14 - 3k \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 + k \\ 8 - 2k \end{bmatrix}$$

Hence, $x = k - 2$

$\qquad y = 8 - 2k$

$\qquad z = k$

Solution of Simultaneous Linear Equations Ex 8.1 Q3(vi)

This system can be written as

$$\begin{bmatrix} 2 & 2 & -2 \\ 4 & 4 & -1 \\ 6 & 6 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

or $\quad A\,X = B$

$$|A| = 2(14) - 2(14) - 2(0) = 0$$

So, A is singular and the system has either no solution or infinite solutions according as

$$(\text{Adj}\, A) \times (B) \neq 0 \quad \text{or} \quad (\text{Adj}\, A) \times (B) = 0$$

Let C_{ij} be the co-factor of a_{ij} in A

$$\begin{array}{lll} C_{11} = 14 & C_{21} = -16 & C_{31} = 6 \\ C_{12} = -14 & C_{22} = 16 & C_{32} = -6 \\ C_{13} = 0 & C_{23} = 0 & C_{33} = 0 \end{array}$$

$$\text{adj}\, A = \begin{bmatrix} 14 & -14 & 0 \\ -16 & 16 & 0 \\ 6 & -6 & 0 \end{bmatrix}^T = \begin{bmatrix} 14 & -16 & 6 \\ -14 & 16 & -6 \\ 0 & 0 & 0 \end{bmatrix}$$

$$(\text{adj}\, A) \times B = \begin{bmatrix} 14 & -16 & 6 \\ -14 & 16 & -6 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 14 - 32 + 18 \\ -14 + 32 - 18 \\ 0 + 0 + 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

So, $AX = B$ has infinite solutions.

Now, $\quad$ let $z = k$

So, $\quad 2x + 2y = 1 + 2k$

$\quad\quad 4x + 4y = 2 + k$

Which can be written as

$$\begin{bmatrix} 2 & 2 \\ 4 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 + 2k \\ 2 + k \end{bmatrix}$$

or $\quad A\,X = B$

$$|A| = 0, \; z = 0$$

Again,

$$2x + 2y = 1$$
$$4x + 4y = 2$$

Solution of Simultaneous Linear Equations Ex 8.1 Q4(i)

The above system can be written as

$$\begin{bmatrix} 2 & 5 \\ 6 & 15 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 13 \end{bmatrix}$$

or $\quad A X = B$

$$|A| = 0$$

So, A is singular, and the above system will be inconsistent if

$$(\text{adj } A) \times B \neq 0$$

Now, $\quad C_{11} = 15$

$C_{12} = -6$

$C_{21} = -5$

$C_{22} = 2$

$$\text{adj } A = \begin{bmatrix} 15 & -6 \\ -5 & 2 \end{bmatrix}^T = \begin{bmatrix} 15 & -5 \\ -6 & 2 \end{bmatrix}$$

$$(\text{adj } A) \times (B) = \begin{bmatrix} 15 & -5 \\ -6 & 2 \end{bmatrix}\begin{bmatrix} 7 \\ 13 \end{bmatrix}$$

$$= \begin{bmatrix} 105 - 65 \\ -42 + 26 \end{bmatrix}$$

$$= \begin{bmatrix} 40 \\ -16 \end{bmatrix}$$

$$\neq 0$$

Hence, the above system is inconsistent

Solution of Simultaneous Linear Equations Ex 8.1 Q4(ii)

This system can be written as

$$\begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

or $\quad A X = B$

$$|A| = 0$$

So, the above system will be inconsistent, if

$$(\text{adj } A) \times B \neq 0$$

$C_{11} = 9$

$C_{12} = -6$

$C_{21} = -3$

$C_{22} = 2$

$$\text{adj } A = \begin{bmatrix} 9 & -6 \\ -3 & 2 \end{bmatrix}^T = \begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix}$$

$$(\text{adj } A) \times B = \begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} 45 - 30 \\ -30 + 20 \end{bmatrix}$$

$$= \begin{bmatrix} 15 \\ -10 \end{bmatrix}$$

$$\neq 0$$

Hence, the above system is inconsistent

Solution of Simultaneous Linear Equations Ex 8.1 Q4(iii)

This system can be written as

$$\begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

or $\quad A\,X = B$

$$|A| = -12 + 12 = 0$$

So, A is singular. Now system will be inconsistent, if

$$(\text{adj } A) \times B \neq 0$$

$$C_{11} = -3$$
$$C_{12} = -6$$
$$C_{21} = 2$$
$$C_{22} = 4$$

$$\text{adj } A = \begin{bmatrix} -3 & -6 \\ 2 & 4 \end{bmatrix}^T = \begin{bmatrix} -3 & 2 \\ -6 & 4 \end{bmatrix}$$

$$\begin{aligned}(\text{adj } A) \times (B) &= \begin{bmatrix} -3 & 2 \\ -6 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix} \\ &= \begin{bmatrix} -9 + 10 \\ -18 + 20 \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ 2 \end{bmatrix} \\ &\neq 0\end{aligned}$$

Hence, the above system is inconsistent

Solution of Simultaneous Linear Equations Ex 8.1 Q4(iv)

The above system can be written as

$$\begin{bmatrix} 4 & -5 & -2 \\ 5 & -4 & 2 \\ 2 & 2 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}$$

or $\quad A\,X = B$

$$\begin{aligned}|A| &= 4(-36) + 5(36) - 2(18) \\ &= -144 + 180 - 36 \\ &= 0\end{aligned}$$

So, A is singular and the above system will be inconsistent, if

$$(\text{adj } A) \times B \neq 0$$

$$\begin{array}{lll} C_{11} = -36 & C_{21} = 36 & C_{31} = -18 \\ C_{12} = -36 & C_{22} = 36 & C_{32} = -18 \\ C_{13} = 18 & C_{23} = -18 & C_{33} = 9 \end{array}$$

$$(\text{adj } A) = \begin{bmatrix} -36 & -36 & 18 \\ 36 & 36 & -18 \\ -18 & -18 & 9 \end{bmatrix}^T = \begin{bmatrix} -36 & 36 & -18 \\ -36 & 36 & -18 \\ 18 & -18 & 9 \end{bmatrix}$$

$$(\text{adj } A) \times (B) = \begin{bmatrix} -36 & 36 & -18 \\ -36 & 36 & -18 \\ 18 & -18 & 9 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix} = \begin{bmatrix} -72 - 72 + 18 \\ -72 - 72 + 18 \\ +36 + 36 - 9 \end{bmatrix} \neq 0$$

Hence, the above system is inconsistent.

Solution of Simultaneous Linear Equations Ex 8.1 Q4(v)

The above system can be written as

$$\begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

or $\quad A\,X = B$

$$|A| = 3(-5) + 1(3) - 2(-6) = -15 + 3 + 12 = 0$$

So, A is singular and the above system of equations will be inconsistent, if

$$(\text{adj } A) \times B \neq 0$$

$C_{11} = -5 \qquad C_{21} = +10 \qquad C_{31} = 5$

$C_{12} = 3 \qquad C_{22} = 6 \qquad C_{32} = 3$

$C_{13} = -6 \qquad C_{23} = 12 \qquad C_{33} = 6$

$$(\text{adj } A) = \begin{bmatrix} -5 & 3 & -6 \\ 10 & 6 & 12 \\ 5 & 3 & 6 \end{bmatrix}^T = \begin{bmatrix} -5 & 10 & 5 \\ 3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}$$

$$(\text{adj } A) \times (B) = \begin{bmatrix} -5 & 10 & 5 \\ 3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -10 - 10 + 15 \\ 6 - 6 + 9 \\ -12 - 12 + 18 \end{bmatrix} \neq 0$$

Hence, the given system of equations is inconsistent.

Solution of Simultaneous Linear Equations Ex 8.1 Q4(vi)

The above system can be written as

$$\begin{bmatrix} 1 & 1 & -2 \\ 1 & -2 & 1 \\ -2 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}$$

or $\quad A\,X = B$

$$|A| = 1(-3) - 1(3) - 2(-3) = -3 - 3 + 6 = 0$$

So, A is singular. Now the system can be inconsistent, if

$$(\text{adj } A) \times B \neq 0$$

$C_{11} = -3 \qquad C_{21} = -3 \qquad C_{31} = -3$

$C_{12} = -3 \qquad C_{22} = -3 \qquad C_{32} = -3$

$C_{13} = -3 \qquad C_{23} = -3 \qquad C_{33} = -3$

$$(\text{adj } A) = \begin{bmatrix} -3 & -3 & -3 \\ -3 & -3 & -3 \\ -3 & -3 & -3 \end{bmatrix}^T = \begin{bmatrix} -3 & -3 & -3 \\ -3 & -3 & -3 \\ -3 & -3 & -3 \end{bmatrix}$$

$$(\text{adj } A) \times (B) = \begin{bmatrix} -3 & -3 & -3 \\ -3 & -3 & -3 \\ -3 & -3 & -3 \end{bmatrix}\begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix} = \begin{bmatrix} -15 + 6 - 12 \\ -15 + 6 - 12 \\ -15 + 6 - 12 \end{bmatrix}$$

$$= \begin{bmatrix} -21 \\ -21 \\ -21 \end{bmatrix}$$

$$\neq 0$$

Hence, the given system is inconsistent.

Solution of Simultaneous Linear Equations Ex 8.1 Q5

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 2+4+0 & 2-2+0 & -4+4+0 \\ 4-12+8 & 4+6-4 & -8-12+20 \\ 0-4+4 & 0+2-2 & 0-4+10 \end{bmatrix}$$

$$AB = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$AB = 6I$, where I is a 3×3 unit matrix

or $A^{-1} = \frac{1}{6}B$ [By def. of inverse]

$$= \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

Now, the ginven system of equations can be written as

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

or $AX = B$

or $X = A^{-1}B$

$$= \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 6+34-28 \\ -12+34-28 \\ 6-17+35 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

Hence, $x = 2, y = -1, z = 4$

Solution of Simultaneous Linear Equations Ex 8.1 Q6

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$$|A| = 2(0) + 3(-2) + 5(1) = -1 \neq 0$$

Also, $C_{11} = 0$ $\quad C_{21} = -1$ $\quad C_{31} = 2$

$C_{12} = 2$ $\quad C_{22} = -9$ $\quad C_{32} = 23$

$C_{13} = 1$ $\quad C_{23} = -5$ $\quad C_{33} = 13$

$$(\text{adj } A) = \begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

The given system of equations can be written as

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ -1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

or $\quad A X = B$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} -5+6 \\ -22+45+69 \\ -11-25+39 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence, $x = 1, y = 2, z = 3$

Solution of Simultaneous Linear Equations Ex 8.1 Q7

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$$

$$|A| = 1(1+3) - 2(-1+2) + 5(5) = 4 - 2 + 25 = 27 \neq 0$$

$C_{11} = 4$ $\quad$ $C_{21} = 17$ $\quad$ $C_{31} = 3$
$C_{12} = -1$ $\quad$ $C_{22} = -11$ $\quad$ $C_{32} = 6$
$C_{13} = 5$ $\quad$ $C_{23} = 1$ $\quad$ $C_{33} = -3$

$$A^{-1} = \frac{1}{|A|} \times \text{adj}\, A = \frac{1}{27}\begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}$$

Now, the given set of equations can be represented as

$x + 2y + 5z = 10$
$x - y - z = -2$
$2x + 3y - z = -11$

or $$\begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ -2 \\ -11 \end{bmatrix}$$

or $$X = A^{-1} \times B$$
$$= \frac{1}{27}\begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}\begin{bmatrix} 10 \\ -2 \\ -11 \end{bmatrix}$$
$$= \frac{1}{27}\begin{bmatrix} 40 - 34 - 33 \\ -10 + 22 - 66 \\ 50 - 2 + 33 \end{bmatrix} = \frac{1}{27}\begin{bmatrix} -27 \\ -54 \\ 81 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix}$$

Hence, $x = -1, y = -2, z = 3$

Solution of Simultaneous Linear Equations Ex 8.1 Q8

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$$

$$|A| = 1(7) + 2(2) = 11$$

$$\begin{array}{lll} C_{11} = 7 & C_{21} = 2 & C_{31} = -6 \\ C_{12} = -2 & C_{22} = 1 & C_{32} = -3 \\ C_{13} = -4 & C_{23} = 2 & C_{33} = 5 \end{array}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{11} \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$$

Now, $x - 2y = 10$
$2x + y + 3z = 8$
$-2y + z = 7$

or $$\begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

or $$X = A^{-1} \times B$$
$$= \frac{1}{11} \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$
$$= \frac{1}{11} \begin{bmatrix} 70 + 16 - 42 \\ -20 + 8 - 21 \\ -40 + 16 + 35 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11} \begin{bmatrix} 44 \\ -33 \\ 11 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$$

Hence, $x = 4, y = -3, z = 1$

Solution of Simultaneous Linear Equations Ex 8.1 Q8(ii)

$$A = \begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

$$|A| = 3(3) + 4(-3) + 2(-3) = -9$$

$$\begin{array}{lll} C_{11} = 3 & C_{21} = 4 & C_{31} = -26 \\ C_{12} = 3 & C_{22} = 1 & C_{32} = -11 \\ C_{13} = -3 & C_{23} = -4 & C_{33} = 17 \end{array}$$

$$A^{-1} = \frac{1}{|A|} \text{adj} A = \frac{1}{-9} \begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix}$$

Now,
$3x - 4y + 2z = -1$
$2x + 3y + 5z = 7$
$x + z = 2$

Or $$\begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix}$$

$$X = A^{-1} \times B$$
Or $$= \frac{1}{-9} \begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix} \begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-9} \begin{bmatrix} -27 \\ -18 \\ 9 \end{bmatrix}$$

Hence $x = 3, y = 2, z = -1$

Solution of Simultaneous Linear Equations Ex 8.1 Q8(iii)

$$A=\begin{bmatrix}1 & -2 & 0\\ 2 & 1 & 3\\ 0 & -2 & 1\end{bmatrix} \text{ and } B=\begin{bmatrix}7 & 2 & -6\\ -2 & 1 & -3\\ -4 & 2 & 5\end{bmatrix}$$

$$A\times B=\begin{bmatrix}11 & 0 & 0\\ 0 & 11 & 0\\ 0 & 0 & 11\end{bmatrix}$$

$AB=11I$, where I is a 3×3 unit matrix

$$A^{-1}=\frac{1}{11}B \qquad \text{[By def. of inverse]}$$

Or

$$=\frac{1}{11}\begin{bmatrix}7 & 2 & -6\\ -2 & 1 & -3\\ -4 & 2 & 5\end{bmatrix}$$

Now, the given system of equations can be written as

$$\begin{bmatrix}1 & -2 & 0\\ 2 & 1 & 3\\ 0 & -2 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}10\\ 8\\ 7\end{bmatrix}$$

Or $\quad AX=B$

$$X=A^{-1}B$$

Or

$$=\frac{1}{11}\begin{bmatrix}7 & 2 & -6\\ -2 & 1 & -3\\ -4 & 2 & 5\end{bmatrix}\begin{bmatrix}10\\ 8\\ 7\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{11}\begin{bmatrix}44\\ -33\\ 11\end{bmatrix}=\begin{bmatrix}4\\ -3\\ 1\end{bmatrix}$$

Hence, $x=4, y=-3, z=1$

Solution of Simultaneous Linear Equations Ex 8.1 Q9

Let the numbers are x, y, z.

$$x + y + z = 2 \qquad --- (1)$$

Also, $2y + (x + z) = 1$

$$x + 2y + z = 1 \qquad --- (2)$$

Again,

$$x + z + 5(x) = 6$$

$$5x + y + z = 6 \qquad --- (3)$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 5 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$$

or $\quad A X = B$

$$|A| = 1(1) - 1(-4) + 1(-9)$$
$$= 1 + 4 - 9 = -4 \neq 0$$

Hence, the unique solutions given by $x = A^{-1}B$

$$C_{11} = 1 \qquad C_{21} = 0 \qquad C_{31} = -1$$
$$C_{12} = 4 \qquad C_{22} = -4 \qquad C_{32} = 0$$
$$C_{13} = -9 \qquad C_{23} = 4 \qquad C_{33} = 1$$

or $\quad X = A^{-1}B = \frac{1}{|A|}(\text{adj } A) \times B = \frac{1}{-4}\begin{bmatrix} 1 & 0 & -1 \\ 4 & -4 & 0 \\ -9 & 4 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$

$$= \frac{-1}{4}\begin{bmatrix} 2 - 6 \\ 8 - 4 \\ -18 + 4 + 6 \end{bmatrix} = \frac{-1}{4}\begin{bmatrix} -4 \\ 4 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

Hence, $x = 1, y = -1, z = 2$

Solution of Simultaneous Linear Equations Ex 8.1 Q10

Let the three investments are x, y, z

$$x + y + z = 10,000 \qquad \text{...... (1)}$$

Also

$$\frac{10}{100}x + \frac{12}{100}y + \frac{15}{100}z = 1310$$
$$0.1x + 0.12y + 0.15z = 1310 \qquad \text{...... (2)}$$

Also

$$\frac{10}{100}x + \frac{12}{100}y = \frac{15}{100}z - 190$$
$$0.1x + 0.12y - 0.15z = -190 \qquad \text{...... (3)}$$

The above system can be written as

$$\begin{bmatrix} 1 & 1 & 1 \\ 0.1 & 0.12 & 0.15 \\ 0.1 & 0.12 & -0.15 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10000 \\ 1310 \\ -190 \end{bmatrix}$$

Or $\quad AX = B$

$$|A| = 1(-0.036) - 1(-0.03) + 1(0) = -0.006 \neq 0$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$$\begin{array}{lll} C_{11} = -0.036 & C_{21} = 0.27 & C_{31} = 0.03 \\ C_{12} = 0.03 & C_{22} = -0.25 & C_{32} = -0.05 \\ C_{13} = 0 & C_{23} = -0.02 & C_{33} = 0.02 \end{array}$$

$$\text{adj}A = \begin{bmatrix} -0.036 & 0.03 & 0 \\ 0.27 & -0.25 & -0.02 \\ 0.03 & -0.05 & 0.02 \end{bmatrix}^T = \begin{bmatrix} -0.036 & 0.27 & 0.03 \\ 0.03 & -0.25 & -0.05 \\ 0 & -0.02 & 0.02 \end{bmatrix}$$

Now,

$$X = A^{-1}B = \frac{1}{|A|}(\text{Adj}A) \times B$$

$$= \frac{1}{-0.006} \begin{bmatrix} -0.036 & 0.27 & 0.03 \\ 0.03 & -0.25 & -0.05 \\ 0 & -0.02 & 0.02 \end{bmatrix} \begin{bmatrix} 10000 \\ 1310 \\ -190 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-0.006} \begin{bmatrix} -12 \\ -18 \\ -30 \end{bmatrix} = \begin{bmatrix} 2000 \\ 3000 \\ 5000 \end{bmatrix}$$

Hence, $x = \text{Rs}\,2000, y = \text{Rs}\,3000, z = \text{Rs}\,5000$

Solution of Simultaneous Linear Equations Ex 8.1 Q11

$$x + y + z = 45 \qquad ---(1)$$
$$z = x + 8 \qquad ---(2)$$
$$x + z = 2y \qquad ---(3)$$

or $$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix}$$

$$\begin{aligned}|A| &= 1(2) - 1(-2) + 1(2) \\ &= 2 + 2 + 2 = 6 \neq 0\end{aligned}$$

$$\begin{array}{lll} C_{11} = 2 & C_{21} = -3 & C_{31} = 1 \\ C_{12} = 2 & C_{22} = 0 & C_{32} = -2 \\ C_{13} = 2 & C_{23} = +3 & C_{33} = 1 \end{array}$$

$$\begin{aligned}X = A^{-1} \times B &= \frac{1}{|A|}(\text{adj } A) \times B \\ &= \frac{1}{6}\begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix}\begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix} \\ &= \frac{1}{6}\begin{bmatrix} 90 - 24 \\ 90 \\ 90 + 24 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 66 \\ 90 \\ 114 \end{bmatrix}\end{aligned}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ 15 \\ 19 \end{bmatrix}$$

Hence, $x = 11, y = 15, z = 19$

Solution of Simultaneous Linear Equations Ex 8.1 Q12

The given problem can be modelled using the following system of equations

$$3x + 5y - 4z = 6000$$
$$2x - 3y + z = 5000$$
$$-x + 4y + 6z = 13000$$

Which can write as $Ax = B$,

Where

$$A = \begin{bmatrix} 3 & 5 & -4 \\ 2 & -3 & 1 \\ -1 & 4 & 6 \end{bmatrix}, \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 6000 \\ 5000 \\ 13000 \end{bmatrix}$$

Now

$$|A| = 3(-18-4) - 2(30+16) - 1(5-12)$$
$$= 3(-22) - 2(46) + 7$$
$$= -66 - 92 + 7$$
$$= -151 \neq 0$$

$\therefore$ A^{-1} exists.

Now $Ax = B \Rightarrow \quad x = A^{-1}B$

$$A^{-1} = \frac{adj\,(A)}{|A|}$$

Cofators of A are

$$C_{11} = -22 \qquad C_{21} = -13 \qquad C_{31} = 5$$
$$C_{12} = -46 \qquad C_{22} = 14 \qquad C_{32} = -17$$
$$C_{13} = -7 \qquad C_{23} = -11 \qquad C_{33} = -19$$

$$\therefore \quad adj\,(A) = \begin{bmatrix} -22 & -46 & -7 \\ -13 & +14 & -11 \\ 5 & -17 & -19 \end{bmatrix}$$

Hence,

$$X = \frac{1}{|A|} adj\,(A)(B)$$
$$= \frac{1}{-151}\begin{bmatrix} -22 & -46 & -7 \\ -13 & +14 & -11 \\ 5 & -17 & -19 \end{bmatrix}\begin{bmatrix} 6000 \\ 5000 \\ 13000 \end{bmatrix}$$
$$= \frac{1}{-151}\begin{bmatrix} -132000 & -23000 & -91000 \\ -78000 & +70000 & -143000 \\ -3000 & -85000 & -247000 \end{bmatrix}$$
$$= \begin{bmatrix} 3000 \\ 1000 \\ 2000 \end{bmatrix}$$

$\therefore$ $x = 3000, y = 1000$ and $z = 2000$.

Solution of Simultaneous Linear Equations Ex 8.1 Q13

From the given data, we get
the following three equations:
$x + y + z = 12$
$2x + 3y + 3z = 33$
$x - 2y + z = 0$
This system of equations can be written
in the matrix form as

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix} \ldots.(1)$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}$$

$$|A| = 1(9) - 1(-1) + 1(-7) = 3$$

$$cofA = \begin{bmatrix} 9 & 1 & -7 \\ -3 & 0 & 3 \\ 0 & -1 & 1 \end{bmatrix}$$

$$adjA = [cofA]^T = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{adjA}{|A|} = \frac{1}{3} \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix} \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 11 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 36 - 33 + 0 \\ 4 + 0 + 0 \\ -28 + 33 + 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

An award for organising different festivals in the colony
can be included by the management.

Solution of Simultaneous Linear Equations Ex 8.1 Q14

Let X, Y and Z be the cash awards for
Honesty, Regularity and Hard work respectively.
As per the data in the question, we get

$X + Y + Z = 6000$

$X + 3Z = 11000$

$X - 2Y + Z = 0$

The above three simulataneous equations
can be written in the matrix form as

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix} \ldots(1)$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix}$$

$$|A| = 1(6) - 1(-2) + 1(-2) = 6$$

$$cofA = \begin{bmatrix} 6 & 2 & -2 \\ -3 & 0 & 3 \\ 3 & -2 & -1 \end{bmatrix}$$

Solution of Simultaneous Linear Equations Ex 8.1 Q15

Let x, y and z be teh prize amount per person for
Resourcefulness, Competence and Determination respectively.
As per the data in the question, we get

$4x + 3y + 2z = 37000$

$5x + 3y + 4z = 47000$

$x + y + z = 12000$

The above three simulataneous equations
can be written in matrix form as

$$\begin{bmatrix} 4 & 3 & 2 \\ 5 & 3 & 4 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 37000 \\ 47000 \\ 12000 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 4 & 3 & 2 \\ 5 & 3 & 4 \\ 1 & 1 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 37000 \\ 47000 \\ 12000 \end{bmatrix} \ldots(1)$$

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 5 & 3 & 4 \\ 1 & 1 & 1 \end{bmatrix}$$

$$|A| = 4(-1) - 3(1) + 2(2) = -3$$

$$cofA = \begin{bmatrix} -1 & -1 & 2 \\ -1 & 2 & -1 \\ 6 & -6 & -3 \end{bmatrix}$$

$$adjA = (cofA)^T = \begin{bmatrix} -1 & -1 & 6 \\ -1 & 2 & -6 \\ 2 & -1 & -3 \end{bmatrix}$$

$$A^{-1} = \frac{adjA}{|A|} = \frac{1}{-3}\begin{bmatrix} -1 & -1 & 6 \\ -1 & 2 & -6 \\ 2 & -1 & -3 \end{bmatrix}$$

From (1)

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{1}{-3}\begin{bmatrix} -1 & -1 & 6 \\ -1 & 2 & -6 \\ 2 & -1 & -3 \end{bmatrix}\begin{bmatrix} 37000 \\ 47000 \\ 12000 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{1}{-3}\begin{bmatrix} -12000 \\ -15000 \\ -9000 \end{bmatrix} = \begin{bmatrix} 4000 \\ 5000 \\ 3000 \end{bmatrix}$$

The values x, y and z describe the amount of prizes per person for resourcefulness, competence and determination.

Solution of Simultaneous Linear Equations Ex 8.1 Q16

Let x, y and z be the prize amount per person for adaptibility, carefulness and calmness respectively.

As per the given data, we get

$2x + 4y + 3z = 29000$

$5x + 2y + 3z = 30500$

$x + y + z = 9500$

The above three simulataneous equations can be written in the matrix form as

$$\begin{bmatrix} 2 & 4 & 3 \\ 5 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 29000 \\ 30500 \\ 9500 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 2 & 4 & 3 \\ 5 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 29000 \\ 30500 \\ 9500 \end{bmatrix} \ldots(1)$$

$$A = \begin{bmatrix} 2 & 4 & 3 \\ 5 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$|A| = 2(-1) - 4(2) + 3(3) = -1$$

$$cofA = \begin{bmatrix} -1 & -2 & 3 \\ -1 & -1 & 2 \\ 6 & 9 & -16 \end{bmatrix}$$

$$adjA = (cofA)^T = \begin{bmatrix} -1 & -1 & 6 \\ -2 & -1 & 9 \\ 3 & 2 & -16 \end{bmatrix}$$

$$A^{-1} = \frac{adjA}{|A|} = \frac{\begin{bmatrix} -1 & -1 & 6 \\ -2 & -1 & 9 \\ 3 & 2 & -16 \end{bmatrix}}{-1} = \begin{bmatrix} 1 & 1 & -6 \\ 2 & 1 & -9 \\ -3 & -2 & 16 \end{bmatrix}$$

From (1)

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1 & 1 & -6 \\ 2 & 1 & -9 \\ -3 & -2 & 16 \end{bmatrix}\begin{bmatrix} 29000 \\ 30500 \\ 9500 \end{bmatrix} \ldots(1)$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 2500 \\ 3000 \\ 4000 \end{bmatrix}$$

Solution of Simultaneous Linear Equations Ex 8.1 Q17

Let x, y and z be the prize amount per student for sincerity, truthfulness and helpfulness respectively.
As per the data in the question, we get

$3x + 2y + z = 1600$

$4x + y + 3z = 2300$

$x + y + z = 900$

The above three simulataneous equations can be written in matrix form as

$$\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix} \ldots(1)$$

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$|A| = 3(-2) - 2(1) + 1(3) = -5$$

$$cofA = \begin{bmatrix} -2 & -1 & 3 \\ -1 & 2 & -1 \\ 5 & -5 & -5 \end{bmatrix}$$

$$adjA = (cofA)^T = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{adjA}{|A|} = \frac{\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}}{-5}$$

From (1)

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}}{-5}\begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}\begin{bmatrix} -320 \\ -460 \\ -180 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 200 \\ 300 \\ 400 \end{bmatrix}$$

Excellence in extra-curricular activities should be another value considered for an award.

Solution of Simultaneous Linear Equations Ex 8.1 Q18

x, y and z be prize amount per student for
Discipline, Politeness and Punctuality respectively.
As per the data in the question, we get

$3x+2y+z=1000$

$4x+y+3z=1500$

$x+y+z=600$

The above three simulataneous equations
can be written in matrix form as

$$\begin{bmatrix}3&2&1\\4&1&3\\1&1&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1000\\1500\\600\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}X\\Y\\Z\end{bmatrix}=\begin{bmatrix}3&2&1\\4&1&3\\1&1&1\end{bmatrix}^{-1}\begin{bmatrix}1000\\1500\\600\end{bmatrix}\ldots(1)$$

$$A=\begin{bmatrix}3&2&1\\4&1&3\\1&1&1\end{bmatrix}$$

$$|A|=3(-2)-2(1)+1(3)=-5$$

$$cofA=\begin{bmatrix}-2&-1&3\\-1&2&-1\\5&-5&-5\end{bmatrix}$$

$$adjA=(cofA)^T=\begin{bmatrix}-2&-1&5\\-1&2&-5\\3&-1&-5\end{bmatrix}$$

$$A^{-1}=\frac{adjA}{|A|}=\frac{\begin{bmatrix}-2&-1&5\\-1&2&-5\\3&-1&-5\end{bmatrix}}{-5}$$

From (1)

$$\begin{bmatrix}X\\Y\\Z\end{bmatrix}=\frac{\begin{bmatrix}-2&-1&5\\-1&2&-5\\3&-1&-5\end{bmatrix}}{-5}\begin{bmatrix}1000\\1500\\600\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}X\\Y\\Z\end{bmatrix}=\begin{bmatrix}-2&-1&5\\-1&2&-5\\3&-1&-5\end{bmatrix}\begin{bmatrix}-200\\-300\\-120\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}X\\Y\\Z\end{bmatrix}=\begin{bmatrix}100\\200\\300\end{bmatrix}$$

Solution of Simultaneous Linear Equations Ex 8.1 Q19

x, y and z be prize amount per student for
Tolerance, Kindness and Leadership respectively.
As per the data in the question, we get

$3x+2y+z=2200$

$4x+y+3z=3100$

$x+y+z=1200$

The above three simulataneous equations
can be written in matrix form as

$$\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix} \ldots(1)$$

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$|A| = 3(-2) - 2(1) + 1(3) = -5$$

$$cofA = \begin{bmatrix} -2 & -1 & 3 \\ -1 & 2 & -1 \\ 5 & -5 & -5 \end{bmatrix}$$

$$adjA = (cofA)^T = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{adjA}{|A|} = \frac{\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}}{-5}$$

From (1)

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}}{-5}\begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}\begin{bmatrix} -440 \\ -620 \\ -240 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 300 \\ 400 \\ 500 \end{bmatrix}$$

Solution of Simultaneous Linear Equations Ex 8.1 Q20

Let the amount deposited be x, y and z respectively.
As per the data in the question, we get

$x + y + z = 7000$

$5\%x + 8\%y + 8.5\%z = 550$

$\Rightarrow 5x + 8y + 8.5z = 55000$

$x - y = 0$

The above equations can be written in matrix form as

$$\begin{bmatrix} 1 & 1 & 1 \\ 5 & 8 & 8.5 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7000 \\ 55000 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 5 & 8 & 8.5 \\ 1 & -1 & 0 \end{bmatrix}^{-1}\begin{bmatrix} 7000 \\ 55000 \\ 0 \end{bmatrix} \ldots(1)$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 5 & 8 & 8.5 \\ 1 & -1 & 0 \end{bmatrix}$$

$$|A| = 1(8.5) - 1(-8.5) + 1(-13) = 4$$

$$cofA = \begin{bmatrix} 8.5 & 8.5 & -13 \\ -1 & -1 & 2 \\ 0.5 & -3.5 & 3 \end{bmatrix}$$

$$adjA = (cofA)^T = \begin{bmatrix} 8.5 & 8.5 & -13 \\ -1 & -1 & 2 \\ 0.5 & -3.5 & 3 \end{bmatrix}^T$$

$$adjA = (cofA)^T = \begin{bmatrix} 8.5 & 8.5 & -13 \\ -1 & -1 & 2 \\ 0.5 & -3.5 & 3 \end{bmatrix}^T$$

$$= \begin{bmatrix} 8.5 & -1 & 0.5 \\ 8.5 & -1 & -3.5 \\ -13 & 2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{adjA}{|A|} = \frac{1}{4}\begin{bmatrix} 8.5 & -1 & 0.5 \\ 8.5 & -1 & -3.5 \\ -13 & 2 & 3 \end{bmatrix}$$

From (1)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 8.5 & -1 & 0.5 \\ 8.5 & -1 & -3.5 \\ -13 & 2 & 3 \end{bmatrix}\begin{bmatrix} 7000 \\ 55000 \\ 0 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 4500 \\ 4500 \\ 19000 \end{bmatrix} = \begin{bmatrix} 1125 \\ 1125 \\ 4750 \end{bmatrix}$$

Hence, the amounts deposited in the three accounts are 1125, 1125 and 4750 respectively.

Ex 8.2

Solution of Simultaneous Linear Equations Ex 8.2 Q1

$2x - y + z = 0$
$3x + 2y - z = 0$
$x + 4y + 3z = 0$

The systm can be written as

$$\begin{bmatrix} 2 & -1 & 1 \\ 3 & 2 & -1 \\ 1 & 4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
$$A \quad X = 0$$

Now $|A| = 2(10) + 1(10) + 1(10)$
$= 40$
$\neq 0$

Since $|A| \neq 0$, hence $x = y = z = 0$ is the only solution of this homogeneous system.

Solution of Simultaneous Linear Equations Ex 8.2 Q2

$2x - y + 2z = 0$
$5x + 3y - z = 0$
$x + 5y - 5z = 0$

$$\begin{bmatrix} 2 & -1 & 2 \\ 5 & 3 & -1 \\ 1 & 5 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

or $A \quad x = 0$

$$|A| = 2(-10) + 1(-24) + 2(22)$$
$$= -20 - 24 + 44$$
$$= 0$$

Hence, the system has infinite solutions.

Let $\quad z = k$

$2x - y = -2k$

$5x + 3y = k$

$$\begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2k \\ k \end{bmatrix}$$

$A \quad x = B$

$|A| = 6 + 5 = 11 \neq 0$ so A^{-1} exist

Now $adj\ A = \begin{bmatrix} 3 & -5 \\ 1 & 2 \end{bmatrix}' = \begin{bmatrix} 3 & 1 \\ -5 & 2 \end{bmatrix}$

$$x = A^{-1}.B = \frac{1}{|A|}(adj\ A)B = \frac{1}{11}\begin{bmatrix} 3 & 1 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} -2k \\ k \end{bmatrix} = \begin{bmatrix} \frac{-5k}{11} \\ \frac{12k}{11} \end{bmatrix}$$

Hence, $x = \frac{-5k}{11}$, $y = \frac{12k}{11}$, $z = k$

Solution of Simultaneous Linear Equations Ex 8.2 Q3

$3x - y + 2z = 0$
$4x + 3y + 3z = 0$
$5x + 7y + 4z = 0$

$$|A| = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 3 & 3 \\ 5 & 7 & 4 \end{bmatrix}$$

$$= B(-9) + 1(1) + 2(13) = -27 + 1 + 26 = -27 + 27$$
$$= 0$$

Hence, it has infinite solutions.

Let $\quad z = k$

$3x - y = -2k$

$4x + 3y = -3k$

or $\quad \begin{bmatrix} 3 & -1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2k \\ -3k \end{bmatrix}$

or $\quad A \quad x = B$

$|A| = 9 + 4 = 13 \neq 0$ hence A^{-1} exists

$adj\ A = \begin{bmatrix} 3 & -4 \\ 1 & 3 \end{bmatrix}' = \begin{bmatrix} 3 & 1 \\ -4 & 3 \end{bmatrix}$

Now $x = A^{-1}B = \frac{1}{|A|}(adj\ A)B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{13}\begin{bmatrix} 3 & 1 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} -2k \\ -3k \end{bmatrix} = \frac{1}{13}\begin{bmatrix} -9k \\ -k \end{bmatrix}$$

Hence, $x = \frac{-9k}{13}$, $y = \frac{-k}{13}$, $z = k$

Solution of Simultaneous Linear Equations Ex 8.2 Q4

$x+y-6z=0$
$x-y+2z=0$
$-3x+y+2z=0$

$$\text{Hence, } |A| = \begin{bmatrix} 1 & 1 & -6 \\ 1 & -1 & 2 \\ -3 & 1 & 2 \end{bmatrix}$$

$$= 1(-4) - 1(8) - 6(-2)$$
$$= -4 - 8 + 12$$
$$= 0$$

Hence, the system has infinite solutions.

Let $z = k$
$x + y = 6k$
$x - y = -2k$

$$\text{or} \quad \begin{bmatrix} 1 & +1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6k \\ -2k \end{bmatrix}$$

or $A \quad X = B$

$|A| = -1 - 1 = -2 \neq 0$ hence A^{-1} exists.

$$adj\ A = \begin{bmatrix} -1 & -1 \\ -1 & +1 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{|A|}(adj\ A)B = \frac{1}{-2}\begin{bmatrix} -1 & -1 \\ -1 & +1 \end{bmatrix}\begin{bmatrix} 6k \\ -2k \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-2}\begin{bmatrix} -6k+2k \\ -6k-2k \end{bmatrix} = \left(\frac{1}{-2}\right)\begin{bmatrix} -4k \\ -8k \end{bmatrix} = \frac{-1}{2}\begin{bmatrix} -4k \\ -8k \end{bmatrix} = \begin{bmatrix} 2k \\ 4k \end{bmatrix}$$

Hence, $x = 2k,\ y = 4k, z = k$

Solution of Simultaneous Linear Equations Ex 8.2 Q5

$x+y+z=0$
$x-y-5z=0$
$x+2y+4z=0$

$$|A| = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -5 \\ 1 & 2 & 4 \end{bmatrix}$$
$$= 1(6) - 1(9) + 1(3) = 9 - 9 = 0$$

Hence, the system has infinite solutions.

Let $z = k$
$x + y = -k$
$x - y = 5k$

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -k \\ 5k \end{bmatrix}$$

or $A \quad X = B$

$|A| = -2 \neq 0$, hence A^{-1} exists.

$$adj\ A = \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\text{so, } X = A^{-1}B = \frac{1}{|A|}(adj\ A)B = \frac{1}{-2}\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} -k \\ 5k \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \left(\frac{1}{-2}\right)\begin{bmatrix} k-5k \\ k+5k \end{bmatrix} = \begin{bmatrix} 2k \\ -3k \end{bmatrix}$$

$x = 2k,\ y = -3k, z = k$

Solution of Simultaneous Linear Equations Ex 8.2 Q6

$x+y-z=0$
$x-2y+z=0$
$3x+6y-5z=0$

$$\text{Hence, } |A| = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -2 & 1 \\ 3 & 6 & -5 \end{bmatrix}$$
$$= 1(4) - 1(-8) - 1(12)$$
$$= 4 + 8 - 12 = 0$$

Hence, the system will have infinite solutions.

Let $z = k$
$x + y = -k$
$x - 2y = -k$

or $\begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} k \\ -k \end{bmatrix}$

or $A \quad x = B$

$|A| = -3 \neq 0$, hence A^{-1} exists.

Now, $adj\ A = \begin{bmatrix} -2 & -1 \\ -1 & 1 \end{bmatrix}' = \begin{bmatrix} -2 & -1 \\ -1 & 1 \end{bmatrix}$

Next $x = A^{-1}B$

$$= \frac{1}{|A|}(adj\ A)(B) = \frac{1}{-3}\begin{bmatrix} -2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} k \\ -k \end{bmatrix}$$

$$= \frac{-1}{3}\begin{bmatrix} -2k + k \\ -2k \end{bmatrix}$$

$$= \frac{-1}{3}\begin{bmatrix} -k \\ -2k \end{bmatrix} = \begin{bmatrix} \frac{k}{3} \\ \frac{2k}{3} \end{bmatrix}$$

Hence, $x = \frac{k}{3}$, $y = \frac{2k}{3}$, $z = k$

or $x = k$, $y = 2k$, $z = 3k$

Solution of Simultaneous Linear Equations Ex 8.2 Q7

$3x + y - 2z = 0$
$x + y + z = 0$
$x - 2y + z = 0$

$$\text{Hence, } |A| = \begin{bmatrix} 3 & 1 & -2 \\ 1 & 1 & 1 \\ 1 & -2 & 1 \end{bmatrix}$$
$$|A| = B(1+2) - 1(1-1) - 2(-3)$$
$$= 9 - 0 + 6$$
$$= 15 \neq 0$$

Hence, the given system has only trivial solutions given by $x = y = z = 0$

Solution of Simultaneous Linear Equations Ex 8.2 Q8

$2x + 3y - z = 0$
$x - y - 2z = 0$
$3x + y + 3z = 0$

Hence, $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & -1 & -2 \\ 3 & 1 & 3 \end{bmatrix}$

$$|A| = \begin{bmatrix} 2 & 3 & -1 \\ 1 & -1 & -2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$\begin{aligned} &= 2(-3+2) - 3(3+6) - 1(4) \\ &= -2 - 27 - 4 \\ &\neq 0 \end{aligned}$$

Hence, the system has only trivial solutions given by $x = y = z = 0$

Ex 9.1

Continuity Ex 9.1 Q1

We have to check the continuity of function at $x = 0$.

$$\text{L.H.L} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} \frac{-h}{|-h|} = \lim_{h\to 0} \frac{-h}{h} = -1$$

$$\text{R.H.L} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} \frac{h}{|h|} = 1$$

Thus, LHL $\neq$ R.H.L

So, the given function in discontinuous and the discontinuity is of first kind.

Continuity Ex 9.1 Q2

We have, to check the continuity at $x = 3$.

$$\text{L.H.L} = \lim_{x\to 3^-} f(x) = \lim_{h\to 0} f(3-h) = \lim_{h\to 0} \frac{(3-h)^2-(3-h)-6}{(3-h)-3} = \lim_{h\to 0} \frac{h^2-5h}{-h} = \lim_{h\to 0} -h+5 = 5$$

$$\text{R.H.L} = \lim_{x\to 3^+} f(x) = \lim_{h\to 0} f(3+h) = \lim_{h\to 0} \frac{(3+h)^2-(3+h)-6}{(3+h)-3} = \lim_{h\to 0} \frac{h^2+5h}{h} = \lim_{h\to 0} h+5 = 5$$

$f(3) = 5$

Thus, we have, LHL = RHL = $f(3) = 5$

So, The function is continuous at $x = 3$

Continuity Ex 9.1 Q3

We have, to check the continuity of the function at $x = 3$.

$$\text{LHL} = \lim_{x\to 3^-} f(x) = \lim_{h\to 0} f(3-h) = \lim_{h\to 0}\frac{(3-h)^2-9}{(3-h)-3} = \lim_{h\to 0}\frac{h^2-6h}{-h} = \lim_{h\to 0} -h+6 = 6$$

$$\text{RHL} = \lim_{x\to 3^+} f(x) = \lim_{h\to 0} f(3+h) = \lim_{h\to 0}\frac{(3+h)^2-9}{(3+h)-3} = \lim_{h\to 0}\frac{h^2+6h}{h} = \lim_{h\to 0} h+6 = 6$$

$f(3) = 6$

Thus, we have, $\quad \text{LHL} = \text{RHL} = f(3) = 6$

So, the given function is continuous at $x = 3$.

Continuity Ex 9.1 Q4

We want, to check the continuity of the function at $x = 1$.

$$\text{LHL} = \lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h) = \lim_{h\to 0}\frac{(1-h)^2-1}{(1-h)-1} = \lim_{h\to 0}\frac{h^2-2h}{-h} = \lim_{h\to 0} -h+2 = 2$$

$$\text{RHL} = \lim_{x\to 1^+} f(x) = \lim_{h\to 0} f(1+h) = \lim_{h\to 0}\frac{(1+h)^2-1}{(1+h)-1} = \lim_{h\to 0}\frac{h^2+2h}{h} = \lim_{h\to 0} h+2 = 2$$

$f(1) = 2$

we find that $\text{LHL} = \text{RHL} = f(1) = 2$

Hence, $f(x)$ is continuous at $x = 1$.

Continuity Ex 9.1 Q5

We have, to check the continuty of the function at $x = 0$.

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\frac{\sin 3(-h)}{-h} = \lim_{h\to 0} -\frac{\sin 3h}{-h} = 3$$

$$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0}\frac{\sin 3h}{h} = 3$$

$f(0) = 1$

$\text{LHL} = \text{RHL} \neq f(0)$

$\Rightarrow$ Function is discontinuous at $x = 0$. It is removable discontinuty.

Continuity Ex 9.1 Q6

We have, to check the continuity of the function at $x = 0$.

$$\text{L.H.L} = \lim_{x\to 0^-} f(x) = \lim_{x\to 0} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} e^{1/-h} = e^{-\infty} = 0$$

$$\text{R.H.L} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} e^{1/h} = e^{\infty} = \infty$$

So, $\text{LHL} \neq \text{RHL}$

Hence, the function is discontinuous at $x = 0$. This is discontinuty of I^{st} kind.

Continuity Ex 9.1 Q7

We want, to check the continuity of the given function at $x = 0$.

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\frac{1-\cos(-h)}{(-h)^2}$$

$$= \lim_{h\to 0}\frac{1-\cos h}{h^2}\ [\because \cos(-\theta) = \cos\theta]$$

$$= \lim_{h\to 0}\frac{2\sin^2\frac{h}{2}}{h^2}\ \left[\because 1-\cos\theta = 2\sin^2\frac{\theta}{2}\right]$$

$$= \lim_{h\to 0} 2.\left(\frac{\sin\frac{h}{2}}{h}\right)^2 = 2\times\frac{1}{4} = \frac{1}{2}$$

$$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0}\frac{1-\cos h}{h^2} = \lim_{h\to 0}\frac{2\sin^2\frac{h}{2}}{h^2} = \lim_{h\to 0} 2\left(\frac{\sin^2\frac{h}{2}}{h}\right)^2 = 2\times\frac{1}{4} = \frac{1}{2}$$

$f(0) = 1$

$\text{LHL} = \text{RHL} \neq f(0)$

Hence, the function is discontinuous at $x = 0$

This is removable discontinuty.

Continuity Ex 9.1 Q8

We want, to check the continuty of the function at $x = 0$.

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\frac{-h-|-h|}{2} = \lim_{h\to 0}\frac{-h-h}{2} = 0$$

$$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0}\frac{h-(|h|)}{2} = 0$$

$f(0) = 2$

Thus, $\text{LHL} = \text{RHL} \neq f(0)$

Hence, The function is discontinuous at $x = 0$

This is removable discontinuty.

Continuity Ex 9.1 Q9

We want, to check the continuty of the function at $x = a$.

$$\text{LHL} = \lim_{x\to a^-} f(x) = \lim_{h\to 0} f(a-h) = \lim_{h\to 0}\frac{|a-h-a|}{a-h-a} = \lim_{h\to 0}\frac{h}{-h} = -1$$

$$\text{RHL} = \lim_{x\to a^+} f(x) = \lim_{h\to 0} f(a+h) = \lim_{h\to 0}\frac{|a+h-a|}{a+h-a} = \lim_{h\to 0}\frac{h}{h} = 1$$

Thus, $\text{LHL} \neq \text{RHL}$

Hence, function is discontinuous at $x = a$. And the discontinuty is of first kind.

Continuity Ex 9.1 Q10(i)

We want, to check the continuity at $x = 0$.

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}|-h|\cos\left(\frac{1}{-h}\right) = \lim_{h\to 0} h\cos\left(\frac{1}{h}\right) = 0$$

$$\text{RHL} = \lim_{x\to 0^+} = \lim_{h\to 0} f(0+h) = \lim_{h\to 0}|h|\cos\left(\frac{1}{h}\right) = 0$$

$f(0) = 0$

Thus, $\text{LHL} = \text{RHL} = f(0) = 0$

Hence, function is continuous at $x = 0$.

Continuity Ex 9.1 Q10(ii)

We want, to check the continuity at $x = 0$.

LHL = $\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} (-h)^2 \sin\left(\frac{1}{-h}\right) = 0$

RHL = $\lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} h^2 \sin\left(\frac{1}{h}\right) = 0$

$f(0) = 0$

Thus, LHL = RHL = $f(0) = 0$

Hence, the function is continuous at $x = 0$.

Continuity Ex 9.1 Q10(iii)

We want, to check the continuity of the function at $x = a$.

LHL = $\lim_{x\to a^-} f(x) = \lim_{h\to 0} f(a-h) = \lim_{h\to 0} (a-h-a)\sin\left(\frac{1}{a-h-a}\right) = \lim_{h\to 0} -h\sin\left(\frac{-1}{h}\right) = 0$

RHL = $\lim_{x\to a^+} f(x) = \lim_{h\to 0} f(a+h) = \lim_{h\to 0} (a+h-a)\sin\left(\frac{1}{a+h-a}\right) = \lim_{h\to 0} h\sin\left(\frac{1}{h}\right) = 0$

$f(a) = 0$

Thus, LHL $\neq$ RHL = $f(a) = 0$

Hence, the function is continuous at $x = a$.

Continuity Ex 9.1 Q10(iv)

We want, to check the continuity of the function at $x = 0$.

LHL = $\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} \frac{e^{-h}-1}{\log(1+2(-h))} = \lim_{h\to 0} \frac{e^{-h}-1}{\log(1-2h)} = DNE$

RHL = $\lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} \frac{e^{h}-1}{\log(1+2h)} = DNE$

Thus, Both LHL and RHL do not exist

$\therefore$ Function is discontinuous and the discontinutiy is of IInd kind.

Continuity Ex 9.1 Q10(v)

We want, to check the continuity at $x = 1$

LHL = $\lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h) = \lim_{h \to 0} \frac{1-(1-h)^n}{1-(1-h)} = \lim_{h \to 0} \frac{1 - \left[1 - nh + \frac{n(n-1)}{2!}h^2 + \ldots\right]}{h}$

$= \lim_{h \to 0} n - \frac{n(n-1)}{2!}h + \ldots$

$= n$

RHL = $\lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1+h) = \lim_{h \to 0} \frac{1-(1+h)^n}{1-(1+h)} = \lim_{h \to 0} \frac{1 - \left[1 + nh + \frac{n(n-1)}{2!}h^2 + \ldots\right]}{-h}$

$= \lim_{h \to 0} n + \frac{n(n-1)}{2!}h + \ldots$

$= n$

$f(1) = n - 1$

Thus, LHL = RHL $\neq f(1)$

Hence, function is discontinuous at $x = 1$

This is removable discontinuity.

Continuity Ex 9.1 Q10(vi)

We want, to check the continuity at $x = 1$.

LHL = $\lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h) = \lim_{h \to 0} \frac{\left|(1-h)^2 - 1\right|}{(1-h) - 1} \quad = \lim_{h \to 0} \frac{\left|h^2 - 2h\right|}{-h} = \lim_{h \to 0} -(h-2) = 2$

RHL = $\lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1+h) = \lim_{h \to 0} \frac{\left|(1+h)^2 - 1\right|}{1 + h - 1} \quad = \lim_{h \to 0} \frac{h^2 + 2h}{h} = 2$

$f(1) = 2$

$\therefore$ LHL = RHL = $f(1) = 2$

Hence, function is continuous.

Continuity Ex 9.1 Q10(vii)

LHL = $\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{2(|-h|) + (-h)^2}{-h} = \lim_{h \to 0} \frac{2h + h^2}{-h} = -2$

RHL = $\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{2 \times |h| + h^2}{h} = 2$

Thus, LHL $\neq$ RHL

Function is not continuous at $x = 0$

This is discontinuity of I^{st} kind.

Continuity Ex 9.1 Q11

We want to check the continuity at $x = 1$.

LHL = $\lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h) = \lim_{h \to 0} 1 + (1-h)^2 = \lim_{h \to 0} 1 + 1 - 2h + h^2 = 2$

RHL = $\lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1+h) = \lim_{h \to 0} 2 - (1+h) = 1$

LHL $\neq$ RHL

Hence, the function is discontinuous at $x = 1$

This is discontinuity of I^{st} kind.

Continuity Ex 9.1 Q12

We want to check the continuity at $x = 0$.

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\frac{\sin(3\times(-h))}{\tan(2\times(-h))} = \lim_{h\to 0}\frac{-\sin 3h}{-\tan 2h} = \lim_{h\to 0}\frac{\frac{\sin 3h}{3h}\times 3h}{\frac{\tan 2h}{2h}\times 2h} = \frac{3}{2}$$

$$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0}\frac{\log(1+3h)}{e^{2h}-1} = \lim_{h\to 0}\frac{\frac{\log(1+3h)}{3h}\times 3h}{\frac{e^{2h}-1}{2h}\times 2h} = \frac{3}{2}$$

$f(0) = \frac{3}{2}$

Thus, $\text{LHL} = \text{RHL} = f(0) = \frac{3}{2}$

Hence, the function is continuous at $x = 0$

Continuity Ex 9.1 Q13

We want to check the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} 2(-h) - |-h| = \lim_{h\to 0} -2h - h = 0$$

$$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} 2h - |h| = 0$$

$f(0) = 0$

Thus, $\text{LHL} = \text{RHL} = f(0) = 0$

Hence, the function is continuous at $x = 0$

Continuity Ex 9.1 Q14

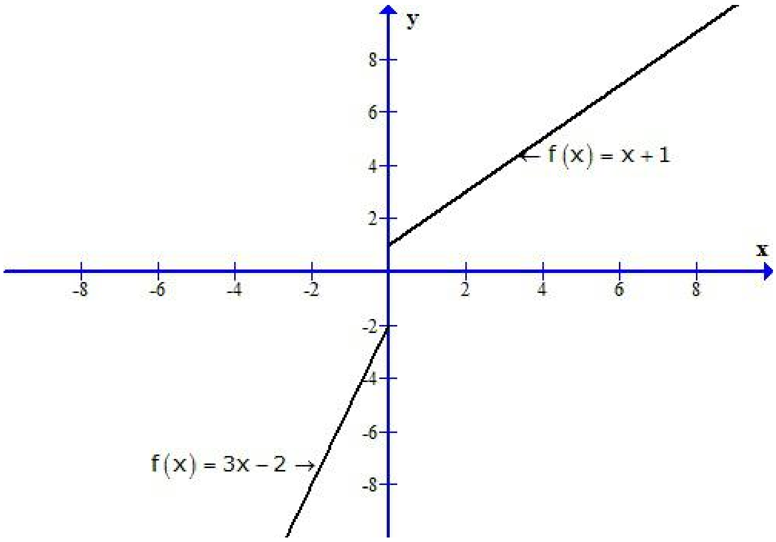
We want to check the continuty of the function at $x = 0$.

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} 3(-h) - 2 = \lim_{h\to 0} -3h - 2 = -2$$

$$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} h + 1 = 1 = 0$$

$\text{LHL} \neq \text{RHL}$

So, the function is discontinuous



Continuity Ex 9.1 Q15

We want to discuss the continuity of the function at $x = 0$.

LHL = $\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} -(-h) = 0$

RHL = $\lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} h = 0$

$f(0) = 1$

Thus, LHL = RHL $\neq f(0)$
Hence, the function is discontinuous at $x = 0$. And this is removable discontinuity.

Continuity Ex 9.1 Q16

We want to discuss the continuity of the function at $x = \frac{1}{2}$.

LHL = $\lim_{x\to \frac{1}{2}^-} f(x) = \lim_{h\to 0} f\left(\frac{1}{2}-h\right) = \lim_{h\to 0} \frac{1}{2} - h = \frac{1}{2}$

RHL = $\lim_{x\to \left(\frac{1}{2}\right)^+} = \lim_{h\to 0} f\left(\frac{1}{2}+h\right) = \lim_{h\to 0} 1 - \left(\frac{1}{2}+h\right) = \frac{1}{2}$

$f\left(\frac{1}{2}\right) = \frac{1}{2}$

Thus, LHL = RHL = $f\left(\frac{1}{2}\right) = \frac{1}{2}$

Hence, the function is continuous at $x = \frac{1}{2}$.

Continuity Ex 9.1 Q17

We want to check the continuity of the function at $x = 0$.

LHL = $\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} 2(-h) - 1 = -1$

RHL = $\lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} 2h + 1 = 1$

Thus, LHL $\neq$ RHL
Hence, the function is discontinuous at $x = 0$. This is discontinuity of I^{st} kind.

Continuity Ex 9.1 Q18

We have given that the function is continuous at $x = 1$
LHL = RHL = $f(1)$ ..... (1)

Now, LHL = $\lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h) = \lim_{h\to 0} \frac{(1-h)^2 - 1}{(1-h) - 1} = \lim_{h\to 0} \frac{h^2 - 2h}{-h} = 2$

$f(1) = k$

From (1), LHL = $f(1)$
$\therefore 2 = k$

Continuity Ex 9.1 Q19

We have that the function is continuous at $x = 1$
$\therefore$ LHL = RHL = $f(1)$ .... (1)

Now,

LHL = $\lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h) = \lim_{h\to 0} \frac{(1-h)^2 - 3(1-h) + 2}{(1-h) - 1} = \lim_{h\to 0} \frac{h^2 + h}{-h} = \lim_{h\to 0} -h - 1 = -1$

$f(1) = k$
From (1), we get,
$k = -1$

Continuity Ex 9.1 Q20

We know that a function is continuous at 0 if

LHL = RHL = $f(0)$ ....(1)

Now,

LHL = $\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} \frac{\sin 5(-h)}{3(-h)} = \lim_{h\to 0} \frac{-\sin 5h}{-3h} = \lim_{h\to 0} \frac{\sin 5h}{5h} \times \frac{5h}{3h} = \frac{5}{3}$

$f(0) = k$

Thus, from (1),

$k = \frac{5}{3}$

Continuity Ex 9.1 Q21

The given function is $f(x) = \begin{cases} kx^2, & \text{if } x \le 2 \\ 3, & \text{if } x > 2 \end{cases}$

The given function f is continuous at $x = 2$, if f is defined at $x = 2$ and if the value of f at $x = 2$ equals the limit of f at $x = 2$

It is evident that f is defined at $x = 2$ and $f(2) = k(2)^2 = 4k$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

$\Rightarrow \lim_{x\to 2^-} (kx^2) = \lim_{x\to 2^+} (3) = 4k$

$\Rightarrow k \times 2^2 = 3 = 4k$

$\Rightarrow 4k = 3 = 4k$

$\Rightarrow 4k = 3$

$\Rightarrow k = \frac{3}{4}$

Therefore, the required value of k is $\frac{3}{4}$.

Continuity Ex 9.1 Q22

We have given that the function is continuous at $x = 0$

So, LHL = RHL = $f(0)$.....(1)

Now,

LHL = $\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} \frac{\sin 2(-h)}{5(-h)} = \lim_{h\to 0} \frac{-\sin 2h}{-5h} = \lim_{h\to 0} \frac{\sin 2h}{2h} \times \frac{2h}{5h} = \frac{2}{5}$

$f(0) = k$

Using (1), $k = \frac{2}{5}$

Continuity Ex 9.1 Q23

We have given that the function is continuous at $x = 2$

LHL = RHL = $f(2)$.....(1)

Now,

LHL = $\lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2-h) = \lim_{h \to 0} a(2-h) + 5 = 2a + 5$

$f(2) = 2a + 5$

RHL = $\lim_{x \to 2^+} f(x) = \lim_{h \to 0} f(2+h) = \lim_{h \to 0} 2 + h - 1 = 1$

$\therefore$ Using (1),

$2a + 5 = 1 \Rightarrow a = -2$

Continuity Ex 9.1 Q24

We have, at $x = 0$

LHL = $\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{-h}{|-h| + 2(-h)^2} = \lim_{h \to 0} \frac{-h}{h + 2h^2} = \lim_{h \to 0} \frac{-1}{1 + 2h} = -1$

$f(0) = k$

RHL = $\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{h}{|h| + 2h^2} = \lim_{h \to 0} \frac{1}{1 + 2h} = 1$

Since, LHL $\neq$ RHL, function will remain discontinuous at $x = 0$, regardless the choice of k.

Continuity Ex 9.1 Q25

Since $f(x)$ is continuous at $x = \frac{\pi}{2}$, L.H.Limit = R.H.Limit.

$$\Rightarrow \lim_{x \to \frac{\pi}{2}^-} f(x) = \lim_{x \to \frac{\pi}{2}^+} f(x) = \lim_{x \to \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \lim_{x \to \frac{\pi}{2}^-} \frac{k\cos x}{\pi - 2x} = 3$$

$$\Rightarrow k \lim_{x \to \frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right)}{2\left(\frac{\pi}{2} - x\right)} = 3$$

$$\Rightarrow \frac{k}{2} \lim_{x \to \frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right)}{\left(\frac{\pi}{2} - x\right)} = 3$$

$$\Rightarrow \frac{k}{2} = 3$$

$$\Rightarrow k = 6$$

Continuity Ex 9.1 Q26

We have given that the function is continuous at $x = 0$

LHL = RHL = $f(0)$....(1)

$f(0) = c$

LHL = $\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{\sin(a+1)(-h) + \sin(-h)}{-h} = \lim_{h \to 0} \frac{-\sin(ah+h) - \sin h}{-h}$

$$= \lim_{h \to 0} \frac{\sin(a+h)h}{h} + \lim_{h \to 0} \frac{\sin h}{h}$$
$$= a + 1 + 1 = a + 2$$

RHL = $\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{\sqrt{h+bh^2} - \sqrt{h}}{bh^{\frac{3}{2}}}$

$$= \lim_{h \to 0} \frac{\sqrt{h+bh^2} - \sqrt{h}}{bh^{\frac{3}{2}}} \times \frac{\sqrt{h+bh^2} + \sqrt{h}}{\sqrt{h+bh^2} + \sqrt{h}}$$
$$= \lim_{h \to 0} \frac{h+bh^2-h}{bh^{\frac{3}{2}}\left(\sqrt{h+bh^2} + \sqrt{h}\right)} = \lim_{h \to 0} \frac{bh^2}{bh^2\left(\sqrt{1+bh} + 1\right)} = \frac{1}{2}$$

$\therefore$ from (1),

$a + 2 = \frac{1}{2} \Rightarrow a = \frac{-3}{2}$

$c = \frac{1}{2}$ and

$b \in R - \{0\}$

Hence, $a = \frac{-3}{2}$, $b \in R - \{0\}$, $c = \frac{1}{2}$

Continuity Ex 9.1 Q27

We have given that the function is continuous at $x = 0$

$\therefore$ LHL = RHL = $f(0)$.....(1)

$f(0) = \frac{1}{2}$

LHL = $\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{1 - \cos k(-h)}{-h\sin(-h)} = \lim_{h \to 0} \frac{1 - \cos kh}{+h\sin h}$

$$= \lim_{h \to 0} \frac{2\sin^2 \frac{kh}{2}}{h.2\sin\frac{h}{2}.\cos\frac{h}{2}}$$
$$= \lim_{h \to 0} \left(\frac{\sin\frac{kh}{2}}{\frac{kh}{2}}\right)^2 \times \frac{\frac{k^2h^2}{4}}{\frac{\sin\frac{h}{2}}{\frac{h}{2}} \times \frac{h}{2}} \cdot \frac{1}{h}$$
$$= \lim_{h \to 0} \left(\frac{\sin\frac{kh}{2}}{\frac{kh}{2}}\right)^2 \cdot \frac{\frac{k^2}{4}}{\frac{\sin\frac{h}{2}}{\frac{h}{2}} \cdot \frac{1}{2}}$$
$$= \frac{k^2}{2}$$

$\therefore$ Using (1) we get,

$\frac{k^2}{2} = \frac{1}{2} \Rightarrow k = \pm 1$

Continuity Ex 9.1 Q28

We have given that the function is continuous at $x = 4$

$\therefore$ LHL = RHL = $f(4)\ldots.(1)$

$f(4) = a + b\ldots..(A)$

LHL = $\lim\limits_{x\to 4^-} f(x) = \lim\limits_{h\to 0} f(4-h) = \lim\limits_{h\to 0}\dfrac{(4-h)-4}{|(4-h)-4|} + a = \lim\limits_{h\to 0}\dfrac{-h}{h} + a = a-1 \qquad \ldots.(B)$

RHL = $\lim\limits_{x\to 4^+} f(x) = \lim\limits_{h\to 0} f(4+h) = \lim\limits_{h\to 0}\dfrac{(4+h)-4}{|(4+h)-4|} + b = \lim\limits_{h\to 0}\dfrac{h}{h} + b = b+1 \qquad \ldots.(C)$

$\therefore$ from (1)

$a - 1 = b + 1 \Rightarrow a - b = 2 \;\ldots..(D)$

from (A) and (B)

$a + b = a - 1 \Rightarrow b = -1$

from (A) and (C)

$a + b = b + 1 \Rightarrow a = 1$

Thus, $a = 1$ and $b = -1$

Continuity Ex 9.1 Q29

We have given that the function is continuous at $x = 0$

$\therefore$ LHL = RHL = $f(0)\ldots..(1)$

$f(0) = k$

LHL = $\lim\limits_{x\to 0} f(x) = \lim\limits_{h\to 0} f(0-h) = \lim\limits_{h\to 0}\dfrac{\sin 2(0-h)}{-h} = \lim\limits_{h\to 0}\dfrac{-\sin 2h}{-h} = 2$

$\therefore$ using (1), we get $k = 2$

Continuity Ex 9.1 Q30

We know that a function is continuous at $x = 0$ if,

LHL = RHL = $f(0)\ldots.(1)$

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\frac{\log\left(1-\frac{h}{a}\right) - \log\left(1+\frac{h}{b}\right)}{(-h)} = \lim_{h\to 0}\frac{\log\left(1+\left(\frac{-h}{a}\right)\right)}{\left(\frac{-h}{a}\right)\times a} + \frac{\log\left(1+\frac{h}{b}\right)}{h}$$

$$= \frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}$$

from (1),

$f(0) = \dfrac{a+b}{ab}$

Continuity Ex 9.1 Q31

We are given that the function is continuous at $x = 2$

$\therefore$ LHL = RHL = $f(2)$ .....(1)

Now,

$f(2) = k$ ....(A)

$$\text{LHL} = \lim_{x\to 2^-} f(x) = \lim_{h\to 0} f(2-h) = \lim_{h\to 0}\frac{2^{(2-h)+2}-16}{4^{(2-h)}-16} = \lim_{h\to 0}\frac{2^{4-h}-16}{4^{2-h}-16}$$

$$= \lim_{h\to 0}\frac{2^4.2^{-h}-16}{4^2.4^{-h}-16}$$

$$= \lim_{h\to 0}\frac{16.2^{-h}-16}{16.4^{-h}-16}$$

$$= \lim_{h\to 0}\frac{16\left(2^{-h}-1\right)}{16\left(4^{-h}-1\right)}$$

$$= \lim_{h\to 0}\frac{2^{-h}-1}{\left(2^{-h}\right)-1^2} \qquad \left[\because 2^{-2h} = \left(2^{-h}\right)^2 = 4^{-h}\right]$$

$$= \lim_{h\to 0}\frac{2^{-h}-1}{\left(2^{-h}-1\right)\left(2^{-h}+1\right)} = \frac{1}{2} \quad \text{......(B)}$$

$\therefore$ Using (1) from (A) & (B)

$k = \frac{1}{2}$

Continuity Ex 9.1 Q33

We know that a function is said to be continuous at $x = \pi$ if

LHL = RHL = value of the function at $x = \pi$ .....(1)

$$\text{LHL} = \lim_{x\to \pi^-} f(x) = \lim_{h\to 0} f(\pi-h) = \lim_{h\to 0}\frac{1-\cos 7(\pi-h-\pi)}{5\left((\pi-h)-\pi\right)^2} = \lim_{h\to 0}\frac{1-\cos 7h}{5h^2}$$

$$= \lim_{h\to 0}\frac{2\sin^2\frac{7}{2}h}{5h^2}$$

$$= \lim_{h\to 0}\frac{2}{5}\left(\frac{\sin\frac{7}{2}h}{\frac{7}{2}h}\right)^2\times\left(\frac{7}{2}\right)^2$$

$$= \frac{2}{5}\times\frac{49}{4} = \frac{49}{10} \quad \text{....(B)}$$

Thus, using (1) we get,

$f(\pi) = \frac{49}{10}$

Continuity Ex 9.1 Q34

It is given that the function is continuous at $x = 0$

$\therefore$ LHL = RHL = $f(0)$....(1)

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\frac{2(-h)+3\sin(-h)}{3(-h)+2\sin(-h)} = \lim_{h\to 0}\frac{-2h-3\sin h}{-3h-2\sin h}$$

$$= \lim_{h\to 0}\frac{\frac{2h+3\sin h}{h}}{\frac{3h+2\sin h}{h}}$$

$$= \lim_{h\to 0}\frac{2+3\frac{\sin h}{h}}{3+2\frac{\sin h}{h}} = \frac{2+3}{3+2} = 1 \qquad \left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1\right]$$

Using (1) we get,

$f(0) = 1$

Continuity Ex 9.1 Q35

It is given that the function is continuous at $x = 0$.
LHL = RHL = $f(0)$ .... (1)

$f(0) = k$ ....(A)

LHL = $\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{1 - \cos 4(-h)}{8(-h)^2} = \lim_{h \to 0} \frac{1 - \cos 4h}{8h^2} = \lim_{h \to 0} \frac{2\sin^2 2h}{8h^2} = \lim_{h \to 0} \left(\frac{\sin 2h}{2h}\right)^2 = 1$

Thus, using (1) we get,
$k = 1$

Continuity Ex 9.1 Q36

The given function will be continuous at $x = 0$ if
LHL = RHL = $f(0)$ .... (1)

$f(0) = 8$ .....(A)

LHL = $\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{1 - \cos 2k(-h)}{(-h)^2} = \lim_{h \to 0} \frac{1 - \cos 2kh}{h^2} = \lim_{h \to 0} \frac{2\sin^2 kh}{h^2}$

$= \lim_{h \to 0} 2\left(\frac{\sin kh}{kh}\right)^2 .k^2$

$= 2k^2$

Thus, using (1) we get,
$2k^2 = 8 \quad \Rightarrow k^2 = 4 \quad \Rightarrow k = \pm 2$

Hence, $k = \pm 2$

Let $x-1=y$

$\Rightarrow x=y+1$

Thus,

$$\lim_{x\to 1}(x-1)\tan\frac{\pi x}{2}=\lim_{y\to 0}y\tan\frac{\pi(y+1)}{2}$$

$$=\lim_{y\to 0}y\tan\left(\frac{\pi y}{2}+\frac{\pi}{2}\right)$$

$$=-\lim_{y\to 0}y\cot\frac{\pi y}{2}$$

$$=-\lim_{y\to 0}y\frac{\cos\frac{\pi y}{2}}{\sin\frac{\pi y}{2}}$$

$$=-\lim_{y\to 0}y\frac{\cos\frac{\pi y}{2}}{\frac{\left(\sin\frac{\pi y}{2}\right)\frac{\pi}{2}}{\frac{\pi}{2}}}$$

$$=-\lim_{y\to 0}\frac{\cos\frac{\pi y}{2}}{\frac{\left(\sin\frac{\pi y}{2}\right)\frac{\pi}{2}}{\frac{\pi y}{2}}}$$

$$=-\lim_{y\to 0}\frac{2}{\pi}\frac{\cos\frac{\pi y}{2}}{\frac{\left(\sin\frac{\pi y}{2}\right)}{\frac{\pi y}{2}}}$$

$$=-\frac{2}{\pi}\lim_{y\to 0}\cos\frac{\pi y}{2}$$

$$=-\frac{2}{\pi}$$

Since the function is continuous, $L.H.Limit=R.H.Limit$

Thus, $k=-\frac{2}{\pi}$

Since the function is continuous at every point, therefore

$$LHL=RHL=f(0)$$

Now

$$f(0)=\cos 0$$
$$=1$$

Again

$$LHL=\lim_{x\to 0}k\left(x^2-2x\right)$$
$$=\lim_{h\to 0^-}k\left(h^2-2h\right)$$
$$=0$$

Therefore there is no value of k

Since the function is continuous at every point, therefore

$$LHL=RHL=f(\pi)$$

Now

$$f(\pi)=k\pi+1$$

Again

$$RHL=\lim_{x\to\pi^+}\cos x$$
$$=\lim_{h\to 0^+}\cos(\pi-h)$$
$$=-\lim_{h\to 0^+}\cosh$$
$$=-1$$

Therefore we can write

$$k\pi+1=-1$$

$$k=-\frac{2}{\pi}$$

We are given that function is continuous at $x = 5$.
$\therefore$ LHL = RHL = $f(5)$....(1)

$f(5) = 5k + 1$

LHL = $\lim_{x \to 5^+} f(x) = \lim_{h \to 0} f(5+h) = \lim_{h \to 0} 3(5+h) - 5 = 10$

Thus, using (1), we get,
$5k + 1 = 10$
$5k = 9$
$k = \frac{9}{5}$

We know that the function will be continuous at $x = 5$. if
LHL = RHL = $f(5)$.....(1)

$f(5) = k$

LHL = $\lim_{x \to 5^-} f(x) = \lim_{h \to 0} f(5-h) = \lim_{h \to 0} \frac{(5-h)^2 - 25}{(5-h) - 5} = \lim_{h \to 0} \frac{h^2 - 10h}{-h} = \lim_{h \to 0} -h + 10 = 10$

Thus, using (1), we get,
$k = 10$

We know that a function will be continuous at $x = 1$. if
LHL = RHL = $f(1)$ ....(1)

$f(1) = k.1^2 = k$

LHL = $\lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h) = \lim_{h \to 0} 4 = 4$

Thus, using (1), we get,
$k = 4$

We know that a function will be continuous at $x = 0$. if
LHL = RHL = $f(0)$.....(1)

$f(0) = k(0+2) = 2k$

LHL = $\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} 3(h) + 1 = 1$

Thus, using (1), we get,
$2k = 1$
$k = \frac{1}{2}$

Continuity Ex 9.1 Q37

It is given that the function is continuous at $x = 3$ and at $x = 5$

$\therefore$ LHL = RHL = $f(3)$.....(1) and
LHL = RHL = $f(5)$.....(2)

Now,
$f(3) = 1$
RHL = $\lim_{x \to 3^+} f(x) = \lim_{h \to 0} f(3+h) = \lim_{h \to 0} a(3+h) + b = 3a + b$

Thus, using (1), we get,
$3a + b = 1$....(3)

$f(5) = 7$
LHL = $\lim_{x \to 5^-} f(x) = \lim_{h \to 0} f(5-h) = \lim_{h \to 0} a(5-h) + b = 5a + b$

Thus, using (2), we get
$5a + b = 7$ .....(4)

Now, solving (3) and (4) we get,
$a = 3$ and $b = -8$

Continuity Ex 9.1 Q38

We want to discuss the continuity of the function at $x = 1$

We need to prove that

LHL = RHL = $f(1)$

$f(1) = \frac{1^2}{2} = \frac{1}{2}$

LHL = $\lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h) = \lim_{h \to 0} \frac{(1-h)^2}{2} = \frac{1}{2}$

RHL = $\lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1+h) = \lim_{h \to 0} 2(1+h)^2 - 3(1+h) + \frac{3}{2} = 2 - 3 + \frac{3}{2} = \frac{1}{2}$

Thus, LHL = RHL = $f(1) = \frac{1}{2}$
Hence, function is continuous at $x = 1$

Continuity Ex 9.1 Q39

We want to discuss the continuity at $x = 0$ and $x = 1$

Now,
$f(0) = 1$
LHL = $\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} |-h| + |-h-1| = 1.$
RHL = $\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} |h| + |h-1| = 1$

$\because$ LHL = RHL = $f(0) = 1$, function is continuous at $x = 0$.

For $x = 1$.

$f(1) = 1$
LHL = $\lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h) = \lim_{h \to 0} |1-h| + |1-h-1| = 1$
RHL = $\lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1+h) = \lim_{h \to 0} |1+h| + |1+h-1| = 1$

$\because$ LHL = RHL = $f(1) = 1$ function is continuous at $x = 1$.

For $x = -1$

$f(-1) = |-1-1| + |-1+1| = 2$

LHL $= \lim\limits_{x \to 1^-} f(x) = \lim\limits_{h \to 0} f(-1-h) = \lim\limits_{h \to 0} |-1-h-1| + |-1-h+1| = 2$

RHL $= \lim\limits_{x \to 1^+} f(x) = \lim\limits_{h \to 0} f(-1+h) = \lim\limits_{h \to 0} |-1+h-1| + |-1+h+1| = 2$

Thus, LHL = RHL $= f(-1) = 2$
Hence, function is continuous at $x = -1$

For $x = 1$
$f(1) = |1-1| + |1+1| = 2$

LHL $= \lim\limits_{x \to 1^-} f(x) = \lim\limits_{h \to 0} f(1-h) = \lim\limits_{h \to 0} |1-h-1| + |1-h+1| = 2$

RHL $= \lim\limits_{x \to 1^+} f(x) = \lim\limits_{h \to 0} f(1+h) = \lim\limits_{h \to 0} |1+h-1| + |1+h+1| = 2$

Thus, LHL = RHL $= f(1) = 2$
Hence, function is continuous at $x = 1$

Continuity Ex 9.1 Q40

Since $f(x)$ *is continuous at* $x = 0$, *L.H.Limit = R.H.Limit.*
Thus, we have

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$$

$$\Rightarrow \lim_{x \to 0^-} a \sin \frac{\pi}{2}(x+1) = \lim_{x \to 0^+} \frac{\tan x - \sin x}{x^3}$$

$$\Rightarrow a \times 1 = \lim_{x \to 0} \frac{\tan x - \sin x}{x^3}$$

$$\Rightarrow a = \lim_{x \to 0} \frac{\frac{\sin x}{\cos x} - \sin x}{x^3}$$

$$\Rightarrow a = \lim_{x \to 0} \frac{\frac{\sin x}{x}\left(\frac{1}{\cos x} - 1\right)}{x^2}$$

$$\Rightarrow a = \lim_{x \to 0} \frac{\frac{\sin x}{x}\left(\frac{1 - \cos x}{\cos x}\right)}{x^2}$$

$$\Rightarrow a = \lim_{x \to 0} \frac{\sin x}{x} \times \lim_{x \to 0} \frac{1}{\cos x} \times \lim_{x \to 0} \frac{1 - \cos x}{x^2}$$

$$\Rightarrow a = 1 \times 1 \times \lim_{x \to 0} \frac{1 - \cos x}{x^2}$$

$$\Rightarrow a = \lim_{x \to 0} \frac{1 - \cos x}{x^2} \times \frac{1 + \cos x}{1 + \cos x}$$

$$\Rightarrow a = \lim_{x \to 0} \frac{1 - \cos^2 x}{x^2(1 + \cos x)}$$

$$\Rightarrow a = \lim_{x \to 0} \frac{\sin^2 x}{x^2(1 + \cos x)}$$

$$\Rightarrow a = \lim_{x \to 0} \frac{\sin^2 x}{x^2} \times \lim_{x \to 0} \frac{1}{1 + \cos x}$$

$$\Rightarrow a = 1 \times \lim_{x \to 0} \frac{1}{1 + \cos x}$$

$$\Rightarrow a = 1 \times \frac{1}{1+1}$$

$$\Rightarrow a = \frac{1}{2}$$

Continuity Ex 9.1 Q41

It is given that function is continuous at $x = 0$. then,
LHL = RHL = $f(0)$ .... (1)

Now,
$f(0) = 2 \cdot 0 + k = k$

LHL = $\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} -2(-h)^2 + k = k$

RHL = $\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} 2(h^2) + k = k$

Thus, the function will be continuous for any $k \in R$.

Continuity Ex 9.1 Q42

The given function f is $f(x)=\begin{cases}\lambda\left(x^2-2x\right), & \text{if } x\le 0\\ 4x+1, & \text{if } x>0\end{cases}$

If f is continuous at $x=0$, then

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)=f(0)$$
$$\Rightarrow \lim_{x\to 0^-} \lambda\left(x^2-2x\right)=\lim_{x\to 0^+}(4x+1)=\lambda\left(0^2-2\times 0\right)$$
$$\Rightarrow \lambda\left(0^2-2\times 0\right)=4\times 0+1=0$$
$$\Rightarrow 0=1=0, \text{ which is not possible}$$

Therefore, there is no value of λ for which f is continuous at $x=0$

At $x=1$,

$f(1)=4x+1=4\times 1+1=5$

$$\lim_{x\to 1}(4x+1)=4\times 1+1=5$$
$$\therefore \lim_{x\to 1} f(x)=f(1)$$

Therefore, for any values of λ, f is continuous at $x=1$

Continuity Ex 9.1 Q43

The function will be continuous at $x=2$
if LHL = RHL = $f(2)$ ....(1)

Now,
$f(2)=k$

LHL = $\lim\limits_{x\to 2^-} f(x)=\lim\limits_{h\to 0} f(2-h)=\lim\limits_{h\to 0} 2(2-h)+1=5.$

Thus, using (1) we get,
$k=5$

Continuity Ex 9.1 Q44

It is given that the function is continuous at $x = \frac{\pi}{2}$

$\therefore$ LHL = RHL = $f\left(\frac{\pi}{2}\right)$ .....(1)

Now,

$f\left(\frac{\pi}{2}\right) = a$

$$\text{LHL} = \lim_{x\to\frac{\pi^-}{2}} f(x) = \lim_{h\to0} f\left(\frac{\pi}{2}-h\right) = \lim_{h\to0}\frac{1-\sin^3\left(\frac{\pi}{2}-h\right)}{3\cos^2\left(\frac{\pi}{2}-h\right)} = \lim_{h\to0}\frac{1-\cos^3 h}{3\sin^2 h}$$

$$= \lim_{h\to0}\frac{(1-\cos h)\left(1+\cos^2 h+\cos h\right)}{3\sin^2 h}$$

$$= \lim_{h\to0}\frac{2\sin^2\frac{h}{2}\left(1+\cos^2 h+\cos h\right)}{3\sin^2 h}$$

$$= \lim_{h\to0}\frac{2\left(\dfrac{\sin\frac{h}{2}}{\frac{h}{2}}\right)^2\times\dfrac{h^2}{4}.\left(1+\cos^2 h+\cos h\right)}{3\left(\dfrac{\sin h}{h}\right)^2.h^2}$$

$$= \lim_{h\to0}\frac{2.\frac{1}{4}\left(1+\cos^2 h+\cos h\right)}{3} = \frac{1}{2}$$

$$\text{RHL} = \lim_{x\to\frac{\pi^+}{2}} f(x) = \lim_{h\to0} f\left(\frac{\pi}{2}+h\right) = \lim_{h\to0}\frac{b\left(1-\sin\left(\frac{\pi}{2}+h\right)\right)}{\left(\pi-2\left(\frac{\pi}{2}+h\right)\right)^2} = \lim_{h\to0}\frac{b(1-\cos h)}{(\pi-\pi-2h)^2}$$

$$= \lim_{h\to0}\frac{b.2\sin^2\frac{h}{2}}{(2h)^2}$$

$$= \lim_{h\to0}\frac{b}{2}\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)^2\times\frac{1}{4}$$

$$= \lim_{h\to0}\frac{b}{8} = \frac{b}{8}$$

Thus, using (1) we get,

$a = \frac{1}{2}$

And

$\frac{b}{8} = \frac{1}{2} \Rightarrow b = 4$

Thus, $a = \frac{1}{2}$ and $b = 4$

Continuity Ex 9.1 Q45

It is given that the function is continuous at $x = 0$, then

LHL = RHL = $f(0)$.....(1)

Now,

$f(0) = k$

$$\text{RHL} = \lim_{x\to0^+} f(x) = \lim_{h\to0} f(0+h) = \lim_{h\to0}\frac{h}{|h|} = 1....(B)$$

Thus, using (1) we get,

$k = 1$

Continuity Ex 9.1 Q46

Since the function is continuous at $x=3$, therefore

$$LHL = RHL = f(3)$$

Now

$$\begin{aligned} RHL &= \lim_{x\to 3^+} f(x) \\ &= \lim_{h\to 0} f(3+h) \\ &= \lim_{h\to 0} b(3+h)+3 \\ &= \lim_{h\to 0} 3b+3h+3 \\ &= 3b+3 \end{aligned}$$

Again

$$\begin{aligned} f(3) &= a(3)+1 \\ &= 3a+1 \end{aligned}$$

Thus we can write

$$f(3) = RHL$$

$$3a+1 = 3b+3$$

$$3a-3b = 2$$

Ex 9.2

Chapter 9 Continuity Ex 9.2 Q1

When $x < 0$, we have, $f(x) = \frac{\sin x}{x}$

We know that $\sin x$ and the identity function x both are everywhere continuous.
So, the quotient function $\frac{\sin x}{x} = f(x)$ is continuous for $x < 0$

When $x > 0$, we have $f(x) = x + 1$, which being a polynomial, is continuous for $x > 0$

Let us now consider $x = 0$

LHL= $\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} \frac{\sin(-h)}{-h} = \lim_{h\to 0} \frac{-\sin h}{-h} = 1$

$RHL = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} \frac{\sin h}{h} = 1$

$f(0) = 0 + 1 = 1$

Thus, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0) = 1$

$\therefore$ $f(x)$ is contiunuos at $x = 0$

Hence, $f(x)$ is contiunuos everywhere.

Chapter 9 Continuity Ex 9.2 Q2

When $x \neq 0$,

$$f(x) = \frac{x}{|x|} = \begin{cases} \frac{-x}{x} = -1 \ ; \ x < 0 \\ \frac{x}{|x|} = 1 \ ; x > 0 \end{cases}$$

So, $f(x)$ is a constant function when $x \neq 0$

hence, is continuous for all $x < 0$ and $x > 0$

Now,

Consider the point $x = 0$.

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{-h}{|-h|} = -1$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{h}{|h|} = 1$$

So, LHL $\neq$ RHL

Hence, function is discontinuous at $x = 0$

Chapter 9 Continuity Ex 9.2 Q3(i)

When $x \neq 1$

$f(x) = x^3 - x^2 + 2x - 2$ is a polynomial, so is continuous for $x < 1$ and $x > 1$

Now, consider the point $x = 1$

$$\text{LHL} = \lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h) = \lim_{h \to 0} (1-h)^3 - (1-h)^2 + 2(1-h) - 2 = 1 - 1 + 2 - 2 = 0$$

$$\text{RHL} = \lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1+h) = \lim_{h \to 0} (1+h)^3 - (1+h)^2 + 2(1+h) - 2 = 1 - 1 + 2 - 2 = 0$$

$f(1) = 4$

LHL = RHL $\neq f(1)$

Thus, function is not discontinuous at $x = 1$

Chapter 9 Continuity Ex 9.2 Q3(ii)

When $x \neq 2$, we have,

$$f(x) = \frac{x^4 - 16}{x - 2} = \frac{(x^2+4)(x^2-4)}{x-2} = \frac{(x^2+4)(x+2)(x-2)}{x-2} = f(x) = (x^2+4)(x+2)$$

which is a polynomial, so the function is continuous when $x < 2$ or $x > 2$

Now, consider the point $x = 2$

$$\text{LHL} = \lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2-h) = \lim_{h \to 0} \frac{(2-h)^4 - 16}{(2-h) - 2}$$

$$= \lim_{h \to 0} \frac{2^4 - 4.8h + 6.4h^2 - 4.2h^3 + h^4 - 16}{-h}$$

$$= \lim_{h \to 0} \frac{16 - 32h + 24h^2 - 8h^3 + h^4 - 16}{-h}$$

$$= \lim_{h \to 0} 32 - 24h + 8h^2 - h^3 = 32$$

$$\text{RHL} = \lim_{x \to 2^+} f(x) = \lim_{h \to 0} f(2+h) = \lim_{h \to 0} \frac{(2+h)^4 - 16}{(2+h) - 2} = \lim_{h \to 0} \frac{16 + 32h + 24h^2 + 8h^3 + h^4 - 16}{h}$$

$$= \lim_{h \to 0} 32 + 24h + 8h^2 + h^3$$

$$= 32$$

Also, $f(2) = 16$

Thus, LHL = RHL $\neq f(2)$

Hence, the function is discontinuous at $x = 2$

Chapter 9 Continuity Ex 9.2 Q3(iii)

When $x < 0$, we have, $f(x) = \frac{\sin x}{x}$

We know that $\sin x$ and the identity function continuous for $x < 0$, so the quotient function $f(x) = \frac{\sin x}{x}$ is continuous for $x < 0$.

When $x > 0$ $f(x) = 2x + 3$, which is a polynomial of degree 1 so $f(x) = 2x + 3$ is continuous for $x > 0$.

Now, consider the point $x = 0$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{\sin(-h)}{-h} = \lim_{h \to 0} \frac{-\sin h}{-h} = 1$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} \frac{\sin h}{h} = 1$$

$f(0) = 2 \times 0 + 3 = 3$

Thus, L.H.L = R.H.L $\neq f(0)$

Hence, $f(x)$ is discontinuous at $x = 0$

Chapter 9 Continuity Ex 9.2 Q3(iv)

When $x \neq 0$ $f(x) = \frac{\sin 3x}{x}$

We know that $\sin 3x$ and the identity function x are continuous for $x < 0$ and $x > 0$.

So, the quotient function $f(x) = \frac{\sin 3x}{x}$ is continuous for $x < 0$ and $x > 0$.

Now, consider the point $x = 0$

$$\text{LHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{\sin 3(-h)}{-h} = \lim_{h \to 0} \frac{-\sin 3h}{-h} = 3$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} \frac{\sin 3h}{h} = 3$$

$f(0) = 4$

Thus, LHL = RHL $\neq f(0)$

Hence, $f(x)$ is discontinuous at $x = 0$

Chapter 9 Continuity Ex 9.2 Q3(v)

When $x \neq 0$, we have, $f(x) = \frac{\sin x}{x} + \cos x$

We know that

$\sin x$ and $\cos x$ is continuous for $x < 0$ and $x > 0$.

The identity function x is also continuous for $x < 0$ and $x > 0$.

$\therefore$ The quotient function $f(x) = \frac{\sin x}{x}$ is continuous for $x < 0$ and $x > 0$.

And, the sum $\frac{\sin x}{x} + \cos x$ is also continuous for each $x < 0$ and $x > 0$.

Now, consider the point $x = 0$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{\sin(-h)}{-h} + \cos(-h) = \lim_{h \to 0} \frac{-\sin h}{-h} + \cos h = 1 + 1 = 2$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} \frac{\sin h}{h} + \cos h = 1 + 1 = 2$$

$f(0) = 5$

Thus, LHL = RHL $\neq f(0)$

Hence, $f(x)$ is discontinuous at $x = 0$

Chapter 9 Continuity Ex 9.2 Q3(vi)

When $x \neq 0$, we have, $f(x) = \dfrac{x^4 + x^3 + 2x^2}{\tan^{-1} x}$

We know that a polynomial is continuous for $x < 0$ and $x > 0$, Also the inverse trignometric function is continuous in its domain.

Here, $x^4 + x^3 + 2x^2$ is polynomial, so is continuous for $x < 0$ and $x > 0$ and $\tan^{-1} x$ is also continuous for $x < 0$ and $x > 0$

So, the quotient function $f(x) = \dfrac{x^4 + x^3 + 2x^2}{\tan^{-1} x}$ is continuous for each $x < 0$ and $x > 0$.

Now, consider the point $x = 0$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{(-h)^4 + (-h)^3 + 2(-h)^2}{\tan^{-1}(-h)} = \lim_{h \to 0} \frac{h^4 - h^3 + 2h^2}{\tan^{-1} h} = 0$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} \frac{h^4 + h^3 + 2h^2}{\tan^{-1} h} = 0$$

$f(0) = 10$

Thus, LHL = RHL $\neq f(0)$

Hence, the function is not continuous at $x = 0$

Chapter 9 Continuity Ex 9.2 Q3(vii)

When $x \neq 0$, we have,

$$f(x) = \frac{e^x - 1}{\log_e(1 + 2x)}$$

We know that e^x and the constant function is continuous for $x < 0$ and $x > 0$

$\Rightarrow$ $e^x - 1$ is continuous for $x < 0$ and $x > 0$

Again, logarithmic function is continuous for $x < 0$ and $x > 0$

$\Rightarrow$ $\log_e(1 + 2x)$ is continuous for $x > 0$ and $x < 0$

So, the quotient function $f(x) = \dfrac{e^x - 1}{\log_e(1 + 2x)}$ is continuous for each $x < 0$ and $x > 0$.

Now, consider the point $x = 0$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{e^{-h} - 1}{\log_e(1 - 2h)} = \lim_{h \to 0} \frac{\frac{e^{-h} - 1}{-h}}{\frac{\log_e(1 - 2h)}{-2h} \times -2} = \frac{1}{2}$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} \frac{e^{h} - 1}{\log_e(1 + 2h)} = \lim_{h \to 0} \frac{\frac{e^{h} - 1}{h}}{\frac{\log_e(1 + 2h)}{2h} \times 2} = \frac{1}{2}$$

$f(0) = 7$

Thus, LHL = RHL $\neq f(0)$

Hence, $f(x)$ is not continuous at $x = 0$

Chapter 9 Continuity Ex 9.2 Q3(viii)

We know that

(i) The absolute value function $g(x) = |x|$ is continuous on IR

(ii) Polynomial function are every where continuous.
So, the only possible point of discontinuity of $f(x)$ can be $x = 1$

Now

$$f(1) = |1-3| = |-2| = 2$$
$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} |x-3| = 2$$
$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}\left(\frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}\right)$$
$$= \frac{1}{4} - \frac{3}{2} + \frac{13}{4} = \frac{8}{4} = 2$$

Since
$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1) = 2$$

$\therefore$ $f(x)$ is continuous at x

Hence $f(x)$ has no point of discontinuity.

Chapter 9 Continuity Ex 9.2 Q3(ix)

When $x < -3$,
$f(x) = |x| + 3$
We know that $|x|$ is continuous for $x < -3$
$\therefore$ $|x| + 3$ is continuous for $x < -3$

When $x > 3$
$f(x) = 6x + 2$ which is a polynomial of degree 1, so $f(x) = 6x + 2$ is continuous for $x > 3$

When $-3 < x < 3$
$f(x) = -2x$ which is again a polynomial so, it is continuous for $-3 < x < 3$

Now, consider the point $x = -3$
$$\text{LHL} = \lim_{x\to -3^-} f(x) = \lim_{h\to 0} f(-3-h) = \lim_{h\to 0} |-3-h| + 3 = \lim_{h\to 0} |3+h| + 3 = 6$$

$$\text{RHL} = \lim_{x\to -3^+} f(x) = \lim_{h\to 0} f(-3+h) = \lim_{h\to 0} -2(-3+h) = 6$$
$$f(-3) = |-3| + 3 = 6$$

Thus, LHL = RHL = $f(-3) = 6$
So, the function is continuous at $x = -3$

Now, consider the point $x = 3$

$$\text{LHL} = \lim_{x\to 3^-} f(x) = \lim_{h\to 0} f(3-h) = \lim_{h\to 0} -2(3-h) = -6$$

Chapter 9 Continuity Ex 9.2 Q3(xi)

The given function is $f(x)=\begin{cases}2x, & \text{if } x<0\\ 0, & \text{if } 0\le x\le 1\\ 4x, & \text{if } x>1\end{cases}$

The given function is defined at all points of the real line.

Let c be a point on the real line.

Case I:

$$\text{If } c<0,\ \text{then } f(c)=2c$$
$$\lim_{x\to c} f(x)=\lim_{x\to c}(2x)=2c$$
$$\therefore \lim_{x\to c} f(x)=f(c)$$

Therefore, f is continuous at all points x, such that $x<0$

Chapter 9 Continuity Ex 9.2 Q3(xii)

The given function f is $f(x)=\begin{cases}\sin x-\cos x, & \text{if } x\ne 0\\ -1 & \text{if } x=0\end{cases}$

It is evident that f is defined at all points of the real line.

Let c be a real number.

Case I:

$$\text{If } c\ne 0,\ \text{then } f(c)=\sin c-\cos c$$
$$\lim_{x\to c} f(x)=\lim_{x\to c}(\sin x-\cos x)=\sin c-\cos c$$
$$\therefore \lim_{x\to c} f(x)=f(c)$$

Therefore, f is continuous at all points x, such that $x\ne 0$

Chapter 9 Continuity Ex 9.2 Q3(xiii)

The given function f is $f(x)=\begin{cases}-2, & \text{if } x\le -1\\ 2x, & \text{if } -1<x\le 1\\ 2, & \text{if } x>1\end{cases}$

The given function is defined at all points of the real line.

Let c be a point on the real line.

Case I:

$$\text{If } c<-1,\ \text{then } f(c)=-2 \text{ and } \lim_{x\to c} f(x)=\lim_{x\to c}(-2)=-2$$
$$\therefore \lim_{x\to c} f(x)=f(c)$$

Therefore, f is continuous at all points x, such that $x<-1$

Case II:

If $c=-1$, then $f(c)=f(-1)=-2$

The left hand limit of f at $x=-1$ is,

$$\lim_{x\to-1^-} f(x)=\lim_{x\to-1^-}(-2)=-2$$

The right hand limit of f at $x=-1$ is,

$$\lim_{x\to-1^+} f(x)=\lim_{x\to-1^+}(2x)=2\times(-1)=-2$$

$$\therefore \lim_{x\to-1} f(x)=f(-1)$$

Therefore, f is continuous at $x=-1$

Case III:

If $-1<c<1$, then $f(c)=2c$

$$\lim_{x\to c} f(x)=\lim_{x\to c}(2x)=2c$$

$$\therefore \lim_{x\to c} f(x)=f(c)$$

Therefore, f is continuous at all points of the interval $(-1, 1)$.

Case IV:

If $c=1$, then $f(c)=f(1)=2\times1=2$

The left hand limit of f at $x=1$ is,

$$\lim_{x\to1^-} f(x)=\lim_{x\to1^-}(2x)=2\times1=2$$

The right hand limit of f at $x=1$ is,

$$\lim_{x\to1^+} f(x)=\lim_{x\to1^+}2=2$$

$$\therefore \lim_{x\to1} f(x)=f(c)$$

Therefore, f is continuous at $x=2$

Case V:

If $c>1$, then $f(c)=2$ and $\lim_{x\to c} f(x)=\lim_{x\to c}(2)=2$

$$\lim_{x\to c} f(x)=f(c)$$

Therefore, f is continuous at all points x, such that $x>1$

Thus, from the above observations, it can be concluded that f is continuous at all points of the real line

Chapter 9 Continuity Ex 9.2 Q4(i)

We have given that the function is continuous at $x = 0$

$\therefore$ LHL = RHL = $f(0)$ ....(1)

$$\text{LHL} = \lim_{x\to0^-} f(x) = \lim_{h\to0} f(0-h) = \lim_{h\to0}\frac{\sin(-2h)}{5(-h)} = \lim_{h\to0}\frac{-\sin 2h}{-5h} = \lim_{h\to0}\frac{\sin 2h}{2h}\times\frac{2h}{5h} = \frac{2}{5}$$

$f(0) = 3k$

So, using (1) we get,

$$\frac{2}{5} = 3k$$

$$k = \frac{2}{15}$$

Chapter 9 Continuity Ex 9.2 Q4(ii)

It is given that the function is continuous

$\therefore$ LHL = RHL = $f(2)$ ....(1)

LHL = $\lim_{x\to 2^-} f(x) = \lim_{h\to 0} f(2-h) = \lim_{h\to 0} k(2-h)+5 \quad = 2k+5$

RHL = $\lim_{x\to 2^+} f(x) = \lim_{h\to 0} f(2+h) = \lim_{h\to 0} (2+h)-1 = 1$

Thus, using (1), we get,

$2k+5 = 1$

$k = -2$

Chapter 9 Continuity Ex 9.2 Q4(iii)

It is given that the function is continuous

LHL = RHL = $f(0)$....(1)

LHL = $\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} k\left((-h)^2+3(-h)\right) = \lim_{h\to 0} k\left(h^2-3h\right) = 0$

$f(0) = \cos 2\times 0 = \cos 0^\circ = 1$

LHL $\neq f(0)$

Hence, no value of k can make f continuous

Chapter 9 Continuity Ex 9.2 Q4(iv)

First check the continuity of the function at $x = 3$

$f(3) = 2$ .....(A)

RHL = $\lim_{x\to 3^+} f(x) = \lim_{h\to 0} f(3+h) = \lim_{h\to 0} a(3+h)+b \quad = 3a+b$.....(B)

$\therefore$ $f(x)$ will be continuous at $x = 3$ if $3a+b = 2$.....(1)

Now, check the continuity at $x = 5$

$f(5) = 9$ ....(C)

LHL = $\lim_{x\to 5^-} f(x) = \lim_{h\to 0} f(5-h) = \lim_{h\to 0} a(5-h)+b = 5a+b$

$f(x)$ will be continuous at $x = 5$ if $5a+b = 9$....(2)

Solving (1) & (2), we get

$a = \frac{7}{2}$ and $b = \frac{-17}{2}$

Chapter 9 Continuity Ex 9.2 Q4(v)

It is given that the function is continuous

At $x = -1$

$f(-1) = 4$

RHL = $\lim_{x\to -1^+} f(x) = \lim_{h\to 0} f(-1+h) = \lim_{h\to 0} a(-1+h)^2+b = a+b$

Since, $f(x)$ is continuous at $x = -1$

$\therefore$ $a+b = 4$ ....(A)

Now, at $x = 0$,

$f(0) = \cos 0^\circ = 1$

LHL = $\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} a(-h)^2+b = b$

Since, $f(x)$ is continuous at $x = 0$

$\therefore$ $f(0) =$ LHL

$\Rightarrow b = 1$

$\therefore$ from (A)

$a = 3$

Thus, $a = 3$, $b = 1$

Chapter 9 Continuity Ex 9.2 Q4(vi)

It is given that the function is continuous.

At $x = 0$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{\sqrt{1-Ph}-\sqrt{1+Ph}}{-h} = \lim_{h \to 0} \frac{\left(\sqrt{1-Ph}-\sqrt{1+Ph}\right)}{-h} \times \frac{\left(\sqrt{1-Ph}+\sqrt{1+Ph}\right)}{\left(\sqrt{1-Ph}+\sqrt{1+Ph}\right)}$$

$$= \lim_{h \to 0} \frac{(1-Ph)-(1+Ph)}{-h\left(\sqrt{1-Ph}+\sqrt{1+Ph}\right)} = \frac{2P}{2} = P$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{2h+1}{h-2} = \frac{-1}{2}$$

Since, $f(x)$ is continuous so,

$$P = \frac{-1}{2}$$

Chapter 9 Continuity Ex 9.2 Q4(vii)

The given function f is $f(x) = \begin{cases} 5, & \text{if } x \le 2 \\ ax+b, & \text{if } 2 < x < 10 \\ 21, & \text{if } x \ge 10 \end{cases}$

It is evident that the given function f is defined at all points of the real line.

If f is a continuous function, then f is continuous at all real numbers.

In particular, f is continuous at $x = 2$ and $x = 10$

Since f is continuous at $x = 2$, we obtain

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$$
$$\Rightarrow \lim_{x \to 2^-} (5) = \lim_{x \to 2^+} (ax+b) = 5$$
$$\Rightarrow 5 = 2a+b = 5$$
$$\Rightarrow 2a+b = 5 \qquad ...(1)$$

Since f is continuous at $x = 10$, we obtain

$$\lim_{x \to 10^-} f(x) = \lim_{x \to 10^+} f(x) = f(10)$$
$$\Rightarrow \lim_{x \to 10^-} (ax+b) = \lim_{x \to 10^+} (21) = 21$$
$$\Rightarrow 10a+b = 21 = 21$$
$$\Rightarrow 10a+b = 21 \qquad ...(2)$$

On subtracting equation (1) from equation (2), we obtain

$8a = 16$

$a = 2$

By putting $a = 2$ in equation (1), we obtain

$2 \times 2 + b = 5$

$4 + b = 5$

$b = 1$

Therefore, the values of a and b for which f is a continuous function are 2 and 1 respectively.

Chapter 9 Continuity Ex 9.2 Q4(viii)

Since the function is continuous at $x=\frac{\pi}{2}$ therefore

LHL of $f(x)$ at $x=\frac{\pi}{2}$ is

$$=\lim_{x\to\frac{\pi}{2}^-} f(x)$$

$$=\lim_{h\to 0} f\left(h-\frac{\pi}{2}\right)$$

$$=\lim_{h\to 0}\frac{k\cos\left(h-\frac{\pi}{2}\right)}{\pi-2\left(h-\frac{\pi}{2}\right)}$$

$$=\lim_{h\to 0}\frac{k\sinh}{2\pi-2h}$$

$$=\frac{k}{2}\lim_{h\to 0}\frac{\sin(\pi-h)}{(\pi-h)}$$

$$=\frac{k}{2}$$

Again

$$f\left(\frac{\pi}{2}\right)=3$$

Hence

$$LHL=f\left(\frac{\pi}{3}\right)$$

$$\frac{k}{2}=3$$

$$k=6$$

Chapter 9 Continuity Ex 9.2 Q5

We have given that $f(x)$ is continuous on $[0.\infty]$

$\therefore$ $f(x)$ is continuous at $x = 1$ and $x = \sqrt{2}$

$\therefore$ At $x = 1$, LHL = RHL = $f(1)$ .....(A)

$f(1) = a$ .....(1)

LHL = $\lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h) = \lim_{h\to 0}\frac{(1-h)^2}{a} = \frac{1}{a}$

Using (A) we get,

$a = \frac{1}{a} \Rightarrow a^2 = 1 \Rightarrow a = \pm 1$

At $x = \sqrt{2}$ LHL = RHL = $f(\sqrt{2})$ .....(B)

$f(\sqrt{2}) = \frac{2b^2-4b}{(\sqrt{2})^2} = \frac{2b^2-4b}{2} = b^2-2b$ ......(2)

LHL = $\lim_{x\to\sqrt{2}^-} f(x) = \lim_{h\to 0} f(\sqrt{2}-h) = \lim_{h\to 0} a = a.$

So, using (B), we get,

$b^2 - 2b = a$

For $a = 1$, $b^2 - 2b - 1 = 0$

$\Rightarrow b = \frac{2\pm\sqrt{4+4}}{2} = 1\pm\sqrt{2}$

For $a = -1$ $b^2 - 2b + 1 = 0$

$\Rightarrow (b-1)^2 = 0 \Rightarrow b = 1$

Thus, $a = -1$, $b = 1$ or $a = 1$, $b = 1\pm\sqrt{2}$

Chapter 9 Continuity Ex 9.2 Q6

Since, $f(x)$ is continuous on $[0,\pi]$

$f(x)$ is continuous at $x = \frac{\pi}{4}$ and $x = \frac{\pi}{2}$

At $x = \frac{\pi}{4}$,

LHL = RHL = $f\left(\frac{\pi}{4}\right)$.....(A)

Now, $f\left(\frac{\pi}{4}\right) = 2\frac{\pi}{4}.cot\left(\frac{\pi}{4}\right) + b = \frac{\pi}{2}.1 + b = \frac{\pi}{2} + b$ ....(1)

LHL = $\lim_{x\to\frac{\pi}{4}^-} f(x) = \lim_{h\to 0} f\left(\frac{\pi}{4} - h\right) = \lim_{h\to 0}\left(\frac{\pi}{4} - h\right) + a\sqrt{2}\,sin\left(\frac{\pi}{4} - h\right) = \frac{\pi}{4} + a\sqrt{2}.\frac{1}{\sqrt{2}} = \frac{\pi}{4} + a$

Thus, using (A)

$\frac{1}{2} + b = \frac{\pi}{4} + a$

$a - b = \frac{\pi}{4}$.....(B)

At $x = \frac{\pi}{2}$

LHL = RHL = $f\left(\frac{\pi}{2}\right)$.....(C)

Now, $f\left(\frac{\pi}{2}\right) = a\cos 2.\frac{\pi}{2} - b\,sin\frac{\pi}{2} = -a - b$....(2)

LHL = $\lim_{x\to\frac{\pi}{2}^-} f(x) = \lim_{h\to 0} f\left(\frac{\pi}{2} - h\right)$

$= \lim_{h\to 0} 2\left(\frac{\pi}{2} - h\right)cot\left(\frac{\pi}{2} - h\right) + b = \pi \times 0 + b = b$

using (C), we get,

$-a - b = b \quad \Rightarrow \; 2b = -a \quad \Rightarrow \; b = \frac{-a}{2}$

from (B), $a + \frac{a}{2} = \frac{\pi}{4}$

$\Rightarrow \qquad \frac{3}{2}a = \frac{\pi}{4}$

$\Rightarrow \qquad a = \frac{\pi}{6}$

and $b = \frac{-a}{2} = \frac{-\pi}{12}$

Thus, $a = \frac{\pi}{6}$, $b = \frac{-\pi}{12}$

Chapter 9 Continuity Ex 9.2 Q7

It is given that the $f(x)$ is continuous on $[0,8]$

$f(x)$ is continuous at $x = 2$ and $x = 4$.

Now, At $x = 2$

LHL = RHL = $f(2)$....(A)

$f(2) = 3 \times 2 + 2 = 8$....(1)

LHL = $\lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2-h) = \lim_{h \to 0} (2-h)^2 + a(2-h) + b = 4 + 2a + b$

from (A)

$4 + 2a + b = 8$

$2a + b = 4$.....(B)

Now, At $x = 4$

LHL = RHL = $f(4)$....(C)

$f(4) = 3 \times 4 + 2 = 14$.....(2)

RHL = $\lim_{x \to 4^+} f(x) = \lim_{h \to 0} f(4+h) = \lim_{h \to 0} 2a(4+h) + 5b = 8a + 5b$

From (C), we get,

$8a + 5b = 14$....(D)

Solving (B) and (D), we get,

$a = 3$ and $b = -2$

Chapter 9 Continuity Ex 9.2 Q8

The function will be continuous on $\left[0, \frac{\pi}{2}\right]$ if it is continuous at every point in $\left[0, \frac{\pi}{2}\right]$

Let us consider the point $x = \frac{\pi}{4}$,

We must have,

LHL = RHL = $f\left(\frac{\pi}{4}\right)$....(A)

LHL = $\lim_{x \to \frac{\pi}{4}^-} f(x) = \lim_{h \to 0} f\left(\frac{\pi}{4} - h\right) = \lim_{h \to 0} \dfrac{tan\left(\frac{\pi}{4} - \frac{\pi}{4} + h\right)}{cot + 2\left(\frac{\pi}{4} - h\right)} = \lim_{h \to 0} \dfrac{tan\, h}{tan\, 2h}$ $\left[\because cot\left(\frac{\pi}{2} - \theta\right) = tan\,\theta\right]$

$$= \lim_{h \to 0} \frac{\frac{tan\, h}{h}}{\frac{tan\, 2h}{h}} = \frac{1}{2}$$

Thus, using (A) we get,

$f\left(\frac{\pi}{4}\right) = \frac{1}{2}$

Hence, $f(x)$ will be continuous on $\left[0, \frac{\pi}{2}\right]$ if $f\left(\frac{\pi}{4}\right) = \frac{1}{2}$.

Chapter 9 Continuity Ex 9.2 Q9

When $x < 2$, we have

$f(x) = 2x - 1$, which is a polynomial of degree 1.

So, $f(x)$ is continuous for $x < 2$.

When $x > 2$, we have

$f(x) = \frac{3x}{2}$, which is again a polynomial of degree 1.

So, $f(x)$ is continuous for $x > 2$.

Now, consider the point $x = 2$

$f(2) = \frac{3 \times 2}{2} = 3$

LHL $= \lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2-h) = \lim_{h \to 0} 2(2-h) - 1 = 3$

RHL $= \lim_{x \to 2^+} f(x) = \lim_{h \to 0} f(2+h) = \lim_{h \to 0} \frac{3(2+h)}{2} = 3$

LHL = RHL $= f(2) = 3$

Thus, $f(x)$ is continuous at $x = 2$

Hence, $f(x)$ is continuous every where.

Chapter 9 Continuity Ex 9.2 Q10

Let $f(x) = \sin|x|$

This function f is defined for every real number and f can be written as the composition of two functions as,

$f = g\ o\ h$, where $g(x) = |x|$ and $h(x) = \sin x$

$\left[\because (goh)(x) = g(h(x)) = g(\sin x) = |\sin x| = f(x)\right]$

It has to be proved first that $g(x) = |x|$ and $h(x) = \sin x$ are continuous functions.

$g(x) = |x|$ can be written as

$$g(x) = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

Clearly, g is defined for all real numbers.

Let c be a real number.

Case I:

If $c < 0$, then $g(c) = -c$ and $\lim_{x \to c} g(x) = \lim_{x \to c}(-x) = -c$

$\therefore \lim_{x \to c} g(x) = g(c)$

Therefore, g is continuous at all points x, such that $x < 0$

Case II:

If $c > 0$, then $g(c) = c$ and $\lim_{x \to c} g(x) = \lim_{x \to c} x = c$

$\therefore \lim_{x \to c} g(x) = g(c)$

Therefore, g is continuous at all points x, such that $x > 0$

Case III:

If $c=0$, then $g(c)=g(0)=0$

$$\lim_{x\to 0^-} g(x)=\lim_{x\to 0^-}(-x)=0$$
$$\lim_{x\to 0^+} g(x)=\lim_{x\to 0^+}(x)=0$$
$$\therefore \lim_{x\to 0^-} g(x)=\lim_{x\to 0^+}(x)=g(0)$$

Therefore, g is continuous at $x=0$

From the above three observations, it can be concluded that g is continuous at all points.

$h(x)=\sin x$

It is evident that $h(x)=\sin x$ is defined for every real number.

Let c be a real number. Put $x=c+k$

If $x\to c$, then $k\to 0$

$h(c)=\sin c$

$$\begin{aligned}
h(c)&=\sin c\\
\lim_{x\to c} h(x)&=\lim_{x\to c}\sin x\\
&=\lim_{k\to 0}\sin(c+k)\\
&=\lim_{k\to 0}[\sin c\cos k+\cos c\sin k]\\
&=\lim_{k\to 0}(\sin c\cos k)+\lim_{h\to 0}(\cos c\sin k)\\
&=\sin c\cos 0+\cos c\sin 0\\
&=\sin c+0\\
&=\sin c
\end{aligned}$$
$$\therefore \lim_{x\to c} h(x)=g(c)$$

Therefore, h is a continuous function.

It is known that for real valued functions g and h,such that $(g \circ h)$ is defined at c, if g is continuous at c and if f is continuous at $g(c)$, then $(f \circ g)$ is continuous at c.

Therefore, $f(x)=(goh)(x)=g(h(x))=g(\sin x)=|\sin x|$ is a continuous function.

Chapter 9 Continuity Ex 9.2 Q11

When $x < 0$, we have,

$f(x) = \frac{\sin x}{x}$

We know that the $\sin x$ and the identity function x are continuous for $x < 0$.

So, the quotient function $f(x) = \frac{\sin x}{x}$ is continuous for $x < 0$.

When $x > 0$, we have,

$f(x) = x + 1$, which is a polynomial of degree 1. So, $f(x)$ is continuous for $x > 0$

Now, consider the point $x = 0$.

$f(0) = 0 + 1 = 1$.

LHL $= \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{\sin(-h)}{-h} = \lim_{h \to 0} \frac{-\sin h}{-h} = 1$

RHL $= \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} h + 1 = 1$

Thus, LHL = RHL $= f(0) = 1$

So, $f(x)$ is continuous at $x = 0$.

Hence, $f(x)$ is continuous everywhere

Chapter 9 Continuity Ex 9.2 Q12

The given function is $g(x) = x - [x]$

It is evident that g is defined at all integral points.

Let n be an integer.

Then,

$g(n) = n - [n] = n - n = 0$

The left hand limit of f at $x = n$ is,

$\lim_{x \to n^-} g(x) = \lim_{x \to n^-} (x - [x]) = \lim_{x \to n^-} (x) - \lim_{x \to n^-} [x] = n - (n-1) = 1$

The right hand limit of f at $x = n$ is,

$\lim_{x \to n^+} g(x) = \lim_{x \to n^+} (x - [x]) = \lim_{x \to n^+} (x) - \lim_{x \to n^+} [x] = n - n = 0$

It is observed that the left and right hand limits of f at $x = n$ do not coincide.

Therefore, f is not continuous at $x = n$

Hence, g is discontinuous at all integral points

Chapter 9 Continuity Ex 9.2 Q13

It is known that if g and h are two continuous functions, then
$g+h,\ g-h,$ and $g.h$ are also continuous.

It has to proved first that $g\ (x) = \sin x$ and $h\ (x) = \cos x$ are continuous functions.

Let $g\ (x) = \sin x$

It is evident that $g\ (x) = \sin x$ is defined for every real number.

Let c be a real number. Put $x = c + h$

If $x \to c$, then $h \to 0$

$$g(c) = \sin c$$

$$\begin{aligned}\lim_{x\to c} g(x) &= \lim_{x\to c} \sin x \\ &= \lim_{h\to 0} \sin(c+h) \\ &= \lim_{h\to 0}[\sin c \cos h + \cos c \sin h] \\ &= \lim_{h\to 0}(\sin c \cos h) + \lim_{h\to 0}(\cos c \sin h) \\ &= \sin c \cos 0 + \cos c \sin 0 \\ &= \sin c + 0 \\ &= \sin c\end{aligned}$$

$$\therefore \lim_{x\to c} g(x) = g(c)$$

Therefore, g is a continuous function.

Let $h\ (x) = \cos x$

It is evident that $h\ (x) = \cos x$ is defined for every real number.

Let c be a real number. Put $x = c + h$

If $x \to c$, then $h \to 0$

$h\ (c) = \cos c$

$$\begin{aligned}\lim_{x\to c} h(x) &= \lim_{x\to c} \cos x \\ &= \lim_{h\to 0} \cos(c+h) \\ &= \lim_{h\to 0}[\cos c \cos h - \sin c \sin h] \\ &= \lim_{h\to 0} \cos c \cos h - \lim_{h\to 0} \sin c \sin h \\ &= \cos c \cos 0 - \sin c \sin 0 \\ &= \cos c \times 1 - \sin c \times 0 \\ &= \cos c\end{aligned}$$

$$\therefore \lim_{x\to c} h(x) = h(c)$$

Therefore, h is a continuous function.

Therefore, it can be concluded that

(a) $f\ (x) = g\ (x) + h\ (x) = \sin x + \cos x$ is a continuous function

(b) $f\ (x) = g\ (x) - h\ (x) = \sin x - \cos x$ is a continuous function

(c) $f\ (x) = g\ (x) \times h\ (x) = \sin x \times \cos x$ is a continuous function

Chapter 9 Continuity Ex 9.2 Q14

The given function is $f(x) = \cos(x^2)$

This function f is defined for every real number and f can be written as the composition of two functions as,

$f = g \circ h$, where $g(x) = \cos x$ and $h(x) = x^2$

$$\left[\because (goh)(x) = g(h(x)) = g(x^2) = \cos(x^2) = f(x)\right]$$

It has to be first proved that $g(x) = \cos x$ and $h(x) = x^2$ are continuous functions.

It is evident that g is defined for every real number.

Let c be a real number.

Then, $g(c) = \cos c$

Put $x = c + h$

If $x \to c$, then $h \to 0$

$$\begin{aligned}
\lim_{x\to c} g(x) &= \lim_{x\to c} \cos x \\
&= \lim_{h\to 0} \cos(c+h) \\
&= \lim_{h\to 0}\left[\cos c \cos h - \sin c \sin h\right] \\
&= \lim_{h\to 0} \cos c \cos h - \lim_{h\to 0} \sin c \sin h \\
&= \cos c \cos 0 - \sin c \sin 0 \\
&= \cos c \times 1 - \sin c \times 0 \\
&= \cos c
\end{aligned}$$

$$\therefore \lim_{x\to c} g(x) = g(c)$$

Therefore, $g(x) = \cos x$ is continuous function.

$h(x) = x^2$

Clearly, h is defined for every real number.

Let k be a real number, then $h(k) = k^2$

$$\lim_{x\to k} h(x) = \lim_{x\to k} x^2 = k^2$$

$$\therefore \lim_{x\to k} h(x) = h(k)$$

Therefore, h is a continuous function.

It is known that for real valued functions g and h, such that $(g \circ h)$ is defined at c, if g is continuous at c and if f is continuous at $g(c)$, then $(f \circ g)$ is continuous at c.

Therefore, $f(x) = (goh)(x) = \cos(x^2)$ is a continuous function.

Chapter 9 Continuity Ex 9.2 Q15

The given function is $f(x) = |\cos x|$

This function f is defined for every real number and f can be written as the composition of two functions as,

$f = g \circ h$, where $g(x) = |x|$ and $h(x) = \cos x$

$$\left[\because (goh)(x) = g(h(x)) = g(\cos x) = |\cos x| = f(x)\right]$$

It has to be first proved that $g(x) = |x|$ and $h(x) = \cos x$ are continuous functions.

$g(x) = |x|$ can be written as

$$g(x) = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

Clearly, g is defined for all real numbers.

Let c be a real number.

Case I:

$$\text{If } c<0, \text{ then } g(c)=-c \text{ and } \lim_{x\to c} g(x)=\lim_{x\to c}(-x)=-c$$
$$\therefore \lim_{x\to c} g(x)=g(c)$$

Therefore, g is continuous at all points x, such that $x < 0$

Case II:

$$\text{If } c>0, \text{ then } g(c)=c \text{ and } \lim_{x\to c} g(x)=\lim_{x\to c} x=c$$
$$\therefore \lim_{x\to c} g(x)=g(c)$$

Therefore, g is continuous at all points x, such that $x > 0$

Case III:

$$\text{If } c=0, \text{ then } g(c)=g(0)=0$$

$$\lim_{x\to 0^-} g(x)=\lim_{x\to 0^-}(-x)=0$$
$$\lim_{x\to 0^+} g(x)=\lim_{x\to 0^+}(x)=0$$
$$\therefore \lim_{x\to 0^-} g(x)=\lim_{x\to 0^+}(x)=g(0)$$

Therefore, g is continuous at $x = 0$

From the above three observations, it can be concluded that g is continuous at all points.

$h(x) = \cos x$

It is evident that $h(x) = \cos x$ is defined for every real number.

Let c be a real number. Put $x = c + h$

If $x \to c$, then $h \to 0$

$h(c) = \cos c$

$$\begin{aligned}
\lim_{x\to c} h(x) &= \lim_{x\to c} \cos x \\
&= \lim_{h\to 0} \cos(c+h) \\
&= \lim_{h\to 0} [\cos c \cos h - \sin c \sin h] \\
&= \lim_{h\to 0} \cos c \cos h - \lim_{h\to 0} \sin c \sin h \\
&= \cos c \cos 0 - \sin c \sin 0 \\
&= \cos c \times 1 - \sin c \times 0 \\
&= \cos c
\end{aligned}$$
$$\therefore \lim_{x\to c} h(x) = h(c)$$

Therefore, $h(x) = \cos x$ is a continuous function.

It is known that for real valued functions g and h,such that $(g \circ h)$ is defined at c, if g is continuous at c and if f is continuous at $g(c)$, then $(f \circ g)$ is continuous at c.

Therefore, $f(x)=(goh)(x)=g(h(x))=g(\cos x)=|\cos x|$ is a continuous function

Chapter 9 Continuity Ex 9.2 Q16

The given function is $f(x)=|x|-|x+1|$

The two functions, g and h, are defined as

$g(x)=|x|$ and $h(x)=|x+1|$

Then, $f = g - h$

The continuity of g and h is examined first.

$g(x)=|x|$ can be written as

$$g(x)=\begin{cases}-x, & \text{if } x<0\\ x, & \text{if } x\geq 0\end{cases}$$

Clearly, g is defined for all real numbers.

Let c be a real number.

Case I:

If $c<0$, then $g(c)=-c$ and $\lim\limits_{x\to c} g(x)=\lim\limits_{x\to c}(-x)=-c$

$\therefore \lim\limits_{x\to c} g(x)=g(c)$

Therefore, g is continuous at all points x, such that $x < 0$

Case II:

If $c>0$, then $g(c)=c$ and $\lim\limits_{x\to c} g(x)=\lim\limits_{x\to c} x=c$

$\therefore \lim\limits_{x\to c} g(x)=g(c)$

Therefore, g is continuous at all points x, such that $x > 0$

Case III:

If $c=0$, then $g(c)=g(0)=0$

$$\lim_{x\to 0^-} g(x)=\lim_{x\to 0^-}(-x)=0$$
$$\lim_{x\to 0^+} g(x)=\lim_{x\to 0^+}(x)=0$$
$$\therefore \lim_{x\to 0^-} g(x)=\lim_{x\to 0^+}(x)=g(0)$$

Therefore, g is continuous at $x=0$

From the above three observations, it can be concluded that g is continuous at all points.

$h(x)=|x+1|$ can be written as

$$h(x)=\begin{cases}-(x+1), & \text{if, } x<-1\\ x+1, & \text{if } x\geq -1\end{cases}$$

Clearly, h is defined for every real number.

Let c be a real number.

Case I:

If $c<-1$, then $h(c)=-(c+1)$ and $\lim_{x\to c} h(x)=\lim_{x\to c}[-(x+1)]=-(c+1)$
$\therefore \lim_{x\to c} h(x)=h(c)$

Therefore, h is continuous at all points x, such that $x<-1$

Case II:

If $c>-1$, then $h(c)=c+1$ and $\lim_{x\to c} h(x)=\lim_{x\to c}(x+1)=c+1$
$\therefore \lim_{x\to c} h(x)=h(c)$

Therefore, h is continuous at all points x, such that $x>-1$

Case III:

If $c=-1$, then $h(c)=h(-1)=-1+1=0$

$$\lim_{x\to -1^-} h(x)=\lim_{x\to -1^-}[-(x+1)]=-(-1+1)=0$$

$$\lim_{x\to -1^+} h(x)=\lim_{x\to -1^+}(x+1)=(-1+1)=0$$
$$\therefore \lim_{x\to -1^-} h(x)=\lim_{h\to -1^+} h(x)=h(-1)$$

Therefore, h is continuous at $x=-1$

From the above three observations, it can be concluded that h is continuous at all points of the real line.

g and h are continuous functions. Therefore, $f=g-h$ is also a continuous function.

Therefore, f has no point of discontinuity.

Chapter 9 Continuity Ex 9.2 Q17

He given function f is $f(x)=\begin{cases} x^2\sin\frac{1}{x}, & \text{if } x\neq 0 \\ 0, & \text{if } x=0 \end{cases}$

It is evident that f is defined at all points of the real line.

Let c be a real number.

Case I:

If $c\neq 0$, then $f(c)=c^2\sin\frac{1}{c}$

$$\lim_{x\to c} f(x)=\lim_{x\to c}\left(x^2\sin\frac{1}{x}\right)=\left(\lim_{x\to c} x^2\right)\left(\lim_{x\to c}\sin\frac{1}{x}\right)=c^2\sin\frac{1}{c}$$
$$\therefore \lim_{x\to c} f(x)=f(c)$$

Therefore, f is continuous at all points $x\neq 0$

Case II:

If $c=0$, then $f(0)=0$

$$\lim_{x\to0^-} f(x)=\lim_{x\to0^-}\left(x^2\sin\frac{1}{x}\right)=\lim_{x\to0}\left(x^2\sin\frac{1}{x}\right)$$

It is known that, $-1\le\sin\frac{1}{x}\le1,\ x\ne0$

$$\Rightarrow -x^2\le\sin\frac{1}{x}\le x^2$$

$$\Rightarrow \lim_{x\to0}\left(-x^2\right)\le\lim_{x\to0}\left(x^2\sin\frac{1}{x}\right)\le\lim_{x\to0}x^2$$

$$\Rightarrow 0\le\lim_{x\to0}\left(x^2\sin\frac{1}{x}\right)\le0$$

$$\Rightarrow \lim_{x\to0}\left(x^2\sin\frac{1}{x}\right)=0$$

$$\therefore \lim_{x\to0^-} f(x)=0$$

Similarly, $\lim_{x\to0^+} f(x)=\lim_{x\to0^+}\left(x^2\sin\frac{1}{x}\right)=\lim_{x\to0}\left(x^2\sin\frac{1}{x}\right)=0$

$$\therefore \lim_{x\to0^-} f(x)=f(0)=\lim_{x\to0^+} f(x)$$

Therefore, f is continuous at $x=0$

From the above observations, it can be concluded that f is continuous at every point of the real line.

Thus, f is a continuous function.

Chapter 9 Continuity Ex 9.2 Q18

$$f(x)=\frac{1}{x+2}$$

$$\lim_{x\to-2^-} f(x)=\lim_{h\to0}\frac{1}{-2-h+2}=\lim_{h\to0}-\frac{1}{h}\to-\infty$$

$$\lim_{x\to-2^+} f(x)=\lim_{h\to0}\frac{1}{-2+h+2}=\lim_{h\to0}\frac{1}{h}\to\infty$$

$\therefore$ $f(x)$ is discontinuous at $x = -2$

Let $g(x)=f(f(x))=\dfrac{x+2}{2x+5}$

$$\lim_{x\to-\frac{5}{2}^-} g(x)=\lim_{h\to0}\frac{-\frac{5}{2}-h+2}{-5-h+5}=\lim_{h\to0}-\frac{-\frac{5}{2}-h+2}{h}\to-\infty$$

$$\lim_{x\to-\frac{5}{2}^+} f(x)=\lim_{h\to0}\frac{-\frac{5}{2}+h+2}{-5+h+5}=\lim_{h\to0}\frac{-\frac{5}{2}-h+2}{h}\to\infty$$

$\therefore$ $g(x)$ is discontinuous at $x = -\frac{5}{2}$

$\therefore$ $f(f(x))$ is discontinuous at $x = -\frac{5}{2}$

$\therefore$ $f(x)$ is discontinuous at $x = -2$ and $-\frac{5}{2}$.

Chapter 9 Continuity Ex 9.2 Q19

$f(t) = \frac{1}{t^2 + t - 2}$, where $t = \frac{1}{x - 1}$

Clearly $t = \frac{1}{x-1}$ is discontinuous at $x = 1$.

For $x \neq 1$, we have

$f(t) = \frac{1}{t^2 + t - 2} = \frac{1}{(t+2)(t-1)}$

This is discontinuous at $t = -2$ and $t = 1$

For $t = -2$, $t = \frac{1}{x - 1} \Rightarrow x = \frac{1}{2}$

For $t = 1$, $t = \frac{1}{x - 1} \Rightarrow x = 2$

Hence f is discontinuous at $x = \frac{1}{2}$, $x = 1$ and $x = 2$.

Ex 10.1

Chapter 10 Differentiability Ex 10.1 Q1

$$f(x) = |x-3|$$
$$= \begin{cases} -(x-3), & \text{if } x < 3 \\ x-3, & \text{if } x \ge 3 \end{cases}$$
$$f(3) = 3 - 3 = 0$$

$$\text{LHL} = \lim_{x \to 3^-} f(x)$$
$$= \lim_{h \to 0} f(3-h)$$
$$= \lim_{h \to 0} 3 - (3-h)$$
$$= \lim_{h \to 0} 0$$

$$\text{RHL} = \lim_{x \to 3^+} f(x)$$
$$= \lim_{h \to 0} f(3+h)$$
$$= \lim_{h \to 0} 3 + h - 3$$
$$= 0$$

$$\text{LHL} = f(3) = \text{RHL}$$
$\therefore$ $f(x)$ is continuous at $x = 3$

$$(\text{LHD at } x = 3) = \lim_{x \to 3^-} \frac{f(x) - f(3)}{x - 3}$$
$$= \lim_{h \to 0} \frac{f(3-h) - f(3)}{3 - h - 3}$$
$$= \lim_{h \to 0} \frac{3 - (3-h) - 0}{-h}$$
$$= \lim_{h \to 0} \frac{h}{-h}$$
$$= -1$$

$$(\text{RHD at } x = 3) = \lim_{x \to 3^+} \frac{f(x) - f(3)}{x - 3}$$
$$= \lim_{h \to 0} \frac{f(3+h) - f(3)}{3 + h - 3}$$
$$= \lim_{h \to 0} \frac{3 + h - 3 - 0}{h}$$
$$= \lim_{h \to 0} \frac{h}{h}$$
$$= 1$$

$$(\text{LHD at } x = 3) \ne (\text{RHD at } x = 3)$$
$\therefore$ $f(x)$ is continuous but not differentiable at $x = 3$.

Chapter 10 Differentiability Ex 10.1 Q2

$$f(x) = x^{\frac{1}{3}}$$

$$(\text{LHD at } x = 0) = \lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0}$$
$$= \lim_{h \to 0} \frac{f(0-h) - f(0)}{0 - h - 0}$$
$$= \lim_{h \to 0} \frac{(-h)^{\frac{1}{3}} - 0}{-h}$$
$$= \lim_{h \to 0} \frac{(-h)^{\frac{1}{3}}}{-h}$$
$$= \lim_{h \to 0} \frac{(-1)^{\frac{1}{3}} h^{\frac{1}{3}}}{(-1)h}$$
$$= \lim_{h \to 0} (-1)^{\frac{-2}{3}} h^{\frac{-2}{3}}$$
$$= \text{Not defined}$$

$$(\text{RHD at } x = 0) = \lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0}$$
$$= \lim_{h \to 0} \frac{f(0+h) - f(0)}{0 + h - 0}$$
$$= \lim_{h \to 0} \frac{h^{\frac{1}{3}} - 0}{h}$$
$$= \lim_{h \to 0} h^{\frac{-2}{3}}$$
$$= \text{Not defined}$$

Since,

LHD and RHD does not exists at $x = 0$

$\therefore \quad f(x)$ is not differentiable at $x = 0$

Chapter 10 Differentiability Ex 10.1 Q3

$$f(x) = \begin{cases} 12x - 13, & \text{if } x \le 3 \\ 2x^2 + 5, & \text{if } x > 3 \end{cases}$$

$$(\text{LHD at } x = 3) = \lim_{x \to 3^-} \frac{f(x) - f(3)}{x - 3}$$
$$= \lim_{h \to 0} \frac{f(3-h) - f(3)}{3 - h - 3}$$
$$= \lim_{h \to 0} \frac{[12(3-h) - 13] - [12(3) - 13]}{-h}$$
$$= \lim_{h \to 0} \frac{36 - 12h - 13 - 36 + 13}{-h}$$
$$= \lim_{h \to 0} \frac{-12h}{-h}$$
$$= 12$$

$$(\text{RHD at } x = 3) = \lim_{x \to 3^+} \frac{f(x) - f(3)}{x - 3}$$
$$= \lim_{h \to 0} \frac{f(3+h) - f(3)}{x - 3}$$
$$= \lim_{h \to 0} \frac{\left[2\left(3 + h^2\right) + 5\right] - [12(3) - 13]}{3 + h - 3}$$
$$= \lim_{h \to 0} \frac{18 + 12h + 2h^2 + 5 - 36 + 13}{h}$$
$$= \lim_{h \to 0} \frac{2h^2 + 12h}{h}$$
$$= \lim_{h \to 0} \frac{h(2h + 12)}{h}$$
$$= 12$$

Now,

$(\text{LHD at } x = 3) = (\text{RHD at } x = 3)$

$\therefore \quad f(x)$ is differentiable at $x = 3$

$\quad f'(x) = 12$

Chapter 10 Differentiability Ex 10.1 Q4

$$f(x) = \begin{cases} 3x - 2 & , \ 0 < x \le 1 \\ 2x^2 - x & , \ 1 < x \le 2 \\ 5x - 4 & , \ x > 2 \end{cases}$$

$$f(2) = 2(2)^2 - 2$$
$$= 8 - 2 = 6$$

$$\text{LHL} = \lim_{x \to 2^-} f(x)$$
$$= \lim_{h \to 0} f(2 - h)$$
$$= \lim_{h \to 0} \left[2(2 - h)^2 - (2 - h)\right]$$
$$= 8 - 2$$
$$= 6$$

$$\text{RHL} = \lim_{x \to 2^+} f(x)$$
$$= \lim_{h \to 0} f(2 + h)$$
$$= \lim_{h \to 0} 5(2 + h) - 4$$
$$= 6$$

$$\text{LHL} = f(2) = \text{RHL}$$

$f(x)$ is continuous at $x = 2$

$$(\text{LHD at } x = 2) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$$
$$= \lim_{h \to 0} \frac{f(2 - h) - f(2)}{2 - h - 2}$$
$$= \lim_{h \to 0} \frac{\left[2(2 - h)^2 - (2 - h)\right] - [8 - 2]}{-h}$$
$$= \lim_{h \to 0} \frac{8 - 8h + 2h^2 - h - 6}{-h}$$
$$= \lim_{h \to 0} \frac{2h^2 - 6h}{-h}$$
$$= \lim_{h \to 0} \frac{h(2h - 6)}{-h}$$
$$= \lim_{h \to 0} (6 - 2h)$$
$$= 6$$

$$(\text{RHD at } x = 2) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2}$$
$$= \lim_{h \to 0} \frac{f(2 + h) - f(2)}{2 + h - 2}$$
$$= \lim_{h \to 0} \frac{[5(2 + h) - 4] - [8 - 2]}{h}$$
$$= \lim_{h \to 0} \frac{10 + 5h - 4 - 6}{h}$$
$$= 5$$

Chapter 10 Differentiability Ex 10.1 Q5

$f(x) = |x|+|x-1|$ in theinterval $(-1, 2)$.

$$f(x) = \begin{cases} x + x + 1 & -1 < x < 0 \\ 1 & 0 \le x \le 1 \\ -x - x + 1 & 1 < x < 2 \end{cases}$$

$$f(x) = \begin{cases} 2x + 1 & -1 < x < 0 \\ 1 & 0 \le x \le 1 \\ -2x + 1 & 1 < x < 2 \end{cases}$$

We know that a polynomial and a constant function is continuous and differentiable everywhere.
So, $f(x)$ is continuous and differentiable for $x \in (-1, 0)$, $x \in (0, 1)$ and $(1, 2)$.

We need to check continuity and differentiability at $x = 0$ and $x = 1$.

Continuity at $x = 0$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} 2x + 1 = 1$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} 1 = 1$$

$$f(0) = 1$$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$$
$\therefore$ $f(x)$ is continuous at $x = 0$.

Continuity at $x = 1$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} 1 = 1$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 1 = 1$$

$$f(1) = 1$$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(1)$$
$\therefore$ $f(x)$ is continuous at $x = 1$.

Differentiability at $x = 0$
$$(\text{LHD at } x = 0) = \lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^-} \frac{2x + 1 - 1}{x - 0} = \lim_{x \to 0^-} \frac{2x}{x} = 2$$
$$(\text{RHD at } x = 0) = \lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^+} \frac{1 - 1}{x} = \lim_{x \to 0^+} \frac{0}{x} = 0$$

$\therefore$ $(\text{LHD at } x = 0) \ne (\text{RHD at } x = 0)$
So, $f(x)$ is differentiable at $x = 0$.

Differentiability at $x = 1$
$$(\text{LHD at } x = 1) = \lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^-} \frac{1 - 1}{x - 1} = 0$$
$$(\text{RHD at } x = 1) = \lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^+} \frac{-2x + 1 - 1}{x - 1} \to \infty$$

$\therefore$ $(\text{LHD at } x = 1) \ne (\text{RHD at } x = 1)$
So, $f(x)$ is not differentiable at $x = 1$.

So, $f(x)$ is continuous on $(-1, 2)$ but not differentiable at $x = 0, 1$.

Chapter 10 Differentiability Ex 10.1 Q6

$$f(x)=\begin{cases} x, & x\le 1 \\ 2-x, & 1\le x\le 2 \\ -2+3x-x^2, & x>2 \end{cases}$$

Differentiability at x = 1

$$(\text{LHD at } x=1)=\lim_{x\to1^-}\frac{f(x)-f(1)}{x-1}=\lim_{x\to1^-}\frac{x-1}{x-1}=1$$

$$(\text{RHD at } x=1)=\lim_{x\to1^+}\frac{f(x)-f(1)}{x-1}=\lim_{x\to1^+}\frac{2-x-1}{x-1}=\lim_{x\to1^+}\frac{1-x}{x-1}=-1$$

$\therefore$ (LHD at x = 1) $\neq$ (RHD at x = 1)

So, $f(x)$ is not differentiable at x = 1.

Differentiability at x = 2

$$(\text{LHD at } x=2)=\lim_{x\to2^-}\frac{f(x)-f(2)}{x-2}=\lim_{x\to2^-}\frac{2-x-0}{x-2}=\lim_{x\to2^-}\frac{2-x}{x-2}=-1$$

$$(\text{RHD at } x=2)=\lim_{x\to2^+}\frac{f(x)-f(2)}{x-2}=\lim_{x\to2^+}\frac{-2+3x-x^2-0}{x-2}=\lim_{x\to2^+}\frac{(1-x)(x-2)}{x-2}=-1$$

$\therefore$ (LHD at x = 2) = (RHD at x = 2)

So, $f(x)$ is differentiable at x = 2.

Chapter 10 Differentiability Ex 10.1 Q7(i)

$$f(x)=\begin{cases} x^m \sin\left(\frac{1}{x}\right), & x\ne 0 \\ 0, & x=0 \end{cases}$$

$$(\text{LHD at } x=0)=\lim_{x\to0^-}\frac{f(x)-f(0)}{x-0}$$

$$=\lim_{h\to0}\frac{f(0-h)-f(0)}{(0-h)-0}$$

$$=\lim_{h\to0}\frac{(0-h)^m \sin\left(\frac{1}{-h}\right)-0}{-h}$$

$$=\lim_{h\to0}\frac{(-h)^m \sin\left(-\frac{1}{h}\right)}{-h}$$

$$=\lim_{h\to0}(-h)^{m-1}\sin\left(-\frac{1}{h}\right)$$

$$=\lim_{h\to0}-(-h)^{m-1}\sin\left(\frac{1}{h}\right)$$

$$=0\times k \qquad [\text{When } -1\le k\le 1]$$

$$=0$$

$$(\text{RHD at } x=0)=\lim_{x\to0^+}\frac{f(x)-f(0)}{x-0}$$

$$=\lim_{h\to0}\frac{f(0+h)-f(0)}{(0+h)-0}$$

$$=\lim_{h\to0}\frac{h^m \sin\left(\frac{1}{h}\right)-0}{h}$$

$$=\lim_{h\to0}\left(h^{m-1}\right)\sin\left(\frac{1}{h}\right)$$

$$=0\times k' \qquad [\text{Since } -1\le k'\le 1]$$

$$=0$$

(LHD at $x=0$) = (RHD at $x=0$)

$\therefore$ $f(x)$ is differentiable at $x=0$

Chapter 10 Differentiability Ex 10.1 Q7(ii)

LHL $= \lim_{x\to 0^-} f(x)$

$= \lim_{h\to 0} f(0-h)$

$= \lim_{h\to 0} (-h)^m \sin\left(-\frac{1}{h}\right)$

$= -\lim_{h\to 0} (-h)^m \sin\left(\frac{1}{h}\right)$

$= 0 \times k$ $\qquad$ $[\text{When } -1 \le k \le 1]$

$= 0$

RHL $= \lim_{x\to 0^+} f(x)$

$= \lim_{h\to 0} f(0+h)$

$= \lim_{h\to 0} (0+h)^m \sin\frac{1}{(0+h)}$

$= \lim_{h\to 0} h^m \sin\left(\frac{1}{h}\right)$

$= 0 \times k'$ $\qquad$ $[\text{Where } -1 \le k' \le 1]$

$= 0$

LHL $= f(0) =$ RHL

$\therefore$ $f(x)$ is continuons at $x = 0$

For differentiablility at $x = 0$

$(\text{LHD at } x = 0) = \lim_{x\to 0^{-1}} \frac{f(x) - f(0)}{x - 0}$

$= \lim_{h\to 0} \frac{(0-h) - f(0)}{(0-h) - 0}$

$= \lim_{h\to 0} \frac{(-h)^m \sin\left(-\frac{1}{h}\right)}{-h}$

$= \lim_{h\to 0} - (-h)^{m-1} \sin\left(\frac{1}{h}\right)$

$=$ Not definded $\qquad$ $[\text{Since } 0 < m < 1]$

$(\text{RHD at } x = 0) = \lim_{x\to 0^+} \frac{f(x) - f(0)}{x - 0}$

$= \lim_{h\to 0} \frac{f(0+h) - f(0)}{0 + h - 0}$

$= \lim_{h\to 0} \frac{h^m \sin\left(\frac{1}{h}\right)}{h}$

Chapter 10 Differentiability Ex 10.1 Q7(iii)

LHL $= \lim_{x\to 0^-} f(x)$

$= \lim_{h\to 0} f(0-h)$

$= \lim_{h\to 0} (-h)^m \sin\left(-\frac{1}{h}\right)$

= Not defined as $m \le 0$

RHL $= \lim_{x\to 0^+} f(x)$

$= \lim_{h\to 0} f(0+h)$

$= \lim_{h\to 0} h^m \sin\left(\frac{1}{h}\right)$

= Not defined, as $m \le 0$

Since RHL and LHL are not difined, so $f(x)$ is not continuous

Let $x = 0$ for $m \le 0$.

Now,

$(\text{LHD at } x = 0) = \lim_{x\to 0^-} \frac{f(x) - f(0)}{x - 0}$

$= \lim_{h\to 0} \frac{f(0-h) - 0}{0 - h - 0}$

$= \lim_{h\to 0} \frac{(-h)^m \sin\left(-\frac{1}{h}\right)}{-h}$

$= \lim_{h\to 0} - (-h)^{m-1} \sin\left(\frac{1}{h}\right)$

= Not defined, as $m \le 0$

RHD $= \lim_{x\to 0^+} \frac{f(x) - f(0)}{x - 0}$

$= \lim_{h\to 0} \frac{f(0+h) - f(0)}{0 + h - 0}$

$= \lim_{h\to 0} \frac{h^m \sin\left(\frac{1}{h}\right)}{h}$

$= \lim_{h\to 0} \left(h^{m-1}\right) \sin\left(\frac{1}{h}\right)$

= Not defined, as $m \le 0$

Thus,

$f(x)$ is neither continuous not differentiable at $x = 0$ for $m \le 0$.

Chapter 10 Differentiability Ex 10.1 Q8

$$f(x) = \begin{cases} x^2 + 3x + a, & \text{if } x \le 1 \\ bx + 2 \quad , & \text{if } x > 1 \end{cases}$$

$$\begin{aligned}
(\text{LHD at } x = 1) &= \lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} \\
&= \lim_{h \to 0} \frac{f(1-h) - f(1)}{1 - h - 1} \\
&= \lim_{h \to 0} \frac{\left[(1-h)^2 + 3(1-h) + a\right] - [1 + 3 + a]}{-h} \\
&= \lim_{h \to 0} \frac{h^2 - 5h}{-h} \\
&= -5
\end{aligned}$$

$$\begin{aligned}
(\text{RHD at } x = 1) &= \lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1} \\
&= \lim_{h \to 0} \frac{f(1+h) - f(1)}{1 + h - 1} \\
&= \lim_{h \to 0} \frac{[b(1+h) + 2] - (b + 2)}{h} \\
&= \lim_{h \to 0} \frac{b + bh + 2 - b - 2}{h} \\
&= b
\end{aligned}$$

Since $f(x)$ is differentiable, so

$$(\text{LHD at } x = 1) = (\text{RHD at } x = 1)$$

$$5 = b$$

$$\begin{aligned}
f(1) &= 1 + 3 + a \\
&= 4 + a
\end{aligned}$$

$$\begin{aligned}
\text{LHL} \quad &= \lim_{x \to 1^{-1}} f(x) \\
&= \lim_{h \to 0} f(1-h) \\
&= \lim_{h \to 0} (1-h)^2 + 3(1-h) + a \\
&= 4 + a
\end{aligned}$$

$$\begin{aligned}
\text{RHL} \quad &= \lim_{x \to 1^+} f(x) \\
&= \lim_{h \to 0} f(1+h) \\
&= \lim_{h \to 0} b(1+b) + 2 \\
&= b + 2
\end{aligned}$$

Chapter 10 Differentiability Ex 10.1 Q9

$$f(x) = \begin{cases} |2x-3|\,[x], & x \ge 1 \\ \sin\left(\frac{\pi x}{2}\right), & x < 1 \end{cases}$$

$$f(x) = \begin{cases} (2x-3)[x], & x \ge \frac{3}{2} \\ -(2x-3), & 1 \le x \le \frac{3}{2} \\ \sin\left(\frac{\pi x}{2}\right), & x < 1 \end{cases}$$

For continuity at $x = 1$

$f(1) = -(2.1-3) = 1$

LHL $= \lim\limits_{x \to 1^{-1}} f(x)$

$= \lim\limits_{h \to 0} f(1-h)$

$= \lim\limits_{h \to 0} \sin\left(\frac{\pi(1-h)}{2}\right)$

$= \sin\frac{\pi}{2}$

$= 1$

RHL $= \lim\limits_{x \to 0^+} f(x)$

$= \lim\limits_{h \to 0} f(1+h)$

$= \lim\limits_{h \to 0} -(2(1+h)-3)$

$= -1(-1)$

$= 1$

$LHL = f(1) = RHL$

So, $f(x)$ is continuous at $x = 1$

For differentiablility at $x = 1$

$$(\text{LHD at } x = 1) = \lim_{x\to 0^-} \frac{f(x) - f(1)}{x - 1}$$

$$= \lim_{h\to 0} \frac{f(1-h) - 1}{1 - h - 1}$$

$$= \lim_{h\to 0} \frac{\sin\left(\frac{\pi(1-h)}{2}\right) - 1}{-h}$$

$$= \lim_{h\to 0} \frac{\sin\left(\frac{\pi}{2} - \frac{\pi}{2}h\right) - 1}{-h}$$

$$= \lim_{h\to 0} \frac{\cos\left(\frac{\pi}{2}h\right) - 1}{\frac{-h}{2}}$$

$$= \lim_{h\to 0} \frac{2\sin^2\left(\frac{\pi}{4}h\right)}{h} \times \frac{\left(\frac{\pi}{4}h\right)^2}{\left(\frac{\pi}{4}h\right)^2}$$

$$= 0$$

$$(\text{RHD at } x = 1) = \lim_{x\to 1^+} \frac{f(x) - f(1)}{x - 1}$$

$$= \lim_{h\to 0} \frac{f(1+h) - f(1)}{1 + h - 1}$$

$$= \lim_{h\to 0} \frac{-\left[2(1+h) - 3\right] - 1}{h}$$

$$= \lim_{h\to 0} \frac{-2 - 2h + 3 - 1}{h}$$

$$= \lim_{h\to 0} \frac{-2h}{h}$$

$$= -2$$

$(\text{LHD at } x = 1) \neq (\text{RHD at } x = 1)$

$\therefore$ $f(x)$ is continuous but differentiable at $x = 1$.

Chapter 10 Differentiability Ex 10.1 Q10

$$\text{Her,e} \quad f(x) = \begin{cases} ax^2 - b & , \text{ if } |x| < 1 \\ \frac{1}{|x|} & , \text{ if } |x| \geq 1 \end{cases}$$

$$= \begin{cases} -\frac{1}{x} & , \text{ if } x \leq -1 \\ ax^2 - b & , \text{ if } -1 < x < 1 \\ \frac{1}{x} & , \text{ if } x \geq 1 \end{cases}$$

$$\text{LHL} = \lim_{x\to 1^-} f(x)$$

$$= \lim_{h\to 0} f(1-h)$$

$$= \lim_{h\to 0} a(1-h)^2 - b$$

$$= a - b$$

$$\text{RHL} = \lim_{x\to 1^+} f(x)$$

$$= \lim_{h\to 0} f(1+h)$$

$$= \lim_{h\to 0} \frac{1}{1+h}$$

$$= 1$$

Since, $f(x)$ is continuous, so

LHS = RHS

$a - b = 1$ ---(i)

$$(\text{LHD at } x = 1) = \lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1}$$
$$= \lim_{h \to 0} \frac{f(1-h) - 1}{1 - h - 1}$$
$$= \lim_{h \to 0} \frac{a(1-h)^2 - b - 1}{-h}$$
$$= \lim_{h \to 0} \frac{a(1-h)^2 - (a-1) - 1}{-h}$$

Using equation (i),
$$= \lim_{h \to 0} \frac{a + ah^2 - 2ah - a + 1 - 1}{-h}$$
$$= \lim_{h \to 0} \frac{ah^2 - 2ah}{-h}$$
$$= \lim_{h \to 0} (2a - ah)$$
$$= 2a$$

$$(\text{RHD at } x = 1) = \lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1}$$
$$= \lim_{h \to 0} \frac{f(1+h) - f(1)}{1 + h - 1}$$
$$= \lim_{h \to 0} \frac{\frac{1}{1+h} - 1}{h}$$
$$= \lim_{h \to 0} \frac{1 - 1 - h}{(1+h)h}$$
$$= \lim_{h \to 0} \frac{-1}{1+h}$$
$$= -1$$

Since $f(x)$ is differentiable at $x = 1$,
$$(\text{LHD at } x = 1) = (\text{RHD at } x = 1)$$
$$2a = -1$$
$$a = \frac{-1}{2}$$

Put $a = \frac{-1}{2}$ in equation (i),
$$a - b = 1$$
$$\left(\frac{-1}{2}\right) - b = 1$$
$$b = \frac{-1}{2} - 1$$
$$b = \frac{-3}{2}$$
$$a = \frac{-1}{2}$$

Ex 10.2

Differentiability Ex 10.2 Q1

Here, $f(x) = x^2$ is a polynomial function so, it is differentiable at $x = 2$.

$$f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$$
$$= \lim_{h\to 0} \frac{(2+h)^2 - (2)^2}{h}$$
$$= \lim_{h\to 0} \frac{4 + h^2 + 4h - 4}{h}$$
$$= \lim_{h\to 0} \frac{4h + h^2}{h}$$
$$= \lim_{h\to 0} (4+h)$$
$$= 4$$
$$\therefore \quad f'(2) = 4$$

Chapter 10 Differentiability Ex 10.2 Q2

$f(x) = x^2 - 4x + 7$ is a polynomial function, So it is differentiable everywhere.

$$f'(5) = \lim_{h\to 0} \frac{f(5+h) - f(5)}{h}$$
$$= \lim_{h\to 0} \frac{\left\{(5+h)^2 - 4(5+h) + 7\right\} - [25 - 20 + 7]}{h}$$
$$= \lim_{h\to 0} \frac{h^2 + 25 + 10h - 20 - 4h + 7 - 12}{h}$$
$$= \lim_{h\to 0} \frac{h^2 + 6h}{h}$$
$$= \lim_{h\to 0} (h+6)$$
$$= 6$$

$$f'\left(\frac{7}{2}\right) = \lim_{h\to 0} \frac{f\left(\frac{7}{2}+h\right) - f\left(\frac{7}{2}\right)}{h}$$
$$= \lim_{h\to 0} \frac{\left[\left(\frac{7}{2}+h\right)^2 - 4\left(\frac{7}{2}+h\right) + 7\right] - \left[\left(\frac{7}{2}\right)^2 - 4\left(\frac{7}{2}\right) + 7\right]}{h}$$
$$= \lim_{h\to 0} \frac{\left[\frac{49}{2} + h^2 + 7h - 14 - 4h + 7\right] - \left[\frac{49}{4} - 14 + 7\right]}{h}$$
$$= \lim_{h\to 0} \frac{\frac{49}{4} + h^2 + 7h - 14 - 4h + 7 - \frac{49}{4} + 14 - 7}{h}$$
$$= \lim_{h\to 0} \frac{h^2 + 3h}{h}$$
$$= \lim_{h\to 0} (h+3)$$
$$= 3$$

Now,

$$f'(5) = 6$$
$$= 2(3)$$
$$f'(5) = 2f'\left(\frac{7}{2}\right)$$

Chapter 10 Differentiability Ex 10.2 Q3

We know that, $f(x) = 2x^3 - 9x^2 + 12x + 9$ is a polynomial function. So, it is differentiable every where. For $x = 1$

$$f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h\to 0} \frac{\left[2(1+h)^3 - 9(1+h)^2 + 12(1+h) + 9\right] - \left[2 - 9 + 12 + 9\right]}{h}$$

$$= \lim_{h\to 0} \frac{\left[2\left(1+h^3+3h^2+3h\right) - 9\left(1+h^2+2h\right) + 12 + 12h + 9\right] - [14]}{h}$$

$$= \lim_{h\to 0} \frac{\left[2 + 2h^3 + 6h^2 + 6h - 9 - 9h^2 - 18h + 12 + 12h + 9 - 14\right]}{h}$$

$$= \lim_{h\to 0} \frac{2h^3 - 3h^2}{h}$$

$$= \lim_{h\to 0} \frac{h^2(2h-3)}{h}$$

$$= \lim_{h\to 0} h(2h-3)$$

$$f'(1) = 0 \qquad \text{---(i)}$$

For $x = 2$

$$f'(2) = \lim_{h\to 0} \frac{\left[2(2+h)^3 - 9(2+h)^2 + 12(12+h) + 9\right] - \left[16 - 36 + 24 + 9\right]}{h}$$

$$= \lim_{h\to 0} \frac{\left[2\left(8+h^3+12h+6h^2\right) - 9\left(4+h^2+4h\right) + 24 + 12h + 9\right] - [13]}{h}$$

$$= \lim_{h\to 0} \frac{\left[16 + 2h^3 + 24h + 12h^2 - 36 - 9h^2 - 36h + 33 + 12h - 13\right]}{h}$$

$$= \lim_{h\to 0} \frac{2h^3 + 3h^2}{h}$$

$$= \lim_{h\to 0} \frac{h^2(2h+3)}{h}$$

$$= \lim_{h\to 0} h(2h+3)$$

$$f'(2) = 0 \qquad \text{---(ii)}$$

From equation (i) and (ii),

$$f'(1) = f'(2)$$

Chapter 10 Differentiability Ex 10.2 Q4

$$\Phi(x) = \lambda x^2 + 7x - 4 \text{ and } \Phi'(5) = 97$$

$$\Phi'(5) = \lim_{h\to 0} \frac{\left[\lambda(5+h)^2 + 7(5+h) - 4\right] - \left[25\lambda + 35 - 4\right]}{h}$$

$$97 = \lim_{h\to 0} \frac{\lambda\left(25 + h^2 + 10h\right) + 35 + 7h - 4 - 25\lambda - 35 + 4}{h}$$

$$= \lim_{h\to 0} \frac{25\lambda + \lambda h^2 + 10\lambda h - 25\lambda + 7h}{h}$$

$$= \lim_{h\to 0} \frac{\lambda h^2 + h(10\lambda + 7)}{h}$$

$$= \lim_{h\to 0} \frac{f(\lambda h + 10\lambda + 7)}{h}$$

$$97 = 10\lambda + 7$$

$$10\lambda = 97 + 7$$

$$\lambda = \frac{90}{10}$$

$$\lambda = 9$$

Chapter 10 Differentiability Ex 10.2 Q5

$f(x) = x^3 + 7x^2 + 8x - 9$ is a polynomial function. So, it is differentiable every where.

$$f'(4) = \lim_{h \to 0} \frac{f(4+h) - h(4)}{h}$$

$$= \lim_{h \to 0} \frac{\left[(4+h)^3 + 7(4+h)^2 + 8(4+h) - 9\right] - \left[64 + 112 + 32 - 9\right]}{h}$$

$$= \lim_{h \to 0} \frac{\left[64 + h^3 + 48h + 12h^2 + 112 + 7h^2 + 56h + 32 + 8h - 9\right] - \left[210 - 9\right]}{h}$$

$$= \lim_{h \to 0} \frac{h^3 + 19h^2 + 112h + 210 - 9 - 210 + 9}{h}$$

$$= \lim_{h \to 0} \frac{h^3 + 19h^2 + 112h}{h}$$

$$= \lim_{h \to 0} \frac{h\left(h^2 + 19h + 112\right)}{h}$$

$$f'(4) = 112$$

Chapter 10 Differentiability Ex 10.2 Q6

$$f(x) = mx + c$$

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{f(h) - h(0)}{h}$$

$$= \lim_{h \to 0} \frac{(mh + c) - (m \times 0 + c)}{h}$$

$$= \lim_{h \to 0} \frac{mh + c - c}{h}$$

$$= \lim_{h \to 0} \frac{mh}{h}$$

$$= m$$

$$f'(0) = m$$

Chapter 10 Differentiability Ex 10.2 Q7

$$f(x) = \begin{cases} 2x + 3, \text{ if} -3 \le x < -2 \\ x + 1, \text{ if} -2 \le x < 0 \\ x + 2, \text{ if } 0 \le x \le 1 \end{cases}$$

We know that polynomial funtiona are continuous and differentiable everywhere.
So f(x) is differentiable on $x \in [-3,2)$, $x \in (-2,0)$ and $x \in (0,1]$.
We need to check the differentiability at x = -2 and x = 0

Differentiability at x = -2

$$(\text{LHD at } x = -2) = \lim_{x \to -2^-} \frac{f(x) - f(-2)}{x - (-2)} = \lim_{x \to -2^-} \frac{2x + 3 + 1}{x + 2} = \lim_{x \to -2^-} \frac{2(x+2)}{x+2} = 2$$

$$(\text{RHD at } x = -2) = \lim_{x \to -2^+} \frac{f(x) - f(-2)}{x - (-2)} = \lim_{x \to -2^+} \frac{x + 1 + 1}{x + 2} = \lim_{x \to -2^+} \frac{x+2}{x+2} = 1$$

$\therefore$ (LHD at x = -2) $\ne$ (RHD at x = -2)
So, $f(x)$ is not differentiable at x = -2.

Differentiability at x = 0

$$(\text{LHD at } x = 0) = \lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^-} \frac{x + 1 - 2}{x} = \lim_{x \to 0^-} \frac{x - 1}{x} \to \infty$$

$$(\text{RHD at } x = 0) = \lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^+} \frac{x + 2 - 2}{x} = \lim_{x \to 0^+} \frac{x}{x} = 1$$

$\therefore$ (LHD at x = 0) $\ne$ (RHD at x = 0)
So, $f(x)$ is not differentiable at x = 0.

Chapter 10 Differentiability Ex 10.2 Q8

We know that, modulus function

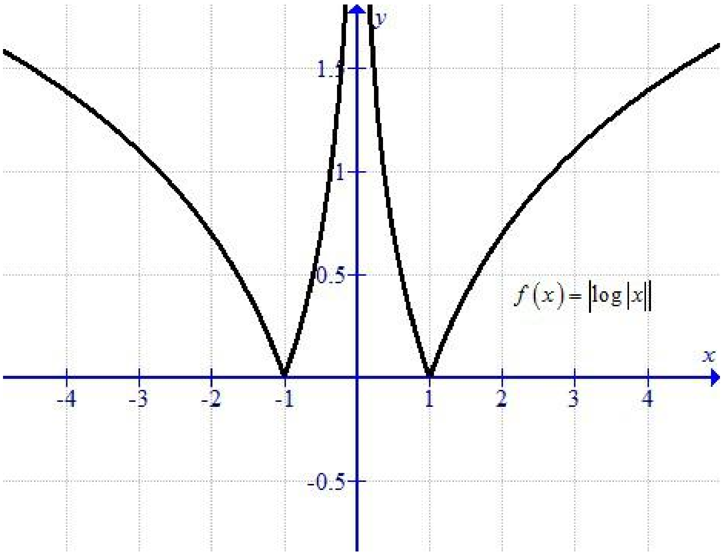
$f(x) = |x|$ is continuous but bot differentiable at $x = 0$,

So,

$f(x) = |x| + |x-1| + |x-2| + |x-3| + |x-4|$ is continuous but not differentiable

$x = 0, 1, 2, 3, 4.$

Chapter 10 Differentiability Ex 10.2 Q9



$f(x) = |\log|x||$

Since, it is an absolute function. So, it is continuous function.

The graph of the function is as below:-

Chapter 10 Differentiability Ex 10.2 Q10

$$f(x) = e^{|x|}$$

$$f(x) = \begin{cases} e^{-x} & , x < 0 \\ e^{x} & , x \geq 0 \end{cases}$$

For continuity at $x = 0$

$$\begin{aligned}
\text{RHL} &= \lim_{x \to 0^+} f(x) \\
&= \lim_{h \to 0} f(0+h) \\
&= \lim_{h \to 0} e^{(0+h)} \\
&= \lim_{h \to 0} e^{h} \\
&= e^{0} \\
\text{RHL} &= 1 \\
\text{LHL} &= \lim_{x \to 0^-} f(x) \\
&= \lim_{h \to 0} f(0-h) \\
&= \lim_{h \to 0} e^{-(0-h)} \\
&= \lim_{h \to 0} e^{h} \\
\text{LHL} &= 1
\end{aligned}$$

$$f(0) = e^{0} = 1$$

Now,

$$\text{LHL} = f(0) = \text{RHL}$$

So, $f(x)$ is continuous at $x = 0$

For differentiablility at $x = 0$

$$\begin{aligned}
\text{LHD} &= \lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0} \\
&= \lim_{h \to 0} \frac{f(0-h) - e^{0}}{(0-h) - 0} \\
&= \lim_{h \to 0} \frac{e^{-(0-h)} - 1}{-h} \\
&= \lim_{h \to 0} \frac{e^{h} - 1}{-h} \\
&= 1 \qquad \left[\text{Since } \lim_{x \to 0} \frac{e^{x} - 1}{x} = 1\right]
\end{aligned}$$

$$\begin{aligned}
\text{RHD} &= \lim_{hx \to 0^+} \frac{f(x) - f(0)}{x - 0} \\
&= \lim_{h \to 0} \frac{f(0+h) - f(0)}{(0+h) - 0} \\
&= \lim_{h \to 0} \frac{e^{x} - e^{0}}{h} \\
&= \lim_{h \to 0} \frac{e^{h} - 1}{h} \\
&= 1 \qquad \left[\text{Since } \lim_{x \to 0} \frac{e^{x} - 1}{x} = 1\right]
\end{aligned}$$

Clearly,

$$\text{LHD} \neq \text{RHD}$$

So,

$f(x)$ is not differentiable at $x = 0$.

Differentiability Ex 10.2 Q11

$$f(x)=\begin{cases}(x-c)\cos\frac{1}{(x-c)} & ,x\neq c\\ 0 & ,x=c\end{cases}$$

$$(\text{LHL at } x=c)=\lim_{x\to c^-} f(x)$$
$$=\lim_{h\to 0} f(-h)$$
$$=\lim_{h\to 0}(c-h-c)\cos\left(\frac{1}{c-h-c}\right)$$
$$=\lim_{h\to 0}-h\cos\left(-\frac{1}{h}\right)$$
$$=\lim_{h\to 0}-h\cos\left(\frac{1}{h}\right)$$
$$=0$$

$$(\text{RHL at } x=c)=\lim_{x\to c^+} f(x)$$
$$=\lim_{h\to 0} f(c+h)$$
$$=\lim_{h\to 0}(c+h-c)\cos\left(\frac{1}{c+h-c}\right)$$
$$=\lim_{h\to 0}h\cos\left(\frac{1}{h}\right)$$
$$=0$$

$$f(e)=0$$

Since, LHL $= f(x) =$ RHL at $x=c$

$\Rightarrow$ $f(x)$ is continuous at $x=c$

$$(\text{LHD at } x=c)=\lim_{h\to 0}\frac{f(c-h)-f(c)}{-h}$$
$$=\lim_{h\to 0}\frac{(c-h-c)\cos\left(\frac{1}{c-h-c}\right)-0}{-h}$$
$$=\lim_{h\to 0}\cos\left(-\frac{1}{h}\right)$$
$$=\lim_{h\to 0}\cos\left(\frac{1}{h}\right)$$
$$=k$$

$$(\text{RHD at } x=c)=\lim_{h\to 0}\frac{f(c+h)-f(c)}{h}$$
$$=\lim_{h\to 0}\frac{(c+h-c)\cos\left(\frac{1}{c+h-c}\right)-0}{h}$$
$$=\lim_{h\to 0}\frac{h\cos\left(\frac{1}{h}\right)}{h}$$
$$=\lim_{h\to 0}\cos\left(\frac{1}{h}\right)$$
$$=k$$

$$(\text{LHD at } x=c)=(\text{RHD at } x=c)$$

So,

$f(x)$ is differentiable and continuous at $x=c$.

Differentiability Ex 10.2 Q12

$$f(x) = |\sin x| = \begin{cases} -\sin x &, \; x < n\pi \\ \sin x &, \; x \ge n\pi \end{cases}$$

For $x = n\pi$ (n even)

$$(\text{LHD at } x = n\pi) = \lim_{x \to n\pi^-} \frac{f(x) - f(n\pi)}{x - n\pi}$$

$$= \lim_{h \to 0} \frac{-\sin(n\pi - h) - \sin n\pi}{n\pi - h - n\pi}$$

$$= \lim_{h \to 0} \frac{\sin h - 0}{-h}$$

$$= -1$$

$$(\text{RHD at } x = n\pi) = \lim_{h \to 0} \frac{\sin(n\pi + h) - \sin n\pi}{h}$$

$$= \lim_{h \to 0} \frac{\sin h}{h}$$

$$= 1$$

$$(\text{LHD at } x = n\pi) \ne (\text{RHD at } x = n\pi)$$

For $x = n\pi$ (n is odd)

$$(\text{LHD at } x = n\pi) = \lim_{h \to 0} \frac{-\sin(n\pi - h) - \sin n\pi}{-h}$$

$$= \lim_{h \to 0} \frac{-\sin h}{-h}$$

$$= 1$$

$$(\text{RHD at } x = n\pi) = \lim_{h \to 0} \frac{\sin(n\pi + h) - \sin n\pi}{h}$$

$$= \lim_{h \to 0} \frac{-\sin h - 0}{h}$$

$$= -1$$

$$(\text{LHD at } x = n\pi) \ne (\text{RHD at } x = n\pi)$$

Thus,

$f(x) = |\sin x|$ is not differentiable at $x = n\pi$

$f(x) = \cos|x|$

Since, $\cos(-x) = \cos x$

$\Rightarrow \quad f(x) = \cos x$

$\Rightarrow \quad f(x) = \cos|x|$ is differnetiable everywhere.

Ex 11.1

Differentiation Ex 11.1 Q1

Let $f(x) = e^{-x}$

$\Rightarrow \quad f(x+h) = e^{-(x+h)}$

$$\frac{d}{dx}\left(f(x)\right) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{e^{-(x+h)} - e^{-x}}{h}$$
$$= \lim_{h\to 0} \frac{e^{-x} \times e^{-h} - e^{-x}}{h}$$
$$= \lim_{h\to 0} e^{-x} \left\{\frac{\left(e^{-h} - 1\right)}{-h}\right\} \times (-1)$$
$$= -e^{-x} \qquad \left[\text{Since, } \lim_{h\to 0} \frac{e^{h} - 1}{h} = 1\right]$$

So,

$$\frac{d}{dx}\left(e^{-x}\right) = -e^{-x}$$

Differentiation Ex 11.1 Q2

Let $f(x) = e^{3x}$

$\Rightarrow \quad f(x+h) = e^{3(x+h)}$

$$\frac{d}{dx}\left(f(x)\right) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{e^{3(x+h)} - e^{3x}}{h}$$
$$= \lim_{h\to 0} \frac{e^{3x}e^{3h} - e^{3x}}{h}$$
$$= \lim_{h\to 0} e^{3x} \left\{\frac{\left(e^{3h} - 1\right)}{3h}\right\} \times 3$$
$$= 3e^{3x} \qquad \left[\text{Since, } \lim_{x\to 0} \frac{e^{x} - 1}{x} = 1\right]$$

Hence,

$$\frac{d}{dx}\left(e^{3x}\right) = 3e^{3x}$$

Differentiation Ex 11.1 Q3

Let $f(x) = e^{ax+b}$

$\Rightarrow$ $f(x+h) = e^{a(x+h)+b}$

$$\therefore \quad \frac{d}{dx}\left(f(x)\right) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{e^{a(x+h)+b} - e^{(ax+b)}}{h}$$

$$= \lim_{h\to 0} \frac{e^{ax+b}e^{ah} - e^{ax+b}}{h}$$

$$= \lim_{h\to 0} e^{ax+b}\left\{\frac{\left(e^{ah} - 1\right)}{ah}\right\}\times a$$

$$= ae^{ax+b} \qquad \left[\text{Since, } \lim_{x\to 0} \frac{e^x - 1}{x} = 1\right]$$

So,

$$\frac{d}{dx}\left(e^{ax+b}\right) = ae^{ax+b}$$

Differentiation Ex 11.1 Q4

Let $f(x) = e^{\cos x}$

$\Rightarrow$ $f(x+h) = e^{\cos(x+h)}$

$$\therefore \quad \frac{d}{dx}\left(f(x)\right) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{e^{\cos(x+h)} - e^{\cos x}}{h}$$

$$= \lim_{h\to 0} e^{\cos x}\left[\frac{e^{\cos(x+h)-\cos x} - 1}{h}\right]$$

$$= \lim_{h\to 0} e^{\cos x}\left[\frac{e^{\cos(x+h)-\cos x} - 1}{\cos(x+h) - \cos x}\right]\times\frac{\cos(x+h) - \cos x}{h}$$

$$= \lim_{h\to 0} e^{\cos x}\times\left(\frac{\cos(x+h) - \cos x}{h}\right) \qquad \left[\text{Since, } \lim_{h\to 0} \frac{e^x - 1}{x} = 1\right]$$

$$= \lim_{h\to 0} e^{\cos x}\times\left(\frac{-2\sin\frac{x+h+x}{2}\times\sin\frac{x+h-x}{2}}{h}\right) \qquad \left[\text{Since, } \cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}\right]$$

$$= e^{\cos x}\lim_{h\to 0} \frac{-2\sin\left(\frac{2x+h}{2}\right)}{2}\times\frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}$$

$$= e^{\cos x}\lim_{h\to 0} -2\sin\left(\frac{2x+h}{2}\right)\times\frac{1}{2} \qquad \left[\text{since, } \lim_{x\to 0} \frac{\sin x}{x} = 1\right]$$

$$= e^{\cos x}\left(-\sin x\right)$$

$$= -\sin x e^{\cos x}$$

Hence,

$$\frac{d}{dx}\left(e^{\cos x}\right) = -\sin x e^{\cos x}$$

Differentiation Ex 11.1 Q5

Let $\quad f(x) = e^{\sqrt{2x}}$

$\Rightarrow \quad f(x+h) = e^{\sqrt{2(x+h)}}$

$$\frac{d}{dx}(f(x)) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{e^{\sqrt{2(x+h)}} - e^{\sqrt{2x}}}{h}$$

$$= \lim_{h\to 0} e^{\sqrt{2x}} \frac{e^{\sqrt{2(x+h)}-\sqrt{2x}} - 1}{h}$$

$$= e^{\sqrt{2x}} \lim_{h\to 0} \left(\frac{\left(e^{\sqrt{2(x+h)}-\sqrt{2x}} - 1\right)}{\sqrt{2(x+h)} - \sqrt{2x}}\right)\left(\frac{\sqrt{2(x+h)} - \sqrt{2x}}{h}\right)$$

$$= e^{\sqrt{2x}} \lim_{h\to 0} \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h} \qquad \left[\text{Since, } \lim_{h\to 0} \frac{e^h - 1}{h} = 1\right]$$

$$= e^{\sqrt{2x}} \lim_{h\to 0} \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h} \times \frac{\sqrt{2(x+h)} + \sqrt{2x}}{\sqrt{2(x+h)} + \sqrt{2x}} \qquad [\text{Rationalizing the numerator}]$$

$$= de^{\sqrt{2x}} \lim_{h\to 0} \frac{2(x+h) - 2x}{h\left(\sqrt{2(x+h)} + \sqrt{2x}\right)}$$

$$= e^{\sqrt{2x}} \lim_{h\to 0} \frac{2x + 2h - 2x}{h\left(\sqrt{2(x+h)} + \sqrt{2x}\right)}$$

$$= e^{\sqrt{2x}} \lim_{h\to 0} \frac{2h}{h\left(\sqrt{2(x+h)} + \sqrt{2x}\right)}$$

$$= e^{\sqrt{2x}} \lim_{h\to 0} \frac{2}{\left(\sqrt{2(x+h)} + \sqrt{2x}\right)}$$

$$= \frac{e^{\sqrt{2x}}}{\sqrt{2x}}$$

So,

$$\frac{d}{dx}\left(e^{\sqrt{2x}}\right) = \frac{e^{\sqrt{2x}}}{\sqrt{2x}}$$

Differentiation Ex 11.1 Q6

Let $\quad f(x) = \log\cos x$

$\Rightarrow \quad f(x+h) = \log\cos(x+h)$

$$\therefore \quad \frac{d}{dx}(f(x)) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\log\cos(x+h) - \log\cos x}{h}$$

$$= \lim_{h\to 0} \frac{\log\frac{\cos(x+h)}{\cos x}}{h} \qquad \left[\text{Since, } \log A - \log B = \log\frac{A}{B}\right]$$

$$= \lim_{h\to 0} \frac{\log\left\{1 + \frac{\cos(x+h)}{\cos x} - 1\right\}}{h}$$

$$= \lim_{h\to 0} \frac{\log\left\{1 + \frac{\cos(x+h)}{\cos x}\right\}}{\left(\frac{\cos(x+h)}{\cos x}\right) h \times \left(\frac{\cos x}{\cos(x+h) - \cos x}\right)}$$

$$= \lim_{h\to 0} \frac{\cos(x+h) - \cos x}{\cos x \times h} \qquad \left[\text{Since, } \lim_{x\to 0} \frac{\log(1+x)}{x} = 1\right]$$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{x+h+x}{2}\right)\sin\left(\frac{x+h-x}{2}\right)}{\cos x \times h}$$

$$= -2\lim_{h\to 0} \frac{\sin\left(\frac{2x+h}{2}\right) \times \left(\sin\frac{h}{2}\right)}{2\cos x\left(\frac{h}{2}\right)}$$

$$= \frac{-2\sin x}{2\cos x} \qquad \left[\text{Since, } \lim_{x\to 0} \frac{\sin x}{x} = 1\right]$$

$$= -\tan x$$

So,

$$\frac{d}{dx}(\log\cos x) = -\tan x$$

Differentiation Ex 11.1 Q7

Let $\quad f(x) = e^{\sqrt{\cot x}}$

$\Rightarrow \quad f(x+h) = e^{\sqrt{\cot(x+h)}}$

$$\frac{d}{dx}(f(x)) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{e^{\sqrt{\cot(x+h)}} - e^{\sqrt{\cot x}}}{h}$$
$$= \lim_{h\to 0} \frac{e^{\sqrt{\cot x}}\left(e^{\sqrt{\cot(x+h)}-\sqrt{\cot x}} - 1\right)}{h}$$
$$= e^{\sqrt{\cot x}} \lim_{h\to 0} \left(\frac{e^{\sqrt{\cot(x+h)}-\sqrt{\cot x}} - 1}{\sqrt{\cot(x+h)} - \sqrt{\cot x}}\right) \times \left(\frac{\sqrt{\cot(x+h)} - \sqrt{\cot x}}{h}\right)$$
$$= e^{\sqrt{\cot x}} \lim_{h\to 0} \frac{\left(\sqrt{\cot(x+h)} - \sqrt{\cot x}\right)}{h} \times \frac{\sqrt{\cot(x+h)} + \sqrt{\cot x}}{\sqrt{\cot(x+h)} + \sqrt{\cot x}}$$

$\left[\text{Since, } \lim_{x\to 0} \frac{e^x - 1}{x} = 1 \text{ and rationalizing numerator}\right]$

$$= e^{\sqrt{\cot x}} \lim_{h\to 0} \frac{\cot(x+h) - \cot x}{h\left(\sqrt{\cot(x+h)} + \sqrt{\cot x}\right)}$$
$$= e^{\sqrt{\cot x}} \lim_{h\to 0} \frac{\frac{\cot(x+h)\cot x + 1}{\cot(x+h-x)}}{h\left(\sqrt{\cot(x+h)} + \sqrt{\cot x}\right)} \qquad \left[\text{Since, } \cot(A-B) = \frac{\cot A \cot B + 1}{\cot A - \cot B}\right]$$
$$= e^{\sqrt{\cot x}} \lim_{h\to 0} \frac{\cot(x+h)\cot x + 1}{\text{cothx}\, h\left(\sqrt{\cot(x+h)} + \sqrt{\cot x}\right)}$$
$$= e^{\sqrt{\cot x}} \lim_{h\to 0} \frac{\left(\cot(x+h)\cot x + 1\right)}{\left(\frac{h}{\tan h}\right)\left(\sqrt{\cot(x+h)} + \sqrt{\cot x}\right)}$$
$$= \frac{e^{\sqrt{\cot x}} \times \left(\cot^2 x + 1\right)}{2\sqrt{\cot x}} \qquad \left[\text{Since, } \lim_{x\to 0} \frac{\tan x}{x} = 1\right]$$
$$= \frac{e^{\sqrt{\cot x}} \times \text{cosec}^2 x}{2\sqrt{\cot x}} \qquad \left[\text{Since, } \left(1 + \cot^2 x\right) = \text{cosec}^2 x\right]$$

So,

$$\frac{d}{dx}\left(e^{\sqrt{\cot x}}\right) = \frac{e^{\sqrt{\cot x}} \times \text{cosec}^2 x}{2\sqrt{\cot x}}$$

Differentiation Ex 11.1 Q8

Let $\quad f(x) = x^2 e^x$

$\Rightarrow \quad f(x+h) = (x+h)^2 e^{(x+h)}$

$$\therefore \quad \frac{d}{dx}(f(x)) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{(x+h)^2 e^{(x+h)} - x^2 e^x}{h}$$
$$= \lim_{h\to 0} \left(\frac{x^2 e^{(x+h)} - x^2 e^x}{h} + \frac{2xhe^{(x+h)}}{h} + \frac{h^2 e^{(x+h)}}{h}\right)$$
$$= \lim_{h\to 0} \left(\frac{x^2 e^x \left(e^{(x+h)-x} - 1\right)}{h} + 2xe^{(x+h)} + he^{(x+h)}\right)$$
$$= \lim_{h\to 0} \left[x^2 e^x \frac{\left(e^h - 1\right)}{h} + 2xe^{(x+h)} + he(x+h)\right]$$
$$= x^2 e^x + 2xe^x + 0 \times e^x \qquad \left[\text{Since, } \lim_{x\to 0} \frac{e^x - 1}{x} = 1\right]$$

So,

$$\frac{d}{dx}\left(x^2 e^x\right) = e^x\left(x^2 + 2x\right)$$

Differentiation Ex 11.1 Q9

Let $\quad f(x) = \log \operatorname{cosec} x$

$\Rightarrow \quad f(x+h) = \log \operatorname{cosec}(x+h)$

$$\frac{d}{dx}(f(x)) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\log \operatorname{cosec}(x+h) - \log \operatorname{cosec} x}{h}$$

$$= \lim_{h\to 0} \frac{\log\left(\frac{\operatorname{cosec}(x+h)}{\operatorname{cosec} x}\right)}{h}$$

$$= \lim_{h\to 0} \frac{\log\left(1 + \left(\frac{\sin x}{\sin(x+h)} - 1\right)\right)}{h}$$

$$= \lim_{h\to 0} \left\{\frac{\log\left(1 + \left(\frac{\sin x - \sin(x+h)}{\sin(x+h)}\right)\right)}{\left(\frac{\sin x - \sin(x+h)}{\sin(x+h)}\right)}\right\} \frac{\left(\frac{\sin x - \sin(x+h)}{\sin(x+h)}\right)}{h}$$

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{x+x+h}{2}\right)\sin\left(\frac{x-x-h}{2}\right)}{\sin(x+h)h}$$

$$\left[\text{Since, } \lim_{x\to 0} \frac{\log(1+x)}{x} = 1 \text{ and } \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{2x+h}{2}\right)}{\sin(x+h)(-2)} \left\{\frac{\sin\left(-\frac{h}{2}\right)}{-\frac{h}{2}}\right\} \qquad \left[\text{Since, } \lim_{x\to 0} \frac{\sin x}{x} = 1\right]$$

$$= -\cot x$$

So,

$$\frac{d}{dx}(\log \operatorname{cosec} x) = -\cot x.$$

Differentiation Ex 11.1 Q10

Let $\quad f(x) = \sin^{-1}(2x+3)$

$\Rightarrow \quad f(x+h) = \sin^{-1}(2(x+h)+3)$

$\Rightarrow \quad f(x+h) = \sin^{-1}(2x+2h+3)$

$$\therefore \quad \frac{d}{dx}(f(x)) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin^{-1}(2x+2h+3) - \sin^{-1}(2x+3)}{h}$$

$$= \lim_{h\to 0} \frac{\sin^{-1}\left[(2x+2h+3)\sqrt{1-(2x+3)^2} - (2x+3)\sqrt{1-(2x+2h+3)^2}\right]}{h}$$

$$\left[\text{Since, } \sin^{-1}x - \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} - y\sqrt{1-x^2}\right]\right]$$

$$= \lim_{h\to 0} \frac{\sin^{-1}z}{z} \times \frac{z}{h}$$

Where, $z = (2x+2h+3)\sqrt{1-(2x+3)^2} - (2x+3)\sqrt{1-(2x+2h+3)^2}$ and $\lim_{h\to 0} \frac{\sin^{-1}h}{h} = 1$

$$= \lim_{h\to 0} \frac{z}{h}$$

$$= \lim_{h\to 0} \frac{(2x+2h+3)\sqrt{1-(2x+3)^2} - (2x+3)\sqrt{1-(2x+2h+3)^2}}{h}$$

$$= \lim_{h\to 0} \frac{(2x+2h+3)^2 - (2x+3)^2 - (2x+3)^2\left(1-(2x+2h+3)^2\right)}{h\left\{(2x+2h+3)\sqrt{1-(2x+3)^2} + (2x+3)\sqrt{1-(2x+2h+3)^2}\right\}}$$

[Since, rationalizing numerator]

$$= \lim_{h\to 0} \frac{\left[(2x+3)^2 + 4h^2 + 4h(2x+3)\right]\left(1-(2x+3)^2\right) - (2x+3)^2\left[1-(2x+3)^2 - 4h^2 - 4h(2x+3)\right]}{h\left\{(2x+2h+3)\sqrt{1-(2x+3)^2} + (2x+3)\sqrt{1-(2x+2h+3)^2}\right\}}$$

$$= \lim_{h\to 0} \frac{\left[\begin{array}{r}(2x+3)^2 + 4h^2 + 4h(2x+3) - (2x+3)^4 - 4h^2(2x+3)^2 - 4h(2x+3)^3 - (2x+3)^2 \\ + (2x+3)^4 + 4h^2(2x+3)^2 + 4h(2x+3)^3\end{array}\right]}{h\left\{(2x+2h+3)\sqrt{1-(2x+3)^2} + (2x+3)\sqrt{1-(2x+2h+3)^2}\right\}}$$

$$= \lim_{h\to 0} \frac{4h\left[h + (2x+3)\right]}{h\left\{(2x+2h+3)\sqrt{1-(2x+3)^2} + (2x+3)\sqrt{1-(2x+2h+3)^2}\right\}}$$

$$= \frac{4(2x+3)}{(2x+3)\sqrt{1-(2x+3)^2} + (2x+3)\sqrt{1-(2x+3)^2}}$$

$$= \frac{4(2x+3)}{2(2x+3)\sqrt{1-(2x+3)^2}}$$

$$= \frac{2}{\sqrt{1-(2x+3)^2}}$$

So,

$$\frac{d}{dx}\left(\sin^{-1}(2x+3)\right) = \frac{2}{\sqrt{1-(2x+3)^2}}$$

Ex 11.2

Differentiation Ex 11.2 Q1

Let,

$$y = \sin(3x+5)$$

Differentiate y with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sin(3x+5)\right)$$
$$= \cos(3x+5)\frac{d}{dx}(3x+5) \quad \text{[using chain rule]}$$
$$= \cos(3x+5)\times[3(1)+0]$$
$$= 3\cos(3x+5)$$

So,

$$\frac{d}{dx}\left(\sin(3x+5)\right) = 3\cos(3x+5).$$

Differentiation Ex 11.2 Q2

Let,

$$y = \tan^2 x$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = 2\tan x\frac{d}{dx}(\tan x) \quad \text{[using chain rule]}$$
$$= 2\tan x \times \sec^2 x$$

So,

$$\frac{d}{dx} = \left(\tan^2 x\right) = 2\tan x\sec^2 x.$$

Differentiation Ex 11.2 Q3

Let,

$$y = \tan\left(x° + 45°\right)$$
$$y = \tan\left\{\left(x° + 45°\right)\frac{\pi}{180°}\right\}$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\tan\left\{\left(x° + 45°\right)\frac{\pi}{180°}\right\}$$
$$= s\,ec^2\left\{\left(x° + 45°\right)\frac{\pi}{180°}\right\} \times \frac{d}{dx}\left(x° + 45°\right)\frac{\pi}{180°} \qquad \text{[Using chain rule]}$$
$$= \frac{\pi}{180°}\sec^2\left(x° + 45°\right)$$

So,

$$\frac{d}{dx} = \left(\tan\left(x° + 45°\right)\right) = \frac{\pi}{180°}\sec^2\left(x° + 45°\right).$$

Differentiation Ex 11.2 Q4

Let,

$$y = \sin\left(\log x\right)$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\sin\left(\log x\right)$$
$$= \cos\left(\log x\right)\frac{d}{dx}\left(\log x\right) \qquad \text{[Using chain rule]}$$
$$= \frac{1}{x}\cos\left(\log x\right)$$

So,

$$\frac{d}{dx} = \left(\sin\left(\log x\right)\right) = \frac{1}{x}\cos\left(\log x\right).$$

Differentiation Ex 11.2 Q5

Let,

$$y = e^{\sin\sqrt{x}}$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\sin\sqrt{x}}\right)$$
$$= e^{\sin\sqrt{x}}\frac{d}{dx}\left(\sin\sqrt{x}\right) \qquad \text{[Using chain rule]}$$
$$= e^{\sin\sqrt{x}} \times \cos\sqrt{x}\,\frac{d}{dx}\sqrt{x} \qquad \text{[Using chain rule]}$$
$$= e^{\sin\sqrt{x}} \times \cos\sqrt{x} \times \frac{1}{2\sqrt{x}}$$
$$= \frac{1}{2\sqrt{x}}\cos\sqrt{x} \times e^{\sin\sqrt{x}}$$

So,

$$\frac{d}{dx} = \left(e^{\sin\sqrt{x}}\right) = \frac{1}{2\sqrt{x}}\cos\sqrt{x} \times e^{\sin\sqrt{x}}.$$

Differentiation Ex 11.2 Q6

Let,

$$y = e^{\tan x}$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\tan x}\right)$$
$$= e^{\tan x}\frac{d}{dx}\left(\tan x\right) \qquad \text{[Using chain rule]}$$
$$= e^{\tan x} \times \sec^2 x$$

So,

$$\frac{d}{dx} = \left(e^{\tan x}\right) = \sec^2 x \times e^{\tan x}.$$

Differentiation Ex 11.2 Q7

Let,

$$y = \sin^2(2x+1)$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\sin^2(2x+1)\right]$$

$$= 2\sin(2x+1)\frac{d}{dx}\sin(2x+1) \qquad \text{[Using chain rule]}$$

$$= 2\sin(2x+1)\cos(2x+1)\frac{d}{dx}(2x+1) \qquad \text{[Using chain rule]}$$

$$= 4\sin(2x+1)\cos(2x+1)$$

$$= 2\sin 2(2x+1) \qquad \left[\text{Since, } \sin^2 A = 2\sin A\cos A\right]$$

$$= 2\sin(4x+2)$$

So,

$$\frac{d}{dx}\left(\sin^2(2x+1)\right) = 2\sin(4x+2).$$

Differentiation Ex 11.2 Q8

Let,

$$y = \log_7(2x-3)$$

$$\Rightarrow \quad y = \frac{\log(2x-3)}{\log 7} \qquad \left[\text{Since, } \log_a{}^b = \frac{\log b}{\log a}\right]$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{1}{\log 7}\frac{d}{dx}\left(\log(2x-3)\right)$$

$$= \frac{1}{\log 7}\times\frac{1}{(2x-3)}\frac{d}{dx}(2x-3) \qquad \text{[Using chain rule]}$$

$$= \frac{2}{(2x-3)\log 7}$$

Hence,

$$\frac{d}{dx}\left(\log_7(2x-3)\right) = \frac{2}{(2x-3)\log 7}.$$

Differentiation Ex 11.2 Q9

Let,

$$y = \tan 5x^\circ$$

$$\Rightarrow \quad y = \tan\left(5x^\circ\times\frac{\pi}{180^\circ}\right)$$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\tan\left(5x^\circ\times\frac{\pi}{180^\circ}\right)$$

$$= \sec^2 x\left(5x^\circ\times\frac{\pi}{180^\circ}\right)\frac{d}{dx}\left(5x^\circ\frac{\pi}{180^\circ}\right) \qquad \text{[Using chain rule]}$$

$$= \left(\frac{5\pi}{180^\circ}\right)\sec^2\left(5x^\circ\frac{\pi}{180^\circ}\right)$$

$$= \frac{5\pi}{180^\circ}\sec^2(5x^\circ)$$

Hence,

$$\frac{d}{dx}\left(\tan(5x^\circ)\right) = \frac{5\pi}{180^\circ}\sec^2(5x^\circ).$$

Differentiation Ex 11.2 Q10

Let,

$$y = 2^{x^3}$$

Differentiate with respect to x,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(2^{x^3}\right)\\ &= 2^{x^3} \times \log_2 \frac{d}{dx}\left(x^3\right) && \text{[Using chain rule]}\\ &= 3x^2 \times 2^{x^3} \times \log_2\end{aligned}$$

So,

$$\frac{d}{dx}\left(2^{x^3}\right) = 3x^2 \times 2^{x^3} \log_2 .$$

Differentiation Ex 11.2 Q11

Let, $y = 3^{e^x}$

Differentiate it with respect to x,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(3^{e^x}\right)\\ &= 3^{e^x} \log 3 \frac{d}{dx}\left(e^x\right) && \text{[Using chain rule]}\\ &= e^x \times 3^{e^x} \log 3\end{aligned}$$

So,

$$\frac{\mathrm{d}}{\mathrm{dx}}\left(3^{e^x}\right) = e^x \times 3^{e^x} \log 3.$$

Differentiation Ex 11.2 Q12

Let $\quad y = \log_x 3$

$\Rightarrow \quad y = \dfrac{\log 3}{\log x} \qquad \left[\text{Since, } \log_a{}^b = \dfrac{\log b}{\log a}\right]$

Differentiate with respect to x,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(\frac{\log 3}{\log x}\right)\\ &= \log 3 \frac{d}{dx}(\log x)^{-1}\\ &= \log 3 \times \left[-1(\log x)^{-2}\right]\frac{d}{dx}(\log x) && \text{[Using chain rule]}\\ &= -\frac{\log 3}{(\log x)^2} \times \frac{1}{x}\\ &= -\left(\frac{\log 3}{\log x}\right)^2 \times \frac{1}{x} \times \frac{1}{\log 3}\\ &= -\frac{1}{x \log 3 (\log_3 x)^2} && \left[\text{Since, } \frac{\log b}{\log a} = loa_a{}^b\right]\end{aligned}$$

So,

$$\frac{d}{dx}(\log_x 3) = -\frac{1}{x \log 3 (\log_3 x)^2}.$$

Differentiation Ex 11.2 Q13

Let $y = 3^{x^2+2x}$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(3^{x^2+2x}\right)$$
$$= 3^{x^2+2x} \times \log 3 \frac{d}{dx}\left(x^2 + 2x\right) \qquad \left[\text{Using chain rule}\right]$$
$$= (2x + 2)\log 3 \times 3^{x^2+2x}$$

So,

$$\frac{d}{dx}\left(3^{x^2+2x}\right) = (2x + 2)\log 3 \times 3^{x^2+2x}.$$

Differentiation Ex 11.2 Q14

Let $\quad y = \sqrt{\frac{a^2 - x^2}{a^2 + x^2}}$

$\Rightarrow \quad y = \left(\frac{a^2 - x^2}{a^2 + x^2}\right)^{\frac{1}{2}}$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{a^2 - x^2}{a^2 + x^2}\right)^{\frac{1}{2}}$$
$$= \frac{1}{2}\left(\frac{a^2 - x^2}{a^2 + x^2}\right)^{\frac{1}{2}-1} \times \frac{d}{dx}\left(\frac{a^2 - x^2}{a^2 + x^2}\right) \qquad \left[\text{Using chain rule}\right]$$
$$= \frac{1}{2}\left(\frac{a^2 - x^2}{a^2 + x^2}\right)^{\frac{-1}{2}} \times \left\{\frac{\left(a^2 + x^2\right)\frac{d}{dx}\left(a^2 - x^2\right) - \left(a^2 - x^2\right)\frac{d}{dx}\left(a^2 + x^2\right)}{\left(a^2 + x^2\right)^2}\right\} \qquad \left[\text{Using chain rule}\right]$$
$$= \frac{1}{2}\left(\frac{a^2 + x^2}{a^2 - x^2}\right)^{\frac{1}{2}}\left\{\frac{-2x\left(a^2 + x^2\right) - 2x\left(a^2 - x^2\right)}{\left(a^2 + x^2\right)^2}\right\}$$
$$= \frac{1}{2}\left(\frac{a^2 + x^2}{a^2 - x^2}\right)^{\frac{1}{2}}\left\{\frac{-2xa^2 - 2x^3 - 2xa^2 + 2x^3}{\left(a^2 + x^2\right)^2}\right\}$$
$$= \frac{1}{2}\left(\frac{a^2 + x^2}{a^2 - x^2}\right)^{\frac{1}{2}}\left(\frac{-4xa^2}{\left(a^2 + x^2\right)^2}\right)$$
$$= \frac{-2xa^2}{\sqrt{a^2 - x^2}\left(a^2 + x^2\right)^{\frac{3}{2}}}$$

So,

$$\frac{d}{dx}\left(\sqrt{\frac{a^2 - x^2}{a^2 + x^2}}\right) = \frac{-2a^2x}{\sqrt{a^2 - x^2}\left(a^2 + x^2\right)^{\frac{3}{2}}}.$$

Differentiation Ex 11.2 Q15

Let $y = 3^{x\log x}$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(3^{x\log x}\right)$$
$$= 3^{x\log x} \times \log 3 \frac{d}{dx}(x\log x) \qquad \text{[Using chain rule]}$$
$$= 3^{x\log x} \times \log 3\left[x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(x)\right] \qquad \text{[Using chain rule]}$$
$$= 3^{x\log x} \times \log 3\left[\frac{x}{x} + \log x\right]$$
$$= 3^{x\log x}(1+\log x) \times \log 3$$

So,

$$\frac{d}{dx}\left(3^{x}\log x\right) = \log 3 \times 3^{x\log x}(1+\log x).$$

Differentiation Ex 11.2 Q16

Let $y = \sqrt{\frac{1+\sin x}{1-\sin x}}$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1+\sin x}{1-\sin x}\right)^{\frac{1}{2}}$$
$$= \frac{1}{2}\left(\frac{1+\sin x}{1-\sin x}\right)^{\frac{1}{2}-1}\frac{d}{dx}\left(\frac{1+\sin x}{1-\sin x}\right)$$
$$= \frac{1}{2}\left(\frac{1-\sin x}{1+\sin x}\right)^{\frac{1}{2}}\left[\frac{(1-\sin x)(\cos x)-(1+\sin x)(-\cos x)}{(1-\sin x)^2}\right]$$
$$= \frac{1}{2}\frac{(1-\sin x)^{\frac{1}{2}}}{(1+\sin x)^{\frac{1}{2}}}\left[\frac{\cos x - \cos x\sin x + \cos x + \sin x\cos x}{(1-\sin x)^2}\right]$$
$$= \frac{1}{2}\times\frac{2\cos x}{\sqrt{1+\sin x}\,(1-\sin x)^{\frac{3}{2}}}$$
$$= \frac{\cos x}{\sqrt{1+\sin x}\,(1-\sin x)^{\frac{3}{2}}}$$
$$= \frac{\cos x}{\sqrt{1+\sin x}\sqrt{1-\sin x}\,(1-\sin x)}$$
$$= \frac{\cos x}{\sqrt{1-\sin^2 x}\,(1-\sin x)}$$
$$= \frac{\cos x}{\cos x\,(1-\sin x)} \qquad \left[\text{Using } 1-\sin^2 x = \cos^2 x\right]$$
$$= \frac{1}{(1-\sin x)}\times\frac{(1+\sin x)}{(1+\sin x)}$$
$$= \frac{(1+\sin x)}{\left(1-\sin^2 x\right)}$$
$$= \frac{1+\sin x}{\cos^2 x}$$

Thus, $\frac{dy}{dx} = \frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}$

$\Rightarrow \frac{dy}{dx} = sec^2 x + \tan x sec x$

$\Rightarrow \frac{dy}{dx} = sec x[\tan x + sec x]$

Differentiation Ex 11.2 Q17

Let $\quad y = \sqrt{\dfrac{1-x^2}{1+x^2}}$

$$y = \left(\frac{1-x^2}{1+x^2}\right)^{\frac{1}{2}}$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(\frac{1-x^2}{1+x^2}\right)^{\frac{1}{2}-1} \frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right) \qquad \text{[Using chain rule]}$$

$$= \frac{1}{2}\left(\frac{1-x^2}{1+x^2}\right)^{\frac{-1}{2}}\left[\frac{\left(1+x^2\right)\frac{d}{dx}\left(1-x^2\right)-\left(1-x^2\right)\frac{d}{dx}\left(1+x^2\right)}{\left(1+x^2\right)^2}\right] \qquad \text{[Using quotient rule]}$$

$$= \frac{1}{2}\left(\frac{1+x^2}{1-x^2}\right)^{\frac{1}{2}}\left[\frac{\left(1+x^2\right)(-2x)-\left(1-x^2\right)(2x)}{\left(1+x^2\right)^2}\right]$$

$$= \frac{1}{2}\left(\frac{1+x^2}{1-x^2}\right)^{\frac{1}{2}}\left[\frac{-2x-2x^3-2x+2x^3}{\left(1+x^2\right)^2}\right]$$

$$= \frac{1}{2}\frac{-4x}{\sqrt{1-x^2}\left(1+x^2\right)^{\frac{3}{2}}}$$

So,

$$\frac{d}{dx}\left(\sqrt{\frac{1-x^2}{1+x^2}}\right) = \frac{-2x}{\sqrt{1-x^2}\left(1+x^2\right)^{\frac{3}{2}}}.$$

Differentiation Ex 11.2 Q18

Let $\quad y = (\log\sin x)^2$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}(\log\sin x)^2$$

$$= 2(\log\sin x)\frac{d}{dx}(\log\sin x) \qquad \text{[Using chain rule]}$$

$$= 2(\log\sin x)\times\frac{1}{\sin x}\frac{d}{dx}(\log x)$$

$$= 2(\log\sin x)\times\frac{1}{\sin x}\times\frac{1}{x}$$

$$= \frac{2\log\sin x}{x\sin x}$$

So,

$$\frac{d}{dx}(\log\sin x)^2 = \frac{2\log\sin x}{x\sin x}$$

Differentiation Ex 11.2 Q19

Let $y = \sqrt{\frac{1+x}{1-x}}$

$\Rightarrow \quad y = \left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}-1}\frac{d}{dx}\left(\frac{1+x}{1-x}\right) \qquad \text{[Using chain rule]}$$

$$= \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{\frac{-1}{2}}\left[\frac{(1-x)\frac{d}{dx}(1+x)-(1+x)\frac{d}{dx}(1-x)}{(1-x)^2}\right] \qquad \text{[Using chain rule]}$$

$$= \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{\frac{1}{2}}\left[\frac{(1-x)(1)-(1+x)(-1)}{(1-x)^2}\right]$$

$$= \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{\frac{1}{2}}\left[\frac{1-x+1+x}{(1-x)^2}\right]$$

$$= \frac{1}{2}\frac{(1-x)^{\frac{1}{2}}}{(1+x)^{\frac{1}{2}}} \times \frac{2}{(1-x)^2}$$

$$= \frac{1}{\sqrt{1+x}\,(1-x)^{3/2}}$$

So,

$$\frac{d}{dx}\left(\sqrt{\frac{1+x}{1-x}}\right) = \frac{1}{\sqrt{1+x}\,(1-x)^{3/2}}$$

Differentiation Ex 11.2 Q20

Let $y = \sin\left(\frac{1+x^2}{1-x^2}\right)$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sin\left(\frac{1+x^2}{1-x^2}\right)\right)$$

$$= \cos\left(\frac{1+x^2}{1-x^2}\right)\frac{d}{dx}\left(\frac{1+x^2}{1-x^2}\right) \qquad \text{[Using chain rule]}$$

$$= \cos\left(\frac{1+x^2}{1-x^2}\right)\left[\frac{\left(1-x^2\right)\frac{d}{dx}\left(1+x^2\right)-\left(1+x^2\right)\frac{d}{dx}\left(1-x^2\right)}{(1-x)^2}\right] \qquad \text{[Using quotient rule]}$$

$$= \cos\left(\frac{1+x^2}{1-x^2}\right)\left[\frac{\left(1-x^2\right)(2x)-\left(1+x^2\right)(-2x)}{\left(1-x^2\right)^2}\right]$$

$$= \cos\left(\frac{1+x^2}{1-x^2}\right)\left[\frac{2x-2x^3+2x+2x^3}{\left(1-x^2\right)^2}\right]$$

$$= \frac{4x}{\left(1-x^2\right)^2}\cos\left(\frac{1+x^2}{1-x^2}\right)$$

So,

$$\frac{d}{dx}\left(\sin\left(\frac{1+x^2}{1-x^2}\right)\right) = \frac{4x}{\left(1-x^2\right)^2}\cos\left(\frac{1+x^2}{1-x^2}\right).$$

Differentiation Ex 11.2 Q21

Let $y = e^{3x}\cos 2x$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{3x}\cos 2x\right)$$
$$= e^{3x} \times \frac{d}{dx}(\cos 2x) + \cos 2x \frac{d}{dx}\left(e^{3x}\right) \qquad \text{[Using product rule]}$$
$$= e^{3x} \times (-\sin 2x)\frac{d}{dx}(2x) + \cos 2x e^{3x}\frac{d}{dx}(3x) \qquad \text{[Using chain rule]}$$
$$= -2e^{3x}\sin 2x + 3e^{3x}\cos 2x$$
$$= e^{3x}(3\cos 2x - 2\sin 2x)$$

so,

$$\frac{d}{dx}\left(e^{3x}\cos 2x\right) = e^{3x}(3\cos 2x - 2\sin 2x).$$

Differentiation Ex 11.2 Q22

Let $y = \sin(\log\sin x)$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\sin(\log\sin x)$$
$$= \cos(\log\sin x)\frac{d}{dx}(\log\sin x) \qquad \text{[Using chain rule]}$$
$$= \cos(\log\sin x) \times \frac{1}{\sin x}\frac{d}{dx}0\sin x$$
$$= \cos(\log\sin x)\frac{\cos x}{\sin x}$$
$$= \cos(\log s\sin x) \times \cot x$$

Hence,

$$\frac{d}{dx}\left(\sin(\log\sin x)\right) = \cos(\log\sin x) \times \cot x.$$

Differentiation Ex 11.2 Q23

Let $y = e^{\tan 3x}$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\tan 3x}\right)$$
$$= e^{\tan 3x}\frac{d}{dx}(\tan 3x) \qquad \text{[Using chain rule]}$$
$$= e^{\tan 3x} \times \sec^2 3x \times \frac{d}{dx}(3x)$$
$$=$$

So,

$$\frac{d}{dx}\left(e^{\tan 3x}\right) = 3e^{\tan 3x} \times \sec^2 3x$$

Differentiation Ex 11.2 Q24

Let $\quad y = e^{\sqrt{\cot x}}$

$\Rightarrow \quad y = e^{(\cot x)^{\frac{1}{2}}}$

Differentiate it with respect to x,

$$\begin{aligned}
\frac{dy}{dx} &= \frac{d}{dx}\left(e^{(\cot x)^{\frac{1}{2}}}\right) \\
&= e^{(\cot x)^{\frac{1}{2}}} \times \frac{d}{dx}(\cot x)^{\frac{1}{2}} && \text{[Using chain rule]} \\
&= e^{\sqrt{\cot x}} \times \frac{1}{2}(\cot x)^{\frac{1}{2}-1}\frac{d}{dx}(\cot x) \\
&= -\frac{e^{\sqrt{\cot x}} \times \operatorname{cosec}^2 x}{2\sqrt{\cot x}}
\end{aligned}$$

So,

$$\frac{d}{dx}\left(e^{\sqrt{\cot x}}\right) = -\frac{e^{\sqrt{\cot x}} \times \operatorname{cosec}^2 x}{2\sqrt{\cot x}}$$

Differentiation Ex 11.2 Q25

Let $\quad y = \log\left(\frac{\sin x}{1+\cos x}\right)$

Differentiating with respect to x,

$$\begin{aligned}
\frac{dy}{dx} &= \frac{d}{dx}\log\left(\frac{\sin x}{1+\cos x}\right) \\
&= \frac{1}{\left(\frac{\sin x}{1+\cos x}\right)} \times \frac{d}{dx}\left(\frac{\sin x}{1+\cos x}\right) && \text{[Using chain rule]} \\
&= \left(\frac{1+\cos x}{\sin x}\right)\left[\frac{(1+\cos x)\frac{d}{dx}(\sin x) - \sin x\frac{d}{dx}(1+\cos x)}{(1+\cos x)^2}\right] && \text{[Using quotient rule]} \\
&= \frac{(1+\cos x)}{\sin x}\left[\frac{(1+cox)(\cos x) - \sin x(-\sin x)}{(1+\cos x)^2}\right] \\
&= \frac{(1+\cos x)}{\sin x}\left[\frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}\right] \\
&= \frac{(1+\cos x)}{\sin x}\left[\frac{(1+\cos x)}{(1+\cos x)^2}\right] \\
&= \frac{1}{\sin x} \\
&= \operatorname{cosec} x
\end{aligned}$$

So,

$$\frac{d}{dx}\left(\log\left(\frac{\sin x}{1+\cos x}\right)\right) = \operatorname{cosec} x.$$

Differentiation Ex 11.2 Q26

Let $y = \log\sqrt{\frac{1-\cos x}{1+\cos x}}$

$\Rightarrow \quad y = \log\left(\frac{1-\cos x}{1+\cos x}\right)^{\frac{1}{2}}$

$\Rightarrow \quad y = \frac{1}{2}\log\left(\frac{1-\cos x}{1+\cos x}\right)$ $\quad\left[\text{Using } \log a^b = b\log a\right]$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left\{\frac{1}{2}\log\left(\frac{1-\cos x}{1+\cos x}\right)\right\}$$

$$= \frac{1}{2}\times\frac{1}{\left(\frac{1-\cos x}{1+\cos x}\right)}\times\frac{d}{dx}\left(\frac{1-\cos x}{1+\cos x}\right) \quad [\text{Using chain rule}]$$

$$= \frac{1}{2}\left(\frac{1+\cos x}{1-\cos x}\right)\left[\frac{(1+\cos x)\frac{d}{dx}(1-\cos x)-(1-\cos x)\frac{d}{dx}(1+\cos x)}{(1+\cos x)^2}\right] \quad \left[\begin{array}{l}\text{Using}\\ \text{quotient}\\ \text{rule}\end{array}\right]$$

$$= \frac{1}{2}\left(\frac{1+\cos x}{1-\cos x}\right)\left[\frac{(1+\cos x)(\sin x)-(1-\cos x)(-\sin x)}{(1+\cos x)^2}\right]$$

$$= \frac{1}{2}\frac{(1+\cos x)}{(1-\cos x)}\left[\frac{\sin x+\sin x\cos x+\sin x-\sin x\cos x}{(1+\cos x)^2}\right]$$

$$= \frac{1}{2}\frac{(1+\cos x)}{(1-\cos x)}\left[\frac{2\sin x}{(1+\cos x)^2}\right]$$

$$= \frac{\sin x}{(1-\cos x)(1+\cos x)}$$

$$= \frac{\sin x}{1-\cos^2 x}$$

$$= \frac{\sin x}{\sin^2 x} \quad \left[\text{Sicne } 1-\cos^2 x = \sin^2 x\right]$$

$$= \frac{1}{\sin x}$$

$$= \operatorname{cosec} x$$

So,

$$\frac{d}{dx}\left(\log\sqrt{\frac{1-\cos x}{1+\cos x}}\right) = \operatorname{cosec} x.$$

Differentiation Ex 11.2 Q27

Let $y = \tan\left(e^{\sin x}\right)$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\tan e^{\sin x}\right]$$

$$= \sec^2\left(e^{\sin x}\right)\frac{d}{dx}\left(e^{\sin x}\right) \quad [\text{Using chain rule}]$$

$$= \sec^2\left(e^{\sin x}\right)\times e^{\sin x}\times\frac{d}{dx}(\sin x)$$

$$= \cos x\sec^2\left(e^{\sin x}\right)\times e^{\sin x}$$

So,

$$\frac{d}{dx}\left(\tan e^{\sin x}\right) = \sec^2\left(e^{\sin x}\right)\times e^{\sin x}\times\cos x.$$

Differentiation Ex 11.2 Q28

Let $\quad y = \log\left(x + \sqrt{x^2+1}\right)$

Differentiate with respect to x,

$$\begin{aligned}
\frac{dy}{dx} &= \frac{d}{dx}\log\left(x + \sqrt{x^2+1}\right) \\
&= \frac{1}{x+\sqrt{x^2+1}}\frac{d}{dx}\left(x + \left(x^2+1\right)^{\frac{1}{2}}\right) && \text{[Using chain rule]} \\
&= \frac{1}{x+\sqrt{x^2+1}}\left[1 + \frac{1}{2}\left(x^2+1\right)^{\frac{1}{2}-1}\frac{d}{dx}\left(x^2+1\right)\right] \\
&= \frac{1}{x+\sqrt{x^2+1}}\left[1 + \frac{1}{2\sqrt{x^2+1}}\times 2x\right] \\
&= \frac{1}{x+\sqrt{x^2+1}}\left[\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}\right] \\
&= \frac{1}{\sqrt{x^2+1}}
\end{aligned}$$

So,

$$\frac{d}{dx}\left(\log\left(x+\sqrt{x^2+1}\right)\right) = \frac{1}{\sqrt{x^2+1}}.$$

Differentiation Ex 11.2 Q29

Let $\quad y = \frac{e^x \log x}{x^2}$

Differentiate with respect to x,

$$\begin{aligned}
\frac{dy}{dx} &= \frac{x^2\frac{d}{dx}\left(e^x\log x\right) - \left(e^x\log x\right)\frac{d}{dx}\left(x^2\right)}{\left(x^2\right)^2} && \text{[Using quotient rule]} \\
&= \frac{x^2\left[e^x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}\left(e^x\right) - e^x\log x \times 2x\right]}{x^4} && \text{[Using product rule]} \\
&= \frac{x^2\left[\frac{e^x}{x} + e^x\log x\right] - 2xe^x\log x}{x^4} \\
&= \frac{\frac{x^2e^x\left(1+x\log x\right)}{x} - 2xe^x\log x}{x^4} \\
&= \frac{xe^x\left[1 + x\log x - 2\log x\right]}{x^4} \\
&= \frac{xe^x}{x^3}\left[\frac{1}{x} + \frac{x\log x}{x} - \frac{2\log x}{x}\right] \\
&= e^x x^{-2}\left[\frac{1}{x} + \log x - \frac{2}{x}\log x\right]
\end{aligned}$$

So,

$$\frac{d}{dx}\left[\frac{e^x\log x}{x^2}\right] = e^x x^{-2}\left[\frac{1}{x} + \log x - \frac{2}{x}\log x\right].$$

Differentiation Ex 11.2 Q30

Let $\quad y = \log(\operatorname{cosec}x - \cot x)$

Differentiating with respect to x,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\log(\operatorname{cosec}x - \cot x)\\ &= \frac{1}{(\operatorname{cosec}x - \cot x)}\frac{d}{dx}(\operatorname{cosec}x - \cot x) \qquad \text{[Using chain rule]}\\ &= \frac{1}{(\operatorname{cosec}x - \cot x)}\times\left(-\operatorname{cosec}x\cot x + \operatorname{cosec}^2x\right)\\ &= \frac{\operatorname{cosec}x\,(\operatorname{cosec}x - \cot x)}{(\operatorname{cosec}x - \cot x)}\\ &= \operatorname{cosec}x\end{aligned}$$

So,

$$\frac{d}{dx}\left(\log(\operatorname{cosec}x - \cot x)\right) = \operatorname{cosec}x.$$

Differentiation Ex 11.2 Q31

Let $\quad y = \dfrac{e^{2x}+e^{-2x}}{e^{2x}-e^{-2x}}$

Differentiating with respect to x,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left[\frac{e^{2x}+e^{-2x}}{e^{2x}-e^{-2x}}\right]\\ &= \left[\frac{\left(e^{2x}-e^{-2x}\right)\frac{d}{dx}\left(e^{2x}+e^{-2x}\right)-\left(e^{2x}+e^{-2x}\right)\frac{d}{dx}\left(e^{2x}-e^{-2x}\right)}{\left(e^{2x}-e^{-2x}\right)^2}\right] \qquad \begin{bmatrix}\text{Using quotient rule}\\ \text{and chain rule}\end{bmatrix}\\ &= \frac{\left(e^{2x}-e^{-2x}\right)\left[e^{2x}\frac{d}{dx}(2x)+e^{-2x}\frac{d}{dx}(-2x)\right]-\left(e^{2x}+e^{-2x}\right)\begin{bmatrix}e^{2x}\frac{d}{dx}(2x)-\\ e^{-2x}\frac{d}{dx}(-2x)\end{bmatrix}}{\left(e^{2x}-e^{-2x}\right)^2}\\ &= \frac{\left(e^{2x}-e^{-2x}\right)\left(2e^{2x}-2e^{-2x}\right)-\left(e^{2x}+e^{-2x}\right)\left(2e^{2x}+2e^{-2x}\right)}{\left(e^{2x}-e^{-2x}\right)^2}.\\ &= \frac{2\left(e^{2x}-e^{-2x}\right)^2-2\left(e^{2x}+e^{-2x}\right)^2}{\left(e^{2x}-e^{-2x}\right)^2}\\ &= \frac{2\left[e^{4x}+e^{-4x}-2e^{2x}e^{-2x}-e^{4x}-e^{-4x}-2e^{2x}e^{-2x}\right]}{\left(e^{2x}-e^{-2x}\right)^2}\\ &= \frac{-8}{\left(e^{2x}-e^{-2x}\right)^2}\end{aligned}$$

So,

$$\frac{d}{dx}\left(\frac{e^{2x}+e^{-2x}}{e^{2x}-e^{-2x}}\right) = \frac{-8}{\left(e^{2x}-e^{-2x}\right)^2}.$$

Differentiation Ex 11.2 Q32

Let $\quad y = \log\left(\frac{x^2+x+1}{x^2-x+1}\right)$

Differentiating with respect to x,.

$$\frac{dy}{dx} = \frac{d}{dx}\left[\log\left(\frac{x^2+x+1}{x^2-x+1}\right)\right]$$

$$= \frac{1}{\left(\frac{x^2+x+1}{x^2-x+1}\right)}\frac{d}{dx}\left(\frac{x^2+x+1}{x^2-x+1}\right) \qquad \text{[Using chain rule and quotient rule]}$$

$$= \left(\frac{x^2-x+1}{x^2+x+1}\right)\left[\frac{\left(x^2-x+1\right)\frac{d}{dx}\left(x^2+x+1\right)-\left(x^2+x+1\right)\frac{d}{dx}\left(x^2-x+1\right)}{\left(x^2-x+1\right)^2}\right]$$

$$= \left(\frac{x^2-x+1}{x^2+x+1}\right)\left[\frac{\left(x^2-x+1\right)(2x+1)-\left(x^2+x+1\right)(2x-1)}{\left(x^2-x+1\right)^2}\right]$$

$$= \frac{\left(x^2-x+1\right)}{\left(x^2+x+1\right)}\left[\frac{2x^3-2x^2+2x+x^2-x+1-2x^3-2x^2-2x+x^2+x+1}{\left(x^2-x+1\right)^2}\right]$$

$$= \frac{-4x^2+2x^2+2}{\left(x^2+x+1\right)\left(x^2-x+1\right)}$$

$$= \frac{-2\left(x^2-1\right)}{x^4+1+2x^2-x^2}$$

$$= \frac{-2\left(x^2-1\right)}{x^4+x^2+1}$$

So,

$$\frac{d}{dx}\left(\log\frac{x^2+x+1}{x^2-x+1}\right) = \frac{-2\left(x^2-1\right)}{x^4+x^2+1}$$

Differentiation Ex 11.2 Q33

Let $\quad y = \tan^{-1}\left(e^x\right)$

Differentiate it with respect to x,.

$$\frac{dy}{dx} = \frac{d}{dx}\left(\tan^{-1}e^x\right)$$

$$= \frac{1}{1+\left(e^x\right)^2}\frac{d}{dx}\left(e^x\right) \qquad \text{[Using chain rule]}$$

$$= \frac{1}{1+e^{2x}} \times e^x$$

$$= \frac{e^x}{1+e^{2x}}$$

So,

$$\frac{d}{dx}\left(\tan^{-1}e^x\right) = \frac{e^x}{1+e^{2x}}.$$

Differentiation Ex 11.2 Q34

Let $y = e^{\sin^{-1} 2x}$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\sin^{-1} 2x}\right)$$
$$= e^{\sin^{-1} 2x} \times \frac{d}{dx}\left(\sin^{-1} 2x\right) \qquad \text{[Using chain rule]}$$
$$= e^{\sin^{-1} 2x} \times \frac{1}{\sqrt{1-(2x)^2}} \frac{d}{dx}(2x)$$
$$= \frac{2e^{\sin-1} 2x}{\sqrt{1-4x^2}}$$

So,

$$\frac{d}{dx}\left(e^{\sin-1} 2x\right) = \frac{2e \sin^{-1} 2x}{\sqrt{1-4x^2}}.$$

Differentiation Ex 11.2 Q35

Let $y = \sin\left(2\sin^{-1} x\right)$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sin\left(2\sin^{-1} x\right)\right)$$
$$= \cos\left(2\sin^{-1} x\right) \frac{d}{dx}\left(2\sin^{-1} x\right) \qquad \text{[Using chain rule]}$$
$$= \cos\left(2\sin^{-1} x\right) \times 2 \frac{1}{\sqrt{1-x^2}}$$
$$= \frac{2\cos\left(2\sin^{-1} x\right)}{\sqrt{1-x^2}}$$

So,

$$\frac{d}{dx}\left(\sin\left(2\sin^{-1} x\right)\right) = \frac{2\cos\left(2\sin^{-1} x\right)}{\sqrt{1-x^2}}.$$

Differentiation Ex 11.2 Q36

Let $y = e^{\tan^{-1}\sqrt{x}}$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\tan^{-1}}\sqrt{x}\right)$$
$$= e^{\tan^{-1}\sqrt{x}} \frac{d}{dx}\left(\tan^{-1}\sqrt{x}\right) \qquad \text{[Using chain rule]}$$
$$= e^{\tan^{-1}\sqrt{x}} \times \frac{1}{1+\left(\sqrt{x}\right)^2} \frac{d}{dx}\left(\sqrt{x}\right)$$
$$= \frac{e^{\tan^{-1}\sqrt{x}}}{1+x} \times \frac{1}{2\sqrt{x}}$$
$$= \frac{e^{\tan-1}\sqrt{x}}{2\sqrt{x}\,(1+x)}$$

So,

$$\frac{d}{dx}\left(e^{\tan-1}\sqrt{x}\right) = \frac{e^{\tan^{-1}\sqrt{x}}}{2\sqrt{x}\,(1+x)}.$$

Differentiation Ex 11.2 Q37

Let $\quad y = \sqrt{\tan^{-1}\left(\frac{x}{2}\right)}$

$\Rightarrow \quad y = \left(\tan^{-1}\left(\frac{x}{2}\right)\right)^{\frac{1}{2}}$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\tan^{-1}\left(\frac{x}{2}\right)\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(\tan^{-1}\frac{x}{2}\right)^{\frac{1}{2}-1}\frac{d}{dx}\left(\tan^{-1}\frac{x}{2}\right) \qquad \text{[Using chain rule]}$$

$$= \frac{1}{2}\left(\tan^{-1}\frac{x}{2}\right)^{\frac{-1}{2}} \times \frac{1}{1+\left(\frac{x}{2}\right)^2} \times \frac{d}{dx}\left(\frac{x}{2}\right)$$

$$= \frac{4}{4\sqrt{\tan^{-1}\left(\frac{x}{2}\right)}\left(4+x^2\right)}$$

$$= \frac{1}{\left(4+x^2\right)\sqrt{\tan^{-1}\left(\frac{x}{2}\right)}}$$

So,

$$\frac{d}{dx}\left(\sqrt{\tan^{-1}\left(\frac{x}{2}\right)}\right) = \frac{1}{\left(4+x^2\right)\sqrt{\tan^{-1}\left(\frac{x}{2}\right)}}.$$

Differentiation Ex 11.2 Q38

Let $\quad y = \log\left(\tan^{-1}x\right)$

Differentiate with respect to x,.

$$\frac{dy}{dx} = \frac{d}{dx}\log\left(\tan^{-1}x\right)$$

$$= \frac{1}{\tan^{-1}x} \times \frac{d}{dx}\left(\tan^{-1}x\right) \qquad \text{[Using chain rule]}$$

$$= \frac{1}{\left(1+x^2\right)\tan^{-1}x}$$

So,

$$\frac{d}{dx}\left(\log\tan^{-1}x\right) = \frac{1}{\left(1+x^2\right)\tan^{-1}x}.$$

Differentiation Ex 11.2 Q39

Let $y = \dfrac{2^x \cos x}{\left(x^2+3\right)^2}$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{2^x \cos x}{\left(x^2+3\right)^2}\right]$$

$$= \left[\frac{\left(x^2+3\right)^2 \frac{d}{dx}\left(2^x \cos x\right) - \left(2^x \cos x\right)\frac{d}{dx}\left(x^2+3\right)^2}{\left[\left(x^2+3\right)^2\right]^2}\right]$$

[Using quotient rule, product rule and chain rule]

$$= \left[\frac{\left(x^2+3\right)^2\left[2^x \frac{d}{dx}\cos x + \cos x \frac{d}{dx}2^x\right] - \left(2^x \cos x\right)2\left(x^2+3\right)\frac{d}{dx}\left(x^2+3\right)}{\left(x^2+3\right)^4}\right]$$

$$= \left[\frac{\left(x^2+3\right)^2\left[-2^x \sin x + \cos x\, 2^x \log 2\right] - 2\left(2^x \cos x\right)\left(x^2+3\right)(2x)}{\left(x^2+3\right)^4}\right]$$

$$= \left[\frac{2^x\left(x^2+3\right)\left[\left(x^2+3\right)\{\cos x \log 2 - \sin x\}\right] - 4x\cos x}{\left(x^2+3\right)^4}\right]$$

$$= \frac{2^x}{\left(x^2 3\right)^2}\left[\cos x \log 2 - \sin x - \frac{4x\cos x}{\left(x^2+3\right)}\right]$$

So,

$$\frac{d}{dx}\left(\frac{2^x \cos x}{\left(x^2+3\right)^2}\right) = \frac{2^x}{\left(x^2+3\right)^2}\left[\cos x \log 2 - \sin x - \frac{4x\cos x}{\left(x^2+3\right)}\right].$$

Differentiation Ex 11.2 Q40

Let $y = x\sin 2x + 5^x + k^k + \left(\tan^2 x\right)^3$

Dfifferentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[x\sin 2x + 5^x + k^k + \left(\tan^6 x\right)\right]$$

$$= \frac{d}{dx}(x\sin 2x) + \frac{d}{dx}\left(5^x\right) + \frac{d}{dx}\left(k^k\right) + \frac{d}{dx}\left(\tan^6 x\right)$$

$$= \left[x\frac{d}{dx}(\sin 2x) + \sin 2x \frac{d}{dx}(x)\right] + 5^x \log 5 + 0 + 6\tan^5 x \frac{d}{dx}(\tan x)$$

[Using product rule and chain rule]

$$= \left[x\cos 2x \frac{d}{dx}(2x) + \sin 2x\right] + 5^x \log 5 + 6\tan^5 x \sec^2 x$$

$$= 2x\cos 2x + \sin 2x + 5^x \log 5 + 6\tan^5 x \sec^2 x$$

so,

$$\frac{d}{dx}\left(x\sin 2x + 5^x + k^k + \left(\tan^2 x\right)^3\right) = 2x\cos 2x + \sin 2x + 5^x \log 5 + 6\tan^5 x \sec^2 x.$$

Differentiation Ex 11.2 Q41

Let $\quad y = \log(3x+2) - x^2\log(2x-1)$

Differentiate with respect to x,

$$\begin{aligned}
\frac{dy}{dx} &= \frac{d}{dx}\left[\log(3x+2) - x^2\log(2x-1)\right] \\
&= \frac{d}{dx}\log(3x+2) - \frac{d}{dx}\left(x^2\log(2x-1)\right) \\
&= \frac{1}{(3x+2)}\frac{d}{dx}(3x+2) - \left[x^2\frac{d}{dx}\log(2x-1) + \log(2x-1)\frac{d}{dx}\left(x^2\right)\right] \\
&\quad \text{[Using product rule and chain rule]} \\
&= \frac{3}{3x+2} - \left[x^2 \times \frac{1}{(2x-1)}\frac{d}{dx}(2x-1) + \log(2x-1)\times 2x\right] \\
&= \frac{3}{3x+2} - \frac{2x^2}{(2x-1)} - 2x\log(2x-1)
\end{aligned}$$

So,

$$\frac{d}{dx}\left(\log(3x+2) - x^2\log(2x-1)\right) = \frac{3}{3x+2} - \frac{2x^2}{(2x-1)} - 2x\log(2x-1).$$

Differentiation Ex 11.2 Q42

Let $\quad y = \dfrac{3x^2\sin x}{\sqrt{7-x^2}}$

Differentiate it with respect to x,

$$\begin{aligned}
\frac{dy}{dx} &= \frac{d}{dx}\left(\frac{3x^2\sin x}{\left(7-x^2\right)^{\frac{1}{2}}}\right) \\
&= \frac{\left(7-x^2\right)^{\frac{1}{2}} \times \frac{d}{dx}\left(3x^2\sin x\right) - 3x^2\sin x\,\frac{d}{dx}\left(7-x^2\right)^{\frac{1}{2}}}{\left[\left(7-x^2\right)^{\frac{1}{2}}\right]^2} \\
&\quad \text{[Using quotient rule, chain and product rule]} \\
&= \left[\frac{\left(7-x^2\right)^{\frac{1}{2}} \times 3 \times \left[x^2\frac{d}{dx}\sin x + \sin x\frac{d}{dx}x^2\right] - 3x^2\sin x \times \frac{1}{2}\left(7-x^2\right)^{\frac{-1}{2}}\frac{d}{dx}\left(7-x^2\right)}{\left(7-x^2\right)}\right] \\
&= \left[\frac{\left(7-x^2\right)^{\frac{1}{2}} 3\left(x^2\cos x + 2x\sin x\right) - 3x^2\sin x \times \frac{1}{2}\left(7-x^2\right)^{\frac{-1}{2}}(-2x)}{\left(7-x^2\right)}\right] \\
&= \left[\frac{\left(7-x^2\right)^{\frac{1}{2}} \times 3\left(x^2\cos x + 2x\sin x\right)}{\left(7-x^2\right)} + \frac{3x^3\sin x\left(7-x^2\right)^{\frac{-1}{2}}}{\left(7-x^2\right)}\right] \\
&= \left[\frac{6x\sin x + 3x^2\cos x}{\sqrt{\left(7-x^2\right)}} + \frac{3x^3\sin x}{\left(7-x^2\right)^{\frac{3}{2}}}\right]
\end{aligned}$$

So,

$$\frac{d}{dx}\left(\frac{3x^2\sin x}{\sqrt{7-x^2}}\right) = \left[\frac{6x\sin x + 3x^2\cos x}{\sqrt{7-x^2}} + \frac{3x^3\sin x}{\left(7-x^2\right)^{\frac{3}{2}}}\right].$$

Differentiation Ex 11.2 Q43

Let $\quad y = \sin^2\left[\log(2x+3)\right]$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\sin^2\left(\log(2x+3)\right)\right]$$
$$= 2\sin\left(\log(2x+3)\right)\frac{d}{dx}\sin\left(\log(2x+3)\right) \quad \text{Using chain rule}$$
$$= 2\sin\left(\log(2x+3)\right)\cos\left(\log(2x+3)\right)\frac{d}{dx}\log(2x+3)$$
$$= \sin\left(2\log(2x+3)\right)\times\frac{1}{(2x+3)}\frac{d}{dx}(2x+3)$$

$$\left[\text{Since, } 2\sin A\cos A = \sin^2 A\right]$$

$$= \sin\left(2\log(2x+3)\right)\times\frac{2}{(2x+3)}$$

So,
$$\frac{d}{dx}\left(\sin^2\log(2x+3)\right) = \sin\left(2\log(2x+3)\right)\times\frac{2}{(2x+3)}.$$

Differentiation Ex 11.2 Q44

Let $\quad y = e^x\log\sin 2x$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[e^x\log\sin 2x\right]$$
$$= e^x\frac{d}{dx}\log\sin 2x + \log\sin 2x\frac{d}{dx}\left(e^x\right)$$

$$\left[\text{Using product rule and chain rule}\right]$$

$$= e^x\frac{1}{\sin 2x}\frac{d}{dx}(\sin 2x) + \log\sin 2x\left(e^x\right)$$
$$= \frac{e^x}{\sin 2x}\cos 2x\frac{d}{dx}(2x) + e^x\log\sin 2x$$
$$= \frac{2\cos 2xe^x}{\sin 2x} + e^x\log\sin 2x$$
$$= e^x\left(2\cot 2x + \log\sin 2x\right)$$

so,
$$\frac{d}{dx}\left(e^x\log\sin 2x\right) = e^x\left(2\cot 2x + \log\sin 2x\right).$$

Differentiation Ex 11.2 Q45

Let $\quad y = \dfrac{\sqrt{x^2+1}+\sqrt{x^2-1}}{\sqrt{x^2+1}-\sqrt{x^2-1}}$

$$\Rightarrow \quad y = \frac{\left(x^2+1\right)^{\frac{1}{2}}+\left(x^2-1\right)^{\frac{1}{2}}}{\left(x^2+1\right)^{\frac{1}{2}}-\left(x^2-1\right)^{\frac{1}{2}}}$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{\left(x^2+1\right)^{\frac{1}{2}}+\left(x^2-1\right)^{\frac{1}{2}}}{\left(x^2+1\right)^{\frac{1}{2}}-\left(x^2-1\right)^{\frac{1}{2}}}\right]$$

$$= \left[\frac{\left\{\left(x^2+1\right)^{\frac{1}{2}}-\left(x^2-1\right)^{\frac{1}{2}}\right\}\frac{d}{dx}\left\{\left(x^2+1\right)^{\frac{1}{2}}+\left(x^2-1\right)^{\frac{1}{2}}\right\}-\left\{\left(x^2+1\right)^{\frac{1}{2}}+\left(x^2-1\right)^{\frac{1}{2}}\right\}\frac{d}{dx}\left\{\left(x^2+1\right)^{\frac{1}{2}}-\left(x^2-1\right)^{\frac{1}{2}}\right\}}{\left\{\left(x^2+1\right)^{\frac{1}{2}}-\left(x^2-1\right)^{\frac{1}{2}}\right\}^2}\right]$$

[Using quotient rule and chian rule]

$$= \left[\frac{\left\{\left(x^2+1\right)^{\frac{1}{2}}-\left(x^2-1\right)^{\frac{1}{2}}\right\}\left[\frac{1}{2}\left(x^2+1\right)^{\frac{-1}{2}}\frac{d}{dx}\left(x^2+1\right)+\frac{1}{2}\left(x^2-1\right)^{\frac{-1}{2}}\frac{d}{dx}\left(x^2-1\right)\right]}{\left[\left(x^2+1\right)+\left(x^2-1\right)-2\sqrt{x^4-1}\right]}\right]$$

$$-\left[\frac{\left\{\left(x^2+1\right)^{\frac{1}{2}}+\left(x^2-1\right)^{\frac{1}{2}}\right\}\frac{1}{2}\left[\left(x^2+1\right)^{\frac{-1}{2}}\frac{d}{dx}\left(x^2+1\right)-\frac{1}{2}\left(x^2-1\right)^{\frac{-1}{2}}\frac{d}{dx}\left(x^2-1\right)\right]}{\left[\left(x^2+1\right)\left(x^2-1\right)-2\sqrt{x^4-1}\right]}\right]$$

$$= \left[\frac{\left(\sqrt{x^2+1}-\sqrt{x^2-1}\right)\left(\frac{2x}{2\sqrt{x^2+1}}+\frac{2x}{2\sqrt{x^2-1}}\right)}{\left[2x^2-2\sqrt{x^4-1}\right]}\right] - \left[\frac{\left(\sqrt{x^2+1}+\sqrt{x^2-1}\right)\left(\frac{2x}{2\sqrt{x^2+1}}-\frac{2x}{2\sqrt{x^2-1}}\right)}{\left[2x^2-2\sqrt{x^4-1}\right]}\right]$$

$$= \left[\frac{x\left(\sqrt{x^2+1}-\sqrt{x^2-1}\right)\left(\sqrt{x^2-1}+\sqrt{x^2+1}\right)}{2\left[x^2-\sqrt{x^4-1}\right]\left(\sqrt{x^2+1}\sqrt{x^2-1}\right)}\right] - \left[\frac{x\left(\sqrt{x^2+1}+\sqrt{x^2-1}\right)\left(\sqrt{x^2-1}-\sqrt{x^2+1}\right)}{2\left[x^2-\sqrt{x^4-1}\right]\left(\sqrt{x^2+1}\sqrt{x^2-1}\right)}\right]$$

$$= \left[\frac{x\left(x^2+1-x^2+1\right)-x\left(x^2-1-x^2-1\right)}{2\left[x^2-\sqrt{x^4-1}\right]\sqrt{x^4-1}}\right]$$

$$= \left[\frac{4x}{2\left(x^2-\sqrt{x^4-1}\right)\sqrt{x^4-1}}\right]$$

$$= 2x\left[\frac{1\times\left(x^2+\sqrt{x^4-1}\right)}{\left(x^2-\sqrt{x^4-1}\sqrt{x^4-1}\times\left(x^2+\sqrt{x^4-1}\right)\right)}\right]$$

Multiplying and divide by $\left(x^2+\sqrt{x^4-1}\right)$,

$$= 2x\left[\frac{x^2+\sqrt{x^4-1}}{\left(x^4-x^4+1\right)\sqrt{x^4-1}}\right]$$

$$= 2x\left[\frac{x^2+\sqrt{x^4-1}}{\sqrt{x^4-1}}\right]$$

$$= \frac{2x^3}{\sqrt{x^4-1}}+2x$$

So,

$$\frac{d}{dx}\left[\frac{\sqrt{x^2+1}+\sqrt{x^2-1}}{\sqrt{x^2+1}-\sqrt{x^2-1}}\right] = \frac{2x^3}{\sqrt{x^4-1}}+2x.$$

Differentiation Ex 11.2 Q46

Let $\quad y = \log\left[x + 2 + \sqrt{x^2 + 4x + 1}\right]$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\log\left[x + 2 + \sqrt{x^2 + 4x + 1}\right]$$

$$= \frac{1}{\left[x + 2 + \sqrt{x^4 + 4x + 1}\right]}\frac{d}{dx}\left[x + 2 + \left(x^2 + 4x + 1\right)^{\frac{1}{2}}\right]$$

[using chain rule]

$$= \frac{1}{\left[x + 2 + \sqrt{x^4 + 4x + 1}\right]} \times \left[1 + 0 + \frac{1}{2}\left(x^2 + 4x + 1\right)^{\frac{-1}{2}}\frac{d}{dx}\left(x^2 + 4x + 1\right)\right]$$

$$= \frac{1 + \frac{(2x + 4)}{2\left(\sqrt{x^2 + 4x + 1}\right)}}{\left[x + 2 + \sqrt{x^4 + 4x + 1}\right]}$$

$$= \frac{\sqrt{x^2 + 4x + 1} + x + 2}{\left[x + 2 + \sqrt{x^2 + 4x + 1}\right] \times \sqrt{x^2 + 4x + 1}}$$

$$= \frac{1}{\sqrt{x^2 + 4x + 1}}$$

So,

$$\frac{d}{dx}\left[\log\left\{x + 2 + \sqrt{x^2 + 4x + 1}\right\}\right] = \frac{1}{\sqrt{x^2 + 4x + 1}}.$$

Differentiation Ex 11.2 Q47

Let $\quad y = \left(\sin^{-1} x^4\right)^4$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sin^{-1} x^4\right)^4$$

$$= 4\left(\sin^{-1} x^4\right)\frac{d}{dx}\left(\sin^{-1} x^4\right) \qquad \text{[Using chain rule]}$$

$$= 4\left(\sin^{-1} x^4\right)^3 \frac{1}{\sqrt{1 - \left(x^4\right)^2}}\frac{d}{dx}\left(x^4\right)$$

$$= 4\left(\sin^{-1} x^4\right)^3 \frac{4x^3}{\sqrt{1 - x^8}}$$

$$= \frac{16x^3\left(\sin^{-1} x^4\right)^3}{\sqrt{1 - x^8}}$$

So,

$$\frac{d}{dx}\left(\sin^{-1} x^4\right) = \frac{16x^3\left(\sin^{-1} x^4\right)^3}{\sqrt{1 - x^8}}.$$

Differentiation Ex 11.2 Q48

Let $y = \sin^{-1}\left(\frac{x}{\sqrt{x^2+a^2}}\right)$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\sin^{-1}\left(\frac{x}{\sqrt{x^2+a^2}}\right)$$

$$= \frac{1}{\sqrt{1-\left(\frac{x}{\sqrt{x^2+a^2}}\right)^2}} \times \frac{d}{dx}\left(\frac{x}{\sqrt{x^2+a^2}}\right) \qquad \text{[Using chain rule and quotient rule]}$$

$$= \frac{1}{\sqrt{1-\left(\frac{x}{\sqrt{x^2+a^2}}\right)^2}} \times \left[\frac{\left(x^2+a^2\right)^{\frac{1}{2}}\frac{d}{dx}(x) - \frac{d}{dx}\left(x^2+a^2\right)^{\frac{1}{2}}}{\left[\left(x^2+a^2\right)^{\frac{1}{2}}\right]^2}\right]$$

$$= \frac{\sqrt{x^2+a^2}}{\sqrt{x^2+a^2-x^2}}\left[\frac{\sqrt{x^2+a^2} - x \times \frac{1}{2\sqrt{x^2+a^2}}\frac{d}{dx}\left(x^2+a^2\right)}{\left(x^2+a^2\right)}\right]$$

$$= \frac{\sqrt{x^2+a^2}}{a\left(x^2+a^2\right)}\left[\sqrt{x^2+a^2} - \frac{x}{2\sqrt{x^2+a^2}} \times 2x\right]$$

$$= \frac{\sqrt{x^2+a^2}}{a\left(x^2+a^2\right)}\left[\frac{x^2+a^2-x^2}{\sqrt{x^2+a^2}}\right]$$

$$= \frac{a^2}{a\left(x^2+a^2\right)}$$

$$= \frac{a}{\left(a^2+x^2\right)}$$

So,

$$\frac{d}{dx}\left(\sin^{-1}\frac{x}{\sqrt{x^2+a^2}}\right) = \frac{a}{a^2+x^2}$$

Differentiation Ex 11.2 Q49

Consider

$$y = \frac{e^x \sin x}{\left(x^2+2\right)^3}$$

Differentiating it with respect toxand applying the chain and product rule, we get

$$\frac{dy}{dx} = \frac{\left(x^2+2\right)^3\frac{d}{dx}\left(e^x \sin x\right) - e^x \sin x\frac{d}{dx}\left(x^2+2\right)^3}{\left[\left(x^2+2\right)^3\right]^2}$$

$$= \frac{\left(x^2+2\right)^3\left[e^x \cos x + \sin x e^x\right] - e^x \sin x 3\left(x^2+2\right)^2(2x)}{\left(x^2+2\right)^6}$$

$$= \frac{\left(x^2+2\right)^3\left[e^x \cos x + \sin x e^x\right] - 6xe^x \sin x\left(x^2+2\right)^2}{\left(x^2+2\right)^6}$$

$$= \frac{\left(x^2+2\right)^2\left[\left(x^2+2\right)\left(e^x \cos x + \sin x e^x\right) - 6xe^x \sin x\right]}{\left(x^2+2\right)^6}$$

$$= \frac{x^2e^x \cos x + x^2 \sin x e^x + 2e^x \cos x + 2\sin x e^x - 6xe^x \sin x}{\left(x^2+2\right)^4}$$

$$= \frac{e^x \sin x}{\left(x^2+2\right)^3} + \frac{e^x \cos x}{\left(x^2+2\right)^3} - \frac{6xe^x \sin x}{\left(x^2+2\right)^4}$$

Therefore,

$$\frac{dy}{dx} = \frac{e^x \sin x}{\left(x^2+2\right)^3} + \frac{e^x \cos x}{\left(x^2+2\right)^3} - \frac{6xe^x \sin x}{\left(x^2+2\right)^4}$$

Differentiation Ex 11.2 Q50

Consider

$y = 3e^{-3x}\log(1+x)$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx} = 3\frac{d}{dx}\left[e^{-3x}\log(1+x)\right]$$

$$\frac{dy}{dx} = 3\left(e^{-3x}\frac{1}{1+x} + \log(1+x)\left(-3e^{-3x}\right)\right)$$

$$= 3\left(\frac{e^{-3x}}{1+x} - 3e^{-3x}\log(1+x)\right)$$

$$= 3e^{-3x}\left(\frac{1}{1+x} - 3\log(1+x)\right)$$

Differentiation Ex 11.2 Q51

Consider

$$y = \frac{x^2+2}{\sqrt{\cos x}}$$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx} = \frac{\sqrt{\cos x}\frac{d}{dx}\left(x^2+2\right) - \left(x^2+2\right)\frac{d}{dx}\sqrt{\cos x}}{\left(\sqrt{\cos x}\right)^2}$$

$$= \frac{2x\sqrt{\cos x} - \left(x^2+2\right)\left(-\frac{1}{2}\frac{\sin x}{\sqrt{\cos x}}\right)}{\cos x}$$

$$= \frac{2x\sqrt{\cos x} + \frac{\left(x^2+2\right)\sin x}{2\sqrt{\cos x}}}{\cos x}$$

$$= \frac{4x\cos x + \left(x^2+2\right)\sin x}{2(\cos x)^{\frac{3}{2}}}$$

$$= \frac{2x}{\sqrt{\cos x}} + \frac{1}{2}\frac{\left(x^2+2\right)\sin x}{(\cos x)^{\frac{3}{2}}}$$

Therefore,

$$\frac{dy}{dx} = \frac{2x}{\sqrt{\cos x}} + \frac{1}{2}\frac{\left(x^2+2\right)\sin x}{(\cos x)^{\frac{3}{2}}}$$

Differentiation Ex 11.2 Q52

Consider

$$y = \frac{x^2\left(1-x^2\right)^3}{\cos 2x}$$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx} = \frac{\cos 2x\frac{d}{dx}x^2\left(1-x^2\right)^3 - x^2\left(1-x^2\right)^3\frac{d}{dx}\cos 2x}{\cos^2 2x}$$

$$= \frac{\cos 2x\left[x^2\frac{d}{dx}\left(1-x^2\right)^3 + \left(1-x^2\right)^3\frac{d}{dx}x^2 - x^2\left(1-x^2\right)^3(-2\sin 2x)\right]}{\cos^2 2x}$$

$$= \frac{\cos 2x\left[-6x^3\left(1-x^2\right)^2 + \left(1-x^2\right)^3 2x + 2x^2\left(1-x^2\right)^3\sin 2x\right]}{\cos^2 2x}$$

$$= \frac{2x\left(1-x^2\right)^2}{\cos 2x} - \frac{6x^3\left(1-x^2\right)^2}{\cos 2x} + \frac{2x^2\left(1-x^2\right)^3\sin 2x}{\cos^2 2x}$$

$$= 2x\left(1-x^2\right)\sec 2x\left\{1-4x^2+x\left(1-x^2\right)\tan 2x\right\}$$

Therefore,

$$\frac{dy}{dx} = 2x\left(1-x^2\right)\sec 2x\left\{1-4x^2+x\left(1-x^2\right)\tan 2x\right\}$$

Differentiation Ex 11.2 Q53

Consider

$$y = \log(3x+2) - x^2\log(2x-1)$$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[\log(3x+2) - x^2\log(2x-1)\right]$$

$$\frac{dy}{dx} = \frac{3}{3x+2} - \left[x^2\frac{d}{dx}\log(2x-1) + \log(2x-1)\frac{d}{dx}x^2\right]$$

$$\frac{dy}{dx} = \frac{3}{3x+2} - \left(\frac{2x^2}{2x-1} + 2x\log(2x-1)\right)$$

$$\frac{dy}{dx} = \frac{3}{3x+2} - \frac{2x^2}{2x-1} - 2x\log(2x-1)$$

Therefore,

$$\frac{dy}{dx} = \frac{3}{3x+2} - \frac{2x^2}{2x-1} - 2x\log(2x-1)$$

Differentiation Ex 11.2 Q54

Consider

$$y = e^{ax}\sec x\tan 2x$$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{ax}\sec x\tan 2x\right)$$

$$= e^{ax}\frac{d}{dx}\sec x\tan 2x + \sec x\tan 2x\frac{d}{dx}e^{ax}$$

$$= e^{ax}\left[\sec x\tan x\tan 2x + \left(2+2\tan^2 2x\right)\sec x\right] + ae^{ax}\sec x\tan 2x$$

$$= e^{ax}\left[\sec x\tan x\tan 2x + 2\sec x + 2\tan^2 2x\sec x\right] + ae^{ax}\sec x\tan 2x$$

$$= ae^{ax}\sec x\tan 2x + e^{ax}\sec x\tan x\tan 2x + e^{ax}\sec x\left(2+2\tan^2 2x\right)$$

$$\frac{dy}{dx} = e^{ax}\sec x\left\{a\tan 2x + \tan x\tan 2x + 2\sec^2 2x\right\}$$

Therefore,

$$\frac{dy}{dx} = e^{ax}\sec x\left\{a\tan 2x + \tan x\tan 2x + 2\sec^2 2x\right\}$$

Differentiation Ex 11.2 Q55

Consider

$$y = \log\left(\cos x^2\right)$$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx} = \frac{d}{dx}\log\left(\cos x^2\right)$$

$$= \frac{-2x\sin x^2}{\cos x^2}$$

$$\frac{dy}{dx} = -2x\tan x^2$$

Therefore,

$$\frac{dy}{dx} = -2x\tan x^2$$

Differentiation Ex 11.2 Q56

Consider

$$y = \cos\left(\log x\right)^2$$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx} = \frac{d}{dx}\cos\left(\log x\right)^2$$

$$= -\sin\left(\log x\right)^2\frac{d}{dx}\left(\log x\right)^2$$

$$= -\sin\left(\log x\right)^2\frac{2\log x}{x}$$

$$\frac{dy}{dx} = \frac{-2\log x\sin\left(\log x\right)^2}{x}$$

Therefore,

$$\frac{dy}{dx} = \frac{-2\log x\sin\left(\log x\right)^2}{x}$$

Differentiation Ex 11.2 Q57

Consider

$$y = \log\sqrt{\frac{x-1}{x+1}}$$

Differentiating it with respect to x and applying the chain and product rule, we get

$$y = \log\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$$

$$y = \frac{1}{2}\log\left(\frac{x-1}{x+1}\right)$$

$$y = \frac{1}{2}\left[\log\left(x-1\right) - \log\left(x+1\right)\right]$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{d}{dx}\log\left(x-1\right) - \frac{d}{dx}\log\left(x+1\right)\right]$$

$$= \frac{1}{2}\left(\frac{1}{x-1} - \frac{1}{x+1}\right)$$

$$= \frac{1}{2}\left(\frac{2}{x^2-1}\right)$$

$$\frac{dy}{dx} = \frac{1}{x^2-1}$$

Therefore,

$$\frac{dy}{dx} = \frac{1}{x^2-1}$$

Differentiation Ex 11.2 Q58

Here $y=\log\left\{\sqrt{x-1}-\sqrt{x+1}\right\}$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx}=\frac{d}{dx}\log\left\{\sqrt{x-1}-\sqrt{x+1}\right\}$$

$$\frac{dy}{dx}=\frac{1}{\left\{\sqrt{x-1}-\sqrt{x+1}\right\}}\frac{d}{dx}\left(\sqrt{x-1}-\sqrt{x+1}\right)$$

$$=\frac{1}{\left\{\sqrt{x-1}-\sqrt{x+1}\right\}}\left[\frac{d}{dx}\sqrt{x-1}-\frac{d}{dx}\sqrt{x+1}\right]$$

$$=\frac{1}{\left\{\sqrt{x-1}-\sqrt{x+1}\right\}}\left[\frac{1}{2}(x-1)^{-\frac{1}{2}}-\frac{1}{2}(x+1)^{-\frac{1}{2}}\right]$$

$$=\frac{1}{2}\frac{1}{\left\{\sqrt{x-1}-\sqrt{x+1}\right\}}\left(\frac{1}{\sqrt{x-1}}-\frac{1}{\sqrt{x+1}}\right)$$

$$=\frac{1}{2}\frac{1}{\cancel{\left\{\sqrt{x-1}-\sqrt{x+1}\right\}}}\left(\frac{-\cancel{\left\{\sqrt{x-1}-\sqrt{x+1}\right\}}}{\left(\sqrt{x-1}\right)\left(\sqrt{x+1}\right)}\right)$$

$$=\frac{-1}{2}\left(\frac{1}{\left(\sqrt{x-1}\right)\left(\sqrt{x+1}\right)}\right)$$

$$\frac{dy}{dx}=\frac{-1}{2\sqrt{x^2-1}}$$

Therefore,

$$\frac{dy}{dx}=\frac{-1}{2\sqrt{x^2-1}}$$

Differentiation Ex 11.2 Q59

Here $y=\sqrt{x+1}+\sqrt{x-1}$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx}=\frac{d}{dx}\sqrt{x+1}+\frac{d}{dx}\sqrt{x-1}$$

$$=\frac{1}{2}(x+1)^{-\frac{1}{2}}+\frac{1}{2}(x-1)^{-\frac{1}{2}}$$

$$=\frac{1}{2}\left(\frac{1}{\sqrt{x+1}}+\frac{1}{\sqrt{x-1}}\right)$$

$$=\frac{1}{2}\left(\frac{\sqrt{x-1}+\sqrt{x+1}}{\left(\sqrt{x+1}\right)\left(\sqrt{x-1}\right)}\right)$$

$$\frac{dy}{dx}=\frac{1}{2}\left(\frac{y}{\left(\sqrt{x^2-1}\right)}\right)$$

$$\sqrt{x^2-1}\frac{dy}{dx}=\frac{1}{2}y$$

Differentiation Ex 11.2 Q60

Here $y=\frac{x}{x+2}$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x}{x+2}\right)$$

$$=\frac{(x+2)\frac{dx}{dx}-x\frac{d}{dx}(x+2)}{(x+2)^2}$$

$$=\frac{x+2-x}{(x+2)^2}$$

$$=\frac{x+2}{(x+2)^2}-\frac{x}{(x+2)^2}$$

$$=\frac{1}{x+2}-\frac{xy^2}{x^2}\qquad\left[\text{Since } x+2=\frac{x}{y}\right]$$

$$=\frac{y}{x}-\frac{y^2}{x}$$

$$\frac{dy}{dx}=\frac{1}{x}y(1-y)$$

$$x\frac{dy}{dx}=(1-y)y$$

Differentiation Ex 11.2 Q61

Here $y=\log\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx}=\frac{d}{dx}\log\left(x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)$$

$$=\frac{1}{x^{\frac{1}{2}}+x^{-\frac{1}{2}}}\frac{d}{dx}\left(x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)$$

$$=\frac{1}{\sqrt{x}+\frac{1}{\sqrt{x}}}\left(\frac{1}{2}x^{-\frac{1}{2}}-\frac{1}{2}x^{-\frac{1}{2}}x^{-1}\right)$$

$$=\frac{1}{2}\frac{\sqrt{x}}{x+1}\left(\frac{1}{\sqrt{x}}-\frac{1}{x\sqrt{x}}\right)$$

$$=\frac{1}{2}\frac{\cancel{\sqrt{x}}}{x+1}\left(\frac{x-1}{x\cancel{\sqrt{x}}}\right)$$

$$\frac{dy}{dx}=\frac{x-1}{2x(x+1)}$$

Differentiation Ex 11.2 Q62

Given, $y=\sqrt{\frac{1+e^x}{1-e^x}}$

Differentiate with respect to x,

$$\frac{dy}{dx}=\frac{d}{dx}\left(\sqrt{\frac{1+e^x}{1-e^x}}\right)$$

$$=\frac{1}{2\sqrt{\left(\frac{1+e^x}{1-e^x}\right)}}\times\frac{d}{dx}\left(\frac{1+e^x}{1-e^x}\right) \qquad \text{[Using chain rule, quotient rule]}$$

$$=\frac{1}{2}\times\sqrt{\frac{1-e^x}{1+e^x}}\left[\frac{\left(1-e^x\frac{d}{dx}\left(1+e^x\right)-\left(1+e^x\right)\frac{d}{dx}\left(1-e^x\right)\right)}{\left(1-e^x\right)^2}\right]$$

$$=\frac{1}{2}\sqrt{\frac{1-e^x}{1+e^x}}\left[\frac{\left(1-e^x\right)e^x+\left(1+e^x\right)e^x}{\left(1-e^x\right)2}\right]$$

$$=\frac{1}{2}\sqrt{\frac{1-e^x}{1+e^x}}\times\left[\frac{2e^x}{\left(1-e^x\right)^2}\right]$$

$$=\frac{e^x}{\sqrt{\left(1+e^x\right)}\sqrt{\left(1-e^x\right)}}\frac{1}{\left(1-e^x\right)}$$

$$\frac{dy}{dx}=\frac{e^x}{\left(1-e^x\right)\sqrt{1-e^{2x}}}.$$

Differentiation Ex 11.2 Q63

Given, $y = \sqrt{x} + \frac{1}{\sqrt{x}}$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$$
$$= \frac{d}{dx}\left(\sqrt{x}\right) + \frac{d}{dx}\left(x^{-\frac{1}{2}}\right)$$
$$= \frac{1}{2\sqrt{x}} + \left(-\frac{1}{2} \times x^{-\frac{1}{2}-1}\right)$$
$$= \frac{1}{2\sqrt{x}} - \frac{1}{2x\sqrt{x}}$$
$$\frac{dy}{dx} = \frac{x-1}{2x\sqrt{x}}$$
$$2x\frac{dy}{dx} = \frac{x-1}{\sqrt{x}}$$
$$\Rightarrow \quad 2x\frac{dy}{dx} = \frac{x}{\sqrt{x}} - \frac{1}{\sqrt{x}}$$
$$\Rightarrow \quad 2x\frac{dy}{dx} = \sqrt{x} - \frac{1}{\sqrt{x}}.$$

Differentiation Ex 11.2 Q64

Given, $y = \frac{x\sin^{-1}x}{\sqrt{1-x^2}}$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x\sin^{-1}x}{\sqrt{1-x^2}}\right)$$
$$= \left[\frac{\sqrt{1-x^2}\frac{d}{dx}\left(x\sin^{-1}x\right) - \left(x\sin^{-1}x\right)\frac{d}{dx}\left(\sqrt{1-x^2}\right)}{\left(\sqrt{1-x^2}\right)^2}\right]$$

[Using quotient rule, product rule, chain rule]

$$= \left[\frac{\sqrt{1-x^2}\left\{x\frac{d}{dx}\sin^{-1}x + \sin^{-1}x\frac{d}{dx}(x)\right\} - \left(x\sin^{-1}x\right)\frac{1}{2\sqrt{1-x^2}}\frac{d}{dx}\left(1-x^2\right)}{\left(1-x^2\right)}\right]$$
$$= \left[\frac{\sqrt{1-x^2}\left\{\frac{x}{\sqrt{1-x^2}} + \sin^{-1}x\right\} - \frac{x\sin^{-1}x\,(-2x)}{2\sqrt{1-x^2}}}{\left(1-x^2\right)}\right]$$
$$= \left[\frac{x + \sqrt{1-x^2}\sin^{-1}x + \frac{x^2\sin^{-1}x}{\sqrt{1-x^2}}}{\left(1-x^2\right)}\right]$$
$$\left(1-x^2\right)\frac{dy}{dx} = x + \frac{\sqrt{1-x^2}\sin^{-1}x}{1} + \frac{x^2\sin^{-1}x}{\sqrt{1-x^2}}$$
$$\Rightarrow \quad \left(1-x^2\right)\frac{dy}{dx} = x + \left(\frac{\left(1-x^2\right)\sin^{-1}x + x^2\sin^{-1}x}{\sqrt{1-x^2}}\right)$$
$$\Rightarrow \quad \left(1-x^2\right)\frac{dy}{dx} = x + \left(\frac{\sin^{-1}x - x^2\sin^{-1}x + x^2\sin^{-1}x}{\sqrt{1-x^2}}\right)$$
$$\Rightarrow \quad \left(1-x^2\right)\frac{dy}{dx} = x + \left(\frac{\sin^{-1}x}{\sqrt{1-x^2}}\right)$$
$$\left(1-x^2\right)\frac{dy}{dx} = x + \frac{y}{x} \qquad \left\{\text{Since, given } y = \frac{x\sin^{-1}x}{\sqrt{1-x^2}}\right\}.$$

Differentiation Ex 11.2 Q65

Given, $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)$$

$$= \left[\frac{\left(e^x + e^{-x}\right)\frac{d}{dx}\left(e^x - e^{-x}\right) - \left(e^x - e^{-x}\right)\frac{d}{dx}\left(e^x + e^{-x}\right)}{\left(e^x + e^{-x}\right)^2}\right]$$

[Using quotient rule and chain rule]

$$= \left[\frac{\left(e^x + e^{-x}\right)\left[e^x - e^{-x}\frac{d}{dx}(-x) - \left(e^x - e^{-x}\right)\left(e^x + e^{-x}\frac{d}{dx}(-x)\right)\right]}{\left(e^x + e^{-x}\right)^2}\right]$$

$$= \left[\frac{\left(e^x + e^{-x}\right)\left(e^x + e^{-x}\right) - \left(e^x - e^{-x}\right)\left(e^x - e^{-x}\right)}{\left(e^x + e^{-x}\right)^2}\right]$$

$$= \left[\frac{e^{2x} + e^{-2x} + 2e^x \times e^{-x} - e^{2x} - e^{-2x} + 2e^x e^{-x}}{\left(e^x + e^{-x}\right)2}\right]$$

$$\frac{dy}{dx} = \left[\frac{4}{\left(e^x + e^{-x}\right)2}\right] \qquad ---(i)$$

Now,

$$1 - y^2 = 1 - \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)^2$$

$$= 1 - \frac{\left(e^x - e^{-x}\right)^2}{\left(e^x + e^{-x}\right)^2}$$

$$= \frac{\left(e^x + e^{-x}\right)^2 - \left(e^x - e^{-x}\right)^2}{\left(e^x + e^{-x}\right)^2}$$

$$= \frac{4}{\left(e^x + e^{-x}\right)2}$$

Differentiation Ex 11.2 Q66

Given, $y = (x-1)\log(x-1) - (x+1)\log(x+1)$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[(x-1)\log(x-1) - (x+1)\log(x+1)\right]$$

$$= \left[(x-1)\frac{d}{dx}\log(x-1) + \log(x-1)\frac{d}{dx}(x-1)\right] -$$
$$\left[(x+1)\frac{d}{dx}\log(x+1) + \log(x+1)\frac{d}{dx}(x+1)\right]$$

[Using product rule, chain rule]

$$= \left[(x-1) \times \frac{1}{(x-1)}\frac{d}{dx}(x-1) + \log(x-1) \times (1)\right] -$$
$$\left[(x+1)\frac{1}{(x+1)} \times \frac{d}{dx}(x+1) + \log(x+1)(1)\right]$$

$$= \left[(1) + \log(x-1)\right] - \left[1 + \log(x+1)\right]$$

$$= \log(x-1) - \log(x+1)$$

$$\frac{dy}{dx} = \log\frac{(x-1)}{(x+1)} \qquad \left[\text{Since, } \log\left(\frac{a}{b}\right) = \log a - \log b\right].$$

Differentiation Ex 11.2 Q67

Given, $y = e^x \cos x$

Differentiating with respect to x,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(e^x \cos x\right)\\ &= e^x \frac{d}{dx}\cos x + \cos x \frac{d}{dx}e^x && \text{[Using product rule]}\\ &= e^x\left(-\sin x\right) + e^x \cos x\\ &= e^x\left(\cos x - \sin x\right)\\ &= \sqrt{2}e^x\left(\frac{\cos x}{\sqrt{2}} - \frac{\sin x}{\sqrt{2}}\right) && \left[\text{Multiplying and dividing by } \sqrt{2}\right]\\ &= \sqrt{2}e^x\left(\cos\frac{\pi}{4}\cos x - \sin\frac{\pi}{4}\sin x\right)\end{aligned}$$

$$\frac{dy}{dx} = \sqrt{2}e^x \cos\left(x + \frac{\pi}{4}\right).$$

Differentiation Ex 11.2 Q68

Given, $y = \frac{1}{2}\log\left(\frac{1-\cos 2x}{1+\cos 2x}\right)$

$$\Rightarrow \quad y = \frac{1}{2}\log\left(\frac{2\sin^2 x}{2\cos^2 x}\right) \qquad \left[\begin{aligned}\text{Since, } 1-\cos 2x &= 2\sin^2 x,\\ 1+co2x &= 2\cos^2 x\end{aligned}\right]$$

$$\Rightarrow \quad y = \frac{1}{2}\log\left(\tan^2 x\right)$$

$$\Rightarrow \quad y = \frac{2}{2}\log\tan x \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

$$\Rightarrow \quad y = \log\tan x$$

Differentiate it with respect to x,

$$\begin{aligned}\frac{dy}{dx} &= \left(\log\tan x\right)\\ &= \frac{1}{\tan x}\times\frac{d}{dx}\left(\tan x\right) && \text{[Using chain rule]}\\ &= \frac{\sec^2 x}{\tan x}\\ &= \frac{1}{\cos^2 x \times \frac{\sin x}{\cos x}}\\ &= \frac{1}{\sin x\cos x}\\ &= \frac{2}{2\sin x\cos x}\\ &= \frac{2}{\sin 2x} && \left[\text{Since, } \frac{1}{\sin x = \operatorname{cosec} x}\right]\end{aligned}$$

So,

$$\frac{dy}{dx} = 2\operatorname{cosec} 2x.$$

Differentiation Ex 11.2 Q69

Here, $y = x\sin^{-1}x + \sqrt{1-x^2}$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[x\sin^{-1}x + \sqrt{1-x^2}\right]$$
$$= \frac{d}{dx}\left(x\sin^{-1}x\right) + \frac{d}{dx}\left(\sqrt{1-x^2}\right)$$
$$= \left[x\frac{d}{dx}\sin^{-1}x + \sin^{-1}x\frac{d}{dx}(x)\right] + \frac{1}{2\sqrt{1-x^2}}\frac{d}{dx}\left(1-x^2\right)$$

[Using product rule and chain rule]

$$= \left[\frac{x}{\sqrt{1-x^2}} + \sin^{-1}x\right] - \frac{2x}{2\sqrt{1-x^2}}$$
$$= \frac{x}{\sqrt{1-x^2}} + \sin^{-1}x - \frac{x}{\sqrt{1-x^2}}$$
$$= \sin^{-1}x$$

So,

$$\frac{dy}{dx} = \sin^{-1}x.$$

Differentiation Ex 11.2 Q70

Here, $y = \sqrt{x^2+a^2}$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{x^2+a^2}\right)$$
$$= \frac{1}{2\sqrt{x^2+a^2}}\frac{d}{dx}\left(x^2+a^2\right) \qquad \text{[Using chain rule]}$$
$$= \frac{1}{2\sqrt{x^2+a^2}}\times(2x)$$
$$= \frac{x}{\sqrt{x^2+a^2}}$$
$$\frac{dy}{dx} = \frac{x}{y} \qquad \left[\text{Since } \sqrt{x^2+a^2} = y\right]$$
$$\Rightarrow \quad y\frac{dy}{dx} = x$$
$$\Rightarrow \quad y\frac{dy}{dx} - x = 0.$$

Differentiation Ex 11.2 Q71

Here, $y = e^x + e^{-x}$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^x + e^{-x}\right)$$
$$= \frac{d}{dx}e^x + \frac{d}{dx}e^{-x}$$
$$= e^x + e^{-x}\frac{d}{dx}(-x) \qquad \text{[Using chian rule]}$$
$$= e^x + e^{-x}(-1)$$
$$= \left(e^x - e^{-x}\right)$$
$$= \sqrt{\left(e^x + e^{-x}\right)^2 - 4e^x \times e^{-x}} \qquad \left[\text{Since } (a-b) = \sqrt{(a+b)^2 - 4ab}\right]$$
$$= \sqrt{y^2 - 4} \qquad \left[\text{Since } e^x + e^{-x} = y\right]$$

So,

$$\frac{dy}{dx} = \sqrt{y^2-4}.$$

Differentiation Ex 11.2 Q72

Given, $y = \sqrt{a^2 - x^2}$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{a^2 - x^2}\right)$$

$$= \frac{1}{2\sqrt{a^2 - x^2}}\frac{d}{dx}\left(a^2 - x^2\right) \qquad \left[\text{Using chain rule}\right]$$

$$= \frac{1}{2\sqrt{a^2 - x^2}}(-2x)$$

$$= \frac{-x}{\sqrt{a^2 - x^2}}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-x}{y} \qquad \left[\text{Since } \sqrt{a^2 - x^2} = y\right]$$

$$\Rightarrow \quad y\frac{dy}{dx} = -x$$

$$y\frac{dy}{dx} + x = 0$$

Differentiation Ex 11.2 Q73

Here, $xy = 4$

$$\Rightarrow \quad y = \frac{4}{x}$$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{4}{x}\right)$$

$$= 4\frac{d}{dx}\left(x^{-1}\right)$$

$$= 4\left(-1 \times x^{-1-1}\right)$$

$$= 4\left(-\frac{1}{x^2}\right)$$

$$= \frac{-4}{x^2}$$

$$= -\frac{4}{\left(\frac{x}{y}\right)^2} \qquad \left[\text{Since } x = \frac{4}{y}\right]$$

$$= -\frac{4y^2}{16}$$

$$\frac{dy}{dx} = -\frac{y^2}{4}$$

$$\Rightarrow \quad 4\frac{dy}{dx} = -y^2$$

$$\Rightarrow \quad 4\frac{dy}{dx} = 3y^2 - 4y^2$$

$$\Rightarrow \quad 4\frac{dy}{dx} + 4y^2 = 3y^2$$

$$\Rightarrow \quad 4\left(\frac{dy}{dx} + y^2\right) = 3y^2$$

Dividing both the sides by x,

$$\Rightarrow \quad \frac{4}{x}\left(\frac{dy}{dx} + y^2\right) = \frac{3y^2}{x}$$

$$\Rightarrow \quad y\left(\frac{dy}{dx} + y^2\right) = \frac{3y^2}{x} \qquad \left[\text{Since } \frac{4}{x} = y\right]$$

$$\Rightarrow \quad x\left(\frac{dy}{dx} + y^2\right) = \frac{3y^2}{y}$$

$$\Rightarrow \quad x\left(\frac{dy}{dx} + y^2\right) = 3y.$$

Differentiation Ex 11.2 Q74

$$\frac{d}{dx}\left\{\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right\}=\sqrt{a^2-x^2}$$

$$\text{LHS} = \frac{d}{dx}\left\{\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right\}$$

$$= \frac{d}{dx}\left(\frac{x}{2}\sqrt{a^2-x^2}\right)+\frac{d}{dx}\left(\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right)$$

$$= \frac{1}{2}\left[x\frac{d}{dx}\sqrt{a^2-x^2}+\sqrt{a^2-x^2}\frac{d}{dx}(x)\right]+\frac{a^2}{2}\times\frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}}\times\frac{d}{dx}\left(\frac{x}{a}\right)$$

[Using product rule, chain rule]

$$= \frac{1}{2}\left[x\times\frac{1}{2\sqrt{a^2-x^2}}\frac{d}{dx}\left(a^2-x^2\right)+\sqrt{a^2-x^2}\right]+\left(\frac{a^2}{2}\right)\times\frac{1}{\sqrt{\frac{a^2-x^2}{a^2}}}\times\left(\frac{1}{a}\right)$$

$$= \frac{1}{2}\left[\frac{x(-2x)}{2\sqrt{a^2-x^2}}+\sqrt{a^2-x^2}\right]+\left(\frac{a^2}{2}\right)\frac{a}{\sqrt{a^2-x^2}}\times\left(\frac{1}{a}\right)$$

$$= \frac{1}{2}\left[\frac{-2x^2+2\left(a^2-x^2\right)}{2\sqrt{a^2-x^2}}\right]+\frac{a^2}{2\sqrt{a^2-x^2}}$$

$$= \frac{1}{2}\left[\frac{2\left(a^2-2x^2\right)}{2\sqrt{a^2-x^2}}\right]+\frac{a^2}{2\sqrt{a^2-x^2}}$$

$$= \frac{a^2-2x^2}{2\sqrt{a^2-x^2}}+\frac{a^2}{2\sqrt{a^2-x^2}}$$

$$= \frac{a^2-2x^2+a^2}{2\sqrt{a^2-x^2}}$$

$$= \frac{2a^2-2x^2}{2\sqrt{a^2-x^2}}$$

$$= \frac{2\left(a^2-x^2\right)}{2\sqrt{a^2-x^2}}$$

$$= \frac{\left(a^2-x^2\right)}{\sqrt{a^2-x^2}}$$

$$= \sqrt{a^2-x^2}$$

$= \text{RHS}$

Differentiation Ex 11.3 Q1

Ex 11.3

Let $y = \cos^{-1}\left\{2x\sqrt{1-x^2}\right\}$

Put $x = \cos\theta$

$$y = \cos^{-1}\left\{2\cos\theta\sqrt{1-\cos^2\theta}\right\}$$
$$= \cos^{-1}\{2\cos\theta\sin\theta\}$$
$$y = \cos^{-1}\{\sin 2\theta\} \qquad \left[\text{Since } \sin 2\theta = 2\sin\theta\cos\theta, \sin^2\theta + \cos^2\theta = 1\right]$$
$$y = \cos^{-1}\left[\cos\left(\frac{\pi}{2} - \theta\right)\right] \qquad ---(i)$$

Now,

$$\frac{1}{\sqrt{2}} < x < 1$$
$$\Rightarrow \quad \frac{1}{\sqrt{2}} < \cos\theta < 1$$
$$\Rightarrow \quad 0 < \theta < \frac{\pi}{4}$$
$$\Rightarrow \quad 0 < 2\theta < \frac{\pi}{2}$$
$$\Rightarrow \quad 0 > -2\theta > -\frac{\pi}{2}$$
$$\Rightarrow \quad \frac{\pi}{2} > \left(\frac{\pi}{2} - 2\theta\right) > 0$$

Hence, from equation (i),

$$y = \frac{\pi}{2} - 2\theta \qquad \left[\text{Since } \cos^{-1}(\cos\theta) = \theta, \text{if } \theta \in [0, \pi]\right]$$
$$y = \frac{\pi}{2} - 2\cos^{-1}x \qquad [\text{Since } x = \cos\theta]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\pi}{2}\right) - 2\frac{d}{dx}\left(\cos^{-1}x\right)$$
$$= 0 - 2\left(\frac{-1}{\sqrt{1-x^2}}\right)$$
$$\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q2

Let $y = \cos^{-1}\left\{\sqrt{\frac{1+x}{2}}\right\}$

Put $x = \cos 2\theta$

$$y = \cos^{-1}\left\{\sqrt{\frac{1+\cos 2\theta}{2}}\right\}$$
$$= \cos^{-1}\left\{\sqrt{\frac{2\cos^2\theta}{2}}\right\}$$
$$y = \cos^{-1}\{\cos\theta\} \qquad ---(i)$$

Here, $-1 < x < 1$
$$\rightarrow \quad -1 < \cos 2\theta < 1$$
$$\Rightarrow \quad 0 < 2\theta < \pi$$
$$\Rightarrow \quad 0 < \theta < \frac{\pi}{2}$$

So, from equation (i),

$$y = \theta \qquad \left[\text{Since } \cos^{-1}(\cos\theta) = \theta \text{ if } \theta \in [0, \pi]\right]$$
$$y = \frac{1}{2}\cos^{-1}x \qquad [\text{Since } x = \cos 2\theta]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = -\frac{1}{2\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q3

Let $\quad y = \sin^{-1}\left\{\sqrt{\frac{1-x}{2}}\right\}$

Let $\quad x = \cos 2\theta$

$$y = \sin^{-1}\left\{\sqrt{\left(\frac{1-\cos 2\theta}{2}\right)}\right\}$$

$$= \sin^{-1}\left\{\sqrt{\frac{2\sin^2\theta}{2}}\right\}$$

$$y = \sin^{-1}(\sin\theta) \qquad \text{---(i)}$$

Here, $\quad 0 < x < 1$

$\Rightarrow \quad 0 < \cos 2\theta < 1$

$\Rightarrow \quad 0 < 2\theta < \frac{\pi}{2}$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{4}$

so, from equation (i),

$$y = \theta \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$y = \frac{1}{2}\cos^{-1} x \qquad \left[\text{Since, } x = \cos 2\theta\right]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = -\frac{1}{2\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q4

Let $\quad y = \sin^{-1}\left\{\sqrt{1-x^2}\right\}$

Let $\quad x = \cos\theta$

$$y = \sin^{-1}\left\{\sqrt{1-\cos^2\theta}\right\}$$

$$y = \sin^{-1}(\sin\theta) \qquad \text{---(i)}$$

Here, $\quad 0 < x < 1$

$\Rightarrow \quad 0 < \cos\theta < 1$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{2}$

From equatoin (i),

$$y = \theta \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$y = \cos^{-1} x \qquad \left[\text{Since } x = \cos\theta\right]$$

Differentiating with respect to x,

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q5

Let $y = \tan^{-1}\left\{\dfrac{x}{\sqrt{a^2 - x^2}}\right\}$

Let $x = a\sin\theta$

$$y = \tan^{-1}\left\{\frac{a\sin\theta}{\sqrt{a^2 - a^2\sin^2\theta}}\right\}$$

$$y = \tan^{-1}\left\{\frac{a\sin\theta}{\sqrt{a^2\left(1 - \sin^2\theta\right)}}\right\}$$

$$y = \tan^{-1}\left\{\frac{a\sin\theta}{a\cos\theta}\right\}$$

$$y = \tan^{-1}(\tan\theta) \qquad \text{---(i)}$$

Here, $-a < x < a$

$\Rightarrow \quad -1 < \dfrac{x}{a} < 1$

$\Rightarrow \quad -\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$

From equatoin(i),

$y = \theta \qquad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]\right]$

$y = \sin^{-1}\left(\dfrac{x}{a}\right) \qquad [\text{Since } x = a\sin\theta]$

Differentiating it with respect to x,

Using chain rule,

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - \left(\frac{x}{a}\right)^2}}\frac{d}{dx}\left(\frac{x}{a}\right)$$

$$= \frac{a}{\sqrt{a^2 - x^2}} \times \left(\frac{1}{a}\right)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{a^2 - x^2}}.$$

Differentiation Ex 11.3 Q6

Let $y = \sin^{-1}\left\{\dfrac{x}{\sqrt{x^2 + a^2}}\right\}$

Put $x = a\tan\theta$

$$y = \sin^{-1}\left\{\frac{a\tan\theta}{\sqrt{a^2 + \tan^2\theta + a^2}}\right\}$$

$$= \sin^{-1}\left\{\frac{a\tan\theta}{\sqrt{a^2\left(\tan^2\theta + 1\right)}}\right\}$$

$$= \sin^{-1}\left\{\frac{a\tan\theta}{a\sec\theta}\right\}$$

$$= \sin^{-1}\{\sin\theta\}$$

$$= \theta$$

$y = \tan^{-1}\left(\dfrac{x}{a}\right) \qquad [x = a\tan\theta]$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{1}{1 + \left(\frac{x}{a}\right)^2}\frac{d}{dx}\left(\frac{x}{a}\right)$$

$$= \frac{a^2}{a^2 + x^2} \times \left(\frac{1}{a}\right)$$

$$\frac{dy}{dx} = \frac{a}{a^2 + x^2}.$$

Differentiation Ex 11.3 Q7

Let $y = \sin^{-1}\left\{2x^2 - 1\right\}$

Let $x = \cos\theta$

$$y = \sin^{-1}\left(2\cos^2\theta - 1\right)$$

$$= \sin^{-1}(\cos 2\theta)$$

$$y = \sin^{-1}\left\{\sin\left(\frac{\pi}{2} - 2\theta\right)\right\} \quad \text{---(i)}$$

Here, $0 < x < 1$

$\Rightarrow \quad 0 < \cos\theta < 1$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{2}$

$\Rightarrow \quad 0 < 2\theta < \pi$

$\Rightarrow \quad 0 > -2\theta > -\pi$

$\Rightarrow \quad \frac{\pi}{2} > \left(\frac{\pi}{2} - 2\theta\right) > -\frac{\pi}{2}$

So, from equatoin (i),

$$y = \frac{\pi}{2} - 2\theta \qquad \left[\text{Since, } \sin^{-1}(\cos\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$y = \frac{\pi}{2} - 2\cos^{-1}x \qquad [\text{Since } x = \cos\theta]$$

$$\frac{dy}{dx} = 0 - 2\frac{d}{dx}\left(\cos^{-1}x\right)$$

$$= -2\left(-\frac{1}{\sqrt{1-x^2}}\right)$$

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q8

Let $y = \sin^{-1}\left\{1 - 2x^2\right\}$

Let $x = \sin\theta$, So,

$$y = \sin^{-1}\left(1 - 2\sin^2\theta\right)$$

$$= \sin^{-1}(\cos 2\theta)$$

$$y = \sin^{-1}\left\{\sin\left(\frac{\pi}{2} - 2\theta\right)\right\} \quad \text{---(i)}$$

Here, $0 < x < 1$

$\Rightarrow \quad 0 < \sin\theta < 1$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{2}$

$\Rightarrow \quad 0 < 2\theta < \pi$

$\Rightarrow \quad 0 > -2\theta > -\pi$

$\Rightarrow \quad \frac{\pi}{2} > \left(\frac{\pi}{2} - 2\theta\right) > \frac{\pi}{2} - \pi$

$\Rightarrow \quad \frac{\pi}{2} > \left(\frac{\pi}{2} - 2\theta\right) > \left(-\frac{\pi}{2}\right)$

So, from equatoin (i),

$$y = \frac{\pi}{2} - 2\theta \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$y = \frac{\pi}{2} - 2\sin^{-1}x \qquad [\text{Since } x = \sin\theta]$$

Differentiating with respect to x,

$$\frac{dy}{dx} = 0 - 2\left(\frac{1}{\sqrt{1-x^2}}\right)$$

$$\frac{dy}{dx} = -\frac{2}{\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q9

Let $\quad y = \cos^{-1}\left\{\frac{x}{\sqrt{x^2 + a^2}}\right\}$

Put $\quad x = a\cot\theta,$

$$y = \cos^{-1}\left\{\frac{a\cot\theta}{\sqrt{a^2\cot^2\theta + a^2}}\right\}$$

$$= \cos^{-1}\left\{\frac{a\cot\theta}{a\,\operatorname{cosec}\theta}\right\}$$

$$= \cos^{-1}\left\{\frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}\right\}$$

$$= \cos^{-1}(\cos\theta)$$

$$= \theta$$

$$y = \cot^{-1}\left(\frac{x}{a}\right) \qquad \left[\text{Since, } a\cot\theta = x\right]$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{-1}{1 + \left(\frac{x}{a}\right)^2}\frac{d}{dx}\left(\frac{x}{a}\right)$$

$$= \frac{-a^2}{a^2 + x^2} \times \left(\frac{1}{a}\right)$$

$$\frac{dy}{dx} = \frac{-a}{a^2 + x^2}.$$

Differentiation Ex 11.3 Q10

Let $\quad y = \sin^{-1}\left\{\frac{\sin x + \cos x}{\sqrt{2}}\right\}$

$$= \sin^{-1}\left\{\sin x\left(\frac{1}{\sqrt{2}}\right) + \cos x \times \left(\frac{1}{\sqrt{2}}\right)\right\}$$

$$= \sin^{-1}\left\{\sin x \cos\frac{\pi}{4} + \cos x \times \sin\frac{\pi}{4}\right\}$$

$$y = \sin^{-1}\left\{\sin\left(x + \frac{\pi}{4}\right)\right\}$$

Here, $\quad \frac{-3\pi}{4} < x < \frac{\pi}{4}$

$$\Rightarrow \quad \left(\frac{-3\pi}{4} + \frac{\pi}{4}\right) \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]\right]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 1 + 0$$

$$\frac{dy}{dx} = 1$$

Differentiation Ex 11.3 Q11

Let $y = \cos^{-1}\left\{\frac{\cos x + \sin x}{\sqrt{2}}\right\}$

$$y = \cos^{-1}\left\{\cos x\left(\frac{1}{\sqrt{2}}\right) + \sin x\left(\frac{1}{\sqrt{2}}\right)\right\}$$

$$= \cos^{-1}\left\{\cos x \cos\left(\frac{\pi}{4}\right) + \sin x \sin x\left(\frac{\pi}{4}\right)\right\}$$

$$y = \cos^{-1}\left[\cos\left(x - \frac{\pi}{4}\right)\right] \qquad \text{---(i)}$$

Here, $-\frac{\pi}{4} < x < \frac{\pi}{4}$

$\Rightarrow \quad \left(-\frac{\pi}{4} - \frac{\pi}{4}\right) < \left(x - \frac{\pi}{4}\right) < \left(\frac{\pi}{4} - \frac{\pi}{4}\right)$

$\Rightarrow \quad -\frac{\pi}{2} < \left(x - \frac{\pi}{4}\right) < 0$

So, from equation (i),

$$y = -\left(x - \frac{\pi}{4}\right) \qquad \left[\text{Since, } \cos^{-1}(\cos\theta) = -\theta, \text{ if } \theta \in [-\pi, 0]\right]$$

$$y = -x + \frac{\pi}{4}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = -1.$$

Differentiation Ex 11.3 Q12

Let $y = \tan^{-1}\left\{\frac{x}{1 + \sqrt{1 - x^2}}\right\}$

Put $x = \sin\theta$, so

$$y = \tan^{-1}\left\{\frac{\sin\theta}{1 + \sqrt{1 - \sin^2\theta}}\right\}$$

$$= \tan^{-1}\left\{\frac{\sin\theta}{1 + \cos\theta}\right\}$$

$$= \tan^{-1}\left\{\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}\right\}$$

$$y = \tan^{-1}\left\{\tan\frac{\theta}{2}\right\} \qquad \text{--- (i)}$$

Here, $-1 < x < 1$

$\Rightarrow \quad -1 < \sin\theta < 1$

$\Rightarrow \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\Rightarrow \quad -\frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{4}$

So, from equation (i),

$$y = \frac{\theta}{2} \qquad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$y = \frac{1}{2}\sin^{-1} x \qquad [\text{Since, } x = \sin\theta]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1 - x^2}}.$$

Differentiation Ex 11.3 Q13

Let $\quad y = \tan^{-1}\left\{\frac{x}{a + \sqrt{a^2 - x^2}}\right\}$

Put $\quad x = a\sin\theta$, so

$$y = \tan^{-1}\left\{\frac{a\sin\theta}{a + \sqrt{a^2 - a^2\sin^2\theta}}\right\}$$

$$= \tan^{-1}\left\{\frac{a\sin\theta}{a + \sqrt{a^2\left(1 - \sin^2\theta\right)}}\right\}$$

$$= \tan^{-1}\left\{\frac{a\sin\theta}{a + a\cos\theta}\right\}$$

$$= \tan^{-1}\left\{\frac{a\sin\theta}{a\left(1 + \cos\theta\right)}\right\}$$

$$= \tan^{-1}\left(\frac{\sin\theta}{1 + \cos\theta}\right)$$

$$= \tan^{-1}\left(\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}\right)$$

$$y = \tan^{-1}\left(\tan\frac{\theta}{2}\right) \qquad \text{---(i)}$$

Here, $\quad -a < x < a$

$\Rightarrow \quad -1 < \frac{x}{a} < 1$

$\Rightarrow \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\Rightarrow \quad -\frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{4}$

So, from equation (i),

$$y = \frac{\theta}{2} \qquad \left[\text{Since, } \tan^{-1}\left(\tan\theta\right) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$y = \frac{1}{2} + \sin^{-1}\left(\frac{x}{a}\right) \qquad \left[\text{Since, } x = a\sin\theta\right]$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{\sqrt{1 - \left(\frac{x}{a}\right)^2}} \frac{d}{dx}\left(\frac{x}{a}\right)$$

$$= \frac{a}{2\sqrt{a^2 - x^2}} \times \left(\frac{1}{a}\right)$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{a^2 - x^2}}.$$

Differentiation Ex 11.3 Q14

Let $\quad y = \sin^{-1}\left\{\frac{x + \sqrt{1 - x^2}}{\sqrt{2}}\right\}$

Put $\quad x = \sin\theta$, so

$$= \sin^{-1}\left\{\frac{\sin\theta + \sqrt{1 - \sin^2\theta}}{\sqrt{2}}\right\}$$

$$= \sin^{-1}\left\{\frac{\sin\theta + \cos\theta}{\sqrt{2}}\right\}$$

$$= \sin^{-1}\left\{\sin\theta\left(\frac{1}{\sqrt{2}}\right) + \cos\theta\left(\frac{1}{\sqrt{2}}\right)\right\}$$

$$= \sin^{-1}\left\{\sin\theta\cos\frac{\pi}{4} + \cos\theta\sin\frac{\pi}{4}\right\}$$

$$y = \sin^{-1}\left\{\sin\left(\theta + \frac{\pi}{4}\right)\right\} \qquad \text{---(i)}$$

Here, $\quad -1 < x < 1$

$\Rightarrow \quad -1 < \sin\theta < 1$

$\Rightarrow \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\Rightarrow \quad \left(-\frac{\pi}{2} + \frac{\pi}{4}\right) < \left(\frac{\pi}{4} + \theta\right) < \frac{3\pi}{4}$

So, from equation (i),

$$y = \theta + \frac{\pi}{4} \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ as } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$y = \sin^{-1}x + \frac{\pi}{4} \qquad [\text{Since, } \sin\theta = x]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}} + 0$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}}.$$

Differentiation Ex 11.3 Q15

Let $\quad y = \cos^{-1}\left\{\frac{x+\sqrt{1-x^2}}{\sqrt{2}}\right\}$

Put $\quad x = \sin\theta$, so

$$y = \cos^{-1}\left\{\frac{\sin\theta+\sqrt{1-\sin^2\theta}}{\sqrt{2}}\right\}$$

$$= \cos^{-1}\left\{\frac{\sin\theta+\cos\theta}{\sqrt{2}}\right\}$$

$$= \cos^{-1}\left\{\sin\theta\left(\frac{1}{\sqrt{2}}\right)+\cos\theta\left(\frac{1}{\sqrt{2}}\right)\right\}$$

$$= \cos^{-1}\left\{\sin\theta\times\sin\frac{\pi}{4}+\cos\theta\times\cos\frac{\pi}{4}\right\}$$

$$y = \cos^{-1}\left\{\cos\left(\theta-\frac{\pi}{4}\right)\right\} \qquad \text{---(i)}$$

Here, $\quad -1 < x < 1$

$\Rightarrow \quad -1 < \sin\theta < 1$

$\Rightarrow \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\Rightarrow \quad -\frac{\pi}{2}+\frac{\pi}{4} < \left(\theta-\frac{\pi}{4}\right) < \frac{\pi}{2}-\frac{\pi}{4}$

$\Rightarrow \quad \left(-\frac{3\pi}{4}\right) < \left(\theta-\frac{\pi}{4}\right) < \left(\frac{\pi}{4}\right)$

So, from equation (i),

$$y = -\left(\theta-\frac{\pi}{4}\right) \qquad \left[\text{Since, } \cos^{-1}(\cos\theta) = -\theta, \text{ if } \theta\in[-\pi,0]\right]$$

$$y = -\theta+\frac{\pi}{4}$$

$$y = -\sin^{-1}x+\frac{\pi}{4} \qquad [\text{Since, } x = \sin\theta]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}+0$$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q16

Let $\quad y = \tan^{-1}\left\{\frac{4x}{1-4x^2}\right\}$

Put $\quad 2x = \tan\theta$, so

$$y = \tan^{-1}\left\{\frac{2\tan\theta}{1-\tan^2\theta}\right\}$$

$$y = \tan^{-1}\{\tan 2\theta\} \qquad \text{---(i)}$$

Here, $\quad -\frac{1}{2} < x < \frac{1}{2}$

$\Rightarrow \quad -1 < 2x < 1$

$\Rightarrow \quad -1 < \tan\theta < 1$

$\Rightarrow \quad -\frac{\pi}{4} < \theta < \frac{\pi}{4}$

$\Rightarrow \quad -\frac{\pi}{2} < (2\theta) < \frac{\pi}{2}$

So, from equation (i),

$$y = 2\theta \qquad \left[\text{Sicne, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$y = 2\tan^{-1}(2x) \qquad [\text{Since, } 2x = \tan\theta]$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = 2\left(\frac{1}{1+(2x)^2}\right)\frac{d}{dx}(2x)$$

$$\frac{dy}{dx} = \frac{4}{1+4x^2}.$$

Differentiation Ex 11.3 Q17

Let $\quad y = \tan^{-1}\left\{\frac{2^{x+1}}{1-4^x}\right\}$

Put $\quad 2^x = \tan\theta$, so,

$$= \tan^{-1}\left\{\frac{2^x \times 2}{1-(2^x)^2}\right\}$$

$$= \tan^{-1}\left\{\frac{2\tan\theta}{1-\tan^2\theta}\right\}$$

$$y = \tan^{-1}\{\tan(2\theta)\} \qquad \text{---(i)}$$

Here, $\quad -\infty < x < 0$

$\Rightarrow \quad 2^{-\infty} < 2^x < 2^{\circ}$

$\Rightarrow \quad 0 < 2^x < 1$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{4}$

$\Rightarrow \quad 0 < (2\theta) < \frac{\pi}{2}$

From equatoin (i),

$$y = 2\theta \qquad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$$

$$y = 2\tan^{-1}(2^x)$$

Differentiate it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{2}{1+(2^x)^2}\frac{d}{dx}(2^x)$$

$$= \frac{2 \times 2^x \log 2}{1+4^x}$$

$$\frac{dy}{dx} = \frac{2^{x+1}\log 2}{1+4^x}.$$

Differentiation Ex 11.3 Q18

Let $\quad y = \tan^{-1}\left\{\frac{2a^x}{1-a^{2x}}\right\}$

Put $\quad a^x = \tan\theta,$

$$y = \tan^{-1}\left\{\frac{2\tan\theta}{1-\tan^2\theta}\right\}$$

$$y = \tan^{-1}\left\{\tan(2\theta)\right\} \qquad ---(i)$$

Here, $\quad -\infty < x < 0$

$\Rightarrow \quad a^{-\infty} < a^x < 2^\circ$

$\Rightarrow \quad 0 < \tan\theta < 1$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{4}$

$\Rightarrow \quad 0 < (2\theta) < \frac{\pi}{2}$

From equatoin (i),

$$y = 2\theta \qquad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$$

$$y = 2\tan^{-1}\left(a^x\right)$$

Differentiate it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{2}{1+\left(a^x\right)^2}\frac{d}{dx}\left(a^x\right)$$

$$\frac{dy}{dx} = \frac{2a^x \log a}{1+a^{2x}}.$$

Differentiation Ex 11.3 Q19

Let $\quad y = \sin^{-1}\left\{\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right\}$

Put $\quad x = \cos 2\theta, so,$

$$= \sin^{-1}\left\{\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{2}\right\}$$

$$= \sin^{-1}\left\{\frac{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}{2}\right\}$$

$$= \sin^{-1}\left\{\frac{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}{2}\right\}$$

$$= \sin^{-1}\left\{\cos\theta\left(\frac{1}{\sqrt{2}}\right)+\left(\frac{1}{\sqrt{2}}\right)\sin\theta\right\}$$

$$= \sin^{-1}\left\{\cos\theta\sin\left(\frac{\pi}{4}\right)+\cos\frac{\pi}{4}\sin\theta\right\}$$

$$y = \sin^{-1}\left\{\sin\left(\theta+\frac{\pi}{4}\right)\right\} \qquad \text{---(i)}$$

Here, $\quad 0 < x < 1$

$\Rightarrow \quad 0 < \cos 2\theta < 1$

$\Rightarrow \quad 0 < 2\theta < \frac{\pi}{2}$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{4}$

$\Rightarrow \quad \frac{\pi}{4} < \left(\theta+\frac{\pi}{4}\right) < \frac{\pi}{2}$

So, from equatoin (i),

$$y = \theta + \frac{\pi}{4} \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2},\frac{\pi}{2}\right)\right]$$

$$y = \frac{1}{2}\cos^{-1}x + \frac{\pi}{4}$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{-1}{\sqrt{1-x^2}}\right) + 0$$

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q20

Let $\quad y = \tan^{-1}\left(\frac{\sqrt{1+a^2x^2}-1}{ax}\right)$

Put $\quad ax = \tan\theta$

$$y = \tan^{-1}\left(\frac{\sqrt{1+a^2x^2}-1}{ax}\right)$$

$$= \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$

$$y = \tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$= \frac{\theta}{2}$$

$$y = \frac{1}{2}\tan^{-1}(ax)$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{1}{2}\times\left(\frac{1}{1+(ax)^2}\right)\frac{d}{dx}(ax)$$

$$\frac{dy}{dx} = \frac{1}{2\left(1+a^2x^2\right)}(a)$$

$$\frac{dy}{dx} = \frac{a}{2\left(1+a^2x^2\right)}.$$

Differentiation Ex 11.3 Q21

Let $f(x) = \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$

This function is defined for all real numbers where $\cos x \neq 1$
i.e at all odd multiples of π

$$f(x) = \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$$

$$= \tan^{-1}\left[\frac{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}{2\cos^2\left(\frac{x}{2}\right)}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{x}{2}\right)\right] = \frac{x}{2}$$

Thus, $f'(x) = \frac{d}{dx}\left(\frac{x}{2}\right) = \frac{1}{2}$

Differentiation Ex 11.3 Q22

Let $\quad y = \sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

Put $\quad x = \cot\theta$

$$y = \sin^{-1}\left(\frac{1}{\sqrt{1+\cot^2\theta}}\right)$$

$$= \sin^{-1}\left(\frac{1}{\sqrt{\operatorname{cosec}^2\theta}}\right)$$

$$= \sin^{-1}(\sin\theta)$$

$$= \theta$$

$$y = \cot^{-1}x \qquad \left[\text{Since, } \cot\theta = x\right]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = -\frac{1}{\left(1+x^2\right)}.$$

Differentiation Ex 11.3 Q23

Let $\quad y = \cos^{-1}\left(\frac{1-x^{2n}}{1+x^{2n}}\right)$

Put $\quad x^n = \tan\theta, so,$

$$y = \cos^{-1}\left(\frac{1-\left(x^n\right)^2}{1+\left(x^n\right)^2}\right)$$

$$= \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$y = \cos^{-1}\left(\cos 2\theta\right) \qquad \text{---(i)}$$

Here, $\quad 0 < x < \infty$

$\Rightarrow \quad 0 < x^n < \infty$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{2}$

$\Rightarrow \quad 0 < \left(2\theta\right) < \pi$

So, from equation (i),

$$y = 2\theta \qquad \left[\text{Sicne, } \cos^{-1}\left(\cos\theta\right) = \theta, \text{ if } \theta \in \left[0, \pi\right]\right]$$

$$y = 2\tan^{-1}\left(x^n\right)$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = 2\left(\frac{1}{1+\left(x^n\right)^2}\right)\frac{d}{dx}\left(x^n\right)$$

$$= \frac{2}{1+x^{2n}} \times \left(nx^{n-1}\right)$$

$$\frac{dy}{dx} = \frac{2nx^{n-1}}{1+x^{2n}}.$$

Differentiation Ex 11.3 Q24

Let $\quad y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right) + \sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$

$$= \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) \qquad \left[\text{Since, } \sec^{-1}x = \cos^{-1}\left(\frac{1}{x}\right)\right]$$

$$y = \frac{\pi}{2} \qquad \left[\text{Since, } \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 0.$$

Differentiation Ex 11.3 Q25

Let $\quad y = \tan^{-1}\left(\frac{a+x}{1-ax}\right)$

$$y = \tan^{-1}a + \tan^{-1}x \qquad \left[\text{Since, } \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\tan^{-1}a\right) + \frac{d}{dx}\left(\tan^{-}x\right)$$

$$= 0 + \frac{1}{1+x^2}$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}.$$

Differentiation Ex 11.3 Q26

Let $y = \tan^{-1}\left(\frac{\sqrt{x}+\sqrt{a}}{1-\sqrt{xa}}\right)$

$y = \tan^{-1}\sqrt{x} + \tan^{-1}\sqrt{a}$ $\left[\text{Since, } \tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\tan^{-1}\sqrt{x}\right) + \frac{d}{dx}\left(\tan^{-1}\sqrt{a}\right)$$

$$= \frac{1}{1+\left(\sqrt{x}\right)^2}\frac{d}{dx}\left(\sqrt{x}\right) + 0$$

$$= \left(\frac{1}{1+x}\right)\left(\frac{1}{2\sqrt{x}}\right)$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}\,(1+x)}.$$

Differentiation Ex 11.3 Q27

Let $y = \tan^{-1}\left[\frac{a+b\tan x}{b-a\tan x}\right]$

$$= \tan^{-1}\left[\frac{\frac{a+b\tan x}{b}}{\frac{b-a\tan x}{b}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{a}{b}+\tan x}{1+\frac{a}{b}\tan x}\right]$$

$$= \tan^{-1}\left[\frac{\tan\left(\tan^{-1}\frac{a}{b}\right)+\tan x}{1-\tan\left(\tan^{-1}\frac{a}{b}\right)+\tan x}\right]$$

$$= \tan^{-1}\left[\tan\left(\tan^{-1}\frac{a}{b} \;+x.\right)\right]$$

$$y = \tan^{-1}\left(\frac{a}{b}\right)+x$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = 0+1$$

$$\frac{dy}{dx} = 1.$$

Differentiation Ex 11.3 Q28

Let $y = \tan^{-1}\left(\frac{a+bx}{b-ax}\right)$

$$= \tan^{-1}\left(\frac{\frac{a+bx}{b}}{\frac{b-ax}{b}}\right)$$

$$= \tan^{-}\left(\frac{\frac{a}{b}+\frac{bx}{b}}{\frac{b}{a}-\frac{ax}{b}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{a}{b}+x}{1-\left(\frac{a}{b}\right)x}\right)$$

$y = \tan^{-1}\left(\frac{a}{b}\right)+\tan^{-1}x$ $\left[\text{Since, } \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 0+\frac{1}{1+x^2}$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}.$$

Differentiation Ex 11.3 Q29

Let $\quad y = \tan^{-1}\left(\frac{x-a}{x+a}\right)$

$$= \tan^{-1}\left(\frac{\frac{x-a}{x}}{\frac{x+a}{x}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{x}{x}-\frac{a}{x}}{\frac{x}{x}+\frac{a}{x}}\right)$$

$$= \tan^{-1}\left(\frac{1-\frac{a}{x}}{1+1\times\frac{a}{x}}\right)$$

$$y = \tan^{-1}(1) - \tan^{-1}\left(\frac{a}{x}\right)$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = 0 - \frac{1}{1+\left(\frac{a}{x}\right)^2}\frac{d}{dx}\left(\frac{a}{x}\right)$$

$$= -\frac{x^2}{x^2+a^2}\left(\frac{-a}{x^2}\right)$$

$$\frac{dy}{dx} = \frac{a}{a^2+x^2}.$$

Differentiation Ex 11.3 Q30

Let $\quad y = \tan^{-1}\left(\frac{x}{1+6x^2}\right)$

$$= \tan^{-1}\left(\frac{3x-2x}{1+(3x)(2x)}\right)$$

$$y = \tan^{-1}3x - \tan^{-1}2x \qquad \left[\text{Since, } \tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{1}{1+(3x)^2}\frac{d}{dx}(3x) - \frac{1}{1+(2x)^2}\frac{d}{dx}(2x)$$

$$= \frac{1}{1+9x^2}(3) - \frac{1}{1+4x^2}(2)$$

$$\frac{dy}{dx} = \frac{3}{1+9x^2} - \frac{2}{1+4x^2}.$$

Differentiation Ex 11.3 Q31

Let $\quad y = \tan^{-1}\left(\frac{5x}{1-6x^2}\right)$

$$= \tan^{-1}\left(\frac{3x+2x}{1-(3x)(2x)}\right)$$

$$y = \tan^{-1}(3x) + \tan^{-1}(2x) \qquad \left[\text{Since, } \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{1}{1+(3x)^2}\frac{d}{dx}(3x) + \frac{1}{1+(2x)^2}\frac{d}{dx}(2x)$$

$$= \frac{1}{1+9x^2}(3) + \frac{1}{1+4x^2}(2)$$

$$\frac{dy}{dx} = \frac{3}{1+9x^2} + \frac{2}{1+4x^2}.$$

Differentiation Ex 11.3 Q32

Let $y = \tan^{-1}\left[\frac{\cos x + \sin x}{\cos x - \sin x}\right]$

$$= \tan^{-1}\left[\frac{\frac{\cos x + \sin x}{\cos x}}{\frac{\cos x - \sin x}{\cos x}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}}\right]$$

$$= \tan^{-1}\left[\frac{1 + \tan x}{1 - \tan x}\right]$$

$$= \tan^{-1}\left[\frac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4}\tan x}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4} + x\right)\right]$$

$$y = \frac{\pi}{4} + x$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 0 + 1$$

$$\frac{dy}{dx} = 1.$$

Differentiation Ex 11.3 Q33

Let $y = \tan^{-1}\left[\frac{x^{\frac{1}{3}} + a^{\frac{1}{3}}}{1 - (ax)^{\frac{1}{3}}}\right]$

$$y = \tan^{-1}\left(x^{\frac{1}{3}}\right) + \tan^{-1}\left(a^{\frac{1}{3}}\right) \qquad \left[\text{Since, } \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{1}{1 + \left(x^{\frac{1}{3}}\right)^2} \times \frac{d}{dx}\left(x^{\frac{1}{3}}\right) + 0$$

$$= \frac{\left(\frac{1}{3} \times x^{\frac{1}{3}-1}\right)}{1 + x^{\frac{2}{3}}}$$

$$\frac{dy}{dx} = \frac{1}{3x^{\frac{2}{3}}\left(1 + x^{\frac{2}{3}}\right)}.$$

Differentiation Ex 11.3 Q34

Let $f(x) = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$

To find the domain, we need to find all x such that

$-1 \le \frac{2^{x+1}}{1+4^x} \le 1$

Since the quantity in the middle is always positive, we need

to find all x such that $\frac{2^{x+1}}{1+4^x} \le 1$

i.e all x such that $2^{x+1} \le 1+4^x$

We may rewrite as $2 \le \frac{1}{2^x} + 2^x$, which is true for all x

Hence the function is defined at all real numbers.

Putting $2^x = \tan\theta$

$$f(x) = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right) = \sin^{-1}\left(\frac{2^x.2}{1+(2^x)^2}\right)$$

$$= \sin^{-1}\left[\frac{2\tan\theta}{1+\tan^2\theta}\right] = \sin^{-1}(\sin 2\theta) = 2\theta = 2\tan^{-1}(2^x)$$

Thus, $f'(x) = 2.\frac{1}{1+(2^x)^2}\frac{d}{dx}(2^x)$

$$= \frac{2}{1+4^x}.(2^x)\log 2 = \frac{2^{x+1}\log 2}{1+4^x}$$

Differentiation Ex 11.3 Q35

Let $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) + \sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$

$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Put, $x = \tan\theta$

$y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) + \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$y = \sin^{-1}(\sin 2\theta) + \cos^{-1}(\cos 2\theta)$ ---(i)

Here, $0 < x < 1$

$\Rightarrow \quad 0 < \tan\theta < 1$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{4}$

$\Rightarrow \quad 0 < (2\theta) < \frac{\pi}{2}$

So, from eqauation (i),

$y = 2\theta + 2\theta$ $\left[\begin{array}{l}\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \\ \cos^{-1}(\cos\theta) = \theta, \text{ if } \theta \in [0, \pi]\end{array}\right]$

$y = 4\theta$

$y = 4\tan^{-1}x$ [Since, $x = \tan\theta$]

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{4}{1+x^2}.$$

Differentiation Ex 11.3 Q36

Here, $y = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) + \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

Put $x = \tan\theta$

$$y = \sin^{-1}\left(\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}\right) + \cos^{-1}\left(\frac{1}{\sqrt{1+\tan^2\theta}}\right)$$

$$= \sin^{-1}\left(\frac{\frac{\sin\theta}{\cos\theta}}{\sec\theta}\right) + \cos^{-1}\left(\frac{1}{\sec\theta}\right)$$

$$= \sin^{-1}\left(\frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}\right) + \cos^{-1}(\cos\theta)$$

$$y = \sin^{-1}(\sin\theta) + \cos^{-1}(\cos\theta) \qquad \text{---(i)}$$

Here, $0 < x < \infty$

$\Rightarrow \quad 0 < \tan\theta < \infty$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{2}$

So, from eqation (i),

$$y = \theta + \theta \qquad \left[\begin{array}{l}\text{Since, } \sin^{-1}(\sin\theta)\theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \\ \text{and } \cos^{-1}(\cos\theta) = \theta, \text{ if } \theta \in [0, \pi]\end{array}\right]$$

$$= 2\theta$$

$$y = 2\tan^{-1} x \qquad [\text{Since, } x = \tan\theta]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{2}{1+x^2}.$$

Differentiation Ex 11.3 Q37

Let $f(x) = \cos^{-1}(\sin x)$

We observe that this function is defined for all real numbers.

$$f(x) = \cos^{-1}(\sin x)$$

$$= \cos^{-1}\left[\cos\left(\frac{\pi}{2} - x\right)\right] = \frac{\pi}{2} - x$$

Thus, $f'(x) = \frac{d}{dx}\left(\frac{\pi}{2} - x\right) = -1$

Let $y = \cot^{-1}\left(\frac{1-x}{1+x}\right)$

Put $x = \tan\theta$, so,

$$y = \cot^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$$

$$= \cot^{-1}\left(\frac{\tan\frac{\pi}{4} - \tan\theta}{1 + \tan\frac{\pi}{4}\tan\theta}\right)$$

$$= \cot^{-1}\left[\tan\left(\frac{\pi}{4} - \theta\right)\right]$$

$$= \cot^{-1}\left[\cot\left(\frac{\pi}{2} - \frac{\pi}{4} + \theta\right)\right]$$

$$= \frac{\pi}{4} + \theta$$

$$y = \frac{\pi}{4} + \tan^{-1} x \qquad [\text{Since } x = \tan\theta]$$

Differentiating it with respect do x,

$$\frac{dy}{dx} = 0 + \frac{1}{1+x^2}$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}.$$

Differentiation Ex 11.3 Q38

Let $y=\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$ ...(1)

Then, $\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}$

$$=\frac{\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)^2}{\left(\sqrt{1+\sin x}-\sqrt{1-\sin x}\right)\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}$$

$$=\frac{(1+\sin x)+(1-\sin x)+2\sqrt{(1-\sin x)(1+\sin x)}}{(1+\sin x)-(1-\sin x)}$$

$$=\frac{2+2\sqrt{1-\sin^2 x}}{2\sin x}$$

$$=\frac{1+\cos x}{\sin x}$$

$$=\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$=\cot\frac{x}{2}$$

Therefore, equation (1) becomes

$$y=\cot^{-1}\left(\cot\frac{x}{2}\right)$$

$$\Rightarrow y=\frac{x}{2}$$

$$\therefore \frac{dy}{dx}=\frac{1}{2}\frac{d}{dx}(x)$$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{2}$$

Differentiation Ex 11.3 Q39

Here, $y=\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$

$$y=\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

Put $x=\tan\theta$,

$$y=\tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)+\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$y=\tan^{-1}(\tan 2\theta)+\cos^{-1}(\cos 2\theta) \quad \text{---(i)}$$

Here, $< x < \infty$

$\Rightarrow \quad 0<\tan\theta<\infty$

$\Rightarrow \quad 0<\theta<\frac{\pi}{2}$

$\Rightarrow \quad 0<2\theta<\pi$

So, from equation (i),

$$y=2\theta+2\theta \qquad \left[\begin{array}{l}\text{Since, } \tan^{-1}(\tan\theta)=\theta, \text{ if } \theta\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\\ \text{and } \cos^{-1}(\cos\theta)=\theta, \text{ if } \theta\in[0,\pi]\end{array}\right]$$

$$y=4\theta$$

$$y=4\tan^{-1}x \qquad [\text{Using } x=\tan\theta]$$

Differentiating it with respect to x,

$$\frac{dy}{dx}=\frac{4}{1+x^2}$$

Differentiation Ex 11.3 Q40

Here, $y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$

$y = \cos^{-1}\left(\frac{x-1}{x+1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$ $\quad\left[\text{Since, } \sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)\right]$

$y = \frac{\pi}{2}$ $\quad\left[\text{Since, } \cos^{-1}x + \sin^{-1}x = \frac{\pi}{2}\right]$

Differentiating with respect to x,

$$\frac{dy}{dx} = 0$$

Differentiation Ex 11.3 Q41

Here, $y = \sin\left[2\tan^{-1}\left[\sqrt{\frac{1-x}{1+x}}\right]\right]$

Put $x = \cos 2\theta$, so,

$$y = \sin\left[2\tan^{-1}\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}\right]$$

$$= \sin\left[2\tan^{-1}\sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}}\right]$$

$$= \sin\left[2\tan^{-1}\sqrt{\tan^2\theta}\right]$$

$$= \sin\left[2\tan^{-1}(\tan\theta)\right]$$

$$= \sin(2\theta)$$

$$= \sin\left[2\times\frac{1}{2}\cos^{-1}x\right] \quad [\text{Since, } x = \cos 2\theta]$$

$$= \sin\left(\sin^{-1}\sqrt{1-x^2}\right)$$

$$y = \sqrt{1-x^2}$$

Differentiating with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}\frac{d}{dx}\left(1-x^2\right).$$

Differentiation Ex 11.3 Q42

Here, $y = \cos^{-1}(2x) + 2\cos^{-1}\sqrt{1-4x^2}$

Put $2x = \cos\theta$, so

$$y = \cos^{-1}(\cos\theta) + 2\cos^{-1}\sqrt{1-\cos^2\theta}$$
$$= \cos^{-1}(\cos\theta) + 2\cos^{-1}(\sin\theta)$$
$$= \cos^{-1}(\cos\theta) + 2\cos^{-1}\left(\cos\left(\frac{\pi}{2}-\theta\right)\right) \quad ---(i)$$

Here, $0 < x < \frac{1}{2}$

$\Rightarrow \quad 0 < 2x < 1$

$\Rightarrow \quad 0 < \cos\theta < 1$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{2}$

and

$\Rightarrow \quad 0 > -\theta > -\frac{\pi}{2}$

$\Rightarrow \quad \frac{\pi}{2} > \left(\frac{\pi}{2}-\theta\right) > 0$

So, from equation (i),

$$y = \theta + 2\left(\frac{\pi}{2}-\theta\right) \quad \left[\text{Since, } \cos^{-1}(\cos(\theta)) = \theta, \text{ if } \theta \in [0,\pi]\right]$$
$$= \theta + \pi - 2\theta$$
$$y = \pi - \theta$$
$$y = \pi - \cos^{-1}(2x) \quad [\text{Since, } 2x = \cos\theta]$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = 0 - \left[\frac{-1}{\sqrt{1-(2x)^2}}\right]\frac{d}{dx}(2x)$$
$$= \frac{1}{\sqrt{1-4x^2}}(2)$$
$$\frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}}.$$

Differentiation Ex 11.3 Q43

Here, $\frac{d}{dx}\left[\tan^{-1}(a+bx)\right] = 1 \text{ at } x = 0$

So, using chain rule,

$$\left[\left\{\frac{1}{1+(a+bx)^2}\right\}\frac{d}{dx}(a+bx)\right]_{x=0} = 1$$

$$\left[\frac{1}{1+(a+bx)^2} \times (b)\right]_{x=0} = 1$$

$\Rightarrow \quad \frac{b}{1+(a+0)^2} = 1$

$\Rightarrow \quad b = 1 + a^2.$

Differentiation Ex 11.3 Q44

Here, $y = \cos^{-1}(2x) + 2\cos^{-1}\sqrt{1-4x^2}$

Put $2x = \cos\theta$, so,

$$y = \cos^{-1}(\cos\theta) + 2\cos^{-1}\sqrt{1-\cos^2\theta}$$

$$= \cos^{-1}(\cos\theta) + 2\cos^{-1}(\sin\theta)$$

$$y = \cos^{-1}(\cos\theta) + 2\cos^{-1}\left(\cos\left(\frac{\pi}{2}-\theta\right)\right) \qquad \text{---(i)}$$

Now, $-\frac{1}{2} < x < 0$

$\Rightarrow \quad -1 < 2x < 0$

$\Rightarrow \quad -1 < \cos\theta < 0$

$\Rightarrow \quad \frac{\pi}{2} < \theta < \pi$

And

$\Rightarrow \quad -\frac{\pi}{2} > -\theta > -\pi$

$\Rightarrow \quad \left(\frac{\pi}{2}-\frac{\pi}{2}\right) > \left(\frac{\pi}{2}-\theta\right) > \left(\frac{\pi}{2}-\pi\right)$

$\Rightarrow \quad 0 > \left(\frac{\pi}{2}-\theta\right) > -\frac{\pi}{2}$

So, from equation (i),

$$y = \theta + 2\left[-\left(\frac{\pi}{2}-\theta\right)\right] \qquad \left[\begin{array}{l}\text{Since, } \cos^{-1}\cos(\theta) = \theta, \text{if } \theta \in [0,\pi] \\ \cos^{-1}\cos(\theta) = -\theta, \text{ if } \theta \in [-\pi, 0]\end{array}\right]$$

$$y = \theta - 2\times\frac{\pi}{2} + 2\theta$$

$$y = -\pi + 3\theta$$

$$y = -\pi + 3\cos^{-1}(2x) \qquad [\text{Since, } 2x = \cos\theta]$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = 0 + 3\left(\frac{-1}{\sqrt{1-(2x)^2}}\right)\frac{d}{dx}(2x)$$

$$= \frac{-3}{\sqrt{1-4x^2}}(2)$$

$$\frac{dy}{dx} = -\frac{6}{\sqrt{1-4x^2}}$$

Differentiation Ex 11.3 Q45

Here, $y = \tan^{-1}\left(\dfrac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right)$

Put $x = \cos 2\theta$, so

$$y = \tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta} - \sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{2\cos^2\theta} - \sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta} + \sqrt{2\sin^2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{2}\,(\cos\theta - \sin\theta)}{\sqrt{2}\,(\cos\theta + \sin\theta)}\right)$$

$$= \tan^{\{1\}}\left(\frac{\frac{\cos\theta - \sin\theta}{\cos\theta}}{\frac{\cos\theta + \sin\theta}{\cos\theta}}\right)$$

[Dividing numerator and denominator by $\cos\theta$]

$$= \tan^{-}1\left(\frac{\frac{\cos\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}\right)$$

$$= \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{\frac{\tan\pi}{4} - \tan\theta}{1 + \frac{\tan\pi}{4} + \tan\theta}\right)$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4} - \theta\right)\right]$$

$$= \frac{\pi}{4} - \theta$$

$$y = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x \qquad [\text{Using } x = \cos 2\theta]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 0 - \frac{1}{2}\left(\frac{-1}{\sqrt{1-x^2}}\right)$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q46

Here, $y = \cos^{-1}\left\{\frac{2x - 3\sqrt{1-x^2}}{\sqrt{13}}\right\}$

Let $x = \cos\theta$, so,

$$y = \cos^{-1}\left\{\frac{2\cos\theta - 3\sqrt{1-\cos^2\theta}}{\sqrt{13}}\right\}$$

$$= \cos^{-1}\left\{\frac{2}{\sqrt{13}}\cos\theta - \frac{3}{\sqrt{13}}\sin\theta\right\}$$

Let $\cos\phi = \frac{2}{\sqrt{13}}$

$\Rightarrow$ $\sin\phi = \sqrt{1-\cos^2\phi}$

$$= \sqrt{1-\left(\frac{2}{\sqrt{13}}\right)^2}$$

$$= \sqrt{\frac{13-4}{13}}$$

$$= \sqrt{\frac{9}{13}}$$

$$\sin\phi = \frac{3}{\sqrt{13}}$$

So,

$$y = \cos^{-1}\{\cos\phi\cos\theta - \sin\phi\sin\theta\}$$

$$= \cos^{-1}[\cos(\theta+\phi)]$$

$$y = \phi + \theta$$

$$y = \cos^{-1}\left(\frac{2}{\sqrt{13}}\right) + \cos^{-1}x \qquad \left[\text{Since, } x = \cos\theta, \cos\phi = \frac{2}{\sqrt{13}}\right]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 0 + \left(-\frac{1}{\sqrt{1-x^2}}\right)$$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q47

Consider the given expression:

$$y = \sin^{-1}\left\{\frac{2^{x+1} \times 3^x}{1+(36)^x}\right\}$$

$$= \sin^{-1}\left\{\frac{2 \times 2^x \times 3^x}{1+(6^2)^x}\right\}$$

$$y = \sin^{-1}\left\{\frac{2 \times 6^x}{1+6^{2x}}\right\} \ldots(1)$$

Substituting $6^x = \tan\theta$ in the above equation, we get,

$$y = \sin^{-1}\left\{\frac{2 \times 6^x}{1+6^{2x}}\right\}$$

$$= \sin^{-1}\left\{\frac{2 \times \tan\theta}{1+\tan^2\theta}\right\}$$

$$= \sin^{-1}(\sin 2\theta)$$

$$= 2\theta$$

$$= 2\tan^{-1}(6^x)$$

Differentiating the above function with respect to x, we have,

$$\frac{d}{dx}\left[\sin^{-1}\left\{\frac{2^{x+1} \times 3^x}{1+(36)^x}\right\}\right] = \frac{d}{dx}\left[2\tan^{-1}(6^x)\right]$$

$$= 2 \times \frac{1}{1+(6^x)^2} \times 6^x \log 6$$

$$= \frac{2 \times 6^x \log 6}{1+6^{2x}}$$

Ex 11.4

Differentiation Ex 11.4 Q1

Given,

$$xy = c^2$$

Differentiate with respect to x,

$$\frac{d}{dx}(xy) = \frac{d}{dx}(c^2)$$

$$\Rightarrow \quad x\frac{dy}{dx} + y\frac{d}{dx}(x) = 0 \qquad \text{[Using product rule]}$$

$$\Rightarrow \quad x\frac{dy}{dx} + y = 0$$

$$\Rightarrow \quad x\frac{dy}{dx} = -y$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{y}{x}$$

Differentiation Ex 11.4 Q2

Here, $y^3 - 3xy^2 = x^3 + 3x^2y$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{d}{dx}(y^3) - \frac{d}{dx}(3xy^2) = \frac{d}{dx}(x^3) + \frac{d}{dx}(3x^2y)$$

$$\Rightarrow \quad 3y^2\frac{dy}{dx} - 3\left[x\frac{d}{dx}y^2\frac{d}{dx}(x)\right] = 3x^2 + 3\left[x^2\frac{d}{dx}(y) + y\frac{d}{dx}(x^2)\right] \qquad \text{[Using product rule]}$$

$$\Rightarrow \quad 3y^2\frac{dy}{dx} - 3\left[x(2y)\frac{dy}{dx} + y^2\right] = 3x^2 + 3\left[x^2\frac{dy}{dx} + y(2x)\right]$$

$$\Rightarrow \quad 3y^2\frac{dy}{dx} - 6xy\frac{dy}{dx} - 3y^2 = 3x^2 + 3x^2\frac{dy}{dx} + 6xy$$

$$\Rightarrow \quad 3y^2\frac{dy}{dx} - 6xy\frac{dy}{dx} - 3x^2\frac{dy}{dx} = 3x^2 + 6xy + 3y^2$$

$$\Rightarrow \quad 3\frac{dy}{dx}(y^2 - 2xy - x^2) = 3(x^2 + 2xy + y^2)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{3(x+y)^2}{3(y^2 - 2xy - x^2)}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{(x+y)^2}{y^2 - 2xy - x^2}$$

Differentiation Ex 11.4 Q3

Here, $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$

Differentiate it with respect to x,

$$\frac{d}{dx}\left(x^{\frac{2}{3}}\right) + \frac{d}{dx}\left(y^{\frac{2}{3}}\right) = \frac{d}{dx}\left(a^{\frac{2}{3}}\right)$$

$$\Rightarrow \quad \frac{2}{3}x^{\left(\frac{2}{3}-1\right)} + \frac{2}{3}y^{\left(\frac{2}{3}-1\right)}\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{2}{3}x^{\frac{-1}{3}} + \frac{2}{3}y^{\frac{-1}{3}}\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{2}{3}y^{\frac{-1}{3}}\frac{dy}{dx} = -\frac{2}{3}x^{\frac{-1}{3}}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{2}{3}x^{\frac{-1}{3}} \times \frac{3}{2y^{\frac{-1}{3}}}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{x^{\frac{-1}{3}}}{y^{\frac{-1}{3}}}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{y^{\frac{1}{3}}}{x^{\frac{1}{3}}}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$$

Differentiation Ex 11.4 Q4

Given, $4x+3y=\log(4x-3y)$

Differentiating with respect to x,

$$\frac{d}{dx}(4x)+\frac{d}{dx}(3y)=\frac{d}{dx}(\log(4x-3y))$$

$$\Rightarrow \quad 4+3\frac{dy}{dx}=\frac{1}{(4x-3y)}\frac{d}{dx}(4x-3y) \qquad \text{[Using chain rule]}$$

$$\Rightarrow \quad 4+3\frac{dy}{dx}=\frac{1}{(4x-3y)}\left(4-3\frac{dy}{dx}\right)$$

$$\Rightarrow \quad 4+3\frac{dy}{dx}=\frac{4}{(4x-3y)}-\frac{3}{(4x-3y)}\frac{dy}{dx}$$

$$\Rightarrow \quad 3\frac{dy}{dx}+\frac{3}{(4x-3y)}\frac{dy}{dx}=\frac{4}{(4x-3y)}-4$$

$$\Rightarrow \quad 3\frac{dy}{dx}\left(1+\frac{1}{(4x-3y)}\right)=4\left(\frac{1}{(4x-3y)}-1\right)$$

$$\Rightarrow \quad 3\frac{dy}{dx}\left[\frac{4x-3y+1}{(4x-3y)}\right]=4\left[\frac{1-4x+3y}{(4x-3y)}\right]$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{4}{3}\left[\frac{1-4x+3y}{(4x-3y)}\right]\left[\frac{4x-3y}{4x-3y+1}\right]$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{4}{3}\left(\frac{1-4x+3y}{4x-3y+1}\right)$$

Differentiation Ex 11.4 Q5

Given,

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Differentiating with respect to x,

$$\frac{d}{dx}\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)=\frac{d}{dx}(1)$$

$$\Rightarrow \quad \frac{d}{dx}\left(\frac{x^2}{a^2}\right)+\frac{d}{dx}\left(\frac{y^2}{b^2}\right)=0$$

$$\Rightarrow \quad \frac{1}{a^2}(2x)+\frac{1}{b^2}(2y)\frac{dy}{dx}=0$$

$$\Rightarrow \quad \frac{2y}{b^2}\frac{dy}{dx}=-\frac{2x}{a^2}$$

$$\Rightarrow \quad \frac{dy}{dx}=-\left(\frac{2x}{a^2}\right)\left(\frac{b^2}{2y}\right)$$

$$\Rightarrow \quad \frac{dy}{dx}=-\frac{b^2x}{a^2y}$$

Differentiation Ex 11.4 Q6

Given,

$$x^5+y^5=5xy$$

Differentiating with respect to x,

$$\frac{d}{dx}\left(x^5\right)+\frac{d}{dx}\left(y^5\right)=\frac{d}{dx}(5xy)$$

$$\Rightarrow \quad 5x^4+5y^4\frac{dy}{dx}=5\left[x\frac{dy}{dx}+y\frac{dy}{dx}(x)\right] \qquad \text{[Using product rule]}$$

$$\Rightarrow \quad 5x^4+5y^4\frac{dy}{dx}=5\left[x\frac{dy}{dx}+y\,(1)\right]$$

$$\Rightarrow \quad 5x^4+5y^4\frac{dy}{dx}=5x\frac{dy}{dx}+5y$$

$$\Rightarrow \quad 5y^4\frac{dy}{dx}-5x\frac{dy}{dx}=5y-5x^4$$

$$\Rightarrow \quad 5\frac{dy}{dx}\left(y^4-x\right)=5\left(y-x^4\right)$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{5\left(y-x^4\right)}{5\left(y^4-x\right)}$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{y-x^4}{y^4-x}$$

Differentiation Ex 11.4 Q7

Given,

$$(x+y)^2 = 2axy$$

Differentiating with respect to x,

$\Rightarrow \quad \frac{d}{dx}(x+y)^2 = \frac{d}{dx}(2axy)$

$\Rightarrow \quad 2(x+y)\frac{d}{dx}(x+y) = 2a\left[x\frac{dy}{dx} + y\frac{d}{dx}(x)\right]$ [Using chain rule and product rule]

$\Rightarrow \quad 2(x+y)\left[1+\frac{dy}{dx}\right] = 2a\left[x\frac{dy}{dx} + y\,(1)\right]$

$\Rightarrow \quad 2(x+y) + 2(x+y)\frac{dy}{dx} = 2ax\frac{dy}{dx} + 2ay$

$\Rightarrow \quad \frac{dy}{dx}\left[2(x+y) - 2ax\right] = 2ay - 2(x+y)$

$\Rightarrow \quad \frac{dy}{dx} = \frac{2[ay-x-y]}{2[x+y-ax]}$

$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{ay-x-y}{x+y-ax}\right)$

Differentiation Ex 11.4 Q8

Given,

$$\left(x^2+y^2\right)^2 = xy$$

Differentiating with respect to x,

$\frac{d}{dx}\left(\left(x^2+y^2\right)^2\right) = \frac{d}{dx}(xy)$

$\Rightarrow \quad 2\left(x^2+y^2\right)\frac{d}{dx}\left(x^2+y^2\right) = x\frac{dy}{dx} + y\frac{d}{dx}(x)$ [Using chain rule and product rule]

$\Rightarrow \quad 2\left(x^2+y^2\right)\left(2x+2y\frac{dy}{dx}\right) = x\frac{dy}{dx} + y\,(1)$

$\Rightarrow \quad 4x\left(x^2+y^2\right) + 4y\left(x^2+y^2\right)\frac{dy}{dx} = x\frac{dy}{dx} + y$

$\Rightarrow \quad 4y\left(x^2+y^2\right)\frac{dy}{dx} - x\frac{dy}{dx} = y - 4x\left(x^2+y^2\right)$

$\Rightarrow \quad \frac{dy}{dx}\left[4yx^2 + 4y^3 - x\right] = y - 4x^3 - 4xy^2$

$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{y-4x^3-4xy^2}{4yx^2+4y^3-x}\right)$

Differentiation Ex 11.4 Q9

Here,

$$\tan^{-1}\left(x^2+y^2\right) = a$$

Differentiating with respect to x,

$\frac{d}{dx}\left(\tan^{-1}\left(x^2+y^2\right)\right) = \frac{d}{dx}(a)$

$\Rightarrow \quad \frac{1}{1+\left(x^2+y^2\right)^2} \times \frac{d}{dx}\left(x^2+y^2\right) = 0$ [Using chain rule]

$\Rightarrow \quad \left[\frac{1}{1+\left(x^2+y^2\right)^2}\right]\left(2x+2y\frac{dy}{dx}\right) = 0$

$\Rightarrow \quad \left\{\frac{2x}{1+\left(x^2+y^2\right)^2}\right\} + \left\{\frac{2y}{1+\left(x^2+y^2\right)^2}\cdot\right\}\frac{dy}{dx} = 0$

$\Rightarrow \quad \frac{2y}{1+\left(x^2+y^2\right)^2}\frac{dy}{dx} = -\frac{2x}{1+\left(x^2+y^2\right)^2}$

$\Rightarrow \quad \frac{dy}{dx} = -\left(\frac{2x}{1+\left(x^2+y^2\right)^2}\right)\left(\frac{1+\left(x^2+y^2\right)^2}{2y}\right)$

$\Rightarrow \quad \frac{dy}{dx} = -\left(\frac{x}{y}\right)$

Differentiation Ex 11.4 Q10

Given,

$$e^{x-y} = \log\left(\frac{x}{y}\right)$$

Differentiating with respect to x,

$$\frac{d}{dx}\left(e^{x-y}\right) = \frac{d}{dx}\log\left(\frac{x}{y}\right)$$

$$\Rightarrow \quad e^{(x-y)}\frac{d}{dx}(x-y) = \frac{1}{\left(\frac{x}{y}\right)}\times\frac{d}{dx}\left(\frac{x}{y}\right) \qquad \text{[Using chain rule and quotient rule]}$$

$$\Rightarrow \quad e^{(x-y)}\left(1-\frac{dy}{dx}\right) = \frac{y}{x}\left[\frac{y\frac{d}{dx}(x) - x\frac{dy}{dx}}{y^2}\right]$$

$$\Rightarrow \quad e^{(x-y)} - e^{(x-y)}\frac{dy}{dx} = \frac{1}{xy}\left[y(1) - x\frac{dy}{dx}\right]$$

$$\Rightarrow \quad e^{(x-y)} - e^{(x-y)}\frac{dy}{dx} = \frac{y}{xy} - \frac{x}{xy}\frac{dy}{dx}$$

$$\Rightarrow \quad e^{(x-y)} - e(x-y)\frac{dy}{dx} = \frac{1}{x} - \frac{1}{y}\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} - e^{(x-y)}\frac{dy}{dx} = \frac{1}{x} - e^{(x-y)}$$

$$\Rightarrow \quad \frac{dy}{dx}\left[\frac{1}{y} - \frac{e^{(x-y)}}{1}\right] = \frac{1}{x} - \frac{e^{(x-y)}}{1}$$

$$\Rightarrow \quad \frac{dy}{dx}\left[\frac{1-ye^{(x-y)}}{y}\right] = \frac{\left(1-xe^{(x-y)}\right)}{x}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x}\left[\frac{1-xe^{(x-y)}}{1-ye^{(x-y)}}\right]$$

$$= \frac{-y}{-x}\left[\frac{xe^{(x-y)}-1}{ye^{(x-y)}-1}\right]$$

$$\frac{dy}{dx} = \frac{y}{x}\left[\frac{xe^{(x-y)}-1}{ye^{(x-y)}-1}\right]$$

Differentiation Ex 11.4 Q11

Given,

$$\sin xy + \cos(x+y) = 1$$

Differentiating with respect to x,

$$\frac{d}{dx}\sin xy + \frac{d}{dx}\cos(x+y) = \frac{d}{dx}(1)$$

$$\Rightarrow \quad \cos xy\frac{d}{dx}(xy) - \sin(x+y)\frac{d}{dx}(x+y) = 0 \qquad \text{[Using chain rule and product rule]}$$

$$\Rightarrow \quad \cos(xy)\left[x\frac{dy}{dx} + y\frac{d}{dx}(x)\right] - \sin(x+y)\left[1+\frac{dy}{dx}\right] = 0$$

$$\Rightarrow \quad \cos(xy)\left[x\frac{dy}{dx} + y(1)\right] - \sin(x+y) + \sin(x+y)\frac{dy}{dx} = 0$$

$$\Rightarrow \quad x\cos(xy)\frac{dy}{dx} + y\cos(xy) - \sin(x+y) + \sin(x+y)\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \left[x\cos(xy) + \sin(x+y)\right]\frac{dy}{dx} = \left[\sin(x+y) - y\cos xy\right]$$

$$\Rightarrow \quad \frac{dy}{dx} = \left[\frac{\sin(x+y) - y\cos xy}{x\cos xy + \sin(x+y)}\right]$$

Differentiation Ex 11.4 Q12

Here,

$$\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$$

Let $\quad x=\sin A, y=\sin B,$ so

$\Rightarrow \quad \sqrt{1-\sin^2 A}+\sqrt{1-\sin^2 B}=a(\sin A-\sin B)$

$\Rightarrow \quad \cos A+\cos B=a(\sin A-\sin B) \qquad \left[\text{Since } \left(1-\sin^2\theta\right)=\cos^2\theta\right]$

$\Rightarrow \quad a=\dfrac{\cos A+\cos B}{\sin A-\sin B}$

$\Rightarrow \quad a=\dfrac{2\cos\frac{A+B}{2}\times\cos\frac{A-B}{2}}{2\cos\frac{A+B}{2}\times\sin\frac{A-B}{2}} \qquad \left[\begin{array}{l}\text{Since, } \sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) \\ \qquad \cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\end{array}\right]$

$\Rightarrow \quad a=\cot\left(\dfrac{A-B}{2}\right)$

$\Rightarrow \quad \cot^{-1}a=\dfrac{A-B}{2}$

$\Rightarrow \quad 2\cot^{-1}a=A-B$

$\Rightarrow \quad 2\cot^{-1}a=\sin^{-1}x-\sin^{-1}y \qquad [\text{Since } x=\sin A, y=\sin B]$

Differentiating with respect to x,

$$\frac{d}{dx}\left(2\cot^{-1}a\right)=\frac{d}{dx}\left(\sin^{-1}x\right)-\frac{d}{dx}\left(\sin^{-1}y\right)$$

$\Rightarrow \quad 0=\dfrac{1}{\sqrt{1-x^2}}-\dfrac{1}{\sqrt{1-y^2}}\dfrac{dy}{dx}$

$\Rightarrow \quad \dfrac{1}{\sqrt{1-y^2}}\dfrac{dy}{dx}=\dfrac{1}{\sqrt{1-x^2}}$

$\Rightarrow \quad \dfrac{dy}{dx}=\dfrac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

$$\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$$

Differentiation Ex 11.4 Q13

Here,

$$y\sqrt{1-x^2}+x\sqrt{1-y^2}=1$$

Let $\quad x=\sin A, y=\sin B$

$\Rightarrow \quad \sin B\sqrt{1-\sin^2 A}+\sin A\sqrt{1-\sin^2 B}=1$

$\Rightarrow \quad \sin B\cos A+\sin A\cos B=1 \qquad \left[\begin{array}{l}\text{since } 1-\sin^2\theta=\cos^2\theta \text{ and} \\ \sin(x+y)=\sin x\cos y+\cos x\sin y\end{array}\right]$

$\Rightarrow \quad \sin(A+B)=1$

$\Rightarrow \quad A+B=\sin^{-1}(1)$

$\Rightarrow \quad \sin^{-1}x+\sin^{-1}y=\dfrac{\pi}{2} \qquad [\text{Since } x=\sin A, y=\sin B]$

Differentiating with respect to x,

$\Rightarrow \quad \dfrac{d}{dx}\left(\sin^{-1}x\right)+\dfrac{d}{dx}\left(\sin^{-1}y\right)=\dfrac{d}{dx}\left(\dfrac{\pi}{2}\right)$

$\Rightarrow \quad \dfrac{1}{\sqrt{1-x^2}}+\dfrac{1}{\sqrt{1-y^2}}\dfrac{dy}{dx}=0$

$\Rightarrow \quad \dfrac{dy}{dx}=-\sqrt{\dfrac{1-y^2}{1-x^2}}$

Differentiation Ex 11.4 Q14

Here,

$$xy = 1 \qquad \text{---(i)}$$

Differentiating with respect to x,

$$\frac{d}{dx}(xy) = \frac{d}{dx}(1)$$

$$\Rightarrow \quad x\frac{dy}{dx} + y\frac{d}{dx}(x) = 0 \qquad \text{[Using product rule]}$$

$$\Rightarrow \quad x\frac{dy}{dx} + y(1) = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{y}{x} \qquad \left[\text{Put } x = \frac{1}{y} \text{ from equation (i)}\right]$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{y}{\frac{1}{y}}$$

$$\Rightarrow \quad \frac{dy}{dx} = -y^2$$

$$\Rightarrow \quad \frac{dy}{dx} + y^2 = 0$$

Differentiation Ex 11.4 Q15

Here,

$$xy^2 = 1$$

Differentiating with respect to x,

$$\frac{d}{dx}\left(xy^2\right) = \frac{d}{dx}(1)$$

$$\Rightarrow \quad x\frac{d}{dx}\left(y^2\right) + y^2\frac{d}{dx}(x) = 0 \qquad \text{[Using product rule]}$$

$$\Rightarrow \quad x(2y)\frac{dy}{dx} + y^2(1) = 0$$

$$\Rightarrow \quad 2xy\frac{dy}{dx} = -y^2$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-y^2}{2xy}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-y}{2x}$$

Put $x = \frac{1}{y^2}$ from equation (i)

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-y}{2\left(\frac{1}{y^2}\right)}$$

$$\Rightarrow \quad 2\frac{dy}{dx} = -y^3$$

$$2\frac{dy}{dx} + y^3 = 0$$

Differentiation Ex 11.4 Q16

Given,

$$x\sqrt{1+y}+y\sqrt{1+x}=0$$

$$\Rightarrow \quad x\sqrt{1+y}=-y\sqrt{1+x}$$

Squaring both the sides,

$$\Rightarrow \quad \left(x\sqrt{1+y}\right)^2=\left(-y\sqrt{1+x}\right)^2$$

$$\Rightarrow \quad x^2(1+y)=y^2(1+x)$$

$$\Rightarrow \quad x^2+x^2y=y^2+y^2x$$

$$\Rightarrow \quad x^2-y^2=y^2x-x^2y$$

$$\Rightarrow \quad (x-y)(x+y)=xy(y-x)$$

$$\Rightarrow \quad (x+y)=-xy$$

$$\Rightarrow \quad y+xy=-x$$

$$\Rightarrow \quad y(1+x)=-x$$

$$\Rightarrow \quad y=\frac{-x}{(1+x)}$$

Differentiating with respect to x using quotient rule,

$$\Rightarrow \quad \frac{dy}{dx}=\left[\frac{-(1+x)\frac{d}{dx}(x)+(-x)\frac{d}{dx}(x+1)}{(1+x)^2}\right]$$

$$\Rightarrow \quad \frac{dy}{dx}=\left[\frac{-(1+x)(1)+x(1)}{(1+x)^2}\right]$$

$$\Rightarrow \quad \frac{dy}{dx}=\left[\frac{-1-x+x}{(1+x)^2}\right]$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{-1}{(1+x)^2}$$

$$\Rightarrow \quad (1+x)^2\frac{dy}{dx}=-1$$

$$\Rightarrow \quad (1+x)^2\frac{dy}{dx}+1=0$$

Differentiation Ex 11.4 Q17

Here,

$$\log\sqrt{x^2+y^2}=\tan^{-1}\left(\frac{x}{y}\right)$$

$$\Rightarrow \quad \log\left(x^2+y^2\right)^{\frac{1}{2}}=\tan^{-1}\left(\frac{y}{x}\right)$$

$$\Rightarrow \quad \frac{1}{2}\log\left(x^2+y^2\right)=\tan^{-1}\left(\frac{y}{x}\right)$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{1}{2}\frac{d}{dx}\log\left(x^2+y^2\right)=\frac{d}{dx}\tan^{-1}\left(\frac{y}{x}\right)$$

$$\Rightarrow \quad \frac{1}{2}\times\left(\frac{1}{x^2+y^2}\right)\frac{d}{dx}\left(x^2+y^2\right)=\frac{1}{1+\left(\frac{y}{x}\right)^2}\frac{d}{dx}\left(\frac{y}{x}\right) \qquad \text{[Using chain rule, quotient rule]}$$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{1}{x^2+y^2}\right)\left[2x+2y\frac{dy}{dx}\right]=\frac{x^2}{\left(x^2+y^2\right)}\left[\frac{x\frac{dy}{dx}-y\frac{d}{dx}(x)}{x^2}\right]$$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{1}{x^2+y^2}\right)\times 2\left(x+y\frac{dy}{dx}\right)=\frac{x^2}{\left(x^2+y^2\right)}\left[\frac{x\frac{dy}{dx}-y(1)}{x^2}\right]$$

$$\Rightarrow \quad x+y\frac{dy}{dx}=x\frac{dy}{dx}-y$$

$$\Rightarrow \quad y\frac{dy}{dx}-x\frac{dy}{dx}=-y-x$$

$$\Rightarrow \quad \frac{dy}{dx}(y-x)=-(y+x)$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{-(y+x)}{y-x}$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{x+y}{x-y}$$

Differentiation Ex 11.4 Q18

Here,

$$\sec\left(\frac{x+y}{x-y}\right) = a$$

$$\Rightarrow \quad \frac{x+y}{x-y} = \sec^{-1}(a)$$

Differentiating with respect to x,

$$\Rightarrow \quad \left[\frac{(x-y)\frac{d}{dx}(x+y) - (x+y)\frac{d}{dx}(x-y)}{(x-y)^2}\right] = 0 \qquad [\text{Using quotient rule}]$$

$$\Rightarrow \quad (x-y)\left(1+\frac{dy}{dx}\right) - (x+y)\left(1-\frac{dy}{dx}\right) = 0$$

$$\Rightarrow \quad (x-y) + (x-y)\frac{dy}{dx} - (x+y) + (x+y)\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx}[x-y+x+y] = x+y-x+y$$

$$\Rightarrow \quad \frac{dy}{dx}(2x) = 2y$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x}$$

Differentiation Ex 11.4 Q19

Here,

$$\tan^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = a$$

$$\Rightarrow \quad \frac{x^2-y^2}{x^2+y^2} = \tan a$$

$$\Rightarrow \quad x^2-y^2 = \tan a\left(x^2+y^2\right)$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{d}{dx}\left(x^2-y^2\right) = \tan a\frac{d}{dx}\left(x^2+y^2\right)$$

$$\Rightarrow \quad \left(2x-2y\frac{dy}{dx}\right) = \tan a\left(2x+2y\frac{dy}{dx}\right)$$

$$\Rightarrow \quad 2x-2y\frac{dy}{dx} = 2x\tan a + 2t\tan a\frac{dy}{dx}$$

$$\Rightarrow \quad 2y\tan a\frac{dy}{dx} + 2y\frac{dy}{dx} = 2x - 2x\tan a$$

$$\Rightarrow \quad 2y\frac{dy}{dx}(1+\tan a) = 2x(1-\tan a)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x}{y}\left(\frac{1-\tan a}{1+\tan a}\right)$$

Differentiation Ex 11.4 Q20

Here,

$$xy\log(x+y)=1$$

Differentiating it with respect to x,

$$\Rightarrow \quad \frac{d}{dx}\left[xy\log(x+y)\right]=\frac{d}{dx}(1)$$

$$\Rightarrow \quad xy\frac{d}{dx}\log(x+y)+x\log(x+y)\frac{dy}{dx}+y\log(x+y)\frac{d}{dx}(x)=0$$

[Using chain rule and product rule]

$$\Rightarrow \quad xy\times\left(\frac{1}{x+y}\right)\frac{d}{dx}(x+y)+x\log(x+y)\frac{dy}{dx}+y\log(x+y)(1)=0$$

$$\Rightarrow \quad \left(\frac{xy}{x+y}\right)\left(1+\frac{dy}{dx}\right)+x\log(x+y)\frac{dy}{dx}+y\log(x+y)=0$$

$$\Rightarrow \quad \left(\frac{xy}{x+y}\right)\frac{dy}{dx}+\left(\frac{xy}{x+y}\right)+x\left(\frac{1}{xy}\right)\frac{dy}{dx}+y\left(\frac{1}{xy}\right)=0$$

$$\left[\text{Sicne from equation (i) } \log(x+y)=\frac{1}{xy}\right]$$

$$\Rightarrow \quad \frac{dy}{dx}\left[\frac{xy}{x+y}+\frac{1}{y}\right]=-\left[\frac{1}{x}+\frac{xy}{x+y}\right]$$

$$\frac{dy}{dx}\left[\frac{xy^2+x+y}{(x+y)y}\right]=-\left[\frac{x+y+x^2y}{x(x+y)}\right]$$

$$\frac{dy}{dx}=-\left(\frac{x+y+x^2y}{x(x+y)}\right)\left(\frac{y(x+y)}{xy^2+x+y}\right)$$

$$=-\frac{y}{x}\left(\frac{x+y+x^2y}{x+y+xy^2}\right)$$

So,

$$\frac{dy}{dx}=-\frac{y}{x}\left(\frac{x^2y+x+y}{xy^2+x+y}\right)$$

Differentiation Ex 11.4 Q21

Here,

$$y=x\sin(a+y) \qquad \text{---(i)}$$

Differentiating with respect to y,

$$\Rightarrow \quad \frac{dy}{dx}=\frac{d}{dx}\left[x\sin(a+y)\right]$$

$$\Rightarrow \quad \frac{dy}{dx}=x\frac{d}{dx}\sin(a+y)+\sin(a+y)\frac{d}{dx}(x) \qquad \text{[Using product rule, chain ruel]}$$

$$\Rightarrow \quad \frac{dy}{dx}=x\cos(a+y)\frac{d}{dx}(a+y)+\sin(a+y)(1)$$

$$=x\cos(a+y)\left(0+\frac{dy}{dx}\right)+\sin(a+y)$$

$$\Rightarrow \quad \frac{dy}{dx}\left(1-x\cos(a+y)\right)=\sin(a+y)$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{\sin(a+y)}{1-x\cos(a+y)}$$

Put x from equation (i), $x=\dfrac{y}{\sin(a+y)}$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{\sin(a+y)}{1-\dfrac{y}{\sin(a+y)}\cos(a+y)}$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin(a+y)-y\cos(a+y)}$$

Differentiation Ex 11.4 Q22

Here,

$$x\sin(a+y)+\sin a\cos(a+y)=0 \qquad \text{---(i)}$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{d}{dx}\left[x\sin(a+y)\right]+\frac{d}{dx}\left[\sin a\cos(a+y)\right]=0$$

$$\Rightarrow \quad \left[x\frac{d}{dx}\sin(a+y)+\sin(a+y)\frac{d}{dx}(x)\right]+\sin a\frac{d}{dx}\cos(a+y)=0$$

[Using product rule and chain rule]

$$\Rightarrow \quad \left[x\cos(a+y)\frac{d}{dx}(a+y)+\sin(a+y)(1)\right]+\sin a\left[-\sin(a+y)\frac{d}{dx}(a+y)\right]=0$$

$$\Rightarrow \quad \left[x\cos(a+y)\left(0+\frac{dy}{dx}\right)+\sin(a+y)\right]-\sin a\sin(a+y)\left(0+\frac{dy}{dx}\right)=0$$

$$\Rightarrow \quad x\cos(a+y)\frac{dy}{dx}+\sin(a+y)-\sin a\sin(a+y)\frac{dy}{dx}=0$$

$$\Rightarrow \quad \frac{dy}{dx}\left[x\cos(a+y)-\sin a\sin(a+y)\right]=-\sin(a+y)$$

Put $x=-\sin a\dfrac{\cos(a+y)}{\sin(a+y)}$ from equation (i),

$$\Rightarrow \quad \frac{dy}{dx}\left[-\sin a\frac{\cos^2(a+y)}{\sin(a+y)}-\sin a\sin(a+y)\right]=-\sin(a+y)$$

$$\Rightarrow \quad -\frac{dy}{dx}\left[\frac{\sin a\cos^2(a+y)+\sin a\sin^2(a+y)}{\sin(a+y)}\right]=-\sin(a+y)$$

$$\Rightarrow \quad \frac{dy}{dx}=\sin(a+y)^2\left[\frac{\sin(a+y)}{\sin a\left(\cos^2(a+y)+\sin^2(a+y)\right)}\right]$$

$$\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a} \qquad \left[\text{Since } \sin^2\theta+\cos^2\theta=1\right]$$

Differentiation Ex 11.4 Q23

Here,

$$y=x\sin y$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{dy}{dx}=\frac{d}{dx}(x\sin y)$$

$$\Rightarrow \quad \frac{dy}{dx}=x\frac{d}{dx}(\sin y)+\sin y\frac{d}{dx}(x) \qquad \text{[Using product rule]}$$

$$\Rightarrow \quad \frac{dy}{dx}=x\cos\frac{dy}{dx}+\sin y\,(1)$$

$$\Rightarrow \quad \frac{dy}{dx}(1-x\cos y)=\sin y$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{\sin y}{1-x\cos y}$$

Differentiation Ex 11.4 Q24

Here,

$$y\sqrt{x^2+1} = \log\left(\sqrt{x^2+1}-x\right)$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{d}{dx}\left(y\sqrt{x^2+1}\right) = \frac{d}{dx}\log\left(\sqrt{x^2+1}-x\right) \qquad \text{[Using product rule and chain rule]}$$

$$\Rightarrow \quad y\frac{d}{dx}\left(\sqrt{x^2+1}\right)+\sqrt{x^2+1}\frac{dy}{dx} = \frac{1}{\left(\sqrt{x^2+1}-x\right)}\times\frac{d}{dx}\left(\sqrt{x^2+1}-x\right)$$

$$\Rightarrow \quad y\frac{1}{2\sqrt{x^2+1}}\times\frac{d}{dx}\left(x^2+1\right)+\sqrt{x^2+1}\frac{dy}{dx} = \frac{1}{\left(\sqrt{x^2+1}-x\right)}\times\left[\frac{1}{2\sqrt{x^2+1}}\frac{d}{dx}\left(x^2+1\right)-1\right]$$

$$\Rightarrow \quad \frac{2xy}{2\sqrt{x^2+1}}+\sqrt{x^2+1}\frac{dy}{dx} = \frac{1}{\left(\sqrt{x^2+1}-x\right)}\left[\frac{2x}{2\sqrt{x^2+1}}-1\right]$$

$$\Rightarrow \quad \sqrt{x^2+1}\frac{dy}{dx} = \left[\frac{1}{\sqrt{x^2+1}-x}\right]\left[\frac{x-\sqrt{x^2+1}}{\sqrt{x^2+1}}\right]-\frac{xy}{\sqrt{x^2+1}}$$

$$\Rightarrow \quad \sqrt{x^2+1}\frac{dy}{dx} = \frac{-1}{\sqrt{x^2+1}}-\frac{xy}{\sqrt{x^2+1}}$$

$$\Rightarrow \quad \sqrt{x^2+1}\frac{dy}{dx} = \frac{-(1+xy)}{\sqrt{x^2+1}}$$

$$\Rightarrow \quad \left(x^2+1\right)\frac{dy}{dx} = -(1+xy)$$

$$\Rightarrow \quad \left(x^2+1\right)\frac{dy}{dx}+1+xy = 0$$

Differentiation Ex 11.4 Q25

Here,

$$y = [\log_{\cos x}\sin x][\log_{\sin x}\cos x]^{-1}+\sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$y = [\log_{\cos x}\sin x][\log_{\cos x}\sin x]+\sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$\left[\text{Since, } \log_a a = (\log_b a)^{-1}\right]$$

$$y = \left[\frac{\log\sin x}{\log\cos x}\right]^2+\sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$\left[\text{Sicne, } \log_a b = \frac{\log b}{\log a}\right]$$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{\log\sin x}{\log\cos x}\right]^2+\frac{d}{dx}\left(\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right)$$

$$= 2\left[\frac{\log\sin x}{\log\cos x}\right]\frac{d}{dx}\left(\frac{\log\sin x}{\log\cos x}\right)+\frac{1}{\sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}}\times\frac{d}{dx}\left[\frac{2x}{1+x^2}\right]$$

$$\frac{dy}{dx} = 2\left[\frac{\log\sin x}{\log\cos x}\right]\left(\frac{(\log\cos x)\frac{d}{dx}(\log\sin x)-\log\sin x\frac{d}{dx}(\log\cos x)}{(\log\cos x)^2}\right)+$$

$$\text{[Using chain rule, quotient rule]} \quad \left(\frac{\left(1+x^2\right)}{\sqrt{1+x^4-2x^2}}\right)\left(\frac{\left(1+x^2\right)(2)-(2x)(2x)}{\left(1+x^2\right)^2}\right)$$

$$= 2\left(\frac{\log\sin x}{\log\cos x}\right)\left(\frac{\log\cos x\times\frac{1}{\sin x}\frac{d}{dx}(\sin x)-\log\sin x\times\frac{1}{\cos x}\frac{d}{dx}(\cos x)}{(\log\cos x)^2}\right)+$$

$$\left(\frac{\left(1+x^2\right)}{\sqrt{1+x^4-2x^2}}\right)\left(\frac{\left(1+x^2\right)(2)-(2x)(2x)}{\left(1+x^2\right)^2}\right)$$

$$= 2\left(\frac{\log\sin x}{\log\cos x}\right)\left(\frac{\log\cos x\left(\frac{\cos x}{\sin x}\right)+\log\sin x\left(\frac{\sin x}{\cos x}\right)}{(\log\cos x)^2}\right)+$$

$$\left(\frac{1+x^2}{\sqrt{\left(1-x^2\right)^2}}\right)\left(\frac{2+2x^2-4x^2}{\left(1+x^2\right)^2}\right)$$

$$\frac{dy}{dx} = 2\frac{\log\sin x}{(\log\cos x)^3}(\cot x\log\cos x+\tan x\log\sin x)+\frac{2}{1+x^2}$$

Put $x = \frac{\pi}{4}$

$$\frac{dy}{dx} = 2\left(\frac{\log\sin\frac{\pi}{4}}{\left(\log\cos\frac{\pi}{4}\right)^3}\right)\left(\cot\frac{\pi}{4}\log\cos\frac{\pi}{4}+\tan\frac{\pi}{4}\log\sin\frac{\pi}{4}\right)+2\left(\frac{1}{1+\left(\frac{\pi}{4}\right)^2}\right)$$

$$= 2\left(\frac{1}{\left(\log\frac{1}{\sqrt{2}}\right)^2}\right)\left(1\times\log\frac{1}{\sqrt{2}}+1\times\log\frac{1}{\sqrt{2}}\right)+2\left(\frac{16}{16+\pi}\right)$$

$$= 2\times\frac{2\log\left(\frac{1}{\sqrt{2}}\right)}{\left(\log\left(\frac{1}{\sqrt{2}}\right)\right)}+\frac{32}{16+\pi^2}$$

$$= 4\frac{1}{\log\left(\frac{1}{\sqrt{2}}\right)}+\frac{32}{16+\pi^2}$$

$$= 4\frac{1}{-\frac{1}{2}\log^2}+\frac{32}{16+\pi^2}$$

$$= -\frac{8}{\log 2}+\frac{32}{16+\pi^2}$$

$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}} = 8\left[\frac{4}{16+\pi^2}-\frac{1}{\log 2}\right]$$

Differentiation Ex 11.4 Q26

Here,

$$\sin(xy)+\frac{y}{x} = x^2-y^2$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{d}{dx}(\sin xy)+\frac{d}{dx}\left(\frac{y}{x}\right) = \frac{d}{dx}\left(x^2\right)-\frac{d}{dx}\left(y^2\right)$$

$$\Rightarrow \quad \cos(xy)\frac{d}{dx}(xy)+\left[\frac{x\frac{dy}{dx}-y\frac{d}{dx}(x)}{x^2}\right] = 2x-2y\frac{dy}{dx} \qquad \left[\begin{array}{l}\text{Using chain rule, quotient ruel,}\\ \text{product rule}\end{array}\right]$$

$$\Rightarrow \quad \cos(xy)\left[x\frac{dy}{dx}+y\frac{d}{dx}(x)\right]+\left[\frac{x\frac{dy}{dx}-y(1)}{x^2}\right] = 2x-2y\frac{dy}{dx}$$

$$\Rightarrow \quad \cos(xy)\left[x\frac{dy}{dx}+y(1)\right]+\frac{1}{x^2}\left(x\frac{dy}{dx}-y\right) = 2x-2y\frac{dy}{dx}$$

$$\Rightarrow \quad x\cos(xy)\frac{dy}{dx}+y\cos(xy)+\frac{1}{x}\frac{dy}{dx}-\frac{y}{x^2} = 2x-2y\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx}\left[x\cos(xy)+\frac{1}{x}+2y\right] = \frac{y}{x^2}-y\cos(xy)+2x$$

$$\Rightarrow \quad \frac{dy}{dx}\left[\frac{x^2\cos(xy)+1+2xy}{x}\right] = \frac{1}{x^2}\left(y-x^2y\cos xy+2x^3\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2x^3+y-x^2y\cos(xy)}{x\left(x^2\cos xy+1+2xy\right)}$$

Differentiation Ex 11.4 Q27

Here,

$$\sqrt{y+x}+\sqrt{y-x}=c$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{d}{dx}\left(\sqrt{y+x}\right)+\frac{d}{dx}\sqrt{y-x}=\frac{d}{dx}(c)$$

$$\Rightarrow \quad \frac{1}{2\sqrt{y+x}}\frac{d}{dx}(y+x)+\frac{1}{2\sqrt{y-x}}\frac{d}{dx}(y-x)=0$$

Using chain rule

$$\Rightarrow \quad \frac{1}{2\sqrt{y+x}}\left[\frac{dy}{dx}+1\right]+\frac{1}{2\sqrt{y-x}}\left[\frac{dy}{dx}-1\right]=0$$

$$\Rightarrow \quad \frac{dy}{dx}\left(\frac{1}{2\sqrt{y+x}}\right)+\frac{dy}{dx}\left(\frac{1}{2\sqrt{y-x}}\right)=\frac{1}{2\sqrt{y-x}}-\frac{1}{2\sqrt{y+x}}$$

$$\Rightarrow \quad \frac{dy}{dx}\times\frac{1}{2}\left[\frac{1}{\sqrt{y+x}}+\frac{1}{\sqrt{y-x}}\right]=\frac{1}{2}\left[\frac{\sqrt{y+x}-\sqrt{y-x}}{\sqrt{y-x}\sqrt{y+x}}\right]$$

$$\Rightarrow \quad \frac{dy}{dx}\left[\frac{\sqrt{y-x}+\sqrt{y+x}}{\sqrt{y+x}\sqrt{y-x}}\right]=\left[\frac{\sqrt{y+x}-\sqrt{y-x}}{\sqrt{y-x}\sqrt{y+x}}\right]$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{\sqrt{y+x}-\sqrt{y-x}}{\sqrt{y-x}+\sqrt{y+x}}\times\frac{\left(\sqrt{y+x}-\sqrt{y-x}\right)}{\left(\sqrt{y+x}-\sqrt{y-x}\right)}$$

[rationalizing the denominator]

$$\Rightarrow \quad \frac{dy}{dx}=\frac{(y+x)+(y-x)-2\sqrt{y+x}\sqrt{y-x}}{y+x-y+x}$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{2y-2\sqrt{y^2-x^2}}{2x}$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{2y}{2x}-\frac{2\sqrt{y^2-x^2}}{2x}$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{y}{x}-\sqrt{\frac{y^2-x^2}{x^2}}$$

$$\frac{dy}{dx}=\frac{y}{x}-\sqrt{\frac{y^2}{x^2}-1}$$

Differentiation Ex 11.4 Q28

Here,

$$\tan(x+y)+\tan(x-y)=1$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{d}{dx}\tan(x+y)+\frac{d}{dx}\tan(x-y)=\frac{d}{dx}(1)$$

$$\Rightarrow \quad \sec^2(x+y)\frac{d}{dx}(x+y)+\sec^2(x-y)\frac{d}{dx}(x-y)=0 \qquad \text{[Using chain rule]}$$

$$\Rightarrow \quad \sec^2(x+y)\left[1+\frac{dy}{dx}\right]+\sec^2(x-y)\left[1-\frac{dy}{dx}\right]=0$$

$$\Rightarrow \quad \sec^2(x+y)\frac{dy}{dx}-\sec^2(x-y)\frac{dy}{dx}=-\left[\sec^2(x+y)+\sec^2(x-y)\right]$$

$$\Rightarrow \quad \frac{dy}{dx}\left[\sec^2(x+y)-\sec^2(x-y)\right]=-\left[\sec^2(x+y)+\sec^2(x-y)\right]$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{\sec^2(x+y)+\sec^2(x-y)}{\sec^2(x-y)-\sec^2(x+y)}$$

Differentiation Ex 11.4 Q29

Here,

$$e^x + e^y = e^{x+y}$$

Differentiating with respect to x using chain rule,

$$\Rightarrow \quad \frac{d}{dx}\left(e^x\right) + \frac{d}{dx}e^y = \frac{d}{dx}\left(e^{x+y}\right)$$

$$\Rightarrow \quad e^x + e^y \frac{dy}{dx} = e^{x+y} \frac{d}{dx}(x+y)$$

$$\Rightarrow \quad e^x + e^y \frac{dy}{dx} = e^{x+y}\left[1 + \frac{dy}{dx}\right]$$

$$\Rightarrow \quad e^y \frac{dy}{dx} - e^{x+y}\frac{dy}{dx} = e^{x+y} - e^x$$

$$\Rightarrow \quad \frac{dy}{dx}\left(e^y - e^{x+y}\right) = e^{x+y} - e^x$$

$$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{e^x \times e^y - e^x}{e^y - e^x \times e^y}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{e^x\left(e^y - 1\right)}{e^y\left(1 - e^x\right)}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{e^x\left(e^y - 1\right)}{e^y\left(e^x - 1\right)}$$

Differentiation Ex 11.4 Q30

It is given that, $\cos y = x\cos(a+y)$

$$\therefore \frac{d}{dx}[\cos y] = \frac{d}{dx}[x\cos(a+y)]$$

$$\Rightarrow -\sin y\frac{dy}{dx} = \cos(a+y)\cdot\frac{d}{dx}(x) + x\cdot\frac{d}{dx}[\cos(a+y)]$$

$$\Rightarrow -\sin y\frac{dy}{dx} = \cos(a+y) + x\cdot[-\sin(a+y)]\frac{dy}{dx}$$

$$\Rightarrow [x\sin(a+y) - \sin y]\frac{dy}{dx} = \cos(a+y) \qquad ...(1)$$

Since $\cos y = x\cos(a+y)$, $x = \dfrac{\cos y}{\cos(a+y)}$

Then, equation (1) reduces to

$$\left[\frac{\cos y}{\cos(a+y)}\cdot\sin(a+y) - \sin y\right]\frac{dy}{dx} = \cos(a+y)$$

$$\Rightarrow [\cos y\cdot\sin(a+y) - \sin y\cdot\cos(a+y)]\cdot\frac{dy}{dx} = \cos^2(a+y)$$

$$\Rightarrow \sin(a+y-y)\frac{dy}{dx} = \cos^2(a+b)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos^2(a+b)}{\sin a}$$

Hence, proved.

Ex 11.5

Differentiation Ex 11.5 Q1

Let $y = x^{\frac{1}{x}}$ ---(i)

Taking log on both the sides,

$$\Rightarrow \quad \log y = \log x^{\frac{1}{x}}$$

$$\Rightarrow \quad \log y = \frac{1}{x}\log x \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = \frac{1}{x}\frac{d}{dx}(\log x) + \log x\frac{d}{dx}\left(x^{-1}\right) \qquad \left[\text{Using product rule}\right]$$

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = \frac{1}{x}\times\frac{1}{x} + (\log x)\times\left(-\frac{1}{x^2}\right)$$

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2} - \frac{\log x}{x^2}$$

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = \frac{(1-\log x)}{x^2}$$

$$\Rightarrow \quad \frac{dy}{dx} = y\left[\frac{1-\log x}{x^2}\right]$$

Put the value of y from equation (i),

$$\frac{dy}{dx} = x^{\frac{1}{x}}\left[\frac{1-\log x}{x}\right]$$

Differentiation Ex 11.5 Q2

Let $y = x^{\sin x}$ ---(i)

Taking log on both the sides,

$$\log y = \log x^{\sin x}$$

$$\log y = \sin x \log x \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating with respect to x,

$$\frac{1}{y}\frac{dy}{dx} = \sin x\frac{d}{dx}\log x + \log x\frac{d}{dx}\sin x \qquad \left[\text{Using product rule}\right]$$

$$\frac{1}{y}\frac{dy}{dx} = \sin x\left(\frac{1}{x}\right) + \log x(\cos x)$$

$$\frac{dy}{dx} = y\left[\frac{\sin x}{x} + (\log x)(\cos x)\right]$$

Put the value of y,

$$\frac{dy}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + (\log x)(\cos x)\right]$$

Differentiation Ex 11.5 Q3

Let $y = (1+\cos x)^x$ ---(i)

Taking log on both the sides,

$$\log y = \log(1+\cos x)^x$$
$$\log y = x\log(1+\cos x)$$

Differentiating with respect to x,

$$\frac{1}{y}\frac{dy}{dx} = x\frac{d}{dx}\log(1+\cos x) + \log(1+\cos x)\frac{d}{dx}(x) \qquad \text{[Using product rule and chain rule]}$$
$$\frac{1}{y}\frac{dy}{dx} = x\frac{1}{(1+\cos x)}\frac{d}{dx}(1+\cos x) + \log(1+\cos x)(1)$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{x}{(1+\cos x)}(0-\sin x) + \log(1+\cos x)$$
$$\frac{1}{y}\frac{dy}{dx} = \log(1+\cos x) - \frac{xsomx}{(1+\cos x)}$$
$$\frac{dy}{dx} = y\left[\log(1+\cos x) - \frac{x\sin x}{1+\cos x}\right]$$
$$\frac{dy}{dx} = (1+\cos x)^x\left[\log(1+\cos x) - \frac{x\sin x}{(1+\cos x)}\right] \qquad \text{[Using equation (i)]}$$

Differentiation Ex 11.5 Q4

Let $y = x^{\cos^{-1}x}$ ---(i)

Taking log on both the sides,

$$\log y = \log x^{\cos^{-1}x}$$
$$\log y = \cos^{-1}x\log x \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating it with respect to x using product rule,

$$\frac{1}{y}\frac{dy}{dx} = \cos^{-1}x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}\left(\cos^{-1}x\right)$$
$$= \cos^{-1}x\left(\frac{1}{x}\right) + \log x\left(\frac{-1}{\sqrt{1-x^2}}\right)$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{\cos^{-1}x}{x} - \frac{\log x}{\sqrt{1-x^2}}$$
$$\frac{dy}{dx} = y\left[\frac{\cos^{-1}x}{x} - \frac{\log x}{\sqrt{1-x^2}}\right]$$
$$\frac{dy}{dx} = x^{\cos^{-1}x}\left[\frac{\cos^{-1}x}{x} - \frac{\log x}{\sqrt{1-x^2}}\right] \qquad \text{[Using equation (i)]}$$

Differentiation Ex 11.5 Q5

Let $y = (\log x)^x$ ---(i)

Taking log on both the sides,

$$\log y = \log(\log x)^x$$
$$\log y = x\log(\log x) \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating with respect to x, using product rule, chain rule,

$$\frac{1}{y}\frac{dy}{dx} = x\frac{d}{dx}\log(\log x) + \log\log x\frac{d}{dx}(x)$$
$$= x\frac{1}{\log x}\frac{d}{dx}(\log x) + \log\log x\,(1)$$
$$= \frac{x}{\log x}\left(\frac{1}{x}\right) + \log\log x$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{\log x} + \log\log x$$
$$\frac{dy}{dx} = y\left[\frac{1}{\log x} + \log\log x\right]$$
$$\frac{dy}{dx} = (\log x)^x\left[\frac{1}{\log x} + \log\log x\right] \qquad \text{[Using equation (i)]}$$

Differentiation Ex 11.5 Q6

Let $\quad y = (\log x)^{\cos x} \quad$ ---(i)

Taking log on both the sides,

$$\log y = \log(\log x)^{\cos x}$$
$$\log y = \cos x \log(\log x) \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating with respect to x, using product rule, chain rule,

$$\frac{1}{y}\frac{dy}{dx} = \cos x \frac{d}{dx}\log(\log x) + \log\log x \frac{d}{dx}(\cos x)$$
$$= \frac{\cos x}{\log x}\frac{d}{dx}(\log x) + \log\log x \times (-\sin x)$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{\cos x}{\log x} \times \left(\frac{1}{x}\right) - \sin x \log\log x$$
$$\frac{dy}{dx} = y\left[\frac{\cos x}{x\log x} - \sin x \log\log x\right]$$
$$\frac{dy}{dx} = (\log x)^{\cos x}\left[\frac{\cos x}{x\log x} - \sin x \log\log x\right] \qquad [\text{Using equation (i)}]$$

Differentiation Ex 11.5 Q7

Let $\quad y = (\sin x)^{\cos x} \quad$ ---(i)

Taking log on both the sides,

$$\log y = \log(\sin x)^{\cos x}$$
$$\log y = \cos x \log \sin x \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating with respect to x, using product rule, chain rule,

$$\frac{1}{y}\frac{dy}{dx} = \cos x \frac{d}{dx}\log \sin x + \log \sin x \frac{d}{dx}\cos x$$
$$= \cos x \frac{1}{\sin x}\frac{d}{dx}(\sin x) + \log \sin x(-\sin x)$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{\cos x}{\sin x}(\cos x) - \sin x \log \sin x$$
$$\frac{dy}{dx} = y\left[\cos x \cot x - \sin x \log \sin x\right]$$
$$\frac{dy}{dx} = (\sin x)^{\cos x}\left[\cos x \cot x - \sin x \log \sin x\right]$$

Differentiation Ex 11.5 Q8

Let $\quad y = e^{x\log x}$

$\Rightarrow \quad y = e^{\log x^x} \qquad \left[\text{Since, } \log a^b = b\log a\right]$

$\Rightarrow \quad y = x^x \quad$ ---(i) $\qquad \left[\text{Since, } e^{\log a} = a\right]$

Taking log both the sides,

$$\log y = \log x^x$$
$$\log y = x\log x$$

Differentiating with respect to x, using product rule,

$$\frac{1}{y}\frac{dy}{dx} = x\frac{d}{dx}(\log x) + \log x \frac{d}{dx}(x)$$
$$= x\left(\frac{1}{x}\right) + \log x(1)$$
$$\frac{1}{y}\frac{dy}{dx} = 1 + \log x$$
$$\frac{dy}{dx} = y\left[1 + \log x\right]$$
$$\frac{dy}{dx} = x^x(1 + \log x) \qquad [\text{Using equation (i)}]$$

Differentiation Ex 11.5 Q9

Let $y = (\sin x)^{\log x}$ ---(i)

Taking log on both the sides,

$$\log y = \log(\sin x)^{\log x}$$
$$\log y = \log x \log(\sin x) \qquad \left[\text{Using } \log a^b = b\log a\right]$$

Differentiating with respect to x, using product rule and chain rule,

$$\frac{1}{y}\frac{dy}{dx} = \log x \frac{d}{dx}(\log \sin x) + \log \sin x \frac{d}{dx}(\log x)$$
$$= \log x \left(\frac{1}{\sin x}\right)\frac{d}{dx}(\sin x) + \log \sin x \left(\frac{1}{x}\right)$$
$$= \frac{\log x}{\sin x} \times \cos x + \frac{\log \sin x}{x}$$
$$\frac{1}{y}\frac{dy}{dx} = \log x \cot x + \frac{\log \sin x}{x}$$
$$\frac{dy}{dx} = y\left[\log x \cot x + \frac{\log \sin x}{x}\right]$$
$$\frac{dy}{dx} = (\sin x)^{\log x}\left[\log x \cot x + \frac{\log \sin x}{x}\right] \qquad \left[\text{Using equation (i)}\right]$$

Differentiation Ex 11.5 Q10

Let $y = 10^{\log \sin x}$ ---(i)

Taking log on both the sides,

$$\log y = \log 10^{\log \sin x}$$
$$\log y = \log \sin x \log 10 \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating with respect to x, using chain rule,

$$\frac{1}{y}\frac{dy}{dx} = \log 10 \frac{d}{dx}(\log \sin x)$$
$$= \log 10\left(\frac{1}{\sin x}\right)\frac{d}{dx}(\sin x)$$
$$\frac{1}{y}\frac{dy}{dx} = \log 10\left(\frac{1}{\sin x}\right)(\cos x)$$
$$\frac{dy}{dx} = y\left[\log 10 \cot x\right]$$
$$\frac{dy}{dx} = 10^{\log \sin x}\left[\log 10 \times \cot x\right] \qquad \left[\text{Using equation (i)}\right]$$

Differentiation Ex 11.5 Q11

Let $y = (\log x)^{\log x}$

Taking logarithm on both the sides, we obtain

$$\log y = \log x \cdot \log(\log x)$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}\left[\log x \cdot \log(\log x)\right]$$
$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \log(\log x).\frac{d}{dx}(\log x) + \log x \cdot \frac{d}{dx}\left[\log(\log x)\right]$$
$$\Rightarrow \frac{dy}{dx} = y\left[\log(\log x).\frac{1}{x} + \log x \cdot \frac{1}{\log x} \cdot \frac{d}{dx}(\log x)\right]$$
$$\Rightarrow \frac{dy}{dx} = y\left[\frac{1}{x}\log(\log x) + \frac{1}{x}\right]$$
$$\therefore \frac{dy}{dx} = (\log x)^{\log x}\left[\frac{1}{x} + \frac{\log(\log x)}{x}\right]$$

Differentiation Ex 11.5 Q12

Let $\quad y = 10^{(10x)}$ ---(i)

Taking log on both the siedes,

$$\log y = \log 10^{(10x)}$$
$$\log y = 10^x \log 10$$

Differentiating it with respect to x,

$$\frac{1}{y}\frac{dy}{dx} = \log 10 \times 10^x \log 10$$
$$\frac{1}{y}\frac{dy}{dx} = 10^x \times (\log 10)^2$$
$$\frac{dy}{dx} = 10^{(10^x)} \times 10^x (\log 10)^2 \qquad \text{[Using equation (i)]}$$

Differentiation Ex 11.5 Q13

Let $\quad y = \sin x^x$

$\Rightarrow \quad \sin^{-1} y = x^x$

Taking log on both the siedes,

$$\log\left(\sin^{-1} y\right) = \log x^x$$
$$\log\left(\sin^{-1} y\right) = x \log x \qquad \left[\text{Since, } \log a^b = b \log a\right]$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{1}{\sin^{-1} y}\frac{dy}{dx} = \left(\sin^{-1} y\right) = x\frac{d}{dx}(\log x) + \log x \frac{d}{dx}(x)$$
$$\frac{1}{\sin^1 y} \times \left(\frac{1}{\sqrt{1-y^2}}\right)\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \log x\,(1)$$
$$\frac{dy}{dx} = \sin^{-1} y\sqrt{1-y^2}\,(1+\log x)$$
$$= \sin^{-1} \quad \left(\sin x^x\right)\sqrt{1-\left(\sin x^x\right)^2}\,(1+\log x)$$
$$= x^x\sqrt{\cos^2 x^x}\,(1+\log x) \qquad \text{[Using equation (i)]}$$

$$\frac{dy}{dx} = x^x \cos x^x\,(1+\log x)$$

Differentiation Ex 11.5 Q14

Let $\quad y = \left(\sin^{-1} x\right)^x$

Taking log on both the sides,

$$\log y = \log\left(\sin^{-1} x\right)^x$$
$$\log y = x \log\left(\sin^{-1} x\right) \qquad \left[\text{Since, } \log a^b = b \log a\right]$$

Differentiating it with respect to x using product rule and chain rule,

$$\frac{1}{y}\frac{dy}{dx} = x\frac{d}{dx}\left(\log \sin^{-1} x\right) + \log \sin^{-1} x \frac{d}{dx}(x)$$
$$= x \times \frac{1}{\sin^{-1} x}\frac{d}{dx}\left(\sin^{-1} x\right) + \log \sin^{-1} x\,(1)$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{x}{\sin^{-1} x}\left(\frac{1}{\sqrt{1-x^2}}\right) + \log \sin^{-1} x$$
$$\frac{dy}{dx} = y\left[\log \sin^{-1} x + \frac{x}{\sin^{-1} x\left(\sqrt{1-x^2}\right)}\right]$$
$$\frac{dy}{dx} = \left(\sin^{-1} x\right)^2\left[\log \sin^{-1} x + \frac{x}{\sin^{-1} x\sqrt{1-x^2}}\right] \qquad \text{[Using equation (i)]}$$

Differentiation Ex 11.5 Q15

Let $\quad y = x^{\sin^{-1} x}$ ---(i)

Taking log on both the sides,

$$\log y = \log x^{\sin^{-1} x}$$
$$\log y = \sin^{-1} x \log x \qquad \left[\text{Since, } \log a^b = b \log a\right]$$

Differentiating it with respect to x using product rule,

$$\frac{1}{y}\frac{dy}{dx} = \sin^{-1} x \frac{d}{dx}(\log x) + (\log x)\frac{d}{dx}\left(\sin^{-1} x\right)$$
$$\frac{1}{y}\frac{dy}{dx} = \sin^{-1} x \left(\frac{1}{x}\right) + (\log x)\left(\frac{1}{\sqrt{1-x^2}}\right)$$
$$\frac{dy}{dx} = y\left[\frac{\sin^{-1} x}{x} + \frac{\log x}{\sqrt{1-x^2}}\right]$$

$$\frac{dy}{dx} = x^{\sin^{-1} x}\left[\frac{\sin^{-1} x}{x} + \frac{\log x}{\sqrt{1-x^2}}\right] \qquad \left[\text{Using equation (i)}\right]$$

Differentiation Ex 11.5 Q16

Let $\quad y = (\tan x)^{\frac{1}{x}}$ ---(i)

Taking log on both the sides,

$$\log y = \log(\tan x)^{\frac{1}{x}}$$
$$\log y = \frac{1}{x}\log(\tan x) \qquad \left[\text{Since, } \log a^b = b \log a\right]$$

Differentiating it with respect to x using product rule and chain rule,

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x}\frac{d}{dx}\log(\tan x) + \log(\tan x)\frac{d}{dx}\left(\frac{1}{x}\right)$$
$$= \frac{1}{x} \times \frac{1}{\tan x}\frac{d}{dx}(\tan x) + \log(\tan x)\left(-\frac{1}{x^2}\right)$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x \tan x}\left(\sec^2 x\right) - \frac{\log(\tan x)}{x^2}$$

$$\frac{dy}{dx} = y\left[\frac{\sec^2 x}{x \tan x} - \frac{\log(\tan x)}{x^2}\right]$$
$$\frac{dy}{dx} = (\tan x)^{\frac{1}{x}}\left[\frac{\sec^2 x}{x \tan x} - \frac{\log \tan x}{x^2}\right] \qquad \left[\text{Using equation (i)}\right]$$

Differentiation Ex 11.5 Q17

Let $y = x^{\tan^{-1} x}$ ---(i)

Taking log on both the sides,

$$\log y = \log x^{\tan^{-1} x}$$
$$\log y = \tan^{-1} x \log x \qquad \left[\text{Since, } \log a^b = b \log a\right]$$

Differentiating it with respect to x using product rule,

$$\frac{1}{y}\frac{dy}{dx} = \tan^{-1} x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}\left(\tan^{-1} x\right)$$
$$\frac{1}{y}\frac{dy}{dx} = \tan^{-1} x \left(\frac{1}{x}\right) + \log x \left(\frac{1}{1+x^2}\right)$$
$$\frac{dy}{dx} = y\left[\frac{\tan^{-1} x}{x} + \frac{\log x}{1+x^2}\right]$$

$$\frac{dy}{dx} = x^{\tan^{-1} x}\left[\frac{\tan^{-1} x}{x} + \frac{\log x}{1+x^2}\right] \qquad \left[\text{Using equation (i)}\right]$$

Differentiation Ex 11.5 Q18(i)

Let $y = x^x \sqrt{x}$ ---(i)

Taking log on both the sides,

$$\log y = \log\left(x^x \sqrt{x}\right)$$
$$= \log x^x + \log x^{\frac{1}{2}} \qquad \left[\text{Since, } \log^{(ab)} = \log a + \log b\right]$$
$$\log y = x \log x + \frac{1}{2}\log x \qquad \left[\text{Since, } \log a^b = b \log a\right]$$

Differentiating it with respect to x using product rule,

$$\frac{1}{y}\frac{dy}{dx} = x\frac{d}{dx}(\log x) + \log x \frac{d}{dx}(x) + \frac{1}{2}\frac{d}{dx}(\log x)$$
$$= x\left(\frac{1}{x}\right) + \log x\,(1) + \frac{1}{2}\left(\frac{1}{x}\right)$$
$$\frac{1}{y}\frac{dy}{dx} = 1 + \log x + \frac{1}{2x}$$
$$\frac{dy}{dx} = y\left(1 + \log x + \frac{1}{2x}\right)$$

$$\frac{dy}{dx} = x^x\sqrt{x}\left(1 + \log x + \frac{1}{2x}\right) \qquad \left[\text{Using equation (i)}\right]$$

Differentiation Ex 11.5 Q18(ii)

Let $y = x^{(\sin x - \cos x)} + \left(\frac{x^2-1}{x^2+1}\right)$

$y = e^{\log x^{(\sin x - \cos x)}} + \left(\frac{x^2-1}{x^2+1}\right)$

$y = e^{(\sin x - \cos x)\log x} + \left(\frac{x^2-1}{x^2+1}\right)$ $\qquad \left[\text{Since, } e^{\log a} = a,\ \log a^b = b\log a\right]$

Differentiating it with respect to x using chain rule and quotient rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left[e^{(\sin x - \cos x)\log x}\right] + \frac{d}{dx}\left[\frac{x^2-1}{x^2+1}\right]$$

$$= e^{(\sin x - \cos x)\log x}\frac{d}{dx}\{(\sin x - \cos x)\log x\} + \left[\frac{(x^2+1)\frac{d}{dx}(x^2-1) - (x^2-1)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}\right]$$

$$= e^{\log x^{(\sin x - \cos x)}}\left[(\sin x - \cos x)\frac{d}{dx}(\log x) + (\log x)\frac{d}{dx}(\sin x - \cos x)\right] + \left[\frac{(x^2+1)(2x) - (x^2-1)(2x)}{(x^2+1)^2}\right]$$

$$= x^{(\sin x - \cos x)}\left[(\sin x - \cos x)\left(\frac{1}{x}\right) + \log x(\sin x + \cos x)\right] + \left[\frac{2x^3 + 2x - 2x^3 + 2x}{(x^2+1)^2}\right]$$

$$\frac{dy}{dx} = x^{(\sin x - \cos x)}\left[\frac{(\sin x - \cos x)}{x} + \log x(\sin x + \cos x)\right] + \frac{4x}{(x^2+1)^2}$$

Differentiation Ex 11.5 Q18(iii)

Let $y = x^{x\cos x} + \dfrac{x^2+1}{x^2-1}$

Also, let $u = x^{x\cos x}$ and $v = \dfrac{x^2+1}{x^2-1}$

$\therefore y = u + v$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{du}{dx} + \dfrac{dv}{dx}$ $\qquad ...(1)$

$u = x^{x\cos x}$

$\Rightarrow \log u = \log\left(x^{x\cos x}\right)$

$\Rightarrow \log u = x\cos x\log x$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(x)\cdot\cos x\cdot\log x + x\cdot\frac{d}{dx}(\cos x)\cdot\log x + x\cos x\cdot\frac{d}{dx}(\log x)$$

$$\Rightarrow \frac{du}{dx} = u\left[1\cdot\cos x\cdot\log x + x\cdot(-\sin x)\log x + x\cos x\cdot\frac{1}{x}\right]$$

$$\Rightarrow \frac{du}{dx} = x^{x\cos x}(\cos x\log x - x\sin x\log x + \cos x)$$

$$\Rightarrow \frac{du}{dx} = x^{x\cos x}\left[\cos x(1 + \log x) - x\sin x\log x\right] \qquad ...(2)$$

$v = \dfrac{x^2+1}{x^2-1}$

$\Rightarrow \log v = \log\left(x^2+1\right) - \log\left(x^2-1\right)$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{v}\frac{dv}{dx}=\frac{2x}{x^2+1}-\frac{2x}{x^2-1}$$

$$\Rightarrow \frac{dv}{dx}=v\left[\frac{2x\left(x^2-1\right)-2x\left(x^2+1\right)}{\left(x^2+1\right)\left(x^2-1\right)}\right]$$

$$\Rightarrow \frac{dv}{dx}=\frac{x^2+1}{x^2-1}\times\left[\frac{-4x}{\left(x^2+1\right)\left(x^2-1\right)}\right]$$

$$\Rightarrow \frac{dv}{dx}=\frac{-4x}{\left(x^2-1\right)^2} \qquad ...(3)$$

From (1), (2), and (3), we obtain

$$\frac{dy}{dx}=x^{x\cos x}\left[\cos x\left(1+\log x\right)-x\sin x\log x\right]-\frac{4x}{\left(x^2-1\right)^2}$$

Differentiation Ex 11.5 Q18(iv)

Let $y=\left(x\cos x\right)^x+\left(x\sin x\right)^{\frac{1}{x}}$

Also, let $u=\left(x\cos x\right)^x$ and $v=\left(x\sin x\right)^{\frac{1}{x}}$

$\therefore y=u+v$

$$\Rightarrow \frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx} \qquad ...(1)$$

$$u=\left(x\cos x\right)^x$$

$$\Rightarrow \log u=\log\left(x\cos x\right)^x$$

$$\Rightarrow \log u=x\log\left(x\cos x\right)$$

$$\Rightarrow \log u=x\left[\log x+\log\cos x\right]$$

$$\Rightarrow \log u=x\log x+x\log\cos x$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{u}\frac{du}{dx}=\frac{d}{dx}\left(x\log x\right)+\frac{d}{dx}\left(x\log\cos x\right)$$

$$\Rightarrow \frac{du}{dx}=u\left[\left\{\log x\cdot\frac{d}{dx}(x)+x\cdot\frac{d}{dx}(\log x)\right\}+\left\{\log\cos x\cdot\frac{d}{dx}(x)+x\cdot\frac{d}{dx}(\log\cos x)\right\}\right]$$

$$\Rightarrow \frac{du}{dx}=\left(x\cos x\right)^x\left[\left(\log x\cdot 1+x\cdot\frac{1}{x}\right)+\left\{\log\cos x\cdot 1+x\cdot\frac{1}{\cos x}\cdot\frac{d}{dx}(\cos x)\right\}\right]$$

$$\Rightarrow \frac{du}{dx}=\left(x\cos x\right)^x\left[\left(\log x+1\right)+\left\{\log\cos x+\frac{x}{\cos x}\cdot(-\sin x)\right\}\right]$$

$$\Rightarrow \frac{du}{dx}=\left(x\cos x\right)^x\left[\left(1+\log x\right)+\left(\log\cos x-x\tan x\right)\right]$$

$$\Rightarrow \frac{du}{dx}=\left(x\cos x\right)^x\left[1-x\tan x+\left(\log x+\log\cos x\right)\right]$$

$$\Rightarrow \frac{du}{dx}=\left(x\cos x\right)^x\left[1-x\tan x+\log\left(x\cos x\right)\right] \qquad ...(2)$$

$$v=(x\sin x)^{\frac{1}{x}}$$

$$\Rightarrow \log v=\log(x\sin x)^{\frac{1}{x}}$$

$$\Rightarrow \log v=\frac{1}{x}\log(x\sin x)$$

$$\Rightarrow \log v=\frac{1}{x}(\log x+\log\sin x)$$

$$\Rightarrow \log v=\frac{1}{x}\log x+\frac{1}{x}\log\sin x$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{v}\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{x}\log x\right)+\frac{d}{dx}\left[\frac{1}{x}\log(\sin x)\right]$$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx}=\left[\log x\cdot\frac{d}{dx}\left(\frac{1}{x}\right)+\frac{1}{x}\cdot\frac{d}{dx}(\log x)\right]+\left[\log(\sin x)\cdot\frac{d}{dx}\left(\frac{1}{x}\right)+\frac{1}{x}\cdot\frac{d}{dx}\{\log(\sin x)\}\right]$$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx}=\left[\log x\cdot\left(-\frac{1}{x^2}\right)+\frac{1}{x}\cdot\frac{1}{x}\right]+\left[\log(\sin x)\cdot\left(-\frac{1}{x^2}\right)+\frac{1}{x}\cdot\frac{1}{\sin x}\cdot\frac{d}{dx}(\sin x)\right]$$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx}=\frac{1}{x^2}(1-\log x)+\left[-\frac{\log(\sin x)}{x^2}+\frac{1}{x\sin x}\cdot\cos x\right]$$

$$\Rightarrow \frac{dv}{dx}=(x\sin x)^{\frac{1}{x}}\left[\frac{1-\log x}{x^2}+\frac{-\log(\sin x)+x\cot x}{x^2}\right]$$

$$\Rightarrow \frac{dv}{dx}=(x\sin x)^{\frac{1}{x}}\left[\frac{1-\log x-\log(\sin x)+x\cot x}{x^2}\right]$$

$$\Rightarrow \frac{dv}{dx}=(x\sin x)^{\frac{1}{x}}\left[\frac{1-\log(x\sin x)+x\cot x}{x^2}\right] \qquad ...(3)$$

From (1), (2), and (3), we obtain

$$\frac{dy}{dx}=(x\cos x)^x\left[1-x\tan x+\log(x\cos x)\right]+(x\sin x)^{\frac{1}{x}}\left[\frac{x\cot x+1-\log(x\sin x)}{x^2}\right]$$

Differentiation Ex 11.5 Q18(v)

Let $y=\left(x+\frac{1}{x}\right)^{x}+x^{\left(1+\frac{1}{x}\right)}$

Also, let $u=\left(x+\frac{1}{x}\right)^{x}$ and $v=x^{\left(1+\frac{1}{x}\right)}$

$\therefore y=u+v$

$\Rightarrow \frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx} \qquad ...(1)$

Then, $u=\left(x+\frac{1}{x}\right)^{x}$

$\Rightarrow \log u=\log\left(x+\frac{1}{x}\right)^{x}$

$\Rightarrow \log u=x\log\left(x+\frac{1}{x}\right)$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{u}\cdot\frac{du}{dx}=\frac{d}{dx}(x)\times\log\left(x+\frac{1}{x}\right)+x\times\frac{d}{dx}\left[\log\left(x+\frac{1}{x}\right)\right]$$

$$\Rightarrow \frac{1}{u}\frac{du}{dx}=1\times\log\left(x+\frac{1}{x}\right)+x\times\frac{1}{\left(x+\frac{1}{x}\right)}\cdot\frac{d}{dx}\left(x+\frac{1}{x}\right)$$

$$\Rightarrow \frac{du}{dx}=u\left[\log\left(x+\frac{1}{x}\right)+\frac{x}{\left(x+\frac{1}{x}\right)}\times\left(1-\frac{1}{x^2}\right)\right]$$

$$\Rightarrow \frac{du}{dx}=\left(x+\frac{1}{x}\right)^{x}\left[\log\left(x+\frac{1}{x}\right)+\frac{\left(x-\frac{1}{x}\right)}{\left(x+\frac{1}{x}\right)}\right]$$

$$\Rightarrow \frac{du}{dx}=\left(x+\frac{1}{x}\right)^{x}\left[\log\left(x+\frac{1}{x}\right)+\frac{x^2-1}{x^2+1}\right]$$

$$\Rightarrow \frac{du}{dx}=\left(x+\frac{1}{x}\right)^{x}\left[\frac{x^2-1}{x^2+1}+\log\left(x+\frac{1}{x}\right)\right]$$

$$v = x^{\left(1+\frac{1}{x}\right)}$$

$$\Rightarrow \log v = \log\left[x^{\left(1+\frac{1}{x}\right)}\right]$$

$$\Rightarrow \log v = \left(1+\frac{1}{x}\right)\log x$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{v}\cdot\frac{dv}{dx} = \left[\frac{d}{dx}\left(1+\frac{1}{x}\right)\right]\times\log x + \left(1+\frac{1}{x}\right)\cdot\frac{d}{dx}\log x$$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \left(-\frac{1}{x^2}\right)\log x + \left(1+\frac{1}{x}\right)\cdot\frac{1}{x}$$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx} = -\frac{\log x}{x^2} + \frac{1}{x} + \frac{1}{x^2}$$

$$\Rightarrow \frac{dv}{dx} = v\left[\frac{-\log x + x + 1}{x^2}\right]$$

$$\Rightarrow \frac{dv}{dx} = x^{\left(1+\frac{1}{x}\right)}\left(\frac{x+1-\log x}{x^2}\right) \qquad ...(3)$$

Therefore, from (1), (2), and (3), we obtain

$$\frac{dy}{dx} = \left(x+\frac{1}{x}\right)^x\left[\frac{x^2-1}{x^2+1} + \log\left(x+\frac{1}{x}\right)\right] + x^{\left(1+\frac{1}{x}\right)}\left(\frac{x+1-\log x}{x^2}\right)$$

Differentiation Ex 11.5 Q18(vi)

Let $y = e^{\sin x} + (\tan x)^x$

$y = e^{\sin x} + e^{\log(\tan x)^x}$

$y = e^{\sin x} + e^{x\log(\tan x)}$ $\qquad \left[\text{Since, } \log a^b = b\log a, e^{\log a} = a\right]$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\sin x}\right) + \frac{d}{dx}\left(e^{x\log(\tan x)}\right)$$

$$= e^{\sin x}\frac{d}{dx}(\sin x) + e^{x\log(\tan x)}\times\frac{d}{dx}(x\log\tan x)$$

$$= e^{\sin x}(\cos x) + e^{\log(\tan x)^x}\left[x\frac{d}{dx}\log\tan x + \log\tan x\frac{d}{dx}(x)\right]$$

$$= e^{\sin x}(\cos x) + (\tan x)^x\left[\frac{x}{\tan x}\frac{d}{dx}(\tan x) + \log\tan x\,(1)\right]$$

$$\frac{dy}{dx} = \cos x e^{\sin x} + (\tan x)^x\left[\frac{x}{\tan x}\left(\sec^2 x\right) + \log\tan x\right]$$

Differentiation Ex 11.5 Q18(vii)

Let $\quad y = (\cos x)^x + (\sin x)^{\frac{1}{x}}$

$$y = e^{\log(\cos x)^x} + e^{\log(\sin)^{\frac{1}{x}}} \qquad \left[\text{Since, } \log a^b = b\log a, e^{\log a} = a\right]$$

$$y = e^{x\log(\cos x)} + e^{\frac{1}{x}\log \sin x}$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}e^{x\log\cos x} + \frac{d}{dx}e^{\frac{1}{x}\log\sin x}$$

$$= e^{x\log\cos x} \times \frac{d}{dx}(x\log x) + e^{\frac{1}{x}\log\sin x}\frac{d}{dx}\left(\frac{1}{x}\log\sin x\right)$$

$$= e^{\log(\cos x)} \times \left[x\frac{d}{dx}\log\cos x + \log\cos x \times \frac{d}{dx}(x)\right] + e^{\log(\sin x)^{\frac{1}{x}}} \times \left[\frac{1}{x}\frac{d}{dx}\log\sin x + \log\sin x\frac{d}{dx}\left(\frac{1}{x}\right)\right]$$

$$= (\cos x)^x\left[x \times \left(\frac{1}{\cos x}\right)\frac{d}{dx}\cos x + \log\cos x\,(1)\right] + (\sin)^{\frac{1}{x}}\left[\frac{1}{x} \times \frac{1}{\sin x} \times \frac{d}{dx}(\sin x) + \log\sin x\left(-\frac{1}{x^2}\right)\right]$$

$$= (\cos x)^x\left[x\left(\frac{1}{\cos x}\right)(-\sin x) + \log\cos x\right] + (\sin x)^{\frac{1}{x}}\left[\frac{1}{x} \times \frac{1}{\sin x}(\cos x) - \frac{1}{x^2}\log\sin x\right]$$

$$\frac{dy}{dx} = (\cos x)^x[\log\cos x - x\tan x] + (\sin x)^{\frac{1}{x}}\left[\frac{\cot x}{x} - \frac{1}{x^2}\log\sin x\right]$$

Differentiation Ex 11.5 Q18(viii)

Let $y = x^{x^2-3} + (x-3)^{x^2}$

Also, let $u = x^{x^2-3}$ and $v = (x-3)^{x^2}$

$\therefore y = u + v$

Differentiating both sides with respect to x, we obtain

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad \ldots(1)$$

$u = x^{x^2-3}$

$\therefore \log u = \log\left(x^{x^2-3}\right)$

$\log u = \left(x^2-3\right)\log x$

Differentiating with respect to x, we obtain

$$\frac{1}{u}\cdot\frac{du}{dx} = \log x\cdot\frac{d}{dx}\left(x^2-3\right) + \left(x^2-3\right)\cdot\frac{d}{dx}(\log x)$$

$$\Rightarrow \frac{1}{u}\frac{du}{dx} = \log x\cdot 2x + \left(x^2-3\right)\cdot\frac{1}{x}$$

$$\Rightarrow \frac{du}{dx} = x^{x^2-3}\cdot\left[\frac{x^2-3}{x} + 2x\log x\right]$$

Also,

$v = (x-3)^{x^2}$

$\therefore \log v = \log(x-3)^{x^2}$

$\Rightarrow \log v = x^2\log(x-3)$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{v}\cdot\frac{dv}{dx}=\log(x-3)\cdot\frac{d}{dx}(x^2)+x^2\cdot\frac{d}{dx}[\log(x-3)]$$
$$\Rightarrow\frac{1}{v}\frac{dv}{dx}=\log(x-3)\cdot 2x+x^2\cdot\frac{1}{x-3}\cdot\frac{d}{dx}(x-3)$$
$$\Rightarrow\frac{dv}{dx}=v\left[2x\log(x-3)+\frac{x^2}{x-3}\cdot 1\right]$$
$$\Rightarrow\frac{dv}{dx}=(x-3)^{x^2}\left[\frac{x^2}{x-3}+2x\log(x-3)\right]$$

Substituting the expressions of $\frac{du}{dx}$ and $\frac{dv}{dx}$ in equation (1), we obtain

$$\frac{dy}{dx}=x^{x^2-3}\left[\frac{x^2-3}{x}+2x\log x\right]+(x-3)^{x^2}\left[\frac{x^2}{x-3}+2x\log(x-3)\right]$$

Differentiation Ex 11.5 Q19

Here,

$$y=e^x+10^x+x^x$$
$$=e^x+10^x+e^{\log x^x} \qquad \left[\text{Since, } e^{\log_e a}=a, \log a^b=b\log a\right]$$
$$y=e^x+10^x+e^{x\log x}$$

Differentiating it with respect to x using product rule, chain rule,

$$\frac{dy}{dx}=\frac{d}{dx}(e^x)+\frac{d}{dx}(10^x)+\frac{d}{dx}(e^{x\log x})$$
$$=e^x+10^x\log 10+e^{x\log x}\frac{d}{dx}(x\log x)$$
$$=e^x+10^x\log 10+e^{x\log x}\left[x\times\frac{d}{dx}(\log x)+\log x\frac{d}{dx}(x)\right]$$
$$=e^x+10^x\log 10+e^{\log x^x}\left[x\left(\frac{1}{x}\right)+\log x(1)\right]$$
$$=e^x+10^x\log 10+x^x[1+\log x]$$
$$=e^x+10^x\log 10+x^x[\log e+\log x] \qquad [\text{Since, } \log_e e=1]$$

$$\frac{dy}{dx}=e^x+10^x\log 10+x^x(\log ex) \qquad [\text{Sicne } \log A+\log B=\log AB]$$

Differentiation Ex 11.5 Q20

Here,

$$y=x^n+n^x+x^x+n^n$$
$$y=x^n+n^n+e^{\log x^x}+n^n \qquad \left[\text{Since, } e^{\log_e a}=a \text{ and } \log a^b=b\log a\right]$$
$$y=x^n+n^x+e^{x\log x}+n^n$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx}=\frac{d}{dx}(x^n)+\frac{d}{dx}(n^x)+\frac{d}{dx}(e^{x\log x})+\frac{d}{dx}(n^n)$$
$$=nx^{n-1}+n^x\log n+e^{\log x^x}\left[a\frac{d}{dx}\log x+\log x\frac{d}{dx}(1)\right]$$
$$=nx^{n-1}+n^x\log n+x^x\left[x\left(\frac{1}{x}\right)+\log x\right]$$
$$=nx^{n-1}+n^x\log n+x^x[1+\log x]$$
$$=nx^{n-1}+n^x\log n+x^x[\log e+\log x] \qquad [\text{Since, } \log_e e=1 \text{ and } \log A+\log B=\log(AB)]$$

$$\frac{dy}{dx}=nx^{n-1}+n^x\log n+x^x\log(ex)$$

Differentiation Ex 11.5 Q21

Here,

$$y=\frac{\left(x^2-1\right)^3(2x-1)}{\sqrt{(x-3)(4x-1)}} \qquad \text{---(i)}$$

$$y=\frac{\left(x^2-1\right)^3(2x-1)}{(x-3)^{\frac{1}{2}}(4x-)^{\frac{1}{2}}}$$

Taking log on both the sides,

$$\log y=\log\left[\frac{\left(x^2-1\right)^3(2x-1)}{(x-3)^{\frac{1}{2}}(4x-1)^{\frac{1}{2}}}\right]$$

$$=\log\left(x^2-1\right)^3+\log(2x-1)-\log(x-3)^{\frac{1}{2}}-\log(4x-1)^{\frac{1}{2}}$$

$$\left[\text{Since, } \log(AB)=\log A+\log B, \log\left(\frac{A}{B}\right)=\log A-\log B\right]$$

$$=3\log\left(x^2-1\right)+\log(2x-1)-\frac{1}{2}\log(x-3)-\frac{1}{2}\log(4x-1)$$

Differentiating it with respect to x using chain rule,

$$\frac{1}{y}\frac{dy}{dx}=3\frac{d}{dx}\log\left(x^2-1\right)+\frac{d}{dx}\log(2x-1)-\frac{1}{2}\frac{d}{dx}\log(x-3)-\frac{1}{2}\log(4x-1)$$

$$=3\left(\frac{1}{x^2-1}\right)\frac{d}{dx}\left(x^2-1\right)+\frac{1}{(2x-1)}\frac{d}{dx}(2x-1)-\frac{1}{2}\left(\frac{1}{x-3}\right)\frac{d}{dx}(x-3)-\frac{1}{2}\frac{1}{(4x-1)}\frac{d}{dx}(4x-1)$$

$$=3\left(\frac{1}{x^2-1}\right)(2x)+\frac{1}{(2x-1)}(2)-\frac{1}{2}\left(\frac{1}{x-3}\right)(1)-\frac{1}{2}\left(\frac{1}{4x-1}\right)(4)$$

$$\frac{1}{y}\frac{dy}{dx}=\left[\frac{6x}{x^2-1}+\frac{2}{2x-1}-\frac{1}{2(x-3)}-\frac{2}{4x-1}\right]$$

$$\frac{dy}{dx}=y\left[\frac{6x}{x^2-1}+\frac{2}{2x-1}-\frac{1}{2(x-3)}-\frac{2}{4x-1}\right]$$

$$\frac{dy}{dx}=\frac{\left(x^2-1\right)^3(2x-1)}{\sqrt{(x-3)(4x-1)}}\left[\frac{6x}{x^2-1}+\frac{2}{2x-1}-\frac{1}{2(x-3)}-\frac{2}{4x-1}\right] \qquad \left[\text{Using equaiton (i)}\right]$$

Differentiation Ex 11.5 Q22

Here,

$$y=\frac{e^{ax}\sec^x\times\log x}{\sqrt{1-2x}} \qquad \text{---(i)}$$

$$\Rightarrow \quad y=\frac{e^{ax}\times\sec^x\times\log x}{(1-2x)^{\frac{1}{2}}}$$

Taking log on both the sides,

$$\log y=\log e^{ax}+\log^{\sec x}+\log\log x-\frac{1}{2}\log(1-2x) \qquad \left[\begin{array}{l}\text{Since, } \log\left(\frac{A}{B}\right)=\log A-\log B,\\ \log(AB)=\log A+\log B\end{array}\right]$$

$$\log y=ax+\log^{\sec x}+\log\log x-\frac{1}{2}\log(1-2x) \qquad \left[\text{Since, } \log a^b=b\log a \text{ and } \log_e e=1\right]$$

Differentiating it with respect to x using chain rule,

$$\frac{1}{y}\frac{dy}{dx}=\frac{d}{dx}(ax)+\frac{d}{dx}(\log\sec x)+\frac{d}{dx}(\log\log x)-\frac{1}{2}\log(1-2x)$$

$$\frac{1}{y}\frac{dy}{dx}=a+\frac{1}{\sec x}\frac{d}{dx}(\sec x)+\frac{1}{\log x}\frac{d}{dx}(\log x)-\frac{1}{2}\left(\frac{1}{1-2x}\right)\frac{d}{dx}(1-2x)$$

$$\frac{1}{y}\frac{dy}{dx}=a+\frac{\sec x\tan x}{\sec x}+\frac{1}{(\log x)}\left(\frac{1}{x}\right)-\frac{1}{2}\left(\frac{1}{1-2x}\right)(-2)$$

$$\frac{dy}{dx}=y\left[a+\tan x+\frac{1}{x\log x}+\frac{1}{1-2x}\right]$$

$$\frac{dy}{dx}=\frac{e^{ax}\sec x\log x}{\sqrt{1-2x}}\left[a+\tan x+\frac{1}{x\log x}+\frac{1}{1-2x}\right] \qquad \left[\text{Using equaton (i)}\right]$$

Differentiation Ex 11.5 Q23

Here,

$$y = e^{3x} \times \sin 4x \times 2^x \qquad \text{---(i)}$$

Taking log on both the sides,

$$\log y = \log e^{3x} + \log \sin 4x + \log 2^x \qquad [\text{Since, } \log(AB) = \log A + \log B]$$
$$\log y = 3x \log e + \log \sin 4x + x \log 2 \qquad [\text{Since, } \log_e e = 1, \log a^b = b \log a]$$
$$\log y = 3x + \log \sin 4x + x \log 2$$

Differentiating it with respect to x,

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}(3x) + \frac{d}{dx}(\log \sin 4x) + \frac{d}{dx}(x \log 2)$$
$$= 3 + \frac{1}{\sin 4x}\frac{d}{dx}(\sin 4x) + \log 2(1)$$
$$= 3 + \frac{1}{\sin 4x}(\cos 4x)\frac{d}{dx}(4x) + \log 2$$
$$= 3 + \cot x(4) + \log 2$$
$$\frac{1}{y}\frac{dy}{dx} = 3 + 4\cot 4x + \log 2$$
$$\frac{dy}{dx} = y[3 + 4\cot 4x + \log 2]$$

$$\frac{dy}{dx} = e^{3x} \times \sin 4x \times 2^x [3 + 4\cot 4x + \log 2]$$

Differentiation Ex 11.5 Q24

Here,

$$y = \sin x \sin 2x \sin 3x \sin 4x \qquad \text{---(i)}$$

Taking log on both the sides,

$$\log y = \log(\sin x \sin 2x \sin 3x \sin 4x)$$
$$\log y = \log \sin x + \log \sin 2x + \log \sin 3x + \log \sin 4x$$

Differentiating it with respect to x using chain rule,

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}\log \sin x + \frac{d}{dx}\log \sin 2x + \frac{d}{dx}\log \sin 3x + \frac{d}{dx}\log \sin 4x$$
$$= \frac{1}{\sin x}\frac{d}{dx}(\sin x) + \frac{1}{\sin 2x}\frac{d}{dx}(\sin 2x) + \frac{1}{\sin 3x}\frac{d}{dx}(\sin 3x) + \frac{1}{\sin 4x}\frac{d}{dx}(\sin 4x)$$
$$= \frac{1}{\sin x}(\cos x) + \frac{1}{\sin 2x}(\cos 2x)\frac{d}{dx}(2x) + \frac{1}{\sin 3x}(\cos 3x)\frac{d}{dx}(3x) + \frac{1}{\sin 4x}(\cos 4x)\frac{d}{dx}(4x)$$
$$\frac{1}{y}\frac{dy}{dx} = [\cot x + \cot 2x(2) + \cot 3x(3) + \cot 4x(4)]$$
$$\frac{dy}{dx} = y[\cot x + 2\cot 2x + 3\cot x 3x + 4\cot 4x]$$

$$\frac{dy}{d} = (\sin x \sin 2x \sin 3x \sin 4x)[\cot x + 2\cot 2x + 3\cot x 3x + 4\cot 4x] \qquad [\text{Using equation (i)}]$$

Differentiation Ex 11.5 Q25

Let $y = x^{\sin x} + (\sin x)^x$

Also, let $u = x^{\sin x}$ and $v = (\sin x)^x$

$\therefore y = u + v$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad ...(1)$$

$u = x^{\sin x}$

$\Rightarrow \log u = \log\left(x^{\sin x}\right)$

$\Rightarrow \log u = \sin x \log x$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(\sin x)\cdot \log x + \sin x \cdot \frac{d}{dx}(\log x)$$

$$\Rightarrow \frac{du}{dx} = u\left[\cos x \log x + \sin x \cdot \frac{1}{x}\right]$$

$$\Rightarrow \frac{du}{dx} = x^{\sin x}\left[\cos x \log x + \frac{\sin x}{x}\right] \qquad ...(2)$$

$v = (\sin x)^x$

$\Rightarrow \log v = \log(\sin x)^x$

$\Rightarrow \log v = x \log(\sin x)$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{v}\frac{dv}{dx} = \frac{d}{dx}(\ x) \times \log(\sin x) + \ x \times \frac{d}{dx}\left[\log(\sin x)\right]$$

$$\Rightarrow \frac{dv}{dx} = v\left[\log(\sin x) + x \cdot \frac{1}{\sin x} \cdot \frac{d}{dx}(\sin x)\right]$$

$$\Rightarrow \frac{dv}{dx} = (\sin x)^x \left[\log \sin x + \frac{x}{\sin x}\cos x\right]$$

$$\Rightarrow \frac{dv}{dx} = (\sin x)^x \left[\log \sin x + x \cot x\right]$$

$$\Rightarrow \frac{dv}{dx} = (\sin x)^x \left[\log \sin x + x \cot x\right] \qquad ...(3)$$

From (1), (2), and (3), we obtain

$$\frac{dy}{dx} = x^{\sin x}\left(\cos x \log x + \frac{\sin x}{x}\right) + (\sin x)^x \left[\log \sin x + x \cot x\right]$$

Differentiation Ex 11.5 Q26

Here,

$$y=(\sin x)^{\cos x}+(\cos x)^{\sin x}$$
$$y=e^{\log(\sin x)^{\cos x}}+e^{\log(\cos x)^{\sin x}}$$
$$y=e^{\cos x\log\sin x}+e^{\sin x\log\cos x} \qquad \left[\text{Since, } \log_e e=1 \text{ and } \log a^b=b\log a\right]$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx}=\frac{d}{dx}\left(e^{\cos x\log\sin x}\right)+\frac{d}{dx}\left(e^{\sin x\log\cos x}\right)$$
$$=e^{\cos x\log\sin x}\frac{d}{dx}(\cos x\log\sin x)+e^{\sin x\log\cos x}\frac{d}{dx}(\sin x\log\cos x)$$
$$=e^{\log(\sin x)^{\cos x}}\left[\cos x\frac{d}{dx}\log\sin x+\log\sin x\frac{d}{dx}(\cos x)\right]+e^{\log(\cos x)^{\sin x}}\left[\sin x\frac{d}{dx}\log\cos x+\log\cos x\frac{d}{dx}(\sin x)\right]$$
$$=(\sin)^{\cos x}\left[\cos x\left(\frac{1}{\sin x}\right)\frac{d}{dx}(\sin x)+\log\sin x\times(-\sin x)\right]+(\cos x)^{\sin x}\left[\sin x\left(\frac{1}{\cos x}\right)\frac{d}{dx}(\cos x)+\log\cos x\,(\cos x)\right]$$
$$=(\sin x)^{\cos x}\left[\cot x\times\cos x-\sin x\log\sin x\right]+(\cos x)^{\sin x}\left[\tan x\,(-\sin x)+\cos x\log\cos x\right]$$

$$\frac{dy}{dx}=(\sin x)^{\cos x}\left[\cot x\times\cos x-\sin x\log\sin x\right]+(\cos x)^{\sin x}\left[\cos x\log\cos x-\sin x\tan x\right]$$

Differentiation Ex 11.5 Q27

Here,

$$y=(\tan x)^{\cot x}+(\cot x)^{\tan x}$$
$$y=e^{\log(\tan x)^{\cot x}}+e^{\log(\cot x)^{\tan x}} \qquad \left[\text{Since, } \log_e e=1,\log a^b=b\log a\right]$$
$$y=e^{\cot x\log\tan x}+e^{\tan x\log(\cot x)}$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx}=\frac{d}{dx}\left(e^{\cot x\log\tan x}\right)+\frac{d}{dx}\left(e^{\tan x\log\cot x}\right)$$
$$=e^{\cot x\log\tan x}\frac{d}{dx}(\cot x\log\tan x)+e^{\tan x\log\cot x}\frac{d}{dx}(\tan x\log\cot x)$$
$$=e^{\log(\tan x)^{\cot x}}\left[\cot x\frac{d}{dx}\log\tan x+\log\tan x\frac{d}{dx}\cot x\right]+e^{\log(\cot x)^{\tan x}}\begin{bmatrix}\tan x\frac{d}{dx}\log\cot x+\\ \log\cot x\frac{d}{dx}(\tan x)\end{bmatrix}$$
$$=(\tan x)^{\cot x}\left[\cot x\times\left(\frac{1}{\tan x}\right)\frac{d}{dx}(\tan x)+\log\tan x\left(-\operatorname{cosec}^2 x\right)\right]+(\cot x)^{\tan x}\begin{bmatrix}\tan x\left(\frac{1}{\cot x}\right)\frac{d}{dx}(\cot x)\\ +\log\cot x\left(\sec^2 x\right)\end{bmatrix}$$
$$=\tan x^{\cot x}\left[(1)\left(\sec^2 x\right)-\operatorname{cosec}^2 x\log\tan x\right]+(\cot)^{\tan x}\left[(1)\left(-\operatorname{cosec}^2 x\right)+\sec^2 x\log\cot x\right]$$

$$\frac{dy}{dx}=(\tan)^{\cot x}\left[\sec^{2x}-\operatorname{cosec}^2 x\log\tan x\right]+(\cot x)^{\tan x}\left[\sec^2 x\log\cot x-\operatorname{cosec}^2 x\right]$$

Differentiation Ex 11.5 Q28

Here,

$$y=(\sin x)^x+\sin^{-1}\sqrt{x}$$
$$=e^{\log(\sin x)^x}+\sin^{-1}\sqrt{x}$$
$$y=e^{x\,\text{lgo}\,\sin x}+\sin^{-1}\sqrt{x} \qquad \left[\text{Since, } \log_e e=1,\log a^b=b\log a\right]$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx}=\frac{d}{dx}\left(e^{x\log\sin x}\right)+\frac{d}{dx}\sin^{-1}\left(\sqrt{x}\right)$$
$$=e^{x\log\sin x}\frac{d}{dx}(x\log\sin x)+\frac{1}{\sqrt{1-\left(\sqrt{x}\right)^2}}\frac{d}{dx}\left(\sqrt{x}\right)$$
$$=e^{\log(\sin x)^x}\left[x\frac{d}{dx}\log\sin x+\log\sin x\frac{d}{dx}(x)+\frac{1}{\sqrt{1-x}}\times\frac{1}{2\sqrt{x}}\right]$$
$$=(\sin x)^x\left[x\times\frac{1}{\sin x}\frac{d}{dx}(\sin x)+\log\sin x\,(1)\right]+\frac{1}{2\sqrt{x-x^2}}$$
$$=(\sin x)^x\left[\frac{x}{\sin x}(\cos x)+\log\sin x\right]+\frac{1}{2\sqrt{x-x^2}}$$

$$\frac{dy}{dx}=(\sin x)^x\left[x\cot x+\log\sin x\right]+\frac{1}{2\sqrt{x-x^2}}$$

Differentiation Ex 11.5 Q29

Here,

$$y = x^{\cos x} + (\sin x)^{\tan x}$$
$$y = e^{\log x^{\cos x}} + e^{\log(\sin x)\tan x} \qquad \left[\text{Since, } e^{\log_e a} = a \text{ and } \log a^b = b\log a\right]$$
$$y = e^{\cos x \log x} + e^{\tan x \log \sin x}$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\cos x \log x}\right) + \frac{d}{dx}\left(e^{\tan x \log \sin x}\right)$$
$$= e^{\cos x \log x}\frac{d}{dx}(\cos x \log x) + e^{\tan x \log \sin x} \times \frac{d}{dx}(\tan x \log \sin x)$$
$$= e^{\log x^{\cos x}}\left[\cos x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(\cos x)\right] + e^{\log(\sin x)^{\tan x}}\left[\tan x \frac{d}{dx}\log \sin x + \log \sin x \frac{d}{dx}(\tan x)\right]$$
$$= x^{\cos x}\left[\cos x\left(\frac{1}{x}\right) + \log x(-\sin x)\right] + (\sin x)^{\tan x}\left[\tan x\left(\frac{1}{\sin x}\right)\frac{d}{dx}(\sin x) + \log \sin x\left(\sec^2 x\right)\right]$$
$$= x^{\cos x}\left[\frac{\cos x}{x} - \sin x \log x\right] + (\sin x)^{\tan x}\left[\tan x\left(\frac{1}{\sin x}\right)(\cos x) + \sec^2 x \log \sin x\right]$$

$$\frac{dy}{dx} = x^{\cos x}\left[\frac{\cos x}{x} - \sin x \log x\right] + (\sin x)^{\tan x}\left[1 + \sec^2 x \log \sin x\right]$$

Here,

$$y = x^x + (\sin x)^x$$
$$= e^{\log x^x} + e^{\log(\sin x)^x}$$
$$y = e^{x\log x} + e^{x \log \sin x} \qquad \left[\text{Using } e^{\log a} = a \text{ and } \log a^b = b\log a\right]$$

Differentiating with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{x\log x}\right) + \frac{d}{dx}\left(e^{x\log \sin x}\right)$$
$$= e^{x\log x}\frac{d}{dx}(x\log x) + e^{x\log \sin x}\frac{d}{dx}(x\log \sin x)$$
$$= e^{\log x^x}\left[x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(x)\right] + e^{\log(\sin x)^x}\left[x\frac{d}{dx}(\log \sin x) + \log \sin x\frac{d}{dx}(x)\right]$$
$$= x^x\left[x\left(\frac{1}{x}\right) + \log x(1)\right] + (\sin x)^x\left[x \times \left(\frac{1}{\sin x}\right)\frac{d}{dx}(\sin x) + \log \sin x(1)\right]$$
$$= x^x[1 + \log x] + (\sin x)^x\left[x\left(\frac{1}{\sin x}\right)(\cos x) + \log \sin x\right]$$

$$\frac{dy}{dx} = x^x(1 + \log x) + (\sin x)^x[x\cot x + \log \sin x]$$

Differentiation Ex 11.5 Q30

Here,

$$y = (\tan x)^{\log x} + \cos^2\left(\frac{\pi}{4}\right)$$
$$y = e^{\log(\tan x)^{\cos x}} + \cos^2\left(\frac{\pi}{4}\right)$$
$$y = e^{\log x \log \tan x} + \cos x^2\left(\frac{\pi}{4}\right) \qquad \left[\text{Since, } e^{\log a} = a \text{ and } \log a^b = b\log a\right]$$

Differentiating it using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\log x \log \tan x}\right) + \frac{d}{dx}\cos^2\left(\frac{\pi}{4}\right)$$
$$= e^{\log x \log \tan x}\frac{d}{dx}(\log x \log \tan x) + 0$$
$$= e^{\log(\tan x)^{\cos x}}\left[\log x\frac{d}{dx}(\log \tan x) + \log \tan x\frac{d}{dx}(\log x)\right]$$
$$= (\tan x)^{\log x}\left[\log x\left(\frac{1}{\tan x}\right)\frac{d}{dx}(\tan x) + \log \tan x\left(\frac{1}{x}\right)\right]$$
$$= (\tan x)^{\log x}\left[\log x\left(\frac{1}{\tan x}\right)\left(\sec^2 x\right) + \frac{\log \tan x}{x}\right]$$

$$\frac{dy}{dx} = (\tan x)^{\log x}\left[\log x\left(\frac{\sec^2 x}{\tan x}\right) + \frac{\log \tan x}{x}\right]$$

Differentiation Ex 11.5 Q31

Here,

$$y = x^x + x^{\frac{1}{x}}$$

$$= e^{\log x^x} + e^{\log x^{\frac{1}{x}}}$$

$$y = e^{x\log x} + e^{\left(\frac{1}{x}\log x\right)} \qquad \left[\text{Since, } e^{\log a} = a, \log a^b = b\log a\right]$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{x\log x}\right) + \frac{d}{dx}\left(e^{\frac{1}{x}\log x}\right)$$

$$= e^{x\log x} + \frac{d}{dx}(x\log x) + e^{\frac{1}{x}\log x}\frac{d}{dx}\left(\frac{1}{x}\log x\right)$$

$$= e^{\log x^x}\left[x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(x)\right] + e^{\log x^{\frac{1}{x}}}\left[\frac{1}{x}\frac{d}{dx}(\log x) + \log x\frac{d}{dx}\left(\frac{1}{x}\right)\right]$$

$$= x^x\left[x\left(\frac{1}{x}\right) + \log x\,(1)\right] + x^{\frac{1}{x}}\left[\left(\frac{1}{x}\right)\left(\frac{1}{x}\right) + \log x\left(-\frac{1}{x^2}\right)\right]$$

$$= x^x\left[1 + \log x\right] + x^{\frac{1}{x}}\left(\frac{1}{x^2} - \frac{1}{x^2}\log x\right)$$

$$\frac{dy}{dx} = x^x\left[1 + \log x\right] + x^{\frac{1}{x}}\frac{(1 - \log x)}{x^2}$$

Differentiation Ex 11.5 Q32

Let $y = (\log x)^x + x^{\log x}$

Also, let $u = (\log x)^x$ and $v = x^{\log x}$

$\therefore y = u + v$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad ...(1)$$

$u = (\log x)^x$

$$\Rightarrow \log u = \log\left[(\log x)^x\right]$$

$$\Rightarrow \log u = x\log(\log x)$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(x)\times\log(\log x) + x\cdot\frac{d}{dx}\left[\log(\log x)\right]$$

$$\Rightarrow \frac{du}{dx} = u\left[1\times\log(\log x) + x\cdot\frac{1}{\log x}\cdot\frac{d}{dx}(\log x)\right]$$

$$\Rightarrow \frac{du}{dx} = (\log x)^x\left[\log(\log x) + \frac{x}{\log x}\cdot\frac{1}{x}\right]$$

$$\Rightarrow \frac{du}{dx} = (\log x)^x\left[\log(\log x) + \frac{1}{\log x}\right]$$

$$\Rightarrow \frac{du}{dx} = (\log x)^x\left[\frac{\log(\log x)\cdot\log x + 1}{\log x}\right]$$

$$\frac{1}{v}\cdot\frac{dv}{dx}=\frac{d}{dx}\left[(\log x)^2\right]$$

$$\Rightarrow\frac{1}{v}\cdot\frac{dv}{dx}=2(\log x)\cdot\frac{d}{dx}(\log x)$$

$$\Rightarrow\frac{dv}{dx}=2v(\log x)\cdot\frac{1}{x}$$

$$\Rightarrow\frac{dv}{dx}=2x^{\log x}\frac{\log x}{x}$$

$$\Rightarrow\frac{dv}{dx}=2x^{\log x-1}\cdot\log x \qquad ...(3)$$

Therefore, from (1), (2), and (3), we obtain

$$\frac{dy}{dx}=(\log x)^{x-1}\left[1+\log x.\log(\log x)\right]+2x^{\log x-1}\cdot\log x$$

Differentiation Ex 11.5 Q33

Here,

$$x^{13}y^7=(x+y)^{20}$$

Taking log on both the sides,

$$\log\left(x^{13}y^7\right)=\log(x+y)^{20}$$

$$13\log x+7\log y=20\log(x+y) \qquad \left[\text{Since, }\log(AB)=\log A+\log B,\log a^b=b\log a\right]$$

Differentiating it with respect to x using chain rule,

$$13\frac{d}{dx}(\log x)+7\frac{d}{dx}(\log y)=20\frac{d}{dx}\log(x+y)$$

$$\frac{13}{x}+\frac{7}{y}\frac{dy}{dx}=\frac{20}{x+y}\frac{d}{dx}(x+y)$$

$$\frac{13}{x}+\frac{7}{y}\frac{dy}{dx}=\frac{20}{(x+y)}\left[1+\frac{dy}{dx}\right]$$

$$\frac{7}{y}\frac{dy}{dx}-\frac{20}{(x+y)}=\frac{20}{(x+y)}-\frac{13}{x}$$

$$\frac{dy}{dx}\left[\frac{7}{y}-\frac{20}{(x+y)}\right]=\frac{20}{(x+y)}-\frac{13}{x}$$

$$\frac{dy}{dx}\left[\frac{7(x+y)-20y}{y(x+y)}\right]=\left[\frac{20x-13(x+y)}{x(x+y)}\right]$$

$$\frac{dy}{dx}=\left[\frac{20x-13x-13y}{x(x+y)}\right]\left(\frac{y(x+y)}{7x+7y-20y}\right)$$

$$=\frac{y}{x}\left(\frac{7x-13y}{7x-13y}\right)$$

$$\frac{dy}{dx}=\frac{y}{x}$$

Differentiation Ex 11.5 Q34

Here,

$$x^{16}y^{9} = \left(x^2 + y\right)^{17}$$

Taking log on both the sides,

$$\log\left(x^{16} \times y^{9}\right) = \log\left(x^2 + y\right)^{17} \qquad \left[\text{Since, } \log(AB) = \log A + \log B, \log a^{b} = b\log a\right]$$

$$16\log x + 9\log y = 17\log\left(x^2 + y\right)$$

Differentiating it with respect to x using chain rule,

$$16\frac{d}{dx}(\log x) + 9\frac{d}{dx}(\log y) = 17\frac{d}{dx}\log\left(x^2 + y\right)$$

$$\frac{16}{x} + \frac{9}{y}\frac{dy}{dx} = 17\frac{1}{\left(x^2 + y\right)}\frac{d}{dx}\left(x^2 + y\right)$$

$$\frac{16}{x} + \frac{9}{y}\frac{dy}{dx} = \frac{17}{x^2 + y}\left[2x + \frac{dy}{dx}\right]$$

$$\frac{9}{y}\frac{dy}{dx} - \frac{17}{\left(x^2 + y\right)}\frac{dy}{dx} = \left(\frac{34x}{x^2 + y}\right) - \frac{16}{x}$$

$$\frac{dy}{dx}\left[\frac{9}{y} - \frac{17}{\left(x^2 + y\right)}\right] = \frac{34x^2 - 16x^2 - 16y}{x\left(x^2 + y\right)}$$

$$\frac{dy}{dx}\left[\frac{9x^2 + 9y - 17y}{y\left(x^2 + y\right)}\right] = \frac{18x^2 - 16y}{x\left(x^2 + y\right)}$$

$$\frac{dy}{dx} = \frac{y}{x}\left(\frac{2\left(9x^2 - 8y\right)}{9x^2 - 8y}\right)$$

$$\frac{dy}{dx} = \frac{2y}{x}$$

$$x\frac{dy}{dx} = 2y$$

Differentiation Ex 11.5 Q35

Here,

$$y = \sin\left(x^x\right) \qquad ---(i)$$

Let $\quad u = x^x \qquad ---(ii)$

Taking log on both the sides,

$$\log u = \log x^x$$
$$\log u = x\log x$$

Differentiating it with respect to x,

$$\frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(x\log x)$$
$$= x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(x)$$
$$= x\left(\frac{1}{x}\right) + \log x\,(1)$$
$$\frac{1}{u}\frac{du}{dx} = 1 + \log x$$
$$\frac{du}{dx} = u\,(1 + \log x)$$
$$\frac{du}{dx} = x^x\,(1 + \log x) \qquad ---(iii)\ \left[\text{Using equation (ii)}\right]$$

Now, using equation (ii) in equation (i),

$$y = \sin u$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}(\sin u)$$
$$= \cos u\frac{du}{dx}$$

Using equation (ii) and (iii),

$$\frac{dy}{dx} = \cos\left(x^x\right)\times x^x\,(1 + \log x)$$

Differentiation Ex 11.5 Q36

Here,

$$x^x + y^x = 1$$
$$e^{\log x^x} + e^{\log y^x} = 1$$
$$e^{x\log x} + e^{x\log y} = 1 \qquad \left[\text{Since, } e^{\log a} = a, \log a^b = b\log a\right]$$

Differentiating it with respect to x using product rule and chain rule,

$$\frac{d}{dx}\left(e^{x\log x}\right) + \frac{d}{dx}\left(e^{x\log y}\right) = \frac{d}{dx}(1)$$
$$e^{x\log x}\frac{d}{dx}(x\log x) + e^{x\log y}\frac{d}{dx}(x\log y) = 0$$
$$e^{\log x^x}\left[x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(x)\right] + e^{\log y^x}\left[x\frac{d}{dx}(\log y) + \log y\frac{d}{dx}(x)\right] = 0$$
$$x^x\left[x\left(\frac{1}{x}\right) + \log x\,(1)\right] + y^x\left[x\left(\frac{1}{y}\right)\frac{dy}{dx} + \log y\,(1)\right] = 0$$
$$x^x\left[1 + \log x\right] + y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) = 0$$
$$y^x\times\frac{x}{y}\frac{dy}{dx} = -\left[x^x(1 + \log x) + y^x\log y\right]$$
$$\left(xy^{x-1}\right)\frac{dy}{dx} = -\left[x^x(1 + \log x) + y^x\log y\right]$$
$$\frac{dy}{dx} = -\left[\frac{x^x(1 + \log x) + y^x\log y}{xy^{x-1}}\right]$$

Differentiation Ex 11.5 Q37

Here,

$$x^y \times y^x = 1$$

Taking on both sides,

$$\log\left(x^y \times y^x\right) = \log(1)$$

$$y = \log x + x\log y = \log 1 \qquad \left[\text{Since, } \log(AB) = \log A + \log B, \log a^b = b\log a\right]$$

Differentiating it with respect to x using product rule,

$$\frac{d}{dx}(y\log x) + \frac{d}{dx}(x\log y) = \frac{d}{dx}(\log 1)$$

$$\left[y\frac{d}{dx}(\log x) + \log x\frac{dy}{dx}\right] + \left[x\frac{d}{dx}(\log y) + \log y\frac{d}{dx}(x)\right] = 0$$

$$\left[y\left(\frac{1}{x}\right) + \log x\frac{dy}{dx}\right] + \left[x\left(\frac{1}{y}\frac{dy}{dx}\right) + \log y\,(1)\right] = 0$$

$$\frac{y}{x} + \log x\frac{dy}{dx} + \frac{x}{y}\frac{dy}{dx} + \log y = 0$$

$$\frac{dy}{dx}\left(\log x + \frac{x}{y}\right) = -\left[\log y + \frac{y}{x}\right]$$

$$\frac{dy}{dx}\left[\frac{y\log x + x}{y}\right] = -\left[\frac{x\log y + y}{x}\right]$$

$$\frac{dy}{dx} = -\frac{y}{x}\left[\frac{x\log y + y}{y\log x + x}\right]$$

Differentiation Ex 11.5 Q38

Here,

$$x^y + y^x = (x+y)^{x+y}$$

$$e^{\log x^y} + e^{\log y^x} = e^{\log(x+y)^{(x+y)}}$$

$$e^{y\log x} + e^{x\log y} = e^{(x+y)\log(x+y)} \qquad \left[\text{Since, } e^{\log a} = a, \log a^b = b\log a\right]$$

Differentiating it with respect to x using chain rule, product rule,

$$\Rightarrow \quad \frac{d}{dx}\left(e^{y\log x}\right) + \frac{d}{dx}\left(e^{x\log y}\right) = \frac{d}{dx}e^{(x+y)\log(x+y)}$$

$$\Rightarrow \quad e^{y\log x}\left[y\frac{d}{dx}(\log x) + \log x\frac{dy}{dx}\right] + e^{x\log y}\left[x\frac{d}{dx}\log y + \log y\frac{d}{dx}(x)\right] = e^{(x+y)\log(x+y)}\frac{d}{dx}\left[(x+y)\log(x+y)\right]$$

$$\Rightarrow \quad e^{\log x^y}\left[y\left(\frac{1}{x}\right) + \log x\frac{dy}{dx}\right] + e^{\log x}\left[\frac{x}{y}\frac{dy}{dx} + \log y\,(1)\right] = e^{\log(x+y)^{(x+y)}}\left[(x+y)\frac{d}{dx}\log(x+y) + \log(x+y)\frac{d}{dx}(x+y)\right]$$

$$\Rightarrow \quad x^y\left[\frac{y}{x} + \log x\frac{dy}{dx}\right] + y^x\left[\frac{x}{y}\frac{dy}{dx} + \log y\right] = (x+y)^{(x+y)}\left[(x+y)\frac{1}{(x+y)}\frac{d}{dx}(x+y) + \log(x+y)\left(1 + \frac{dy}{dx}\right)\right]$$

$$\Rightarrow \quad x^y \times \frac{y}{x} + x^y\log x\frac{dy}{dx} + y^x \times \frac{x}{y}\frac{dy}{dx} + y^x\log y = (x+y)^{(x+y)}\left[1 \times \left(1 + \frac{dy}{dx}\right) + \log(x+y)\left(1 + \frac{dy}{dx}\right)\right]$$

$$\Rightarrow \quad x^{y-1} \times y + x^y\log x\frac{dy}{dx} + y^{x-1} \times x\frac{dy}{dx} + y^x\log y = (x+y)^{(x+y)} + (x+y)^{(x+y)}\frac{dy}{dx} + (x+y)^{(x+y)}\log(x+y) + (x+y)^{(x+y)}\log(x+y)\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx}\left[x^y\log x + xy^{x-1} - (x+y)^{(x+y)}\left(1 + \log(x+y)\right)\right] = (x+y)^{(x+y)}\left(1 + \log(x+y)\right) - x^{y-1} \times y - y^x\log y$$

$$\Rightarrow \quad \frac{dy}{dx} = \left[\frac{(x+y)^{(x+y)}\left(1 + \log(x+y)\right) - x^{y-1} \times y - y^x\log y}{x^y\log x + xy^{x-1} - (x+y)^{(x+y)}\left(1 + \log(x+y)\right)}\right]$$

Differentiation Ex 11.5 Q39

Here,

$$x^m y^n = 1$$

Taking log on both the side,

$$\log\left(x^m y^n\right) = \log(1)$$
$$m\log x + n\log y = \log(1)$$

Differentiating it with respect to x,

$$\frac{dy}{dx}(m\log x) + \frac{d}{dx}(n\log y) = \frac{d}{dx}(\log(1))$$
$$\frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = -\frac{m}{x}\times\frac{y}{n}$$

$$\frac{dy}{dx} = -\frac{my}{nx}$$

Differentiation Ex 11.5 Q40

Here,

$$y^x = e^{y-x}$$

Taking log on both the sides,

$$\log y^x = \log e^{(y-x)} \qquad \left[\text{Since, } \log a^b = b\log a \text{ and } \log_e e = 1\right]$$
$$x\log y = (y-x)\log e$$
$$x\log y = y - x \qquad \text{---(i)}$$

Differentiating it with respect to x using product rule,

$$\frac{d}{dx}(x\log y) = \frac{d}{dx}(y-x)$$
$$\left[x\frac{d}{dx}(\log y) + \log y\frac{d}{dx}(x)\right] = \frac{dy}{dx} - 1$$
$$x\left(\frac{1}{y}\right)\frac{dy}{dx} + \log y\,(1) = \frac{dy}{dx} - 1$$
$$\frac{dy}{dx}\left(\frac{x}{y} - 1\right) = -1 - \log y$$
$$\frac{dy}{dx}\left(\frac{y}{(1+\log y)\,y}\right) = -(1+\log y) \qquad \left[\text{Since, from equation (i), } x = \frac{y}{(1+\log y)}\right]$$

$$\frac{dy}{dx}\left[\frac{1-1-\log y}{(1+\log y)}\right] = -(1+\log y)$$
$$\frac{dy}{dx} = -\frac{(1+\log y)^2}{-\log y}$$

$$\frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$$

Differentiation Ex 11.5 Q41

Here,

$$(\sin x)^y = (\cos y)^x$$

Taking log on both the sides,

$$\log(\sin x)^y = \log(\cos y)^x \qquad \left[\text{Using } \log a^b = b\log a\right]$$
$$y\log(\sin x) = x\log(\cos y)$$

Differentiating it with respect to x using product rule and chain rule,

$$\frac{d}{dx}\left[y\log\sin x\right] = \frac{d}{dx}\left[x\log\cos y\right]$$
$$y\frac{d}{dx}(\log\sin x) + \log\sin x\frac{dy}{dx} = x\frac{dy}{dx}\log\cos y + \log\cos y\frac{d}{dx}(x)$$
$$y\left(\frac{1}{\sin x}\right)\frac{d}{dx}(\sin x) + \log\sin x\frac{dy}{dx} = \frac{x}{\cos y}\frac{d}{dx}(\cos y) + \log\cos y\,(1)$$
$$\frac{y}{\sin x}(\cos x) + \log\sin x\frac{dy}{dx} = \frac{x}{\cos y}(-\sin y)\frac{dy}{dx} + \log\cos y$$
$$y\cot x + \log\sin x\frac{dy}{dx} = -x\tan y\frac{dy}{dx} + \log\cos y$$
$$\frac{dy}{dx}(\log\sin x + x\tan y) = \log\cos y - y\cot x$$

$$\frac{dy}{dx} = \frac{\log\cos y - y\cot x}{\log\sin x + x\tan y}$$

Differentiation Ex 11.5 Q42

Here,

$$(\cos x)^y = (\tan y)^x$$

Taking log on both the sides,

$$\log(\cos x)^y = \log(\tan y)^x$$
$$y\log\cos x = x\log\tan y \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{d}{dx}(y\log\cos x) = \frac{d}{dx}(x\log\tan y)$$
$$\left(y\frac{d}{dx}\log\cos x + \log\cos x\frac{dy}{dx}\right) = \left(x\frac{d}{dx}\log\tan y + \log\tan y\frac{d}{dx}(x)\right)$$
$$\left(y\left(\frac{1}{\cos x}\right)\frac{d}{dx}(\cos x) + \log\cos x\frac{dy}{dx}\right) = \left(x\frac{1}{\tan y}\frac{d}{dx}(\tan y) + \log\tan y\,(1)\right)$$
$$\left(\frac{y}{\cos x}(-\sin x) + \log\cos x\frac{dy}{dx}\right) = \left(\frac{x}{\tan y}\left(\sec^2 y\right)\right)\frac{dy}{dx} + \log\tan y - y\tan x + \log\cos x\frac{dy}{dx}$$
$$= \left(\sec y\ \mathrm{cosec}\,y \times x\frac{dy}{dx} + \log\tan y\right)$$

$$\frac{dy}{dx}\left[\log\cos x - x\sec y\ \mathrm{cosec}\,y\right] = \log\tan y + y\tan x$$

$$\frac{dy}{dx} = \left[\frac{\log\tan y + y\tan x}{\log\cos x - x\sec y\ \mathrm{cosec}\,y}\right]$$

Differentiation Ex 11.5 Q43

Here,

$$e^x + e^y = e^{x+y} \qquad \text{---(i)}$$

Differentiating both the sides using chain rule,

$$\frac{d}{dx}\left(e^x\right) + \frac{d}{dx}\left(e^y\right) = \frac{d}{dx}\left(e^{x+y}\right)$$
$$e^x + e^y \frac{dy}{dx} = e^{x+y} \frac{d}{dx}(x+y)$$
$$e^x + e^y \frac{dy}{dx} = e^{x+y}\left[1 + \frac{dy}{dx}\right]$$
$$e^y \frac{dy}{dx} - e^{x+y} \frac{dy}{dx} = e^{x+y} - e^x$$
$$\frac{dy}{dx} = \frac{e^{x+y} - e^x}{e^y - e^{x+y}}$$
$$= \left(\frac{e^x + e^y - e^x}{e^y - e^x - e^y}\right) \qquad \left[\text{Using equation (i)}\right]$$
$$\frac{dy}{dx} = -e^{y-x}$$

$$\frac{dy}{dx} + e^{y-x} = 0$$

Differentiation Ex 11.5 Q44

Here,

$$e^y = y^x$$

Taking log on both the sides,

$$\log e^y = \log y^x$$
$$y \log e = x \log y \qquad \left[\text{Since, } \log a^b = b \log a, \log_e e = 1\right]$$
$$y = x \log y \qquad \text{---(i)}$$

Differentiating it with respect to x using product rule,

$$\frac{dy}{dx} = \frac{d}{dx}(x \log y)$$
$$= x \frac{dy}{dx}(\log y) + \log y \frac{d}{dx}(x)$$
$$\frac{dy}{dx} = \frac{x}{y}\frac{dy}{dx} + \log y\,(1)$$
$$\frac{dy}{dx}\left(1 - \frac{x}{y}\right) = \log y$$
$$\frac{dy}{dx}\left(\frac{y-x}{y}\right) = \log y$$
$$\frac{dy}{dx} = \frac{y \log y}{y - x}$$
$$\frac{dy}{dx} = \frac{y \log y}{\left(y - \frac{y}{\log y}\right)} \qquad \left[\text{Since, using equation (i)}\right]$$
$$= \frac{y \log y \times \log y}{y \log y - y}$$
$$= \frac{y (\log y)^2}{y (\log y - 1)}$$

$$\frac{dy}{dx} = \frac{(\log y)^2}{(\log y - 1)}$$

Differentiation Ex 11.5 Q45

Here,

$$e^{x+y} - x = 0$$
$$e^{x+y} = x \qquad ---(i)$$

Differentiating it with respect to x using chain rule,

$$\frac{d}{dx}\left(e^{x+y}\right) = \frac{d}{dx}(x)$$
$$e^{x+y}\frac{d}{dx}(x+y) = 1$$
$$x\left[1+\frac{dy}{dx}\right] = 1 \qquad \left[\text{Using equatoin (i)}\right]$$
$$1+\frac{dy}{dx} = \frac{1}{x}$$
$$\frac{dy}{dx} = \frac{1}{x} - 1$$

$$\frac{dy}{dx} = \frac{1-x}{x}$$

Differentiation Ex 11.5 Q46

Here $y = x\sin(a+y)$

Differentiating it with respect to x using the chain rule and product rule,

$$\frac{dy}{dx} = x\frac{d}{dx}\sin(a+y) + \sin(a+y)\frac{dx}{dx}$$
$$\frac{dy}{dx} = x\cos(a+y)\frac{dy}{dx} + \sin(a+y)$$
$$(1 - x\cos(a+y))\frac{dy}{dx} = \sin(a+y)$$
$$\frac{dy}{dx} = \frac{\sin(a+y)}{(1-x\cos(a+y))}$$
$$\frac{dy}{dx} = \frac{\sin(a+y)}{\left(1-\frac{y}{\sin(a+y)}\cos(a+y)\right)} \qquad \left[\text{Since } \frac{y}{\sin(a+y)} = x\right]$$
$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin(a+y) - y\cos(a+y)}$$

Differentiation Ex 11.5 Q47

Here $x\sin(a+y) + \sin a\cos(a+y) = 0$

Differentiating it with respect to x using the chain rule and product rule,

$$\frac{d}{dx}\left[x\sin(a+y) + \sin a\cos(a+y)\right] = 0$$
$$x\frac{d}{dx}\sin(a+y) + \sin(a+y)\frac{dx}{dx} + \sin a\frac{d}{dx}\cos(a+y) + \cos(a+y)\frac{d}{dx}\sin a = 0$$
$$x\cos(a+y)\left(0+\frac{dy}{dx}\right) + \sin(a+y) + \sin a\left(-\sin(a+y)\frac{dy}{dx}\right) + 0 = 0$$
$$\left[x\cos(a+y) - \sin a\sin(a+y)\right]\frac{dy}{dx} + \sin(a+y) = 0$$

$$\frac{dy}{dx} = -\frac{\sin(a+y)}{x\cos(a+y) - \sin a\sin(a+y)}$$
$$\frac{dy}{dx} = \frac{-\sin(a+y)}{\left(-\frac{\sin a\cos(a+y)}{\sin(a+y)}\right)\cos(a+y) - \sin a\sin(a+y)} \qquad \left[\text{Since } x = -\frac{\sin a\cos(a+y)}{\sin(a+y)}\right]$$
$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{(\sin a)\cos^2(a+y) + \sin a\sin^2(a+y)}$$
$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{(\sin a)\left[\cos^2(a+y) + \sin^2(a+y)\right]}$$
$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{(\sin a)} \qquad \left[\text{Since } \cos^2(a+y) + \sin^2(a+y) = 1\right]$$

Differentiation Ex 11.5 Q48

Here,

$$(\sin x)^y = x + y$$

Taking log on both the sides,

$$\log(\sin x)^y = \log(x + y)$$
$$y\log(\sin x) = \log(x + y) \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating it with respect to x using chain rule, product rule,

$$\frac{d}{dx}\left(y\log(\sin x)\right) = \frac{d}{dx}\log(x + y)$$
$$y\frac{d}{dx}\log\sin x + \log\sin x\frac{dy}{dx} = \frac{1}{x+y}\frac{d}{dx}(x + y)$$
$$\frac{y}{\sin x}\frac{d}{dx}(\sin x) + \log\sin x\frac{dy}{dx} = \frac{1}{(x+y)}\left[1 + \frac{dy}{dx}\right]$$
$$\frac{y(\cos x)}{(\sin x)} + \log\sin x\frac{dy}{dx} = \frac{1}{(x+y)} + \frac{1}{(x+y)}\frac{dy}{dx}$$
$$\frac{dy}{dx}\left(\log\sin x - \frac{1}{x+y}\right) = \frac{1}{(x+y)} - y\cot x$$
$$\frac{dy}{dx}\left(\frac{(x+y)\log\sin x - 1}{(x+y)}\right) = \left(\frac{1 - y(x+y)\cot x}{x+y}\right)$$

$$\frac{dy}{dx} = \left(\frac{1 - y(x+y)\cot x}{(x+y)\log\sin x - 1}\right)$$

Differentiation Ex 11.5 Q49

Here,

$$xy\log(x + y) = 1 \qquad \text{---(i)}$$

Differentiating with respect to x using chain rule, product rule,

$$\frac{dy}{dx}\left(xy\log(x + y)\right) = \frac{d}{dx}(1)$$
$$xy\frac{d}{dx}\log(x + y) + x\log(x + y)\frac{dy}{dx} + y\log(x + y)\frac{d}{dx}(x) = 0$$
$$\frac{xy}{(x+y)}\left(1 + \frac{dy}{dx}\right) + x\log(x + y)\frac{dy}{dx} + y\log(x + y)(1) = 0$$
$$\left(\frac{xy}{x+y}\right)\left(1 + \frac{dy}{dx}\right) + x\log(x + y)\frac{dy}{dx} + y\log(x + y) = 0$$
$$\left(\frac{xy}{x+y}\right)\frac{dy}{dx} + \frac{xy}{x+y} + x\left(\frac{1}{xy}\right)\frac{dy}{dx} + y\left(\frac{1}{xy}\right) = 0 \qquad \left[\text{Using equation (i)}\right]$$
$$\frac{dy}{dx}\left[\frac{xy}{x+y} + \frac{1}{y}\right] = -\left[\frac{1}{x} + \frac{xy}{x+y}\right]$$
$$\frac{dy}{dx}\left[\frac{xy^2 + x + y}{(x+y)y}\right] = -\left[\frac{x + y + x^2y}{x(x+y)}\right]$$

$$\frac{dy}{dx} = -\frac{y}{x}\left(\frac{x + y + x^2y}{x + y + xy^2}\right)$$

Differentiation Ex 11.5 Q50

Here,

$$y = x \sin y \qquad \text{---(i)}$$

Differentiating it with respect to x using product rule,

$$\frac{dy}{dx} = \frac{d}{dx}(x \sin y)$$
$$= x \frac{d}{dx}(\sin y) + \sin y \frac{d}{dx}(x)$$
$$= x \cos y \frac{dy}{dx} + \sin y\,(1)$$
$$\frac{dy}{dx} - x \cos y \frac{dy}{dx} = \sin y$$
$$\frac{dy}{dx}(1 - x \cos y) = \sin y$$
$$\frac{dy}{dx} = \frac{\sin y}{(1 - x \cos y)}$$

Put the value of $\sin y = \frac{y}{x}$ form equation (i),

$$\frac{dy}{dx} = \frac{y}{x\,(1 - x \cos y)}$$

Differentiation Ex 11.5 Q51

Here,

$$f(x) = (1+x)\left(1+x^2\right)\left(1+x^4\right)\left(1+x^8\right)$$

Differentiating with respect to x using product rule and chain rule,

$$\Rightarrow \quad f'(x) = (1+x)\left(1+x^2\right)\frac{d}{dx}\left(1+x^8\right) + (1+x)\left(1+x^2\right)\left(1+x^8\right)\frac{d}{dx}\left(1+x^4\right) + (1+x)\left(1+x^4\right)\left(1+x^8\right)$$
$$\frac{d}{dx}\left(1+x^2\right) + \left(1+x^2\right)\left(1+x^4\right)\left(1+x^8\right)\frac{d}{dx}(1+x)$$

$$\Rightarrow \quad f'(x) = (1+x)\left(1+x^2\right)\left(1+x^4\right)8x^7 + (1+x)\left(1+x^2\right)\left(1+x^8\right)\left(4x^3\right) + (1+x)\left(1+x^4\right)\left(1+x^8\right)(2x)$$
$$+\left(1+x^2\right)\left(1+x^4\right)\left(1+x^8\right)(1)$$

$$f'(1) = (1+1)(1+1)(8) + (1+1)(1+1)(1+1)(4) + (1+1)(1+1)(1+1)(2) + (1+1)(1+1)(1+1)$$

$$f'(1) = (2)(2)(2)(8) + (2)(2)(2)(4) + (2)(2)(2)(2) + (2)(2)(2)$$
$$= 64 + 32 + 16 + 8$$
$$= 120$$

So,

$$f'(1) = 120$$

Differentiation Ex 11.5 Q52

Here,

$$y = \log\left(\frac{x^2+x+}{x^2-x+1}\right)+\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}x}{1-x^2}\right)$$

Differentiating it with respect to x using chain rule and quotient rule,

$$\frac{dy}{dx} = \frac{d}{dx}\log\left(\frac{x^2+x+1}{x^2-x+1}\right)+\frac{2}{\sqrt{3}}\frac{d}{dx}\tan^{-1}\left(\frac{\sqrt{3}x}{1-x^2}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{\left(\frac{x^2+x+1}{x^2-x+1}\right)}\frac{d}{dx}\left(\frac{x^2+x+1}{x^2-x+1}\right)+\frac{2}{\sqrt{3}}\left\{\frac{1}{1+\left(\frac{\sqrt{3}x}{1-x^2}\right)}\right\}\frac{d}{dx}\left(\frac{\sqrt{3}x}{1-x^2}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{x^2-x+1}{x^2+x+1}\right)\left(\frac{\left(x^2-x+1\right)\frac{d}{dx}\left(x^2+x+1\right)-\left(x^2+x+1\right)\frac{d}{dx}\left(x^2-x+1\right)}{\left(x^2-x+1\right)^2}\right)+\frac{2}{\sqrt{3}}\left\{\frac{(1-x)^2}{1+x^4-2x^2+3x^2}\right\}$$

$$\left\{\frac{\left(1-x^2\right)^2\frac{d}{dx}\left(\sqrt{3}x\right)-\sqrt{3}x\frac{d}{dx}(1-x)^2}{\left(1-x^2\right)^2}\right\}$$

$$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{1}{x^2+x+1}\right)\left(\frac{\left(x^2-x+1\right)(2x+1)-\left(x^2+x+1\right)(2x-1)}{\left(x^2-x+1\right)}\right)+\frac{2}{\sqrt{3}}\left(\frac{\left(1-x^2\right)^2}{1+x^2+x^4}\right)\left(\frac{\left(1-x^2\right)\left(\sqrt{3}\right)-\sqrt{3}x(-2x)}{\left(1-x^2\right)^2}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{2x^3-2x^2+2x+x^2-x+1-2x^3-2x^2-2x+x^2+x+1}{x^4+2x^2+1-x^2}\right)+\frac{2}{\sqrt{3}}\left(\frac{\sqrt{3}-\sqrt{3}x^2+2\sqrt{3}x^2}{1+x^2+x^4}\right)$$

$$= \left(\frac{-2x^2+2}{x^4+x^2+1}\right)+\frac{2\sqrt{3}\left(x^2+1\right)}{\sqrt{3}\left(1+x^2+x^4\right)}$$

$$= \frac{2\left(1-x^2\right)}{\left(x^4+x^2+1\right)}+\frac{2\left(x^2+1\right)}{1+x^2+x^4}$$

$$= \frac{2\left(1-x^2+x^2+1\right)}{1+x^2+x^4}$$

$$\frac{dy}{dx} = \frac{4}{1+x^2+x^4}$$

Differentiation Ex 11.5 Q53

Here,

$$y = (\sin x - \cos x)^{(\sin x-\cos x)}$$

Takig log on both the sides,

$$\Rightarrow \quad \log y = \log(\sin x - \cos x)^{(\sin x-\cos x)}$$

$$\Rightarrow \quad \log y = (\sin x - \cos x)\log(\sin x - \cos x)$$

Differentiating it with respect to x using product rule, chain rule,

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = \log(\sin x - \cos x)\frac{d}{dx}(\sin x - \cos x)+(\sin x - \cos x)\frac{d}{dx}\log(\sin x - \cos x)$$

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = \log(\sin x - \cos x)\times(\cos x + \sin x)+\frac{(\sin x - \cos x)}{(\sin x - \cos x)}\frac{d}{dx}(\sin x - \cos x)$$

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = (\cos x + \sin x)\log(\sin x - \cos x)+(\cos x + \sin x)$$

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = (\cos x + \sin x)\left(1+\log(\sin x - \cos x)\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = y\left[(\cos x + \sin x)\left(1+\log(\sin x - \cos x)\right)\right]$$

Using equation (i),

$$\frac{dy}{dx} = (\sin x - \cos x)^{(\sin x-\cos x)}\left[(\cos x + \sin x)\left(1+\log(\sin x - \cos x)\right)\right]$$

Differentiation Ex 11.5 Q54

The given function is $xy = e^{(x-y)}$

Taking logarithm on both the sides, we obtain

$$\log(xy) = \log\left(e^{x-y}\right)$$
$$\Rightarrow \log x + \log y = (x-y)\log e$$
$$\Rightarrow \log x + \log y = (x-y)\times 1$$
$$\Rightarrow \log x + \log y = x - y$$

Differentiating both sides with respect to x, we obtain

$$\frac{d}{dx}(\log x) + \frac{d}{dx}(\log y) = \frac{d}{dx}(x) - \frac{dy}{dx}$$
$$\Rightarrow \frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = 1 - \frac{dy}{dx}$$
$$\Rightarrow \left(1 + \frac{1}{y}\right)\frac{dy}{dx} = 1 - \frac{1}{x}$$
$$\Rightarrow \left(\frac{y+1}{y}\right)\frac{dy}{dx} = \frac{x-1}{x}$$
$$\therefore \frac{dy}{dx} = \frac{y(x-1)}{x(y+1)}$$

Differentiation Ex 11.5 Q55

Given that $y^x + x^y + x^x = a^b$.

Putting $u = y^x$, $v = x^y$ and $w = x^x$, we get $u + v + w = a^b$

Therefore $\frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} = 0$ ...(1)

Now, $u = y^x$. Taking logrithm on both sides, we have

$$\log u = x \log y$$

Differentiating both sides w.r.t. x, we have

$$\frac{1}{u}\cdot\frac{du}{dx} = x\frac{d}{dx}(\log y) + \log y \frac{d}{dx}(x)$$
$$= x\frac{1}{y}\cdot\frac{dy}{dx} + \log y.1$$

So $$\frac{du}{dx} = u\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) = y^x\left[\frac{x}{y}\frac{dy}{dx} + \log y\right] \quad ...(2)$$

Also $v = x^y$

Taking logarithm on both sides, we have

$$\log v = y \log x$$

Differentiating both sides w.r.t. x, we have

$$\frac{1}{v}\cdot\frac{dv}{dx} = y\frac{d}{dx}(\log x) + \log x\frac{dy}{dx}$$
$$= y.\frac{1}{x} + \log x.\frac{dy}{dx}$$

So $$\frac{dv}{dx} = v\left[\frac{y}{x} + \log x\frac{dy}{dx}\right]$$
$$= x^y\left[\frac{y}{x} + \log x\frac{dy}{dx}\right] \quad ...(3)$$

Again $w = x^x$

Takinglogarithm on both sides, we have

$$\log w = x \log x.$$

Differentiating both sides w.r.t. x, we have

$$\frac{1}{w}\cdot\frac{dw}{dx} = x\frac{d}{dx}(\log x) + \log x.\frac{d}{dx}(x)$$
$$= x.\frac{1}{x} + \log x \,.\, 1$$

i.e. $$\frac{dw}{dx} = w(1 + \log x)$$
$$= x^x(1 + \log x) \quad ...(4)$$

From (1), (2), (3), (4), we have

$$y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) + x^y\left(\frac{y}{x} + \log x\frac{dy}{dx}\right) + x^x(1 + \log x) = 0$$

or $$\left(x.y^{x-1} + x^y \,.\, \log x\right)\frac{dy}{dx} = -x^x(1 + \log x) - y.\,x^{y-1} - y^x \log y$$

Therefore $$\frac{dy}{dx} = \frac{-\left[y^x \log y + y.x^{y-1} + x^x(1 + \log x)\right]}{x.y^{x-1} + x^y \log x}$$

Differentiation Ex 11.5 Q56

Here $(\cos x)^y = (\cos y)^x$

Taking log on both sides,

$$\log(\cos x)^y = \log(\cos y)^x$$

$$y \log \cos x = x \log \cos y$$

Differentiating it with respect to x using the chain rule and product rule,

$$\frac{d}{dx}(y \log \cos x) = \frac{d}{dx}(x \log \cos y)$$

$$y\frac{d}{dx}\log \cos x + \log \cos x \frac{dy}{dx} = x\frac{d}{dx}\log \cos y + \log \cos y \frac{d}{dx}x$$

$$y\frac{1}{\cos x}(-\sin x) + \log \cos x \frac{dy}{dx} = x\frac{1}{\cos y}(-\sin y)\frac{dy}{dx} + \log \cos y$$

$$\left(\log \cos x + \frac{x \sin y}{\cos y}\right)\frac{dy}{dx} = \log \cos y + y\frac{\sin y}{\cos y}$$

$$(\log \cos x + x \tan y)\frac{dy}{dx} = \log \cos y + y \tan y$$

$$\frac{dy}{dx} = \frac{\log \cos y + y \tan y}{(\log \cos x + x \tan y)}$$

Differentiation Ex 11.5 Q57

Consider the given function,

$\cos y = x \cos(a+y)$, where $\cos a \neq \pm 1$

Differentiating both sides w.r.t. 'x' we get

$$-\sin y\frac{dy}{dx} = x\left(-\sin(a+y)\frac{dy}{dx}\right) + \cos(a+y)$$

$$\Rightarrow \frac{dy}{dx}\left[x\sin(a+y) - \sin y\right] = \cos(a+y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos(a+y)}{x\sin(a+y) - \sin y}$$

Multiplying the numerator and the denominator by $\cos(a+y)$ on the R.H.S., we have,

$$\frac{dy}{dx} = \frac{\cos^2(a+y)}{x\cos(a+y)\sin(a+y) - \cos(a+y)\sin y}$$

$$= \frac{\cos^2(a+y)}{\cos y\sin(a+y) - \cos(a+y)\sin y} \quad [\because \cos y = x\cos(a+y), \text{ given function}]$$

$$= \frac{\cos^2(a+y)}{\sin[(a+y)-y]} = \frac{\cos^2(a+y)}{\sin a}$$

Differentiation Ex 11.5 Q58

Consider the given function, $(x-y)e^{\frac{x}{x-y}} = a$.

We need to prove that $y\frac{dy}{dx} + x = 2y$.

Differentiating the given equation w.r.t. 'x' we get

$$(x-y)\left[e^{\frac{x}{x-y}}\left(\frac{(x-y) - x\left(1-\frac{dy}{dx}\right)}{(x-y)^2}\right)\right] + e^{\frac{x}{x-y}}\left(1-\frac{dy}{dx}\right) = 0$$

$$\Rightarrow \frac{(x-y) - x\left(1-\frac{dy}{dx}\right)}{(x-y)} + \left(1-\frac{dy}{dx}\right) = 0$$

$$\Rightarrow \left(1-\frac{dy}{dx}\right)\left(1-\frac{x}{x-y}\right) + 1 = 0$$

$$\Rightarrow \left(1-\frac{dy}{dx}\right)\left(\frac{-y}{x-y}\right) + 1 = 0$$

$$\Rightarrow -y + y\frac{dy}{dx} + x - y = 0$$

$$\Rightarrow y\frac{dy}{dx} + x = 2y$$

Differentiation Ex 11.5 Q59

$$x = e^{x/y}$$

$$\log x = \frac{x}{y}\ldots\ldots\ldots(i)$$

$$y = \frac{x}{\log x}$$

$$\frac{dy}{dx} = \frac{\log x\frac{d}{dx}(x) - x\frac{d}{dx}(\log x)}{(\log x)^2}$$

$$\frac{dy}{dx} = \frac{\log x - x \times \frac{1}{x}}{(\log x)^2}$$

$$\frac{dy}{dx} = \frac{\log x - 1}{(\log x)^2}$$

$$\frac{dy}{dx} = \frac{\frac{x}{y} - 1}{(\log x)^2}\ldots\ldots[\text{from (i)}]$$

$$\frac{dy}{dx} = \frac{x - y}{y(\log x)^2}$$

$$\frac{dy}{dx} = \frac{x - y}{x(\log x)}\ldots\ldots[\text{from (i)}]$$

Differentiation Ex 11.5 Q60

$$y = x^{\tan x} + \sqrt{\frac{x^2+1}{2}}$$

$$y = e^{\tan x \log x} + e^{\frac{1}{2}\log\left(\frac{x^2+1}{2}\right)}$$

$$\frac{dy}{dx} = e^{\tan x \log x}\frac{d}{dx}(\tan x \log x) + e^{\frac{1}{2}\log\left(\frac{x^2+1}{2}\right)}\frac{d}{dx}\left(\frac{1}{2}\log\left(\frac{x^2+1}{2}\right)\right)$$

$$\frac{dy}{dx} = x^{\tan x}\left[\frac{\tan x}{x} + \sec^2 x \log x\right] + \sqrt{\frac{x^2+1}{2}}\left(\frac{1}{2} \times \frac{2}{x^2+1} \times (x)\right)$$

$$\frac{dy}{dx} = x^{\tan x}\left[\frac{\tan x}{x} + \sec^2 x \log x\right] + \sqrt{\frac{x^2+1}{2}}\left(\frac{x}{x^2+1}\right)$$

$$\frac{dy}{dx} = x^{\tan x}\left[\frac{\tan x}{x} + \sec^2 x \log x\right] + \frac{x}{\sqrt{2(x^2+1)}}$$

Differentiation Ex 11.5 Q61

$$y = 1 + \frac{\alpha}{\left(\frac{1}{x} - \alpha\right)} + \frac{\beta/x}{\left(\frac{1}{x} - \alpha\right)\left(\frac{1}{x} - \beta\right)} + \frac{\gamma/x^2}{\left(\frac{1}{x} - \alpha\right)\left(\frac{1}{x} - \beta\right)\left(\frac{1}{x} - \gamma\right)}$$

$$\left\{\begin{array}{l} \text{Using the theorem,} \\ \text{If } y = 1 + \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + \frac{c}{(x-c)} \text{ then,} \\ \frac{dy}{dx} = \frac{y}{x}\left\{\frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x}\right\} \end{array}\right\}$$

Here we have $\frac{1}{x}$ instead of x.

So using above theorem we get,

$$\frac{dy}{dx} = \frac{\alpha}{\left(\frac{1}{x} - \alpha\right)} + \frac{\beta}{\left(\frac{1}{x} - \beta\right)} + \frac{\gamma}{\left(\frac{1}{x} - \gamma\right)}$$

Ex 11.6

Differentiation Ex 11.6 Q1

Here,

$$y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots \text{ to } \infty}}}$$

$$y = \sqrt{x + y}$$

Squaring both the sides,

$$y^2 = x + y$$

Differentiating it with respect to x,

$$2y\,\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$\frac{dy}{dx}(2y - 1) = 1$$

$$\frac{dy}{dx} = \frac{1}{2y - 1}$$

Differentiation Ex 11.6 Q2

Here,

$$y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \ldots \text{to } \infty}}}$$

$$y = \sqrt{\cos x + y}$$

squaring both the sides,

$$y^2 = \cos x + y$$

Differntiating it with respect to x,

$$2y\frac{dy}{dx} = -\sin x + \frac{dy}{dx}$$

$$\frac{dy}{dx}(2y - 1) = -\sin x$$

$$\frac{dy}{dx} = \frac{-\sin x}{(2y - 1)}$$

$$\frac{dy}{dx} = \frac{\sin x}{1 - 2y}$$

Differentiation Ex 11.6 Q3

Here,

$$y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \ldots \text{to } \infty}}}$$
$$y = \sqrt{\log x + y}$$

Squaring both sides,

$$y^2 = \log x + y$$

Differentiating it with respect to x,

$$2y \frac{dy}{dx} = \frac{1}{x} + \frac{dy}{dx}$$
$$\frac{dy}{dx}(2y - 1) = \frac{1}{x}$$

Differentiation Ex 11.6 Q4

Here,

$$y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + \ldots \text{to } \infty}}}$$
$$y = \sqrt{\tan x + y}$$

Squaring both the sides,

$$y^2 = \tan x + y$$

Differentiating it with respect to x,

$$2y \frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$$

$$\frac{dy}{dx}(2y - 1) = \sec^2 x$$

$$\frac{dy}{dx} = \frac{\sec^2 x}{2y - 1}$$

Differentiation Ex 11.6 Q5

Here,

$$y = (\sin x)^{(\sin x)^{(\sin x)^{\cdots\infty}}}$$

$$\Rightarrow \quad y = (\sin x)^{y}$$

Taking log on both the sides,

$$\log y = \log(\sin x)^{y}$$
$$\log y = y\,(\log \sin x)$$

Differentiating it with respect to x, using product rule,

$$\frac{1}{y}\frac{dy}{dx} = y\frac{d}{dx}(\log \sin x) + \log \sin x \frac{dy}{dx}$$
$$\frac{1}{y}\frac{dy}{dx} = y\frac{1}{\sin x}\frac{d}{dx}(\sin x) + \log \sin x \frac{dy}{dx}$$
$$\frac{dy}{dx}\left(\frac{1}{y} - \log \sin x\right) = \frac{y}{\sin x}(\cot x)$$
$$\frac{dy}{dx}\left(\frac{1 - y\log \sin x}{y}\right) = y\cot x$$
$$\frac{dy}{dx} = \frac{y^{2}\cot x}{(1 - y\log \sin x)}$$

Differentiation Ex 11.6 Q6

Here,

$$y = (\tan x)^{(\tan x)^{(\tan x)^{\cdots\infty}}}$$
$$y = (\tan x)^{y}$$

Taking log on both the sides,

$$\log y = \log(\tan x)^{y}$$
$$\log y = y\log \tan x$$

Differentiating with respect to x using product rule and chain rule,

$$\frac{1}{y}\frac{dy}{dx} = y\frac{d}{dx}\log \tan x + \log \tan \frac{dy}{dx}$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{y}{\tan x}\frac{d}{dx}(\tan x) + \log \tan x\frac{dy}{dx}$$
$$\frac{dy}{dx}\left(\frac{1}{y} - \log \tan x\right) = \frac{y}{\tan x}\sec^{2} x$$
$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}} = \frac{y\sec^{2}\left(\frac{\pi}{4}\right)}{\tan\left(\frac{\pi}{4}\right)} * \frac{y}{1 - y\log \tan\left(\frac{\pi}{4}\right)}$$
$$\left(\frac{dy}{dx}\right)_{\frac{\pi}{4}} = \frac{y^{2}\left(\sqrt{2}\right)^{2}}{1(1 - y\log \tan 1)}$$
$$= \frac{2(1)^{2}}{(1-0)}$$

$$\left(\frac{dy}{dx}\right)_{\frac{\pi}{4}} = 2 \qquad \left\{\begin{array}{l}\text{since,}\\ (y)_{\frac{\pi}{4}} = \left(\tan\frac{\pi}{4}\right)^{\left(\tan\frac{\pi}{4}\right)^{\left(\tan\frac{\pi}{4}\right)^{\cdots\infty}}}\\ \Rightarrow y = (1)^{\infty}\\ \Rightarrow y = 1\end{array}\right\}$$

Differentiation Ex 11.6 Q7

Here,

$$y = e^{x^{e^x}} + x^{e^{e^x}} + e^{x^{x^e}}$$
$$y = u + v + w$$
$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} \qquad \text{---(i)}$$

Were $u = e^{x^{e^x}}, v = x^{e^{e^x}}, w = e^{x^{x^e}}$

Now, $u = e^{x^{e^x}} \qquad \text{---(ii)}$

Taking log on both the sides,

$$\log x = \log e^{x^{e^x}}$$
$$\log x = x^{e^x} \log e$$
$$\log x = x^{e^x} \qquad \text{---(iii)} \qquad \left\{\begin{matrix}\text{since } \log e - 1, \\ \log a^b = b \log a\end{matrix}\right\}$$

Taking log on both the sides,

$$\log\log x = \log x^{e^x}$$
$$\log\log x = e^x \log x$$

Differentiating it with respect to x,

$$\frac{1}{\log x}\frac{d}{dx}(\log x) = e^x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}\left(e^x\right)$$
$$\frac{1}{\log x}\frac{1}{4}\frac{du}{dx} = \frac{e^x}{x} + e^x \log x$$
$$\frac{du}{dx} = 4\log x\left[\frac{e^x}{x} + e^x \log x\right]$$
$$\frac{du}{dx} = e^{x^{e^x}} * x^{e^x}\left[\frac{e^x}{x} + e^x \log x\right] \qquad \text{---(A)}$$

Using equqtion (ii) and (iii)

Now

$$v = x^{e^{e^x}} \qquad \text{---(iv)}$$

Taking log on both the sides,

$$\log v = \log x^{e^{e^x}}$$
$$\log v = e^{e^x} \log x$$

Differentiating it with respect to x,

$$\frac{1}{v}\frac{dv}{dx} = e^{e^x}\frac{d}{dx}(\log x) + \log x \frac{d}{dx}\left(e^{e^x}\right)$$
$$\frac{1}{v}\frac{dv}{dx} = e^{e^x}\left(\frac{1}{x}\right) + \log x e^{e^x}\frac{d}{dx}\left(e^x\right)$$
$$\frac{dv}{dx} = v[e^{e^x}\left(\frac{1}{x}\right) + \log x e^{e^x} e^x]$$
$$\frac{dv}{dx} = x^{e^{e^x}} * e^{e^x}[\frac{1}{x} + e^x \log x] \qquad \text{---(B)}$$

$\left\{\text{sinx using equation}(4)\right\}$

Now, $w = e^{x^{x^e}} \qquad \text{---(v)}$

Taking log on both the sides,

$$\log w = \log e^{x^{x^e}}$$
$$\log w = x^{x^e} \log e$$
$$\log w = x^{x^e} \qquad \text{---(vi)}$$

Taking log on both the sides,

$$\log\log w = \log x^{x^e}$$
$$\log\log w = x^e \log x$$

Differentiating it with respect to x,

$$\frac{1}{\log w}\frac{d}{dx}(\log w) = x^{e}\frac{d}{dx}(\log x) + \log x\frac{d}{dx}\left(x^{e}\right)$$

$$\frac{1}{\log w}\left(\frac{1}{w}\right)\frac{dw}{dx} = x^{e}\left(\frac{1}{x}\right) + \log e x^{e-1}$$

$$\frac{dw}{dx} = w\log w[x^{e-1} + e\log x x^{e-1}]$$

$$\frac{dw}{dx} = e^{x^{x^{e}}} x^{x^{e}} x^{e-1}(1 + e\log x) \qquad \text{---(C)} \quad \{\text{Using equation } (v),(vi)\}$$

Using equation $(A),(B)$ and (C) in equation (i),

$$\frac{dy}{dx} = e^{x^{e^{x}}} x^{e^{x}}\left[\frac{e^{x}}{x} + e^{x}\log x\right] + x^{e^{e^{x}}} e^{e^{x}}\left[\frac{1}{x} + e^{x}\log x\right]$$
$$+ e^{x^{x^{e}}} x^{x^{e}} x^{e-1}(1 + e\log x)$$

Differentiation Ex 11.6 Q8

Here,

$$y = (\cos x)^{(\cos x)^{(\cos x)^{\cdots \infty}}}$$

$$y = (\cos x)^{y}$$

Taking log on both the sides,

$$\log y = \log(\cos x)^{y}$$

$$\log y = y\log(\cos x), \{\text{since } \log a^{b} = b\log a\}$$

Differentiating it with respect to x using product rule and chain rule,

$$\frac{1}{y}\frac{dy}{dx} = y\frac{d}{dx}\log(\cos x) + \log\cos x\frac{dy}{dx}$$

$$\frac{1}{y}\frac{dy}{dx} = y\left(\frac{1}{\cos x}\right)\frac{d}{dx}(\cos x) + \log\cos x\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(\frac{1}{y} - \log\cos x\right) = \frac{y}{\cos x}(-\sin x)$$

$$\frac{dy}{dx}\left(\frac{1 - y\log\cos x}{y}\right) = -y\tan x$$

$$\frac{dy}{dx} = -\frac{y^{2}\tan x}{(1 - y\log\cos x)}$$

Ex 11.7

Differentiation Ex 11.7 Q1

Given that $x = at^2$, $y = 2at$

So, $\frac{dx}{dt} = \frac{d}{dt}\left(at^2\right) = 2at$

$\frac{dy}{dt} = \frac{d}{dt}(2at) = 2a$

Therefore, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$

Differentiation Ex 11.7 Q2

Here,

$$x = a(\theta + \sin\theta)$$

Differentiating it with respect to θ,

$$\frac{dx}{d\theta} = a(1 + \cos\theta) \qquad \text{---(i)}$$

And,

$$y = a(1 - \cos\theta)$$

Differentiating it with respect to θ,

$$\frac{dy}{d\theta} = a(\theta + \sin\theta)$$

$$\frac{dy}{d\theta} = a\sin\theta \qquad \text{---(ii)}$$

Using equation (i) and (ii),

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$= \frac{a\sin\theta}{a(1 - \cos\theta)}$$

$$= \frac{2\frac{\sin\theta}{2}\frac{\cos\theta}{2}}{\frac{2\sin^2\theta}{2}}, \qquad \left\{\begin{array}{l}\text{Since, } 1 - \cos\theta = \frac{2\sin^{2\theta}}{2}, \\ 2\frac{\sin\theta}{2}\frac{\cos\theta}{2} = \sin\theta\end{array}\right\}$$

$$= \frac{dy}{dx} = \frac{\tan\theta}{2}$$

Differentiation Ex 11.7 Q3

Here $x = a\cos\theta$ and $y = b\sin\theta$

Then,

$$\frac{dx}{d\theta} = \frac{d}{d\theta}(a\cos\theta) = -a\sin\theta$$

$$\frac{dy}{d\theta} = \frac{d}{d\theta}(b\sin\theta) = b\cos\theta$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{b\cos\theta}{-a\sin\theta} = -\frac{b}{a}\cot\theta$$

Differentiation Ex 11.7 Q4

Here,

$$x = ae^{\theta}(\sin\theta - \cos\theta)$$

Differentiating it with respcet to θ,

$$\frac{dx}{d\theta} = a\left[e^{\theta}\frac{d}{d\theta}(\sin\theta - \cos\theta) + (\sin\theta - \cos\theta)\frac{d}{d\theta}(e^{\theta})\right]$$
$$= a\left[e^{\theta}(\cos\theta + \sin\theta) + (\sin\theta - \cos\theta)e^{\theta}\right]$$
$$\frac{dx}{d\theta} = a\left[2e^{\theta}\sin\theta\right] \qquad \text{---(i)}$$

And, $y = ae^{\theta}(\sin\theta + \cos\theta)$

Differentiating it with respect to θ,

$$\frac{dy}{d\theta} = a\left[e^{\theta}\frac{d}{d\theta}(\sin\theta + \cos\theta) + (\sin\theta + \cos\theta)\frac{d}{d\theta}(e^{\theta})\right]$$
$$= a\left[e^{\theta}(\cos\theta - \sin\theta) + (\sin\theta + \cos\theta)e^{\theta}\right]$$
$$\frac{dy}{d\theta} = a\left[2e^{\theta}\cos\theta\right] \qquad \text{---(ii)}$$

Dividing equation (ii) by equation (i),

$$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\left(2e^{\theta}\cos\theta\right)}{a\left(2e^{\theta}\sin\theta\right)}$$
$$\frac{dy}{dx} = \cot\theta$$

Differentiation Ex 11.7 Q5

Here $x = b\sin^2\theta$ and $y = a\cos^2\theta$

Then,

$$\frac{dx}{d\theta} = \frac{d}{d\theta}\left(b\sin^2\theta\right) = 2b\sin\theta\cos\theta$$
$$\frac{dy}{d\theta} = \frac{d}{d\theta}\left(a\cos^2\theta\right) = -2a\cos\theta\sin\theta$$
$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-2a\cos\theta\sin\theta}{2b\sin\theta\cos\theta} = -\frac{a}{b}$$

Differentiation Ex 11.7 Q6

Here $x = a(1-\cos\theta)$ and $y = a(\theta + \sin\theta)$

Then,

$$\frac{dx}{d\theta} = \frac{d}{d\theta}\left[a(1-\cos\theta)\right] = a(\sin\theta)$$
$$\frac{dy}{d\theta} = \frac{d}{d\theta}\left[a(\theta + \sin\theta)\right] = a(1+\cos\theta)$$
$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \left.\frac{a(1+\cos\theta)}{a(\sin\theta)}\right|_{\theta=\frac{\pi}{2}} = \frac{a(1+0)}{a} = 1$$

Differentiation Ex 11.7 Q7

Here,

$$x = \frac{e^t + e^{-t}}{2}$$

Differentiating it with respect to t,

$$\frac{dx}{dt} = \frac{1}{2}\left[\frac{d}{dt}\left(e^t\right) + \frac{d}{dt}\left(e^{-t}\right)\right]$$

$$= \frac{1}{2}\left[e^t + e^{-t}\frac{d}{dt}(-t)\right]$$

$$\frac{dx}{dt} = \frac{1}{2}\left(e^t - e^{-t}\right) = y \qquad \text{---(i)}$$

And, $\quad y = \dfrac{e^t - e^{-t}}{2}$

Differentiating it with respect to t,

$$\frac{dy}{dt} = \frac{1}{2}\left[\frac{d}{dt}\left(e^t\right) - \frac{d}{dt}e^{-t}\right]$$

$$= \frac{1}{2}\left[e^t - e^{-t}\frac{d}{dt}\left(e^{-t}\right)\right]$$

$$= \frac{1}{2}\left(e^t - e^{-t}(-1)\right)$$

$$\frac{dy}{dt} = \frac{1}{2}\left(e^t + e^{-t}\right) = x \qquad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{x}{y}$$

$$\frac{dy}{dt} = \frac{x}{y}$$

Differentiation Ex 11.7 Q8

Here,

$$x = \frac{3at}{1+t^2}$$

Differentiating it with respect to t using quotiont rule,

$$\frac{dx}{dt} = \left[\frac{\left(1+t^2\right)\frac{d}{dt}(3at) - 3at\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{\left(1+t^2\right)(3a) - 3at(2t)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{3a+3at^2-6at^2}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{3a-3at^2}{\left(1-t^2\right)^2}\right]$$

$$\frac{dx}{dt} = \frac{3a\left(1-t^2\right)}{\left(1+t^2\right)^2} \qquad \text{---(i)}$$

And, $\quad y = \frac{3at^2}{1+t^2}$

Differentiating it with respect to t using quotient rule,

$$\frac{dy}{dt} = \left[\frac{\left(1+t^2\right)\frac{d}{dt}\left(3at^2\right) - 3at^2\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}\right]$$

$$\frac{dy}{dt} = \left[\frac{\left(1+t^2\right)(6at) - \left(3at^2\right)(2t)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{6at+6at^3-6at^3}{\left(1+t^2\right)^2}\right]$$

$$\frac{dy}{dt} = \frac{6at}{\left(1+t^2\right)^2} \qquad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{6at}{\left(1+t^2\right)^2} \times \frac{\left(1+t^2\right)^2}{3a\left(1-t^2\right)}$$

$$\frac{dy}{dx} = \frac{2t}{1-t^2}$$

Differentiation Ex 11.7 Q9

The given equations are $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$

Then, $\frac{dx}{d\theta} = a\left[\frac{d}{d\theta}\cos\theta + \frac{d}{d\theta}(\theta\sin\theta)\right] = a\left[-\sin\theta + \theta\frac{d}{d\theta}(\sin\theta) + \sin\theta\frac{d}{d\theta}(\theta)\right]$

$$= a\left[-\sin\theta + \theta\cos\theta + \sin\theta\right] = a\theta\cos\theta$$

$$\frac{dy}{d\theta} = a\left[\frac{d}{d\theta}(\sin\theta) - \frac{d}{d\theta}(\theta\cos\theta)\right] = a\left[\cos\theta - \left\{\theta\frac{d}{d\theta}(\cos\theta) + \cos\theta\cdot\frac{d}{d\theta}(\theta)\right\}\right]$$

$$= a\left[\cos\theta + \theta\sin\theta - \cos\theta\right]$$

$$= a\theta\sin\theta$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta$$

Differentiation Ex 11.7 Q10

Here,

$$x = e^{\theta}\left(\theta + \frac{1}{\theta}\right)$$

Differentiating it with respect to θ using product rule,

$$\frac{dx}{d\theta} = e^{\theta}\frac{d}{d\theta}\left(\theta + \frac{1}{\theta}\right) + \left(\theta + \frac{1}{\theta}\right)\frac{d}{d\theta}\left(e^{\theta}\right)$$

$$= e^{\theta}\left(1 - \frac{1}{\theta^2}\right) + \left(\frac{\theta^2 + 1}{\theta}\right)e^{\theta}$$

$$= e^{\theta}\left(1 - \frac{1}{\theta^2} + \frac{\theta^2 + 1}{\theta}\right)$$

$$= e^{\theta}\left(\frac{\theta^2 - 1 + \theta^3 + \theta}{\theta^2}\right)$$

$$\frac{dx}{d\theta} = \frac{e^{\theta}\left(\theta^3 + \theta^2 + \theta - 1\right)}{\theta^2} \qquad \text{---(i)}$$

And, $y = e^{\theta}\left(\theta - \frac{1}{\theta}\right)$

Differentiating it with respect to θ using product rule and chain rule,

$$\frac{dy}{d\theta} = e^{-\theta}\frac{d}{d\theta}\left(\theta - \frac{1}{\theta}\right) + \left(\theta - \frac{1}{\theta}\right)\frac{d}{d\theta}\left(e^{-\theta}\right)$$

$$= e^{-\theta}\left(1 + \frac{1}{\theta^2}\right) + \left(\theta - \frac{1}{\theta}\right)e^{-\theta}\frac{d}{d\theta}(-\theta)$$

$$= e^{-\theta}\left(1 + \frac{1}{\theta^2}\right) + \left(\theta - \frac{1}{\theta}\right)e^{-\theta}(-1)$$

$$\frac{dy}{d\theta} = e^{-\theta}\left[1 + \frac{1}{\theta^2} - \theta + \frac{1}{\theta}\right]$$

$$= e^{-\theta}\left[\frac{\theta^2 + 1 - \theta^3 + \theta}{\theta^2}\right]$$

$$\frac{dy}{d\theta} = e^{-\theta}\left[\frac{-\theta^3 + \theta^2 + \theta + 1}{\theta^2}\right] \qquad \text{---(ii)}$$

Differentiation Ex 11.7 Q11

Here,

$$x = \frac{2t}{1+t^2}$$

Differentiating it with respect to t using quotient rule,

$$\frac{dy}{dx} = \left[\frac{\left(1+t^2\right)\frac{d}{dt}(2t) - 2t\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{\left(1+t^2\right)(2) - 2t(2t)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{2+2t^2-4t^2}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{2-2t^2}{\left(1+t^2\right)^2}\right]$$

$$\frac{dx}{dt} = \frac{2\left(1-t^2\right)}{\left(1+t^2\right)^2} \quad \text{---(i)}$$

And, $y = \frac{1-t^2}{1+t^2}$

Differentiating it with respect to t using quotient rule,

$$\frac{dy}{dt} = \left[\frac{\left(1+t^2\right)\frac{d}{dt}\left(1-t^2\right) - \left(1-t^2\right)\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{\left(1+t^2\right)(-2t) - \left(1-t^2\right)(2t)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{-2t-2t^3-2t+2t^3}{\left(1+t^2\right)^2}\right]$$

$$\frac{dy}{dt} = \left[\frac{-4t}{\left(1+t^2\right)^2}\right] \quad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-4t}{\left(1+t^2\right)^2} \times \frac{\left(1+t^2\right)^2}{2\left(1-t^2\right)}$$

$$= \frac{-2t}{1-t^2}$$

$$\frac{dy}{dx} = -\frac{x}{y} \qquad \left[\text{Sicne, } \frac{x}{y} = \frac{2t}{1+t^2} \times \frac{1+t^2}{1-t^2} = \frac{2t}{1-t^2}\right]$$

Differentiation Ex 11.7 Q12

Here,

$$x = \cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$$

Differentiating it with respect to t using chain rule,

$$\frac{dx}{dt} = \frac{-1}{\sqrt{1-\left(\frac{1}{1+t^2}\right)^2}}\frac{d}{dt}\left(\frac{1}{\sqrt{1+t^2}}\right)$$

$$= \frac{-1}{\sqrt{1-\frac{1}{\left(1+t^2\right)}}}\left\{\frac{-\ 1}{2\left(1+t^2\right)^{\frac{3}{2}}}\right\}\frac{d}{dt}\left(1+t^2\right)$$

$$= \frac{\left(1+t^2\right)^{\frac{1}{2}}}{\sqrt{1+t^2-1}} \times \frac{-\ 1}{2\left(1+t^2\right)^{\frac{3}{2}}}(2t)$$

$$= \frac{-\ t}{\sqrt{t^2} \times \left(1+t^2\right)}$$

$$\frac{dx}{dt} = \frac{-1}{1+t^2} \qquad \text{---(i)}$$

Now, $y = \sin^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$

Differentiating it with respect to t using chain rule,

$$\frac{dy}{dt} = \frac{1}{\sqrt{1-\frac{1}{\left(\sqrt{1+t^2}\right)^2}}} \times \frac{d}{dt}\left(\frac{1}{\sqrt{1+t^2}}\right)$$

$$= \frac{\left(1+t^2\right)^{\frac{1}{2}}}{\sqrt{1+t^2-1}} \times \left(\frac{-1}{2\left(1+t^2\right)^{\frac{3}{2}}}\right)\frac{d}{dt}\left(1+t^2\right)$$

$$= \frac{-1}{2\sqrt{t^2}\left(1+t^2\right)} \times (2t)$$

$$\frac{dy}{dt} = \frac{-1}{\left(1+t^2\right)} \qquad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = -\frac{1}{\left(1+t^2\right)} \times \frac{\left(1+t^2\right)}{-1}$$

$$\frac{dy}{dx} = 1$$

Differentiation Ex 11.7 Q13

Here,

$$x = \frac{1-t^2}{1+t^2}$$

Differentiating it with respect to t using quotient rule,

$$\frac{dx}{dt} = \left[\frac{\left(1+t^2\right)\frac{d}{dt}\left(1-t^2\right) - \left(1-t^2\right)\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{\left(1+t^2\right)(-2t) - \left(1-t^2\right)(2t)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{-2t - 2t^3 - 2t + 2t^3}{\left(1+t^2\right)^2}\right]$$

$$\frac{dx}{dt} = \left(\frac{-4t}{\left(1+t^2\right)^2}\right) \qquad \text{---(i)}$$

And, $y = \dfrac{2t}{1+t^2}$

Differentiating it with respect to t using quotient rule,

$$\frac{dy}{dt} = \left[\frac{\left(1+t^2\right)\frac{d}{dt}(2t) - (2t)\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}\right]$$

$$\frac{dy}{dt} = \left[\frac{\left(1+t^2\right)(2) - (2t)(2t)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{2 + 2t^2 - 4t^2}{\left(1+t^2\right)^2}\right]$$

$$\frac{dy}{dt} = \frac{2\left(1-t^2\right)}{\left(1+t^2\right)^2} \qquad \text{---(ii)}$$

Differentiation Ex 11.7 Q14

Here, x= 2cosθ – cos2θ

Differentiating it with respect to θ using chain rule,

$$\frac{dx}{d\theta} = 2(-\sin\theta) - (-\sin 2\theta)\frac{d}{d\theta}(2\theta)$$
$$= -2\sin\theta + 2\sin 2\theta$$
$$\frac{dx}{d\theta} = 2(\sin 2\theta - \sin\theta) \qquad \text{---(i)}$$

And, $y = 2\sin\theta - \sin 2\theta$

Differentiating it with respect to θ using chain rule,

$$\frac{dy}{d\theta} = 2\cos\theta - \cos 2\theta \frac{d}{d\theta}(2\theta)$$
$$= 2\cos\theta - \cos 2\theta(2)$$
$$= 2\cos\theta - 2\cos 2\theta$$
$$\frac{dy}{d\theta} = 2(\cos\theta - \cos 2\theta) \qquad \text{---(ii)}$$

Dividing equation (ii) by equation (i),

$$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{2(\cos\theta - \cos 2\theta)}{2(\sin 2\theta - \sin\theta)}$$
$$= \frac{\cos\theta - \cos 2\theta}{\sin 2\theta - \sin\theta}$$
$$\frac{dy}{dx} = \frac{-2\sin\left(\frac{\theta + 2\theta}{2}\right)\sin\left(\frac{\theta - 2\theta}{2}\right)}{2\cos\left(\frac{2\theta + \theta}{2}\right)\sin\left(\frac{2\theta - \theta}{2}\right)} \qquad \left[\begin{aligned} &\text{Since, } \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) \\ &\qquad \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\end{aligned}\right]$$
$$= \frac{-\sin\left(\frac{3\theta}{2}\right)\left(\sin\left(\frac{-\theta}{2}\right)\right)}{\cos\left(\frac{3\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)}$$

$$= \frac{-\sin\left(\frac{3\theta}{2}\right)\left(-\sin\frac{\theta}{2}\right)}{\cos\left(\frac{3\theta}{2}\right)\left(\sin\frac{\theta}{2}\right)}$$
$$= \frac{\sin\left(\frac{3\theta}{2}\right)}{\cos\left(\frac{3\theta}{2}\right)}$$

$$\frac{dy}{dx} = \tan\left(\frac{3\theta}{2}\right)$$

Differentiation Ex 11.7 Q15

Here,

$$x = e^{\cos 2t}$$

Differentiating it with respect to t using chain rule,

$$\begin{aligned}
\frac{dx}{dt} &= \frac{d}{dt}\left(e^{\cos 2t}\right) \\
&= e^{\cos 2t}\frac{d}{dt}(\cos 2t) \\
&= e^{\cos 2t}(-\sin 2t)\frac{d}{dt}(2t) \\
&= -\sin 2te^{\cos 2t}(2) \\
\frac{dx}{dt} &= -2\sin 2te^{\cos 2t} \qquad \text{---(i)}
\end{aligned}$$

And, $y = e^{\sin 2t}$

Differentiating it with respect to t using chain rule,

$$\begin{aligned}
\frac{dy}{dt} &= \frac{d}{dt}\left(e^{\sin 2t}\right) \\
&= e^{\sin 2t}\frac{d}{dt}(\sin 2t) \\
&= e^{\sin 2t}(\cos 2t)\frac{d}{dt}(2t) \\
&= e^{\sin 2t}(\cos 2t)(2) \\
\frac{dy}{dt} &= 2\cos 2te^{\sin 2t} \qquad \text{---(ii)}
\end{aligned}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2\cos 2te^{\sin 2t}}{-2\sin 2te^{\cos 2t}}$$

$$\frac{dy}{dx} = -\frac{y\log x}{x\log y} \qquad \left[\begin{aligned}\text{Since, } x = e^{\cos 2t} &\Rightarrow \log x = \cos 2t \\ y = e^{\sin 2t} &\Rightarrow \log y = \sin 2t\end{aligned}\right]$$

Differentiation Ex 11.7 Q16

Here,

$$x = \cos t$$

Differentiating it with respect to t,

$$\frac{dx}{dt} = \frac{d}{dt}(\cos t)$$

$$\frac{dx}{dt} = -\sin t \qquad \text{---(i)}$$

and, $y = \sin t$

Differentiating it with respect to t,

$$\frac{dy}{dt} = \frac{d}{dt}(\sin t)$$

$$\frac{dy}{dt} = \cos t \qquad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos t}{-\sin t}$$

$$\frac{dy}{dx} = -\cot t$$

$$\left(\frac{dy}{dx}\right) = -\cot\left(\frac{2\pi}{3}\right)$$

$$= -\cot\left(\pi - \frac{\pi}{3}\right)$$

$$= -\left[-\cot\left(\frac{\pi}{3}\right)\right]$$

$$= \cot\left(\frac{\pi}{3}\right)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{3}}$$

Differentiation Ex 11.7 Q17

Here,

$$x = a\left(t + \frac{1}{t}\right)$$

Differentiating it with respect to t,

$$\frac{dx}{dt} = a\frac{d}{dt}\left(t + \frac{1}{t}\right)$$

$$= a\left(1 - \frac{1}{t^2}\right)$$

$$\frac{dx}{dt} = a\left(\frac{t^2 - 1}{t^2}\right) \qquad \text{---(i)}$$

And, $y = a\left(t - \frac{1}{t}\right)$

Differentiating it with respect to t,

$$\frac{dy}{dt} = a\frac{d}{dt}\left(t - \frac{1}{t}\right)$$

$$= a\left(1 + \frac{1}{t^2}\right)$$

$$\frac{dy}{dt} = a\left(\frac{t^2 + 1}{t^2}\right) \qquad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = a\frac{\left(t^2 + 1\right)}{t^2} \times \frac{t^2}{a\left(t^2 - 1\right)}$$

$$\frac{dy}{dx} = \frac{t^2 + 1}{t^2 - 1}$$

$$\frac{dy}{dx} = \frac{x}{y} \qquad \left[\text{Since, } \frac{x}{y} = \frac{a\left(t^2 + 1\right)}{t} \times \frac{t}{a\left(t^2 - 1\right)} = \left(\frac{t^2 + 1}{t^2 - 1}\right)\right]$$

Differentiation Ex 11.7 Q18

Here,

$$x = \sin^{-1}\left(\frac{2t}{1+t^2}\right)$$

Put $\quad t = \tan\theta$

$$x = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$= \sin^{\{1\}}(\sin 2\theta)$$

$$= 2\theta \qquad \left[\text{Since, } \sin 2x = \frac{2\tan x}{1+\tan^2 x}\right]$$

$$x = 2\left(\tan^{-1} t\right) \qquad [\text{Since, } t = \sin\theta]$$

Differentiating it with respect to t,

$$\frac{dx}{dt} = \frac{2}{1+t^2} \qquad \text{---(i)}$$

Now,

$$y = \tan^{-1}\left(\frac{2t}{1-t^2}\right)$$

Put $\quad t = \tan\theta$

$$y = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$$

$$= \tan^{-1}(\tan 2\theta) \qquad \left[\text{Sicne, } \tan 2x = \frac{2\tan x}{1-\tan^2 x}\right]$$

$$= 2\theta$$

$$y = 2\tan^{-1} t \qquad [\text{Since, } t = \tan\theta]$$

Differentiating it with respect to t,

$$\frac{dy}{dt} = \frac{2}{1+t^2} \qquad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2}{1+t^2} \times \frac{1+t^2}{2}$$

$$\frac{dy}{dx} = 1$$

Differentiation Ex 11.7 Q19

The given equations are $x = \dfrac{\sin^3 t}{\sqrt{\cos 2t}}$ and $y = \dfrac{\cos^3 t}{\sqrt{\cos 2t}}$

$$\text{Then, } \frac{dx}{dt} = \frac{d}{dt}\left[\frac{\sin^3 t}{\sqrt{\cos 2t}}\right]$$

$$= \frac{\sqrt{\cos 2t} \cdot \frac{d}{dt}\left(\sin^3 t\right) - \sin^3 t \cdot \frac{d}{dt}\sqrt{\cos 2t}}{\cos 2t}$$

$$= \frac{\sqrt{\cos 2t} \cdot 3\sin^2 t \cdot \frac{d}{dt}(\sin t) - \sin^3 t \times \frac{1}{2\sqrt{\cos 2t}} \cdot \frac{d}{dt}(\cos 2t)}{\cos 2t}$$

$$= \frac{3\sqrt{\cos 2t} \cdot \sin^2 t\cos t - \frac{\sin^3 t}{2\sqrt{\cos 2t}} \cdot (-2\sin 2t)}{\cos 2t}$$

$$= \frac{3\cos 2t\sin^2 t\cos t + \sin^3 t\sin 2t}{\cos 2t\sqrt{\cos 2t}}$$

$$\frac{dy}{dt}=\frac{d}{dt}\left[\frac{\cos^3 t}{\sqrt{\cos 2t}}\right]$$

$$=\frac{\sqrt{\cos 2t}.\frac{d}{dt}\left(\cos^3 t\right)-\cos^3 t\cdot\frac{d}{dt}\left(\sqrt{\cos 2t}\right)}{\cos 2t}$$

$$=\frac{\sqrt{\cos 2t}.3\cos^2 t\cdot\frac{d}{dt}\left(\cos t\right)-\cos^3 t.\frac{1}{2\sqrt{\cos 2t}}\cdot\frac{d}{dt}\left(\cos 2t\right)}{\cos 2t}$$

$$=\frac{3\sqrt{\cos 2t}.\cos^2 t\left(-\sin t\right)-\cos^3 t\cdot\frac{1}{2\sqrt{\cos 2t}}.\left(-2\sin 2t\right)}{\cos 2t}$$

$$=\frac{-3\cos 2t.\cos^2 t.\sin t+\cos^3 t\sin 2t}{\cos 2t\cdot\sqrt{\cos 2t}}$$

$$\therefore\frac{dy}{dx}=\frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}=\frac{-3\cos 2t.\cos^2 t.\sin t+\cos^3 t\sin 2t}{3\cos 2t\sin^2 t\cos t+\sin^3 t\sin 2t}$$

$$=\frac{-3\cos 2t.\cos^2 t.\sin t+\cos^3 t\left(2\sin t\cos t\right)}{3\cos 2t\sin^2 t\cos t+\sin^3 t\left(2\sin t\cos t\right)}$$

$$=\frac{\sin t\cos t\left[-3\cos 2t.\cos t+2\cos^3 t\right]}{\sin t\cos t\left[3\cos 2t\sin t+2\sin^3 t\right]}$$

$$=\frac{\left[-3\left(2\cos^2 t-1\right)\cos t+2\cos^3 t\right]}{\left[3\left(1-2\sin^2 t\right)\sin t+2\sin^3 t\right]}\qquad\left[\begin{array}{l}\cos 2t=\left(2\cos^2 t-1\right),\\ \cos 2t=\left(1-2\sin^2 t\right)\end{array}\right]$$

$$=\frac{-4\cos^3 t+3\cos t}{3\sin t-4\sin^3 t}$$

$$=\frac{-\cos 3t}{\sin 3t}\qquad\left[\begin{array}{l}\cos 3t=4\cos^3 t-3\cos t,\\ \sin 3t=3\sin t-4\sin^3 t\end{array}\right]$$

$$=-\cot 3t$$

Differentiation Ex 11.7 Q20

Here,

$$x = \left(t + \frac{1}{t}\right)^{a}$$

Differentiating it with respect to t using chain rule,

$$\frac{dx}{dt} = \frac{d}{dt}\left(\left(t + \frac{1}{t}\right)^{a}\right)$$

$$= a\left(t + \frac{1}{t}\right)^{a-1} \frac{d}{dt}\left(t + \frac{1}{t}\right)$$

$$\frac{dx}{dt} = a\left(t + \frac{1}{t}\right)^{1-1}\left(1 - \frac{1}{t^2}\right) \qquad \text{---(i)}$$

And, $y = a^{\left(t+\frac{1}{t}\right)}$

Differentiating it with respect to t using chain rule,

$$\frac{dy}{dt} = \frac{d}{dt}\left(a^{\left(t+\frac{1}{t}\right)}\right)$$

$$= a^{\left(t+\frac{1}{t}\right)} \times \log a \frac{d}{dt}\left(t + \frac{1}{t}\right)$$

$$\frac{dy}{dt} = a^{\left(t+\frac{1}{t}\right)} \times \log a \left(1 - \frac{1}{t^2}\right) \qquad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a^{\left(t+\frac{1}{t}\right)} \times \log a \left(1 - \frac{1}{t^2}\right)}{a\left(t + \frac{1}{t}\right)^{a-1}\left(1 - \frac{1}{t^2}\right)}$$

$$\frac{dy}{dx} = \frac{a^{\left(t+\frac{1}{t}\right)} \times \log a}{a\left(t + \frac{1}{t}\right)^{a-1}}$$

Differentiation Ex 11.7 Q21

Here,

$$x = a\left(\frac{1+t^2}{1-t^2}\right)$$

Differentiating it with respect to t using chain rule,

$$\frac{dx}{dt} = a\left[\frac{\left(1+t^2\right)\frac{d}{dt}\left(1+t^2\right)-\left(1+t^2\right)\frac{d}{dt}\left(1-t^2\right)}{\left(1-t^2\right)^2}\right]$$

$$= a\left[\frac{\left(1-t^2\right)(2t)-\left(1+t^2\right)(-2t)}{\left(1-t^2\right)^2}\right]$$

$$= a\left[\frac{2t-2t^2+2t+2t^3}{\left(1-t^2\right)^2}\right]$$

$$\frac{dy}{dt} = \frac{4at}{\left(1-t^2\right)^2} \qquad \text{---(i)}$$

And, $y = \dfrac{2t}{1-t^2}$

Differentiating it with respect to t using quotient rule,

$$\frac{dy}{dt} = 2\left[\frac{\left(1-t^2\right)\frac{d}{dt}(t)-t\frac{d}{dt}\left(1-t^2\right)}{\left(1-t^2\right)^2}\right]$$

$$= 2\left[\frac{\left(1-t^2\right)(1)-t(-2t)}{\left(1-t^2\right)^2}\right]$$

$$= 2\left[\frac{1-t^2+2t^2}{\left(1-t^2\right)^2}\right]$$

$$\frac{dy}{dt} = \frac{2\left(1+t^2\right)}{\left(1-t^2\right)} \qquad \text{---(ii)}$$

Differentiation Ex 11.7 Q22

It is given that, $y = 12(1-\cos t), x = 10(t-\sin t)$

$$\therefore \frac{dx}{dt} = \frac{d}{dt}\left[10(t-\sin t)\right] = 10\cdot\frac{d}{dt}(t-\sin t) = 10(1-\cos t)$$

$$\frac{dy}{dt} = \frac{d}{dt}\left[12(1-\cos t)\right] = 12\cdot\frac{d}{dt}(1-\cos t) = 12\cdot\left[0-(-\sin t)\right] = 12\sin t$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{12\sin t}{10(1-\cos t)} = \frac{12\cdot 2\sin\frac{t}{2}\cdot\cos\frac{t}{2}}{10\cdot 2\sin^2\frac{t}{2}} = \frac{6}{5}\cot\frac{t}{2}$$

Differentiation Ex 11.7 Q23

Here $x = a(\theta-\sin\theta)$ and $y = a(1+\cos\theta)$

Then,

$$\frac{dx}{d\theta} = \frac{d}{d\theta}\left[a(\theta-\sin\theta)\right] = a(1-\cos\theta)$$

$$\frac{dy}{d\theta} = \frac{d}{d\theta}\left[a(1+\cos\theta)\right] = a(-\sin\theta)$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \left.\frac{-a\sin\theta}{a(1-\cos\theta)}\right|_{\theta=\frac{\pi}{3}} = -\frac{\sin\frac{\pi}{3}}{1-\cos\frac{\pi}{3}} = \frac{\frac{\sqrt{3}}{2}}{1-\frac{1}{2}} = -\sqrt{3}$$

Differentiation Ex 11.7 Q24

Consider the given functions,

$x = a \sin 2t\ (1 + \cos 2t)$ and $y = b \cos 2t\ (1-\cos 2t)$

Rewriting the above function, we have,

$x = a \sin 2t + \frac{a}{2}\sin 4t$

Differentiating the above function w.r.t. 't', we have,

$\frac{dx}{dt} = 2a\cos 2t + 2a\cos 4t \ldots (1)$

$y = b \cos 2t\ (1-\cos 2t)$

$y = b \cos 2t - b\cos^2 2t$

$\frac{dy}{dt} = -2b\sin 2t + 2b\cos 2t \sin 2t = -2b\sin 2t + b\sin 4t \ldots (2)$

From (1) and (2),

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2b\sin 2t + b\sin 4t}{2a\cos 2t + 2a\cos 4t}$$

$$\therefore \left.\frac{dy}{dx}\right|_{\pi/4} = \left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right|_{t=\pi/4} = \frac{-2b}{-2a} = \frac{b}{a}$$

Differentiation Ex 11.7 Q25

Consider the given functions,

$x = \cos t\left(3 - 2\cos^2 t\right)$

$x = 3\cos t - 2\cos^3 t$

$\frac{dx}{dt} = -3\sin t + 6\cos^2 t \sin t \ldots .(1)$

$y = \sin t\ \left(3 - 2\sin^2 t\right)$

$y = 3\sin t - 2\sin^3 t$

$\frac{dy}{dt} = 3\cos t - 6\sin^2 t\cos t \ldots .(2)$

$$\frac{dy}{dx} = \left(\frac{dy}{dt}\right)/\left(\frac{dx}{dt}\right) \ldots .\text{[From equations (1) and (2)]}$$

$$= \frac{3\cos t - 6\sin^2 t\cos t}{-3\sin t + 6\cos^2 t\sin t}$$

$$= \frac{3\cos t\left(1 - 2\sin^2 t\right)}{3\sin t\left(2\cos^2 t - 1\right)}$$

$$= \cot t\frac{\left(1 - 2\left(1 - \cos^2 t\right)\right)}{\left(2\cos^2 t - 1\right)}$$

$$= \cot t$$

$$\left.\frac{dy}{dx}\right|_{\pi/4} = \cot\frac{\pi}{4} = 1$$

Differentiation Ex 11.7 Q26

$x = \frac{1 + \log t}{t^2}, y = \frac{3 + 2\log t}{t}$

$$\frac{dx}{dt} = \frac{t^2\left(\frac{1}{t}\right) - (1 + \log t)(2t)}{t^4} = \frac{t - 2t - 2t\log t}{t^4} = \frac{-2\log t - 1}{t^3}$$

$$\frac{dy}{dt} = \frac{t\left(\frac{2}{t}\right) - (3 + 2\log t)(1)}{t^2} = \frac{2 - 3 - 2\log t}{t^2} = \frac{-2\log t - 1}{t^2}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{-2\log t - 1}{t^2}}{\frac{-2\log t - 1}{t^3}} = t$$

Differentiation Ex 11.7 Q27

$$x = 3\sin t - \sin 3t,\ y = 3\cos t - \cos 3t$$

$$\frac{dx}{dt} = 3\cos t - 3\cos 3t$$

$$\frac{dy}{dt} = -3\sin t + 3\sin 3t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-3\sin t + 3\sin 3t}{3\cos t - 3\cos 3t}$$

When $t = \frac{\pi}{3}$

$$\frac{dy}{dx} = \frac{-3\sin\left(\frac{\pi}{3}\right) + 3\sin(\pi)}{3\cos\left(\frac{\pi}{3}\right) - 3\cos(\pi)} = \frac{-3\times\frac{\sqrt{3}}{2} + 0}{3\times\frac{1}{2} - 3(-1)} = -\frac{1}{\sqrt{3}}$$

Differentiation Ex 11.7 Q28

$$\sin x = \frac{2t}{1+t^2},\ \tan y = \frac{2t}{1-t^2}$$

$$\Rightarrow x = \sin^{-1}\left(\frac{2t}{1+t^2}\right) \text{ and } y = \tan^{-1}\left(\frac{2t}{1-t^2}\right)$$

$$\frac{dx}{dt} = \frac{1}{\sqrt{1-\left(\frac{2t}{1+t^2}\right)^2}} \times \frac{2\left(1+t^2\right) - (2t)(2t)}{\left(1+t^2\right)^2}$$

$$\frac{dx}{dt} = \frac{2}{\left(1+t^2\right)}$$

$$\frac{dy}{dt} = \frac{1}{\left(\frac{2t}{1-t^2}\right)^2 + 1} \times \frac{2\left(1-t^2\right) - (2t)(-2t)}{\left(1-t^2\right)^2}$$

$$\frac{dy}{dt} = \frac{2}{\left(1+t^2\right)}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{2}{\left(1+t^2\right)}}{\frac{2}{\left(1+t^2\right)}} = 1$$

Ex 11.8

Differentiation Ex 11.8 Q1

Let $u = x^2, \; v = x^3$

Differentiating u with respect to x,

$$\frac{du}{dx} = 2x \qquad ---(i)$$

Differentiating v with respect to x,

$$\frac{dv}{dx} = 3x^2 \qquad ---(ii)$$

Dividing equation (i) by (ii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2x}{3x^2}$$

$$\frac{du}{dv} = \frac{2}{3x}$$

Differentiation Ex 11.8 Q2

Let $u = \log\left(1 + x^2\right)$

Differentiating it with respect to x using chain rule,

$$\frac{du}{dx} = \frac{1}{\left(1+x^2\right)} \frac{d}{dx}\left(1+x^2\right)$$

$$= \frac{1}{\left(1+x^2\right)}(2x)$$

$$\frac{du}{dx} = \frac{2x}{\left(1+x^2\right)} \qquad ---(i)$$

Let $v = \tan^{-1} x$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{1}{1+x^2} \qquad ---(ii)$$

Dividing equation (i) by (ii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2x}{\left(1+x^2\right)} \times \frac{\left(1+x^2\right)}{1}$$

$$\frac{du}{dv} = 2x$$

Differentiation Ex 11.8 Q3

Let $\quad u = (\log x)^x$

Taking log on both the sides,

$$\log u = \log(\log x)^x$$
$$\log u = x\log(\log x) \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating it with respect to x using chain rule, product rule,

$$\frac{1}{u}\frac{du}{dx} = x\frac{d}{dx}\log(\log x) + \log(\log x)\frac{d}{dx}(x)$$
$$\frac{1}{u}\frac{du}{dx} = x\left(\frac{1}{\log x}\right)\frac{d}{dx}(\log x) + \log\log x\,(1)$$
$$\frac{du}{dx} = u\left[\frac{x}{\log x}\left(\frac{1}{x}\right) + \log\log x\right]$$
$$\frac{du}{dx} = (\log x)^x\left[\frac{1}{\log x} + \log\log x\right] \qquad \text{---(i)}$$

Again, let $\quad v = \log x$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{1}{x} \qquad \text{---(ii)}$$

Dividing equation (i) by (ii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{(\log x)^x\left[\frac{1}{\log x} + \log\log x\right]}{\frac{1}{x}}$$
$$\frac{du}{dv} = \frac{(\log x)^x\left[\frac{1 + \log x \times \log\log x}{\log x}\right]}{\frac{1}{x}}$$

$$\frac{du}{dv} = (\log x)^{x-1}\,(1 + \log x \times \log\log x) \times x$$

Differentiation Ex 11.8 Q4(i)

Let $\quad u = \sin^{-1}\sqrt{1-x^2}$

Put $\quad x = \cos\theta$, so,

$$u = \sin^{-1}\sqrt{1-\cos^2\theta}$$

$$u = \sin^{-1}(\sin\theta) \qquad \text{---(i)}$$

And, $\quad v = \cos^{-1}x \qquad$ ---(ii)

Now, $x \in (0,1)$

$\Rightarrow \quad \cos\theta \in (0,1)$

$\Rightarrow \quad \theta \in \left(0, \frac{\pi}{2}\right)$

So, from equation (i),

$$u = \theta \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$$

$$u = \cos^{-1}x \qquad [\text{Since, } \cos\theta = x]$$

Differentiating it with respect to x,

$$\frac{du}{dx} = \frac{-1}{\sqrt{1-x^2}} \qquad \text{---(ii)}$$

From equation (ii),

$$v = \cos^{-1}x$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{-1}{\sqrt{1-x^2}} \qquad \text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{-1}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{-1}$$

$$\frac{du}{dv} = 1$$

Differentiation Ex 11.8 Q4(ii)

Let $\quad u = \sin^{-1}\sqrt{1-x^2}$

Put $\quad x = \cos\theta$, so,

$$u = \sin^{-1}\sqrt{1-\cos^2\theta}$$

$$u = \sin^{-1}(\sin\theta) \qquad \text{---(i)}$$

And, $\quad v = \cos^{-1}x \qquad \text{---(ii)}$

Here,

$$x \in (-1, 0)$$

$$\Rightarrow \quad \cos\theta \in (-,10)$$

$$\Rightarrow \quad \theta \in \left(\frac{\pi}{2}, \pi\right)$$

So, from equation (i),

$$u = \pi - \theta \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \pi - \theta, \theta \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)\right]$$

$$u = \pi - \cos^{-1}x \qquad [\text{Since, } x = \cos\theta]$$

Differentiating it with respect to x,

$$\frac{du}{dx} = 0 - \left(\frac{-1}{\sqrt{1-x^2}}\right)$$

$$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}} \qquad \text{---(v)}$$

And, from equation (ii),

$$v = \cos^{-1}x$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{-1}{\sqrt{1-x^2}} \qquad \text{---(vi)}$$

Dividing equation (v) by (vi)

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{1}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{-1}$$

$$\frac{du}{dv} = -1$$

Differentiation Ex 11.8 Q5(i)

Let $\quad u = \sin^{-1}\left(4x\sqrt{1-4x^2}\right)$

Put $\quad 2x = \cos\theta$, so

$$u = \sin^{-1}\left(2 \times \cos\theta\sqrt{1-\cos^2\theta}\right)$$

$$= \sin^{-1}(2\cos\theta\sin\theta)$$

$$u = \sin^{-1}(\sin 2\theta) \qquad \text{---(i)}$$

Let $\quad v = \sqrt{1-4x^2} \qquad$ ---(ii)

Here,

$$x \in \left(-\frac{1}{2}, -\frac{1}{2\sqrt{2}}\right)$$

$$\Rightarrow \quad 2x \in \left(-1, -\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \quad \theta \in \left(\frac{3\pi}{4}, \pi\right)$$

So, from equation (i),

$$u = \pi - 2\theta \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \pi - \theta \text{ if } \theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]\right]$$

$$u = \pi - 2\cos^{-1}(2x) \qquad [\text{Since, } 2x = \cos\theta]$$

Differentiating it with respect to x using chain rule,

$$\frac{du}{dx} = 0 - 2\left(\frac{-1}{\sqrt{1-(2x)^2}}\right)\frac{d}{dx}(2x)$$

$$= \frac{2}{\sqrt{1-4x^2}}(2)$$

$$\frac{du}{dx} = \frac{4}{\sqrt{1-4x^2}} \qquad \text{---(vi)}$$

From equation (iv)

$$\frac{dv}{dx} = \frac{-4x}{\sqrt{1-4x^2}}$$

but, $\quad x \in \left(-\frac{1}{2}, -\frac{1}{2\sqrt{2}}\right)$

$$\frac{dv}{dx} = \frac{-4(-x)}{\sqrt{1-4(-x)^2}}$$

$$\frac{dv}{dx} = \frac{4x}{\sqrt{1-4x^2}} \qquad \text{---(vii)}$$

Differentiating equation (ii) with respect to x using chain rule,

$$\frac{dv}{dx} = \frac{1}{2\sqrt{1-4x^2}}\frac{d}{dx}\left(1-4x^2\right)$$

$$= \frac{1}{2\sqrt{1-4x^2}}(-8x)$$

$$\frac{dv}{dx} = \frac{-4x}{\sqrt{1-4x^2}} \qquad \text{---(iv)}$$

Divide equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{4}{\sqrt{1-4x^2}} \times \frac{\sqrt{1-4x^2}}{-4x}$$

$$\frac{du}{dv} = -\frac{1}{x}$$

Differentiation Ex 11.8 Q5(ii)

Let $\quad u = \sin^{-1}\left(4x\sqrt{1-4x^2}\right)$

Put $\quad 2x = \cos\theta$, so

$$u = \sin^{-1}\left(2 \times \cos\theta\sqrt{1-\cos^2\theta}\right)$$
$$= \sin^{-1}(2\cos\theta\sin\theta)$$
$$u = \sin^{-1}(\sin 2\theta) \qquad \text{---(i)}$$

Let $\quad v = \sqrt{1-4x^2} \qquad$ ---(ii)

Here,

$$x \in \left(\frac{1}{2\sqrt{2}}, \frac{1}{2}\right)$$
$$\Rightarrow \quad 2x \in \left(\frac{1}{2\sqrt{2}}, 1\right)$$
$$\Rightarrow \quad \cos\theta \in \left(\frac{1}{\sqrt{2}}, 1\right)$$
$$\Rightarrow \quad \theta \in \left(0, \frac{\pi}{4}\right)$$

So, from equation (i)

$$u = 2\theta \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$
$$u = 2\cos^{-1}(2x) \qquad [\text{Since, } 2x = \cos\theta]$$

Differentiate it with respect to x using chain rule,

$$\frac{du}{dx} = 2\left(\frac{-1}{\sqrt{1-(2x)^2}}\right)\frac{d}{dx}(2x)$$
$$= \left(\frac{-2}{\sqrt{1-4x^2}}(2)\right)$$
$$\frac{du}{dx} = \frac{-4}{\sqrt{1-4x^2}} \qquad \text{---(v)}$$

Dividing equation (v) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{-4}{\sqrt{1-4x^2}} \times \frac{\sqrt{1-4x^2}}{-4x}$$

$$\frac{du}{dv} = \frac{1}{x}$$

Differentiation Ex 11.8 Q5(iii)

Let $\quad u = \sin^{-1}\left(4x\sqrt{1-4x^2}\right)$

Put $\quad 2x = \cos\theta$, so

$$u = \sin^{-1}\left(2\times\cos\theta\sqrt{1-\cos^2\theta}\right)$$

$$= \sin^{-1}\left(2\cos\theta\sin\theta\right)$$

$$u = \sin^{-1}\left(\sin 2\theta\right) \qquad \text{---(i)}$$

Let $\quad v = \sqrt{1-4x^2} \qquad$ ---(ii)

Here,

$$x \in \left(-\frac{1}{2}, -\frac{1}{2\sqrt{2}}\right)$$

$$\Rightarrow \quad 2x \in \left(-1, -\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \quad \theta \in \left(\frac{3\pi}{4}, \pi\right)$$

So, from equation (i),

$$u = \pi - 2\theta \qquad \left[\text{Since, } \sin^{-1}\left(\sin\theta\right) = \pi - \theta \text{ if } \theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]\right]$$

$$u = \pi - 2\cos^{-1}\left(2x\right) \qquad \left[\text{Since, } 2x = \cos\theta\right]$$

Differentiating it with respect to x using chain rule,

$$\frac{du}{dx} = 0 - 2\left(\frac{-1}{\sqrt{1-(2x)^2}}\right)\frac{d}{dx}(2x)$$

$$= \frac{2}{\sqrt{1-4x^2}}(2)$$

$$\frac{du}{dx} = \frac{4}{\sqrt{1-4x^2}} \qquad \text{---(vi)}$$

From equation (iv)

$$\frac{dv}{dx} = \frac{-4x}{\sqrt{1-4x^2}}$$

but, $\quad x \in \left(-\frac{1}{2}, -\frac{1}{2\sqrt{2}}\right)$

$$\frac{dv}{dx} = \frac{-4(-x)}{\sqrt{1-4(-x)^2}}$$

$$\frac{dv}{dx} = \frac{4x}{\sqrt{1-4x^2}} \qquad \text{---(vii)}$$

Dividing equation (vi) by (vii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{4}{\sqrt{1-4x^2}} \times \frac{\sqrt{1-4x^2}}{4x}$$

$$\frac{du}{dv} = \frac{1}{x}$$

Differentiation Ex 11.8 Q6

Let $\quad u = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

Put $\quad x = \tan\theta$, so

$$u = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{\sin\theta}{2}}{\frac{\cos\theta}{2}}\right)$$

$$u = \tan^{-1}\left(\frac{\tan\theta}{2}\right) \qquad \text{---(i)}$$

And,

Let $v = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$$= \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$v = \sin^{-1}(\sin 2\theta) \qquad \text{---(ii)}$$

Here,

$-1 < x < 1$

$\Rightarrow \quad -1 < \tan\theta < 1$

$\Rightarrow \quad -\frac{\pi}{4} < \theta < \frac{\pi}{4} \qquad$ ---(A)

So, from equation (i),

$$u = \frac{\theta}{2} \qquad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$u = \frac{1}{2}\tan^{-1}x \qquad [\text{Since, } x = \tan\theta]$$

Differentiating it with respect to x,

$$\frac{du}{dx} = \frac{1}{2}\left(\frac{1}{1+x^2}\right)$$

$$\frac{du}{dx} = \frac{1}{2\left(1+x^2\right)} \qquad \text{---(iii)}$$

Now, from equation (ii) and (A)

$$v = 2\theta \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$v = 2\tan^{-1}x \qquad [\text{Since, } x = \tan\theta]$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = 2\left(\frac{1}{1+x^2}\right) \qquad \text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{1}{2\left(1+x^2\right)} \times \frac{\left(1+x^2\right)}{2}$$

$$\frac{du}{dv} = \frac{1}{4}$$

Differentiation Ex 11.8 Q7(i)

Let $\quad u = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

Put $\quad x = \sin\theta$, so

$$u = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$$
$$= \sin^{-1}\left(2\sin\theta\cos\theta\right)$$
$$u = \sin^{-1}\left(\sin 2\theta\right) \qquad \text{---(i)}$$

And,

Let $\quad v = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

$$= \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right)$$
$$= \sec^{-1}\left(\frac{1}{\cos\theta}\right)$$
$$= \sec^{-1}\left(\sec\theta\right)$$
$$= \cos^{-1}\left(\frac{1}{\frac{1}{\cos\theta}}\right) \qquad \left[\text{Since, } \sec^{-1}x = \cos^{-1}\left(\frac{1}{x}\right)\right]$$
$$v = \cos^{-1}\left(\cos\theta\right) \qquad \text{---(ii)}$$

Here,

$$x \in \left(0, \frac{1}{\sqrt{2}}\right)$$
$$\Rightarrow \quad \sin\theta \in \left(0, \frac{1}{\sqrt{2}}\right)$$
$$\Rightarrow \quad \theta \in \left(0, \frac{\pi}{4}\right)$$

So, from equation (i),

$$u = 2\theta \qquad \left[\text{Since, } \sin^{-1}\left(\sin\theta\right) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

Let $\quad u = 2\sin^{-1}x \qquad [\text{Since, } x = \sin\theta]$

Differentiating it with respect to x,

$$\frac{du}{dx} = 2\left(\frac{1}{\sqrt{1-x^2}}\right)$$
$$\frac{du}{dx} = \frac{2}{\sqrt{1-x^2}} \qquad \text{---(iii)}$$

And, from equation (ii),

$$v = \theta \qquad \left[\text{Since, } \cos^{-1}\left(\cos\theta\right) = \theta, \text{ if } \theta \in [0, \pi]\right]$$
$$v = \sin^{-1}x \qquad [\text{Since, } x = \sin\theta]$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}} \qquad \text{---(iv)}$$

Dividing equation (iii) by (iv),3

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{1}$$

$$\frac{du}{dv} = 2$$

Differentiation Ex 11.8 Q7(ii)

Let $u = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

Put $x = \sin\theta$, so

$$u = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$$

$$= \sin^{-1}(2\sin\theta\cos\theta)$$

$$u = \sin^{-1}(\sin 2\theta) \qquad \text{---(i)}$$

And,

Let $v = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

$$= \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right)$$

$$= \sec^{-1}\left(\frac{1}{\cos\theta}\right)$$

$$= \sec^{-1}(\sec\theta)$$

$$= \cos^{-1}\left(\frac{1}{\sec\theta}\right) \qquad \left[\text{Since, } \sec^{-1}x = \cos^{-1}\left(\frac{1}{x}\right)\right]$$

$$v = \cos^{-1}(\cos\theta) \qquad \text{---(ii)}$$

Here,

$$x \in \left(\frac{1}{\sqrt{2}}, 1\right)$$

$$\Rightarrow \quad \sin\theta \in \left(\frac{1}{\sqrt{2}}, 1\right)$$

$$\Rightarrow \quad \theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$

So, from equation (i),

$$u = 2\theta \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$u = 2\sin^{-1}x \qquad [\text{Since, } x = \sin\theta]$$

Differentiating it with respect to x,

$$\frac{du}{dx} = 2\left(\frac{1}{\sqrt{1-x^2}}\right) \qquad \text{---(iv)}$$

From equation (ii)

$$v = \theta \qquad \left[\text{Since, } \cos^{-1}(\cos\theta) = \theta, \text{ if } \theta \in [0, \pi]\right]$$

$$v = \sin^{-1}x$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}} \qquad \text{---(v)}$$

Dividing equation (iv) by (v),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{1}$$

$$\frac{du}{dv} = 2$$

Differentiation Ex 11.8 Q8

Let $u = (\cos x)^{\sin x}$

Taking on both the sides,

$$\log u = \log(\cos x)^{\sin x}$$
$$\log u = \sin x \log(\cos x)$$

Differentiating it with respect to x using product and chain rule,

$$\frac{1}{u}\frac{du}{dx} = \sin x \frac{d}{dx}(\log \cos x) + \log \cos x \frac{d}{dx}(\sin x)$$
$$\frac{1}{u}\frac{du}{dx} = \sin x \left(\frac{1}{\cos x}\right)\frac{d}{dx}(\cos x) + \log \cos x\,(\cos x)$$
$$\frac{du}{dx} = 4\left[(\tan x) \times (-\sin x) + \log\log x \times (\cos x)\right]$$
$$\frac{du}{dx} = (\cos x)^{\sin x}\left[\cos x \log \cos x - \sin x \tan x\right] \quad \text{---(i)}$$

Let $v = (\sin x)^{\cos x}$

Taking log on both the sides,

$$\log v = \log(\sin x)^{\cos x}$$
$$\log v = \cos x \log(\sin x)$$

Differentiating it with respect to x using product rule and chain rule,

$$\frac{1}{v}\frac{dv}{dx} = \cos x \frac{d}{dx}(\log \sin x) + \log \sin x \frac{d}{dx}(\cos x)$$
$$\frac{1}{v}\frac{dv}{dx} = \cos x \left(\frac{1}{\sin x}\right)\frac{d}{dx}(\sin x) + \log \sin x\,(-\sin x)$$
$$\frac{dv}{dx} = v\left[\cot x\,(\cos x) - \sin x \log \sin x\right]$$
$$\frac{dv}{dx} = (\sin x)^{\cos x}\left[\cot x\,(\cos x) - \sin x \log \sin x\right] \quad \text{---(ii)}$$

Dividing equation (i) by (ii),

$$\frac{du}{dv} = \frac{(\cos x)^{\sin x}\left[\cos x \log \cos x - \sin x \tan x\right]}{(\sin x)^{\cos x}\left[\cot x\,(\cos x) - \sin x \log \sin x\right]}$$

Differentiation Ex 11.8 Q9

Let $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Put $x = \tan\theta$,

$u = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$u = \sin^{-1}(\sin 2\theta)$ ---(i)

Let $v = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$= \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$v = \cos^{-1}(\cos 2\theta)$ ---(ii)

Here, $0 < x < 1$

$\Rightarrow$ $0 < \tan\theta < 1$

$\Rightarrow$ $0 < \theta < \frac{\pi}{4}$

So, from equation (i),

$u = 2\theta$ $\left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$

$u = 2\tan^{-1}x$ $\left[\text{Since, } x = \tan\frac{\pi}{2}\right]$

Differentiating it with respect to x,

$$\frac{du}{dx} = \frac{2}{1+x^2} \quad \text{---(iii)}$$

From equation (ii),

$v = 2\theta$ $\left[\text{Since, } \cos^{-1}(\cos\theta) = \theta, \text{ if } \theta \in [0, \pi]\right]$

$v = 2\tan^{-1}x$ $[\text{Since, } x = \tan\theta]$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{2}{1+x^2} \quad \text{---(iv)}$$

Differentiation Ex 11.8 Q10

Let $\quad u = \tan^{-1}\left(\frac{1+ax}{1-ax}\right)$

Put $\quad ax = \tan\theta$

$$u = \tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{\frac{\tan\pi}{4}+\tan\theta}{1-\frac{\tan\pi}{4}\tan\theta}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\theta\right)\right)$$

$$= \frac{\pi}{4}+\theta$$

$$u = \frac{\pi}{4}+\tan^{-1}(ax) \qquad \left[\text{Since, } \tan\theta = ax\right]$$

Differentiate it with respect to x using chain rule,

$$\frac{du}{dx} = 0 + \frac{1}{1+(ax)^2}\frac{d}{dx}(ax)$$

$$\frac{du}{dx} = \frac{a}{1+a^2x^2} \qquad \text{---(i)}$$

Now,

Let $\quad v = \sqrt{1+a^2x^2}$

Differentiating it with respect to x using chain rule,

$$\frac{dv}{dx} = \frac{1}{2\sqrt{1+a^2x^2}}\frac{d}{dx}\left(1+a^2x^2\right)$$

$$= \frac{1}{2\sqrt{1+a^2x^2}}\left(2a^2x\right)$$

$$\frac{dv}{dx} = \frac{a^2x}{\sqrt{1+a^2x^2}} \qquad \text{---(ii)}$$

Differentiation Ex 11.8 Q11

Let $\quad u = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

Put $\quad x = \sin\theta,$

$$u = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$$

$$= \sin^{-1}\left(2\sin\theta\cos\theta\right)$$

$$u = \sin^{-1}\left(\sin 2\theta\right) \qquad \text{---(i)}$$

Let $\quad v = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

$$= \tan^{-1}\left(\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)$$

$$v = \tan^{-1}\left(\tan\theta\right) \qquad \text{---(ii)}$$

Here, $\quad -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

$\Rightarrow \quad -\frac{1}{\sqrt{2}} < \sin\theta < \frac{1}{\sqrt{2}}$

$\Rightarrow \quad \left(-\frac{\pi}{4}\right) < \theta < \left(\frac{\pi}{4}\right)$

So, from equation (i),

$$u = 2\theta \qquad \left[\text{Since, } \sin^{-1}\left(\sin\theta\right) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$u = 2\sin^{-1}x$$

Differentiating it with respect to x,

$$\frac{du}{dx} = \frac{2}{\sqrt{1-x^2}} \qquad \text{---(iii)}$$

From equation (ii),

$$v = \theta \qquad \left[\text{Since, } \tan^{-1}\left(\tan\theta\right) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$v = \sin^{-1}x \qquad \left[\text{Since, } x = \sin\theta\right]$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}} \qquad \text{---(iv)}$$

Dividing equation (iii) by (iv)

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{1}$$

$$\frac{du}{dv} = 2$$

Differentiation Ex 11.8 Q12

Let $u = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

Put $x = \tan\theta$, so

$$u = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$$

$$u = \tan^{-1}(\tan 2\theta) \qquad \text{---(i)}$$

Let $v = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$$= \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$v = \cos^{-1}(\cos 2\theta) \qquad \text{---(ii)}$$

Here, $0 < x < 1$

$\Rightarrow \quad 0 < \tan\theta < 1$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{4}$

So, from equation (i),

$$u = 2\theta \qquad \left[\text{Since, } tab^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$u = 2\tan^{-1}x \qquad [\text{Since, } x = \tan\theta]$$

Differentiating it with respect to x,

$$\frac{du}{dx} = \frac{2}{1+x^2} \qquad \text{---(iii)}$$

From equation (ii),

$$v = \theta \qquad \left[\text{Since, } \cos^{-1}(\cos\theta) = \theta, \text{ if } \theta \in [0, \pi]\right]$$

$$v = 2\tan^{-1}x \qquad [\text{Since, } x = \tan\theta]$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{2}{1+x^2} \qquad \text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2}{1+x^2} \times \frac{1+x^2}{2}$$

$$\frac{du}{dv} = 1$$

Differentiation Ex 11.8 Q13

Let $u = \tan^{-1}\left(\frac{x-1}{x+1}\right)$

Put $x = \tan\theta$, so

$$u = \tan^{-1}\left(\frac{\tan\theta - 1}{\tan\theta + 1}\right)$$

$$= \tan^{-1}\left(\frac{\tan\theta - \frac{\tan\pi}{4}}{1 + \tan\theta\frac{\tan\pi}{4}}\right)$$

$$u = \tan^{-1}\left(\tan\left(\theta - \frac{\pi}{4}\right)\right) \quad \text{---(i)}$$

Here, $-\frac{1}{2} < x < \frac{1}{2}$

$\Rightarrow \quad -\frac{1}{2} < \tan\theta < \frac{1}{2}$

$\Rightarrow \quad -\tan^{-1}\left(\frac{1}{2}\right) < \theta < \tan^{-1}\left(\frac{1}{2}\right)$

So,

$$u = \theta - \frac{\pi}{4} \quad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$u = \tan^{-1} x - \frac{\pi}{4}$$

Differentiating it with respect to x,

$$\frac{du}{dx} = \frac{1}{1+x^2} - 0$$

$$\frac{du}{dx} = \frac{1}{1+x^2} \quad \text{---(ii)}$$

And,

Let $v = \sin^{-1}\left(3x - 4x^3\right)$

Put $x = \sin\theta$, so

$$v = \sin^{-1}\left(3\sin\theta - 4\sin^3\theta\right)$$

$$v = \sin^{-1}(\sin 3\theta) \quad \text{---(iii)}$$

Now, $-\frac{1}{2} < x < \frac{1}{2}$

$\Rightarrow \quad -\frac{1}{2} < \sin\theta < \frac{1}{2}$

$\Rightarrow \quad -\frac{1}{6} < \theta < \frac{\pi}{6}$

So, from equation (iii),

$$v = 3\theta \quad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$v = 3\sin^{-1} x \quad \left[\text{Since, } x = \sin\theta\right]$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{3}{\sqrt{1-x^2}} \quad \text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{1}{1+x^2} \times \frac{\sqrt{1-x^2}}{3}$$

$$\frac{du}{dv} = \frac{\sqrt{1-x^2}}{3\left(1+x^2\right)}$$

Differentiation Ex 11.8 Q14

Let $\quad u = \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$

$$= \tan^{-1}\left(\frac{\frac{\cos^2 x}{2} - \frac{\sin^2 x}{2}}{\frac{\cos^2 x}{2} + \frac{\sin^2 x}{2} + \frac{2\sin x}{2}\frac{\cos x}{2}}\right)$$

$$= \tan^{-1}\left(\frac{\left(\frac{\cos x}{2} + \frac{\sin x}{2}\right)\left(\frac{\cos x}{2} - \frac{\sin x}{2}\right)}{\left(\frac{\cos x}{2} + \frac{\sin x}{2}\right)^2}\right)$$

$$= \tan^{-1}\left(\frac{\frac{\cos x}{2} - \frac{\sin x}{2}}{\frac{\cos x}{2} + \frac{\sin x}{2}}\right)$$

$$= \tan^{-1}\left[\frac{\frac{\frac{\cos x}{2}}{\frac{\cos x}{2}} - \frac{\frac{\sin x}{2}}{\frac{\cos x}{2}}}{\frac{\frac{\cos x}{2}}{\frac{\cos x}{2}} + \frac{\frac{\sin x}{2}}{\frac{\cos x}{2}}}\right]$$

$$= \tan^{-1}\left[\frac{1 - \frac{\tan x}{2}}{1 + \frac{\tan x}{2}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{\tan \pi}{4} - \frac{\tan x}{2}}{1 + \frac{\tan \pi}{4} \times \frac{\tan x}{2}}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right]$$

$$u = \frac{\pi}{4} - \frac{x}{2}$$

Differentiating it with respect to x,

$$\frac{du}{dx} = 0 - \left(\frac{1}{2}\right)$$

$$\frac{du}{dx} = -\frac{1}{2} \qquad \text{---(i)}$$

Let $\quad v = \sec^{-1} x$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{1}{x\sqrt{x^2-1}} \qquad \text{---(ii)}$$

Dividing equation (i) by (ii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = -\frac{1}{2} \times \frac{x\sqrt{x^2-1}}{1}$$

$$\frac{du}{dv} = \frac{-x\sqrt{x^2-1}}{2}$$

Differentiation Ex 11.8 Q15

Let $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1} x$, so

$$u = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$u = \sin^{-1}(\sin 2\theta) \qquad \text{---(i)}$$

Let $v = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$$= \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$$

$$v = \tan^{-1}(\tan 2\theta) \qquad \text{---(ii)}$$

Here, $-1 < x < 1$

$\Rightarrow \quad -1 < \tan\theta < 1$

$\Rightarrow \quad -\frac{\pi}{4} < \theta < \frac{\pi}{4}$

So, from equation (i),

$$u = 2\theta \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$u = 2\tan^{-1} x$$

Differentiating it with respect to x,

$$\frac{du}{dx} = \frac{2}{(1+x^2)} \qquad \text{---(iii)}$$

From equation (ii),

$$v = 2\theta \qquad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$$

$$v = 2\tan^{-1} x$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{2}{1+x^2} \qquad \text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2}{1+x^2} \times \frac{1+x^2}{2}$$

$$\frac{du}{dv} = 1$$

Differentiation Ex 11.8 Q16

Let $\quad u = \cos^{-1}\left(4x^3 - 3x\right)$

Put $\quad x = \cos\theta \Rightarrow \theta = \cos^{-1}x$, so

$$u = \cos^{-1}\left(4\cos^3\theta - 3\cos\theta\right)$$

$$u = \cos^{-1}\left(\cos 3\theta\right) \qquad \text{---(i)}$$

Let $\quad v = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

$$= \tan^{-1}\left(\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}\right)$$

$$= \tan^{-1}\left(\frac{\sin\theta}{\cot\theta}\right)$$

$$v = \tan^{-1}\left(\tan\theta\right) \qquad \text{---(ii)}$$

Here,

$$\frac{1}{2} < x < 1$$

$$\Rightarrow \quad \frac{1}{2} < \cos\theta < 1$$

$$\Rightarrow \quad 0 < \theta < \frac{\pi}{3}$$

So, from equation (i),

$$u = 3\theta \qquad \left[\text{Since, } \cos^{-1}\left(\cos\theta\right) = \theta, \text{ if } \theta \in [0, \pi]\right]$$

$$u = 3\cos^{-1}x$$

Differentiating it with respect to x,

$$\frac{du}{dx} = \frac{-3}{\sqrt{1-x^2}} \qquad \text{---(iii)}$$

From equation (ii),

$$v = \theta \qquad \left[\text{Since, } \tan^{-1}\left(\tan\theta\right) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$$

$$v = \cos^{-1}x$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{-1}{\sqrt{1-x^2}} \qquad \text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \left(\frac{-3}{\sqrt{1-x^2}}\right)\left(-\frac{\sqrt{1-x^2}}{1}\right)$$

$$\frac{du}{dv} = 3$$

Differentiation Ex 11.8 Q17

Let $u = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

Put $x = \cos\theta \Rightarrow \theta = \sin^{-1} x$, so

$$u = \tan^{-1}\left(\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}\right)$$
$$= \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)$$
$$u = \tan^{-1}(\tan\theta) \quad \text{---(i)}$$

And,

Let $v = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

$$v = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$$
$$= \sin^{-1}(2\sin\theta\cos\theta)$$
$$v = \sin^{-1}(\sin 2\theta) \quad \text{---(ii)}$$

Here, $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

$\Rightarrow \quad -\frac{1}{\sqrt{2}} < \sin\theta < \frac{1}{\sqrt{2}}$

$\Rightarrow \quad -\frac{\pi}{4} < \theta < \frac{\pi}{4}$

So, from equation (i),

$$u = \theta \quad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$$

$$u = \sin^{-1} x$$

Differentiation Ex 11.8 Q18

Let $u = \sin^{-1}\left(\sqrt{1-x^2}\right)$

Put $x = \cos\theta \Rightarrow \theta = \cos^{-1} x$, so

$$u = \sin^{-1}(\sin\theta) \quad \text{---(i)}$$

And,

Let $v = \cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

$$= \cot^{-1}\left(\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}\right)$$
$$= \cot^{-1}\left(\frac{\cos\theta}{\sin\theta}\right)$$
$$v = \cot^{-1}(\cot\theta) \quad \text{---(ii)}$$

Here, $0 < x < 1$

$\Rightarrow \quad 0 < \cos\theta < 1$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{2}$

So, from equation (i),

$$u = \theta \quad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$u = \cos^{-1} x$$

Differentiation Ex 11.8 Q19

Let $u = \sin^{-1}\left\{2ax\sqrt{1-a^2x^2}\right\}$

Put $ax = \sin\theta \Rightarrow \theta = \sin^{-1}(ax)$

$$u = \sin^{-1}\left\{2\sin\theta\sqrt{1-\sin^2\theta}\right\}$$
$$= \sin^{-1}\{2\sin\theta\cos\theta\}$$
$$u = \sin^{-1}(\sin 2\theta) \qquad \text{---(i)}$$

And,

Let $v = \sqrt{1-a^2x^2}$

Differentiating it with respect to x using chain rule,

$$\frac{dv}{dx} = \frac{1}{2\sqrt{1-a^2x^2}} \times \frac{d}{dx}\left(1-a^2x^2\right)$$
$$= \left(\frac{0-2a^2x}{2\sqrt{1-a^2x^2}}\right)$$
$$\frac{dv}{dx} = \frac{-a^2x}{\sqrt{1-a^2x^2}} \qquad \text{---(ii)}$$

Here, $-\frac{1}{\sqrt{2}} < ax < \frac{1}{\sqrt{2}}$

$\Rightarrow \quad -\frac{1}{\sqrt{2}} < \sin\theta < \frac{1}{\sqrt{2}}$

$\Rightarrow \quad -\frac{\pi}{4} < \theta < \frac{\pi}{4}$

So, from equation (i),

$$u = 2\theta \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$
$$u = 2\sin^{-1} ax$$

Differentiation Ex 11.8 Q20

Let $\quad u = \tan^{-1}\left(\frac{1-x}{1+x}\right)$

Put $\quad x = \tan\theta \Rightarrow \theta = \tan^{-1}x$, so

$$u = \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{\frac{\tan\pi}{4} - \tan\theta}{1+\frac{\tan\pi}{4}\tan\theta}\right)$$

$$u = \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\theta\right)\right) \qquad \text{---(i)}$$

Here, $\quad -1 < x < 1$

$\Rightarrow \quad -1 < \tan\theta < 1$

$\Rightarrow \quad -\frac{\pi}{4} < \theta < \frac{\pi}{4}$

So, from equation (i),

$$u = \left(\frac{\pi}{4}-\theta\right) \qquad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$$

$$u = \frac{\pi}{4} - \tan^{-1}x$$

Differentiating it with respect to x,

$$\frac{du}{dx} = 0 - \left(\frac{1}{1+x^2}\right)$$

$$\frac{du}{dx} = -\frac{1}{1+x^2} \qquad \text{---(ii)}$$

And,

Let $\quad v = \sqrt{1-x^2}$

Differentiating it with respect to x using chain rule,

$$\frac{dv}{dx} = \frac{1}{2\sqrt{1-x^2}} \times \frac{d}{dx}\left(1-x^2\right)$$

$$= \frac{1}{2\sqrt{1-x^2}}(-2x)$$

$$\frac{dv}{dx} = \frac{-x}{\sqrt{1-x^2}} \qquad \text{---(iii)}$$

Dividing equation (ii) by (iii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = -\frac{1}{\left(1+x^2\right)} \times \frac{\sqrt{1-x^2}}{-x}$$

$$\frac{du}{dv} = \frac{\sqrt{1-x^2}}{x\left(1+x^2\right)}$$

Ex 12.1

Higher Order Derivatives Ex 12.1 Q1(i)

We have $f(x) = x^3 + \tan x$

$\Rightarrow \quad f'(x) = 3x^2 + \sec^2 x$

$\Rightarrow \quad f''(x) = 6x + 2\sec x \times \sec x \tan x$

$\Rightarrow \quad f''(x) = 6x + 2\sec^2 x \tan x.$

Higher Order Derivatives Ex 12.1 Q1(ii)

Let $y = \sin(\log x)$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\sin(\log x)\right] = \cos(\log x)\cdot\frac{d}{dx}(\log x) = \frac{\cos(\log x)}{x}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{\cos(\log x)}{x}\right]$$

$$= \frac{x\cdot\frac{d}{dx}\left[\cos(\log x)\right] - \cos(\log x)\cdot\frac{d}{dx}(x)}{x^2}$$

$$= \frac{x\cdot\left[-\sin(\log x)\cdot\frac{d}{dx}(\log x)\right] - \cos(\log x).1}{x^2}$$

$$= \frac{-x\sin(\log x).\frac{1}{x} - \cos(\log x)}{x^2}$$

$$= \frac{-\left[\sin(\log x) + \cos(\log x)\right]}{x^2}$$

Higher Order Derivatives Ex 12.1 Q1(iii)

Let $y = \log(\sin x)$

Differentiating with repect to x, we get,

$$\frac{dy}{dx} = \frac{\cos x}{\sin x}$$

Again differentiating with respect to x, we get,

$$\frac{d^2y}{dx^2} = \frac{-\sin x \times \sin x - \cos x \times \cos x}{\sin^2 x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-1}{\sin^2 x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\operatorname{cosec}^2 x$$

Higher Order Derivatives Ex 12.1 Q1(iv)

Let $y = e^x \sin 5x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^x \sin 5x\right) = \sin 5x \cdot \frac{d}{dx}\left(e^x\right) + e^x \frac{d}{dx}(\sin 5x)$$

$$= \sin 5x \cdot e^x + e^x \cdot \cos 5x \cdot \frac{d}{dx}(5x) = e^x \sin 5x + e^x \cos 5x \cdot 5$$

$$= e^x\left(\sin 5x + 5\cos 5x\right)$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left[e^x\left(\sin 5x + 5\cos 5x\right)\right]$$

$$= \left(\sin 5x + 5\cos 5x\right) \cdot \frac{d}{dx}\left(e^x\right) + e^x \cdot \frac{d}{dx}\left(\sin 5x + 5\cos 5x\right)$$

$$= \left(\sin 5x + 5\cos 5x\right)e^x + e^x\left[\cos 5x \cdot \frac{d}{dx}(5x) + 5(-\sin 5x) \cdot \frac{d}{dx}(5x)\right]$$

$$= e^x\left(\sin 5x + 5\cos 5x\right) + e^x\left(5\cos 5x - 25\sin 5x\right)$$

$$= e^x\left(10\cos 5x - 24\sin 5x\right) = 2e^x\left(5\cos 5x - 12\sin 5x\right)$$

Higher Order Derivatives Ex 12.1 Q1(v)

Let $y = e^{6x}\cos 3x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{6x} \cdot \cos 3x\right) = \cos 3x \cdot \frac{d}{dx}\left(e^{6x}\right) + e^{6x} \cdot \frac{d}{dx}(\cos 3x)$$

$$= \cos 3x \cdot e^{6x} \cdot \frac{d}{dx}(6x) + e^{6x} \cdot (-\sin 3x) \cdot \frac{d}{dx}(3x)$$

$$= 6e^{6x}\cos 3x - 3e^{6x}\sin 3x \qquad \ldots(1)$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left(6e^{6x}\cos 3x - 3e^{6x}\sin 3x\right) = 6 \cdot \frac{d}{dx}\left(e^{6x}\cos 3x\right) - 3 \cdot \frac{d}{dx}\left(e^{6x}\sin 3x\right)$$

$$= 6 \cdot \left[6e^{6x}\cos 3x - 3e^{6x}\sin 3x\right] - 3 \cdot \left[\sin 3x \cdot \frac{d}{dx}\left(e^{6x}\right) + e^{6x} \cdot \frac{d}{dx}(\sin 3x)\right] \qquad [\text{Using } (1)]$$

$$= 36e^{6x}\cos 3x - 18e^{6x}\sin 3x - 3\left[\sin 3x \cdot e^{6x} \cdot 6 + e^{6x} \cdot \cos 3x \cdot 3\right]$$

$$= 36e^{6x}\cos 3x - 18e^{6x}\sin 3x - 18e^{6x}\sin 3x - 9e^{6x}\cos 3x$$

$$= 27e^{6x}\cos 3x - 36e^{6x}\sin 3x$$

$$= 9e^{6x}\left(3\cos 3x - 4\sin 3x\right)$$

Higher Order Derivatives Ex 12.1 Q1(vi)

Let $y = x^3 \log x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}\left[x^3 \log x\right] = \log x \cdot \frac{d}{dx}\left(x^3\right) + x^3 \cdot \frac{d}{dx}(\log x)$$
$$= \log x \cdot 3x^2 + x^3 \cdot \frac{1}{x} = \log x \cdot 3x^2 + x^2$$
$$= x^2(1 + 3\log x)$$
$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left[x^2(1+3\log x)\right]$$
$$= (1+3\log x) \cdot \frac{d}{dx}\left(x^2\right) + x^2 \frac{d}{dx}(1+3\log x)$$
$$= (1+3\log x) \cdot 2x + x^2 \cdot \frac{3}{x}$$
$$= 2x + 6x\log x + 3x$$
$$= 5x + 6x\log x$$
$$= x(5 + 6\log x)$$

Higher Order Derivatives Ex 12.1 Q1(vii)

Let $y = \tan^{-1} x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$$
$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{1+x^2}\right) = \frac{d}{dx}\left(1+x^2\right)^{-1} = (-1) \cdot \left(1+x^2\right)^{-2} \cdot \frac{d}{dx}\left(1+x^2\right)$$
$$= \frac{-1}{\left(1+x^2\right)^2} \times 2x = \frac{-2x}{\left(1+x^2\right)^2}$$

Higher Order Derivatives Ex 12.1 Q1(viii)

Let $y = x \cdot \cos x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}(x \cdot \cos x) = \cos x \cdot \frac{d}{dx}(x) + x\frac{d}{dx}(\cos x) = \cos x \cdot 1 + x(-\sin x) = \cos x - x\sin x$$
$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left[\cos x - x\sin x\right] = \frac{d}{dx}(\cos x) - \frac{d}{dx}(x\sin x)$$
$$= -\sin x - \left[\sin x \cdot \frac{d}{dx}(x) + x \cdot \frac{d}{dx}(\sin x)\right]$$
$$= -\sin x - (\sin x + x\cos x)$$
$$= -(x\cos x + 2\sin x)$$

Higher Order Derivatives Ex 12.1 Q1(ix)

Let $y = \log(\log x)$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\log(\log x)\right] = \frac{1}{\log x} \cdot \frac{d}{dx}(\log x) = \frac{1}{x\log x} = (x\log x)^{-1}$$
$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left[(x\log x)^{-1}\right] = (-1) \cdot (x\log x)^{-2} \cdot \frac{d}{dx}(x\log x)$$
$$= \frac{-1}{(x\log x)^2} \cdot \left[\log x \cdot \frac{d}{dx}(x) + x \cdot \frac{d}{dx}(\log x)\right]$$
$$= \frac{-1}{(x\log x)^2} \cdot \left[\log x \cdot 1 + x \cdot \frac{1}{x}\right] = \frac{-(1+\log x)}{(x\log x)^2}$$

Higher Order Derivatives Ex 12.1 Q2

$$y = e^{-x} \cos x$$

differentiating both sides w.r.t x

$$\Rightarrow \quad \frac{dy}{dx} = e^{-x}(-\sin x) + (\cos x)(-e^{-x})$$

$$\Rightarrow \quad \frac{dy}{dx} = -e^{-x} \sin x - e^{-x} \cos x = -e^{-x}(\sin x + \cos x)$$

again differentiating both sides w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -e^{-x}(\cos x - \sin x) + e^{-x}(\sin x + \cos x)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 2e^{-x} \sin x$$

Higher Order Derivatives Ex 12.1 Q3

$$y = x + \tan x$$

differentiating both sides w.r.t x

$$\Rightarrow \quad \frac{dy}{dx} = 1 + \sec^2 x$$

differentiating w.r.t x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 0 + 2\sec^2 x \tan x$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{2\sin x}{\cos^3 x}$$

$$\Rightarrow \quad \cos^2 x \frac{d^2y}{dx^2} = 2\tan x + 2x - 2x$$

$$\Rightarrow \quad \cos^2 x \frac{d^2y}{dx^2} = 2(x + \tan x) - 2x$$

$$\Rightarrow \quad \cos^2 x \frac{d^2y}{dx^2} = 2y - 2x$$

$$\Rightarrow \quad \cos^2 x \frac{d^2y}{dx^2} - 2y + 2x = 0$$

Higher Order Derivatives Ex 12.1 Q4

$$y = x^3 \log x$$

differentiating w.r.t x

$$\Rightarrow \quad \frac{dy}{dx} = 3x^2 \log x + \frac{x^3}{x}$$

$$\Rightarrow \quad \frac{dy}{dx} = 3x^2 \log x + x^2$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = (\log x)(3 \times 2x) + \frac{3x^2}{x} + 2x$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 6x \log x + 5x$$

differentiating w.r.t x

$$\Rightarrow \quad \frac{d^3y}{dx^3} = \frac{6x}{x} + 6\log x + 5$$

$$\Rightarrow \quad \frac{d^3y}{dx^3} = 6\log x + 11$$

differentiating w.r.t x

$$\Rightarrow \quad \frac{d^4y}{dx^4} = \frac{6}{x} + 0$$

$$\Rightarrow \quad \frac{d^4y}{dx^4} = \frac{6}{x}$$

Higher Order Derivatives Ex 12.1 Q5

$y = log\left(sin\,x\right)$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = \frac{d\left(log\left(sin\,x\right)\right)}{d\left(sin\,x\right)} \times \frac{d\left(sin\,x\right)}{dx} \text{(chain rule)}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{sin\,x} \times cos\,x = cot\,x$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -cosec^2x$$

differentiating w.r.t x

$$\Rightarrow \quad \frac{d^3y}{dx^3} = \left(-2\,cosecx\right) \times \left(-cot\,x\,cosecx\right)$$

$$\Rightarrow \quad \frac{d^3y}{dx^3} = \frac{2\,cosec^2\,cos\,x}{sin\,x}$$

$$\Rightarrow \quad \frac{d^3y}{dx^3} = 2\,cosec^3x\,cos\,x$$

Higher Order Derivatives Ex 12.1 Q6

$y = 2\,sin\,x + 3\,cos\,x$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = 2\,cos\,x + 3\left(-sin\,x\right) = 2\,cos\,x - 3\,sin\,x$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 2\left(-sin\,x\right) - 3\,cos\,x = -\left(2\,sin\,x + 3\,cos\,x\right) = -y$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + y = 0$$

Higher Order Derivatives Ex 12.1 Q7

$y = \frac{log\,x}{x}$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x\left(\frac{1}{x}\right) - \left(log\,x\right)\left(i\right)}{x^2}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1 - log\,x}{x^2}$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{x^2\left(-\frac{1}{x}\right) - \left(1 - log\,x\right)\left(2x\right)}{x^4}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-x - 2x + 2x\,log\,x}{x^4} = \frac{x\left(2\,log\,x - 3\right)}{x^4}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{2\,log\,x - 3}{x^3}$$

Higher Order Derivatives Ex 12.1 Q8

$x = a\sec\theta \qquad y = b\tan\theta$
differentiating both w.r.t.θ

$$\Rightarrow \quad \frac{dx}{d\theta} = a\sec\theta\tan\theta \qquad \ldots\ldots\ldots(1)$$
$$\Rightarrow \quad \frac{dy}{d\theta} = b\sec^2\theta \qquad \ldots\ldots(2)$$

Dividing (2) by (1)

$$\Rightarrow \quad \frac{dy}{dx} = \frac{b\sec^2\theta}{a\sec\theta\tan\theta} \qquad \ldots\ldots(3)$$

Differentiating (3) w.r.t.θ

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{d\theta} = \frac{b}{a}\left[\frac{\tan\theta\left(\sec\theta\tan\theta\right) - \sec\theta\left(\sec^2\theta\right)}{\tan^2\theta}\right]$$

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{d\theta} = \frac{b}{a}\left[\frac{\sec\theta\left(\tan^2\theta\right) - \sec^2\theta}{\tan^2\theta}\right] \ldots\ldots(4)$$

Dividing (4) by (1)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{b\sec\theta\left(\tan^2\theta - \sec^2\theta\right)}{a \times a\sec\theta\tan\theta \times \tan^2\theta}$$

Multiplying & dividing RHS by b^3

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-b^4}{a^2 \times b^3\tan^3\theta}$$
$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3}$$

Higher Order Derivatives Ex 12.1 Q9

It is given that, $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$

$$\therefore \frac{dx}{dt} = a \cdot \frac{d}{dt}(\cos t + t\sin t)$$
$$= a\left[-\sin t + \sin t \cdot \frac{d}{dt}(t) + t \cdot \frac{d}{dt}(\sin t)\right]$$
$$= a\left[-\sin t + \sin t + t\cos t\right] = at\cos t$$
$$\frac{dy}{dt} = a \cdot \frac{d}{dt}(\sin t - t\cos t)$$
$$= a\left[\cos t - \left\{\cos t \cdot \frac{d}{dt}(t) + t \cdot \frac{d}{dt}(\cos t)\right\}\right]$$
$$= a\left[\cos t - \left\{\cos t - t\sin t\right\}\right] = at\sin t$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{at\sin t}{at\cos t} = \tan t$$

Then, $$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(\tan t) = \sec^2 t \cdot \frac{dt}{dx}$$
$$= \sec^2 t \cdot \frac{1}{at\cos t} \qquad \left[\frac{dx}{dt} = at\cos t \Rightarrow \frac{dt}{dx} = \frac{1}{at\cos t}\right]$$
$$= \frac{\sec^3 t}{at}, 0 < t < \frac{\pi}{2}$$

Higher Order Derivatives Ex 12.1 Q10

$y = e^x \cos x$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = e^x(-\sin x) + e^x \cos x = e^x(\cos x - \sin x)$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = e^x(-\cos x - \sin x) + e^x(\cos x - \sin x)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -2e^x \sin x$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 2e^x \cos\left(x + \frac{\pi}{2}\right)$$

Higher Order Derivatives Ex 12.1 Q11

$x = a\cos\theta$

differentiating w.r.t. θ

$$\Rightarrow \quad \frac{dy}{d\theta} = -a\sin\theta \quad .....(1)$$

$$\Rightarrow \quad \frac{dy}{d\theta} = b\cos\theta \quad .....(2)$$

Dividing (2) by (1)

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-b\cos\theta}{a\sin\theta} \quad ......(3)$$

differentiating (3) w.r.t. θ

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{d\theta} = \frac{-b}{a}\left\{\frac{\sin\theta(-\sin\theta) - \cos\theta(\cos\theta)}{\sin^2\theta}\right\} = \frac{b}{a}\frac{\left(\sin^2\theta + \cos^2\theta\right)}{\sin^2\theta} = \frac{b}{a\sin^2\theta} \quad ......(4)$$

Dividing (4) by (1)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-b}{a^2\sin^3\theta} \times \frac{b^3}{b^3}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3}$$

Higher Order Derivatives Ex 12.1 Q12

$x = a\left(1 - \cos^3\theta\right); \quad y = a\sin^3\theta$

differentiating both w.r.t.θ

$$\Rightarrow \quad \frac{dx}{d\theta} = a\left(0 - 3\cos^2\theta(-\sin\theta)\right); \quad \frac{dy}{d\theta} = a\left(3\sin^2\theta \times \cos\theta\right) ......(2)$$

$$\Rightarrow \quad \frac{dy}{d\theta} = 3a\sin\theta\cos^2\theta; \quad \frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{3a\sin^2\theta\cos\theta}{3a\sin\theta\cos^2\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

Differentiating w.r.t.θ

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{d\theta} = \sec^2\theta \quad ....(3)$$

Dividing (3) by (1)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{\sec^2\theta}{3a\sin\theta\cos^2\theta}$$

Putting $\theta = \pi/6$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{\frac{2}{\sqrt{3}} \times \frac{2}{\sqrt{3}}}{3a \times \frac{1}{2} \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}} = \frac{2^5}{3a \times \left(\sqrt{3}\right)^4} = \frac{32}{27a}$$

Higher Order Derivatives Ex 12.1 Q13

$x = a(\theta + \sin\theta)$; $y = a(1 + \cos\theta)$

differentiating both w.r.t.θ

$$\Rightarrow \quad \frac{dx}{d\theta} = a(1+\cos\theta); \quad (1)$$

$$\Rightarrow \quad \frac{dy}{d\theta} = a(0-\sin\theta) \quad (2)$$

Dividing (2) by (1)

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{-a\sin\theta}{a(1+\cos\theta)}$$

Differentiating w.r.t.θ

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{d\theta} = -\left\{\frac{(1+\cos\theta)(\cos\theta) - (\sin\theta)(0-\sin\theta)}{(1+\cos\theta)^2}\right\} = -\left\{\frac{\cos\theta + \cos^2\theta + \sin^2\theta}{(1+\cos\theta)^2}\right\}$$

$$= -\left\{\frac{\cos\theta + 1}{(\cos + 1)^2}\right\}$$

$$= \frac{-1}{1+\cos\theta} \quad \ldots\ldots(3)$$

dividing (3) by (1)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-1 \times a}{a(1+\cos\theta)^2 \times a} = \frac{-a}{y^2}$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q14

$x = a(\theta - \sin\theta);\ y = a(1 + \cos\theta)$

Diiferentiating the above functions with respect to θ, *we get*,

$$\frac{dx}{d\theta} = a(1 - \cos\theta) \quad \ldots(1)$$

$$\frac{dy}{d\theta} = a(-\sin\theta) \qquad \ldots(2)$$

Dividing equation (2) *by* (1), *we have*,

$$\frac{dy}{dx} = \frac{a(-\sin\theta)}{a(1 - \cos\theta)} = \frac{-\sin\theta}{1 - \cos\theta}$$

Differentiating with respect to θ, *we have*,

$$\frac{d\left(\frac{dy}{dx}\right)}{d\theta} = \frac{(1 - \cos\theta)(-\cos\theta) + \sin\theta(\sin\theta)}{(1 - \cos\theta)^2}$$

$$= \frac{-\cos\theta + \cos^2\theta + \sin^2\theta}{(1 - \cos\theta)^2}$$

$$= \frac{1 - \cos\theta}{(1 - \cos\theta)^2}$$

$$\frac{d\left(\frac{dy}{dx}\right)}{d\theta} = \frac{1}{1 - \cos\theta} \ldots(3)$$

Dividing equation (3) *by* (1), *we have*,

$$\frac{d^2y}{dx^2} = \frac{1}{1 - \cos\theta} \times \frac{1}{a(1 - \cos\theta)}$$

$$= \frac{1}{a(1 - \cos\theta)^2}$$

$$= \frac{1}{a\left(2\sin^2\frac{\theta}{2}\right)^2}$$

$$= \frac{1}{4a\sin^4\left(\frac{\theta}{2}\right)}$$

$$= \frac{1}{4a}\operatorname{cosec}^4\left(\frac{\theta}{2}\right)$$

Higher Order Derivatives Ex 12.1 Q15

$x = a(1 - \cos\theta)$; $y = a(\theta + \sin\theta)$

Differentiating both w.r.t.θ

$$\Rightarrow \quad \frac{dx}{d\theta} = a(0 + \sin\theta); \quad \frac{dy}{d\theta} = a(1 + \cos\theta)$$

Dividing (2) by (1)

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{a(1 + \cos\theta)}{a\sin\theta}$$

Differentiating w.r.t.θ

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{d\theta} = \frac{\sin\theta(0 - \sin\theta) - (1 + \cos\theta)\cos\theta}{\sin^2\theta} = -\frac{\sin^2\theta - \cos\theta - \cos^2\theta}{\sin^2\theta}$$

$$= -\frac{(1 + \cos\theta)}{\sin^2\theta} \quad \ldots\ldots(3)$$

dividing (3) by (1)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\frac{(1 + \cos\theta)}{\sin^2\theta \times a\sin\theta}$$

Putting $\theta = \pi/2$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\frac{1}{a}$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q17

$x = \cos\theta;\ y = \sin^3\theta$

Differentiating both w.r.t. θ

$$\Rightarrow \quad \frac{dx}{d\theta} = -\sin\theta; \qquad (1)$$

$$\frac{dy}{d\theta} = 3\sin^2\theta\cos\theta \qquad (2)$$

Dividing (2) by (1)

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dy}{d\theta}\times\frac{d\theta}{dx} = -\frac{3\sin^2\theta\cos\theta}{\sin\theta} = -3\sin\theta\cos\theta$$

Differentiating w.r.t. θ

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{d\theta} = -3\left\{\sin\theta\left(-\sin\theta\right)+\cos\theta\left(\cos\theta\right)\right\} = -3\left(\cos^2\theta-\sin^2\theta\right)\ldots\ldots\ldots(3)$$

Dividing (3) by (1)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{+3\left(\cos^2\theta-\sin^2\theta\right)}{\sin\theta}\times\frac{\sin^2\theta}{\sin^2\theta}$$

$$\Rightarrow \quad \sin^3\theta\frac{d^2y}{dx^2} = 3\sin^2\theta\left(\cos^2\theta-\sin^2\theta\right)$$

$$\Rightarrow \quad y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2 = 3\sin^2\theta\left(\cos^2\theta-\sin^2\theta\right)+\left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow \quad y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2 = 3\sin^2\theta\cos^2\theta-3\sin^4\theta+9\sin^2\theta\cos^2\theta$$

adding and subtracting $3\sin^2\theta\cos^2\theta$ on RHS

$$\Rightarrow \quad y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2 = 12\sin^2\theta\cos^2\theta-3\sin^4\theta+3\sin^2\theta\cos^2\theta-3\sin^2\theta\cos^2\theta$$

$$\Rightarrow \quad y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2 = 15\sin^2\theta\cos^2\theta-3\sin^2\theta\left(\sin^2\theta+\cos^2\theta\right)$$

$$= 15\sin^2\theta\cos^2\theta-3\sin^2\theta$$

$$\Rightarrow \quad y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2 = 3\sin^2\theta\left\{5\cos^2\theta-1\right\}$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q18

$y = \sin(\sin x)$
differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = \frac{d(\sin(\sin x))}{d(\sin x)} \times \frac{d(\sin x)}{dx} = \cos(\sin x) \times \cos x$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = (\cos(\sin x))(-\sin x) + (\cos x)(-\sin(\sin x))(\cos x)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\sin x \cos(\sin x) \times \frac{\cos x}{\cos x} - y\cos^2 x$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\tan x \frac{dy}{dx} - y\cos^2 x$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y\cos^2 x = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q19

$x = \sin t\,; \quad y = \sin pt$
differentiating both w.r.t. t

$$\Rightarrow \quad \frac{dy}{dt} = \cos t \;\ldots\ldots(1)\,; \quad \frac{dy}{dt} = p\cos pt \;\ldots\ldots(2)$$

dividing (2) by (1)

$$\Rightarrow \quad \frac{dy}{dx} = p\frac{\cos pt}{\cos t}$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{dt} = p\left\{\frac{p\cos t(-\sin pt) - (\cos pt)(-\sin t)}{\cos^2 t}\right\}$$

$$= p\left\{\frac{\sin t\cos pt - p\cos t\sin pt(-\sin t)}{\cos^2 t}\right\}\ldots\ldots(3)$$

$\Rightarrow$ dividing (3) by (1)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = p\left\{\frac{\sin t\cos pt - p\cos t\sin pt}{\cos^3 t}\right\} = \left\{\frac{\tan t\cos t - p\sin pt}{\cos^2 t}\right\}$$

$\because \quad \sin^2 t + \cos^2 t = 1$

$\Rightarrow \quad 1 - \sin^2 t = \cos^2 t$

$\Rightarrow \quad 1 - x^2 = \cos^2 t$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = p\left\{\frac{\tan t\cos pt - p\sin pt}{1 - x^2}\right\}$$

$$\Rightarrow \quad (1 - X^2)\frac{d^2y}{dx^2} = p\frac{\sin t\cos pt}{\cos t} - p^2\sin pt = \frac{dy}{dx} - p^2y$$

$$\Rightarrow \quad (1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q20

$y = e^{\tan^{-1}x}$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = e^{\tan^{-1}x}\left(\frac{1}{1+x^2}\right)$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{\left(1+x^2\right)\left(e^{\tan^{-1}x}\right)\times\frac{1}{1+x^2} - e^{\tan^{-1}x}\left(2x\right)}{\left(1+x^2\right)^2}$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{d^2y}{dx^2} = \frac{e^{\tan^{-1}x} - 2xe^{\tan^{-1}x}}{1+x^2}$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{d^2y}{dx^2} = \frac{e^{\tan^{-1}x}}{1+x^2}\left(1-2x\right) = \frac{dy}{dx}\left(1-2x\right)$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{d^2y}{dx^2} + \left(2x-1\right)\frac{dy}{dx} = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q21

$y = e^{\tan^{-1}x}$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = e^{\tan^{-1}x}\left(\frac{1}{1+x^2}\right)$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{\left(1+x^2\right)\left(e^{\tan^{-1}x}\right)\times\frac{1}{1+x^2} - e^{\tan^{-1}x}\left(2x\right)}{\left(1+x^2\right)^2}$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{d^2y}{dx^2} = \frac{e^{\tan^{-1}x} - 2xe^{\tan^{-1}x}}{1+x^2}$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{d^2y}{dx^2} = \frac{e^{\tan^{-1}x}}{1+x^2}\left(1-2x\right) = \frac{dy}{dx}\left(1-2x\right)$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{d^2y}{dx^2} + \left(2x-1\right)\frac{dy}{dx} = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q22

It is given that, $y = 3\cos(\log x) + 4\sin(\log x)$

Then,

$$y_1 = 3 \cdot \frac{d}{dx}[\cos(\log x)] + 4 \cdot \frac{d}{dx}[\sin(\log x)]$$

$$= 3 \cdot \left[-\sin(\log x) \cdot \frac{d}{dx}(\log x)\right] + 4 \cdot \left[\cos(\log x) \cdot \frac{d}{dx}(\log x)\right]$$

$$\therefore y_1 = \frac{-3\sin(\log x)}{x} + \frac{4\cos(\log x)}{x} = \frac{4\cos(\log x) - 3\sin(\log x)}{x}$$

$$\therefore y_2 = \frac{d}{dx}\left(\frac{4\cos(\log x) - 3\sin(\log x)}{x}\right)$$

$$= \frac{x\{4\cos(\log x) - 3\sin(\log x)\}' - \{4\cos(\log x) - 3\sin(\log x)\}(x)'}{x^2}$$

$$= \frac{x\left[4\{\cos(\log x)\}' - 3\{\sin(\log x)\}'\right] - \{4\cos(\log x) - 3\sin(\log x)\}.1}{x^2}$$

$$= \frac{x\left[-4\sin(\log x).(\log x)' - 3\cos(\log x).(\log x)'\right] - 4\cos(\log x) + 3\sin(\log x)}{x^2}$$

$$= \frac{x\left[-4\sin(\log x).\frac{1}{x} - 3\cos(\log x).\frac{1}{x}\right] - 4\cos(\log x) + 3\sin(\log x)}{x^2}$$

$$= \frac{-4\sin(\log x) - 3\cos(\log x) - 4\cos(\log x) + 3\sin(\log x)}{x^2}$$

$$= \frac{-\sin(\log x) - 7\cos(\log x)}{x^2}$$

$$\therefore x^2 y_2 + x y_1 + y$$

$$= x^2\left(\frac{-\sin(\log x) - 7\cos(\log x)}{x^2}\right) + x\left(\frac{4\cos(\log x) - 3\sin(\log x)}{x}\right) + 3\cos(\log x) + 4\sin(\log x)$$

$$= -\sin(\log x) - 7\cos(\log x) + 4\cos(\log x) - 3\sin(\log x) + 3\cos(\log x) + 4\sin(\log x)$$

$$= 0$$

Hence, proved.

Higher Order Derivatives Ex 12.1 Q23

$y = e^{2x}(ax + b)$

differentiating w.r.t. x

$\Rightarrow \quad \frac{dy}{dx} = e^{2x}(a) + 2(ax + b)(e^{2x})$

$\Rightarrow \quad \frac{dy}{dx} = ae^{2x} + 2y$

differentiating w.r.t. x

$\Rightarrow \quad \frac{d^2y}{dx^2} = 2ae^{2x} + 2\frac{dy}{dx}$

$\Rightarrow \quad \frac{d^2y}{dx^2} = 2\frac{dy}{dx} + 2ae^{2x} + 4y - 4y = 2\frac{dy}{dx} + 2\frac{dy}{dx} - 4y$

$\Rightarrow \quad \frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$

$\Rightarrow \quad y_2 - 4y_1 + 4y = 0$

Hence proved!

Higher Order Derivatives Ex 12.1 Q24

$$x = \sin\left(\frac{1}{a}\log y\right)$$

$$\Rightarrow \quad \sin^{-1} x = \frac{1}{a}\log y$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{1}{\sqrt{1-x^2}} = \frac{1}{ay}\frac{dy}{dx}$$

$$\Rightarrow \quad y_1 = \frac{dy}{dx} = \frac{ay}{\sqrt{1-x^2}}$$

differentiating w.r.t.x

$$\Rightarrow \quad y_2 = \frac{d^2y}{dx^2} = a\left[\frac{\sqrt{1-x^2}\frac{dy}{dx} + \frac{y \times 2x}{2\sqrt{1-x^2}}}{1-x^2}\right]$$

$$\Rightarrow \quad \left(1-x^2\right)y_2 = a\sqrt{1-x^2}\frac{dy}{dx} + \frac{ayx}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad \left(1-x^2\right)y_2 = x\frac{dy}{dx} + a\sqrt{1-x^2} \times \frac{ay}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad \left(1-x^2\right)y_2 - xy_1 - a^2y = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q25

$$\log y = \tan^{-1} x$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = \frac{1}{1+x^2}$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{dy}{dx} = y$$

differentiating w.r.t.x

$$\Rightarrow \quad \left(1+x^2\right)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = \frac{dy}{dx}$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{d^2y}{dx^2} + \left(2x-1\right)\frac{dy}{dx} = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q26

$$y = \tan^{-1} x$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{1+x^2}$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{dy}{dx} = 1$$

differentiating w.r.t.x

$$\Rightarrow \quad \left(1+x^2\right)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q27

$$y = \left[\log\left(x+\sqrt{1+x^2}\right)\right]^2$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = 2\log\left(x+\sqrt{1+x^2}\right)\times\frac{1}{x+\sqrt{1+x^2}}\times\left(1+\frac{1\times 2x}{2\sqrt{1+x^2}}\right)$$

$$\Rightarrow \quad y_1 = \frac{2\log\left(x+\sqrt{1+x^2}\right)}{x+\sqrt{1+x^2}}\times\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}} = \frac{2\log\left(x+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}$$

squaring both sides

$$\Rightarrow \quad (y_1)^2 = \frac{4}{1+x^2}\left[\log\left(x+\sqrt{1+x^2}\right)\right]^2 = \frac{4y}{1+x^2}$$

$$\Rightarrow \quad \left(1+x^2\right)(y_1)^2 = 4y$$

differentiating w.r.t. x

$$\Rightarrow \quad \left(1+x^2\right)2y_1y_2 + 2x\,(y_1)^2 = 4y_1$$

$$\Rightarrow \quad \left(1+x^2\right)y_2 + xy_1 = 2$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q28

The given relationship is $y = \left(\tan^{-1} x\right)^2$

Then,

$$y_1 = 2\tan^{-1} x \frac{d}{dx}\left(\tan^{-1} x\right)$$

$$\Rightarrow y_1 = 2\tan^{-1} x . \frac{1}{1+x^2}$$

$$\Rightarrow \left(1+x^2\right)y_1 = 2\tan^{-1} x$$

Again differentiating with respect to x on both the sides, we obtain

$$\left(1+x^2\right)y_2 + 2xy_1 = 2\left(\frac{1}{1+x^2}\right)$$

$$\Rightarrow \left(1+x^2\right)^2 y_2 + 2x\left(1+x^2\right)y_1 = 2$$

Hence, proved.

Higher Order Derivatives Ex 12.1 Q29

$$y = \cot x$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = -\operatorname{cosec}^2 x$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\left[2\operatorname{cosec} x\left(-\operatorname{cosec} x \cot x\right)\right] = 2\operatorname{cosec}^2 x \cot x = -2\frac{dy}{dx}.y$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + 2y\frac{dy}{dx} = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q30

$$y = \log\left(\frac{x^2}{e^2}\right)$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{x^2/e^2}\times\frac{1}{e^2}\times 2x = \frac{2}{x}$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 2\left(\frac{-1}{x^2}\right) = \frac{-2}{x^2}$$

Higher Order Derivatives Ex 12.1 Q31

$y = ae^{2x} + be^{-x}$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = 2ae^{2x} + be^{-x}(-1) = 2ae^{2x} - be^{-x}$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 2ae^{2x}(2) - be^{-x}(-1) = 4ae^{2x} + be^{-x}$$

Adding and subtracting be^{-x} on RHS

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 4ae^{2x} + 2be^{-x} - be^{-x} = 2\left(ae^{2x} + be^{-x}\right) + 2ae^{2x} - be^{-x} = 2y + \frac{dy}{dx}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$$

Higher Order Derivatives Ex 12.1 Q32

$$y = e^x(\sin x + \cos x)$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = e^x(\cos x - \sin x) + (\sin x + \cos x)e^x$$

$$\Rightarrow \quad \frac{dy}{dx} = y + e^x(\cos x - \sin x)$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{dy}{dx} + e^x(-\sin x - \cos x) + (\cos x - \sin x)e^x$$

$$= \frac{dy}{dx} - y + (\cos x - \sin x)e^x$$

Adding and subtracting y on RHS

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{dy}{dx} - y + (\cos x - \sin x)e^x + y - y = 2\frac{dy}{dx} - 2y$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q33

It is given that, $y = \cos^{-1} x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1-x^2}} = -\left(1-x^2\right)^{\frac{-1}{2}}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[-\left(1-x^2\right)^{\frac{-1}{2}}\right]$$

$$= -\left(-\frac{1}{2}\right)\cdot\left(1-x^2\right)^{\frac{-3}{2}}\cdot\frac{d}{dx}\left(1-x^2\right)$$

$$= \frac{1}{2\sqrt{\left(1-x^2\right)^3}} \times (-2x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-x}{\sqrt{\left(1-x^2\right)^3}} \qquad \ldots\text{(i)}$$

$y = \cos^{-1} x \Rightarrow x = \cos y$

Putting $x = \cos y$ in equation (i), we obtain

$$\frac{d^2y}{dx^2} = \frac{-\cos y}{\sqrt{\left(1-\cos^2 y\right)^3}}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-\cos y}{\sqrt{\left(\sin^2 y\right)^3}}$$

$$= \frac{-\cos y}{\sin^3 y}$$

$$= \frac{-\cos y}{\sin y} \times \frac{1}{\sin^2 y}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\cot y \cdot \operatorname{cosec}^2 y$$

Higher Order Derivatives Ex 12.1 Q34

It is given that, $y = e^{a\cos^{-1} x}$

Taking logarithm on both the sides, we obtain

$\log\ y = a\cos^{-1} x \log e$

$\log\ y = a\cos^{-1} x$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{y}\frac{dy}{dx} = a \times \frac{-1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-ay}{\sqrt{1-x^2}}$$

By squaring both the sides, we obtain

$$\left(\frac{dy}{dx}\right)^2 = \frac{a^2y^2}{1-x^2}$$

$$\Rightarrow \left(1-x^2\right)\left(\frac{dy}{dx}\right)^2 = a^2y^2$$

$$\left(1-x^2\right)\left(\frac{dy}{dx}\right)^2 = a^2y^2$$

Again differentiating both sides with respect to x, we obtain

$$\left(\frac{dy}{dx}\right)^2 \frac{d}{dx}\left(1-x^2\right) + \left(1-x^2\right) \times \frac{d}{dx}\left[\left(\frac{dy}{dx}\right)^2\right] = a^2\frac{d}{dx}\left(y^2\right)$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2(-2x) + \left(1-x^2\right) \times 2\frac{dy}{dx}.\frac{d^2y}{dx^2} = a^2.2y.\frac{dy}{dx}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2(-2x) + \left(1-x^2\right) \times 2\frac{dy}{dx}.\frac{d^2y}{dx^2} = a^2.2y.\frac{dy}{dx}$$

$$\Rightarrow -x\frac{dy}{dx} + \left(1-x^2\right)\frac{d^2y}{dx^2} = a^2.y \qquad \left[\frac{dy}{dx} \neq 0\right]$$

$$\Rightarrow \left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$$

Hence, proved.

Higher Order Derivatives Ex 12.1 Q35

It is given that, $y = 500e^{7x} + 600e^{-7x}$

Then,

$$\frac{dy}{dx} = 500.\frac{d}{dx}\left(e^{7x}\right) + 600.\frac{d}{dx}\left(e^{-7x}\right)$$
$$= 500 \cdot e^{7x} \cdot \frac{d}{dx}(7x) + 600 \cdot e^{-7x} \cdot \frac{d}{dx}(-7x)$$
$$= 3500e^{7x} - 4200e^{-7x}$$
$$\therefore \frac{d^2y}{dx^2} = 3500 \cdot \frac{d}{dx}\left(e^{7x}\right) - 4200.\frac{d}{dx}\left(e^{-7x}\right)$$
$$= 3500 \cdot e^{7x}.\frac{d}{dx}(7x) - 4200 \cdot e^{-7x} \cdot \frac{d}{dx}(-7x)$$
$$= 7 \times 3500 \cdot e^{7x} + 7 \times 4200 \cdot e^{-7x}$$
$$= 49 \times 500e^{7x} + 49 \times 600e^{-7x}$$
$$= 49\left(500e^{7x} + 600e^{-7x}\right)$$
$$= 49y$$

Hence, proved

Higher Order Derivatives Ex 12.1 Q36

$$y = 2\cos t - \cos 2t; \quad y = 2\sin t - \sin 2t$$

differentiating w.r.t. t

$$\Rightarrow \quad \frac{dy}{dt} = 2(-\sin t) - 2(-\sin 2t); \ \frac{dy}{dt} = 2\cos t - 2\cos 2t$$

dividing (2) by (1)

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2(\cos t - \cos 2t)}{2(\sin 2t - \sin t)}$$

differentiating w.r.t. t

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{dt} = \frac{(\sin 2t - \sin t)(-\sin t + 2\sin 2t) - (\cos t - \cos 2t)(2\cos 2t - \cos t)}{(\sin 2t - \sin t)^2} \quad \ldots\ldots..(3)$$

dividing (3) by (1)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{(\sin 2t - \sin t)(2\sin 2t - \sin t) - (\cos t - \cos 2t)(2\cos 2t - \cos t)}{2(\sin 2t - \sin t)^3}$$

Putting $t = \pi/2$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{(0-1)(0-1) - (0-(-1))(2(-1)-0)}{2(0-1)^3} = \frac{1+2}{-2} = \frac{-3}{2}$$

Higher Order Derivatives Ex 12.1 Q37

$$x = 4z^2 + 5 \qquad y = 6z^2 + 7z + 3$$

differentiating both w.r.t.z

$$\Rightarrow \quad \frac{dx}{dz} = 8z + 0 \qquad \frac{dy}{dz} = 12z + 7$$

$$\Rightarrow \quad \frac{dx}{dz} = \frac{12z+7}{8z} = \frac{12z}{8z} + \frac{7}{8z}$$

differentiating w.r.t.z

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{dz} = 0 + \frac{7}{8}\left(\frac{-1}{z^2}\right) \qquad \ldots..(3)$$

dividing (3) by (1)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-7}{8z^2 \times 8z} = \frac{-7}{64z^3}$$

Higher Order Derivatives Ex 12.1 Q38

$$y = \log\left(1+\cos x\right)$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{1+\cos x} \times -\sin x = \frac{-\sin x}{1+\cos x}$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\left[\frac{\left(1+\cos x\right)\cos x - \sin x\left(-\sin x\right)}{\left(1+\cos x\right)^2}\right]$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\left[\frac{\cos x + \cos^2 x + \sin^2 x}{\left(1+\cos x\right)^2}\right] = -\left[\frac{1+\cos x}{\left(1+\cos x\right)^2}\right] = \frac{-1}{1+\cos x}$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^3y}{dx^3} = -\left(\frac{+1}{\left(1+\cos x\right)^2} \times +\sin x\right) = -\left(\frac{-\sin x}{1+\cos x}\right)\times\left(\frac{-1}{1+\cos x}\right) = -\frac{dy}{dx}\cdot\frac{d^2y}{dx^2}$$

$$\Rightarrow \quad \frac{d^3y}{dx^3} + \frac{d^2y}{dx^2}\cdot\frac{dy}{dx} = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q39

$$y = \sin\left(\log x\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \cos\left(\log x\right) \times \frac{1}{x}$$

$$\Rightarrow \quad x\frac{dy}{dx} = \cos\left(\log x\right)$$

$$\Rightarrow \quad x\frac{d^2y}{dx^2} + \frac{dy}{dx} = -\sin\left(\log x\right) \times \frac{1}{x}$$

$$\Rightarrow \quad x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -y$$

$$\Rightarrow \quad x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q40

Given $y = 3e^{2x} + 2e^{3x}$

Then, $\frac{dy}{dx} = 6e^{2x} + 6e^{3x} = 6\left(e^{2x} + e^{3x}\right)$

$\therefore \quad \frac{d^2y}{dx^2} = 12e^{2x} + 18e^{3x} = 6\left(2e^{2x} + 3e^{3x}\right)$

Hence,

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 6\left(2e^{2x} + 3e^{3x}\right) - 30\left(e^{2x} + e^{3x}\right) + 6\left(3e^{2x} + 2e^{3x}\right)$$
$$= 0$$

Higher Order Derivatives Ex 12.1 Q41

$$y = \left(\cot^{-1} x\right)^2$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = y_1 = 2\cot^{-1} x\,\frac{-1}{1+x^2}$$

$$= \frac{-2\cot^{-1} x}{1+x^2} \text{(chain rule)}$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{dy}{dx} = -2\cot^{-1} x$$

differentiating w.r.t. x

$$\Rightarrow \quad \left(1+x^2\right)y_2 + 2xy_1 = +2\left(\frac{+1}{1+x^2}\right)$$

(multiplication rule on LHS)

$$\Rightarrow \quad \left(1+x^2\right)^2 y_2 + 2x\left(1+x^2\right)y_1 = 2$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q42

We know that, $\frac{d}{dx}\left(\operatorname{cosec}^{-1}x\right)=\frac{-1}{|x|\sqrt{x^2-1}}$

Let $y=\operatorname{cosec}^{-1}x$

$$\frac{dy}{dx}=\frac{-1}{|x|\sqrt{x^2-1}}$$

Since $x>1,\ |x|=x$

Thus,

$$\frac{dy}{dx}=\frac{-1}{x\sqrt{x^2-1}}\ ...(1)$$

Differentiating the above function with respect to x, *we have,*

$$\frac{d^2y}{dx^2}=\frac{x\frac{2x}{2\sqrt{x^2-1}}+\sqrt{x^2-1}}{x^2\left(x^2-1\right)}$$

$$=\frac{\frac{x^2}{\sqrt{x^2-1}}+\sqrt{x^2-1}}{x^2\left(x^2-1\right)}$$

$$=\frac{x^2+x^2-1}{x^2\left(x^2-1\right)^{\frac{3}{2}}}$$

$$=\frac{2x^2-1}{x^2\left(x^2-1\right)^{\frac{3}{2}}}$$

Thus, $x\left(x^2-1\right)\frac{d^2y}{dx^2}=\frac{2x^2-1}{x\sqrt{x^2-1}}\ ...(2)$

Similarly, from (1), *we have*

$$\left(2x^2-1\right)\frac{dy}{dx}=\frac{-2x^2+1}{x\sqrt{x^2-1}}\ ...(3)$$

Thus, from (2) *and* (3), *we have,*

$$x\left(x^2-1\right)\frac{d^2y}{dx^2}+\left(2x^2-1\right)\frac{dy}{dx}=\frac{2x^2-1}{x\sqrt{x^2-1}}+\left(\frac{-2x^2+1}{x\sqrt{x^2-1}}\right)=0$$

Hence proved.

Higher Order Derivatives Ex 12.1 Q43

Given that, $x = \cos t + \log\tan\frac{t}{2},\ y = \sin t$

Differentiating with respect to t, we have,

$$\frac{dx}{dt} = -\sin t + \frac{1}{\tan\frac{t}{2}} \times sec^2\frac{t}{2} \times \frac{1}{2}$$

$$= -\sin t + \frac{1}{\frac{\sin\frac{t}{2}}{\cos\frac{t}{2}}} \times \frac{1}{\cos^2\frac{t}{2}} \times \frac{1}{2}$$

$$= -\sin t + \frac{1}{\frac{\sin\frac{t}{2}}{\cos\frac{t}{2}}} \times \frac{1}{\cos^2\frac{t}{2}} \times \frac{1}{2}$$

$$= -\sin t + \frac{1}{2\sin\frac{t}{2}\cos\frac{t}{2}}$$

$$= -\sin t + \frac{1}{\sin t}$$

$$= \frac{1-\sin^2 t}{\sin t}$$

$$= \frac{\cos^2 t}{\sin t}$$

$$= \cos t \times \cot t$$

Now find the value of $\frac{dy}{dt}$:

$$\frac{dy}{dt} = \cos t$$

Thus, $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \cos t \times \frac{1}{\cos t \times \cot t}$

$$\Rightarrow \frac{dy}{dx} = \tan t$$

Since $\frac{dy}{dt} = \cos t$, *we have* $\frac{d^2y}{dt^2} = -\sin t$

At $t = \frac{\pi}{4}, \left(\frac{d^2y}{dt^2}\right)_{t=\frac{\pi}{4}} = -\sin\left(\frac{\pi}{4}\right) = \frac{-1}{\sqrt{2}}$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

$$= \frac{\frac{d}{dt}(\tan t)}{\cos t \times \cot t}$$

$$= \frac{sec^2 t}{\cos t \times \cot t}$$

$$= \frac{sec^2 t}{\cos t \times \frac{\cos t}{\sin t}}$$

$$= \frac{sec^2 t}{\cos^2 t} \times \sin t$$

$$= sec^4 t \times \sin t$$

Thus, $\left(\frac{d^2y}{dx^2}\right)_{t=\frac{\pi}{4}} = sec^4\left(\frac{\pi}{4}\right) \times \sin\frac{\pi}{4} = 2$

Higher Order Derivatives Ex 12.1 Q44

$$x = a\sin t \text{ and } y = a\left(\cos t + \log\tan\frac{t}{2}\right)$$

$$\frac{dx}{dt} = a\cos t$$

$$\frac{d^2x}{dt^2} = -a\sin t$$

$$\frac{dy}{dt} = -a\sin t + a\frac{1}{\tan\frac{t}{2}} \times \sec^2\frac{t}{2} \times \frac{1}{2}$$

$$= -a\sin t + a\frac{1}{2\sin\frac{t}{2}\cos\frac{t}{2}}$$

$$= -a\sin t + a\,\text{cosec}\,t$$

$$\frac{d^2y}{dt^2} = -a\cos t - a\,\text{cosec}\,t\cot t$$

$$\frac{d^2y}{dx^2} = \frac{\frac{dx}{dt}\frac{d^2y}{dt^2} - \frac{dy}{dt}\frac{d^2x}{dt^2}}{\left(\frac{dx}{dt}\right)^3}$$

$$= \frac{a\cos t\left(-a\cos t - a\,\text{cosec}\,t\cot t\right) - \left(-a\sin t + a\,\text{cosec}\,t\right)\left(-a\sin t\right)}{\left(a\cos t\right)^3}$$

$$= \frac{-a^2\cos^2 t - a^2\cot^2 t - a^2\sin^2 t + a^2}{a^3\cos^3 t}$$

$$= \frac{-a^2\cos^2 t - a^2\sin^2 t - a^2\cot^2 t + a^2}{a^3\cos^3 t}$$

$$= \frac{-a^2\left(\cos^2 t + \sin^2 t\right) - a^2\cot^2 t + a^2}{a^3\cos^3 t}$$

$$= -\frac{1}{a\sin^2 t\cos t}$$

Higher Order Derivatives Ex 12.1 Q45

$$x = a\left(\cos t + t\sin t\right)$$

$$\frac{dx}{dt} = -a\sin t + at\cos t + a\sin t$$

$$= at\cos t$$

$$\frac{d^2x}{dt^2} = -at\sin t + a\cos t$$

$$y = a\left(\sin t - t\cos t\right)$$

$$\frac{dy}{dt} = a\cos t - a\cos t + at\sin t$$

$$= at\sin t$$

$$\frac{d^2y}{dt^2} = at\cos t + a\sin t$$

$$\frac{d^2y}{dx^2} = \frac{\frac{dx}{dt}\frac{d^2y}{dt^2} - \frac{dy}{dt}\frac{d^2x}{dt^2}}{\left(\frac{dx}{dt}\right)^3}$$

$$= \frac{at\cos t\left(at\cos t + a\sin t\right) - at\sin t\left(-at\sin t + a\cos t\right)}{\left(at\cos t\right)^3}$$

$$= \frac{a^2t^2\cos^2 t + a^2t\cos t\sin t + a^2t^2\sin^2 t - a^2t\sin t\cos t}{\left(at\cos t\right)^3}$$

$$= \frac{a^2t^2}{a^3t^3\cos^3 t} = \frac{1}{at\cos^3 t}$$

$$\left.\frac{d^2y}{dx^2}\right|_{t=\frac{\pi}{4}} = \frac{1}{a \times \frac{\pi}{4}\cos^3\frac{\pi}{4}} = \frac{8\sqrt{2}}{\pi a}$$

Higher Order Derivatives Ex 12.1 Q46

$$x=a\left(\cos t+\log\tan\frac{t}{2}\right) \text{ and } y=a\sin t$$

$$\frac{dx}{dt}=-a\sin t+a\frac{1}{\tan\frac{t}{2}}\times\sec^2\frac{t}{2}\times\frac{1}{2}$$

$$=-a\sin t+a\frac{1}{2\sin\frac{t}{2}\cos\frac{t}{2}}$$

$$=-a\sin t+a\operatorname{cosec} t$$

$$\frac{d^2x}{dt^2}=-a\cos t-a\operatorname{cosec} t\cot t$$

$$\frac{dy}{dt}=a\cos t$$

$$\frac{d^2y}{dt^2}=-a\sin t$$

$$\frac{d^2y}{dx^2}=\frac{\frac{dx}{dt}\frac{d^2y}{dt^2}-\frac{dy}{dt}\frac{d^2x}{dt^2}}{\left(\frac{dx}{dt}\right)^3}$$

$$=\frac{(-a\sin t+a\operatorname{cosec} t)(-a\sin t)-(a\cos t)(-a\cos t-a\operatorname{cosec} t\cot t)}{(-a\sin t+a\operatorname{cosec} t)^3}$$

$$=\frac{a^2\sin^2 t+a^2\cos^2 t-a^2+a^2\cot^2 t}{\left(-a\sin t+\frac{a}{\sin t}\right)^3}$$

$$=\frac{a^2\cot^2 t}{a^3\cos^6 t}\times\sin^3 t=\frac{1}{a}\times\frac{\sin t}{\cos^4 t}$$

$$\left.\frac{d^2y}{dx^2}\right|_{t=\frac{\pi}{3}}=\frac{1}{a}\times\frac{\sin\frac{\pi}{3}}{\cos^4\frac{\pi}{3}}=\frac{8\sqrt{3}}{a}$$

Higher Order Derivatives Ex 12.1 Q47

$$x=a(\cos 2t+2t\sin 2t)$$

$$\frac{dx}{dt}=-2a\sin 2t+2a\sin 2t+4at\cos 2t=4at\cos 2t$$

$$y=a(\sin 2t-2t\cos 2t)$$

$$\frac{dy}{dt}=2a\cos 2t-2a\cos 2t+4at\sin 2t=4at\sin 2t$$

$$\frac{dy}{dx}=\tan 2t$$

$$\frac{d^2y}{dx^2}=\frac{d}{dx}(\tan 2t)$$

$$\frac{d^2y}{dx^2}=\sec^2 2t\frac{d}{dx}(2t)$$

$$\frac{d^2y}{dx^2}=2\sec^2 2t\frac{d}{dx}(t)$$

$$\frac{d^2y}{dx^2}=2\sec^2 2t\times\frac{1}{4at\cos 2t}$$

$$\frac{d^2y}{dx^2}=\frac{1}{2a}\sec^3 2t$$

Higher Order Derivatives Ex 12.1 Q48

$x = a\sin t - b\cos t; \quad y = a\cos t + b\sin t$

Differentiating both w.r.t.t

$$\Rightarrow \quad \frac{dx}{dt} = a\cos t + b\sin t; \quad \frac{dy}{dt} = -a\sin t + b\cos t$$

$$\Rightarrow \quad \frac{dx}{dt} = y \ \ldots\ldots(1) \quad ; \quad \frac{dy}{dt} = -x \ \ldots\ldots(2)$$

Dividing (2) by (1)

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} == -\frac{x}{y}$$

Differentiating w.r.t.t

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{dt} = -\left\{\frac{y\frac{dx}{dt} - x\frac{dy}{dt}}{y^2}\right\}$$

Putting values from (1) and (2)

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{dt} = -\left\{\frac{y^2 + x^2}{y^2}\right\} \ldots\ldots\ldots(3)$$

Dividing (3) by (1)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\left\{\frac{y^2 + x^2}{y^2 \times y}\right\} = -\left\{\frac{x^2 + y^2}{y^3}\right\}$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q49

$y = A \sin 3x + B \cos 3x$
differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = 3A \cos 3x + 3B\left(-\sin 3x\right)$$

again differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 3A\left(-\sin 3x\right) \times 3 - 3B\left(\cos 3x\right) \times 3$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -9\left(A \sin 3x + B \cos 3x\right) = -9y$$

Now adding $\frac{d^2y}{dx^2} + \frac{4dy}{dx} + 3y$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + \frac{4dy}{dx} + 3y = -9y + 4\left(3A \cos 3x - 3B \sin 3x\right) + 3y$$

$$= 12\left(A \cos 3x - B \sin 3x\right) - 6\left(A \sin 3x + B \cos 3x\right)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + \frac{4dy}{dx} + 3y = \left(12A - 6B\right)\cos 3x - \left(12B + 6A\right)\sin 3x$$

But given,

$$\frac{d^2y}{dx^2} + \frac{4dy}{dx} + 3y = 10 \cos 3x$$

Thus, $12A - 6B = 10$ ....(1)
and $-\left(12B + 6A\right) = 0$ ....(2)

solving (2)
$$12B + 6A = 0 \Rightarrow 6A = -12B \Rightarrow A = -2B$$

Putting value of A in (1)

$$\Rightarrow \quad 12\left(-2B\right) - 63 = 10$$
$$\Rightarrow \quad -24B - 6B = 10$$
$$\Rightarrow \quad -30B = 10$$
$$\Rightarrow \quad B = \frac{-1}{3}$$
$$\Rightarrow \quad \therefore \quad A = -2 \times \frac{-1}{3} = \frac{2}{3}$$

and $A = \frac{2}{3}$; $\quad B = \frac{-1}{3}$

Higher Order Derivatives Ex 12.1 Q50

$$y = Ae^{-kt}\cos(pt+c)$$

differentiating w.r.t. t

$$\Rightarrow \quad \frac{dy}{dt} = A\left\{e^{-kt}\left(-\sin(pt+c)\times p\right)+\left(\cos(pt+c)\right)\left(-re^{-kt}\right)\right\}$$

$$\Rightarrow \quad -Ape^{-kt}\sin(pt+c) - kAe^{-kt}\cos(pt+c)$$

$$\Rightarrow \quad \frac{dy}{dt} = -Ape^{-kt}\sin(pt+c) - ky$$

differentiating w.r.t. t

$$\Rightarrow \quad \frac{d^2y}{dt^2} = -Ap\left\{e^{-kt}\left(\cos(pt+c)\times p\right)+\left(\sin(pt+c)\right)\left(e^{-kt}\times -R\right) - ky^1\right\}$$

$$= -p^2y + Apke^{-kt}\sin(pt+c) - ky^1$$

Adding & subtracting ky^1 on RHS

$$\Rightarrow \quad \frac{d^2y}{dt^2} = +Apke^{-kt}\sin(pt+c) - p^2y - 2ky^1 + ky^1$$

$$\frac{d^2y}{dt^2} = Apke^{-kt}\sin(pt+c) - p^2y - 2ky^1 - kApe^{-kt}\sin(pt+c) - k^2y$$

$$\Rightarrow \quad \frac{d^2y}{dt^2} = -\left(p^2+k^2\right)y - 2k\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{d^2y}{dt^2} + 2k\frac{dy}{dt} + n^2y = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q51

$$y = x^n\left\{a\cos(\log x) + b\sin(\log x)\right\}$$

$$y = ax^n\cos(\log x) + bx^n\sin(\log x)$$

$$\frac{dy}{dx} = anx^{n-1}\cos(\log x) - ax^{n-1}\sin(\log x) + bnx^{n-1}\sin(\log x) + bx^{n-1}\cos(\log x)$$

$$\frac{dy}{dx} = x^{n-1}\cos(\log x)(na+b) + x^{n-1}\sin(\log x)(bn-a)$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(x^{n-1}\cos(\log x)(na+b) + x^{n-1}\sin(\log x)(bn-a)\right)$$

$$\frac{d^2y}{dx^2} = (na+b)\left[(n-1)x^{n-2}\cos(\log x) - x^{n-2}\sin(\log x)\right] + (bn-a)\left[(n-1)x^{n-2}\sin(\log x) + x^{n-2}\cos(\log x)\right]$$

$$\frac{d^2y}{dx^2} = (na+b)x^{n-2}\left[(n-1)\cos(\log x) - \sin(\log x)\right] + (bn-a)x^{n-2}\left[(n-1)\sin(\log x) + \cos(\log x)\right]$$

$$x^2\frac{d^2y}{dx^2} + (1-2n)\frac{dy}{dx} + \left(1+n^2\right)y$$

$$= (na+b)x^n\left[(n-1)\cos(\log x) - \sin(\log x)\right] + (bn-a)x^n\left[(n-1)\sin(\log x) + \cos(\log x)\right]$$

$$+ (1-2n)x^{n-1}\cos(\log x)(na+b) + (1-2n)x^{n-1}\sin(\log x)(bn-a)$$

$$+ a\left(1+n^2\right)x^n\cos(\log x) + b\left(1+n^2\right)x^n\sin(\log x)$$

$$= 0$$

Higher Order Derivatives Ex 12.1 Q52

$$y = a\left\{x+\sqrt{x^2+1}\right\}^n + b\left\{x-\sqrt{x^2+1}\right\}^{-n},$$

$$\frac{dy}{dx} = na\left\{x+\sqrt{x^2+1}\right\}^{n-1}\left[1+x\left(x^2+1\right)^{-\frac{1}{2}}\right] - nb\left\{x-\sqrt{x^2+1}\right\}^{-n-1}\left[1-x\left(x^2+1\right)^{-\frac{1}{2}}\right]$$
$$\frac{dy}{dx} = \frac{na}{\sqrt{x^2+1}}\left\{x+\sqrt{x^2+1}\right\}^n + \frac{nb}{\sqrt{x^2+1}}\left\{x-\sqrt{x^2+1}\right\}^{-n}$$
$$\frac{dy}{dx} = \frac{n}{\sqrt{x^2+1}}\left[a\left\{x+\sqrt{x^2+1}\right\}^n + b\left\{x-\sqrt{x^2+1}\right\}^{-n}\right]$$

$$x\frac{dy}{dx} = \frac{nx}{\sqrt{x^2+1}}y$$

$$\frac{d^2y}{dx^2} = \frac{nx}{\sqrt{x^2+1}}\frac{dy}{dx} + y\left[\frac{\sqrt{x^2+1} - x^2\left(x^2+1\right)^{-\frac{1}{2}}}{x^2+1}\right]$$
$$\frac{d^2y}{dx^2} = \frac{n^2x^2}{x^2+1} + y\left[\frac{1}{\left(x^2+1\right)\sqrt{x^2+1}}\right]$$
$$\frac{d^2y}{dx^2} = \frac{n^2x^2\left(\sqrt{x^2+1}\right)+y}{\left(x^2+1\right)\sqrt{x^2+1}}$$

$$\left(x^2-1\right)\frac{d^2y}{dx^2} = \frac{n^2x^4\left(\sqrt{x^2+1}\right)+x^2y}{\left(x^2+1\right)\sqrt{x^2+1}} - \frac{n^2x^2\left(\sqrt{x^2+1}\right)+y}{\left(x^2+1\right)\sqrt{x^2+1}}$$

Now
$$\left(x^2-1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - ny$$
$$= \frac{n^2x^4\left(\sqrt{x^2+1}\right)+x^2y}{\left(x^2+1\right)\sqrt{x^2+1}} - \frac{n^2x^2\left(\sqrt{x^2+1}\right)+y}{\left(x^2+1\right)\sqrt{x^2+1}} + \frac{nx}{\sqrt{x^2+1}}y - ny$$
$$= 0$$

Ex 13.1

Derivatives as a Rate Measurer Ex 13.1 Q1

Let total suface area of the cylinder be A

$$A = 2\pi r\left(h + r\right)$$

Differentiating it with respect to r as r varies

$$\frac{dA}{dr} = 2\pi r\left(0 + 1\right) + \left(h + r\right)2\pi$$

$$= 2\pi r + 2\pi h + 2\pi r$$

$$\frac{dA}{dr} = 4\pi r + 2\pi h$$

Derivatives as a Rate Measurer Ex 13.1 Q2

Let D be the diatmeter and r be the radius of sphere,

So, volume of sphere $= \frac{4}{3}\pi r^2$

$$v = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3$$

$$v = \frac{4}{24}\pi D^3$$

Differentiating it with respect to D.

$$\frac{dv}{dD} = \frac{12}{24}\pi D^2$$

$$\frac{dv}{dD} = \frac{\pi D^2}{2}$$

Derivatives as a Rate Measurer Ex 13.1 Q3

Given, radius of sphere $(r) = 2\text{cm}$.

We know that,

$$v = \frac{4}{3}\pi r^2$$

$$\frac{dv}{dr} = 4\pi r^2 \qquad \text{---(i)}$$

And $A = 4\pi r^2$

$$\frac{dA}{dr} = 8\pi r^2 \qquad \text{---(ii)}$$

Dividing equation (i) by (ii),

$$\frac{\frac{dv}{dr}}{\frac{dA}{dr}} = \frac{4\pi r^2}{8\pi r}$$

$$\frac{dv}{dA} = \frac{r}{2}$$

$$\left(\frac{dv}{dA}\right)_{r=2} = 1$$

Derivatives as a Rate Measurer Ex 13.1 Q4

Let r be two radius of circular disc.

We know that,

Area $A = \pi r^2$

$$\frac{dA}{dr} = 2\pi r \qquad \text{---(i)}$$

Circumference $C = 2\pi r$

$$\frac{dc}{dr} = 2\pi \qquad \text{---(ii)}$$

Dividing equation (i) by (ii),

$$\frac{\frac{dA}{dr}}{\frac{dc}{dr}} = \frac{2\pi r}{2\pi}$$

$$\frac{dA}{dc} = r$$

$$\left(\frac{dA}{dc}\right)_{r=3} = 3$$

Derivatives as a Rate Measurer Ex 13.1 Q5

Let r be the radius, v be the volume of cone and h be height

$$v = \frac{1}{3}\pi r^2 h$$

$$\frac{dv}{dr} = \frac{2}{3}\pi rh.$$

Derivatives as a Rate Measurer Ex 13.1 Q6

Let r be radius and A be area of circle, so

$$A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r$$

$$\left(\frac{dA}{dr}\right)_{r=5} = 2\pi\,(5)$$

$$\left(\frac{dA}{dr}\right)_{r=5} = 10\pi$$

Derivatives as a Rate Measurer Ex 13.1 Q7

Here, $r = 2$ cm

$$v = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$\left(\frac{dV}{dr}\right)_{r=2} = 4\pi(2)^2$$

$$\left(\frac{dV}{dr}\right)_{r=2} = 16\pi$$

Derivatives as a Rate Measurer Ex 13.1 Q8

Marginal cost is the rate of change of total cost with respect to output.

$$\therefore \text{Marginal cost (MC)} = \frac{dC}{dx} = 0.007\left(3x^2\right) - 0.003\left(2x\right) + 15$$

$$= 0.021x^2 - 0.006x + 15$$

When $x = 17$, MC $= 0.021\ (17^2) - 0.006\ (17) + 15$

$= 0.021(289) - 0.006(17) + 15$

$= 6.069 - 0.102 + 15$

$= 20.967$

Hence, when 17 units are produced, the marginal cost is Rs. 20.967

Derivatives as a Rate Measurer Ex 13.1 Q9

Marginal revenue is the rate of change of total revenue with respect to the number of units sold.

$$\therefore \text{Marginal Revenue (MR)} = \frac{dR}{dx} = 13(2x) + 26 = 26x + 26$$

When $x = 7$,

MR $= 26(7) + 26 = 182 + 26 = 208$

Hence, the required marginal revenue is Rs 208.

Derivatives as a Rate Measurer Ex 13.1 Q10

$$R(x) = 3x^2 + 36x + 5$$

$$\frac{dR}{dx} = 6x + 36$$

$$\left.\frac{dR}{dx}\right|_{x=5} = 6 \times 5 + 36$$

$$= 30 + 36$$

$$= 66$$

This, as per the question, indicates the money to be spent
on the welfare of the employess, when the number of employees is 5.

Ex 13.2

Derivatives as a Rate Measurer Ex 13.2 Q1

Let x be the side of square.

Given, $\frac{dx}{dt} = 4$ cm/min, $x = 8$ cm

We know that

$$\text{Area } (A) = x^2$$
$$\frac{dA}{dt} = 2x\frac{dx}{dt}$$
$$\left(\frac{dA}{dt}\right)_{8 \text{ cm}} = 2 \times (8)(4)$$
$$\frac{dA}{dt} = 64 \text{ cm}^2/\text{min}$$

Area increases at a rate of 64 cm^2/min.

Derivatives as a Rate Measurer Ex 13.2 Q2

Let edge of the cube is x cm.

$$\frac{dx}{dt} = 3 \text{ cm/sec}, x = 10 \text{ cm}$$

Let V be volume of cube,

$$V = x^3$$
$$\frac{dV}{dt} = 3x^2\frac{dx}{dt}$$
$$= 3(10)^2 \times (3)$$
$$= 900 \text{ cm}^3/\text{sec}$$

So,

Volume increases at a rate of 900 cm^3/sec.

Derivatives as a Rate Measurer Ex 13.2 Q3

Let x be the side of the square.

Here, $\frac{dx}{dt} = 0.2$ cm/sec.

$$P = 4x$$

$$\frac{dP}{dt} = 4\frac{dx}{dt}$$

$$= 4 \times (0.2)$$

$$\frac{dP}{dt} = 0.8 \text{ cm/sec}$$

So, perimeter increases at the rate of 0.8 cm /sec.

Derivatives as a Rate Measurer Ex 13.2 Q4

The circumference of a circle (C) with radius (r) is given by

$C = 2\pi r$.

Therefore, the rate of change of circumference (C) with respect to time (t) is given by,

$$\frac{dC}{dt} = \frac{dC}{dr} \cdot \frac{dr}{dt} \text{ (By chain rule)}$$

$$= \frac{d}{dr}(2\pi r)\frac{dr}{dt}$$

$$= 2\pi \cdot \frac{dr}{dt}$$

It is given that $\frac{dr}{dt} = 0.7$ cm/s.

Hence, the rate of increase of the circumference is $2\pi(0.7) = 1.4\pi$ cm/s.

Derivatives as a Rate Measurer Ex 13.2 Q5

Let r be the radius of the spherical soap bubble.

Here, $\frac{dr}{dt} = 0.2$ cm/sec, $r = 7$ cm

Surface Area $(A) = 4\pi r^2$

$$\frac{dA}{dt} = 4\pi(2r)\frac{dr}{dt}$$

$$\left(\frac{dA}{dt}\right)_{r=7} = 4\pi(2 \times 7) \times 0.$$

$$= 11.2\pi \text{ cm}^2\text{/sec.}$$

So, area of bubble increases at the rate of 11.2π cm^2/sec.

Derivatives as a Rate Measurer Ex 13.2 Q6

The volume of a sphere (V) with radius (r) is given by,

$$V = \frac{4}{3}\pi r^3$$

$\therefore$Rate of change of volume (V) with respect to time (t) is given by,

$$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt} \text{ [By chain rule]}$$

$$= \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right) \cdot \frac{dr}{dt}$$

$$= 4\pi r^2 \cdot \frac{dr}{dt}$$

It is given that $\frac{dV}{dt} = 900 \text{ cm}^3/\text{s}$.

$$\therefore 900 = 4\pi r^2 \cdot \frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt} = \frac{900}{4\pi r^2} = \frac{225}{\pi r^2}$$

Therefore, when radius = 15 cm,

$$\frac{dr}{dt} = \frac{225}{\pi(15)^2} = \frac{1}{\pi}$$

Hence, the rate at which the radius of the balloon increases when the radius is 15 cm is $\frac{1}{\pi}$ cm/s.

Derivatives as a Rate Measurer Ex 13.2 Q7

Let r be the radius of the air bubble.

Here, $\frac{dr}{dt} = 0.5$ cm/sec, $r = 1$ cm

Volume $(V) = \frac{4}{3}\pi r^3$

$$\frac{dV}{dt} = \frac{4}{3}\pi\left(3r^2\right)\frac{dr}{dt}$$

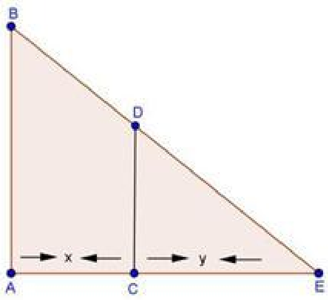
$$= 4\pi r^2 \frac{dr}{dt}$$

$$= 4\pi\left(1\right)^2 \times \left(0.5\right)$$

$$\frac{dV}{dt} = 2\pi \text{ cm}^3\text{/sec.}$$

So, volume of air bubble increases at the rate of 2π cm^3/sec.

Derivatives as a Rate Measurer Ex 13.2 Q8



Let AB be the lamp-post. Suppose at time t, the man CD is at a distance of x meters from the lamp-post and y meters be the length of his shadow CB.

Here, $\frac{dx}{dt} = 5$ km/hr

$CD = 2$ m, $AB = 6$ m

Here, ΔABE and ΔCDE are similar, so

$$\frac{AB}{CD} = \frac{AE}{CE}$$

$$\frac{6}{2} = \frac{x+y}{y}$$

$$3y = x + y$$

$$2y = x$$

$$2\frac{dy}{dt} = \frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{5}{2} \text{ km/hr}$$

So, the length of his shadow increases at the rate of $\frac{5}{2}$ km/hr.

Derivatives as a Rate Measurer Ex 13.2 Q9

The area of a circle (A) with radius (r) is given by $A = \pi r^2$.

Therefore, the rate of change of area (A) with respect to time (t) is given by,

$$\frac{dA}{dt} = \frac{d}{dt}\left(\pi r^2\right) = \frac{d}{dr}\left(\pi r^2\right)\frac{dr}{dt} = 2\pi r\frac{dr}{dt} \text{ [By chain rule]}$$

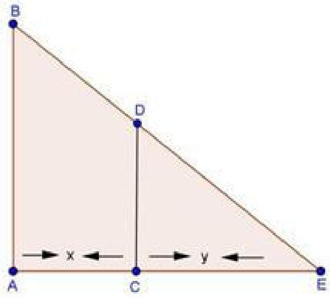
It is given that $\frac{dr}{dt} = 4$ cm/s.

Thus, when $r = 10$cm,

$$\frac{dA}{dt} = 2\pi(10)(4) = 80\pi$$

Hence, when the radius of the circular wave is 8 cm, the enclosed area is increasing at the rate of 80π cm^2/s

Derivatives as a Rate Measurer Ex 13.2 Q10



Let AB be the height of pole. Suppose at time t, the man CD is at a distance of x meters from the lamp-post and y meters be the length of his shadow CE, then

$$\frac{dx}{dt} = 1.1 \text{ m/sec}$$

ΔABE is similar to ΔCDE,

$$\frac{AB}{CD} = \frac{AE}{CE}$$

$$\frac{600}{160} = \frac{x+y}{y}$$

$$\frac{15}{4} = \frac{x+y}{y}$$

$$15y = 4x + 4y$$

$$11y = 4x$$

$$11\frac{dy}{dx} = 4\frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{4}{11}(1.1)$$

$$\frac{dy}{dt} = 0.4 \text{ m/sec}$$

Rate of increasing of shadow = 0.4 m/sec.

Derivatives as a Rate Measurer Ex 13.2 Q11

Let AB be the height of source of light. Suppose at time t, the man CD is at a distance of x meters from the lamp-post and y meters be the length of his shadow CE, then

$$\frac{dx}{dt} = 2 \text{ m/sec}$$

ΔABE is similar to ΔCDE,

$$\frac{AB}{CD} = \frac{AE}{CE}$$

$$\frac{900}{180} = \frac{x+y}{y}$$

$$5y = x + y$$

$$4y = x$$

$$4\frac{dy}{dt} = \frac{dx}{dt}$$

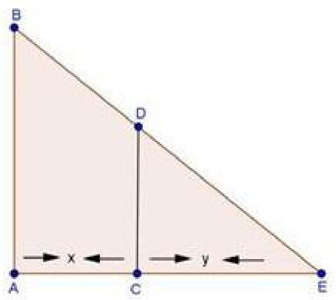
$$\frac{dy}{dt} = \frac{2}{4}$$

$$= \frac{1}{2}$$

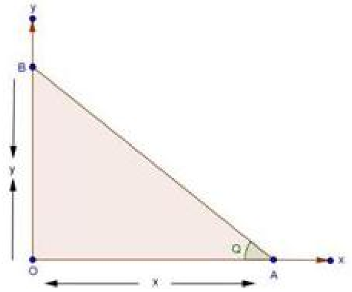
$$\frac{dy}{dt} = 0.5 \text{ m/sec}$$

So, rate of increase of shadow is 0.5 m/sec.

The diagram of the problem is shown below



Derivatives as a Rate Measurer Ex 13.2 Q12



Let AB be the position of the ladder, at time t, such that $OA = x$ and $OB = y$

Here,

$$OA^2 + OB^2 = AB^2$$
$$x^2 + y^2 = (13)^2$$
$$x^2 + y^2 = 169 \qquad \text{---(i)}$$

And $\frac{dx}{dt} = 1.5 \text{ m/sec}$

From figure, $\tan\theta = \frac{y}{x}$

Differentiating equation (i) with respect to t,

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$
$$2(1.5)x + 2y\frac{dy}{dt} = 0$$
$$3x + 2y\frac{dy}{dt} = 0$$
$$\frac{dy}{dt} = -\frac{3x}{2y}$$

Differentiating equation (ii) with respect to t,

$$\sec^2\theta\frac{d\theta}{dt} = \frac{d\frac{dy}{dt} - y\frac{dx}{dt}}{x^2}$$
$$= \frac{x \times \left(-\frac{3x}{2y}\right) - y(1.5)}{x^2}$$
$$= \frac{-1.5x^2 - 1.5y^2}{yx^2}$$
$$\frac{d\theta}{dt} = \frac{-1.5\left(x^2 + y^2\right)}{x^2 y \sec^2\theta}$$
$$= \frac{-1.5\left(x^2 + y^2\right)}{x^2 y\left(1 + \tan^2\theta\right)}$$
$$\frac{d\theta}{dt} = \frac{-1.5\left(x^2 + y^2\right)}{x^2 y\left(1 + \frac{y^2}{x^2}\right)}$$
$$= \frac{-1.5\left(x^2 + y^2\right) \times x^2}{x^2 y\left(x^2 + y^2\right)}$$
$$= \frac{-1.5}{y}$$
$$= \frac{-1.5}{\sqrt{169 - x^2}}$$
$$= \frac{-1.5}{\sqrt{169 - 144}}$$
$$= \frac{-1.5}{5}$$
$$= -0.3 \text{ radian/sec}$$

So, angle between ladder and ground is decreasing at the rate of 0.3 radian/sec.

Derivatives as a Rate Measurer Ex 13.2 Q13

Here, curve is

$$y = x^2 + 2x$$

And $\quad \frac{dy}{dt} = \frac{dx}{dt} \qquad$ ---(i)

$$y = x^2 + 2x$$

$\Rightarrow \quad \frac{dy}{dt} = 2x\frac{dx}{dt} + 2\frac{dx}{dt}$

$\Rightarrow \quad \frac{dy}{dt} = \frac{dx}{dt}(2x + 2)$

Using equation (i),

$$2x + 2 = 1$$
$$2x = -1$$
$$x = -\frac{1}{2}$$

So, $\quad y = x^2 + 2x$

$$= \left(-\frac{1}{2}\right)^2 + 2\left(-\frac{1}{2}\right)$$
$$= \frac{1}{4} - 1$$
$$y = -\frac{3}{4}$$

So, required points is $\left(-\frac{1}{2}, -\frac{3}{4}\right)$.

Derivatives as a Rate Measurer Ex 13.2 Q14

Here,

$\frac{dx}{dt} = 4$ units/sec, and $x = 2$

And, $\quad y = 7x - x^3$

Slope of the curve$(S) = \frac{dy}{dx}$

$$S = 7 - 3x^2$$
$$\frac{ds}{dt} = -6x\frac{dx}{dt}$$
$$= -6(2)(4)$$
$$= -48 \text{ units/sec}$$

So, slope is decreasing at the rate of 48 units/sec.

Derivatives as a Rate Measurer Ex 13.2 Q15

Here,

$\frac{dy}{dt} = 3\frac{dx}{dt} \qquad$ ---(i)

And, $\quad y = x^3$

$$\frac{dy}{dt} = 3x^2\frac{dx}{dt}$$

$3\frac{dx}{dt} = 3x^2\frac{dx}{dt} \qquad$ [Using equation (i)]

$$3x^2 = 3$$

$$x^2 = 1$$

$$x = \pm 1$$

Put $x = 1 \Rightarrow y = (1)^3 = 1$

Put $x = -1 \Rightarrow y = (-1)^3 = -13$

So, the required points are $(1,1)$ and $(-1,-1)$.

Derivatives as a Rate Measurer Ex 13.2 Q16(i)

Here,

$$2\frac{d(\sin\theta)}{dt} = \frac{d\theta}{dt}$$

$$2 \times \cos\theta\frac{d\theta}{dt} = \frac{d\theta}{dt}$$

$$2\cos\theta = 1$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}.$$

Derivatives as a Rate Measurer Ex 13.2 Q16(ii)

$$\frac{d\theta}{dt} = -2\frac{d}{dt}(\cos\theta)$$

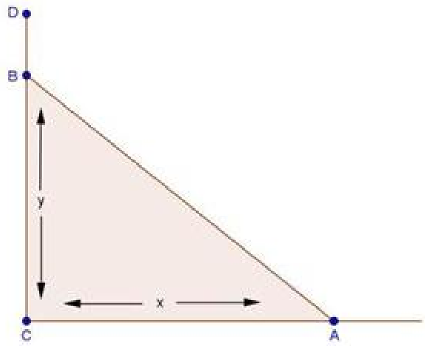
$$\frac{d\theta}{dt} = -2(-\sin\theta)\frac{d\theta}{dt}$$

$$1 = 2\sin\theta$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}$$

Derivatives as a Rate Measurer Ex 13.2 Q17



Let CD be the wall and AB is the ladder

Here, $AB = 6$ meter and $\left(\frac{dx}{dt}\right)_{x=4} = 0.5$ m/sec.

From figure,

$$AB^2 = x^2 + y^2$$

$$(6)^2 = x^2 + y^2$$

$$36 = x^2 + y^2$$

Differentiating it with respect to t,

$$0 = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt} \qquad \text{---(i)}$$

$$\left(\frac{dy}{dt}\right)_{x=4} = \frac{4(0.5)}{\sqrt{36 - x^2}}$$

$$= -\frac{2}{\sqrt{36 - 16}}$$

$$= -\frac{2}{2\sqrt{5}}$$

$$= -\frac{1}{\sqrt{5}} \text{ m/sec.}$$

So, ladder top is sliding at the rate of $\frac{1}{\sqrt{5}}$ m/sec.

Now, to find x when $\frac{dx}{dt} = -\frac{dy}{dt}$

From equation (i),

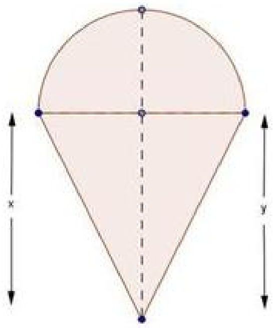
$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$
$$-\frac{dx}{dt} = -\frac{x}{y}\frac{dx}{dt}$$
$$x = y$$

Now,

$$36 = x^2 + y^2$$
$$36 = x^2 + x^2$$
$$2x^2 = 36$$
$$x^2 = 18$$
$$x = 3\sqrt{2} \text{ m}$$

When foot and top are moving at the same rate, foot of wall is $3\sqrt{2}$ meters away from the wall

Derivatives as a Rate Measurer Ex 13.2 Q18



Let height of the cone is x cm. and radius of sphere is r cm.

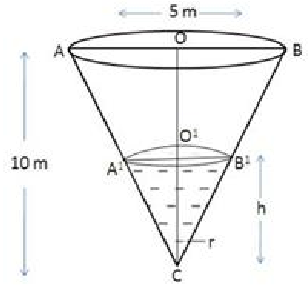
Here given,

$$x = 2r \qquad \text{---(i)}$$
$$h = x + r$$
$$h = 2r + r$$
$$h = 3r \qquad \text{---(ii)}$$

$$v = \text{volume of cone} + \text{volume of hemisphere}$$
$$= \frac{1}{3}\pi r^2 x + \frac{2}{3}\pi r^3$$
$$= \frac{1}{3}\pi r^2(2r) + \frac{2}{3}\pi r^3 \qquad [\text{Using equation (ii)}]$$
$$v = \frac{2}{3}\pi r^3 + \frac{2}{3}\pi r^3$$
$$= \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi\left(\frac{h}{3}\right)^3$$
$$v = \frac{4}{81}\pi h^3$$
$$\frac{dv}{dh} = \frac{4}{81}\pi \times 3h^2$$
$$\left(\frac{dv}{dh}\right)_{h=9} = \frac{12}{81}\pi(9)^2$$
$$\left(\frac{dv}{dh}\right)_{h=9} = 12\pi \text{ cm}^2$$

Volume is changing at the rate 12π cm^2 with respect to total height.

Derivatives as a Rate Measurer Ex 13.2 Q19



Let α be the semi vertical angle of the cone CAB whose height CO is 10 m and radius OB = 5 m.

Now,

$$\tan\alpha = \frac{OB}{CO}$$
$$= \frac{5}{10}$$
$$\tan\alpha = \frac{1}{2}$$

Let V be the volume of the water in the cone, then

$$v = \frac{1}{3}\pi\left(O'B'\right)^2\left(CO'\right)$$
$$= \frac{1}{3}\pi\left(h\tan\alpha\right)^2(h)$$
$$v = \frac{1}{3}\pi h^3\tan^2\alpha$$
$$v = \frac{\pi}{12}h^2 \qquad \left[\because \tan\alpha = \frac{1}{2}\right]$$
$$\frac{dv}{dt} = \frac{\pi}{12}3h^2\frac{dh}{dt}$$
$$\pi = \frac{\pi}{4}h^2\frac{dh}{dt} \qquad \left[\because \frac{dV}{dt} = m^3/\text{min}\right]$$
$$\frac{dh}{dt} = \frac{4}{h^2}$$
$$\left(\frac{dh}{dt}\right)_{2.5} = \frac{4}{(2.5)^2} \qquad \left[\because h = 10 - 7.5 = 2.5 \text{ m}\right]$$
$$= \frac{4}{6.25}$$
$$= 0.64 \text{ m/min}$$

So, water level is rising at the rate of 0.64 m/min.

Derivatives as a Rate Measurer Ex 13.2 Q20

Let AB be the lamp-post. Suppose at time t, the man CD is at a distance x m. from the lamp-post and y m be the length of the shadow CE.

Here, $\frac{dx}{dt} = 6$ km/hr

$CD = 2$ m, $AB = 6$ m

Here, ΔABE and ΔCDE are similar

So, $\frac{AB}{CD} = \frac{AE}{CE}$

$\frac{6}{2} = \frac{x+y}{y}$

$3y = x + y$

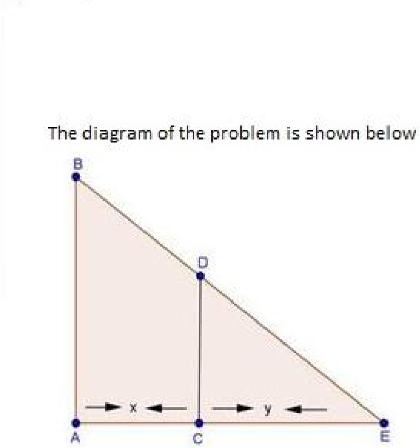
$2y = x$

$2\frac{dy}{dt} = \frac{dx}{dt}$

$2\frac{dy}{dt} = 6$

$\frac{dy}{dt} = 3$ km/hr

So, length of his shadow increases at the rate of 3 km/hr.



Derivatives as a Rate Measurer Ex 13.2 Q21

Here, $\frac{dA}{dt} = 2$ cm^2/sec

To find $\frac{dV}{dt}$ at $r = 6$ cm

$A = 4\pi r^2$

$\frac{dA}{dt} = 8\pi r\frac{dr}{dt}$

$2 = 8\pi r\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1}{4\pi r}$ cm/sec

Now, $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$

$= 4\pi r^2\left(\frac{1}{4\pi r}\right)$

$= r$

$\frac{dV}{dt} = 6$ cm^3/sec

So, volume of bubble is increasing at the rate of 6 cm^3/sec.

Derivatives as a Rate Measurer Ex 13.2 Q22

Here, $\frac{dr}{dt} = 2$ cm/sec, $\frac{dh}{dt} = -3$ cm/sec

To find $\frac{dV}{dt}$ when $r = 3$ cm, $h = 5$ cm

Now, V = volume of cylinder

$$V = \pi r^2 h$$
$$\frac{dV}{dt} = \pi\left[2r\frac{dr}{dt}\times h + r^2\frac{dh}{dt}\right]$$
$$= \pi\left[2(3)(2)(5) + (3)^2(-3)^2\right]$$
$$= \pi[60 - 27]$$
$$\frac{dV}{dt} = 33\pi \text{ cm}^3/\text{sec}$$

So, volume of cylinder is increasing at the rate of 33π cm^3/sec.

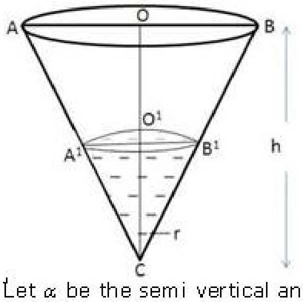
Derivatives as a Rate Measurer Ex 13.2 Q23

Let V be volume of sphere with miner radius r and onter radius R, then

$$V = \frac{4}{3}\pi\left(R^3 - r^3\right)$$
$$\frac{dV}{dt} = \frac{4}{3}\pi\left(3R^2\frac{dR}{dt} - 3r^2\frac{dr}{dt}\right)$$
$$0 = \frac{4\pi}{3}3\left(R^2\frac{dR}{dt} - r^2\frac{dr}{dt}\right) \qquad [\text{Since volume } V \text{ is constant}]$$
$$R^2\frac{dR}{dt} = r^2\frac{dr}{dt}$$
$$(8)^2\frac{dR}{dt} = (4)^2(1)$$
$$\frac{dR}{dt} = \frac{16}{64}$$
$$\frac{dR}{dt} = \frac{1}{4} \text{ cm/sec}$$

Rate of increasing of onter radius = $\frac{1}{4}$ cm/sec.

Derivatives as a Rate Measurer Ex 13.2 Q24



Let α be the semi vertical angle of the cone CAB whose height CO is half of radius OB.

Now,

$$\tan\alpha = \frac{OB}{CO}$$

$$= \frac{OB}{2OB} \qquad [\because CO = 2OB]$$

$$\tan\alpha = \frac{1}{2}$$

Let V be the volume of the sand in the cone

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$$

$$= \frac{\pi}{12}h^3$$

$$\frac{dV}{dt} = \frac{3\pi}{12}h^2\frac{dh}{dt}$$

$$50 = \frac{3\pi}{12}h^2\frac{dh}{dt} \qquad \left[\because \frac{dV}{dt} = 50\ \text{cm}^3/\text{min}\right]$$

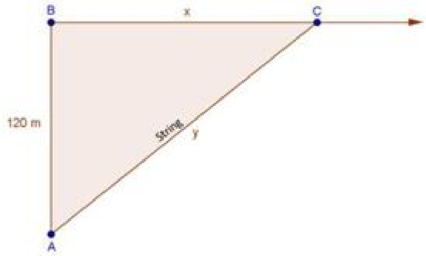
$$\frac{dh}{dt} = \frac{200}{\pi h^2}$$

$$= \frac{200}{\pi(5)^2}$$

$$\frac{dh}{dt} = \frac{8}{3.14}\ \text{cm/min}$$

Rate of increasing of height $= \frac{8}{\pi}$ cm/min

Derivatives as a Rate Measurer Ex 13.2 Q25



Let C be the position of kite and AC be the string.

Here, $y^2 = x^2 + (120)^2$ ---(i)

$$2y\frac{dy}{dt} = 2x\frac{dx}{dt}$$

$$y\frac{dy}{dt} = x\frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{x}{y}(52) \quad \text{---(ii)} \quad \left[\because \frac{dx}{dt} = 52 \text{ m/sec}\right]$$

From equation (i),

$$y^2 = x^2 + (120)^2$$

$$(130)^2 = x^2 + (120)^2$$

$$x^2 = 16900 - 14400$$

$$x^2 = 2500$$

$$x = 50$$

Using equation (ii),

$$\frac{dy}{dt} = \frac{x}{y}(52)$$

$$= \frac{50}{130}(52)$$

$$= 20 \text{ m/sec}$$

So, string is being paid out at the rate of 20 m/sec.

Derivatives as a Rate Measurer Ex 13.2 Q26

Here,

$$\frac{dy}{dt} = 2\frac{dx}{dt} \quad \text{---(i)}$$

and $y = \left(\frac{2}{3}\right)x^3 + 1$

$$\frac{dy}{dt} = \frac{2}{3} \times 3x^2\frac{dx}{dt}$$

$$2\frac{dx}{dt} = 2x^2\frac{dx}{dt} \quad [\text{Using equation (i)}]$$

$$2 = 2x^2$$

$\Rightarrow$ $x = \pm 1$

$$y = \left(\frac{2}{3}\right)x^3 + 1$$

Put $x = 1$, $y = \frac{2}{3} + 1 = \frac{5}{3}$

Put $x = -1$, $y = \frac{2}{3}(-1) + 1 = \frac{1}{3}$

So, required point $\left(1, \frac{5}{3}\right)$ and $\left(-1, \frac{1}{3}\right)$.

Derivatives as a Rate Measurer Ex 13.2 Q27

Here,

$$\frac{dx}{dt} = \frac{dy}{dt} \qquad ---(i)$$

and curve is

$$y^2 = 8x$$

$$2y\frac{dy}{dt} = 8\frac{dx}{dt}$$

$$2y = 8 \qquad \left[\text{using equation (i)}\right]$$

$$y = 4$$

$$\Rightarrow \quad (4)^2 = 8x$$

$$\Rightarrow \quad x = 2$$

So, required point = $(2, 4)$.

Derivatives as a Rate Measurer Ex 13.2 Q28

Let edge of cube be x cm

Here,

$$\frac{dV}{dt} = 9 \text{ cm}^3/\text{sec}$$

To find $\frac{dA}{dt}$ when $x = 10$ cm

We know that

$$V = x^3$$

$$\frac{dV}{dt} = 3x^2\left(\frac{dx}{dt}\right)$$

$$9 = 3(10)^2\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{3}{100} \text{ cm/sec}$$

Now, $A = 6x^2$

$$\frac{dA}{dt} = 12x\frac{dx}{dt}$$

$$= 12(10)\left(\frac{3}{100}\right)$$

$$\frac{dA}{dt} = 3.6 \text{ cm}^2/\text{sec.}$$

Derivatives as a Rate Measurer Ex 13.2 Q29

Given, $\frac{dV}{dt} = 25 \text{ cm}^3/\text{sec}$

To find $\frac{dA}{dt}$ when $r = 5$ cm

We know that,

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{4}{3}\pi\left(3r^2\right)\frac{dr}{dt}$$

$$25 = 4\pi(5)^2\frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{4\pi} \text{ cm/sec}$$

Now, $A = 4\pi r^2$

$$\frac{dA}{dt} = 8\pi r\frac{dr}{dt}$$

$$= 8\pi(5)\left(\frac{1}{4\pi}\right)$$

$$\frac{dA}{dt} = 10 \text{ cm}^2/\text{sec.}$$

Derivatives as a Rate Measurer Ex 13.2 Q30

Given,

$$\frac{dx}{dt} = -5 \text{ cm/min}$$
$$\frac{dy}{dt} = 4 \text{ cm/min}$$

(i) To find $\frac{dP}{dt}$ when $x = 8$ cm, $y = 6$ cm

$$P = 2(x + y)$$
$$\frac{dP}{dt} = 2\left(\frac{dx}{dt} + \frac{dy}{dt}\right)$$
$$= 2(-5 + 4)$$
$$\frac{dP}{dt} = -2 \text{ cm/min}$$

(ii) To find $\frac{dA}{dt}$ when $x = 8$ cm and $y = 6$ cm

$$A = xy$$
$$\frac{dA}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt}$$
$$= (8)(4) + (6)(-5)$$
$$= 32 - 30$$

$$\frac{dA}{dt} = 2 \text{ cm}^2\text{/min.}$$

Derivatives as a Rate Measurer Ex 13.2 Q31

Let r be the radius of the given disc and A be its area.

Then, $A = \pi r^2$

$\therefore \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ [by chain rule]

Now, the approximate increase ofradius = $dr = \frac{dr}{dt}\Delta t = 0.05 \text{cm /sec}$

$\therefore$ the approximate rate of increase in areais given by

$$dA = \frac{dA}{dt}(\Delta t) = 2\pi r\left(\frac{dr}{dt}\Delta t\right) = 2\pi(3.2)(0.05) = 0.320\pi \text{ cm}^3/\text{s}$$

Ex 14.1

Differentials Errors and Approximation Ex 14.1 Q1

Let $x = \frac{\pi}{2}$, $x + \Delta x = \frac{22}{14}$

$$\Delta x = \frac{22}{14} - x$$

$$\Delta x = \left(\frac{22}{14} - \frac{\pi}{2}\right)$$

$$y = \sin x$$

$$\frac{dy}{dx} = \cos x$$

$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}} = \frac{\cos \pi}{2}$$

$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}} = 0$$

$$\therefore \quad \Delta y = \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}} \times \Delta ax$$

$$= 0 \times \left(\frac{22}{14} - \frac{\pi}{2}\right)$$

$$\Delta y = 0$$

So, there is no change in y.

Differentials Errors and Approximation Ex 14.1 Q2

Let $x = 10,\ x + \Delta x = 9.8$

$$\Delta x = 9.8 - x$$
$$= 9.8 - 10$$
$$\Delta x = -0.2$$

$$y = \frac{4}{3}\pi x^3 \qquad \left[\text{volume of sphere}\right]$$
$$\frac{dy}{dx} = 4\pi r^2$$
$$\left(\frac{dy}{dx}\right)_{x=10} = 4\pi\left(10\right)^2$$
$$\left(\frac{dy}{dx}\right)_{x=10} = 400\pi \ \text{cm}^2$$

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=10} \times \Delta x$$
$$= 400\pi \times \left(-0.2\right)$$
$$\Delta y = -80\pi \ \text{cm}^3$$

So, approximate diecocase in volume is 80π cm^3.

Differentials Errors and Approximation Ex 14.1 Q3

Let $x = 10,\ x + \Delta x = 10 + \frac{k}{100} \times 10$

$$x + \Delta x = 10 + 0.k$$
$$\Rightarrow \quad \Delta x = 10 + 0.k - 10$$
$$\Delta x = 0.k$$

$$y = \pi r^2$$
$$\frac{dy}{dx} = 2\pi r$$
$$\left(\frac{dy}{dx}\right)_{x=10} = 2\pi\left(10\right)$$
$$\left(\frac{dy}{dx}\right)_{x=10} = 20\pi \ \text{cm}$$

So,

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=10} \times \Delta x$$
$$= \left(20\pi\right) \times \left(0.k\right)$$
$$\Delta y = 2k\pi \text{cm}^2$$

Area of the plate increases by $2k\pi$ cm^2.

Differentials Errors and Approximation Ex 14.1 Q4

Let $\quad \text{length}\left(\text{L}\right) = x$

$$x + \Delta x = x + \frac{x}{100}$$
$$\Delta x = 0.01x$$

Now,

$$y = 6x^2$$
$$\frac{dy}{dx} = 12x \ \text{cm}$$

So,

$$\Delta y = \left(\frac{dy}{dx}\right) \times \Delta x$$
$$= \left(12x\right)\left(0.01x\right)$$
$$\Delta y = 0.12x^2 \ \ \text{cm}^2$$
$$= 6\left(0.02\right)x^2$$
$$= 2\% \text{ of } 6x^2$$

Percentage error in area is 2%.

Differentials Errors and Approximation Ex 14.1 Q5

Let x be the radius of sphere,

$$\Delta x = 0.1\% \text{ of } x$$
$$\Delta x = 0.001x$$

Now,

Let y = volume of sphere

$$y = \frac{4}{3}\pi x^3$$
$$\frac{dy}{dx} = 4\pi x^2$$
$$\Delta y = \left(\frac{dy}{dx}\right) \times \Delta x$$
$$= 4\pi x^2 \times 0.001x$$
$$= \frac{4}{3}\pi x^3 (0.003)$$
$$= \frac{0.3}{100} \times y$$
$$\Delta y = 0.3\% \text{ of y}$$

So, percentage error in volume of error = 0.3%.

Differentials Errors and Approximation Ex 14.1 Q6

Given, $\Delta v = -\frac{1}{2}\%$

$$= -0.5\%$$
$$\Delta v = -0.005$$

Here,

$$pv^{1.4} = k$$

Taking log on both the sides,

$$\log\left(pv^{1.4}\right) = \log k$$
$$\log p + 1.4 \log v = \log k$$

Differentiate it with respect to v,

$$\frac{1}{p}\frac{dp}{dv} + \frac{1.4}{v} = 0$$
$$\frac{dp}{dv} = -\frac{1.4}{v}p$$

$$\Delta p = \left(\frac{dp}{dv}\right)\Delta v$$
$$= -\frac{1.4p}{v} \times (-0.005)$$
$$\Delta p = \frac{1.4p(0.005)}{v}$$
$$\Delta p \text{ in } \% = \frac{\Delta p}{p} \times 100$$
$$= \frac{1.4p(0.005)}{p} \times 100$$
$$= 0.7\%$$

So, percentage error in p = 0.7%.

Differentials Errors and Approximation Ex 14.1 Q7

Let h be the height of the cone, and α be the semivertide angle.

Here vertgide angle α is fixed.

$$\Delta h = k\% \text{ of } h$$
$$= \frac{k}{100} \times h$$
$$\Delta h = (0.0k)h$$

(i) $A = \pi r(r + l)$

$$= \pi(r^2 + rl)$$
$$= \pi(r^2) + r\sqrt{h^2 + r^2} \qquad \left[\text{Since, in a cone } l^2 = h^2 + r^2\right]$$

$$r = h\tan\alpha \qquad [\text{from figure}]$$
$$A = \pi\left[h^2\tan^2\alpha + h\tan\alpha\sqrt{h^2 + h^2\tan^2\alpha}\right]$$
$$= \pi\left[h^2\tan^2\alpha + h\tan\alpha\sqrt{h^2(1 + \tan^2\alpha)}\right]$$
$$= \pi\left[h^2\tan^2\alpha + h\tan\alpha \times h\sec\alpha\right]$$
$$= \pi h^2\left[\frac{\sin^2\alpha}{\cos^2\alpha} + \frac{\sin\alpha}{\cos^2\alpha}\right]$$
$$A = \pi h^2\frac{\sin\alpha(\sin\alpha + 1)}{\cos^2\alpha}$$

Differentiating with respect to h as α is fixed.

$$\frac{dA}{dh} = 2\pi h\frac{\sin\alpha(\sin\alpha + 1)}{\cos^2\alpha}$$

So,

$$\Delta A = \frac{dA}{dh} \times \Delta h$$
$$\Delta a = \frac{2\pi h(0.0kh)\sin\alpha(\sin\alpha + 1)}{\cos^2\alpha}$$
$$\Delta A \text{ in \% of } A = \frac{2\pi h(0.0kh)\sin\alpha(\sin\alpha + 1)}{\cos^2\alpha} \times \frac{100}{A}$$
$$= \frac{2\pi kh^2 \times \sin\alpha(\sin\alpha + 1)}{\cos^2\alpha} \times \frac{\cos^2\alpha}{\pi h^2\sin\alpha(\sin\alpha + 1)}$$
$$= 2k\%$$

So, percentage increase in area $= 2k\%$.

(ii)

Let v = volume of cone

$$= \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi(h\tan\alpha)^2 h$$
$$v = \frac{\pi}{3}\tan^2\alpha h^2$$

Differentiating it with respect to h treating α as constant,

$$\frac{dv}{dh} = \pi\tan^2\alpha \times h^2$$
$$\Delta v = \left(\frac{dv}{dh}\right)\Delta h$$
$$= \pi\tan^2\alpha h^2 \times (0.0kh)$$
$$\Delta v = 0.0k\pi h^3\tan^2\alpha$$

$$\text{Percentage increase in } v = \frac{\Delta v \times 100}{v}$$
$$= \frac{0.0k\pi h^3\tan^2\alpha \times 100}{\frac{\pi}{3}\tan^2\alpha \times h^3}$$
$$= 3k\%$$

So, percentage increase in volume $= 3k\%$.

Differentials Errors and Approximation Ex 14.1 Q8

Let error in radius $(r) = x\%$ of r

$\Delta r = 0.0xr$

Let $v =$ volume of sphere

$v = \frac{4}{3}\pi r^3$

Differentiating it with respect to r,

$\frac{dv}{dr} = 4\pi r^2$

So,

$$\Delta v = \left(\frac{dv}{dr}\right) \times \Delta r$$

$$= \left(4\pi r^2\right)(0.0x)r$$

$$\Delta v = 0.0x \times 4\pi r^3$$

$$\text{Percentage of error in volume} = \frac{\Delta v \times 100}{v}$$

$$= \frac{(0.0x)\, 4\pi r^3 \times 100}{\frac{4}{3}\pi r^3}$$

$$= 3x\%$$

Percentage of error in volume $= 3$ (percentage of error in radius).

Differentials Errors and Approximation Ex 14.1 Q9(i)

Let $x = 25,\ x + \Delta x = 25.02$

$\Delta x = 25.02 - 25$

$\Delta x = 0.02$

Let $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\left(\frac{dy}{dx}\right)_{x=25} = \frac{1}{2\sqrt{25}}$$

$$\left(\frac{dy}{dx}\right)_{x=25} = \frac{1}{10}$$

Now,

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=25} \times x$$

$$= \frac{1}{10}(0.02)$$

$$\Delta y = 0.002$$

$$\sqrt{25.02} = y + \Delta y$$

$$= \sqrt{25} + 0.002$$

$$= 5 + 0.002$$

$$\sqrt{25.02} = 5.002$$

Differentials Errors and Approximation Ex 14.1 Q9(ii)

Let $x = 0.008,\ x + \Delta x = 0.009$
$\Delta x = 0.009 - 0.008$
$\Delta x = 0.001$

Let $y = x^{\frac{1}{3}}$

$$\frac{dy}{dx} = \frac{1}{3x^{\frac{2}{3}}}$$

$$\left(\frac{dy}{dx}\right)_{x=0.008} = \frac{1}{3\left(0.008\right)^{\frac{2}{3}}}$$
$$= \frac{1}{3\left(0.04\right)}$$
$$= \frac{100}{12}$$
$$= 0.8333$$

So,

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=0.008} \times \Delta x$$
$$= \left(0.8333\right)\left(0.001\right)$$
$$\Delta y = 0.008333$$
$$\left(0.009\right)^{\frac{1}{3}} = y + \Delta y$$
$$= \left(x\right)^{\frac{1}{3}} + 0.008333$$
$$= \left(0.008\right)^{\frac{1}{3}} + 0.008333$$
$$= 0.52 + 0.008333$$

$$\left(0.009\right)^{\frac{1}{3}} = 0.208333$$

Differentials Errors and Approximation Ex 14.1 Q9(iii)

Let $x = 0.008,\ x + \Delta x = 0.007$
$\Delta x = 0.007 - 0.008$
$\Delta x = -0.001$

Let $y = x^{\frac{1}{3}}$

$$\frac{dy}{dx} = \frac{1}{3\left(x\right)^{\frac{2}{3}}}$$

$$\left(\frac{dy}{dx}\right)_{x=0.008} = \frac{1}{3\left(0.008\right)^{\frac{2}{3}}}$$
$$= \frac{100}{12}$$
$$\left(\frac{dy}{dx}\right)_{x=0.008} = 8.333$$

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=0.008} \times \Delta x$$
$$= \left(8.333\right)\left(-0.001\right)$$
$$\Delta y = -0.008333$$
$$\left(0.007\right)^{\frac{1}{3}} = y + \Delta y$$
$$= x^{\frac{1}{3}} - 0.008333$$
$$= \left(0.008\right)^{\frac{1}{3}} - 0.008333$$
$$= 0.2 - 0.008333$$

$$\left(0.007\right)^{\frac{1}{3}} = 0.191667$$

Differentials Errors and Approximation Ex 14.1 Q9(iv)

Let $x = 400,\ x + \Delta x = 401$

$\Delta x = 401 - 400$

$\Delta x = 1$

Let $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\left(\frac{dy}{dx}\right)_{x=400} = \frac{1}{2\sqrt{400}} = \frac{1}{40}$$

$$\left(\frac{dy}{dx}\right)_{x=400} = 0.025$$

So,

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=400} \times \Delta x$$

$$= (0.025)(1)$$

$$= 0.025$$

$$\sqrt{401} = y + \Delta y$$

$$= \sqrt{x} + 0.025$$

$$= \sqrt{400} + 0.025$$

$$= 20 + 0.025$$

$$\sqrt{401} = 20.025$$

Differentials Errors and Approximation Ex 14.1 Q9(v)

Let $x = 16,\ x + \Delta x = 15$

$\Delta x = 15 - 16$

$\Delta x = -1$

Let $y = x^{\frac{1}{4}}$

$$\frac{dy}{dx} = \frac{1}{4x^{\frac{3}{4}}}$$

$$\left(\frac{dy}{dx}\right)_{x=16} = \frac{1}{4(16)^{\frac{3}{4}}} = \frac{1}{32} = 0.03125$$

Now,

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=16} \times \Delta x$$

$$= (0.03125)(-1)$$

$$\Delta y = -0.03125$$

$$(15)^{\frac{1}{4}} = y + \Delta y$$

$$= (x)^{\frac{1}{4}} - 0.03125$$

$$= (16)^{\frac{1}{4}} - 0.03125$$

$$= 2 - 0.03125$$

$$(15)^{\frac{1}{4}} = 1.96875$$

Differentials Errors and Approximation Ex 14.1 Q9(vi)

Let $x = 256,\ x + \Delta x = 255$

$\Delta x = 255 - 256$

$\Delta x = -1$

Let $y = x^{\frac{1}{4}}$

$$\frac{dy}{dx} = \frac{1}{4x^{\frac{3}{4}}}$$

$$\left(\frac{dy}{dx}\right)_{x=256} = \frac{1}{4(256)^{\frac{3}{4}}}$$

$$= \frac{1}{256}$$

$$= 0.00391$$

Now,

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=256} \times \Delta x$$

$$= (0.00391)(-1)$$

$$\Delta y = -0.00391$$

$$(255)^{\frac{1}{4}} = y + \Delta y$$

$$= (x)^{\frac{1}{4}} + (-0.00391)$$

$$= (256)^{\frac{1}{4}} - 0.00391$$

$$= 4 - 0.00391$$

$$(255)^{\frac{1}{4}} = 3.99609$$

Differentials Errors and Approximation Ex 14.1 Q9(vii)

Let $x = 2,\ x + \Delta x = 2.002$

$\Delta x = 2.002 - 2$

$\Delta x = 0.002$

Let $y = \frac{1}{x^2}$

$$\frac{dy}{dx} = -\frac{2}{x^3}$$

$$\left(\frac{dy}{dx}\right)_{x=2} = -\frac{2}{8}$$

$$= -0.25$$

Now,

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=2} \times \Delta x$$

$$= (-0.25)(0.002)$$

$$\Delta y = -0.0005$$

Now,

$$\frac{1}{(2.002)^3} = y + \Delta y$$

$$= \frac{1}{x^2} + (-0.0005)$$

$$= \frac{1}{4} - 0.0005$$

$$= 0.25 - 0.0005$$

$$\frac{1}{(2.002)^3} = 0.2495$$

Differentials Errors and Approximation Ex 14.1 Q9(viii)

Let $x = 4,\ x + \Delta x = 4.04$

$\Delta x = 4.04 - 4$

$\Delta x = 0.04$

Let $y = \log x$

$\frac{dy}{dx} = \frac{1}{x}$

$\left(\frac{dy}{dx}\right)_{x=4} = \frac{1}{4}$

$= 0.25$

Now,

$\Delta y = \left(\frac{dy}{dx}\right)_{x=4} \times \Delta x$

$= (0.25)(0.04)$

$\Delta y = 0.01$

$\log_e 4.04 = y + \Delta y$

$= \log x + (0.01)$

$= \log_e 4 + 0.01$

$= \frac{\log_e 4}{\log_{10} e} + 0.01$

$= \frac{0.6021}{0.4343} + 0.01$ $\left[\text{Since, } \log_a b = \frac{\log_c b}{\log_c a}\right]$

$= 1.38637 + 0.01$

$\log_e 4.04 = 1.39637$

Differentials Errors and Approximation Ex 14.1 Q9(ix)

Let $x = 10,\ x + \Delta x = 10.02$

$\Delta x = 10.02 - 10$

$\Delta x = 0.02$

Let $y = \log_e x$

$\frac{dy}{dx} = \frac{1}{x}$

$\left(\frac{dy}{dx}\right)_{x=10} = \frac{1}{10}$

$\left(\frac{dy}{dx}\right)_{x=10} = 0.1$

Now,

$\Delta y = \left(\frac{dy}{dx}\right)_{x=10} \times \Delta x$

$= (0.1)(0.02)$

$\Delta y = 0.002$

$\log_e (10.02) = y + \Delta y$

$= \log_e x + 0.002$

$= \log_e 10 + 0.002$

$= 2.3026 + 0.002$

$\log_e (10.02) = 2.3046$

Differentials Errors and Approximation Ex 14.1 Q9(x)

Let $x = 10,\ x + \Delta x = 10.1$

$\Delta x = 10.1 - 10$

$\Delta x = 0.1$

Let $y = \log_{10} x$

$= \dfrac{\log_e x}{\log_e 10}$ $\left[\text{Since, } \log_a b = \dfrac{\log_c a}{\log_c b}\right]$

$$\left(\frac{dy}{dx}\right) = \frac{1}{x \log_e 10}$$

$$\left(\frac{dy}{dx}\right)_{x=10} = \frac{1}{10 \log_e 10}$$

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=10} \times \Delta x$$

$$= \frac{1}{10(\log_e 10)} \times 0.1$$

$$\Delta y = \frac{0.01}{(\log_e 10)}$$

$$\log_{10}(10.1) = y + \Delta y$$

$$= \log_{10} x + \frac{0.01}{\log_e 10}$$

$$= \log_{10} 10 + 0.01 \log_{10} e \qquad \left[\text{Since, } \log_a b = \frac{1}{\log_b a}\right]$$

$$= 1 + (0.01)(0.4343)$$

$$\log_{10}(10.1) = 1.004343$$

Differentials Errors and Approximation Ex 14.1 Q9(xi)

Let $x = 60°,\ x + \Delta x = 61°$

$\Delta x = 61° - 60°$

$\Delta x = 1° = \dfrac{\pi}{18°} = 0.01745$

Let $y = \cos x$

$$\frac{\mathrm{dy}}{\mathrm{dx}} = -\sin x$$

$$\left(\frac{dy}{dx}\right)_{x=60°} = -\sin\left(60^{\circ}\right)$$

$$= -\frac{\sqrt{3}}{2}$$

$$= -0.866$$

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=60°} \times (\Delta x)$$

$$= (-0.866)(0.01745)$$

$$= -0.01511$$

So,

$$\cos 61° = y + \Delta y$$

$$= \cos 60° - 0.01511$$

$$= \frac{1}{2} - 0.01511$$

$$= 0.5 - 0.01511$$

$$\cos 61° = 0.48489$$

Differentials Errors and Approximation Ex 14.1 Q9(xii)

Let $x = 25,\ x + \Delta x = 25.1$

$\Delta x = 25.1 - 25$

$\Delta x = 0.1$

Let $y = \dfrac{1}{\sqrt{x}}$

$$\frac{dy}{dx} = \frac{2}{2x^{\frac{3}{2}}}$$

$$\left(\frac{dy}{dx}\right)_{x=25} = -\frac{1}{2(25)^{\frac{3}{2}}}$$

$$= -\frac{1}{250}$$

$$= -0.004$$

Now,

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=25} \times (\Delta x)$$

$$= (-0.004)(0.1)$$

$$= -0.0004$$

$$\frac{1}{\sqrt{25.1}} = y + \Delta y$$

$$= \frac{1}{\sqrt{x}} + (-0.0004)$$

$$= \frac{1}{\sqrt{25}} - 0.0004$$

$$= \frac{1}{5} - 0.0004$$

$$= 0.2 - 0.0004$$

$$\frac{1}{\sqrt{25.1}} = 0.1996$$

Differentials Errors and Approximation Ex 14.1 Q9(xiii)

Let $x = \dfrac{\pi}{2},\ x + \Delta x = \dfrac{22}{14}$

$$\Delta x = \left(\frac{22}{14} - \frac{\pi}{2}\right)$$

$$\Delta x = \sin x$$

Let $y = \sin x$

$$\frac{dy}{dx} = \cos x$$

$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}} = \cos\frac{\pi}{2}$$

$$\left(\frac{dy}{dx}\right)_{\frac{\pi}{2}} = 0$$

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}} \times (\Delta x)$$

$$= 0 \times \left(\frac{22}{14} - \frac{\pi}{2}\right)$$

$$= 0$$

So,

$$\sin\left(\frac{22}{14}\right) = y + \Delta y$$

$$= \sin x + 0$$

$$= \sin\left(\frac{\pi}{2}\right)$$

$$\sin\left(\frac{22}{14}\right) = 1$$

Differentials Errors and Approximation Ex 14.1 Q9(xiv)

Let $x = \frac{\pi}{3}$, $x + \Delta x = \frac{11\pi}{36}$

$$\Delta x = \frac{11\pi}{36} - \frac{\pi}{3}$$
$$= -\frac{\pi}{36}$$
$$= -\frac{22}{7 \times 36}$$
$$= -0.0873$$

Let $y = \cos x$

$$\frac{dy}{dx} = -\sin x$$
$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{3}} = -\sin\frac{\pi}{3}$$
$$= -\frac{\sqrt{3}}{2}$$
$$= -0.866$$
$$\Delta y = \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{3}} \times (\Delta x)$$
$$= (-0.866)(-0.0873)$$
$$= 0.0756$$

$$\cos\left(\frac{11\pi}{36}\right) = y + \Delta y$$
$$= \cos x + (0.0756)$$
$$= \cos\frac{\pi}{3} + 0.0756$$
$$= \frac{1}{2} + 0.0756$$
$$= 0.5 + 0.0756$$

$$\cos\frac{11\pi}{36} = 0.7546$$

Differentials Errors and Approximation Ex 14.1 Q9(xv)

Let $x = 36$, $x + \Delta x = 37$

$$\Delta x = 37 - 36$$
$$= 1$$

Let $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$
$$\left(\frac{dy}{dx}\right)_{x=36} = \frac{1}{2\sqrt{36}}$$
$$= \frac{1}{12}$$
$$= 0.0833$$

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=36} \times (\Delta x)$$
$$= (0.0833)(1)$$
$$= 0.0833$$

$$\sqrt{37} = y + \Delta y$$
$$= \sqrt{x} + 0.0833$$
$$= \sqrt{36} + 0.0833$$

$$\sqrt{37} = 6.0833$$

Differentials Errors and Approximation Ex 14.1 Q9(xvi)

Let $\quad x = 81,\ x + \Delta x = 80$

$\Delta x = 80 - 81$

$= -1$

Let $\quad y = x^{\frac{1}{4}}$

$$\frac{dy}{dx} = \frac{1}{4(81)^{\frac{3}{4}}}$$

$$= \frac{1}{108}$$

$$= 0.00926$$

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=81} \times (\Delta x)$$

$$= (0.00926)(-1)$$

$$= -0.00926$$

$$(80)^{\frac{1}{4}} = y + \Delta y$$

$$= x^{\frac{1}{4}} - 0.00926$$

$$= (81)^{\frac{1}{4}} - 0.00926$$

$$= 3 - 0.00926$$

$$(80)^{\frac{1}{4}} = 2.99074$$

Differentials Errors and Approximation Ex 14.1 Q9(xvii)

Let $\quad x = 27,\ x + \Delta x = 29$

$\Delta x = 29 - 27$

$= 2$

Let $\quad y = x^{\frac{1}{3}}$

$$\frac{dy}{dx} = \frac{1}{3(x)^{\frac{2}{3}}}$$

$$\left(\frac{dy}{dx}\right)_{x=27} = \frac{1}{3(27)^{\frac{2}{3}}}$$

$$= \frac{1}{27}$$

$$= 0.03704$$

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=27} \times (\Delta x)$$

$$= (0.03704)(2)$$

$$\Delta y = 0.07408$$

$$(28)^{\frac{1}{3}} = y + \Delta y$$

$$= x^{\frac{1}{3}} + 0.07408$$

$$= (27)^{\frac{1}{3}} + 0.07408$$

$$= 3 + 0.07408$$

$$(29)^{\frac{1}{3}} = 3.07408$$

Differentials Errors and Approximation Ex 14.1 Q9(xviii)

Let $x = 64,\ x + \Delta x = 66$

$\Delta x = 66 - 64$

$= 2$

Let $y = x^{\frac{1}{3}}$

$\frac{dy}{dx} = \frac{1}{3(x)^{\frac{2}{3}}}$

$\left(\frac{dy}{dx}\right)_{x=64} = \frac{1}{3(64)^{\frac{2}{3}}}$

$= \frac{1}{48}$

$= 0.020833$

$\Delta y = \left(\frac{dy}{dx}\right)_{x=64} \times (\Delta x)$

$= (0.020833)(2)$

$= 0.041666$

$(66)^{\frac{1}{3}} = y + \Delta y$

$= x^{\frac{1}{3}} + 0.041666$

$= (64)^{\frac{1}{3}} + 0.041666$

$= 4 + 0.041666$

$(66)^{\frac{1}{3}} = 4.041666$

Differentials Errors and Approximation Ex 14.1 Q9(xix)

Let $x = 25,\ x + \Delta x = 26$

$\Delta x = 26 - 25$

$= 1$

Let $y = \sqrt{x}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

$\left(\frac{dy}{dx}\right)_{x=25} = \frac{1}{2\sqrt{25}}$

$= \frac{1}{10}$

$= 0.1$

$\Delta y = \left(\frac{dy}{dx}\right)_{x=25} \times (\Delta x)$

$= (0.1)(1)$

$= 0.1$

$\sqrt{26} = y + \Delta y$

$= \sqrt{x} + 0.01$

$= \sqrt{25} + 0.1$

$\sqrt{26} = 5.1$

Differentials Errors and Approximation Ex 14.1 Q9(xx)

Let $x = 0.49,\ x + \Delta x = 0.487$

$\Delta x = 0.48 - 0.49$

$= -0.01$

Let $y = \sqrt{x}$

$\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{1}{2\sqrt{x}}$

$\left(\frac{dy}{dx}\right)_{x=0.49} = \frac{1}{2\sqrt{0.49}}$

$= \frac{1}{1.4}$

$= 0.71428$

$\Delta y = \left(\frac{dy}{dx}\right)_{x=0.49} \times (\Delta x)$

$= (0.71428)(-0.01)$

$\Delta y = -0.0071428$

$\sqrt{37} = y + \Delta y$

$= \sqrt{0.49} - 0.0071428$

$= 0.7 - 0071428$

$\sqrt{0.48} = 0.6928572$

Differentials Errors and Approximation Ex 14.1 Q9(xxi)

Let $x = 81,\ x + \Delta x = 82$

$\Delta x = 82 - 81$

$= 1$

Let $y = x^{\frac{1}{4}}$

$\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{1}{4x^{\frac{3}{4}}}$

$\left(\frac{dy}{dx}\right)_{x=81} = \frac{1}{4(81)^{\frac{3}{4}}}$

$= \frac{1}{108}$

$= 0.009259$

$\Delta y = \left(\frac{dy}{dx}\right)_{x=81} \times (\Delta x)$

$= (0.008259)(1)$

$= 0.009259$

$(82)^{\frac{1}{4}} = y + \Delta y$

$= x^{\frac{1}{4}} + 0.009259$

$= (81)^{\frac{1}{4}} + 0.009259$

$(82)^{\frac{1}{4}} = 3.009259$

Differentials Errors and Approximation Ex 14.1 Q9(xxii)

Let $x = \frac{16}{81}$, $x + \Delta x = \frac{17}{81}$

$$\Delta x = \frac{17}{81} - \frac{16}{81}$$
$$= \frac{1}{81}$$

Let $y = x^{\frac{1}{4}}$

$$\frac{dy}{dx} = \frac{1}{4x^{\frac{3}{4}}}$$
$$\left(\frac{dy}{dx}\right)_{x=\frac{16}{81}} = \frac{1}{4\left(\frac{16}{81}\right)^{\frac{3}{4}}}$$
$$= \frac{27}{32}$$
$$= 0.84375$$

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=\frac{16}{81}} \times (\Delta x)$$
$$= (0.84375)\left(\frac{1}{81}\right)$$
$$= 0.01041$$

$$\left(\frac{17}{81}\right)^{\frac{1}{4}} = y + \Delta y$$
$$= \left(\frac{16}{81}\right)^{\frac{1}{4}} + 0.01041$$
$$= 0.6666 + 0.01041$$

$$\left(\frac{17}{81}\right)^{\frac{1}{4}} = 0.67707$$

Differentials Errors and Approximation Ex 14.1 Q9(xxiii)

Let $x = 32$, $x + \Delta x = 33$

$$\Delta x = 33 - 32$$
$$= 1$$

Let $y = x^{\frac{1}{5}}$

$$\frac{dy}{dx} = \frac{1}{5x^{\frac{4}{5}}}$$
$$\left(\frac{dy}{dx}\right)_{x=32} = \frac{1}{5(32)^{\frac{4}{5}}}$$
$$= \frac{1}{80}$$
$$= 0.0125$$

$\therefore$
$$\Delta y = \left(\frac{dy}{dx}\right)_{x=32} \times (\Delta x)$$
$$= (0.0125)(1)$$
$$\Delta y = 0.0125$$

$$(33)^{\frac{1}{5}} = y + \Delta y$$
$$= x^{\frac{1}{5}} + 0.0125$$
$$= (32)^{\frac{1}{5}} + 0.0125$$

$$(33)^{\frac{1}{5}} = 2.0125$$

Differentials Errors and Approximation Ex 14.1 Q9(xxiv)

Let $x = 36,\ x + \Delta x = 36.6$

$\Delta x = 36.6 - 36$

$= 0.6$

Let $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\left(\frac{dy}{dx}\right)_{x=36} = \frac{1}{2\sqrt{36}}$$

$$= \frac{1}{12}$$

$$= 0.0833$$

$\therefore$ $$\Delta y = \left(\frac{dy}{dx}\right)_{x=36} \times (\Delta x)$$

$$= (0.0833)(0.6)$$

$$= 0.04998$$

$$\sqrt{36.6} = y + \Delta y$$

$$= \sqrt{x} + 0.04998$$

$$= \sqrt{36} + 0.04998$$

$$\sqrt{36.6} = 6.04998$$

Differentials Errors and Approximation Ex 14.1 Q9(xxv)

Let $x = 27,\ x + \Delta x = 25$

$\Delta x = 25 - 27$

$= -2$

Let $y = x^{\frac{1}{3}}$

$$\frac{dy}{dx} = \frac{1}{3x^{\frac{2}{3}}}$$

$$\left(\frac{dy}{dx}\right)_{x=27} = \frac{1}{3(27)^{\frac{2}{3}}}$$

$$= \frac{1}{27}$$

$$= 0.037$$

$\therefore$ $$\Delta y = \left(\frac{dy}{dx}\right)_{x=27} \times (\Delta x)$$

$$= (0.037)(-2)$$

$$= -0.074$$

$$(25)^{\frac{1}{3}} = y + \Delta y$$

$$= x^{\frac{1}{3}} + (-0.074)$$

$$= (27)^{\frac{1}{3}} - 0.074$$

$$= 3 - 0.074$$

$$(25)^{\frac{1}{3}} = 2.926$$

Differentials Errors and Approximation Ex 14.1 Q9(xxvi)

Let $y = f(x) = \sqrt{x}$, $x = 49$ and $x + \Delta x = 49.5$

Then $\Delta x = 0.5$

For $x = 49$ we have

$y = \sqrt{49} = 7$

$dx = \Delta x = 0.5$

$y = \sqrt{x}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=49} = \frac{1}{2 \times 7} = \frac{1}{14}$

$\therefore dy = \frac{dy}{dx}dx$

$\Rightarrow dy = \frac{1}{14}(0.5) = \frac{5}{140}$

$\Rightarrow \Delta y = \frac{1}{28}$

Hence,

$\sqrt{49.5} = y + \Delta y = 7 + \frac{1}{28} = 7 + 0.0357 = 7.0357$

Differentials Errors and Approximation Ex 14.1 Q9(xxvii)

Define a function $y = x^{3/2}$

For $x = 4$, $y = 8$

$x + \Delta x = 3.968 \Rightarrow \Delta x = 3.968 - 4 = -0.032$

$\frac{dy}{dx} = \frac{3}{2}x^{1/2}$

$\Rightarrow dy = \left(\frac{3}{2}x^{1/2}\right)dx$

$\Rightarrow \Delta y|_{x=4} \simeq (3)\Delta x$

$\Rightarrow \Delta y|_{x=4} \simeq 3 \times (-0.032) = -0.096$

$(3.968)^{3/2} = y + \Delta y = 8 - 0.096$

$= 7.904$

Differentials Errors and Approximation Ex 14.1 Q9(xxviii)

Let $y = f(x) = x^5$, $x = 2$ and $x + \Delta x = 1.999$

Then $\Delta x = -0.001$

For $x = 2$ we have

$y = (2)^5 = 32$

$dx = \Delta x = -0.001$

$y = x^5$

$\Rightarrow \frac{dy}{dx} = 5x^4$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=1} = 5(2)^4 = 80$

$\therefore dy = \frac{dy}{dx}dx$

$\Rightarrow dy = 80(-0.001) = -0.080$

$\Rightarrow \Delta y = -0.080$

Hence,

$(1.999)^5 = y + \Delta y = 32 - 0.080 = 31.920$

Differentials Errors and Approximation Ex 14.1 Q9(xxix)

Let $y = f(x) = \sqrt{x}$, $x = 0.09$ and $x + \Delta x = 0.082$
Then $\Delta x = -0.008$
For $x = 0.09$ we have
$y = \sqrt{0.09} = 0.3$

$dx = \Delta x = -0.008$
$y = \sqrt{x}$
$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$
$\Rightarrow \left(\frac{dy}{dx}\right)_{x=1} = \frac{1}{2 \times \sqrt{0.09}} = \frac{1}{2 \times 0.3} = \frac{1}{0.6}$
$\therefore dy = \frac{dy}{dx} dx$
$\Rightarrow dy = \frac{1}{0.6}(-0.008)$
$\Rightarrow \Delta y = -\frac{8}{600}$
Hence,
$\sqrt{0.082} = y + \Delta y = 0.3 - \frac{8}{600} = 0.3 - 0.0133 = 0.2867$

Differentials Errors and Approximation Ex 14.1 Q10

Let $x = 2$ and $\Delta x = 0.01$. Then, we have:

$f(2.01) = f(x + \Delta x) = 4(x + \Delta x)^2 + 5(x + \Delta x) + 2$

Now, $\Delta y = f(x + \Delta x) - f(x)$

$\therefore f(x + \Delta x) = f(x) + \Delta y$

$\approx f(x) + f'(x) \cdot \Delta x \qquad (\text{as } dx = \Delta x)$

$$\begin{aligned}
\Rightarrow f(2.01) &\approx (4x^2 + 5x + 2) + (8x + 5)\Delta x \\
&= \left[4(2)^2 + 5(2) + 2\right] + \left[8(2) + 5\right](0.01) \qquad [\text{as } x = 2,\ \Delta x = 0.01] \\
&= (16 + 10 + 2) + (16 + 5)(0.01) \\
&= 28 + (21)(0.01) \\
&= 28 + 0.21 \\
&= 28.21
\end{aligned}$$

Hence, the approximate value of $f(2.01)$ is 28.21.

Differentials Errors and Approximation Ex 14.1 Q11

Let $x = 5$ and $\Delta x = 0.001$. Then, we have:

$f(5.001) = f(x + \Delta x) = (x + \Delta x)^3 - 7(x + \Delta x)^2 + 15$
Now, $\Delta y = f(x + \Delta x) - f(x)$
$\therefore f(x + \Delta x) = f(x) + \Delta y$
$\approx f(x) + f'(x) \cdot \Delta x \qquad (\text{as } dx = \Delta x)$

$$\begin{aligned}
\Rightarrow f(5.001) &\approx (x^3 - 7x^2 + 15) + (3x^2 - 14x)\Delta x \\
&= \left[(5)^3 - 7(5)^2 + 15\right] + \left[3(5)^2 - 14(5)\right](0.001) \qquad [x = 5, \Delta x = 0.001] \\
&= (125 - 175 + 15) + (75 - 70)(0.001) \\
&= -35 + (5)(0.001) \\
&= -35 + 0.005 \\
&= -34.995
\end{aligned}$$

Hence, the approximate value of $f(5.001)$ is -34.995.

Differentials Errors and Approximation Ex 14.1 Q12

Let $x = 1000,\ x + \Delta x = 1005$

$\Delta x = 1005 - 1000$

$= 5$

Let $y = \log_{10} x$

$$\frac{dy}{dx} = \frac{\log_e x}{\log_e 10} \qquad \left[\because \log_a b = \frac{\log_e b}{\log_e a}\right]$$

$$\frac{dy}{dx} = \frac{1}{x \log_e 10}$$

$$\left(\frac{dy}{dx}\right)_{x=1000} = \frac{\log_{10} e}{1000} \qquad \left[\because \log_a b = \frac{1}{\log_b a}\right]$$

$$= \frac{0.4343}{1000}$$

$$= (0.0004343)$$

$$\therefore \quad \Delta y = \left(\frac{dy}{dx}\right)_{x=1000} \times (\Delta x)$$

$$= (0.0004343)(5)$$

$$= 0.0021715$$

$$\log_{10} 1005 = y + \Delta y$$

$$= \log_{10} x + 0.0021715$$

$$= \log_{10} 1000 + 0.0021715$$

$$= \log_{10} 10^3 + 0.0021715$$

$$= 3\log_{10} 10 + 0.0021715$$

$$\log_{10} 1005 = 3.0021715$$

Differentials Errors and Approximation Ex 14.1 Q13

Let r be the radius of the sphere and Δr be the error in measuring the radius.

Then,

$r = 9$ m and $\Delta r = 0.03$ m

Now, the surface area of the sphere (S) is given by,

$S = 4\pi r^2$

$$\therefore \frac{dS}{dr} = 8\pi r$$

$$\therefore dS = \left(\frac{dS}{dr}\right)\Delta r$$

$$= (8\pi r)\Delta r$$

$$= 8\pi(9)(0.03) \text{ m}^2$$

$$= 2.16\pi \text{ m}^2$$

Hence, the approximate error in calculating the surface area is 2.16π m^2.

Differentials Errors and Approximation Ex 14.1 Q14

The surface area of a cube (S) of side x is given by $S = 6x^2$.

$$\therefore \frac{dS}{dx} = \left(\frac{dS}{dx}\right)\Delta x$$

$$= (12x)\Delta x$$

$$= (12x)(0.01x) \qquad [\text{as } 1\% \text{ of } x \text{ is } 0.01x]$$

$$= 0.12x^2$$

Hence, the approximate change in the surface area of the cube is $0.12x^2$ m^2.

Differentials Errors and Approximation Ex 14.1 Q15

Let r be the radius of the sphere and Δr be the error in measuring the radius.

Then,

$r = 7$ m and $\Delta r = 0.02$ m

Now, the volume V of the sphere is given by,

$$V = \frac{4}{3}\pi r^3$$
$$\therefore \frac{dV}{dr} = 4\pi r^2$$
$$\therefore dV = \left(\frac{dV}{dr}\right)\Delta r$$
$$= \left(4\pi r^2\right)\Delta r$$
$$= 4\pi(7)^2(0.02)\ \text{m}^3 = 3.92\pi\ \text{m}^3$$

Hence, the approximate error in calculating the volume is $3.92\ \pi\ \text{m}^3$.

Differentials Errors and Approximation Ex 14.1 Q16

The volume of a cube (V) of side x is given by $V = x^3$.

$$\therefore dV = \left(\frac{dV}{dx}\right)\Delta x$$
$$= \left(3x^2\right)\Delta x$$
$$= \left(3x^2\right)(0.01x) \qquad [\text{as } 1\% \text{ of } x \text{ is } 0.01x]$$
$$= 0.03x^3$$

Hence, the approximate change in the volume of the cube is $0.03x^3\ \text{m}^3$.

Ex 15.1

Mean Value Theorems Ex 15.1 Q1(i)

$f(x) = 3 + (x-2)^{\frac{2}{3}}$ on $[1, 3]$

Differentiating it with respect to x,

$$f'(x) = \frac{2}{3} \times \frac{1}{(x-2)^{\frac{1}{3}}}$$

Clearly, $\lim_{x \to 2} = \frac{2}{3} \times \frac{1}{(x-2)^{\frac{1}{3}}}$

Thus, $f(x)$ is not differentiable at $x = 2 \in (1, 3)$

Hene, Rolle's theorem is not applicable for $f(x)$ in $x \in [1, 3]$.

Mean Value Theorems Ex 15.1 Q1(ii)

Here, $f(x) = [x]$ and $x \in [-1, 1]$, at $n = 1$

$$\begin{aligned} \text{LHL} &= \lim_{x \to (1-h)} [x] \\ &= \lim_{h \to 0} [1 - h] \\ &= 0 \\ \text{RHL} &= \lim_{x \to (1+h)} [x] \\ &= \lim_{h \to 0} [1 + h] \\ &= 1 \\ & \text{LHL} \neq RHL \end{aligned}$$

So, $f(x)$ is not continuous at $1 \in [-1, 1]$

Hence, rolle's theorem is not applicable on $f(x)$ in $[-1, 1]$.

Mean Value Theorems Ex 15.1 Q1(iii)

Here, $f(x) = \sin\left(\frac{1}{x}\right)$, $x \in [-1,1]$, at $n = 0$

$$\text{LHS} = \lim_{x\to(0-h)} \sin\left(\frac{1}{x}\right)$$
$$= \lim_{h\to 0} \sin\left(\frac{1}{0-h}\right)$$
$$= \lim_{h\to 0} \sin\left(\frac{-1}{h}\right)$$
$$= -\lim_{h\to 0} \sin\left(\frac{1}{h}\right)$$
$$= -k \qquad \left[\text{Let } \lim_{h\to 0} \sin\left(\frac{1}{h}\right) = k \text{ as } k \in [-1,1]\right]$$

$$\text{RHS} = \lim_{x\to(0+h)} \sin\left(\frac{1}{x}\right)$$
$$= \lim_{h\to 0} \sin\left(\frac{1}{h}\right)$$
$$= k$$

$\Rightarrow$ LHS $\neq$ RHS

$\Rightarrow$ $f(x)$ is not continuous at $n = 0$

So, rolle's theorem is not applicable on $f(x)$ in $[-1,1]$

Mean Value Theorems Ex 15.1 Q1(iv)

Here, $f(x) = 2x^2 - 5x + 3$ on $[1,3]$

$f(x)$ is continuous in $[1,3]$ and $f(x)$ is differentiable is $(1,3)$ since it is a polynomial function.

Now,

$$f(x) = 2x^2 - 5x + 3$$
$$f(1) = 3(1)^2 - 5(1) + 3$$
$$= 2 - 5 + 3$$
$$f(1) = 0 \qquad \text{---(i)}$$
$$f(3) = 2(3)^2 - 5(3) + 3$$
$$= 18 - 15 + 3$$
$$f(3) = 6 \qquad \text{---(ii)}$$

From equation (i) and (ii),

$$f(1) \neq f(3)$$

So, rolle's theorem is not applicable on $f(x)$ in $[1,3]$.

Mean Value Theorems Ex 15.1 Q1(v)

Here, $f(x) = x^{\frac{2}{3}}$ on $[-1,1]$

$$f'(x) = \frac{2}{3x^{\frac{1}{3}}}$$
$$f'(0) = \frac{2}{3(0)^{\frac{1}{3}}}$$
$$f'(0) = \infty$$

So, $f'(x)$ does not exist at $x = 0 \in (-1,1)$

$\Rightarrow$ $f(x)$ is not differentiatable in $x \in (-1,1)$

So, rolle's theorem is not applicable on $f(x)$ in $[-1,1]$.

Mean Value Theorems Ex 15.1 Q1(vi)

Here, $f(x) = \begin{cases} -4x+5, & 0 \le x \le 1 \\ 2x-3, & 1 < x \le 2 \end{cases}$

For $n = 1$

LHS $= \lim_{x \to (1-h)} (-4x+5)$

$= \lim_{h \to 0} [-4(1-h)+5]$

$= -4+5$

LHS $= 1$

RHS $= \lim_{x \to (1+h)} (2x-3)$

$= \lim_{h \to 0} [2(1+h)-3]$

$= 2-3$

RHS $= -1$

So, LHS $\ne$ RHS

$\Rightarrow$ $f(x)$ is not contiuuous at $x = 1 \in [0,2]$

$\Rightarrow$ Rolle's theorem is not applicable on $f(x)$ in $[0,2]$.

Mean Value Theorems Ex 15.1 Q2(i)

Here,

$f(x) = x^2 - 8x + 12$ on $[2,6]$

$f(x)$ is continuous is $[2,6]$ and differentiable is $(2,6)$ as it is a polynomial function

And $f(2) = (2)^2 - 8(2) + 12 = 0$

$f(6) = (6)^2 - 8(6) + 12 = 0$

$\Rightarrow$ $f(2) = f(6)$

So, Rolle's theorem is applicable, therefore we show have $f'(c) = 0$ such that $c \in (2,6)$

So, $f(x) = x^2 - 8x + 12$

$\Rightarrow$ $f'(x) = 2x - 8$

So, $f'(c) = 0$

$2c - 8 = 0$

$c = 4 \in (2,6)$

Therefore, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q2(ii)

The given function is $f(x) = x^2 - 4x + 3$

f, being a polynomial function, is continuous in $[1, 4]$ and is differentiable in $(1, 4)$ whose derivative is $2x - 4$.

$f(1) = 1^2 - 4\times1 + 3 = 0$, $f(4) = 4^2 - 4\times4 + 3 = 3$

$\therefore \frac{f(b)-f(a)}{b-a} = \frac{f(4)-f(1)}{4-1} = \frac{3-(0)}{3} = \frac{3}{3} = 1$

Mean Value Theorem states that there is a point $c \in (1, 4)$ such that $f'(c) = 1$

$f'(c) = 1$

$\Rightarrow 2c - 4 = 1$

$\Rightarrow c = \frac{5}{2}$, where $c = \frac{5}{2} \in (1, 4)$

Hence, Mean Value Theorem is verified for the given function

Mean Value Theorems Ex 15.1 Q2(iii)

Here,

$$f(x) = (x-1)(x-2)^2 \text{ on } (1,2)$$

$f(x)$ is cantinuous is $[1,2]$ and differentiable is $(1,2)$ since it is a polynomial function.

And $\quad f(1) = (1-1)(1-2)^2 = 0$

$$f(2) = (2-1)(2-2)^2 = 0$$

$$\Rightarrow \quad f(1) = f(2)$$

So, Rolle's theorem is applicable on $f(x)$ in $[1,2]$, therefore, there exist a $c \in (1,2)$ such that $f'(c) = 0$

Now,

$$f(x) = (x-1)(x-2)^2$$

$$f'(x) = (x-1) \times 2(x-2) + (x-2)^2$$

$$f'(x) = (x-2)(3x-4)$$

So, $\quad f'(c) = 0$

$$(c-2)(3c-4) = 0$$

$$\Rightarrow \quad c = 2 \text{ or } c = \frac{4}{3} \in (1,2)$$

Thus, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q2(iv)

Here,

$$f(x) = x(x-1)^2 \text{ on } [0,1]$$

$f(x)$ is continuous on $[0,1]$ and differentiable on $(0,1)$ as it is a polynomial function.

Now,

$$f(0) = 0(0-1)^2 = 0$$

$$f(1) = 1(1-1)^2 = 0$$

$$\Rightarrow \quad f(0) = f(1)$$

So, Rolle's theorem is applicable on $f(x)$ in $[0,1]$ therefore, we should show that there exist a $c \in (0,1)$ such that $f'(c) = 0$

Now,

$$f(x) = x(x-1)^2$$

$$f'(x) = (x-1)^2 + x \times 2(x-1)$$

$$= (x-1)(x-1+2x)$$

$$f'(x) = (x-1)(3x-1)$$

So, $\quad f'(c) = 0$

$$(c-1)(3c-1) = 0$$

$$\Rightarrow \quad c = 1 \text{ or } c = \frac{1}{3} \in (0,1)$$

Thus, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q2(v)

Here,

$$f(x) = (x^2 - 1)(x - 2) \text{ on } [-1, 2]$$

$f(x)$ is continuous is $[-1, 2]$ and differentiable in $(-1, 2)$ as it is a polynomial functions.

Now,

$$f(-1) = (1 - 1)(-1 - 2) = 0$$
$$f(2) = (4 - 1)(2 - 2) = 0$$
$$\Rightarrow \quad f(-1) = f(2)$$

So, Rolle's theorem is applicable on $f(x)$ is $[-1, 2]$ therefore, we have to show that there exist a $c \in (-1, 2)$ such that $f'(c) = 0$

Now,

$$f(x) = (x^2 - 1)(x - 2)$$
$$f'(x) = 2x(x - 2) + (x^2 - 1)$$
$$= 2x^2 - 4 + x^2 - 1$$
$$f'(x) = 3x^2 - 5$$

Now,

$$f'(c) = 0$$
$$\Rightarrow \quad 3x^2 - 5 = 0$$
$$\Rightarrow \quad x = -\sqrt{\frac{5}{3}} \text{ or } x = \sqrt{\frac{5}{3}} \in (-1, 2)$$

Thus, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q2(vi)

Here, $f(x) = x(x - 4)^2$ on $[0, 4]$

$f(x)$ is continuous is $[0, 4]$ and differentiable is $(0, 4)$ since $f(x)$ is a polynomial function.

Now,

$$f(x) = x(x - 4)^2$$
$$f(0) = 0(0 - 4)^2$$
$$f(0) = 0 \qquad ---(i)$$
$$f(4) = 4(4 - 4)^2$$
$$f(4) = 0 \qquad ---(ii)$$

From equation (i) and (ii),

$$f(0) = f(4)$$

So, Rolle's theorem is applicable, therefore, we have to show that $f'(c) = 0$ for $c \in (0, 4)$

$$f'(x) = x \times 2(x - 4) + (x - 4)^2$$
$$= 2x^2 - 8x + x^2 + 16 - 8x$$

So, $\quad f'(c) = 3c^2 - 16c + 16$

$$0 = 3c^2 - 12c - 4c + 16$$
$$0 = 3c(c - 4) - 4(c - 4)$$
$$0 = (c - 4)(3c - 4)$$
$$\Rightarrow \quad c = 4 \text{ or } c = \frac{4}{3} \in (0, 4)$$

So, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q2(vii)

Here, $f(x) = x(x-2)^2$ on $[0,2]$
$f(x)$ is continuous is $[0,2]$ and differentiable is $(0,2)$
as it is a polynomial function.

And $f(0) = 0(0-2)^2 = 0$

$f(2) = 2(2-2)^2 = 0$

$\Rightarrow$ $f(0) = f(2)$

So, Rolle's theorem is applicable on $f(x)$ is $[0,2]$, therefore,
we have to show that $f'(c) = 0$ as $c \in (0,2)$

$$
\begin{aligned}
& f(x) = x(x-2)^2 \\
& f'(x) = x \times 2(x-2) + (x-2) \\
& f'(x) = 2x(x-2) + (x-2) \\
\Rightarrow\quad & f'(c) = 0 \\
& 2c(c-2) + (c-2) = 0 \\
& (c-2)(2c+1) = 0 \\
& c = 2 \text{ or } c = -\frac{1}{2} \\
\Rightarrow\quad & c = 2 \in (0,2)
\end{aligned}
$$

So, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q2(viii)

Here, $f(x) = x^2 + 5x + 6$ on $[-3,-2]$
$f(x)$ is continuous is $[-3,-2]$ and $f(x)$ is differentiable is $(-3,-2)$
since it is a polynomial function.

Now,

$$
\begin{aligned}
& f(x) = x^2 + 5x + 6 \\
& f(-3) = (-3)^2 + 5(-3) + 6 \\
& \quad\;\; = 9 - 15 + 6 \\
& f(-3) = 0 \qquad\qquad\qquad ---\text{(i)} \\
& f(-2) = (-2)^2 + 5(-2) + 6 \\
& \quad\;\; = 4 - 10 + 6 \\
& f(-2) = 20 \qquad\qquad\qquad ---\text{(ii)}
\end{aligned}
$$

From equation (i) and (ii),

$$f(-3) = f(-2)$$

So, Rolle's theorem is applicable is $[-3,-2]$, we have to show that
$f'(c) = 0$ as $c \in (-3,-2)$.

Now,

$$
\begin{aligned}
& f(x) = x^2 + 5x + 6 \\
& f'(x) = 2x + 5 \\
\Rightarrow\quad & f'(c) = 0 \\
& 2c + 5 = 0 \\
& c = \frac{-5}{2} \in (-3,-2)
\end{aligned}
$$

So, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(i)

Here,

$$f(x) = \cos 2\left(x - \frac{\pi}{4}\right) \text{ on } \left[0, \frac{\pi}{2}\right]$$

We know that cosine function is continuous and differentiable every where, so $f(x)$ is continuous is $\left[0, \frac{\pi}{2}\right]$ and differentiable is $\left[0, \frac{\pi}{2}\right]$.

Now,

$$f(0) = \cos 2\left(0 - \frac{\pi}{4}\right) = 0$$

$$f\left(\frac{\pi}{2}\right) = \cos 2\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = 0$$

$$\Rightarrow \quad f(0) = f\left(\frac{\pi}{2}\right)$$

So, Rolle's theorem is applicable.

Hence, there must exists a $c \in \left(0, \frac{\pi}{2}\right)$ such that $f'(c) = 0$.

Now,

$$f'(x) = -\sin 2\left(x - \frac{\pi}{4}\right) \times 2$$

$$f'(x) = -2\sin\left(2x - \frac{\pi}{2}\right)$$

$$\Rightarrow \quad -2\sin\left(2c - \frac{\pi}{2}\right) = 0$$

$$\Rightarrow \quad \sin\left(2c - \frac{\pi}{2}\right) = \sin 0$$

$$\Rightarrow \quad 2c - \frac{\pi}{2} = 0$$

$$c = \frac{\pi}{4} \in \left(0, \frac{\pi}{2}\right)$$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(ii)

Here,

$$f(x) = \sin 2x \text{ on } \left[0, \frac{\pi}{2}\right]$$

We know that $\sin x$ is a continuous and differentiable every where. So, $f(x)$ is continuous in $\left[0, \frac{\pi}{2}\right]$ and differentiable is $\left(0, \frac{\pi}{2}\right)$.

Now,

$$f(0) = \sin 0 = 0$$

$$f\left(\frac{\pi}{2}\right) = \sin \pi = 0$$

$$\Rightarrow \quad f(0) = f\left(\frac{\pi}{2}\right)$$

So, Rolle's theorem is applicable, so, there must exist a $c \in \left(0, \frac{\pi}{2}\right)$ such that $f'(c) = 0$

Now,

$$f'(x) = 2\cos 2x$$

$$f'(c) = 2\cos 2c = 0$$

$$\Rightarrow \quad \cos 2c = 0$$

$$\Rightarrow \quad 2c = \frac{\pi}{2}$$

$$\Rightarrow \quad c = \frac{\pi}{4} \in \left(0, \frac{\pi}{4}\right)$$

Thus, Rolle's theorem verified.

Mean Value Theorems Ex 15.1 Q3(iii)

Here,

$$f(x) = \cos 2x \text{ on } \left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$$

We know that $\cos x$ is a continuous and differentiable every where. So, $f(x)$ is continuous in $\left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$ and differentiable is $\left(\frac{-\pi}{4}, \frac{\pi}{4}\right)$.

Now, $f\left(-\frac{\pi}{4}\right) = \cos 2\left(-\frac{\pi}{4}\right) = \cos\left(-\frac{\pi}{2}\right) = 0$

$f\left(\frac{\pi}{4}\right) = \cos 2\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{2}\right) = 0$

$\Rightarrow \quad f\left(-\frac{\pi}{4}\right) = f\left(\frac{\pi}{4}\right)$

So, Rolle's theorem is applicable, so, there must exist a $c \in \left(0, \frac{\pi}{2}\right)$ such that $f'(c) = 0$

Now,

$f'(x) = 2 \sin 2x$

$f'(c) = 2 \sin 2c = 0$

$\Rightarrow \quad \sin 2c = 0$

$\Rightarrow \quad 2c = 0$

$\Rightarrow \quad c = 0 \in \left(\frac{-\pi}{4}, \frac{\pi}{4}\right)$

Thus, Rolle's theorem verified.

Mean Value Theorems Ex 15.1 Q3(iv)

Here,

$$f(x) = e^x \times \sin x \quad \text{on } [0, \pi]$$

We know that since and expential function are continuous and differentiable every where so, $f(x)$ is continuous is $[0, \pi]$ and differentiable is $(0, \pi)$.

Now,

$f(0) = e^0 \sin 0 = 0$

$f(\pi) = e^\pi \sin \pi = 0$

$\Rightarrow \quad f(0) = f(\pi)$

So, Rolle's theorem is applicable, so there must exist a point $c \in (0, \pi)$ such that $f'(c) = 0$.

Now,

$f(x) = e^x \sin x$

$f'(x) = e^x \cos x + e^x \sin x$

Now, $f'(c) = 0$

$e^c (\cos c + \sin c) = 0$

$\Rightarrow \quad e^c = 0$ or $\cos c = -\sin c$

$\Rightarrow \quad e^c = 0$ gives no value of c or $\tan c = -1$

$\Rightarrow \quad \tan c = \tan\left(\pi - \frac{\pi}{4}\right)$

$c = \frac{3\pi}{4} \in (0, \pi)$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(v)

Here,

$$f(x) = e^x \cos x \text{ on } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

We know that expontial and cosine function are continuous and differentiable every where so, $f(x)$ is continuous is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and differentiable is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Now,

$$f\left(-\frac{\pi}{2}\right) = e^{\frac{\pi}{2}} \cos\left(-\frac{\pi}{2}\right) = 0$$

$$f\left(\frac{\pi}{2}\right) = e^{\frac{\pi}{2}} \cos\left(\frac{\pi}{2}\right) = 0$$

$$\Rightarrow \quad f\left(-\frac{\pi}{2}\right) = f\left(\frac{\pi}{2}\right)$$

So, Rolle's theorem is applicable, so there must exist a point $c \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $f'(c) = 0$.

Now,

$$f(x) = e^x \cos x$$

$$f'(x) = -e^x \sin x + e^x \cos x$$

So, $\quad f'(c) = 0$

$$e^c(-\sin c + \cos c) = 0$$

$\Rightarrow \quad e^c = 0$ gives no value of c

$$\Rightarrow \quad -\sin c + \cos c = 0$$

$$\Rightarrow \quad \tan c = 1$$

$$\Rightarrow \quad c = \frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(vi)

Here,

$$f(x) = \cos 2x \text{ on } [0, \pi]$$

We know that, cosine function is contiuous and differentiable every where, so $f(x)$ is continuous is $[0, \pi]$ and differentiable is $(0, \pi)$.

Now,

$$f(0) = \cos 0 = 1$$

$$f(\pi) = \cos(2\pi) = 1$$

$$\Rightarrow \quad f(0) = f(\pi)$$

So, Rolle's theorem is applicable, so there must exist a point $c \in (0, \pi)$ such that $f'(c) = 0$.

Now,

$$f(x) = \cos 2x$$

$$f'(x) = -2\sin 2x$$

So, $\quad f'(c) = 0$

$$\Rightarrow \quad -2\sin 2c = 0$$

$$\Rightarrow \quad \sin 2c = 0$$

$$\Rightarrow \quad 2c = 0 \quad \text{or} \quad 2c = \pi$$

$$\Rightarrow \quad c = 0 \quad \text{or} \quad c = \frac{\pi}{2} \in (0, \pi)$$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(vii)

$f(x) = \dfrac{\sin x}{e^x}$ *on* $x \in [0, \pi]$

We know that, exponential and sine *both functions are continuous and differentiable every where, so* $f(x)$ *is continuous is* $[0, \pi]$ *and differentiable is* $[0, \pi]$

Now,

$$f(0) = \frac{\sin 0}{e^0} = 0$$

$$f(\pi) = \frac{\sin \pi}{e^\pi} = 0$$

$\Rightarrow f(0) = f(\pi)$

Since Rolle's theorem applicable, therefore there must exist a point $c \in [0, \pi]$ such that $f'(c) = 0$

Now,

$$f(x) = \frac{\sin x}{e^x}$$

$$\Rightarrow f'(x) = \frac{e^x(\cos x) - e^x(\sin x)}{\left(e^x\right)^2}$$

Now,

$$f'(c) = 0$$
$$\Rightarrow e^c(\cos c - \sin c) = 0$$
$$\Rightarrow e^c \neq 0 \text{ and } \cos c - \sin c = 0$$
$$\Rightarrow \tan c = 1$$
$$c = \frac{\pi}{4} \in [0, \pi]$$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(viii)

Here,

$$f(x) = \sin 3x \text{ on } [0, \pi]$$

We know that, sine function is continuous and differentiable every where. So, $f(x)$ is continuous is $(0, \pi)$ and differentiable is $(0, \pi)$.

Now,

$$f(0) = \sin 0 = 0$$
$$f(\pi) = \sin 3\pi = 0$$
$$\Rightarrow \quad f(0) = f(\pi)$$

So, Rolle's theorem is applicable, so there must exists a point $c \in (0, \pi)$ such that $f'(c) = 0$.

Now,

$$f(x) = \sin 3x$$
$$f'(x) = 3\cos 3x$$

Now,

$$f'(c) = 0$$
$$\Rightarrow \quad 3\cos 3x = 0$$
$$\Rightarrow \quad \cos 3x = 0$$
$$\Rightarrow \quad 3x = \frac{\pi}{2}$$
$$\Rightarrow \quad x = \frac{\pi}{6} \in (0, \pi)$$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(ix)

Here,

$$f(x) = e^{1-x^2} \text{ on } [-1,1]$$

We know that, exponential function is continuous and differentiable every where. So, $f(x)$ is continuous is $[-1,1]$ and differentiable is $(-1,1)$.

Now,

$$f(-1) = e^{1-1} = 1$$

$$f(1) = e^{1-1} = 1$$

$$\Rightarrow \quad f(-1) = 1$$

So, Rolle's theorem is applicable, so there must exist a point $c \in (-1,1)$ such that $f'(c) = 0$.

Now,

$$f(x) = e^{1-x^2}$$

$$f'(x) = e^{1-x^2}(-2x)$$

Now,

$$f'(c) = 0$$

$$-2ce^{1-c^2} = 0$$

$$\Rightarrow \quad c = 0 \quad \text{or} \quad e^{1-c^2} = 0$$

$$\Rightarrow \quad c = 0 \in (-1,1)$$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(x)

Here,

$$f(x) = \log\left(x^2+2\right) - \log 3 \text{ on } [-1,1]$$

We know that, logarithmic function is continuous and differentiable is its domain, so $f(x)$ is continuous is $[-1,1]$ and differentiable is $(-1,1)$.

Now,

$$f(-1) = \log(1+2) - \log 3 = 0$$

$$f(1) = \log(1+2) - \log 3 = 0$$

$$\Rightarrow \quad f(-1) = f(1)$$

So, Rolle's theorem is applicable, so there must exist a point $c \in (-1,1)$ such that $f'(c) = 0$.

Now,

$$f(x) = \log\left(x^2+2\right) - \log 3$$

$$f'(x) = \frac{(2x)}{x^2+2}$$

Now,

$$f'(c) = 0$$

$$\frac{2c}{c^2+2} = 0$$

$$\Rightarrow \quad c = 0 \in (-1,1)$$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(xi)

Here,

$$f(x) = \sin x + \cos x \text{ on } \left[0, \frac{\pi}{2}\right]$$

We know that $\sin x$ and $\cos x$ are continuous and differentiable every where, so $f(x)$ is continuous is $\left[0, \frac{\pi}{2}\right]$ and differentiable is $\left(0, \frac{\pi}{2}\right)$.

Now,

$$f(0) = \sin 0 + \cos c0 = 1$$

$$f\left(\frac{\pi}{2}\right) = \frac{\sin \pi}{2} + \frac{\cos \pi}{2} = 1$$

$$\Rightarrow \quad f(0) = f\left(\frac{\pi}{2}\right)$$

So, Rolle's theorem is applicable, so there must exist a point $c \in \left(0, \frac{\pi}{2}\right)$ such that $f'(c) = 0$.

Now,

$$f(x) = \sin x + \cos x$$

$$f'(x) = \cos x - \sin x$$

Now,

$$f'(c) = 0$$

$$\cos c - \sin c = 0$$

$$\Rightarrow \quad \tan c = 1$$

$$\Rightarrow \quad c = \frac{\pi}{4} \in \left(0, \frac{\pi}{2}\right)$$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(xii)

Here,

$$f(x) = 2\sin x + \sin 2x \text{ on } [0, \pi]$$

We know that sine function is continuous and differentiable every where, so $f(x)$ is continuous is $[0, \pi]$ and differentiable is $(0, \pi)$.

Now,

$$f(0) = 2\sin 0 + \sin 0 = 0$$

$$f(\pi) = 2\sin \pi + \sin 2\pi = 0$$

$$\Rightarrow \quad f(0) = f(\pi)$$

So, Rolle's theorem is applicable, so there must exist a point $c \in (0, \pi)$ such that $f'(c) = 0$.

Now,

$$f(x) = 2\sin x + \sin 2x$$

$$f'(x) = 2\cos x + 2\cos 2x$$

Now,

$$f'(c) = 0$$

$$2\cos c + 2\cos 2c = 0$$

$$\Rightarrow \quad 2\left(\cos c + 2\cos^2 c - 1\right) = 0$$

$$\Rightarrow \quad \left(2\cos^2 + 2\cos c - \cos c - 1\right) = 0$$

$$\Rightarrow \quad (2\cos c - 1)(\cos c + 1) = 0$$

$$\Rightarrow \quad \cos c = \frac{1}{2}, \quad \cos c = -1$$

$$\Rightarrow \quad \tan c = 1$$

$$c = \frac{\pi}{3} \in (0, \pi), \quad c = \pi$$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(xiii)

Here,

$$f(x) = \frac{x}{2} - \sin\frac{\pi x}{6} \text{ on } [-1, 0]$$

We know that sine function is continuous and differentiable every where, so $f(x)$ is continuous is $[-1, 0]$ and differentiable is $(-1, 0)$.

Now,

$$f(-1) = \frac{-1}{2} - \sin\left(-\frac{\pi}{6}\right)$$
$$= -\frac{1}{2} + \sin\frac{\pi}{6}$$
$$= -\frac{1}{2} + \frac{1}{2}$$
$$f(-1) = 0 \qquad \text{---(i)}$$

And $\quad f(0) = 0 - \sin 0$

$$f(0) = 0 \qquad \text{---(ii)}$$

From equation (i) and (ii),

$$f(-1) = f\binom{0}{}$$

So, Rolle's theorem is applicable, so there must exist a point $c \in (-1, 0)$ such that $f'(c) = 0$.

Now,

$$f(x) = \frac{x}{2} - \sin\left(\frac{\pi x}{6}\right)$$
$$f'(x) = \frac{1}{2} - \frac{\pi}{6}\cos\left(\frac{\pi x}{6}\right)$$

Now,

$$f'(c) = 0$$
$$\frac{1}{2} - \frac{\pi}{6}\cos\left(\frac{\pi c}{6}\right) = 0$$
$$\Rightarrow \quad -\frac{\pi}{6}\cos\left(\frac{\pi c}{6}\right) = -\frac{1}{2}$$
$$\Rightarrow \quad \cos\left(\frac{\pi c}{6}\right) = 3\pi$$
$$\Rightarrow \quad \frac{\pi c}{6} = \cos^{-1}\left(\frac{66}{7}\right)$$
$$\Rightarrow \quad c = \frac{6}{\pi}\cos^{-1}\left(\frac{66}{7}\right)$$
$$\Rightarrow \quad c = \frac{21}{11}\cos^{-1}\left(\frac{66}{7}\right)$$
$$\Rightarrow \quad c \in \left(-\frac{21}{11}, \frac{21}{11}\right) \qquad \left[\text{since, } \cos^{-1}x \in [-1, 1]\right]$$
$$\Rightarrow \quad c \in (-1.9, 1.9)$$
$$\Rightarrow \quad c \in (-1, 0)$$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(xiv)

Here,

$$f(x) = \frac{6x}{\pi} - 4\sin^2 x \text{ on } \left[0, \frac{\pi}{6}\right]$$

We know that sine and its square function is continuous and differentiable every where, so $f(x)$ is continuous is $\left[0, \frac{\pi}{6}\right]$ and differentiable is $\left(0, \frac{\pi}{6}\right)$.

Now,

$$f(0) = 0 - 0 = 0$$

$$f\left(\frac{\pi}{6}\right) = 1 - 1 = 0$$

$$\Rightarrow \quad f(0) = f\left(\frac{\pi}{6}\right)$$

So, Rolle's theorem is applicable, so there must exist a point $c \in \left(0, \frac{\pi}{6}\right)$ such that $f'(c) = 0$.

Now,

$$f(x) = \frac{6x}{\pi} - 4\sin^2 x$$

$$f'(x) = \frac{6}{\pi} - 8\sin x \cos x$$

$$f'(x) = \frac{6}{\pi} - 4\sin 2x$$

Now,

$$f'(c) = 0$$

$$\frac{6}{\pi} - 4\sin 2c = 0$$

$$\Rightarrow \quad 4\sin 2c = \frac{6}{\pi}$$

$$\Rightarrow \quad \sin 2c = \frac{3}{2\pi}$$

$$\Rightarrow \quad 2c = \sin^{-1}\left(\frac{21}{44}\right)$$

$$\Rightarrow \quad c = \frac{1}{2}\sin^{-1}\left(\frac{21}{44}\right)$$

$$\Rightarrow \quad c \in \left(-\frac{1}{2}, \frac{1}{2}\right) \qquad \left[\text{since, } \sin^{-1} x \in [-1, 1]\right]$$

$$\Rightarrow \quad c \in \left(0, \frac{11}{21}\right)$$

$$\Rightarrow \quad c \in \left(0, \frac{\pi}{6}\right)$$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(xv)

Here,

$$f(x) = 4^{\sin x} \text{ on } [0, \pi]$$

We know that exponential and $\sin x$ both are continuous and differentiable, so $f(x)$ is continuous is $[0, \pi]$ and differentiable is $(0, \pi)$.

Now,

$$f(0) = 4^{\sin 0} = 4^0 = 1$$

$$f(\pi) = 4^{\sin \pi} = 4^0 = 1$$

$$\Rightarrow \quad f(0) = f(\pi)$$

So, Rolle's theorem is applicable, so there must exist a point $c \in (0, \pi)$ such that $f'(c) = 0$.

Now,

$$f(x) = 4^{\sin x}$$

$$f'(x) = 4^{\sin x} \log 4 \times \cos x$$

Now,

$$f'(c) = 0$$

$$4^{\sin c} \times \cos x c \log 4 = 0$$

$$\Rightarrow \quad \cos c = 0$$

$$\Rightarrow \quad c = \frac{\pi}{2} \in (0, \pi)$$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(xvi)

Here,

$$f(x) = x^2 - 5x + 4 \text{ on } [1, 4]$$

$f(x)$ is continuous and differentiable as it is a polynomial funciton.

Now,

$$f(1) = (1)^2 - 5(1) + 4 = 0$$

$$f(4) = (4)^2 - 5(4) + 4 = 0$$

$$\Rightarrow \quad f(1) = f(4)$$

So, Rolle's theorem is applicable, so there must exist a point $c \in (1, 4)$ such that $f'(c) = 0$.

Now,

$$f(x) = x^2 - 5x + 4$$

$$f'(x) = 2x - 5$$

So,

$$f'(c) = 0$$

$$\Rightarrow \quad 2c - 5 = 0$$

$$\Rightarrow \quad c = \frac{5}{2} \in (1, 4)$$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(xvii)

Here,

$$f(x) = \sin^4 x + \cos^4 x \text{ on } \left[0, \frac{\pi}{2}\right]$$

We know that *sine* and *cosine* function are differentiable and continuous.

So, $f(x)$ is continuous is $\left[0, \frac{\pi}{2}\right]$ and it is differentiable is $\left(0, \frac{\pi}{2}\right)$.

Now,

$$f(0) = \sin^4(0) + \cos^4(0) = 1$$

$$f\left(\frac{\pi}{2}\right) = \sin^4\left(\frac{\pi}{2}\right) + \cos^4\left(\frac{\pi}{2}\right) = 1$$

$$\Rightarrow \quad f(0) = f\left(\frac{\pi}{2}\right)$$

So, Rolle's theorem is applicable, so there must exist a point $c \in \left(0, \frac{\pi}{2}\right)$ such that $f'(c) = 0$.

Now,

$$f(x) = \sin^4 x + \cos^4 x$$

$$f'(x) = 4\sin^3 x \cos x - 4\cos^3 x \sin x$$

$$= -2(2\sin x \cos x)\left(\cos^2 x - \cos^2 x\right)$$

$$= -2\sin 2x \cos 2x$$

$$f'(x) = -\sin 4x$$

Now,

$$f'(c) = 0$$

$$-\sin 4x = 0$$

$$\sin 4x = 0$$

$$\Rightarrow \quad 4x = 0 \quad \text{or} \quad 4x = \pi$$

$$\Rightarrow \quad x = 0 \quad \text{or} \quad x = \frac{\pi}{4} \in \left(0, \frac{\pi}{2}\right)$$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(xvii)

Since trigonometric functions are differentiable and continuous,

the given function, $f(x) = \sin x - \sin 2x$ is also continuous and differentiable.

Now $f(0) = \sin 0 - \sin 2 \times 0 = 0$

and

$f(\pi) = \sin\pi - \sin 2 \times \pi = 0$

$\Rightarrow f(0) = f(\pi)$

Thus, $f(x)$ satisfies conditions of the Rolle's Theorem on $[0, \pi]$.

Therefore, there exists $c \in [0, \pi]$ such that $f'(c) = 0$

Now $f(x) = \sin x - \sin 2x$

$\Rightarrow f'(x) = \cos x - 2\cos 2x = 0$

$\Rightarrow \cos x = 2\cos 2x$

$\Rightarrow \cos x = 2\left(2\cos^2 x - 1\right)$

$\Rightarrow \cos x = 4\cos^2 x - 2$

$\Rightarrow 4\cos^2 x - \cos x - 2 = 0$

$\Rightarrow \cos x = \dfrac{1 \pm \sqrt{33}}{8} = 0.8431 \text{ or } -0.5931$

$\Rightarrow x = \cos^{-1}(0.8431) \text{ or } \cos^{-1}(-0.5931)$

$\Rightarrow x = \cos^{-1}(0.8431) \text{ or } 180^\circ - \cos^{-1}(0.5931) \quad \left[\because \cos^{-1}(-x) = \pi - \cos^{-1}(x)\right]$

$\Rightarrow x = 32^\circ 32' \text{ or } x = 126^\circ 23'$

Both $32^\circ 32'$ and $126^\circ 23' \in [0, \pi]$ such that $f'(c) = 0$.

Hence Rolle's Theorem is verified.

Mean Value Theorems Ex 15.1 Q3(xviii)

Since trigonometric functions are differentiable and continuous,
the given function, $f(x) = \sin x - \sin 2x$ is also continuous and differentiable.

Now $f(0) = \sin 0 - \sin 2 \times 0 = 0$

and

$f(\pi) = \sin \pi - \sin 2 \times \pi = 0$

$\Rightarrow f(0) = f(\pi)$

Thus, $f(x)$ satisfies conditions of the Rolle's Theorem on $[0, \pi]$.

Therefore, there exists $c \in [0, \pi]$ such that $f'(c) = 0$

Now $f(x) = \sin x - \sin 2x$

$\Rightarrow f'(x) = \cos x - 2\cos 2x = 0$

$\Rightarrow \cos x = 2\cos 2x$

$\Rightarrow \cos x = 2(2\cos^2 x - 1)$

$\Rightarrow \cos x = 4\cos^2 x - 2$

$\Rightarrow 4\cos^2 x - \cos x - 2 = 0$

$\Rightarrow \cos x = \frac{1 \pm \sqrt{33}}{8} = 0.8431 \text{ or } -0.5931$

$\Rightarrow x = \cos^{-1}(0.8431) \text{ or } \cos^{-1}(-0.5931)$

$\Rightarrow x = \cos^{-1}(0.8431) \text{ or } 180° - \cos^{-1}(0.5931) \quad [\because \cos^{-1}(-x) = \pi - \cos^{-1}(x)]$

$\Rightarrow x = 32°32' \text{ or } x = 126°23'$

Both $32°32'$ and $126°23' \in [0, \pi]$ such that $f'(c) = 0$.

Hence Rolle's Theorem is verified.

Mean Value Theorems Ex 15.1 Q7

Let $f(x) = 16 - x^2$, then $f'(x) = -2x$

$f(x)$ is continuous on $[-1, 1]$ because it is a polynomial function.

Also $f(-1) = 16 - (-1)^2 = 15$

$f(1) = 16 - (1)^2 = 15$

$f(-1) = f(1)$

There exists a $c \in [-1, 1]$ such that $f'(c) = 0$

$\Rightarrow -2c = 0$

$\Rightarrow c = 0$

Thus, at $0 \in [-1, 1]$ the tangent is parallel to the x-axis.

Mean Value Theorems Ex 15.1 Q8(i)

Let $f(x) = x^2$, then $f'(x) = 2x$

$f(x)$ is continuous on $[-2, 2]$ because it is a polynomial function.

$f(x)$ is differentiable on $(-2, 2)$ as it is a polynomial function.

Also $f(-2) = (-2)^2 = 4$

$f(2) = 2^2 = 4$

$\Rightarrow \quad f(-2) = f(2)$

$\therefore \quad$ There exists $c \in (-2, 2)$ such that $f'(c) = 0$

$\Rightarrow \quad 2c = 0$

$\Rightarrow \quad c = 0$

Thus, at $0 \in [-2, 2]$ the tangent is parallel to the x-axis.

x = 0, then y = 0

Therefore, the point is (0, 0)

Mean Value Theorems Ex 15.1 Q8(ii)

Let $f(x) = e^{1-x^2}$ on $[-1,1]$

Since, $f(x)$ is a composition of two continuous functions, it is continuous on $[-1,1]$

Also $f(x) = -2xe^{1-x^2}$

$f(2) = 2^2 = 4$

$\therefore$ $f'(x)$ exists for every value of x in $(-1,1)$

$\Rightarrow$ $f(x)$ is differentiable on $(-1,1)$

By rolle's theorem, there exists $c \in (-1,1)$ such that $f'(c) = 0$

$\Rightarrow$ $-2ce^{1-c^2} = 0$

$\Rightarrow$ $c = 0$

Thus, at $c = 0 \in [-1,1]$ the tangent is parallel to the x-axis.

x = 0, then y = e

Therefore, the point is (0, e)

Mean Value Theorems Ex 15.1 Q8(iii)

Let $f(x) = 12(x+1)(x-2)$

Since, $f(x)$ is a polynomial function, it is continuous on $[-1,2]$ and differentiable on $(-1,2)$

Also $f'(x) = 12[(x-2)+(x+1)] = 12[2x-1]$

By rolle's theorem, there exists $c \in (-1,2)$ such that $f'(c) = 0$

$\Rightarrow$ $12(2c-1) = 0$

$\Rightarrow$ $c = \frac{1}{2}$

Thus, at $c = \frac{1}{2} \in (-1,2)$ the tangent to $y = 12(x+1)(x-2)$ is parallel to x-axis

Mean Value Theorems Ex 15.1 Q9

It is given that $f : [-5,5] \to \mathbf{R}$ is a differentiable function.

Since every differentiable function is a continuous function, we obtain

(a) f is continuous on $[-5, 5]$.

(b) f is differentiable on $(-5, 5)$.

Therefore, by the Mean Value Theorem, there exists c □ $(-5, 5)$ such that

$$f'(c) = \frac{f(5)-f(-5)}{5-(-5)}$$

$$\Rightarrow 10f'(c) = f(5)-f(-5)$$

It is also given that $f'(x)$ does not vanish anywhere.

$$\therefore f'(c) \neq 0$$

$$\Rightarrow 10f'(c) \neq 0$$

$$\Rightarrow f(5)-f(-5) \neq 0$$

$$\Rightarrow f(5) \neq f(-5)$$

Hence, proved.

Mean Value Theorems Ex 15.1 Q10

By Rolle's Theorem, for a function $f:[a, b] \to \mathbf{R}$, if

(a) f is continuous on $[a, b]$

(b) f is differentiable on (a, b)

(c) $f(a) = f(b)$

then, there exists some $c \in (a, b)$ such that $f'(c) = 0$

Therefore, Rolle's Theorem is not applicable to those functions that do not satisfy any of the three conditions of the hypothesis.

(i) $f(x) = [x]$ for $x \in [5, 9]$

It is evident that the given function $f(x)$ is not continuous at every integral point.

In particular, $f(x)$ is not continuous at $x = 5$ and $x = 9$

$f(x)$ is not continuous in $[5, 9]$.

Also, $f(5) = [5] = 5$ and $f(9) = [9] = 9$

$\therefore f(5) \neq f(9)$

The differentiability of f in $(5, 9)$ is checked as follows.

Let n be an integer such that $n \in (5, 9)$.

The left hand limit of f at $x = n$ is,

$$\lim_{h \to 0^-} \frac{f(n+h) - f(n)}{h} = \lim_{h \to 0^-} \frac{[n+h] - [n]}{h} = \lim_{h \to 0^-} \frac{n-1-n}{h} = \lim_{h \to 0^-} \frac{-1}{h} = \infty$$

The right hand limit of f at $x = n$ is,

$$\lim_{h \to 0^+} \frac{f(n+h) - f(n)}{h} = \lim_{h \to 0^+} \frac{[n+h] - [n]}{h} = \lim_{h \to 0^+} \frac{n-n}{h} = \lim_{h \to 0^+} 0 = 0$$

Since the left and right hand limits of f at $x = n$ are not equal, f is not differentiable at $x = n$

f is not differentiable in $(5, 9)$.

It is observed that f does not satisfy all the conditions of the hypothesis of Rolle's Theorem.

Hence, Rolle's Theorem is not applicable for $f(x) = [x]$ for $x \in [5, 9]$.

(ii) $f(x)=[x]$ for $x \in [-2,\ 2]$

It is evident that the given function $f(x)$ is not continuous at every integral point.

In particular, $f(x)$ is not continuous at $x=-2$ and $x=2$

$f(x)$ is not continuous in $[-2, 2]$.

Also, $f(-2)=[-2]=-2$ and $f(2)=[2]=2$

$\therefore f(-2) \neq f(2)$

The differentiability of f in $(-2, 2)$ is checked as follows.

Let n be an integer such that $n \in (-2, 2)$.

The left hand limit of f at $x=n$ is,

$$\lim_{h\to 0^-} \frac{f(n+h)-f(n)}{h} = \lim_{h\to 0^-} \frac{[n+h]-[n]}{h} = \lim_{h\to 0^-} \frac{n-1-n}{h} = \lim_{h\to 0^-} \frac{-1}{h} = \infty$$

The right hand limit of f at $x=n$ is,

$$\lim_{h\to 0^+} \frac{f(n+h)-f(n)}{h} = \lim_{h\to 0^+} \frac{[n+h]-[n]}{h} = \lim_{h\to 0^+} \frac{n-n}{h} = \lim_{h\to 0^+} 0 = 0$$

Since the left and right hand limits of f at $x=n$ are not equal, f is not differentiable at $x=n$

f is not differentiable in $(-2, 2)$.

It is observed that f does not satisfy all the conditions of the hypothesis of Rolle's Theorem.

Hence, Rolle's Theorem is not applicable for $f(x)=[x]$ for $x \in [-2,\ 2]$.

Mean Value Theorems Ex 15.1 Q11

It is given that the Rolle's Theorem holds for the function $f(x) = x^3 + bx^2 + cx,\ x \in [1,2]$ at the point $x = \frac{4}{3}$.

We need to find the values of b and c.

$f(x) = x^3 + bx^2 + cx$

Since it satisfies the rolle's theorem, we have,

$f(1) = f(2)$

$\Rightarrow 1^3 + b \times 1^2 + c \times 1 = 2^3 + b \times 2^2 + c \times 2$

$\Rightarrow 1 + b + c = 8 + 4b + 2c$

$\Rightarrow 3b + c = -7 \ldots(1)$

Differentiating the given function, we have,

$f'(x) = 3x^2 + 2bx + c$

$f'\left(\frac{4}{3}\right) = 3 \times \left(\frac{4}{3}\right)^2 + 2b \times \left(\frac{4}{3}\right) + c$

$\Rightarrow 0 = \frac{16}{3} + \frac{8b}{3} + c \ldots(2)$

Solving the equations (1) and (2), we have,

b = −5 and c = 8

Ex 15.2

Mean Value Theorems Ex 15.2 Q1(i)

Here,

$$f(x) = x^2 - 1 \text{ on } [2,3]$$

It is a polynomial function so it is continuous in $[2,3]$ and differentiable in $(2,3)$. So, both conditions of Lagrange's mean value theorem are satisfied.

Therefore, there exist a point $c \in (2,3)$ such that

$$f'(c) = \frac{f(3) - f(2)}{3-2}$$

$$2c = \frac{\left((3)^2 - 1\right) - \left((2)^2 - 1\right)}{1}$$

$$2c = (8-3)$$

$$c = \frac{5}{2} \in (2,3)$$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(ii)

Here,

$$f(x) = x^3 - 2x^2 - x + 3 \text{ on } [0,1]$$

Since, $f(x)$ is a polynomial function. So, $f(x)$ is continuous in $[0,1]$ and differentiable in $(0,1)$. So, Lagrange's mean value theorem is applicable. Thus, there exists a point $c \in (0,1)$ such that

$$f'(c) = \frac{f(1) - f(0)}{1 - 0}$$

$$\Rightarrow \quad 3c^2 - 4c - 1 = \frac{\left[(1)^3 - 2(1)^2 - (1) + 3\right] - 3}{1}$$

$$\Rightarrow \quad 3c^2 - 4c - 1 = 1 - 3$$

$$\Rightarrow \quad 3c^2 - 4c + 1 = 0$$

$$\Rightarrow \quad 3c^2 - 3c - c + 1 = 0$$

$$\Rightarrow \quad 3c(c - 1) - 1(c - 1) = 0$$

$$\Rightarrow \quad (3c - 1)(c - 1) = 0$$

$$\Rightarrow \quad c = \frac{1}{3} \in (0,1)$$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(iii)

Here,

$$f(x) = x(x - 1)$$

$$f(x) = x^2 - x \quad \text{on } [1,2]$$

We know that, polynomial function is continuous and differentiable. So, $f(x)$ is continuous in $[1,2]$ and $f(x)$ is differentiable in $(1,2)$. So, Lagrange's mean value theorem is applicable. Thus, there exists a point $c \in (1,2)$ such that

$$f'(c) = \frac{f(2) - f(1)}{2 - 1}$$

$$\Rightarrow \quad 2c - 1 = \frac{(4 - 2) - (1 - 1)}{1}$$

$$\Rightarrow \quad 2c - 1 = \frac{2 - 0}{1}$$

$$\Rightarrow \quad 2c = 3$$

$$\Rightarrow \quad c = \frac{3}{2} \in (1,2)$$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(iv)

Here,

$$f(x) = x^2 - 3x + 2 \text{ on } [-1,2]$$

We know that, polynomial function is continuous and differentiable. So, $f(x)$ is continuous in $[-1,2]$ and differentiable in $(-1,2)$. So, Lagrange's mean value theorem is applicable, so there exist a point $c \in (-1,2)$ such that

$$f'(c) = \frac{f(2) - f(-1)}{2 + 1}$$

$$\Rightarrow \quad 2c - 3 = \frac{(4 - 6 + 2) - (1 + 3 + 2)}{3}$$

$$\Rightarrow \quad 2c - 3 = -\frac{6}{3}$$

$$\Rightarrow \quad 2c = 1$$

$$\Rightarrow \quad c = \frac{1}{2} \in (-1,2)$$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(v)

Here,

$$f(x) = 2x^2 - 3x + 1 \text{ on } [1,3]$$

We know that, polynomial function is continuous and differentiable. So, $f(x)$ is continuous in $[1,3]$ and $f(x)$ is differentiable in $(1,3)$. So, Lagrange's mean value theorem is applicable, so there exist a point $c \in (1,3)$ such that

$$f'(c) = \frac{f(3) - f(1)}{3-1}$$

$$\Rightarrow \quad 4c - 3 = \frac{\left(2(3)^2 - 3(3) + 1\right) - (2 - 3 + 1)}{3-1}$$

$$\Rightarrow \quad 4c - 3 = \frac{10}{2}$$

$$\Rightarrow \quad 4c = 5 + 3$$

$$\Rightarrow \quad 4c = 8$$

$$\Rightarrow \quad c = 2 \in (1,3)$$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(vi)

Here,

$$f(x) = x^2 - 2x + 4 \text{ on } [1,5]$$

We know that, polynomial is always continuous and differentiable. So, $f(x)$ is continuous in $[1,5]$ and it is differentiable in $(1,5)$. So, Lagrange's mean value theorem is applicable. Thus, there exists a point $c \in (1,5)$ such that

$$f'(c) = \frac{f(5) - f(1)}{5-1}$$

$$\Rightarrow \quad 2c - 2 = \frac{\left((5)^2 - 2(5) + 4\right) - (1 - 2 + 4)}{4}$$

$$\Rightarrow \quad 2c - 2 = \frac{19-3}{4}$$

$$\Rightarrow \quad 2c - 2 = 4$$

$$\Rightarrow \quad 2c = 6$$

$$\Rightarrow \quad c = 3 \in (1,5)$$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(vii)

Here,

$$f(x) = 2x - x^2 \text{ on } [0,1]$$

We know that, polynomial is continuous and differentiable. So, $f(x)$ is continuous in $[0,1]$ and differentiable in $(0,1)$. So, Lagrange's mean value theorem is applicable. Thus, there exists a point $c \in (0,1)$ such that

$$f'(c) = \frac{f(1) - f(0)}{1-0}$$

$$\Rightarrow \quad 2 - 2c = \frac{\left(2(1) - (1)^2\right) - (0)}{1}$$

$$\Rightarrow \quad 2 - 2c = 1$$

$$\Rightarrow \quad 1 = 2c$$

$$\Rightarrow \quad c = \frac{1}{2} \in (0,1)$$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(viii)

$$f(x) = (x-1)(x-2)(x-3) \text{ on } [0,4]$$

We know that, polynomial is continuous and differentiable every where. So, $f(x)$ is continuous in $[0,4]$ and differentiable in $(0,4)$. So, Lagrange's mean value theorem is applicable. Thus, there exists a point $c \in (0,4)$ such that

$$f'(c) = \frac{f(4) - f(0)}{4-0}$$

$$\Rightarrow \quad (c-1)(c-2) + (c-2)(c-3) + (c-1)(c-3) = \frac{(3)(2)(1) - (-1)(-2)(-3)}{4-0}$$

$$\Rightarrow \quad c^2 - 3c + 2 + c^2 + 5c + 6 + c^2 - 4c + 3 = \frac{6+6}{4}$$

$$\Rightarrow \quad 3c^2 - 12c + 11 = 3$$

$$\Rightarrow \quad 3c^2 = 12c + 8 = 0$$

$$\Rightarrow \quad c = \frac{-(-12) \pm \sqrt{144 - 4 \times 3 \times 8}}{6}$$

$$\Rightarrow \quad c = \frac{12 \pm \sqrt{48}}{6}$$

$$\Rightarrow \quad c = 2 \pm \frac{2\sqrt{3}}{3} \in (0,4)$$

$$\Rightarrow \quad c = 2 \pm \frac{2}{\sqrt{3}} \in (0,4)$$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(ix)

Here,

$$f(x) = \sqrt{25 - x^2} \text{ on } [-3,4]$$

Given function is continuous as it has unique value for each $x \in [-3,4]$ and

$$f'(x) = \frac{-2x}{2\sqrt{25 - x^2}}$$

$$f'(x) = \frac{-x}{\sqrt{25 - x^2}}$$

So, $f'(x)$ exists for all values for $x \in (-3,4)$ so, $f(x)$ is differentiable in $(-3,4)$. So, Lagrange's mean value theorem is applicable. Thus, there exists a point $c \in (-3,4)$ such that

$$f'(c) = \frac{f(4) - f(-3)}{4+3}$$

$$\Rightarrow \quad \frac{-2c}{2\sqrt{25 - c^2}} = \frac{\sqrt{9} - \sqrt{16}}{7}$$

$$\Rightarrow \quad -7c = -\sqrt{25 - c^2}$$

Squaring both the sides,

$$49c^2 = 25 - c^2$$

$$c^2 = \frac{1}{2}$$

$$c = \pm\frac{1}{\sqrt{2}} \in (-3,4)$$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(x)

Here,

$$f(x) = \tan^{-1} x \text{ on } [0,1]$$

We know that, $\tan^{-1} x$ has unique value in $[0,1]$ so, it is continuous in $[0,1]$

$$f'(x) = \frac{1}{1+x^2}$$

So, $f'(x)$ exists for each $x \in (0,1)$

So, $f'(x)$ is differentiable in $(0,1)$, thus Lagrange's mean value theorem is applicable, so there exist a point $c \in (0,1)$ such that

$$f'(c) = \frac{f(1) - f(0)}{1-0}$$

$$\Rightarrow \quad \frac{1}{1+c^2} = \frac{\tan^{-1}(1) - \tan^{-1}(0)}{1}$$

$$\Rightarrow \quad \frac{1}{1+c^2} = \frac{\frac{\pi}{4} - 0}{1}$$

$$\Rightarrow \quad \frac{4}{\pi} = 1 + c^2$$

$$\Rightarrow \quad c = \sqrt{\frac{4}{\pi} - 1}$$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(xi)

Here,

$$f(x) = x + \frac{1}{x} \text{ on } [1,3]$$

$f(x)$ attiams a unique value for each $x \in [1,3]$, so it is continuous

$f'(x) = 1 - \frac{1}{x^2}$ is definded for each $x \in (1,3)$

$\Rightarrow \quad f(x)$ is differentiable in $(1,3)$, so Lagrange's mean value theorem is a applicable, so there exist a point $c \in (1,3)$ such that

$$f'(c) = \frac{f(3) - f(1)}{3-1}$$

$$\Rightarrow \quad 1 - \frac{1}{c^2} = \frac{\left(3 + \frac{1}{3} - (1+1)\right)}{2}$$

$$\Rightarrow \quad 1 - \frac{1}{c^2} = \frac{\frac{10}{3} - 2}{2}$$

$$\Rightarrow \quad 1 - \frac{1}{c^2} = \frac{4}{3 \times 2}$$

$$\Rightarrow \quad 1 - \frac{2}{3} = \frac{1}{c^2}$$

$$\Rightarrow \quad c^2 = 3$$

$$\Rightarrow \quad c = \sqrt{3} \in (1,3)$$

So, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(xii)

Here,

$$f(x) = x(x+4)^2 \text{ on } [0,4]$$

We know that every polynomial function is continuous and differentiable every wher, so, $f(x)$ is continuous in $[0,4]$ and differentiable in $(0,4)$, so, Lagrange's mean value theorem is applicable, thus there exist a point $c \in (0,4)$ such that

$$f'(c) = \frac{f(4) - f(0)}{4 - 0}$$

$\Rightarrow \quad 3c^2 + 16c + 16 = \dfrac{4 \times (8)^2 - 0}{4}$

$\Rightarrow \quad 3c^2 + 16c + 16 = 64$

$\Rightarrow \quad 3c^2 + 16c - 48 = 0$

$\Rightarrow \quad c = \dfrac{-16 \pm \sqrt{256 + 576}}{6}$

$\Rightarrow \quad = \dfrac{-16 \pm \sqrt{832}}{6}$

$\Rightarrow \quad = \dfrac{-16 \pm 8\sqrt{13}}{6}$

$\Rightarrow \quad c = \dfrac{-8 \pm 4\sqrt{13}}{3}$

$$c = \frac{-8 + 4\sqrt{13}}{3} \in (0,4)$$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(xiii)

Here,

$$f(x) = x\sqrt{x^2 - 4} \text{ on } [2,4]$$

$f(x)$ is continuous at it attains a unique value for each $x \in [2,4]$ and

$$f'(x) = \frac{2x}{2\sqrt{x^2 - 4}}$$

$$f'(x) = \frac{x}{\sqrt{x^2 - 4}}$$

$\Rightarrow \quad f'(x)$ exists for each $x \in (2,4)$

$\Rightarrow \quad f(x)$ is differentiable in $(2,4)$, so

Lagrange's mean value theorem is applicable, so there exist a $c \in (2,4)$ such that

$$f'(c) = \frac{f(4) - f(2)}{4 - 2}$$

$\Rightarrow \quad \dfrac{c}{\sqrt{c^2 - 4}} = \dfrac{\sqrt{12} - 0}{2}$

Squarintg both the sides,

$\Rightarrow \quad \dfrac{c^2}{c^2 - 4} = \dfrac{12}{4}$

$\Rightarrow \quad 4c^2 = 12c^2 - 48$

$\Rightarrow \quad 8c^2 = 48$

$\Rightarrow \quad c^2 = 6$

$\Rightarrow \quad c = \sqrt{6} \in (2,4)$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(xiv)

Here,

$$f(x) = x^2 + x - 1 \text{ on } [0, 4]$$

$f(x)$ is polynomial, so it is continuous is $[0, 4]$ and differentiable in $(0, 4)$
as every polynomial is continuous and differentiable every where. So,
Lagrange's mean value theorem is applicable, so there exists a point $c \in [0, 4]$ such that

$$f'(c) = \frac{f(4) - f(0)}{4 - 0}$$

$$\Rightarrow \quad 2c + 1 = \frac{\left((4)^2 + 4 - 1\right) - (0 - 1)}{4}$$

$$\Rightarrow \quad 2c + 1 = \frac{19 + 1}{4}$$

$$\Rightarrow \quad 2c + 1 = 5$$

$$\Rightarrow \quad c = 2 \in (0, 4)$$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(xv)

Here,

$$f(x) = \sin x - \sin 2x - x \text{ on } [0, \pi]$$

We know that $\sin x$ and polynomial is continuous and differentiable every where so,
$f(x)$ is continuous in $[0, \pi]$ and differentiable in $[0, \pi]$. So, Lagrange's mean value theorem is applicable. So, there exist a point $c \in (0, \pi)$ such that

$$f'(c) = \frac{f(\pi) - f(0)}{\pi - 0}$$

$$\Rightarrow \quad \cos c - 2\cos 2c - 1 = \frac{(\sin \pi - \sin 2\pi - \pi) - (0)}{\pi}$$

$$\Rightarrow \quad \cos c - 2\cos 2c = -1 + 1$$

$$\Rightarrow \quad \cos c - 2\left(2\cos^2 c - 1\right) = 0$$

$$\Rightarrow \quad 4\cos^2 c - \cos c - 2 = 0$$

$$\Rightarrow \quad \cos c - \frac{-(-1) \pm \sqrt{1 - 4 \times 4 \times (-2)}}{8}$$

$$\Rightarrow \quad \cos c = \frac{1 \pm \sqrt{33}}{8}$$

$$\Rightarrow \quad c = \cos^{-1}\left(\frac{1 \pm \sqrt{33}}{8}\right) \in (0, \pi)$$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(xvi)

The given function is $f(x) = x^3 - 5x^2 - 3x$, f being a polynomial function, is continuous in [1,3] and is differentiable in [1,3] whose derivative is $3x^2 - 10x - 3$.

$$f(1) = 1^3 - 5(1)^2 - 3(1) = -7$$

$$f(3) = 3^3 - 5(3)^2 - 3(3) = 27 - 45 - 9 = -27$$

$$\therefore \frac{f(b) - f(a)}{b - a} = \frac{f(3) - f(1)}{3 - 1} = \frac{-27 + 7}{2} = -10$$

Mean value theorem states that there is a point $c(1,3)$ such that $f'(c) = 3c^2 - 10c - 3$

$$f'(c) = -10$$

$$3c^2 - 10c - 3 = -10$$

$$3c^2 - 10c + 7 = 0$$

$$3c^2 - 3c - 7c + 7 = 0$$

$$c = \frac{7}{3}, \text{ where } c = \frac{7}{3} \in (1, 3)$$

Hence, Mean value theorem is verified for the given function.

Mean Value Theorems Ex 15.2 Q2

Here,

$$f(x) = |x| \text{ on } [-1,1]$$

$$f(x) = \begin{cases} -x, & x < 0 \\ x, & x \ge 0 \end{cases}$$

For differentiability at $x = 0$

$$\text{LHD} = \lim_{x \to 0^-} \frac{f(0-h) - f(0)}{-h}$$

$$= \lim_{h \to 0} \frac{-(0-h) - 0}{-h}$$

$$= \lim_{h \to 0} \frac{h}{-h}$$

$$\text{LHD} = -1$$

$$\text{RHD} = \lim_{x \to 0^+} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{(0+h) - 0}{h}$$

$$= \lim_{h \to 0} \frac{h}{h}$$

$$= 1$$

$\therefore$ LHD $\neq$ RHD

$\Rightarrow$ $f(x)$ is not differentiable at $x = 0 \in (-1,1)$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q3

Here,

$$f(x) = \frac{1}{x} \text{ on } [-1,1]$$

$$f'(x) = -\frac{1}{x^2}$$

$\Rightarrow$ $f'(x)$ doesnot exist at $x = 0 \in (-1,1)$

$\Rightarrow$ $f(x)$ is not differentiable in $(-1,1)$

Hence, LMVT is verified

Mean Value Theorems Ex 15.2 Q4

Here,

$$f(x) = \frac{1}{4x-1}, x \in [1,4]$$

f(x) attain unique value for each $x \in [1,4]$, so f(x) is continuous in [1, 4].

$$f'(x) = -\frac{4}{(4x-1)^2}$$

$\Rightarrow f'(x)$ exists for each $x \in (1, 4)$

$\Rightarrow f'(x)$ is differentiable in $(1, 4)$

So, Lagranges mean value theroem is applicable.

So, there exist a point $c \in (1, 4)$ such that,

$$f'(c) = \frac{f(4) - f(1)}{4-1}$$

$$\Rightarrow -\frac{4}{(4x-1)^2} = \frac{\frac{1}{15} - \frac{1}{3}}{3}$$

$$\Rightarrow -\frac{4}{(4x-1)^2} = -\frac{4}{45}$$

$$\Rightarrow (4x-1)^2 = 45$$

$$\Rightarrow 4x - 1 = \pm 3\sqrt{5}$$

$$\Rightarrow x = \frac{3\sqrt{5}+1}{4} \in [1,4]$$

Mean Value Theorems Ex 15.2 Q5

Here,

$$\text{curve is } y = (x-4)^2$$

Since, it a polynomial function so it is differentiable and continuous. So, it Lagrange's mean value theorem is applicable, so, there exist a point c such that,

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$\Rightarrow \quad 2(c-4) = \dfrac{f(5)-f(4)}{5-4}$

$\Rightarrow \quad 2c-8 = \dfrac{1-0}{1}$

$\Rightarrow \quad 2c = 9$

$\Rightarrow \quad c = \dfrac{9}{2}$

$\Rightarrow \quad y = \left(\dfrac{9}{2}-4\right)^2$

$\quad y = \dfrac{1}{4}$

Thus, $(c, y) = \left(\dfrac{9}{2}, \dfrac{1}{4}\right)$ is required point.

Mean Value Theorems Ex 15.2 Q6

Here,

$$y = x^2 + x$$

Since, y is a polynomial function, so it continuous differentiable,

$\Rightarrow$ Lagrange's mean value theorem is applicable, so, there exist a point c such that,

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$\Rightarrow \quad 2c+1 = \dfrac{f(1)-f(0)}{1-0}$

$\Rightarrow \quad 2c+1 = 2$

$\Rightarrow \quad c = \dfrac{1}{2}$

$\Rightarrow \quad y = \left(\dfrac{1}{2}\right)^2 + \dfrac{1}{2}$

$\Rightarrow \quad y = \dfrac{3}{4}$

So, $(c, y) = \left(\dfrac{1}{2}, \dfrac{3}{4}\right)$ is the required point.

Mean Value Theorems Ex 15.2 Q7

Here,

$$y = (x-3)^2$$

Since, y is a polynomial function, so it continuous differentiable,

$\Rightarrow$ Lagrange's mean value theorem is applicable

$\Rightarrow$ There exist a point c such that,

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$\Rightarrow \quad 2(c-3) = \dfrac{f(4)-f(3)}{4-3}$

$\Rightarrow \quad 2c-6 = \dfrac{1-0}{1}$

$\Rightarrow \quad 2c = 7$

$\Rightarrow \quad c = \dfrac{7}{2}$

$\Rightarrow \quad y = \left(\dfrac{7}{2}-3\right)^2$

$\Rightarrow \quad y = \dfrac{1}{4}$

So, $(c, y) = \left(\dfrac{7}{2}, \dfrac{1}{4}\right)$ is the required point.

Mean Value Theorems Ex 15.2 Q8

Here,

$$y = x^3 - 3x$$

y is a polynomial function, so it is continuous differentiable, so

Lagrange's mean value theorem is applicable thus there exists a point c such that,

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\Rightarrow \quad 3c^2 - 3 = \frac{f(2) - f(1)}{2 - 1}$$

$$\Rightarrow \quad 3c^2 - 3 = \frac{2 + 2}{1}$$

$$\Rightarrow \quad 3c^2 = 7$$

$$\Rightarrow \quad c = \pm\sqrt{\frac{7}{3}}$$

$$\Rightarrow \quad y = \mp\frac{2}{3}\sqrt{\frac{7}{3}}$$

So, $(c, y) = \left(\pm\sqrt{\frac{7}{3}}, \mp\frac{2}{3}\sqrt{\frac{7}{3}}\right)$ is the required point.

Mean Value Theorems Ex 15.2 Q9

Here,

$$y = x^3 + 1$$

It is a polynomial function, so it is continuous differentiable.

$\Rightarrow$ Lagrange's mean value theorem is applicable, so there exists a point c such that,

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\Rightarrow \quad 3c^2 = \frac{f(3) - f(1)}{3 - 1}$$

$$\Rightarrow \quad 3c^2 = \frac{28 - 2}{2}$$

$$\Rightarrow \quad c^2 = \frac{13}{3}$$

$$\Rightarrow \quad c = \sqrt{\frac{13}{3}}$$

$$\Rightarrow \quad y = \left(\frac{13}{3}\right)^{\frac{3}{2}} + 1$$

So, $(c, y) = \left(\sqrt{\frac{13}{3}}, \left(\frac{13}{3}\right)^{\frac{3}{2}} + 1\right)$ is the required point.

Mean Value Theorems Ex 15.2 Q10

Trigonometric functions are continuous and differentiable.

Thus, the curve C is continuous between the points $(a,0)$ and $(0,a)$and is differentiable on $[a,a]$

Therefore, by Lagrange's Mean Value Theorem, there exists a real number $c \in (a,a)$ such that

$$f'(c) = \frac{a-0}{0-a} = -1$$

Now consider the parametric functions of the given function

$x = a\cos^3\theta$

and

$y = a\sin^3\theta$

$$\Rightarrow \frac{dx}{d\theta} = 3a\cos^2\theta(-\sin\theta)$$

and

$$\Rightarrow \frac{dy}{d\theta} = 3a\sin^2\theta(\cos\theta)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{3a\sin^2\theta(\cos\theta)}{3a\cos^2\theta(-\sin\theta)}$$

$$\Rightarrow \frac{dy}{dx} = -\tan\theta$$

Slope of the chord joining the points $(a,0)$ and $(0,a)$

=Slope of the tangent at $(c,f(c))$, where c lies on the curve

$$\Rightarrow \frac{a-0}{0-a} = -\tan\theta$$

$$\Rightarrow -1 = -\tan\theta$$

$$\Rightarrow \tan\theta = 1$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

Now substituting $\theta = \frac{\pi}{4}$, in the parametric representations, we have,

$x = a\cos^3\theta, y = a\sin^3\theta$

$$\Rightarrow x = a\cos^3\left(\frac{\pi}{4}\right), y = a\sin^3\left(\frac{\pi}{4}\right)$$

$$\Rightarrow x = \frac{a}{2\sqrt{2}}, y = \frac{a}{2\sqrt{2}}$$

Thus, $P\left(\frac{a}{2\sqrt{2}}, \frac{a}{2\sqrt{2}}\right)$ is a point on C, where the tangent is parallel to the chord joining the points $(a,0)$ and $(0,a)$.

Mean Value Theorems Ex 15.2 Q11

Consider the function as

$$f(x) = \tan x, \qquad \left\{x \in [a,b] \text{ such that } 0 < a < b < \frac{\pi}{2}\right\}$$

We know that $\tan x$ is continuous and differentiable in $\left(0, \frac{\pi}{2}\right)$, so, Lagrange's mean value theorem is applicable on (a,b), so there exists a point c such that,

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\Rightarrow \quad \sec^2 c = \frac{\tan b - \tan a}{b - a} \qquad \text{---(i)}$$

Now,

$$c \in (a,b)$$

$$\Rightarrow \quad a < c < b$$

$$\Rightarrow \quad \sec^2 a < \sec^2 c < \sec^2 b$$

$$\Rightarrow \quad \sec^2 a < \left(\frac{\tan b - \tan a}{b - a}\right) < \sec^2 b$$

Using equation (i),

$$\Rightarrow \quad (b - a)\sec^2 a < (\tan b - \tan a) < (b - a)\sec^2 b$$

Ex 16.1

Tangents and Normals Ex 16.1 Q1(i)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad \text{---(A)}$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad \text{---(B)}$$

Now,

$$y = \sqrt{x^3}$$

$$\therefore \quad \frac{dy}{dx} = \frac{3x^2}{2\sqrt{x^3}}$$

$\therefore$ Slope of tangent at $x = 4$ is

$$\left(\frac{dy}{dx}\right)_{x=4} = \frac{3.16}{2\sqrt{64}} = \frac{48}{16} = 3$$

Slope of normal at $x = 4$ is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} = \frac{-1}{3}$$

Tangents and Normals Ex 16.1 Q1(ii)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad \text{---(A)}$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad \text{---(B)}$$

$y = \sqrt{x}$

$$\therefore \quad \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$\therefore$ Slope oftangent at $x = 9$.

$$\therefore \quad \left(\frac{dy}{dx}\right)_{x=9} = \frac{1}{2\sqrt{9}} = \frac{1}{6}$$

Also, the slope of normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} = -6$$

Tangents and Normals Ex 16.1 Q1(iii)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad \text{---(A)}$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad \text{---(B)}$$

$y = x^3 - x$

$$\therefore \quad \frac{dy}{dx} = 3x^2 - 1$$

$\therefore$ Slope of tangent at $x = 2$ is

$$\left(\frac{dy}{dx}\right)_{x=2} = 3.2^2 - 1 = 11$$

Slope of normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} = \frac{-1}{11}$$

Tangents and Normals Ex 16.1 Q1(iv)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad \text{---(A)}$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad \text{---(B)}$$

$y = 2x^2 + 3\sin x$

$$\therefore \quad \frac{dy}{dx} = 4x + 3\cos x$$

So, slope of tangent of $x = 0$ is

$$\left(\frac{dy}{dx}\right)_{x=0} = 4.0 + 3\cos 0° = 3$$

And slope of normanl is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} = \frac{-1}{3}$$

Tangents and Normals Ex 16.1 Q1(v)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad \text{---(A)}$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad \text{---(B)}$$

$x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$

Now,

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-a\sin\theta}{a(1-\cos\theta)}$$

$\therefore$ Slope of tangent of $\theta = -\frac{\pi}{2}$

$$\left(\frac{dy}{dx}\right)_{\theta=-\frac{\pi}{2}} = \frac{-a\sin\left(-\frac{\pi}{2}\right)}{a\left(1-\cos\left(-\frac{\pi}{2}\right)\right)}$$

$$= \frac{a}{a(1-0)} = 1$$

Also, the slope of normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} = -1$$

Tangents and Normals Ex 16.1 Q1(vi)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad \text{---(A)}$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad \text{---(B)}$$

$x = a\cos^3\theta$, $y = a\sin^3\theta$

$\therefore$ $\frac{dx}{d\theta} = 3a\cos^2\theta \times (-\sin\theta) = -3a\sin\theta \times \cos^2\theta$

and $\frac{dy}{dx} = 3a\sin^2\theta \times \cos\theta$

Now,

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3a\sin^2\theta \times \cos\theta}{-3a\sin\theta \times \cos^2\theta}$$

$$= -\tan\theta$$

$\therefore$ Slope of tangent at $\theta = \frac{\pi}{4}$ is

$$\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{4}} = 1$$

Also, the slope of normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} = -1$$

Tangents and Normals Ex 16.1 Q1(vii)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad ---(A)$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad ---(B)$$

$x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$

$\therefore \quad \frac{dx}{d\theta} = a(1 - \cos\theta)$, $\frac{dy}{d\theta} = a(0 + \sin\theta) = a\sin\theta$

Now,

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\sin\theta}{a(1 - \cos\theta)}$$

Now, the slope of tangent at $\theta = \frac{\pi}{2}$ is

$$\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = \frac{a\sin\frac{\pi}{2}}{a\left(1 - \cos\frac{\pi}{2}\right)} = \frac{a}{a} = 1$$

And, the slope of normal is

$$\frac{-1}{\frac{dy}{dx}} = -1$$

Tangents and Normals Ex 16.1 Q1(viii)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad ---(A)$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad ---(B)$$

$y = (\sin 2x + \cot x + 2)^2$

$\therefore \quad \frac{dy}{dx} = 2(\sin 2x + \cot x + 2)\left(2\cos 2x - \operatorname{cosec}^2 x\right)$

$\therefore \quad$ Slope of tangent of $x = \frac{\pi}{2}$ is

$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}} = 2\left(\sin\pi + \cos\frac{\pi}{2} + 2\right)\left(2\cos\pi - \operatorname{cosec}^2\frac{\pi}{2}\right)$$
$$= 2(0 + 0 + 2)(-2 - 1)$$
$$= -12$$

$\therefore \quad$ Slope of normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{1}{12}$$

Tangents and Normals Ex 16.1 Q1(ix)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad \text{---(A)}$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad \text{---(B)}$$

$x^2 + 3y + y^2 = 5$

Differentiating with respect to x, we get

$$2x + 3\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx}(3 + 2y) = -2x$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-2x}{3 + 2y}$$

So, the slope of tangent at $(1,1)$ is

$$\frac{dy}{dx} = \frac{-2.1}{3 + 2.1} = \frac{-2}{5}$$

The slope of normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{5}{2}$$

Tangents and Normals Ex 16.1 Q1(x)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad \text{---(A)}$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad \text{---(B)}$$

$xy = 6$

Differentiating with respect to x, we get

$$y + x\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-y}{x}$$

$\therefore$ Slope of tangent at $(1,6)$ is

$$\frac{dy}{dx} = -6 \text{ and}$$

Slope of normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{1}{6}$$

Tangents and Normals Ex 16.1 Q2

Differentiating with respect to x, we get

$$y + x\frac{dy}{dx} + a + b\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx}(x+b) = -(a+y)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-(a+y)}{x+b}$$

$$\therefore \quad \text{Slope of tangent} = \left(\frac{dy}{dx}\right)_{x=1,\, y=1} = \frac{-(a+1)}{b+1} = 2 \qquad [\text{given}]$$

$$\Rightarrow \quad -(a+1) = 2b+2$$

$$\Rightarrow \quad 2b + a = -3 \qquad ---(\text{i})$$

Also, $(1,1)$ lies on the curve, so x = 1, y = 1 satisfies the equation

$$xy + ax + by = 2$$

$$\Rightarrow \quad 1 + a + b = 2$$

$$\Rightarrow \quad a + b = 1 \qquad ---(\text{ii})$$

Solving (i) and (ii), we get

$$a = 5,\ b = -4$$

Tangents and Normals Ex 16.1 Q3

We have,

$$y = x^3 + ax + b \qquad ---(\text{i})$$

$$x - y + 5 = 0 \qquad ---(\text{ii})$$

Now,

Point $(1,-6)$ lies on (i), so,

$$-6 = 1 + a + b$$

$$\Rightarrow \quad a + b = -7 \qquad ---(\text{iii})$$

Also,

Slope of tangent to (i) is

$$\frac{dy}{dx} = 3x^2 + a$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(1,-6)} = 3 + a$$

And slope of tangent to (ii) is

$$\frac{dy}{dx} = 1$$

According to the question slope of (i) and (ii) are parallel

$$\therefore \quad 3 + a = 1$$

$$\Rightarrow \quad a = -2$$

From (iii)

$$b = -5$$

Tangents and Normals Ex 16.1 Q4

We have,

$$y = x^3 - 3x \qquad \text{---(i)}$$

$\therefore$ Slope of (i) is

$$\frac{dy}{dx} = 3x^2 - 3 \qquad \text{---(ii)}$$

Also,

The slope of the chord obtained by joining the points $(1,-2)$ and $(2,2)$ is

$$\frac{2-(-2)}{2-1} \qquad \left[\text{Slope } \frac{y_2 - y_1}{x_2 - x_1}\right]$$

$$= 4$$

According to the question slope of tangent to (i) and the chord are parallel

$\therefore \quad 3x^2 - 3 = 4$

$\Rightarrow \quad 3x^2 = 7$

$\Rightarrow \quad x = \pm\sqrt{\frac{7}{3}}$

From (i)

$$y = \pm\sqrt{\frac{7}{3}} \mp 3\sqrt{\frac{7}{3}}$$

$$= \mp\frac{2}{3}\sqrt{\frac{7}{3}}$$

Thus, the required point is

$$\pm\sqrt{\frac{7}{3}}, \quad \mp\frac{2}{3}\sqrt{\frac{7}{3}}$$

Tangents and Normals Ex 16.1 Q5

The given equations are

$$y = x^3 - 2x^2 - 2x \qquad \text{---(i)}$$

$$y = 2x - 3 \qquad \text{---(ii)}$$

Slope to the tangents of (i) and (ii) are

$$\frac{dy}{dx} = 3x^2 - 4x - 2 \qquad \text{---(iii)}$$

and $\quad \frac{dy}{dx} = 2 \qquad \text{---(iv)}$

According to the question slope to (i) and (ii) are parallel, so

$3x^2 - 4x - 2 = 2$

$\Rightarrow \quad 3x^2 - 4x - 4 = 0$

$\Rightarrow \quad 3x^2 - 6x + 2x - 4 = 0$

$\Rightarrow \quad 3x(x-2) + 2(x-2) = 0$

$\Rightarrow \quad (3x+2)(x-2) = 0$

$\Rightarrow \quad x = \frac{-2}{3} \text{ or } 2$

From (i)

$$y = \frac{4}{27} \text{ or } -4$$

Thus, the points are

$$\left(\frac{-2}{3}, \frac{4}{27}\right) \text{ and } (2,-4)$$

Tangents and Normals Ex 16.1 Q6

We have,

$$y^2 = 2x^3 \qquad \text{---(i)}$$

Differentiating (i) with respect to x, we get

$$2y\frac{dy}{dx} = 6x^2$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{3x^2}{y} \qquad \text{---(ii)}$$

According to the question

$$\frac{3x^2}{y} = 3$$

$$\Rightarrow \quad x^2 = y \qquad \text{---(iii)}$$

From (i) and (ii)

$$\left(x^2\right)^2 = 2x^3$$

$$\Rightarrow \quad x^4 - 2x^3 = 0$$

$$\Rightarrow \quad x^3\left(x - 2\right) = 0$$

$$\Rightarrow \quad x = 0 \text{ or } 2$$

If $x = 0$, then

$\frac{dy}{dx} = \frac{3x^2}{y} \Rightarrow \frac{dy}{dx} = 0$

Which is not possible.

$\therefore$ x = 2.

Putting x = 2 in the equation of the curve $y^2 = 2x^3$, we get y = 4.

Hence the required point is $(2, 4)$

Tangents and Normals Ex 16.1 Q7

We know that the slope to any curve is $\frac{dy}{dx} = \tan\theta$ where θ is the angle with possitive direction of x-axis.

Now,

The given curve is

$$xy + 4 = 0 \qquad \text{---(i)}$$

Differentiating with respect to x, we get

$$y + x\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-y}{x} \qquad \text{---(ii)}$$

Also,

$$\frac{dy}{dx} = \tan 45° = 1 \qquad \text{---(iii)}$$

$\therefore$ From (ii) and (iii)

$$\frac{-y}{x} = 1$$

$$\Rightarrow \quad x = -y \qquad \text{---(iv)}$$

From (i) and (iv), we get

$$-y^2 + 4 = 0$$

$$\Rightarrow \quad y = \pm 2$$

$$\therefore \quad x = \mp 2$$

Thus, the points are

$(2, -2)$ and $(-2, 2)$

Tangents and Normals Ex 16.1 Q8

The given equation of the curve is

$$y = x^2 \qquad ---(i)$$

$\therefore$ Slope of tangent to (i) is

$$\frac{dy}{dx} = 2x \qquad ---(ii)$$

According to the question

$$\frac{dy}{dx} = x \qquad ---(iii) \qquad [\text{Slope} = x\text{-coordinate}]$$

From (ii) and (iii)

$$2x = x$$

$$\Rightarrow \quad x = 0 \ \& \ y = 0$$

Thus, the required point is $(0,0)$

Tangents and Normals Ex 16.1 Q9

The given equation of the curve is

$$x^2 + y^2 - 2x - 4y + 1 = 0 \qquad ---(i)$$

Differentiating with respect is x, we get

$$2x + 2y\frac{dy}{dx} - 2 - 4\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx}(2y - 4) = 2 - 2x$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2(1-x)}{2(y-2)} \qquad ---(ii)$$

According to the question the tangent is parallel to x-axis, so $\theta = 0°$

$$\therefore \quad \text{Slope} = \tan\theta = \tan 0° = 0 \qquad ---(iii)$$

From (ii) and (iii), we get

$$\frac{1-x}{y-2} = 0$$

$$\Rightarrow \quad 1 - x = 0$$

$$\Rightarrow \quad x = 1$$

$\therefore$ from (i)

$$y = 0, 4$$

Thus, the points are $(1,0)$ and $(1,4)$

Tangents and Normals Ex 16.1 Q10

The given equation of curve is

$$y = x^2 \qquad ---(i)$$

$$\therefore \quad \text{Slope} = \frac{dy}{dx} = 2x \qquad ---(ii)$$

As per question

$$\text{slope} = \tan 45° = 1 \qquad ---(iii)$$

From (ii) and (iii), we have

$$2x = 1$$

$$\Rightarrow \quad x = \frac{1}{2}$$

$\therefore$ From (i)

$$y = \frac{1}{4}$$

Thus, the required point is

$$\left(\frac{1}{2}, \ \frac{1}{4}\right)$$

Tangents and Normals Ex 16.1 Q11

The given equation of the curve is

$$y = 3x^2 - 9x + 8 \qquad \text{---(i)}$$

$$\text{Slope} = \frac{dy}{dx} = 6x - 9 \qquad \text{---(ii)}$$

As per question

The tangent is equally inclined to the axes

$$\therefore \quad \theta = \frac{\pi}{4} \text{ or } \frac{-\pi}{4}$$

$$\therefore \quad \text{Slope} = \tan\theta$$

$$= \tan\frac{\pi}{4} \text{ or } \tan\left(\frac{-\pi}{4}\right)$$

$$= 1 \text{ or } -1 \qquad \text{---(iii)}$$

From (ii) and (iii), we have,

$$6x - 9 = 1 \quad \text{or} \quad 6x - 9 = -1$$

$$\Rightarrow \quad x = \frac{5}{3} \quad \text{or} \quad x = \frac{4}{3}$$

So, from (i)

$$y = \frac{4}{3} \quad \text{or} \quad y = \frac{4}{3}$$

Thus, the points are

$$\left(\frac{5}{3}, \frac{4}{3}\right) \text{ or } \left(\frac{4}{3}, \frac{4}{3}\right)$$

Tangents and Normals Ex 16.1 Q12

The given equation are

$$y = 2x^2 - x + 1 \qquad \text{---(i)}$$

$$y = 3x + 4 \qquad \text{---(ii)}$$

Slope to (i) is

$$\frac{dy}{dx} = 4x - 1 \qquad \text{---(iii)}$$

Slope to (ii) is

$$\frac{dy}{dx} = 3 \qquad \text{---(iv)}$$

According to the question

$$4x - 1 = 3$$

$$\Rightarrow \quad x = 1$$

Thus from (i)

$$y = 2$$

Hence, the point is $(1,2)$.

Tangents and Normals Ex 16.1 Q13

The given equation of curve is

$$y = 3x^2 + 4 \qquad \text{---(i)}$$

$$\text{Slope} = m_1 = \frac{dy}{dx} = 6x \qquad \text{---(ii)}$$

Now,

The given slope $m_2 = \frac{-1}{6}$

We have,

tangent to (i) is perpendicular to the tangent whose slope is $\frac{-1}{6}$

$\therefore \quad m_1 \times m_2 = -1$

$\Rightarrow \quad 6x \times \frac{-1}{6} = -1$

$\Rightarrow \quad x = 1$

From (i)

$y = 7$

Thus, the required point is $(1, 7)$.

Tangents and Normals Ex 16.1 Q14

The given equation of curve and the line is

$$x^2 + y^2 = 13 \qquad \text{---(i)}$$

and $\quad 2x + 3y = 7 \qquad \text{---(ii)}$

Slope $= m_1$ for (i)

$$m_1 = \frac{dy}{dx} = \frac{-x}{y} \qquad \text{---(iii)}$$

Slope $= m_2$ for (ii)

$$m_2 = \frac{dy}{dx} = \frac{-2}{3} \qquad \text{---(iv)}$$

According to the question

$m_1 = m_2$

$\Rightarrow \quad \frac{-x}{y} = \frac{-2}{3}$

$\Rightarrow \quad x = \frac{2}{3}y$

From (i)

$\frac{4}{9}y^2 + y^2 = 13$

$\Rightarrow \quad \frac{13y^2}{9} = 13$

$\Rightarrow \quad y = \pm 3$

$\therefore \quad x = \pm 2$

Thus, the points are $(2, 3)$ and $(-2, -3)$.

Tangents and Normals Ex 16.1 Q15

The given equation of the curve is

$$2a^2y = x^3 - 3ax^2 \qquad ---(i)$$

Differentiating with respect to x, we get

$$2a^2\frac{dy}{dx} = 3x^2 - 6ax$$

$$\therefore \text{ Slope } m_1 = \frac{dy}{dx} = \frac{1}{2a^2}\left[3x^2 - 6ax\right] \qquad ---(ii)$$

Also,

$$\text{Slope } m_2 = \frac{dy}{dx} = \tan\theta$$

$$= \tan 0° = 0 \qquad [\because \text{Slope is parallel to } x\text{-axis}]$$

$$\therefore \quad m_1 = m_2$$

$$\Rightarrow \quad \frac{1}{2a^2}\left[3x^2 - 6ax\right] = 0$$

$$\Rightarrow \quad 3x\left[x - 2a\right] = 0$$

$$\Rightarrow \quad x = 0 \text{ or } 2a$$

$\therefore$ From (i)

$$y = 0 \text{ or } -2a$$

Thus, the required points are $(0,0)$ or $(2a,-2a)$.

Tangents and Normals Ex 16.1 Q16

The given equations of curve and the line are

$$y = x^2 - 4x + 5 \qquad ---(i)$$

$$2y + x = 7 \qquad ---(ii)$$

Slope of the tangent to (i) is

$$m_1 = \frac{dy}{dx} = 2x - 4 \qquad ---(iii)$$

Slope of the line (ii) is

$$m_2 = \frac{dy}{dx} = \frac{-1}{2} \qquad ---(iv)$$

We have given that slope of (i) and (ii) are perpendicular to each other.

$$\therefore \quad m_1 \times m_2 = -1$$

$$\Rightarrow \quad (2x - 4)\left(\frac{-1}{2}\right) = -1$$

$$\Rightarrow \quad -2x + 4 = -2$$

$$\Rightarrow \quad x = 3$$

From (i)

$$y = 2$$

Thus, the required point is $(3,2)$.

Tangents and Normals Ex 16.1 Q17

Differentiating $\frac{x^2}{4}+\frac{y^2}{25}=1$ with respect to x, we get

$$\frac{x}{2}+\frac{2y}{25}\cdot\frac{dy}{dx}=0$$

or
$$\frac{dy}{dx}=\frac{-25}{4}\cdot\frac{x}{y}$$

(i) Now, the tangent is parallel to the x – axis if the slope of the tangent is zero.

$$\therefore \quad \frac{-25}{4}\cdot\frac{x}{y}=0$$

This is possible if x = 0.

Then $\frac{x^2}{4}+\frac{y^2}{25}=1$ for x = 0 gives $y^2 = 25$

$$\therefore \quad y=\pm 5$$

Thus, the points at which the tangents are parallel to the x – axis are (0, 5) and (0, –5).

(ii) Now, the tangent is parallel to the y – axis if the slope of the normal is zero.

$$\therefore \quad \frac{4y}{25x}=0$$

This is possible if y = 0.

Then $\frac{x^2}{4}+\frac{y^2}{25}=1$ for y = 0 gives $x^2 = 4$

$$\therefore \quad x=\pm 2$$

Thus, the points at which the tangents are parallel to the y – axis are (2, 0) and (–2, 0).

Tangents and Normals Ex 16.1 Q18

The equation of the given curve is $x^2+y^2-2x-3=0$.

On differentiating with respect to x, we have:

$$2x+2y\frac{dy}{dx}-2=0$$
$$\Rightarrow y\frac{dy}{dx}=1-x$$
$$\Rightarrow \frac{dy}{dx}=\frac{1-x}{y}$$

Now, the tangents are parallel to the x-axis if the slope of the tangent is 0.

$$\therefore \frac{1-x}{y}=0\Rightarrow 1-x=0\Rightarrow x=1$$

But, $x^2+y^2-2x-3=0$ for $x=1$.

$$\Rightarrow y^2=4\Rightarrow y=\pm 2$$

Hence, the points at which the tangents are parallel to the x-axis are (1, 2) and (1, −2)

(b)
Now, the tangents are parallel to the x-axis if the slope of the tangents is 0

$$\frac{y}{1-x}=0$$
$$y=0$$

But,
$$x^2+y^2-2x-3=0 \text{ for } y=0$$
$$x^2-2x-3=0$$
$$x=-1,3$$

Hence, the points at which the tangents are parallel to the y-axis are,
$(-1,0),(3,0)$

Tangents and Normals Ex 16.1 Q19

The equation of the given curve is $\frac{x^2}{9}+\frac{y^2}{16}=1$.

On differentiating both sides with respect to x, we have:

$$\frac{2x}{9}+\frac{2y}{16}\cdot\frac{dy}{dx}=0$$
$$\Rightarrow \frac{dy}{dx}=\frac{-16x}{9y}$$

(i) The tangent is parallel to the x-axis if the slope of the tangent is i.e., $0\frac{-16x}{9y}=0$, which is possible if $x = 0$.

Then, $\frac{x^2}{9}+\frac{y^2}{16}=1$ for $x = 0$

$$\Rightarrow y^2=16 \Rightarrow y=\pm 4$$

Hence, the points at which the tangents are parallel to the x-axis are

(0, 4) and (0, − 4).

(ii) The tangent is parallel to the y-axis if the slope of the normal is 0, which gives $\frac{-1}{\left(\frac{-16x}{9y}\right)}=\frac{9y}{16x}=0 \Rightarrow y=0$.

Then, $\frac{x^2}{9}+\frac{y^2}{16}=1$ for $y = 0$.

$$\Rightarrow x=\pm 3$$

Hence, the points at which the tangents are parallel to the y-axis are

(3, 0) and (− 3, 0).

Tangents and Normals Ex 16.1 Q20

The equation of the given curve is $y = 7x^3 + 11$.

$$\therefore \frac{dy}{dx}=21x^2$$

The slope of the tangent to a curve at (x_0, y_0) is $\left.\frac{dy}{dx}\right]_{(x_0,y_0)}$.

Therefore, the slope of the tangent at the point where $x = 2$ is given by,

$$\left.\frac{dy}{dx}\right]_{x=-2}=21(2)^2=84$$

It is observed that the slopes of the tangents at the points where $x = 2$ and $x = -2$ are equal.

Hence, the two tangents are parallel.

Tangents and Normals Ex 16.1 Q21

The given equation of curve is

$$y = x^3 \qquad \text{---(i)}$$

Slope of the tangent to (i) is

$$m_1 = \frac{dy}{dx} = 3x^2 \qquad \text{---(ii)}$$

Also,

given that slope of the tangent is parallel to x-coordinate of the point.

$$\therefore \quad m_2 = \frac{dy}{dx} = x \qquad \text{---(iii)}$$

From (ii) and (iii)

$$m_1 = m_2$$

$$\Rightarrow \quad 3x^2 = x$$

$$\Rightarrow \quad 3x^2 - x = 0$$

$$\Rightarrow \quad x(3x - 1) = 0$$

$$\Rightarrow \quad x = 0 \quad \text{or} \quad \frac{1}{3}$$

$\therefore$ From (i)

$$y = 0 \quad \text{or} \quad \frac{1}{27}$$

Thus, the required point is $(0,0)$ or $\left(\frac{1}{3}, \frac{1}{27}\right)$.

Ex 16.2

Tangents and Normals Ex 16.2 Q1

The given equation of the curve is

$$\sqrt{x} + \sqrt{y} = a \qquad \text{---(i)}$$

Differentiating with respect to x, we get

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \times \frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$$

$$\therefore \quad m = \left(\frac{dy}{dx}\right)_{\left(\frac{a^2}{4},\, \frac{a^2}{4}\right)} = -\frac{\frac{a}{2}}{\frac{a}{2}} = -1$$

Thus,

the equation of tangent is

$$y - y_1 = m\left(x - x_1\right)$$

$$\Rightarrow \quad y - \frac{a^2}{4} = (-1)\left(x - \frac{a^2}{4}\right)$$

$$\Rightarrow \quad x + y = \frac{a^2}{4} + \frac{a^2}{4}$$

$$\Rightarrow \quad x + y = \frac{a^2}{2}$$

Tangents and Normals Ex 16.2 Q2

The equation of the curve is

$$y = 2x^3 - x^2 + 3 \qquad \text{---(i)}$$

$$\text{Slope} = m = \frac{dy}{dx} = 6x^2 - 2x$$

$$\therefore \quad m = \left(\frac{dy}{dx}\right)_{(1,4)} = 4$$

Now,

The equation of normal is (i) is

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$$\Rightarrow \quad (y - 4) = \frac{-1}{4}(x - 1)$$

$$\Rightarrow \quad x + 4y = 16 + 1$$

$$\Rightarrow \quad x + 4y = 17$$

Tangents and Normals Ex 16.2 Q3(i)

(i) The equation of the curve is $y = x^4 - 6x^3 + 13x^2 - 10x + 5$.

On differentiating with respect to x, we get:

$$\frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$$

$$\left.\frac{dy}{dx}\right]_{(0,\ 5)} = -10$$

Thus, the slope of the tangent at (0, 5) is -10. The equation of the tangent is given as:

$$y - 5 = -10(x - 0)$$

$$\Rightarrow y - 5 = -10x$$

$$\Rightarrow 10x + y = 5$$

The slope of the normal at (0, 5) is $\dfrac{-1}{\text{Slope of the tangent at }(0,\ 5)} = \dfrac{1}{10}$.

Therefore, the equation of the normal at (0, 5) is given as:

$$y - 5 = \frac{1}{10}(x - 0)$$

$$\Rightarrow 10y - 50 = x$$

$$\Rightarrow x - 10y + 50 = 0$$

Tangents and Normals Ex 16.2 Q3(ii)

(ii) The equation of the curve is $y = x^4 - 6x^3 + 13x^2 - 10x + 5$.

On differentiating with respect to x, we get:

$$\frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$$
$$\left.\frac{dy}{dx}\right]_{(1,\ 3)} = 4 - 18 + 26 - 10 = 2$$

Thus, the slope of the tangent at (1, 3) is 2. The equation of the tangent is given as:

$$y - 3 = 2(x - 1)$$
$$\Rightarrow y - 3 = 2x - 2$$
$$\Rightarrow y = 2x + 1$$

The slope of the normal at (1, 3) is $\dfrac{-1}{\text{Slope of the tangent at }(1,\ 3)} = \dfrac{-1}{2}$.

Therefore, the equation of the normal at (1, 3) is given as:

$$y - 3 = -\frac{1}{2}(x - 1)$$
$$\Rightarrow 2y - 6 = -x + 1$$
$$\Rightarrow x + 2y - 7 = 0$$

Tangents and Normals Ex 16.2 Q3(iii)

The equation of the curve is $y = x^2$.

On differentiating with respect to x, we get:

$$\frac{dy}{dx} = 2x$$
$$\left.\frac{dy}{dx}\right]_{(0,\ 0)} = 0$$

Thus, the slope of the tangent at (0, 0) is 0 and the equation of the tangent is given as:

$$y - 0 = 0\ (x - 0)$$

$$\Rightarrow y = 0$$

The slope of the normal at (0, 0) is $\dfrac{-1}{\text{Slope of the tangent at }(0,\ 0)} = -\dfrac{1}{0}$, which is not defined.

Therefore, the equation of the normal at $(x_0, y_0) = (0, 0)$ is given by

$$x = x_0 = 0.$$

Tangents and Normals Ex 16.2 Q3(iv)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad (A) \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad (B) \quad \text{Normal}$$

Where m is the slope

We have,

$$y = 2x^2 - 3x - 1 \qquad P = (1, -2)$$

Slope $m = \frac{dy}{dx} = 4x - 3$

$$m = \left(\frac{dy}{dx}\right)_P = 1$$

$\therefore$ equation of tangent from (A)

$$(y + 2) = 1(x - 1)$$

$$\Rightarrow \quad x - y = 3$$

And equation of normal from (B)

$$(y + 2) = -1(x - 1)$$

$$\Rightarrow \quad x + y + 1 = 0$$

Tangents and Normals Ex 16.2 Q3(v)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad (A) \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad (B) \quad \text{Normal}$$

Where m is the slope

We have,

$$y^2 = \frac{x^3}{4 - x} \qquad P - (2, -2)$$

Differentiating with respect to x, we get

$$2y\frac{dy}{dx} = \frac{3x^2(4 - x) + x^3}{(4 - x)^2}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{3x^2(4 - x) + x^3}{2y(4 - x)^2}$$

$$\therefore \text{ Slope } m = \left(\frac{dy}{dx}\right)_P = \frac{3 \times 4(4 - 2) + 8}{-2 \times 2(4 - 2)^2}$$

$$= \frac{32}{-16} = -2$$

From (A)

Equation of tangent is

$$(y + 2) = -2(x - 2)$$

$$\Rightarrow \quad 2x + y = 2$$

From (B)

Equation of Normal is

$$(y + 2) = \frac{1}{2}(x - 2)$$

$$\Rightarrow \quad x - 2y = 6$$

Tangents and Normals Ex 16.2 Q3(vi)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad \text{(A)} \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{(B)} \quad \text{Normal}$$

Where m is the slope

We have,

$$y = x^2 + 4x + 1 \qquad \text{and} \qquad P = (x = 3)$$

$$\text{Slope} = \frac{dy}{dx} = 2x + 4$$

$$\therefore \qquad m = \left(\frac{dy}{dx}\right)_P = 10$$

From (A)

Equation of tangent is

$$(y - 22) = 10(x - 3)$$

$$\Rightarrow \qquad 10x - y = 8$$

From (B)

Equation of normal is

$$(y - 22) = \frac{-1}{10}(x - 3)$$

$$\Rightarrow \qquad x + 10y = 223$$

Tangents and Normals Ex 16.2 Q3(vii)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad \text{(A)} \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{(B)} \quad \text{Normal}$$

Where m is the slope

We have,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{and} \quad P = (a\cos\theta, b\sin\theta)$$

Differentiating with respect to x, we get

$$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{xb^2}{ya^2}$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = \frac{-a\cos\theta b^2}{b\sin\theta a^2}$$

$$= \frac{-b}{a}\cot\theta$$

From (A)

Equation of tangent is,

$$(y - b\sin\theta) = \frac{-b}{a}\cot\theta(x - a\cos\theta)$$

$$\Rightarrow \quad \frac{b}{a}x\cot\theta + y = b\sin\theta + b\cot\theta \times \cos\theta$$

$$\Rightarrow \quad \frac{x}{a}\cot\theta + \frac{y}{b} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta}$$

$$\Rightarrow \quad \frac{x}{a}\cot\theta + \frac{y}{b} = \frac{1}{\sin\theta}$$

$$\Rightarrow \quad \frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$$

From (B)

Equation of normal is

$$(y - b\sin\theta) = \frac{a}{b}\frac{\sin\theta}{\cos\theta}(x - a\cos\theta)$$

$$\Rightarrow \quad \frac{a}{b}x\tan\theta - y = \frac{a^2}{b}\sin\theta - b\sin\theta$$

$$\Rightarrow \quad \frac{a}{b}x\tan\theta - y = \frac{a^2 - b^2}{b}\sin\theta$$

$$\Rightarrow \quad \frac{a}{b}x\sec\theta - y\,\text{cosec}\theta = \frac{a^2 - b^2}{b}$$

$$\Rightarrow \quad ax\sec\theta - by\,\text{cosec}\theta = a^2 - b^2$$

Tangents and Normals Ex 16.2 Q3(viii)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad (A) \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad (B) \quad \text{Normal}$$

Where m is the slope

We have,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad P = (a\sec\theta, b\tan\theta)$$

Differentiating with respect to x, we get

$$\frac{2x}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{xb^2}{ya^2}$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = \frac{a\sec\theta b^2}{b\tan\theta a^2}$$

$$= \frac{b}{a\sin\theta}$$

From (A)

Equation of tangent is,

$$(y - b\tan\theta) = \frac{b}{a\sin\theta}(x - a\sec\theta)$$

$$\Rightarrow \quad \frac{b}{a}\frac{x}{\sin\theta} - y = \frac{b\sec\theta}{\sin\theta} - b\tan\theta$$

$$\Rightarrow \quad \frac{bx}{a\sin\theta} - y = \frac{b\sec\theta}{\sin\theta}\left(1 - \sin^2\theta\right)$$

$$\Rightarrow \quad \frac{x}{a} - \frac{y}{b}\sin\theta = \cos\theta$$

$$\Rightarrow \quad \frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$$

From (B)

Equation of normal is

$$y - b\tan\theta = \frac{-a\sin\theta}{b}(x - a\sec\theta)$$

$$\Rightarrow \quad ax\sin\theta + by = b^2\tan\theta + a^2\tan\theta$$

$$\Rightarrow \quad ax\cos\theta + by\cot\theta = a^2 + b^2$$

Tangents and Normals Ex 16.2 Q3(ix)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad \text{(A)} \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{(B)} \quad \text{Normal}$$

Where m is the slope

We have,

$$y^2 = 4ax \qquad P\left(\frac{a}{m^2}, \frac{2a}{m}\right)$$

Differentiating with respect to x, we get

$$2y\frac{dy}{dx} = 4a$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2a}{y}$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = m$$

From (A)

Equation of tangent is

$$\left(y - \frac{2a}{m}\right) = m\left(x - \frac{a}{m^2}\right)$$

$$\Rightarrow \quad m^2x - my = 2a - a$$

$$\Rightarrow \quad m^2x - my = a$$

From (B)

Equation of normal is

$$\left(y - \frac{2a}{m}\right) = \frac{-1}{m}\left(x - \frac{a}{m^2}\right)$$

$$\Rightarrow \quad (my - 2a) = \frac{-m^2x + a}{m^2}$$

$$\Rightarrow \quad m^2x + m^3y = 2am^2 + a$$

$$\Rightarrow \quad m^2x + m^3y - 2am^2 - a = 0$$

Tangents and Normals Ex 16.2 Q3(x)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad \text{(A)} \qquad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{(B)} \qquad \text{Normal}$$

Where m is the slope

We have,

$$c^2\left(x^2 + y^2\right) = x^2y^2 \qquad P = \left(\frac{c}{\cos\theta}, \frac{c}{\sin\theta}\right)$$

Differentiating with respect to x, we get

$$c^2\left(2x + 2y\frac{dy}{dx}\right) = 2xy^2 + 2x^2y\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx}\left(2yc^2 - 2x^2y\right) = 2xy^2 - 2xc^2$$

$$\therefore \quad \frac{dy}{dx} = \frac{x\left(y^2 - c^2\right)}{y\left(c^2 - x^2\right)}$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = \frac{\frac{c}{\cos\theta}\left(\frac{c^2}{\sin^2\theta} - c^2\right)}{\frac{c}{\sin\theta}\left(c^2 - \frac{c^2}{\cos^2\theta}\right)}$$

$$= \frac{c^2\tan\theta\left(1 - \sin^2\theta\right)}{c^2\tan^2\theta\left(\cos^2\theta - 1\right)}$$

$$= \frac{1}{-\tan\theta} \times \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \frac{-\cos^3\theta}{\sin^3\theta}$$

From (A)

Equation of tangent is

$$\left(y - \frac{c}{\sin\theta}\right) = \frac{-\cos^3\theta}{\sin^3\theta}\left(x - \frac{c}{\cos\theta}\right)$$

$$\Rightarrow \quad x\cos^3\theta + y\sin^3\theta = c\sin^2\theta + c\cos^2\theta$$

$$\Rightarrow \quad x\cos^3\theta + y\sin^3\theta = c$$

From (B)

Equation of normal is

$$\left(y - \frac{c}{\sin\theta}\right) = \frac{\sin^3\theta}{\cos^3\theta}\left(x - \frac{c}{\cos\theta}\right)$$

$$\Rightarrow \quad x\sin^3\theta - y\cos^3\theta = \frac{c\sin^3\theta}{\cos\theta} - \frac{c\cos^3\theta}{\sin\theta}$$

$$\Rightarrow \quad x\sin^3\theta - y\cos^3\theta = \frac{c\left(\sin^4\theta - \cos^4\theta\right)}{\cos\theta \times \sin\theta}$$

$$= \frac{c\left(\sin^2\theta - \cos^2\theta\right)\left(\sin^2\theta + \cos^2\theta\right)}{\frac{1}{2}\sin 2\theta}$$

$$= \frac{-2c\cos 2\theta}{\sin 2\theta} = -2c\cot 2\theta$$

$$\therefore \quad x\sin^3\theta - y\cos^3\theta + 2c\cot 2\theta = 0$$

Tangents and Normals Ex 16.2 Q3(xi)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad \text{(A)} \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{(B)} \quad \text{Normal}$$

Where m is the slope

We have,

$$xy = c^2 \qquad P = \left(ct, \frac{c}{t}\right)$$

Differentiating with respect to x, we get

$$y + x\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{y}{x}$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = \frac{\frac{-c}{t}}{ct} = \frac{-1}{t^2}$$

From (A)

Equation of tangent is

$$\left(y - \frac{c}{t}\right) = \frac{-1}{t^2}(x - ct)$$

$$\Rightarrow \quad x + t^2y = tc + ct$$

$$\Rightarrow \quad x + t^2y = 2ct$$

From (B)

Equation of normal is

$$\left(y - \frac{c}{t}\right) = t^2(x - ct)$$

$$\Rightarrow \quad xt^3 - ty = ct^3 \times t - c$$

$$\Rightarrow \quad xt^3 - ty = ct^4 - c$$

Tangents and Normals Ex 16.2 Q3(xii)

We know that the equation of tangent and the normal to any curve is given by

$y - y_1 = m(x - x_1)$ (A) Tangent

$y - y_1 = \frac{-1}{m}(x - x_1)$ (B) Normal

Where m is the slope

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{---(i)} \qquad P = (x_1, y_1)$$

Differentiating with resect to x, we get

$$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{xb^2}{ya^2}$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = -\frac{x_1 b^2}{y_1 a^2}$$

From (A)

Equation of tangent is

$$(y - y_1) = -\frac{x_1 b^2}{y_1 a^2}(x - x_1)$$

$$\Rightarrow \quad xx_1 b^2 + yy_1 a^2 = x_1^2 b^2 + y_1^2 a^2$$

Divide by $a^2 b^2$ both side

$$\Rightarrow \quad \frac{xx_1}{a^2} + \frac{yy_1}{b^2} = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2}$$

$$= 1 \qquad [\because (x_1, y_1) \text{ lies on (i)}]$$

$$\therefore \quad \frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

From (B)

Equation of normal is

$$(y - y_1) = \frac{y_1 a^2}{x_1 b^2}(x - x_1)$$

$$xy_1 a^2 - yx_1 b^2 = x_1 y_1 a^2 - y_1 x_1 b^2$$

Dividing by $x_1 y_1$ both side

$$\frac{xa^2}{x_1} - \frac{yb^2}{y_1} = a^2 - b^2$$

Tangents and Normals Ex 16.2 Q3(xiii)

Differentiating $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with respect to x, we have:

$$\frac{2x}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{2y}{b^2}\frac{dy}{dx} = \frac{2x}{a^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{b^2 x}{a^2 y}$$

Therefore, the slope of the tangent at (x_0, y_0) is $\left.\frac{dy}{dx}\right]_{(x_0, y_0)} = \frac{b^2 x_0}{a^2 y_0}$.

Then, the equation of the tangent at (x_0, y_0) is given by,

$$y - y_0 = \frac{b^2 x_0}{a^2 y_0}(x - x_0)$$

$$\Rightarrow a^2 yy_0 - a^2 y_0^2 = b^2 xx_0 - b^2 x_0^2$$

$$\Rightarrow b^2 xx_0 - a^2 yy_0 - b^2 x_0^2 + a^2 y_0^2 = 0$$

Tangents and Normals Ex 16.2 Q3(xiv)

Differentiating $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2$ with respect to x, we get

$$\frac{2}{3}x^{\frac{-1}{3}} + \frac{2}{3}y^{\frac{-1}{3}}\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$$

Therefore, the slope of the tangent at $(1,1)$ is $\left.\frac{dy}{dx}\right]_{(1,1)} = -1$

So, the equation of the tangent at $(1,1)$ is

$$y - 1 = -1(x - 1)$$

$$\Rightarrow \quad y + x - 2 = 0$$

Also, the slope of the normal at $(1,1)$ is given by $\frac{-1}{\text{slope of tangent at } (1,1)} = 1$

$\therefore$ the equation of the normal at $(1,1)$ is

$$y - 1 = 1(x - 1)$$

$$\Rightarrow \quad y - x = 0$$

Tangents and Normals Ex 16.2 Q3(xv)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad (A) \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad (B) \quad \text{Normal}$$

Where m is the slope

We have,

$$x^2 = 4y \qquad P = (2,1)$$

$$\therefore \quad 2x = \frac{4dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x}{2}$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = 1$$

From (A)

Equation of tangent is

$$y - 1 = 1(x - 2)$$

$$\Rightarrow \quad x - y = 1$$

From (B)

Equation of normal is

$$(y - 1) = -1(x - 2)$$

$$\Rightarrow \quad x + y = 3$$

Tangents and Normals Ex 16.2 Q3(vi)

The equation of the given curve is $y^2 = 4x$.

Differentiating with respect to x, we have:

$$2y\frac{dy}{dx} = 4$$
$$\Rightarrow \frac{dy}{dx} = \frac{4}{2y} = \frac{2}{y}$$
$$\therefore \left.\frac{dy}{dx}\right]_{(1,2)} = \frac{2}{2} = 1$$

Now, the slope at point (1, 2) is $\frac{-1}{\left.\frac{dy}{dx}\right]_{(1,2)}} = \frac{-1}{1} = -1$.

$\therefore$Equation of the tangent at (1, 2) is $y - 2 = -1(x - 1)$.

$\Rightarrow y - 2 = -x + 1$

$\Rightarrow x + y - 3 = 0$

Equation of the normal is ,

$$y-2=-(-1)(x-1)$$
$$y-2=x-1$$
$$x-y+1=0$$

Tangents and Normals Ex 16.2 Q3(xix)

Let $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be the equation of the curve.

Rewriting the above equation as,

$$\frac{y^2}{b^2} = \frac{x^2}{a^2} - 1$$
$$\Rightarrow y^2 = \frac{b^2}{a^2}x^2 - b^2$$

Differentiating the above function w.r.t. x, we get,

$$2y\frac{dy}{dx} = \frac{b^2}{a^2}2x$$
$$\Rightarrow \frac{dy}{dx} = \frac{b^2}{a^2}\frac{x}{y}$$

$$\Rightarrow \left[\frac{dy}{dx}\right]_{(\sqrt{2}a,b)} = \frac{b^2}{a^2}\frac{\sqrt{2}a}{b} = \frac{\sqrt{2}b}{a}$$

Slope of the tangent $m = \frac{\sqrt{2}b}{a}$

Equation of the tangent is

$$(y - y_1) = m(x - x_1)$$
$$\Rightarrow (y - b) = \frac{\sqrt{2}b}{a}(x - \sqrt{2}a)$$
$$\Rightarrow a(y - b) = \sqrt{2}b(x - \sqrt{2}a)$$
$$\Rightarrow \sqrt{2}bx - ay + ab - 2ab = 0$$
$$\Rightarrow \sqrt{2}bx - ay - ab = 0$$

Slope of the normal is $-\frac{1}{\frac{\sqrt{2}b}{a}} = -\frac{a}{b\sqrt{2}}$

Equation of the normal is

$$(y - y_1) = m(x - x_1)$$
$$\Rightarrow (y - b) = \frac{-a}{\sqrt{2}b}(x - \sqrt{2}a)$$
$$\Rightarrow \sqrt{2}b(y - b) = -a(x - \sqrt{2}a)$$
$$\Rightarrow ax + \sqrt{2}by - \sqrt{2}b^2 - \sqrt{2}a^2 = 0$$
$$\Rightarrow ax + \sqrt{2}by - \sqrt{2}(a^2 + b^2) = 0$$

Tangents and Normals Ex 16.2 Q4

The given equations are,

$$x=\theta+\sin\theta \quad , \quad y=1+\cos\theta$$

$$\frac{dx}{d\theta}=1+\cos\theta \quad , \quad \frac{dy}{d\theta}=-\sin\theta$$

$$\therefore \quad \frac{dx}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{-\sin\theta}{1+\cos\theta}$$

Slope,

$$m=\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{4}}=\frac{-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}$$

$$=-1+\frac{1}{\sqrt{2}}$$

Thus, equation of tangent is,

$$y-y_1=m(x-x_1)$$

$$y-\left(1+\frac{1}{\sqrt{2}}\right)=\left(-1+\frac{1}{\sqrt{2}}\right)\left(x-\left(\frac{\pi}{4}+\frac{1}{\sqrt{2}}\right)\right)$$

$$y-1-\frac{1}{\sqrt{2}}=\left(-1+\frac{1}{\sqrt{2}}\right)\left(x-\frac{\pi}{4}-\frac{1}{\sqrt{2}}\right)$$

Tangents and Normals Ex 16.2 Q5(i)

We know that the equation of tangent and normal to any curve at the point (x_1, y_1) is

$$y-y_1=m(x-x_1) \qquad \text{---(A) Tangent}$$

$$y-y_1=\frac{-1}{m}(x-x_1) \qquad \text{---(B) Normal}$$

Where m is slope.

$x=\theta+\sin\theta, \quad y=1+\cos\theta, \quad \theta=\frac{\pi}{2}$

$$\therefore \quad P=\left[\left(\frac{\pi}{2}+1\right),1\right]$$

and $\frac{dx}{d\theta}=1+\cos\theta, \frac{dy}{d\theta}=-\sin\theta$

$$\therefore \quad \text{Slope } m=\left(\frac{dy}{dx}\right)_P=\left(\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}\right)=\frac{-1}{+1}=-1$$

Equation of tangent from (A)

$$(y-1)=-1\left(x-\left(\frac{\pi}{2}+1\right)\right)$$

$$\Rightarrow \quad x+y=\frac{\pi}{2}+1+1$$

$$\Rightarrow \quad 2(x+y)=\pi+4$$

From (B)

Equation of normal is

$$(y-1)=1\left(x-\left(\frac{\pi}{2}+1\right)\right)$$

$$\Rightarrow \quad 2(x-y)=\pi$$

Tangents and Normals Ex 16.2 Q5(ii)

We know that the equation of tangent and normal to any curve at the point (x_1, y_1) is

$$y - y_1 = m(x - x_1) \qquad \text{---(A) Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{---(B) Normal}$$

Where m is slope.

$$x = \frac{2at^2}{1+t^2}, \quad y = \frac{2at^3}{1+t^2}, \quad t = \frac{1}{2}$$

$$\therefore \quad P = \left(x = \frac{a}{2+\frac{1}{2}} = \frac{2a}{5}, y = \frac{a}{4+1} = \frac{a}{5} \right)$$

Now,

$$\frac{dx}{dt} = \frac{4a + \left(1+t^2\right) - 2at^2\,(2t)}{\left(1+t^2\right)^2}$$

$$= \frac{4at}{\left(1+t^2\right)^2}$$

$$\frac{dy}{dt} = \frac{6at^2\left(1+t^2\right) - \left(2at^3\right)(2t)}{\left(1+t^2\right)^2}$$

$$= \frac{6at^2 - 2at^4}{\left(1+t^2\right)^2}$$

$$\therefore \quad \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{6at^2 + 2at^4}{4at}$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = \frac{\frac{3a}{2} + \frac{a}{8}}{2a} = \frac{13}{16}$$

From (A)
Equation of tangent is,

$$\left(y - \frac{a}{5}\right) = \frac{13}{16}\left(x - \frac{2a}{5}\right)$$

$$16y - \frac{16a}{5} = 13x - \frac{26a}{5}$$

$$13x - 16y - 2a = 0$$

Equation of normal is,

$$\left(y - \frac{a}{5}\right) = -\frac{16}{13}\left(x - \frac{2a}{5}\right)$$

$$13y - \frac{13a}{5} = -16x + \frac{32a}{5}$$

$$16x + 13y - 9a = 0$$

Tangents and Normals Ex 16.2 Q5(iii)

We know that the equation of tangent and normal to any curve at the point (x_1, y_1) is

$$y - y_1 = m(x - x_1) \qquad \text{---(A) Tangent}$$
$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{---(B) Normal}$$

Where m is slope.

$x = at^2, \qquad y = 2at, \qquad t = 1$

$\therefore \qquad P = (a, 2a)$

and

$$\frac{dx}{dt} = 2at, \qquad \frac{dy}{dt} = 2a$$

$$\therefore \qquad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2a} = 1$$

From (A)

Equation of tangent is

$$(y - 2a) = 1(x - a)$$
$$\Rightarrow \qquad x - y + a = 0$$

From (B)

Equation of normaol is

$$(y - 2a) = -1(x - a)$$
$$\Rightarrow \qquad x + y = 3a$$

Tangents and Normals Ex 16.2 Q5(iv)

We know that the equation of tangent and normal to any curve at the point (x_1, y_1) is

$$y - y_1 = m(x - x_1) \qquad \text{---(A) Tangent}$$
$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{---(B) Normal}$$

Where m is slope.

$x = a\sec t, \qquad y = b\tan t, \qquad t = t$

$$\therefore \qquad \frac{dx}{dt} = a\sec t \times \tan t$$

and

$$\frac{dy}{dt} = b\sec^2 t$$

$$\therefore \qquad \text{Slope } m = \frac{dy}{dx} = \frac{b\sec^2 t}{a\sec t \times \tan t}$$
$$= \frac{b}{a}\operatorname{cosec} t$$

From (A)

Equatin of tangent

$$(y - b\tan t) = \frac{b}{a}\operatorname{cosec} t\,(x - a\sec t)$$
$$\Rightarrow \qquad bx\operatorname{cosec} t - ay = ab\operatorname{cosec} t \times \sec t - ab\tan t$$
$$= \frac{ab\left[1 - \sin^2 t\right]}{\sin t \times \cos t}$$
$$= \frac{ab\cos t}{\sin t}$$
$$\Rightarrow \qquad bx\sec t - ay\tan t = ab$$

From (B)

Equation of normal is

$$(y - b\tan t) = \frac{-a\sin t}{b}(x - a\sec t)$$
$$\Rightarrow \qquad ax\sin t + by = a^2\tan t + b^2\tan t$$

$$\Rightarrow \qquad ax\cos t + by\cot t = a^2 + b^2$$

Tangents and Normals Ex 16.2 Q5(v)

We know that the equation of tangent and normal to any curve at the point (x_1, y_1) is

$$y - y_1 = m(x - x_1) \qquad \text{---(A) Tangent}$$
$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{---(B) Normal}$$

Where m is slope.

$$x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$$
$$\frac{dx}{d\theta} = a(1 + \cos\theta), \frac{dy}{d\theta} = a\sin\theta$$

$$\therefore \quad \text{Slope } m = \frac{dy}{dx} = \frac{\sin\theta}{1 + \cos\theta} = \frac{\frac{2\sin\theta}{2} \times \frac{\cos\theta}{2}}{\frac{2\cos^2\theta}{2}}$$
$$= \frac{\tan\theta}{2}$$

Now,
From (A)
Equation of tangent

$$y - a(1 - \cos\theta) = \frac{\tan\theta}{2}(x - a(\theta + \sin\theta))$$
$$\Rightarrow \quad \frac{x\tan\theta}{2} - y = a(\theta + \sin\theta)\frac{\tan\theta}{2} - a(1 - \cos\theta)$$

From (B)
Equation of normal is

$$y - a(1 - \cos\theta) = \frac{-\cot\theta}{2}(x - a(\theta + \sin\theta))$$

$$\Rightarrow \quad (y - 2a)\frac{\tan\theta}{2} + x - a\theta = 0$$

Tangents and Normals Ex 16.2 Q5(vi)

$$x = 3\cos\theta - \cos^3\theta, \; y = 3\sin\theta - \sin^3\theta$$
$$\Rightarrow \frac{dx}{d\theta} = -3\sin\theta + 3\cos^2\theta\sin\theta \text{ and } \frac{dy}{d\theta} = 3\cos\theta - 3\sin^2\theta\cos\theta$$
$$\Rightarrow \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{3\cos\theta - 3\sin^2\theta\cos\theta}{-3\sin\theta + 3\cos^2\theta\sin\theta} = \frac{\cos\theta(1 - \sin^2\theta)}{-\sin\theta(1 - \cos^2\theta)} = \frac{\cos^3\theta}{-\sin^3\theta} = -\tan^3\theta$$

So equation of the tangent at θ is

$$y - 3\sin\theta + \sin^3\theta = -\tan^3\theta(x - 3\cos\theta + \cos^3\theta)$$
$$\Rightarrow 4(y\cos^3\theta - x\sin^3\theta) = 3\sin 4\theta$$

So equation of normal at θ is

$$y - 3\sin\theta + \sin^3\theta = \frac{1}{\tan^3\theta}(x - 3\cos\theta + \cos^3\theta)$$
$$\Rightarrow y\cos^3\theta - x\cos^3\theta = 3\sin^4\theta - \sin^6\theta - 3\cos^4\theta + \cos^6\theta$$
$$\Rightarrow y\sin^3\theta - x\cos^3\theta = 3\sin^4\theta - \sin^6\theta - 3\cos^4\theta + \cos^6\theta$$

Tangents and Normals Ex 16.2 Q6

The given equation of curve is

$$x^2 + 2y^2 - 4x - 6y + 8 = 0 \qquad \text{---(i) at } x = 2$$

Differentiating with respect to x, we get

$$2x + 4y\frac{dy}{dx} - 4 - 6\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx}[4y - 6] = 4 - 2x$$

$$\therefore \quad \frac{dy}{dx} = \frac{2 - x}{2y - 3}$$

Now,
From (i) at $x = 2$

$$4 + 2y^2 - 8 - 6y + 8 = 0$$

$$\Rightarrow \quad 2y^2 - 6y + 4 = 0$$

$$\Rightarrow \quad y^2 - 3y + 2 = 0$$

$$\Rightarrow \quad (y - 2)(y - 1) = 0$$

$$\Rightarrow \quad y = 2, 1$$

Thus,

$$\text{Slope } m_1 = \left(\frac{dy}{dx}\right)_{(2,2)} = 0$$

$$m_2 = \left(\frac{dy}{dx}\right)_{(2,1)} = 0$$

Thus, the equation of normal is

$$(y - y_1) = \frac{-1}{0}(x - 2)$$

$$\Rightarrow \quad x = 2$$

Tangents and Normals Ex 16.2 Q7

The equation of the given curve is $ay^2 = x^3$.

On differentiating with respect to x, we have:

$$2ay\frac{dy}{dx} = 3x^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{3x^2}{2ay}$$

The slope of a tangent to the curve at (x_0, y_0) is $\left.\frac{dy}{dx}\right]_{(x_0, y_0)}$.

$\Rightarrow$ The slope of the tangent to the given curve at (am^2, am^3) is

$$\left.\frac{dy}{dx}\right]_{(am^2, am^3)} = \frac{3(am^2)^2}{2a(am^3)} = \frac{3a^2m^4}{2a^2m^3} = \frac{3m}{2}.$$

$\therefore$ Slope of normal at (am^2, am^3)

$$= \frac{-1}{\text{slope of the tangent at } (am^2, am^3)} = \frac{-2}{3m}$$

Hence, the equation of the normal at (am^2, am^3) is given by,

$$y - am^3 = \frac{-2}{3m}(x - am^2)$$

$$\Rightarrow 3my - 3am^4 = -2x + 2am^2$$

$$\Rightarrow 2x + 3my - am^2(2 + 3m^2) = 0$$

Tangents and Normals Ex 16.2 Q8

The given equations are

$$y^2 = ax^3 + b \quad \text{---(i)}$$

$$y = 4x - 5 \quad \text{---(ii)} \qquad P = (2,3)$$

Differentiating (i) with respect to x, we get

$$2y\frac{dy}{dx} = 3ax^2$$

$$\therefore \quad \frac{dy}{dx} = \frac{3ax^2}{2y}$$

$$\therefore \quad m_1 = \left(\frac{dy}{dx}\right)_P = \frac{12a}{6} = 2a$$

$$m_2 = \text{slope of (ii)} = 4$$

According to the question

$$m_1 = m_2 \Rightarrow 2a = 4 \Rightarrow a = 2$$

From (i)

$$y^2 = 2 \times 2^3 + b$$

$$\Rightarrow \quad 9 = 16 + b$$

$$\Rightarrow \quad b = -7$$

Thus,

$$a = 2, b = -7$$

Tangents and Normals Ex 16.2 Q9

The given equatioins are,

$$y = x^2 + 4x - 16 \quad \text{---(i)}$$

$$3x - y + 1 = 0 \quad \text{---(ii)}$$

Slope m_1 of (i)

$$m_1 = \frac{dy}{dx} = 2x + 4$$

Slope m_2 of (ii)

$$m_2 = 3$$

As per question

$$m_1 = m_2$$

$$\Rightarrow \quad 2x + 4 = 3$$

$$\Rightarrow \quad x = \frac{-1}{2}$$

From (i)

$$y = \frac{1}{4} - 2 - 16 = -\frac{71}{4}$$

$$\therefore \quad P = \left(\frac{-1}{2}, \frac{-71}{4}\right)$$

Thus, the equation of tangent

$$\left(y + \frac{71}{4}\right) = 3\left(x + \frac{1}{2}\right)$$

$$\Rightarrow \quad 3x - y = \frac{71}{4} - \frac{3}{2}$$

$$\Rightarrow \quad 3x - y = \frac{65}{4}$$

$$\Rightarrow \quad 12x - 4y - 65 = 0$$

Tangents and Normals Ex 16.2 Q10

The given equation is

$$y = x^3 + 2x + 6 \qquad \text{---(i)}$$
$$x + 14y + 4 = 0 \qquad \text{---(ii)}$$

Slope m_1 of (i)

$$m_1 = \frac{dy}{dx} = 3x^2 + 2$$

Slope m_2 of (ii)

$$m_2 = \frac{-1}{14}$$

$\therefore$ Slope of normal to (i) is

$$\frac{-1}{m_1} = \frac{-1}{3x^2 + 2}$$

According to the question

$$\frac{-1}{3x^2 + 2} = \frac{-1}{14}$$

$\Rightarrow \quad 3x^2 2 = 14$

$\Rightarrow \quad x^2 = 4$

$\Rightarrow \quad x = \pm 2$

From (i)

$$y = 8 + 4 + 6 \quad \text{or} \quad -8 - 4 + 6$$
$$= 18 \quad \text{or} \quad -6$$

so, $\quad P = (2, 18)$ and $Q = (-2, -6)$

Thus, the equation of normal is

$$(y - 18) = \frac{-1}{14}(x - 2) \quad \Rightarrow \quad x + 14y + 86 = 0$$

or $\quad (y + 6) = \frac{-1}{14}(x + 2) \quad \Rightarrow \quad x + 14y - 254 = 0$

Tangents and Normals Ex 16.2 Q11

The given equations are,

$$y = 4x^3 - 3x + 5 \qquad \text{---(i)}$$
$$9y + x + 3 = 0 \qquad \text{---(ii)}$$

Slope m_1 of (i)

$$m_1 = \frac{dy}{dx} = 12x^2 - 3$$

Slope m_2 of (ii)

$$m_2 = \frac{-1}{9}$$

According to the question

$$m_1 \times m_2 = -1$$

$\Rightarrow \quad (12x^2 - 3)\left(-\frac{1}{9}\right) = -1$

$\Rightarrow \quad 4x^2 - 1 = 3$

$\Rightarrow \quad x^2 = 1$

$\Rightarrow \quad x = \pm 1$

From (i)

$$y = 4 - 3 + 5 \quad \text{or} \quad -4 + 3 + 5$$
$$= 6 \quad \text{or} \quad 4$$

$\therefore \quad P = (1, 6)$ or $Q = (-, 1, 4)$

Thus, the equation of tangent is

$$(y - 6) = 9(x - 1) \quad \Rightarrow \quad 9x - y - 3 = 0$$
$$(y - 4) = 9(x + 1) \quad \Rightarrow \quad 9x - y + 13 = 0$$

Tangents and Normals Ex 16.2 Q12

The given equations are,

$y = x \log_e x$ ---(i)

$2x - 2y + 3 = 0$ ---(ii)

Slope m_1 of (i)

$$m_1 = \frac{dy}{dx} = \log_e x + 1$$

slope m_2 of (ii)

$$m_2 = 1$$

Tangents and Normals Ex 16.2 Q13

The equation of the given curve is $y = x^2 - 2x + 7$.

On differentiating with respect to x, we get:

$$\frac{dy}{dx} = 2x - 2$$

(a) The equation of the line is $2x - y + 9 = 0$.

$2x - y + 9 = 0 \Rightarrow y = 2x + 9$

This is of the form $y = mx + c$.

$\therefore$Slope of the line $= 2$

If a tangent is parallel to the line $2x - y + 9 = 0$, then the slope of the tangent is equal to the slope of the line.

Therefore, we have:

$2 = 2x - 2$

$\Rightarrow 2x = 4$

$\Rightarrow x = 2$

Now, $x = 2$

$\Rightarrow y = 4 - 4 + 7 = 7$

Thus, the equation of the tangent passing through (2, 7) is given by,

$$y - 7 = 2(x - 2)$$
$$\Rightarrow y - 2x - 3 = 0$$

Hence, the equation of the tangent line to the given curve (which is parallel to line $2x - y + 9 = 0$) is $y - 2x - 3 = 0$.

(b) The equation of the line is $5y - 15x = 13$.

$$5y - 15x = 13 \Rightarrow y = 3x + \frac{13}{5}$$

This is of the form $y = mx + c$.

$\therefore$Slope of the line = 3

If a tangent is perpendicular to the line $5y - 15x = 13$, then the slope of the tangent is $\frac{-1}{\text{slope of the line}} = \frac{-1}{3}$.

$$\Rightarrow 2x - 2 = \frac{-1}{3}$$

$$\Rightarrow 2x = \frac{-1}{3} + 2$$
$$\Rightarrow 2x = \frac{5}{3}$$
$$\Rightarrow x = \frac{5}{6}$$
$$\text{Now}, x = \frac{5}{6}$$
$$\Rightarrow y = \frac{25}{36} - \frac{10}{6} + 7 = \frac{25 - 60 + 252}{36} = \frac{217}{36}$$

Thus, the equation of the tangent passing through $\left(\frac{5}{6}, \frac{217}{36}\right)$ is given by,

$$y - \frac{217}{36} = -\frac{1}{3}\left(x - \frac{5}{6}\right)$$
$$\Rightarrow \frac{36y - 217}{36} = \frac{-1}{18}(6x - 5)$$
$$\Rightarrow 36y - 217 = -2(6x - 5)$$
$$\Rightarrow 36y - 217 = -12x + 10$$
$$\Rightarrow 36y + 12x - 227 = 0$$

Hence, the equation of the tangent line to the given curve (which is perpendicular to line $5y - 15x = 13$) is $36y + 12x - 227 = 0$.

Tangents and Normals Ex 16.2 Q14

The equation of the given curve is $y = \frac{1}{x-3}$, $x \neq 3$.

The slope of the tangent to the given curve at any point (x, y) is given by,

$$\frac{dy}{dx} = \frac{-1}{(x-3)^2}$$

If the slope of the tangent is 2, then we have:

$$\frac{-1}{(x-3)^2} = 2$$

$$\Rightarrow 2(x-3)^2 = -1$$
$$\Rightarrow (x-3)^2 = \frac{-1}{2}$$

This is not possible since the L.H.S. is positive while the R.H.S. is negative.

Hence, there is no tangent to the given curve having slope 2.

Tangents and Normals Ex 16.2 Q15

The slope of the tangent to the given curve at any point (x, y) is given by,

$$\frac{dy}{dx} = \frac{-(2x-2)}{(x^2-2x+3)^2} = \frac{-2(x-1)}{(x^2-2x+3)^2}$$

If the slope of the tangent is 0, then we have:

$$\frac{-2(x-1)}{(x^2-2x+3)^2} = 0$$

$$\Rightarrow -2(x-1) = 0$$
$$\Rightarrow x = 1$$

When $x = 1$, $y = \frac{1}{1-2+3} = \frac{1}{2}$.

$\therefore$The equation of the tangent through $\left(1, \frac{1}{2}\right)$ is given by,

$$y - \frac{1}{2} = 0(x-1)$$
$$\Rightarrow y - \frac{1}{2} = 0$$
$$\Rightarrow y = \frac{1}{2}$$

Hence, the equation of the required line is $y = \frac{1}{2}$.

Tangents and Normals Ex 16.2 Q16

The equation of the given curve is $y=\sqrt{3x-2}$.

The slope of the tangent to the given curve at any point (x, y) is given by,

$$\frac{dy}{dx}=\frac{3}{2\sqrt{3x-2}}$$

The equation of the given line is $4x-2y+5=0$.

$4x-2y+5=0 \Rightarrow\ y=2x+\frac{5}{2}$ (which is of the form $y=mx+c$)

$\therefore$Slope of the line = 2

Now, the tangent to the given curve is parallel to the line $4x-2y-5=0$ if the slope of the tangent is equal to the slope of the line.

$$\frac{3}{2\sqrt{3x-2}}=2$$
$$\Rightarrow \sqrt{3x-2}=\frac{3}{4}$$
$$\Rightarrow 3x-2=\frac{9}{16}$$
$$\Rightarrow 3x=\frac{9}{16}+2=\frac{41}{16}$$
$$\Rightarrow x=\frac{41}{48}$$

When $x=\frac{41}{48}$, $y=\sqrt{3\left(\frac{41}{48}\right)-2}=\sqrt{\frac{41}{16}-2}=\sqrt{\frac{41-32}{16}}=\sqrt{\frac{9}{16}}=\frac{3}{4}$.

$\therefore$Equation of the tangent passing through the point $\left(\frac{41}{48}, \frac{3}{4}\right)$ is given by,

$$y-\frac{3}{4}=2\left(x-\frac{41}{48}\right)$$

$$\Rightarrow \frac{4y-3}{4}=2\left(\frac{48x-41}{48}\right)$$
$$\Rightarrow 4y-3=\frac{48x-41}{6}$$
$$\Rightarrow 24y-18=48x-41$$
$$\Rightarrow 48x-24y=23$$

Hence, the equation of the required tangent is $48x-24y=23$

Tangents and Normals Ex 16.2 Q17

The given equations are,

$$x^2 + 3y - 3 = 0 \qquad \text{---(i)}$$
$$y = 4x - 5 \qquad \text{---(ii)}$$

Slope m_1 of (i)

$$m_1 = \frac{dy}{dx} = -\frac{2x}{3}$$

Slope m_2 of (ii)

$$m_2 = 4$$

According to the question

$$m_1 = m_2$$
$$\Rightarrow \quad -\frac{2x}{3} = 4$$
$$\Rightarrow \quad x = -6$$

From (i)

$$36 + 3y - 3 = 0$$
$$\Rightarrow \quad 3y = -33$$
$$\therefore \quad y = -11$$

So, $P = (-6, -11)$

Thus, the equation of tangent is

$$(y + 11) = 4(x + 6)$$
$$\Rightarrow \quad 4x - y + 13 = 0$$

Tangents and Normals Ex 16.2 Q18

The equations are

$$\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2 \qquad \text{---(i)}$$
$$\frac{x}{a} + \frac{y}{b} = 2 \qquad \text{---(ii)}$$

$$P = (a, b)$$

We need to prove (ii) is the tangent to (i)

Differentiating (i) with respect to x, we get

$$n\left(\frac{x}{a}\right)^n \times \frac{1}{a} + n\left(\frac{y}{b}\right)^{n-1} \times \frac{1}{b} \times \frac{dy}{dx} = 0$$
$$\Rightarrow \quad \frac{x^{n-1}}{a^n} + \frac{y^{n-1}}{b^n} \times \frac{dy}{dx} = 0$$
$$\Rightarrow \quad \frac{dy}{dx} = -\left(\frac{x}{y}\right)^{n-1} \times \left(\frac{b}{a}\right)^n$$
$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = -\left(\frac{a}{b}\right)^{n-1} \times \left(\frac{b}{a}\right)^n$$
$$= -\frac{b}{a}$$

Thus, the equation of tangent is

$$(y - b) = -\frac{b}{a}(x - a)$$
$$\Rightarrow \quad bx + ay = ab + ab$$
$$\Rightarrow \quad bx + ay = 2ab$$
$$\Rightarrow \quad \frac{x}{a} + \frac{y}{b} = 2$$

Tangents and Normals Ex 16.2 Q19

We have,

$$x = \sin 3t, \qquad y = \cos 2t, \qquad t = \frac{\pi}{4}$$

$$\therefore \qquad P = \left(x = \frac{1}{\sqrt{2}}, y = 0\right)$$

Now,

$$\frac{dx}{dt} = 3\cos 3t, \quad \frac{dy}{dt} = -2\sin 2t$$

$$\therefore \quad \text{Slope } m = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2\sin 2t}{3\cos 3t}$$

$$= \frac{-2}{-3 \times \frac{1}{\sqrt{2}}}$$

$$= \frac{+2\sqrt{2}}{3}$$

Thus, equation of tangent is

$$(y - 0) = \frac{+2\sqrt{2}}{3}\left(x - \frac{1}{\sqrt{2}}\right)$$

$$2\sqrt{2}x - 3y = 2$$

Ex 16.3

Tangents and Normals Ex 16.3 Q1(i)

We know that angle of intersection of two curves is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \qquad ---(A)$$

Where m_1 and m_2 are slopes of curves.

The given equations are

$$y^2 = x \qquad ---(i)$$

$$x^2 = y \qquad ---(ii)$$

$$m_1 = \frac{dy}{dx} = \frac{1}{2y}$$

$$m_2 = \frac{dy}{dx} = 2x$$

Solving (i) and (ii)

$$x^4 - x = 0 \qquad \Rightarrow \qquad x\left(x^3 - 1\right) = 0$$

and $\quad y = 0, 1$

$\therefore \quad m_1 = \frac{1}{2}, \infty \quad$ and $\quad m_2 = 0$ or 2

$$\therefore \quad \tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \frac{3}{4}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{3}{4}\right)$$

and $\quad \tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \infty$

$$\theta = \frac{\pi}{2}$$

Tangents and Normals Ex 16.3 Q1(ii)

We know that angle of intersection of two curves is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \qquad \text{---(A)}$$

Where m_1 and m_2 are slopes of curves.

$y = x^2$ ---(i)

$x^2 + y^2 = 20$ ---(ii)

Solving (i) and (ii)

$y + y^2 = 20$

$\Rightarrow \quad y^2 + y - 20 = 0$

$\Rightarrow \quad (y + 5)(y - 4) = 0$

$\Rightarrow \quad y = -5, 4$

$\therefore \quad x = \sqrt{-5}, \pm 2$

$\therefore$ Points are $P = (2, 4)$, $Q = (-2, 4)$

Now,

Slope m_1 for (i)

$m_1 = 2x = 4$

Slope m_2 for (ii)

$$m_2 = \frac{dy}{dx} = \frac{-x}{y} = \frac{-1}{2}$$

Now,

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{\frac{-1}{2} - 4}{1 - \frac{1}{2} \times 4}\right|$$

$$= \frac{9}{2}$$

$$\therefore \quad \theta = \tan^{-1}\frac{9}{2}$$

Tangents and Normals Ex 16.3 Q1(iii)

We know that angle of intersection of two curves is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \qquad \text{---(A)}$$

Where m_1 and m_2 are slopes of curves.

$$2y^2 = x^3 \qquad \text{---(i)}$$
$$y^2 = 32x \qquad \text{---(ii)}$$

Solving (i) and (ii)

$$x^3 = 64x$$
$$\Rightarrow \quad x\left(x^2 - 64\right) = 0$$
$$\Rightarrow \quad x(x+8)(x-8) = 0$$
$$\Rightarrow \quad x = 0, -8, 8$$

$$\therefore \quad y = 0, -, 16$$

$$\therefore \quad P = (0,0),\ Q = (8,16)$$

Now,

$$m_1 = \frac{dy}{dx} = \frac{3x^2}{4y} = 0 \text{ or } 3$$
$$m_2 = \frac{dy}{dx} = \frac{32}{2y} = \infty \text{ or } 1$$

From (A)

$$\tan\theta = \left|\frac{\infty - 0}{10}\right| = \infty \Rightarrow \theta = \frac{\pi}{2}$$

and $$\tan\theta = \left|\frac{3-1}{13} = \frac{2}{4} = \frac{1}{2}\right|$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{1}{2}\right)$$

Thus,

$$\theta = \frac{\pi}{2} \text{ and } \tan^{-1}\left(\frac{1}{2}\right)$$

Tangents and Normals Ex 16.3 Q1(iv)

We have,

$$x^2 + y^2 - 4x - 1 = 0 \qquad \text{---(i)}$$

and $$x^2 + y^2 - 2y - 9 = 0 \qquad \text{---(ii)}$$

Equation (i) can be written as

$$(x-2)^2 + y^2 - 5 = 0 \qquad \text{---(iii)}$$

Subtracting (ii) from (i), we get

$$-4x + 2y + 8 = 0$$

$$\Rightarrow \quad y = 2x - 4$$

Substituting in (iii), we get

$$(x-2)^2 + (2x-4)^2 - 5 = 0$$

$$\Rightarrow \quad (x-2)^2 + 4(x-2)^2 - 5 = 0$$

$$\Rightarrow \quad (x-2)^2 = 1$$

$$\Rightarrow \quad x - 2 = 1,\ x - 2 = -1$$

$$\Rightarrow \quad x = 3 \text{ or } x = 1$$

$$\therefore \quad y = 2(3) - 4 = 2 \text{ or } y = -2$$

$\therefore$ The points of intersection of the two curves are $(3,2)$ and $(-1,-2)$

Differentiation (i) and (ii), w.r.t x we get

$$2x + 2y\frac{dy}{dx} - 4 = 0 \qquad \text{---(iv)}$$

and $$2x + 2y\frac{dy}{dx} - 2\frac{dy}{dx} = 0 \qquad \text{---(v)}$$

$\therefore$ At $(3,2)$, from equation (iv) we have,

$$\left(\frac{dy}{dx}\right)_{C_1} = \frac{4-2(3)}{2(2)} = \frac{-1}{2}$$

$$\left(\frac{dy}{dx}\right)_{C_2} = \frac{-2(3)}{(2\times 2-3)} = \frac{-6}{2} = -3$$

$\therefore$ If φ is the angle between the curves

Then,

$$\tan\varphi = \frac{\left(\frac{dy}{dx}\right)_{C_1} - \left(\frac{dy}{dx}\right)_{C_2}}{1 + \left(\frac{dy}{dx}\right)_{C_1}\left(\frac{dy}{dx}\right)_{C_2}}$$

Tangents and Normals Ex 16.3 Q1(v)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad \text{---(i)}$$

$$x^2+y^2=ab \qquad \text{---(ii)}$$

From (ii), we get

$$y^2=ab-x^2$$

$\therefore$ From (i), we get

$$\frac{x^2}{a^2}+\frac{ab-x^2}{b^2}=1$$

$\Rightarrow \quad b^2x^2+a^3b-a^2x^2=a^2b^2$

$\Rightarrow \quad \left(b^2-a^2\right)x^2=a^2b^2-a^3b$

$$\Rightarrow \quad x^2=\frac{a^2b^2-a^3b}{b^2-a^2}$$

$$=\frac{a^2b\left(b-a\right)}{\left(b-a\right)\left(b+a\right)}$$

$$=\frac{a^2b}{b+a}$$

$$\therefore \quad x=\pm\sqrt{\frac{a^2b}{a+b}}$$

$$\therefore \quad y^2=ab-x^2=ab-\frac{a^2b}{a+b}$$

$$=\frac{a^2b+ab^2-a^2b}{a+b}=\frac{ab^2}{a+b}$$

Differentiating (i) and (ii) w.r.t x we get

$$\frac{2x}{a^2}+\frac{2y}{b^2}\times\left(\frac{dy}{dx}\right)_{C_1}=0$$

and $\quad 2x+2y\left(\frac{dy}{dx}\right)_{C_2}=0$

$$\therefore \quad \left(\frac{dy}{dx}\right)_{C_1}=\frac{-x}{a^2}\times\frac{b^2}{y}=\frac{-b^2x}{a^2y}$$

$$\left(\frac{dy}{dx}\right)_{C_2}=\frac{-x}{y}$$

At $\left(\pm\sqrt{\frac{a^2b}{a+b}},\pm\sqrt{\frac{ab^2}{a+b}}\right)$ we get

$$\left(\frac{dy}{dx}\right)_{C_1}=\frac{-b^2}{a^2}\sqrt{\frac{a}{b}}=\frac{-b^2\sqrt{a}}{a^2\sqrt{b}}$$

$$\left(\frac{dy}{dx}\right)_{C_2}=-\sqrt{\frac{a}{b}}$$

Tangents and Normals Ex 16.3 Q1(vi)

We know that angle of intersection of two curves is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \qquad \text{---(A)}$$

Where m_1 and m_2 are slopes of curves.

$$x^2 + 4y^2 = 8 \qquad \text{---(i)}$$
$$x^2 - 2y^2 = 2 \qquad \text{---(ii)}$$

Solving (i) and (ii)

$$6y^2 = 6 \Rightarrow \quad y = \pm 1$$
$$\therefore \quad x^2 = 2 + 2 \Rightarrow \quad x = \pm 2$$

$\therefore$ Point of intersection are

$$P = (2, 1) \text{ and } (-2, -1)$$

Now,

Slope m_1 for (i)

$$8y\frac{dy}{dx} = -2x \Rightarrow \frac{dy}{dx} = -\frac{x}{4y}$$
$$\therefore \quad m_1 = \frac{1}{2}$$

Slope m_2 for (ii)

$$4y\frac{dy}{dx} = 2x \Rightarrow \frac{dy}{dx} = \frac{x}{2y}$$
$$\therefore \quad m_2 = 1$$

From (A)

$$\tan\theta = \left|\frac{1 - \frac{1}{2}}{1 + 1 \times \frac{1}{2}}\right| = \frac{1}{3}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{1}{3}\right)$$

Tangents and Normals Ex 16.3 Q1(vii)

We know that angle of intersection of two curves is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \qquad \text{---(A)}$$

Where m_1 and m_2 are slopes of curves.

$$x^2 = 27y \qquad \text{---(i)}$$
$$y^2 = 8x \qquad \text{---(ii)}$$

Solving (i) and (ii) are

$$\frac{y^4}{64} = 27y$$

$\Rightarrow \quad y\left(y^3 - 27 \times 64\right) = 0$

$\Rightarrow \quad y = 0 \text{ or } 12$

$\therefore \quad x = 0 \text{ or } 18$

$\therefore$ Points or intersection is $(0,0)$ and $(18,12)$

Now,

Slope of (i)

$$m_1 = \frac{2x}{27} = \frac{12}{9} = \frac{4}{3}$$

Slope of (ii)

$$m_2 = \frac{8}{2y} = \frac{8}{24} = \frac{1}{3}$$

From (A)

$$\tan\theta = \left|\frac{\frac{4}{3} - \frac{1}{3}}{1 + \frac{4}{3} \times \frac{1}{3}}\right| = \frac{9}{13}$$

$\therefore \quad \theta = \tan^{-1}\left(\frac{9}{13}\right)$

Tangents and Normals Ex 16.3 Q1(viii)

We know that angle of intersection of two curves is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \qquad ---(A)$$

Where m_1 and m_2 are slopes of curves.

$$x^2 + y^2 = 2x \qquad ---(i)$$
$$y^2 = x \qquad ---(ii)$$

Solving (i) and (ii)

$$x^2 + x = 2x$$
$$\Rightarrow \quad x^2 - x = 0$$
$$\Rightarrow \quad x(x-1) = 0$$
$$\Rightarrow \quad x = 0, 1$$

$\therefore \quad y = 0$ or 1

$\therefore$ The points of intersection is $P = (0,0)$, $Q = (1,1)$

$\therefore$ Slope of (i)

$$2y\frac{dy}{dx} = 2 - 2x$$
$$\therefore \quad \frac{dy}{dx} = \frac{2-2x}{2y} = \frac{1-x}{y}$$
$$\therefore \quad m_1 = 0$$

Slope of (ii)

$$m_2 = \frac{1}{2y} = \frac{1}{2}$$

From (A)

$$\tan\theta = \left|\frac{\frac{1}{2} - 0}{1 + \frac{1}{2}\times 0}\right| = \frac{1}{2}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{1}{2}\right)$$

Tangents and Normals Ex 16.3 Q1(ix)

$y = 4 - x^2$......(i)
$y = x^2$.......(ii)

Substituting eq (ii) in (i) we get,

$x^2 = 4 - x^2$

$\Rightarrow x^2 = 2$

$\Rightarrow x = \pm\sqrt{2}$

From(i) when $x = \sqrt{2}$,we get $y = 2$ and when $x = -\sqrt{2}$,we get $y = 2$

Thus the two curves intersect at $\left(\sqrt{2},\ 2\right)$ and $\left(-\sqrt{2}, 2\right)$.

Differnentiating (i) wrt x, we get

$$\frac{dy}{dx} = 0 - 2x = -2x$$

Differnentiating (ii) wrt x, we get

$$\frac{dy}{dx} = 2x$$

Angle of intersection at $\left(\sqrt{2},\ 2\right)$

$$m_1 = \left(\frac{dy}{dx}\right)_{\left(\sqrt{2},\ 2\right)} = -2\sqrt{2}$$

Angle of intersection at $\left(-\sqrt{2},\ 2\right)$

$$m_2 = \left(\frac{dy}{dx}\right)_{\left(-\sqrt{2},\ 2\right)} = 2\sqrt{2}$$

Let θ be the angle of intersection of the two curves.

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{2\sqrt{2} + 2\sqrt{2}}{1 + \left(2\sqrt{2}\right)\left(-2\sqrt{2}\right)}\right| = \left|\frac{4\sqrt{2}}{-7}\right| = \frac{4\sqrt{2}}{7}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{4\sqrt{2}}{7}\right)$$

Tangents and Normals Ex 16.3 Q2(i)

We know that two curves intersects orthogonally if

$$m_1 \times m_2 = -1 \qquad ---(A)$$

Where m_1 and m_2 are the slopes of two curves

$$y = x^3 \qquad ---(i)$$

$$6y = 7 - x^2 \qquad ---(ii)$$

Slope of (i)

$$\frac{dy}{dx} = 3x^2 = m_1$$

Slope of (ii)

$$\frac{dy}{dx} = -\frac{2}{6}x = m_2$$

Point of intersection of (i) and (ii) is

$$6x^3 = 7 - x^2$$

$$\Rightarrow \quad 6x^3 + x^2 - 7 = 0$$

$$\Rightarrow \quad x = 1$$

$$\therefore \quad y = 1$$

$$\therefore \quad P = (1,1)$$

$$\therefore \quad m_1 = 3 \text{ and } m_2 = -\frac{1}{3}$$

Now,

$$m_1 \times m_2 = 3 \times -\frac{1}{3} = -1$$

$\therefore$ (i) and (ii) cuts orthogonally.

Tangents and Normals Ex 16.3 Q2(ii)

We know that two curves intersects orthogonally if

$$m_1 \times m_2 = -1 \qquad \text{---(A)}$$

Where m_1 and m_2 are the slopes of two curves

$$x^3 - 3xy^2 = -2 \qquad \text{---(i)}$$
$$3x^2y - y^3 = 2 \qquad \text{---(ii)}$$

Point of intersection of (i) and (ii)

$$(i) + (ii)$$
$$\Rightarrow \quad x^3 - 3xy^2 + 3x^2y - y^3 = 0$$
$$\Rightarrow \quad (x - y)^3 = 0$$
$$\Rightarrow \quad x = y$$

$\therefore$ from (i)

$$x^3 - 3x^2 = -2$$
$$\Rightarrow \quad -2x^3 = -2$$
$$\Rightarrow \quad x = 1$$

$\therefore$ $P = (1,1)$ is the point of intersection

Now,

Slope of (i)

$$3x^2 - 3y^2 - 6xy\frac{dy}{dx} = 0$$
$$\therefore \quad m_1 = \frac{dy}{dx} = \frac{3(x^2 - y^2)}{6xy}$$

Slope of (ii)

$$6xy + 3x^2\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 0$$
$$\Rightarrow \quad m_2 = \frac{dy}{dx} = \frac{-6xy}{3(x^2 - y^2)}$$

$$\therefore \quad m_1 \times m_2 = \frac{(x^2 - y^2)}{2xy} \times \frac{-2xy}{(x^2 - y^2)} = -1$$

Tangents and Normals Ex 16.3 Q2(iii)

We know that two curves intersects orthogonally if

$$m_1 \times m_2 = -1 \qquad ---(A)$$

Where m_1 and m_2 are the slopes of two curves

$$x^2 + 4y^2 = 8 \qquad ---(i)$$

$$x^2 - 2y^2 = 4 \qquad ---(ii)$$

Point of intersection of (i) and (ii) is (i) − (ii), we get

$$6y^2 = 4$$

$$\Rightarrow \quad y = \sqrt{\frac{2}{3}}$$

$$\therefore \quad x^2 = 4 + \frac{8}{6}$$

$$x^2 = \frac{32}{6}$$

$$\Rightarrow \quad x = \frac{4}{\sqrt{3}}$$

Now,

Slope of (i)

$$2x + 8y\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{x}{4y}$$

$$\Rightarrow \quad m_1 = -\frac{1}{4} \times \frac{4}{\sqrt{2}} = -\frac{1}{\sqrt{2}} \qquad \left[\because \frac{x}{y} = \frac{4}{\sqrt{2}}\right]$$

Slope of (ii)

$$2x - 4y\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x}{2y}$$

$$\Rightarrow \quad m_2 = \frac{1}{2} \times \frac{4}{\sqrt{2}} = \sqrt{2}$$

$$\therefore \quad m_1 \times m_2 = -\frac{1}{\sqrt{2}} \times \sqrt{2} = -1$$

$\therefore$ (i) and (ii) cuts orthogonally.

Tangents and Normals Ex 16.3 Q3(i)

We have,

$$x^2 = 4y \qquad ---(i)$$

$$4y + x^2 = 8 \qquad ---(ii) \qquad P = (2,1)$$

Slope of (i)

$$2x = 4\frac{dy}{dx}$$

$$\therefore \quad m_1 = \left(\frac{dy}{dx}\right)_P = \left(\frac{x}{2}\right)_P = 1$$

Slope of (ii)

$$4\frac{dy}{dx} + 2x = 0$$

$$\therefore \quad m_2 = \left(\frac{dy}{dx}\right)_P = \left(-\frac{x}{2}\right)_P = -1$$

$$\therefore \quad m_1 \times m_2 = 1 \times -1 = -1$$

Hence the result.

Tangents and Normals Ex 16.3 Q3(ii)

We have,

$$x^2 = y \qquad \text{---(i)}$$

$$x^3 + 6y = 7 \qquad \text{---(ii)} \qquad P = (1,1)$$

Slope of (i)

$$2x = \frac{dy}{dx}$$

$$\therefore \quad m_1 = \left(\frac{dy}{dx}\right)_P = 2$$

Slope of(ii)

$$3x^2 + 6\frac{dy}{dx} = 0$$

$$\therefore \quad m_2 = \left(\frac{dy}{dx}\right)_P = \left(-\frac{x^2}{2}\right)_P = \frac{-1}{2}$$

$$\therefore \quad m_1 \times m_2 = 2 \times \frac{-1}{2} = -1$$

Tangents and Normals Ex 16.3 Q3(iii)

We have,

$$y^2 = 8x \qquad \text{---(i)}$$

$$2x^2 + y^2 = 10 \qquad \text{---(ii)} \qquad P\left(1, 2\sqrt{2}\right)$$

Slope of (i)

$$2y\frac{dy}{dx} = 8$$

$$\therefore \quad m_1 = \left(\frac{dy}{dx}\right)_P = \left(\frac{4}{y}\right)_P = \sqrt{2}$$

Slope of (ii)

$$4x + 2y\frac{dy}{dx} = 0$$

$$\therefore \quad m_2 = \left(\frac{dy}{dx}\right)_P = \left(-\frac{2x}{y}\right)_P = \frac{-1}{\sqrt{2}}$$

$$\therefore \quad m_1 \times m_2 = \sqrt{2} \times \frac{-1}{\sqrt{2}} = -1$$

Tangents and Normals Ex 16.3 Q4

We have,

$$4x = y^2 \qquad \text{---(i)}$$

$$4xy = k \qquad \text{---(ii)}$$

Slope of (i)

$$4 = 2y\frac{dy}{dx}$$

$$\Rightarrow \quad m_1 = \frac{dy}{dx} = \frac{2}{y}$$

Slope of (ii)

$$y + x\frac{dy}{dx} = 0$$

$$\Rightarrow \quad m_2 = \frac{dy}{dx} = \frac{-y}{x}$$

Solving (i) and (ii)

$$\frac{k}{y} = y^2$$

$$\Rightarrow \quad y^3 = k$$

$$k = \frac{k^{\frac{2}{3}}}{4}$$

$\therefore$ (i) and (ii) cuts orthogonolly

$$\therefore \quad m_1 \times m_2 = -1$$

$$\Rightarrow \quad \frac{2}{y} \times \frac{-y}{x} = -1$$

$$\Rightarrow \quad \frac{2}{x} = 1$$

$$\Rightarrow \quad x = 2$$

$$\Rightarrow \quad \frac{k^{\frac{2}{3}}}{4} = 2$$

$$\Rightarrow \quad k^{\frac{2}{3}} = 8$$

$$\therefore \quad k^2 = 512$$

Tangents and Normals Ex 16.3 Q5

We have,

$$2x = y^2 \qquad \text{---(i)}$$
$$2xy = k \qquad \text{---(ii)}$$

Slope of (i)

$$2 = 2y\frac{dy}{dx}$$
$$\Rightarrow \quad m_1 = \frac{dy}{dx} = \frac{1}{y}$$

Slope of (ii)

$$y + x\left(\frac{dy}{dx}\right) = 0$$
$$\therefore \quad m_2 = \frac{dy}{dx} = \frac{-y}{x}$$

Now,
Solving (i) and (ii)

$$\frac{k}{y} = y^2$$
$$\Rightarrow \quad y^3 = k$$
$$\therefore \quad x = \frac{y^2}{2} = \frac{k^{\frac{2}{3}}}{2}$$

$\therefore$ (i) and (ii) cuts orthogonolly

$$\therefore \quad m_1 \times m_2 = -1$$
$$\frac{1}{y} \times \frac{-y}{x} = -1$$
$$\Rightarrow \quad \frac{1}{x} = 1$$
$$\Rightarrow \quad x = 1$$
$$\Rightarrow \quad \frac{k^{\frac{2}{3}}}{2} = 1$$
$$\Rightarrow \quad k^{\frac{2}{3}} = 2$$

Closing both side, we get

$$k^2 = 8$$

Tangents and Normals Ex 16.3 Q6

$xy = 4$

$\Rightarrow x = \frac{4}{y}$......(i)

$x^2 + y^2 = 8$.......(ii)

Substituting eq (i) in (ii) we get,

$x^2 + y^2 = 8$

$\Rightarrow \left(\frac{4}{y}\right)^2 + y^2 = 8$

$\Rightarrow 16 + y^4 = 8y^2$

$\Rightarrow y^4 - 8y^2 + 16 = 0$

$\Rightarrow \left(y^2 - 4\right)^2 = 0$

$\Rightarrow y^2 = 4$

$\Rightarrow y = \pm 2$

From(i) when $y = 2$,we get $x = 2$ and when $y = -2$,we get $x = -2$

Thus the two curves intersect at $(2, 2)$ and $(-2, 2)$.

Differnentiating (i) wrt x, we get

$y + x\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$

Differnentiating (i) wrt x, we get

$y + x\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$

Differnentiating (ii) wrt x, we get

$2x + 2y\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{x}{y}$

At $(2, 2)$

$\left(\frac{dy}{dx}\right)_{C_1} = -1$

$\left(\frac{dy}{dx}\right)_{C_2} = -1$

Clearly $\left(\frac{dy}{dx}\right)_{C_1} = \left(\frac{dy}{dx}\right)_{C_2}$ at $(2, 2)$

So given two curves touch each other at $(2, 2)$.

Simillarly, it can be seen that two curves touch each other at $(-2, -2)$.

Tangents and Normals Ex 16.3 Q7

$y^2 = 4x......(i)$
$x^2 + y^2 - 6x + 1 = 0.......(ii)$

Differnentiating (i) wrt x, we get

$$2y\frac{dy}{dx} = 4$$
$$\Rightarrow \frac{dy}{dx} = \frac{2}{y}$$

Differnentiating (ii) wrt x, we get

$$2x + 2y\frac{dy}{dx} - 6 + 0 = 0$$
$$\Rightarrow \frac{dy}{dx} = \frac{3-x}{y}$$

At (1, 2)

$$\left(\frac{dy}{dx}\right)_{C_1} = \frac{2}{2} = 1$$
$$\left(\frac{dy}{dx}\right)_{C_2} = \frac{3-1}{2} = \frac{2}{2} = 1$$

Clearly $\left(\frac{dy}{dx}\right)_{C_1} = \left(\frac{dy}{dx}\right)_{C_2}$ at (1, 2)

So given two curves touch each other at (1, 2).

Tangents and Normals Ex 16.3 Q8

We have,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \text{---(i)}$$
$$xy = c^2 \qquad \text{---(ii)}$$

Slope of (i)

$$\frac{2x}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0$$
$$\therefore \quad m_1 = \frac{dy}{dx} = \frac{x}{y} \times \frac{b^2}{a^2}$$

Slope of (ii)

$$y + x\frac{dy}{dx} = 0$$
$$\therefore \quad m_2 = \frac{dy}{dx} = \frac{-y}{x}$$

$\therefore$ (i) and (ii) cuts orthogonally

$$\therefore \quad m_1 \times m_2 = -1$$
$$\Rightarrow \quad \frac{x}{y} \times \frac{-y}{x} \times \frac{a^2}{b^2} = -1$$
$$\Rightarrow \quad a^2 = b^2$$

We have,

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad \text{---(i)}$$

$$\frac{x^2}{A^2}-\frac{y^2}{B^2}=1 \qquad \text{---(ii)}$$

Slope of (i)

$$\frac{2x}{a^2}+\frac{2y}{b^2}\times\frac{dy}{dx}=0$$

$$\therefore \quad m_1=\frac{dy}{dx}=-\frac{x}{y}\frac{b^2}{a^2}$$

Slope of (ii)

$$\frac{2x}{A^2}-\frac{2y}{B^2}\times\frac{dy}{dx}=0$$

$$\therefore \quad m_2=\frac{dy}{dx}=\frac{x}{y}\times\frac{B^2}{A^2}$$

$\therefore$ (i) and (ii) cuts orthogonally

$$\therefore \quad m_1\times m_2=-1$$

$$\therefore \quad \frac{-x}{y}\frac{b^2}{a^2}\times\frac{x}{y}\times\frac{B^2}{A^2}=-1$$

$$\Rightarrow \quad \frac{x^2}{y^2}\times\frac{b^2B^2}{a^2A^2}=1$$

$$\Rightarrow \quad \frac{x^2}{y^2}=\frac{a^2A^2}{b^2B^2} \qquad \text{---(iii)}$$

Now,

(i) − (ii) gives

$$x^2\left[\frac{1}{a^2}-\frac{1}{A^2}\right]+y^2\left[\frac{1}{b^2}+\frac{1}{B^2}\right]=0$$

$$\Rightarrow \quad \frac{x^2}{y^2}=\frac{B^2+b^2}{b^2B^2}\times\frac{a^2A^2}{a^2-A^2}$$

Put in (iii), we get

$$\frac{\left(B^2+b^2\right)}{b^2B^2}\times\frac{a^2A^2}{\left(a^2-A^2\right)}=\frac{a^2A^2}{b^2B^2}$$

$$\Rightarrow \quad B^2+b^2=a^2-A^2$$

$$\Rightarrow \quad a^2-b^2=A^2+B^2$$

Tangents and Normals Ex 16.3 Q9

We have,

$$\frac{x^2}{a^2+\lambda_1}+\frac{y^2}{b^2+\lambda_1}=1 \qquad ---(i)$$

$$\frac{x^2}{a^2+\lambda_2}+\frac{y^2}{b^2+\lambda_2}=1 \qquad ---(ii)$$

slope of (i)

$$\frac{2x}{a^2+\lambda_1}+\frac{2y}{b^2+\lambda_1}\times\frac{dy}{dx}=0$$

$$\Rightarrow \qquad m_1=\frac{dy}{dx}=\frac{-x}{y}\times\frac{b^2+\lambda_1}{a^2+\lambda_1}$$

Slope of (ii)

$$\frac{2x}{a^2+\lambda_2}+\frac{2y}{b^2+\lambda_2}\times\frac{dy}{dx}=0$$

$$\Rightarrow \qquad m_2=\frac{dy}{dx}=\frac{-x}{y}\times\frac{b^2+\lambda_2}{a^2+\lambda_2}$$

Now,

Subtracting (ii) from (i), we get

$$x^2\left[\frac{1}{a^2+\lambda_1}-\frac{1}{a^2+\lambda_2}\right]+y^2\left[\frac{1}{b^2+\lambda_1}-\frac{1}{b^2+\lambda_2}\right]=0$$

$$\Rightarrow \qquad \frac{x^2}{y^2}=\frac{\lambda_2-\lambda_1}{\left(b^2+\lambda_1\right)\left(b^2+\lambda_2\right)}\times\frac{1}{\frac{\lambda_1-\lambda_2}{\left(a^2+\lambda_1\right)\left(a^2+\lambda_2\right)}}$$

Now,

$$m_1\times m_2=\frac{x^2}{y^2}\times\frac{\left(b^2+\lambda_1\right)\left(b^2+\lambda_2\right)}{\left(a^2+\lambda_1\right)\left(a^2+\lambda_2\right)}$$

$$=\frac{\left(\lambda_2-\lambda_1\right)}{\left(b^2+\lambda_1\right)\left(b^2+\lambda_2\right)}\times-\frac{\left(a^2+\lambda_1\right)\left(a^2+\lambda_2\right)}{\lambda_2-\lambda_1}\times\frac{\left(b^2+\lambda_1\right)\left(b^2+\lambda_2\right)}{\left(a^2+\lambda_1\right)\left(a^2+\lambda_2\right)}$$

$$=-1$$

$\therefore$ (i) and (ii) cuts orthogonolly

Tangents and Normals Ex 16.3 Q10

Suppose the straight line $x\cos\alpha+y\sin\alpha=p$ touches the curve at $Q(x_1,y_1)$.

But equation of tangent to $\frac{x^2}{a^2}+\frac{y}{b^2}=1$ at $Q(x_1,y_1)$ is

$$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$$

Thus equation $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$ and $x\cos\alpha+y\sin\alpha=p$ represent the same line.

$$\therefore \frac{x_1/a^2}{\cos\alpha}+\frac{y_1/b^2}{\sin\alpha}=\frac{1}{p}$$

$$\Rightarrow x_1=\frac{a^2\cos\alpha}{p},\quad y_1=\frac{b^2\sin\alpha}{p}\ldots\ldots\ldots\ldots(i)$$

The point $Q(x_1,y_1)$ lies on the curve $\frac{x^2}{a^2}+\frac{y}{b^2}=1$

$$\therefore \frac{a^4\cos^2\alpha}{p^2a^2}+\frac{b^4\sin^2\alpha}{p^2b^2}=1$$

$$\Rightarrow a^2\cos^2\alpha-b^2\sin^2\alpha=p^2$$

Ex 17.1

Increasing and Decreasing Functions Ex 17.1 Q1

Let $x_1,\ x_2 \in (0,\infty)$

We have,

$$x_1 < x_2$$
$$\Rightarrow \quad \log_e x_1 < \log_e x_2$$
$$\Rightarrow \quad f(x_1) < f(x_2)$$

So, $f(x)$ is increasing in $(0,\infty)$.

Increasing and Decreasing Functions Ex 17.1 Q2

Case I

When $a > 1$

Let $x_1, x_2 \in (0,\infty)$

We have

$$x_1 < x_2$$
$$\Rightarrow \quad \log_a x_1 < \log_a x_2$$
$$\Rightarrow \quad f(x_1) < f(x_2)$$

Thus, $f(x)$ is increasing on $(0,\infty)$

Case II

When $0 < a < 1$

$$f(x) = \log_a x = \frac{\log x}{\log a}$$

When $a < 1 \Rightarrow \log a < 0$

Let $x_1 < x_2$

$$\Rightarrow \quad \log x_1 < \log x_2$$
$$\Rightarrow \quad \frac{\log x_1}{\log a} > \frac{\log x_2}{\log a} \qquad [\because \log a < 0]$$
$$\Rightarrow \quad f(x_1) > f(x_2)$$

So, $f(x)$ is decreasing on $(0,\infty)$.

Increasing and Decreasing Functions Ex 17.1 Q3

We have,

$$f(x) = ax + b,\ a > 0$$

Let $x_1, x_2 \in R$ and $x_1 > x_2$

$\Rightarrow \quad ax_1 > ax_2$ for some $a > 0$

$\Rightarrow \quad ax_1 + b > ax_2 + b$ for some b

$\Rightarrow \quad f(x_1) > f(x_2)$

$\therefore\ f(x)$ is increasing function of R.

Increasing and Decreasing Functions Ex 17.1 Q4

We have,

$$f(x) = ax + b,\ a < 0$$

Let $x_1, x_2 \in R$ and $x_1 > x_2$

$\Rightarrow \quad ax_1 < ax_2$ for some $a < 0$

$\Rightarrow \quad ax_1 + b < ax_2 + b$ for some b

$\Rightarrow \quad f(x_1) < f(x_2)$

Hence, $x_1 > x_2 \Rightarrow f(x_1) < f(x_2)$

$\therefore\ f(x)$ is decreasing function of R.

Increasing and Decreasing Functions Ex 17.1 Q5

We have,

$$f(x) = \frac{1}{x}$$

Let $x_1, x_2 \in (0, \infty)$ and $x_1 > x_2$

$\Rightarrow \quad \frac{1}{x_1} < \frac{1}{x_2}$

$\Rightarrow \quad f(x_1) < f(x_2)$

Thus, $x_1 > x_2 \Rightarrow f(x_1) < f(x_2)$

So, $f(x)$ is decreasing function.

Increasing and Decreasing Functions Ex 17.1 Q6

We have,

$$f(x) = \frac{1}{1 + x^2}$$

Case I

When $x \in [0, \infty)$

Let $x_1, x_2 \in (0, \infty]$ and $x_1 > x_2$

$\Rightarrow \quad {x_1}^2 > {x_2}^2$

$\Rightarrow \quad 1 + {x_1}^2 > 1 + {x_2}^2$

$\Rightarrow \quad \frac{1}{1 + {x_1}^2} < \frac{1}{1 + {x_2}^2}$

$\Rightarrow \quad f(x_1) < f(x_2)$

So, $f(x)$ is decreasing on $[0, \infty)$

Case II

When $x \in (-\infty, 0]$

Let $x_1 > x_2$

$\Rightarrow \quad {x_1}^2 < {x_2}^2 \qquad [\because -2 > -3 \Rightarrow 4 < 9]$

$\Rightarrow \quad 1 + {x_1}^2 < 1 + {x_2}^2$

$\Rightarrow \quad \frac{1}{1 + {x_1}^2} > \frac{1}{1 + {x_2}^2}$

$\Rightarrow \quad f(x_1) > f(x_2)$

So, $f(x)$ is increasing on $(-\infty, 0]$

Increasing and Decreasing Functions Ex 17.1 Q7

We have,

$$f(x) = \frac{1}{1+x^2}$$

Case I

When $x \in [0,\infty)$

Let $x_1 > x_2$

$\Rightarrow \quad x_1^2 > x_2^2$

$\Rightarrow \quad 1+x_1^2 > 1+x_2^2$

$\Rightarrow \quad \frac{1}{1+x_1^2} < \frac{1}{1+x_2^2}$

$\Rightarrow \quad f(x_1) < f(x_2)$

$\therefore \; f(x)$ is decreasing on $[0,\infty)$.

Case II

When $x \in (-\infty,0]$

Let $x_1 > x_2$

$\Rightarrow \quad x_1^2 < x_2^2$

$\Rightarrow \quad 1+x_1^2 < 1+x_2^2$

$\Rightarrow \quad \frac{1}{1+x_1^2} > \frac{1}{1+x_2^2}$

$\Rightarrow \quad f(x_1) > f(x_2)$

So, $f(x)$ is increasing on $(-\infty,0]$

Thus, $f(x)$ is neither increasing nor decreasing on R.

Increasing and Decreasing Functions Ex 17.1 Q8

We have,

$$f(x) = |x| = \begin{cases} x, & x > 0 \\ -x, & x < 0 \end{cases}$$

(a)

Let $x_1,\ x_2 \in (0,\infty)$ and $x_1 > x_2$

$\Rightarrow \quad f(x_1) > f(x_2)$

So, $f(x)$ is increasing in $(0,\infty)$

(b)

Let $x_1,\ x_2 \in (-\infty,0)$ and $x_1 > x_2$

$\Rightarrow \quad -x_1 < -x_2$

$\Rightarrow \quad f(x_1) < f(x_2)$

$\therefore \; f(x)$ is strictly decreasing on $(-\infty,0)$.

Increasing and Decreasing Functions Ex 17.1 Q9

$$f(x) = 7x - 3$$

Let $x_1,\ x_2 \in R$ and $x_1 > x_2$

$\Rightarrow \quad 7x_1 > 7x_2$

$\Rightarrow \quad 7x_1 - 3 > 7x_2 - 3$

$\Rightarrow \quad f(x_1) > f(x_2)$

$\therefore \; f(x)$ is strictly increasing on R.

Ex 17.2

Increasing and Decreasing Functions Ex 17.2 Q1(i)

We have,

$f(x) = 10 - 6x - 2x^2$

$\therefore f'(x) = -6 - 4x$

Now,

$f'(x) = 0 \Rightarrow x = -\frac{3}{2}$

The point $x = -\frac{3}{2}$ divides the real line into two disjoint intervals i.e., $\left(-\infty, -\frac{3}{2}\right)$ and $\left(-\frac{3}{2}, \infty\right)$.

In interval $\left(-\infty, -\frac{3}{2}\right)$ i.e., when $x < -\frac{3}{2}$, $f'(x) = -6 - 4x < 0$.

$\therefore f$ is strictly increasing for $x < -\frac{3}{2}$.

In interval $\left(-\frac{3}{2}, \infty\right)$ i.e., when $x > -\frac{3}{2}$, $f'(x) = -6 - 4x < 0$.

$\therefore f$ is strictly decreasing for $x > -\frac{3}{2}$.

Increasing and Decreasing Functions Ex 17.2 Q1(ii)

We have,

$f(x)=x^2+2x-5$

$\therefore f'(x)=2x+2$

Now,

$f'(x)=0 \Rightarrow x=-1$

Point $x=-1$ divides the real line into two disjoint intervals i.e., $(-\infty,-1)$ and $(-1,\infty)$.

In interval $(-\infty,-1)$, $f'(x)=2x+2<0$.

$\therefore f$ is strictly decreasing in interval $(-\infty,-1)$.

Thus, f is strictly decreasing for $x<-1$.

In interval $(-1,\infty)$, $f'(x)=2x+2>0$.

$\therefore f$ is strictly increasing in interval $(-1,\infty)$.

Thus, f is strictly increasing for $x>-1$.

Increasing and Decreasing Functions Ex 17.2 Q1(iii)

We have,

$f(x)=6-9x-x^2$

$\therefore f'(x)=-9-2x$

Now,

$f'(x)=0$ gives $x=-\dfrac{9}{2}$

The point $x=-\dfrac{9}{2}$ divides the real line into two disjoint intervals i.e., $\left(-\infty,-\dfrac{9}{2}\right)$ and $\left(-\dfrac{9}{2},\infty\right)$.

In interval $\left(-\infty,-\dfrac{9}{2}\right)$ i.e., for $x<-\dfrac{9}{2}$, $f'(x)=-9-2x>0$.

$\therefore f$ is strictly increasing for $x<-\dfrac{9}{2}$.

In interval $\left(-\dfrac{9}{2},\infty\right)$ i.e., for $x>-\dfrac{9}{2}$, $f'(x)=-9-2x<0$.

$\therefore f$ is strictly decreasing for $x>-\dfrac{9}{2}$.

Increasing and Decreasing Functions Ex 17.2 Q1(iv)

$f(x) = 2x^3 - 12x^2 + 18x + 15$

$\therefore \quad f'(x) = 6x^2 - 24x + 18$

$= 6(x^2 - 4x + 3)$

$= 6(x - 3)(x - 1)$

Critical point

$f'(x) = 0$

$\Rightarrow \quad 6(x - 3)(x - 1) = 0$

$\Rightarrow \quad x = 3,\ 1$

Clearly, $f(x) > 0$ if $x < 1$ and $x > 3$

and $f(x) < 0$ if $1 < x < 3$

Thus, $f(x)$ increases on $(-\infty, 1) \cup (3, \infty)$, decreases on $(1, 3)$.

Increasing and Decreasing Functions Ex 17.2 Q1(v)

We have,

$f(x) = 5 + 36x + 3x^2 - 2x^3$

$\therefore \quad f'(x) = 36 + 6x - 6x^2$

Critical point

$f'(x) = 0$

$\Rightarrow \quad 36 + 6x - 6x^2 = 0$

$\Rightarrow \quad -6(x^2 - x - 6) = 0$

$\Rightarrow \quad (x - 3)(x + 2) = 0$

$\therefore \quad x = 3,\ -2$

Clearly, $f'(x) > 0$ if $-2 < x < 3$

Also $f'(x) < 0$ if $x < -2$ and $x > 3$

Thus, increases if $x \in (-2, 3)$, decreases if $x \in (-\infty, -2) \cup (3, \infty)$

Increasing and Decreasing Functions Ex 17.2 Q1(vi)

We have,

$f(x) = 8 + 36x + 3x^2 - 2x^3$

$\therefore \quad f'(x) = 36 + 6x - 6x^2$

Critical points

$f'(x) = 0$

$\Rightarrow \quad 6(6 + x - x^2) = 0$

$\Rightarrow \quad (3 - x)(2 + x) = 0$

$\Rightarrow \quad x = 3,\ -2$

Clearly, $f'(x) > 0$ if $-2 < x < 3$

and $f'(x) < 0$ if $-\infty < x < -2$ and $3 < x < \infty$

Thus, increases in $(-2, 3)$, decreases in $(-\infty, -2) \cup (3, \infty)$

Increasing and Decreasing Functions Ex 17.2 Q1(vii)

We have,

$f(x) = 5x^3 - 15x^2 - 120x + 3$

$\therefore \quad f'(x) = 15x^2 - 30x - 120$

Critical points

$f'(x) = 0$

$\Rightarrow \quad 15(x^2 - 2x - 8) = 0$

$\Rightarrow \quad (x - 4)(x + 2) = 0$

$\Rightarrow \quad x = 4,\ -2$

Clearly, $f'(x) > 0$ if $x < -2$ and $x > 4$

and $f'(x) < 0$ if $-2 < x < 4$

Thus, increases in $(-\infty, -2) \cup (4, \infty)$, decreases in $(-2, 4)$

Increasing and Decreasing Functions Ex 17.2 Q1(viii)

$f(x) = x^3 - 6x^2 - 36x + 2$

$\therefore \quad f'(x) = 3x^2 - 12x - 36$

Critical point

$f'(x) = 0$

$\Rightarrow \quad 3(x^2 - 4x - 12) = 0$

$\Rightarrow \quad (x-6)(x+2) = 0$

$\Rightarrow \quad x = 6, \ -2$

Clearly, $f'(x) > 0$ if $x < -2$ and $x > 6$

$f'(x) < 0$ if $-2x < x < 6$

Thus, increases in $(-\infty,-2) \cup (6,\infty)$, decreases in $(-2,6)$.

Increasing and Decreasing Functions Ex 17.2 Q1(ix)

We have,

$f(x) = 2x^3 - 15x^2 + 36x + 1$

$\therefore \quad f'(x) = 6x^2 - 30x + 36$

Critical points

$\Rightarrow \quad 6(x^2 - 5x + 6) = 0$

$\Rightarrow \quad (x-3)(x-2) = 0$

$\Rightarrow \quad x = 3, \ 2$

Clearly, $f'(x) > 0$ if $x < 2$ and $x > 3$

$f'(x) < 0$ if $2 < x < 3$

Thus, $f(x)$ increases in $(-\infty,2) \cup (3,\infty)$, decreases in $(2,3)$.

Increasing and Decreasing Functions Ex 17.2 Q1(x)

We have,

$f(x) = 2x^3 + 9x^2 + 12x - 1$

$\therefore \quad f'(x) = 6x^2 + 18x + 12$

Critical ponts

$f'(x) = 0$

$\Rightarrow \quad 6(x^2 + 3x + 2) = 0$

$\Rightarrow \quad (x+2)(x+1) = 0$

$\Rightarrow \quad x = -2, \ -1$

Increasing and Decreasing Functions Ex 17.2 Q1(xi)

We have,

$f(x) = 2x^3 - 9x^2 + 12x - 5$

$\therefore \quad f'(x) = 6x^2 - 18x + 12$

Critical points

$f'(x) = 0$

$\Rightarrow \quad 6(x^2 - 3x + 2) = 0$

$\Rightarrow \quad (x-2)(x-1) = 0$

$\Rightarrow \quad x = 2, \ 1$

Clearly, $f'(x) > 0$ if $x < 1$ and $x > 2$

$f'(x) < 0$ if $1 < x < 2$

Thus, $f(x)$ increases in $(-\infty,1) \cup (2,\infty)$, decreases in $(1,2)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xii)

We have,

$$f(x) = 6 + 12x + 3x^2 - 2x^3$$

$\therefore \quad f'(x) = 12 + 6x - 6x^2$

Critical points

$$f'(x) = 0$$

$\Rightarrow \quad 6\left(2 + x - x^2\right) = 0$

$\Rightarrow \quad (2 - x)(1 + x) = 0$

$\Rightarrow \quad x = 2, \; -1$

Clearly, $f'(x) > 0$ if $-1 < x < 2$

$f'(x) < 0$ if $x < -1$ and $x > 2$.

Thus, $f(x)$ increases in $(-1, 2)$, decreases in $(-\infty, -1) \cup (2, \infty)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xiii)

We have,

$$f(x) = 2x^3 - 24x + 107$$

$\therefore \quad f'(x) = 6x^2 - 24$

Critical points

$$f'(x) = 0$$

$\Rightarrow \quad 6\left(x^2 - 4\right) = 0$

$\Rightarrow \quad (x - 2)(x + 2) = 0$

$\Rightarrow \quad x = 2, -2$

Clearly, $f'(x) > 0$ if $x < -2$ and $x > 2$

$f'(x) < 0$ if $-2 < x < 2$

Thus, $f(x)$ increases in $(-\infty, -2) \cup (2, \infty)$, decreases in $(-2, 2)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xiv)

We have

$$f(x) = -2x^3 - 9x^2 - 12x + 1$$

$$f'(x) = -6x^2 - 18x - 12$$

Critical points

$$f'(x) = 0$$

$$-6x^2 - 18x - 12 = 0$$

$$x^2 + 3x + 2 = 0$$

$$(x+2)(x+1) = 0$$

$$x = -2, -1$$

Clearly, $f'(x) > 0$ if $x < -1$ and $x < -2$

$f'(x) < 0$ if $-2 < x < -1$

Thus, $f(x)$ is increasing in $(-2, -1)$, decreasing in $(-\infty, -2) \cup (-1, \infty)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xv)

We have,

$$f(x) = (x - 1)(x - 2)^2$$

$\therefore \quad f'(x) = (x - 2)^2 + 2(x - 1)(x - 2)$

$f'(x) = (x - 2)(x - 2 + 2x - 2)$

$\Rightarrow \quad f'(x) = (x - 2)(3x - 4)$

Critical points

$$f'(x) = 0$$

$\Rightarrow \quad (x - 2)(3x - 4) = 0$

$\Rightarrow \quad x = 2, \; \frac{4}{3}$

Clearly, $f'(x) > 0$ if $x < \frac{4}{3}$ and $x > 2$

$f'(x) < 0$ if $\frac{4}{3} < x < 2$

Thus, $f(x)$ increases in $\left(-\infty, \frac{4}{3}\right) \cup (2, \infty)$, decreases in $\left(\frac{4}{3}, 2\right)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xvi)

We have,

$$f(x) = x^3 - 12x^2 + 36x + 17$$

$$\therefore \quad f'(x) = 3x^2 - 24x + 36$$

Critical points

$$f'(x) = 0$$

$$\Rightarrow \quad 3(x^2 - 8x + 12) = 0$$

$$\Rightarrow \quad (x-6)(x-2) = 0$$

$$\Rightarrow \quad x = 6, 2$$

Clearly, $f'(x) > 0$ if $x < 2$ and $x > 6$

$f'(x) < 0$ if $2 < x < 6$

Thus, $f(x)$ increases in $(-\infty, 2) \cup (6, \infty)$, decreases in $(2, 6)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xvii)

We have

$$f(x) = 2x^3 - 24x + 7$$

$$f'(x) = 6x^2 - 24$$

Critical points

$$f'(x) = 0$$

$$6x^2 - 24 = 0$$

$$6x^2 = 24$$

$$x^2 = 4$$

$$x = 2, -2$$

Clearly, $f'(x) > 0$ if $x > 2$ and $x < -2$

$f'(x) < 0$ if $-2 \le x \le 2$

Thus, $f(x)$ is increasing in $(-\infty, -2) \cup (2, \infty)$, decreasing in $(-2, 2)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xviii)

We have $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$

$$\therefore \quad f'(x) = \frac{3}{10}(4x^3) - \frac{4}{5}(3x^2) - 3(2x) + \frac{36}{5}$$

$$= \frac{6}{5}(x-1)(x+2)(x-3)$$

Now $f'(x) = 0$

$\Rightarrow \frac{6}{5}(x-1)(x+2)(x-3) = 0$

$\Rightarrow x = 1, -2$ or 3

The points $x = 1, -2$ and 3 divide the number line into four disjoint intervals namely, $(-\infty, -2)$, $(-2, 1)$, $(1, 3)$ and $(3, \infty)$.

Consider the interval $(-\infty, -2)$, i.e $-\infty < x < -2$

In this case, we have $x - 1 < 0$, $x + 2 < 0$ and $x - 3 < 0$

$\therefore \quad f'(x) < 0$ when $-\infty < x < -2$

Thus, the function f is strictly decreasing in $(-\infty, -2)$

Consider the interval $(-2, 1)$, i.e $-2 < x < 1$

In this case, we have $x - 1 < 0$, $x + 2 > 0$ and $x - 3 < 0$

$\therefore \quad f'(x) > 0$ when $-2 < x < 1$

Thus, the function f is strictly increasing in $(-2, 1)$

Now, consider the interval $(1, 3)$, i.e $1 < x < 3$

In this case, we have $x - 1 > 0$, $x + 2 > 0$ and $x - 3 < 0$

$\therefore \quad f'(x) < 0$ when $1 < x < 3$

Thus, the function f is strictly decreasing in $(1, 3)$

Finally consider the interval $(3, \infty)$, i.e $3 < x < \infty$

In this case, we have $x - 1 > 0$, $x + 2 > 0$ and $x - 3 > 0$

$\therefore \quad f'(x) > 0$ when $x > 3$

Thus, the function f is strictly increasing in $(3, \infty)$

Increasing and Decreasing Functions Ex 17.2 Q1(xix)

We have,

$$f(x) = x^4 - 4x$$

$\therefore \quad f'(x) = 4x^3 - 4$

Critical points,

$$f'(x) = 0$$

$\Rightarrow \quad 4\left(x^3 - 1\right) = 0$

$\Rightarrow \quad x = 1$

Clearly, $f'(x) > 0$ if $x > 1$

$f'(x) < 0$ if $x < 1$

Thus, $f(x)$ increases in $(1, \infty)$, decreases in $(-\infty, 1)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xx)

$f(x) = \frac{x^4}{4} + \frac{2}{3}x^3 - \frac{5}{2}x^2 - 6x + 7$

$\therefore \quad f'(x) = x^3 + 2x^2 - 5x - 6$

Critical points

$$f'(x) = 0$$

$\Rightarrow \quad x^3 + 2x^2 - 5x - 6 = 0$

$\Rightarrow \quad (x+1)(x+3)(x-2) = 0$

$\Rightarrow \quad x = -1, \ -3, \ 2$

Clearly, $f'(x) > 0$ if $-3 < x < -1$ and $x > 2$

$f'(x) < 0$ if $x < -3$ and $-1 < x < 2$

Thus, $f(x)$ increases in $(-3, -1) \cup (2, \infty)$, decreases in $(-\infty, -3) \cup (-1, 2)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xxi)

$f(x) = x^4 - 4x^3 + 4x^2 + 15$

$\therefore \quad f'(x) = 4x^3 - 12x^2 + 8x$

Critical points

$$f'(x) = 0$$

$\Rightarrow \quad 4x\left(x^2 - 3x + 2\right) = 0$

$\Rightarrow \quad 4x(x-2)(x-1) = 0$

$\Rightarrow \quad x = 0, \ 2, \ 1$

Clealry, $f'(x) > 0$ if $0 < x < 1$ and $x > 2$

$f'(x) < 0$ if $x < 0$ and $1 < x < 2$

Thus, $f(x)$ increases in $(0, 1) \cup (2, \infty)$, decreases in $(-\infty, 0) \cup (1, 2)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xxii)

We have,

$$f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}}; \ x > 0$$

$\therefore \quad f'(x) = \frac{15}{2}x^{\frac{1}{2}} - \frac{15}{2}x^{\frac{3}{2}}$

Critical points

$$f'(x) = 0$$

$\Rightarrow \quad \frac{15}{2}x^{\frac{1}{2}} - \frac{15}{2}x^{\frac{3}{2}} = 0$

$\Rightarrow \quad \frac{15}{2}x^{\frac{1}{2}}(1 - x) = 0$

$\Rightarrow \quad x = 0, \ 1$

Clearly, $f'(x) > 0$ if $0 < x < 1$

and $f'(x) < 0$ if $x > 1$

Thus, $f(x)$ increases in $(0, 1)$, decreases in $(1, \infty)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xxiii)

We have,

$$f(x) = x^8 + 6x^2$$

$\therefore$ $$f'(x) = 8x^7 + 12x$$

Critical points

$$f'(x) = 0$$

$\Rightarrow$ $$8x^7 + 12x = 0$$

$\Rightarrow$ $$4x(2x^6 + 3) = 0$$

$\Rightarrow$ $$x = 0$$

Clearly, $f'(x) > 0$ if $x > 0$

$f'(x) < 0$ if $x < 0$

Thus, $f(x)$ increases in $(0, \infty)$, decreases in $(-\infty, 0)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xxiv)

We have,

$$f(x) = x^3 - 6x^2 + 9x + 15$$

$\therefore$ $$f'(x) = 3x^2 - 12x + 9$$

Critical points

$$f'(x) = 0$$

$\Rightarrow$ $$3(x^2 - 4x + 3) = 0$$

$\Rightarrow$ $$(x - 3)(x - 1) = 0$$

$\Rightarrow$ $$x = 3,\ 1$$

Clearly, $f'(x) > 0$ if $x < 1$ and $x > 3$

$f'(x) < 0$ if $1 < x < 3$

Thus, $f(x)$ increases in $(-\infty, 1) \cup (3, \infty)$, decreases in $(1, 3)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xxv)

We have,

$$y = [x(x-2)]^2 = [x^2 - 2x]^2$$

$$\therefore \frac{dy}{dx} = y' = 2(x^2 - 2x)(2x - 2) = 4x(x-2)(x-1)$$

$$\therefore \frac{dy}{dx} = 0 \Rightarrow x = 0, x = 2, x = 1.$$

The points $x = 0$, $x = 1$, and $x = 2$ divide the real line into four disjoint intervals i.e., $(-\infty, 0)$, $(0, 1)$ $(1, 2)$, and $(2, \infty)$.

In intervals $(-\infty, 0)$ and $(1, 2)$, $\frac{dy}{dx} < 0$.

$\therefore y$ is strictly decreasing in intervals $(-\infty, 0)$ and $(1, 2)$.

However, in intervals (0, 1) and (2, ∞), $\frac{dy}{dx} > 0$.

$\therefore y$ is strictly increasing in intervals (0, 1) and (2, ∞).

$\therefore$ y is strictly increasing for $0 < x < 1$ and $x > 2$.

Increasing and Decreasing Functions Ex 17.2 Q1(xxvi)

Consider the given function

$f(x)=3x^4-4x^3-12x^2+5$

$\Rightarrow f'(x)=12x^3-12x^2-24x$

$\Rightarrow f'(x)=12x(x^2-x-2)$

$\Rightarrow f'(x)=12x(x+1)(x-2)$

For $f(x)$ to be increasing, we must have,

$f'(x)>0$

$\Rightarrow 12x(x+1)(x-2)>0$

$\Rightarrow x(x+1)(x-2)>0$

$\Rightarrow -1<x<0$ or $2<x<\infty$

$\Rightarrow x\in(-1,0)\cup(2,\infty)$

So, $f(x)$ is increasing in $(-1,0)\cup(2,\infty)$

For $f(x)$ to be decreasing, we must have,

$f'(x)<0$

$\Rightarrow 12x(x+1)(x-2)<0$

$\Rightarrow x(x+1)(x-2)<0$

$\Rightarrow -\infty<x<-1$ or $0<x<2$

$\Rightarrow x\in(-\infty,-1)\cup(0,2)$

So, $f(x)$ is decreasing in $(-\infty,-1)\cup(0,2)$

Increasing and Decreasing Functions Ex 17.2 Q1(xxvii)

Consider the given function

$f(x)=\frac{3}{2}x^4-4x^3-45x^2+51$

$\Rightarrow f'(x)=4\times\frac{3}{2}x^3-12x^2-90x$

$\Rightarrow f'(x)=6x^3-12x^2-90x$

$\Rightarrow f'(x)=6x(x^2-2x-15)$

$\Rightarrow f'(x)=6x(x+3)(x-5)$

For $f(x)$ to be increasing, we must have,

$f'(x)>0$

$\Rightarrow 6x(x+3)(x-5)>0$

$\Rightarrow x(x+3)(x-5)>0$

$\Rightarrow -3<x<0$ or $5<x<\infty$

$\Rightarrow x\in(-3,0)\cup(5,\infty)$

So, $f(x)$ is increasing in $(-3,0)\cup(5,\infty)$

For $f(x)$ to be decreasing, we must have,

$f'(x)<0$

$\Rightarrow 6x(x+3)(x-5)<0$

$\Rightarrow x(x+3)(x-5)<0$

$\Rightarrow -\infty<x<-3$ or $0<x<5$

$\Rightarrow x\in(-\infty,-3)\cup(0,5)$

So, $f(x)$ is decreasing in $(-\infty,-3)\cup(0,5)$

Increasing and Decreasing Functions Ex 17.2 Q1(xxviii)

Consider the given function

$$f(x) = \log(2+x) - \frac{2x}{2+x}, x \in R$$

$$\Rightarrow f'(x) = \frac{1}{2+x} - \frac{(2+x)2 - 2x \times 1}{(2+x)^2}$$

$$\Rightarrow f'(x) = \frac{1}{2+x} - \frac{4+2x-2x}{(2+x)^2}$$

$$\Rightarrow f'(x) = \frac{1}{2+x} - \frac{4}{(2+x)^2}$$

$$\Rightarrow f'(x) = \frac{2+x-4}{(2+x)^2}$$

$$\Rightarrow f'(x) = \frac{x-2}{(2+x)^2}$$

For $f(x)$ to be increasing, we must have,

$f'(x) > 0$

$\Rightarrow x - 2 > 0$

$\Rightarrow 2<x<\infty$

$\Rightarrow x \in (2,\infty)$

So, $f(x)$ *is increasing* in $(2,\infty)$

For $f(x)$ to be decreasing, we must have,

$f'(x) < 0$

$\Rightarrow x - 2 < 0$

$\Rightarrow -\infty<x<2$

$\Rightarrow x \in (-\infty,2)$

So, $f(x)$ is decreasing in $(-\infty,2)$

Increasing and Decreasing Functions Ex 17.2 Q2

We have,

$$f(x) = x^2 - 6x + 9$$

$\therefore \quad f'(x) = 2x - 6$

Critical points

$$f'(x) = 0$$

$\Rightarrow \quad 2(x-3) = 0$

$\Rightarrow \quad x = 3$

Clearly, $f'(x) > 0$ if $x > 3$

$$f'(x) < 0 \text{ if } x < 3$$

Thus, $f(x)$ increases in $(3, \infty)$, decreases in $(-\infty, 3)$

IInd part

The given equation of curves

$$y = x^2 - 6x + 9 \qquad \text{---(i)}$$

$$y = x + 5 \qquad \text{---(ii)}$$

Slope of (i)

$$m_1 = \frac{dy}{dx} = 2x - 6$$

Slope of (ii)

$$m_2 = 1$$

Given that slope of normal to (i) is parallelt to (ii)

$\therefore \quad \frac{-1}{2x-6} = 1$

$\Rightarrow \quad 2x - 6 = -1$

$\Rightarrow \quad x = \frac{5}{2}$

From (i)

$$y = \frac{25}{4} - 15 + 9$$

$$= \frac{25}{4} - 6$$

$$= \frac{1}{4}$$

Thus, the required point is $\left(\frac{5}{2}, \frac{1}{4}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q3

We have,

$$f(x) = \sin x - \cos x, \quad 0 < x < 2\pi$$

$\therefore \quad f'(x) = \cos x + \sin x$

Critical points

$$f'(x) = 0$$

$\Rightarrow \quad \cos x + \sin x = 0$

$\Rightarrow \quad \tan x = -1$

$\Rightarrow \quad x = \frac{3\pi}{4}, \ \frac{7\pi}{4}$

Clearly, $f'(x) > 0$ if $0 < x < \frac{3\pi}{4}$ and $\frac{7\pi}{4} < x < 2\pi$

$$f'(x) < 0 \text{ if } \frac{3\pi}{4} < x < \frac{7\pi}{4}$$

Thus, $f(x)$ increases in $\left(0, \frac{3\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right)$, decreases in $\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q4

We have,

$$f(x) = e^{2x}$$

$$\therefore \quad f'(x) = 2e^{2x}$$

We know that

$f(x)$ is increasing if $f'(x) > 0$

$$\Rightarrow \quad 2e^{2x} > 0$$

$$\Rightarrow \quad e^{2x} > 0$$

Since, the value of e lies between 2 and 3
So, any power of e will be greater than zero.

Thus, $f(x)$ is increasing on R.

Increasing and Decreasing Functions Ex 17.2 Q5

We have,

$$f(x) = e^{\frac{1}{x}}, \qquad x \neq 0$$

$$f'(x) = e^{\frac{1}{x}} \times \left(\frac{-1}{x^2}\right)$$

$$\therefore \quad f'(x) = -\frac{e^{\frac{1}{x}}}{x^2}$$

Now,

$$x \in R,\ x \neq 0$$

$$\Rightarrow \quad \frac{1}{x^2} > 0 \text{ and } e^{\frac{1}{x}} > 0$$

$$\Rightarrow \quad \frac{e^{\frac{1}{x}}}{x^2} > 0$$

$$\Rightarrow \quad -\frac{e^{\frac{1}{x}}}{x^2} < 0$$

$$\Rightarrow \quad f'(x) < 0$$

Hence, $f(x)$ is a decreasing function for all $x \neq 0$.

Increasing and Decreasing Functions Ex 17.2 Q6

We have,

$$f(x) = \log_a x, \quad 0 < a < 1$$

$$\Rightarrow \quad f'(x) = \frac{1}{x \log a}$$

$$\therefore \quad 0 < a < 1$$

$$\Rightarrow \quad \log a < 0$$

Now,

$$x > 0$$

$$\Rightarrow \quad \frac{1}{x} > 0$$

$$\Rightarrow \quad \frac{1}{x \log a} < 0$$

$$\Rightarrow \quad f'(x) < 0$$

Thus, $f(x)$ is a decreasing function for $x > 0$.

Increasing and Decreasing Functions Ex 17.2 Q7

The given function is $f(x) = \sin x$.

$\therefore f'(x) = \cos x$

(a) Since for each $x \in \left(0, \frac{\pi}{2}\right)$, $\cos x > 0$, we have $f'(x) > 0$.

Hence, f is strictly increasing in $\left(0, \frac{\pi}{2}\right)$.

(b) Since for each $x \in \left(\frac{\pi}{2}, \pi\right)$, $\cos x < 0$, we have $f'(x) < 0$.

Hence, f is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.

(c) From the results obtained in (a) and (b), it is clear that f is neither increasing nor decreasing in $(0, \pi)$.

Increasing and Decreasing Functions Ex 17.2 Q8

We have,

$f(x) = \log \sin x$

$\therefore f'(x) = \frac{1}{\sin x} \cos x = \cot x$

In interval $\left(0, \frac{\pi}{2}\right)$, $f'(x) = \cot x > 0$.

$\therefore f$ is strictly increasing in $\left(0, \frac{\pi}{2}\right)$.

In interval $\left(\frac{\pi}{2}, \pi\right)$, $f'(x) = \cot x < 0$.

$\therefore f$ is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.

Increasing and Decreasing Functions Ex 17.2 Q9

We have,

$$f(x) = x - \sin x$$

$$\therefore \quad f'(x) = 1 - \cos x$$

Now,

$$x \in R$$

$$\Rightarrow \quad -1 < \cos x < 1$$

$$\Rightarrow \quad -1 > \cos x > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ is increasing for all $x \in R$.

Increasing and Decreasing Functions Ex 17.2 Q10

We have,

$$f(x) = x^3 - 15x^2 + 75x - 50$$

$\therefore \quad f'(x) = 3x^2 - 30x + 75$

$\Rightarrow \quad f'(x) = 3(x^2 - 10x + 25)$

$\qquad = 3(x-5)^2$

Now,

$x \in R$

$\Rightarrow \quad (x-5)^2 > 0$

$\Rightarrow \quad 3(x-5)^2 > 0$

$\Rightarrow \quad f'(x) > 0$

Hence, $f(x)$ is an increasing function for all $x \in R$.

Increasing and Decreasing Functions Ex 17.2 Q11

We have,

$$f(x) = \cos^2 x$$

$\therefore \quad f'(x) = 2\cos x(-\sin x)$

$\Rightarrow \quad f'(x) = -2\sin x \cos x$

$\Rightarrow \quad f'(x) = -\sin 2x$

Now,

$x \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow \quad 2x \in (0, \pi)$

$\Rightarrow \quad \sin 2x > 0$ when $2x \in (0, \pi)$

$\Rightarrow \quad -\sin 2x < 0$

$\Rightarrow \quad f'(x) < 0$

Hence, $f(x)$ is a decreasing function on $\left(0, \frac{\pi}{2}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q12

We have

$f(x) = \sin x$

$f'(x) = \cos x$

Now,

$x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\Rightarrow \cos x > 0$

$\Rightarrow f'(x) > 0$

Therefore, $f(x) = \sin x$ is an increasing function on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q13

We have,

$$f(x) = \cos x$$

$\therefore \quad f'(x) = -\sin x$

Now,

If $x \in (0, \pi)$

$\Rightarrow \quad \sin x > 0$

$\Rightarrow \quad -\sin x < 0$

Hence, $f(x)$ is decreasing function on $(0, \pi)$

If $x \in (-\pi, 0)$

$\Rightarrow \quad \sin x < 0 \qquad [\because \sin(-\theta) = -\sin\theta]$

$\Rightarrow \quad -\sin x > 0$

Hence, $f(x)$ is increasing function on $(-\pi, 0)$

If $x \in (-\pi, \pi)$

Thus, $\sin x > 0$ for $x \in (0, \pi)$

and $\quad \sin x < 0$ for $x \in (-\pi, 0)$

$\Rightarrow \quad -\sin x < 0$ for $x \in (0, \pi)$

and $\quad -\sin x > 0$ for $x \in (-\pi, 0)$

Hence, $f(x)$ is neither increasing nor decreasing on $(-\pi, \pi)$.

Increasing and Decreasing Functions Ex 17.2 Q14

We have,

$$f(x) = \tan x$$

$\therefore \quad f'(x) = \sec^2 x$

Now,

$$x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$$

$\Rightarrow \quad \sec^2 x > 0$

$\Rightarrow \quad f'(x) > 0$

Hence, $f(x)$ is increasing function on $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q15

We have,

$$f(x) = \tan^{-1}(\sin x + \cos x)$$

$$\therefore \quad f'(x) = \frac{1}{1 + (\sin x + \cos x)^2} \times (\cos x - \sin x)$$

$$= \frac{\cos x - \sin x}{1 + \sin^2 x + \cos^2 x + 2\sin x \cos x}$$

$$= \frac{\cos x - \sin x}{2(1 + \sin x \cos x)}$$

Now,

$$x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$

$\Rightarrow \quad \cos x - \sin x < 0$

$\Rightarrow \quad \dfrac{\cos x - \sin x}{2(1 + \sin x \cos x)} < 0 \qquad [\because 2(1 + \sin x \cos x) > 0]$

$\Rightarrow \quad f'(x) < 0$

Hence, $f(x)$ is decreasing function on $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q16

We have,

$$f(x) = \sin\left(2x + \frac{\pi}{4}\right)$$

$$\therefore \quad f'(x) = \cos\left(2x + \frac{\pi}{4}\right) \times 2$$

$$\therefore \quad f'(x) = 2\cos\left(2x + \frac{\pi}{4}\right)$$

Now,

$$x \in \left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$$

$$\Rightarrow \quad \frac{3\pi}{8} < x < \frac{5\pi}{8}$$

$$\Rightarrow \quad \frac{3\pi}{4} < 2x < \frac{5\pi}{4}$$

$$\Rightarrow \quad \pi < 2x < \frac{\pi}{4} < \frac{3\pi}{2}$$

$$\Rightarrow \quad 2x + \frac{\pi}{4} \text{ lies in IIIrd quadrant}$$

$$\Rightarrow \quad \cos\left(2x + \frac{\pi}{4}\right) < 0$$

$$\Rightarrow \quad 2\cos\left(2x + \frac{\pi}{4}\right) < 0$$

$$\Rightarrow \quad f'(x) < 0$$

Hence, $f(x)$ is decreasing on $\left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q17

We have,

$$f(x) = \tan x - 4x$$

$$\therefore \quad f'(x) = \sec^2 x - 4$$

$$= \frac{1 - 4\cos^2 x}{\cos^2 x}$$

$$= \frac{(1 + 2\cos x)(1 - 2\cos x)}{\cos^2 x}$$

$$= 4\sec^2 x\left(\frac{1}{2} + \cos x\right)\left(\frac{1}{2} - \cos x\right)$$

Now,

$$x \in \left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$$

$$\Rightarrow \quad -\frac{\pi}{3} < x < \frac{\pi}{3}$$

$$\Rightarrow \quad \cos x > \frac{1}{2}$$

$$\Rightarrow \quad \left(\frac{1}{2} - \cos x\right) < 0$$

$$\Rightarrow \quad 4\sec^2 x\left(\frac{1}{2} + \cos x\right)\left(\frac{1}{2} - \cos x\right) < 0$$

$$\Rightarrow \quad f'(x) < 0$$

Hence, $f(x)$ is decreasing function on $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q18

We have,

$$f(x) = (x - 1)e^x + 1$$

$$\therefore \quad f'(x) = e^x + (x - 1)e^x$$

$$\Rightarrow \quad f'(x) = e^x(1 + x - 1) = xe^x$$

Now,

$$x > 0$$

$$\Rightarrow \quad e^x > 0$$

$$\Rightarrow \quad xe^x > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ is an increasing function for all $x > 0$.

Increasing and Decreasing Functions Ex 17.2 Q19

We have,

$$f(x) = x^2 - x + 1$$

$$\therefore \quad f'x = 2x - 1$$

Now,

$$x \in (0,1)$$

$$\Rightarrow \quad 2x - 1 > 0 \text{ if } x > \frac{1}{2}$$

$$\text{and} \quad 2x - 1 < 0 \text{ if } x < \frac{1}{2}$$

$$\Rightarrow \quad f'(x) > 0 \text{ if } x > \frac{1}{2}$$

$$\text{and} \quad f'(x) < 0 \text{ if } x < \frac{1}{2}$$

Thus, $f(x)$ is neither increasing nor decreasing on $(0,1)$.

Increasing and Decreasing Functions Ex 17.2 Q20

We have,

$$f(x) = x^9 + 4x^7 + 11$$

$$f'(x) = 9x^8 + 28x^6$$

$$= x^6\left(9x^2 + 28\right)$$

Now,

$$x \in R$$

$$\Rightarrow \quad x^6 > 0 \text{ and } 9x^2 + 28 > 0$$

$$\Rightarrow \quad x^6\left(9x^2 + 28\right) > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Thus, $f(x)$ is an increasing function for $x \in R$.

Increasing and Decreasing Functions Ex 17.2 Q21

We have,

$$f(x) = x^3 - 6x^2 + 12x - 18$$

$$\therefore \quad f'(x) = 3x^2 - 12x + 12$$

$$= 3\left(x^2 - 4x + 4\right)$$

$$= 3(x-2)^2$$

Now,

$$x \in R$$

$$\Rightarrow \quad (x-2)^2 > 0$$

$$\Rightarrow \quad 3(x-2)^2 > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Thus, $f(x)$ is on increasing function for $x \in R$.

Increasing and Decreasing Functions Ex 17.2 Q22

A function $f(x)$ is said to be increasing on $[a,b]$ if $f(x) > 0$

Now, we have,

$$f(x) = x^2 - 6x + 3$$

$$\therefore \quad f'(x) = 2x - 6$$

$$= 2(x-3)$$

Again,

$$x \in [4,6]$$

$$\Rightarrow \quad 4 \le x \le 6$$

$$\Rightarrow \quad 1 \le x - 3 \le 3$$

$$\Rightarrow \quad (x-3) > 0$$

$$\Rightarrow \quad 2(x-3) > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ is an increasing function for $x \in [4,6]$.

Increasing and Decreasing Functions Ex 17.2 Q23

We have,

$$f(x) = \sin x - \cos x$$

$$\therefore \quad f'(x) = \cos x + \sin x$$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x\right)$$

$$= \sqrt{2}\left(\frac{\sin \pi}{4}\cos x + \frac{\cos \pi}{4}\sin x\right)$$

$$= \sqrt{2}\ \sin\left(\frac{\pi}{4} + x\right)$$

Now,

$$x \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$$

$$\Rightarrow \quad -\frac{\pi}{4} < x < \frac{\pi}{4}$$

$$\Rightarrow \quad 0 < \frac{\pi}{4} + x < \frac{\pi}{2}$$

$$\Rightarrow \quad \sin 0^\circ < \sin\left(\frac{\pi}{4} + x\right) < \sin\frac{\pi}{2}$$

$$\Rightarrow \quad 0 < \sin\left(\frac{\pi}{4} + x\right) < 1$$

$$\Rightarrow \quad \sqrt{2}\sin\left(\frac{\pi}{4} + x\right) > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ is an increasing function on $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q24

We have,

$$f(x) = \tan^{-1} x - x$$

$$\therefore \quad f'(x) = \frac{1}{1 + x^2} - 1$$

$$= \frac{-x^2}{1 + x^2}$$

Now,

$$x \in R$$

$$\Rightarrow \quad x^2 > 0 \text{ and } 1 + x^2 > 0$$

$$\Rightarrow \quad \frac{x^2}{1 + x^2} > 0$$

$$\Rightarrow \quad \frac{-x^2}{1 + x^2} < 0$$

$$\Rightarrow \quad f'(x) < 0$$

Hence, $f(x)$ is a decreasing function for $x \in R$.

Increasing and Decreasing Functions Ex 17.2 Q25

We have,

$$f(x) = -\frac{x}{2} + \sin x$$

$$\therefore \quad f'(x) = -\frac{1}{2} + \cos x$$

Now,

$$x \in \left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$$

$$\Rightarrow \quad -\frac{\pi}{3} < x < \frac{\pi}{3}$$

$$\Rightarrow \quad \cos\left(-\frac{\pi}{3}\right) < \cos x < \cos\frac{\pi}{3}$$

$$\Rightarrow \quad \cos\frac{\pi}{3} < \cos x < \cos\frac{\pi}{3}$$

$$\Rightarrow \quad \frac{1}{2} < \cos x < \frac{1}{2}$$

$$\Rightarrow \quad -\frac{1}{2} + \cos x + 0$$

$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ is an increasing function on $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q26

We have,

$$f(x) = \log(1+x) - \frac{x}{1+x}$$

$$\therefore \quad f'(x) = \frac{1}{1+x} - \left(\frac{(1+x) - x}{(1+x)^2}\right)$$

$$= \frac{1}{1+x} - \frac{1}{(1+x)^2}$$

$$= \frac{x}{(1+x)^2}$$

Critical points

$$f'(x) = 0$$

$$\Rightarrow \quad \frac{x}{(1+x)^2} = 0$$

$$\Rightarrow \quad x = 0, \; -1$$

Clearly, $f'(x) > 0$ if $x > 0$

and $\quad f'(x) < 0$ if $-1 < x < 0$ or $x < -1$

Hence, $f(x)$ increases in $(0, \infty)$, decreases in $(-\infty, -1) \cup (-1, 0)$.

Increasing and Decreasing Functions Ex 17.2 Q27

We have,

$$f(x) = (x+2)e^{-x}$$

$$\therefore \quad f'(x) = e^{-x} - e^{-x}(x+2)$$

$$= e^{-x}(1 - x - 2)$$

$$= -e^{-x}(x+1)$$

Critical points

$$f'(x) = 0$$

$$\Rightarrow \quad -e^{-x}(x+1) = 0$$

$$\Rightarrow \quad x = -1$$

Clearly, $f'(x) > 0$ if $x < -1$

$f'(x) < 0$ if $x > -1$

Hence, $f(x)$ increases in $(-\infty, -1)$, decreases in $(-1, \infty)$

.

Increasing and Decreasing Functions Ex 17.2 Q28

We have,

$$f(x) = 10^x$$

$$\therefore \quad f'(x) = 10^x \times \log 10$$

Now,

$$x \in R$$

$$\Rightarrow \quad 10^x > 0$$

$$\Rightarrow \quad 10^x \log 10 > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ in an increasing function for all x.

Increasing and Decreasing Functions Ex 17.2 Q29

We have,

$$f(x) = x - [x]$$

$$\therefore \quad f'(x) = 1 > 0$$

$\therefore$ $f(x)$ in an increasing function on $(0, 1)$.

Increasing and Decreasing Functions Ex 17.2 Q30

We have,

$$f(x) = 3x^5 + 40x^3 + 240x$$

$$\therefore \quad f'(x) = 15x^4 + 120x^2 + 240$$

$$= 15\left(x^4 + 8x^2 + 16\right)$$

$$= 15\left(x^2 + 4\right)^2$$

Now,

$$x \in R$$

$$\Rightarrow \quad \left(x^2 + 4\right)^2 > 0$$

$$\Rightarrow \quad 15\left(x^2 + 4\right)^2 > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ is an increasing function for all x.

Increasing and Decreasing Functions Ex 17.2 Q31

We have,

$$f(x) = \log \cos x$$

$$\therefore f'(x) = \frac{1}{\cos x}(-\sin x) = -\tan x$$

In interval $\left(0, \frac{\pi}{2}\right)$, $\tan x > 0 \Rightarrow -\tan x < 0$.

$$\therefore f'(x) < 0 \text{ on } \left(0, \frac{\pi}{2}\right)$$

$\therefore f$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$.

In interval $\left(\frac{\pi}{2}, \pi\right)$, $\tan x < 0 \Rightarrow -\tan x > 0$.

$$\therefore f'(x) > 0 \text{ on } \left(\frac{\pi}{2}, \pi\right)$$

Increasing and Decreasing Functions Ex 17.2 Q32

Given $f(x) = x^3 - 3x^2 + 4x$

$\therefore \quad f'(x) = 3x^2 - 6x + 4$

$= 3\left(x^2 - 2x + 1\right) + 1$

$= 3(x-1)^2 + 1 > 0, \quad \text{for all } x \in \mathbf{R}$

Hence, f is strictly increasing on $\mathbf{R}$.

Increasing and Decreasing Functions Ex 17.2 Q33

Given $f(x) = \cos x$

$\therefore \quad f'(x) = -\sin x$

(i) Since for each $x \in (0, \pi)$, $\sin x > 0$

$\Rightarrow \quad f'(x) < 0$

So f is strictly decreasing in $(0, \pi)$

(ii) Since for each $x \in (\pi, 2\pi)$, $\sin x < 0$

$\Rightarrow \quad f'(x) > 0$

So f is strictly increasing in $(\pi, 2\pi)$

(iii) Clearly from (i) & (ii) above, f is neither increasing nor decreasing in $(0, 2\pi)$

Increasing and Decreasing Functions Ex 17.2 Q34

We have,

$f(x) = x^2 - x\sin x$

$\therefore \quad f'(x) = 2x - \sin x - x\cos x$

Now,

$x \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow \quad 0 \le \sin x \le 1, \quad 0 \le \cos x \le 1$

$\Rightarrow \quad 2x - \sin x - x\cos x > 0$

$\Rightarrow \quad f'(x) \ge 0$

Hence, $f(x)$ is an increasing function on $\left(0, \frac{\pi}{2}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q35

We have,

$f(x) = x^3 - ax$

$\therefore \quad f'(x) = 3x^2 - a$

Given that $f(x)$ is on increasing function

$\therefore \quad f'(x) > 0 \quad \text{for all } x \in R$

$\Rightarrow \quad 3x^2 - a > 0 \quad \text{for all } x \in R$

$\Rightarrow \quad a < 3x^2 \quad \text{for all } x \in R$

But the last value of $3x^2 = 0$ for $x = 0$

$\therefore \quad a \le 0$

Increasing and Decreasing Functions Ex 17.2 Q36

We have,

$f(x) = \sin x - bx + c$

$\therefore \quad f'(x) = \cos x - b$

Given that $f(x)$ is a decreasing function on R

$\therefore \quad f'(x) < 0 \quad \text{for all } x \in R$

$\Rightarrow \quad \cos x - b < 0 \quad \text{for all } x \in R$

$\Rightarrow \quad b > \cos x \quad \text{for all } x \in R$

But man value of $\cos x$ in 1

$\therefore \quad b \ge 1$

Increasing and Decreasing Functions Ex 17.2 Q37

We have,

$$f(x) = x + \cos x - a$$

$$\therefore \quad f'(x) = 1 - \sin x = \frac{2\cos^2 x}{2}$$

Now,

$$x \in R$$

$$\Rightarrow \quad \frac{\cos^2 x}{2} > 0$$

$$\Rightarrow \quad \frac{2\cos^2 x}{2} > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ is an increasing function for $x \in R$.

Increasing and Decreasing Functions Ex 17.2 Q38

As $f(0) = f(1)$ and f is differentiable, hence by Rolles theorem:
$f'(c) = 0$ for some $c \in [0, 1]$

Let us now apply LMVT (as function is twice differentiable) for point c and $x \in [0,1]$, hence

$$\frac{|f'(x) - f(c)|}{x - c} = f''(d)$$

$$\Rightarrow \frac{|f'(x) - 0|}{x - c} = f''(d)$$

$$\Rightarrow \frac{|f'(x)|}{x - c} = f''(d)$$

As given that $|f''(d)| \le 1$ for $x \in [0,1]$

$$\Rightarrow \frac{|f'(x)|}{x - c} \le 1$$

$$\Rightarrow |f'(x)| \le x - c$$

Now as both x and c lie in $[0, 1]$, hence $x - c \in (0, 1)$

$$\Rightarrow |f'(x)| < 1 \text{ for all } x \in [0,1]$$

Increasing and Decreasing Functions Ex 17.2 Q39(i)

Consider the given function,

$$f(x) = x|x|, \ x \in R$$

$$\Rightarrow f(x) = \begin{cases} -x^2, & x<0 \\ x^2, & x>0 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} -2x, & x<0 \\ 2x, & x>0 \end{cases}$$

$$\Rightarrow f'(x) > 0, \text{ for values of } x$$

Therefore, $f(x)$ is an increasing function for all real values.

Increasing and Decreasing Functions Ex 17.2 Q39(ii)

Consider the function

$f(x) = \sin x + |\sin x|,\ 0 < x \le 2\pi$

$\Rightarrow f(x) = \begin{cases} 2\sin x, & 0 < x < \pi \\ 0, & \pi < x < 2\pi \end{cases}$

$\Rightarrow f'(x) = \begin{cases} 2\cos x, & 0 < x < \pi \\ 0, & \pi < x < 2\pi \end{cases}$

The function 2cosx will be positive between $\left(0, \frac{\pi}{2}\right)$.

Hence the function $f(x)$ is increasing in the interval $\left(0, \frac{\pi}{2}\right)$.

The function 2cosx will be negative between $\left(\frac{\pi}{2}, \pi\right)$.

Hence the function $f(x)$ is decreasing in the interval $\left(\frac{\pi}{2}, \pi\right)$.

The value of $f'(x) = 0$, when $\pi \le x < 2\pi$.

Therefore, the function $f(x)$ is neither increasing nor decreasing in the interval $(\pi, 2\pi)$

Increasing and Decreasing Functions Ex 17.2 Q39(iii)

Consider the function,

$f(x) = \sin x(1 + \cos x),\ 0 < x < \frac{\pi}{2}$

$\Rightarrow f'(x) = \cos x + \sin x(-\sin x) + \cos x(\cos x)$

$\Rightarrow f'(x) = \cos x - \sin^2 x + \cos^2 x$

$\Rightarrow f'(x) = \cos x + (\cos^2 x - 1) + \cos^2 x$

$\Rightarrow f'(x) = \cos x + 2\cos^2 x - 1$

$\Rightarrow f'(x) = 2\cos^2 x + \cos x - 1$

$\Rightarrow f'(x) = 2\cos^2 x + 2\cos x - \cos x - 1$

$\Rightarrow f'(x) = 2\cos x(\cos x + 1) - 1(\cos x + 1)$

$\Rightarrow f'(x) = (2\cos x - 1)(\cos x + 1)$

For f(x) to be increasing, we must have,

$f'(x) > 0$

$\Rightarrow f'(x) = (2\cos x - 1)(\cos x + 1) > 0$

$\Rightarrow 0 < x < \frac{\pi}{3}$

$\Rightarrow x \in \left(0, \frac{\pi}{3}\right)$

So, f(x)is*increa*sing in $\left(0, \frac{\pi}{3}\right)$

For f(x) to be decreasing, we must have,

$f'(x) < 0$

$\Rightarrow f'(x) = (2\cos x - 1)(\cos x + 1) < 0$

$\Rightarrow \frac{\pi}{3} < x < \frac{\pi}{2}$

$\Rightarrow x \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

So, f(x)is decreasing in $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

Ex 18.1

Maxima and Minima 18.1 Q1

$f(x) = 4x^2 - 4x + 4$ on R

$= 4x^2 - 4x + 1 + 3$

$= (2x-1)^2 + 3$

$\because \quad (2x-1)^2 \ge 0$

$\Rightarrow \quad (2x-1)^2 + 3 \ge 3$

$\Rightarrow \quad f(x) \ge f\left(\frac{1}{2}\right)$

Thus, the minimum value of $f(x)$ is 3 at $x = \frac{1}{2}$

Since, $f(x)$ can be made as large as we please. Therefore maximum value does not exist

Maxima and Minima 18.1 Q2

The given function is $f(x) = -(x-1)^2 + 2$

It can be observed that $(x-1)^2 \ge 0$ for every $x \in \mathbf{R}$.

Therefore, $f(x) = -(x-1)^2 + 2 \le 2$ for every $x \in \mathbf{R}$.

The maximum value of f is attained when $(x-1) = 0$.

$(x-1) = 0 \Rightarrow x = 1$

$\therefore$Maximum value of $f = f(1) = -(1-1)^2 + 2 = 2$

Hence, function f does not have a minimum value.

Maxima and Minima 18.1 Q3

$f(x) = |x+2|$ on R

$\because \quad |x+2| \geq 0$ for $x \in R$

$\Rightarrow \quad f(x) \geq 0$ for all $x \in R$

So, the minimum value of $f(x)$ is 0, which attains at $x = -2$

Clearly, $f(x) = |x+2|$ does not have the maximum value.

Maxima and Minima 18.1 Q4

$$h(x) = \sin 2x + 5$$

We know that $-1 \leq \sin 2x \leq 1$.

$\Rightarrow -1 + 5 \leq \sin 2x + 5 \leq 1 + 5$

$\Rightarrow 4 \leq \sin 2x + 5 \leq 6$

Hence, the maximum and minimum values of h are 6 and 4 respectively.

Maxima and Minima 18.1 Q5

$$f(x) = |\sin 4x + 3|$$

We know that $-1 \leq \sin 4x \leq 1$.

$\Rightarrow 2 \leq \sin 4x + 3 \leq 4$

$\Rightarrow 2 \leq |\sin 4x + 3| \leq 4$

Hence, the maximum and minimum values of f are 4 and 2 respectively.

Maxima and Minima 18.1 Q6

$f(x) = 2x^3 + 5$ on R

Here, we observe that the values of $f(x)$ increase when the values of x are increased and $f(x)$ can be made as large as possible, we please.

So, $f(x)$ does not have the maximum value.

Similarly $f(x)$ can be made as small as we please by giving smaller values to x.

So, $f(x)$ does not have the minimum value.

Maxima and Minima 18.1 Q7

$$g(x) = -|x+1| + 3$$

We know that $-|x+1| \leq 0$ for every $x \in \mathbf{R}$.

Therefore, $g(x) = -|x+1| + 3 \leq 3$ for every $x \in \mathbf{R}$.

The maximum value of g is attained when $|x+1| = 0$

$|x+1| = 0$

$\Rightarrow x = -1$

$\therefore$ Maximum value of $g = g(-1) = -|-1+1| + 3 = 3$

Hence, function g does not have a minimum value.

Maxima and Minima 18.1 Q8

$$f(x) = 16x^2 - 16x + 28 \text{ on } R$$
$$= 16x^2 - 16x + 4 + 24$$
$$= (4x - 2)^2 + 24$$

Now,

$$(4x - 2)^2 \geq 0 \text{ for all } x \in R$$
$$\Rightarrow \quad (4x - 2)^2 + 24 \geq 24 \text{ for all } x \in R$$
$$\Rightarrow \quad f(x) \geq f\left(\frac{1}{2}\right)$$

Thus, the minimum value of $f(x)$ is 24 at $x = \frac{1}{2}$

Since $f(x)$ can be made as large as possible by giving difference values to x.

Thus, maximum values does not exist.

Maxima and Minima 18.1 Q9

$f(x) = x^3 - 1$ on R

Here, we observe that the values of $f(x)$ increases when the values of x are increased and $f(x)$ can be made as large as we please by giving large values to x.

So, $f(x)$ does not have the maximum value.

Similarly, $f(x)$ can be made as small as we please by giving smaller values to x.

So, $f(x)$ does not have the minimum value.

Ex 18.2

Maxima and Minima Ex 18.2 Q1

$f(x) = (x-5)^4$

$\therefore \quad f'(x) = 4(x-5)^3$

For local maxima and minima

$f'(x) = 0$

$\Rightarrow \quad 4(x-5)^3 = 0$

$\Rightarrow \quad x - 5 = 0$

$\Rightarrow \quad x = 5$

$f'(x)$ changes from $-$ve to $+$ve as passes through 5.

So, $x = 5$ is the point of local minima

Thus, local minimum value is $f(5) = 0$

Maxima and Minima Ex 18.2 Q2

$g(x) = x^3 - 3x$

$\therefore g'(x) = 3x^2 - 3$

Now,

$g'(x) = 0 \Rightarrow 3x^2 = 3 \Rightarrow x = \pm 1$

$g''(x) = 6x$

$g''(1) = 6 > 0$

$g''(-1) = -6 < 0$

By second derivative test, $x = 1$ is a point of local minima and local minimum value of g at $x = 1$ is $g(1) = 1^3 - 3 = 1 - 3 = -2$. However,

$x = -1$ is a point of local maxima and local maximum value of g at

$x = -1$ is $g(1) = (-1)^3 - 3(-1) = -1 + 3 = 2$.

Maxima and Minima Ex 18.2 Q3

$f(x) = x^3 (x-1)^2$

$$\begin{aligned}
\therefore \quad f'(x) &= 3x^2 (x-1)^2 + 2x^3 (x-1) \\
&= (x-1)\left(3x^2 (x-1) + 2x^3\right) \\
&= (x-1)\left(3x^3 - 3x^2 + 2x^3\right) \\
&= (x-1)\left(5x^3 - 3x^2\right) \\
&= x^2 (x-1)(5x-3)
\end{aligned}$$

For all maxima and minima,

$f'(x) = 0$

$\Rightarrow \quad x^2 (x-1)(5x-3) = 0$

$\Rightarrow \quad x = 0, 1, \frac{3}{5}$

At $x = \frac{3}{5}$ $f'(x)$ changes from $+$ve to $-$ve

$\therefore \quad x = \frac{3}{5}$ is point of minima.

At $x = 1$ $f'(x)$ changes from $-$ve to $+$ve

$\therefore \quad x = 1$ is point of maxima

Maxima and Minima Ex 18.2 Q4

$f(x) = (x-1)(x+2)^2$

$$\begin{aligned}
\therefore \quad f'(x) &= (x+2)^2 + 2(x-1)(x+2) \\
&= (x+2)(x+2+2x-2) \\
&= (x+2)(3x)
\end{aligned}$$

For point of maxima and minima

$f'(x) = 0$

$\Rightarrow \quad (x+2) \times 3x = 0$

$\Rightarrow \quad x = 0, -2$

At $x = -2$ $f'(x)$ changes from $+$ve to $-$ve

$\therefore \quad x = -2$ is point of local maxima

At $x = 0$ $f'(x)$ changes from $-$ve to $+$ve

$\therefore \quad x = 0$ is point of local minima

Thus, local min value $= f(0) = -4$

local max value $= f(-2) = 0$.

Maxima and Minima Ex 18.2 Q5

$f(x) = (x-1)^3(x+1)^2$

$\therefore \quad f'(x) = 3(x-1)^2(x+1)^2 + 2(x-1)^3(x+1)$

$= (x-1)^2(x+1)\{3(x+1) + 2(x-1)\}$

$= (x-1)^2(x+1)(5x+1)$

For the point of local maxima and minima,

$f'(x) = 0$

$\Rightarrow \quad (x-1)^2(x+1)(5x+1) = 0$

$\Rightarrow \quad x = 1, -1, -\frac{1}{5}$

Here,

At $x = -1$ $f'(x)$ changes from $+$ve to $-$ve so $x = -1$ is point of maxima.

At $x = -\frac{1}{5}$, $f'(x)$ changes from $-$ve to $+$ve so $x = -\frac{1}{5}$ is point of minima

Hence, local max value $= 0$

local min value $= -\frac{3456}{3125}$.

Maxima and Minima Ex 18.2 Q6

$f(x) = x^3 - 6x^2 + 9x + 15$

$\therefore \quad f'(x) = 3x^2 - 12x + 9$

$= 3(x^2 - 4x + 3)$

$= 3(x-3)(x-1)$

For, the point of local maxima and minima,

$f'(x) = 0$

$\Rightarrow \quad 3(x-3)(x-1) = 0$

$\Rightarrow \quad x = 3, 1$

At $\quad x = -1$, $f'(x)$ changes from $+$ve to $-$ve

$\therefore \quad x = 1$ is point of local maxima

At $\quad x = 3$, $f'(x)$ changes from $-$ve to $+$ve

$\therefore \quad x = 3$ is point of local manima

Hence, local max value $= f(1) = 19$

local min value $= f(3) = 15$.

Maxima and Minima Ex 18.2 Q7

$f(x) = \sin 2x, \ 0 < x, \pi$

$\therefore \quad f'(x) = 2\cos 2x$

For, the point of local maxima and minima,

$f'(x) = 0$

$\Rightarrow \quad 2x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\Rightarrow \quad x = \frac{\pi}{4}, \frac{3\pi}{4}$

At $\quad x = \frac{\pi}{4}$, $f'(x)$ changes from $+$ve to $-$ve

$\therefore \quad x = \frac{\pi}{4}$ is point of local maxima

At $\quad x = \frac{3\pi}{4}$, $f'(x)$ changes from $-$ve to $+$ve

$\therefore \quad x = \frac{3\pi}{4}$ is point of local minima,

Hence, local max value $= f\left(\frac{\pi}{4}\right) = 1$

local min value $= f\left(\frac{3\pi}{4}\right) = -1$.

Maxima and Minima Ex 18.2 Q8

$f(x) = \sin x - \cos x,\ 0 < x < 2\pi$

$\therefore f'(x) = \cos x + \sin x$

$f'(x) = 0 \Rightarrow \cos x = -\sin x \Rightarrow \tan x = -1 \Rightarrow x = \frac{3\pi}{4}, \frac{7\pi}{4} \in (0, 2\pi)$

$f''(x) = -\sin x + \cos x$

$f''\left(\frac{3\pi}{4}\right) = -\sin\frac{3\pi}{4} + \cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = -\sqrt{2} < 0$

$f''\left(\frac{7\pi}{4}\right) = -\sin\frac{7\pi}{4} + \cos\frac{7\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2} > 0$

Therefore, by second derivative test, $x = \frac{3\pi}{4}$ is a point of local maxima and the local maximum value of f at $x = \frac{3\pi}{4}$ is

$f\left(\frac{3\pi}{4}\right) = \sin\frac{3\pi}{4} - \cos\frac{3\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$. However, $x = \frac{7\pi}{4}$ is a point of local minima and the local minimum value of f at $x = \frac{7\pi}{4}$ is $f\left(\frac{7\pi}{4}\right) = \sin\frac{7\pi}{4} - \cos\frac{7\pi}{4} = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = -\sqrt{2}$.

Maxima and Minima Ex 18.2 Q9

$f(x) = \cos x,\ 0 < x < \pi$

$\therefore \quad f'(x) = -\sin x$

For, the point of local maxima and minima,

$f'(x) = 0$

$\Rightarrow \quad -\sin x = 0$

$\Rightarrow \quad x = 0$, and π

But, these two points lies outside the interval $(0, \pi)$

So, no local maxima and minima will exist in the interval $(0, \pi)$.

Maxima and Minima Ex 18.2 Q10

$\therefore \quad f'(x) = 2\cos 2x - 1$

For, the point of local maxima and minima,

$f'(x) = 0$

$\Rightarrow \quad 2\cos 2x - 1 = 0$

$\Rightarrow \quad \cos 2x = \frac{1}{2} = \cos\frac{\pi}{3}$

$\Rightarrow \quad 2x = \frac{\pi}{3}, -\frac{\pi}{3}$

$\Rightarrow \quad x = \frac{\pi}{6}, -\frac{\pi}{6}$

At $\quad x = -\frac{\pi}{6}$, $f'(x)$ changes from $-$ve to $+$ ve

$\therefore \quad x = -\frac{\pi}{6}$ is point of local manima

At $\quad x = \frac{\pi}{6}$, $f'(x)$ changes from $+$ve to $-$ve

$\therefore \quad x = \frac{\pi}{6}$ is point of local maxima

Hence, local max value $= f\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} - \frac{\pi}{6}$

local min value $= f\left(-\frac{\pi}{6}\right) = \frac{-\sqrt{3}}{2} + \frac{\pi}{6}$.

Maxima and Minima Ex 18.2 Q11

$f(x) = 2\sin x - x, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

For checking the minima and maxima, we have
$f'(x) = 2\cos x - 1 = 0$

$\Rightarrow \cos x = \frac{1}{2} = \cos\frac{\pi}{3}$

$\Rightarrow x = -\frac{\pi}{3}, \frac{\pi}{3}$

At $x = -\frac{\pi}{3}$, $f(x)$ *changes from* $-ve$ *to* $+ve$

$\Rightarrow x = -\frac{\pi}{3}$ *is point of local minima with value* $= -\sqrt{3} - \frac{\pi}{3}$

At $x = \frac{\pi}{3}$, $f(x)$ *changes from* $+ve$ *to* $+ve$

$\Rightarrow x = \frac{\pi}{3}$ *is point of local maxima with value* $= \sqrt{3} - \frac{\pi}{3}$

Maxima and Minima Ex 18.2 Q12

$$\therefore f'(x) = \sqrt{1-x} + x \cdot \frac{1}{2\sqrt{1-x}}(-1) = \sqrt{1-x} - \frac{x}{2\sqrt{1-x}}$$

$$= \frac{2(1-x)-x}{2\sqrt{1-x}} = \frac{2-3x}{2\sqrt{1-x}}$$

$$f'(x) = 0 \Rightarrow \frac{2-3x}{2\sqrt{1-x}} = 0 \Rightarrow 2-3x = 0 \Rightarrow x = \frac{2}{3}$$

$$f''(x) = \frac{1}{2}\left[\frac{\sqrt{1-x}(-3)-(2-3x)\left(\frac{-1}{2\sqrt{1-x}}\right)}{1-x}\right]$$

$$= \frac{\sqrt{1-x}(-3)+(2-3x)\left(\frac{1}{2\sqrt{1-x}}\right)}{2(1-x)}$$

$$= \frac{-6(1-x)+(2-3x)}{4(1-x)^{\frac{3}{2}}}$$

$$= \frac{3x-4}{4(1-x)^{\frac{3}{2}}}$$

$$f''\left(\frac{2}{3}\right) = \frac{3\left(\frac{2}{3}\right)-4}{4\left(1-\frac{2}{3}\right)^{\frac{3}{2}}} = \frac{2-4}{4\left(\frac{1}{3}\right)^{\frac{3}{2}}} = \frac{-1}{2\left(\frac{1}{3}\right)^{\frac{3}{2}}} < 0$$

Therefore, by second derivative test, $x = \frac{2}{3}$ is a point of local maxima and the local maximum value of f at $x = \frac{2}{3}$ is

$$f\left(\frac{2}{3}\right) = \frac{2}{3}\sqrt{1-\frac{2}{3}} = \frac{2}{3}\sqrt{\frac{1}{3}} = \frac{2}{3\sqrt{3}} = \frac{2\sqrt{3}}{9}.$$

Maxima and Minima Ex 18.2 Q13

We have,

$$f(x) = x^3(2x-1)^3$$

$$\therefore \quad f'(x) = 3x^2(2x-1)^3 + 3x^3(2x-1)^2 \times 2$$

$$= 3x^2(2x-1)^2(2x-1+2x)$$

$$= 3x^2(4x-1)$$

For, the point of local maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \quad 3x^2(4x-1) = 0$$

$$\Rightarrow \quad x = 0, \frac{1}{4}$$

At $\quad x = \frac{1}{4}$, $f'(x)$ changes from $-$ ve to $+$ ve

$\therefore \quad x = \frac{1}{4}$ is the point of local minima,

$\therefore \quad$ local min value $= f\left(\frac{1}{4}\right) = \frac{-1}{512}$.

Maxima and Minima Ex 18.2 Q14

We have,

$$f(x) = \frac{x}{2} + \frac{2}{x}, \ x > 0$$

$$\therefore \quad f'(x) = \frac{1}{2} - \frac{2}{x^2}$$

For the point of local maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \quad \frac{1}{2} - \frac{2}{x^2} = 0$$

$$\Rightarrow \quad x^2 - 4 = 0$$

$$\Rightarrow \quad x = \sqrt{4}, -\sqrt{4}$$

$$\Rightarrow \quad x = 2, -2$$

At $\quad x = 2$, $f'(x)$ changes from $-$ ve to $+$ ve

$\therefore \quad x = 2$ is point of local minima.

$\therefore \quad$ local min value $= f(2) = 2$.

Maxima and Minima Ex 18.2 Q15

$$g(x) = \frac{1}{x^2+2}$$

$$\therefore g'(x) = \frac{-(2x)}{(x^2+2)^2}$$

$$g'(x) = 0 \Rightarrow \frac{-2x}{(x^2+2)^2} = 0 \Rightarrow x = 0$$

Now, for values close to $x = 0$ and to the left of 0, $g'(x) > 0$. Also, for values close to $x = 0$ and to the right of 0, $g'(x) < 0$.

Therefore, by first derivative test, $x = 0$ is a point of local maxima and the local maximum value of $g(0)$ is $\frac{1}{0+2} = \frac{1}{2}$..

Ex 18.3

Maxima and Minima 18.3 Q1(i)

$f(x) = x^4 - 62x^2 + 120x + 9$

$\therefore \quad f'(x) = 4x^3 - 124x + 120 = 4(x^3 - 31x + 30)$

$\quad f''(x) = 12x^2 - 124 = 4(3x^2 - 31)$

For maxima and minima,

$\quad f'(x) = 0$

$\Rightarrow \quad 4(x^3 - 31x + 30) = 0$

$\Rightarrow \quad 4(x^3 - 31x + 30) = 0$

$\Rightarrow \quad x = 5, 1, -6$

Now,

$\quad f''(5) = 176 > 0$

$\Rightarrow \quad x = 5$ is point of local minima

$\quad f''(1) = -112 < 0$

$\Rightarrow \quad x = 1$ is point of local maxima

$\quad f''(-6) = 308 > 0$

$\Rightarrow \quad x = -6$ is point of local minima

$\therefore \quad$ local max value $= f(1) = 68$

local min value $= f(5) = -316$

and $= f(-6) = -1647$.

Maxima and Minima 18.3 Q1(ii)

We have,

$$f(x) = x^3 - 6x^2 + 9x + 15$$

$\therefore \quad f'(x) = 3x^2 - 12x + 9$

$$= 3\left(x^2 - 4x + 3\right)$$

$$f''(x) = 6x - 12$$

$$= 6(x - 2)$$

For maxima and minima,

$$f'(x) = 0$$

$\Rightarrow \quad 3\left(x^2 - 4x + 3\right) = 0$

$\Rightarrow \quad 3(x - 3)(x - 1) = 0$

$\Rightarrow \quad x = 3, 1$

Now,

$$f''(3) = 6 > 0$$

$\therefore \quad x = 3$ is point of local minima

$$f''(1) = -6 < 0$$

$\therefore \quad x = 1$ is point of local maxima

$\therefore \quad$ local max value $= f(1) = 19$

local min value $= f(3) = 15.$

Maxima and Minima 18.3 Q1(iii)

We have,

$$f(x) = (x - 1)(x + 2)^2$$

$\therefore \quad f'(x) = (x + 2)^2 + 2(x - 1)(x + 2)$

$$= (x + 2)(x + 2 + 2x - 2)$$

$$= (x + 2)(3x)$$

and, $\quad f''(x) = 3(x + 2) + 3x$

$$= 6x + 6$$

For maxima and minima,

$$f'(x) = 0$$

$\Rightarrow \quad 3x(x + 2) = 0$

$\Rightarrow \quad x = 0, -2$

Now,

$$f''(0) = 6 > 0$$

$\therefore \quad x = 0$ is point of local minima

$$f''(-2) = -6 < 0$$

$\therefore \quad x = -2$ is point of local maxima

$\therefore \quad$ local max value $= f(-2) = 0$

local min value $= f(0) = -4.$

Maxima and Minima 18.3 Q1(iv)

We have,

$$f(x) = \frac{2}{x} - \frac{2}{x^2},\ x > 0$$

$\therefore \quad f'(x) = \frac{-2}{x^2} + \frac{4}{x^3}$

and, $\quad f''(x) = \frac{+4}{x^3} - \frac{12}{x^4}$

For maxima and minima,

$$f'(x) = 0$$

$\Rightarrow \quad \frac{-2}{x^2} + \frac{4}{x^3} = 0$

$\Rightarrow \quad \frac{-2(x-2)}{x^3} = 0$

$\Rightarrow \quad x = 2$

Now,

$$f''(2) = \frac{4}{8} - \frac{12}{6} = \frac{1}{2} - \frac{3}{4} = \frac{-1}{4} < 0$$

$\therefore \quad x = 2$ is point of local maxima

local max value $= f(2) = \frac{1}{2}$.

Maxima and Minima 18.3 Q1(v)

We have,

$$f(x) = xe^x$$

$\therefore \quad f'(x) = e^x + xe^x = e^x(x+1)$

$$f''(x) = e^x(x+1) + e^x$$

$$= e^x(x+2)$$

For maxima and minima,

$$f'(x) = 0$$

$\Rightarrow \quad e^x(x+1) = 0$

$\Rightarrow \quad x = -1$

Now,

$$f''(-1) = e^{-1} = \frac{1}{e} > 0$$

$\therefore \quad x = -1$ is point of local minima

Hence,

local min value $= f(-1) = \frac{-1}{e}$.

Maxima and Minima 18.3 Q1(vi)

We have,

$$f(x) = \frac{x}{2} + \frac{2}{x},\ x > 0$$

$\therefore \quad f'(x) = \frac{1}{2} - \frac{2}{x^2}$

and, $\quad f''(x) = \frac{4}{x^3}$

For maxima and minima,

$$f'(x) = 0$$

$\Rightarrow \quad \frac{1}{2} - \frac{2}{x^2} = 0$

$\Rightarrow \quad \frac{x^2 - 4}{2x^2} = 0$

$\Rightarrow \quad x = 2, -2$

Now,

$$f''(2) = \frac{1}{2} > 0$$

$\therefore \quad x = 2$ is point of minima

We will not consider $x = -2$ as $x > 0$

$\therefore \quad$ local min value $= f(2) = 2$.

Maxima and Minima 18.3 Q1(vii)

We have,

$$f(x) = (x+1)(x+2)^{\frac{1}{3}},\ x \ge -2$$

$$\therefore \quad f'(x) = (x+2)^{\frac{1}{3}} + \frac{1}{3}(x+1)(x+2)^{\frac{-2}{3}}$$

$$= (x+2)^{\frac{-2}{3}}\left(x+2+\frac{1}{3}(x+1)\right)$$

$$= \frac{1}{3}(x+2)^{\frac{-2}{3}}(4x+7)$$

and, $$f''(x) = -\frac{2}{9}(x+2)^{\frac{-5}{3}}(4x+7) + \frac{1}{3}(x+2)^{\frac{-2}{3}} \times 4$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \quad \frac{1}{3}(x+2)^{\frac{-2}{3}}(4x+7) = 0$$

$$\Rightarrow \quad x = -\frac{7}{4}$$

Now,

$$f''\left(-\frac{7}{4}\right) = \frac{4}{3}\left(-\frac{7}{4}+2\right)^{\frac{-2}{3}}$$

$$\therefore \quad x = \frac{-7}{4} \text{ is point of minima}$$

$$\therefore \quad \text{local min value} = f\left(\frac{-7}{4}\right) = \frac{-3}{4^{\frac{4}{3}}}.$$

Maxima and Minima 18.3 Q1(viii)

We have,

$$f(x) = x\sqrt{32-x^2}, -5 \le x \le 5$$

$$\therefore \quad f'(x) = \sqrt{32-x^2} + \frac{x}{2\sqrt{32-x^2}} \times (-2x)$$

$$= \frac{2\left(32-x^2\right)-2x^2}{2\sqrt{32-x^2}}$$

$$= \frac{64-4x^2}{2\sqrt{32-x^2}}$$

and, $$f''(x) = \frac{2\sqrt{32-x^2} \times (-8x)\dfrac{-2\left(64-4x^2\right)}{2\sqrt{32-x^2}} \times (-2x)}{4\left(32-x^2\right)}$$

$$= \frac{-4\left(32-x^2\right) \times 8x + 4x\left(64-x^2\right)}{8\left(32-x^2\right)^{\frac{3}{2}}}$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \quad \frac{4\left(16-x^2\right)}{2\sqrt{32-x^2}} = 0$$

$$\Rightarrow \quad x = \pm 4$$

Now,

$$f''(4) = \frac{4 \times 4(64-16-8\times 32+8\times 16)}{8(32-16)^{\frac{3}{2}}} < 0$$

$$\therefore \quad x = 4 \text{ is point of maxima}$$

Maxima and Minima 18.3 Q1(ix)

$Local\ Maximum\ value\ = f(4)$

$$= 4\sqrt{32-4^2}$$
$$= 4\sqrt{32-16}$$
$$= 4\sqrt{16}$$
$$= 16$$

$Local\ minimum\ at\ x = -4;$

$Local\ Minimum\ value\ = f(-4)$

$$= -4\sqrt{32-(-4)^2}$$
$$= -4\sqrt{32-16}$$
$$= -4\sqrt{16}$$
$$= -16$$

Maxima and Minima 18.3 Q1(x)

$$f(x) = x + \frac{a^2}{x}$$

$\therefore \quad f'(x) = 1 - \frac{a^2}{x^2}$

$$f''(x) = \frac{2a^2}{x^3}$$

For maxima and minima,

$$f'(x) = 0$$

$\Rightarrow \quad 1 - \frac{a^2}{x^2} = 0$

$\Rightarrow \quad x^2 - a^2 = 0$

$\Rightarrow \quad x = \pm a$

Now,

$$f''(a) = \frac{2}{a} > 0 \text{ as } a > 0$$

$\therefore \quad x = a$ is point of minima

$$f''(-a) = \frac{-2}{a} < 0 \text{ as } a > 0$$

$\therefore \quad x = -a$ is point of maxima

Hence,

local max value = $f(-a) = -2a$

local min value = $f(a) = 2a$.

Maxima and Minima 18.3 Q1(xi)

$$f(x) = x\sqrt{2-x^2}$$

$$\therefore \quad f'(x) = \sqrt{2-x^2} - \frac{2x^2}{2\sqrt{2-x^2}}$$

$$= \frac{2\left(2-x^2\right) - 2x^2}{2\sqrt{2-x^2}}$$

$$= \frac{2-2x^2}{\sqrt{2-x^2}}$$

$$f''(x) = \frac{\sqrt{2-x^2}\,(-4x) + \dfrac{\left(2-2x^2\right)2x}{\sqrt{2-x^2}}}{\left(\sqrt{2-x^2}\right)^2}$$

$$= \frac{-\left(2-x^2\right)4x + 4x - 4x^3}{\left(2-x^2\right)^{\frac{3}{2}}}$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \quad \frac{2\left(1-x^2\right)}{\sqrt{2-x^2}} = 0$$

$$\Rightarrow \quad x = \pm 1$$

Now,

$$f''(1) < 0$$

$\Rightarrow$ $x = 1$ is point of local maxima

$$f''(-1) > 0$$

$\Rightarrow$ $x = -1$ is point of local minima

Hence,

local max value $= f(1) = 1$

local min value $= f(-1) = -1$.

Maxima and Minima 18.3 Q1(xii)

$$f(x) = x + \sqrt{1-x}$$

$$\therefore \quad f'(x) = 1 - \frac{1}{2\sqrt{1-x}} = \frac{2\sqrt{1-x} - 1}{2\sqrt{1-x}}$$

$$\therefore \quad f'(x) = \frac{2\sqrt{1-x}\left(\dfrac{-1}{\sqrt{1-x}}\right) + \dfrac{\left(2\sqrt{1-x} - 1\right)}{\sqrt{1-x}}}{4(1-x)}$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \quad \frac{2\sqrt{1-x} - 1}{2\sqrt{1-x}} = 0$$

$$\Rightarrow \quad \sqrt{1-x} = \frac{1}{2}$$

$$\Rightarrow \quad x = 1 - \frac{1}{4} = \frac{3}{4}$$

Now,

$$f''\left(\frac{3}{4}\right) < 0$$

$\Rightarrow$ $x = \frac{3}{4}$ is point of local maxima

Hence,

local max value $= f\left(\frac{3}{4}\right) = \frac{5}{4}$.

Maxima and Minima 18.3 Q2(i)

$$f(x) = (x-1)(x-2)^2$$

$$\therefore \quad f'(x) = (x-2)^2 + 2(x-1)(x-2)$$

$$= (x-2)(x-2+2x-2)$$

$$= (x-2)(3x-4)$$

$$f''(x) = (3x-4) + 3(x-2)$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \quad (x-2)(3x-4) = 0$$

$$\Rightarrow \quad x = 2, \frac{4}{3}$$

Now,

$$f''(2) > 0$$

$\therefore$ $x = 2$ is local minima

$$f''\left(\frac{4}{3}\right) = -2 < 0$$

$\therefore$ $x = \frac{4}{3}$ is point of local maxima

$\therefore$ local max value $= f\left(\frac{4}{3}\right) = \frac{4}{27}$

local min value $= f(2) = 0$.

Maxima and Minima 18.3 Q2(ii)

$$f(x) = x\sqrt{1-x}$$

$$\therefore \quad f'(x) = \sqrt{1-x} + \frac{x}{2\sqrt{1-x}}(-1)$$

$$= \frac{2(1-x) - x}{2\sqrt{1-x}}$$

$$= \frac{2-3x}{2\sqrt{1-x}}$$

$$f''(x) = \frac{2\sqrt{1-x}(-3) + \frac{(2-3x)}{\sqrt{1-x}}}{4(1-x)}$$

For maximum and minimum,

$$f'(x) = 0$$

$$\Rightarrow \quad \frac{2-3x}{2\sqrt{1-x}} = 0$$

$$\Rightarrow \quad x = \frac{2}{3}$$

Now,

$$f''\left(\frac{2}{3}\right) < 0$$

$\therefore$ $x = \frac{2}{3}$ is point ofmaxima

$\therefore$ local max value $= f\left(\frac{2}{3}\right) = \frac{2}{3\sqrt{3}}$.

Maxima and Minima 18.3 Q2(iii)

$$f(x) = -(x-1)^3(x+1)^2$$

$\therefore \quad f'(x) = -3(x-1)^2(x+1)^2 - 2(x-1)^3(x+1)$

$$= -(x-1)^2(x+1)(3x+3+2x-2)$$

$$= -(x-1)^2(x+1)(5x+1)$$

$\therefore \quad f''(x) = -2(x-1)(x+1)(5x+1) - (x-1)^2(5x+1) - 5(x-1)^2(x+1)$

For maximum and minimum value,

$$f'(x) = 0$$

$\Rightarrow \quad -(x-1)^2(x+1)(5x+1) = 0$

$\Rightarrow \quad x = 1, -1, -\frac{1}{5}$

Now,

$$f''(1) = 0$$

$\therefore \quad x = 1$ is inflection point

$$f''(-1) = -4 \times -4 = 16 > 0$$

$\therefore \quad x = -1$ is point of minima

$$f''\left(\frac{-1}{5}\right) = -5\left(\frac{36}{25}\right) \times \frac{4}{5} = \frac{-144}{25} < 0$$

$\therefore \quad x = \frac{-1}{5}$ is point of maxima

Hence,

local max value $= f\left(-\frac{1}{5}\right) = \frac{3456}{3125}$

local min value $= f(-1) = 0.$

Maxima and Minima 18.3 Q3

We have,

$$y = a\log x + bx^2 + x$$

$\therefore \quad \frac{dy}{dx} = \frac{a}{x} + 2bx + 1$

and $\quad \frac{d^2y}{dx^2} = \frac{-a}{x^2} + 2b$

For maximum and minimum value,

$$\frac{dy}{dx} = 0$$

$\Rightarrow \quad \frac{a}{x} + 2bx + 1 = 0$

Given that extreme value exist at $x = 1, 2$

$\Rightarrow \quad a + 2b = -1 \qquad$ ---(i)

$$\frac{a}{2} + 4b = -1$$

$\Rightarrow \quad a + 8b = -2 \qquad$ ---(ii)

Solving (i) and (ii), we get

$$a = \frac{-2}{3},\ b = \frac{-1}{6}.$$

Maxima and Minima 18.3 Q4

The given function is $f(x)=\dfrac{\log x}{x}$.

$$f'(x)=\frac{x\left(\frac{1}{x}\right)-\log x}{x^2}=\frac{1-\log x}{x^2}$$

Now, $f'(x)=0$

$\Rightarrow 1-\log x=0$

$\Rightarrow \log x=1$
$\Rightarrow \log x=\log e$
$\Rightarrow x=e$

$$\text{Now, } f''(x)=\frac{x^2\left(-\frac{1}{x}\right)-(1-\log x)(2x)}{x^4}$$
$$=\frac{-x-2x(1-\log x)}{x^4}$$
$$=\frac{-3+2\log x}{x^3}$$

$$\text{Now, } f''(e)=\frac{-3+2\log e}{e^3}=\frac{-3+2}{e^3}=\frac{-1}{e^3}<0$$

Therefore, by second derivative test, f is the maximum at $x=e$.

Maxima and Minima 18.3 Q5

$$f(x)=\frac{4}{x+2}+x$$

$$\therefore \quad f'(x)=\frac{-4}{(x+2)^2}+1$$

$$f''(x)=\frac{8}{(x+2)^3}$$

For maximum and minimum value,

$f'(x)=0$

$\Rightarrow \quad \dfrac{-4}{(x+2)^2}+1=0$

$\Rightarrow \quad (x+2)^2=4$

$\Rightarrow \quad x^2+4x=0$

$\Rightarrow \quad x(x+4)=0$

$x=0,-4$

Now,

$f''(0)=1>0$

$\therefore \quad x=0$ is point of minima

$f''(-4)=-1<0$

$\therefore \quad x=-4$ is point of maxima

$\therefore \quad$ local max value $=f(-4)=-6$

local min value $=f(0)=2$.

Maxima and Minima 18.3 Q6

We have,

$$y = \tan x - 2x$$

$\therefore \quad y' = \sec^2 x - 2$

$$y'' = 2\sec^2 x \tan x$$

For maximum and minimum value,

$$y' = 0$$

$\Rightarrow \quad \sec^2 x = 2$

$\Rightarrow \quad \sec x = \pm\sqrt{2}$

$\Rightarrow \quad x = \frac{\pi}{4}, \frac{3\pi}{4}$

$\therefore \quad y''\left(\frac{\pi}{4}\right) = 4 > 0$

$\therefore \quad x = \frac{\pi}{4}$ is point of minima

$$y''\left(\frac{3\pi}{4}\right) = -4 < 0$$

$\therefore \quad x = \frac{3\pi}{4}$ is point of maxima

Hence,

$$\text{max value} = f\left(\frac{3\pi}{4}\right) = -1 - \frac{3\pi}{2}$$

$$\text{min value} = f\left(\frac{\pi}{4}\right) = 1 - \frac{\pi}{2}.$$

Maxima and Minima 18.3 Q7

Consider the function

$f(x) = x^3 + ax^2 + bx + c$

Then $f'(x) = 3x^2 + 2ax + b$

It is given that $f(x)$ is maximum at $x = -1$.

$\therefore f'(-1) = 3(-1)^2 + 2a(-1) + b = 0$

$\Rightarrow f'(-1) = 3 - 2a + b = 0 \ldots(1)$

It is given that $f(x)$ is minimum at $x = 3$.

$\therefore f'(3) = 3(3)^2 + 2a(3) + b = 0$

$\Rightarrow f'(3) = 27 + 6a + b = 0 \ldots(2)$

Solving equations (1) and (2), we have,

$a = -3$ and $b = -9$

Since $f'(x)$ is independent of constant c, it can be any real number.

Ex 18.4

Maxima and Minima 18.4 Q1(i)

The given function is $f(x) = 4x - \frac{1}{2}x^2$.

$\therefore f'(x) = 4 - \frac{1}{2}(2x) = 4 - x$

Now,

$f'(x) = 0 \Rightarrow x = 4$

Then, we evaluate the value of f at critical point $x = 4$ and at the end points of the interval $\left[-2, \frac{9}{2}\right]$.

$f(4) = 16 - \frac{1}{2}(16) = 16 - 8 = 8$

$f(-2) = -8 - \frac{1}{2}(4) = -8 - 2 = -10$

$f\left(\frac{9}{2}\right) = 4\left(\frac{9}{2}\right) - \frac{1}{2}\left(\frac{9}{2}\right)^2 = 18 - \frac{81}{8} = 18 - 10.125 = 7.875$

Hence, we can conclude that the absolute maximum value of f on $\left[-2, \frac{9}{2}\right]$ is 8 occurring at $x = 4$ and the absolute minimum value of f on $\left[-2, \frac{9}{2}\right]$ is -10 occurring at $x = -2$.

Maxima and Minima 18.4 Q1(ii)

The given function is $f(x)=(x-1)^2+3$.

$\therefore f'(x)=2(x-1)$

Now,

$f'(x)=0 \Rightarrow 2(x-1)=0 \Rightarrow x=1$

Then, we evaluate the value of f at critical point $x = 1$ and at the end points of the interval $[-3, 1]$.

$$f(1)=(1-1)^2+3=0+3=3$$
$$f(-3)=(-3-1)^2+3=16+3=19$$

Hence, we can conclude that the absolute maximum value of f on $[-3, 1]$ is 19 occurring at $x = -3$ and the minimum value of f on $[-3, 1]$ is 3 occurring at $x = 1$.

Maxima and Minima 18.4 Q1(iii)

Let $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$.

$$\begin{aligned}\therefore f'(x) &= 12x^3-24x^2+24x-48\\ &=12\left(x^3-2x^2+2x-4\right)\\ &=12\left[x^2(x-2)+2(x-2)\right]\\ &=12(x-2)\left(x^2+2\right)\end{aligned}$$

Now, $f'(x)=0$ gives $x = 2$ or $x^2 + 2 = 0$ for which there are no real roots.

Therefore, we consider only $x = 2 \in [0, 3]$.

Now, we evaluate the value of f at critical point $x = 2$ and at the end points of the interval $[0, 3]$.

$$\begin{aligned}f(2) &= 3(16)-8(8)+12(4)-48(2)+25\\ &=48-64+48-96+25\\ &=-39\\ f(0) &= 3(0)-8(0)+12(0)-48(0)+25\\ &=25\\ f(3) &= 3(81)-8(27)+12(9)-48(3)+25\\ &=243-216+108-144+25=16\end{aligned}$$

Hence, we can conclude that the absolute maximum value of f on $[0, 3]$ is 25 occurring at $x = 0$ and the absolute minimum value of f at $[0, 3]$ is -39 occurring at $x = 2$.

Maxima and Minima 18.4 Q1(iv)

$f(x) = (x-2)\sqrt{x-1}$

$\Rightarrow \quad f'(x) = \sqrt{x-1} + (x-2)\frac{1}{2\sqrt{x-1}}$

Put $f'(x) = 0$

$\Rightarrow \quad \sqrt{x-1} + \frac{x-2}{2\sqrt{x-1}} = 0$

$\Rightarrow \quad \frac{2(x-1)+(x-2)}{2\sqrt{x-1}} = 0$

$\Rightarrow \quad \frac{3x-4}{2\sqrt{x-1}} = 0$

$\Rightarrow \quad x = \frac{4}{3}$

Now,

$f(1) = 0$

$f\left(\frac{4}{3}\right) = \left(\frac{4}{3}-2\right)\sqrt{\frac{4}{3}-1} = \frac{4-6}{3\sqrt{3}} = \frac{-2}{3\sqrt{3}} = \frac{-2\sqrt{3}}{9}$

$f(9) = (9-2)\sqrt{9-1} = 7\sqrt{8} = 14\sqrt{2}$

$\therefore$ The absolute maximum value of $f(x)$ is $14\sqrt{2}$ at $x = 9$ and the absolute minimum value is $\frac{-2\sqrt{3}}{9}$ at $x = \frac{4}{3}$.

Maxima and Minima 18.4 Q2

Let $f(x) = 2x^3 - 24x + 107$.

$\therefore f'(x) = 6x^2 - 24 = 6(x^2-4)$

Now,

$f'(x) = 0 \Rightarrow 6(x^2-4) = 0 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

We first consider the interval [1, 3].

Then, we evaluate the value of f at the critical point $x = 2 \in [1, 3]$ and at the end points of the interval [1, 3].

$f(2) = 2(8) - 24(2) + 107 = 16 - 48 + 107 = 75$

$f(1) = 2(1) - 24(1) + 107 = 2 - 24 + 107 = 85$

$f(3) = 2(27) - 24(3) + 107 = 54 - 72 + 107 = 89$

Hence, the absolute maximum value of $f(x)$ in the interval [1, 3] is 89 occurring at $x = 3$.

Next, we consider the interval [−3, −1].

Evaluate the value of f at the critical point $x = -2 \in [-3, -1]$ and at the end points of the interval [1, 3].

$f(-3) = 2(-27) - 24(-3) + 107 = -54 + 72 + 107 = 125$

Maxima and Minima 18.4 Q3

$$f(x) = \cos^2 x + \sin x$$
$$f'(x) = 2\cos x(-\sin x) + \cos x$$
$$= -2\sin x \cos x + \cos x$$

Now, $f'(x) = 0$

$$\Rightarrow 2\sin x \cos x = \cos x \Rightarrow \cos x(2\sin x - 1) = 0$$
$$\Rightarrow \sin x = \frac{1}{2} \text{ or } \cos x = 0$$
$$\Rightarrow x = \frac{\pi}{6}, \text{ or } \frac{\pi}{2} \text{ as } x \in [0, \pi]$$

Now, evaluating the value of f at critical points $x = \frac{\pi}{2}$ and $x = \frac{\pi}{6}$ and at the end points of the interval $[0, \pi]$ (i.e., at $x = 0$ and $x = \pi$), we have:

$$f\left(\frac{\pi}{6}\right) = \cos^2\frac{\pi}{6} + \sin\frac{\pi}{6} = \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{1}{2} = \frac{5}{4}$$
$$f(0) = \cos^2 0 + \sin 0 = 1 + 0 = 1$$
$$f(\pi) = \cos^2 \pi + \sin \pi = (-1)^2 + 0 = 1$$
$$f\left(\frac{\pi}{2}\right) = \cos^2\frac{\pi}{2} + \sin\frac{\pi}{2} = 0 + 1 = 1$$

Hence, the absolute maximum value of f is $\frac{5}{4}$ occurring at $x = \frac{\pi}{6}$ and the absolute minimum value of f is 1 occurring at $x = 0, \frac{\pi}{2}$, and π.

Maxima and Minima 18.4 Q4

We have

$$f(x) = 12x^{\frac{4}{3}} - 6x^{\frac{1}{3}}$$

$$\therefore \quad f'(x) = 16x^{\frac{1}{3}} - \frac{2}{x^{\frac{2}{3}}} = \frac{2(8x-1)}{x^{\frac{2}{3}}}$$

Thus, $f'(x) = 0$

$$\Rightarrow \quad x = \frac{1}{8}$$

Further note that $f'(x)$ is not defined at $x = 0$.

So, the critical points are $x = 0$ and $x = \frac{1}{8}$.

Evaluating the value of f at critical points $x = 0, \frac{1}{8}$ and at end points of the interval $x = -1$ and $x = 1$

$$f(-1) = 12(-1)^{4/3} - 6(-1)^{1/3} = 18$$
$$f(0) = 12(0) - 6(0) = 0$$
$$f\left(\frac{1}{8}\right) = 12\left(\frac{1}{8}\right)^{4/3} - 6\left(\frac{1}{8}\right)^{1/3} = \frac{-9}{4}$$
$$f(1) = 12(1)^{4/3} - 6(1)^{1/3} = 6$$

Hence we conclude that absolute maximum value of f is 18 at $x = -1$ and absolute minimum value of f is $\frac{-9}{4}$ at $x = \frac{1}{8}$.

Maxima and Minima 18.4 Q5

Given,

$$f(x) = 2x^3 - 15x^2 + 36x + 1$$

$$\therefore \quad f'(x) = 6x^2 - 30x + 36 = 6\left(x^2 - 5x + 6\right) = 6(x-2)(x-3)$$

Note that $f'(x) = 0$ gives $x = 2$ and $x = 3$

We shall now evaluate the value of f at these points
and at the end points of the interval [1,5],
i.e at x= 1, 2, 3 and 5

At $x = 1$, $f(1) = 2\left(1^3\right) - 15\left(1^2\right) + 36(1) + 1 = 24$

At $x = 2$, $f(2) = 2\left(2^3\right) - 15\left(2^2\right) + 36(2) + 1 = 29$

At $x = 3$, $f(3) = 2\left(3^3\right) - 15\left(3^2\right) + 36(3) + 1 = 28$

At $x = 5$, $f(5) = 2\left(5^3\right) - 15\left(5^2\right) + 36(5) + 1 = 56$

Thus we conclude that the absolute maximum value of f on [1,5] is 56,
occurring at x=5, and absolute minimum value of f on [1,5] is 24 which
occurs at x=1.

Ex 18.5

Maxima and Minima 18.5 Q1

Let x and y be the two numbers.

Given that $x + y = 16$ ---(i)

$Let \quad s = x^2 + y^2$ ---(ii)

From (i) and (ii)

$$S = x^2 + (15 - x)^2$$

$$\therefore \quad \frac{ds}{dx} = 2x + 2(15 - x)(-1)$$

$$= 2x - 30 + 2x$$

$$= 4x - 30$$

Now, $\frac{ds}{dx} = 0$

$\Rightarrow \quad 4x - 30 = 0$

$\Rightarrow \quad x = \frac{15}{2}$

Since,

$$\frac{d^2s}{dx^2} = 4 > 0$$

$\therefore \quad x = \frac{15}{2}$ is the point of local minima.

So, from (i)

$$y = 15 - \frac{15}{2} = \frac{15}{2}$$

Hence, the required numbers are $\frac{15}{2}, \frac{15}{2}$.

Maxima and Minima 18.5 Q2

Let x and y be the two parts of 64.

$\therefore \quad x + y = 64 \qquad$ ---(i)

Let $\quad S = x^3 + y^3 \qquad$ ---(ii)

From (i) and (ii), we get

$$S = x^3 + (64 - x)^3$$

$$\therefore \quad \frac{dS}{dx} = 3x^2 + 3(64 - x)^2 \times (-1)$$

$$= 3x^2 - 3\left(4096 - 128x + x^2\right)$$

$$= -3(4096 - 128x)$$

For maxima and minima,

$$\frac{dS}{dx} = 0$$

$$\Rightarrow \quad -3(4096 - 128x) = 0$$

$$\Rightarrow \quad x = 32$$

Now,

$$\frac{d^2s}{dx^2} = 384 > 0$$

$\therefore \quad x = 32$ is the point of local minima.

Thus, the two parts of 64 are $(32, 32)$.

Maxima and Minima 18.5 Q3

Let x and y be the two numbers, such that, $x, y \geq -2$ and

$$x + y = \frac{1}{2} \qquad \text{---(i)}$$

Let $S = x + y^3$ ---(ii)

From (i) and (ii), weget

$$S = x + \left(\frac{1}{2} - x\right)^3$$

$$\therefore \quad \frac{dS}{dx} = 1 + 3\left(\frac{1}{2} - x\right)^2 \times (-1)$$

$$= 1 - 3\left(\frac{1}{4} - x + x^2\right)$$

$$= \frac{1}{4} + 3x - 3x^2$$

For maximum and minimum,

$$\frac{dS}{dx} = 0$$

$$\Rightarrow \quad \frac{1}{4} + 3x - 3x^2 = 0$$

$$\Rightarrow \quad 1 + 12x - 12x^2 = 0$$

$$\Rightarrow \quad 12x^2 - 12x - 1 = 0$$

$$\Rightarrow \quad x = \frac{12 \pm \sqrt{144 + 48}}{24}$$

$$\Rightarrow \quad x = \frac{1}{2} \pm \frac{8\sqrt{3}}{24}$$

$$\Rightarrow \quad x = \frac{1}{2} \pm \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad x = \frac{1}{2} - \frac{1}{\sqrt{3}}, \frac{1}{2} + \frac{1}{\sqrt{3}}$$

Now,

$$\frac{d^2S}{dx^2} = 3 - 6x$$

At $x = \frac{1}{2} - \frac{1}{\sqrt{3}}$, $\frac{d^2S}{dx^2} = 3\left(1 - 2\left(\frac{1}{2} - \frac{1}{\sqrt{3}}\right)\right)$

$$= 3\left(+\frac{2}{\sqrt{3}}\right) = 2\sqrt{3} > 0$$

$\therefore \quad x = \frac{1}{2} - \frac{1}{\sqrt{3}}$ is point of local minima

$\therefore$ from (i)

$$y = \frac{1}{2} - \left(\frac{1}{2} - \frac{1}{\sqrt{3}}\right) = \frac{1}{\sqrt{3}}$$

Hence, the required numbers are $\frac{1}{2} - \frac{1}{\sqrt{3}}$, $\frac{1}{\sqrt{3}}$.

Maxima and Minima 18.5 Q4

Let x and y be the two parts of 15, such that

$\therefore \quad x + y = 15 \qquad$ ---(i)

Also, $S = x^2y^3 \qquad$ ---(ii)

From (i) and (ii), weget

$$S = x^2(15-x)^3$$

$$\therefore \quad \frac{dS}{dx} = 2x(15-x)^3 - 3x^2(15-x)^2$$
$$= (15-x)^2\left[30x - 2x^2 - 3x^2\right]$$
$$= 5x(15-x)^2(6-x)$$

For maxima and minima,

$$\frac{dS}{dx} = 0$$
$$\Rightarrow \quad 5x(15-x)^2(6-x) = 0$$
$$\Rightarrow \quad x = 0,\ 15,\ 6$$

Now,

$$\frac{d^2S}{dx^2} = 5(15-x)^2(6-x) - 5x \times 2(15-x)(6-x) - 5x(15-x)^2$$

$\therefore \quad$ At $x = 0$, $\frac{dS^2}{dx^2} = 1125 > 0$

$\therefore \quad x = 0$ is point of local minima

At $\quad x = 15$, $\frac{d^2s}{dx^2} = 0$

$\therefore \quad x = 15$ is an inflection point.

At $\quad x = 6$, $\frac{ds^2}{dx^2} = -2430 < 0$

$\therefore \quad x = 6$ is the point of local maxima

Thus the numbers are 6 and 9.

Maxima and Minima 18.5 Q5

Let r and h be the radius and height of the cylinder respectively.

Then, volume (V) of the cylinder is given by,

$$V = \pi r^2 h = 100 \qquad \text{(given)}$$
$$\therefore h = \frac{100}{\pi r^2}$$

Surface area (S) of the cylinder is given by,

$$S = 2\pi r^2 + 2\pi rh = 2\pi r^2 + \frac{200}{r}$$
$$\therefore \frac{dS}{dr} = 4\pi r - \frac{200}{r^2},\ \frac{d^2S}{dr^2} = 4\pi + \frac{400}{r^3}$$
$$\frac{dS}{dr} = 0 \ \Rightarrow \ 4\pi r = \frac{200}{r^2}$$
$$\Rightarrow r^3 = \frac{200}{4\pi} = \frac{50}{\pi}$$
$$\Rightarrow r = \left(\frac{50}{\pi}\right)^{\frac{1}{3}}$$

Now, it is observed that when $r = \left(\frac{50}{\pi}\right)^{\frac{1}{3}}$, $\frac{d^2S}{dr^2} > 0$.

$\therefore$By second derivative test, the surface area is the minimum when the radius of the cylinder is $\left(\frac{50}{\pi}\right)^{\frac{1}{3}}$ cm.

When $r = \left(\frac{50}{\pi}\right)^{\frac{1}{3}}$, $h = \frac{100}{\pi\left(\frac{50}{\pi}\right)^{\frac{2}{3}}} = \frac{2\times 50}{(50)^{\frac{2}{3}}(\pi)^{1-\frac{2}{3}}} = 2\left(\frac{50}{\pi}\right)^{\frac{1}{3}}$ cm.

Hence, the required dimensions of the can which has the minimum surface area is given by radius $= \left(\frac{50}{\pi}\right)^{\frac{1}{3}}$ cm and height $= 2\left(\frac{50}{\pi}\right)^{\frac{1}{3}}$ cm.

Maxima and Minima 18.5 Q6

We are given that the bending moment M at a distance x from one end of the beam is given by

(i) $M = \frac{WL}{2}x - \frac{W}{2}x^2$

$\therefore \quad \frac{dM}{dx} = \frac{WL}{2} - Wx$

For maxima and minima,

$$\frac{dM}{dx} = 0 \Rightarrow \quad \frac{WL}{2} - Wx = 0 \Rightarrow \quad x = \frac{L}{2}$$

Now,

$$\frac{d^2M}{dx^2} = -W < 0$$

$\therefore \quad x = \frac{L}{2}$ is point of local maxima.

(ii) $M = \frac{Wx}{3} - \frac{Wx^3}{3L^2}$

$\therefore \quad \frac{dM}{dx} = \frac{W}{3} - \frac{Wx^2}{L^2}$

For maxima and minima,

$$\frac{dM}{dx} = 0 \Rightarrow \quad \frac{W}{3} - \frac{Wx^2}{L^2} = 0 \Rightarrow \quad x = \frac{L}{\sqrt{3}}$$

Now,

$$\frac{d^2M}{dx^2} = -\frac{2xW}{L^2}$$

At $\quad x = \frac{L}{\sqrt{3}}, \ \frac{d^2M}{dx^2} = -\frac{2W}{\sqrt{3}L} < 0$

$\therefore \quad x = \frac{L}{\sqrt{3}}$ is point of local maxima

$$\Rightarrow \quad \frac{d^2s}{dx^2} = -\frac{\sqrt{2}r}{\frac{r^2}{2}}$$

$$= \frac{2\sqrt{2}}{r} < 0$$

$\therefore \quad x = \frac{r}{\sqrt{2}}$ is the point of local maxima

From (i)

$$y = \frac{r}{\sqrt{2}}$$

Hence, $x = \frac{r}{\sqrt{2}}, y = \frac{r}{\sqrt{2}}$ is the required number.

Maxima and Minima 18.5 Q7

Let a piece of length l be cut from the given wire to make a square.

Then, the other piece of wire to be made into a circle is of length $(28 - l)$ m.

Now, side of square $= \frac{l}{4}$.

Let r be the radius of the circle. Then, $2\pi r = 28 - l \Rightarrow r = \frac{1}{2\pi}(28 - l)$.

The combined areas of the square and the circle (A) is given by,

$$A = (\text{side of the square})^2 + r^2$$
$$= \frac{l^2}{16} + \pi\left[\frac{1}{2\pi}(28-l)\right]^2$$
$$= \frac{l^2}{16} + \frac{1}{4\pi}(28-l)^2$$
$$\therefore \frac{dA}{dl} = \frac{2l}{16} + \frac{2}{4\pi}(28-l)(-1) = \frac{l}{8} - \frac{1}{2\pi}(28-l)$$
$$\frac{d^2A}{dl^2} = \frac{1}{8} + \frac{1}{2\pi} > 0$$
$$\text{Now, } \frac{dA}{dl} = 0 \Rightarrow \frac{l}{8} - \frac{1}{2\pi}(28-l) = 0$$
$$\Rightarrow \frac{\pi l - 4(28-l)}{8\pi} = 0$$
$$\Rightarrow (\pi+4)l - 112 = 0$$
$$\Rightarrow l = \frac{112}{\pi+4}$$

Thus, when $l = \frac{112}{\pi+4}, \frac{d^2A}{dl^2} > 0$.

$\therefore$ By second derivative test, the area (A) is the minimum when $l = \frac{112}{\pi+4}$.

Hence, the combined area is the minimum when the length of the wire in making the square is $\frac{112}{\pi+4}$ cm while the length of the wire in making the circle is $28 - \frac{112}{\pi+4} = \frac{28\pi}{\pi+4}$ cm.

Maxima and Minima 18.5 Q8

Let the wire of length 20 m be cut into x cm and y cm and bent into a square and equilateral triangle, so that the sum of area of square and triangle is minimum.

Now,

$$x + y = 20 \quad \text{---(i)}$$
$$x = 4l \text{ and } y = 3a$$

Let s = sum of area of square and triangle

$$s = l^2 + \frac{\sqrt{3}}{4}a^2 \quad \text{---(ii)}$$

$$\left[\because \text{area of equilateral } \Delta = \frac{\sqrt{3}}{4}(\text{one side})^2\right]$$

We have, $4l+3a=20$

$\Rightarrow 4l=20-3a$

$\Rightarrow l=\dfrac{20-3a}{4}$

From (i), we have,

$$s=\left(\frac{20-3a}{4}\right)^2+\frac{\sqrt{3}}{4}a^2$$

$$\frac{ds}{da}=2\left(\frac{20-3a}{4}\right)\left(\frac{-3}{4}\right)+2a\times\frac{\sqrt{3}}{4}$$

To find the maximum or minimum, $\dfrac{ds}{da}=0$

$$\Rightarrow 2\left(\frac{20-3a}{4}\right)\left(\frac{-3}{4}\right)+2a\times\frac{\sqrt{3}}{4}=0$$

$\Rightarrow -3(20-3a)+4a\sqrt{3}=0$

$\Rightarrow -60+9a+4a\sqrt{3}=0$

$\Rightarrow 9a+4a\sqrt{3}=60$

$\Rightarrow a\left(9+4\sqrt{3}\right)=60$

$\Rightarrow a=\dfrac{60}{9+4\sqrt{3}}$

Differentiating once again, we have,

$$\frac{d^2s}{da^2}=\frac{9+4\sqrt{3}}{8}>0$$

Thus, the sum of the areas of the square and triangle is minimum when $a=\dfrac{60}{9+4\sqrt{3}}$

We know that, $l=\dfrac{20-3a}{4}$

$$\Rightarrow l=\frac{20-3\left(\frac{60}{9+4\sqrt{3}}\right)}{4}$$

$$\Rightarrow l=\frac{180+80\sqrt{3}-180}{4\left(9+4\sqrt{3}\right)}$$

$$\Rightarrow l=\frac{20\sqrt{3}}{9+4\sqrt{3}}$$

Maxima and Minima 18.5 Q9

Let r be the radius of the circle and a be the side of the square.

Then, we have:

$2\pi r+4a=k$ (where k is constant)

$\Rightarrow a=\dfrac{k-2\pi r}{4}$

The sum of the areas of the circle and the square (A) is given by,

$$A=\pi r^2+a^2=\pi r^2+\frac{(k-2\pi r)^2}{16}$$

$$\therefore \frac{dA}{dr}=2\pi r+\frac{2(k-2\pi r)(-2\pi)}{16}=2\pi r-\frac{\pi(k-2\pi r)}{4}$$

Now, $\dfrac{dA}{dr}=0$

$\Rightarrow 2\pi r=\dfrac{\pi(k-2\pi r)}{4}$

$8r=k-2\pi r$

$$\Rightarrow (8+2\pi)r = k$$

$$\Rightarrow r = \frac{k}{8+2\pi} = \frac{k}{2(4+\pi)}$$

Now, $\frac{d^2A}{dr^2} = 2\pi + \frac{\pi^2}{2} > 0$

$\therefore$ When $r = \frac{k}{2(4+\pi)}$, $\frac{d^2A}{dr^2} > 0.$

$\therefore$ The sum of the areas is least when $r = \frac{k}{2(4+\pi)}$.

When $r = \frac{k}{2(4+\pi)}$, $a = \frac{k - 2\pi\left[\frac{k}{2(4+\pi)}\right]}{4} = \frac{k(4+\pi)-\pi k}{4(4+\pi)} = \frac{4k}{4(4+\pi)} = \frac{k}{4+\pi} = 2r.$

Hence, it has been proved that the sum of their areas is least when the side of the square is double the radius of the circle.

Maxima and Minima 18.5 Q10

ABC is a right angled triangle. Hypotenuse $h = AC = 5$ cm.

Let x and y one the other two side of the triangle.

$\therefore \quad x^2 + y^2 = 25 \quad$ ---(i)

$\therefore \quad$ Area of $\Delta ABC = \frac{1}{2} BC \times AB$

$\Rightarrow \quad S = \frac{1}{2}xy \quad$ ---(ii)

From (i) and (ii)

$$S = \frac{1}{2}x\sqrt{25-x}$$

$$\therefore \quad \frac{ds}{dx} = \frac{1}{2}\left[\sqrt{25-x^2} - \frac{2x^2}{2\sqrt{25-x^2}}\right]$$

$$= \frac{1}{2}\frac{\left[25-x^2-x^2\right]}{\sqrt{25-x^2}}$$

$$= \frac{1}{2}\left[\frac{25-2x^2}{\sqrt{25-x^2}}\right]$$

For maxima and minima,

$$\frac{ds}{dx} = 0$$

$$\Rightarrow \quad \frac{1}{2}\left[\frac{25-2x^2}{\sqrt{25-x^2}}\right] = 0$$

$$\Rightarrow \quad x = 5\sqrt{2}$$

Now,

$$\frac{d^2s}{dx^2} = \frac{1}{2}\frac{\sqrt{25-x^2}\times(-4x) + \frac{(25-2x^2)2x}{2\sqrt{25-x^2}}}{(25-x^2)}$$

At $\quad x = \frac{5}{\sqrt{2}}$, $\frac{d^2s}{dx^2} = \frac{1}{2}\frac{\left[-\frac{25}{\sqrt{2}}\times\frac{5}{\sqrt{2}} + 0\right]}{\frac{25}{2}}$

$$= -\frac{5}{2} < 0$$

$\therefore \quad x = \frac{5}{\sqrt{2}}$ is a point local maxima,

Maxima and Minima 18.5 Q11

ABC is a given triangle with $AB = a, BC = b$ and $\angle ABC = \theta$.
AD in perpendicular to BC.

$\therefore \quad BD = a\sin\theta$

Now,

$$\text{Area of } \Delta ABC = \frac{1}{2} \times BC \times AD$$

$$\Rightarrow \quad A = \frac{1}{2} b \times a \sin\theta \qquad \text{---(i)}$$

$$\therefore \quad \frac{dA}{d\theta} = \frac{1}{2} ab\cos\theta$$

For maxima and minima,

$$\frac{dA}{d\theta} = 0$$

$$\Rightarrow \quad \frac{1}{2} ab\cos\theta = 0$$

$$\Rightarrow \quad \cos\theta = 0$$

$$\Rightarrow \quad \theta = \frac{\pi}{2}$$

Now,

$$\frac{d^2A}{d\theta^2} = -\frac{1}{2} ab\sin\theta$$

$$\text{At} \quad \theta = \frac{\pi}{2}, \quad \frac{d^2A}{d\theta^2} = -\frac{1}{2} ab < 0$$

$$\therefore \quad \theta = \frac{\pi}{2} \text{ is point of local maxima}$$

$$\therefore \text{ Maximum area of } \Delta = \frac{1}{2} ab \sin\frac{\pi}{2} = \frac{1}{2} ab.$$

Maxima and Minima 18.5 Q12

Let the side of the square to be cut off be x cm. Then, the length and the breadth of the box will be $(18 - 2x)$ cm each and the height of the box is x cm.

Therefore, the volume $V(x)$ of the box is given by,

$$V(x) = x(18 - 2x)^2$$

$$\begin{aligned} \therefore V'(x) &= (18-2x)^2 - 4x(18-2x) \\ &= (18-2x)[18-2x-4x] \\ &= (18-2x)(18-6x) \\ &= 6\times 2(9-x)(3-x) \\ &= 12(9-x)(3-x) \end{aligned}$$

$$\begin{aligned} \text{And, } V''(x) &= 12[-(9-x)-(3-x)] \\ &= -12(9-x+3-x) \\ &= -12(12-2x) \\ &= -24(6-x) \end{aligned}$$

$$\text{Maximum volume is } V_{x=3} = 3\times(18-2\times 3)^2$$

$$\Rightarrow V = 3\times 12^2$$

$$\Rightarrow V = 3\times 144$$

$$\Rightarrow V = 432\ cm^3$$

Maxima and Minima 18.5 Q13

Let the side of the square to be cut off be x cm. Then, the height of the box is x, the length is $45 - 2x$, and the breadth is $24 - 2x$.

Therefore, the volume $V(x)$ of the box is given by,

$$
\begin{aligned}
V(x) &= x(45-2x)(24-2x) \\
&= x(1080-90x-48x+4x^2) \\
&= 4x^3-138x^2+1080x \\
\therefore V'(x) &= 12x^2-276x+1080 \\
&= 12(x^2-23x+90) \\
&= 12(x-18)(x-5) \\
V''(x) &= 24x-276 = 12(2x-23)
\end{aligned}
$$

Now, $V'(x) = 0 \Rightarrow x = 18$ and $x = 5$

It is not possible to cut off a square of side 18 cm from each corner of the rectangular sheet. Thus, x cannot be equal to 18.

$\therefore x = 5$

Now, $V''(5) = 12(10-23) = 12(-13) = -156 < 0$

$\therefore$ By second derivative test, $x = 5$ is the point of maxima.

Hence, the side of the square to be cut off to make the volume of the box maximum possible is 5 cm.

Maxima and Minima 18.5 Q14

Let l, b, and h represent the length, breadth, and height of the tank respectively.

Then, we have height $(h) = 2$ m

Volume of the tank = 8m^3

Volume of the tank = $l \times b \times h$

$\therefore 8 = l \times b \times 2$

$\Rightarrow lb = 4 \Rightarrow b = \frac{4}{l}$

Now, area of the base = $lb = 4$

Area of the 4 walls $(A) = 2h(l + b)$

$$
\begin{aligned}
&\therefore A = 4\left(l+\frac{4}{l}\right) \\
&\Rightarrow \frac{dA}{dl} = 4\left(1-\frac{4}{l^2}\right)
\end{aligned}
$$

Now, $\frac{dA}{dl} = 0$

$$
\begin{aligned}
&\Rightarrow 1-\frac{4}{l^2} = 0 \\
&\Rightarrow l^2 = 4
\end{aligned}
$$

$\Rightarrow l = \pm 2$

However, the length cannot be negative.

Therefore, we have $l = 4$.

$\therefore b = \frac{4}{l} = \frac{4}{2} = 2$

Now, $\frac{d^2 A}{dl^2} = \frac{32}{l^3}$

When $l = 2$, $\frac{d^2 A}{dl^2} = \frac{32}{8} = 4 > 0$.

Thus, by second derivative test, the area is the minimum when $l = 2$.

We have $l = b = h = 2$.

$\therefore$Cost of building the base = Rs 70 × (lb) = Rs 70 (4) = Rs 280

Cost of building the walls = Rs $2h$ ($l + b$) × 45 = Rs 90 (2) (2 + 2)

= Rs 8 (90) = Rs 720

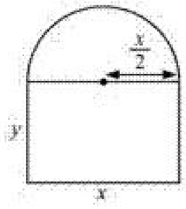
Required total cost = Rs (280 + 720) = Rs 1000

Hence, the total cost of the tank will be Rs 1000.

Maxima and Minima 18.5 Q15

Maxima and Minima 18.5 Q15

Radius of the semicircular opening $= \frac{x}{2}$



It is given that the perimeter of the window is 10 m.

$$\therefore x+2y+\frac{\pi x}{2}=10$$
$$\Rightarrow x\left(1+\frac{\pi}{2}\right)+2y=10$$
$$\Rightarrow 2y=10-x\left(1+\frac{\pi}{2}\right)$$
$$\Rightarrow y=5-x\left(\frac{1}{2}+\frac{\pi}{4}\right)$$

$\therefore$Area of the window (A) is given by,

$$A=xy+\frac{\pi}{2}\left(\frac{x}{2}\right)^2$$
$$=x\left[5-x\left(\frac{1}{2}+\frac{\pi}{4}\right)\right]+\frac{\pi}{8}x^2$$
$$=5x-x^2\left(\frac{1}{2}+\frac{\pi}{4}\right)+\frac{\pi}{8}x^2$$
$$\therefore \frac{dA}{dx}=5-2x\left(\frac{1}{2}+\frac{\pi}{4}\right)+\frac{\pi}{4}x$$
$$=5-x\left(1+\frac{\pi}{2}\right)+\frac{\pi}{4}x$$
$$\therefore \frac{d^2A}{dx^2}=-\left(1+\frac{\pi}{2}\right)+\frac{\pi}{4}=-1-\frac{\pi}{4}$$

Now, $\dfrac{dA}{dx}=0$

$$\Rightarrow 5-x\left(1+\frac{\pi}{2}\right)+\frac{\pi}{4}x=0$$

$$\Rightarrow 5-x-\frac{\pi}{4}x=0$$

$$\Rightarrow x\left(1+\frac{\pi}{4}\right)=5$$

$$\Rightarrow x=\frac{5}{\left(1+\frac{\pi}{4}\right)}=\frac{20}{\pi+4}$$

Thus, when $x=\dfrac{20}{\pi+4}$ then $\dfrac{d^2A}{dx^2}<0$.

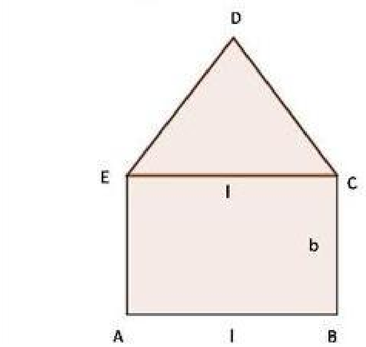
Therefore, by second derivative test, the area is the maximum when length $x=\dfrac{20}{\pi+4}$ m.

Now,

$$y=5-\frac{20}{\pi+4}\left(\frac{2+\pi}{4}\right)=5-\frac{5(2+\pi)}{\pi+4}=\frac{10}{\pi+4}\text{ m}$$

Hence, the required dimensions of the window to admit maximum light is given by length $=\dfrac{20}{\pi+4}$ m and breadth $=\dfrac{10}{\pi+4}$ m.

Maxima and Minima 18.5 Q16



The perimeter of the window = 12 m

$\Rightarrow \quad (l + 2b) + (l + l) = 12$

$\Rightarrow \quad 3l + 2b = 12 \quad$ ----- (i)

Let S = Area of the rectangle + Area of the equilateral Δ

From (i),

$$S = l\left(\frac{12-3l}{2}\right) + \frac{\sqrt{3}}{4}l^2$$

$$\therefore \quad \frac{dS}{dl} = 6 - 3l + \frac{\sqrt{3}}{2}l = 6 - \sqrt{3}\left(\sqrt{3} - \frac{1}{2}\right)l$$

For maxima and minima,

$$\frac{dS}{dl} = 0$$

$$\Rightarrow \quad 6 - \sqrt{3}\left(\sqrt{3} - \frac{1}{2}\right)l = 0$$

$$\Rightarrow \quad l = \frac{6}{\sqrt{3}\left(\sqrt{3} - \frac{1}{2}\right)} = \frac{12}{6 - \sqrt{3}}$$

Now, $\dfrac{d^2S}{dl^2} = -\sqrt{3}\left(\sqrt{3} - \dfrac{1}{2}\right) = -3 + \dfrac{\sqrt{3}}{2} < 0$

$\therefore \quad l = \dfrac{12}{6 - \sqrt{3}}$ is the point of local maxima

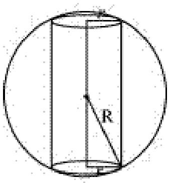
From (i),

$$b = \frac{12 - 3l}{2} = \frac{12 - 3\left(\frac{12}{6-\sqrt{3}}\right)}{2} = \frac{24 - 6\sqrt{3}}{6 - \sqrt{3}}$$

Maxima and Minima 18.5 Q17

A sphere of fixed radius (R) is given.

Let r and h be the radius and the height of the cylinder respectively.



From the given figure, we have $h = 2\sqrt{R^2 - r^2}$.

The volume (V) of the cylinder is given by,

$$V = \pi r^2 h = 2\pi r^2\sqrt{R^2 - r^2}$$

$$\therefore \frac{dV}{dr} = 4\pi r\sqrt{R^2 - r^2} + \frac{2\pi r^2(-2r)}{2\sqrt{R^2 - r^2}}$$

$$= 4\pi r\sqrt{R^2 - r^2} - \frac{2\pi r^3}{\sqrt{R^2 - r^2}}$$

$$= \frac{4\pi r\left(R^2 - r^2\right) - 2\pi r^3}{\sqrt{R^2 - r^2}}$$

$$= \frac{4\pi r R^2 - 6\pi r^3}{\sqrt{R^2 - r^2}}$$

Now, $\dfrac{dV}{dr} = 0 \Rightarrow 4\pi r R^2 - 6\pi r^3 = 0$

$$\Rightarrow r^2 = \frac{2R^2}{3}$$

$$\text{Now, } \frac{d^2V}{dr^2} = \frac{\sqrt{R^2-r^2}\left(4\pi R^2 - 18\pi r^2\right) - \left(4\pi rR^2 - 6\pi r^3\right)\frac{(-2r)}{2\sqrt{R^2-r^2}}}{\left(R^2-r^2\right)}$$

$$= \frac{\left(R^2-r^2\right)\left(4\pi R^2 - 18\pi r^2\right) + r\left(4\pi rR^2 - 6\pi r^3\right)}{\left(R^2-r^2\right)^{\frac{3}{2}}}$$

$$= \frac{4\pi R^4 - 22\pi r^2 R^2 + 12\pi r^4 + 4\pi r^2 R^2}{\left(R^2-r^2\right)^{\frac{3}{2}}}$$

Now, it can be observed that at $r^2 = \frac{2R^2}{3}, \frac{d^2V}{dr^2} < 0$.

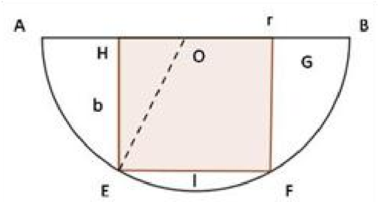
$\therefore$The volume is the maximum when $r^2 = \frac{2R^2}{3}$.

When $r^2 = \frac{2R^2}{3}$, the height of the cylinder is $2\sqrt{R^2 - \frac{2R^2}{3}} = 2\sqrt{\frac{R^2}{3}} = \frac{2R}{\sqrt{3}}$.

Hence, the volume of the cylinder is the maximum when the height of the cylinder is $\frac{2R}{\sqrt{3}}$.

Maxima and Minima 18.5 Q18

Let $EFGH$ be a rectangle inscribed in a semi-circle with radius r.



Let l and b are the length and width of rectangle.
In ΔOHE

$$HE^2 = OE^2 - OH^2$$

$$\Rightarrow \quad HE = b = \sqrt{r^2 - \left(\frac{r}{2}\right)^2} \qquad \text{---(i)}$$

Let S = Area of rectangle

$$= lb = l \times \sqrt{r^2 - \left(\frac{l}{2}\right)^2}$$

$$\therefore \quad S = \frac{1}{2} l \sqrt{4r^2 - l^2}$$

$$\therefore \quad \frac{ds}{dl} = \frac{1}{2}\left[\sqrt{4r^2 - l^2} - \frac{l^2}{\sqrt{4r^2 - l^2}}\right]$$

$$= \frac{1}{2}\left[\frac{4r^2 - l^2 - l^2}{\sqrt{4r^2 - l^2}}\right]$$

$$= \frac{2r^2 - l^2}{\sqrt{4r^2 - l^2}}$$

For maxima and minima,

$$\frac{ds}{dl = 0}$$

$$\Rightarrow \quad \frac{2r^2 - l^2}{\sqrt{4r^2 - l^2}} = 0$$

$$\Rightarrow \quad l = \pm\sqrt{2}r$$

Also,

$$\frac{d^2s}{dl^2} = 0 \text{ at } l = \sqrt{2}r$$

So, the dimension of the rectangle

$$l = \sqrt{2}r, \; b = \sqrt{r^2 - \left(\frac{l}{2}\right)^2} = \frac{r}{\sqrt{2}}$$

$$\text{Area of rectangle} = lb = \sqrt{2}r \times \frac{r}{\sqrt{2}}$$

$$= r^2.$$

Maxima and Minima 18.5 Q19

Let r and h be the radius and the height (altitude) of the cone respectively.

Then, the volume (V) of the cone is given as:

$$V = \frac{1}{3\pi}\pi r^2 h \Rightarrow h = \frac{3V}{r^2}$$

The surface area (S) of the cone is given by,

$S = \pi rl$ (where l is the slant height)

$$= \pi r\sqrt{r^2 + h^2}$$

$$= \pi r\sqrt{r^2 + \frac{9V^2}{\pi^2 r^4}} = \frac{\pi r\sqrt{\pi^2 r^6 + 9V^2}}{\pi r^2}$$

$$= \frac{1}{r}\sqrt{\pi^2 r^6 + 9V^2}$$

$$\therefore \frac{dS}{dr} = \frac{r \cdot \frac{6\pi^2 r^5}{2\sqrt{\pi^2 r^6 + 9V^2}} - \sqrt{\pi^2 r^6 + 9V^2}}{r^2}$$

$$= \frac{3\pi^2 r^6 - \pi^2 r^6 - 9V^2}{r^2\sqrt{\pi^2 r^6 + 9V^2}}$$

$$= \frac{2\pi^2 r^6 - 9V^2}{r^2\sqrt{\pi^2 r^6 + 9V^2}}$$

$$= \frac{2\pi^2 r^6 - 9V^2}{r^2\sqrt{\pi^2 r^6 + 9V^2}}$$

Now, $\frac{dS}{dr} = 0 \Rightarrow 2\pi^2 r^6 = 9V^2 \Rightarrow r^6 = \frac{9V^2}{2\pi^2}$

Thus, it can be easily verified that when $r^6 = \frac{9V^2}{2\pi^2}$, $\frac{d^2S}{dr^2} > 0$.

$\therefore$ By second derivative test, the surface area of the cone is the least when $r^6 = \frac{9V^2}{2\pi^2}$.

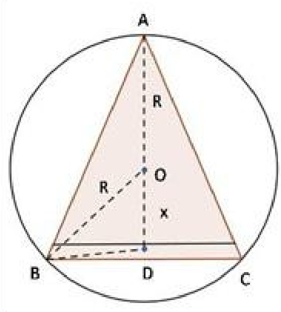
When $r^6 = \frac{9V^2}{2\pi^2}$, $h = \frac{3V}{\pi r^2} = \frac{3}{\pi r^2}\left(\frac{2\pi^2 r^6}{9}\right)^{\frac{1}{2}} = \frac{3}{\pi r^2} \cdot \frac{\sqrt{2}\pi r^3}{3} = \sqrt{2}r.$

Hence, for a given volume, the right circular cone of the least curved surface has an altitude equal to $\sqrt{2}$ times the radius of the base.

Maxima and Minima 18.5 Q20

We have a cone, which is inscribed in a sphere.

Let v be the volume of greatest cone ABC. If is obvious that, for maximum volume the axis of the cone must be along the diameter of sphere.



Let $OD = x$ and $AO = OB = R$

$\Rightarrow \quad BD = \sqrt{R^2 - x^2}$ and $AD = R + x$

Now,

$$v = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi BD^2 \times AD$$
$$= \frac{1}{3}\pi\left(R^2 - x^2\right) \times (R + x)$$

$$\therefore \quad \frac{dv}{dx} = \frac{\pi}{3}\left[-2x(R + x) + R^2 - x^2\right]$$
$$= \frac{\pi}{3}\left[R^2 - 2xR - 3x^2\right]$$

For maximum and minimum

$$\frac{dv}{dx} = 0$$
$$\Rightarrow \quad \frac{\pi}{3}\left[R^2 - 2xR - 3x^2\right] = 0$$
$$\Rightarrow \quad \frac{\pi}{3}\left[(R - 3x)(R + x)\right] = 0$$
$$\Rightarrow \quad R - 3x = 0 \text{ or } x = -R$$
$$\Rightarrow \quad x = \frac{R}{3} \qquad \left[\because x = -R \text{ is not possible as, } x = -R \text{ will make the altitude } 0\right]$$

Now,

$$\frac{d^2v}{dx^2} = \frac{\pi}{3}[-2R - 6x]$$

At $\quad x = \frac{R}{3}, \quad \frac{d^2v}{dx^2} = \frac{\pi}{3}[-2R - 2R]$

$$= \frac{-4\pi R}{3} < 0$$

$\therefore \quad x = \frac{R}{3}$ is the point of local maxima.

Maxima and Minima 18.5 Q21

Volume of the cone$=\frac{1}{3}\pi r^2 h$

$\Rightarrow V = \frac{1}{3}\pi r^2 h$

Squaring both the sides, we have,

$V^2 = \left(\frac{1}{3}\pi r^2 h\right)^2$

$= \frac{1}{9}\pi^2 r^4 h^2 \ldots(1)$

$\Rightarrow \pi^2 r^2 h^2 = \frac{9V^2}{r^2} \ldots(2)$

Consider the curved surface area of the cone.

Thus,

$C=\pi rl$

Squaring both the sides, we have,

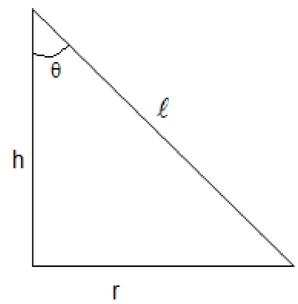
$C^2=\pi^2 r^2 l^2$

We know that $l^2 = r^2 + h^2$

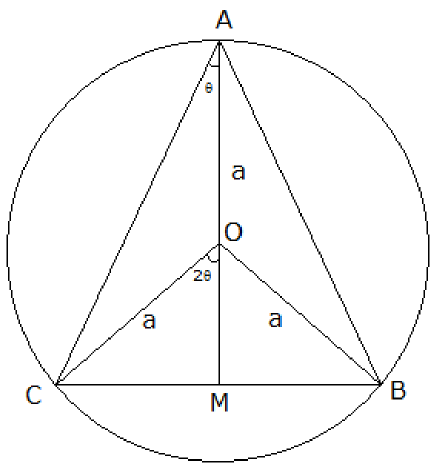
$\Rightarrow C^2=\pi^2 r^2 (r^2 + h^2)$

$\Rightarrow C^2=\pi^2 r^4 + \pi^2 r^2 h^2$

$\Rightarrow C^2=\pi^2 r^4 + \frac{9V^2}{r^2} \ldots$(From equation (2))



Maxima and Minima 18.5 Q22



ABC is an isosceles triangle such that AB = AC.
The vertical angle $\angle BAC = 2\theta$.
Triangle is inscribed in the circle with center O and radius a.

Draw AM perpendicular to BC.
$\because$ ΔABC is an isoscales triangle the circumcentre of the circle will lie or the perpendicular from A to BC.

Let O be the circumcentre.
$\angle BOC = 2 \times 2\theta = 4\theta$ ....[Using central angle theorem]
$\angle COM = 2\theta$ ....[$\because$ ΔOMB and ΔOMC are congruent triangles]
$OA = OB = OC = a$.......[Radius of the circle]

In ΔOMC,
$CM = a\sin 2\theta$ and $OM = a\cos 2\theta$
$BC = 2CM$...[Perpendicular from the center bisects the chord]
$BC = 2a\sin 2\theta$..........(1)
Height of $\Delta ABC = AM = AO + OM$
$AM = a + a\cos 2\theta$........(2)

Area of ΔABC is,
$A = \frac{1}{2} \times BC \times AM$
Differentiating equation (3) with respect to θ.

$$\frac{dA}{d\theta} = a^2\left(2\cos 2\theta + \frac{1}{2} \times 4\cos 4\theta\right)$$

$$\frac{dA}{d\theta} = 2a^2(\cos 2\theta + \cos 4\theta)$$

Differentiating agin with respect to θ.

$$\frac{d^2A}{d\theta^2} = 2a^2(-2\sin 2\theta - 4\sin 4\theta)$$

For maximum value of area equating $\frac{dA}{d\theta} = 0$

$$2a^2(\cos 2\theta + \cos 4\theta) = 0$$
$$\cos 2\theta + \cos 4\theta = 0$$
$$\cos 2\theta + 2\cos^2 2\theta - 1 = 0$$
$$(2\cos 2\theta - 1)(2\cos 2\theta + 1) = 0$$
$$\cos 2\theta = \frac{1}{2} \text{ or } \cos 2\theta = -1$$
$$2\theta = \frac{\pi}{3} \text{ or } 2\theta = \pi$$
$$\theta = \frac{\pi}{6} \text{ or } \theta = \frac{\pi}{2}$$

If $2\theta = \pi$ it will not form a triangle.
$\therefore \theta = \frac{\pi}{6}$
Also $\frac{d^2A}{d\theta^2}$ is negative for $\theta = \frac{\pi}{6}$.

Thus the area of the triangle is maximum when $\theta = \frac{\pi}{6}$.

Maxima and Minima 18.5 Q23

Here, $ABCD$ is a rectangle with width $AB = x$ cm and length $AD = y$ cm.

The rectangle is rotated about AD. Let v be the volume of the cylinder so formed.

$\therefore \quad v = \pi r^2 y \qquad$ ---(i)

Again,

Perimeter of $ABCD = 2(l + b) = 2(x + y) \qquad$ ---(ii)

$\Rightarrow \quad 36 = 2(x + y)$

$\Rightarrow \quad y = 18 - x \qquad$ ---(iii)

From (i) and (ii), we get

$$v - \pi r^2 (18 - x) = \pi (18x^2 - x^3)$$

$$\Rightarrow \quad \frac{dv}{dx} = \pi (36x - 3x^2)$$

For maxima or minima, we have,

$$\frac{dv}{dx} = 0$$

$$\Rightarrow \quad \pi (36x - 3x^2) = 0$$

$$\Rightarrow \quad 3\pi (12x - x^2) = 0$$

$$\Rightarrow \quad x(12 - x) = 0$$

$\Rightarrow \quad x = 0$ (Not possible) or 12

$\therefore \quad x = 12$ cm

From (iii)

$y = 18 - 12 = 6$ cm

Now,

$$\frac{d^2v}{dx^2} = \pi (36 - 6x)$$

At $\quad (x = 12, y = 6) \dfrac{d^2v}{dx^2} = \pi (36 - 72) = -36\pi < 0$

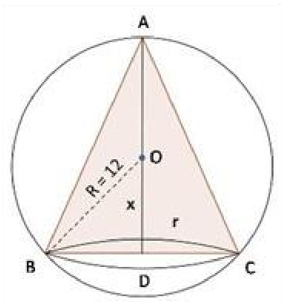
$\therefore \quad (x = 12, y = 6)$ is the point of local maxima,

Hence,

The dimension of rectangle, which wiout maximum value, when revolved about one of its side is width = 12 cm and length = 6 cm.

Maxima and Minima 18.5 Q24

Let r and h be the radius of the base of cone and height of the cone respectively.



Let $OD = x$

It is abvious that the axis of cone must be along the diameter of shpere for maximum volume of cone.

Now,

$$\text{In } \Delta BOD,\ BD = \sqrt{R^2 - x^2}$$
$$= \sqrt{144 - x^2}$$
$$AD = AO + OD = R + x = 12 + x$$
$$v = \text{volume of cone} = \frac{1}{3}\pi r^2 h$$
$$\Rightarrow \quad v = \frac{1}{3}\pi BD^2 \times AD$$
$$= \frac{1}{3}\pi\left(144 - x^2\right)\left(2 + x\right)$$
$$= \frac{1}{3}\pi\left(1728 + 144x - 12x^2 - x^3\right)$$
$$\therefore \quad \frac{dv}{dx} = \frac{1}{3}\pi\left(144 - 24x - 3x^2\right)$$

For maximum and minimum of v.

$$\frac{dv}{dx} = 0$$
$$\Rightarrow \quad \frac{1}{3}\pi\left(144 - 24x - 3x^2\right) = 0$$
$$\Rightarrow \quad x = -12, 4$$
$$x = -12 \text{ is not possible}$$
$$\therefore \quad x = 4$$

Now,

$$\frac{d^2v}{dx^2} = \frac{\pi}{3}\left(-24 - 6x\right)$$
$$\text{At} \quad x = 4, \frac{d^2v}{dx^2} = -2\pi\left(4 + x\right)$$
$$= -2\pi \times 8 = -16\pi < 0$$
$$\therefore \quad x = 4 \text{ is point of local maxima.}$$

Hence,

$$\text{Height of cone of maximum volume} = R + x$$
$$= 12 + 4$$
$$= 16 \text{ cm}.$$

Maxima and Minima 18.5 Q25

We have, a closed cylinder whose volume $v = 2156 \text{ cm}^3$

Let r and h be the radius and the height of the cylinder. Then,

$$\therefore \quad v = \pi r^2 h = 2156 \qquad \text{---(i)}$$

Total surface area = $S = 2\pi rh + 2\pi r^2$

$$\Rightarrow \quad S = 2\pi r\left(h + r\right) \qquad \text{---(ii)}$$

From (i) and (ii)

$$S = \frac{2156 \times 2}{r} + 2\pi r^2$$

$$\therefore \quad \frac{ds}{dr} = -\frac{4312}{r^2} + 4\pi r$$

For maximum and minimum

$$\frac{ds}{dr} = 0$$

$$\Rightarrow \quad \frac{-4312 + 4\pi r^3}{r^2} = 0$$

$$\Rightarrow \quad r^3 = \frac{4312}{4\pi}$$

$$\Rightarrow \quad r = 7$$

Now,

$$\frac{d^2s}{dr^2} = \frac{8624}{r^3} + 4\pi > 0 \text{ for } r = 7.$$

$\therefore$ $r = 7$ is the point of local minima

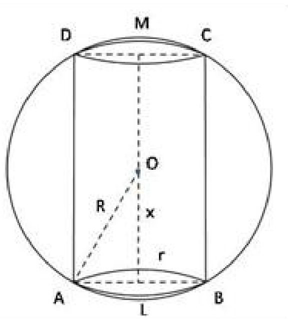
Hence,

The total surface area of closed cylinder will be munimum at $r = 7$ cm.

Maxima and Minima 18.5 Q26

Let r be the radius of the base of the cylinder and h be the height of the cylinder.

$\therefore$ $LM = h$.



Let $R = 5\sqrt{3}$ cm be the radius of the sphere.

It is obvious, that for maximum volume of cylinder $ABCD$, the axis of cylinder must be along the diameter of sphere.

Let $OL = x$
$\therefore$ $h = 2x$

Now,

$$\text{In } \Delta AOL,\ AL = \sqrt{AO^2 - OL^2}$$
$$= \sqrt{75 - x^2}$$

Now,

$$v = \text{volume of cylinder} = \pi r^2 h$$
$$\Rightarrow v = \pi AL^2 \times ML$$
$$= \pi\left(75 - x^2\right) \times 2x$$

For maxima and minima of v, we must have,

$$\frac{dv}{dx} = \pi\left[150 - 6x^2\right] = 0$$
$$\Rightarrow x = 5 \text{ cm}$$

Also, $\frac{d^2v}{dx^2} = -12\pi x$

At $x = 5$, $\frac{d^2v}{dx^2} = -60\pi x < 0$

$\therefore$ $x = 5$ is point of local maxima.

Hence,

The maximum volume of cylinder is $= \pi\left(75 - 25\right) \times 10 = 500\pi \text{ cm}^3$.

Maxima and Minima 18.5 Q27

Let x and y be two positive numbers with

$$x^2 + y^2 = r^2 \quad \text{---(i)}$$

Let $S = x + y$ ---(ii)

$\therefore$ $S = x + \sqrt{r^2 - x^2}$ from (ii)

$\therefore$ $\frac{dS}{dx} = 1 - \frac{x}{\sqrt{r^2 - x^2}}$

For maxima and minima,

$$\frac{dS}{dx} = 0$$

$\Rightarrow$ $1 - \frac{x}{\sqrt{r^2 - x^2}} = 0$

$\Rightarrow$ $x = \sqrt{r^2 - x^2}$

$\Rightarrow$ $2x^2 = r^2$

$\Rightarrow$ $x = \frac{r}{\sqrt{2}}, \frac{-r}{\sqrt{2}}$

$\therefore$ x & y are positive numbers

$\therefore$ $x = \frac{r}{\sqrt{2}}$

Also, $\frac{d^2S}{dx^2} = \frac{-\left(\sqrt{r^2 - x^2} + \frac{x^2}{\sqrt{r^2 - x^2}}\right)}{r^2 - x^2}$

At, $x = \frac{r}{\sqrt{2}}, \frac{d^2S}{dx^2} = -\left[\frac{\frac{r}{\sqrt{2}} + \frac{\frac{r^2}{2}}{\frac{r}{\sqrt{2}}}}{\frac{r^2}{2}}\right] < 0$

Since $\frac{d^2S}{dx^2} < 0$, *the sum is largest when* $x = y = \frac{r}{\sqrt{2}}$

Maxima and Minima 18.5 Q28

The given equation of parabola is

$$x^2 = 4y \qquad \text{---(i)}$$

Let $P(x, y)$ be the nearest point on (i) from the point $A(0, 5)$

Let S be the square of the distance of P from A.

$$\therefore \quad S = x^2 + (y-5)^2 \qquad \text{---(ii)}$$

From (i),

$$S = 4y + (y-5)^2$$

$$\Rightarrow \quad \frac{dS}{dy} = 4 + 2(y-5)$$

For maxima or minima, we have

$$\frac{dS}{dy} = 0$$

$$\Rightarrow \quad 4 + 2(y-5) = 0$$

$$\Rightarrow \quad 2y = 6$$

$$\Rightarrow \quad y = 3$$

From (i)

$$x^2 = 12$$

$$\therefore \quad x = \pm 2\sqrt{3}$$

$$\Rightarrow \quad P = \left(2\sqrt{3}, 3\right) \text{ and } P' = \left(-2\sqrt{3}, 3\right)$$

Now,

$$\frac{d^2S}{dy^2} = 2 > 0$$

$\therefore$ P and P' are the point of local minima

Hence, the nearest points are $P\left(2\sqrt{3}, 3\right)$ and $P'\left(-2\sqrt{3}, 3\right)$.

Maxima and Minima 18.5 Q29

Let $P(x, y)$ be a point on

$y^2 = 4x$ ---(i)

Let S be the square of the distance between $A(2, -8)$ and P.

$\therefore$ $S = (x-2)^2 + (y+8)^2$ ---(ii)

Using (i),

$$S = \left(\frac{y^2}{4} - 2\right)^2 + (y+8)^2$$

$$\therefore \quad \frac{dS}{dy} = 2\left(\frac{y^2}{4} - 2\right) \times \frac{y}{2} + 2(y+8)$$

$$= \frac{y^3 - 8y}{4} + 2y + 16$$

$$= \frac{y^3}{4} + 16$$

For maxima and minima,

$$\frac{dS}{dy} = 0$$

$$\Rightarrow \quad \frac{y^3}{4} + 16 = 0$$

$$\Rightarrow \quad y = -4$$

Now,

$$\frac{d^2S}{dy^2} = \frac{3y^2}{4}$$

At $y = -4$, $\dfrac{d^2S}{dy^2} = 12 > 0$

$\therefore$ $y = -4$ is the point of local minima

From (i)

$$x = \frac{y^2}{4} = 4$$

Thus, the required point is $(4, -4)$ nearest to $(2, -8)$.

Maxima and Minima 18.5 Q30

Let $P(x, y)$ be a point on the curve,

$$x^2 = 8y \qquad \text{---(i)}$$

Let $A = (2, 4)$ be a point and

let $S =$ square of the distance between P and A

$$\therefore \quad S = (x-2)^2 + (y-4)^2 \qquad \text{---(ii)}$$

Using (i), we get

$$S = (x-2)^2 + \left(\frac{x^2}{8} - 4\right)^2$$

$$\therefore \quad \frac{dS}{dy} = 2(x-2) + 2\left(\frac{x^2}{8} - 4\right) \times \frac{2x}{8}$$

$$= 2(x-2) + \frac{(x^2-32)x}{16}$$

Also, $$\frac{d^2S}{dx^2} = 2 + \frac{1}{16}\left[x^2 - 32 + 2x^2\right]$$

$$= 2 + \frac{1}{16}\left[3x^2 - 32\right]$$

For maxima and minima,

$$\frac{dS}{dx} = 0$$

$$\Rightarrow \quad 2(x-2) + \frac{x(x^2-32)}{16} = 0$$

$$\Rightarrow \quad 32x - 64 + x^3 - 32x = 0$$

$$\Rightarrow \quad x^3 - 64 = 0$$

$$\Rightarrow \quad x = 4$$

Now,

At $x = 4$, $\frac{d^2S}{dx^2} = 2 + \frac{1}{16}\left[16 \times 3 - 32\right] = 2 + 1 = 3 > 0$

$\therefore$ $x = 4$ is point of local minima

From (i)

$$y = \frac{x^2}{8} = 2$$

Thus, $P(4, 2)$ is the nearest point.

Maxima and Minima 18.5 Q31

Let $P(x, y)$ be a point on the curve $x^2 = 2y$ which is closest to $A(0, 5)$

Let $\quad S$ = square of the length of AP

$\Rightarrow \quad S = x^2 + (y-5)^2 \qquad$ ---(ii)

Using (i),

$$S = 2y + (y-5)^2$$

$$\therefore \quad \frac{dS}{dy} = 2 + 2(y-5)$$

For maxima and minima,

$$\frac{dS}{dy} = 0$$

$\Rightarrow \quad 2 + 2y - 10 = 0$

$\Rightarrow \quad y = 4$

Now,

$$\frac{d^2S}{dy^2} = 2 > 0$$

$\therefore \quad y = 4$ is the point of local minima

From (i)

$$r = \pm 2\sqrt{2}$$

Hence, $\left(\pm 2\sqrt{2}, 4\right)$ is the closest point on the curve to $A(0, 5)$.

Maxima and Minima 18.5 Q32

The given equations are

$y = x^2 + 7x + 2 \qquad$ ---(i)

and $\quad y = 3x - 3 \qquad$ ---(ii)

Let $P(x, y)$ be the point on parabola (i) which is closest to the line (ii)

Let S be the perpendicular distance from P to the line (ii).

$$\therefore \quad S = \frac{|y - 3x + 3|}{\sqrt{1^2 + (-3)^2}}$$

$$\Rightarrow \quad S = \frac{|x^2 + 7x + 2 - 3x + 3|}{\sqrt{10}} \qquad \text{---(iii)}$$

$$\Rightarrow \quad \frac{dS}{dx} = \frac{2x + 4}{\sqrt{10}}$$

For maxima or minima, we have

$$\frac{dS}{dx} = 0$$

$$\Rightarrow \quad \frac{2x + y}{\sqrt{10}} = 0$$

$\Rightarrow \quad x = -2$

From (i)

$$y = 4 - 14 + 2 = -8$$

Now,

$$\frac{d^2S}{dx^2} = \frac{2}{\sqrt{10}} > 0$$

$\therefore .(x = -2, y = -8)$ is the point of local minima,

Hence,

The closest point on the parabola to the line $y = 3x - 3$ is $(-2, -8)$.

Maxima and Minima 18.5 Q33

Let $P(x, y)$ be a point on the curve $y^2 = 2x$ which is minimum distance from the point $A(1, 4)$.

Let

S = square of the length of AP

$$S = (x-1)^2 + (y-4)^2$$

Using this equation, we have

$$S = x^2 + 1 - 2x + y^2 + 16 - 8y$$

$$S = x^2 - \cancel{2x} + \cancel{2x} + 17 - 8y$$

$$S = \frac{y^4}{4} - 8y + 17 \qquad \left[\text{Since } x = \frac{y^2}{2}\right]$$

$$\frac{dS}{dy} = y^3 - 8$$

For maxima and minima, we have

$$\frac{dS}{dy} = 0$$

$$y^3 - 8 = 0$$

$$y^3 = 2^3$$

$$y = 2$$

Now,

$$\frac{d^2S}{dy^2} = 3y^2$$

$$\frac{d^2S}{dy^2} = 12 > 0$$

$\therefore y = 2$ is minimum point

We have

$$x = \frac{y^2}{2}$$

$$= \frac{4}{2}$$

$$= 2$$

Hence, $\boxed{(2,2)}$ is at a minimum distance from the point $(1, 4)$.

Maxima and Minima 18.5 Q34

The given equation of curve is

$$y = x^3 + 3x^2 + 2x - 27 \qquad \text{---(i)}$$

Slope of (i)

$$m = \frac{dy}{dx} = -3x^2 + 6x + 2 \qquad \text{---(ii)}$$

Now,

$$\frac{dm}{dx} = -6x + 6$$

and $\frac{d^2m}{dx^2} = -6 < 0$

For maxima and minima,

$$\frac{dm}{dx} = 0$$

$\Rightarrow \quad -6x + 6 = 0$

$\Rightarrow \quad x = 1$

$\therefore \quad \frac{d^2m}{dx^2} = -6 < 0$

$\therefore \quad x = 1$ is point of local maxima

Hence, maximum slope $= -3 + 6 + 2 = 5$

Maxima and Minima 18.5 Q35

We have,

Cost of producing x radio sets is Rs. $\frac{x^2}{4} + 35x + 25$

Selling price of x radio is Rs. $x\left(50 - \frac{x}{2}\right)$

So,

Profit on x radio sets is

$$P = \text{Rs}\ \left(50x - \frac{x^2}{2} - \frac{x^2}{4} - 35x - 25\right)$$

$$\therefore \quad \frac{dP}{dx} = 50 - x - \frac{x}{2} - 35$$

$$= 15 - \frac{3}{2}x$$

For maxima and minima,

$$\frac{dP}{dx} = 0$$

$$\Rightarrow \quad 15 - \frac{3}{2}x = 0$$

$$\Rightarrow \quad x = 10$$

Also,

$$\frac{d^2P}{dx^2} = \frac{-3}{2} < 0$$

$\therefore$ $x = 10$ is the point of local maxima

Hence, the daily output should be 10 radio sets.

Maxima and Minima 18.5 Q35

We have,

Cost of producing x radio sets is Rs. $\frac{x^2}{4} + 35x + 25$

Selling price of x radio is Rs. $x\left(50 - \frac{x}{2}\right)$

So,

Profit on x radio sets is

$$P = \text{Rs}\ \left(50x - \frac{x^2}{2} - \frac{x^2}{4} - 35x - 25\right)$$

$$\therefore \quad \frac{dP}{dx} = 50 - x - \frac{x}{2} - 35$$

$$= 15 - \frac{3}{2}x$$

For maxima and minima,

$$\frac{dP}{dx} = 0$$

$$\Rightarrow \quad 15 - \frac{3}{2}x = 0$$

$$\Rightarrow \quad x = 10$$

Also,

$$\frac{d^2P}{dx^2} = \frac{-3}{2} < 0$$

$\therefore$ $x = 10$ is the point of local maxima

Hence, the daily output should be 10 radio sets.

Maxima and Minima 18.5 Q36

Let $S(x)$ be the selling price of x items and let $C(x)$ be the cost price of x items.

Then, we have $S(x) = \left(5 - \frac{x}{100}\right)x = 5x - \frac{x^2}{100}$

and $C(x) = \frac{x}{5} + 500$

Thus, the profit function $P(x)$ is given by

$$P(x) = S(x) - C(x) = 5x - \frac{x^2}{100} - \frac{x}{5} - 500 = \frac{24}{5}x - \frac{x^2}{100} - 500$$

$\therefore \quad P'(x) = \frac{24}{5} - \frac{x}{50}$

Now, $P'(x) = 0$

$\Rightarrow \frac{24}{5} - \frac{x}{50} = 0$

$\Rightarrow \quad x = \frac{24}{5} \times 50 = 240$

Also $P''(x) = -\frac{1}{50}$

So, $\quad P''(240) = -\frac{1}{50} < 0$

Thus, $x = 240$ is a point of maxima.
Hence, the manufacturer can earn maximum profit,
if he sells 240 items.

Maxima and Minima 18.5 Q37

Let l be the length of side of square base of the tank and h be the height of tank.
Then,

Volume of tank $(v) = l^2h$

Total surface area $(s) = l^2 + 4lh$

Since the tank holds a given quantity of water the volume (v) is constant.

$\therefore \quad v = l^2h \qquad$ ---(i)

Also, cost of lining with lead will be least if the total surface area is least.
So we need to minimise the surface area.

$\therefore \quad S = l^2 + 4lh \qquad$ ---(ii)

Now,

From (i) and (ii)

$S = l^2 + \frac{4v}{l}$

$\therefore \quad \frac{ds}{dl} = 2l - \frac{4v}{l^2}$

For maximum and minimum

$\frac{ds}{dl} = 0$

$\Rightarrow \quad 2l - \frac{4v}{l^2} = 0$

$\Rightarrow \quad 2l^3 - 4v = 0$

$\Rightarrow \quad l^3 = 2v = 2t^2h$

$\Rightarrow \quad l^2[l - 2h] = 0$

$\Rightarrow \quad l = 0$ or $2h$

$l = 0$ is not possible.

$\therefore \quad l = 2h$

Now,

$\frac{d^2s}{dl^2} = 2 + \frac{8v}{l^3}$

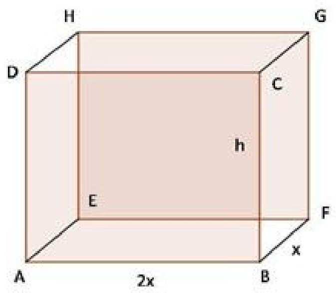
At $\quad l = 2h, \ \frac{d^2s}{dl^2} > 0 \qquad$ for all h.

$\therefore \quad l = 2h$ is point of local minima

$\therefore \quad S$ is minimum when $l = 2h$

Maxima and Minima 18.5 Q38

Let *ABCDEFGH* be a box of constant volume *c*. We are given that the box is twice as long as its width.



$\therefore$ Let $BF = x$

$\Rightarrow$ $AB = 2x$

Cost of material of top and front side = 3 x cost of material of the bottom of the box.

$\Rightarrow$ $2x \times x + xh + xh + 2xh + 2xh = 3 \times 2x^2$

$\Rightarrow$ $2x^2 + 2xh + 4xh = 6x^2$

$\Rightarrow$ $4x^2 - 6xh = 0$

$\Rightarrow$ $2x(2x - 3h) = 0$

$\Rightarrow$ $x = \frac{3h}{2}$ or $h = \frac{2x}{3}$

Volume of box $= 2x \times x \times h$

$\Rightarrow$ $c = 2x^2h$

$\Rightarrow$ $h = \frac{c}{2x^2}$ ---(ii)

Now,

S = Surface area of box $= 2\left(2x^2 + 2xh + xh\right)$

$\Rightarrow$ $S = 2\left(2x^2 + 3xh\right)$

From (i)

$S = 2\left(2x^2 + \frac{3xc}{2x^2}\right)$

$\Rightarrow$ $S = 2\left(2x^2 + \frac{3}{2}\frac{c}{x}\right)$

For maxima and minima,

$\frac{dS}{dx} = 2\left(4x - \frac{3}{2}\frac{c}{x^2}\right) = 0$

$\Rightarrow$ $8x^3 - 3c = 0$

$\Rightarrow$ $x = \left(\frac{3c}{8}\right)^{\frac{1}{3}}$

Now,

$\frac{d^2s}{dx^2} = 2\left(4 + 3\frac{c}{x^3}\right) > 0$ as $x = \left(\frac{3c}{8}\right)^{\frac{1}{3}}$

$x = \left(\frac{3c}{8}\right)^{\frac{1}{3}}$ is point of local minima

$\therefore$ Most economic dimension will be

x = width $= \left(\frac{3c}{8}\right)^{\frac{1}{3}}$

$2x$ = length $= 2\left(\frac{3c}{8}\right)^{\frac{1}{3}}$

h = height $= \frac{2x}{3} = \frac{2}{3}\left(\frac{3c}{8}\right)^{\frac{1}{3}}$.

Maxima and Minima 18.5 Q39

Let s be the sum of the surface areas of a sphere and a cube.

$\therefore \quad s = 4\pi r^2 + 6l^2 \qquad$ ---(i)

Let v = volume of sphere + volume of cube

$\Rightarrow \quad v = \frac{4}{3}\pi r^3 + l^3 \qquad$ ---(ii)

From (i)

$$l = \sqrt{\frac{s - 4\pi r^2}{6}}$$

$$\therefore \quad v = \frac{4}{3}\pi r^2 + \left(\frac{s - 4\pi r^2}{6}\right)^{\frac{3}{2}}$$

$$\therefore \quad \frac{dV}{dr} = 4\pi r^2 + \frac{3}{2}\left(\frac{s - 4\pi r^2}{6}\right)^{\frac{1}{2}} \times \left(\frac{-4\pi}{6}\right)^{2r}$$

For maxima and minima,

$$\frac{dv}{dr} = 0$$

$$\Rightarrow \quad 4\pi r^2 = \frac{\pi}{6}\left(s - 4\pi r^2\right)^{\frac{1}{2}} \times 2r = 0$$

$$\Rightarrow \quad 2r\pi\left[2r - l\right] = 0$$

$$\therefore \quad r = 0, \ \frac{l}{2}$$

Now,

$$\frac{d^2v}{dr^2} = 8\pi r - \frac{2\pi}{\sqrt{6}}\left[\left(s - 4\pi r^2\right)\right]^{\frac{1}{2}} - \frac{8\pi r^2}{2\left(s - 4\pi r^2\right)^{\frac{1}{2}}}$$

At $\quad r = \frac{l}{2}$

$$\frac{d^2v}{dr^2} = \pi\frac{l}{2} - \frac{2\pi}{\sqrt{6}}\left[\sqrt{6}l - \frac{8\pi\frac{l^2}{4}}{2\sqrt{6}l}\right] = 4\pi l - \frac{2\pi}{\sqrt{6}}\left[\frac{12l^2 - 2\pi l^2}{2\sqrt{6}l}\right]$$

Maxima and Minima 18.5 Q40

Let ABCDEF be a half cylinder with rectangular base and semi-circular ends.
Here AB = height of the cylinder
AB = h
Let r be the radius of the cylinder.
Volume of the half cylinder is $V = \frac{1}{2}\pi r^2 h$

$\Rightarrow \frac{2v}{\pi r^2} = h$

$\therefore$ TSA of the half cylinder is
S = LSA of the half cylinder + area of two semi-circular ends + area of the rectangle (base)

$S = \pi rh + \frac{\pi r^2}{2} + \frac{\pi r^2}{2} + h \times 2r$

$S = (\pi r + 2r)h + \pi r^2$

$S = (\pi r + 2r)\frac{2v}{\pi r^2} + \pi r^2$

$S = (\pi + 2)\frac{2v}{\pi r} + \pi r^2$

Differentiate S wrt r we get,

$\frac{ds}{dr} = \left[(\pi + 2) \times \frac{2v}{\pi}\left(\frac{-1}{r^2}\right) + 2\pi r\right]$

For maximum and minimum values of S, we have $\frac{ds}{dr} = 0$

$\Rightarrow (\pi + 2) \times \frac{2v}{\pi}\left(\frac{-1}{r^2}\right) + 2\pi r = 0$

$\Rightarrow (\pi + 2) \times \frac{2v}{\pi r^2} = 2\pi r$

But 2r = D

$\therefore h : D = \pi : \pi + 2$

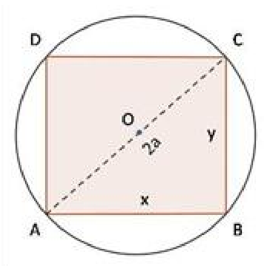
Differentiate $\frac{ds}{dr}$ wrt r we get,

$\frac{d^2s}{dr^2} = (\pi + 2)\frac{V}{\pi} \times \frac{2}{r^3} + 2\pi > 0$

Thus S will be minimum when h : 2r is π : π - 12.
Height of the cylinder : Diameter of the circular end
$\pi : \pi + 2$

Maxima and Minima 18.5 Q41

Let *ABCD* be the cross-sectional area of the beam which is cut from *a* circular log of radius *a*.



$\therefore \quad AO = a \Rightarrow AC = 2a$

Let x be the width of log and y be the depth of log $ABCD$

Let S be the strength of the beam according to the question,

$$S = xy^2 \qquad \text{---(i)}$$

In ΔABC

$$x^2 + y^2 = (2a)^2$$

$$\Rightarrow \quad y = (2a)^2 - x^2 \qquad \text{---(ii)}$$

From (i) and (ii), we get

$$S = x\left((2a)^2 - x^2\right)$$

$$\Rightarrow \quad \frac{dS}{dx} = \left(4a^2 - x^2\right) - 2x^2$$

$$\Rightarrow \quad \frac{dS}{dx} = 4a^2 - 3x^2$$

For maxima or minima

$$\frac{dS}{dx} = 0$$

$$\Rightarrow \quad 4a^2 - 3x^2 = 0$$

$$\Rightarrow \quad x^2 = \frac{4a^2}{3}$$

$$\therefore \quad x = \frac{2a}{\sqrt{3}}$$

From (ii),

$$y^2 = 4a^2 - \frac{4a^2}{3} = \frac{8a^2}{3}$$

$$\therefore \quad y = 2a \times \sqrt{\frac{2}{3}}$$

Now,

$$\frac{d^2S}{dx^2} = -6x$$

At $\quad x = \frac{2a}{\sqrt{3}},\ y = \sqrt{\frac{2}{3}}2a,\ \frac{d^2S}{dx^2} = -\frac{12a}{\sqrt{3}} < 0$

$\therefore \quad \left(x = \frac{2a}{\sqrt{3}}, y = \sqrt{\frac{2}{3}}2a\right)$ is the point of local maxima.

Hence,

The dimension of strongest beam is width $= x = \frac{2a}{\sqrt{3}}$ and depth $= y = \sqrt{\frac{2}{3}}2a$.

Maxima and Minima 18.5 Q42

Let l be a line through the point $P(1,4)$ that cuts the x-axis and y-axis.

Now, equation of l is

$$y - 4 = m(x - 1)$$

$\therefore$ x – Intercept is $\frac{m-4}{m}$ and y – Intercept is $4 - m$

Let $S = \frac{m-4}{m} + 4 - m$

$\therefore$ $\frac{dS}{dm} = +\frac{4}{m^2} - 1$

For maxima and minima,

$$\frac{dS}{dm} = 0$$

$\Rightarrow$ $\frac{4}{m^2} - 1 = 0$

$\Rightarrow$ $m = \pm 2$

Now,

$$\frac{d^2S}{dm^2} = -\frac{8}{m^3}$$

At $m = 2$, $\frac{d^2S}{dm^2} = -1 < 0$

$m = -2$ $\frac{d^2S}{dm^2} = 1 > 0$

$\therefore$ $m = -2$ is point of local minima.

$\therefore$ least value of sum of intercept is

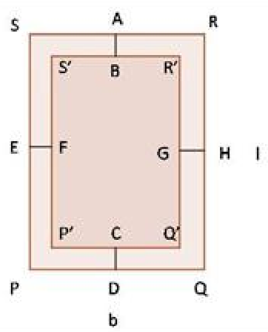
$$\frac{m-4}{m} + 4 - m$$
$$= 3 + 6 = 9$$

Maxima and Minima 18.5 Q43

The area of the page $PQRS$ in 150 cm^2

Also, $AB + CD = 3$ cm

$EF + GH = 2$ Cm



Let x and y be the combined width of margin at the top and bottom and the sides respectively.

$\therefore \quad x = 3$ cm and $y = 2$ cm.

Now, area of printed matter = area of $P'Q'R'S'$

$\Rightarrow \quad A = P'Q' \times Q'R'$

$\Rightarrow \quad A = (b - y)(l - x)$

$\Rightarrow \quad A = (b - 2)(l - 3) \qquad$ ---(i)

Also,

Area of $PQRS = 150$ cm^2

$\Rightarrow \quad lb = 150 \qquad$ ---(ii)

From (i) and (ii)

$$A = (b - 2)\left(\frac{150}{b} - 3\right)$$

$\therefore \quad$ For maximum and minimum,

$$\frac{dA}{db} = \left(\frac{150}{b} - 3\right) + (b - 2)\left(-\frac{150}{b^2}\right) = 0$$

$\Rightarrow \quad \frac{(150 - 3b)}{b} + (-150)\frac{(b - 2)}{b^2} = 0$

$\Rightarrow \quad 150b - 3b^2 - 150b + 300 = 0$

$\Rightarrow \quad -3b^2 + 300 = 0$

$\Rightarrow \quad b = 10$

From (ii)

$l = 15$

Now,

$$\frac{d^2A}{db^2} = \frac{-150}{b^2} - 150\left[-\frac{1}{b^2} + \frac{4}{b^3}\right]$$

At $\quad b = 10$

$$\frac{d^2A}{db^2} = -\frac{15}{10} - 150\left[-\frac{1}{100} + \frac{4}{1000}\right]$$

$= -1.5 - .15[-10 + 4]$

$= -1.5 + .9$

$= -0.6 < 0$

$\therefore \quad b = 10$ is point of local maxima.

Hence,

The required dimension will be $l = 15$ cm, $b = 10$ cm.

Maxima and Minima 18.5 Q44

The space s described in time t by a moving particle is given by

$$s = t^5 - 40t^3 + 30t^2 + 80t - 250$$

$\therefore$ velocity $= \dfrac{ds}{dt} = 5t^4 - 120t^2 + 60t + 80$

Acceleration $= a = \dfrac{d^2s}{dt^2} = 20t^3 - 240t + 60t$ ---(i)

Now,

$$\frac{da}{dt} = 60t^2 - 240$$

For maxima and minima,

$$\frac{da}{dt} = 0$$

$\Rightarrow$ $60t^2 - 240 = 0$

$\Rightarrow$ $60\left(t^2 - 4\right) = 0$

$\Rightarrow$ $t = 2$

Now,

$$\frac{d^2a}{dt^2} = 120t$$

At $t = 2$, $\dfrac{d^2a}{dt^2} = 240 > 0$

$\therefore$ $t = 2$ is point of local minima

Hence, minimum acceleration is $160 - 480 + 60 = -260$.

Maxima and Minima 18.5 Q45

We have,

$$\text{Distance, } s = \frac{t^4}{4} - 2t^3 + 4t^2 - 7$$

$$\text{Velocity,} v = \frac{ds}{dt} = t^3 - 6t^2 + 8t$$

$$\text{Acceleration,} a = \frac{d^2s}{dt^2} = 3t^2 - 12t + 8$$

For velocity to be maximum and minimum,

$$\frac{dv}{dt} = 0$$

$\Rightarrow \quad 3t^2 - 12t + 8 = 0$

$\Rightarrow \quad t = \dfrac{12 \pm \sqrt{144 - 96}}{6}$

$$= 2 \pm \frac{4\sqrt{3}}{6}$$

$\therefore \quad t = 2 + \dfrac{2}{\sqrt{3}},\ 2 - \dfrac{2}{\sqrt{3}}$

Now,

$$\frac{d^2v}{dt^2} = 6t - 12$$

At $\quad t = 2 - \dfrac{2}{\sqrt{3}},\ \dfrac{d^2v}{dt^2} = 6\left(2 - \dfrac{2}{\sqrt{3}}\right) - 12 = \dfrac{-12}{\sqrt{3}} < 0$

$\quad t = 2 + \dfrac{2}{\sqrt{3}},\ \dfrac{d^2r}{dt^2} = 6\left(2 + \dfrac{2}{\sqrt{3}}\right) - 12 = \dfrac{12}{\sqrt{3}} > 0$

$\therefore \quad$ At $t = 2 - \dfrac{2}{\sqrt{3}}$, velocity is maximum

For acceleration to be maximum and minimum

$$\frac{da}{dt} = 0$$

$\Rightarrow \quad 6t - 12 = 0$

$\Rightarrow \quad t = 2$

Now,

$$\frac{d^2a}{dt^2} = 6 > 0$$

$\therefore$ At, $t = 2$ Acceleration is minimum.

Ex 19.1

Indefinite Integrals Ex 19.1 Q1

(i)

$$\int x^4 dx = \frac{x^{4+1}}{4+1} + c$$
$$= \frac{x^5}{5} + c$$

(ii)

$$\int x^{\frac{5}{4}} dx = \frac{x^{\frac{5}{4}+1}}{\frac{5}{4}+1} + c$$
$$= \frac{x^{\frac{5+4}{4}}}{\frac{5+4}{4}} + c$$
$$= \frac{4x^{\frac{9}{4}}}{9} + c$$

(iii)

$$\int \frac{1}{x^5} dx = \int x^{-5} dx$$
$$= \frac{x^{-5+1}}{-5+1} + c$$
$$= \frac{x^{-4}}{-4} + c$$
$$= \frac{-1}{4x^4} + c$$

(iv)

$$\int \frac{1}{x^{\frac{3}{2}}} dx = \int x^{\frac{-3}{2}} dx$$
$$= \int x^{\frac{-3}{2}} dx$$
$$= \frac{x^{\frac{-3}{2}+1}}{\frac{-3}{2}+1} + c$$
$$= \frac{x^{\frac{-1}{2}}}{\frac{-1}{2}} + c$$
$$= -2 \times \frac{1}{\sqrt{x}} + c$$
$$= \frac{-2}{\sqrt{x}} + c$$

(v)

$$\int 3^x dx = \frac{3^x}{\log 3} + c \qquad \left[\because \int a^x dx = \frac{a^x}{\log a} + c\right]$$

(vi)

$$\int \frac{1}{\sqrt[3]{x^2}}dx = \int \frac{1}{x^{\frac{2}{3}}}dx$$
$$= \int x^{\frac{-2}{3}}dx$$
$$= \frac{d^{\frac{-2}{3}+1}}{\frac{-2}{3}+1}+c$$
$$= \frac{x^{\frac{1}{3}}}{\frac{1}{3}}+c$$
$$= 3\sqrt[3]{x}+c$$

(vii)

$$\int 3^{2\log_3 x}dx = \int 3^{\log_3 x^2}dx$$
$$= \int x^2 dx \qquad \left[\because a^{\log_a x} = x\right]$$
$$= \frac{x^3}{3}+c$$

(viii)

$$\int \log_x x\,dx = \int 1dx$$
$$= x + c.$$

Indefinite Integrals Ex 19.1 Q2

(i)

$$\int \sqrt{\frac{1+\cos 2x}{2}}dx = \int \sqrt{\frac{2\cos^2 x}{2}}dx \qquad \left[\because \cos 2x = 2\cos^2 - 1\right]$$
$$= \int \cos x dx$$
$$= \sin x + c$$

(ii)
$$\int \sqrt{\frac{1-\cos 2x}{2}}dx = \int \sqrt{\frac{2\sin^2 x}{2}}dx$$
$$= \int \sin x dx$$
$$= -\cos x + c$$

Indefinite Integrals Ex 19.1 Q3

Evaluate the integral as follows

$$\int \frac{e^{6\log_e x}-e^{5\log_e x}}{e^{4\log_e x}-e^{3\log_e x}}dx = \int \frac{x^6 - x^5}{x^4 - x^3}dx$$
$$= \int \frac{x^5(x-1)}{x^3(x-1)}dx$$
$$= \int x^2 dx$$
$$= \frac{x^3}{3}+C$$

Indefinite Integrals Ex 19.1 Q4

$$\int \frac{1}{a^x b^x}dx = \int a^{-x}b^{-x}dx$$
$$= \int (ab)^{-x}dx$$
$$= \frac{(ab)^{-x}}{\log_e (ab)^{-1}}+c$$
$$= \frac{(ab)^{-x}}{-\log_e (ab)}+c$$
$$= \frac{a^{-x}b^{-x}}{-\log_e (ab)}+c$$

Indefinite Integrals Ex 19.1 Q5

(i) $\int \frac{\cos 2x + 2\sin^2 x}{\sin^2 x}dx$

$$= \int \frac{\cos 2x + 2\sin^2 x}{\sin^2 x}dx$$

$$= \int \frac{1 - 2\sin^2 x + 2\sin^2 x}{\sin^2 x}dx$$

$$= \int \frac{1}{\sin^2 x}dx$$

$$= \int \operatorname{cosec}^2 x\,dx$$

$$= -\cot x + c$$

(ii) $\int \frac{2\cos^2 x - \cos 2x}{\cos^2 x}dx$

$$= \int \frac{2\cos^2 x - \left(2\cos^2 x - 1\right)}{\cos^2 x}dx$$

$$= \int \frac{2\cos^2 x - 2\cos^2 x + 1}{\cos^2 x}dx$$

$$= \int \frac{1}{\cos^2 x}dx$$

$$= \int \sec^2 x\,dx$$

$$= \tan x + c$$

Indefinite Integrals Ex 19.1 Q6

$$\int \frac{e^{\log\sqrt{x}}}{x}dx = \int \frac{\sqrt{x}}{x}dx$$

$$= \int x^{\frac{1}{2}} \times x^{-1}dx$$

$$= \int x^{\frac{1}{2}-1}dx$$

$$= \int x^{\frac{-1}{2}}dx$$

$$= \frac{x^{\frac{-1}{2}+1}}{\frac{-1}{2}+1} + c$$

$$= \frac{x^{\frac{1}{2}}}{\frac{1}{2}}$$

$$= 2\sqrt{x} + c$$

Ex 19.2

Indefinite Integrals Ex 19.2 Q1

$$\int\left(3x\sqrt{5}+4\sqrt{x}+5\right)dx$$

$$= \int 3x\sqrt{5}dx + \int 4\sqrt{x}dx + \int 5dx$$

$$= \int 3x^{\frac{3}{2}}dx + 4\int x^{\frac{1}{2}}dx + 5\int dx$$

$$= \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \frac{4x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + 5x + c$$

$$= \frac{6}{5}x^{\frac{5}{2}} + \frac{8}{3}x^{\frac{3}{2}} + 5x + c$$

Indefinite Integrals Ex 19.2 Q2

$$\int\left(2^x + \frac{5}{x} - \frac{1}{x^{\frac{1}{3}}}\right)dx$$

$$= \int 2^x dx + 5\int\frac{1}{x}dx - \int\frac{1}{x^{\frac{1}{3}}}dx$$

$$= \frac{2^x}{\log 2} + 5\log x - \frac{3}{2}x^{\frac{2}{3}} + c$$

Indefinite Integrals Ex 19.2 Q3

$$\int\left\{\sqrt{x}\left(ax^2+bx+c\right)\right\}dx$$

$$= \int\sqrt{x}\times ax^2dx + \int\sqrt{x}\times bxdx + \int c\sqrt{x}dx$$

$$= \int ax^{\frac{5}{2}}dx + \int bx^{\frac{3}{2}}dx + \int cx^{\frac{1}{2}}dx$$

$$= \frac{ax^{\frac{5}{2}+1}}{\frac{5}{2}+1} + \frac{bx^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \frac{cx^{\frac{1}{2}+1}}{\frac{1}{2}+1} + d$$

$$= \frac{2ax^{\frac{7}{2}}}{7} + \frac{2bx^{\frac{5}{2}}}{5} + \frac{2cx^{\frac{3}{2}}}{3} + d$$

Indefinite Integrals Ex 19.2 Q4

$$\int(2-3x)(3+2x)(1-2x)\,dx$$

$$= \int\left(6+4x-9x-6x^2\right)(1-2x)dx$$

$$= \int\left(-6x^2-5x+6\right)(1-2x)dx$$

$$= \int\left(-6x^2+12x^3-5x+10x^2+6-12x\right)dx$$

$$= \int\left(4x^2+12x^3-17x+6\right)dx$$

$$= \int\left(12x^3+4x^2-17x+6\right)dx$$

$$= \frac{12}{4}x^4 + \frac{4}{3}x^3 - \frac{17}{2}x^2 + 6x + c$$

$$= 3x^4 + \frac{4}{3}x^3 - \frac{17}{2}x^2 + 6x + c$$

Indefinite Integrals Ex 19.2 Q5

$$\int\left(\frac{m}{x}+\frac{x}{m}+m^x+x^m+mx\right)dx$$

$$= m\int\frac{1}{x}dx + \frac{1}{m}\int xdx + \int m^xdx\int x^mdx + m\int xdx$$

$$= m\log|x| + \frac{x^2}{2m} + \frac{m^x}{\log m} + \frac{x^{m+1}}{m+1} + \frac{mx^2}{2} + c$$

Indefinite Integrals Ex 19.2 Q6

$$\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx$$

$$=\int\left(x+\frac{1}{x}-2\right)dx$$

$$=\int x dx+\int\frac{1}{x}dx-2\int 1.dx$$

$$=\frac{x^2}{2}+\log|x|-2x+C$$

Indefinite Integrals Ex 19.2 Q7

$$\int\frac{(1+x)^3}{\sqrt{x}}dx$$

$$=\int\frac{1+x^3+3x^2+3x}{5x}dx$$

$$=\int\frac{1}{\sqrt{x}}dx+\int\frac{x^3}{\sqrt{x}}dx+\int\frac{3x^2}{\sqrt{x}}dx+\int\frac{3x}{\sqrt{x}}dx$$

$$=\int x^{\frac{-1}{2}}dx+\int x^{\frac{5}{2}}dx+3\int x^{\frac{3}{2}}dx+3\int x^{\frac{1}{2}}dx$$

$$=\frac{x^{\frac{-1}{2}+1}}{\frac{-1}{2}+1}+\frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1}+\frac{3x^{\frac{3}{2}+1}}{\frac{3}{2}+1}+3\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+c$$

$$=\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+\frac{x^{\frac{7}{2}}}{\frac{7}{2}}+\frac{3x^{\frac{5}{2}}}{\frac{5}{2}}+3\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+c$$

$$=2x^{\frac{1}{2}}+\frac{2}{7}x^{\frac{7}{2}}+\frac{6}{5}x^{\frac{5}{2}}+\frac{6}{3}x^{\frac{3}{2}}+c$$

$$=2x^{\frac{1}{2}}+\frac{2}{7}x^{\frac{7}{2}}+\frac{6}{5}x^{\frac{5}{2}}+2x^{\frac{3}{2}}+c$$

$$\therefore\quad \int\frac{(1+x)^3}{\sqrt{x}}dx=2x^{\frac{1}{2}}+\frac{2}{7}x^{\frac{7}{2}}+\cdot\frac{6}{5}\,x^{\frac{5}{2}}+2x^{\frac{3}{2}}+c$$

Indefinite Integrals Ex 19.2 Q8

$$\int\left\{x^2+e^{\log x}+\left(\frac{e}{2}\right)^x\right\}dx$$

$$=\int x^2dx+\int e^{\log x}dx+\int\left(\frac{e}{2}\right)^x dx$$

$$=\frac{x^3}{3}+\int x dx+\int\left(\frac{e}{2}\right)dx$$

$$=\frac{x^3}{3}+\frac{x^2}{2}+\frac{1}{\log\left(\frac{e}{2}\right)}\times\left(\frac{e}{2}\right)^x+c$$

Indefinite Integrals Ex 19.2 Q9

$$\int\left(x^e+e^x+e^e\right)dx$$

$$=\int x^e dx+\int e^x dx+\int e^e dx$$

$$=\frac{x^{e+1}}{e+1}+e^x+e^e x+c \qquad [\because e \text{ is constant}]$$

$$\therefore\quad \int\left(x^e+e^x+e^e\right)dx=\frac{x^{e+1}}{e+1}+e^x+e^e x+c$$

Indefinite Integrals Ex 19.2 Q10

$$\int \sqrt{x}\left(x^3 - \frac{2}{x}\right)dx = \int x^{\frac{7}{2}}dx - 2\int x^{-\frac{1}{2}}dx$$
$$= \frac{x^{\frac{7}{2}+1}}{\frac{7}{2}+1} - 2\frac{x^{\frac{-1}{2}+1}}{\frac{-1}{2}+1} + c$$
$$= \frac{x^{\frac{9}{2}}}{\frac{9}{2}} - \frac{2x^{\frac{-1}{2}}}{\frac{-1}{2}} + c$$
$$= \frac{2}{9}x^{\frac{9}{2}} - 4x^{\frac{-1}{2}} + c$$
$$\therefore \quad \int \sqrt{x}\left(x^3 - \frac{2}{x}\right) = \frac{2}{9}x^{\frac{9}{2}} - 4\sqrt{x} + c$$

Indefinite Integrals Ex 19.2 Q11

$$\int \frac{1}{\sqrt{x}}\left(1 + \frac{1}{x}\right)dx$$
$$= \int\left(\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{x} \times x}\right)dx$$
$$= \int x^{\frac{-1}{2}} + \int x^{\frac{-3}{2}}dx$$
$$= 2x^{\frac{1}{2}} - 2x^{\frac{-1}{2}} + c$$
$$= 2\sqrt{x} - \frac{2}{\sqrt{x}} + c$$
$$\therefore \quad \int \frac{1}{\sqrt{x}}\left(1 + \frac{1}{x}\right)dx = 2\sqrt{x} - \frac{2}{\sqrt{x}} + c$$

Indefinite Integrals Ex 19.2 Q12

$$\int \frac{x^6 + 1}{x^2 + 1}dx$$
$$= \int \frac{\left(x^2\right)^3 + (1)^3}{x^2 + 1}dx$$
$$= \int \frac{\left(x^2 + 1\right)\left(x^4 + 1 - x^2\right)}{x^2 + 1}dx$$
$$= \int \left(x^4 - x^2 + 1\right)dx$$
$$= \int x^4 dx - \int x^2 dx + \int 1 dx$$
$$= \frac{x^5}{5} - \frac{x^3}{3} + x + c$$

Indefinite Integrals Ex 19.2 Q13

$$\int \frac{x^{\frac{-1}{3}} + \sqrt{x} + 2}{\sqrt[3]{x}}dx$$
$$= \int \frac{x^{\frac{-1}{3}}dx}{x^{\frac{1}{3}}} + \int \frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}}dx + \int \frac{2}{x^{\frac{1}{3}}}dx$$
$$= \int x^{\frac{-2}{3}}dx + \int x^{\frac{1}{6}}dx + 2\int x^{\frac{-1}{3}}dx$$
$$= 3x^{\frac{1}{3}} + \frac{6}{7}x^{\frac{7}{6}} + 3x^{\frac{2}{3}} + c$$

Indefinite Integrals Ex 19.2 Q14

$$\int \frac{\left(1+\sqrt{x}\right)^2}{\sqrt{x}}dx$$

$$= \int \frac{1+x+2\sqrt{x}}{x^{\frac{1}{2}}}dx$$
$$= \int x^{\frac{-1}{2}} + \int x^{\frac{1}{2}}dx + 2\int dx$$
$$= 2\sqrt{x} + \frac{2}{3}x^{\frac{3}{2}} + 2x + c$$

$$\therefore \quad \int \frac{\left(1+\sqrt{x}\right)^2}{\sqrt{x}}dx = 2\sqrt{x} + 2x + \frac{2}{3}x^{\frac{3}{2}} + c$$

Indefinite Integrals Ex 19.2 Q15

$$\int \sqrt{x}\,(3-5x)dx$$

$$= 3\int \sqrt{x}dx - 5\int x^{\frac{3}{2}}dx$$
$$= \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - 5\frac{x^{\frac{5}{2}}}{\frac{5}{2}} + c$$

$$= 2x^{\frac{3}{2}} - 2x^{\frac{5}{2}} + c$$

Indefinite Integrals Ex 19.2 Q16

$$\int \frac{(x+1)(x-2)}{\sqrt{x}}dx$$

$$= \int \frac{x^2-2x+x-2}{x^{\frac{1}{2}}}dx$$
$$= \int \frac{x^2-x-2}{x^{\frac{1}{2}}}dx$$
$$= \int \frac{x^2}{x^{\frac{1}{2}}}dx - \int x^{\frac{1}{2}}dx - 2\int x^{\frac{-1}{2}}dx$$
$$= \frac{2x^{\frac{5}{2}}}{5} - \frac{2x^{\frac{3}{2}}}{3} - 4x^{\frac{1}{2}} + c$$

$$\therefore \quad \int \frac{(x+1)(x-2)}{\sqrt{x}}dx = \frac{2}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} - 4x^{\frac{1}{2}} + c$$
$$= \frac{2}{5}x^{\frac{5}{2}} - \frac{2x^{\frac{3}{2}}}{3} - 4\sqrt{x} + c$$

Indefinite Integrals Ex 19.2 Q17

$$\int \frac{x^5+x^{-2}+2}{x^2}dx$$

$$= \int \left(\frac{x^5}{x^2} + \frac{x^{-2}}{x^2} + \frac{2}{x^2}\right)dx$$
$$= \int x^3dx + \int x^{-4} + 2\int x^{-2}dx$$
$$= \frac{x^4}{4} + \frac{x^{-3}}{-3} + \frac{2x^{-1}}{-1} + c$$
$$= \frac{x^4}{4} - \frac{x^{-3}}{3} - \frac{2}{x} + c$$

Indefinite Integrals Ex 19.2 Q18

$\int(3x+4)^2dx$

$$= \int\left(9x^2+16+24x\right)dx$$
$$= 9\int x^2dx + 16\int dx + 24\int xdx$$
$$= 9\frac{x^3}{3} + 16x + 24\frac{x^2}{2} + c$$
$$= 3x^3 + 16x + 12x^2 + c$$

$\therefore \quad \int(3x+4)^2 = 3x^3 + 12x^2 + 16x + c$

Indefinite Integrals Ex 19.2 Q19

$\int\frac{2x^4+7x^3+6x^2}{x^2+2x}dx$

$$= \int\frac{x\left(2x^3+7x^2+6x\right)}{x(x+2)}dx$$
$$= \int\frac{2x^3+7x^2+6c}{x+2}dx$$
$$= \int\frac{2x^3+4x^2+3x^2+6x}{(x+2)}dx$$
$$= \int\frac{2x^2(x+2)+3x(x+2)}{(x+2)}dx$$
$$= \int\frac{(x+2)\left(2x^2+3x\right)}{x+2}dx$$
$$= \int\left(2x^2+3x\right)dx$$
$$= \int 2x^2dx + \int 3xdx$$
$$= \frac{2}{3}x^3 + \frac{3}{2}x^2 + c$$

Indefinite Integrals Ex 19.2 Q20

$\int\frac{5x^4+12x^3+7x^2}{x^2+x}dx$

$$= \int\frac{5x^4+7x^3+5x^3+7x^2}{x^2+x}dx$$
$$= \int\frac{5x^3+7x^2+5x^2+7x}{x+1}dx$$
$$= \int\frac{5x^2(x+1)+7x(x+1)}{x+1}dx$$
$$= \int\left(5x^2+7x\right)dx$$
$$= \frac{5x^3}{3} + \frac{7x^2}{2} + C$$

Indefinite Integrals Ex 19.2 Q21

$\int\frac{\sin^2 x}{1+\cos x}dx$

$$= \int\frac{1-\cos^2 x}{1+\cos x}dx$$
$$= \int\frac{(1-\cos x)(1+\cos x)}{(1+\cos x)}dx$$
$$= \int(1-\cos x)dx$$
$$= x - \sin x + c$$

Indefinite Integrals Ex 19.2 Q22

$$\int\left(\sec^2 x + \operatorname{cosec}^2 x\right)dx$$

$$= \int \sec^2 x dx + \int \operatorname{cosec}^2 x dx$$
$$= \tan x - \cot x + c$$

$$\therefore \quad \int\left(\sec^2 x + \operatorname{cosec}^2 x\right)dx = \tan x - \cot x + c$$

Indefinite Integrals Ex 19.2 Q23

Evaluate the integral as follows

$$\int \frac{\sin^3 x - \cos^3 x}{\sin^2 x \cos^2 x}dx = \int\left(\frac{\sin^3 x}{\sin x^2 \cos^2 x} - \frac{\cos^3 x}{\sin x^2 \cos^2 x}\right)dx$$
$$= \int\left(\sin x \sec^2 x - \cos x \operatorname{cosec}^2 x\right)dx$$
$$= \int\left(\tan x \sec x - \cot x \operatorname{cosec} x\right)dx$$
$$= \sec x + \operatorname{cosec} x + C$$

Indefinite Integrals Ex 19.2 Q24

$$\mathrm{I} \int \frac{5\cos^3 x + 6\sin^3 x}{2\sin^2 x \cos^2 x}dx$$

Now,

$$\mathrm{I} = \int \frac{5\cos^3 x + \sin^3 x}{2\sin^2 x \cos^2 x}dx$$
$$= \int \frac{5\cos^3 x}{2\sin^2 x \cos^2 x}dx + \int \frac{6\sin^3 x}{2\sin^2 x \cos^2 x}dx$$
$$= \frac{5}{2}\int \frac{\cos x}{\sin^2 x}dx + 3\int \frac{\sin x}{\cos^2 x}dx$$
$$= \frac{5}{2}\int \cot x \operatorname{cosec} x dx + 3\int \sec x \tan x dx$$
$$= + \frac{-5}{2}\operatorname{cosec} x + 3\sec x + c$$

$$\therefore \quad \mathrm{I} = \frac{-5}{2}\cos\sec x + 3\sec x + c$$

Indefinite Integrals Ex 19.2 Q25

$$\int\left(\tan x + \cot x\right)^2 dx$$

$$= \int\left(\tan^2 x + \cot^2 x + 2\tan x \cot x\right)dx$$
$$= \int\left(\sec^2 x - 1 + \operatorname{cosec}^2 x - 1 + \frac{2\times 1}{\cot x}\cot x\right)dx$$
$$= \int\left(\sec^2 x + \operatorname{cosec}^2 x\right)dx$$
$$= \int \sec^2 x dx + \int \operatorname{cosec}^2 x$$
$$= \tan x - \cot x + c$$

$$\therefore \quad \int\left(\tan x + \cot x\right)^2 = \tan x - \cot x + c$$

Indefinite Integrals Ex 19.2 Q26

$$\int \frac{1-\cos 2x}{1+\cos 2x}dx$$

$$= \int \frac{2\sin^2 x}{2\cos^2 x}dx$$
$$= \int \tan^2 x dx$$
$$= \int\left(\sec^2 x - 1\right)dx$$
$$= \int \sec^2 x dx - \int dx$$
$$= \tan x - x + c$$

$$\therefore \quad \int \frac{1-\cos 2x}{1+\cos 2x}dx = \tan x - x + c$$

Indefinite Integrals Ex 19.2 Q27

$$\int \frac{\cos x}{1-\cos x}dx$$

$$= \int \frac{\cos x\left(1+\cos x\right)}{\left(1-\cos x\right)\left(1+\cos x\right)}dx$$
$$= \int \frac{\cos x+\cos^2 x}{1-\cos^2 x}dx$$
$$= \int \frac{\cos x+\cos^2 x}{\sin^2 x}dx$$
$$= \int \frac{\cos x}{\sin^2 x}dx + \int \frac{\cos^2 x}{\sin^2 x}dx$$
$$= \int \cot x \times \operatorname{cosec} x dx + \int \left(\operatorname{cosec}^2 x - 1\right)dx$$
$$= -\operatorname{cosec} x - \cot x - x + c$$

$$\therefore \quad \int \frac{\cos x}{1-\cos x} \times dx = -\operatorname{cosec} x - \cot x - x + c$$

Indefinite Integrals Ex 19.2 Q28

$$\int \frac{\cos^2 x - \sin^2 x}{\sqrt{1+\cos 4x}}dx$$

$$= \int \frac{\cos^2 x - \sin^2 x}{\sqrt{2\cos^2 2x}}dx$$
$$= \frac{1}{\sqrt{2}}\int \frac{\cos^2 x - \sin^2 x}{\cos 2x}dx$$
$$= \frac{1}{\sqrt{2}}\int \frac{\cos^2 x - \sin^2 x}{\cos^2 x - \sin^2 x}dx \qquad \left[\because \cos 2x = \cos^2 x - \sin^2 x\right]$$
$$= \frac{1}{\sqrt{2}}\int 1 \times dx$$
$$= \frac{x}{\sqrt{2}} + c$$

$$\therefore \quad \int \frac{\cos^2 x - \sin^2 x}{\sqrt{1+\cos 4x}} \times dx = \frac{x}{\sqrt{2}} + c$$

Indefinite Integrals Ex 19.2 Q29

$$\int \frac{1}{1-\cos x}dx$$

$$= \int \frac{1}{1-\cos x} \times \frac{1+\cos x}{1+\cos x} \times dx$$
$$= \int \frac{1+\cos x}{1-\cos^2 x} \times dx$$
$$= \int \frac{1+\cos x}{\sin^2 x} \times dx$$
$$= \int \frac{1}{\sin^2 x}dx + \int \frac{\cos x}{\sin^2 x}dx$$
$$= \int \operatorname{cosec}^2 x dx + \int \cot x \times \operatorname{cosec} x dx$$
$$= -\cot x - \operatorname{cosec} x + c$$

$$\therefore \quad \int \frac{1}{1-\cos x}dx = -\cot x - \operatorname{cosec} x + c.$$

Indefinite Integrals Ex 19.2 Q30

$$\int \frac{1}{1-\sin x}dx$$

$$= \int \frac{1}{1-\sin x} \times \frac{1+\sin x}{1+\sin x} \times dx$$
$$= \int \frac{1+\sin x}{1-\sin^2 x} \times dx$$
$$= \int \frac{1+\sin x}{\cos^2 x} \times dx$$
$$= \int \left(\frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}\right) \times dx$$
$$=$$
$$= \int \frac{1}{\cos^2 x}dx + \int \frac{\sin x}{\cos^2 x} \times dx$$
$$= \int \sec^2 x dx + \int \tan x \sec x dx$$
$$= \tan x + \sec x + c$$

$$\therefore \quad \int \frac{1}{1-\sin x} \times dx = \tan x + \sec x + c.$$

Indefinite Integrals Ex 19.2 Q31

$$\int \frac{\tan x}{\sec x + \tan x} \times dx$$

$$= \int \frac{\tan x}{\sec x + \tan x} \times \frac{\sec x - \tan x}{\sec x - \tan x} \times dx$$
$$= \int \frac{\tan x \left(\sec x - \tan x\right)}{\sec^2 x - \tan^2 x} \times dx$$
$$= \int \left(\tan x \sec x - \tan^2 x\right) dx$$
$$= \int \sec \tan x dx - \int \left(\sec^2 x - 1\right) dx$$
$$= \int \sec x \tan x dx - \int \sec^2 x dx + \int dx$$
$$= \sec x - \tan x + x + c$$

$$\therefore \quad \int \frac{\tan x}{\sec x + \tan x} \times dx = \sec x - \tan x + x + c.$$

Indefinite Integrals Ex 19.2 Q32

$$\int \frac{\operatorname{cosec} x}{\operatorname{cosec} x - \cot x} \times dx$$

$$= \int \frac{\operatorname{cosec} x}{\operatorname{cosec} x - \cot x} \times \frac{\operatorname{cosec} x + \cot x}{\operatorname{cosec} x + \cot x} \times dx$$
$$= \int \frac{\operatorname{cosec} x \left(\operatorname{cosec} x + \cot x\right)}{\operatorname{cosec}^2 x - \cot^{2x}} \times dx$$
$$= \int \left(\operatorname{cosec}^2 x + \operatorname{cosec} x \cot x\right) dx$$
$$= \int \operatorname{cosec}^2 x dx + \int \operatorname{cosec} x dx$$
$$= -\cot x - \operatorname{cosec} x + c$$

$$\therefore \quad \int \frac{\operatorname{cosec} x}{\operatorname{cosec} x - \cot x} \times dx = -\cot x - \operatorname{cosec} x + c.$$

Indefinite Integrals Ex 19.2 Q33

$$\int \frac{1}{1+\cos 2x} \times dx$$

$$= \int \frac{1}{2\cos^2 x} \times dx$$
$$= \frac{1}{2}\int \sec^2 x \times dx$$
$$= \frac{1}{2} \times \tan x + c$$
$$= \frac{\tan x}{2} + c$$

$$\therefore \quad \int \frac{1}{1+\cos 2x} \times dx = \frac{1}{2}\tan x + c.$$

Indefinite Integrals Ex 19.2 Q34

$$\int \frac{1}{1-\cos 2x}dx$$

$$= \int \frac{1}{2\sin^2 x} \times dx$$
$$= \frac{1}{2}\int co\sec^2 x \times dx$$
$$= \frac{-1}{2} \times \cot x + c$$
$$= \frac{-1\cot x}{2} + c$$

$$\therefore \quad \int \frac{1}{1-\cos 2x} = \frac{-1}{2}\cot x + c.$$

Indefinite Integrals Ex 19.2 Q35

$$\int \tan^{-1}\left[\frac{\sin 2x}{1+\cos 2x}\right]dx$$

$$= \int \tan^{-1}\left[\frac{2\sin x\cos x}{2\cos^2 x}\right]dx$$
$$= \int \tan^{-1}\left[\frac{\sin x}{\cos x}\right]dx$$
$$= \int \tan^{-1}(\tan x)\,dx$$
$$= \int x dx$$
$$= \frac{x^2}{2} + c$$

$$\therefore \quad \int \tan^{-1}\left[\frac{\sin 2x}{1+\cos 2x}\right]dx = \frac{x^2}{2} + c.$$

Indefinite Integrals Ex 19.2 Q36

$$\int \cos^{-1}(\sin x)dx$$

$$= \int \cos^{-1}\left[\cos\left(\frac{\pi}{2} - x\right)\right]dx$$
$$= \int\left(\frac{\pi}{2} - x\right)dx$$
$$= \frac{\pi}{2}\int dx - \int x dx$$
$$= \frac{\pi}{2} \times x - \frac{x^2}{2} + c$$

$$\therefore \quad \int \cos^{-1}(\sin x)dx = \frac{\pi}{2} \times x - \frac{x^2}{2} + c.$$

Indefinite Integrals Ex 19.2 Q37

$$\int \cos^{-1}(\sin x)dx$$

$$= \int \cot^{-1}\left[\frac{\sin 2x}{1-\cos 2x}\right]dx$$
$$= \int \cot^{-1}\left(\frac{\cos x}{\sin x}\right)dx$$
$$= \int \cot^{-1}(\cot x)dx$$
$$= \int x dx$$
$$= \frac{x^2}{2} + c$$

$$\therefore \quad \int \cot^{-1}\left[\frac{\sin 2x}{1-\cos 2x}\right]dx = \frac{x^2}{2} + c.$$

Indefinite Integrals Ex 19.2 Q38

$$\int \sin^{-1}\left(\frac{2\tan x}{1+\tan^2 x}\right)dx$$

$$= \int \sin^{-1}(\sin 2x)dx \qquad \left[\because \sin 2x = \frac{2\tan x}{1+\tan^2 x}\right]$$
$$= \int 2x dx$$
$$= 2\int x\, dx$$
$$= \frac{2x^2}{2} + c$$
$$= x^2 + c$$

$$\therefore \quad \int \sin^{-1}\left(\frac{2\tan x}{1+\tan^2 x}\right) = x^2 + c.$$

Indefinite Integrals Ex 19.2 Q39

$$\int \frac{\left(x^3+8\right)(x-1)}{x^2-2x+4}dx$$

$$= \int \frac{(x+2)\left(x^2-2x+4\right)(x-1)}{x^2-2x+4}dx$$
$$= \int (x+2)(x-1)\,dx$$
$$= \int \left(x^2 - x + 2x - 2\right)dx$$
$$= \int \left(x^2 + x - 2\right)dx$$
$$= \frac{x^3}{3} + \frac{x^2}{2} - 2x + c$$

$$\therefore \quad \int \frac{\left(x^3+8\right)(x-1)}{x^2-2x+4}dx = \frac{x^3}{3} + \frac{x^2}{2} - 2x + c.$$

Indefinite Integrals Ex 19.2 Q40

$$\int (a\tan x + b\cot x)^2 dx$$

$$= \int \left(a^2\tan^2 x + b^2\cot^2 x + 2ab\tan x\cot x\right)dx$$
$$= \int \left[a^2\left(\sec^2 x - 1\right) + b^2\left(\operatorname{cosec}^2 x - 1\right) + 2ab\right]dx$$
$$= \int \left[a^2\sec^2 x - a^2 + b^2\operatorname{cosec}^2 x - b^2 + 2ab\right]dx$$
$$= a^2\tan x - a^2 x - b^2\cot x - b^2 x + 2abx + c$$
$$= a^2\tan x + -b^2\cot x - \left(a^2 + b^2 - 2ab\right)x + c$$

$$\therefore \quad \int (a\tan x + b\cot x)^2 dx = a^2\tan x - b^2\cot x - \left(a^2 + b^2 - 2ab\right)x + c.$$

Indefinite Integrals Ex 19.2 Q41

$$\int \frac{x^3 - 3x^2 + 5x - 7 + x^2 a^x}{2x^2}dx$$

$$= \frac{1}{2}\int \frac{x^3}{x^2}dx - \frac{3}{2}\int \frac{x^2}{x^2}dx + \frac{5}{2}\int x\,\frac{x}{x^2}dx - \frac{7}{2}\int x^{-2}dx + \frac{1}{2}\int \frac{x^2 a^x}{x^2}dx$$
$$= \frac{1}{2}\times\frac{x^2}{2} - \frac{3}{2}x + \frac{5}{2}\log x - \frac{7}{2}x^{-1} + \frac{1}{2}\frac{a^x}{\log a} + c$$
$$= \frac{1}{2}\left[\frac{x^2}{2} - 3x + 5\log x + \frac{7}{x} + \frac{a^x}{\log a}\right] + c$$

$$\therefore \quad \int \frac{x^3 - 3x^2 + 5x - 7 + x^2 a^x}{2x^2}dx = \frac{1}{2}\left[\frac{x^2}{2} - 3x + 5\log x + \frac{7}{x} + \frac{a^x}{\log a}\right] + c$$

Indefinite Integrals Ex 19.2 Q42

$$\frac{\cos x}{1+\cos x}=\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}} \qquad \left[\cos x=\cos^2\frac{x}{2}-\sin^2\frac{x}{2} \text{ and } \cos x=2\cos^2\frac{x}{2}-1\right]$$

$$=\frac{1}{2}\left[1-\tan^2\frac{x}{2}\right]$$

$$\therefore \int\frac{\cos x}{1+\cos x}dx=\frac{1}{2}\int\left(1-\tan^2\frac{x}{2}\right)dx$$

$$=\frac{1}{2}\int\left(1-\sec^2\frac{x}{2}+1\right)dx$$

$$=\frac{1}{2}\int\left(2-\sec^2\frac{x}{2}\right)dx$$

$$=\frac{1}{2}\left[2x-\frac{\tan\frac{x}{2}}{\frac{1}{2}}\right]+C$$

$$=x-\tan\frac{x}{2}+C$$

Indefinite Integrals Ex 19.2 Q43

$$\frac{1-\cos x}{1+\cos x}=\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}} \qquad \left[2\sin^2\frac{x}{2}=1-\cos x \text{ and } 2\cos^2\frac{x}{2}=1+\cos x\right]$$

$$=\tan^2\frac{x}{2}$$

$$=\left(\sec^2\frac{x}{2}-1\right)$$

$$\therefore \int\frac{1-\cos x}{1+\cos x}dx=\int\left(\sec^2\frac{x}{2}-1\right)dx$$

$$=\left[\frac{\tan\frac{x}{2}}{\frac{1}{2}}-x\right]+C$$

$$=2\tan\frac{x}{2}-x+C$$

Indefinite Integrals Ex 19.2 Q44

$$\int\left\{3\sin x-4\cos x+\frac{5}{\cos^2 x}-\frac{6}{\sin^2 x}+\tan^2 x-\cot^2 x\right\}dx$$

$$=3\int\sin x dx-4\int\cos x dx+5\int\sec^2 dx-6\int\operatorname{cosec}^2 x+\int\tan^2 x dx-\int\cot^2 x dx$$

$$=3\int\sin x dx-4\int\cos x dx+5\int\sec^2 x dx-6\int\operatorname{cosec}^2 x+\int\left(\sec^2 x-1\right)dx-\int\left(\operatorname{cosec}^2 x-1\right)dx$$

$$=3\int\sin x dx-4\int\cos x dx+6\int\sec^2 x dx-7\int\operatorname{cosec}^2 x dx$$

$$=-3\cos x-4\sin x+6\tan x+7\cot x+c$$

Indefinite Integrals Ex 19.2 Q45

It is given that $f'(x) = x - \frac{1}{x^2}$

$$\therefore \quad \int f'(x) = \int \left(x - \frac{1}{x^2}\right)dx$$

$$\Rightarrow \quad f(x) = \int x dx - \int \frac{1}{x^2}dx$$

$$= \frac{x^2}{2} + x^{-1} + c$$

$$= \frac{x^2}{2} + \frac{1}{x} + c$$

$$\Rightarrow \quad f(x) = \frac{x^2}{2} + \frac{1}{x} + c \qquad \text{---(i)}$$

Now,

$$f(1) = \frac{1}{2} \qquad [\text{given}]$$

$$\Rightarrow \quad \frac{1^2}{2} + \frac{1}{1} + c = \frac{1}{2}$$

$$\Rightarrow \quad c = -1$$

Putting $c = -1$ in (i), we get

$$f(x) = \frac{x^2}{2} + \frac{1}{x} - 1.$$

Indefinite Integrals Ex 19.2 Q46

It is given that $f'(x) = x + b$

$\therefore \quad \int f'(x) = \int (x+b)dx$

$\Rightarrow \quad f(x) = \frac{x^2}{2} + bx + c \qquad$ ---(i)

Since,

$f(1) = 5$

$\therefore \quad \frac{1^2}{2} + b \times 1 + c = 5$

$\Rightarrow \quad \frac{1}{2} + b + c = 5$

$\Rightarrow \quad b + c = \frac{9}{2} \qquad$ ---(ii)

and, $\quad f(2) = 13$

$\Rightarrow \quad \frac{(2)^2}{2} + b \times 2 + c = 13$

$\Rightarrow \quad 2 + 2b + c = 13$

$\Rightarrow \quad 2b + c = 11 \qquad$ ---(iii)

Subtracting equation (ii) from equatio n(iii), we get

$b = 11 - \frac{9}{2}$

$\Rightarrow \quad b = \frac{13}{2}$

Putting $b = \frac{13}{2}$ in equation (ii), we get

$\frac{13}{2} + c = \frac{9}{2}$

$\Rightarrow \quad c = \frac{9}{2} - \frac{13}{2}$

$\Rightarrow \quad c = \frac{9-13}{2} = \frac{-4}{2} = -2$

Putting $b = \frac{13}{2}$ and $c = -2$ in equation (i), we get

$f(x) = \frac{x^2}{x} + \frac{13}{2}x - 2$

$\therefore \quad f(x) = \frac{x^2}{2} + \frac{13}{2}x - 2$

Indefinite Integrals Ex 19.2 Q47

We have,

$f'(x) = 8x^3 - 2x$

$\Rightarrow \quad f(x) = \int f'(x)dx = \int (8x^3 - 2x)dx$

$\Rightarrow \quad f(x) = \int (8x^3 - 2x)dx$

$= \int 8x^3 dx - \int 2x dx$

$= \frac{8x^4}{4} - \frac{2x^2}{2} + c$

$= 2x^4 - x^2 + c$

$\Rightarrow \quad f(x) = 2x^4 - x^2 + c \qquad$ ---(i)

Since, $f(2) = 8$

$\therefore \quad f(2) = 2(2)^4 - (2)^2 + c = 8$

$\Rightarrow \quad 32 - 4 + c = 8$

$\Rightarrow \quad 28 + c = 8$

$\Rightarrow \quad c = -20$

Putting $c = -20$ in equation (i), we get

$f(x) = 2x^4 - x^2 - 20$

Hence, $f(x) = 2x^4 - x^2 - 20$.

Indefinite Integrals Ex 19.2 Q48

We have,

$$f(x) = \int f'(x)\,dx$$

$$\Rightarrow \quad f(x) = \int (a\sin x + b\cos x)\,dx$$

$$= -a\cos x + b\sin x + c$$

$$\therefore \quad f(x) = -a\cos x + b\sin x + c \qquad \text{---(i)}$$

Since,

$$f'(0) = 4$$

$$\therefore \quad f'(0) = a\sin 0 + b\cos 0 = 4$$

$$\Rightarrow \quad a \times 0 + b \times 1 = 4$$

$$\Rightarrow \quad b = 4$$

Now,

$$f(0) = 3$$

$$\therefore \quad f(0) = -a\cos 0 + b\sin 0 + c = 3$$

$$\Rightarrow \quad -a + 0 + c = 3$$

$$\Rightarrow \quad c - a = 3 \qquad \text{---(ii)}$$

and, $f\left(\frac{\pi}{2}\right) = 5$

$$\therefore \quad f\left(\frac{\pi}{2}\right) = -a\cos\left(\frac{\pi}{2}\right) + b\sin\left(\frac{\pi}{2}\right) + c = 5$$

$$\Rightarrow \quad -a \times 0 + b \times 1 + c = 5$$

$$\Rightarrow \quad b + c = 5$$

$$\Rightarrow \quad 4 + c = 5 \qquad [\because b = 4]$$

$$\Rightarrow \quad c = 5 - 4$$

$$\Rightarrow \quad c = 1$$

Putting $c = 1$ in equation (ii), we get

$$1 - a = 3$$

$$\Rightarrow \quad -a = 3 - 1$$

$$\Rightarrow \quad -a = 2$$

$$\Rightarrow \quad a = -2$$

Putting $a = -2$, $b = 4$ and $c = 1$ in equatio (i), we get

$$f(x) = -(-2)\cos x + 4\sin x + 1$$

$$\Rightarrow \quad f(x) = 2\cos x + 4\sin x + 1$$

Hence, $f(x) = 2\cos x + 4\sin x + 1$

Indefinite Integrals Ex 19.2 Q49

We have,

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$$

$$\Rightarrow \quad \int f(x) = \int\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)dx$$

$$= \int x^{\frac{1}{2}}dx + \int x^{\frac{-1}{2}}dx$$

$$= \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$$

Hence, the primitive or anti-derivative of $f(x) = \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$.

Ex 19.3

Indefinite Integrals Ex 19.3 Q1

Let $I = \int [2x-3]^5 + \sqrt{3x+2}dx$. Then,

$$I = \int (2x-3)^5 dx + \int (3x+2)^{\frac{1}{2}} dx$$
$$= \frac{(2x-3)^6}{2\times 6} + \frac{(3x+2)^{\frac{3}{2}}}{3\times\frac{3}{2}} + c$$
$$= \frac{(2x-3)^6}{12} + \frac{2}{9}(3x+2)^{\frac{3}{2}} + c$$

$$\therefore \quad I = \frac{(2x-3)^6}{12} + \frac{2}{9}(3x+2)^{\frac{3}{2}} + c$$

Indefinite Integrals Ex 19.3 Q2

Let $I = \int \left[\frac{1}{(7x-5)^3} + \frac{1}{\sqrt{5x-4}}\right]dx$. Then,

$$I = \int (7x-5)^{-3} dx + \int (5x-4)^{\frac{-1}{2}} dx$$
$$= \frac{(7x-5)^{-2}}{7\times(-2)} + \frac{(5x-4)^{\frac{1}{2}}}{5\times\frac{1}{2}} + c$$
$$= -\frac{(7x-5)^{6-2}}{14} + \frac{2}{5}\sqrt{(5x-4)} + c$$

$$\therefore \quad I = \frac{-1}{14}(7x-5)^{-2} + \frac{2}{5}\times\sqrt{5x-4} + c.$$

Indefinite Integrals Ex 19.3 Q3

Let $I = \int \frac{1}{2-3x} + \frac{1}{\sqrt{3x-2}}dx$. Then,

$$I = \int \frac{1}{2-3x}dx + \int \frac{1}{\sqrt{3x-2}}dx$$
$$= \frac{\log|2-3x|}{-3} + \frac{2}{3}(3x-2)^{\frac{1}{2}} c$$
$$= \frac{-1}{3}\times\log|2x-3| + \frac{2}{3}\times\sqrt{3x-2} + c$$

Indefinite Integrals Ex 19.3 Q4

Let $I = \int \frac{x+3}{(x+1)^4}dx$. Then,

$$I = \int \frac{x+1+2}{(x+1)^4}dx$$
$$= \int \frac{x+1}{(x+1)^4}\times dx + 2\int \frac{1}{(x+1)^4}\times dx$$
$$= \int \frac{1}{(x+1)^3}\times dx + 2\int \frac{1}{(x+1)^4}\times dx$$
$$= \int (x+1)^{-3}\times dx + 2\int (x-1)^{-4} dx$$
$$= \frac{(x+1)^{-2}}{-2} + 2\frac{(x+1)^{-3}}{-3} + c$$
$$= -\frac{1}{2}\times\frac{1}{(x+1)^2} - \frac{2}{3}\times\frac{1}{(x+1)^3} + c$$

$$\therefore \quad I = \frac{-1}{2(x+1)^2} - \frac{2}{3(x+1)^3} + c$$

Indefinite Integrals Ex 19.3 Q5

Let $I = \int \frac{1}{\sqrt{x+1}+\sqrt{x}}\,dx$. Then,

$$I = \int \frac{1}{\sqrt{x+1}+\sqrt{x}} \times \frac{\sqrt{x+1}-\sqrt{x}}{\sqrt{x+1}-\sqrt{x}} \times dx$$
$$= \int \frac{\sqrt{x+1}-\sqrt{x}}{\left(\sqrt{x+1}\right)^2-\left(\sqrt{x}\right)^2} \times dx$$
$$= \int \frac{\sqrt{x+1}-\sqrt{x}}{x+1-x} \times dx$$
$$= \int \left(\sqrt{x+1}-\sqrt{x}\right) \times dx$$
$$= \int (x+1)^{\frac{1}{2}}\,dx - \int x^{\frac{1}{2}}dx$$
$$= \frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{2}{3}x^{\frac{3}{2}} + c$$

$$\therefore \quad I = \frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{2}{3}x^{\frac{3}{2}} + c.$$

Indefinite Integrals Ex 19.3 Q6

Let $I = \int \frac{1}{\sqrt{2x+3}+\sqrt{2x-3}}\,dx$. Then,

$$I = \int \frac{1}{\sqrt{2x+3}+\sqrt{2x-3}} \times \frac{\sqrt{2x+3}-\sqrt{2x-3}}{\sqrt{2x+3}-\sqrt{2x-3}} \times dx$$
$$= \int \frac{\sqrt{2x+3}-\sqrt{2x-3}}{\left(\sqrt{2x+3}\right)^2-\left(\sqrt{2x-3}\right)^2} \times dx$$
$$= \int \frac{\sqrt{2x+3}-\sqrt{2x-3}}{2x+3-2x+3} \times dx$$
$$= \frac{1}{6}\int (2x+3)^{\frac{1}{2}}dx - \frac{1}{6}\int (2x-3)^{\frac{1}{2}}\,dx$$
$$= \frac{1}{6} \times \frac{(2x+3)^{\frac{3}{2}}}{\frac{3}{2}\times 2} - \frac{1}{6}\,\frac{(2x-3)^{\frac{3}{2}}}{\frac{3}{2}\times 2} + c$$
$$= \frac{1}{18} \times (2x+3)^{\frac{3}{2}} - \frac{1}{18}(2x-3)^{\frac{3}{2}} + c$$

$$\therefore \quad I = \frac{1}{18}(2x+3)^{\frac{3}{2}} - \frac{1}{18}(2x-3)^{\frac{3}{2}} + c.$$

Indefinite Integrals Ex 19.3 Q7

Let $I = \int \frac{2x}{(2x+1)^2}\,dx$. Then,

$$I = \int \frac{2x+1-1}{(2x+1)^2} \times dx$$
$$= \int \frac{2x+1}{(2x+1)^2} \times dx - \int \frac{1}{(2x+1)^2} \times dx$$
$$= \int \frac{1}{2x+1} \times dx - \int (2x+1)^{-2} \times dx$$
$$= \frac{1}{2}\log|2x+1| - \frac{(2x+1)^{-1}}{-1\times 2} + c$$
$$= \frac{1}{2}\log|2x+1| + \frac{1}{2} \times \frac{1}{2x+1} + c$$

$$\therefore \quad I = \frac{1}{2}\log|2x+1| + \frac{1}{2(2x+1)} + c.$$

Indefinite Integrals Ex 19.3 Q8

Let $I = \int \frac{1}{\sqrt{x+a}+\sqrt{x+b}} dx$. Then,

$$I = \int \frac{1}{\sqrt{x+a}+\sqrt{x+b}} \times \frac{\sqrt{x+a}-\sqrt{x+b}}{\sqrt{x+a}-\sqrt{x+b}} \times dx$$
$$= \int \frac{\sqrt{x+a}-\sqrt{x+b}}{x+a-x-b} \times dx$$
$$= \int \frac{\sqrt{x+a}-\sqrt{x+b}}{a-b} \times dx$$
$$= \frac{1}{a-b}\left[\frac{2}{3}(x+a)^{\frac{3}{2}} - \frac{2}{3}(x+b)^{\frac{3}{2}}\right] + c$$
$$= \frac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}}\right] + c$$

$$\therefore \quad I = \frac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}}\right] + c.$$

Indefinite Integrals Ex 19.3 Q9

Let $I = \int \sin\sqrt{1+\cos 2x}\ dx$

$$I = \int \sin x \times \sqrt{2\cos^2 x} \times dx$$
$$= \int \sin x \times \sqrt{2} \times \cos x \times dx$$
$$= \sqrt{2}\int \sin x \times \cos x \times dx$$
$$= \frac{\sqrt{2}}{2}\int 2\sin x \times \cos x \times dx$$
$$= \frac{\sqrt{2}}{2}\int \sin 2x dx$$
$$= \frac{\sqrt{2}}{2} \times \frac{-\cos 2x}{2} + c$$
$$= \frac{-1}{2\sqrt{2}} \times \cos 2x + c$$

$$\therefore \quad I = \frac{-1}{2\sqrt{2}} \times \cos 2x + c$$

Indefinite Integrals Ex 19.3 Q10

Let $I = \int \frac{1+\cos x}{1-\cos x} dx$. Then,

$$I = \int \frac{2\cos^2\frac{x}{2}}{2\sin^2\frac{x}{2}} \times dx$$
$$= \int \frac{\cos^2\frac{x}{2}}{\sin^2\frac{x}{2}} \times dx$$
$$= \int \cot^2\frac{x}{2} \times dx$$
$$= \int \left(\operatorname{cosec}^2\frac{x}{2} - 1\right) dx$$
$$= \frac{-\cot\frac{x}{2}}{\frac{1}{2}} - x + c$$
$$= -2\cot\frac{x}{2} - x + c$$

Indefinite Integrals Ex 19.3 Q11

Let $I = \int \frac{1-\cos x}{1+\cos x} \times dx$. Then,

$$I = \int \frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}} \times dx$$

$$= \int \frac{\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}} \times dx$$

$$= \int \tan^2\frac{x}{2} dx$$

$$= \int \left(\sec^2\frac{x}{2} - 1\right) dx$$

$$= \frac{\tan\frac{x}{2}}{\frac{1}{2}} - x + c$$

$$= 2\tan\frac{x}{2} - x + c$$

Indefinite Integrals Ex 19.3 Q12

Let $I = \int \frac{1}{1-\sin\frac{x}{2}}$. Then,

$$I = \int \frac{1}{1-\sin\frac{x}{2}} \times \frac{1+\sin\frac{x}{2}}{1+\sin\frac{x}{2}} dx$$

$$= \int \frac{1+\sin\frac{x}{2}}{1-\sin^2\frac{x}{2}} \times dx$$

$$= \int \frac{1+\sin\frac{x}{2}}{\cos^2\frac{x}{2}} \times dx$$

$$= \int \frac{1}{\cos^2\frac{x}{2}} dx + \int \frac{\sin\frac{x}{2}}{\cos^2\frac{x}{2}} dx$$

$$= \int \sec^2\frac{x}{2} dx + \int \sec\frac{x}{2}\tan\frac{x}{2} dx$$

$$= \frac{\tan\frac{x}{2}}{\frac{1}{2}2} + + \frac{\sec\frac{x}{2}}{\frac{1}{2}} + c$$

$$= 2\tan\frac{x}{2} + 2\sec\frac{x}{2} + c$$

$$\therefore \quad I = 2\left(\tan\frac{x}{2} + \sec\frac{x}{2}\right) + c$$

Indefinite Integrals Ex 19.3 Q13

Let $I = \int \frac{1}{1+\cos 3x} \times dx$. Then,

$$I = \int \frac{1}{1+\cos 3x} \times \frac{1-\cos 3x}{1-\cos 3x} \times dx$$

$$= \int \frac{1-\cos 3x}{1-\cos^2 3x} \times dx$$

$$= \int \frac{1-\cos 3x}{\sin^2 3x} \times dx$$

$$= \int \left(\frac{1}{\sin^2 3x} - \frac{\cos 3x}{\sin^2 3x}\right) dx$$

$$= \int \left(\operatorname{cosec}^2 3x - \operatorname{cosec} 3x \cot 3x\right) dx$$

$$= \frac{-\cot 3x}{3} + \frac{\operatorname{cosec} 3x}{3} + c$$

$$= \frac{-1}{3} \times \frac{\cos 3x}{\sin 3x} + \frac{1}{3} \times \frac{1}{\sin 3x} + c$$

$$= \frac{1-\cos 3x}{3\sin 3x} + c$$

$$\therefore \quad I = \frac{1-\cos 3x}{3\sin 3x} + c.$$

Indefinite Integrals Ex 19.3 Q14

Consider $I = \int (e^x+1)^2 e^x dx$

let $(e^x+1) = t \to e^x dx = dt$

$$I = \int (e^x+1)^2 e^x dx$$

$$= \int (t)^2 dt$$

$$= \frac{t^3}{3} + C$$

$$= \frac{(e^x+1)^3}{3} + C$$

Indefinite Integrals Ex 19.3 Q15

Let $I = \int \left(e^x + \frac{1}{e^x}\right)^2 dx$. Then,

$$I = \int \left(e^x + \frac{1}{e^x}\right)^2 \times dx$$

$$= \int \left(e^{2x} + \frac{1}{e^{2x}} + 2\right) dx$$

$$= \frac{e^{2x}}{2} - \frac{1}{2}e^{-2x} + 2x + c$$

$$\therefore \quad I = \frac{1}{2} \times e^{2x} + 2x - \frac{1}{2} \times e^{-2x} + c$$

Indefinite Integrals Ex 19.3 Q16

Let $I = \int \frac{1+\cos 4x}{\cot x - \tan x} \times dx$. Then,

$$I = \int \frac{2\cos^2 2x}{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}} dx$$

$$= \int \frac{2\cos^2 2x}{\frac{\cos^2 x - \sin^2 x}{\sin x \cos x}} dx$$

$$= \int \frac{2\cos^2 2x \times \sin x \cos x}{\cos^2 x - \sin^2 x} dx$$

$$= \int \frac{\cos^2 2x \times \sin 2x}{\cos^2 2x} dx$$

$$= \int \cos 2x \times \sin 2x \times dx$$

$$= \frac{1}{2} \int 2 \sin 2x \cos 2x dx$$

$$= \frac{1}{2} \int \left[\sin(2x+2x) + \sin(2x-2x)\right] dx$$

$$= \frac{1}{2} \int (\sin 4x + \sin 0) dx$$

$$= \frac{1}{2} \int (\sin 4x + 0) dx$$

$$= \frac{1}{2} \int \sin 4x$$

$$= -\frac{1}{2} \times \frac{\cos 4x}{4} + c$$

$$= -\frac{1}{8} \times \cos 4x + c$$

Indefinite Integrals Ex 19.3 Q17

Let $I = \int \frac{1}{\sqrt{x+3} - \sqrt{x+2}} \, dx$. Then,

$$I = \int \frac{1}{\sqrt{x+3} - \sqrt{x+2}} \times \frac{\sqrt{x+3} + \sqrt{x+2}}{\sqrt{x+3} + \sqrt{x+2}} dx$$

$$= \int \frac{\sqrt{x+3} + \sqrt{x+2}}{x+3-x-2} dx$$

$$= \int \left[(x+3)^{\frac{1}{2}} + (x+2)^{\frac{1}{2}}\right] dx$$

$$= \frac{(x+3)^{\frac{3}{2}}}{\frac{3}{2}} + \frac{(x+2)^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{2}{3} \times (x+3)^{\frac{3}{2}} + \frac{2}{3}(x+2)^{\frac{3}{2}} + c$$

$$= \frac{2}{3}\left\{(x+3)^{\frac{3}{2}} + (x+2)^{\frac{3}{2}}\right\} + c$$

$$\therefore \quad I = \frac{2}{3}\left\{(x+3)^{\frac{3}{2}} + (x+2)^{\frac{3}{2}}\right\} + c$$

Indefinite Integrals Ex 19.3 Q18

$\tan^2(2x-3) = \sec^2(2x-3) - 1$

Let $2x - 3 = t$

$\Rightarrow 2dx = dt$

$$\Rightarrow \int \tan^2(2x-3) dx = \int \left[\left(\sec^2(2x-3)\right) - 1\right] dx$$

$$= \frac{1}{2} \int \left(\sec^2 t\right) dt - \int 1 dx$$

$$= \frac{1}{2} \int \sec^2 t \, dt - \int 1 dx$$

$$= \frac{1}{2} \tan t - x + C$$

$$= \frac{1}{2} \tan(2x-3) - x + C$$

Indefinite Integrals Ex 19.3 Q19

Consider $I = \int \frac{1}{\cos^2 x\left(1-\tan x\right)^2} dx$

$$= \int \frac{1}{\cos^2 x\left(1-\frac{\sin x}{\cos x}\right)^2} dx$$

$$= \int \frac{1}{\left(\cos x-\sin x\right)^2} dx$$

$$= \int \frac{1}{1-\sin 2x} dx$$

$$= \int \frac{1}{1+\cos\left(\frac{\pi}{2}+2x\right)} dx$$

$$= \int \frac{1}{2\cos^2\left(\frac{\pi}{4}+x\right)} dx$$

$$= \frac{1}{\;}\int \sec^2\left(\frac{\pi}{\;}+x\right) dx$$

Ex 19.4

Indefinite Integrals Ex 19.4 Q1

Let $I = \int \frac{x^2 + 5x + 2}{x + 2} dx$

Using long division method, we have

$$\frac{x^2 + 5x + 2}{x + 2} = x + 3 - \frac{4}{x + 2}$$

$$\therefore \quad I = \int \frac{x^2 + 5x + 2}{x + 2} = \int \left(x + 3 - \frac{4}{x + 2}\right) dx$$

$$\Rightarrow \quad I = \int x dx + 3\int dx - 4\int \frac{1}{x + 2} dx$$

$$= \frac{x^2}{2} + 3x - 4\log|x + 2| + c$$

$$\therefore \quad I = \frac{x^2}{2} + 3x - 4\log|x + 2| + c$$

Indefinite Integrals Ex 19.4 Q2

Let $I = \int \frac{x^3}{x - 2} dx$

Using long division method, we have

$$\frac{x^3}{x - 2} = x^2 + 2x + 4 + \frac{8}{x - 2}$$

$$\therefore \quad I = \int \left(x^2 + 2x + 4 + \frac{8}{x - 2}\right) dx$$

$$= \int x^2 dx + 2\int x dx + 4\int dx + 8\int \frac{1}{x - 2} dx$$

$$= \frac{x^3}{3} + \frac{2x^2}{2} + 4x + 8\log|x - 2| + c$$

$$= \frac{x^3}{3} + x^2 + 4x + 8\log|x - 2| + c$$

$$\therefore \quad I = \frac{x^3}{3} + x^2 + 4x + 8\log|x - 2| + c$$

Indefinite Integrals Ex 19.4 Q3

Let $I = \int \frac{x^2 + x + 5}{3x + 2} dx$

Using long division method, we have

$$\frac{x^2 + x + 5}{3x + 2} = \frac{x}{3} + \frac{1}{9} + \frac{43}{9} \times \frac{1}{3x + 2}$$

$$\therefore \quad I = \int \left(\frac{x}{3} + \frac{1}{9} + \frac{43}{9} \times \frac{1}{3x + 2}\right) dx$$

$$= \frac{x^2}{6} + \frac{1}{9} \times x + \frac{43}{9 \times 3} |3x + 2| + c$$

$$= \frac{x^2}{6} + \frac{1}{9} \times x + \frac{43}{27} \times \log|3x + 2| + c$$

$$\therefore \quad I = \frac{x^2}{6} + \frac{1}{9} \times x + \frac{43}{27} \times \log|3x + 2| + c$$

Indefinite Integrals Ex 19.4 Q4

Let $I = \int \frac{2x+3}{(x-1)^2} dx$. Then,

$$I = \int \frac{2x+2-2+3}{(x-1)^2} \times dx$$
$$= \int \frac{2x-2+5}{(x-1)^2} \times dx$$
$$= 2\int \frac{(x-1)}{(x-1)^2} \times dx + 5\int \frac{1}{(x-1)^2} dx$$
$$= 2\int \frac{1}{x-1} \times dx + 5\int (x-1)^{-2} \times dx$$
$$= 2\log|x-1| + 5 \times \frac{(x-1)^{-1}}{-1} + c$$
$$= 2\log|x-1| - \frac{5}{x-1} + c$$

$$\therefore \quad I = 2\log|x-1| - \frac{5}{x-1} + c.$$

Indefinite Integrals Ex 19.4 Q5

Let $I = \int \frac{x^2+3x-1}{(x+1)^2} dx$. Then,

$$I = \int \frac{x^2+x+2x-1}{(x+1)^2} dx$$
$$= \int \frac{x(x+1)+2x-1}{(x+1)^2} dx$$
$$= \int \frac{x(x+1)}{(x+1)^2} dx + \int \frac{2x-1}{(x+1)^2} dx$$
$$= \int \frac{x}{x+1} dx + \int \frac{\sqrt{2x+2-2-1}}{(x+1)^2} dx$$
$$= \int \frac{x+1-1}{x+1} dx + \int \frac{2(x+1)-3}{(x+1)^2} dx$$
$$= \int \frac{x+1}{x+1} dx - \int \frac{1}{x+1} dx + \int \frac{2(x+1)}{(x+1)^2} dx - 3\int \frac{1}{(x+1)^2} dx$$
$$= \int dx - \int \frac{1}{x+1} dx + 2\int \frac{1}{x+1} dx - 3\int (x+1)^{-2} dx$$
$$= x - \log|x+1| + 2\log|x+1| + \frac{3}{x+1} + c$$
$$= x + \log|x+1| + \frac{3}{x+1} + c$$

$$\therefore \quad I = x + \log|x+1| + \frac{3}{x+1} + c$$

Indefinite Integrals Ex 19.4 Q6

Let $I = \int \frac{2x-1}{(x-1)^2} dx$. Then,

$$I = \int \frac{2x-1+2-2}{(x-1)^2} dx$$
$$= \int \frac{2x-2+1}{(x-1)^2} dx$$
$$= \int \frac{2(x-1)}{(x-1)^2} dx + 1\int \frac{1}{(x-1)^2} dx$$
$$= 2\int \frac{1}{x-1} dx + \int (x-1)^2 dx$$
$$= 2\log|x-1| - (x-1)^{-1} + c$$
$$= 2\log|x-1| - \frac{1}{x-1} + c$$

$$\therefore \quad I = \frac{-1}{x-1} + 2\log|x-1| + c.$$

Ex 19.5

Indefinite Integrals Ex 19.5 Q1

Let $I = \int \frac{x+1}{\sqrt{2x+3}}dx$

Let $x + 1 = \lambda(2x+3) + \mu$

On equating the coefficients of like powers of x on bothy sides, we get

$$I = 2\lambda,\ 3\lambda + \mu = 1$$

$$\Rightarrow \quad \lambda = \frac{1}{2} \text{ and } 3 \times \frac{1}{2} + \mu = 1$$

$$\Rightarrow \quad \lambda = \frac{1}{2} \text{ and } \mu = \frac{-1}{2}$$

Replacing $x + 1$ by $\lambda(2x+3) + \mu$ in the given integral, we get

$$I = \int \frac{\lambda(2x+3)+\mu}{\sqrt{2x+3}}dx$$

$$= \int \frac{\lambda(2x+3)}{\sqrt{2x+3}}dx + \mu\int \frac{1}{\sqrt{2x+3}}dx$$

$$= \lambda\int (2x+3)^{\frac{1}{2}}dx + \mu\int (2x+3)^{\frac{-1}{2}}dx$$

$$= \lambda\frac{(2x+3)^{\frac{3}{2}}}{\frac{3}{2}\times 2} + \mu\frac{(2x+3)^{\frac{1}{2}}}{2\times\frac{1}{2}} + c$$

$$= \frac{1}{2}\times\frac{(2x+3)^{\frac{3}{2}}}{3} + \left(\frac{-1}{2}\right)\times(2x+3)^{\frac{1}{2}} + c \qquad \left[\because \lambda = \frac{1}{2}, \mu = \frac{-1}{2}\right]$$

$$= \frac{(2x+3)^{\frac{3}{2}}}{6} - \frac{(2x+3)^{\frac{1}{2}}}{2} + c$$

$$\therefore \quad I = \frac{1}{6}\times(2x+3)^{\frac{3}{2}} - \frac{1}{2}(2x+3)^{\frac{1}{2}} + c.$$

Indefinite Integrals Ex 19.5 Q2

Let $I = \int x\sqrt{x+2}dx$. Then,

$$I = \int\{(x+2)-2\}\sqrt{x+2}dx \qquad [\because x = (x+2)-2]$$

$$\Rightarrow \quad I = f\left\{(x+2)^{\frac{3}{2}} - 2(x+2)^{\frac{1}{2}}\right\}dx$$

$$\Rightarrow \quad I = \frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + c$$

Indefinite Integrals Ex 19.5 Q3

Let $I = \int \frac{x-1}{\sqrt{x+4}}dx$. Then,

$$I = \int \frac{x+4-4-1}{\sqrt{x+4}}$$

$$= \int \frac{x+4-5}{\sqrt{x+4}}dx$$

$$= \int \frac{x+4}{\sqrt{x+4}}dx - 5\int \frac{1}{\sqrt{x+4}}dx$$

$$= \int (x+4)^{\frac{1}{2}}dx - 5\int (x+4)^{\frac{-1}{2}}dx$$

$$= \frac{(x+4)^{\frac{3}{2}}}{\frac{3}{2}} - 5\frac{(x+4)^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= \frac{2}{3}\times(x+4)^{\frac{3}{2}} - 10(x+4)^{\frac{1}{2}} + c$$

$$\therefore \quad I = \frac{2}{3}\times(x+4)^{\frac{3}{2}} - 10(x+4)^{\frac{1}{2}} + c$$

Indefinite Integrals Ex 19.5 Q4

Let $I = \int (x+2)\sqrt{3x+5}\,dx$

Let $x + 2 = \lambda(3x+5) + \mu$ on equating the coefficients of like powers of x on both sides, we get

$$3\lambda = 1 \quad \text{and} \quad 5\lambda + \mu = 2$$

$$\Rightarrow \quad \lambda = \frac{1}{3} \quad \text{and} \quad 5 \times \frac{1}{3} + \mu = 2$$

$$\Rightarrow \quad \lambda = \frac{1}{3} \quad \text{and} \quad \mu = \frac{1}{3}$$

$$\therefore \quad I = \int \{\lambda(3x+5) + \mu\}\sqrt{3x+5}\,dx$$

$$= \lambda\int (3x+5)\sqrt{3x+5}\,dx + \mu\int \sqrt{3x+5}$$

$$= \lambda\int (3x+5)^{\frac{3}{2}}\,dx + \mu\int (3x+5)^{\frac{1}{2}}dx$$

$$= \lambda \times \frac{(3x+5)^{\frac{5}{2}}}{\frac{5}{2}\times 3} + \mu\frac{(3x+5)^{\frac{3}{2}}}{3\times\frac{3}{2}} + c$$

$$= \frac{1}{3}\times\frac{2}{15}\times(3x+5)^{\frac{5}{2}} + \frac{1}{3}\times\frac{2}{9}(3x+5)^{\frac{3}{2}} + c$$

$$= \frac{2}{45}\times(3x+5)^{\frac{5}{2}} + \frac{2}{27}\times(3x+5)^{\frac{3}{2}} + c$$

$$= \frac{2}{9}\times(3x+5)^{\frac{3}{2}}\left[\frac{1}{5}\times(3x+5)^{1} + \frac{1}{3}\right] + c$$

$$= \frac{2}{9}\times(3x+5)^{\frac{3}{2}}\left[\frac{3(3x+5)+5}{15}\right] + c$$

$$= \frac{2}{9}\times(3x+5)^{\frac{3}{2}}\frac{(9x+15+5)}{15} + c$$

$$= \frac{2}{135}\times(3x+5)^{\frac{3}{2}}(9x+20) + c$$

$$\therefore \quad I = \frac{2}{135}\times(9x+20)(3x+5)^{\frac{3}{2}} + c.$$

Indefinite Integrals Ex 19.5 Q5

Let $I = \int \frac{2x+1}{\sqrt{3x+2}}dx$

Let $2x+1 = \lambda(3x+2)+\mu$ on equating the coefficients of like powers of x on both sides, we get

$$3\lambda = 2 \quad \text{and} \quad 2\lambda+\mu = 1$$

$$\Rightarrow \quad \lambda = \frac{2}{3} \quad \text{and} \quad 2\times\frac{2}{3}+\mu = 1$$

$$\Rightarrow \quad \lambda = \frac{2}{3} \quad \text{and} \quad \mu = \frac{-1}{3}$$

$$\therefore \quad I = \int \frac{\lambda(3x+2)+\mu}{\sqrt{3x+2}}dx$$

$$= \lambda\int \frac{3x+2}{\sqrt{3x+2}}dx + \mu\int \frac{1}{\sqrt{3x+2}}dx$$

$$= \lambda\int (3x+2)^{\frac{1}{2}}dx + \mu\int (3x+2)^{\frac{-1}{2}}dx$$

$$= \lambda \times \frac{(3x+2)^{\frac{3}{2}}}{\frac{3}{2}\times 3} + \mu\frac{(3x+2)^{\frac{1}{2}}}{\frac{1}{2}\times 3} + c$$

$$= \frac{2}{3}\times\frac{2}{9}\times(3x+2)^{\frac{3}{2}} - \frac{1}{3}\times\frac{2}{3}(3x+2)^{\frac{1}{2}} + c$$

$$= \frac{4}{27}\times(3x+2)^{\frac{3}{2}} - \frac{2}{9}\times(3x+2)^{\frac{1}{2}} + c$$

$$= \frac{2}{9}\times\sqrt{3x+2}\left[\frac{2}{3}\times(3x+2)-1\right]+c$$

$$= \frac{2}{9}\times\sqrt{3x+2}\left[\frac{6x+4-3}{3}\right]+c$$

$$= \frac{2}{27}\times\sqrt{3x+2}\,(6x+1)+c$$

$$\therefore \quad I = \frac{2}{27}\times(6x+1)\sqrt{3x+2}+c.$$

Indefinite Integrals Ex 19.5 Q6

Let $I = \int \frac{3x+5}{\sqrt{7x+9}}dx$

Let $3x+5 = \lambda(7x+9)+\mu$ on equating the coefficients of like powers of x on both sides, we get

$$7\lambda = 3 \quad \text{and} \quad 9\lambda+\mu = 5$$

$$\Rightarrow \quad \lambda = \frac{3}{7} \quad \text{and} \quad 9\times\frac{3}{7}+\mu = 5$$

$$\Rightarrow \quad \lambda = \frac{3}{7} \quad \text{and} \quad \mu = \frac{8}{7}$$

$$\therefore \quad I = \int \frac{\lambda(7x+9)+\mu}{\sqrt{7x+9}}dx$$

$$= \lambda\int \frac{7x+9}{\sqrt{7x+9}}dx + \mu\int \frac{1}{\sqrt{7x+9}}dx$$

$$= \lambda\int (7x+9)^{\frac{1}{2}}dx + \mu\int (7x+9)^{\frac{-1}{2}}dx$$

$$= \lambda \times \frac{(7x+9)^{\frac{3}{2}}}{\frac{3}{2}\times 7} + \mu\frac{(7x+9)^{\frac{1}{2}}}{\frac{1}{2}\times 7} + c$$

$$= \frac{3}{7}\times\frac{2}{21}\times(7x+9)^{\frac{3}{2}} + \frac{8}{7}\times\frac{2}{7}(7x+9)^{\frac{1}{2}} + c$$

$$= \frac{2}{49}\times(7x+9)^{\frac{3}{2}} + \frac{16}{49}\times(7x+9)^{\frac{1}{2}} + c$$

$$= \frac{2}{49}\times(7x+9)^{\frac{1}{2}}[7x+9+8]+c$$

$$= \frac{2}{49}\times(7x+9)^{\frac{1}{2}}[7x+17]+c$$

$$= \frac{2}{49}\times(7x+17)\sqrt{7x+9}+c$$

Indefinite Integrals Ex 19.5 Q7

Let $I = \int \frac{x}{\sqrt{x+4}}dx$. Then,

$$I = \int \frac{x+4-4}{\sqrt{x+4}}dx$$
$$= \int \frac{x+4}{\sqrt{x+4}}dx - 4\int \frac{1}{\sqrt{x+4}}dx$$
$$= \int (x+4)^{\frac{1}{2}}dx - 4\int (x+4)^{\frac{-1}{2}}dx$$
$$= \frac{(x+4)^{\frac{3}{2}}}{\frac{3}{2}} - 4\frac{(x+4)^{\frac{1}{2}}}{\frac{1}{2}} + c$$
$$= \frac{2(x+4)^{\frac{3}{2}}}{3} - 8(x+4)^{\frac{1}{2}} + c$$
$$= 2(x+4)^{\frac{1}{2}}\left[\frac{1}{3}(x+4) - 4\right] + c$$
$$= 2(x+4)^{\frac{1}{2}}\left[\frac{(x+4)-12}{3}\right] + c$$
$$= \frac{2}{3}(x+4)^{\frac{1}{2}}[x-8] + c$$

$$\therefore \quad I = \frac{2}{3} \times (x-8)\sqrt{x+4} + c.$$

Indefinite Integrals Ex 19.5 Q8

Let $I = \int \frac{2-3x}{\sqrt{1+3x}} \times dx$. Then,

$$I = \int \frac{2-3x-1+1}{\sqrt{1+3x}} \times dx$$
$$= \int \frac{-3x-1+3}{\sqrt{1+3x}} \times dx$$
$$= \int -\frac{(3x+1)}{\sqrt{1+3x}} \times dx + 3\int \frac{1}{\sqrt{1+3x}}dx$$
$$= -1\int \frac{1+3x}{\sqrt{1+3x}} \times dx + 3\int \frac{1}{\sqrt{1+3x}}dx$$
$$= -1\int (1+3x)^{\frac{1}{2}}dx + 3\int (1+3x)^{\frac{-1}{2}}dx$$
$$= -1 \times \frac{(1+3x)^{\frac{3}{2}}}{\frac{3}{2} \times 3} + 3 \times \frac{(1+3x)^{\frac{1}{2}}}{\frac{1}{2} \times 3} + c$$
$$= -\frac{2}{9} \times (1+3x)^{\frac{3}{2}} + 2(1+3x)^{\frac{1}{2}} + c$$
$$= 2(1+3x)^{\frac{1}{2}}\left[-\frac{1}{9}(1+3x)^1 + 1\right] + c$$
$$= 2(1+3x)^{\frac{1}{2}}\left[\frac{-1-3x+9}{9}\right] + c$$
$$= 2(1+3x)^{\frac{1}{2}}\left[\frac{8-3x}{9}\right] + c$$
$$= \frac{2}{9}\sqrt{1+3x}\,(8-3x) + c$$

$$\therefore \quad I = \frac{2}{9}(8-9x)\sqrt{1+3x} + c$$

Indefinite Integrals Ex 19.5 Q9

Let $I = \int 5x + 3\sqrt{2x-1}\, dx$

Let $5x + 3 = \lambda(2x-1) + \mu$ comparing both sides, we get

$$2\lambda = 5 \quad \text{and} \quad -\lambda + \mu = 3$$

$$\Rightarrow \quad \lambda = \frac{5}{2} \quad \text{and} \quad \frac{-5}{2} + \mu = 3$$

$$\Rightarrow \quad \lambda = \frac{5}{2} \quad \text{and} \quad \mu = \frac{11}{2}$$

$$\therefore \quad I = \int \left\{\lambda(2x-1) + \mu\right\}\sqrt{2x-1}dx$$

$$= \lambda\int (2x-1)\sqrt{2x-1}dx + \mu\int \sqrt{2x-1}dx$$

$$= \lambda\int (2x-1)^{\frac{3}{2}}dx + \mu\int (2x-1)^{\frac{1}{2}}dx$$

$$= \lambda\frac{(2x-1)^{\frac{5}{2}}}{\frac{5}{2}\times 2} + \mu\frac{(2x-1)^{\frac{3}{2}}}{\frac{3}{2}\times 2}$$

$$= \lambda\frac{(2x-1)^{\frac{5}{2}}}{5} + \mu\frac{(2x-1)^{\frac{3}{2}}}{3} + c$$

$$= \frac{5}{2}\times\frac{(2x-1)^{\frac{5}{2}}}{5} + \frac{11}{2}\times\frac{(2x-1)^{\frac{3}{2}}}{3} + c$$

$$= \frac{(2x-1)^{\frac{5}{2}}}{2} + \frac{11}{6}\times(2x-1)^{\frac{3}{2}} + c$$

$$= \frac{1}{2}(2x-1)^{\frac{3}{2}}\left[(2x-1) + \frac{11}{3}\right] + c$$

$$= \frac{1}{2}\times(2x-1)^{\frac{3}{2}}\left[\frac{6x+8}{3}\right] + c$$

$$= \frac{1}{2}\times(2x-1)^{\frac{3}{2}}\times 2\frac{(3x+4)}{3} + c$$

$$= (2x-1)^{\frac{3}{2}}\times\frac{(3x+4)}{3} + c$$

$$= \frac{1}{3}\times(3x+4)(2x-1)^{\frac{3}{2}} + c$$

$$\therefore \quad I = \frac{1}{3}\times(3x+4)(2x-1)^{\frac{3}{2}} + c.$$

Ex 19.6

Indefinite Integrals Ex 19.6 Q1

$$\sin^2(2x+5)=\frac{1-\cos 2(2x+5)}{2}=\frac{1-\cos(4x+10)}{2}$$

$$\Rightarrow \int \sin^2(2x+5)\,dx=\int\frac{1-\cos(4x+10)}{2}dx$$

$$=\frac{1}{2}\int 1\,dx-\frac{1}{2}\int\cos(4x+10)\,dx$$

$$=\frac{1}{2}x-\frac{1}{2}\left(\frac{\sin(4x+10)}{4}\right)+C$$

$$=\frac{1}{2}x-\frac{1}{8}\sin(4x+10)+C$$

Indefinite Integrals Ex 19.6 Q2

We need to evaluate $\int \sin^3(2x+1)dx$

by using the formula $\rightarrow \sin 3\theta=-4\sin^3\theta+3\sin\theta$

$$\therefore \sin^3(2x+1)=\frac{3\sin(2x+1)-\sin 3(2x+1)}{4}$$

$$\int \sin^3(2x+1)dx$$

$$=\int\frac{3\sin(2x+1)-\sin 3(2x+1)}{4}dx$$

$$=\int\frac{3\sin(2x+1)-\sin 3(2x+1)}{4}dx$$

$$=-\frac{3}{8}\cos(2x+1)+\frac{1}{24}\cos 3(2x+1)+C$$

Indefinite Integrals Ex 19.6 Q3

Evaluate the integral as follows

$$\text{I}\quad \int\cos^4 2x\,dx=\int\left(\cos^2 2x\right)^2dx$$

$$=\int\left(\frac{1}{2}(\cos 4x+1)\right)^2dx$$

$$=\int\left(\frac{1}{4}(\cos^2 4x)+\frac{1}{4}+\frac{\cos 4x}{2}\right)dx$$

$$=\int\left(\frac{1}{4}\left(\frac{1}{2}(\cos 8x+1)\right)+\frac{1}{4}+\frac{\cos 4x}{2}\right)dx$$

$$=\int\frac{1}{8}\left(\cos 8x+\frac{3}{8}+\frac{\cos 4x}{2}\right)dx$$

$$=\frac{1}{64}\sin 8x+\frac{3}{8}x+\frac{1}{8}\sin 4x+C$$

Indefinite Integrals Ex 19.6 Q4

Let $I=\int \sin^2 bx\,dx$. Then,

$$I=\int\frac{1-\cos 2bx}{2}\times dx$$

$$=\frac{1}{2}\int dx-\frac{1}{2}\int\cos 2bx\,dx$$

$$=\frac{1}{2}x-\frac{1}{2}\frac{\sin(2bx)}{2b}+c$$

$$\therefore\qquad I=\frac{x}{2}-\frac{\sin 2bx}{4b}+c$$

Indefinite Integrals Ex 19.6 Q5

Let $I = \int \sin^2 \frac{x}{2} dx$. Then,

$$I = \frac{1}{2}\int 2\sin^2 \frac{x}{2} dx$$
$$= \frac{1}{2}\int (1 - \cos x) dx \qquad \left[\because \cos 2x = 1 - 2\sin^2 x\right]$$
$$= \frac{1}{2}\int dx - \frac{1}{2}\int \cos x dx$$
$$= \frac{1}{2} \times x - \frac{1}{2} \times \sin x + c$$
$$= \frac{1}{2}(x - \sin x) + c$$

$$\therefore \qquad I = \frac{1}{2}(x - \sin x) + c.$$

Indefinite Integrals Ex 19.6 Q6

We have,

$$\int \cos^2 \frac{x}{2} dx = \frac{1}{2}\int 2\cos^2 \frac{x}{2} dx$$
$$= \frac{1}{2}\int 1 + \cos x dx$$
$$= \frac{1}{2}\int dx + \frac{1}{2}\int \cos x dx$$
$$= \frac{1}{2} \times x + \frac{1}{2}\sin x + c$$
$$= \frac{1}{2}(x + \sin x) + c$$

$$\therefore \qquad \int \cos^2 \frac{x}{2} = \frac{1}{2}(x + \sin x) + c.$$

Indefinite Integrals Ex 19.6 Q7

Let $I = \int \cos^2 nx dx$. Then,

$$I = \frac{1}{2}\int 2\cos^2 nx dx$$
$$= \frac{1}{2}\int [1 + \cos 2nx] dx$$
$$= \frac{1}{2}\int \left[x + \frac{\sin 2nx}{2n}\right] + c$$
$$= \frac{x}{2} + \frac{1}{4n} \times \sin 2nx + c$$

$$\therefore \qquad I = \frac{x}{2} + \frac{1}{4n} \times \sin 2nx + c.$$

Indefinite Integrals Ex 19.6 Q8

Let $I = \int \sin\sqrt{1 - \cos 2x}\ dx$. Then,

$$
\begin{aligned}
I &= \int \sin x \times \sqrt{2\sin^2 x} \times dx \\
&= \int \sin x \times \sqrt{2} \times \sin x dx \\
&= \sqrt{2}\int \sin^2 x \times dx \\
&= \frac{\sqrt{2}}{2}\int 2\sin^2 x \times dx \\
&= \frac{\sqrt{2}}{2}\left[x - \frac{\sin 2x}{2}\right] + c \\
&= \frac{\sqrt{2}x}{2} - \frac{\sqrt{2}}{4} \times \sin 2x + c \\
&= \frac{1}{\sqrt{2}} \times x - \frac{\sin 2x}{2\sqrt{2}} + c
\end{aligned}
$$

$$\therefore \quad I = \frac{1}{\sqrt{2}} \times x - \frac{\sin 2x}{2\sqrt{2}} + c.$$

Ex 19.7

Indefinite Integrals Ex 19.7 Q1

Let $I = \int \sin 4x \cos 7x dx$. Then,

$$I = \frac{1}{2}\int 2\sin 4x \times \cos 7x dx$$
$$= \frac{1}{2}\int \left(\sin 11x + \sin(-3x)\right)dx$$
$$= \frac{1}{2}\int \sin 11x dx - \frac{1}{2}\int \sin 3x dx$$
$$= \frac{-1}{2\times 11}\times \cos 11x + \frac{1}{2\times 3}\cos 3x + c$$
$$= -\frac{1}{22}\times \cos 11x + \frac{1}{6}\times \cos 3x + c$$

$$\therefore \quad I = \frac{-1}{22}\times \cos 11x + \frac{1}{6}\times \cos 3x + c.$$

Indefinite Integrals Ex 19.7 Q2

Let $I = \int \cos 3x \cos 4x dx$. Then,

$$I = \frac{1}{2}\int \left(2\cos 3x \cos 4x\right)\times dx$$
$$= \frac{1}{2}\int \left(\cos 7x + \cos(-x)\right)dx$$
$$= \frac{1}{2}\int \cos 7x + \frac{1}{2}\int \cos dx \qquad \left[\because \cos(-0) = \cos 0\right]$$
$$= \frac{\sin 7x}{2\times 7} + \frac{\sin x}{2} + c$$
$$= \frac{1}{14}\times \sin 7x + \frac{1}{2}\sin x + c$$

$$\therefore \quad I = \frac{1}{14}\times \sin 7x + \frac{1}{2}\times \sin x + c.$$

Indefinite Integrals Ex 19.7 Q3

Let $I = \int \cos mx \cos nx \ dx \ \ m \neq n$. Then,

$$I = \frac{1}{2}\int 2\cos mx \cos nx \ dx$$
$$= \frac{1}{2}\int \left[\cos(m+n)x + \cos(m-n)x\right]dx$$
$$= \frac{1}{2}\times \frac{\sin(m+n)x}{m+n} + \frac{1}{2}\times \frac{\sin(m-n)x}{m-n} + c$$

$$\therefore \quad I = \frac{1}{2}\left[\frac{\sin(m+n)x}{m+n} + \frac{\sin(m-n)x}{m-n}\right] + c.$$

Indefinite Integrals Ex 19.7 Q4

We have,

$$\int \sin mx \cos nx \ dx, \ m \neq n$$
$$= \frac{1}{2}\int 2\sin mx \cos nx \ dx$$
$$= \frac{1}{2}\int \left[\sin(m+n)x + \sin(m-n)x\right]dx$$
$$= \frac{1}{2}\times \left[\frac{-\cos(m+n)}{m+n} - \frac{\cos(m-n)}{m-n}\right] + c$$

$$\therefore \quad \int \sin mx \cos nx = \frac{1}{2}\left[\frac{-\cos(m+n)x}{m+n} - \frac{\cos(m-n)x}{m-n}\right] + c.$$

Ex 19.8

Indefinite Integrals Ex 19.8 Q1

We have,

$$\int \frac{1}{\sqrt{1-\cos 2x}}dx = \int \frac{1}{\sqrt{2\sin^2 x}}dx$$

$$= \int \frac{1}{\sqrt{2}\sin x}dx$$

$$= \frac{1}{\sqrt{2}}\int \operatorname{cosec} x\, dx$$

$$= \frac{1}{\sqrt{2}}\log\left|\tan\frac{x}{2}\right| + c$$

$$\because \quad \int \frac{1}{\sqrt{1-\cos 2x}}dx = \frac{1}{\sqrt{2}}\log\left|\tan\frac{x}{2}\right| + c$$

Indefinite Integrals Ex 19.8 Q2

We have,

$$\int \frac{1}{\sqrt{1+\cos x}}dx = \int \frac{1}{\sqrt{2\cos^2\frac{x}{2}}}dx$$

$$= \int \frac{1}{\sqrt{2}\cos\frac{x}{2}}dx$$

$$= \frac{1}{\sqrt{2}}\int \sec\frac{x}{2}dx$$

$$= \frac{1}{\sqrt{2}}\int \operatorname{cosec}\left(\frac{\pi}{2}+\frac{x}{2}\right)dx$$

$$= \frac{2}{\sqrt{2}}\log\left|\tan\left(\frac{\pi}{4}+\frac{x}{4}\right)\right| + c$$

$$\because \quad \int \frac{1}{\sqrt{1+\cos x}}dx = \sqrt{2}\log\left|\tan\left(\frac{\pi}{4}+\frac{x}{4}\right)\right| + c$$

Indefinite Integrals Ex 19.8 Q3

Let $I = \int \sqrt{\frac{1+\cos 2x}{1-\cos 2x}}dx$ then,

$$I = \int \sqrt{\frac{2\cos^2 x}{2\sin^2 x}}dx$$

$$= \int \sqrt{\cot^2 x}\,dx$$

$$= \int \cot x\,dx$$

$$= \log|\sin x| + c \qquad \left[\because \quad \int \cot x = \log|\sin x| + c\right]$$

$$I = \log|\sin x| + c$$

Indefinite Integrals Ex 19.8 Q4

Let $I = \int \sqrt{\frac{1-\cos x}{1+\cos x}}dx$ then,

$$I = \int \sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}dx$$

$$= \int \sqrt{\tan^2\frac{x}{2}}dx$$

$$= \int \tan\frac{x}{2}dx$$

$$= -2\log\left|\cos\frac{x}{2}\right| + c \qquad \left[\because \quad \int \tan x\,dx = \log|\cos x| + c\right]$$

$$\because \quad I = -2\log\left|\cos\frac{x}{2}\right| + c$$

Indefinite Integrals Ex 19.8 Q5

Let $I = \int \frac{\sec x}{\sec 2x} dx$, then,

$$I = \int \frac{\frac{1}{\cos x}}{\frac{1}{\cos 2x}} dx$$

$$= \int \frac{\cos 2x}{\cos x} dx$$

$$= \int \frac{2\cos^2 x - 1}{\cos x} dx$$

$$= \int 2\cos x \, dx - \int \frac{1}{\cos x} dx$$

$$= 2\int \cos dx - \int \sec x \, dx$$

$$= 2\sin x - \log\left|\sec x + \tan x\right| + c$$

$$\therefore \quad I = 2\sin x - \log\left|\sec x + \tan x\right| + c$$

Indefinite Integrals Ex 19.8 Q6

$$\frac{\cos 2x}{(\cos x + \sin x)^2} = \frac{\cos 2x}{\cos^2 x + \sin^2 x + 2\sin x \cos x} = \frac{\cos 2x}{1 + \sin 2x}$$

$$\therefore \int \frac{\cos 2x}{(\cos x + \sin x)^2} dx = \int \frac{\cos 2x}{(1 + \sin 2x)} dx$$

Let $1 + \sin 2x = t$

$\Rightarrow 2\cos 2x \, dx = dt$

$$\therefore \int \frac{\cos 2x}{(\cos x + \sin x)^2} dx = \frac{1}{2} \int \frac{1}{t} dt$$

$$= \frac{1}{2}\log|t| + C$$

$$= \frac{1}{2}\log|1 + \sin 2x| + C$$

$$= \frac{1}{2}\log\left|(\sin x + \cos x)^2\right| + C$$

$$= \log|\sin x + \cos x| + C$$

Indefinite Integrals Ex 19.8 Q7

Let $I = \int \frac{\sin(x-a)}{\sin(x-b)} dx$ then

$$I = \int \frac{\sin(x - a + b - b)}{\sin(x-b)} dx$$

$$= \int \frac{\sin(x - b + b - a)}{\sin(x-b)} dx$$

$$= \int \frac{\sin(x-b)\cos(b-a) + \cos(x-b)\sin(b-a)}{\sin(x-b)} dx$$

$$= \int \left(\cos(b-a) + \cot(x-b)\sin(b-a)\right) dx$$

$$= \cos(b-a)\int dx + \sin(b-a)\int \cot(x-b) \, dx$$

$$= x\cos(b-a) + \sin(b-a)\log\left|\sin(x-b)\right| + c$$

$$\therefore \quad I = x\cos(b-a) + \sin(b-a)\log\left|\sin(x-b)\right| + c$$

Indefinite Integrals Ex 19.8 Q8

Let $I = \int \frac{\sin(x-\alpha)}{\sin(x+\alpha)}dx$ then,

$$I = \int \frac{\sin(x-\alpha+\alpha-\alpha)}{\sin(x+\alpha)}dx$$
$$= \int \frac{\sin(x+\alpha-2\alpha)}{\sin(x+\alpha)}dx$$
$$= \int \frac{\sin(x+\alpha)\cos 2\alpha - \cos(x+\alpha)\sin 2\alpha}{\sin(x+\alpha)}dx$$
$$= \int \left[\frac{\sin(x+\alpha)\cos 2\alpha}{\sin(x+\alpha)} - \frac{\cos(x+\alpha)\sin 2\alpha}{\sin(x+\alpha)}\right]dx$$
$$= \int (\cos 2\alpha - \cot(x+\alpha)\sin 2\alpha)dx$$
$$= \cos 2\alpha \int dx - \sin 2\alpha \int \cot(x+\alpha)dx$$
$$= x\cos 2\alpha - \sin 2\alpha \log|\sin(x+\alpha)| + c$$

$\therefore \quad I = x\cos 2\alpha - \sin 2\alpha \log|\sin(x+\alpha)| + c$

Indefinite Integrals Ex 19.8 Q9

Let $I = \int \frac{1+\tan x}{1-\tan x}dx$

$$I = \int \frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}}dx$$
$$= \int \frac{\frac{\cos x+\sin x}{\cos x}}{\frac{\cos x-\sin x}{\cos x}}dx$$

$\Rightarrow \quad I = \int \frac{\cos x+\sin x}{\cos x-\sin x}dx$ - - - - - (i)

Let $\cos x - \sin x = t$ then,

$d(\cos x - \sin x) = dt$

$\Rightarrow \quad (-\sin x - \cos x)dx = dt$

$\Rightarrow \quad -(\sin x + \cos x)dx = dt$

$\Rightarrow \quad dx = -\frac{dt}{\sin x + \cos x}$

Putting $\cos x - \sin x = t$ and $dx = \frac{-dt}{\sin x + \cos x}$ in equation (i), we get

$$I = \int \frac{\cos x + \sin x}{t} \times \frac{-dt}{\sin x + \cos x}$$
$$= -\int \frac{dt}{t}$$
$$= -\log|t| + c$$
$$= -\log|\cos x - \sin x| + c$$

$\therefore \quad I = -\log|\cos x - \sin x| + c$

Indefinite Integrals Ex 19.8 Q10

Let $I = \int \frac{\cos x}{\cos(x-a)} dx$ then,

$$I = \int \frac{\cos(x+a-a)}{\cos(x-a)} dx$$
$$= \int \frac{\cos(x-a+a)}{\cos(x-a)} dx$$
$$= \int \frac{\cos(x-a)\cos a - \sin(x-a)\sin a}{\cos(x-a)} dx$$
$$= \int \frac{\cos(x-a)\cos a}{\cos(x-a)} dx - \int \frac{\sin(x-a)\sin a}{\cos(x-a)} dx$$
$$= \cos a \int dx - \sin a \int \tan(x-a) dx$$

$$= x\cos a - \sin a \log\left|\sec(x-a)\right| + c$$

Indefinite Integrals Ex 19.8 Q11

Let $I = \int \sqrt{\frac{1-\sin 2x}{1+\sin 2x}} dx$ then,

$$I = \int \sqrt{\frac{1-\cos\left(\frac{\pi}{2}-2x\right)}{1+\cos\left(\frac{\pi}{2}-2x\right)}} dx$$
$$= \int \sqrt{\frac{2\sin^2\left(\frac{\pi}{4}-x\right)}{2\cos^2\left(\frac{\pi}{4}-x\right)}} dx$$
$$= \int \sqrt{\tan^2\left(\frac{\pi}{4}-x\right)} dx$$
$$= \int \tan\left(\frac{\pi}{4}-x\right) dx$$
$$= \log\left|\cos\left(\frac{\pi}{4}-x\right)\right| + c$$

Indefinite Integrals Ex 19.8 Q12

Let $I = \int \frac{e^{3x}}{e^{3x}+1} dx$ -----(i)

Let $e^{3x}+1 = t$, then,

$d\left(e^{3x}+1\right) = dt$

$\Rightarrow$ $3e^{3x} dx = dt$

$\Rightarrow$ $dx = \frac{dt}{3e^{3x}}$

Putting $e^{3x}+1 = t$ and $dx = \frac{dt}{3e^{3x}}$ in equation (i), we get

$$I = \int \frac{e^{3x}}{t} \times \frac{dt}{3e^{3x}}$$
$$= \frac{1}{3}\int \frac{dt}{t}$$
$$= \frac{1}{3}\log|t| + c$$

$$= \frac{1}{3}\log\left|3e^{3x}+1\right| + c$$

$$\therefore \quad = \frac{1}{3}\log\left|3e^{3x}+1\right| + c$$

Indefinite Integrals Ex 19.8 Q13

Let $I = \int \frac{\sec x \tan x}{3\sec x + 5} dx \quad -----(i)$

Let $3\sec x + 5 = t$, then,

$\Rightarrow \quad d(3\sec x + 5) = dt$

$\Rightarrow \quad 3\sec x \tan x \, dx = dt$

$\Rightarrow \quad dx = \frac{dt}{3\sec x \tan x}$

Putting $3\sec x \tan x dx = t$ and $dx = \frac{dt}{3\sec x \tan x}$ in equation (i), we get

$$I = \int \frac{\sec x \tan x}{t} \times \frac{dt}{3\sec x \tan x}$$
$$= \frac{1}{3}\int \frac{1}{t} dt$$
$$= \frac{1}{3}\log|t| + c$$

$$= \frac{1}{3}\log|3\sec x + 5| + c$$

Indefinite Integrals Ex 19.8 Q14

Let $I = \int \frac{1 - \cot x}{1 + \cot x} dx$ then,

$$I = \int \frac{1 - \frac{\cos x}{\sin x}}{1 + \frac{\cos x}{\sin x}} dx$$
$$= \int \frac{\frac{\sin x - \cos x}{\sin x}}{\frac{\sin x + \cos x}{\sin x}} dx$$

$\Rightarrow \quad I = \int \frac{\sin x - \cos x}{\sin x + \cos x} dx \quad ----(i)$

Let $\sin x + \cos x = t$. then,

$d(\sin x + \cos x) = dt$

$\Rightarrow \quad (\cos x - \sin x)dx = dt$

$\Rightarrow \quad -(\sin x - \cos x)dx = dt$

$\Rightarrow \quad dx = -\frac{dt}{\sin x - \cos x}$

Putting $\sin x + \cos x = t$ and $dx = -\frac{dt}{\sin x - \cos x}$ in equation (i), we get,

$$I = \int \frac{\sin x - \cos x}{t} \times \frac{-dt}{\sin x - \cos x}$$
$$= \int \frac{-dt}{t}$$
$$= -\log|t| + c$$

$$= -\log|\sin x + \cos x| + c$$

Indefinite Integrals Ex 19.8 Q15

Let $I = \int \frac{\sec x \operatorname{cosec} x}{\log(\tan x)} dx$ then,

Let $\log(\tan x) = t$ then,

$d[\log(\tan x)] = dt$

$\Rightarrow \quad \sec x \operatorname{cosec} x\, dx = dt \qquad \left[\because \quad \frac{d}{dx}(\log\tan x) = \sec x \operatorname{cosec} x\right]$

$\Rightarrow \quad dx = \frac{dt}{\sec x \operatorname{cosec} x}$

Putting $\log(\tan x) = t$ and $dx = \frac{dt}{\sec x \operatorname{cosec} x}$ in equation (i), we get,

$$I = \int \frac{\sec x \operatorname{cosec} x}{t} \times \frac{dt}{\sec x \operatorname{cosec} x}$$
$$= \int \frac{dt}{t}$$
$$= \log|t| + c$$

$$= \log|\log\tan x| + c$$

Indefinite Integrals Ex 19.8 Q16

Let $I = \int \frac{1}{x(3+\log x)} dx \; - - - - - (i)$

Let $3 + \log x = t$ then,

$d(3 + \log x) = dt$

$\Rightarrow \quad \frac{1}{x} dx = dt$

$\Rightarrow \quad dx = x\, dt$

Putting $3 + \log x = t$ and $dx = x\, dt$ in equation (i), we get,

$$I = \int \frac{1}{x \times t} \times x dt$$
$$= \int \frac{dt}{t}$$
$$= \log|t| + c$$

$$= \log|(3 + \log x)| + c$$

$\therefore \quad I = \log|(3 + \log x)| + c$

Indefinite Integrals Ex 19.8 Q17

Let $I = \int \frac{e^x + 1}{e^x + x} dx$ ----- (i)

Let $e^x + x = t$ then,

$d\left(e^x + x\right) = dt$

$\Rightarrow \left(e^x + x\right) dx = dt$

$\Rightarrow dx = \frac{dt}{e^x + 1}$

Putting $e^x + x = t$ and $dx = \frac{dt}{e^x + 1}$ in equation (i), we get,

$I = \int \frac{e^x + 1}{t} \times \frac{dt}{e^x + 1}$

$= \int \frac{dt}{t}$

$= \log|t| + c$

$= \log\left|e^x + x\right| + c$

$\therefore I = \log\left|e^x + x\right| + c$

Indefinite Integrals Ex 19.8 Q18

Let $I = \int \frac{1}{x \log x} dx$ ----- (i)

Let $\log x = t$ then,

$d(\log x) = dt$

$\Rightarrow \frac{1}{x} dx = dt$

$\Rightarrow dx = x\, dt$

Putting $\log x = t$ and $dx = x\, dt$ in equation (i), we get,

$I = \int \frac{1}{x \times t} \times x\, dt$

$= \int \frac{dt}{t}$

$= \log|t| + c$

$= \log|(\log x)| + c$

$\therefore I = \log|(\log x)| + c$

Indefinite Integrals Ex 19.8 Q19

Let $I = \int \frac{\sin 2x}{a\cos^2 x + b\sin^2 x} dx$ - - - - - (i)

Let $a\cos^2 x + b\sin^2 x = t$ then,

$d\left(a\cos^2 x + b\sin^2 x\right) = dt$

$\left[a\left(2\cos x\left(-\sin x\right)\right) + b\left(2\sin x\cos x\right)\right]dx = dt$

$\Rightarrow \left[-a\left(2\sin x\cos x\right) + b\left(2\sin x\cos x\right)\right]dx = dt$

$\Rightarrow \left[-a\sin 2x + b\sin 2x\right]dx = dt$

$\Rightarrow \sin 2x\left(b-a\right)dx = dt$

$\Rightarrow dx = \frac{dt}{\left(b-a\right)\sin 2x}$

Putting $a\cos^2 x + b\sin^2 x = t$ and $dx = \frac{dt}{\left(b-a\right)\sin 2x}$ in equation (i), we get,

$$I = \int \frac{\sin 2x}{t} \times \frac{dt}{\left(b-a\right)\sin 2x}$$

$$= \frac{1}{b-a}\int \frac{dt}{t}$$

$$= \frac{1}{b-a}\log|t| + c$$

$$= \frac{1}{b-a}\log\left|a\cos^2 x + b\sin^2 x\right| + c$$

Indefinite Integrals Ex 19.8 Q20

Let $I = \int \frac{\cos x}{2+3\sin x} dx$ - - - - - (i)

Let $2+3\sin x = t$ then,

$d\left(2+3\sin x\right) = dt$

$\Rightarrow 3\cos x\,dx = dt$

$\Rightarrow dx = \frac{dt}{3\cos x}$

Putting $2+3\sin x = t$ and $dx = \frac{dt}{3\cos x}$ in equation (i), we get,

$$I = \int \frac{\cos x}{t} \times \frac{dt}{3\cos x}$$

$$= \frac{1}{3}\int \frac{dt}{t}$$

$$= \frac{1}{3}\log|t| + c$$

$$= \frac{1}{3}\log\left|2+3\sin x\right| + c$$

Indefinite Integrals Ex 19.8 Q21

Let $I = \int \frac{1-\sin x}{x+\cos x}dx$ - - - - - (i)

Let $x + \cos x = t$ then,

$d\left(x+\cos x\right) = dt$

$\Rightarrow \quad \left(1-\sin x\right)dx = dt$

$\Rightarrow \quad dx = \frac{dt}{1-\sin x}$

Putting $x + \cos x = t$ and $dx = \frac{dt}{1-\sin x}$ in equation (i), we get,

$$I = \int \frac{1-\sin x}{t} \times \frac{dt}{1-\sin x}$$

$$= \int \frac{dt}{t}$$

$$= \log\left|t\right| + c$$

$$= \log\left|x + \cos x\right| + c$$

$\therefore \quad I = \log\left|x+\cos x\right| + c$

Indefinite Integrals Ex 19.8 Q22

Let $I = \int \frac{a}{b+ce^x}dx$ then,

$$I = \int \frac{a}{e^x\left[\frac{b}{e^x}+c\right]}dx$$

$\Rightarrow \quad I = \int \frac{a}{e^x\left[be^{-x}+c\right]}dx$ - - - - - - - (i)

Let $be^{-x} + c = t$ then,

$d\left(be^{-x}+c\right) = dt$

$\Rightarrow \quad -be^{-x}dx = dt$

$\Rightarrow \quad dx = \frac{-dt}{be^{-x}}$

$$= -\frac{e^x dt}{b}$$

Putting $be^{-x} + c = t$ and $dx = \frac{-e^x\,dt}{b}$ in equation (i), we get,

$$I = \int \frac{a}{e^x \times t} \times \frac{-e^x\,dt}{b}$$

$$= -\frac{a}{b}\int \frac{dt}{t}$$

$$= -\frac{a}{b}\log\left|t\right| + c$$

$$= -\frac{a}{b}\log\left|be^{-x}+c\right| + c$$

Indefinite Integrals Ex 19.8 Q23

Let $I = \int \frac{1}{e^x+1}dx$ then,

$$I = \int \frac{1}{e^x\left[1+\frac{1}{e^x}\right]}dx$$

$$\Rightarrow \quad I = \int \frac{1}{e^x\left[1+e^{-x}\right]}dx \text{ - - - - - - - (i)}$$

Let $1+e^{-x} = t$ then,

$$d\left(1+e^{-x}\right) = dt$$

$$\Rightarrow \quad -e^{-x}dx = dt$$

$$\Rightarrow \quad dx = \frac{-dt}{e^{-x}}$$

$$dx = -dt \times e^x$$

Putting $1+e^{-x} = t$ and $dx = -e^x dt$ in equation (i),we get,

$$I = \int \frac{1}{e^x \times t} \times -e^x\, dt$$

$$= -\int \frac{dt}{t}$$

$$= -\log|t| + c$$

$$= -\log\left|1+e^{-x}\right| + c$$

$$\therefore \quad = -\log\left|1+e^{-x}\right| + c$$

Indefinite Integrals Ex 19.8 Q24

Let $I = \int \frac{\cot x}{\log \sin x}dx$ - - - - - - - (i)

Let $\log \sin x = t$ then,

$$d\left(\log \sin x\right) = dt$$

$$\Rightarrow \quad \frac{\cos x}{\sin x}dx = dt$$

$$\Rightarrow \quad \cot x\, dx = dt$$

$$\Rightarrow \quad dx = \frac{dt}{\cot x}$$

Putting $\log \sin x = t$ and $dx = \frac{dt}{\cot x}$ in equation (i),we get,

$$I = \int \frac{\cot x}{t} \times \frac{dt}{\cot x}$$

$$= \int \frac{dt}{t}$$

$$= \log|t| + c$$

$$= \log\left|\log \sin x\right| + c$$

Indefinite Integrals Ex 19.8 Q25

Let I $\quad I = \int \frac{e^{2x}}{e^{2x}-2}dx$ - - - - - - - - (i)

Let $e^{2x} - 2 = t$ then,

$d\left(e^{2x}-2\right) = dt$

$\Rightarrow \quad 2e^{2x}dx = dt$

$\Rightarrow \quad dx = \frac{dt}{2e^{2x}}$

Putting $e^{2x} - 2 = t$ and $dx = \frac{dt}{2e^{2x}}$ in equation (i), we get,

$$I = \int \frac{2e^{2x}}{t} \times \frac{dt}{2e^{2x}}$$
$$= \frac{1}{2}\int \frac{dt}{t}$$
$$= \frac{1}{2}\log|t| + c$$
$$= \frac{1}{2}\log\left|e^{2x}-2\right| + c$$

$\therefore \quad = \frac{1}{2}\log\left|e^{2x}-2\right| + c$

Indefinite Integrals Ex 19.8 Q26

$$\frac{2\cos x - 3\sin x}{6\cos x + 4\sin x} = \frac{2\cos x - 3\sin x}{2(3\cos x + 2\sin x)}$$

Let $3\cos x + 2\sin x = t$

$(-3\sin x + 2\cos x)dx = dt$

$$\int \frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}dx = \int \frac{dt}{2t}$$
$$= \frac{1}{2}\int \frac{1}{t}dt$$
$$= \frac{1}{2}\log|t| + C$$
$$= \frac{1}{2}\log|2\sin x + 3\cos x| + C$$

Indefinite Integrals Ex 19.8 Q27

Let $\quad I = \int \frac{\cos 2x + x + 1}{x^2 + \sin 2x + 2x} dx$ - - - - - - - (i)

Let $x^2 + \sin 2x + 2x = t$ then,

$d\left(x^2 + \sin 2x + 2x\right) = dt$

$\Rightarrow \quad \left(2x + 2\cos 2x + 2\right) dx = dt$

$\Rightarrow \quad 2\left(\cos 2x + x + 1\right) dx = dt$

$\Rightarrow \quad dx = \frac{dt}{2\left(\cos 2x + x + 1\right)}$

Putting $x^2 + \sin 2x + 2x = t$ and $dx = \frac{dt}{2\left(\cos 2x + x + 1\right)}$ in equation (i), we get,

$I = \int \frac{\cos 2x + x + 1}{t} \times \frac{dt}{2\left(\cos 2x + x + 1\right)}$

$= \frac{1}{2} \int \frac{dt}{t}$

$= \frac{1}{2} \log|t| + c$

$= \frac{1}{2} \log\left|x^2 + \sin 2x + 2x\right| + c$

$\therefore \quad I = \frac{1}{2} \log\left|x^2 + \sin 2x + 2x\right| + c$

Indefinite Integrals Ex 19.8 Q29

Let $\quad I = \int \frac{-\sin x + 2\cos x}{2\sin x + \cos x} dx$ - - - - - - - (i)

Let $2\sin x + \cos x = t$ then,

$d\left(2\sin x + \cos x\right) = dt$

$\Rightarrow \quad \left(2\cos x - \sin x\right) dx = dt$

$\Rightarrow \quad dx = \frac{dt}{-\sin x + 2\cos x}$

Putting $2\sin x + \cos x = t$ and $dx = \frac{dt}{-\sin x + 2\cos x}$ in equation (i), we get,

$I = \int \frac{-\sin x + 2\cos x}{t} \times \frac{dt}{-\sin x + 2\cos x}$

$= \int \frac{dt}{t}$

$= \log|t| + c$

$= \log|2\sin x + \cos x| + c$

$\therefore \quad I = \log|2\sin x + \cos x| + c$

Indefinite Integrals Ex 19.8 Q30

$$\int \frac{\cos 4x - \cos 2x}{\sin 4x - \sin 2x} dx$$

$$= -\int \frac{2\sin 3x \sin x}{2\cos 3x \sin x} dx$$
$$= -\int \frac{\sin 3x}{\cos 3x} dx$$

Putting $\cos 3x = t$, and $-3\sin 3x dx = dt$

$$= \frac{1}{3}\int \frac{dt}{t}$$
$$= \frac{1}{3}\log|t| + c$$

$$= \frac{1}{3}\log|\cos 3x| + C$$

Indefinite Integrals Ex 19.8 Q31

Let $\quad I = \int \frac{\sec x}{\log(\sec x + \tan x)} dx$ - - - - - - - (i)

Let $\log(\sec x + \tan x) = t$ then,
$d\left[\log(\sec x + \tan x)\right] = dt$

$\Rightarrow \quad \sec x \, dx = dt \qquad \left[\because \quad \frac{d}{dx}\left(\log(\sec x + \tan x)\right) = \sec x\right]$

$\Rightarrow \quad dx = \frac{dt}{\sec x}$

Putting $\log(\sec x + \tan x) = t$ and $dx = \frac{dt}{\sec x}$ in equation (i), we get,

$$I = \int \frac{\sec x}{t} \times \frac{dt}{\sec x}$$
$$= \int \frac{dt}{t}$$
$$= \log|t| + c$$
$$= \log|\log(\sec x + \tan x)| + c$$

$\therefore \quad I = \log|\log(\sec x + \tan x)| + c$

Indefinite Integrals Ex 19.8 Q32

Let $\quad I = \int \frac{\operatorname{cosec} x}{\log \tan \frac{x}{2}} dx$ - - - - - - - (i)

Let $\log \tan \frac{x}{2} = t$ then,

$d\left[\log \tan \frac{x}{2}\right] = dt$

$\Rightarrow \quad \operatorname{cosec} x \, dx = dt$

$\Rightarrow \quad dx = \frac{dt}{\operatorname{cosec} x}$

Putting $\log \tan \frac{x}{2} = t$ and $dx = \frac{dt}{\operatorname{cosec} x}$ in equation (i),we get,

$I = \int \frac{\operatorname{cosec} x}{t} \times \frac{dt}{\operatorname{cosec} x}$

$= \int \frac{dt}{t}$

$= \log|t| + c$

$= \log\left|\log \tan \frac{x}{2}\right| + c$

$\therefore \quad I = \log\left|\log \tan \frac{x}{2}\right| + c$

Indefinite Integrals Ex 19.8 Q33

Let $\quad I = \int \frac{1}{x \log x \log(\log x)} dx$ - - - - - - - (i)

Let $\log(\log x) = t$ then,

$d[\log(\log x)] = dt$

$\Rightarrow \quad \frac{1}{x} \times \frac{1}{\log x} dx = dt$

$\Rightarrow \quad dx = x \log x \, dt$

Putting $\log(\log x) = t$ and $dx = x \log x \, dt$ in equation (i),we get,

$I = \int \frac{1}{x \log x \, t} \times x \log x \, dt$

$= \int \frac{1}{t} dt$

$= \log|t| + c$

$= \log|\log(\log x)| + c$

$\therefore \quad I = \log|\log(\log x)| + c$

Indefinite Integrals Ex 19.8 Q34

Let $\quad I = \int \frac{\operatorname{cosec}^2 x}{1+\cot x} dx \text{ - - - - - - - } (i)$

Let $1+\cot x = t$ then,

$d\left[1+\cot x\right] = dt$

$\Rightarrow \quad -\operatorname{cosec}^2 x\, dx = dt$

$\Rightarrow \quad dx = -\frac{dt}{\operatorname{cosec}^2 x}$

Putting $1+\cot x = t$ and $dx = \frac{-dt}{\operatorname{cosec}^2 x}$ in equation (i), we get,

$$I = \int \frac{\operatorname{cosec}^2 x}{t} \times -\frac{dt}{\operatorname{cosec}^2 x}$$
$$= -\int \frac{1}{t} dt$$
$$= -\log|t| + c$$
$$= -\log|1+\cot x| + c$$

$\therefore \quad I = -\log|1+\cot x| + c$

Indefinite Integrals Ex 19.8 Q35

Let $\quad I = \int \frac{10x^9 + 10^x \log_e 10}{10^x + x^{10}} dx \text{ - - - - - - - } (i)$

Let $10^x + x^{10} = t$ then,

$d\left(10^x + x^{10}\right) = dt$

$\Rightarrow \quad \left(10^x \log_e 10 + 10x^9\right) dx = dt$

$\Rightarrow \quad dx = \frac{dt}{10x^9 + 10^x \log_e 10}$

Putting $10^x + x^{10} = t$ and $dx = \frac{dt}{10x^9 + 10^x \log_e 10}$ in equation (i), we get,

$$I = \int \frac{10x^9 + 10^x \log_e 10}{t} \times \frac{dt}{10x^9 + 10^x \log_e 10}$$
$$= \int \frac{dt}{t}$$
$$= \log|t| + c$$
$$= \log\left|10^x + x^{10}\right| + c$$

$\therefore \quad I = \log\left|10^x + x^{10}\right| + c$

Indefinite Integrals Ex 19.8 Q36

Let $\quad I = \int \frac{1-\sin 2x}{x+\cos^2 x}dx$ - - - - - (i)

Let $x+\cos^2 x = t$ then,

$d\left(x+\cos^2 x\right) = dt$

$\Rightarrow \quad \left(1-2\cos x \sin x\right)dx = dt$

$\Rightarrow \quad dx = \frac{dt}{1-2\cos x \sin x}$

Putting $x+\cos^2 x = t$ and $dx = \frac{dt}{1-2\cos x \sin x}$ in equation (i), we get

$$I = \int \frac{1-\sin 2x}{t} \times \frac{dt}{1-2\cos x \sin x}$$
$$= \int \frac{1-\sin 2x}{t} \times \frac{dt}{1-\sin 2x}$$
$$= \int \frac{dt}{t}$$
$$= \log|t| + c$$
$$= \log\left|x+\cos^2 x\right| + c$$

$\therefore \quad I = \log\left|x+\cos^2 x\right| + c$

Indefinite Integrals Ex 19.8 Q37

Let $\quad I = \int \frac{1+\tan x}{x+\log\sec x}dx$ - - - - - - - (i)

Let $x+\log\sec x = t$ then,

$d\left(x+\log\sec x\right) = dt$

$\Rightarrow \quad \left(1+\tan x\right)dx = dt \quad \left[\because \quad \frac{d}{dx}\left(\log\sec x\right) = \tan x\right]$

$\Rightarrow \quad dx = \frac{dt}{1+\tan x}$

Putting $x+\log\sec x = t$ and $dx = \frac{dt}{1+\tan x}$ in equation (i), we get,

$$I = \int \frac{1+\tan x}{t} \times \frac{dt}{1+\tan x}$$
$$= \int \frac{dt}{t}$$
$$= \log|t| + c$$

$\Rightarrow \quad I = \log\left|x+\log\sec x\right| + c$

Indefinite Integrals Ex 19.8 Q38

Let $\quad I = \int \frac{\sin 2x}{a^2 + b^2 \sin^2 x} dx$ - - - - - - - - (i)

Let $a^2 + b^2 \sin^2 x = t$ then,

$d\left(a^2 + b^2 \sin^2 x\right) = dt$

$\Rightarrow \quad b^2 (2 \sin x \cos x) dx = dt$

$\Rightarrow \quad dx = \frac{dt}{b^2 (2 \sin x \cos x)}$

$= \frac{dt}{b^2 \sin 2x}$

Putting $a^2 + b^2 \sin^2 x = t$ and $dx = \frac{dt}{b^2 \sin 2x}$ in equation (i), we get,

$I = \int \frac{\sin 2x}{t} \times \frac{dt}{b^2 \sin 2x}$

$= \frac{1}{b^2} \int \frac{dt}{t}$

$= \frac{1}{b^2} \log|t| + c$

$= \frac{1}{b^2} \log\left|a^2 + b^2 \sin^2 x\right| + c$

$\Rightarrow \quad I = \frac{1}{b^2} \log\left|a^2 + b^2 \sin^2 x\right| + c$

Indefinite Integrals Ex 19.8 Q39

Let $\quad I = \int \frac{x + 1}{x(x + \log x)} dx$ - - - - - - - - (i)

Let $(x + \log x) = t$ then,

$d(x + \log x) = dt$

$\Rightarrow \quad \left(1 + \frac{1}{x}\right) dx = dt$

$\Rightarrow \quad \left(\frac{x + 1}{x}\right) dx = dt$

$\Rightarrow \quad dx = \frac{x}{x + 1} dt$

Putting $(x + \log x) = t$ and $dx = \frac{x}{x + 1}$ in equation (i), we get,

$I = \int \frac{x + 1}{x \times t} \times \frac{x}{x + 1} dt$

$= \int \frac{dt}{t}$

$= \log|t| + c$

$= \log|x + \log x| + c$

$\Rightarrow \quad I = \log|x + \log x| + c$

Indefinite Integrals Ex 19.8 Q40

Let $I = \int \frac{1}{\sqrt{1-x^2}\left(2+3\sin^{-1}x\right)}dx$ - - - - - - - (i)

Let $2+3\sin^{-1}x = t$ then,

$d\left(2+3\sin^{-1}x\right) = dt$

$\Rightarrow \quad 3\times\frac{1}{\sqrt{1-x^2}}dx = dt$

$\Rightarrow \quad dx = \frac{\sqrt{1-x^2}}{3}dt$

Putting $2+3\sin^{-1}x = t$ and $dx = \frac{\sqrt{1-x^2}}{3}$ in equation (i), we get,

$$I = \int \frac{\sqrt{1-x^2}}{3}\times\frac{1}{\sqrt{1-x^2}\,t}dt$$
$$= \frac{1}{3}\int\frac{dt}{t}$$
$$= \frac{1}{3}\log|t| + c$$
$$= \frac{1}{3}\log\left|2+3\sin^{-1}x\right| + c$$

$$\Rightarrow \quad I = \frac{1}{3}\log\left|2+3\sin^{-1}x\right| + c$$

Indefinite Integrals Ex 19.8 Q41

Let $I = \int \frac{\sec^2 x}{\tan x + 2}dx$ - - - - - - - (i)

Let $\tan x + 2 = t$ then,

$d\left(\tan x + 2\right) = dt$

$\Rightarrow \quad \sec^2 x dx = dt$

$\Rightarrow \quad dx = \frac{1}{\sec^2 x}dt$

Putting $\tan x + 2 = t$ and $dx = \frac{dt}{\sec^2 x}$ in equation (i), we get,

$$I = \int \frac{\sec^2 x}{t}\times\frac{1}{\sec^2 x}dt$$
$$= \int\frac{dt}{t}$$
$$= \log|t| + c$$
$$= \log|\tan x + 2| + c$$

$$\Rightarrow \quad I = \log|\tan x + 2| + c$$

Indefinite Integrals Ex 19.8 Q42

Let $I = \int \frac{2\cos 2x + \sec^2 x}{\sin 2x + \tan x - 5} dx$ ------- (i)

Let $\sin 2x + \tan x - 5 = t$ then,

$d(\sin 2x + \tan x - 5) = dt$

$\Rightarrow$ $(2\cos 2x + \sec^2 x) dx = dt$

$\Rightarrow$ $dx = \frac{1}{2\cos 2x + \sec^2 x} dt$

Putting $\sin 2x + \tan x - 5 = t$ and $dx = \frac{dt}{2\cos 2x + \sec^2 x}$ in equation (i) ,we get,

$$I = \int \frac{2\cos 2x + \sec^2 x}{t} \times \frac{1}{2\cos 2x + \sec^2 x} dt$$
$$= \int \frac{dt}{t}$$
$$= \log|t| + c$$
$$= \log|\sin 2x + \tan x - 5| + c$$

$\therefore$ $I = \log|\sin 2x + \tan x - 5| + c$

Indefinite Integrals Ex 19.8 Q43

Let $I = \int \frac{\sin 2x}{\sin 5x \sin 3x} dx$ then,

$$I = \int \frac{\sin(5x - 3x)}{\sin 5x \sin 3x} dx$$
$$= \int \frac{\sin 5x \cos 3x - \cos 5x \sin 3x}{\sin 5x \sin 3x} dx$$
$$= \int \frac{\sin 5x \cos 3x}{\sin 5x \sin 3x} dx - \int \frac{\cos 5x \sin 3x}{\sin 5x \sin 3x} dx$$
$$= \int \frac{\cos 3x}{\sin 3x} dx - \int \frac{\cos 5x}{\sin 5x} dx$$
$$= \int \cot 3x \, dx - \int \cot 5x \, dx$$
$$= \frac{1}{3}\log|\sin 3x| - \frac{1}{5}\log|\sin 5x| + c$$

$\therefore$ $I = \frac{1}{3}\log|\sin 3x| - \frac{1}{5}\log|\sin 5x| + c$

Indefinite Integrals Ex 19.8 Q44

Let $I = \int \frac{1 + \cot x}{x + \log \sin x} dx$ ----- (i)

Let $x + \log \sin x = t$ then,

$d(x + \log \sin x) = dt$

$\Rightarrow$ $(1 + \cot x) dx = dt$ $\left[\because \frac{d}{dx}(\log \sin x) = \cot x\right]$

$\Rightarrow$ $dx = \frac{dt}{1 + \cot x}$

Putting $x + \log \sin x = t$ and $dx = \frac{dt}{1 + \cot x}$ in equation (i), we get

$$I = \int \frac{1 + \cot x}{t} \times \frac{dt}{1 + \cot x}$$
$$= \int \frac{dt}{t}$$
$$= \log|t| + c$$
$$= \log|x + \log \sin x| + c$$

$\therefore$ $I = \log|x + \log \sin x| + c$

Indefinite Integrals Ex 19.8 Q45

Let $\quad I = \int \frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)}dx$ - - - - - (i)

Let $\sqrt{x}+1 = t$ then,

$d\left(\sqrt{x}+1\right) = dt$

$\Rightarrow \quad \frac{1}{2\sqrt{x}}dx = dt$

$\Rightarrow \quad dx = 2\sqrt{x}\,dt$

Putting $\sqrt{x}+1 = t$ and $dx = 2\sqrt{x}\,dt$ in equation (i), we get

$$I = \int \frac{1}{\sqrt{x}\,t} \times 2\sqrt{x}\,dt$$
$$= 2\int \frac{dt}{t}$$
$$= 2\log|t| + c$$
$$= 2\log\left|\sqrt{x}+1\right| + c$$
$$\therefore \quad I = 2\log\left|\sqrt{x}+1\right| + c$$

Indefinite Integrals Ex 19.8 Q46

Let $I = \int \tan 2x \tan 3x \tan 5x dx$ - - - - - - - - - (i)

Now,

$$\tan(5x) = \tan(2x+3x)$$
$$= \frac{\tan 2x + \tan 3x}{1 - \tan 2x \tan 3x}$$

$\Rightarrow \quad \tan 5x = \frac{\tan 2x + \tan 3x}{1 - \tan 2x \tan 3x}$

$\Rightarrow \quad \tan 5x - \tan 2x \tan 3x \tan 5x = \tan 2x + \tan 3x$

$\Rightarrow \quad \tan 5x - \tan 2x - \tan 3x = \tan 2x \tan 3x \tan 5x$ - - - - - (ii)

Using equation (i) and equation (ii), we get

$I = \int\left[\tan 5x - \tan 2x - \tan 3x\right]dx$

$= \frac{1}{5}\log|\sec 5x| - \frac{1}{2}\log|\sec 2x| - \frac{1}{3}\log|\sec 3x| + c$

$\therefore \quad I = \frac{1}{5}\log|\sec 5x| - \frac{1}{2}\log|\sec 2x| - \frac{1}{3}\log|\sec 3x| + c$

Indefinite Integrals Ex 19.8 Q47

Since,

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$\therefore \quad \tan(x+\theta-x) = \frac{\tan(x+\theta) - \tan x}{1+\tan(x+\theta)\tan x}$

$\Rightarrow \quad 1+\tan(x+\theta)\tan x = \frac{\tan(x+\theta)-\tan x}{\tan\theta}$

$\Rightarrow \quad \int 1+\tan(x+\theta)\tan x dx$

$= \frac{1}{\tan\theta}\left[\int \tan(x+\theta)dx - \int \tan x dx\right]$

$= \frac{1}{\tan\theta}\left[-\log|\cos(x+\theta)| + \log|\cos x|\right] + C$

$= \frac{1}{\tan\theta}\left[\log|\cos x| - \log|\cos(x+\theta)|\right] + C$

$= \frac{1}{\tan\theta}\log\left|\frac{\cos x}{\cos(x+\theta)}\right| + C$

Indefinite Integrals Ex 19.8 Q48

$$\text{Consider } I = \int \left(\frac{\sin 2x}{\sin\left(x - \frac{\pi}{6}\right)\sin\left(x + \frac{\pi}{6}\right)} \right) dx$$

$$= \int \left(\frac{\sin 2x}{\left(\frac{3}{4}\sin^2 x - \frac{1}{4}\cos^2 x\right)} \right) dx$$

$$= \int \left(\frac{\sin 2x}{\left(\frac{3}{4}(1 - \cos^2 x) - \frac{1}{4}\cos^2 x\right)} \right) dx$$

$$= \int \left(\frac{\sin 2x}{\left(\frac{3}{4} - \cos^2 x\right)} \right) dx$$

$$let \cos^2 x = t \to \sin 2x dx = -dt$$

$$I = \int \left(\frac{-dt}{\left(\frac{3}{4} - t\right)} \right)$$

$$I = \log\left|\sin^2 x - \frac{1}{4}\right| + C$$

Indefinite Integrals Ex 19.8 Q49

$$\int \frac{e^{x-1} + x^{e-1}}{e^x + x^e} dx$$

$$= \frac{1}{e} \int \frac{e^x + ex^{e-1}}{e^x + x^e} dx$$

$$\text{Let } e^x + x^e = u$$

$$\Rightarrow \quad \left(e^x + ex^{e-1}\right) dx = du$$

$$= \frac{1}{e} \int \frac{1}{4} du = \frac{1}{e} \log|u| + C$$

$$= \frac{1}{e} \log\left|e^x + x^e\right| + C$$

Indefinite Integrals Ex 19.8 Q50

$$\text{Let } I = \int \frac{1}{\sin x \cos^2 x} dx, \quad \text{then,}$$

$$I = \int \frac{\sin^2 x + \cos^2 x}{\sin x \cos^2 x} dx$$

$$= \int \frac{\sin^2 x}{\sin x \cos^2 x} dx + \int \frac{\cos^2 x}{\sin x \cos^2 x} dx$$

$$= \int \sec x \tan x \, dx + \int \operatorname{cosec} x \, dx$$

$$= \sec x + \log\left|\tan \frac{x}{2}\right| + c$$

$$\therefore \quad I = \sec x + \log\left|\tan \frac{x}{2}\right| + c$$

Indefinite Integrals Ex 19.8 Q51

$$\text{Let } I = \int \frac{1}{\cos 3x - \cos x} dx, \quad \text{then,}$$

$$I = \int \frac{\sin^2 x + \cos^2 x}{-2\sin 2x \sin x} dx$$

$$= \int \frac{\sin^2 x + \cos^2 x}{-4\sin^2 x \cos x} dx$$

$$= -\frac{1}{4}\int \left[\frac{\sin^2 x}{\sin^2 x \cos x} + \frac{\cos^2 x}{\sin^2 x \cos x}\right] dx$$

$$= -\frac{1}{4}\int \left[\sec x + \operatorname{cosec} x \cot x\right] dx$$

$$= -\frac{1}{4}\left[\log\left|\sec x + \tan x\right| - \operatorname{cosec} x\right] + c$$

$$\therefore \quad I = \frac{1}{4}\left[\operatorname{cosec} x - \log\left|\sec x + \tan x\right|\right] + c$$

EX 19.9

Indefinite Integrals Ex 19.9 Q1

Let $\quad I = \int \frac{\log x}{x} dx$

Let $\log x = t$ then,

$d(\log x) = dt$

$\Rightarrow \quad \frac{1}{x} dx = dt$

$\Rightarrow \quad dx = x\,dt$

Putting $\log x = t$ and $dx = x\,dt$, we get

$$I = \int \frac{t}{x} \times x\,dt$$
$$= \int t\,dt$$
$$= \frac{t^2}{2} + c$$
$$= \frac{(\log x)^2}{2} + c$$

$$\therefore \quad I = \frac{(\log x)^2}{2} + c$$

Indefinite Integrals Ex 19.9 Q2

Let $\quad I = \int \frac{\log\left(1+\frac{1}{x}\right)}{x(1+x)} dx \text{ ------ (i)}$

Let $\log\left(1+\frac{1}{x}\right) = t$ then,

$$d\left[\log\left(1+\frac{1}{x}\right)\right] = dt$$

$$\Rightarrow \quad \frac{1}{1+\frac{1}{x}} \times \frac{-1}{x^2} dx = dt$$

$$\Rightarrow \quad \frac{1}{\frac{x+1}{x}} \times \frac{-1}{x^2} dx = dt$$

$$\Rightarrow \quad \frac{-x}{x^2(x+1)} dx = -dt$$

$$\Rightarrow \quad \frac{dx}{x(x+1)} = -dt$$

Putting $\log\left(1+\frac{1}{x}\right) = t$ and $\frac{dx}{x(x+1)} = -dt$ in equation (i), we get

$$I = \int t \times -dt$$

$$= -\frac{t^2}{2} + c$$

$$= -\frac{1}{2}\left[\log\left(1+\frac{1}{x}\right)\right]^2 + c$$

$$\therefore \quad I = -\frac{1}{2}\left[\log\left(1+\frac{1}{x}\right)\right]^2 + c$$

Let $\quad I = \int \frac{\log\left(1+\frac{1}{x}\right)}{x(1+x)} dx \text{ ------ (i)}$

Let $\log\left(1+\frac{1}{x}\right) = t$ then,

$$d\left[\log\left(1+\frac{1}{x}\right)\right] = dt$$

$$\Rightarrow \quad \frac{1}{1+\frac{1}{x}} \times \frac{-1}{x^2} dx = dt$$

$$\Rightarrow \quad \frac{1}{\frac{x+1}{x}} \times \frac{-1}{x^2} dx = dt$$

$$\Rightarrow \quad \frac{-x}{x^2(x+1)} dx = \ dt$$

$$\Rightarrow \quad \frac{dx}{x(x+1)} = -dt$$

Putting $\log\left(1+\frac{1}{x}\right) = t$ and $\frac{dx}{x(x+1)} = -dt$ in equation (i), we get

$$I = \int t \times -dt$$

$$= -\frac{t^2}{2} + c$$

$$= -\frac{1}{2}\left[\log\left(1+\frac{1}{x}\right)\right]^2 + c$$

$$\therefore \quad I = -\frac{1}{2}\left[\log\left(1+\frac{1}{x}\right)\right]^2 + c$$

Indefinite Integrals Ex 19.9 Q3

Let $\quad I = \int \frac{\left(1+\sqrt{x}\right)^2}{\sqrt{x}}dx$

Let $\left(1+\sqrt{x}\right) = t$ then,

$d\left(1+\sqrt{x}\right) = dt$

$\Rightarrow \quad \frac{1}{2\sqrt{x}}dx = dt$

$\Rightarrow \quad dx = dt \times 2\sqrt{x}$

Putting $\left(1+\sqrt{x}\right) = t$ and $dx = dt \times 2\sqrt{x}$, we get

$$I = \int \frac{t^2}{\sqrt{x}} \times dt \times 2\sqrt{x}$$
$$= 2\int t^2 dt$$
$$= 2 \times \frac{t^3}{3} + c$$
$$= \frac{2}{3}\left[1+\sqrt{x}\right]^3 + c$$

$\therefore \quad I = \frac{2}{3}\left(1+\sqrt{x}\right)^3 + c$

Indefinite Integrals Ex 19.9 Q4

Let $\quad I = \int \sqrt{1+e^x}\, e^x dx \text{ ------ (i)}$

Let $1+e^x = t$ then,

$d\left(1+e^x\right) = dt$

$\Rightarrow \quad e^x dx = dt$

$\Rightarrow \quad dx = \frac{dt}{e^x}$

Putting $1+e^x = t$ and $dx = \frac{dt}{e^x}$ in equation (i), we get

$$I = \int \sqrt{t}\, e^x \frac{dt}{e^x}$$
$$= \int t^{\frac{1}{2}} dt$$
$$= \frac{2}{3} \times \frac{t^{\frac{3}{2}}}{} + c$$

$$= \frac{2}{3}\left(1+e^x\right)^{\frac{3}{2}} + c$$

Indefinite Integrals Ex 19.9 Q5

Let $\quad I = \int \sqrt[3]{\cos^2 x}\,\sin x\,dx - - - - - (i)$

Let $\cos x = t$ then,
$d(\cos x) = dt$

$\Rightarrow \quad -\sin x\,dx = dt$

$\Rightarrow \quad dx = -\frac{dt}{\sin x}$

Putting $\cos x = t$ and $dx = -\frac{dt}{\sin x}$ in equation (i), we get

$$I = \int \sqrt[3]{t^2}\,\sin x \times \frac{-dt}{\sin x}$$
$$= -\int t^{\frac{2}{3}} \sin x\,\frac{dt}{\sin x}$$
$$= -\int t^{\frac{2}{3}}\,dt$$
$$= -\frac{3}{5} \times t^{\frac{5}{3}} + c$$
$$= -\frac{3}{5}(\cos x)^{\frac{5}{3}} + c$$

$\therefore \quad I = -\frac{3}{5}(\cos x)^{\frac{5}{3}} + c$

Indefinite Integrals Ex 19.9 Q6

Let $\quad I = \int \frac{e^x}{(1+e^x)^2}\,dx - - - - - (i)$

Let $1 + e^x = t$ then,
$d(1+e^x) = dt$

$\Rightarrow \quad e^x\,dx = dt$

$\Rightarrow \quad dx = \frac{dt}{e^x}$

Putting $1 + e^x = t$ and $dx = \frac{dt}{e^x}$ in equation (i), we get

$$I = \int \frac{e^x}{t^2} \times \frac{dt}{e^x}$$
$$= \int \frac{dt}{t^2}$$
$$= \int t^{-2}\,dt$$
$$= -t^{-1} + c$$
$$= -\frac{1}{t} + c$$
$$= -\frac{1}{1+e^x} + c$$

$\therefore \quad I = -\frac{1}{1+e^x} + c$

Indefinite Integrals Ex 19.9 Q7

Let $\quad I = \int \cot^3 x \operatorname{cosec}^2 x \, dx - - - - - (i)$

Let $\cot x = t$ then,

$d(\cot x) = dt$

$\Rightarrow \quad -\operatorname{cosec}^2 x dx = dt$

$\Rightarrow \quad dx = -\dfrac{dt}{\operatorname{cosec}^2 x}$

Putting $\cot x = t$ and $dx = -\dfrac{dt}{\operatorname{cosec}^2 x}$ in equation (i), we get

$$I = \int t^3 \operatorname{cosec}^2 x \times \frac{-dt}{\operatorname{cosec}^2 x}$$
$$= -\int t^3 dt$$
$$= -\frac{t^4}{4} + c$$
$$= -\frac{\cot^4 x}{4} + c$$

$\therefore \quad I = -\dfrac{\cot^4 x}{4} + c$

Indefinite Integrals Ex 19.9 Q8

Let $\quad I = \int \dfrac{\left\{e^{\sin^{-1} x}\right\}^2}{\sqrt{1 - x^2}} dx - - - - - (i)$

Let $\sin^{-1} x = t$ then,

$d\left(\sin^{-1} x\right) = dt$

$\Rightarrow \quad \dfrac{1}{\sqrt{1 - x^2}} dx = dt$

$\Rightarrow \quad dx = \sqrt{1 - x^2}\, dt$

Putting $\sin^{-1} x = t$ and $dx = \sqrt{1 - x^2} dt$ in equation (i), we get

$$I = \int \frac{\left(e^t\right)^2}{\sqrt{1 - x^2}} \times \sqrt{1 - x^2} dt$$
$$= \int e^{2t} dt$$
$$= \frac{e^{2t}}{2} + c$$
$$= \frac{e^{2\sin^{-1} x}}{2} + c$$

$\therefore \quad I = \dfrac{\left\{e^{\sin^{-1} x}\right\}^2}{2} + c$

Indefinite Integrals Ex 19.9 Q9

Let $I = \int \frac{1+\sin x}{\sqrt{x-\cos x}}dx$ ----- (i)

Let $x - \cos x = t$ then,

$d(x - \cos x) = dt$

$\Rightarrow$ $[1-(-\sin x)]dx = dt$

$\Rightarrow$ $(1+\sin x)dx = dt$

Putting $x - \cos x = t$ and $(1+\sin x)dx = dt$ in equation (i), we get

$$I = \int \frac{dt}{\sqrt{t}}$$
$$= \int t^{\frac{-1}{2}}dt$$
$$= 2t^{\frac{1}{2}} + c$$
$$= 2(x - \cos x)^{\frac{1}{2}} + c$$

$\therefore$ $I = 2\sqrt{x - \cos x} + c$

Indefinite Integrals Ex 19.9 Q10

Let $I = \int \frac{1}{\sqrt{1-x^2}\left(\sin^{-1}x\right)^2}dx$ ----- (i)

Let $\sin^{-1}x = t$ then,

$d\left(\sin^{-1}x\right) = dt$

$\Rightarrow$ $\frac{1}{\sqrt{1-x^2}}dx = dt$

Putting $\sin^{-1}x = t$ and $\frac{1}{\sqrt{1-x^2}}dx = dt$ in equation (i), we get

$$I = \int \frac{dt}{t^2}$$
$$= \int t^{-2}dt$$
$$= -1t^{-1} + c$$
$$= \frac{-1}{t} + c$$
$$= \frac{-1}{\sin^{-1}x} + c$$

$\therefore$ $I = \frac{-1}{\sin^{-1}x} + c$

Indefinite Integrals Ex 19.9 Q11

Let $\quad I = \int \frac{\cot x}{\sqrt{\sin x}} dx \text{ - - - - - } (i)$

Let $\sin x = t$ then,

$d(\sin x) = dt$

$\Rightarrow \quad \cos x \, dx = dt$

Now, $\quad I = \int \frac{\cot x}{\sqrt{\sin x}} dx$

$= \int \frac{\cos x}{\sin x \sqrt{\sin x}} dx$

$= \int \frac{\cos x}{(\sin x)^{\frac{3}{2}}} dx$

$\Rightarrow \quad = \int \frac{\cos x}{(\sin x)^{\frac{3}{2}}} dx \text{ - - - - - - } (ii)$

Putting $\sin x = t$ and $\cos x \, dx = dt$ in equation (ii), we get

$I = \int \frac{dt}{t^{\frac{3}{2}}}$

$= \int t^{-\frac{3}{2}} dt$

$= -2t^{-\frac{1}{2}} + c$

$= \frac{-2}{\sqrt{t}} + c$

$= \frac{-2}{\sqrt{\sin x}} + c$

$\therefore \quad I = \frac{-2}{\sqrt{\sin x}} + c$

Indefinite Integrals Ex 19.9 Q12

Let $\quad I = \int \frac{\tan x}{\sqrt{\cos x}} dx$

$\therefore \quad I = \int \frac{\sin x}{\cos x \sqrt{\cos x}} dx$

$= \int \frac{\sin x}{(\cos x)^{\frac{3}{2}}} dx$

$\Rightarrow \quad I = \int \frac{\sin x}{(\cos x)^{\frac{3}{2}}} dx \text{ - - - - - - (i)}$

Let $\quad \cos x = t \quad$ then,

$d(\cos x) = dt$

$\Rightarrow \quad -\sin x \, dx = dt$

$\Rightarrow \quad \sin x \, dx = -dt$

Putting $\cos x = t$ and $\sin x \, dx = -dt$ in equation (i), we get

$I = \int \frac{-dt}{t^{\frac{3}{2}}}$

$= -\int t^{-\frac{3}{2}} dt$

$= -\left[-2t^{-\frac{1}{2}}\right] + c$

$= \frac{2}{t^{\frac{1}{2}}} + c$

$= \frac{2}{\sqrt{\cos x}} + c$

$\therefore \quad I = \frac{2}{\sqrt{\cos x}} + c$

Indefinite Integrals Ex 19.9 Q13

Let $\quad I = \int \frac{\cos^3 x}{\sqrt{\sin x}} dx$

$\therefore \quad I = \int \frac{\cos^2 x \cos x}{\sqrt{\sin x}} dx$

$= \int \frac{\left(1 - \sin^2 x\right) \cos x}{\sqrt{\sin x}} dx$

$\therefore \quad I = \int \frac{\left(1 - \sin^2 x\right)}{\sqrt{\sin x}} \cos x \, dx \text{ - - - - - - } (i)$

Let $\quad \sin x = t \quad$ then,

$d\left(\sin x\right) = dt$

$\Rightarrow \quad \cos x \, dx = dt$

Putting $\sin x = t$ and $\cos x \, dx = dt$ in equation (i), we get

$I = \int \frac{1 - t^2}{\sqrt{t}} dt$

$= \int \left(t^{\frac{-1}{2}} - t^2 \times t^{\frac{-1}{2}}\right) dt$

$= \int \left(t^{\frac{-1}{2}} - t^{\frac{3}{2}}\right) dt$

$= 2t^{\frac{1}{2}} - \frac{2}{5} t^{\frac{5}{2}} + c$

$\Rightarrow \quad I = 2\left(\sin x\right)^{\frac{1}{2}} - \frac{2}{5}\left(\sin x\right)^{\frac{5}{2}} + c$

$\therefore \quad I = 2\sqrt{\sin x} - \frac{2}{5}\left(\sin x\right)^{\frac{5}{2}} + c$

Indefinite Integrals Ex 19.9 Q14

Let $I = \int \frac{\sin^3 x}{\sqrt{\cos x}} dx$

$\therefore$ $I = \int \frac{\sin^2 x \sin x}{\sqrt{\cos x}} dx$

$\Rightarrow$ $I = \int \frac{\left(1 - \cos^2 x\right)}{\sqrt{\cos x}} \sin x \, dx$ ------ (i)

Let $\cos x = t$ then,
$d\left(\cos x\right) = dt$

$\Rightarrow$ $-\sin x \, dx = dt$
$\Rightarrow$ $\sin x \, dx = -dt$

Putting $\cos x = t$ and $\sin x \, dx = -dt$ in equation (i), we get

$$I = \int \frac{\left(1 - t^2\right)}{\sqrt{t}} \times -dt$$
$$= \int \frac{t^2 - 1}{\sqrt{t}} dt$$
$$= \int \left(\frac{t^2}{t^{\frac{1}{2}}} - \frac{1}{t^{\frac{1}{2}}}\right) dx$$
$$= \int \left(t^{2-\frac{1}{2}} - t^{\frac{-1}{2}}\right) dt$$
$$= \int \left(t^{\frac{3}{2}} - t^{-\frac{1}{2}}\right) dt$$
$$= \frac{2}{5} t^{\frac{5}{2}} - 2t^{\frac{1}{2}} + c$$

$\therefore$ $I = \frac{2}{5} \cos^{\frac{5}{2}} x - 2 \cos^{\frac{1}{2}} x + c$

$\therefore$ $I = \frac{2}{5} \cos^{\frac{5}{2}} x - 2\sqrt{\cos x} + c$

Indefinite Integrals Ex 19.9 Q15

Let $I = \int \frac{1}{\sqrt{\tan^{-1} x}\left(1 + x^2\right)} dx$ ----- (i)

Let $\tan^{-1} x = t$ then,
$d\left(\tan^{-1} x\right) = dt$

$\Rightarrow$ $\frac{1}{1 + x^2} dx = dt$

Putting $\tan^{-1} x = t$ and $\frac{1}{1 + x^2} dx = dt$ in equation (i), we get

$$I = \int \frac{1}{\sqrt{t}} dt$$
$$= \int t^{-\frac{1}{2}} dt$$
$$= 2t^{\frac{1}{2}} + c$$
$$= 2\sqrt{\tan^{-1} x} + c$$

$\therefore$ $I = 2\sqrt{\tan^{-1} x} + c$

Indefinite Integrals Ex 19.9 Q16

$$\text{Let } I = \int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$$
$$= \int \frac{\sqrt{\tan x} \times \cos x}{\sin x \cos x \times \cos x} dx$$
$$= \int \frac{\sqrt{\tan x}}{\tan x \cos^2 x} dx$$
$$= \int \frac{\sec^2 x \, dx}{\sqrt{\tan x}}$$
$$\text{Let } \tan x = t \Rightarrow \sec^2 x \, dx = dt$$
$$\therefore I = \int \frac{dt}{\sqrt{t}}$$
$$= 2\sqrt{t} + C$$
$$= 2\sqrt{\tan x} + C$$

Indefinite Integrals Ex 19.9 Q17

Let $I = \int \frac{1}{x} (\log x)^2 dx$ ----- (i)

Let $\log x = t$ then,
$d(\log x) = dt$

$\Rightarrow \frac{1}{x} dx = dt$

Putting $\log x = t$ and $\frac{1}{x} dx = dt$ in equation (i), we get

$$I = \int t^2 dt$$
$$= \frac{t^3}{3} + c$$
$$= \frac{(\log x)^3}{3} + c$$

$\therefore I = \frac{1}{3} (\log x)^3 + c$

Indefinite Integrals Ex 19.9 Q18

Let $I = \int \sin^5 x \cos x \, dx$ ----- (i)

Let $\sin x = t$ then,
$d(\sin x) = dt$

$\Rightarrow \cos x \, dx = dt$

Putting $\sin x = t$ and $\cos x \, dx = dt$ in equation (i), we get

$$I = \int t^5 dt$$
$$= \frac{t^6}{6} + c$$
$$= \frac{\sin^6 x}{6} + c$$

$\therefore I = \frac{1}{6} \sin^6 x + c$

Indefinite Integrals Ex 19.9 Q19

Let $\quad I = \int \tan^{\frac{3}{2}} x \sec^2 x \, dx - - - - - (i)$

Let $\quad \tan x = t \quad$ then,

$d(\tan x) = dt$

$\Rightarrow \quad \sec^2 x \, dx = dt$

Putting $\tan x = t$ and $\sec^2 x \, dx = dt$ in equation (i), we get

$$I = \int t^{\frac{3}{2}} dt$$
$$= \frac{2}{5} t^{\frac{5}{2}} + c$$
$$= \frac{2}{5} (\tan x)^{\frac{5}{2}} + c$$

$\therefore \quad I = \frac{2}{5} \tan^{\frac{5}{2}} x + c$

Indefinite Integrals Ex 19.9 Q20

Let $\quad I = \int \frac{x^3}{\left(x^2+1\right)^3} x\,dx$ - - - - - (i)

Let $\quad 1+x^2 = t \quad$ then,

$d\left(1+x^2\right) = dt$

$\Rightarrow \quad 2x\,dx = dt$

$\Rightarrow \quad x\,dx = \frac{dt}{2}$

Putting $1+x^2 = t$ and $x\,dx = \frac{dt}{2}$ in equation (i), we get

$$I = \int \frac{x^2}{t^3} \times \frac{dt}{2}$$

$$= \frac{1}{2} \int \frac{(t-1)}{t^3} dt \qquad \left[\because \quad 1+x^2 = t\right]$$

$$= \frac{1}{2} \int \left[\left(\frac{t}{t^3} - \frac{1}{t^3}\right) dt\right]$$

$$= \frac{1}{2} \int \left(t^{-2} - t^{-3}\right) dt$$

$$= \frac{1}{2}\left[-1t^{-1} - \frac{t^{-2}}{-2}\right] + c$$

$$= \frac{1}{2}\left[-\frac{1}{t} + \frac{1}{2t^2}\right] + c$$

$$= -\frac{1}{2t} + \frac{1}{4t^2} + c$$

$$= -\frac{1}{2\left(1+x^2\right)} + \frac{1}{4\left(1+x^2\right)^2} + c$$

$$= \frac{-2\left(1+x^2\right)+1}{4\left(1+x^2\right)^2} + c$$

$$= \frac{-2-2x^2+1}{4\left(1+x^2\right)^2} + c$$

$$= \frac{-2x^2-1}{4\left(1+x^2\right)^2} + c$$

$$= -\frac{\left(1+2x^2\right)}{4\left(x^2+1\right)^2} + c$$

$$\therefore \quad I = \frac{-\left(1+2x^2\right)}{4\left(x^2+1\right)^2} + c$$

Indefinite Integrals Ex 19.9 Q21

Let $x^2+x+1=t$

$(2x+1)dx = dt$

$$\int(4x+2)\sqrt{x^2+x+1}\ dx$$

$$= \int 2\sqrt{t}\, dt$$

$$= 2\int\sqrt{t}\, dt$$

$$= 2\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)+C$$

$$= \frac{4}{3}\left(x^2+x+1\right)^{\frac{3}{2}}+C$$

Indefinite Integrals Ex 19.9 Q22

Let $\quad I = \int \frac{4x+3}{\sqrt{2x^3+3x+1}}dx \text{ - - - - - (i)}$

Let $\quad 2x^3+3x+1 = t \quad$ then,

$d\left(2x^3+3x+1\right) = dt$

$\Rightarrow \quad (4x+3)dx = dt$

Putting $2x^3+3x+1 = t$ and $(4x+3)dx = dt$ in equation (i), we get

$$I = \int \frac{dt}{\sqrt{t}}$$

$$= \int t^{\frac{-1}{2}}dt$$

$$= 2t^{\frac{1}{2}}+c$$

$$= 2\sqrt{t}+c$$

$\therefore \quad I = 2\sqrt{2x^3+3x+1}+c$

Indefinite Integrals Ex 19.9 Q23

Let $I = \int \frac{1}{1+\sqrt{x}}dx - - - - - (i)$

Let $x = t^2$ then,

$dx = d\left(t^2\right)$

$\Rightarrow \quad dx = 2t\,dt$

Putting $x = t^2$ and $dx = 2t\,dt$ in equation (i), we get

$$I = \int \frac{2t}{1+\sqrt{t^2}}dt$$
$$= \int \frac{2t}{1+t}dt$$
$$= 2\int \frac{t}{1+t}dt$$
$$= 2\int \frac{1+t-1}{1+t}dt$$
$$= 2\int \left[\frac{1+t}{1+t} - \frac{1}{1+t}\right]dt$$
$$= 2\int dt - 2\int \frac{1}{1+t}dt$$
$$= 2t - 2\log\left(1+t\right) + c$$
$$= 2\sqrt{x} - 2\log\left(1+\sqrt{x}\right) + c$$

$\therefore \quad I = 2\sqrt{x} - 2\log\left(1+\sqrt{x}\right) + c$

Indefinite Integrals Ex 19.9 Q24

Let $I = \int e^{\cos^2 x}\sin 2x\,dx - - - - (i)$

Let $\cos^2 x = t$ then,

$d\left(\cos^2 x\right) = dt$

$\Rightarrow \quad -2\cos x \sin x\,dx = dt$

$\Rightarrow \quad -\sin 2x\,dx = dt$

$\Rightarrow \quad \sin 2x\,dx = -dt$

Putting $\cos^2 x = t$ and $\sin 2x\,dx = -dt$ in equation (i), we get

$$I = \int e^t\left(-dt\right)$$
$$= -e^t + c$$
$$= -e^{\cos^2 x} + c$$

$\therefore \quad I = -e^{\cos^2 x} + c$

Indefinite Integrals Ex 19.9 Q25

Let $\quad I = \int \frac{1+\cos x}{(x+\sin x)^3} dx \text{-----} (i)$

Let $\quad x + \sin x = t \quad$ then,

$d(x+\sin x) = dt$

$\Rightarrow \quad (1+\cos x) dx = dt$

Putting $x + \sin x = t$ and $(1+\cos x)dx = dt$ in equation (i), we get

$$I = \int \frac{dt}{t^3}$$
$$= \int t^{-3} dt$$
$$= \frac{t^{-2}}{-2} + c$$
$$= -\frac{1}{2t^2} + c$$
$$= \frac{-1}{2(x+\sin x)^2} + c$$

$$\therefore \quad I = \frac{-1}{2(x+\sin x)^2} + c$$

Indefinite Integrals Ex 19.9 Q26

$$\frac{\cos x - \sin x}{1+\sin 2x} = \frac{\cos x - \sin x}{(\sin^2 x + \cos^2 x) + 2\sin x \cos x}$$

$$\left[\sin^2 x + \cos^2 x = 1; \ \sin 2x = 2\sin x \cos x\right]$$

$$= \frac{\cos x - \sin x}{(\sin x + \cos x)^2}$$

Let $\sin x + \cos x = t$

$\therefore (\cos x - \sin x) dx = dt$

$$\Rightarrow \int \frac{\cos x - \sin x}{1+\sin 2x} dx = \int \frac{\cos x - \sin x}{(\sin x + \cos x)^2} dx$$
$$= \int \frac{dt}{t^2}$$
$$= \int t^{-2} dt$$
$$= -t^{-1} + C$$
$$= -\frac{1}{t} + C$$
$$= \frac{-1}{\sin x + \cos x} + C$$

Indefinite Integrals Ex 19.9 Q27

Let $\quad I = \int \frac{\sin 2x}{(a + b\cos 2x)^2} dx - - - - - (i)$

Let $\quad a + b\cos 2x = t \quad$ then,

$d(a + b\cos 2x) = dt$

$\Rightarrow \quad b(-2\sin 2x)dx = dt$

$\Rightarrow \quad \sin 2x \, dx = -\frac{dt}{2b}$

Putting $a + b\cos 2x = t$ and $\sin 2x \, dx = -\frac{dt}{2b}$ in equation (i), we get

$$I = \int \frac{1}{t^2} \times \frac{-dt}{2b}$$
$$= \frac{-1}{2b} \int t^{-2} dt$$
$$= -\frac{1}{2b}\left(-1t^{-1}\right) + c$$
$$= \frac{1}{2bt} + c$$
$$= \frac{1}{2b(a + b\cos 2x)} + c$$

$$\therefore \quad I = \frac{1}{2b(a + b\cos 2x)} + c$$

Indefinite Integrals Ex 19.9 Q28

Let $\quad I = \int \frac{\log x^2}{x} dx - - - - - (i)$

Let $\quad \log x = t \quad$ then,

$d(\log x) = dt$

$\Rightarrow \quad \frac{1}{x} dx = dt$

$\Rightarrow \quad \frac{dx}{x} = dt$

Now, $\quad I = \int \frac{\log x^2}{x} dx$

$= \int \frac{2\log x}{x} dx$

$= 2\int \frac{\log x}{x} dx - - - - (ii)$

Putting $\log x = t$ and $\frac{dx}{x} = dt$ in equation (ii), we get

$$I = 2\int t \, dt$$
$$= \frac{2t^2}{2} + c$$
$$= t^2 + c$$

$$\therefore \quad I = (\log x)^2 + c$$

Indefinite Integrals Ex 19.9 Q29

Let $\quad I = \int \frac{\sin x}{(1+\cos x)^2} dx - - - - - (i)$

Let $\quad 1+\cos x = t \quad$ then,

$d(1+\cos x) = dt$

$\Rightarrow \quad -\sin x \, dx = dt$

$\Rightarrow \quad \sin x \, dx = -dt$

Putting $1+\cos x = t$ and $\sin dx = -dt$ in equation (ii), we get

$$I = \int \frac{-dt}{t^2}$$
$$= -\int t^{-2} dt$$
$$= -\left(-1t^{-1}\right) + c$$
$$= \frac{1}{t} + c$$
$$= \frac{1}{1+\cos x} + c$$

$\therefore \quad I = \frac{1}{1+\cos x} + c$

Indefinite Integrals Ex 19.9 Q30

Let $\log \sin x = t$

$\Rightarrow \frac{1}{\sin x} \cdot \cos x \; dx = dt$

$\therefore \cot x \; dx = dt$

$$\Rightarrow \int \cot x \; \log \sin x \; dx = \int t \, dt$$
$$= \frac{t^2}{2} + C$$
$$= \frac{1}{2}(\log \sin x)^2 + C$$

Indefinite Integrals Ex 19.9 Q31

Let $\quad I = \int \sec x \cdot \log(\sec x + \tan x) dx - - - - - (i)$

Let $\quad \log(\sec x + \tan x) = t \quad$ then,

$d[\log(\sec x + \tan x)] = dt$

$\Rightarrow \quad \sec x \, dx = dt \qquad \left[\because \quad \frac{d}{dx}(\log(\sec x + \tan x)) = \sec x\right]$

Putting $\log(\sec x + \tan x) = t$ and $\sec x \, dx = dt$ in equation (i), we get

$$I = \int t \, dt$$
$$= \frac{t^2}{2} + c$$
$$= \frac{1}{2}[\log(\sec x + \tan x)]^2 + c$$

$\therefore \quad I = \frac{1}{2}[\log(\sec x + \tan x)]^2 + c$

Indefinite Integrals Ex 19.9 Q32

Let $\quad I = \int \operatorname{cosec} x \log\left(\operatorname{cosec} x - \cot x\right) dx - - - - - (i)$

Let $\quad \log\left(\operatorname{cosec} x - \cot x\right) = t \quad$ then,

$dx\left[\log\left(\operatorname{cosec} x - \cot x\right)\right] = dt$

$\Rightarrow \quad \operatorname{cosec} x \, dx = dt \qquad \left[\because \quad \frac{d}{dx}\left(\log\left(\operatorname{cosec} x - \cot x\right)\right) = \operatorname{cosec} x\right]$

Putting $\log\left(\operatorname{cosec} x - \cot x\right) = t$ and $\operatorname{cosec} x \, dx = dt$ in equation (i), we get

$$I = \int t \, dt$$
$$= \frac{t^2}{2} + c$$

$$\therefore \quad I = \frac{1}{2}\left[\log\left(\operatorname{cosec} x - \cot x\right)\right]^2 + c$$

Indefinite Integrals Ex 19.9 Q33

Let $\quad I = \int x^3 \cos x^4 dx - - - - - (i)$

Let $\quad x^4 = t$ then,

$dx\left(x^4\right) = dt$

$\Rightarrow \quad 4x^3 dx = dt$

$\Rightarrow \quad x^3 = \frac{dt}{4}$

Putting $x^4 = t$ and $x^3 dx = \frac{dt}{4}$ in equation (i), we get

$$I = \int \cos t \frac{dt}{4}$$
$$= \frac{1}{4}\sin t + c$$

$$\therefore \quad I = \frac{1}{4}\sin x^4 + c$$

Indefinite Integrals Ex 19.9 Q34

Let $\quad I = \int x^3 \sin x^4 dx - - - - - (i)$

Let $\quad x^4 = t$ then,

$d\left(x^4\right) = dt$

$\Rightarrow \quad 4x^3 dx = dt$

$\Rightarrow \quad x^3 = \frac{dt}{4}$

Putting $x^4 = t$ and $x^3 dx = \frac{dt}{4}$ in equation (i), we get

$$I = \int \sin t \frac{dt}{4}$$
$$= \frac{1}{4}\int \sin t \, dt$$
$$= -\frac{1}{4}\cos t + c$$
$$= -\frac{1}{4}\cos x^4 + c$$

$$\therefore \quad I = -\frac{1}{4}\cos x^4 + c$$

Indefinite Integrals Ex 19.9 Q35

Let $I = \int \frac{x \sin^{-1} x^2}{\sqrt{1-x^4}} dx ----(i)$

Let $\sin^{-1} x^2 = t$ then,

$d\left(\sin^{-1} x^2\right) = dt$

$\Rightarrow \quad 2x \times \frac{1}{\sqrt{1-x^4}} dx = dt$

$\Rightarrow \quad \frac{x}{\sqrt{1-x^4}} dx = \frac{dt}{2}$

Putting $\sin^{-1} x^2 = t$ and $\frac{x}{\sqrt{1-x^4}} dx = \frac{dt}{2}$ in equation (i), we get

$$I = \int t \frac{dt}{2}$$
$$= \frac{1}{2} \times \frac{t^2}{2} + c$$
$$= \frac{1}{4}\left(\sin^{-1} x^2\right)^2 + c$$

$\therefore \quad I = \frac{1}{4}\left(\sin^{-1} x^2\right)^2 + c$

Indefinite Integrals Ex 19.9 Q36

Let $I = \int x^3 \sin\left(x^4+1\right) dx -----(i)$

Let $x^4 + 1 = t$ then,

$d\left(x^4+1\right) = dt$

$\Rightarrow \quad x^4 dx = dt$

$\Rightarrow \quad x^3 dx = \frac{dt}{4}$

Putting $x^4 + 1 = t$ and $x^3 dx = \frac{dt}{4}$ in equation (i), we get

$$I = \int \sin t \frac{dt}{4}$$
$$= -\frac{1}{4} \cos t + c$$
$$= -\frac{1}{4} \cos\left(x^4+1\right) + c$$

$\therefore \quad I = -\frac{1}{4} \cos\left(x^4+1\right) + c$

Indefinite Integrals Ex 19.9 Q37

Let $I = \int \frac{(x+1)e^x}{\cos^2(xe^x)} dx$ - - - - - (i)

Let $xe^x = t$ then,

$d(xe^x) = dt$

$\Rightarrow \quad (e^x + xe^x)dx = dt$

$\Rightarrow \quad (x+1)e^x dx = dt$

Putting $xe^x = t$ and $(x+1)e^x\, dx = dt$ in equation (i), we get

$$I = \int \frac{dt}{\cos^2 t}$$
$$= \int \sec^2 t\, dt$$
$$= \tan t + c$$
$$= \tan(xe^x) + c$$

$\therefore \quad I = \tan(xe^x) + c$

Indefinite Integrals Ex 19.9 Q38

Let $I = \int x^2 e^{x^3} \cos(e^{x^3}) dx$ - - - - - (i)

Let $e^{x^3} = t$ then,

$d(e^{x^3}) = dt$

$\Rightarrow \quad 3x^2 e^{x^3}\, dx = dt$

$\Rightarrow \quad x^2 e^{x^3} dx = \frac{dt}{3}$

Putting $e^{x^3} = t$ and $x^2 e^{x^3}\, dx = \frac{dt}{3}$ in equation (i), we get

$$I = \int \cos t \frac{dt}{3}$$
$$= \frac{\sin t}{3} + c$$
$$= \frac{\sin(e^{x^3})}{3} + c$$

$\therefore \quad I = \frac{1}{3}\sin(e^{x^3}) + c$

Indefinite Integrals Ex 19.9 Q39

Let $\quad I = \int 2x \sec^3\left(x^2+3\right)\tan\left(x^2+3\right)dx$ - - - - - (i)

Let $\quad \sec\left(x^2+3\right) = t \qquad$ then,

$d\left[\sec\left(x^2+3\right)\right] = dt$

$\Rightarrow \quad 2x\sec\left(x^2+3\right)\tan\left(x^2+3\right)dx = dt$

Putting $\sec\left(x^2+3\right) = t$ and $2x\sec\left(x^2+3\right)\tan\left(x^2+3\right)dx = dt$ in equation (i), we get

$$I = \int t^2 dt$$
$$= \frac{t^3}{3} + c$$
$$= \frac{1}{3}\left[\sec\left(x^2+3\right)\right]^3 + c$$

$\therefore \quad I = \frac{1}{3}\left[\sec\left(x^2+3\right)\right]^3 + c$

Indefinite Integrals Ex 19.9 Q40

$$\frac{(x+1)(x+\log x)^2}{x} = \left(\frac{x+1}{x}\right)(x+\log x)^2 = \left(1+\frac{1}{x}\right)(x+\log x)^2$$

Let $(x+\log x) = t$

$$\Rightarrow \left(1+\frac{1}{x}\right)dx = dt$$

$$\Rightarrow \int\left(1+\frac{1}{x}\right)(x+\log x)^2 dx = \int t^2 dt$$
$$= \frac{t^3}{3} + C$$
$$= \frac{1}{3}(x+\log x)^3 + C$$

Indefinite Integrals Ex 19.9 Q41

Let $\quad I = \int \tan x \sec^2 x \sqrt{1 - \tan^2 x}\, dx - - - - - (i)$

Let $\quad 1 - \tan^2 x = t$ then,

$d\left(1 - \tan^2 x\right) = dt$

$\Rightarrow \quad -2 \tan x \sec^2 x\, dx = dt$

$\Rightarrow \quad \tan x \sec^2 x dx = \frac{-dt}{2}$

Putting $1 - \tan^2 x = t$ and $\tan x \sec^2 x\, dx = -\frac{dt}{2}$ in equation (i),

we get

$$I = \int \sqrt{t} \times \frac{-dt}{2}$$

$$= -\frac{1}{2} \int t^{\frac{1}{2}} dt$$

$$= -\frac{1}{2} \times \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= -\frac{1}{3} t^{\frac{3}{2}} + c$$

$$\therefore \quad I = -\frac{1}{3}\left[1 - \tan^2 x\right]^{\frac{3}{2}} + c$$

Indefinite Integrals Ex 19.9 Q42

Let $\quad I = \int \log x \frac{\sin\left(1 + (\log x)^2\right)}{x} dx - - - - - (i)$

Let $\quad 1 + (\log x)^2 = t$ then,

$d\left(1 + (\log x)^2\right) = dt$

$\Rightarrow \quad 2 \log x \frac{1}{x} dx = dt$

$\Rightarrow \quad \frac{\log x}{x} dx = \frac{dt}{2}$

Putting $1 + (\log x)^2 = t$ and $\frac{\log x}{x} dx = \frac{dt}{2}$ in equation (i),

we get

$$I = \int \sin t \times \frac{dt}{2}$$

$$= \frac{1}{2} \int \sin t\, dt$$

$$\therefore \quad I = -\frac{1}{2} \cos t + c$$

$$= -\frac{1}{2} \cos\left[1 + (\log x)^2\right] + c$$

$$\therefore \quad I = -\frac{1}{2} \cos\left[1 + (\log x)^2\right] + c$$

Indefinite Integrals Ex 19.9 Q43

Let $I = \int \frac{1}{x^2}\cos^2\left(\frac{1}{x}\right)dx$ - - - - - (i)

Let $\frac{1}{x} = t$ then,

$d\left(\frac{1}{x}\right) = dt$

$\Rightarrow$ $\frac{-1}{x^2}dx = dt$

$\Rightarrow$ $\frac{1}{x^2}dx = -dt$

Putting $\frac{1}{x} = t$ and $\frac{1}{x^2}dx = -dt$ in equation (i),

we get

$$I = \int \cos^2 t\ (-dt)$$
$$= -\int \cos^2 t\, dt$$
$$= -\int \frac{\cos^2 2t + 1}{2}dt$$
$$= -\frac{1}{2}\int \cos 2t\, dt - \frac{1}{2}\int dt$$
$$= -\frac{1}{2} \times \frac{\sin 2t}{2} - \frac{1}{2}t + c$$

$\therefore$ $I = -\frac{1}{4}\sin 2t - \frac{1}{2}t + c$

$$= -\frac{1}{4}\sin 2 \times \frac{1}{x} - \frac{1}{2} \times \frac{1}{x} + c$$

$\therefore$ $I = -\frac{1}{4}\sin\left(\frac{2}{x}\right) - \frac{1}{2}\left(\frac{1}{x}\right) + c$

Indefinite Integrals Ex 19.9 Q44

Let $I = \int \sec^4 x \tan x\, dx$ - - - - (i)

Let $\tan x = t$ then,

$d(\tan x) = dt$

$\Rightarrow$ $\sec^2 x\, dx = dt$

$\Rightarrow$ $dx = \frac{dt}{\sec^2 x}$

Putting $\tan x = t$ and $dx = \frac{dt}{\sec^2 x}$ in equation (i),

we get

$$I = \int \sec^4 x \tan x \frac{dt}{\sec^2 x}$$
$$= \int \sec^2 x\, t dt$$
$$= \int \left(1 + \tan^2 x\right) t\, dt$$
$$= \int \left(1 + t^2\right) t\, dt$$
$$= \int \left(t + t^3\right) dt$$
$$= \frac{t^2}{2} + \frac{t^4}{4} + c$$
$$= \frac{\tan^2 x}{2} + \frac{\tan^4 x}{4} + c$$

$\therefore$ $I = \frac{1}{2}\tan^2 x + \frac{1}{4}\tan^4 x + c$

Indefinite Integrals Ex 19.9 Q45

Let $\quad I = \int \frac{e^{\sqrt{x}} \cos\left(e^{\sqrt{x}}\right)}{\sqrt{x}} dx \text{ - - - - - (i)}$

Let $\quad e^{\sqrt{x}} = t \quad$ then,

$d\left(e^{\sqrt{x}}\right) = dt$

$\Rightarrow \quad e^{\sqrt{x}} \frac{1}{2\sqrt{x}} dx = dt$

$\Rightarrow \quad \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2dt$

Putting $e^{\sqrt{x}} = t$ and $\frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2dt$ in equation (i),

we get

$$\begin{aligned}
I &= \int \cos t \times 2dt \\
&= 2\int \cos t \, dt \\
&= 2 \sin t + c \\
&= 2 \sin\left(e^{\sqrt{x}}\right) + c
\end{aligned}$$

$\therefore \quad I = 2\sin\left(e^{\sqrt{x}}\right) + c$

Indefinite Integrals Ex 19.9 Q46

Let $\quad I = \int \frac{\cos^5 x}{\sin x} dx \text{ - - - - (i)}$

Let $\quad \sin x = t \quad$ then,

$d(\sin x) = dt$

$\Rightarrow \quad \cos x \, dx = dt$

$\Rightarrow \quad dx = \frac{dt}{\cos x}$

Putting $\sin x = t$ and $dx = \frac{dt}{\cos x}$ in equation (i),

we get

$$\begin{aligned}
I &= \int \frac{\cos^5 x}{t} \times \frac{dt}{\cos x} \\
&= \int \frac{\cos^4 x}{t} dt \\
&= \int \frac{\left(1 - \sin^2 x\right)^2}{t} dt \\
&= \int \frac{\left(1 - t^2\right)^2}{t} dt \\
&= \int \frac{1 + t^4 - 2t^2}{t} dt \\
&= \int \frac{1}{t} dt + \int \frac{t^4}{t} dt - 2\int \frac{t^2}{t} dt \\
&= \log|t| + \frac{t^4}{4} - \frac{2t^2}{2} + c \\
&= \log|\sin x| + \frac{\sin^4 x}{4} - \sin^2 x + c
\end{aligned}$$

$\therefore \quad I = \frac{1}{4} \sin^4 x - \sin^2 x + \log|\sin x| + c$

Indefinite Integrals Ex 19.9 Q47

Let $\sqrt{x} = t$

$\Rightarrow \frac{1}{2\sqrt{x}}dx = dt$

$\Rightarrow \frac{1}{\sqrt{x}}dx = 2dt$

Therefore,

$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

$=2\int \sin t\,dt$

$= -2\cos t + C$

$= -2\cos\sqrt{x} + C$

Indefinite Integrals Ex 19.9 Q48

Let $I = \int \frac{(x+1)e^x}{\sin^2(xe^x)}dx$ ----- (i)

Let $xe^x = t$ then,

$d(xe^x) = dt$

$\Rightarrow (xe^x + e^x)dx = dt$

$\Rightarrow (x+1)e^x dx = dt$

Putting $xe^x = t$ and $(x+1)e^x dx = dt$ in equation (i), we get

$I = \int \frac{dt}{\sin^2 t}$

$= \int \operatorname{cosec}^2 t\,dt$

$= -\cot t + c$

$= -\cot(xe^x) + c$

$\therefore \quad I = -\cot(xe^x) + c$

Indefinite Integrals Ex 19.9 Q49

Let $I = \int 5^{x+\tan^{-1}x}\left(\frac{x^2+2}{x^2+1}\right)dx$ - - - - - (i)

Let $x + \tan^{-1}x = t$ then,

$d\left(x + \tan^{-1}x\right) = dt$

$\Rightarrow \left(1 + \frac{1}{1+x^2}\right)dx = dt$

$\Rightarrow \left(\frac{1+x^2+1}{1+x^2}\right)dx = dt$

$\Rightarrow \frac{\left(x^2+2\right)}{\left(x^2+1\right)}dx = dt$

Putting $x + \tan^{-1}x = t$ and $\left(\frac{x^2+2}{x^2+1}\right)dx = dt$ in equation (i),

we get

$$I = \int 5^t\,dt$$
$$= \frac{5^t}{\log 5} + c$$
$$= \frac{5^{x+\tan^{-1}x}}{\log 5} + c$$

$$\therefore \quad I = \frac{5^{x+\tan^{-1}x}}{\log 5} + c$$

Indefinite Integrals Ex 19.9 Q50

Let $I = \int \frac{e^{m\sin^{-1}x}}{\sqrt{1-x^2}}dx$ - - - - - (i)

Let $m\sin^{-1}x = t$ then,

$d\left(m\sin^{-1}x\right) = dt$

$\Rightarrow m\frac{1}{\sqrt{1-x^2}}dx = dt$

$\Rightarrow \frac{dx}{\sqrt{1-x^2}} = \frac{dt}{m}$

Putting $m\sin^{-1}x = t$ and $\frac{dx}{\sqrt{1-x^2}} = \frac{dt}{m}$ in equation (i),

we get

$$I = \int e^t\,\frac{dt}{m}$$
$$= \frac{1}{m}e^t + c$$
$$= \frac{1}{m}e^{m\sin^{-1}x} + c$$

$$\therefore \quad I = \frac{1}{m}e^{m\sin^{-1}x} + c$$

Indefinite Integrals Ex 19.9 Q51

Let $\sqrt{x} = t$

$$\Rightarrow \frac{1}{2\sqrt{x}} dx = dt$$

$$\Rightarrow \int \frac{\cos\sqrt{x}}{\sqrt{x}} dx = 2\int \cos t \, dt$$
$$= 2\sin t + C$$
$$= 2\sin\sqrt{x} + C$$

Indefinite Integrals Ex 19.9 Q52

Let $\quad I = \int \frac{\sin\left(\tan^{-1} x\right)}{1+x^2} dx \text{ - - - - - (i)}$

Let $\quad \tan^{-1} x = t \quad$ then,

$d\left(\tan^{-1} x\right) = dt$

$$\Rightarrow \quad \frac{1}{1+x^2} dx = dt$$

Putting $\tan^{-1} x = t$ and $\frac{dx}{1+x^2} = dt$ in equation (i), we get

$$I = \int \sin t \, dt$$
$$= -\cos t + c$$
$$= -\cos\left(\tan^{-1} x\right) + c$$

$$\therefore \quad I = -\cos\left(\tan^{-1} x\right) + c$$

Indefinite Integrals Ex 19.9 Q53

Let $\quad I = \int \frac{\sin(\log x)}{x} dx \text{ - - - - - (i)}$

Let $\quad \log x = t \quad$ then,

$d(\log x) = dt$

$$\Rightarrow \quad \frac{1}{x} dx = dt$$

Putting $\log x = t$ and $\frac{1}{x} dx = dt$ in equation (i), we get

$$I = \int \sin t \, dt$$
$$= -\cos t + c$$
$$= -\cos(\log x) + c$$

$$\therefore \quad I = -\cos(\log x) + c$$

Indefinite Integrals Ex 19.9 Q54

Let $\tan^{-1}x = t$

Differentiating the above function with respect to, w, *we have*,

$$\frac{1}{1+x^2}dx = dt$$

$$\Rightarrow \int \frac{e^{m\tan^{-1}x}}{1+x^2} = \int e^{mt} \times dt$$

$$\Rightarrow \int \frac{e^{m\tan^{-1}x}}{1+x^2} = \frac{e^{mt}}{m}$$

Resubstituting the value of t *in the above solution*, *we have*,

$$\Rightarrow \int \frac{e^{m\tan^{-1}x}}{1+x^2} = \frac{e^{m\tan^{-1}x}}{m} + C$$

Indefinite Integrals Ex 19.9 Q55

Let $\quad I = \int \frac{x}{\sqrt{x^2+a^2}+\sqrt{x^2-a^2}}dx$

$$\therefore \quad I = \int \frac{x}{\sqrt{x^2+a^2}+\sqrt{x^2-a^2}} \times \frac{\sqrt{x^2+a^2}-\sqrt{x^2-a^2}}{\sqrt{x^2+a^2}-\sqrt{x^2-a^2}}dx$$

$$= \int \frac{x\left(\sqrt{x^2+a^2}-\sqrt{x^2-a^2}\right)}{x^2+a^2-x^2+a^2}dx$$

$$= \int \frac{x}{2a^2}\left(\sqrt{x^2+a^2}-\sqrt{x^2-a^2}\right)dx$$

$$\therefore \quad I = \frac{1}{2a^2}\int x\left(\sqrt{x^2+a^2}-\sqrt{x^2-a^2}\right)dx \text{ ------ (i)}$$

Let $\quad x^2 = t \quad$ then,

$$d\left(x^2\right) = dt$$

$$\Rightarrow \quad 2x\,dx = dt$$

$$\Rightarrow \quad x\,dx = \frac{dt}{2}$$

Putting $x^2 = t$ and $x\,dx = \frac{dt}{2}$ in equation (i),

we get

$$I = \frac{1}{2a^2}\int\left(\sqrt{t+a^2}-\sqrt{t-a^2}\right)\frac{dx}{2}$$

$$= \frac{1}{4a^2}\left[\frac{2}{3}\left(t+a^2\right)^{\frac{3}{2}} - \frac{2}{3}\left(t-a^2\right)^{\frac{3}{2}}\right]+c$$

$$\therefore \quad I = \frac{1}{4a^2}\left[\frac{2}{3}\left(x^2+a^2\right)^{\frac{3}{2}} - \frac{2}{3}\left(x^2-a^2\right)^{\frac{3}{2}}\right]+c$$

$$= \frac{1}{6a^2}\left[\left(x^2+a^2\right)^{\frac{3}{2}} - \left(x^2-a^2\right)^{\frac{3}{2}}\right]+c$$

Indefinite Integrals Ex 19.9 Q56

Let $\quad I = \int x \dfrac{\tan^{-1} x^2}{1+x^4} dx \text{ - - - - - (i)}$

Let $\quad \tan^{-1} x^2 = t \qquad$ then,

$d\left(\tan^{-1} x^2\right) = dt$

$\Rightarrow \quad \dfrac{1 \times 2x}{1+\left(x^2\right)^2} dx = dt$

$\Rightarrow \quad \dfrac{1 \times x}{1+x^4} dx = \dfrac{dt}{2}$

Putting $\tan^{-1} x^2 = t$ and $\dfrac{x}{1+x^4} dx = \dfrac{dt}{2}$ in equation (i), we get

$$I = \int t \frac{dx}{2}$$

$$= \frac{1}{2} \int t \, dt$$

$$= \frac{1}{2} \times \frac{t^2}{2} + c$$

$$\therefore \quad I = \frac{t^2}{4} + c$$

$$= \frac{\left(\tan^{-1} x^2\right)^2}{4} + c$$

$$\therefore \quad I = \frac{1}{4}\left(\tan^{-1} x^2\right)^2 + c$$

Indefinite Integrals Ex 19.9 Q57

Let $\quad I = \int \dfrac{\left(\sin^{-1} x\right)^3}{\sqrt{1-x^2}} dx \text{ - - - - (i)}$

Let $\quad \sin^{-1} x = t \qquad$ then,

$d\left(\sin^{-1} x\right) = dt$

$\Rightarrow \quad \dfrac{1}{\sqrt{1-x^2}} dx = dt$

Putting $\sin^{-1} x = t$ and $\dfrac{1}{\sqrt{1-x^2}} dx = dt$ in equation (i), we get

$$I = \int t^3 dt$$

$$= \frac{t^4}{4} + c$$

$$\therefore \quad I = \frac{1}{4}\left(\sin^{-1} x\right)^4 + c$$

Indefinite Integrals Ex 19.9 Q58

Let $\quad I = \int \frac{\sin(2+3\log x)}{x} dx ----(i)$

Let $\quad 2+3\log x = t \quad$ then,

$d(2+3\log x) = dt$

$\Rightarrow \quad 3\frac{1}{x}dx = dt$

$\Rightarrow \quad \frac{dx}{x} = \frac{dt}{3}$

Putting $2+3\log x = t$ and $\frac{dx}{x} = \frac{dt}{3}$ in equation (i),

we get

$$I = \int \sin t \frac{dt}{3}$$
$$= \frac{1}{3}(-\cos t) + c$$
$$= -\frac{1}{3}\cos(2+3\log x) + c$$

$\therefore \quad I = -\frac{1}{3}\cos(2+3\log x) + c$

Indefinite Integrals Ex 19.9 Q59

Let $\quad I = \int x e^{x^2} dx ----(i)$

Let $\quad x^2 = t \quad$ then,

$d(x^2) = dt$

$\Rightarrow \quad 2x\,dx = dt$

$\Rightarrow \quad x\,dx = \frac{dt}{2}$

Putting $x^2 = t$ and $x\,dx = \frac{dt}{2}$ in equation (i),

we get

$$I = \int e^t \frac{dt}{2}$$
$$= \frac{1}{2}e^t + c$$
$$= \frac{1}{2}e^{x^2} + c$$

$\therefore \quad I = \frac{1}{2}e^{x^2} + c$

Indefinite Integrals Ex 19.9 Q60

Let $\quad I = \int \frac{e^{2x}}{1+e^x} dx \text{ ----(i)}$

Let $\quad 1+e^x = t \quad$ then,

$d\left(1+e^x\right) = dt$

$\Rightarrow \quad e^x\, dx = dt$

$\Rightarrow \quad dx = \frac{dt}{e^x}$

Putting $1+e^x = t$ and $dx = \frac{dt}{e^x}$ in equation (i),

we get

$$I = \int \frac{e^{2x}}{t} \times \frac{dt}{e^x}$$
$$= \int \frac{e^x}{t} dt$$
$$= \int \frac{t-1}{t} dt$$
$$= \int \left(\frac{t}{t} - \frac{1}{t}\right) dt$$
$$= t - \log|t| + c$$
$$= \left(1+e^x\right) - \log\left|1+e^x\right| + c$$

$\therefore \quad I = 1+e^x - \log\left|1+e^x\right| + c$

Indefinite Integrals Ex 19.9 Q61

Let $\quad I = \int \frac{\sec^2\sqrt{x}}{\sqrt{x}} dx \text{ ----(i)}$

Let $\quad \sqrt{x} = t$ then,

$d\left(\sqrt{x}\right) = dt$

$\Rightarrow \quad \frac{1}{2\sqrt{x}} dx = dt$

$\Rightarrow \quad dx = 2\sqrt{x}\, dt$

$\Rightarrow \quad dx = 2t\,dt \quad \left[\because \quad \sqrt{x} = t\right]$

Putting $\sqrt{x} = t$ and $dx = 2t\,dt$ in equation (i),

we get

$$I = \int \frac{\sec^2 t}{t} \times 2t\, dt$$
$$= 2\int \sec^2 t\, dt$$
$$= 2\tan t + c$$
$$= 2\tan\sqrt{x} + c$$

$\therefore \quad I = 2\tan\sqrt{x} + c$

Indefinite Integrals Ex 19.9 Q62

$$\tan^3 2x \sec 2x = \tan^2 2x \tan 2x \sec 2x$$
$$= \left(\sec^2 2x - 1\right)\tan 2x \sec 2x$$
$$= \sec^2 2x \cdot \tan 2x \sec 2x - \tan 2x \sec 2x$$
$$\therefore \int \tan^3 2x \sec 2x \; dx = \int \sec^2 2x \tan 2x \sec 2x \; dx - \int \tan 2x \sec 2x \; dx$$
$$= \int \sec^2 2x \tan 2x \sec 2x \; dx - \frac{\sec 2x}{2} + C$$

Let $\sec 2x = t$

$\therefore 2\sec 2x \tan 2x \; dx = dt$

$$\therefore \int \tan^3 2x \sec 2x \; dx = \frac{1}{2}\int t^2 dt - \frac{\sec 2x}{2} + C$$
$$= \frac{t^3}{6} - \frac{\sec 2x}{2} + C$$
$$= \frac{\left(\sec 2x\right)^3}{6} - \frac{\sec 2x}{2} + C$$

Indefinite Integrals Ex 19.9 Q63

Let $\quad I = \int \frac{x+\sqrt{x+1}}{x+2}dx - - - - - (i)$

Let $\quad x+1 = t^2$ then,

$d(x+1) = d(t^2)$

$\Rightarrow \quad dx = 2t\,dt$

Putting $x+1 = t^2$ and $dx = 2t\,dt$ in equation (i), we get

$$\begin{aligned}
I &= \int \frac{x+\sqrt{t^2}}{x+2}2t\,dt \\
&= 2\int \frac{(t^2-1)+t}{(t^2-1)+2}t\,dt \quad \left[\because \quad x+1 = t^2\right] \\
&= 2\int \frac{t^2+t-1}{t^2+1}t\,dt \\
&= 2\int \frac{t^3+t^2-t}{t^2+1}dt \\
&= 2\left[\int \frac{t^3}{t^2+1}dt + \int \frac{t^2}{t^2+1}dt - \int \frac{t}{t^2+1}dt\right]
\end{aligned}$$

$\therefore \quad I = 2\left[\int \frac{t^3}{t^2+1}dt + \int \frac{t^2}{t^2+1}dt - \int \frac{t}{t^2+1}dt\right] - - - - (ii)$

Let $I_1 = \int \frac{t^3}{t^2+1}dt$

$I_2 = \int \frac{t^2}{t^2+1}dt$

and $\quad I_3 = \int \frac{t}{t^2+1}dt$

Now, $I_1 = \int \frac{t^3}{t^2+1}dt$

$= \int \left(t - \frac{t}{t^2+1}\right)dt$

$= \frac{t^2}{2} - \frac{1}{2}\log(t^2+1)$

$\therefore \quad I = \frac{t^2}{2} - \frac{1}{2}\log(t^2+1) + c_1 - - - - (iii)$

Since, $I_2 = \int \frac{t^2}{t^2+1}dt$

$= \int \frac{t^2+1-1}{t^2+1}dt$

$= \int \frac{t^2+1}{t^2+1}dt - \int \frac{1}{t^2+1}dt$

$= \int dt - \int \frac{1}{t^2+1}dt$

$\Rightarrow \quad I_2 = t - \tan^{-1}(t^2) + c_2 - - - - (iv)$

and, $I_3 = \int \frac{t}{t^2+1} dt$

$= \frac{1}{2}\log\left(1+t^2\right) + c_3$ ------ (v)

Using equations (ii), (iii), (iv) and (v), we get

$$I = 2\left[\frac{t^2}{2} - \frac{1}{2}\log\left(t^2+1\right) + c_1 + t - \tan^{-1}\left(t^2\right) + c_2 - \frac{1}{2}\log\left(1+t^2\right) + c_3\right]$$

$$= 2\left[\frac{t^2}{2} + t - \tan^{-1}\left(t^2\right) - \log\left(1+t^2\right) + c_1 + c_2 + c_3\right]$$

$$= 2\left[\frac{t^2}{2} + t - \tan^{-1}\left(t^2\right) - \log\left(1+t^2\right) + c_4\right] \qquad \left[\text{Putting } c_1 + c_2 + c_3 = c_4\right]$$

$$= t^2 + 2t - 2\tan^{-1}\left(t^2\right) - 2\log\left(1+t^2\right) + 2c_4$$

$$= \left(x+1\right) + 2\sqrt{x+1} - 2\tan^{-1}\left(\sqrt{x+1}\right) - 2\log\left(1+x+1\right) + 2c_4$$

$$= \left(x+1\right) + 2\sqrt{x+1} - 2\tan^{-1}\left(\sqrt{x+1}\right) - 2\log\left(x+2\right) + c \qquad \left[\text{Putting } 2c_4 = c\right]$$

$\therefore \quad I = \left(x+1\right) + 2\sqrt{x+1} - 2\tan^{-1}\left(\sqrt{x+1}\right) - 2\log\left(x+2\right) + c$

Indefinite Integrals Ex 19.9 Q64

Let $\quad I = \int 5^{5^{5^x}} 5^{5^x} 5^x dx$ ----- (i)

Let $\quad 5^{5^{5^x}} = t$ then,

$d\left(5^{5^{5^x}}\right) = dt$

$\Rightarrow \quad 5^{5^{5^x}} 5^{5^x} 5^x \left(\log 5\right)^3 dx = dt$

$\Rightarrow \quad 5^{5^{5^x}} 5^{5^{5^x}} 5^x dx = \frac{dt}{\left(\log 5\right)^3}$

Putting $5^{5^{5^x}} = t$ and $5^{5^{5^x}} 5^{5^x} 5^x dx = \frac{dt}{\left(\log 5\right)^3}$ in equation (i), we get

$$I = \int \frac{dt}{\left(\log 5\right)^3}$$

$$= \frac{1}{\left(\log 5\right)^3}\int dt$$

$$= \frac{t}{\left(\log 5\right)^3} + c$$

$\therefore \quad I = \frac{5^{5^{5^x}}}{\left(\log 5\right)^3} + c$

Indefinite Integrals Ex 19.9 Q65

Let $\quad I = \int \frac{1}{x\sqrt{x^4 - 1}} dx \text{ - - - - - (i)}$

Let $\quad x^2 = t$ then,

$d\left(x^2\right) = dt$

$\Rightarrow \quad 2x\,dx = dt$

$\Rightarrow \quad dx = \frac{dt}{2x}$

Putting $x^2 = t$ and $dx = \frac{dt}{2x}$ in equation (i),

we get

$$I = \int \frac{1}{x\sqrt{t^2 - 1}} \times \frac{dt}{2x}$$
$$= \frac{1}{2}\int \frac{1}{x^2\sqrt{t^2 - 1}} dt$$
$$= \frac{1}{2}\int \frac{1}{t\sqrt{t^2 - 1}} dt$$
$$= \frac{1}{2}\sec^{-1} t + c$$
$$= \frac{1}{2}\sec^{-1} x^2 + c$$

$\therefore \quad I = \frac{1}{2}\sec^{-1}\left(x^2\right) + c$

Indefinite Integrals Ex 19.9 Q66

Let $\quad I = \int \sqrt{e^x - 1}\, dx \text{ - - - - - (i)}$

Let $\quad e^x - 1 = t^2 \quad$ then,

$d\left(e^x - 1\right) = dt\left(t^2\right)$

$\Rightarrow \quad e^x\,dx = 2t\,dt$

$\Rightarrow \quad dx = \frac{2t}{e^x} dt$

$\Rightarrow \quad dx = \frac{2t}{t^2 + 1} dt \qquad \left[\because \quad e^x - 1 = t^2\right]$

Putting $e^x - 1 = t^2$ and $dx = \frac{2t\,dt}{t^2 + 1}$ in equation (i),

we get

$$I = \int \sqrt{t^2} \times \frac{2t\,dt}{t^2 + 1}$$
$$= 2\int \frac{t \times t}{t^2 + 1} dt$$
$$= 2\int \frac{t^2}{t^2 + 1} dt$$
$$= 2\int \frac{t^2 + 1 - 1}{t^2 + 1} dt$$
$$= 2\int \left[\frac{t^2 + 1}{t^2 + 1} - \frac{1}{t^2 + 1}\right] dt$$
$$= 2\int dt - 2\int \frac{1}{t^2 + 1} dt$$
$$= 2t - 2\tan^{-1}(t) + c$$
$$= 2\sqrt{\left(e^x - 1\right)} - 2\tan^{-1}\left(\sqrt{e^x - 1}\right) + c$$

$\therefore \quad I = 2\sqrt{e^x - 1} - 2\tan^{-1}\sqrt{e^x - 1} + c$

Indefinite Integrals Ex 19.9 Q67

$$I = \int \frac{1}{(x+1)(x^2+2x+2)} dx$$

$$= \int \frac{1}{(x+1)\left((x+1)^2+1\right)} dx$$

Let $x + 1 = \tan u$

$\Rightarrow \quad dx = \sec^2 u \; du$

$$\therefore \quad I = \int \frac{\sec^2 u}{\tan u\left(\tan^2 u + 1\right)} du$$
$$= \int \frac{\cos u}{\sin u} du$$
$$= \log\left|\sin u\right| + C$$
$$= \log\left|\frac{\tan u}{\sec^2 u}\right| + C$$

$$= \log\left|\frac{x+1}{\sqrt{x^2+2x+2}}\right| + C$$

Indefinite Integrals Ex 19.9 Q68

Let $\quad I = \int \frac{x^5}{\sqrt{1+x^3}} dx \; ----(\text{i})$

Let $\quad 1 + x^3 = t^2 \quad$ then,

$d\left(1 + x^3\right) = d\left(t^2\right)$

$\Rightarrow \quad 3x^2\, dx = dt\, 2t$

$\Rightarrow \quad dx = \frac{dt}{3x^2} 2t$

Putting $1 + x^3 = t^2$ and $dx = \frac{2t}{3x^2} dt$ in equation (i),

we get

$$I = \int \frac{x^5}{\sqrt{t^2}} \times \frac{2t}{3x^2} dt$$
$$= \int \frac{x^5}{t} \times \frac{2t}{3x^2} dt$$
$$= \frac{2}{3} \int x^3 dt$$
$$= \frac{2}{3} \int \left(t^2 - 1\right) dt$$
$$= \frac{2}{3} \times \frac{t^3}{3} - \frac{2}{3} t + c$$

$$\therefore \quad I = \frac{2}{9}\left(1 + x^3\right)^{\frac{3}{2}} - \frac{2}{3}\sqrt{1 + x^3} + c$$

Indefinite Integrals Ex 19.9 Q69

Let $\quad I = \int 4x^3\sqrt{5-x^2}\,dx ----(\text{i})$

Let $\quad 5-x^2 = t^2 \quad$ then,

$d\left(5-x^2\right) = t^2$

$\Rightarrow \quad -2x\,dx = 2t\,dt$

$\Rightarrow \quad dx = \frac{-t}{x}dt$

Putting $5-x^2 = t^2$ and $dx = \frac{-t}{x}dt$ in equation (i),
we get

$$I = \int 4x^3\sqrt{t^2} \times \frac{-t}{x}dt$$
$$= -4\int x^2 t \times t\,dt$$
$$= -4\int\left(5-t^2\right)t^2 dt \qquad \left[\because \quad 5-x^2 = t^2\right]$$
$$= -4\int\left(5t^2 - t^4\right)dt$$
$$= -20 \times \frac{t^3}{3} + 4\frac{t^5}{5} + c$$
$$= \frac{-20}{3} \times t^3 + \frac{4}{5} \times t^5 + c$$
$$= \frac{-20}{3} \times \left(5-x^2\right)^{\frac{3}{2}} + \frac{4}{5} \times \left(5-x^2\right)^{\frac{5}{2}} + c$$

$\therefore \quad I = \frac{-20}{3} \times \left(5-x^2\right)^{\frac{3}{2}} + \frac{4}{5} \times \left(5-x^2\right)^{\frac{5}{2}} + c$

Indefinite Integrals Ex 19.9 Q70

Let $\quad I = \int \frac{1}{\sqrt{x}+x}dx ----(\text{i})$

Let $\quad \sqrt{x} = t \quad$ then,

$d\left(\sqrt{x}\right) = dt$

$\Rightarrow \quad \frac{1}{2\sqrt{x}}dx = dt$

$\Rightarrow \quad dx = 2\sqrt{x}\,dt$

Putting $\sqrt{x} = t$ and $2\sqrt{x}\,dt = dx$ in equation (i),
we get

$$I = \int \frac{1}{t+t^2}2t \times dt \qquad \begin{bmatrix} \because & \sqrt{x} = t \\ \Rightarrow & x = t^2 \end{bmatrix}$$
$$= \int \frac{2t}{t(1+t)}dt$$
$$= 2\int \frac{t}{(1+t)}dt$$
$$= 2\log\left|1+t\right| + c$$
$$= 2\log\left|1+\sqrt{x}\right| + c$$

$\therefore \quad I = 2\log\left|1+\sqrt{x}\right| + c$

Indefinite Integrals Ex 19.9 Q71

$$\frac{1}{x^2\left(x^4+1\right)^{\frac{3}{4}}}$$

Multiplying and dividing by x^{-3}, we obtain

$$\frac{x^{-3}}{x^2\cdot x^{-3}\left(x^4+1\right)^{\frac{3}{4}}}=\frac{x^{-3}\left(x^4+1\right)^{\frac{-3}{4}}}{x^2\cdot x^{-3}}$$

$$=\frac{\left(x^4+1\right)^{\frac{-3}{4}}}{x^5\cdot\left(x^4\right)^{-\frac{3}{4}}}$$

$$=\frac{1}{x^5}\left(\frac{x^4+1}{x^4}\right)^{-\frac{3}{4}}$$

$$=\frac{1}{x^5}\left(1+\frac{1}{x^4}\right)^{-\frac{3}{4}}$$

Let $\frac{1}{x^4}=t \Rightarrow -\frac{4}{x^5}dx=dt \Rightarrow \frac{1}{x^5}dx=-\frac{dt}{4}$

$$\therefore \int\frac{1}{x^2\left(x^4+1\right)^{\frac{3}{4}}}dx=\int\frac{1}{x^5}\left(1+\frac{1}{x^4}\right)^{-\frac{3}{4}}dx$$

$$=-\frac{1}{4}\int(1+t)^{-\frac{3}{4}}dt$$

$$=-\frac{1}{4}\left[\frac{(1+t)^{\frac{1}{4}}}{\frac{1}{4}}\right]+C$$

$$=-\frac{1}{4}\frac{\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}}{\frac{1}{4}}+C$$

$$=-\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}+C$$

Indefinite Integrals Ex 19.9 Q72

Let $I = \int \frac{\sin^5 x}{\cos^4 x} dx ----(\text{i})$

Let $\cos x = t$ then,

$d(\cos x) = dt$

$\Rightarrow \quad -\sin x \, dx = dt$

$\Rightarrow \quad dx = -\frac{dt}{\sin x}$

Putting $\cos x = t$ and $dx = -\frac{dt}{\sin x}$ in equation (i),

we get

$$
\begin{aligned}
I &= \int \frac{\sin^5 x}{t^4} \times -\frac{dt}{\sin x} \\
&= -\int \frac{\sin^4 x}{t^4} dt \\
&= -\int \frac{\left(1-\cos^2 x\right)^2}{t^4} dt \\
&= -\int \frac{\left(1-t^2\right)^2}{t^4} dt \\
&= -\int \frac{1+t^4-2t^2}{t^4} dt \\
&= -\int \left(\frac{1}{t^4} + \frac{t^4}{t^4} - \frac{2t^2}{t^4}\right) dt \\
&= -\int \left(t^{-4} + 1 - 2t^{-2}\right) dt \\
&= -\left[\frac{t^{-3}}{-3} + t - 2\frac{t^{-1}}{-1}\right] + c \\
&= -\left[-\frac{1}{3} \times \frac{1}{t^3} + t + \frac{2}{t}\right] + c \\
&= \frac{1}{3} \times \frac{1}{t^3} - t - \frac{2}{t} + c \\
&= \frac{1}{3} \times \frac{1}{\cos^3 x} - \cos x - \frac{2}{\cos x} + c
\end{aligned}
$$

$\therefore \quad I = -\cos x - \frac{2}{\cos x} + \frac{1}{3\cos^3 x} + c$

Ex 19.10

Indefinite Integrals Ex 19.10 Q1

Let $I = \int x^2\sqrt{x+2}dx$

Substituting $x+2 = t$ and $dx = dt$ we get,

$$I = \int (t-2)^2\sqrt{t}dt$$
$$= \int \left(t^2+4-4t\right)\sqrt{t}dt$$
$$= \int \left(t^{\frac{5}{2}} - 4t^{\frac{3}{2}} + 4t^{\frac{1}{2}}\right)dt$$
$$= \frac{2}{7}t^{\frac{7}{2}} - \frac{8}{5}t^{\frac{5}{2}} + \frac{8}{3}t^{\frac{3}{2}} + c$$
$$= \frac{2}{7}(x+2)^{\frac{7}{2}} - \frac{8}{5}(x+2)^{\frac{5}{2}} + \frac{8}{3}(x+2)^{\frac{3}{2}} + c$$
$$\therefore \quad I = \frac{2}{7}(x+2)^{\frac{7}{2}} - \frac{8}{5}(x+2)^{\frac{5}{2}} + \frac{8}{3}(x+2)^{\frac{3}{2}} + c$$

Indefinite Integrals Ex 19.10 Q2

Let $I = \int \frac{x^2}{\sqrt{x-1}}dx$

Substituting $x-1 = t$ and $dx = dt$ we get,

$$I = \int \frac{(t+1)^2}{\sqrt{t}}dx$$
$$= \int \frac{t^2+1+2t}{\sqrt{t}}dt$$
$$= \int \left(t^{\frac{3}{2}} + t^{\frac{-1}{2}} + 2t^{\frac{-1}{2}}\right)dt$$
$$= \frac{2}{5}t^{\frac{5}{2}} + 2t^{\frac{1}{2}} + \frac{4}{3}t^{\frac{3}{2}} + c$$
$$= \frac{6t^{\frac{5}{2}} + 30t^{\frac{1}{2}} + 20t^{\frac{3}{2}}}{15} + c$$
$$= \frac{2}{15}t^{\frac{1}{2}}\left(3t^2+15+10t\right) + c$$
$$= \frac{2}{15}\sqrt{x-1}\left(3(x-1)^2+15+10(x-1)\right) + x$$
$$= \frac{2}{15}\sqrt{x-1}\left(3\left(x^2+1-2x\right)+15+10x-10\right) + c$$
$$= \frac{2}{15}\sqrt{x-1}\left(3x^2+3-6x+15+10x-10\right) + c$$
$$= \frac{2}{15}\sqrt{x-1}\left(3x^2+4x+8\right) + c$$
$$\therefore \quad I = \frac{2}{15}\left(3x^2+4x+8\right)\sqrt{x-1} + c$$

Indefinite Integrals Ex 19.10 Q3

Let $I = \int \frac{x^2}{\sqrt{3x+4}} dx$

Substituting $3x + 4 = t$ and $dx = \frac{dt}{3}$ we get,

$$I = \int \frac{\left(\frac{t-4}{3}\right)^2}{\sqrt{t}} \times \frac{dt}{3} \qquad \left[\because x = \frac{t-4}{3}\right]$$

$$= \int \frac{(t-4)^2}{9\sqrt{t}x^3} dt$$

$$= \frac{1}{27} \int \frac{t^2 + 16 - 8t}{\sqrt{t}} dt$$

$$= \frac{1}{27} \int \left(t^{\frac{3}{2}} - 8t^{\frac{1}{2}} + 16t^{\frac{1}{2}} 16t^{\frac{-1}{2}} \right) dt$$

$$= \frac{1}{27} \left[\frac{2}{5} t^{\frac{5}{2}} - \frac{16}{3} t^{\frac{3}{2}} + 32t^{\frac{1}{1}} \right] + c$$

$$= \frac{2}{135} (3x+4)^{\frac{5}{2}} - \frac{16}{81} (3x+4)^{\frac{3}{2}} + \frac{32}{27} (3x+4)^{\frac{1}{2}} + c$$

$$\therefore \quad I = \frac{2}{135} (3x+4)^{\frac{5}{2}} - \frac{16}{81} (3x+4)^{\frac{3}{2}} + \frac{32}{27} (3x+4)^{\frac{1}{2}} + c$$

Indefinite Integrals Ex 19.10 Q4

Let $I = \int \frac{2x-1}{(x-1)^2} dx$

Substituting $x - 1 = t$ and $dx = dt$, we get

$$I = \int \frac{2(t+1)}{t^2} dt$$

$$= \int \frac{2t + 2 - 1}{t^2} dt$$

$$= \int \frac{2t+1}{t^2} dt$$

$$= \int \left(\frac{2t}{t^2} + \frac{1}{t^2} \right) dt$$

$$= 2\int \frac{1}{t} dt + \int t^{-2} dt$$

$$= 2\log|t| - t^{-1} + c$$

$$= 2\log|x-1| - \frac{1}{x-1} + c$$

$$\int \frac{2x-1}{(x-1)^2} dx = 2\log|x-1| - \frac{1}{x-1} + c$$

Indefinite Integrals Ex 19.10 Q5

Let $I = \int (2x^2 + 3)\sqrt{x+2} dx$

Substituting $x + 2 = t$ and $dx = dt$, we get

$$I = \int \left[2(t-2)^2 + 3 \right] \sqrt{t} dt$$

$$= \int \left[2(t^2 + 4 - 4t) + 3 \right] \sqrt{t} dt$$

$$= \int \left[2t^2 + 8 - 8t + 3 \right] \sqrt{t} dt$$

$$= \int \left(2t^{\frac{5}{2}} + 11t^{-\frac{1}{2}} - 8t^{\frac{3}{2}} \right) dt$$

$$= \frac{4}{7} t^{\frac{7}{2}} + \frac{22}{3} t^{\frac{3}{2}} - \frac{16}{5} t^{\frac{5}{2}} + c$$

$$= \frac{4}{7} (x+2)^{\frac{7}{2}} - \frac{16}{5} (x+1) \frac{5}{2} + \frac{22}{3} (x+2)^{\frac{3}{2}} + c$$

$$\therefore \quad I = \frac{4}{7} (x+2)^{\frac{7}{2}} - \frac{16}{5} (x+2)^{\frac{5}{2}} + \frac{22}{3} (x+2)^{\frac{3}{2}} + c$$

Indefinite Integrals Ex 19.10 Q6

Let $I = \int \frac{x^2+3x+1}{(x+1)^2}dx$

Substituting $x+1 = t$ and $dx = dt$, we get

$$I = \int \frac{(t-1)^2+3(t-1)+1}{t^2}dt$$
$$= \int \frac{t^2+1-2t+3t-3+1}{t^2}dt$$
$$= \int \frac{t^2+t-1}{t^2}dt$$
$$= \int \left(\frac{t^2}{t^2}+\frac{t}{t^2}-\frac{1}{t^2}\right)dt$$
$$= \int \left(1+\frac{1}{t}-r^{-2}\right)dt$$
$$= t+\log|t|+t^{-1}+c$$
$$= t+\log|t|+\frac{1}{t}+c$$
$$= (x+1)+\log|x+1|+\frac{1}{x+1}+c$$

Indefinite Integrals Ex 19.10 Q7

Let $I = \int \frac{x^2}{\sqrt{1-x}}dx$

Substituting $1-x = t$ and $dx = -dt$, we get

$$I = \int \frac{(1-t)^2}{\sqrt{t}} \times -dt$$
$$= -\int \frac{1+t^2-2t}{\sqrt{t}} \times dt$$
$$= -\int \left(t^{\frac{-1}{2}}+t^{\frac{3}{2}}-2t^{\frac{1}{2}}\right)dt$$
$$= -\left[2t^{\frac{1}{2}}+\frac{2}{5}t^{\frac{5}{2}}-\frac{4}{3}t^{\frac{3}{2}}\right]+c$$
$$= -\left[\frac{30t^{\frac{1}{2}}+6t^{\frac{5}{2}}-20t^{\frac{3}{2}}}{15}\right]+c$$
$$= -\frac{2t^{\frac{1}{2}}}{15}\left[15+3t^2-10t\right]+c$$
$$= -\frac{2}{15}\sqrt{1-x}\left[15+3(1-x)^2-10(1-x)\right]+c$$
$$= -\frac{2}{15}\sqrt{1-x}\left(15+3\left(1+x^2-2x\right)-10+10x\right)+c$$
$$= -\frac{2}{15}\sqrt{1-x}\left(5+3+3x^2-6x+10x\right)+c$$
$$= -\frac{2}{15}\sqrt{1-x}\left(3x^2+4x+8\right)+c$$
$$= -\frac{2}{15}\left(3x^2+4x+8\right)\sqrt{1-x}+c$$

$$\therefore \quad I = -\frac{2}{15}\left(3x^2+4x+8\right)\sqrt{1-x}+c$$

Indefinite Integrals Ex 19.10 Q8

Let $I = \int x\left(1-x\right)^{23} dx$

Substituting $1-x = t$ and $dx = -dt$, we get

$$\begin{aligned}
I &= -\int \left(1-t\right) t^{23} dt \\
&= -\int \left(t^{23} - t^{24}\right) dt \\
&= -\int \left(\frac{t^{24}}{24} - \frac{t^{25}}{25}\right) + c \\
&= \frac{t^{25}}{25} - \frac{t^{24}}{24} + c \\
&= \frac{\left(1-x\right)^{25}}{25} - \frac{\left(1-x\right)^{24}}{24} + c
\end{aligned}$$

$$\begin{aligned}
\therefore \quad I &= \frac{\left(1-x\right)^{25}}{25} - \frac{\left(1-x\right)^{24}}{24} + c. \\
&= \frac{1}{600}\left(1-x\right)^{24}\left[24\left(1-x\right) - 25\right] + c \\
&= \frac{1}{600}\left(1-x\right)^{24}\left[24 - 24x - 25\right] + c \\
&= \frac{1}{600}\left(1-x\right)^{24}\left[-1 - 24x\right] + c \\
&= \frac{1}{600}\left(1-x\right)^{24} \times -\left[1 + 24x\right] + c \\
&= -\frac{1}{600}\left(1-x\right)^{24}\left(1 + 24x\right) + c
\end{aligned}$$

Ex 19.11

Indefinite Integrals Ex 19.11 Q1

Let $I = \int \tan^3 x \sec^2 x dx$ ---(i)

Let $\tan x = t$. Then

$$d(\tan x) = dt$$

$$\Rightarrow \quad \sec^2 x dx = dt$$

$$\Rightarrow \quad dx = \frac{dt}{\sec^2 x}$$

Puting $\tan x = t$ and $dx = \dfrac{dt}{\sec^2 x}$ in equation (i), we get

$$I = \int t^3 \sec^2 x \times \frac{dt}{\sec^2 x}$$

$$= \int t^3 dt$$

$$= \frac{t^{3+1}}{3+1} + c$$

$$= \frac{t^4}{4} + c$$

$$= \frac{(\tan x)^4}{4} + c$$

$$\therefore \quad I = \frac{(\tan x)^4}{4} + c$$

$$= \frac{1}{4} \times \tan^4 x + c.$$

Indefinite Integrals Ex 19.11 Q2

Let $I = \int \tan x \sec^4 x dx$. Then

$$I = \int \tan x \sec^2 x \sec^2 x dx$$

$$= \int \tan x \left(1 + \tan^2 x\right) \sec^2 x dx$$

$$\Rightarrow \quad I = \int \left(\tan x + \tan^3 x\right) \sec^2 x dx$$

Substituting $\tan x = t$ and $\sec^2 x dx = dt$, we get

$$I = \int \left(t + t^3\right) dt$$

$$= \frac{t^2}{2} + \frac{t^4}{4} + c$$

$$= \frac{\tan^2 x}{2} + \frac{\tan^4}{4} + c$$

$$\therefore \quad I = \frac{1}{2} \times \tan^2 x + \frac{1}{4} \times \tan^4 x + c.$$

Indefinite Integrals Ex 19.11 Q3

Let $I = \int \tan^5 x \sec^4 x dx$. Then

$$I = \int \tan^4 x \sec^2 x \sec^2 x dx$$

$$= \int \tan^5 x \left(1 + \tan^2 x\right) \sec^2 x dx$$

$$= \int \left(\tan^5 x + \tan^7 x\right) \sec^2 x dx$$

Substituting $\tan x = t$ and $\sec^2 x dx = dt$, we get

$$I = \int \left(t^5 + t^7\right) dt$$

$$= \frac{t^6}{6} + \frac{t^8}{8} + c$$

$$= \frac{(\tan x)^6}{6} + \frac{(\tan x)^8}{8} + c$$

$$\therefore \quad I = \frac{1}{6} \times \tan^6 x + \frac{1}{8} \times \tan^8 x + c.$$

Indefinite Integrals Ex 19.11 Q4

Let $I = \int \sec^6 x \tan x dx$. Then

$$I = \int \sec^5 x \left(\sec x \tan x\right) dx$$

Substituting $\sec x = t$ and $\sec x \tan x = dt$, we get

$$I = \int t^5 dt$$

$$= \frac{t^6}{6} + c$$

$$= \frac{\left(\sec x\right)^6}{6} + c$$

$$\therefore \quad I = \frac{1}{6}\sec^6 x + c$$

Indefinite Integrals Ex 19.11 Q5

Let $I = \int \tan^5 x dx$. Then

$$I = \int \tan^2 x \tan^3 x dx$$

$$= \int \left(\sec^2 x - 1\right) \tan^3 x dx$$

$$= \int \sec^2 x \tan^3 x dx - \int \tan^3 x dx$$

$$= \int \sec^2 x \tan^3 x dx - \int \left(\sec^2 x - 1\right) \tan x dx$$

$$= \int \sec^2 x \tan^3 x dx - \int \sec^2 x \tan x dx + \int \tan x dx$$

Substituting $\tan x = t$ and $\sec^2 x dx = dt$ in first two integral, we get

$$I = \int t^3 dt - \int t dt + \int \tan x dx$$

$$= \frac{t^4}{4} - \frac{t^2}{2} + \log\left|\sec x\right| + c$$

$$= \frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \log\left|\sec x\right| + c$$

$$\therefore \quad I = \frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \log\left|\sec x\right| + c$$

Indefinite Integrals Ex 19.11 Q6

Let $I = \int \sqrt{\tan x} \sec^4 x dx$. Then

$$I = \int \sqrt{\tan x} \sec^2 x \sec^2 x dx$$

$$= \int \sqrt{\tan x} \left(1 + \tan^2 x\right) \sec^2 x dx$$

$$= \int \tan x^{\frac{1}{2}} \left(1 + \tan^2 x\right) \sec^2 x dx$$

$$\Rightarrow \quad I = \int \left(\tan x^{\frac{1}{2}} + \tan x^{\frac{5}{2}}\right) \sec^2 x dx$$

Substituting $\tan x = t$ and $\sec^2 x dx = dt$, we get

$$I = \int \left(t^{\frac{1}{2}} + t^{\frac{5}{2}}\right) dt$$

$$= \frac{2}{3} t^{\frac{3}{2}} + \frac{2}{7} t^{\frac{7}{2}} + c$$

$$= \frac{2}{3} \left(\tan x\right)^{\frac{3}{2}} + \frac{2}{7} \left(\tan x\right)^{\frac{7}{2}} + c$$

$$\therefore \quad I = \frac{2}{3} \tan^{\frac{3}{2}} x + \frac{2}{7} \tan^{\frac{7}{2}} x + c$$

Indefinite Integrals Ex 19.11 Q7

Let $I = \int \sec^4 2x dx$. Then

$$I = \int \sec^2 2x \sec^2 2x dx$$
$$= \int \left(1 + \tan^2 2x\right) \sec^2 2x dx$$
$$= \int \left(\sec^2 2x + \sec^2 2x \tan^2 2x\right) dx$$

$\Rightarrow \quad I = \int \sec^2 2x dx + \int \sec^2 2x \tan^2 2x dx$

$\Rightarrow \quad I = \int \sec^2 2x \tan^2 2x dx + \int \sec^2 2x dx$

Substituting $\tan 2x = t$ and $\sec^2 2x dx = \frac{dt}{2}$ in first integral, we get

$$I = \int t^2 \frac{dt}{2} + \int \sec^2 2x dx$$
$$= \frac{1}{2} \times \frac{t^3}{3} + \frac{1}{2} \tan 2x + c$$

$\Rightarrow \quad I = \frac{1}{6} \tan^3 2x + \frac{1}{2} \tan 2x + c$

$\therefore \quad I = \frac{1}{2} \tan 2x + \frac{1}{6} \tan^3 2x + c$

Indefinite Integrals Ex 19.11 Q8

Let $I = \int cosec^4 3x dx$. Then

$$I = \int cosec^2 3x \, cosec^2 3x dx$$
$$= \int \left(1 + \cot^2 3x\right) cosec^2 3x dx$$
$$= \int \left(cosec^2 3x + \cot^2 3x \, cosec^2 3x\right) dx$$

$\Rightarrow \quad I = \int cosec^2 3x dx + \int \cot^2 3x \, cosec^2 3x dx$

Substituting $\cot 3x = t$ and $cosec^2 3x dx = -dt$ in 2nd integral, we get

$$I = \int cosec^2 3x dx - \int t^2 \frac{dt}{3}$$
$$= \frac{-1}{3} \cot 3x - \frac{t^3}{9} + c$$
$$= \frac{-1}{3} \cot 3x - \frac{\cot^3 3x}{9} + c$$

$\therefore \quad I = \frac{-1}{3} \cot 3x - \frac{1}{9} \cot^3 3x + c$

Indefinite Integrals Ex 19.11 Q9

Let $I = \int \cot^n x \, cosec^2 x dx$. $n \neq -1$ ---(i)

Let $\cot x = t$. Then

$$d\left(\cot x\right) = dt$$

$\Rightarrow \quad -cosec^2 x dx = dt$

$\Rightarrow \quad cosec^2 x dx = -dt$

Putting $\cot x = t$ and $cosec^2 x dx = -dt$ in equation (i), we get

$$I = \int t^n \times (-dt)$$
$$= -\frac{t^{n+1}}{n+1} + c$$

$\Rightarrow \quad = -\frac{(\cot x)^{n+1}}{n+1} + c$

Indefinite Integrals Ex 19.11 Q10

Let $I = \int \cot^5 x \operatorname{cosec}^4 x dx$. Then,

$$I = \int \cot^5 x \operatorname{cosec}^2 x \operatorname{cosec}^2 x dx$$
$$= \int \cot^5 x \left(1 + \cot^2 x\right) \operatorname{cosec}^2 x dx$$
$$\Rightarrow \quad I = \int \left(\cot^5 x + \cot^7 x\right) \operatorname{cosec}^2 x dx$$

Substituting $\cot x = t$ and $-\operatorname{cosec}^2 x \, dx = dt$, we get

$$I = \int \left(t^5 + t^7\right)(-dt)$$
$$= -\frac{t^6}{6} - \frac{t^8}{8} + c$$
$$= -\frac{\cot^6 x}{6} - \frac{\cot^8 x}{8} + c$$
$$\therefore \quad I = -\frac{\cot^6 x}{6} - \frac{\cot^8 x}{8} + c$$

Indefinite Integrals Ex 19.11 Q11

Let $I = \int \cot^5 x dx$. Then,

$$I = \int \cot^3 x \times \cot^2 x dx$$
$$= \int \cot^3 x \times \left(\operatorname{cosec}^2 x - 1\right) dx$$
$$= \int \cot^3 x \operatorname{cosec}^2 x dx - \int \cot^3 x dx$$
$$= \int \cot^3 x \operatorname{cosec}^2 x dx - \int \left(\operatorname{cosec}^2 x - 1\right) \cot dx$$
$$= \int \cot^3 x \operatorname{cosec}^2 x dx - \int \operatorname{cosec}^2 x \cot x dx + \int \cot x dx$$
$$\Rightarrow \quad I = \int \cot^3 x \operatorname{cosec}^2 x dx - \int \operatorname{cosec}^2 x \cot x dx + \int \cot x dx$$

Substituting $\cot x = t$ and $-\operatorname{cosec}^2 x \, dx = dt$ in first two integral, we get

$$I = \int t^3 (-dt) - \int t \times (-dt) + \int \cot x dx$$
$$= -\frac{t^4}{4} + \frac{t^2}{2} + \log|\sin x| + c$$
$$= -\frac{1}{4}\cot^4 x + \frac{1}{2}\cot^2 x + \log|\sin x| + c$$
$$\therefore \quad I = -\frac{1}{4}\cot^4 x + \frac{1}{2}\cot^2 x + \log|\sin x| + c$$

Indefinite Integrals Ex 19.11 Q12

Let $I = \int \cot^6 x dx$. Then,

$$I = \int \cot^2 x \times \cot^4 x dx$$
$$= \int \left(\operatorname{cosec}^2 x - 1\right) \times \cot^4 x dx$$
$$= \int \left(\operatorname{cosec}^2 x \cot^4 x - \cot^4 x\right) dx$$
$$= \int \operatorname{cosec}^2 x \times \cot^4 x dx - \int \cot^4 dx$$
$$= \int \operatorname{cosec}^2 x \cot^4 x dx - \int \cot^2 x \left(\operatorname{cosec}^2 - 1\right) dx$$
$$= \int \operatorname{cosec}^2 x \cot^4 x dx - \int \cot^2 x \operatorname{cosec}^2 x dx + \int \cot^2 x dx$$
$$\Rightarrow \quad I = \int \operatorname{cosec}^2 x \cot^4 x dx - \int \cot^2 x \operatorname{cosec}^2 x dx + \int \left(\operatorname{cosec}^2 x - 1\right) dx$$

Substituting $\cot x = t$ and $-\operatorname{cosec}^2 x dx = dt$ in first two integral, we get

$$I = \int t^4 (-dt) - \int t^2 (-dt) + \int \operatorname{cosec}^2 x dx - \int dx$$
$$= -\frac{t^5}{5} + \frac{t^3}{3} - \cot x - x + c$$
$$= -\frac{\cot^5 x}{5} + \frac{\cot^3 x}{3} - \cot x - x + c$$
$$\therefore \quad I = -\frac{1}{5} \times \cot^5 x + \frac{1}{3} \times \cot^3 x - \cot x - x + c$$

Ex 19.12

Indefinite Integrals Ex 19.12 Q1

Let $I = \int \sin^4 x \cos^3 x dx$

Here, power of $\cos x$ is odd, so we substitute

$$\sin x = t$$

$$\Rightarrow \quad \cos x dx = dt$$

$$\Rightarrow \quad dx = \frac{dt}{\cos x}$$

$$\therefore \quad I = \int t^4 \cos^3 x \frac{dt}{\cos x}$$

$$= \int t^4 \cos^2 x dt$$

$$= \int t^4 \left(1 - \sin^2 x\right) dt$$

$$= \int t^4 \left(1 - t^2\right) dt$$

$$= \int \left(t^4 - t^6\right) dt$$

$$= \frac{t^5}{5} - \frac{t^7}{7} + c$$

$$\therefore \quad I = \frac{1}{5} \times \sin^5 x - \frac{1}{7} \times \sin^7 x + c$$

Indefinite Integrals Ex 19.12 Q2

Let $I = \int \sin^5 x dx$. Then

$$I = \int \sin^3 x \sin^2 x dx$$

$$= \int \sin^3 x \left(1 - \cos^2 x\right) dx$$

$$= \int \left(\sin^3 x - \sin^3 x \cos^2 x\right) dx$$

$$= \int \left[\sin x \left(1 - \cos^2 x\right) - \sin^3 x \cos^2 x\right] dx$$

$$= \int \left(\sin x - \sin x \cos^2 x - \sin^3 x \cos^2 x\right) dx$$

$$\Rightarrow \quad I = \int \sin x dx - \int \sin x \cos^2 x dx - \int \sin^3 x \cos^2 x dx$$

Putting $\cos x = t$ and $-\sin x dx = dt$ in 2nd and 3rd integrals, we get

$$I = \int \sin x dx - \int t^2 (-dt) + \int \sin^2 x t^2 dt$$

$$= \int \sin x dx + \int t^2 dt + \int \left(1 - \cos^2 x\right) t^2 dt$$

$$= \int sixdx + \int t^2 dt + \int \left(1 - t^2\right) t^2 dt$$

$$= -\cos x + \frac{t^3}{3} + \frac{t^3}{3} - \frac{t^5}{5} + c$$

$$= -\cos x + \frac{2}{3} t^3 - \frac{1}{5} t^5 + c$$

$$= -\cos x + \frac{2}{3}\left(\cos^3 x\right) - \frac{1}{5}\left(\cos^5 x\right) + c$$

$$\therefore \quad I = -\left[\cos x - \frac{2}{3}\cos^3 x + \frac{1}{5}\cos^5 x\right] + c$$

Indefinite Integrals Ex 19.12 Q3

Let $I = \int \cos^5 x dx$. Then

$$I = \int \cos^2 x \cos^3 x dx$$
$$= \int \left(1 - \sin^2 x\right) \cos^3 x dx$$
$$= \int \cos^3 x dx - \int \sin^2 x \cos^3 x dx$$
$$= \int \cos^2 x \cos x dx - \int \sin^2 x \left(1 - \sin^2 x\right) \cos x dx$$
$$= \int \left(\cos x - \sin^2 x \cos x\right) dx - \int \left(\sin^2 x \cos x - \sin^4 x \cos x\right) dx$$

$\Rightarrow \quad I = \int \cos x dx - 2\int \sin^2 x \cos x dx + \int \sin^4 x \cos x dx$

Putting $\sin x = t$ and $\cos x dx = dt$ in 2nd and 3rd integrals, we get

$$I = \int \cos x dx - 2\int t^2 dt + \int t^4 dt$$
$$= \sin x - \frac{2}{3} t^3 + \frac{t^5}{5} + c$$
$$= \sin x - \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^5 x + c$$

$\therefore \quad I = \sin x - \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^5 x + c$

Indefinite Integrals Ex 19.12 Q4

Let $I = \int \sin^5 x \cos x \, dx$ ---(i)

Let $\sin x = t$. Then,

$$d\left(\sin x\right) = dt$$

$\Rightarrow \quad \cos x dx = dt$

Putting $\sin x = t$ and $\cos x dx = dt$ in equation (i), we get

$$I = \int t^5 dt$$
$$= \frac{t^6}{6} + c$$
$$= \frac{\sin^6 x}{6} + c$$

$\therefore \quad I = \frac{1}{6} \sin^6 x + c$

Indefinite Integrals Ex 19.12 Q5

Let $I = \int \sin^3 x \cos^6 x dx$

Here, power of $\sin x$ is odd, so we substitute

$$\cos x = t$$

$\Rightarrow \quad -\sin x dx = dt$

$\therefore \quad I = \int \sin^2 x t^6 \left(-dt\right)$

$$= -\int \left(1 - \cos^2 x\right) t^6 dt$$
$$= -\int \left(t^6 - t^8\right) dt$$
$$= -\frac{t^7}{7} + \frac{t^9}{9} + c$$
$$= -\frac{\cos^7 x}{7} + \frac{\cos^9 x}{9} + c$$

$\therefore \quad I = -\frac{1}{7} \cos^7 x + \frac{1}{9} \cos^9 x + c$

Indefinite Integrals Ex 19.12 Q6

Let $I = \int \cos^7 x dx$. Then

$$I = \int \cos^6 x \cos x dx$$
$$= \int \left(\cos^2 x\right)^3 \cos x dx$$
$$= \int \left(1 - \sin^2 x\right)^3 \cos x dx$$
$$= \int \left[1 - \sin^6 x - 3\sin^2 x + 3\sin^4 x\right] \cos x dx$$
$$= \int \left[\cos x - \sin^6 x \cos x - 3\sin^2 x \cos x + 3\sin^4 x \cos x\right] dx + c$$

$\Rightarrow \quad I = \int \cos x dx - \int \sin^6 x \cos x dx - 3\int \sin^2 x \cos x dx + 3\int \sin^4 x \cos x dx$

Putting $\sin x = t$ and $\cos x dx = dt$ in 2nd and 3rd and 4th integral, we get

$$I = \int \cos x dx - \int t^6 dt - 3\int t^2 dt + 3\int t^4 dt$$
$$= \sin x - \frac{t^7}{7} - \frac{3}{3}t^3 + \frac{3}{5}t^5 + c$$
$$= \sin x - \frac{1}{7}\sin^7 x - \sin^3 x + \frac{3}{5}\sin^5 x + c$$

$\therefore \quad I = \sin x - \sin^3 x + \frac{3}{5}\sin^5 x - \frac{1}{7}\sin^7 x + c$

Indefinite Integrals Ex 19.12 Q7

Let $I = \int x \cos^3 x^2 \sin x^2 dx$

Let $\cos x^2 = t$. Then

$$d\left(\cos x^2\right) = dt$$

$\Rightarrow \quad -2x \sin x^2 x = dt$

$\Rightarrow \quad x \sin x^2 dx = -\frac{dt}{2}$

$\therefore \quad I = \int t^3 \times \frac{-dt}{2}$

$$= -\frac{t^4}{8} + c$$
$$= -\frac{1}{8}\cos^4 x^2 + c$$

$\therefore \quad I = -\frac{1}{8}\cos^4 x^2 + c$

Indefinite Integrals Ex 19.12 Q8

Let $I = \int \sin^7 x dx$. Then

$$I = \int \sin^6 x \sin x dx$$
$$= \int \left(\sin^2 x\right)^3 \sin x dx$$
$$= \int \left(1 - \cos^2 x\right)^3 \sin x dx$$
$$= \int \left(1 - \cos^6 x + 3\cos^4 x - 3\cos^2 x\right) \sin x dx$$

$\Rightarrow \quad I = \int \sin x dx - \int \cos^6 x \sin x dx + 3\int \cos^4 x \sin x dx - 3\int \cos^2 x \sin x dx$

Putting $\cos x = t$ and $-\sin x dx = dt$ in 2ne, 3rd and 4th integral, we get

$$I = \int \sin x dx - \int t^6 (-dt) + 3\int t^4 (-dt) - 3\int t^2 (-dt)$$
$$= -\cos x + \frac{t^7}{7} - \frac{3}{5}t^5 + \frac{3}{3}t^3 + c$$
$$= -\cos x + \frac{\cos^7 x}{7} - \frac{3}{5}\cos^5 x + \cos^3 x + c$$

$\therefore \quad I = -\cos x + \cos^3 x - \frac{3}{5}\cos^5 x + \frac{1}{7}\cos^7 x + c$

Indefinite Integrals Ex 19.12 Q9

Let $I = \int \sin^3 x \cos^5 x dx$. Then

Let $\cos x = t$. Then

$$d(\cos x) = dt$$

$$\Rightarrow \quad -\sin x dx = dt$$

$$\Rightarrow \quad dx = \frac{-dt}{\sin x}$$

$$\therefore \quad I = \int \sin^3 x t^5 \frac{-dt}{\sin x}$$

$$= -\int \sin^2 x t^5 dt$$

$$= -\int \left(1 - \cos^2 x\right) t^5 dt$$

$$= -\int \left(1 - t^2\right) t^5 dt$$

$$= -\int \left(t^5 - t^7\right) dt$$

$$= -\frac{t^6}{6} + \frac{t^8}{8} + c$$

$$= -\frac{1}{6}\cos^6 x + \frac{1}{8}\cos^8 x + c$$

$$\therefore \quad I = \frac{-1}{6}\cos^6 x + \frac{1}{8}\cos^8 x + c$$

Indefinite Integrals Ex 19.12 Q10

$$\text{Let } I = \int \frac{1}{\sin^4 x \cos^2 x} dx \qquad \text{---(i)}$$

Then, $I = \int \sin^{-4} x \cos^{-2} x dx$

Since $-4 - 2 = -6$, which is even integer. So, we divide both numerator and denominator by $\cos^6 x$.

$$\therefore \quad I = \int \frac{\frac{1}{\cos^6 x}}{\frac{\sin^4 x \cos^2 x}{\cos^6 x}} dx$$

$$= \int \frac{\sec^6 x}{\frac{\sin^4 x}{\cos^4 x}} dx$$

$$= \int \frac{\sec^6 x}{\tan^4 x} dx$$

$$= \int \frac{\sec^4 x \times \sec^2 x}{\tan^4 x} dx$$

$$= \int \frac{\left(\sec^2 x\right)^2 \times \sec^2 x}{\tan^4 x} dx$$

$$= \int \frac{\left(1 + \tan^2 x\right)^2 \times \sec^2 x}{\tan^4 x} dx$$

$$\Rightarrow \quad I = \int \frac{\left(1 + \tan^4 x + 2\tan^2 x\right) \times \sec^2 x}{\tan^4 x} dx \qquad \text{---(ii)}$$

Let $\tan x = t$. Then,

$$d(\tan x) = dt$$

$$\Rightarrow \quad \sec^2 x dx = dt$$

$$\Rightarrow \quad dx = \frac{dt}{\sec^2 x}$$

Putting $\tan x = t$ and $dx = \frac{dt}{\sec^2 x}$ in equation (i), we get

$$I = \int \frac{\left(1 + t^4 + 2t^2\right)}{t^4} \times \sec^2 x \times \frac{dt}{\sec^2 x}$$

$$= \int \left(t^{-4} + 1 + 2t^{-2}\right) dt$$

$$= -\frac{t^{-3}}{3} + t - 2t^{-1} + c$$

$$= -\frac{1}{3t^3} + t - \frac{2}{t} + c$$

$$= -\frac{1}{3\tan^3 x} + \tan x - \frac{2}{\tan x} + c$$

$$= -\frac{1}{3} \times \cot^3 x + \tan x - 2 \times \cot x + c$$

$$\therefore \quad I = \frac{-1}{3} \times \cot^3 x - 2\cot x + \tan x + c$$

Indefinite Integrals Ex 19.12 Q11

Let $I = \int \frac{1}{\sin^3 x \cos^5 x} dx$ ---(i)

Then, $I = \int \sin^{-3} x \cos^{-5} x dx$

Since $-3-5=-8$, which is even integer. So, we divide both numerator and denominator by $\cos^8 x$.

$$\therefore \quad I = \int \frac{\frac{1}{\cos^8 x}}{\frac{\sin^3 x \cos^5 x}{\cos^8 x}} dx$$

$$= \int \frac{\sec^8 x}{\tan^3 x} dx$$

$$= \int \frac{\sec^8 x}{\tan^3 x} \sec^2 x dx$$

$$= \int \frac{\left(\sec^2 x\right)^3}{\tan^3 x} \sec^2 x dx$$

$$= \int \frac{\left(1+\tan^2 x\right)^3}{\tan^3 x} \times \sec^2 x dx$$

$$\Rightarrow \quad I = \int \frac{\left(1+\tan^6 x + 3\tan^4 x + 3\tan^2 x\right) \times \sec^2 x}{\tan^3 x} dx \qquad \text{---(ii)}$$

Let $t = \tan x$. Then,

$$d\left(\tan x\right) = dt$$

$$\Rightarrow \quad \sec^2 x dx = dt$$

$$\therefore \quad I = \int \frac{\left(1+t^6+3t^4+3t^2\right)}{t^3} dt$$

$$= \int \left(t^{-3}+t^3+3t+3t^{-1}\right) dt$$

$$= -\frac{t^{-2}}{2} + \frac{t^4}{4} + \frac{3}{2}t^2 + 3\log t + c$$

$$= -\frac{1}{2t^2} + \frac{t^4}{4} + \frac{3}{2}t^2 + 3\log t + c$$

$$= -\frac{1}{2} \times \frac{1}{\tan^2 x} + \frac{\tan^4 x}{4} + \frac{3}{2} \times \tan^2 x + 3\log\left|\tan x\right| + c$$

$$\therefore \quad I = \frac{-1}{2\tan^2 x} + 3\log\left|\tan x\right| + \frac{3}{2}\tan^2 x + \frac{1}{4} \times \tan^4 x + c$$

Indefinite Integrals Ex 19.12 Q12

Let $I = \int \frac{1}{\sin^3 x \cos x} dx$ ---(i)

Then, $I = \int \sin^{-3} x \cos^{-1} x dx$

Since $-3-1=-4$, which is even integer. So, we divide both numerator and denominator by $\cos^4 x$.

$$\therefore \quad I = \int \frac{\frac{1}{\cos^4 x}}{\frac{\sin^3 x \cos x}{\cos^4 x}} dx$$

$$= \int \frac{\sec^4 x}{\tan^3 x} dx$$

$$= \int \frac{\sec^4 x}{\tan^3 x} \sec^2 x dx$$

$$= \int \frac{\left(\sec^2 x\right)^3}{\tan^3 x} \sec^2 x dx$$

$$= \int \frac{\left(1+\tan^2 x\right)}{\tan^3 x} \times \sec^2 x dx$$

Putting $\tan x = t$ and $\sec^2 x dx = dt$, we get

$$I = \int \frac{1+t^2}{t^3} dt$$

$$= \int \left(t^{-3} + \frac{1}{t}\right) dt$$

$$= -\frac{t^{-2}}{2} + \log\left|t\right| + c$$

$$= -\frac{1}{2t^2} + \log\left|t\right| + c$$

$$= -\frac{1}{2\tan^2 x} + \log\left|\tan x\right| + c$$

Indefinite Integrals Ex 19.12 Q13

$$\frac{1}{\sin x\cos^3 x}=\frac{\sin^2 x+\cos^2 x}{\sin x\cos^3 x}$$
$$=\frac{\sin x}{\cos^3 x}+\frac{1}{\sin x\cos x}$$
$$=\tan x\sec^2 x+\frac{\frac{1\cos^2 x}{\sin x\cos x}}{\cos^2 x}$$
$$=\tan x\sec^2 x+\frac{\sec^2 x}{\tan x}$$

$$\therefore\ \int\frac{1}{\sin x\cos^3 x}dx=\int\tan x\sec^2 x\ dx+\int\frac{\sec^2 x}{\tan x}\ dx$$
Let $\tan x=t\Rightarrow\sec^2 x\ dx=dt$
$$\Rightarrow\int\frac{1}{\sin x\cos^3 x}dx=\int t\,dt+\int\frac{1}{t}dt$$
$$=\frac{t^2}{2}+\log|t|+C$$
$$=\frac{1}{2}\tan^2 x+\log|\tan x|+C$$

Ex 19.13

Indefinite Integrals Ex 19.13 Q1

Let $I = \int \frac{x}{\sqrt{x^4 + a^4}} dx$

$= \int \frac{x}{\sqrt{(x^2)^2 + (a^2)^2}} dx$

Let $x^2 = t$

$\Rightarrow \quad 2x\,dx = dt$

$\Rightarrow \quad x\,dx = \frac{dt}{2}$

$I = \frac{1}{2}\int \frac{dt}{\sqrt{t^2 + (a^2)^2}}$

$= \frac{1}{2}\log\left|t + \sqrt{t^2 + (a^2)^2}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2 + a^2}} dx = \log\left|x + \sqrt{x^2 + a^2}\right| + c\right]$

$I = \frac{1}{2}\log\left|x^2 + \sqrt{(x^2)^2 + (a^2)^2}\right| + c$

$I = \frac{1}{2}\log\left|x^2 + \sqrt{x^4 + a^4}\right| + c$

Indefinite Integrals Ex 19.13 Q2

Let $\tan x = t$

$\Rightarrow \sec^2 x\,dx = dt$

$\Rightarrow \int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx = \int \frac{dt}{\sqrt{t^2 + 2^2}}$

$= \log\left|t + \sqrt{t^2 + 4}\right| + C$

$= \log\left|\tan x + \sqrt{\tan^2 x + 4}\right| + C$

Indefinite Integrals Ex 19.13 Q3

Let $I = \int \frac{e^x}{\sqrt{16 - e^{2x}}} dx$

Let $e^x = t$

$\Rightarrow \quad e^x\,dx = dt$

$I = \int \frac{dt}{\sqrt{(4)^2 - t^2}}$

$= \sin^{-1}\left(\frac{t}{4}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$

$I = \sin^{-1}\left(\frac{e^x}{4}\right) + c$

Indefinite Integrals Ex 19.13 Q4

Let $I = \int \frac{\cos x}{\sqrt{4 + \sin^2 x}} dx$

Let $\sin x = t$

$\Rightarrow \quad \cos x\,dx = dt$

$I = \int \frac{dt}{\sqrt{(2)^2 + t^2}}$

$= \log\left|t + \sqrt{(2)^2 + t^2}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 + x^2}} dx = \log\left|x + \sqrt{a^2 + x^2}\right| + c\right]$

$I = \log\left|\sin x + \sqrt{4 + \sin^2 x}\right| + c$

Indefinite Integrals Ex 19.13 Q5

Let $I = \int \frac{\sin x}{\sqrt{4\cos^2 x - 1}} dx$

Let $2\cos x = t$

$\Rightarrow \quad -2\sin x\, dx = dt$

$\Rightarrow \quad \sin x\, dx = -\frac{dt}{2}$

$I = -\frac{1}{2}\int \frac{dt}{\sqrt{t^2 - 1}}$

$= -\frac{1}{2}\log\left|t + \sqrt{t^2 - 1}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log\left|x + \sqrt{x^2 + a^2}\right| + c\right]$

$I = -\frac{1}{2}\log\left|2\cos x + \sqrt{4\cos^2 x - 1}\right| + c$

Indefinite Integrals Ex 19.13 Q6

Let $I = \int \frac{x}{\sqrt{4 - x^4}} dx$

Let $x^2 = t$

$\Rightarrow \quad 2x\, dx = dt$

$\Rightarrow \quad x\, dx = \frac{dt}{2}$

$I = \frac{1}{2}\int \frac{dt}{\sqrt{(2)^2 - t^2}}$

$= \frac{1}{2}\sin^{-1}\left(\frac{t}{2}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$

$I = \frac{1}{2}\sin^{-1}\left(\frac{x^2}{2}\right) + c$

Indefinite Integrals Ex 19.13 Q7

Let $I = \int \frac{1}{x\sqrt{4 - 9(\log x)^2}} dx$

Let $3\log x = t$

$\Rightarrow \quad \frac{3}{x} dx = dt$

$\Rightarrow \quad \frac{1}{x} dx = \frac{dt}{3}$

$I = \frac{1}{3}\int \frac{dt}{\sqrt{(2)^2 - t^2}}$

$= \frac{1}{3}\sin^{-1}\left(\frac{t}{2}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$

$I = \frac{1}{3}\sin^{-1}\left(\frac{3\log x}{2}\right) + c$

Indefinite Integrals Ex 19.13 Q8

Let $I = \int \frac{\sin 8x}{\sqrt{9 + (\sin 4x)^4}} dx$

Let $\sin^2 4x = t$

$\Rightarrow \quad 2\sin 4x.\cos 4x\,(4)dx = dt$

$\Rightarrow \quad 4\sin 8x\, dx = dt$

$\Rightarrow \quad \sin 8x\, dx = \frac{dt}{4}$

$I = \frac{1}{4}\int \frac{dt}{\sqrt{(3)^2 + t^2}}$

$= \frac{1}{4}\log\left|t + \sqrt{(3)^2 + t^2}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 + x^2}} dx = \log\left|x + \sqrt{a^2 + x^2}\right| + c\right]$

$I = \frac{1}{4}\log\left|\sin^2 4x + \sqrt{9 + \sin^4 4x}\right| + c$

Indefinite Integrals Ex 19.13 Q9

Let $I = \int \frac{\cos 2x}{\sqrt{\sin^2 2x + 8}} dx$

Let $\sin 2x = t$

$\Rightarrow \quad 2\cos 2x dx = dt$

$\Rightarrow \quad \cos 2x\, dx = \frac{dt}{2}$

$$I = \frac{1}{2}\int \frac{dt}{\sqrt{t^2 + \left(2\sqrt{2}\right)^2}}$$

$$= \frac{1}{2}\log\left|t + \sqrt{t^2 + \left(2\sqrt{2}\right)^2}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 + x^2}} dx = \log\left|x + \sqrt{x^2 + a^2}\right| + c\right]$$

$$I = \frac{1}{2}\log\left|\sin 2x + \sqrt{\sin^2 2x + 8}\right| + c$$

Indefinite Integrals Ex 19.13 Q10

Let $I = \int \frac{\sin 2x}{\sqrt{\sin^4 x + 4\sin^2 x - 2}} dx$

Let $\sin^2 x = t$

$\Rightarrow \quad 2\sin x \cos x\, dx = dt$

$\Rightarrow \quad \sin 2x\, dx = dt$

$$I = \int \frac{dt}{\sqrt{t^2 + 4t - 2}}$$

$$= \int \frac{dt}{\sqrt{t^2 + 2t(2) + (2)^2 - (2)^2 - 2}} dx$$

$$= \int \frac{dt}{\sqrt{(t+2)^2 - 6}}$$

Let $t + 2 = u$

$dt = du$

$$= \int \frac{du}{\sqrt{u^2 - \left(\sqrt{6}\right)^2}}$$

$$= \log\left|u + \sqrt{u^2 - 6}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c\right]$$

$$= \log\left|t + 2 + \sqrt{(t+2)^2 - 6}\right| + c$$

$$I = \log\left|\sin^2 x + 2 + \sqrt{\sin^4 x + 4\sin^2 x - 2}\right| + c$$

Indefinite Integrals Ex 19.13 Q11

$$\int \frac{\sin 2x}{\sqrt{\cos^4 x - \sin^2 x + 2}} dx =$$

$let\ t = \cos^2 x \to -dt = 2\cos x \sin x dx$

$$\int \frac{\sin 2x}{\sqrt{\cos^4 x - \sin^2 x + 2}} dx = \int \frac{-1}{\sqrt{t^2 - (1-t) + 2}} dt$$

$$= \int \frac{-1}{\sqrt{t^2 + t + 1}} dt = \int \frac{-1}{\sqrt{t^2 + t + \frac{1}{4} + \frac{3}{4}}} dt$$

$$= \int \frac{-1}{\sqrt{\left(t + \frac{1}{2}\right)^2 + \frac{3}{4}}} dt = -\log\left|\left(t + \frac{1}{2}\right) + \sqrt{t^2 + t + 1}\right|$$

$$= -\log\left|\left(\cos^2 x + \frac{1}{2}\right) + \sqrt{\cos^4 x + \cos^2 x + 1}\right| + C$$

Indefinite Integrals Ex 19.13 Q12

Let $I = \int \frac{\cos x}{\sqrt{4 - \sin^2 x}} dx$

Let $\sin x = t$

$\Rightarrow \quad \cos x \, dx = dt$

$$= \int \frac{dt}{\sqrt{(2)^2 - t^2}}$$

$$= \sin^{-1}\left(\frac{t}{2}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \sin^{-1}\left(\frac{\sin x}{2}\right) + c$$

Indefinite Integrals Ex 19.13 Q13

Let $I = \int \frac{1}{x^{\frac{2}{3}}\sqrt{x^{\frac{2}{3}} - 4}} dx$

Let $x^{\frac{1}{3}} = t$

$\Rightarrow \quad \frac{1}{3} x^{\frac{1}{3} - 1} dx = dt$

$\Rightarrow \quad \frac{1}{3} x^{-\frac{2}{3}} dx = dt$

$\Rightarrow \quad \frac{dx}{x^{\frac{2}{3}}} = 3dt$

$$I = 3\int \frac{dt}{\sqrt{t^2 - (2)^2}}$$

$$= 3\log\left|t + \sqrt{t^2 - 4}\right| + c \quad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c\right]$$

$$I == 3\log\left|x^{\frac{1}{3}} + \sqrt{x^{\frac{2}{3}} - 4}\right| + c$$

Indefinite Integrals Ex 19.13 Q14

Let $I = \int \frac{1}{\sqrt{(1 - x^2)\left[9 + \left(\sin^{-1} x\right)^2\right]}} dx$

Let $\sin^{-1} x = t$

$\Rightarrow \quad \frac{1}{\sqrt{1 - x^2}} dx = dt$

$$I = \int \frac{dt}{\sqrt{(3)^2 + t^2}}$$

$$= \log\left|t + \sqrt{9 + t^2}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 + x^2}} dx = \log\left|x + \sqrt{a^2 + x^2}\right| + c\right]$$

$$I = \log\left|\sin^{-1} x + \sqrt{9 + \left(\sin^{-1} x\right)^2}\right| + c$$

Indefinite Integrals Ex 19.13 Q15

Let $I = \int \frac{\cos x}{\sqrt{\sin^2 x - 2\sin x - 3}} dx$

Let $\sin x = t$

$\Rightarrow \quad \cos x \, dx = dt$

$$= \int \frac{dt}{\sqrt{t^2 - 2t - 3}}$$

$$= \int \frac{dt}{\sqrt{t^2 - 2t + (1)^2 - (1)^2 - 3}}$$

$$= \int \frac{dt}{\sqrt{(t-1)^2 - (2)^2}}$$

Let $t - 1 = u$

$\Rightarrow \quad dt = du$

$$I = \int \frac{du}{\sqrt{u^2 - (2)^2}}$$

$$= \log\left|u + \sqrt{u^2 - 4}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c\right]$$

$$= \log\left|t - 1 + \sqrt{(t-1)^2 + 4}\right| + c$$

$$I = \log\left|\sin x - 1 + \sqrt{\sin^2 x - 2\sin x - 3}\right| + c$$

Indefinite Integrals Ex 19.13 Q16

Let $I = \int \sqrt{\operatorname{cosec} x - 1} \, dx$

$$= \int \sqrt{\frac{1 - \sin x}{\sin x}} dx$$

$$= \int \sqrt{\frac{(1 - \sin x) + (1 + \sin x)}{\sin x (1 + \sin x)}} dx$$

$$= \int \sqrt{\frac{\cos^2 x}{\sin^2 x + \sin x}} dx$$

$$= \int \frac{\cos x}{\sqrt{\sin^2 x + \sin x}} dx$$

Let $\sin x = t$

$\Rightarrow \quad \cos x \, dx = dt$

$$= \int \frac{dt}{\sqrt{t^2 + t}}$$

$$= \int \frac{dt}{\sqrt{t^2 + 2t\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}$$

$$== \int \frac{dt}{\sqrt{\left(t + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}$$

Let, $t + \frac{1}{2} = u$

$\Rightarrow \quad dt = du$

$$= \int \frac{du}{\sqrt{u^2 - \left(\frac{1}{2}\right)^2}}$$

$$== \log\left|u + \sqrt{u^2 - \left(\frac{1}{2}\right)^2}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c\right]$$

$$= \log\left|\left(t + \frac{1}{2}\right) + \sqrt{\left(t + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}\right| + c$$

$$I = \log\left|\sin x + \frac{1}{2} + \sqrt{\sin^2 x + \sin x}\right| + c$$

Indefinite Integrals Ex 19.13 Q17

To evaluate the following integral follow the steps:

$$\int \frac{\sin x - \cos x}{\sqrt{\sin 2x}} dx = \int \frac{\sin x - \cos x}{\sqrt{(\sin x + \cos x)^2 - 1}} dx$$

Let $\sin x + \cos x = t$ therefore $(\cos x - \sin x) dx = dt$

Now

$$\begin{aligned}
\int \frac{\sin x - \cos x}{\sqrt{(\sin x + \cos x)^2 - 1}} dx &= -\int \frac{dt}{\sqrt{t^2 - 1}} \\
&= -\ln\left|t + \sqrt{t^2 - 1}\right| + c \\
&= -\ln\left|\sin x + \cos x + \sqrt{\sin 2x}\right| + c
\end{aligned}$$

Ex 19.14

Indefinite Integrals Ex 19.14 Q1

Let $I = \int \frac{1}{a^2 - b^2x^2}dx$

$= \frac{1}{b^2}\int \frac{1}{\frac{a^2}{b^2} - x^2}dx$

$= \frac{1}{b^2}\int \frac{1}{\left(\frac{a}{b}\right)^2 - x^2}dx$

$I = \frac{1}{b^2} \times \frac{1}{2 \times \left(\frac{a}{b}\right)} \log\left|\frac{\frac{a}{b} + x}{\frac{a}{b} - x}\right| + c$ $\qquad \left[\text{Since } \int \frac{1}{a^2 - x^2}dx = \frac{1}{2a}\log\left|\frac{x+a}{x-a}\right| + c\right]$

$I = \frac{1}{2ab}\log\left|\frac{a+bx}{a-bx}\right| + c$

Indefinite Integrals Ex 19.14 Q2

Let $I = \int \frac{1}{a^2x^2 - b^2}dx$

$= \frac{1}{a^2}\int \frac{1}{x^2 - \frac{b^2}{a^2}}dx$

$= \frac{1}{a^2}\int \frac{1}{x^2 - \left(\frac{b}{a}\right)^2}dx$

$I = \frac{1}{a^2} \times \frac{1}{2 \times \left(\frac{b}{a}\right)} \times \log\left|\frac{x - \frac{b}{a}}{x + \frac{b}{a}}\right| + c$ $\qquad \left[\text{Since } \int \frac{1}{x^2 - a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c\right]$

$I = \frac{1}{2ab}\log\left|\frac{ax-b}{ax+b}\right| + c$

Indefinite Integrals Ex 19.14 Q3

Let $I = \int \frac{1}{a^2x^2 + b^2}dx$

$= \frac{1}{a^2}\int \frac{1}{x^2 + \frac{b^2}{a^2}}dx$

$= \frac{1}{a^2}\int \frac{1}{x^2 + \left(\frac{b}{a}\right)^2}dx$

$I = \frac{1}{a^2} \times \frac{1}{\left(\frac{b}{a}\right)} \tan^{-1}\left(\frac{x}{\frac{b}{a}}\right) + c$ $\qquad \left[\text{Since } \int \frac{1}{x^2 + a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c\right]$

$I = \frac{1}{ab}\tan^{-1}\left(\frac{ax}{b}\right) + c$

Indefinite Integrals Ex 19.14 Q4

Let $I = \int \frac{x^2 - 1}{x^2 - 4} dx$

Let $I = \int \frac{x^2 - 1}{x^2 + 4} dx$

$= \int \frac{(x^2 + 4) - 4 - 1}{x^2 + 4} dx$

$= \int \frac{x^2 + 4}{x^2 + 4} dx - \int \frac{5}{x^2 + 4} dx$

$= \int dx - 5\int \frac{1}{x^2 + (2)^2} dx$

$I = x - 5 \times \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + c$ $\qquad \left[\text{Since } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)\right]$

$I = x - \frac{5}{2} \tan^{-1}\left(\frac{x}{2}\right) + c$

Indefinite Integrals Ex 19.14 Q5

Let $2x = t$

$\Rightarrow 2dx = dt$

$\Rightarrow \int \frac{1}{\sqrt{1+4x^2}} dx = \frac{1}{2} \int \frac{dt}{\sqrt{1+t^2}}$

$= \frac{1}{2}\left[\log\left|t + \sqrt{t^2+1}\right|\right] + C$ $\qquad \left[\int \frac{1}{\sqrt{x^2+a^2}} dt = \log\left|x + \sqrt{x^2+a^2}\right|\right]$

$= \frac{1}{2} \log\left|2x + \sqrt{4x^2+1}\right| + C$

Indefinite Integrals Ex 19.14 Q6

Let $I = \int \frac{1}{\sqrt{a^2 + b^2x^2}} dx$

Let $bx = t$

$\Rightarrow \quad bdx = dt$

$dx = \frac{dt}{b}$

$I = \frac{1}{b} \int \frac{1}{\sqrt{a^2 + t^2}} dt$

$I = \frac{1}{b} \log\left|t + \sqrt{a^2 + t^2}\right| + c$ $\qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 + x^2}} dx = \log\left|x + \sqrt{a^2 + x^2}\right| + c\right]$

$I = \frac{1}{b} \log\left|bx + \sqrt{a^2 + b^2x^2}\right| + c$ $\qquad [\text{since } t = bx]$

Indefinite Integrals Ex 19.14 Q7

Let $I = \int \frac{1}{\sqrt{a^2 - b^2x^2}} dx$

Let $bx = t$

$\Rightarrow \quad bdx = dt$

$dx = \frac{dt}{b}$

so, $\quad I = \frac{1}{b} \int \frac{1}{\sqrt{a^2 - t^2}} dt$

$I = \frac{1}{b} \sin^{-1}\left(\frac{t}{a}\right) + c$ $\qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$

$I = \frac{1}{b} \sin^{-1}\left(\frac{bx}{a}\right) + c$ $\qquad [\text{since } bx = t]$

Indefinite Integrals Ex 19.14 Q8

Let $I = \int \frac{1}{\sqrt{(2-x)^2+1}}dx$

Let $2 - x = t$

$\Rightarrow \quad -dx = dt$

$dx = -dt$

so, $I = -\int \frac{1}{\sqrt{t^2+(1)^2}}dt$

$I = -\log\left|t + \sqrt{t^2+1}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2+a^2}}dx = \log\left|x+\sqrt{x^2+a^2}\right|+c\right]$

$I = -\log\left|(2-x)+\sqrt{(2-x)^2+1}\right| + c \qquad \left[\text{since } t = (2-x)\right]$

Indefinite Integrals Ex 19.14 Q9

Let $I = \int \frac{1}{\sqrt{(2-x)^2-1}}dx$

Let $2 - x = t$

$\Rightarrow \quad -dx = dt$

$dx = -dt$

so, $I = -\int \frac{1}{\sqrt{t^2-(1)^2}}dt$

$I = -\log\left|t + \sqrt{t^2-(1)^2}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x+\sqrt{x^2-a^2}\right|+c\right]$

$I = -\log\left|(2-x)+\sqrt{(2-x)^2-1}\right| + c \qquad \left[\text{since } t = (2-x)\right]$

Indefinite Integrals Ex 19.14 Q10

Let $I = \int \frac{x^4+1}{x^2+1}dx$

$I = \int \frac{(x^2+1)^2-2x^2}{x^2+1}dx \qquad \left[a^2+b^2 = (a+b)^2-2ab\right]$

$I = \int \frac{(x^2+1)^2}{x^2+1}dx - \int \frac{2x^2}{(x^2+1)}dx$

$I = \int (x^2+1)dx - \int \frac{2x^2+2-2}{(x^2+1)}dx$

$I = \int (x^2+1)dx - \int \frac{2(x^2+1)}{(x^2+1)}dx + 2\int \frac{1}{x^2+1}dx$

$I = \int (x^2+1)dx - \int 2dx + 2\int \frac{1}{x^2+1}dx$

$I = \frac{x^3}{3} + x - 2x + 2\,x\tan^{-1}(x) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2+1}}dx = \tan^{-1}(x)+c\right]$

$I = \frac{x^3}{3} - x + 2\tan^{-1}(x) + c$

Ex 19.15

Indefinite Integrals Ex 19.15 Q1

Let $I = \int \frac{1}{4x^2 + 12x + 5} dx$

$$= \frac{1}{4} \int \frac{1}{x^2 + 3x + \frac{5}{4}} dx$$

$$= \frac{1}{4} \int \frac{1}{x^2 + 2 \times x \times \left(\frac{3}{2}\right) + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + \frac{5}{4}} dx$$

$$I = \frac{1}{4} \int \frac{1}{\left(x + \frac{3}{2}\right)^2 - 1} dx$$

Let $\left(x + \frac{3}{2}\right) = t$ - - - - - - - (i)

$\Rightarrow$ $dx = dt$

so,

$$I = \frac{1}{4} \int \frac{1}{t^2 - (1)^2} dt$$

$$I = \frac{1}{4} \times \frac{1}{2 \times (1)} \log \left| \frac{t-1}{t+1} \right| + c \qquad \left[\text{Since, } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c\right]$$

$$I = \frac{1}{8} \log \left| \frac{x + \frac{3}{2} - 1}{x + \frac{3}{2} + 1} \right| + c \qquad [\text{using (i)}]$$

$$I = \frac{1}{8} \log \left| \frac{2x+1}{2x+5} \right| + c$$

Indefinite Integrals Ex 19.15 Q2

Let $I = \int \frac{1}{x^2 - 10x + 34} dx$

$$= \int \frac{1}{x^2 - 2x \times 5 + (5)^2 - (5)^2 + 34} dx$$

$$= \int \frac{1}{(x-5)^2 + 9} dx$$

Let $(x-1) = t$ - - - - - - - (i)

$\Rightarrow$ $dx = dt$

so,

$$I = \int \frac{1}{t^2 + (3)^2} dt$$

$$I = \frac{1}{3} \tan^{-1} \left(\frac{t}{3}\right) + c \qquad \left[\text{Since, } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{2}\right) + c\right]$$

$$I = \frac{1}{3} \tan^{-1} \left(\frac{x-5}{3}\right) + c \qquad [\text{using (i)}]$$

Indefinite Integrals Ex 19.15 Q3

$$\int \frac{1}{1+x-x^2}dx = \int \frac{1}{-(x^2-x-1)}dx$$

adding and subtracting $\frac{1}{4}$ in the denominator to make it a perfect square

$$= \int \frac{1}{-\left(x^2-x+\frac{1}{4}-1-\frac{1}{4}\right)}dx$$

$$= \int \frac{1}{-\left(\left[x^2-x+\frac{1}{4}\right]-1-\frac{1}{4}\right)}dx$$

$$= \int \frac{1}{-\left(\left[x-\frac{1}{2}\right]^2-1-\frac{1}{4}\right)}dx = \int \frac{1}{-\left(\left[x-\frac{1}{2}\right]^2-\frac{5}{4}\right)}dx$$

$$= \int \frac{1}{\left(\left(\frac{\sqrt{5}}{2}\right)^2-\left[x-\frac{1}{2}\right]^2\right)}d$$

$$= \frac{1}{2\left(\frac{\sqrt{5}}{2}\right)}\log\left|\frac{\frac{\sqrt{5}}{2}+\left(x-\frac{1}{2}\right)}{\frac{\sqrt{5}}{2}-\left(x-\frac{1}{2}\right)}\right|$$

$$= \frac{1}{\sqrt{5}}\log\left|\frac{\sqrt{5}+2x-1}{\sqrt{5}-2x+1}\right|$$

$$= \frac{1}{\sqrt{5}}\log\left|\frac{\sqrt{5}-1+2x}{\sqrt{5}+1-2x}\right|$$

Indefinite Integrals Ex 19.15 Q4

Let $I = \int \frac{1}{2x^2-x-1}dx$

$$= \frac{1}{2}\int \frac{1}{x^2-\frac{x}{2}-\frac{1}{2}}dx$$

$$= \frac{1}{2}\int \frac{1}{x^2-2x\times\frac{1}{4}+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2-\frac{1}{2}}dx$$

$$= \frac{1}{2}\int \frac{1}{\left(x-\frac{1}{4}\right)^2-\frac{9}{16}}dx$$

Let $x-\frac{1}{4} = t$

$\Rightarrow \quad dx = dt$

$$I = \frac{1}{2}\int \frac{1}{t^2-\left(\frac{3}{4}\right)^2}dt$$

$$I = \frac{1}{2}\times\frac{1}{2\times\left(\frac{3}{4}\right)}\log\left|\frac{t-\frac{3}{4}}{t+\frac{3}{4}}\right|+c \quad \left[\text{Since, } \int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c\right]$$

$$I = \frac{1}{3}\log\left|\frac{x-\frac{1}{4}-\frac{3}{4}}{x-\frac{1}{4}+\frac{3}{4}}\right|+c$$

$$I = \frac{1}{3}\log\left|\frac{x-1}{2x+1}\right|+c$$

Indefinite Integrals Ex 19.15 Q5

We have $x^2 + 6x + 13 = x^2 + 6x + 3^2 - 3^2 + 13 = (x + 3)^2 + 4$

Sol, $\int \frac{dx}{x^2 + 6x + 13} = \int \frac{1}{(x + 3)^2 + 2^2} dx$

Let $x + 3 = t$. Then $dx = dt$

Therefore, $\int \frac{dx}{x^2 + 6x + 13} = \int \frac{dt}{t^2 + 2^2} = \frac{1}{2} \tan^{-1} \frac{t}{2} + C$ [by 7.4 (3)]

$$= \frac{1}{2} \tan^{-1} \frac{x + 3}{2} + C$$

Ex 19.16

Indefinite Integrals Ex 19.16 Q1

Let $I = \int \frac{\sec^2 x}{1-\tan^2 x} dx$

Let $\tan x = t$

$\Rightarrow \quad \sec^2 x\, dx = dt$

$I = \int \frac{dt}{(1)^2 - t^2}$

$= \frac{1}{2(1)} \log\left|\frac{1+t}{1-t}\right| + c \quad \left[\text{Since, } \int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + c\right]$

$I = \frac{1}{2} \log\left|\frac{1+\tan x}{1-\tan x}\right| + c$

Indefinite Integrals Ex 19.16 Q2

Let $I = \int \frac{e^x}{1+e^{2x}} dx$

Let $\tan e^x = t$

$\Rightarrow \quad e^x\, dx = dt$

so, $\quad I = \int \frac{dt}{1+t^2}$

$= \tan^{-1}(t) + c \quad \left[\text{Since, } \int \frac{1}{1+x^2} dx = \tan^{-1} x + c\right]$

$I = \tan^{-1}(e^x) + c$

Indefinite Integrals Ex 19.16 Q3

Let $I = \int \frac{\cos x}{\sin^2 x + 4\sin x + 5} dx$

Let $\sin x = t$

$\Rightarrow \quad \cos x\, dx = dt$

so, $\quad I = \int \frac{dx}{t^2 + 4t + 5}$

$= \int \frac{dt}{t^2 + 2t(2) + (2)^2 - (2)^2 + 5}$

$= \int \frac{dt}{(t+2)^2 + 1}$

Again, Let $(t+2) = u$

$dt = du$

$I = \int \frac{dt}{u^2+1}$

$= \tan^{-1}(u) + c \quad \left[\text{Since, } \int \frac{1}{x^2+1} dx = \tan^{-1} x + c\right]$

$I = \tan^{-1}(t+2) + c$

$I = \tan^{-1}(\sin x + 2) + c$

Indefinite Integrals Ex 19.16 Q4

$$\text{Let } I = \int \frac{e^x}{e^{2x} + 5e^x + 6} dx$$

$$\text{Let } e^x = t$$

$$\Rightarrow \quad e^x\, dx = dt$$

$$\text{so,} \quad I = \int \frac{dt}{t^2 + 5t + 6}$$

$$= \int \frac{dt}{t^2 + 2t\left(\frac{5}{2}\right) + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 6}$$

$$= \int \frac{dt}{\left(t + \frac{5}{2}\right)^2 - \frac{1}{4}}$$

$$\text{Put } \left(t + \frac{5}{2}\right) = u$$

$$\Rightarrow \quad dt = du$$

$$I = \int \frac{dt}{u^2 - \left(\frac{1}{2}\right)^2}$$

$$= \frac{1}{2\left(\frac{1}{2}\right)} \log\left|\frac{u - \frac{1}{2}}{u + \frac{1}{2}}\right| + c \qquad \left[\text{Since, } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a}\left|\frac{x - a}{x + a}\right| + c\right]$$

$$I = \log\left|\frac{2u - 1}{2u + 1}\right| + c$$

$$I = \log\left|\frac{2\left(t + \frac{5}{2}\right) - 1}{2\left(t + \frac{5}{2}\right) + 1}\right| + c$$

$$I = \log\left|\frac{e^x + 2}{e^x + 3}\right| + c$$

Indefinite Integrals Ex 19.16 Q5

$$\text{Let } I = \int \frac{e^{3x}}{4e^{6x} - 9} dx$$

$$\text{Let } e^{3x} = t$$

$$\Rightarrow \quad 3e^{3x}\, dx = dt$$

$$\Rightarrow \quad e^{3x} dx = \frac{dt}{3}$$

$$I = \frac{1}{3} \int \frac{dt}{4t^2 - 9}$$

$$= \frac{1}{12} \int \frac{dt}{t^2 - \frac{9}{4}}$$

$$= \frac{1}{12} \int \frac{dt}{t^2 - \left(\frac{3}{2}\right)^2}$$

$$= \frac{1}{12} \times \frac{1}{2\left(\frac{3}{2}\right)} \log\left|\frac{t - \frac{3}{2}}{t + \frac{3}{2}}\right| + c \qquad \left[\text{Since, } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a}\left|\frac{x - a}{x + a}\right| + c\right]$$

$$I = \frac{1}{36} \log\left|\frac{2t - 3}{2t + 3}\right| + c$$

$$I = \frac{1}{36} \log\left|\frac{2e^{3x} - 3}{2e^{3x} + 3}\right| + c$$

Indefinite Integrals Ex 19.16 Q6

Let $I = \int \frac{dx}{e^x + e^{-x}}$

$= \int \frac{dx}{e^x + \frac{1}{e^x}}$

$= \int \frac{e^x dx}{\left(e^x\right)^2 + 1}$

Let $e^x = t$

$\Rightarrow \quad e^x dx = dt$

$I = \int \frac{dt}{t^2 + 1}$

$I = \tan^{-1} t + c \qquad \left[\text{Since } \int \frac{1}{1+x^2} dx = \tan^{-1} x + c\right]$

$I = \tan^{-1}\left(e^x\right) + c$

Indefinite Integrals Ex 19.16 Q7

Let $I = \int \frac{x}{x^4 + 2x^2 + 3} dx$

Let $x^2 = t$

$\Rightarrow \quad 2x\, dx = dt$

$\Rightarrow \quad x\, dx = \frac{dt}{2}$

$I = \frac{1}{2} \int \frac{dt}{t^2 + 2t + 3}$

$= \frac{1}{2} \int \frac{dt}{t^2 + 2t + 1 - 1 + 3}$

$= \frac{1}{2} \int \frac{dt}{(t+1)^2 + 2}$

put $t + 1 = u$

$\Rightarrow \quad dt = du$

$I = \frac{1}{2} \int \frac{du}{u^2 + \left(\sqrt{2}\right)^2}$

$= \frac{1}{2} \times \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + c \qquad \left[\text{Since } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c\right]$

$I = \frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{t+1}{\sqrt{2}}\right) + c$

$I = \frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right) + c$

Indefinite Integrals Ex 19.16 Q8

Let $I = \int \frac{3x^5}{1 + x^{12}} dx$

$= \int \frac{3x^5}{1 + \left(x^6\right)^2} dx$

Let $x^6 = t$

$\Rightarrow \quad 6x^5\, dx = dt$

$\Rightarrow \quad x^5\, dx = \frac{dt}{6}$

$I = \frac{3}{6} \int \frac{dt}{1 + t^2}$

$= \frac{1}{2} \tan^{-1}(t) + c \qquad \left[\text{Since } \int \frac{1}{x^2 + 1} dx = \tan^{-1} x + c\right]$

$I = \frac{1}{2} \tan^{-1}\left(x^6\right) + c$

Indefinite Integrals Ex 19.16 Q9

Let $I = \int \frac{x^2}{x^6 - a^6} dx$

$= \int \frac{x^2}{\left(x^3\right)^2 - \left(a^3\right)^2} dx$

Let $x^3 = t$

$\Rightarrow \quad 3x^2 dx = dt$

$\Rightarrow \quad x^2 dx = \frac{dt}{3}$

so, $\quad I = \frac{1}{3} \int \frac{dt}{t^2 - \left(a^3\right)^2}$

$= \frac{1}{3} \times \frac{1}{2a^3} \log \left| \frac{t - a^3}{t + a^3} \right| \qquad \left[\text{Since } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x - a}{x + a} \right| + c \right]$

$I = \frac{1}{6a^3} \log \left| \frac{x^3 - a^3}{x^3 + a^3} \right| + c$

Indefinite Integrals Ex 19.16 Q10

Let $I = \int \frac{x^2}{x^6 + \left(a^3\right)^2} dx$

$= \int \frac{x^2}{\left(x^3\right)^2 + \left(a^3\right)^2} dx$

Let $x^3 = t$

$\Rightarrow \quad 3x^2 dx = dt$

$\Rightarrow \quad x^2 dx = \frac{dt}{3}$

so, $\quad I = \frac{1}{3} \int \frac{dt}{t^2 + \left(a^3\right)^2}$

$= \frac{1}{3} \times \frac{1}{\left(a^3\right)} \tan^{-1} \left(\frac{t}{a^3} \right) + c \qquad \left[\text{Since } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c \right]$

$I = \frac{1}{3a^3} \tan^{-1} \left(\frac{x^3}{a^3} \right) + c$

Indefinite Integrals Ex 19.16 Q11

Let $I = \int \frac{1}{x\left(x^6+1\right)}dx$

$= \int \frac{x^5}{x^6\left(x^6+1\right)}dx$

Let $x^6 = t$

$\Rightarrow \quad 6x^5\,dx = dt$

$\Rightarrow \quad x^5\,dx = \frac{dt}{6}$

$I = \frac{1}{6}\int \frac{dt}{t\left(t+1\right)}$

$= \frac{1}{6}\int \frac{dt}{t^2+t}$

$= \frac{1}{6}\int \frac{dt}{t^2+2t\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}$

$= \frac{1}{6}\int \frac{dt}{\left(t+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}$

Let $t+\frac{1}{2} = u$

$\Rightarrow \quad dt = du$

$I = \frac{1}{6}\int \frac{du}{u^2-\left(\frac{1}{2}\right)^2}$

$= \frac{1}{6}\times\frac{1}{2\left(\frac{1}{2}\right)}\log\left|\frac{u-\frac{1}{2}}{u+\frac{1}{2}}\right|+c \qquad \left[\text{Since } \int\frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c\right]$

$I = \frac{1}{6}\log\left|\frac{t+\frac{1}{2}-\frac{1}{2}}{t+\frac{1}{2}+\frac{1}{2}}\right|+c$

$I = \frac{1}{6}\log\left|\frac{x^6}{x^6+1}\right|+c$

Indefinite Integrals Ex 19.16 Q12

Let $I = \int \frac{x}{x^4 - x^2 + 1} dx$

Let $x^2 = t$

$\Rightarrow \quad 2x\,dx = dt$

$\Rightarrow \quad x\,dx = \frac{dt}{2}$

so, $\quad I = \frac{1}{2}\int \frac{dt}{t^2 - t + 1}$

$= \frac{1}{2}\int \frac{dt}{t^2 - 2t \times \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1}$

$= \frac{1}{2}\int \frac{dt}{\left(t - \frac{1}{2}\right)^2 + \left(\frac{3}{4}\right)}$

Let $\quad t - \frac{1}{2} = u$

$\Rightarrow \quad dt = du$

$I = \frac{1}{2}\int \frac{du}{u^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \frac{1}{2} \times \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} \tan^{-1}\left(\frac{u}{\frac{\sqrt{3}}{2}}\right) + c \quad \left[\text{Since} \quad \int \frac{1}{x^2 + a^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c\right]$

$I = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{t - \frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + c$

$I = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2x^2 - 1}{\sqrt{3}}\right) + c$

Indefinite Integrals Ex 19.16 Q13

Let $I = \int \frac{x}{3x^4 - 18x^2 + 11} dx$

$$= \frac{1}{3}\int \frac{x}{x^4 - 6x^2 + \frac{11}{3}} dx$$

Let $x^2 = t$

$\Rightarrow \quad 2x\,dx = dt$

$\Rightarrow \quad x\,dx = \frac{dt}{2}$

$$I = \frac{1}{3} \times \frac{1}{2}\int \frac{dt}{t^2 - 6t + \frac{11}{3}}$$

$$= \frac{1}{6}\int \frac{dt}{t^2 - 2t(3) + (3)^2 - (3)^2 + \frac{11}{3}}$$

$$= \frac{1}{6}\int \frac{dt}{(t-3)^2 - \left(\frac{16}{3}\right)}$$

Let $t-3 = u$

$\Rightarrow \quad dt = du$

$$I = \frac{1}{6}\int \frac{du}{u^2 - \left(\frac{4}{\sqrt{3}}\right)^2}$$

$$= \frac{1}{6} \times \frac{1}{2\left(\frac{4}{\sqrt{3}}\right)} \log\left|\frac{u - \frac{4}{\sqrt{3}}}{u + \frac{4}{\sqrt{3}}}\right| + c \quad \left[\text{Since } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c\right]$$

$$I = \frac{\sqrt{3}}{48}\log\left|\frac{t - 3 - \frac{4}{\sqrt{3}}}{t - 3 + \frac{4}{\sqrt{3}}}\right| + c$$

$$I = \frac{\sqrt{3}}{48}\log\left|\frac{x^2 - 3 - \frac{4}{\sqrt{3}}}{x^2 - 3 + \frac{4}{\sqrt{3}}}\right| + c$$

Indefinite Integrals Ex 19.16 Q14

To evaluate the following integral follow the steps:

Let $e^x = t$ therefore $e^x dx = dt$

Now

$$\int \frac{e^x}{(1+e^x)(2+e^x)} dx = \int \frac{dt}{(1+t)(2+t)}$$

$$= \int \frac{dt}{(1+t)} - \int \frac{dt}{(2+t)}$$

$$= \ln|1+t| - \ln|2+t| + c$$

$$= \ln\left|\frac{1+t}{2+t}\right| + c$$

$$= \ln\left|\frac{1+e^x}{2+e^x}\right| + c$$

Ex 19.17

Indefinite Integrals Ex 19.17 Q1

Let $I = \int \frac{1}{\sqrt{2x - x^2}} dx$

$= \int \frac{1}{\sqrt{-\left[x^2 - 2x\right]}} dx$

$= \int \frac{1}{\sqrt{-\left[x^2 - 2x(1) + 1^2 - 1^2\right]}} dx$

$= \int \frac{1}{\sqrt{-\left[(x-1)^2 - 1\right]}} dx$

$= \int \frac{1}{\sqrt{1 - (x-1)^2}} dx$

Let $(x - 1) = t$

$\Rightarrow \quad dx = dt$

so, $\quad I = \int \frac{1}{\sqrt{1 - t^2}} dt$

$= \sin^{-1} t + c \quad \left[\text{Since } \int \frac{1}{\sqrt{1 - x^2}} dx = \sin^{-1} x + c\right]$

$I = \sin^{-1}(x - 1) + c$

Indefinite Integrals Ex 19.17 Q2

$8 + 3x - x^2$ can be written as $8 - \left(x^2 - 3x + \frac{9}{4} - \frac{9}{4}\right)$.

Therefore,

$8 - \left(x^2 - 3x + \frac{9}{4} - \frac{9}{4}\right)$

$= \frac{41}{4} - \left(x - \frac{3}{2}\right)^2$

$\Rightarrow \int \frac{1}{\sqrt{8 + 3x - x^2}} dx = \int \frac{1}{\sqrt{\frac{41}{4} - \left(x - \frac{3}{2}\right)^2}} dx$

Let $x - \frac{3}{2} = t$

$\therefore \ dx = dt$

$\Rightarrow \int \frac{1}{\sqrt{\frac{41}{4} - \left(x - \frac{3}{2}\right)^2}} dx = \int \frac{1}{\sqrt{\left(\frac{\sqrt{41}}{2}\right)^2 - t^2}} dt$

$= \sin^{-1}\left(\frac{t}{\frac{\sqrt{41}}{2}}\right) + C$

$= \sin^{-1}\left(\frac{x - \frac{3}{2}}{\frac{\sqrt{41}}{2}}\right) + C$

$= \sin^{-1}\left(\frac{2x - 3}{\sqrt{41}}\right) + C$

Indefinite Integrals Ex 19.17 Q3

Let $I = \int \frac{1}{\sqrt{5 - 4x - 2x^2}} dx$

$$= \int \frac{1}{\sqrt{-2\left[x^2 + 2x - \frac{5}{2}\right]}} dx$$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{-\left[x^2 + 2x + (1)^2 - (1)^2 - \frac{5}{2}\right]}} dx$$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{-\left[(x+1)^2 - \frac{7}{2}\right]}} dx$$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\frac{7}{2} - (x+1)^2}} dx$$

Let $(x+1) = t$

$\Rightarrow \quad dx = dt$

so, $\quad I = \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\left(\sqrt{\frac{7}{2}}\right)^2 - t^2}} dt$

$$= \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{t}{\sqrt{\frac{7}{2}}}\right) + c \quad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \frac{1}{\sqrt{2}} \sin^{-1}\left(\sqrt{\frac{2}{7}} \times (x+1)\right) + c$$

Indefinite Integrals Ex 19.17 Q4

Let $I = \int \frac{1}{\sqrt{3x^2 + 5x + 7}} dx$

$$= \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{x^2 + \frac{5}{3}x + \frac{7}{3}}} dx$$

$$= \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{x^2 + 2x\left(\frac{5}{6}\right) + \left(\frac{5}{6}\right)^2 - \left(\frac{5}{6}\right)^2 + \frac{7}{3}}} dx$$

$$= \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{\left(x + \frac{5}{6}\right)^2 - \frac{59}{36}}} dx$$

Let $\left(x + \frac{5}{6}\right) = t$

$\Rightarrow \quad dx = dt$

$$I = \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{t^2 - \left(\frac{\sqrt{59}}{6}\right)^2}} dt$$

$$= \frac{1}{\sqrt{3}} \log\left|t + \sqrt{t^2 - \left(\frac{\sqrt{59}}{6}\right)}\right| + c \quad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c\right]$$

$$I = \frac{1}{\sqrt{3}} \log\left|x + \frac{5}{6} + \sqrt{\left(x + \frac{5}{6}\right)^2 - \left(\frac{\sqrt{59}}{6}\right)^2}\right| + c$$

$$I = \frac{1}{\sqrt{3}} \log\left|x + \frac{5}{6} + \sqrt{x^2 + \frac{5x}{3} + \frac{7}{3}}\right| + c$$

Indefinite Integrals Ex 19.17 Q5

Let $I = \int \frac{1}{\sqrt{(x-\alpha)(\beta-x)}}dx, (\text{as } \beta > \alpha)$

$$= \int \frac{1}{\sqrt{-x^2 - x(\alpha+\beta) - \alpha\beta}}dx$$

$$= \int \frac{1}{\sqrt{-\left[x^2 - 2x\left(\frac{\alpha+\beta}{2}\right) + \left(\frac{\alpha+\beta}{2}\right)^2 - \left(\frac{\alpha+\beta}{2}\right)^2 + \alpha\beta\right]}}dx$$

$$= \int \frac{1}{\sqrt{-\left[\left(x - \frac{\alpha+\beta}{2}\right)^2 - \left(\frac{\alpha+\beta}{2}\right)^2\right]}}dx$$

$$= \int \frac{1}{\sqrt{\left[\left(\frac{\beta-\alpha}{2}\right)^2 - \left(x - \frac{\alpha+\beta}{2}\right)^2\right]}}dx, \quad [\because \quad \beta > \alpha]$$

Let $\left(x - \frac{\alpha+\beta}{2}\right) = t$

$\Rightarrow \quad dx = dt$

$$I = \int \frac{1}{\sqrt{\left(\frac{\beta-\alpha}{2}\right)^2 - t^2}}dt$$

$$= \sin^{-1}\left(\frac{t}{\frac{\beta-\alpha}{2}}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \sin^{-1}\left(\frac{2\left(x - \frac{\alpha+\beta}{2}\right)}{\beta-\alpha}\right) + c$$

$$I = \sin^{-1}\left(\frac{2x - \alpha - \beta}{\beta - \alpha}\right) + c$$

Indefinite Integrals Ex 19.17 Q6

Let $I = \int \frac{1}{\sqrt{7-3x-2x^2}}dx$

$$= \int \frac{1}{\sqrt{-2\left[x^2+\frac{3}{2}x-\frac{7}{2}\right]}}dx$$

$$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{-\left[x^2+2x\left(\frac{3}{4}\right)+\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^2-\frac{7}{2}\right]}}dx$$

$$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{-\left[\left(x-\frac{3}{4}\right)^2-\frac{65}{16}\right]}}dx$$

$$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{\left(\frac{\sqrt{65}}{4}\right)^2-\left(x+\frac{3}{4}\right)^2}}dx$$

Let $\left(x+\frac{3}{4}\right) = t$

$\Rightarrow \quad dx = dt$

$$I = \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{\left(\frac{\sqrt{65}}{4}\right)^2-t^2}}dt$$

$$= \frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{t}{\frac{\sqrt{41}}{4}}\right)+c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right)+c\right]$$

$$I = \frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{4\left(x+\frac{3}{4}\right)}{\sqrt{65}}\right)+c$$

$$I = \frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{4x+3}{\sqrt{65}}\right)+c$$

Indefinite Integrals Ex 19.17 Q7

Let $I = \int \frac{1}{\sqrt{16-6x-x^2}}dx$

$$= \int \frac{1}{\sqrt{-\left[x^2+6x-16\right]}}dx$$

$$= \int \frac{1}{\sqrt{-\left[x^2+2x(3)+(3)^2-(3)^2-16\right]}}dx$$

$$= \int \frac{1}{\sqrt{-\left[(x+3)^2-25\right]}}dx$$

$$= \int \frac{1}{\sqrt{25-(x+3)^2}}dx$$

Let $(x+3) = t$

$\Rightarrow \quad dx = dt$

$$I = \int \frac{1}{\sqrt{5^2-t^2}}dt$$

$$= \sin^{-1}\left(\frac{t}{5}\right)+c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right)+c\right]$$

$$I = \sin^{-1}\left(\frac{x+3}{5}\right)+c$$

Indefinite Integrals Ex 19.17 Q8

$7-6x-x^2$ can be written as $7-\left(x^2+6x+9-9\right)$.

Therefore,

$$7-\left(x^2+6x+9-9\right)$$
$$=16-\left(x^2+6x+9\right)$$
$$=16-\left(x+3\right)^2$$
$$=\left(4\right)^2-\left(x+3\right)^2$$

$$\therefore \int\frac{1}{\sqrt{7-6x-x^2}}dx=\int\frac{1}{\sqrt{\left(4\right)^2-\left(x+3\right)^2}}dx$$

Let $x+3=t$

$\Rightarrow dx=dt$

$$\Rightarrow \int\frac{1}{\sqrt{\left(4\right)^2-\left(x+3\right)^2}}dx=\int\frac{1}{\sqrt{\left(4\right)^2-\left(t\right)^2}}dt$$
$$=\sin^{-1}\left(\frac{t}{4}\right)+C$$
$$=\sin^{-1}\left(\frac{x+3}{4}\right)+C$$

Indefinite Integrals Ex 19.17 Q9

We have $$\int\frac{dx}{\sqrt{5x^2-2x}}=\int\frac{dx}{\sqrt{5\left(x^2-\frac{2x}{5}\right)}}$$
$$=\frac{1}{\sqrt{5}}\int\frac{dx}{\sqrt{\left(x-\frac{1}{5}\right)^2-\left(\frac{1}{5}\right)^2}}\quad\text{(completing the square)}$$

Put $x-\frac{1}{5}=t$. Then $dx=dt$.

Therefore, $$\int\frac{dx}{\sqrt{5x^2-2x}}=\frac{1}{\sqrt{5}}\int\frac{dt}{\sqrt{t^2-\left(\frac{1}{5}\right)^2}}$$
$$=\frac{1}{\sqrt{5}}\log\left|t+\sqrt{t^2-\left(\frac{1}{5}\right)^2}\right|+C\qquad\left[\text{by } 7.4\ (4)\right]$$
$$=\frac{1}{\sqrt{5}}\log\left|x-\frac{1}{5}+\sqrt{x^2-\frac{2x}{5}}\right|+C$$

Ex 19.18

Indefinite Integrals Ex 19.18 Q1

Let $I = \int \frac{x}{\sqrt{x^4 + a^4}} dx$

$= \int \frac{x}{\sqrt{(x^2)^2 + (a^2)^2}} dx$

Let $x^2 = t$

$\Rightarrow \quad 2x\,dx = dt$

$\Rightarrow \quad x\,dx = \frac{dt}{2}$

$I = \frac{1}{2} \int \frac{dt}{\sqrt{t^2 + (a^2)^2}}$

$= \frac{1}{2} \log\left|t + \sqrt{t^2 + (a^2)^2}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2 + a^2}} dx = \log\left|x + \sqrt{x^2 + a^2}\right| + c\right]$

$I = \frac{1}{2} \log\left|x^2 + \sqrt{(x^2)^2 + (a^2)^2}\right| + c$

$I = \frac{1}{2} \log\left|x^2 + \sqrt{x^4 + a^4}\right| + c$

Indefinite Integrals Ex 19.18 Q2

Let $\tan x = t$

$\Rightarrow \sec^2 x\,dx = dt$

$\Rightarrow \int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx = \int \frac{dt}{\sqrt{t^2 + 2^2}}$

$= \log\left|t + \sqrt{t^2 + 4}\right| + C$

$= \log\left|\tan x + \sqrt{\tan^2 x + 4}\right| + C$

Indefinite Integrals Ex 19.18 Q3

Let $I = \int \frac{e^x}{\sqrt{16 - e^{2x}}} dx$

Let $e^x = t$

$\Rightarrow \quad e^x\,dx = dt$

$I = \int \frac{dt}{\sqrt{(4)^2 - t^2}}$

$= \sin^{-1}\left(\frac{t}{4}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$

$I = \sin^{-1}\left(\frac{e^x}{4}\right) + c$

Indefinite Integrals Ex 19.18 Q4

Let $I = \int \frac{\cos x}{\sqrt{4 + \sin^2 x}} dx$

Let $\sin x = t$

$\Rightarrow \quad \cos x\,dx = dt$

$I = \int \frac{dt}{\sqrt{(2)^2 + t^2}}$

$= \log\left|t + \sqrt{(2)^2 + t^2}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 + x^2}} dx = \log\left|x + \sqrt{a^2 + x^2}\right| + c\right]$

$I = \log\left|\sin x + \sqrt{4 + \sin^2 x}\right| + c$

Indefinite Integrals Ex 19.18 Q5

Let $I = \int \frac{\sin x}{\sqrt{4\cos^2 x - 1}} dx$

Let $2\cos x = t$

$\Rightarrow \quad -2\sin x\, dx = dt$

$\Rightarrow \quad \sin x\, dx = -\frac{dt}{2}$

$I = -\frac{1}{2}\int \frac{dt}{\sqrt{t^2 - 1}}$

$= -\frac{1}{2}\log\left|t + \sqrt{t^2 - 1}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log\left|x + \sqrt{x^2 + a^2}\right| + c\right]$

$I = -\frac{1}{2}\log\left|2\cos x + \sqrt{4\cos^2 x - 1}\right| + c$

Indefinite Integrals Ex 19.18 Q6

Let $I = \int \frac{x}{\sqrt{4 - x^4}} dx$

Let $x^2 = t$

$\Rightarrow \quad 2x\, dx = dt$

$\Rightarrow \quad x\, dx = \frac{dt}{2}$

$I = \frac{1}{2}\int \frac{dt}{\sqrt{(2)^2 - t^2}}$

$= \frac{1}{2}\sin^{-1}\left(\frac{t}{2}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$

$I = \frac{1}{2}\sin^{-1}\left(\frac{x^2}{2}\right) + c$

Indefinite Integrals Ex 19.18 Q7

Let $I = \int \frac{1}{x\sqrt{4 - 9(\log x)^2}} dx$

Let $3\log x = t$

$\Rightarrow \quad \frac{3}{x} dx = dt$

$\Rightarrow \quad \frac{1}{x} dx = \frac{dt}{3}$

$I = \frac{1}{3}\int \frac{dt}{\sqrt{(2)^2 - t^2}}$

$= \frac{1}{3}\sin^{-1}\left(\frac{t}{2}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$

$I = \frac{1}{3}\sin^{-1}\left(\frac{3\log x}{2}\right) + c$

Indefinite Integrals Ex 19.18 Q8

Let $I = \int \frac{\sin 8x}{\sqrt{9+(\sin 4x)^4}}dx$

Let $\sin^2 4x = t$

$\Rightarrow \quad 2\sin 4x.\cos 4x\,(4)dx = dt$

$\Rightarrow \quad 4\sin 8x\,dx = dt$

$\Rightarrow \quad \sin 8x\,dx = \frac{dt}{4}$

$I = \frac{1}{4}\int \frac{dt}{\sqrt{(3)^2+t^2}}$

$= \frac{1}{4}\log\left|t+\sqrt{(3)^2+t^2}\right|+c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2+x^2}}dx = \log\left|x+\sqrt{a^2+x^2}\right|+c\right]$

$I = \frac{1}{4}\log\left|\sin^2 4x+\sqrt{9+\sin^4 4x}\right|+c$

Indefinite Integrals Ex 19.18 Q9

Let $I = \int \frac{\cos 2x}{\sqrt{\sin^2 2x+8}}dx$

Let $\sin 2x = t$

$\Rightarrow \quad 2\cos 2x dx = dt$

$\Rightarrow \quad \cos 2x\,dx = \frac{dt}{2}$

$I = \frac{1}{2}\int \frac{dt}{\sqrt{t^2+(2\sqrt{2})^2}}$

$= \frac{1}{2}\log\left|t+\sqrt{t^2+(2\sqrt{2})^2}\right|+c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2+x^2}}dx = \log\left|x+\sqrt{x^2+a^2}\right|+c\right]$

$I = \frac{1}{2}\log\left|\sin 2x+\sqrt{\sin^2 2x+8}\right|+c$

Indefinite Integrals Ex 19.18 Q10

Let $I = \int \frac{\sin 2x}{\sqrt{\sin^4 x+4\sin^2 x-2}}dx$

Let $\sin^2 x = t$

$\Rightarrow \quad 2\sin x\cos x\,dx = dt$

$\Rightarrow \quad \sin 2x\,dx = dt$

$I = \int \frac{dt}{\sqrt{t^2+4t-2}}$

$= \int \frac{dt}{\sqrt{t^2+2t(2)+(2)^2-(2)^2-2}}dx$

$= \int \frac{dt}{\sqrt{(t+2)^2-6}}$

Let $t+2 = u$

$dt = du$

$= \int \frac{du}{\sqrt{u^2-(\sqrt{6})^2}}$

$= \log\left|u+\sqrt{u^2-6}\right|+c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x+\sqrt{x^2-a^2}\right|+c\right]$

$= \log\left|t+2+\sqrt{(t+2)^2-6}\right|+c$

$I = \log\left|\sin^2 x+2+\sqrt{\sin^4 x+4\sin^2 x-2}\right|+c$

Indefinite Integrals Ex 19.18 Q11

$$\int \frac{\sin 2x}{\sqrt{\cos^4 x - \sin^2 x + 2}} dx =$$

$$let\ t = \cos^2 x \rightarrow -dt = 2\cos x \sin x dx$$

$$\int \frac{\sin 2x}{\sqrt{\cos^4 x - \sin^2 x + 2}} dx = \int \frac{-1}{\sqrt{t^2 - (1-t) + 2}} dt$$

$$= \int \frac{-1}{\sqrt{t^2 + t + 1}} dt = \int \frac{-1}{\sqrt{t^2 + t + \frac{1}{4} + \frac{3}{4}}} dt$$

$$= \int \frac{-1}{\sqrt{\left(t + \frac{1}{2}\right)^2 + \frac{3}{4}}} dt = -\log\left|\left(t + \frac{1}{2}\right) + \sqrt{t^2 + t + 1}\right|$$

$$= -\log\left|\left(\cos^2 x + \frac{1}{2}\right) + \sqrt{\cos^4 x + \cos^2 x + 1}\right| + C$$

Indefinite Integrals Ex 19.18 Q12

Let $I = \int \frac{\cos x}{\sqrt{4 - \sin^2 x}} dx$

Let $\sin x = t$

$\Rightarrow \quad \cos x\, dx = dt$

$$= \int \frac{dt}{\sqrt{(2)^2 - t^2}}$$

$$= \sin^{-1}\left(\frac{t}{2}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \sin^{-1}\left(\frac{\sin x}{2}\right) + c$$

Indefinite Integrals Ex 19.18 Q13

Let $I = \int \frac{1}{x^{\frac{2}{3}}\sqrt{x^{\frac{2}{3}} - 4}} dx$

Let $x^{\frac{1}{3}} = t$

$\Rightarrow \quad \frac{1}{3} x^{\frac{1}{3}-1} dx = dt$

$\Rightarrow \quad \frac{1}{3} x^{-\frac{2}{3}} dx = dt$

$\Rightarrow \quad \frac{dx}{x^{\frac{2}{3}}} = 3dt$

$$I = 3\int \frac{dt}{\sqrt{t^2 - (2)^2}}$$

$$= 3\log\left|t + \sqrt{t^2 - 4}\right| + c \quad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c\right]$$

$$I == 3\log\left|x^{\frac{1}{3}} + \sqrt{x^{\frac{2}{3}} - 4}\right| + c$$

Indefinite Integrals Ex 19.18 Q14

Let $I = \int \frac{1}{\sqrt{\left(1-x^2\right)\left[9+\left(\sin^{-1}x\right)^2\right]}}dx$

Let $\sin^{-1}x = t$

$\Rightarrow \quad \frac{1}{\sqrt{1-x^2}}dx = dt$

$I = \int \frac{dt}{\sqrt{(3)^2+t^2}}$

$= \log\left|t+\sqrt{9+t^2}\right|+c \qquad \left[\text{Since} \quad \int \frac{1}{\sqrt{a^2+x^2}}dx = \log\left|x+\sqrt{a^2+x^2}\right|+c\right]$

$I = \log\left|\sin^{-1}x+\sqrt{9+\left(\sin^{-1}x\right)^2}\right|+c$

Indefinite Integrals Ex 19.18 Q15

Let $I = \int \frac{\cos x}{\sqrt{\sin^2 x - 2\sin x - 3}}dx$

Let $\sin x = t$

$\Rightarrow \quad \cos x\, dx = dt$

$= \int \frac{dt}{\sqrt{t^2-2t-3}}$

$= \int \frac{dt}{\sqrt{t^2-2t+(1)^2-(1)^2-3}}$

$= \int \frac{dt}{\sqrt{(t-1)^2-(2)^2}}$

Let $t-1 = u$

$\Rightarrow \quad dt = du$

$I = \int \frac{du}{\sqrt{u^2-(2)^2}}$

$= \log\left|u+\sqrt{u^2-4}\right|+c \qquad \left[\text{Since} \quad \int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x+\sqrt{x^2-a^2}\right|+c\right]$

$= \log\left|t-1+\sqrt{(t-1)^2+4}\right|+c$

$I = \log\left|\sin x - 1+\sqrt{\sin^2 x - 2\sin x - 3}\right|+c$

Indefinite Integrals Ex 19.18 Q16

Let $I = \int \sqrt{\operatorname{cosec} x - 1}\, dx$

$$= \int \sqrt{\frac{1 - \sin x}{\sin x}}\, dx$$

$$= \int \sqrt{\frac{(1 - \sin x) + (1 + \sin x)}{\sin x\,(1 + \sin x)}}\, dx$$

$$= \int \sqrt{\frac{\cos^2 x}{\sin^2 x + \sin x}}\, dx$$

$$= \int \frac{\cos x}{\sqrt{\sin^2 x + \sin x}}\, dx$$

Let $\sin x = t$

$\Rightarrow \quad \cos x\, dx = dt$

$$= \int \frac{dt}{\sqrt{t^2 + t}}$$

$$= \int \frac{dt}{\sqrt{t^2 + 2t\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}$$

$$== \int \frac{dt}{\sqrt{\left(t + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}$$

Let, $t + \frac{1}{2} = u$

$\Rightarrow \quad dt = du$

$$= \int \frac{du}{\sqrt{u^2 - \left(\frac{1}{2}\right)^2}}$$

$$== \log\left|u + \sqrt{u^2 - \left(\frac{1}{2}\right)^2}\right| + c \qquad \left[\text{Since} \quad \int \frac{1}{\sqrt{x^2 - a^2}}\, dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c\right]$$

$$= \log\left|\left(t + \frac{1}{2}\right) + \sqrt{\left(t + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}\right| + c$$

$$I = \log\left|\sin x + \frac{1}{2} + \sqrt{\sin^2 x + \sin x}\right| + c$$

Indefinite Integrals Ex 19.18 Q17

To evaluate the following integral follow the steps:

$$\int \frac{\sin x - \cos x}{\sqrt{\sin 2x}}\, dx = \int \frac{\sin x - \cos x}{\sqrt{(\sin x + \cos x)^2 - 1}}\, dx$$

Let $\sin x + \cos x = t$ therefore $(\cos x - \sin x)\, dx = dt$

Now

$$\int \frac{\sin x - \cos x}{\sqrt{(\sin x + \cos x)^2 - 1}}\, dx = -\int \frac{dt}{\sqrt{t^2 - 1}}$$

$$= -\ln\left|t + \sqrt{t^2 - 1}\right| + c$$

$$= -\ln\left|\sin x + \cos x + \sqrt{\sin 2x}\right| + c$$

Ex 19.19

Indefinite Integrals Ex 19.19 Q1

Let $I = \int \frac{x}{x^2 + 3x + 2} dx$

Let $x = \lambda \frac{d}{dx}\left(x^2 + 3x + 2\right) + \mu$

$= \lambda\left(2x + 3\right) + \mu$

$x = \left(2\lambda\right)x + \left(3\lambda + \mu\right)$

Comparing the coefficients of like powers of x,

$2\lambda = 1 \Rightarrow \quad \lambda = \frac{1}{2}$

$3\lambda + \mu = 0 \quad \Rightarrow \quad 3\left(\frac{1}{2}\right) + \mu = 0$

$\mu = -\frac{3}{2}$

so, $\quad I = \int \frac{\frac{1}{2}\left(2x+3\right) - \frac{3}{2}}{x^2 + 3x + 2} dx$

$I = \frac{1}{2}\int \frac{2x+3}{x^2 + 3x + 2} dx - \frac{3}{2}\int \frac{1}{x^2 + 3x + 2} dx$

$= \frac{1}{2}\int \frac{2x+3}{x^2 + 3x + 2} dx - \frac{3}{2}\int \frac{1}{x^2 + 2x\left(\frac{3}{2}\right) + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 2} dx$

$I = \frac{1}{2}\int \frac{2x+3}{x^2 + 3x + 2} dx - \frac{3}{2}\int \frac{1}{\left(x + \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dx$

$= \frac{1}{2}\log\left|x^2 + 3x + 2\right| - \frac{3}{2} \times \frac{1}{2\left(\frac{1}{2}\right)}\log\left|\frac{x + \frac{3}{2} - \frac{1}{2}}{x + \frac{3}{2} + \frac{1}{2}}\right| + c \qquad \left[\text{since, } \int \frac{1}{a^2 - x^2} dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c\right]$

$I = \frac{1}{2}\log\left|x^2 + 3x + 2\right| - \frac{3}{2}\log\left|\frac{x+1}{x+2}\right| + c$

Indefinite Integrals Ex 19.19 Q2

Let $I = \int \frac{x+1}{x^2+x+3}dx$

Let $x+1 = \lambda \frac{d}{dx}\left(x^2+x+3\right)+\mu$

$x+1 = \lambda\left(2x+1\right)+\mu$

$x+1 = \left(2\lambda\right)x+\left(\lambda+\mu\right)$

Comparing the coefficients of like powers of x,

$2\lambda = 1 \Rightarrow \quad \lambda = \frac{1}{2}$

$\lambda+\mu = 1 \quad \Rightarrow \quad \left(\frac{1}{2}\right)+\mu = 0$

$\mu = \frac{1}{2}$

so, $\quad I = \int \frac{\frac{1}{2}\left(2x+1\right)+\frac{1}{2}}{x^2+x+3}dx$

$$I = \frac{1}{2}\int \frac{2x+1}{x^2+x+3}dx+\frac{1}{2}\int \frac{1}{x^2+2x\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+3}dx$$

$$I = \frac{1}{2}\int \frac{2x+1}{x^2+x+3}dx+\frac{1}{2}\int \frac{1}{\left(x+\frac{1}{2}\right)^2+\left(\frac{11}{4}\right)}dx$$

$$I = \frac{1}{2}\int \frac{2x+1}{x^2+x+3}dx+\frac{1}{2}\int \frac{1}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{11}}{2}\right)^2}dx$$

$$= \frac{1}{2}\log\left|x^2+x+3\right|+\frac{1}{2}\times\frac{1}{\left(\frac{\sqrt{11}}{2}\right)}\tan^{-1}\left(\frac{x+\frac{1}{2}}{\frac{\sqrt{11}}{2}}\right)+c \qquad \left[\text{since, } \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$$

$$I = \frac{1}{2}\log\left|x^2+x+3\right|+\frac{1}{\sqrt{11}}\tan^{-1}\left(\frac{2x+1}{\sqrt{11}}\right)+c$$

Indefinite Integrals Ex 19.19 Q3

Let $I = \int \frac{x-3}{x^2+2x-4}dx$

Let $x-3 = \lambda \frac{d}{dx}\left(x^2+2x-4\right)+\mu$

$= \lambda\left(2x+2\right)+\mu$

$x-3 = \left(2\lambda\right)x+\left(2\lambda+\mu\right)$

Comparing the coefficients of like powers of x,

$2\lambda = 1 \Rightarrow \quad \lambda = \frac{1}{2}$

$2\lambda+\mu = -3 \quad \Rightarrow \quad 2\left(\frac{1}{2}\right)+\mu = -3$

$\mu = -4$

so, $\quad I = \int \frac{\frac{1}{2}\left(2x+2\right)-4}{x^2+2x-4}dx$

$$I = \frac{1}{2}\int \frac{2x+2}{x^2+2x-4}dx-4\int \frac{1}{x^2+2x+\left(1\right)^2-\left(1\right)^2-4}dx$$

$$I = \frac{1}{2}\int \frac{2x+2}{x^2+2x-4}dx-4\int \frac{1}{\left(x+1\right)^2-\left(\sqrt{5}\right)}dx$$

$$I = \frac{1}{2}\log\left|x^2+2x-4\right|-4\times\frac{1}{2\sqrt{5}}\log\left|\frac{x+1-\sqrt{5}}{x+1+\sqrt{5}}\right|+c \qquad \left[\text{since, } \int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c\right]$$

$$I = \frac{1}{2}\log\left|x^2+2x-4\right|-\frac{2}{\sqrt{5}}\log\left|\frac{x+1-\sqrt{5}}{x+1+\sqrt{5}}\right|+c$$

Indefinite Integrals Ex 19.19 Q4

Let $I = \int \frac{2x-3}{x^2+6x+13}dx$

Let $2x-3 = \lambda \frac{d}{dx}\left(x^2+6x+13\right)+\mu$

$= \lambda\left(2x+6\right)+\mu$

$2x-3 = \left(2\lambda\right)x+\left(6\lambda+\mu\right)$

Comparing the coefficients of like powers of x,

$2\lambda = 2 \Rightarrow \quad \lambda = 1$

$6\lambda+\mu = -3 \quad \Rightarrow \quad 6\left(1\right)+\mu = -3$

$\mu = -9$

so, $\quad I = \int \frac{1\left(2x+6\right)-9}{x^2+6x+13}dx$

$I = \int \frac{2x+6}{x^2+6x+13}dx - 9\int \frac{1}{x^2+2x\left(3\right)+\left(3\right)^2-\left(3\right)^2+13}dx$

$I = \int \frac{2x+6}{x^2+6x+13}dx - 9\int \frac{1}{\left(x+3\right)^2+\left(2\right)^2}dx$

$= \log\left|x^2+6x+13\right| - 9\times\frac{1}{2}\tan^{-1}\left(\frac{x+3}{2}\right)+c \quad \left[\text{since, } \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$

$I = \log\left|x^2+6x+13\right| - \frac{9}{2}\tan^{-1}\left(\frac{x+3}{2}\right)+c$

Indefinite Integrals Ex 19.19 Q5

Let $I = \int \frac{x^2}{x^2+7x+10}dx$

$= \int \left\{1-\frac{7x+10}{x^2+7x+10}\right\}dx$

$I = x - \int \frac{7x+10}{x^2+7x+10}dx + c_1 ----\left(\text{i}\right)$

Let $I_1 = \int \frac{7x+10}{x^2+7x+10}dx$

Let $7x+10 = \lambda \frac{d}{dx}\left(x^2+7x+10\right)+\mu$

$= \lambda\left(2x+7\right)+\mu$

$7x+10 = \left(2\lambda\right)x+7\lambda+\mu$

Comparing the coefficients of like powers of x,

$7 = 2\lambda \quad \Rightarrow \quad \lambda = \frac{7}{2}$

$7\lambda+\mu = 10 \quad \Rightarrow \quad 7\left(\frac{7}{2}\right)+\mu = 10$

$\mu = -\frac{29}{2}$

so, $\quad I = \int \frac{\frac{1}{6}\left(6x-4\right)-\frac{1}{3}}{3x^2-4x+3}dx$

$I = \frac{1}{6}\int \frac{6x-4}{3x^2-4x+3}dx - \frac{1}{9}\int \frac{1}{x^2-\frac{4}{3}x+1}dx$

$I = \frac{1}{6}\int \frac{6x-4}{3x^2-4x+3}dx - \frac{1}{9}\int \frac{1}{x^2-2x\left(\frac{2}{3}\right)+\left(\frac{2}{3}\right)^2-\left(\frac{2}{3}\right)^2+\left(2\right)^2}dx$

$I = \frac{1}{6}\int \frac{6x-4}{3x^2-4x+3}dx - \frac{1}{9}\int \frac{1}{\left(x-\frac{2}{3}\right)^2+\left(\frac{\sqrt{5}}{3}\right)^2}dx$

$= \frac{1}{6}\log\left|3x^2-4x+3\right| - \frac{1}{9}\times\frac{1}{\left(\frac{\sqrt{5}}{3}\right)}\tan^{-1}\left(\frac{x-\frac{2}{3}}{\frac{\sqrt{5}}{3}}\right)+c \quad \left[\text{since, } \int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$

$I = \frac{1}{6}\log\left|3x^2-4x+3\right| - \frac{\sqrt{5}}{15}\tan^{-1}\left(\frac{3x-2}{\sqrt{5}}\right)+c$

Indefinite Integrals Ex 19.19 Q6

We need to evaluate the integral $\int \frac{2x}{2+x-x^2}dx$

write the numerator in the following form

$$2x = \lambda\left\{\frac{d}{dx}\left(2+x-x^2\right)\right\}+\mu$$

i.e. $2x = \lambda\{-2x+1\}+\mu$

Equating the coefficients will give the values of λ,μ

$\lambda = -1, \mu = 1$

$$\int \frac{2x}{2+x-x^2}dx = \int \frac{\lambda\{-2x+1\}+\mu}{2+x-x^2}dx$$

$$= \int \frac{-1\{-2x+1\}+1}{2+x-x^2}dx$$

$$= \int \frac{-1\{-2x+1\}}{2+x-x^2}dx + \frac{1}{2+x-x^2}dx$$

$$= -\log\left|2+x-x^2\right| + \int \frac{1}{2+x-x^2}dx$$

$$= -\log\left|2+x-x^2\right| - \int \frac{1}{\left(x^2-x-2\right)}dx$$

$$= -\log\left|2+x-x^2\right| - \int \frac{1}{\left(x^2-x+\frac{1}{4}-2-\frac{1}{4}\right)}dx$$

$$= -\log\left|2+x-x^2\right| - \int \frac{1}{\left(x^2-x+\frac{1}{4}-\frac{9}{4}\right)}dx$$

$$= -\log\left|2+x-x^2\right| - \int \frac{1}{\left(\left(x-\frac{1}{2}\right)^2-\left(\frac{3}{2}\right)^2\right)}dx$$

$$= -\log\left|2+x-x^2\right| - \frac{1}{3}\log\left|\frac{\left(x-\frac{1}{2}\right)-\left(\frac{3}{2}\right)}{\left(x-\frac{1}{2}\right)+\left(\frac{3}{2}\right)}\right|+C$$

$$= -\log\left|2+x-x^2\right| - \frac{1}{3}\log\left|\frac{(x-2)}{(x+1)}\right|+C$$

Indefinite Integrals Ex 19.19 Q7

Let $I = \int \frac{1-3x}{3x^2+4x+2}dx$

Let $1\text{-}3x = \lambda \frac{d}{dx}\left(3x^2+4x+2\right)+\mu$

$= \lambda\left(6x+4\right)+\mu$

$1-3x = \left(6\lambda\right)x+\left(4\lambda+\mu\right)$

Comparing the coefficients of like powers of x,

$6\lambda = -3 \quad \Rightarrow \quad \lambda = -\frac{1}{2}$

$4\lambda+\mu = 1 \quad \Rightarrow \quad 4\left(-\frac{1}{2}\right)+\mu = 1$

$\mu = 3$

so, $I = \int \frac{-\frac{1}{2}\left(6x+4\right)+3}{3x^2+4x+2}dx$

$I = -\frac{1}{2}\int \frac{6x+4}{3x^2+4x+2}dx + 3\int \frac{1}{3x^2+4x+2}dx$

$I = -\frac{1}{2}\int \frac{6x+4}{3x^2+4x+2}dx + \frac{3}{3}\int \frac{1}{x^2+\frac{4}{3}x+\frac{2}{3}}dx$

$I = -\frac{1}{2}\int \frac{6x+4}{3x^2+4x+2}dx + \int \frac{1}{x^2+2x\left(\frac{2}{3}\right)+\left(\frac{2}{3}\right)^2-\left(\frac{2}{3}\right)^2+\frac{2}{3}}dx$

$= -\frac{1}{2}\int \frac{6x+4}{3x^2+4x+2}dx + \int \frac{1}{\left(x+\frac{2}{3}\right)^2+\frac{2}{9}}dx$

$I = -\frac{1}{2}\int \frac{6x+4}{3x^2+4x+2}dx + \int \frac{1}{\left(x+\frac{2}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2}dx$

$= -\frac{1}{2}\log\left|3x^2+4x+2\right| + \frac{3}{\sqrt{2}}\tan^{-1}\left(\frac{x+\frac{2}{3}}{\frac{\sqrt{2}}{3}}\right)+c \qquad \left[\text{since, } \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$

$I = -\frac{1}{2}\log\left|3x^2+4x+2\right| + \frac{3}{\sqrt{2}}\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)+c$

Indefinite Integrals Ex 19.19 Q8

Let $I = \int \frac{2x+5}{x^2-x-2}dx$

Let $2x+5 = \lambda \frac{d}{dx}\left(x^2-x-2\right)+\mu$

$= \lambda\left(2x-1\right)+\mu$

$2x+5 = \left(2\lambda\right)x-\lambda+\mu$

Comparing the coefficients of like powers of x,

$2\lambda = 2 \quad \Rightarrow \quad \lambda = 1$

$-\lambda+\mu = 5 \quad \Rightarrow \quad -1+\mu = 5$

$\mu = 6$

so, $I = \int \frac{\left(2x-1\right)+6}{x^2-x-2}dx$

$$I = \int \frac{\left(2x-1\right)}{x^2-x-2}dx + 6\int \frac{1}{x^2-2x\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2-2}dx$$

$$I = \int \frac{2x-1}{x^2-x-2}dx + 6\int \frac{1}{\left(x-\frac{1}{2}\right)^2-\frac{9}{4}}dx$$

$$I = \int \frac{2x-1}{x^2-x-2}dx + 6\int \frac{1}{\left(x-\frac{1}{2}\right)^2-\left(\frac{3}{2}\right)^2}dx$$

$$I = \log\left|x^2-x-2\right| + \frac{6}{2\left(\frac{3}{2}\right)}\log\left|\frac{x-\frac{1}{2}-\frac{3}{2}}{x-\frac{1}{2}+\frac{3}{2}}\right|+c \qquad \left[\text{since, } \int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c\right]$$

$$I = \log\left|x^2-x-2\right| + 2\log\left|\frac{x-2}{x+1}\right|+c$$

Indefinite Integrals Ex 19.19 Q9

Let $I = \int \frac{ax^3 + bx}{x^4 + c^2} dx$

Let $ax^3 + bx = \lambda \frac{d}{dx}\left(x^4 + c^2\right) + \mu$

$ax^3 + bx = \lambda\left(4x^3\right) + \mu$

Comparing the coefficients of like powers c

$4\lambda = a \quad \Rightarrow \quad \lambda = \frac{a}{4}$

$\mu = 0 \quad \Rightarrow \quad \mu = 0$

so, $I = \int \frac{\frac{a}{4}\left(4x^3\right) + bx}{x^4 + c^2} dx$

$I = \frac{a}{4}\int \frac{4x^3}{x^4 + c^2} dx + b\int \frac{x}{\left(x^2\right)^2 + c^2} dx$

$I = \frac{a}{4}\int \frac{4x^3}{x^4 + c^2} dx + \frac{b}{2}\int \frac{2x}{\left(x^2\right)^2 + c^2} dx$

$= \frac{a}{4}\log\left|x^4 + c^2\right| + \frac{b}{2} I_1 - - - - (i)$

Now,

$I_1 = \int \frac{2x}{\left(x^2\right)^2 + c^2} dx$

Put $x^2 = t$

$\Rightarrow \quad 2x\,dx = dt$

$I_1 = \int \frac{1}{(t)^2 + c^2} dx$

$= \frac{1}{c}\tan^{-1}\left(\frac{t}{c}\right) + c_1$

$I_1 = \frac{1}{c}\tan^{-1}\left(\frac{x^2}{c}\right) + c_1 - - - - - (ii)$

Using equation (ii) in equation (i),

$I = \frac{a}{4}\log\left|x^4 + c^2\right| + \frac{b}{2c}\tan^{-1}\left(\frac{x^2}{c}\right) + k$

K = Integration constant

Indefinite Integrals Ex 19.19 Q10

Let $I = \int \frac{x+2}{2x^2+6x+5}dx$

Let $x + 2 = \lambda \frac{d}{dx}\left(2x^2+6x+5\right) + \mu$

$= \lambda\left(4x+6\right) + \mu$

$x + 2 = \left(4\lambda\right)x + \left(6\lambda + \mu\right)$

Comparing the coefficients of like powers of x,

$4\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{4}$

$6\lambda + \mu = 2 \quad \Rightarrow \quad 6\left(\frac{1}{4}\right) + \mu = 2$

$\mu = \frac{1}{2}$

so, $I = \int \frac{\frac{1}{4}\left(4x+6\right)+\frac{1}{2}}{2x^2+6x+5}dx$

$I = \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5}dx + \frac{1}{2}\int \frac{1}{2x^2+6x+5}dx$

$I = \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5}dx + \frac{1}{4}\int \frac{1}{x^2+3x+\frac{5}{2}}dx$

$I = \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5}dx + \frac{1}{4}\int \frac{1}{x^2+2x\left(\frac{3}{2}\right)+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^3+\frac{5}{2}}dx$

$= \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5}dx + \frac{1}{4}\int \frac{1}{\left(x+\frac{3}{2}\right)^2+\frac{1}{4}}dx$

$I = \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5}dx + \frac{1}{4}\int \frac{1}{\left(x+\frac{3}{2}\right)^2+\left(\frac{1}{2}\right)^2}dx$

$I = \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5}dx + \frac{1}{4}\times\frac{1}{\frac{1}{2}}\tan^{-1}\left(\frac{x+\frac{3}{2}}{\frac{1}{2}}\right) + c\,dx \qquad \left[\text{since, } \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$

$I = \frac{1}{4}\log\left|2x^2+6x+5\right| + \frac{1}{2}\tan^{-1}\left(2x+3\right) + c$

Indefinite Integrals Ex 19.19 Q11

Let $I=\int \frac{(3\sin x-2)\cos x}{5-\cos^2 x-4\sin x}dx$

$\therefore I=\int \frac{(3\sin x-2)\cos x}{5-(1-\sin^2 x)-4\sin x}dx$

$\Rightarrow I=\int \frac{(3\sin x-2)\cos x}{5-1+\sin^2 x-4\sin x}dx$

Substitute $\sin x=t$

$\Rightarrow \cos x\, dx = dt$

Thus,

$I=\int \frac{(3t-2)}{4+t^2-4t}dt$

$\Rightarrow I=\int \frac{(3t-2)}{t^2-4t+4}dt$

$\Rightarrow I=\int \frac{(3t-2)}{(t-2)^2}dt$

Now let us separate the integrand into the simplest form using partial fractions.

$$\frac{(3t-2)}{(t-2)^2}=\frac{A}{(t-2)}+\frac{B}{(t-2)^2}$$

$$=\frac{A(t-2)+B}{(t-2)^2}$$

$$=\frac{At-2A+B}{(t-2)^2}$$

$\Rightarrow 3t-2=At-2A+B$

Comparing the coefficients, we have,

A=3

and

$-2A+B=-2$

Susbtituting the value of A=3 in the above equation, we have,

$\Rightarrow -2\times 3+B=-2$

$\Rightarrow -6+B=-2$

$\Rightarrow B=6-2$

$\Rightarrow B=4$

Thus, $I=\int \frac{(3t-2)}{(t-2)^2}dt$ becomes,

$I=\int \frac{3}{(t-2)}dt+\int \frac{4}{(t-2)^2}dt$

$=3\log|t-2|-4\left(\frac{1}{t-2}\right)+C$

$=3\log|2-t|+4\left(\frac{1}{2-t}\right)+C$

Now substituting t=sinx, we have,

$I=3\log|2-\sin x|+4\left(\frac{1}{2-\sin x}\right)+C$

Indefinite Integrals Ex 19.19 Q12

Let $I = \int \frac{5x-2}{1+2x+3x^2}dx$

Rewriting the numerator we have,

$$5x-2 = A\frac{d}{dx}\left(1+2x+3x^2\right)+B$$

$$\Rightarrow 5x-2 = A\left(2+6x\right)+B$$

$$\Rightarrow 5x-2 = 6xA+2A+B$$

Comparing the coefficients, we have,

6A=5 and $2A+B=-2$

$$\Rightarrow A = \frac{5}{6}$$

Substituting the value of A in $2A+B=-2, we have,$

$$2\times\frac{5}{6}+B=-2$$

$$\Rightarrow \frac{10}{6}+B=-2$$

$$\Rightarrow B=-2-\frac{10}{6}$$

$$\Rightarrow B=\frac{-12-10}{6}$$

$$\Rightarrow B=\frac{-22}{6}$$

$$\Rightarrow B=\frac{-11}{3}$$

$$5x-2=\frac{5}{6}(2+6x)-\frac{11}{3}$$

Thus, $I = \int \frac{5x-2}{1+2x+3x^2}dx$ becomes,

$$I = \int \frac{\left[\frac{5}{6}(2+6x)-\frac{11}{3}\right]}{3x^2+2x+1}dx$$

$$= \frac{5}{6}\int\frac{(2+6x)}{3x^2+2x+1}dx - \frac{11}{3}\int\frac{dx}{3x^2+2x+1}$$

$$= \frac{5}{6}\log\left(3x^2+2x+1\right)-\frac{11}{3\times3}\int\frac{dx}{x^2+\frac{2}{3}x+\frac{1}{3}}+C$$

$$= \frac{5}{6}\log\left(3x^2+2x+1\right)-\frac{11}{9}\int\frac{dx}{x^2+\frac{2}{3}x+\left(\frac{4}{3}\right)^2+\frac{1}{3}-\left(\frac{4}{3}\right)^2}+C$$

$$= \frac{5}{6}\log\left(3x^2+2x+1\right)-\frac{11}{9}\int\frac{dx}{\left(x+\frac{1}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2}+C$$

$$= \frac{5}{6}\log\left(3x^2+2x+1\right)-\frac{11}{9}\times\frac{1}{\frac{\sqrt{2}}{3}}\tan^{-1}\left[\frac{\left(x+\frac{1}{3}\right)}{\frac{\sqrt{2}}{3}}\right]+C$$

$$= \frac{5}{6}\log\left(3x^2+2x+1\right)-\frac{11}{9}\times\frac{3}{\sqrt{2}}\tan^{-1}\left[\frac{\left(\frac{3x+1}{3}\right)}{\frac{\sqrt{2}}{3}}\right]+C$$

$$= \frac{5}{6}\log\left(3x^2+2x+1\right)-\frac{11}{3\sqrt{2}}\tan^{-1}\left[\frac{3x+1}{\sqrt{2}}\right]+C$$

Ex 19.20

Indefinite Integrals Ex 19.20 Q1

Let $I = \int \frac{x^2 + x + 1}{x^2 - x} dx$

$= \int \left[1 + \frac{2x+1}{x^2 - x}\right] dx$

$= x + \int \frac{2x+1}{x^2 - x} dx + c_1 \text{ - - - - (i)}$

$I_1 = \int \frac{2x+1}{x^2 - x} dx$

Let $2x + 1 = \lambda \frac{d}{dx}\left(x^2 - x\right) + \mu$

$= \lambda\left(2x - 1\right) + \mu$

$2x + 1 = \left(2\lambda\right)x - \lambda + \mu$

Comparing the coefficients of like powers of x,

$2 = 2\lambda \quad \Rightarrow \quad \lambda = 1$

$-\lambda + \mu = 1 \quad \Rightarrow \quad \mu = 2$

so, $\quad I_1 = \int \frac{\left(2x - 1\right) + 2}{x^2 - x} dx$

$I = \int \frac{2x - 1}{x^2 - x} dx + 2\int \frac{1}{x^2 - x} dx$

$I = \int \frac{2x - 1}{x^2 - x} dx + 2\int \frac{1}{x^2 - 2x\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dx$

$= \int \frac{2x - 1}{x^2 - x} dx + 2\int \frac{1}{\left(x - \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dx$

$I = \log\left|x^2 - x\right| + 2 \times \frac{1}{2\left(\frac{1}{2}\right)} \log\left|\frac{x - \frac{1}{2} - \frac{1}{2}}{x - \frac{1}{2} + \frac{1}{2}}\right| + c_1 \quad \left[\text{since, } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a}\log\left|\frac{x - a}{x + a}\right| + c\right]$

$I_1 = \log\left|x^2 - x\right| + 2\log\left|\frac{x - 1}{x}\right| + c_2 \text{ - - - - (ii)}$

Using equation (i) and (ii)

$$I = x + \log\left|x^2 - x\right| + 2\log\left|\frac{x-1}{x}\right| + c$$

$x \quad -x$

Let $2x + 1 = \lambda \frac{d}{dx}\left(x^2 - x\right) + \mu$

$$= \lambda\left(2x - 1\right) + \mu$$

$$2x + 1 = \left(2\lambda\right)x - \lambda + \mu$$

Comparing the coefficients of like powers of x,

$$2 = 2\lambda \quad \Rightarrow \quad \lambda = 1$$

$$-\lambda + \mu = 1 \quad \Rightarrow \quad \mu = 2$$

so, $I_1 = \int \frac{\left(2x - 1\right) + 2}{x^2 - x}dx$

$$I = \int \frac{2x - 1}{x^2 - x}dx + 2\int \frac{1}{x^2 - x}dx$$

$$I = \int \frac{2x - 1}{x^2 - x}dx + 2\int \frac{1}{x^2 - 2x\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}dx$$

$$= \int \frac{2x - 1}{x^2 - x}dx + 2\int \frac{1}{\left(x - \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}dx$$

$$I = \log\left|x^2 - x\right| + 2 \times \frac{1}{2\left(\frac{1}{2}\right)}\log\left|\frac{x - \frac{1}{2} - \frac{1}{2}}{x - \frac{1}{2} + \frac{1}{2}}\right| + c_1 \quad \left[\text{since, } \int \frac{1}{x^2 - a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c\right]$$

$$I_1 = \log\left|x^2 - x\right| + 2\log\left|\frac{x-1}{x}\right| + c_2 \text{ - - - - (ii)}$$

Using equation (i) and (ii)

$$I = x + \log\left|x^2 - x\right| + 2\log\left|\frac{x-1}{x}\right| + c$$

Indefinite Integrals Ex 19.20 Q2

Let $I = \int \frac{x^2 + x - 1}{x^2 + x - 6}dx$

$$= \int\left[1 + \frac{5}{x^2 + x - 6}\right]dx$$

$$I = x + \int \frac{5}{x^2 + x - 6}dx + c_1 \text{ - - - - (i)}$$

Let $I_1 = 5\int \frac{1}{x^2 + x - 6}dx$

$$= 5\int \frac{1}{x^2 + 2x\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - 6}dx$$

$$= 5\int \frac{1}{\left(x + \frac{1}{2}\right)^2 - \left(\frac{5}{2}\right)^2}dx$$

$$= 5\frac{1}{2\left(\frac{5}{2}\right)}\log\left|\frac{x + \frac{1}{2} - \frac{5}{2}}{x + \frac{1}{2} + \frac{5}{2}}\right| + c_2 \qquad \left[\text{since, } \int \frac{1}{x^2 - a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c\right]$$

$$I_1 = \log\left|\frac{x-2}{x+3}\right| + c_2 \text{ - - - - (ii)}$$

Using equation (i) and (ii)

$$I = x + \log\left|\frac{x-2}{x+3}\right| + c$$

Indefinite Integrals Ex 19.20 Q3

Let $I = \int \frac{1-x^2}{x(1-2x)}dx$

$$= \int \frac{1-x^2}{x-2x^2}dx$$

$$= \int \frac{x^2-1}{2x^2-x}dx$$

$$= \int \left[\frac{1}{2} + \frac{\frac{x}{2}-1}{2x^2-x}\right]dx$$

$$I = \frac{1}{2}x + \int \frac{\frac{x}{2}-1}{2x^2-x}dx + c_1 \;----(\text{i})$$

Let $I_1 = \int \frac{\frac{x}{2}-1}{2x^2-x}dx$

Let $\frac{x}{2}-1 = \lambda \frac{d}{dx}\left(2x^2-x\right)+\mu$

$$= \lambda(4x-1)+\mu$$

$$\frac{x}{2}-1 = (4\lambda)x - \lambda + \mu$$

Comparing the coefficients of like powers of x,

$$\frac{1}{2} = 4\lambda \quad \Rightarrow \quad \lambda = \frac{1}{8}$$

$$-\lambda+\mu = -1 \quad \Rightarrow \quad -\left(\frac{1}{8}\right)+\mu = -1$$

$$\mu = -\frac{7}{8}$$

so, $I_1 = \int \frac{\frac{1}{8}(4x-1)-\frac{7}{8}}{2x^2-x}dx$

$$I = \frac{1}{8}\int \frac{4x-1}{2x^2-x}dx - \frac{7}{8}\int \frac{1}{2\left(x^2-\frac{x}{2}\right)}dx$$

$$I = \frac{1}{8}\int \frac{4x-1}{2x^2-x}dx - \frac{7}{16}\int \frac{1}{x^2-2x\left(\frac{1}{4}\right)+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2}dx$$

$$= \frac{1}{8}\int \frac{4x-1}{2x^2-x}dx - \frac{7}{16}\int \frac{1}{\left(x-\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2}dx$$

$$I_1 = \frac{1}{8}\log\left|2x^2-x\right| - \frac{7}{16}\times\frac{1}{2\left(\frac{1}{4}\right)}\log\left|\frac{x-\frac{1}{4}-\frac{1}{4}}{x-\frac{1}{4}+\frac{1}{4}}\right| + c_2 \quad \left[\text{since, } \int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c\right]$$

$$I = \frac{1}{8}\log|x| + \frac{1}{8}\log|2x-1| - \frac{7}{8}\log|1-2x| + \frac{7}{8}\log 2 + \frac{7}{8}\log|x| + c_2$$

$$I_1 = \log|x| - \frac{3}{4}\log|1-2x| + c_3 \;----(\text{ii}) \quad \left[\text{say, } c_3 = c_2 + \frac{7}{8}\log 2\right]$$

Using equation (i) and (ii)

$$I = \frac{1}{2}x + \log|x| - \frac{3}{4}\log|1-2x| + c$$

Indefinite Integrals Ex 19.20 Q4

Here the integrand $\frac{x^2+1}{x^2-5x+6}$ isnot proper rational function, so we divide x^2+1 by x^2-5x+6 and find the

$$\frac{x^2+1}{x^2-5x+6} = 1 + \frac{5x-5}{x^2-5x+6} = 1 + \frac{5x-5}{(x-2)(x-3)}$$

Let $\frac{5x-5}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$

So that

Equating the coefficients of x and constant terms on both sides, we get A + B = 5 and 3A + 2B = 5. Solving the we get A = −5 and B = 10

Thus, $\frac{x^2+1}{x^2-5x+6} = 1 - \frac{5}{x-2} + \frac{10}{x-3}$

Therefore, $\int \frac{x^2+1}{(x+1)^2(x+3)}dx = \int dx - 5\int \frac{1}{x-2}dx + 10\int \frac{x}{x-3}$

$$= x - 5\log|x-2| + 10\log|x-3| + C.$$

Indefinite Integrals Ex 19.20 Q5

Let $I = \int \frac{x^2}{x^2+7x+10}dx$

$$= \int \left\{1 - \frac{7x+10}{x^2+7x+10}\right\}dx$$

$$I = x - \int \frac{7x+10}{x^2+7x+10}dx + c_1 - - - - (i)$$

Let $I_1 = \int \frac{7x+10}{x^2+7x+10}dx$

Let $7x+10 = \lambda \frac{d}{dx}\left(x^2+7x+10\right) + \mu$

$$= \lambda\left(2x+7\right) + \mu$$

$$7x+10 = \left(2\lambda\right)x + 7\lambda + \mu$$

Comparing the coefficients of like powers of x,

$$7 = 2\lambda \qquad \Rightarrow \qquad \lambda = \frac{7}{2}$$

$$7\lambda + \mu = 10 \qquad \Rightarrow \qquad 7\left(\frac{7}{2}\right) + \mu = 10$$

$$\mu = -\frac{29}{2}$$

so, $I_1 = \int \frac{\frac{7}{2}\left(2x+7\right) - \frac{29}{2}}{x^2+7x+10}dx$

$$= \frac{7}{2}\int \frac{\left(2x+7\right)}{x^2+7x+10}dx - \frac{29}{2}\int \frac{1}{x^2+2x\left(\frac{7}{2}\right)+\left(\frac{7}{2}\right)^2-\left(\frac{7}{2}\right)^2+10}dx$$

$$I_1 = \frac{7}{2}\int \frac{2x+7}{x^2+7x+10}dx - \frac{29}{2}\int \frac{1}{\left(x+\frac{7}{2}\right)^2-\left(\frac{3}{2}\right)^2}dx$$

$$I_1 = \frac{2}{7}\log\left|x^2+7x+10\right| - \frac{29}{2}\times\frac{1}{2\left(\frac{3}{2}\right)}\log\left|\frac{x+\frac{7}{2}-\frac{3}{2}}{x+\frac{7}{2}+\frac{3}{2}}\right| + c_2 \qquad \left[\text{since, } \int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c\right]$$

$$= \frac{7}{2}\log\left|x^2+7x+10\right| - \frac{29}{6}\log\left|\frac{x+2}{x+5}\right| + c_2 - - - - - - (ii)$$

Using equation (i) and (ii)

$$I = x - \frac{7}{2}\log\left|x^2+7x+10\right| + \frac{29}{6}\log\left|\frac{x+2}{x+5}\right| + c$$

Indefinite Integrals Ex 19.20 Q6

Let $I = \int \frac{x^2+x+1}{x^2-x+1}dx$

$$= \int \left[1 + \frac{2x}{x^2-x+1}\right]dx$$

$$I = x + \int \frac{2x}{x^2-x+1}dx + c_1 \text{ - - - - (i)}$$

Let $I = \int \frac{2x}{x^2-x+1}dx$

Let $2x = \lambda \frac{d}{dx}\left(x^2-x+1\right) + \mu$

$$= \lambda\left(2x-1\right) + \mu$$

$$2x = \left(2\lambda\right)x - \lambda + \mu$$

Comparing the coefficients of like powers of x,

$$2 = 2\lambda \quad \Rightarrow \quad \lambda = 1$$

$$-\lambda + \mu = 0 \quad \Rightarrow \quad -1 + \mu = 0$$

$$\mu = 1$$

so, $I_1 = \int \frac{(2x-1)+1}{x^2-x+1}dx$

$$= \int \frac{(2x-1)}{x^2-x+1}dx + \int \frac{1}{x^2 - 2x\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1}dx$$

$$I_1 = \int \frac{2x-1}{x^2-x+1}dx + \int \frac{1}{\left(x-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}dx$$

$$= \log\left|x^2-x+1\right| + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + c_2 \qquad \left[\text{since, } \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$= \log\left|x^2-x+1\right| + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + c_2 \text{ - - - - - - (ii)}$$

Using equation (i) and (ii)

$$I = x + \log\left|x^2-x+1\right| + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + c$$

Indefinite Integrals Ex 19.20 Q7

Let $I = \int \frac{(x-1)^2}{x^2+2x+2}dx$

$= \int \frac{x^2-2x+1}{x^2+2x+2}dx$

$= \int \left[1 - \frac{4x+1}{x^2+2x+2}\right]dx$

$I = x - \int \frac{4x+1}{x^2+2x+2}dx + c_1 \text{ - - - - (i)}$

Let $I = \int \frac{4x+1}{x^2+2x+2}dx$

Let $4x+1 = \lambda \frac{d}{dx}(x^2+2x+2) + \mu$

$= \lambda(2x+2) + \mu$

$= (2\lambda)x + (2\lambda + \mu)$

Comparing the coefficients of like powers of x,

$4 = 2\lambda \quad \Rightarrow \quad \lambda = 2$

$2\lambda + \mu = 1 \quad \Rightarrow \quad 2(2) + \mu = 1$

$\mu = -3$

so, $I_1 = \int \frac{2(2x+2)-3}{x^2+2x+2}dx$

$= 2\int \frac{(2x+2)}{x^2+2x+2}dx - 3\int \frac{1}{x^2-2x+(1)^2-(1)^2+2}dx$

$I_1 = 2\int \frac{2x+2}{x^2+2x+2}dx - 3\int \frac{1}{(x+1)^2+(1)^2}dx$

$I_1 = 2\log\left|x^2+2x+2\right| - 3\tan^{-1}(x+1) + c_2 \text{ - - - - (ii)} \quad \left[\text{since, } \int \frac{1}{x^2+1}dx = \tan^{-1}x + c\right]$

Using equation (i) and (ii)

$I = x - 2\log\left|x^2+2x+2\right| + 3\tan^{-1}(x+1) + c$

Indefinite Integrals Ex 19.20 Q8

$$\text{Let } I=\int\frac{x^3+x^2+2x+1}{x^2-x+1}dx$$

$$=\int\left[x+2+\frac{3x-1}{x^2-x+1}\right]dx$$

$$I=\frac{x^2}{2}+2x+\int\frac{3x-1}{x^2-x+1}dx+c_1----(i)$$

$$\text{Let } I_1=\int\frac{3x-1}{x^2-x+1}dx$$

$$\text{Let } 3x-1=\lambda\frac{d}{dx}\left(x^2-x+1\right)+\mu$$

$$=\lambda\left(2x-1\right)+\mu$$

$$3x-1=\left(2\lambda\right)x-\lambda+\mu$$

Comparing the coefficients of like powers of x,

$$3=2\lambda \quad\Rightarrow\quad \lambda=\frac{3}{2}$$

$$-\lambda+\mu=-1 \quad\Rightarrow\quad -\left(\frac{3}{2}\right)+\mu=-1$$

$$\mu=\frac{1}{2}$$

$$\text{so,}\quad I_1=\int\frac{\frac{3}{2}\left(2x-1\right)+\frac{1}{2}}{x^2-x+1}dx$$

$$=\frac{3}{2}\int\frac{\left(2x-1\right)}{x^2-x+1}dx+\frac{1}{2}\int\frac{1}{x^2-2x\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1}dx$$

$$I_1=\frac{3}{2}\int\frac{2x-1}{x^2-x+1}dx+\frac{1}{2}\int\frac{1}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dx$$

$$I_1=\frac{3}{2}\log\left|x^2-x+1\right|+\frac{1}{2}\times\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)+c_2 \qquad \left[\text{since, }\int\frac{1}{x^2+a^2}dx=\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$$

$$I_1=\frac{3}{2}\log\left|x^2-x+1\right|+\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+c_2---(ii)$$

Using equation (i) and (ii)

$$I=\frac{x^2}{2}+2x+\frac{3}{2}\log\left|x^2-x+1\right|+\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+c$$

Indefinite Integrals Ex 19.20 Q9

$$\text{Let } I=\int\frac{x^2\left(x^4+4\right)}{\left(x^2+4\right)}dx$$

$$=\int\frac{x^6+4x^2}{\left(x^2+4\right)}dx$$

$$=\int\left[x^4-4x^2+20-\frac{80}{x^2+4}\right]dx$$

$$I=\frac{x^5}{5}-\frac{4x^3}{3}+20x-80\int\frac{1}{x^2+4}dx+c_1----(i)$$

$$\text{Let } I_1=\int\frac{1}{x^2+4}dx$$

$$=\int\frac{1}{x^2+\left(2\right)^2}dx$$

$$I_1=\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+c_2---(ii) \quad \left[\text{Since }\int\frac{1}{x^2+a^2}dx=\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$$

Using equation (i) and (ii)

$$I=\frac{x^5}{5}-\frac{4x^3}{3}+20x-\frac{80}{2}\tan^{-1}\left(\frac{x}{2}\right)+c$$

$$I=\frac{x^5}{5}-\frac{4x^3}{3}+20x-40\tan^{-1}\left(\frac{x}{2}\right)+c$$

Indefinite Integrals Ex 19.20 Q10

Let $I = \int \frac{x^2}{x^2+6x+12}dx$

$= \int \left[1 - \frac{6x+12}{x^2+6x+12}\right]dx$

$= x - \int \frac{6x+12}{x^2+6x+12}dx + c_1$ - - - - (i)

Let $I_1 = \int \frac{6x+12}{x^2+6x+12}dx$

Let $6x + 12 = \lambda \frac{d}{dx}\left(x^2+6x+12\right) + \mu$

$= \lambda\left(2x+6\right) + \mu$

$6x + 12 = \left(2\lambda\right)x + 6\lambda + \mu$

Comparing the coefficients of like powers of x,

$6 = 2\lambda \quad \Rightarrow \quad \lambda = 3$

$6\lambda + \mu = 12 \quad \Rightarrow \quad 6\left(3\right) + \mu = 12$

$\mu = -6$

so, $I_1 = \int \frac{3\left(2x+6\right)-6}{x^2+6x+12}dx$

$= 3\int \frac{\left(2x+6\right)}{x^2+6x+12}dx - 6\int \frac{1}{x^2+2x\left(3\right)+\left(3\right)^2-\left(3\right)^2+12}dx$

$I_1 = 3\int \frac{2x+6}{x^2+6x+12}dx + 6\int \frac{1}{\left(x+3\right)^2+\left(\sqrt{3}\right)^2}dx$

$I_1 = 3\log\left|x^2+6x+12\right| + \frac{6}{\sqrt{3}}\tan^{-1}\left(\frac{x+3}{\sqrt{3}}\right) + c_2$ $\left[\text{since, } \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c\right]$

$I_1 = 3\log\left|x^2+6x+12\right| + 2\sqrt{3}\tan^{-1}\left(\frac{x+3}{\sqrt{3}}\right) + c_2$ - - - (ii)

Using equation (i) and (ii)

$I = x - 3\log\left|x^2+6x+12\right| + 2\sqrt{3}\tan^{-1}\left(\frac{x+3}{\sqrt{3}}\right) + c$

Ex 19.21

Indefinite Integrals Ex 19.21 Q1

Let $I = \int \frac{x}{\sqrt{x^2+6x+10}}dx$

Let $x = \lambda \frac{d}{dx}\left(x^2+6x+10\right)+\mu$

$= \lambda\left(2x+6\right)+\mu$

$x = \left(2\lambda\right)x + 6\lambda + \mu$

Comparing the coefficients of like powers of x,

$2\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{2}$

$6\lambda + \mu = 0 \quad \Rightarrow \quad 6\left(\frac{1}{2}\right)+\mu = 0$

$\mu = -3$

so, $I_1 = \int \frac{\frac{1}{2}\left(2x+6\right)-3}{\sqrt{x^2+6x+10}}dx$

$= \frac{1}{2}\int \frac{\left(2x+6\right)}{\sqrt{x^2+6x+10}}dx - 3\int \frac{1}{\sqrt{x^2+2x\left(3\right)+\left(3\right)^2-\left(3\right)^2+10}}dx$

$I_1 = \frac{1}{2}\int \frac{2x+6}{\sqrt{x^2+6x+10}}dx - 3\int \frac{1}{\sqrt{\left(x+3\right)^2+\left(1\right)^2}}dx$

$I_1 = \frac{1}{2}\left(2\sqrt{x^2+6x+10}\right) - 3\log\left|x+3+\sqrt{\left(x+3\right)^2+1}\right|+c$ $\quad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{x^2+a^2}}dx = \log\left|x+\sqrt{x^2+a^2}\right|+c\right]$

$I = \sqrt{x^2+6x+10} - 3\log\left|x+3+\sqrt{x^2+6x+10}\right|+c$

Indefinite Integrals Ex 19.21 Q2

Let $I = \int \frac{2x+1}{\sqrt{x^2+2x-1}}dx$

Let $2x+1 = \lambda \frac{d}{dx}\left(x^2+2x-1\right)+\mu$

$= \lambda\left(2x+2\right)+\mu$

$2x+1 = \left(2\lambda\right)x + 2\lambda + \mu$

Comparing the coefficients of like powers of x,

$2\lambda = 2 \quad \Rightarrow \quad \lambda = 1$

$2\lambda + \mu = 1 \quad \Rightarrow \quad 2\left(1\right)+\mu = 1$

$\mu = -1$

so, $I = \int \frac{\left(2x+2\right)-1}{\sqrt{x^2+2x-1}}dx$

$= \int \frac{\left(2x+2\right)}{\sqrt{x^2+2x-1}}dx - \int \frac{1}{\sqrt{x^2+2x+\left(1\right)^2-\left(1\right)^2-1}}dx$

$I = \int \frac{2x+2}{\sqrt{x^2+2x-1}}dx - \int \frac{1}{\sqrt{\left(x+1\right)^2-\left(\sqrt{2}\right)^2}}dx$

$I = \left(2\sqrt{x^2+2x-1}\right) - \log\left|\left(x+1\right)+\sqrt{\left(x+1\right)^2-\left(\sqrt{2}\right)^2}\right|+c$ $\quad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x+\sqrt{x^2-a^2}\right|+c\right]$

$I = 2\sqrt{x^2+2x-1} - \log\left|x+1+\sqrt{x^2+2x-1}\right|+c$

Indefinite Integrals Ex 19.21 Q3

Let $I = \int \frac{x+1}{\sqrt{4+5x-x^2}}dx$

Let $x + 1 = \lambda \frac{d}{dx}\left(4+5x-x^2\right) + \mu$

$= \lambda\left(5-2x\right) + \mu$

$x = \left(-2\lambda\right)x + 5\lambda + \mu$

Comparing the coefficients of like powers of x,

$-2\lambda = 1 \qquad \Rightarrow \qquad \lambda = -\frac{1}{2}$

$5\lambda + \mu = 1 \qquad \Rightarrow \qquad 5\left(-\frac{1}{2}\right) + \mu = 1$

$\mu = \frac{7}{2}$

so, $\quad I = \int \frac{-\frac{1}{2}\left(5-2x\right)+\frac{7}{2}}{\sqrt{4+5x-x^2}}dx$

$= -\frac{1}{2}\int \frac{\left(5-2x\right)}{\sqrt{4+5x-x^2}}dx + \frac{7}{2}\int \frac{1}{\sqrt{-\left[x^2-5x-4\right]}}dx$

$I = -\frac{1}{2}\int \frac{5-2x}{\sqrt{4+5x-x^2}}dx + \frac{7}{2}\int \frac{1}{\sqrt{-\left[x^2-2x\left(\frac{5}{2}\right)+\left(\frac{5}{2}\right)^2-\left(\frac{5}{2}\right)^2-4\right]}}dx$

$I = -\frac{1}{2}\int \frac{5-2x}{\sqrt{4+5x-x^2}}dx + \frac{7}{2}\int \frac{1}{\sqrt{-\left[\left(x-\frac{5}{2}\right)^2-\left(\frac{\sqrt{41}}{2}\right)^2\right]}}dx$

$I = -\frac{1}{2}\int \frac{5-2x}{\sqrt{4+5x-x^2}}dx + \frac{7}{2}\int \frac{1}{\sqrt{\left[\left(\frac{\sqrt{41}}{2}\right)^2-\left(x-\frac{5}{2}\right)^2\right]}}dx$

$I = -\frac{1}{2}\left(2\sqrt{4+5x-x^2}\right) + \frac{7}{2}\sin^{-1}\left(\frac{x-\frac{5}{2}}{\frac{\sqrt{41}}{2}}\right) + c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x} + c,\ \int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$

$I = -\sqrt{4+5x-x^2} + \frac{7}{2}\sin^{-1}\left(\frac{2x-5}{\sqrt{41}}\right) + c$

Indefinite Integrals Ex 19.21 Q4

Let $I = \int \frac{6x-5}{\sqrt{3x^2-5x+1}}dx$

Let $\quad 3x^2 - 5x + 1 = t$

$\left(6x-5\right)dx = dt$

$I = \int \frac{dt}{\sqrt{t}}$

$= 2\sqrt{t} + c$

$I = 2\sqrt{3x^2-5x+1} + c$

Indefinite Integrals Ex 19.21 Q5

Let $I = \int \frac{3x+1}{\sqrt{5-2x-x^2}}dx$

Let $3x+1 = \lambda \frac{d}{dx}\left(5-2x-x^2\right) + \mu$

$= \lambda\left(-2-2x\right) + \mu$

$3x+1 = \left(-2\lambda\right)x - 2\lambda + \mu$

Comparing the coefficients of like powers of x,

$-2\lambda = 3 \quad \Rightarrow \quad \lambda = -\frac{3}{2}$

$-2\lambda + \mu = 1 \quad \Rightarrow \quad -2\left(-\frac{3}{2}\right) + \mu = 1$

$\mu = -2$

so, $I = \int \frac{-\frac{3}{2}\left(-2-2x\right) - 2}{\sqrt{5-2x-x^2}}dx$

$= -\frac{3}{2}\int \frac{\left(-2-2x\right)}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{-\left[x^2+2x-5\right]}}dx$

$I = -\frac{3}{2}\int \frac{\left(-2-2x\right)}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{-\left[x^2+2x+\left(1\right)^2-\left(1\right)^2-5\right]}}dx$

$I = -\frac{3}{2}\int \frac{\left(-2-2x\right)}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{-\left[\left(x+1\right)^2-\left(\sqrt{6}\right)^2\right]}}dx$

$I = -\frac{3}{2}\int \frac{\left(-2-2x\right)}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{\left[\left(\sqrt{6}\right)^2-\left(x+1\right)^2\right]}}dx$

$I = -\frac{3}{2}\times 2\sqrt{5-2x-x^2} - 2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right) + c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x} + c,\ \int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$

$I = -3\sqrt{5-2x-x^2} - 2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right) + c$

Indefinite Integrals Ex 19.21 Q6

Let $I = \int \frac{x}{\sqrt{8+x-x^2}}dx$

Let $x = \lambda \frac{d}{dx}\left(8+x-x^2\right) + \mu$

$= \lambda\left(1-2x\right) + \mu$

$x = \left(-2\lambda\right)x + \lambda + \mu$

Comparing the coefficients of like powers of x,

$-2\lambda = 1 \quad \Rightarrow \quad \lambda = -\frac{1}{2}$

$\lambda + \mu = 0 \quad \Rightarrow \quad \left(-\frac{1}{2}\right) + \mu = 0$

$\mu = \frac{1}{2}$

so, $I = \int \frac{-\frac{1}{2}\left(1-2x\right) + \frac{1}{2}}{\sqrt{8+x-x^2}}dx$

$= -\frac{1}{2}\int \frac{\left(1-2x\right)}{\sqrt{8+x-x^2}}dx + \frac{1}{2}\int \frac{1}{\sqrt{-\left[x^2-x-8\right]}}dx$

$I = -\frac{1}{2}\int \frac{\left(1-2x\right)}{\sqrt{8+x-x^2}}dx + \frac{1}{2}\int \frac{1}{\sqrt{-\left[x^2-2x\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2-8\right]}}dx$

$I = -\frac{1}{2}\int \frac{\left(1-2x\right)}{\sqrt{8+x-x^2}}dx + \frac{1}{2}\int \frac{1}{\sqrt{-\left[\left(x-\frac{1}{2}\right)^2-\left(\frac{33}{4}\right)^2\right]}}dx$

$I = -\frac{1}{2}\int \frac{\left(1-2x\right)}{\sqrt{8+x-x^2}}dx + \frac{1}{2}\int \frac{1}{\sqrt{\left[\left(\frac{\sqrt{33}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2\right]}}dx$

$I = -\frac{1}{2}\times 2\sqrt{8+x-x^2} + \frac{1}{2}\sin^{-1}\left(\frac{x-\frac{1}{2}}{\frac{\sqrt{33}}{2}}\right) + c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x} + c,\ \int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$

$I = -\sqrt{8+x-x^2} + \frac{1}{2}\sin^{-1}\left(\frac{2x-1}{\sqrt{33}}\right) + c$

Indefinite Integrals Ex 19.21 Q7

Let $I = \int \frac{x+2}{\sqrt{x^2+2x-1}}dx$

Let $x+2 = \lambda \frac{d}{dx}\left(x^2+2x-1\right)+\mu$

$x+2 = \lambda(2x+2)+\mu$

$x+2 = (2\lambda)x+2\lambda+\mu$

Comparing the coefficients of like powers of x,

$2\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{2}$

$2\lambda+\mu = 2 \quad \Rightarrow \quad 2\left(\frac{1}{2}\right)+\mu = 2$

$\Rightarrow \quad \mu = 1$

so, $I_1 = \int \frac{\frac{1}{2}(2x+2)+1}{\sqrt{x^2+2x-1}}dx$

$= \frac{1}{2}\int \frac{(2x+2)}{\sqrt{x^2+2x-1}}dx + \int \frac{1}{\sqrt{x^2+2x+(1)^2-(1)^2-1}}dx$

$I = \frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x-1}}dx + \int \frac{1}{\sqrt{(x+1)^2-\left(\sqrt{2}\right)^2}}dx$

$I = \frac{1}{2}2\sqrt{x^2+2x-1} + \log\left|x+1+\sqrt{(x+1)^2-\left(\sqrt{2}\right)^2}\right|+c$ $\qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x},\ \int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x+\sqrt{x^2-a^2}\right|+c\right]$

$I = \sqrt{x^2+2x-1} + \log\left|x+1+\sqrt{x^2+2x-1}\right|+c$

Indefinite Integrals Ex 19.21 Q8

Let $x+2 = A\frac{d}{dx}\left(x^2-1\right)+B \qquad \ldots(1)$

$\Rightarrow x+2 = A(2x)+B$

Equating the coefficients of x and constant term on both sides, we obtain

$2A = 1 \Rightarrow A = \frac{1}{2}$

$B = 2$

From (1), we obtain

$(x+2) = \frac{1}{2}(2x)+2$

Then, $\int \frac{x+2}{\sqrt{x^2-1}}dx = \int \frac{\frac{1}{2}(2x)+2}{\sqrt{x^2-1}}dx$

$= \frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx + \int \frac{2}{\sqrt{x^2-1}}dx \qquad \ldots(2)$

In $\frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx$, let $x^2-1 = t \Rightarrow 2xdx = dt$

In $\frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx$, let $x^2-1 = t \Rightarrow 2xdx = dt$

$\frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx = \frac{1}{2}\int \frac{dt}{\sqrt{t}}$

$= \frac{1}{2}\left[2\sqrt{t}\right]$

$= \sqrt{t}$

$= \sqrt{x^2-1}$

Then, $\int \frac{2}{\sqrt{x^2-1}}dx = 2\int \frac{1}{\sqrt{x^2-1}}dx = 2\log\left|x+\sqrt{x^2-1}\right|$

From equation (2), we obtain

$\int \frac{x+2}{\sqrt{x^2-1}}dx = \sqrt{x^2-1}+2\log\left|x+\sqrt{x^2-1}\right|+C$

Indefinite Integrals Ex 19.21 Q9

$$\int \frac{x-1}{\sqrt{x^2+1}}\,dx =$$

$$\int \frac{x-1}{\sqrt{x^2+1}}\,dx = \int \frac{x}{\sqrt{x^2+1}} - \frac{1}{\sqrt{x^2+1}}\,dx = \frac{1}{2}\int \frac{2x}{\sqrt{x^2+1}}\,dx - \int \frac{1}{\sqrt{x^2+1}}\,dx$$

$$= \frac{1}{2}\int \frac{dt}{\sqrt{t}} - \int \frac{1}{\sqrt{x^2+1}}\,dx = \frac{1}{2}\left(2\sqrt{t}\right) - \int \frac{1}{\sqrt{x^2+1}}\,dx = \sqrt{t} - \ln\left|x+\sqrt{x^2+1}\right| + C$$

$$= \sqrt{x^2+1} - \ln\left|x+\sqrt{x^2+1}\right| + C$$

Indefinite Integrals Ex 19.21 Q10

Let $I = \int \frac{x}{\sqrt{x^2+x+1}}dx$

Let $x = \lambda \frac{d}{dx}\left(x^2+x+1\right)+\mu$

$= \lambda\left(2x+1\right)+\mu$

$x = \left(2\lambda\right)x+\lambda+\mu$

Comparing the coefficients of like powers of x,

$2\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{2}$

$\lambda+\mu = 0 \quad \Rightarrow \quad \left(\frac{1}{2}\right)+\mu = 0$

$\Rightarrow \quad \mu = -\frac{1}{2}$

so, $I = \int \frac{\frac{1}{2}\left(2x+1\right)+\frac{1}{2}}{\sqrt{x^2+x+1}}dx$

$= \frac{1}{2}\int \frac{\left(2x+1\right)}{\sqrt{x^2+x+1}}dx - \frac{1}{2}\int \frac{1}{\sqrt{x^2+2x\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1}}dx$

$I = \frac{1}{2}\int \frac{2x+1}{\sqrt{x^2+x+1}}dx - \frac{1}{2}\int \frac{1}{\sqrt{\left(x+\frac{1}{2}\right)^2-\left(\frac{\sqrt{3}}{2}\right)^2}}dx$

$I = \frac{1}{2}\times 2\sqrt{x^2+x+1} - \frac{1}{2}\log\left|x+\frac{1}{2}+\sqrt{\left(x+\frac{1}{2}\right)^2-\left(\frac{\sqrt{3}}{2}\right)^2}\right|+c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x+\sqrt{x^2-a^2}\right|+c\right]$

$I = \sqrt{x^2+x+1} - \frac{1}{2}\log\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right|+c$

Indefinite Integrals Ex 19.21 Q11

Let $I = \int \frac{x+1}{\sqrt{x^2+1}}dx$

Let $x+1 = \lambda \frac{d}{dx}\left(x^2+1\right)+\mu$

$x+1 = \lambda\left(2x\right)+\mu$

Comparing the coefficients of like powers of x,

$2\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{2}$

$\Rightarrow \quad \mu = 1$

so, $I = \int \frac{\frac{1}{2}\left(2x\right)+1}{\sqrt{x^2+1}}dx$

$= \frac{1}{2}\int \frac{\left(2x\right)}{\sqrt{x^2+1}}dx + \int \frac{1}{\sqrt{x^2+1}}dx$

$I = \frac{1}{2}\times 2\sqrt{x^2+1} + \log\left|x+\sqrt{x^2+1}\right|+c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{x^2+1}}dx = \log\left|x+\sqrt{x^2-a^2}\right|+c\right]$

$I = \sqrt{x^2+1} + \log\left|x+\sqrt{x^2+1}\right|+c$

Indefinite Integrals Ex 19.21 Q12

Let $I = \int \frac{2x+5}{\sqrt{x^2+2x+5}}dx$

Let $2x+5 = \lambda \frac{d}{dx}\left(x^2+2x+5\right)+\mu$

$= \lambda\left(2x+2\right)+\mu$

$2x+5 = \left(2\lambda\right)x+2\lambda+\mu$

Comparing the coefficients of like powers of x,

$2\lambda = 2 \quad \Rightarrow \quad \lambda = 1$

$2\lambda+\mu = 5 \quad \Rightarrow \quad 2\left(1\right)+\mu = 5$

$\Rightarrow \quad \mu = 3$

so, $I = \int \frac{\left(2x+2\right)+3}{\sqrt{x^2+2x+5}}dx$

$= \int \frac{\left(2x+3\right)}{\sqrt{x^2+2x+5}}dx + 3\int \frac{1}{\sqrt{x^2+2x+\left(1\right)^2-\left(1\right)^2+5}}dx$

$I = \int \frac{2x+3}{\sqrt{x^2+2x+5}}dx + 3\int \frac{1}{\sqrt{\left(x+1\right)^2+\left(2\right)^2}}dx$

$I = 2\sqrt{x^2+2x+5}+3\log\left|x+1+\sqrt{\left(x+1\right)^2+\left(2\right)^2}\right|+c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{x^2+a^2}}dx = \log\left|x+\sqrt{x^2+a^2}\right|+c\right]$

$I = 2\sqrt{x^2+2x+5}+3\log\left|x+1+\sqrt{x^2+2x+5}\right|+c$

Indefinite Integrals Ex 19.21 Q13

Let $I = \int \frac{3x+1}{\sqrt{5-2x-x^2}}dx$

Let $3x+1 = \lambda \frac{d}{dx}\left(5-2x-x^2\right)+\mu$

$= \lambda\left(-2-2x\right)+\mu$

$3x+1 = \left(-2\lambda\right)x-2\lambda+\mu$

Comparing the coefficients of like powers of x,

$-2\lambda = 3 \quad \Rightarrow \quad \lambda = -\frac{3}{2}$

$-2\lambda+\mu = 1 \quad \Rightarrow \quad -2\left(-\frac{3}{2}\right)+\mu = 1$

$\Rightarrow \quad \mu = -2$

so, $I = \int \frac{-\frac{3}{2}\left(-2-2x\right)-2}{\sqrt{5-2x-x^2}}dx$

$= -\frac{3}{2}\int \frac{\left(-2-2x\right)}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{-\left[x^2+2x-5\right]}}dx$

$I = -\frac{3}{2}\int \frac{-2-2x}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{-\left[x^2+2x+\left(1\right)^2-\left(1\right)^2+5\right]}}dx$

$I = -\frac{3}{2}\int \frac{-2-2x}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{-\left[\left(x+1\right)^2-\left(\sqrt{6}\right)^2\right]}}dx$

$I = -\frac{3}{2}\int \frac{-2-2x}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{\left(\sqrt{6}\right)^2-\left(x+1\right)^2}}dx$

$I = -\frac{3}{2}\times 2\sqrt{5-2x-x^2}-2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right)+c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right)+c\right]$

$I = -3\sqrt{5-2x-x^2}-2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right)+c$

Indefinite Integrals Ex 19.21 Q14

Let $I = \int \sqrt{\frac{1-x}{1+x}}dx$

$= \int \sqrt{\frac{1-x}{1+x} \times \frac{1-x}{1-x}}dx$

$= \int \frac{1-x}{\sqrt{1-x^2}}dx$

Let $1-x = \lambda \frac{d}{dx}\left(1-x^2\right) + \mu$

$= \lambda\left(-2x\right) + \mu$

$1-x = \left(-2\lambda\right)x + \mu$

Comparing the coefficients of like powers of x,

$-2\lambda = -1 \quad \Rightarrow \quad \lambda = \frac{1}{2}$

$\Rightarrow \quad \mu = 1$

so, $I = \int \frac{\frac{1}{2}\left(-2x\right)+1}{\sqrt{1-x^2}}dx$

$= \frac{1}{2}\int \frac{\left(-2x\right)}{\sqrt{1-x^2}}dx + \int \frac{1}{\sqrt{1-x^2}}dx$

$I = \frac{1}{2} \times 2\sqrt{1-x^2} + \sin^{-1}x + c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x} + c,\ \int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}\left(x\right) + c\right]$

$I = \sqrt{1-x^2} + \sin^{-1}x + c$

Indefinite Integrals Ex 19.21 Q15

Let $I = \int \frac{2x+1}{\sqrt{x^2+4x+3}}dx$

Let $2x+1 = \lambda \frac{d}{dx}\left(x^2+4x+3\right) + \mu$

$= \lambda\left(2x+4\right) + \mu$

$2x+1 = \left(2\lambda\right)x + 4\lambda + \mu$

Comparing the coefficients of like powers of x,

$2\lambda = 2 \quad \Rightarrow \quad \lambda = 1$

$4\lambda + \mu = 1 \quad \Rightarrow \quad 4\left(1\right) + \mu = 1$

$\Rightarrow \quad \mu = -3$

so, $I = \int \frac{\left(2x+4\right)-3}{\sqrt{x^2+4x+3}}dx$

$= \int \frac{\left(2x+4\right)}{\sqrt{x^2+4x+3}}dx - 3\int \frac{1}{\sqrt{x^2+2x\left(2\right)+\left(2\right)^2-\left(2\right)^2+3}}dx$

$= \int \frac{\left(2x+4\right)}{\sqrt{x^2+4x+3}}dx - 3\int \frac{1}{\sqrt{\left(x+2\right)^2-1}}dx$

$I = 2\sqrt{x^2+4x+3} - 3\log\left|x+2+\sqrt{\left(x+2\right)^2-1}\right| + c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x} + c,\ \int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x+\sqrt{x^2-a^2}\right| + c\right]$

$I = 2\sqrt{x^2+4x+3} - 3\log\left|x+2+\sqrt{x^2+4x+3}\right| + c$

Indefinite Integrals Ex 19.21 Q16

Let $I = \int \frac{2x+3}{\sqrt{x^2+4x+5}}dx$

Let $2x+3 = \lambda \frac{d}{dx}\left(x^2+4x+5\right) + \mu$

$= \lambda\left(2x+4\right) + \mu$

$2x+3 = \left(2\lambda\right)x + 4\lambda + \mu$

Comparing the coefficients of like powers of x,

$2\lambda = 2 \quad \Rightarrow \quad \lambda = 1$

$4\lambda + \mu = 3 \quad \Rightarrow \quad 4\left(1\right) + \mu = 3$

$\Rightarrow \quad \mu = -1$

so, $I = \int \frac{\left(2x+4\right)-1}{\sqrt{x^2+4x+5}}dx$

$= \int \frac{\left(2x+4\right)}{\sqrt{x^2+4x+5}}dx - \int \frac{1}{\sqrt{x^2+2x\left(2\right)+\left(2\right)^2-\left(2\right)^2+5}}dx$

$= \int \frac{\left(2x+4\right)}{\sqrt{x^2+4x+5}}dx - \int \frac{1}{\sqrt{\left(x+2\right)^2+\left(1\right)^2}}dx$

$I = 2\sqrt{x^2+4x+5} - \log\left|x+2+\sqrt{\left(x+2\right)^2+1}\right| + c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x} + c,\ \int \frac{1}{\sqrt{x^2+a^2}}dx = \log\left|x+\sqrt{x^2+a^2}\right| + c\right]$

$I = 2\sqrt{x^2+4x+5} - \log\left|x+2+\sqrt{x^2+4x+5}\right| + c$

Indefinite Integrals Ex 19.21 Q17

$$\int \frac{5x+3}{\sqrt{x^2+4x+10}}dx$$

$\rightarrow$ let $5x+3 = \lambda(2x+4)+\mu$

$$\lambda = \frac{5}{2}, \mu = -7$$

$$\int \frac{\lambda(2x+4)+\mu}{\sqrt{x^2+4x+10}}dx = \int \frac{\frac{5}{2}(2x+4)-7}{\sqrt{x^2+4x+10}}dx$$

$$= \int \frac{\frac{5}{2}(2x+4)}{\sqrt{x^2+4x+10}}dx - \int \frac{7}{\sqrt{x^2+4x+10}}dx$$

$$= \int \frac{\frac{5}{2}dt}{\sqrt{t}} - \int \frac{7}{\sqrt{(x+2)^2+6}}dx$$

$$= 5\sqrt{x^2+4x+10} - 7\log\left|(x+2)+\sqrt{x^2+4x+10}\right| + C$$

Indefinite Integrals Ex 19.21 Q18

Let $I = \int \frac{x+2}{\sqrt{x^2+2x+3}}$

$$x+2 = A\frac{d}{dx}\left[x^2+2x+3\right]+B$$

$$\Rightarrow x+2 = 2Ax+2A+B$$

Comparing the coefficients, we have,

$2A=1$ and $2A+B=2$

$$\Rightarrow A = \frac{1}{2}$$

Substituting the value of A in $2A+B=2$, we have,

$$2\times\frac{1}{2}+B=2$$

$$\Rightarrow 1+B=2$$

$$\Rightarrow B = 2-1$$

$$\Rightarrow B = 1$$

Thus we have,

$$x+2 = \frac{1}{2}[2x+2]+1$$

Hence,

$$I = \int \frac{x+2}{\sqrt{x^2+2x+3}}dx$$

$$= \int \frac{\left[\frac{1}{2}[2x+2]+1\right]}{\sqrt{x^2+2x+3}}dx$$

$$= \int \frac{\left[\frac{1}{2}[2x+2]\right]}{\sqrt{x^2+2x+3}}dx + \int \frac{dx}{\sqrt{x^2+2x+3}}$$

$$= \frac{1}{2}\int \frac{[2x+2]}{\sqrt{x^2+2x+3}}dx + \int \frac{dx}{\sqrt{x^2+2x+3}}$$

Substituting $t=x^2+2x+3$ and $dt=2x+2$

in the first integrand, we have,

$$I = \frac{1}{2}\int \frac{dt}{\sqrt{t}} + \int \frac{dx}{\sqrt{x^2+2x+3}}$$

$$= \frac{1}{2}\times 2\sqrt{t} + \int \frac{dx}{\sqrt{x^2+2x+1+2}} + C$$

$$= \sqrt{t} + \int \frac{dx}{\sqrt{(x+1)^2+\left(\sqrt{2}\right)^2}} + C$$

$$I = \sqrt{x^2+2x+3} + \log\left[|x+1| + \sqrt{(x+1)^2+\left(\sqrt{2}\right)^2}\right] + C$$

$$\Rightarrow I = \sqrt{x^2+2x+3} + \log\left[|x+1| + \sqrt{x^2+2x+3}\right] + C$$

Ex 19.22

Indefinite Integrals Ex 19.22 Q1

Let $I = \int \frac{1}{4\cos^2 x + 9\sin^2 x} dx$

Diving numerator and denominator by $\cos^2 x$

$$= \int \frac{\frac{1}{\cos^2 x}}{4 + 9\tan^2 x} dx$$

$$I = \int \frac{\sec^2 x}{4 + 9\tan^2 x} dx$$

Let $\tan x = t$

$\sec^2 x\, dx = dt$

$$I = \int \frac{dt}{4 + 9(t)^2}$$

$$= \int \frac{dt}{4 + (3t)^2}$$

Let $3t = u$

$3dt = du$

$$I = \frac{1}{3}\int \frac{du}{(2)^2 + (u)^2}$$

$$= \frac{1}{3} \times \frac{1}{2} \times \tan^{-1}\left(\frac{u}{2}\right) + c$$

$$I = \frac{1}{6}\tan^{-1}\left(\frac{3t}{2}\right) + c$$

$$I = \frac{1}{6}\tan^{-1}\left(\frac{3\tan x}{2}\right) + c$$

Indefinite Integrals Ex 19.22 Q2

Let $I = \int \frac{1}{4\sin^2 x + 5\cos^2 x} dx$

Diving numerator and denominator by $\cos^2 x$

$$I = \int \frac{\frac{1}{\cos^2 x}}{4\tan^2 x + 5} dx$$

$$= \int \frac{\sec^2 x}{4\tan^2 x + 5} dx$$

Let $\tan x = t$

$\sec^2 x\, dx = dt$

$$I = \int \frac{dt}{4 + 9(t)^2}$$

$$= \int \frac{dt}{4t^2 + 5}$$

Let $2t = u$

$2dt = du$

$$I = \frac{1}{2}\int \frac{du}{(4)^2 + (\sqrt{5})^2}$$

$$= \frac{1}{2} \times \frac{1}{\sqrt{5}} \times \tan^{-1}\left(\frac{u}{\sqrt{5}}\right) + c$$

$$= \frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{2t}{\sqrt{5}}\right) + c$$

$$I = \frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{2\tan x}{\sqrt{5}}\right) + c$$

Indefinite Integrals Ex 19.22 Q3

Let $I = \int \frac{2}{2+\sin 2x}\,dx$

$= \int \frac{2}{2+2\sin x\cos x}\,dx$

Diving numerator and denominator by $\cos^2 x$

$$I = \int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\cos^2 x}+\frac{\sin x\cos x}{\cos^2 x}}\,dx$$

$$= \int \frac{\sec^2 x}{\sec^2 x+\tan x}\,dx$$

$$I = \int \frac{\sec^2 x}{1+\tan^2 x+\tan x}\,dx$$

Let $\tan x = t$

$\sec^2 x\,dx = dt$

$$I = \int \frac{dt}{t^2+t+1}$$

$$= \int \frac{dt}{t^2+2t\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1}$$

$$I = \int \frac{dt}{\left(t+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}\left(\frac{t+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)+c$$

$$= \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2t+1}{\sqrt{3}}\right)+c$$

$$I = \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan x+1}{\sqrt{3}}\right)+c$$

Indefinite Integrals Ex 19.22 Q4

Let $\quad I = \int \frac{\cos x}{\cos 3x} dx$

$$= \int \frac{\cos x}{4\cos^3 x - 3\cos x} dx$$

Diving numerator and denominator by $\cos^3 x$

$$I = \int \frac{\frac{\cos x}{\cos^3 x}}{\frac{4\cos^3 x}{\cos^3 x} + \frac{3\cos x}{\cos^3 x}} dx$$

$$= \int \frac{\sec^2 x}{4 - 3\sec^2 x} dx$$

$$= \int \frac{\sec^2 x}{4 - 3\left(1 + \tan^2 x\right)} dx$$

$$= \int \frac{\sec^2 x}{4 - 3 - 3\tan^2 x} dx$$

$$= \int \frac{\sec^2 x}{1 - 3\tan^2 x} dx$$

Let $\quad \tan x = t$

$$\sec^2 x \, dx = dt$$

$$I = \int \frac{dt}{1 - 3t^2}$$

$$= \int \frac{dt}{1 - \left(\sqrt{3}t\right)^2}$$

Let $\quad \sqrt{3}t = u$

$$\sqrt{3}dt = du$$

$$= \int \frac{du}{(1)^2 - (4)^2}$$

$$= \frac{1}{2\sqrt{3}} \log\left|\frac{u+1}{1-u}\right| + c$$

$$= \frac{1}{2\sqrt{3}} \log\left|\frac{\sqrt{3}t+1}{1-\sqrt{3}t}\right| + c$$

$$I = \frac{1}{2\sqrt{3}} \log\left|\frac{1+\sqrt{3}\tan x}{1-\sqrt{3}\tan x}\right| + c$$

Indefinite Integrals Ex 19.22 Q5

Let $\quad I = \int \frac{1}{1 + 3\sin^2 x} dx$

Diving numerator and denominator by $\cos^2 x$

$$I = \int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\cos^2 x} + \frac{3\sin^2 x}{\cos^2 x}} dx$$

$$= \int \frac{\sec^2 x}{\sec^2 x + 3\tan^2 x} dx$$

$$= \int \frac{\sec^2 x}{1 + \tan^2 x + 3\tan^2 x} dx$$

$$= \int \frac{\sec^2 x}{1 + 4\tan^2 x} dx$$

$$= \int \frac{\sec^2 x}{1 + \left(2\tan x\right)^2} dx$$

Let $\quad 2\tan x = t$

$$2\sec^2 x \, dx = dt$$

$$I = \frac{1}{2}\int \frac{dt}{1 + t^2}$$

$$= \frac{1}{2}\tan^{-1} t + c$$

$$I = \frac{1}{2}\tan^{-1}\left(2\tan x\right) + c$$

Indefinite Integrals Ex 19.22 Q6

Let $\quad I = \int \frac{1}{3+2\cos^2 x} dx$

Diving numerator and denominator by $\cos^2 x$

$$I = \int \frac{\frac{1}{\cos^2 x}}{\frac{3}{\cos^2 x} + \frac{2\cos^2 x}{\cos^2 x}} dx$$

$$= \int \frac{\sec^2 x}{3\sec^2 x + 2} dx$$

$$= \int \frac{\sec^2 x}{3\left(1+\tan^2 x\right)+2} dx$$

$$= \int \frac{\sec^2 x}{3+3\tan^2 x+2} dx$$

$$= \int \frac{\sec^2 x}{5+3\tan^2 x} dx$$

Let $\quad \sqrt{3}\tan x = t$

$\sqrt{3}\sec^2 x\, dx = dt$

$$I = \frac{1}{\sqrt{3}} \int \frac{dt}{\left(\sqrt{5}\right)^2 + t^2}$$

$$= \frac{1}{\sqrt{3}\times\sqrt{5}} \tan^{-1}\left(\frac{t}{\sqrt{5}}\right) + c$$

$$I = \frac{1}{\sqrt{15}} \tan^{-1}\left(\frac{\sqrt{3}\tan x}{\sqrt{5}}\right) + c$$

Indefinite Integrals Ex 19.22 Q7

Let $\quad I = \int \frac{1}{\left(\sin x - 2\cos x\right)\left(2\sin x + \cos x\right)} dx$

$$= \int \frac{1}{2\sin^2 x + \sin x\cos x - 4\sin x\cos x - 2\cos^2 x} dx$$

$$= \int \frac{1}{2\sin^2 x - 3\sin x\cos x - 2\cos^2 x} dx$$

Diving numerator and denominator by $\cos^2 x$

$$I = \int \frac{\sec^2 x}{2\tan^2 x - 3\tan x - 2} dx$$

Let $\quad \tan x = t$

$\sec^2 x\, dx = dt$

$$I = \int \frac{dt}{2t^2 - 3t - 2}$$

$$= \frac{1}{2} \int \frac{dt}{t^2 - \frac{3}{2}t - 1}$$

$$= \frac{1}{2} \int \frac{dt}{t^2 - 2t\left(\frac{3}{4}\right) + \left(\frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2 - 1}$$

$$I = \frac{1}{2} \int \frac{dt}{\left(t - \frac{3}{4}\right)^2 - \left(\frac{5}{4}\right)^2}$$

$$= \frac{1}{2} \times \frac{1}{2\left(\frac{5}{4}\right)} \log\left|\frac{t - \frac{3}{4} - \frac{5}{4}}{t - \frac{3}{4} + \frac{5}{4}}\right| + c$$

$$= \frac{1}{5} \log\left|\frac{t-2}{2t+1}\right| + c$$

$$I = \frac{1}{5} \log\left|\frac{\tan x - 2}{2\tan x + 1}\right| + c$$

Indefinite Integrals Ex 19.22 Q8

Let $I = \int \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$

Diving numerator and denominator by $\cos^4 x$

$$I = \int \frac{2\tan x \sec^2 x}{\tan^4 x + 1} dx$$

Let $\tan^2 x = t$

$$2\tan x \sec^2 x\, dx = dt$$

$$I = \int \frac{dt}{t^2 + 1}$$

$$= \tan^{-1} t + c$$

$$I = \tan^{-1}\left(\tan^2 x\right) + c$$

Indefinite Integrals Ex 19.22 Q9

Let $I = \int \frac{1}{\cos x\left(\sin x + 2\cos x\right)} dx$

$$= \int \frac{1}{\sin x \cos x + 2\cos^2 x} dx$$

Diving numerator and denominator by $\cos^2 x$,

$$I = \int \frac{\sec^2 x}{\tan x + 2} dx$$

Let $2 + \tan x = t$

$$\sec^2 x\, dx = dt$$

$$I = \int \frac{dt}{t}$$

$$= \log|t| + c$$

$$I = \log|2 + \tan x| + c$$

Indefinite Integrals Ex 19.22 Q10

Let $I = \int \frac{1}{\sin^2 x + \sin 2x} dx$

$$= \int \frac{1}{\sin^2 x + 2\sin x \cos x} dx$$

Diving numerator and denominator by $\cos^2 x$,

$$I = \int \frac{\sec^2 x}{\tan^2 x + 2\tan x} dx$$

Let $\tan x = t$

$$\sec^2 x\, dx = dt$$

$$= \int \frac{dt}{t^2 + 2t + (1)^2 - (1)^2}$$

$$= \int \frac{dt}{(t+1)^2 - (1)^2}$$

$$= \frac{1}{2}\log\left|\frac{t+1-1}{t+1+1}\right| + c$$

$$= \frac{1}{2}\log\left|\frac{t}{t+2}\right| + c$$

$$I = \frac{1}{2}\log\left|\frac{\tan x}{\tan x + 2}\right| + c$$

Indefinite Integrals Ex 19.22 Q11

Let $\quad I = \int \frac{1}{\cos 2x + 3\sin^2 x} dx$

$= \int \frac{1}{2\cos^2 x - 1 + 3\sin^2 x} dx$

Diving numerator and denominator by $\cos^2 x$,

$I = \int \frac{\sec^2 x}{2 - \sec^2 x + 3\tan^2 x} dx$

$= \int \frac{\sec^2 x}{2 - \left(1 + \tan^2 x\right)^2 + 3\tan^2 x} dx$

$= \int \frac{\sec^2 x}{2 - 1 - \tan^2 x + 3\tan^2 x} dx$

$= \int \frac{dt}{1 + 2\tan^2 x}$

Let $\quad \sqrt{2}\tan x = t$

$\sqrt{2}\sec^2 x\, dx = dt$

$I = \frac{1}{\sqrt{2}} \int \frac{1}{1 + t^2}$

$= \frac{1}{\sqrt{2}} \tan^{-1} t + c$

$I = \frac{1}{\sqrt{2}} \tan^{-1}\left(\sqrt{2}\tan x\right) + c$

Ex 19.23

Indefinite Integrals Ex 19.23 Q1

Let $\quad I = \int \frac{1}{5+4\cos x}dx$

Put $\quad \cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$I = \int \frac{1}{5+4\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}dx$$

$$= \int \frac{1+\tan^2\frac{x}{2}}{5\left(1+\tan^2\frac{x}{2}\right)+4\left(1-\tan^2\frac{x}{2}\right)}dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{5+5\tan^2\frac{x}{2}+4-4\tan^2\frac{x}{2}}dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{9+\tan^2\frac{x}{2}}dx$$

Let $\quad \tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2}dx = dt$$

$$= \int \frac{2dt}{(3)^2+t^2}$$

$$= 2\times\frac{1}{3}\tan^{-1}(t)+c$$

$$I = \frac{2}{3}\tan^{-1}\left(\frac{\tan\frac{x}{2}}{3}\right)+c$$

Indefinite Integrals Ex 19.23 Q2

Let $\quad I = \int \frac{1}{5-4\sin x}dx$

Put $\quad \sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$= \int \frac{1}{5-4\left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}dx$$

$$= \int \frac{1+\tan^2\frac{x}{2}}{5\left(1+\tan^2\frac{x}{2}\right)-4\left(2\tan\frac{x}{2}\right)}dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{5+5\tan^2\frac{x}{2}-8\tan\frac{x}{2}}dx$$

Let $\quad \tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2}dx = dt$$

$$= \int \frac{2dt}{5t^2-8t+5}$$

$$= \frac{2}{5}\int \frac{dt}{t^2-\frac{8}{5}t+1}$$

$$= \frac{2}{5}\int \frac{dt}{t^2-2t\left(\frac{4}{5}\right)+\left(\frac{4}{5}\right)^2-\left(\frac{4}{5}\right)^2+1}$$

$$I = \frac{2}{5}\int \frac{dt}{\left(t-\frac{4}{5}\right)^2+\left(\frac{3}{5}\right)^2}$$

$$= \frac{2}{5}\times\frac{1}{\frac{3}{5}}\tan^{-1}\left(\frac{t-\frac{4}{5}}{\frac{3}{5}}\right)+c$$

$$= \frac{2}{3}\tan^{-1}\left(\frac{5t-4}{3}\right)+c$$

$$I = \frac{2}{3}\tan^{-1}\left(\frac{5\tan\frac{x}{2}-4}{3}\right)+c$$

Indefinite Integrals Ex 19.23 Q3

Let $\quad I = \int \frac{1}{1-2\sin x}dx$

Put $\quad \sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$I = \int \frac{1}{1-2\left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}dx$$

$$= \int \frac{1+\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}-4\tan\frac{x}{2}}dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{\tan^2\frac{x}{2}-4\tan\frac{x}{2}+1}dx$$

Let $\quad \tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2}dx = dt$$

$$I = \int \frac{2dt}{t^2-4t+1}$$

$$= \int \frac{2dt}{t^2-2t(2)+(2)^2-(2)^2+1}$$

$$I = 2\int \frac{dt}{(t-2)^2+3}$$

$$= 2\int \frac{dt}{(t-2)^2+\left(\sqrt{3}\right)^2}$$

$$= 2\times\frac{1}{2\sqrt{3}}\log\left|\frac{t-2-\sqrt{3}}{t-2+\sqrt{3}}\right|+c$$

$$I = \frac{1}{\sqrt{3}}\log\left|\frac{\tan\frac{x}{2}-2-\sqrt{3}}{\tan\frac{x}{2}-2+\sqrt{3}}\right|+c$$

Indefinite Integrals Ex 19.23 Q4

Let $I = \int \frac{1}{4\cos x - 1} dx$

Put $\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

$I = \int \frac{1}{4\left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right) - 1} dx$

$= \int \frac{1 + \tan^2 \frac{x}{2}}{4\left(1 - \tan^2 \frac{x}{2}\right) - \left(1 + \tan^2 \frac{x}{2}\right)} dx$

$= \int \frac{\sec^2 \frac{x}{2}}{4 - 4\tan^2 \frac{x}{2} - 1 - \tan^2 \frac{x}{2}} dx$

$= \int \frac{\sec^2 \frac{x}{2}}{3 - 5\tan^2 \frac{x}{2}} dx$

Let $\sqrt{5} \tan \frac{x}{2} = t$

$\frac{\sqrt{5}}{2} \sec^2 \frac{x}{2} dt = dt$

$I = \int \frac{dt}{\left(\sqrt{3}\right)^2 - t^2}$

$I = \frac{1}{\sqrt{15}} \log \left| \frac{\sqrt{3} + \sqrt{5} \tan \frac{x}{2}}{\sqrt{3} - \sqrt{5} \tan \frac{x}{2}} \right| + c$

Indefinite Integrals Ex 19.23 Q5

Let $I = \int \frac{1}{1 - \sin x + \cos x} dx$

Put $\sin x = \frac{2\tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}, \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

$I = \int \frac{1}{1 - \frac{2\tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} + \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}} dx$

$= \int \frac{1 + \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2} - 2\tan \frac{x}{2} + 1 - \tan^2 \frac{x}{2}} dx$

$= \int \frac{\sec^2 \frac{x}{2}}{2 - 2\tan \frac{x}{2}} dx$

Let $\tan \frac{x}{2} = t$

$\frac{1}{2} \sec^2 \frac{x}{2} dx = dt$

$= \frac{2}{2} \int \frac{dt}{1 - t}$

$= -\log\left|1 - t\right| + c$

$I = -\log\left|1 - \tan \frac{x}{2}\right| + c$

Indefinite Integrals Ex 19.23 Q6

Let $\quad I = \int \frac{1}{3 + 2\sin x + \cos x} dx$

Put $\quad \sin x = \frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}, \quad \cos x = \frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$

$$I = \int \frac{1}{3 + 2\left(\frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right) + \left(\frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right)} dx$$

$$= \int \frac{\left(1 + \tan^2\frac{x}{2}\right)}{3\left(1 + \tan^2\frac{x}{2}\right) + 4\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2}} dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{3 + 3\tan^2\frac{x}{2} + 4\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2}} dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{2\tan^2\frac{x}{2} + 4\tan\frac{x}{2} + 4} dx$$

Let $\quad \tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2} dx = dt$$

$$I = \frac{2}{2}\int \frac{dt}{t^2 + 2t + 2}$$

$$= \int \frac{dt}{t^2 + 2t + 1 - 1 + 2}$$

$$I = \int \frac{dt}{(t+1)^2 + (1)^2}$$

$$= \tan^{-1}(t+1) + c$$

$$I = \tan^{-1}\left(\tan\frac{x}{2} + 1\right) + c$$

Indefinite Integrals Ex 19.23 Q7

Let $\quad I = \int \frac{1}{13 + 3\cos x + 4\sin x} dx$

Put $\quad \sin x = \frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}, \quad \cos x = \frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$

$$I = \int \frac{1}{13 + 3\left(\frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right) + 4\left(\frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right)} dx$$

$$= \int \frac{\left(1 + \tan^2\frac{x}{2}\right)}{13\left(1 + \tan^2\frac{x}{2}\right) + 3 - 3\tan^2\frac{x}{2} + 8\tan\frac{x}{2}} dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{16 + 13\tan^2\frac{x}{2} - 3\tan^2\frac{x}{2} + 8\tan\frac{x}{2}} dx$$

Let $\quad \tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2} dx = dt$$

$$I = \int \frac{2dt}{16 + 10t^2 + 8t}$$

$$= \frac{2}{10}\int \frac{dt}{t^2 + \frac{4}{5}t + \frac{8}{5}}$$

$$I = \frac{1}{5}\int \frac{dt}{t^2 + 2t\left(\frac{2}{5}\right)^2 + \left(\frac{2}{5}\right)^2 - \left(\frac{2}{5}\right)^2 + \frac{8}{5}}$$

$$= \frac{1}{5}\int \frac{dt}{\left(t + \frac{2}{5}\right)^2 + \left(\frac{6}{5}\right)^2}$$

6

$$= \frac{1}{5} \times \frac{1}{\left(\frac{6}{5}\right)} \tan^{-1}\left(\frac{t + \frac{2}{5}}{\frac{6}{5}}\right) + c$$

$$= \frac{1}{6}\tan^{-1}\left(\frac{5t + 2}{6}\right) + c$$

$$I = \frac{1}{6}\tan^{-1}\left(\frac{5\tan\frac{x}{2} + 2}{6}\right) + c$$

Indefinite Integrals Ex 19.23 Q8

Let $\quad I = \int \frac{1}{\cos x - \sin x} dx$

Put $\quad \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}, \sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

$$I = \int \frac{1}{\left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right) - \left(\frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right)} dx$$

$$= \int \frac{\left(1 + \tan^2 \frac{x}{2}\right)}{1 - \tan^2 \frac{x}{2} - 2 \tan \frac{x}{2}} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{1 - \tan^2 \frac{x}{2} - 2 \tan \frac{x}{2}} dx$$

Let $\quad \tan \frac{x}{2} = t$

$$\frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

$$I = \int \frac{2dt}{1 - t^2 - 2t}$$

$$= -\int \frac{2dt}{t^2 + 2t - 1}$$

$$I = -\int \frac{2dt}{t^2 + 2t + 1 - 1 - 1}$$

$$I = -\int \frac{2dt}{(t + 1)^2 - \left(\sqrt{2}\right)^2}$$

$$= \int \frac{2dt}{\left(\sqrt{2}\right)^2 - (t + 1)^2}$$

$$= \frac{2}{2\sqrt{2}} \log \left| \frac{\sqrt{2} + t + 1}{\sqrt{2} - t - 1} \right| + c$$

$$I = \frac{1}{\sqrt{2}} \log \left| \frac{\sqrt{2} + \tan \frac{x}{2} + 1}{\sqrt{2} - \tan \frac{x}{2} - 1} \right| + c$$

Indefinite Integrals Ex 19.23 Q9

Let $\quad I=\int \frac{1}{\sin x+\cos x} d x$

Put $\quad \sin x=\frac{2 \tan \frac{x}{2}}{1+\tan ^2 \frac{x}{2}}, \cos x=\frac{1-\tan ^2 \frac{x}{2}}{1+\tan ^2 \frac{x}{2}}$

$$I=\int \frac{1}{\frac{2 \tan \frac{x}{2}}{1+\tan ^2 \frac{x}{2}}+\frac{1-\tan ^2 \frac{x}{2}}{1+\tan ^2 \frac{x}{2}}} d x$$

$$=\int \frac{1+\tan ^2 \frac{x}{2}}{2 \tan \frac{x}{2}+1-\tan ^2 \frac{x}{2}} d x$$

$$=\int \frac{\sec ^2 \frac{x}{2}}{2 \tan \frac{x}{2}+1-\tan ^2 \frac{x}{2}} d x$$

Let $\quad \tan \frac{x}{2}=t$

$$\frac{1}{2} \sec ^2 \frac{x}{2} d x=d t$$

$$I=\int \frac{2 d t}{2 t+1-t^2}$$

$$=-2 \int \frac{d t}{t^2-2 t-1}$$

$$I=-2 \int \frac{d t}{t^2-2 t+1-1-1}$$

$$I=-2 \int \frac{d t}{(t-1)^2-(\sqrt{2})^2}$$

$$=2 \int \frac{2 d t}{(\sqrt{2})^2-(t-1)^2}$$

$$=\frac{2}{2 \sqrt{2}} \log \left|\frac{\sqrt{2}+t-1}{\sqrt{2}-t+1}\right|+c$$

$$I=\frac{1}{\sqrt{2}} \log \left|\frac{\sqrt{2}+\tan \frac{x}{2}-1}{\sqrt{2}-\tan \frac{x}{2}+1}\right|+c$$

Indefinite Integrals Ex 19.23 Q10

Let $\quad I = \int \frac{1}{5 - 4\cos x}dx$

Put $\quad \cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$I = \int \frac{1}{5 - 4\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}dx$$

$$= \int \frac{\left(1+\tan^2\frac{x}{2}\right)}{5+5\tan^2\frac{x}{2} - 4 + 4\tan^2\frac{x}{2}}dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{9\tan^2\frac{x}{2}+1}dx$$

Let $\quad 3\tan\frac{x}{2} = t$

$$\frac{3}{2}\sec^2\frac{x}{2}dx = dt$$

$$I = \frac{2}{3}\int\frac{dt}{t^2+1}$$

$$= \frac{2}{3}\tan^{-1}(t) + c$$

$$I = \frac{2}{3}\tan^{-1}\left(3\tan\frac{x}{2}\right) + c$$

Indefinite Integrals Ex 19.23 Q11

Let $\quad I = \int \frac{1}{2 + \sin x + \cos x}dx$

Put $\quad \sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}, \cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$I = \int \frac{1}{2 + \left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right) + \left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}dx$$

$$= \int \frac{\left(1+\tan^2\frac{x}{2}\right)}{2+2\tan^2\frac{x}{2}+2\tan\frac{x}{2}+1-\tan^2\frac{x}{2}}dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{\tan^2\frac{x}{2}+2\tan\frac{x}{2}+3}dx$$

Let $\quad \tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2}dx = dt$$

$$I = \int\frac{2dt}{t^2+2t+3}$$

$$= 2\int\frac{dt}{t^2+2t+1-1+3}$$

$$I = 2\int\frac{dt}{(t+1)^2+\left(\sqrt{2}\right)^2}$$

$$= \frac{2}{\sqrt{2}}\tan^{-1}\left(\frac{t+1}{\sqrt{2}}\right)+c$$

$$I = \sqrt{2}\tan^{-1}\left(\frac{\tan\frac{x}{2}+1}{\sqrt{2}}\right)+c$$

Indefinite Integrals Ex 19.23 Q12

Let $\quad I = \int \frac{1}{\sin x + \sqrt{3}\cos x} dx$

Put $\quad \sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}, \cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$I = \int \frac{1}{\left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right) + \sqrt{3}\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)} dx$$

$$= \int \frac{\left(1+\tan^2\frac{x}{2}\right)}{2\tan\frac{x}{2} + \sqrt{3} - \sqrt{3}\tan^2\frac{x}{2}} dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{2\tan\frac{x}{2} + \sqrt{3} - \sqrt{3}\tan^2\frac{x}{2}} dx$$

Let $\quad \tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2} dx = dt$$

$$I = \int \frac{2dt}{2t + \sqrt{3} - \sqrt{3}t^2}$$

$$= -\frac{2}{\sqrt{3}} \int \frac{dt}{t^2 - \frac{2}{\sqrt{3}}t + \left(\frac{1}{\sqrt{3}}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2 - 1}$$

$$I = -\frac{2}{\sqrt{3}} \int \frac{dt}{\left(t - \frac{1}{\sqrt{3}}\right)^2 - \left(\frac{2}{\sqrt{3}}\right)^2}$$

$$= \frac{2}{\sqrt{3}} \int \frac{dt}{\left(\frac{2}{\sqrt{3}}\right)^2 - \left(t - \frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{2}{\sqrt{3}} \times \frac{1}{2\left(\frac{2}{\sqrt{3}}\right)} \log\left|\frac{\frac{2}{\sqrt{3}} + t + \frac{1}{\sqrt{3}}}{\frac{2}{\sqrt{3}} - t + \frac{1}{\sqrt{3}}}\right| + c$$

$$I = \frac{1}{2}\log\left|\frac{\sqrt{3}t + 1}{3 - \sqrt{3}t}\right| + c$$

$$I = \frac{1}{2}\log\left|\frac{1 + \sqrt{3}\tan\frac{x}{2}}{3 - \sqrt{3}\tan\frac{x}{2}}\right| + c$$

Indefinite Integrals Ex 19.23 Q13

Let $\quad I = \int \frac{1}{\sqrt{3}\sin x + \cos x} dx$

Let $\quad \sqrt{3} = r\cos\theta, \text{ and } 1 = r\sin\theta$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = \frac{\pi}{6}$$

$$r = \sqrt{3+1} = 2$$

$$I = \int \frac{1}{r\cos\theta\sin x + r\sin\theta\cos x} dx$$

$$= \frac{1}{r}\int \frac{1}{\sin(x+\theta)} dx$$

$$= \frac{1}{2}\int \operatorname{cosec}(x+\theta) dx$$

$$= \frac{1}{2}\log\left|\tan\left(\frac{x}{2} + \frac{\theta}{2}\right)\right| + c$$

$$I = \frac{1}{2}\log\left|\tan\left(\frac{x}{2} + \frac{\pi}{12}\right)\right| + c$$

Indefinite Integrals Ex 19.23 Q14

Let $\quad I = \int \frac{1}{\sin x - \sqrt{3}\cos x}dx$

Let $\quad 1 = r\cos\theta,\ \text{and } \sqrt{3} = r\sin\theta$

$r = \sqrt{3+1} = 2$

$\tan\theta = \sqrt{3}$

$\theta = \frac{\pi}{3}$

$I = \int \frac{1}{r\cos\theta\sin x - r\sin\theta\cos x}dx$

$= \frac{1}{2}\int \frac{1}{\sin(x-\theta)}dx$

$= \frac{1}{2}\int \operatorname{cosec}(x-\theta)\,dx$

$= \frac{1}{2}\log\left|\tan\left(\frac{x}{2}-\frac{\theta}{2}\right)\right| + c$

$I = \frac{1}{2}\log\left|\tan\left(\frac{x}{2}-\frac{\pi}{6}\right)\right| + c$

Indefinite Integrals Ex 19.23 Q15

Let $\quad I = \int \frac{1}{5+7\cos x + \sin x}dx$

Put $\quad \sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}, \cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

Now,

$$I = \int \frac{1}{5 + \frac{7\left(1-\tan^2\frac{x}{2}\right)}{\left(1+\tan^2\frac{x}{2}\right)} + \left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}dx$$

$$= \int \frac{\left(1+\tan^2\frac{x}{2}\right)}{5\left(1+\tan^2\frac{x}{2}\right) + 7 - 7\tan^2\frac{x}{2} + 2\tan\frac{x}{2}}dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{-2\tan^2\frac{x}{2} + 12 + 2\tan\frac{x}{2}}dx$$

Let $\quad \tan\frac{x}{2} = t$

$\frac{1}{2}\sec^2\frac{x}{2}dx = dt$

$I = \int \frac{2dt}{-2t^2+12+2t}$

$= -\int \frac{dt}{t^2-t-6}$

$$= -\int \frac{dt}{t^2 - 2t\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - 6}$$

$$= -\int \frac{dt}{\left(t-\frac{1}{2}\right)^2 - \left(\frac{5}{2}\right)^2}$$

$$= -\frac{1}{2\left(\frac{5}{2}\right)}\log\left|\frac{t-\frac{1}{2}-\frac{5}{2}}{t-\frac{1}{2}+\frac{5}{2}}\right| + c$$

$= -\frac{1}{5}\log\left|\frac{t-3}{t+2}\right| + c$

$$I = \frac{1}{5}\log\left|\frac{\tan\frac{x}{2}+2}{\tan\frac{x}{2}-3}\right| + c$$

Ex 19.24

Indefinite Integrals Ex 19.24 Q1

Let $I = \int \frac{1}{1-\cot x}dx$

$= \int \frac{1}{1-\frac{\cos x}{\sin x}}dx$

$= \int \frac{\sin x}{\sin x - \cos x}dx$

Let $\sin x = \lambda \frac{d}{dx}(\sin x - \cos x) + \mu(\sin x - \cos x) + \nu$

$\sin x = \lambda \frac{d}{dx}(\cos x + \sin x) + \mu(\sin x - \cos x) + \nu$

$\sin x = \cos(\lambda - \mu) + \sin x(\lambda + \mu) + \nu$

Comparing the coefficients of $\sin x$ & $\cos x$ on the both the sides,

$\lambda + \mu = 1 \quad \ldots(1)$

$\lambda - \mu = 1 \quad \ldots(2)$

$\nu = 0 \quad \ldots(3)$

Equation $(1),(2),(3)$ gives

$\lambda = \frac{1}{2}, \mu = \frac{1}{2}, \nu = 0$

$I = \int \frac{\frac{1}{2}(\cos x + \sin x) + \frac{1}{2}(\sin x - \cos x)}{(\sin x - \cos x)}dx$

$= \frac{1}{2}\int \frac{(\cos x + \sin x)}{(\sin x - \cos x)}dx + \frac{1}{2}\int dx$

$I = \frac{1}{2}\log|\sin x - \cos x| + \frac{1}{2}x + c$

Indefinite Integrals Ex 19.24 Q2

Let $I = \int \frac{1}{1-\tan x}dx$

$= \int \frac{1}{1-\frac{\sin x}{\cos x}}dx$

$= \int \frac{\cos x}{\cos x - \sin x}dx$

Let $\cos x = \lambda \frac{d}{dx}(\cos x - \sin x) + \mu(\cos x - \sin x) + \nu$

$= \lambda \frac{d}{dx}(-\sin x - \cos x) + \mu(\cos x - \sin x) + \nu$

$\cos x = \sin x(-\lambda - \mu) + \cos x(-\lambda + \mu) + \nu$

Camparing the coefficients of $\cos x$ & $\sin x$ on the both the sides,

$-\lambda - \mu = 0 ------(1)$

$-\lambda + \mu = 1 ------(2)$

$\nu = 0 ---------(3)$

Equation $(1),(2),(3)$ gives

$\lambda = -\frac{1}{2}, \mu = \frac{1}{2}, \nu = 0$

$I = \int \frac{-\frac{1}{2}(-\sin x - \cos x) + \frac{1}{2}(\cos x - \sin x)}{(\cos x - \sin x)}dx$

$= \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\cos x - \sin x)}dx + \frac{1}{2}\int dx$

$= -\frac{1}{2}\log|\cos x - \sin x| + \frac{1}{2}x + c$

$I = \frac{1}{2}x - \frac{1}{2}\log|\cos x - \sin x| + c$

Indefinite Integrals Ex 19.24 Q3

Let $I = \int \frac{3 + 2\cos x + 4\sin x}{2\sin x + \cos x + 3} dx$

Let $3 + 2\cos x + 4\sin x = \lambda \frac{d}{dx}(2\sin x + \cos x + 3) + \mu(2\sin x + \cos x + 3) + \nu$

$3 + 2\cos x + 4\sin x = \lambda(2\cos x - \sin x) + \mu(2\sin x + \cos x + 3) + \nu$

$3 + 2\cos x + 4\sin x = (-\lambda + 2\mu)\sin x + (2\lambda + \mu)\cos x + 3\mu + \nu$

Camparing the coefficients of $\sin x$ & $\cos x$ on the both the sides,

$-\lambda + 2\mu = 4 ------(1)$

$2\lambda + \mu = 2 ------(2)$

$2\mu + \nu = 3 ---------(3)$

Solving equation $(1),(2)$ and (3), we get

$\lambda = 0, \mu = 2, \nu = -3$

$I = \int \frac{2(2\sin x + \cos x + 3) - 3}{(2\sin x + \cos x + 3)} dx$

$= 2\int dx - 3\int \frac{1}{2\sin x + \cos x + 3} dx$

$I = 2x - 3I_1 + C_1 -------(4)$

Let $I_1 = \int \frac{1}{2\sin x + \cos x + 3} dx$

Put $\sin x = \frac{2\tan\frac{x}{2}}{\left(1 + \tan^2\frac{x}{2}\right)}$, $\cos x = \frac{1 - \tan^2\frac{x}{2}}{\left(1 + \tan^2\frac{x}{2}\right)}$

$I_1 = \int \frac{1}{2\left(\frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right) + \left(\frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right) + 3} dx$

$= \int \frac{\left(1 + \tan^2\frac{x}{2}\right)}{4\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2} + 3\left(1 + \tan^2\frac{x}{2}\right)} dx$

$= \int \frac{\sec^2\frac{x}{2}}{2\tan^2\frac{x}{2} + 4\tan\frac{x}{2} + 4} dx$

Let $\tan\frac{x}{2} = t$

$\frac{1}{2}\sec^2\frac{x}{2} = dt$

$I_1 = \int \frac{2dt}{2t^2 + 4t + 4}$

$= \frac{2}{2}\int \frac{dt}{t^2 + 2t + 2}$

$= \int \frac{dt}{t^2 + 2t + 1 - 1 + 2}$

$= \int \frac{dt}{(t + 1)^2 + 1}$

$= \tan^{-1}(t + 1) + C_2$

$= \tan^{-1}\left(\tan\frac{x}{2} + 1\right) + C_2$

Now, using equation (1),

$I = 2x - 3\tan^{-1}\left(\tan\frac{x}{2} + 1\right) + C$

Indefinite Integrals Ex 19.24 Q4

Let $\quad I = \int \frac{1}{p + q\tan x} dx$

$= \int \frac{1}{p + q\left(\frac{\sin x}{\cos x}\right)} dx$

$= \int \frac{\cos x}{p\cos x + q\sin x} dx$

Let $\quad \cos x = \lambda \frac{d}{dx}\left(p\cos x + q\sin x\right) + \mu\left(p\cos x + q\sin x\right) + \nu$

$\cos x = \lambda\left(-p\sin x + q\cos x\right) + \mu\left(p\cos x + q\sin x\right) + \nu$

$\cos x = \left(-p\lambda + q\mu\right)\sin x + \left(q\lambda + p\mu\right)\cos x + \nu$

Camparing the coefficients of $\sin x, \cos x$ on the both the sides,

$-p\lambda + q\mu = 0 ------(1)$

$q\lambda + p\mu = 1 -------(2)$

$\nu = 0 ----------(3)$

Solving equation $(1), (2)$ and (3),

$\lambda = \frac{q}{\left(p^2 + q^2\right)}$

$\mu = \frac{p}{\left(p^2 + q^2\right)}$

$\nu = 0$

Now,

$I = \int \frac{q}{\left(p^2 + q^2\right)} \frac{\left(-p\sin x + q\cos x\right)}{\left(p\cos x + q\sin x\right)} dx + \int \frac{p}{\left(p^2 + q^2\right)} \frac{\left(p\cos x + q\sin x\right)}{\left(p\cos x + q\sin x\right)} dx$

$I = \frac{q}{\left(p^2 + q^2\right)}\left(\log\left|p\cos x + q\sin x\right|\right) + \frac{p}{\left(p^2 + q^2\right)} x + c$

Indefinite Integrals Ex 19.24 Q5

Let $\quad I = \int \frac{5\cos x + 6}{2\cos x + \sin x + 3} dx$

Let $\quad \left(5\cos x + 6\right) = \lambda \frac{d}{dx}\left(2\cos x + \sin x + 3\right) + \mu\left(2\cos x + \sin x + 3\right) + \nu$

$\left(5\cos x + 6\right) = \lambda\left(-2\sin x + \cos x\right) + \mu\left(2\cos x + \sin x + 3\right) + \nu$

$\left(5\cos x + 6\right) = \left(-2\lambda + \mu\right)\sin x + \left(\lambda + 2\mu\right)\cos x + \left(3\mu + \nu\right)$

Camparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$-2\lambda + \mu = 0 ------(1)$

$\lambda + 2\mu = 5 -------(2)$

$3\mu + \nu = 6 -------(3)$

Solving equation $(1), (2)$ and (3),

$\lambda = 1$

$\mu = 2$

$\nu = 0$

Now,

$I = \int \frac{\left(-2\sin x + \cos x\right)}{\left(2\cos x + \sin x + 3\right)} dx + 2\int dx$

$I = \log\left|2\cos x + \sin x + 3\right| + 2x + c$

Indefinite Integrals Ex 19.24 Q6

Let $\quad I = \int \frac{2\sin x + 3\cos x}{3\sin x + 4\cos x}dx$

Let $\quad \left(2\sin x + 3\cos x\right) = \lambda \frac{d}{dx}\left(3\sin x + 4\cos x\right) + \mu\left(3\sin x + 4\cos x\right) + \nu$

$\left(2\sin x + 3\cos x\right) = \lambda\left(3\cos x - 4\sin x\right) + \mu\left(3\sin x + 4\cos x\right) + \nu$

$\left(2\sin x + 3\cos x\right) = \left(3\lambda + 4\mu\right)\cos x + \left(-4\lambda + 3\mu\right)\sin x + \nu$

Camparing the coefficients of $\sin x, \cos x$ on the both the sides,

$3\lambda + 4\mu = 3 ------ (1)$

$-4\lambda + 3\mu = 2 ----- (2)$

$\nu = 0 ------- (3)$

Solving the equation $(1), (2)$ and (3),

$\lambda = \frac{1}{25}$

$\mu = \frac{18}{25}$

$\nu = 0$

$I = \frac{1}{25}\int \frac{\left(3\cos x - 4\sin x\right)}{\left(3\sin x + 4\cos x\right)}dx + \frac{18}{25}\int dx$

$I = \frac{1}{25}\log\left|3\sin x + 4\cos x\right| + \frac{18}{25}x + c$

Indefinite Integrals Ex 19.24 Q7

Let $\quad I = \int \frac{1}{3 + 4\cot x}dx$

$= \int \frac{\sin x}{3\sin x + 4\cos x}dx$

Let $\quad \sin x = \lambda \frac{d}{dx}\left(3\sin x + 4\cos x\right) + \mu\left(3\sin x + 4\cos x\right) + \nu$

$\sin x = \lambda\left(3\cos x - 4\sin x\right) + \mu\left(3\sin x + 4\cos x\right) + \nu$

$\sin x = \left(3\lambda + 4\mu\right)\cos x + \left(-4\lambda + 3\mu\right)\sin x + \nu$

Camparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$3\lambda + 4\mu = 0 ------ (1)$

$-4\lambda + 3\mu = 1 ----- (2)$

$\nu = 0 ------- (3)$

Solving the equation $(1), (2)$ and (3), we get

$\lambda = -\frac{4}{25}$

$\mu = \frac{3}{25}$

$\nu = 0$

$I = -\frac{4}{25}\int \frac{\left(3\cos x - 4\sin x\right)}{\left(3\sin x + 4\cos x\right)}dx + \frac{3}{25}\int dx$

$I = -\frac{4}{25}\log\left|3\sin x + 4\cos x\right| + \frac{3}{25}x + c$

Indefinite Integrals Ex 19.24 Q8

Let $I = \int \frac{2\tan x + 3}{3\tan x + 4} dx$

$= \int \frac{2\sin x + 3\cos x}{3\sin x + 4\cos x} dx$

Let $2\sin x + 3\cos x = \lambda \frac{d}{dx}(3\sin x + 4\cos x) + \mu(3\sin x + 4\cos x) + \nu$

$2\sin x + 3\cos x = \lambda(3\cos x - 4\sin x) + \mu(3\sin x + 4\cos x) + \nu$

$2\sin x + 3\cos x = (3\lambda + 4\mu)\cos x + (-4\lambda + 3\mu)\sin x + \nu$

Camparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$3\lambda + 4\mu = 3 ------(1)$

$-4\lambda + 3\mu = 2 -----(2)$

$\nu = 0$

Solving the equation $(1),(2)$ and (3),

$\mu = \frac{18}{25}$

$\lambda = \frac{1}{25}$

$\nu = 0$

$I = \frac{1}{25}\int \frac{(3\cos x - 4\sin x)}{(3\sin x + 4\cos x)} dx + \frac{18}{25}\int dx$

$I = \frac{18}{25}x + \frac{1}{25}\log|3\sin x + 4\cos x| + c$

Indefinite Integrals Ex 19.24 Q9

Let $I = \int \frac{1}{4 + 3\tan x} dx$

$I = \int \frac{\cos x}{4\cos x + 3\sin x} dx$

Let $\cos x = \lambda \frac{d}{dx}(4\cos x + 3\sin x) + \mu(4\cos x + 3\sin x) + \nu$

$\cos x = \lambda(-4\sin x + 3\cos x) + \mu(4\cos x + 3\sin x) + \nu$

$\cos x = (-4\lambda + 3\mu)\sin x + (3\lambda + 4\mu)\cos x + \nu$

Camparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$-4\lambda + 3\mu = 0 -----(1)$

$3\lambda + 4\mu = 1 ------(2)$

$\nu = 0 -------(3)$

Solving the equation $(1),(2)$ and (3),

$\lambda = \frac{3}{25}$

$\mu = \frac{4}{25}$

$\nu = 0$

$I = \int \frac{3}{25} \frac{(-4\sin x + 3\cos x)}{(4\cos x + 3\sin x)} dx + \frac{4}{25}\int dx$

$I = \frac{3}{25}\log|4\cos x + 3\sin x| + \frac{4}{25}x + c$

Indefinite Integrals Ex 19.24 Q10

Let $\quad I = \int \frac{8\cot x + 1}{3\cot x + 2} dx$

$I = \int \frac{8\cos x + \sin x}{3\cos x + 2\sin x} dx$

Let $\quad 8\cos x + \sin x = \lambda \frac{d}{dx}\left(3\cos x + 2\sin x\right) + \mu\left(3\cos x + 2\sin x\right) + \nu$

$8\cos x + \sin x = \lambda\left(-3\sin x + 2\cos x\right) + \mu\left(3\cos x + 2\sin x\right) + \nu$

$8\cos x + \sin x = \left(-3\lambda + 2\mu\right)\sin x + \left(2\lambda + 3\mu\right)\cos x + \nu$

Camparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$2\lambda + 3\mu = 8 - - - - - (1)$

$-3\lambda + 2\mu = 1 - - - - - - (2)$

$\nu = 0 - - - - - - - (3)$

Solving equation $(1), (2)$ and (3),

$\lambda = 1$

$\mu = 2$

$\nu = 0$

$I = \int \frac{\left(-3\sin x + 2\cos x\right)}{\left(3\cos x + 2\sin x\right)} dx + 2\int dx$

$I = \log\left|3\cos x + 2\sin x\right| + 2x + c$

Indefinite Integrals Ex 19.24 Q11

Let $\quad I = \int \frac{4\sin x + 5\cos x}{5\sin x + 4\cos x} dx$

Let $\quad 4\sin x + 5\cos x = \lambda \frac{d}{dx}\left(5\sin x + 4\cos x\right) + \mu\left(5\sin x + 4\cos x\right) + \nu$

$4\sin x + 5\cos x = \lambda\left(5\cos x - 4\sin x\right) + \mu\left(5\sin x + 4\cos x\right) + \nu$

$4\sin x + 5\cos x = \left(5\lambda + 4\mu\right)\cos x + \left(-4\lambda + 5\mu\right)\sin x + \nu$

Camparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$-4\lambda + 5\mu = 4 - - - - - (1)$

$5\lambda + 4\mu = 5 - - - - - - (2)$

$\nu = 0 - - - - - - - (3)$

Solving equation $(1), (2)$ and (3),

$\lambda = \frac{9}{41}$

$\mu = \frac{40}{41}$

$\nu = 0$

Now,

$I = \frac{40}{41}\int dx + \frac{9}{41}\int \frac{\left(5\cos x - 4\sin x\right)}{\left(5\sin x + 4\cos x\right)} dx$

$I = \frac{40}{41}x + \frac{9}{41}\log\left|5\sin x + 4\cos x\right| + c$

Ex 19.25

Indefinite Integrals Ex 19.25 Q1

Let $I = \int x \cos x \, dx$

Using integration by parts,

$$I = x\int \cos x \, dx - \int \left(1 \times \int \cos x \, dx\right) dx + c$$
$$= x \sin x - \int \sin x \, dx + c$$

$$I = x \sin x + \cos x + c$$

Indefinite Integrals Ex 19.25 Q2

Let $I = \int \log(x+1) \, dx$
$= \int 1 \times \log(x+1) \, dx$

Using integration by parts,

$$I = \log(x+1)\int 1\, dx - \int \left(\frac{1}{x+1} \times \int 1\, dx\right) dx + c$$
$$= x \log(x+1) - \int \left(\frac{x}{x+1}\right) dx + c$$
$$= x \log(x+1) - \int \left(1 - \frac{1}{x+1}\right) dx + c$$

$$I = x \log(x+1) - x + \log(x+1) + c$$

Indefinite Integrals Ex 19.25 Q3

Let $I = \int x^3 \log x \, dx$

Using integration by parts,

$$I = \log x \int x^3 \, dx - \int \left(\frac{1}{x} \times \int x^3 \, dx\right) dx + c$$
$$= \frac{x^4}{4}\log x - \int \frac{x^4}{4x} dx + c$$
$$= \frac{x^4}{4}\log x - \frac{1}{4}\int x^3 \, dx + c$$
$$= \frac{x^4}{4}\log x - \frac{1}{4}\int \frac{x^4}{4} dx + c$$

$$I = \frac{x^4}{4}\log x - \frac{1}{16}x^4 + c$$

Indefinite Integrals Ex 19.25 Q4

Take first function as x and second function as e^x. The integral of the second function is e^x.
Therefore, $\int xe^x \, dx = xe^x - \int 1 .\, e^x \, dx = xe^x - e^x + C.$

Indefinite Integrals Ex 19.25 Q5

Let $I = \int xe^{2x} dx$

Using integration by parts,

$$I = x\int e^{2x} \, dx - \int \left(1 \times \int e^{2x} \, dx\right) dx + c$$
$$= \frac{xe^{2x}}{2} - \int \left(\frac{e^{2x}}{2}\right) dx + c$$
$$= \frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + c$$

$$I = \left(\frac{x}{2} - \frac{1}{4}\right)e^{2x} + c$$

Indefinite Integrals Ex 19.25 Q6

Let $I = \int x^2 e^{-x} dx$

Using integration by parts,

$$I = x^2 \int e^{-x}\, dx - \int \left(2x \int e^{-x}\, dx\right)$$
$$= -x^2 e^{-x} - \int (2x)\left(-e^{-x}\right)$$
$$= -x^2 e^{-x} + 2\int x\, e^{-x} dx$$
$$= -x^2 e^{-x} + 2\left[x \int e^{-x} dx - \int \left(1 \times \int e^{-x} dx\right) dx\right]$$
$$= -x^2 e^{-x} + 2\left[x\left(-e^{-x}\right) - \int \left(-e^{-x}\right) dx\right]$$
$$= -x^2 e^{-x} - 2xe^{-x} + 2\int e^{-x} dx$$
$$I = -x^2 e^{-x} - 2xe^{-x} - 2e^{-x} + c$$
$$I = -e^{-x}\left(x^2 + 2x + 2\right) + c$$

Indefinite Integrals Ex 19.25 Q7

Let $I = \int x^2 \cos x\, dx$

Using integration by parts,

$$I = x^2 \int \cos x\, dx - \int \left(2x \int \cos x\, dx\right) dx$$
$$= x^2 \sin x - 2\int (x)(\sin x) dx$$
$$= x^2 \sin x - 2\left[x \int \sin x\, dx - \int \left(1 \times \int \sin x\, dx\right) dx\right]$$
$$= x^2 \sin x - 2\left[x(-\cos x) - \int (-\cos x) dx\right]$$
$$= x^2 \sin x + 2x \cos x - 2\int (\cos x) dx$$

$$I = x^2 \sin x + 2x \cos x - 2 \sin x + c$$

Indefinite Integrals Ex 19.25 Q8

Let $I = \int x^2 \cos 2x\, dx$

Using integration by parts,

$$I = x^2 \int \cos 2x\, dx - \int \left(2x \int \cos 2x\, dx\right) dx$$
$$= x^2 \frac{\sin 2x}{2} - 2\int x \left(\frac{\sin 2x}{2}\right) dx$$
$$= \frac{1}{2} x^2 \sin 2x - \int x \sin 2x\, dx$$
$$= \frac{1}{2} x^2 \sin 2x - \left[x \int \sin 2x\, dx - \int \left(1 \int \sin 2x\, dx\right) dx\right]$$
$$= \frac{1}{2} x^2 \sin 2x - \left[x\left(\frac{-\cos 2x}{2}\right) - \int \left(-\frac{\cos 2x}{2}\right) dx\right]$$
$$= \frac{1}{2} x^2 \sin 2x + \frac{x}{2} \cos 2x - \frac{1}{2} \int (\cos 2x) dx$$

$$I = \frac{1}{2} x^2 \sin 2x + \frac{x}{2} \cos 2x - \frac{1}{4} \sin 2x + c$$

Indefinite Integrals Ex 19.25 Q9

Let $I = \int x \sin 2x\, dx$

Using integration by parts,

$$I = x \int \sin 2x\, dx - \int \left(1 \int \sin 2x\, dx\right) dx$$
$$= x\left(-\frac{\cos 2x}{2}\right) - \int \left(-\frac{\cos 2x}{2}\right) dx$$
$$= -\frac{x}{2} \cos 2x + \frac{1}{2} \int \cos 2x\, dx$$
$$= -\frac{x}{2} \cos 2x + \frac{1}{2} \frac{\sin 2x}{2} + c$$
$$I = -\frac{x}{2} \cos 2x + \frac{1}{4} \sin 2x + c$$

Indefinite Integrals Ex 19.25 Q10

Let $I = \int \frac{\log(\log x)}{x} dx$

$= \int \left(\frac{1}{x}\right)\left(\log(\log x)\right) dx$

Using integration by parts,

$I = \log \log x \int \frac{1}{x} dx - \int \left(\frac{1}{x \log x} \int \frac{1}{x} dx\right) dx$

$= \log x \times \log(\log x) - \int \left(\frac{1}{x \log x} \log x\right) dx$

$= \log x \times \log(\log x) - \int \frac{1}{x} dx$

$= \log x \times \log(\log x) - \log x + c$

$I = \log x\left(\log \log x - 1\right) + c$

Indefinite Integrals Ex 19.25 Q11

Let $I = \int x^2 \cos x \, dx$

Using integration by parts,

$I = x^2 \int \cos x \, dx - \int \left(2x \int \cos x \, dx\right) dx$

$= x^2 \sin x - 2\int x \sin x \, dx$

$= x^2 \sin x - 2\left[x \int \sin x \, dx - \int \left(1 \int \sin x \, dx\right) dx\right]$

$= x^2 \sin x - 2\left[x(-\cos x) - \int (-\cos x) dx\right]$

$= x^2 \sin x + 2x \cos x - 2\int (\cos x) dx$

$I = x^2 \sin x + 2x \cos x - 2\sin x + c$

Indefinite Integrals Ex 19.25 Q12

Let $I = \int x \operatorname{cosec}^2 x \, dx$

Using integration by parts,

$I = x \int \operatorname{cosec}^2 x \, dx - \int \left(\int \operatorname{cosec}^2 x \, dx\right) dx$

$= -x \cot x + \int \cot x \, dx$

$= -x \cot x + \log|\sin x| + c$

Indefinite Integrals Ex 19.25 Q13

Let $I = \int x \cos^2 x \, dx$

Using integration by parts,

$I = x \int \cos^2 x \, dx - \int \left(1 \int \cos^2 x \, dx\right) dx$

$= x \int \left(\frac{\cos 2x + 1}{2}\right) dx - \int \left(\int \left(\frac{1 + \cos 2x}{2}\right) dx\right) dx$

$= \frac{x}{2}\left[\frac{\sin 2x}{2} + x\right] - \frac{1}{2}\int \left(x + \frac{\sin 2x}{2}\right) dx$

$= \frac{x}{4} \sin 2x + \frac{x^2}{2} - \frac{1}{2} \times \frac{x^2}{2} - \frac{1}{4}\left(-\frac{\cos 2x}{2}\right) + c$

$I = \frac{x}{4} \sin 2x + \frac{x^2}{4} + \frac{1}{8} \cos 2x + c$

Indefinite Integrals Ex 19.25 Q14

Let $I = \int x^n \log x \, dx$

Using integration by parts,

$$I = \log x \int x^2 dx - \int \left(\frac{1}{x} \int x^2 dx\right) dx$$

$$= \frac{x^{n+1}}{n+1} \log x - \int \left(\frac{1}{x} \times \frac{x^{n+1}}{n+1}\right) dx$$

$$= \frac{x^{n+1}}{n+1} \log x - \int \left(\frac{x^n}{n+1}\right) dx$$

$$I = \frac{x^{n+1}}{n+1} \log x - \frac{1}{(n+1)^2} x^{n+1} + c$$

Indefinite Integrals Ex 19.25 Q15

$$\int \frac{\log x}{x^n} dx = \int (\log x) \left(\frac{1}{x^n}\right) dx$$

by integration by parts

$$\int (\log x) \left(\frac{1}{x^n}\right) dx = \log x \int \left(\frac{1}{x^n}\right) dx - \int \left(\frac{d(\log x)}{dx}\right) \left(\int \left(\frac{1}{x^n}\right) dx\right) dx$$

$$= \log x \left(\frac{x^{1-n}}{1-n}\right) - \int \frac{1}{x} \left(\frac{x^{1-n}}{1-n}\right) dx = \log x \left(\frac{x^{1-n}}{1-n}\right) - \int \left(\frac{x^{-n}}{1-n}\right) dx$$

$$= \log x \left(\frac{x^{1-n}}{1-n}\right) - \left(\frac{1}{1-n}\right) \left(\frac{x^{1-n}}{1-n}\right) = \log x \left(\frac{x^{1-n}}{1-n}\right) - \left(\frac{x^{1-n}}{[1-n]^2}\right) + C$$

Indefinite Integrals Ex 19.25 Q16

Let $I = \int x^2 \sin^2 x \, dx$

$$= \int x^2 \left(\frac{1-\cos 2x}{2}\right) dx$$

$$= \int \frac{x^2}{2} dx - \int \left(\frac{x^2 \cos 2x}{2}\right) dx$$

$$= \frac{x^3}{6} - \frac{1}{2}\left[\int x^2 \cos 2x \, dx\right]$$

$$= \frac{x^3}{6} - \frac{1}{2}\left[x^2 \int \cos 2x \, dx - \int \left(2x \int \cos 2x \, dx\right) dx\right]$$

$$= \frac{x^3}{6} - \frac{1}{2}\left(x^2 \frac{\sin 2x}{2}\right) + \frac{1}{2} \times 2 \int \left(x \frac{\sin 2x}{2}\right) dx$$

$$= \frac{x^3}{6} - \frac{1}{4} x^2 \sin 2x + \frac{1}{2}\left[x \int \sin 2x \, dx - \int \left(1 \int \sin 2x \, dx\right) dx\right]$$

$$= \frac{x^3}{6} - \frac{1}{4} x^2 \sin 2x + \frac{1}{2}\left[x\left(-\frac{\cos 2x}{2}\right) - \int \left(-\frac{\cos 2x}{2}\right) dx\right]$$

$$= \frac{x^3}{6} - \frac{1}{4} x^2 \sin 2x - \frac{1}{4} x \cos 2x + \frac{1}{4} \frac{\sin 2x}{2} + c$$

$$I = \frac{x^3}{6} - \frac{1}{4} x^2 \sin 2x - \frac{1}{4} x \cos 2x + \frac{1}{8} \sin 2x + c$$

Indefinite Integrals Ex 19.25 Q17

Let $I = \int 2x^3 e^{x^2} x \, dx$

Let $x^2 = t$

$2x \, dx = dt$

$I = \int t \times e^t dt$

Using integration by parts,

$$= t \int e^t dt - \int \left(1 \times \int e^t dt\right) dt$$

$$= te^t - \int e^t dt$$

$$= te^t - e^t + c$$

$$= e^t (t-1) + c$$

$$I = e^{x^2}(x^2 - 1) + c$$

Indefinite Integrals Ex 19.25 Q18

Let $I = \int x^3 \cos x^2 \, dx$

Let $x^2 = t$

$2x\,dx = dt$

$I = \frac{1}{2}\int t \cos t dt$

Using integration by parts,

$= \frac{1}{2}\left[t\int \cos t\,dt - \int\left(1 \times \int \cos t dt\right)dt\right]$

$= \frac{1}{2}\left[t \times \sin t - \int \sin t\,dt\right]$

$= \frac{1}{2}\left[t \sin t + \cos t\right] + c$

$I = \frac{1}{2}\left[x^2 \sin x^2 + \cos x^2\right] + c$

Indefinite Integrals Ex 19.25 Q19

Let $I = \int x \sin x \cos x \, dx$

$= \int \frac{x}{2}\left(2 \sin x \cos x\right)dx$

$= \frac{1}{2}\int x \sin 2x \, dx$

Using integration by parts,

$= \frac{1}{2}\left[x\int \sin 2x\,dx - \int\left(1 \times \int \sin 2x\,dx\right)dx\right]$

$= \frac{1}{2}\left[x\left(\frac{-\cos 2x}{2}\right) - \int\left(\frac{-\cos 2x}{2}\right)dx\right]$

$= -\frac{1}{4}x \cos 2x + \frac{1}{4}\int \cos 2x\,dx$

$I = -\frac{1}{4}x \cos 2x + \frac{1}{8}\sin 2x + c$

Indefinite Integrals Ex 19.25 Q20

Let $I = \int \sin x \left(\log \cos x\right)dx$

Let $\cos x = t$

$-\sin x\,dx = dt$

$I = -\int \log t\,dt$

$= -\int 1 \times \log t\,dt$

Using integration by parts,

$= -\left[\log t\int dt - \int\left(\frac{1}{t} \times \int dt\right)dt\right]$

$= -\left[t \log t - \int \frac{1}{t} \times t\,dt\right]$

$= -\left[t \log t - \int dt\right]$

$= -\left[t \log t - t + c_1\right]$

$= t\left(1 - t \log t\right) + c$

$I = \cos x\left(1 - \log \cos x\right) + c$

Indefinite Integrals Ex 19.25 Q21

Let $I = \int (\log x)^2 x\,dx$

Using integration by parts,

$$= (\log x)^2 \int x\,dx - \int\left(2(\log x)\left(\frac{1}{x}\right)\int x\,dx\right)dx$$

$$= \frac{x^2}{2}(\log x)^2 - 2\int(\log x)\left(\frac{1}{x}\right)\left(\frac{x^2}{2}\right)dx$$

$$= \frac{x^2}{2}(\log x)^2 - \int x(\log x)\,dx$$

$$= \frac{x^2}{2}(\log x)^2 - \left[\log x\int x\,dx - \int\left(\frac{1}{x}\int x\,dx\right)dx\right]$$

$$= \frac{x^2}{2}(\log x)^2 - \left[\frac{x^2}{2}\log x - \int\left(\frac{1}{x}\times\frac{x^2}{2}\right)dx\right]$$

$$= \frac{x^2}{2}(\log x)^2 - \frac{x^2}{2}\log x + \frac{1}{2}\int x\,dx$$

$$= \frac{x^2}{2}(\log x)^2 - \frac{x^2}{2}\log x + \frac{1}{4}x^2 + c$$

$$I = \frac{x^2}{2}\left[(\log x)^2 - \log x + \frac{1}{2}\right] + c$$

Indefinite Integrals Ex 19.25 Q22

Let $I = \int e^{\sqrt{x}}\,dx$

Let $\sqrt{x} = t$

$$x = t^2$$

$$dx = 2t\,dt$$

$$I = 2\int e^t t\,dt$$

$$I = 2\left[t\int e^t dt - \int\left(1\int e^t dt\right)dt\right]$$

$$I = 2\left[te^t - \int e^t dt\right]$$

$$= 2\left[te^t - e^t\right] + c$$

$$= 2e^t(t-1) + c$$

$$I = 2e^{\sqrt{x}}\left(\sqrt{x} - 1\right) + c$$

Indefinite Integrals Ex 19.25 Q23

Let $I = \int \frac{\log(x+2)}{(x+2)^2}dx$

Let $\frac{1}{x+2} = t$

$$-\frac{1}{(x+2)^2}dx = dt$$

$$I = -\int \log\left(\frac{1}{t}\right)dt$$

$$= -\int \log t^{-1}\,dt$$

$$= -\int 1\times\log t\,dt$$

Using integration by parts,

$$I = \log t\int dt - \int\left(\frac{1}{t}\int dt\right)dt$$

$$= t\log t - \int\left(\frac{1}{t}\times t\right)dt$$

$$= t\log t - \int dt$$

$$= t\log t - t + c$$

$$= \frac{1}{x+2}\left(\log(x+2)^{-1} - 1\right) + c$$

$$I = \frac{-1}{x+2} - \frac{\log(x+2)}{x+2} + c$$

Indefinite Integrals Ex 19.25 Q24

Let $I = \int \frac{x + \sin x}{1 + \cos x} dx$

$$= \int \frac{x}{2\cos^2 \frac{x}{2}} dx + \int \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}} dx$$

$$= \frac{1}{2}\int x \sec^2\frac{x}{2} dx + \int \tan\frac{x}{2} dx$$

Using integration by parts,

$$= \frac{1}{2}\left[x\int \sec^2\frac{x}{2} dx - \int\left(1\int \sec^2\frac{x}{2} dx\right)dx\right] + \int \tan\frac{x}{2} dx$$

$$= \frac{1}{2}\left[2x\tan\frac{x}{2} - 2\int \tan\frac{x}{2} dx\right] + \int \tan\frac{x}{2} dx + c$$

$$= x\tan\frac{x}{2} - \int \tan\frac{x}{2} dx + \int \tan\frac{x}{2} dx + c$$

$$I = x\tan\frac{x}{2} + c$$

Indefinite Integrals Ex 19.25 Q25

Let $I = \int \log_{10} x \, dx$

$$= \int \frac{\log x}{\log 10} dx$$

$$= \frac{1}{\log 10}\int 1 \times \log x \, dx$$

Using integration by parts,

$$= \frac{1}{\log 10}\left[\log x \int dx - \int\left(\frac{1}{x}\int dx\right)dx\right]$$

$$= \frac{1}{\log 10}\left[x\log x - \int\left(\frac{x}{x}\right)dx\right]$$

$$= \frac{1}{\log 10}\left[x\log x - x\right]$$

$$I = \frac{x}{\log 10}\left(\log x - 1\right)$$

Indefinite Integrals Ex 19.25 Q26

Let $I = \int \cos\sqrt{x} \, dx$

$\sqrt{x} = t$

$x = t^2$

$dx = 2t\,dt$

$= \int 2t\cos t \, dt$

$I = 2\int t\cos t \, dt$

$I = 2\left[t\int \cos t\,dt - \int\left(1\int \cos t\,dt\right)dt\right]$

$= 2\left[t\sin t - \int \sin t\,dt\right]$

$= 2\left[t\sin t + \cos t\right] + c$

$$I = 2\left[\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}\right] + c$$

Indefinite Integrals Ex 19.25 Q27

$$\text{Let } I=\int \frac{x\cos^{-1}x}{\sqrt{1-x^2}}dx$$

Let us substitute, $t=\cos^{-1}x$

$$\Rightarrow dt = \frac{-1}{\sqrt{1-x^2}}dx$$

Also, $\cos t = x$

Thus,

$$I=-\int t\cos t\,dt$$

Now let us solve this by the 'by parts' method.

Let $u=t$; $du=dt$

$$\int \cos t\,dt = \int dv$$

$$\Rightarrow \sin t = v$$

$$\text{Thus, } I=-\left[t\sin t-\int \sin t\,dt\right]$$

$$\Rightarrow I=-\left[t\sin t+\cos t\right]+C$$

Substituting the value $t=\cos^{-1}x$, we have,

$$I=-\left[\cos^{-1}x\sin t+x\right]+C$$

$$\Rightarrow I=-\left[\cos^{-1}x\sqrt{1-x^2}+x\right]+C$$

Indefinite Integrals Ex 19.25 Q29

$$\text{Let}\quad I=\int \operatorname{cosec}^3 dx$$

$$=\int \operatorname{cosec}x-\operatorname{cosec}^2 x\,dx$$

Using integration by parts,

$$=\operatorname{cosec}x\times\int \operatorname{cosec}^2 x\,dx+\int\left(\operatorname{cosec}x\cot x\int \operatorname{cosec}^2 x\,dx\right)dx$$

$$=\operatorname{cosec}x\times(-\cot x)+\int \operatorname{cosec}x\cot x\,(-\cot x)dx$$

$$=-\operatorname{cosec}x\cot x-\int \operatorname{cosec}x\cot^2 x\,dx$$

$$=-\operatorname{cosec}x\cot x-\int \operatorname{cosec}x\left(\operatorname{cosec}^2 x-1\right)dx$$

$$=-\operatorname{cosec}x\cot x-\int \operatorname{cosec}^3 x\,dx+\int \operatorname{cosec}x\,dx$$

$$I=-\operatorname{cosec}x\cot x-I+\log\left|\tan\frac{x}{2}\right|+c_1$$

$$2I=-\operatorname{cosec}x\cot x+\log\left|\tan\frac{x}{2}\right|+c_1$$

$$I=-\frac{1}{2}\operatorname{cosec}x\cot x+\frac{1}{2}\log\left|\tan\frac{x}{2}\right|+c$$

Indefinite Integrals Ex 19.25 Q30

$$\text{Let}\quad I=\int \sec^{-1}\sqrt{x}\,dx$$

$$\text{Let}\quad \sqrt{x}=t$$

$$x=t^2$$

$$dx=2t\,dt$$

$$I=\int 2t\sec^{-1}t\,dt$$

$$=2\left[\sec^{-1}t\int t\,dt-\int\left(\frac{1}{t\sqrt{t^2-1}}\int t\,dt\right)dt\right]$$

$$=2\left[\frac{t^2}{2}\sec^{-1}-\int\left(\frac{t}{2t\sqrt{t^2-1}}\right)dt\right]$$

$$=t^2\sec^{-1}t-\int\frac{t}{\sqrt{t^2-1}}dt$$

$$=t^2\sec^{-1}t-\frac{1}{2}\int\frac{2t}{\sqrt{t^2-1}}dt$$

$$=t^2\sec^{-1}t-\frac{1}{2}\times 2\sqrt{t^2-1}+c$$

$$I=x\sec^{-1}\sqrt{x}-\sqrt{x-1}+c$$

Indefinite Integrals Ex 19.25 Q31

$$\int \sin^{-1}\sqrt{x}\ dx =$$

let $x = t^2 \rightarrow dx = 2tdt$

$$\int \sin^{-1}\sqrt{x}\ dx = \int \sin^{-1}\sqrt{t^2}\ 2tdt = \int \sin^{-1}t\ 2tdt$$

$$= \sin^{-1}t\int 2tdt - \left(\int \frac{d\sin^{-1}t}{dt}\left(\int 2tdt\right)dt\right)$$

$$= \sin^{-1}t\,(t^2) - \int \frac{1}{\sqrt{1-t^2}}(t^2)\,dt$$

Lets solve $\int \frac{1}{\sqrt{1-t^2}}(t^2)\,dt$

$$\int \frac{1}{\sqrt{1-t^2}}(t^2)\,dt = \int \frac{t^2-1+1}{\sqrt{1-t^2}}dt = \int \frac{t^2-1}{\sqrt{1-t^2}}dt + \int \frac{1}{\sqrt{1-t^2}}dt$$

we know that, value of $\int \frac{1}{\sqrt{1-t^2}}dt = \sin^{-1}t$

Remaining integral to evaluate is $\int \frac{t^2-1}{\sqrt{1-t^2}}dt = \int -\sqrt{1-t^2}\,dt$

sub t=sin u, dt= cos$u\ du$

$$\int -\sqrt{1-t^2}\,dt = \int -\cos^2 u\,du = -\int\left[\frac{1+\cos 2u}{2}\right]du$$

$$= -\frac{u}{2} - \frac{\sin 2u}{4}$$

Substitute back u=$\sin^{-1}t$ and $t = \sqrt{x}$

$$= -\frac{\sin^{-1}\sqrt{x}}{2} - \frac{\sin(2\sin^{-1}\sqrt{x})}{4}$$

$$\int \sin^{-1}\sqrt{x}\ dx = x\sin^{-1}\sqrt{x} - \frac{\sin^{-1}\sqrt{x}}{2} - \frac{\sin(2\sin^{-1}\sqrt{x})}{4}$$

$$\sin(2\sin^{-1}\sqrt{x}) = 2\sqrt{x}\sqrt{1-x}$$

$$\int \sin^{-1}\sqrt{x}\ dx = x\sin^{-1}\sqrt{x} - \frac{\sin^{-1}\sqrt{x}}{2} - \frac{\sqrt{x(1-x)}}{2}$$

Indefinite Integrals Ex 19.25 Q32

Let $I = \int x\tan^2 x\,dx$

$$= \int x\left(\sec^2 x - 1\right)dx$$

$$= \int x\sec^2 x\,dx - \int x\,dx$$

$$= \left[x\int \sec^2 x\,dx - \int\left(1\int \sec^2 x\,dx\right)dx\right] - \frac{x^2}{2}$$

$$= x\tan x - \int \tan x\,dx - \frac{x^2}{2}$$

$$I = x\tan x - \log\sec x - \frac{x^2}{2} + c$$

Indefinite Integrals Ex 19.25 Q33

Let $I = \int x\left(\frac{\sec 2x - 1}{\sec 2x + 1}\right)dx$

$$= \int x\left(\frac{1-\cos 2x}{1+\cos 2x}\right)dx$$

$$= \int x\left(\frac{\sec^2 x}{\cos^2 x}\right)dx$$

$$= \int x\tan^2 x\,dx$$

$$= \int x\left(\sec^2 x - 1\right)dx$$

$$= \int x\sec^2 x\,dx - \int dx$$

$$= \left[x\int \sec^2 x\,dx - \int\left(1\int \sec^2 x\,dx\right)dx\right] - \frac{x^2}{2}$$

$$= x\tan x - \int \tan x\,dx - \frac{x^2}{2}$$

$$I = x\tan x - \log\sec x - \frac{x^2}{2} + c$$

Indefinite Integrals Ex 19.25 Q34

Let $I = \int (x+1) e^x \log\left(xe^x\right) dx$

Let $xe^x = t$

$\left(1 \times e^x + xe^x\right) dx = dt$

$(x+1) e^x dx = dt$

$I = \int \log t \, dt$

$= \int 1 \times \log t \, dt$

$= \log t \int dt - \int \left(\frac{1}{t} \int dt\right) dt$

$= t \log t - \int \left(\frac{1}{t} t\right) dt$

$= t \log t - \int dt$

$= t \log t - t + c$

$= t \left(\log t - 1\right) + c$

$I = xe^x \left(\log xe^x - 1\right) + c$

Indefinite Integrals Ex 19.25 Q35

Let $I = \int \sin^{-1}\left(3x - 4x^3\right) dx$

Let $x = \sin\theta$

$dx = \cos\theta \, d\theta$

$= \int \sin^{-1}\left(3\sin\theta - 4\sin^3\theta\right) \cos\theta \, d\theta$

$= \int \sin^{-1}\left(\sin 3\theta\right) \cos\theta \, d\theta$

$= \int 3\theta \cos\theta \, d\theta$

$= 3\left[\theta \int \cos\theta \, d\theta - \int \left(1 \int \cos\theta \, d\theta\right) d\theta\right]$

$= 3\left[\theta \sin\theta - \int \sin\theta \, d\theta\right]$

$= 3\left[\theta \sin\theta + \cos\theta\right] + c$

$I = 3\left[x \sin^{-1} x + \sqrt{1 - x^2}\right] + c$

Indefinite Integrals Ex 19.25 Q36

Let $x = \tan\theta \Rightarrow dx = \sec^2\theta \, d\theta$

$$\therefore \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}\left(\sin 2\theta\right) = 2\theta$$

$$\Rightarrow \int \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx = \int 2\theta \cdot \sec^2\theta \, d\theta = 2\int \theta \cdot \sec^2\theta \, d\theta$$

Integrating by parts, we obtain

$$2\left[\theta \cdot \int \sec^2\theta \, d\theta - \int \left\{\left(\frac{d}{d\theta}\theta\right) \int \sec^2\theta \, d\theta\right\} d\theta\right]$$

$= 2\left[\theta \cdot \tan\theta - \int \tan\theta \, d\theta\right]$

$= 2\left[\theta \tan\theta + \log\left|\cos\theta\right|\right] + C$

$= 2\left[x \tan^{-1} x + \log\left|\frac{1}{\sqrt{1+x^2}}\right|\right] + C$

$= 2x \tan^{-1} x + 2\log\left(1+x^2\right)^{-\frac{1}{2}} + C$

$= 2x \tan^{-1} x + 2\left[-\frac{1}{2}\log\left(1+x^2\right)\right] + C$

$= 2x \tan^{-1} x - \log\left(1+x^2\right) + C$

Indefinite Integrals Ex 19.25 Q37

Let $I = \int \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)dx$

Let $x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$I = \int \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2 x}\right)\sec^2\theta\, d\theta$

$= \int \tan^{-1}(\tan 3\theta)\sec^2\theta\, d\theta$

$= \int 3\theta \sec^2\theta\, d\theta$

$= 3\left[\theta\int \sec^2\theta\, d\theta - \int\left(1\int \sec^2\theta\, d\theta\right)d\theta\right]$

$= 3\left[\theta\tan\theta - \int \tan\theta\, d\theta\right]$

$= 3\left[\theta\tan\theta + \log\sec\theta\right] + c$

$= 3\left[x\tan^{-1}x - \log\sqrt{1 + x^2}\right] + c$

$I = 3x\tan^{-1}x - \frac{3}{2}\log\left|1 + x^2\right| + c$

Indefinite Integrals Ex 19.25 Q38

Let $I = \int x^2 \sin^{-1}x\, dx$

$I = \sin^{-1}x\int x^2\, dx - \int\left(\frac{1}{\sqrt{1 - x^2}}\int x^2\, dx\right)dx$

$= \frac{x^3}{3}\sin^{-1}x - \int \frac{x^3}{3\sqrt{1 - x^2}}dx$

$I = \frac{x^3}{3}\sin^{-1}x - \frac{1}{3}I_1 + c_1 \text{ ------ } (1)$

$I_1 = \int \frac{x^3}{\sqrt{1 - x^2}}dx$

Let $1 - x^2 = t^2$

$-2x\, dx = 2t\, dt$

$-x\, dx = t\, dt$

$I_1 = -\int \frac{(1 - t^2)\, t\, dt}{t}$

$= \int (t^2 - 1)\, dt$

$= \frac{t^3}{3} - t + c_2$

$= \frac{(1 - x^2)^{\frac{3}{2}}}{3} - (1 - x^2)^{\frac{1}{2}} + c_2$

Now,

$I = \frac{x^3}{3}\sin^{-1}x - \frac{1}{9}(1 - x^2)^{\frac{3}{2}} + \frac{1}{3}(1 - x^2)^{\frac{1}{2}} + c$

Indefinite Integrals Ex 19.25 Q39

Let $I = \int \frac{\sin^{-1} x}{x^2} dx$

$= \int \left(\frac{1}{x^2}\right)\left(\sin^{-1} x\right) dx$

$I = \left[\sin^{-1} x \int \frac{1}{x^2} dx - \int \left(\frac{1}{\sqrt{1-x^2}} \int \frac{1}{x^2} dx\right) dx\right]$

$= \sin^{-1} x \left(-\frac{1}{x}\right) - \int \frac{1}{\sqrt{1-x^2}} \left(-\frac{1}{x}\right) dx$

$I = -\frac{1}{x} \sin^{-1} x + \int \frac{1}{x\sqrt{1-x^2}} dx$

$I = -\frac{1}{x} \sin^{-1} x + I_1 - - - - - - (1)$

Where,

$I_1 = \int \frac{1}{x\sqrt{1-x^2}} dx$

Let $1 - x^2 = t^2$

$-2x\, dx = 2t\, dt$

$I_1 = \int \frac{x}{x^2\sqrt{1-x^2}} dx$

$= -\int \frac{t\,dt}{\left(1-t^2\right)\sqrt{t}}$

$= -\int \frac{dt}{\left(1-t^2\right)}$

$= \int \frac{1}{t^2-1} dt$

$= \frac{1}{2} \log\left|\frac{t-1}{t+1}\right|$

$= \frac{1}{2} \log\left|\frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}+1}\right| + c_1$

Now,

$I = -\frac{\sin^{-1} x}{x} + \frac{1}{2}\log\left|\left(\frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}+1}\right)\left(\frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}-1}\right)\right| + c$

$= -\frac{\sin^{-1} x}{x} + \frac{1}{2}\log\left|\frac{\left(\sqrt{1-x^2}-1\right)^2}{1-x^2-1}\right| + c$

$= -\frac{\sin^{-1} x}{x} + \frac{1}{2}\log\left|\frac{\left(\sqrt{1-x^2}-1\right)^2}{-x^2}\right| + c$

$= -\frac{\sin^{-1} x}{x} + \log\left|\frac{\sqrt{1-x^2}-1}{-x}\right| + c$

$I = -\frac{\sin^{-1} x}{x} + \log\left|\frac{1-\sqrt{1-x^2}}{x}\right| + c$

Indefinite Integrals Ex 19.25 Q40

Let $I = \int \frac{x^2 \tan^{-1} x}{1+x^2} dx$

Let $\tan^{-1} x = t$

$x = \tan t$

$\frac{1}{1+x^2} dx = dt$

$I = \int t \tan^2 t \, dt$

$= \int t \left(\sec^2 t - 1\right) dt$

$= \int \left(t \sec^2 t - t\right) dt$

$= \int t \sec^2 t \, dt - \int t \, dt$

$= \left[t \int \sec^2 t \, dt - \int \left(1 \int \sec^2 t \, dt\right) dt\right] - \frac{t^2}{2}$

$= \left[t \tan t - \int \tan t \, dt\right] - \frac{t^2}{2}$

$= t \tan t - \log \sec t - \frac{t^2}{2} + c$

$= x \tan^{-1} x - \log \sqrt{1+x^2} - \frac{\tan^2 x}{2} + c$

$I = x \tan^{-1} x - \frac{1}{2} \log \left|1+x^2\right| - \frac{\tan^2 x}{2} + c$

Indefinite Integrals Ex 19.25 Q41

Let $I = \int \cos^{-1} \left(4x^3 - 3x\right) dx$

Let $x = \cos\theta$

$dx = -\sin\theta \, d\theta$

$I = -\int \cos^{-1} \left(4\cos^3\theta - 3\cos\theta\right) \sin\theta \, d\theta$

$= -\int \cos^{-1} \left(\cos 3\theta\right) \sin\theta \, d\theta$

$= -\int 3\theta \sin\theta \, d\theta$

$= -3\left[\theta \int \sin\theta \, d\theta - \int \left(1 \int \sin\theta \, d\theta\right) d\theta\right]$

$= -3\left[-\theta\cos\theta + \int \cos\theta \, d\theta\right]$

$= 3\theta\cos\theta - 3\sin\theta + c$

$I = 3x \cos^{-1} x - 3\sqrt{1-x^2} + c$

Indefinite Integrals Ex 19.25 Q42

Let $I = \int \cos^{-1} \left(\frac{1-x^2}{1+x^2}\right) dx$

Let $x = \tan t$

$dx = \sec^2 t \, dt$

$I = \int \cos^{-1} \left(\frac{1-\tan^2 t}{1+\tan^2 t}\right) \sec^2 t \, dt$

$= \int \cos^{-1} \left(\cos 2t\right) \sec^2 t \, dt$

$= \int 2t \sec^2 t \, dt$

$= 2\left[t \int sce^2 t \, dt - \int \left(1 \int \sec^2 t \, dt\right) dt\right]$

$= 2\left[t \tan^2 t - \int \tan t \, dt\right]$

$= 2\left[t \tan^2 t - \log \sec t\right] + c$

$= 2\left[x \tan^{-1} x - \log \sqrt{1+x^2}\right] + c$

$I = 2x \tan^{-1} x - \log \left|1+x^2\right| + c$

Indefinite Integrals Ex 19.25 Q43

Let $I = \int \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$

Let $x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$I = \int \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)\sec^2\theta\, d\theta$

$= \int \tan^{-1}\left(\tan 2\theta\right)\sec^2\theta\, d\theta$

$= \int 2\theta \sec^2\theta\, d\theta$

$= 2\left[\theta\int \sec^2\theta\, d\theta - \int\left(1\int \sec^2\theta\, d\theta\right)d\theta\right]$

$= 2\left[\theta\tan\theta - \int \tan\theta\, d\theta\right]$

$= 2\left[\theta\tan\theta - \log\sec\theta\right] + c$

$= 2\left[x\tan^{-1}x - \log\sqrt{1+x^2}\right] + c$

$I = 2x\tan^{-1}x - \log\left|1+x^2\right| + c$

Indefinite Integrals Ex 19.25 Q44

Let $I = \int (x+1)\log x\, dx$

$= \log x\int (x+1)dx - \int\left(\frac{1}{x}\int (x+1)\, dx\right)dx$

$= \left(\frac{x^2}{2}+x\right)\log x - \int \frac{1}{x}\left(\frac{x^2}{2}+x\right)dx$

$= \left(\frac{x^2}{2}+x\right)\log x - \frac{1}{2}\int x\, dx - \int dx$

$= \left(x+\frac{x^2}{2}\right)\log x - \frac{1}{2}\times\frac{x^2}{2} - x + c$

$I = \left(x+\frac{x^2}{2}\right)\log x - \left(\frac{x^2}{4}+x\right) + c$

Indefinite Integrals Ex 19.25 Q45

Let $I = \int x^2\tan^{-1}x\, dx$

$= \tan^{-1}x\int x^2 dx - \int\left(\frac{1}{1+x^2}\int x^2 dx\right)$

$= \tan^{-1}x\left(\frac{x^3}{3}\right) - \frac{1}{3}\int\frac{x^3}{1+x^2}dx$

$= \frac{1}{3}x^3\tan^{-1}x - \frac{1}{3}\int\left(x - \frac{x}{1+x^2}\right)dx$

$= \frac{1}{3}x^3\tan^{-1}x - \frac{1}{3}\times\frac{x^2}{2} + \frac{1}{3}\int\frac{x}{1+x^2}dx$

$I = \frac{1}{3}x^3\tan^{-1}x - \frac{1}{6}x^2 + \frac{1}{6}\log\left|1+x^2\right| + c$

Indefinite Integrals Ex 19.25 Q46

Let $I = \int \left(e^{\log x} + \sin x\right) \cos x \, dx$

$= \int (x + \sin x) \cos x \, dx$

$= \int x \cos x \, dx + \int \sin x \cos x \, dx$

$= \left[x \int \cos x \, dx - \int \left(1 \int \cos x \, dx\right) dx\right] + \frac{1}{2} \int \sin 2x \, dx$

$= \left[x \sin x - \int \sin x \, dx\right] + \frac{1}{2}\left(-\frac{\cos 2x}{2}\right) + c$

$I = x \sin x + \cos x - \frac{1}{4} \cos 2x + c$

$I = x \sin x + \cos x - \frac{1}{4}\left[1 - 2\sin^2 x\right] + c$

$I = x \sin x + \cos x - \frac{1}{4} + \frac{1}{2} \sin^2 x + c$

$I = x \sin x + \cos x + \frac{1}{2} \sin^2 x + c - \frac{1}{4}$

$I = x \sin x + \cos x + \frac{1}{2} \sin^2 x + k$, where $k = c - \frac{1}{4}$

Indefinite Integrals Ex 19.25 Q47

Let $I = \int \frac{\left(x \tan^{-1} x\right)}{\left(1 + x^2\right)^{\frac{3}{2}}} dx$

Let $\tan^{-1} x = t$

$\frac{1}{1 + x^2} dx = dt$

$I = \int \frac{t \tan t}{\sqrt{1 + \tan^2 t}} dt$

$= \int \frac{t \tan t}{\sec t} dt$

$= \int t \frac{\sin t}{\cos t} \cos t \, dt$

$= \int t \sin t \, dt$

$= \left[t \int \sin t dt - \int \left(1 \int \sin t \, dt\right) dt\right]$

$= \left[-t \cos t + \int \cos t \, dt\right]$

$= \left[-t \cos t + \sin t\right] + c$

$I = -\frac{\tan^{-1} x}{\sqrt{1 + x^2}} + \frac{x}{\sqrt{1 + x^2}} + c$

Indefinite Integrals Ex 19.25 Q48

Let $I = \int \tan^{-1}\left(\sqrt{x}\right) dx$

Let $x = t^2$

$dx = 2t \, dt$

$I = \int 2t \tan^{-1} t \, dt$

$= 2\left[\tan^{-1} t \int t \, dt - \int \left(\frac{1}{1 + t^2} \int t \, dt\right) dt\right]$

$= 2\left[\frac{t^2}{2} \tan^{-1} t - \int \frac{t^2}{2\left(1 + t^2\right)} dt\right]$

$= t^2 \tan^{-1} t - \int \frac{t^2 + 1 - 1}{1 + t^2} dt$

$= t^2 \tan^{-1} t - \int \left(1 - \frac{1}{1 + t^2}\right) dt$

$= t^2 \tan^{-1} t - t + \tan^{-1} t + c$

$= \left(t^2 + 1\right) \tan^{-1} t - t + c$

$I = (x + 1) \tan^{-1} \sqrt{x} - \sqrt{x} + c$

Indefinite Integrals Ex 19.25 Q49

$$\int x^3 \tan^{-1} x\, dx =$$

$$\int x^3 \tan^{-1} x\, dx = \tan^{-1} x \int x^3 dx - \left(\int \frac{d \tan^{-1} x}{dx} \left(\int x^3 dx \right) dx \right)$$

$$= \tan^{-1} x \frac{x^4}{4} - \left(\int \frac{1}{1+x^2} \left(\frac{x^4}{4} \right) dx \right)$$

$$= \tan^{-1} x \frac{x^4}{4} - \left(\int \frac{1}{1+x^2} \left(\frac{x^4}{4} \right) dx \right)$$

$$\int \frac{1}{1+x^2} \left(\frac{x^4}{4} \right) dx = \frac{1}{4} \left[\int \frac{1}{1+x^2} dx + (x^2 - 1) dx \right]$$

$$\int \frac{1}{1+x^2} \left(\frac{x^4}{4} \right) dx = \frac{1}{4} \left[\tan^{-1} x + \frac{x^3}{3} - x \right]$$

$$\int x^3 \tan^{-1} x\, dx = \frac{x^4}{4} \tan^{-1} x - \frac{1}{4} \left[\tan^{-1} x + \frac{x^3}{3} - x \right] + C$$

Indefinite Integrals Ex 19.25 Q50

Let $I = \int x \sin x \cos 2x\, dx$

$$= \frac{1}{2} \int x \left(2 \sin x \cos 2x \right) dx$$

$$= \frac{1}{2} \int x \left(\sin(x + 2x) - \sin(2x - x) \right) dx$$

$$= \frac{1}{2} \int x \left(\sin 3x - \sin x \right) dx$$

$$= \frac{1}{2} \left[x \int (\sin 3x - \sin x) dx - \int \left(1 \int (\sin 3x - \sin x) dx \right) dx \right]$$

$$= \frac{1}{2} \left[x \left(\frac{-\cos 3x}{3} + \cos x \right) - \int \left(-\frac{\cos 3x}{3} + \cos x \right) dx \right]$$

$$I = \frac{1}{2} \left[-x \frac{\cos 3x}{3} + x \cos x + \frac{1}{9} \sin 3x - \sin x \right] + c$$

Indefinite Integrals Ex 19.25 Q51

Let $I = \int \left(\tan^{-1} x^2 \right) x dx$

Let $x^2 = t$

$2x\, dx = dt$

$$I = \frac{1}{2} \int \tan^{-1} t\, dt$$

$$= \frac{1}{2} \int 1 \tan^{-1} t\, dt$$

$$= \frac{1}{2} \left[\tan^{-1} t \int dt - \left(\int \frac{1}{1+t^2} \int dt \right) dt \right]$$

$$= \frac{1}{2} \left[t \tan^{-1} t - \int \frac{t}{1+t^2} dt \right]$$

$$= \frac{1}{2} t \tan^{-1} t - \frac{1}{4} \int \frac{2t}{1+t^2} dt$$

$$= \frac{1}{2} t \tan^{-1} t - \frac{1}{4} \log\left| 1 + t^2 \right| + c$$

$$I = \frac{1}{2} x^2 \tan^{-1} x^2 - \frac{1}{4} \log\left| 1 + x^4 \right| + c$$

Indefinite Integrals Ex 19.25 Q52

Let first function be $\sin^{-1} x$ and second dunction be $\frac{x}{\sqrt{1-x^2}}$.

First we find the intergral of the second function, i.e., $\int \frac{x dx}{\sqrt{1-x^2}}$.

Put $t = 1 - x^2$. Then $dt = -2x\ dx$

Therefore, $\int \frac{x dx}{\sqrt{1-x^2}} = -\frac{1}{2} \int \frac{dt}{\sqrt{t}} = -\sqrt{t} = -\sqrt{1-x^2}$

Hence, $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx = \left(\sin^{-1} x\right)\left(-\sqrt{1-x^2}\right) - \int \frac{1}{\sqrt{1-x^2}} \left(-\sqrt{1-x^2}\right) dx$

$= -\sqrt{1-x^2} \sin^{-1} x + x + C = x - \sqrt{1-x^2} \sin^{-1} x + C$

Indefinite Integrals Ex 19.25 Q53

Let $I = \int \sin^3 \sqrt{x}\, dx$

$\sqrt{x} = t$

$x = t^2$

$dx = 2t\, dt$

$I = 2\int t \sin^3 t\, dt$

$= 2\int t \left(\frac{3\sin t - \sin 3t}{4}\right) dt$

$= \frac{1}{2} \int t \left(3\sin t - \sin 3t\right) dt$

Using integration by parts,

$I = \frac{1}{2}\left[t\left(-3\cos t + \frac{1}{3}\cos 3t\right) - \int\left(-3\cos t + \frac{\cos 3t}{3}\right) dt\right]$

$= \frac{1}{2}\left[\frac{-9t\cos t + t\cos 3t}{3} - \left\{-3\sin t + \frac{\sin 3t}{9}\right\}\right] + c$

$= \frac{1}{2}\left[\frac{-9t\cos t + t\cos 3t}{3} + \frac{27\sin t - 3\sin 3t}{9}\right] + c$

$= \frac{1}{18}\left[-27t\cos t + 3t\cos 3t + 27\sin t - 3\sin 3t\right] + c$

$I = \frac{1}{18}\left[3\sqrt{x}\cos 3\sqrt{x} + 27\sin\sqrt{x} - 27\sqrt{x}\cos\sqrt{x} - 3\sin 3\sqrt{x}\right] + c$

Indefinite Integrals Ex 19.25 Q54

Let $I = \int x \sin^3 x\, dx$

$= \int x \left(\frac{3\sin x - \sin 3x}{4}\right) dx$

$= \frac{1}{4} \int x \left(3\sin x - \sin 3x\right) dx$

Using integration by parts,

$I = \frac{1}{4}\left[x\int\left(3\sin x - \sin 3x\right) dx - \int\left(1\int\left(3\sin x - \sin 3x\right) dx\right) dx\right]$

$= \frac{1}{4}\left[x\left(-3\cos x + \frac{\cos 3x}{3}\right) - \int\left(-3\cos x + \frac{\cos 3x}{3}\right) dx\right]$

$= \frac{1}{4}\left[-3x\cos x + \frac{x\cos 3x}{3} + 3\sin x - \frac{\sin 3x}{9}\right] + c$

$I = \frac{1}{36}\left[3x\cos 3x - 27x\cos x + 27\sin x - \sin 3x\right] + c$

Indefinite Integrals Ex 19.25 Q55

Let $\quad I = \int \cos^3 \sqrt{x}\, dx$

Let $\quad x = t^2$

$dx = 2t\,dt$

$= 2\int t \cos^3 t\, dt$

$= 2\int t \left(\frac{3\cos t + \cos 3t}{4}\right) dt$

$= \frac{1}{2}\int t\left(3\cos t + \cos 3t\right) dt$

Using integration by parts,

$$I = \frac{1}{2}\left[t\left(3\sin t + \frac{1}{3}\sin 3t\right) + \int\left(1\times 3\sin t + \frac{\sin 3t}{3}\right)dt\right]$$

$$= \frac{1}{2}\left[t\left(\frac{9\sin t + \sin 3t}{3}\right) + 3\cos t + \frac{\cos 3t}{9}\right] + c$$

$$= \frac{1}{18}\left[27t\sin t + 3t\sin 3t + 9\cos t + \cos 3t\right] + c$$

$$I = \frac{1}{18}\left[27\sqrt{x}\sin\sqrt{x} + 3\sqrt{x}\sin 3\sqrt{x} + 9\cos\sqrt{x} + \cos 3\sqrt{x}\right] + c$$

Indefinite Integrals Ex 19.25 Q56

Let $\quad I = \int x\cos^3 x\, dx$

$= \int x\left(\frac{3\cos x + \cos 3x}{4}\right) dx$

$= \frac{1}{4}\int x\left(3\cos x + \cos 3x\right) dx$

Using integration by parts,

$$I = \frac{1}{4}\left[x\int\left(3\cos x + \cos 3x\right)dx - \int\left(1\int\left(3\cos x + \cos 3x\right)dx\right)dx\right]$$

$$= \frac{1}{4}\left[x\left(3\sin x + \frac{\sin 3x}{3}\right) - \int\left(3\sin x + \frac{\sin 3x}{3}\right)dx\right]$$

$$= \frac{1}{4}\left[3x\sin x + \frac{x\sin 3x}{3} + 3\cos x + \frac{\cos 3x}{9}\right] + c$$

$$I = \frac{3x\sin x}{4} + \frac{x\sin 3x}{12} + \frac{3\cos x}{4} + \frac{\cos 3x}{36} + c$$

Indefinite Integrals Ex 19.25 Q57

Let I = $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}$ and x = $\cos\theta \Rightarrow dx = -\sin\theta d\theta$

$$\therefore I = \int \tan^{-1}\left(\tan\frac{\theta}{2}\right)(-\sin\theta)d\theta$$

$$= -\frac{1}{2}\int \theta\sin\theta d\theta$$

$$Let\,\theta = u\ and\ \sin\theta d\theta = v\ so\ that\ \sin\theta = \int v\,d\theta$$

$$Then, \int uv\,dx = u\int(v\,dx) - \left(\int\frac{du}{dx}\int v\,dx\right)dx$$

$$Hence, I = -\frac{1}{2}\left(-\theta\cos\theta - \int -\cos\theta d\theta\right)$$

$$= -\frac{1}{2}\left(-\theta\cos\theta + \sin\theta\right) + c$$

$$= -\frac{1}{2}\left(-\theta\cos\theta + \sqrt{1-\cos^2\theta}\right) + c$$

$$= -\frac{1}{2}\left(-x\cos^{-1}x + \sqrt{1-x^2}\right) + c$$

Indefinite Integrals Ex 19.25 Q58

Let $I = \int \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx$

Let $x = a\tan^2\theta$

$dx = 2a\tan\theta\sec^2\theta\,d\theta$

$I = \int\left(\sin^{-1}\sqrt{\frac{a\tan^2\theta}{a+a\tan^2\theta}}\right)\left(2a\tan\theta\sec^2\theta\right)d\theta$

$= \int\left(\sin^{-1}\sqrt{\frac{\tan^2\theta}{\sec^2\theta}}\right)\left(2a\tan\theta\sec^2\theta\right)d\theta$

$= \int\sin^{-1}\left(\sin\theta\right)\left(2a\tan\theta\sec^2\theta\right)d\theta$

$= \int 2\theta a\tan\theta\sec^2\theta d\theta$

$= 2a\int\theta\left(\tan\theta\sec^2\theta\right)d\theta$

$= 2a\left[\theta\int\tan\theta\sec^2\theta d\theta - \int\left(\int\tan\theta\sec^2\theta d\theta\right)d\theta\right]$

$= 2a\left[\theta\frac{\tan^2\theta}{2} - \int\frac{\tan^2\theta}{2}d\theta\right]$

$= a\theta\tan^2\theta - \frac{2a}{2}\int\left(\sec^2\theta - 1\right)d\theta$

$= a\theta\tan^2\theta - a\tan\theta + a\theta + c$

$= a\left(\tan^{-1}\sqrt{\frac{x}{a}}\right)\frac{x}{a} - a\sqrt{\frac{x}{a}} + a\tan^{-1}\sqrt{\frac{x}{a}} + c$

$I = x\tan^{-1}\sqrt{\frac{x}{a}} - \sqrt{ax} + a\tan^{-1}\sqrt{\frac{x}{a}} + c$

Indefinite Integrals Ex 19.25 Q59

Let $I = \int\frac{x^3\sin^{-1}x^2}{\sqrt{1-x^4}}dx$

Let $\sin^{-1}x^2 = t$

$\frac{1}{\sqrt{1-x^4}}\left(2x\right)dx = dt$

$I = \int\frac{x^2\sin^{-1}x^2}{\sqrt{1-x^2}}x\,dx$

$= \int\left(\sin t\right)t\frac{dt}{2}$

$= \frac{1}{2}\int t\sin t\,dt$

$= \frac{1}{2}\left[t\int\sin t\,dt - \int\left(1\int\sin t\,dt\right)dt\right]$

$= \frac{1}{2}\left[t\left(-\cos t\right) - \int\left(-\cos t\right)dt\right]$

$= \frac{1}{2}\left[-t\cos t + \sin t\right] + c$

$I = \frac{1}{2}\left[x^2 - \sqrt{1-x^4}\sin^{-1}x^2\right] + c$

Indefinite Integrals Ex 19.25 Q60

Let $\quad I = \int \frac{x^2 \sin^{-1} x}{\left(1 - x^2\right)^{\frac{3}{2}}} dx$

Let $\quad \sin^{-1} x = t$

$\frac{1}{\sqrt{1 - x^2}} dx = dt$

$I = \int \frac{\sin^2 t \times t}{\left(1 - \sin^2 t\right)} dt$

$= \int \frac{t \sin^2 t}{\cos^2 t} dt$

$= \int t \tan^2 t dt$

$= \int t \left(\sec^2 t - 1\right) dt$

$= \int t \sec^2 t \, dt - \frac{t^2}{2}$

$= t \int \sec^2 t \, dt - \int \left(1 \int \sec^2 t \, dt\right) dt - \frac{t^2}{2}$

$= t \tan t - \int \tan t \, dt - \frac{t^2}{2}$

$= t \tan t - \log \sec t - \frac{t^2}{2} + c$

$I = \frac{x}{\sqrt{1 - x^2}} \sin^{-1} x + \log \left|1 - x^2\right| - \frac{1}{2} \left(\sin^{-1} x\right)^2 + c$

Ex 19.26

Indefinite Integrals Ex 19.26 Q1

$$\text{Let } I = \int e^x \left(\cos x - \sin x\right) dx$$
$$= \int e^x \cos x dx - \int e^x \sin x dx$$

Integrating by parts

$$= e^x \cos x - \int e^x \left(\frac{d}{dx} \cos x\right) dx - \int e^x \sin x dx$$
$$= e^x \cos x + \int e^x \sin x dx - \int e^x \sin x dx$$

$$= e^x \cos x + c$$

$$\therefore \quad \int e^x \left(\cos x - \sin x\right) dx = e^x \cos x + c$$

Indefinite Integrals Ex 19.26 Q2

$$I = \int e^x \left(x^{-2} - 2x^{-3}\right) dx$$

$$= \int e^x x^{-2} dx - 2\int e^x x^{-3} dx$$

Integrating by parts

$$= e^x x^{-2} - \int e^x \left(\frac{d}{dx}\left(x^{-2}\right)\right) dx - 2\int e^x x^{-3} dx$$
$$= e^x x^{-2} + 2\int e^x x^{-3} dx - 2\int e^x x^{-3} dx$$

$$= \frac{e^x}{x^2} + c$$

$$\int e^x \left(\frac{1}{x^2} - \frac{2}{x^3}\right) dx = \frac{e^x}{x^2} + c$$

Indefinite Integrals Ex 19.26 Q3

$$e^x\left(\frac{1+\sin x}{1+\cos x}\right)$$
$$=e^x\left(\frac{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)$$
$$=\frac{e^x\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}{2\cos^2\frac{x}{2}}$$
$$=\frac{1}{2}e^x\cdot\left(\frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\cos\frac{x}{2}}\right)^2$$
$$=\frac{1}{2}e^x\left[\tan\frac{x}{2}+1\right]^2$$
$$=\frac{1}{2}e^2\left(1+\tan\frac{x}{2}\right)^2$$
$$=\frac{1}{2}e^x\left[1+\tan^2\frac{x}{2}+2\tan\frac{x}{2}\right]$$
$$=\frac{1}{2}e^x\left[\sec^2\frac{x}{2}+2\tan\frac{x}{2}\right]$$
$$\frac{e^x(1+\sin x)dx}{(1+\cos x)}=e^x\left[\frac{1}{2}\sec^2\frac{x}{2}+\tan\frac{x}{2}\right] \qquad \ldots(1)$$

Let $\tan\frac{x}{2}=f(x)\Rightarrow f'(x)=\frac{1}{2}\sec^2\frac{x}{2}$

It is known that, $\int e^x\{f(x)+f'(x)\}dx=e^xf(x)+C$

From equation (1), we obtain

$$\int\frac{e^x(1+\sin x)}{(1+\cos x)}dx=e^x\tan\frac{x}{2}+C$$

Indefinite Integrals Ex 19.26 Q4

Let $I=\int e^x\left(\cot x-\operatorname{cosec}^2x\right)dx$

$$=\int e^x\cot x dx-\int e^x\operatorname{cosec}^2x dx$$

Integrating by parts

$$=e^x\cot x-\int e^x\left(\frac{d}{dx}\cot x\right)dx-\int e^x\operatorname{cosec}^2x dx$$
$$=e^x\cot x+\int e^x\operatorname{cosec}^2x dx-\int e^x\operatorname{cosec}^2x dx$$
$$=e^x\cot x+c$$

$$\int e^x\left(\cot x-\operatorname{cosec}^2x\right)dx=e^x\cot x+c$$

Indefinite Integrals Ex 19.26 Q5

Let $I = \int e^x \frac{1}{2x} dx - \int e^x \frac{1}{2x^2} dx$

Integrating by parts

$$= \frac{e^x}{2x} - \int e^x \left(\frac{d}{dx}\left(\frac{1}{2x}\right)\right) dx - \int \frac{e^x}{2x^2} dx$$
$$= \frac{e^x}{2x} + \int \frac{e^x}{2x^2} dx - \int \frac{e^x}{2x^2} dx$$

$$= \frac{e^x}{2x} + c$$

Indefinite Integrals Ex 19.26 Q6

Let $I = \int e^x \sec x \left(1 + \tan x\right) dx$

$$= \int e^x \sec x dx + \int e^x \sec x \tan x dx$$

Integrating by parts

$$= e^x \sec x - \int e^x \left(\frac{d}{dx} \sec x\right) dx + \int e^x \sec x \tan x dx$$
$$= e^x \sec x - \int e^x \sec x \tan x dx + \int e^x \sec x \tan x dx$$

$$= e^x \sec x + c$$

$\therefore \quad \int e^x \sec x \left(1 + \tan x\right) dx = e^x \sec x + c$

Indefinite Integrals Ex 19.26 Q7

Let $I = \int e^x \left(\tan x - \log \cos x\right) dx$

$$= \int e^x \tan x dx - \int e^x \log \cos x dx$$

Integrating by parts

$$= \int e^x \tan x dx - \left\{e^x \log \cos x - \int e^x \left(\frac{d}{dx} \log \cos x\right) dx\right\}$$
$$= \int e^x \tan x dx - \left\{e^x \log \cos x + \int e^x \tan x dx\right\}$$
$$= \int e^x \tan x dx - e^x \log \cos x - \int e^x \tan x dx + c$$

$$= -e^x \log \cos x + c$$

$$= e^x \log \sec x + c \qquad \left[\because \quad \log \sec x = -\log \cos x\right]$$

Indefinite Integrals Ex 19.26 Q8

Let $I = \int e^x \left[\sec x + \log\left(\sec x + \tan x\right)\right] dx$

$$= \int e^x \sec x dx + \int e^x \log\left(\sec x + \tan x\right) dx$$

Integrating by parts

$$= \int e^x \sec x dx + e^x \log\left(\sec x + \tan x\right) - \int e^x \left\{\frac{d}{dx} \log\left(\sec x + \tan x\right)\right\} dx$$
$$= \int e^x \sec x dx + e^x \log\left(\sec x + \tan x\right) - \int e^x \sec x dx$$

$$= e^x \log\left(\sec x + \tan x\right) + c$$

Indefinite Integrals Ex 19.26 Q9

Let $I = \int e^x \left(\cot x + \log \sin x\right) dx$

$$= \int e^x \cot x dx + \int e^x \log \sin x dx$$

Integrating by parts

$$= \int e^x \log \sin x dx + \int e^x \cot x dx$$

$$= \left(\log \sin x\right) e^x - \int e^x \left(\frac{d}{dx} \log \sin x\right) dx + \int e^x \cot x dx$$

$$= e^x \log \sin x - \int e^x \cot x dx + \int e^x \cot x dx$$

$$= e^x \log \sin x + c$$

Indefinite Integrals Ex 19.26 Q10

Let $I = \int e^x \frac{x+1-2}{(x+1)^3}$

$$= \int e^x \left\{\frac{1}{(x+1)^2} + \frac{-2}{(x+1)^3}\right\} dx$$

$$= \int e^x \frac{1}{(x+1)^2} dx + \int e^x \frac{(-2)}{(x+1)^3} dx$$

Integrating by parts

$$= e^x \frac{1}{(x+1)^2} - \int e^x \left(\frac{d}{dx}(x+1)^{-2}\right) dx + \int e^x \frac{(-2)}{(x+1)^3} dx$$

$$= e^x \frac{1}{(x+1)^2} - \int e^x \frac{(-2)}{(x+1)^3} dx + \int e^x \frac{(-2)}{(x+1)^3} dx$$

$$= e^x \frac{1}{(x+1)^2} + c$$

Indefinite Integrals Ex 19.26 Q11

Let $I = \int e^x \left(\frac{\sin 4x - 4}{2 \sin^2 2x}\right) dx$

$$= \int e^x \left\{\frac{2 \sin 2x \cos 2x}{2 \sin^2 2x} - \frac{4}{2 \sin^2 2x}\right\} dx$$

$$= \int e^x \left(\cot 2x - 2\, \text{cosec}^2 2x\right) dx$$

$$= \int e^x \cot 2x dx - 2\int e^x\, \text{cosec}^2 2x dx$$

Integrating by parts

$$= e^x \cot 2x - \int e^x \frac{d}{dx}\left(\cot 2x\right) dx - 2\int e^x\, \text{cosec}^2 2x dx$$

$$= e^x \cot 2x + 2\int e^x\, \text{cosec}^2 2x - 2\int e^x\, \text{cosec}^2 2x dx$$

$$= e^x \cot 2x + c$$

Indefinite Integrals Ex 19.26 Q12

Let $I = \int \frac{2-x}{(1-x)^2} e^x dx$

$= \int e^x \left\{ \frac{(1-x)+1}{(1-x)^2} \right\} dx$

$= \int e^x \left\{ \frac{1}{1-x} + \frac{1}{(1-x)^2} \right\} dx$

Here, $f(x) = \frac{1}{1-x}$ and $f'(x) = \frac{1}{(1-x)^2}$

And we know that,

$\int e^{ax} (af(x) + f'(x)) dx = e^{ax} f(x) + c$

$\therefore \quad \int e^x \left\{ \frac{1}{1-x} + \frac{1}{(1-x)^2} \right\} dx = e^x . \frac{1}{1-x} + c$

Hence,

$I = \frac{e^x}{1-x} + c$

Indefinite Integrals Ex 19.26 Q13

Let $I = \int e^x \frac{1+x}{(2+x)^2} dx$

$= \int e^x \left(\frac{x+2-1}{(2+x)^2} \right) dx$

$= \int e^x \left\{ \frac{1}{x+2} - \frac{1}{(x+2)^2} \right\} dx$

$= \int e^x \frac{1}{x+2} dx - \int e^x \frac{1}{(x+2)^2} dx$

Integrating by parts

$= e^x \frac{1}{x+2} - \int e^x \left(\frac{d}{dx} \left(\frac{1}{x+2} \right) \right) dx - \int e^x \frac{1}{(x+2)^2} dx$

$= e^x \frac{1}{x+2} + \int e^x \frac{1}{(x+2)^2} dx - \int e^x \frac{1}{(x+2)^2} dx$

$= \frac{e^x}{x+2} + c$

Indefinite Integrals Ex 19.26 Q14

Let $I = \int \frac{\sqrt{1-\sin x}}{1+\cos x} e^{-\frac{x}{2}} dx$

Put $= \frac{x}{2} = t$

$\Rightarrow \quad x = 2t$

$dx = 2dt$

$\therefore \quad \int \frac{\sqrt{1-\sin x}}{1+\cos x} e^{-\frac{x}{2}} dx$

$= 2\int \frac{\sqrt{1-\sin 2t}}{1+\cos 2t} e^{-t} dt \qquad \left[\because \sin^2 t + \cos^2 t = 1\right]$

$= 2\int \frac{\sqrt{\sin^2 t + \cos^2 t - 2\sin t \cos t}}{1+\cos 2t} e^{-t} dt$

$= 2\int \frac{\sqrt{\left(\cos t - \sin t\right)^2}}{2\cos^2 t} e^{-t} dt$

$= 2\int \frac{\left(\cos t - \sin t\right)}{2\cos^2 t} e^{-t} dt$

$= \int \left(\sec t - \tan t \sec t\right) e^{-t} dt$

$= \int \sec t\, e^{-t} dt - \int \tan t \sec t\, e^{-t} dt$

Integrating by parts

$= e^{-t} \sec t + \int e^{-t} \frac{d}{dt}\left(\sec t\right) dt - \int \tan t \sec t\, e^{-t} dt$

$= -e^{-t} \sec t + \int e^{-t} \sec t \tan t dt - \int \sec t \tan t e^{-t} dt$

$= -e^{-t} \sec t + c$

Putting the value of t

$= -e^{-\frac{x}{2}} \sec \frac{x}{2} + c$

Indefinite Integrals Ex 19.26 Q15

We have,

$I = \int e^x \left(\log x + \frac{1}{x}\right) dx$

We know that

$\int e^{ax} \left(af\left(x\right) + f'\left(x\right)\right) dx = e^{ax} f\left(x\right) + c$

Here $f\left(x\right) = \log x$ and $f'\left(x\right) = \frac{1}{x}$

$\therefore \quad \int e^x \left(\log x + \frac{1}{x}\right) dx = e^x \log x + c$

Indefinite Integrals Ex 19.26 Q16

We have,

$$I = \int e^x \left(\log x + \frac{1}{x^2}\right) dx$$

$$= \int e^x \left(\log x + \frac{1}{x} - \frac{1}{x} + \frac{1}{x^2}\right) dx$$
$$= \int e^x \left(\log x - \frac{1}{x}\right) dx + \int e^x \left(\frac{1}{x} + \frac{1}{x^2}\right) dx$$

Integrating by parts

$$= e^x \left(\log x - \frac{1}{x}\right) - \int e^x \frac{d}{dx}\left(\log x - \frac{1}{x}\right) dx + \int e^x \left(\frac{1}{x} + \frac{1}{x^2}\right) dx$$
$$= e^x \left(\log x - \frac{1}{x}\right) - \int e^x \left(\frac{1}{x} + \frac{1}{x^2}\right) dx + \int e^x \left(\frac{1}{x} + \frac{1}{x^2}\right) dx$$

$$= e^x \left(\log x - \frac{1}{x}\right) + c$$

Indefinite Integrals Ex 19.26 Q17

We have,

$$I = \int \frac{e^x}{x} \left\{x (\log x)^2 + 2 \log x\right\} dx$$

$$= \int e^x \left\{(\log x)^2 + \frac{2}{x} \log x\right\} dx$$
$$= \int e^x (\log x)^2 + 2 \int \frac{e^x}{x} \log x dx$$

Integrating by parts

$$= e^x (\log x)^2 - \int e^x \frac{d}{dx} (\log x)^2 dx + 2 \int e^x \frac{1}{x} \log x dx$$
$$= e^x (\log x)^2 - \int e^x \frac{2 \log x}{x} dx + 2 \int e^x \frac{\log x}{x} dx$$

$$= e^x (\log x)^2 + c$$

Indefinite Integrals Ex 19.26 Q18

Let $I = \int e^x \left(\sin^{-1} x + \frac{1}{\sqrt{1 - x^2}}\right) dx$

$$I = \int e^x \sin^{-1} x + \int e^x \frac{1}{\sqrt{1 - x^2}} dx$$

Integrating by parts

$$= e^x \sin^{-1} x - \int e^x \left(\frac{d}{dx}\left(\sin^{-1} x\right)\right) dx + \int e^x \frac{1}{\sqrt{1 - x^2}} dx$$
$$= e^x \sin^{-1} x - \int e^x \frac{1}{\sqrt{1 - x^2}} dx + \int e^x \frac{1}{\sqrt{1 - x^2}} dx$$

$$= e^x \sin^{-1} x + c$$

Indefinite Integrals Ex 19.26 Q19

Let $I = \int e^{2x}\left(-\sin x + 2\cos x\right)dx$

$= -\int e^{2x}\sin x dx + 2\int e^{2x}\cos x dx$

Applying by parts in the 2^{nd} integrand

$\therefore \quad I = -\int e^{2x}\sin x dx + 2\left\{\frac{1}{2}e^{2x}\cos x + \int\frac{1}{2}e^{2x}\sin x dx\right\}$

$= -\int e^{2x}\sin x dx + e^{2x}\cos x + \int e^{2x}\sin x dx + c$

$= e^{2x}\cos x + c$

Thus,

$I = e^{2x}\cos x + c$

Indefinite Integrals Ex 19.26 Q20

Let $I = \int e^x\left(\tan^{-1}x + \frac{1}{1+x^2}\right)dx$

Here, $f(x) = \tan^{-1}x$ and $f'(x) = \frac{1}{1+x^2}$

And we know that,

$\int e^{ax}\left(af(x) + f'(x)\right)dx = e^{ax}f(x) + c$

$\therefore \quad \int e^x\left(\tan^{-1}x + \frac{1}{1+x^2}\right)dx = e^x\tan^{-1}x + c$

Thus,

$I = e^x\tan^{-1}x + c$

Indefinite Integrals Ex 19.26 Q21

Let $I = \int e^x\left(\frac{\sin x\cos x - 1}{\sin^2 x}\right)dx$

$= \int e^x\left(\cot x - \operatorname{cosec}^2 x\right)dx$

$= \int e^x\left(\cot x + \left(-\operatorname{cosec}^2 x\right)\right)dx$

$\because \quad \int e^{ax}\left(af(x) + f'(x)\right)dx = e^{ax}f(x) + c$

Here $f(x) = \cot x$

$\Rightarrow \quad f'(x) = -\operatorname{cosec}^2 x$

$\therefore \quad \int e^x\left(\cot x - \operatorname{cosec}^2 x\right)dx = e^x\cot x + c$

Thus,

$I = e^x\cot x + c$

Indefinite Integrals Ex 19.26 Q22

Let $I = \int \left\{ \tan(\log x) + \sec^2(\log x) \right\} dx$

Let $\log x = z$

$\Rightarrow \quad x = e^z$

$\Rightarrow \quad dx = e^z dz$

$\therefore \quad I = \int \left\{ \tan z + \sec^2 z \right\} e^z dz$

Here, $f(z) = \tan z$ and $f'(z) = \sec^2 z$

And we know that

$\int e^{ax} \left(af(x) + f'(x) \right) dx = e^{ax} f(x) + c$

$\therefore \quad \int e^z \left\{ \tan z + \sec^2 z \right\} dz = e^z \tan z + c$

$\therefore \quad I = x \tan(\log x) + c$

Indefinite Integrals Ex 19.26 Q23

Let $I = \int \frac{e^x (x-4)}{(x-2)^3} dx$

$= \int e^x \left\{ \frac{(x-2) - 2}{(x-2)^3} \right\} dx$

$= \int e^x \left\{ \frac{1}{(x-2)^2} - \frac{2}{(x-2)^3} \right\} dx$

Here, $f(x) = \frac{1}{(x-2)^2}$ and $f'(x) = \frac{-2}{(x-2)^3}$

And we know that,

$\int e^{ax} \left(af(x) + f'(x) \right) dx = e^{ax} f(x) + c$

$\therefore \quad \int e^x \left\{ \frac{1}{(x-2)^2} - \frac{2}{(x-2)^3} \right\} dx = \frac{e^x}{(x-2)^2} + c$

$\therefore \quad I = \frac{e^x}{(x-2)^2} + c$

Indefinite Integrals Ex 19.26 Q24

Let $\mathrm{I}=\int e^{2x}\left(\frac{1-\sin 2x}{1-\cos 2x}\right)dx$

We have, $\cos 2x = 1-2\sin^2 x$

$\mathrm{I}=\int e^{2x}\left(\frac{1-\sin 2x}{1-\left(1-2\sin^2 x\right)}\right)dx$

$= \int e^{2x}\left(\frac{1-\sin 2x}{2\sin^2 x}\right)dx$

$= \int e^{2x}\left(\frac{\operatorname{cosec}^2 x}{2} - \frac{2\sin x\cos x}{2\sin^2 x}\right)dx$

$= \int e^{2x}\left(\frac{\operatorname{cosec}^2 x}{2} - \frac{\cos x}{\sin x}\right)dx$

$= \int e^{2x}\left(\frac{\operatorname{cosec}^2 x}{2} - \cot x\right)dx$

$= \frac{1}{2}\int e^{2x}\operatorname{cosec}^2 x dx - \int e^{2x}\cot x dx$

That is

$\mathrm{I}=\mathrm{I}_1 + I_2$, where, $\mathrm{I}_1 = \frac{1}{2}\int e^{2x}\operatorname{cosec}^2 x dx$ and $\mathrm{I}_2 = -\int e^{2x}\cot x dx$

Consider $\mathrm{I}_1 = \frac{1}{2}\int e^{2x}\operatorname{cosec}^2 x dx$

Take e^{2x} as the first function and $\operatorname{cosec}^2 x$ as the second function.

So, $u=e^{2x}$; $du=2e^{2x}dx$

and

$\int \operatorname{cosec}^2 x dx = \int dv$

$\Rightarrow v = -\cot x$

$\mathrm{I}_1 = \frac{1}{2}\left[e^{2x}\left(-\cot x\right) - \int\left(-\cot x\right)2e^{2x}dx\right]$

$\Rightarrow \mathrm{I}_1 = \frac{1}{2}\left[e^{2x}\left(-\cot x\right) + 2\int \cot x e^{2x}dx\right]$

$\Rightarrow \mathrm{I}_1 = \frac{1}{2}\left[e^{2x}\left(-\cot x\right)\right] + \int \cot x e^{2x}dx$

Thus,

$I = \frac{1}{2}\left[e^{2x}\left(-\cot x\right)\right] + \int \cot x e^{2x}dx - \int e^{2x}\cot x dx$

$= I = \frac{1}{2}\left[e^{2x}\left(-\cot x\right)\right] + C$

Ex 19.27

Indefinite Integrals Ex 19.27 Q1

Let $I = \int e^{ax} \cos bx dx$

Intergrating by parts,

$$I = e^{ax} \frac{\sin bx}{b} - a\int e^{ax} \frac{\sin bx}{b} fx$$
$$= \frac{1}{b} e^{ax} \sin bx - \frac{a}{b} \int e^{ax} \sin bx dx$$
$$= \frac{1}{b} e^{ax} \sin bx - \frac{a}{b}\left[-e^{ax} \frac{\cos bx}{b} + \int a e^{ax} \frac{\cos bx}{b} dx\right]$$
$$= \frac{1}{b} e^{ax} \sin bx + \frac{a}{b^2} e^{ax} \cos bx - \frac{a^2}{b^2} \int e^{ax} \cos bx dx$$

$$\Rightarrow \qquad I = \frac{e^{ax}}{b^2}\left[b \sin bx + a \cos bx\right] - \frac{a^2}{b^2} I + c$$
$$\Rightarrow \qquad I.\left\{\frac{a^2+b^2}{b^2}\right\} = \frac{e^{ax}}{b^2}\left[b \cos bx + a \cos bx\right] + c$$

Thus,

$$I = \frac{e^{ax}}{a^2+b^2}\left[a \cos bx + a \cos bx\right] + c$$

Indefinite Integrals Ex 19.27 Q2

Let $I = \int e^{ax} \sin(bx+c) dx$

$$\Rightarrow \qquad -e^{ax} \frac{\cos(bx+c)}{b} + \int a e^{ax} \frac{\cos(bx+c)}{b} dx$$
$$= -\frac{1}{b} e^{ax} \cos(bx+c) + \frac{a}{b} \int e^{ax} \cos(bx+c) dx$$
$$= -\frac{1}{b} e^{ax} \cos(bx+c) + \frac{a}{b}\left[\int e^{ax} \frac{\sin(bx+c)}{b} - \int a e^{ax} \frac{\sin(bx+c)}{b} dx\right] + c_1$$
$$= \frac{e^{ax}}{b^2}\left\{a \sin(bx+c) - b \cos(bx+c)\right\} - \frac{a^2}{b^2} \int e^{ax} \sin(bx+c) dx + c_1$$

$$\Rightarrow \qquad I = \frac{e^{ax}}{b^2}\left\{a \sin(bx+c) - b \cos(bx+c)\right\} - \frac{a^2}{b^2} I + c_1$$
$$\Rightarrow \qquad I = \left\{\frac{a^2+b^2}{b^2}\right\} - \frac{e^{ax}}{b^2}\left\{a \sin(bx+c) - b \cos(bx+c)\right\} + c_1$$

$$\Rightarrow \qquad I = \frac{e^{ax}}{a^2+b^2}\left\{a \sin(bx+c) - b \cos(bx+c)\right\} + c_1$$

Indefinite Integrals Ex 19.27 Q3

Let $\log x = t$

$\Rightarrow \quad \frac{1}{x}dx = dt$

$\Rightarrow \quad dx = x dt$

$\Rightarrow \quad dx = e^t dt$

$\therefore \quad I = \int \cos(\log x)dx = \int e^t \cos t dt$

We know that

$$\int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2}\{a \cos bx + b \sin bx\} + c$$

Here $a = 1$, $b = 1$

So, $I = \frac{e^t}{2}\{\cos t + \sin t\} + c$

Hence,

$$I = \int \cos(\log x)dx = \frac{e^{\log x}}{2}\{\cos(\log x) + \sin(\log x)\} + c$$

$$\Rightarrow \quad I = \frac{x}{2}\{\cos(\log x) + \sin(\log x)\} + c$$

Indefinite Integrals Ex 19.27 Q4

Let $I = \int e^{2x} \cos(3x + 4)dx$

Integrating by parts

$$I = e^{2x} \frac{\sin(3x + 4)}{3} - \int 2e^{2x} \frac{\sin(3x + 4)}{3}dx$$
$$= \frac{1}{3}e^{2x} \sin(3x + 4) - \frac{2}{3}\int e^{2x} \sin(3x + 4)dx$$
$$= \frac{1}{3}e^{2x} \sin(3x + 4) - \frac{2}{3}\left[-e^{2x} \frac{\cos(3x + 4)}{3} + \int 2e^{2x} \frac{\cos(3x + 4)}{3}dx\right] + c$$
$$I = \frac{e^{2x}}{9}\{2\cos(3x + 4) + 3\sin(3x + 4)\} + c$$

Hence,

$$I = \frac{e^{2x}}{13}\{2\cos(3x + 4) + 3\sin(3x + 4)\} + c$$

Indefinite Integrals Ex 19.27 Q5

Let $I = \int e^{2x} \sin x \cos x dx$

$$= \frac{1}{2}\int e^{2x} 2 \sin x \cos x dx$$
$$= \frac{1}{2}\int e^{2x} \sin 2x dx$$

We know that

$$\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2+b^2}\{a \sin bx - b \cos bx\} + c$$

$$\Rightarrow \quad \int e^{2x} \sin 2x dx = \frac{e^{2x}}{8}\{2 \sin 2x - 2 \cos 2x\} + c$$

$$\therefore \quad I = \frac{1}{2}\cdot\frac{e^{2x}}{8}\{2 \sin 2x - 2 \cos 2x\} + c$$

$$\therefore \quad I = \frac{e^{2x}}{8}\{\sin 2x - \cos 2x\} + c$$

Indefinite Integrals Ex 19.27 Q6

Let $I = \int e^{2x} \sin x\, dx \qquad ...(1)$

Integrating by parts, we obtain

$$I = \sin x \int e^{2x} dx - \int\left\{\left(\frac{d}{dx}\sin x\right)\int e^{2x} dx\right\}dx$$
$$\Rightarrow I = \sin x \cdot \frac{e^{2x}}{2} - \int \cos x \cdot \frac{e^{2x}}{2} dx$$
$$\Rightarrow I = \frac{e^{2x}\sin x}{2} - \frac{1}{2}\int e^{2x}\cos x\, dx$$

Again integrating by parts, we obtain

$$I = \frac{e^{2x}\cdot\sin x}{2} - \frac{1}{2}\left[\cos x \int e^{2x} dx - \int\left\{\left(\frac{d}{dx}\cos x\right)\int e^{2x} dx\right\}dx\right]$$
$$\Rightarrow I = \frac{e^{2x}\sin x}{2} - \frac{1}{2}\left[\cos x \cdot \frac{e^{2x}}{2} - \int(-\sin x)\frac{e^{2x}}{2} dx\right]$$
$$\Rightarrow I = \frac{e^{2x}\cdot\sin x}{2} - \frac{1}{2}\left[\frac{e^{2x}\cos x}{2} + \frac{1}{2}\int e^{2x}\sin x dx\right]$$
$$\Rightarrow I = \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4} - \frac{1}{4}I \qquad \text{[From (1)]}$$

$$\Rightarrow I = \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4} - \frac{1}{4}I$$
$$\Rightarrow I + \frac{1}{4}I = \frac{e^{2x}\cdot\sin x}{2} - \frac{e^{2x}\cos x}{4}$$
$$\Rightarrow \frac{5}{4}I = \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4}$$
$$\Rightarrow I = \frac{4}{5}\left[\frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4}\right] + C$$
$$\Rightarrow I = \frac{e^{2x}}{5}\left[2\sin x - \cos x\right] + C$$

Indefinite Integrals Ex 19.27 Q8

Let $I = \int e^x \sin^2 x dx$

$$= \frac{1}{2}\int e^x 2\sin^2 x dx$$
$$= \frac{1}{2}\int e^x \left(1 - \cos 2x\right) dx$$
$$= \frac{1}{2}\int e^x dx - \frac{1}{2}\int e^x \cos 2x dx$$

$\because \quad \int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2}\left\{a \cos bx - b \sin bx\right\} + c$

$\therefore \quad I = \frac{1}{2}\left[e^x - \frac{e^x}{5}\left\{\cos 2x + 2\sin 2x\right\}\right] + c$

$\therefore \quad I = \frac{e^x}{2} - \frac{e^x}{10}\left\{\cos 2x + 2\sin 2x\right\} + c$

Indefinite Integrals Ex 19.27 Q9

Let $I = \int \frac{1}{x^3} \sin\left(\log x\right) dx$

Let $\log x = t$

$\Rightarrow \quad \frac{1}{x} dx = dt$

$\Rightarrow \quad dx = e^t dt$

$\therefore \quad I = \int e^{-2t} \sin t dt$

We know that,

$\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2 + b^2}\left\{a \sin bx - b \cos bx\right\} + c$

$\therefore \quad \int e^{-2t} \sin t\, dt = \frac{e^{-2t}}{5}\left\{-2 \sin t - \cos t\right\} + c$

$\therefore \quad I = \frac{x^{-2}}{5}\left\{-2 \sin\left(\log x\right) - \cos\left(\log x\right)\right\} + c$

Hence,

$\int \frac{1}{x^3} \sin\left(\log x\right) dx = \frac{-1}{5x^2}\left\{2 \sin\left(\log x\right) + \cos\left(\log x\right)\right\} + c$

Indefinite Integrals Ex 19.27 Q10

Let $I = \int e^{2x} \cos^2 x dx$

$$= \frac{1}{2}\int e^{2x} 2\cos^2 x dx$$
$$= \frac{1}{2}\int e^{2x} (1 + \cos 2x) dx$$
$$= \frac{1}{2}\int e^{2x} dx + \frac{1}{2}\int e^{2x} \cos 2x dx$$

$\because \quad \int e^{ax} \cos bx dx = \dfrac{e^{ax}}{a^2 + b^2}\{a\cos bx - b\sin bx\} + c$

$\therefore \quad I = \dfrac{1}{4}e^{2x} + \dfrac{1}{2}\dfrac{e^{2x}}{8}\{2\cos 2x + 2\sin 2x\} + c$

Hence,

$$I = \frac{e^{2x}}{4} + \frac{e^{2x}}{16}\{2\cos 2x + 2\sin 2x\} + c$$
or
$$I = \frac{e^{2x}}{4} + \frac{e^{2x}}{8}\{\cos 2x + \sin 2x\} + c$$

Indefinite Integrals Ex 19.27 Q11

Let $I = \int e^{-2x} \sin x dx$

$\because \quad \int e^{ax} \sin bx = \dfrac{e^{ax}}{a^2 + b^2}\{a\sin bx - b\cos bx\} + c$

$\therefore \quad I = \dfrac{e^{-2x}}{5}\{-2\sin x - \cos x\} + c$

Indefinite Integrals Ex 19.27 Q12

Let $I = \int x^2 e^{x^3} \cos x^3 dx$

Let $x^3 = t$

$\Rightarrow \quad 3x^2 dx = dt$

$\therefore \quad I = \dfrac{1}{3}\int e^t \cos t dt$

$\because \quad \int e^{ax} \cos bx dx = \dfrac{e^{ax}}{a^2 + b^2}\{a\cos bx + b\sin bx\} + c$

$\therefore \quad I = \dfrac{1}{3}\left\{\dfrac{e^t}{2}(\cos t + \sin t)\right\} + c$

$\therefore \quad I = \dfrac{1}{3}\left\{\dfrac{e^{x^3}}{2}\left(\cos x^3 + \sin x^3\right)\right\} + c$

Ex 19.28

Indefinite Integrals Ex 19.28 Q1

$$\int \sqrt{3+2x-x^2}\ dx = \int \sqrt{4-(x-1)^2}\ dx$$

Let $x-1=t$, so that $dx = dt$

Thus, $\int \sqrt{3+2x-x^2}\ dx = \int \sqrt{4-t^2}\ dt$

$$= \frac{1}{2}t\sqrt{4-t^2} + \frac{4}{2}\sin^{-1}\left(\frac{t}{2}\right) + C$$

$$= \frac{1}{2}(x-1)\sqrt{3+2x-x^2} + 2\sin^{-1}\left(\frac{x-1}{2}\right) + C$$

Indefinite Integrals Ex 19.28 Q2

Let $I = \int \sqrt{x^2+x+1}dx$

$$= \int \sqrt{x^2+x+\frac{1}{4}+\frac{3}{4}}dx$$

$$= \int \sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dx$$

$$= \frac{\left(x+\frac{1}{2}\right)}{2}\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} + \frac{\left(\frac{\sqrt{3}}{2}\right)^2}{2}.\log\left|\left(x+\frac{1}{2}\right)+\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right| + c$$

$$= \left(\frac{2x+1}{4}\right)\sqrt{x^2+x+1} + \frac{3}{8}\log\left|\left(\frac{2x+1}{2}\right)+\frac{1}{2}\sqrt{x^2+x+1}\right| + c$$

$$\therefore \quad I = \left(\frac{2x+1}{4}\right)\sqrt{x^2+x+1} + \frac{3}{8}\log\left|2x+1+\sqrt{x^2+x+1}\right| + c$$

Indefinite Integrals Ex 19.28 Q3

Let $I = \int \sqrt{x-x^2}dx$

$$= \int \sqrt{\frac{1}{4}-\frac{1}{4}+x-x^2}dx \qquad \left[\text{Add and subtract } \frac{1}{4}\right]$$

$$= \int \sqrt{\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}-x\right)^2}dx$$

$$= -\left(\frac{1-2x}{4}\right)\sqrt{\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}-x\right)^2} - \frac{\left(\frac{1}{2}\right)^2}{2}\sin^{-1}\left(\frac{\frac{1-2x}{2}}{\frac{1}{2}}\right) + c$$

$$\therefore \quad I = \left(\frac{2x-1}{4}\right)\sqrt{x-x^2} + \frac{1}{8}\sin^{-1}(2x-1) + c$$

Indefinite Integrals Ex 19.28 Q4

Let $I = \int \sqrt{1 + x - 2x^2} dx$

$$= \sqrt{2}\int \sqrt{\frac{1}{2} + \frac{x}{2} - x^2} dx$$

$$= \sqrt{2}\int \sqrt{\frac{9}{16} - \left(\frac{1}{16} - \frac{x}{2} + x^2\right)} dx$$

$$= \sqrt{2}\int \sqrt{\left(\frac{3}{4}\right)^2 - \left(x - \frac{1}{4}\right)^2} dx$$

$$= \sqrt{2}\left\{\frac{\left(x - \frac{1}{4}\right)}{2}\sqrt{\frac{1}{2} + \frac{x}{2} - x^2} + \frac{9}{32}\sin^{-1}\left(\frac{x - \frac{1}{4}}{\frac{3}{4}}\right)\right\} + c$$

$$I = \frac{1}{8}(4x - 1)\sqrt{1 + x - 2x^2} + \frac{9\sqrt{2}}{32}\sin^{-1}\left(\frac{4x - 1}{3}\right) + c$$

Indefinite Integrals Ex 19.28 Q5

Let $I = \int \cos x \sqrt{4 - \sin^2 x} dx$

Let $\sin x = t$

$\Rightarrow \quad \cos x dx = dt$

$\Rightarrow \quad I = \int \sqrt{4 - t^2} dt$

$= \int \sqrt{2^2 - t^2} dt$

$= \frac{t}{2}\sqrt{2^2 - t^2} + \frac{4}{2}\sin^{-1}\frac{t}{2} + c$

$\therefore \quad I = \frac{1}{2}\sin x \sqrt{4 - \sin^2 x} + 2\sin^{-1}\left(\frac{\sin x}{2}\right) + c$

Indefinite Integrals Ex 19.28 Q6

Let $I = \int e^x \sqrt{e^{2x} + 1} dx$

Let $e^x = t$

$\Rightarrow \quad e^x dx = dt$

$\therefore \quad I = \int \sqrt{t^2 + 1^2} dt$

$= \frac{t}{2}\sqrt{t^2 + 1} + \frac{1}{2}\log\left|t + \sqrt{t^2 + 1}\right| + c$

$\therefore \quad I = \frac{e^x}{2}\sqrt{e^{2x} + 1} + \frac{1}{2}\log\left|e^x + \sqrt{e^{2x} + 1}\right| + c$

Indefinite Integrals Ex 19.28 Q7

Let $I = \int \sqrt{3^2 - x^2}$

We know that,

$\int \sqrt{a^2 - x^2} = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$

$\therefore \quad I = \frac{x}{2}\sqrt{9 - x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3} + c$

Indefinite Integrals Ex 19.28 Q8

Let $I = \int \sqrt{16x^2 + 25}dx$

$$= \int \sqrt{(4x)^2 + 5^2}dx$$

$$= 4\int \sqrt{x^2 + \left(\frac{5}{4}\right)^2}dx$$

$$= 4\left\{\frac{x}{2}\sqrt{x^2 + \left(\frac{5}{4}\right)^2} + \frac{\left(\frac{5}{4}\right)^2}{2}\log\left|x + \sqrt{x^2 + \left(\frac{5}{4}\right)^2}\right| + c\right\}$$

$$\therefore \quad I = 2x\sqrt{x^2 + \frac{25}{16}} + \frac{25}{8}\log\left|x + \sqrt{x^2 + \frac{25}{16}}\right| + c$$

Indefinite Integrals Ex 19.28 Q9

Let $I = \int \sqrt{4x^2 - 5}dx$

$$= 2\int \sqrt{x^2 - \left(\frac{\sqrt{5}}{2}\right)^2}dx$$

$$= 2\left\{\frac{x}{2}\sqrt{x^2 - \frac{5}{4}} - \frac{5}{8}\log\left|x + \sqrt{x^2 - \frac{5}{4}}\right| + c\right.$$

$$\therefore \quad I = x\sqrt{x^2 - \frac{5}{4}} - \frac{5}{4}\log\left|x + \sqrt{x^2 - \frac{5}{4}}\right| + c$$

Indefinite Integrals Ex 19.28 Q10

Let $I = \int \sqrt{2x^2 + 3x + 4}dx$

$$= \sqrt{2}\int \sqrt{x^2 + \frac{3}{2}x + 2}dx$$

$$= \sqrt{2}\int \sqrt{x^2 + \frac{3}{2}x + \frac{9}{16} + \frac{23}{16}}dx$$

$$= \sqrt{2}\int \sqrt{\left(x + \frac{3}{4}\right)^2 + \left(\frac{\sqrt{23}}{4}\right)^2}dx$$

$$= \sqrt{2}\left\{\frac{\left(x + \frac{3}{4}\right)}{2}\sqrt{x^2 + \frac{3}{2}x + 2} + \frac{23}{32}.\log\left|\left(x + \frac{3}{4}\right) + \sqrt{x^2 + \frac{3}{2}x + 2}\right| + c\right\}$$

$$\therefore \quad I = \frac{4x + 3}{8}\sqrt{2x^2 + 3x + 4} + \frac{23\sqrt{2}}{32}.\log\left|\left(x + \frac{3}{4}\right) + \sqrt{x^2 + \frac{3}{2}x + 2}\right| + c$$

Indefinite Integrals Ex 19.28 Q11

$$\text{Let } I = \int \sqrt{3 - 2x - 2x^2}\,dx$$

$$= \sqrt{2}\int \sqrt{\frac{3}{2} - x - x^2}\,dx$$

$$= \sqrt{2}\int \sqrt{\frac{7}{4} - \left(\frac{1}{4} + x + x^2\right)}\,dx \qquad \left[\text{Adding and subtracting } \frac{1}{4}\right]$$

$$= \sqrt{2}\int \sqrt{\left(\frac{\sqrt{7}}{2}\right)^2 - \left(x + \frac{1}{2}\right)^2}\,dx$$

$$= \sqrt{2}\left\{\frac{x + \frac{1}{2}}{2}\sqrt{\frac{3}{2} - x - x^2} + \frac{7}{8}\sin^{-1}\left(\frac{x + \frac{1}{2}}{\frac{\sqrt{7}}{2}}\right)\right\} + c$$

$$\therefore \qquad I = \frac{2x+1}{4}\sqrt{3 - 2x - 2x^2} + \frac{7\sqrt{2}}{8}\sin^{-1}\left(\frac{2x+1}{\sqrt{7}}\right) + c$$

Indefinite Integrals Ex 19.28 Q12

$$\text{Let } x^2 = t$$

$$\Rightarrow \qquad 2x\,dx = dt$$

$$\therefore \qquad I = \frac{1}{2}\int \sqrt{t^2 + 1^2}\,dt$$

$$= \frac{1}{2}\left\{\frac{t}{2}\sqrt{t^2+1} + \frac{1}{2}\log\left|t + \sqrt{t^2+1}\right|\right\} + c$$

$$\therefore \qquad I = \frac{1}{2}\left\{\frac{x^2}{2}\sqrt{x^4+1} + \frac{1}{2}\log\left|x^2 + \sqrt{x^4+1}\right|\right\} + c$$

Indefinite Integrals Ex 19.28 Q13

$$\text{Let } I = \int x^2\sqrt{a^6 - x^6}\,dx$$

$$\text{Let } x^3 = t$$

$$\Rightarrow \qquad 3x^2\,dx = dt$$

$$\therefore \qquad I = \frac{1}{3}\int \sqrt{a^6 - t^2}\,dt$$

$$= \frac{1}{3}\left\{\frac{t}{2}\sqrt{a^6 - t^2} + \frac{a^6}{2}\sin^{-1}\left(\frac{t}{a^3}\right)\right\} + c$$

$$\therefore \qquad I = \frac{x^3}{6}\sqrt{a^6 - x^6} + \frac{a^6}{6}\sin^{-1}\left(\frac{x^3}{a^3}\right) + c$$

Indefinite Integrals Ex 19.28 Q14

$$\text{Let } I = \int \frac{\sqrt{16 + (\log x)^2}}{x}\,dx$$

$$\text{Let } \log x = t$$

$$\Rightarrow \qquad \frac{1}{x}\,dx = dt$$

$$\therefore \qquad I = \int \sqrt{16 + t^2}\,dt$$

$$= \int \sqrt{4^2 + t^2}\,dt$$

$$= \frac{t}{2}\sqrt{16 + t^2} + \frac{16}{2}\log\left|t + \sqrt{16 + t^2}\right| + c$$

$$\therefore \qquad I = \frac{\log x}{2}\sqrt{16 + (\log x)^2} + 8\log\left|\log x + \sqrt{16 + (\log x)^2}\right| + c$$

Indefinite Integrals Ex 19.28 Q15

Let $I = \int \sqrt{2ax - x^2}\,dx$

$$= \int \sqrt{a^2 - \left(a^2 - 2ax + x^2\right)}\,dx \qquad \left[\text{Adding and subtracting } a^2\right]$$
$$= \int \sqrt{a^2 - (a - x)^2}\,dx$$
$$= \int \sqrt{a^2 - (x - a)^2}\,dx$$

$$= \frac{(x - a)}{2}\sqrt{2ax - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x - a}{a}\right) + c$$

$$\therefore \qquad I = \frac{1}{2}(x - a)\sqrt{2ax - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x - a}{a}\right) + c$$

Indefinite Integrals Ex 19.28 Q16

Let $I = \int \sqrt{3 - x^2}\,dx$

$$= \int \sqrt{\left(\sqrt{3}\right)^2 - x^2}\,dx$$

$$I = \frac{x}{2}\sqrt{3 - x^2} + \frac{3}{2}\sin^{-1}\left(\frac{x}{\sqrt{3}}\right) + c$$

Ex 19.29

Indefinite Integrals Ex 19.29 Q1

Let $I = \int (x+1)\sqrt{x^2 - x + 1}\,dx \qquad ---(1)$

Let $x + 1 = \lambda \frac{d}{dx}\left(x^2 - x + 1\right) + \mu$

$= \lambda(2x - 1) + \mu$

Equating similar terms, we get,

$2\lambda = 1 \Rightarrow \quad \lambda = \frac{1}{2}$

$-\lambda + \mu = 1$

$\Rightarrow \quad \mu = 1 + \lambda = 1 + \frac{1}{2} = \frac{3}{2} \therefore \quad \mu = \frac{3}{2}$

So,

$I = \int \left(\frac{1}{2}(2x - 1) + \frac{3}{2}\right)\sqrt{x^2 - x + 1}\,dx$

$= \frac{1}{2}\int (2x - 1)\sqrt{x^2 - x + 1}\,dx + \frac{3}{2}\int \sqrt{x^2 - x + 1}\,dx$

Let $x^2 - x + 1 = t$

$\Rightarrow \quad (2x - 1)\,dx = dt$

$= \frac{1}{2}\int \sqrt{t}\,dt + \frac{3}{2}\int \sqrt{\left(x - \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\,dx$

$= \frac{1}{2}\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{3}{2}\left\{\frac{\left(x - \frac{1}{2}\right)}{2}\sqrt{x^2 - x + 1} + \frac{3}{8}\log\left|\left(x - \frac{1}{2}\right) + \sqrt{x^2 - x + 1}\right|\right.$

$\Rightarrow \quad I = \frac{1}{3}t^{\frac{3}{2}} + \frac{3}{8}(2x - 1)\sqrt{x^2 - x + 1} + \frac{9}{16}.\log\left|\left(x - \frac{1}{2}\right) + \sqrt{x^2 - x + 1}\right| + c$

Hence,

$I = \frac{1}{3}\left(x^2 - x + 1\right)^{\frac{3}{2}} + \frac{3}{8}(2x - 1)\sqrt{x^2 - x + 1} + \frac{9}{16}\log\left|\left(x - \frac{1}{2}\right) + \sqrt{x^2 - x + 1}\right| + c$

Indefinite Integrals Ex 19.29 Q2

Let $I = \int (x+1)\sqrt{2x^2+3}\,dx$

Let $x + 1 = \lambda \frac{d}{dx}(2x^2+3) + \mu$

$= \lambda(4x) + \mu$

Equating similar terms, we get,

$4\lambda = 1 \Rightarrow \quad \lambda = \frac{1}{4}$

$\mu = 1$

$\therefore \quad I = \int \frac{1}{4}(4x)\sqrt{2x^2+3}\,dx + \int 1.\sqrt{2x^2+3}\,dx$

Let $2x^2 + 3 = t$

$\Rightarrow \quad 4x\,dx = dt$

$I = \frac{1}{4}\int \sqrt{t}\,dt + \sqrt{2}\int \sqrt{x^2+\frac{3}{2}}\,dx$

$= \frac{1}{4}.\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \sqrt{2}\left\{\frac{x}{2}\sqrt{x^2+\frac{3}{2}} + \frac{3}{4}\log\left|x+\sqrt{x^2+\frac{3}{2}}\right| + c\right.$

Hence,

$I = \frac{1}{6}(2x^2+3)^{\frac{3}{2}} + \frac{x}{2}\sqrt{2x^2+3} + \frac{3}{2\sqrt{2}}\log\left|x+\sqrt{x^2+\frac{3}{2}}\right| + c$

Indefinite Integrals Ex 19.29 Q3

Let $I = \int (2x-5)\sqrt{2+3x-x^2}\,dx$

Let $2x-5 = \lambda \frac{d}{dx}(2+3x-x^2) + \mu$
$= \lambda(3-2x) + \mu$

Equating similar terms, we get,

$-2\lambda = 2 \quad \Rightarrow \quad \lambda = -1$
$3\lambda + \mu = -5 \quad \Rightarrow \quad \mu = -5 - 3\lambda = -2$

$\therefore \quad \mu = -2$

So,
$I = \int (-1(3-2x) - 2)\sqrt{2+3x-x^2}\,dx$
$= -\int (3-2x)\sqrt{2+3x-x^2}\,dx - 2\int \sqrt{2+3x-x^2}\,dx$

Let $2+3x-x^2 = t$
$\Rightarrow \quad (3-2x)\,dx = dt$

$$I = -\int \sqrt{t}\,dt - 2\int \sqrt{\frac{17}{4} - \left(\frac{9}{4} - 3x - x^2\right)}\,dx$$
$$= -\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - 2\int \sqrt{\left(\frac{\sqrt{17}}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2}\,dx$$

$$\Rightarrow \quad I = -\frac{2}{3}(2+3x-x^2)^{\frac{3}{2}} - 2\left\{\frac{\left(x-\frac{3}{2}\right)}{2}\sqrt{2+3x-x^2} + \frac{17}{8}\sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{\sqrt{17}}{2}}\right)\right. + c$$

Hence,

$$I = -\frac{2}{3}(2+3x-x^2)^{\frac{3}{2}} - \frac{(2x-3)}{2}\sqrt{2+3x-x^2} - \frac{17}{8}\sin^{-1}\left(\frac{2x-3}{\sqrt{17}}\right) + c$$

Indefinite Integrals Ex 19.29 Q4

Let $I = \int (x+2)\sqrt{x^2+x+1}dx$

Let $x+2 = \lambda \frac{d}{dx}\left(x^2+x+1\right)+\mu$

$= \lambda\left(2x+1\right)+\mu$

Equating similar terms, we get,

$2\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{2}$

$\lambda+\mu = 2 \quad \Rightarrow \quad \mu = 2-\lambda = \frac{3}{2}$

$\therefore \quad \mu = \frac{3}{2}$

$\therefore \quad I = \int\left(\frac{1}{2}\left(2x+1\right)+\frac{3}{2}\right)\sqrt{x^2+x+1}dx$

$= \frac{1}{2}\int\left(2x+1\right)\sqrt{x^2+x+1}+\frac{3}{2}\int\sqrt{x^2+x+1}dx$

Let $x^2+x+1 = t$

$\left(2x+1\right)dx = dt$

$\therefore \quad I = \frac{1}{2}\int\sqrt{t}dt+\frac{3}{2}\int\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dx$

$\Rightarrow \quad I = \frac{1}{2}\frac{t^{\frac{3}{2}}}{\frac{3}{2}}+\frac{3}{2}\left\{\frac{\left(x+\frac{1}{2}\right)}{2}\sqrt{x^2+x+1}+\frac{3}{8}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x+1}\right|+c\right.$

Hence,

$I = \frac{1}{3}\left(x^2+x+1\right)^{\frac{3}{2}}+\frac{3}{8}\left(2x+1\right)\sqrt{x^2+x+1}+\frac{9}{16}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x+1}\right|+c$

Indefinite Integrals Ex 19.29 Q5

Let $I = \int (4x+1)\sqrt{x^2-x-2}\,dx$

Let $4x+1 = \lambda \frac{d}{dx}\left(x^2-x-2\right)+\mu$
$= \lambda\left(2x-1\right)+\mu$

Equating similar terms, we get,

$2\lambda = 4 \quad \Rightarrow \quad \lambda = 2$
$-\lambda+\mu = 1 \quad \Rightarrow \quad \mu = 3$

So,
$I = \int \left(2\left(2x-1\right)+3\right)\sqrt{x^2-x-2}\,dx$
$= 2\int\left(2x-1\right)\sqrt{x^2-x-2}\,dx + 3\int\sqrt{x^2-x-2}\,dx$

Let $x^2-x-2 = t$
$\left(2x-1\right)dx = dt$

$\therefore \quad I = 2\int\sqrt{t}\,dt + 3\int\sqrt{\left(x-\frac{1}{2}\right)^2-\left(\frac{3}{2}\right)^2}\,dx$

$\Rightarrow \quad I = 2\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + 3\left\{\frac{\left(x-\frac{1}{2}\right)}{2}\sqrt{x^2-x-2}-\frac{9}{8}\log\left|\left(x-\frac{1}{2}\right)+\sqrt{x^2-x-2}\right|+c\right.$

Hence,

$I = \frac{4}{3}\left(x^2-x-2\right)^{\frac{3}{2}}+\frac{3}{4}\left(2x-1\right)\sqrt{x^2-x-2}-\frac{27}{8}\log\left|\left(x-\frac{1}{2}\right)+\sqrt{x^2-x-2}\right|+c$

Indefinite Integrals Ex 19.29 Q6

Let $I = \int (x-2)\sqrt{2x^2 - 6x + 5}dx$

Let $x - 2 = \lambda \frac{d}{dx}(2x^2 - 6x + 5) + \mu$

$= \lambda(4x - 6) + \mu$

Equating similar terms, we get,

$4\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{4}$

$-6\lambda + \mu = -2 \quad \Rightarrow \quad \mu = -2 + 6\lambda = -\frac{2}{4} = -\frac{1}{2}$

$\therefore \quad \mu = -\frac{1}{2}$

So,

$I = \int \left(\frac{1}{4}(4x-6) + \left(-\frac{1}{2}\right)\right)\sqrt{2x^2 - 6x + 5}dx$

$= \frac{1}{4}\int (4x-6)\sqrt{2x^2 - 6x + 5} - \frac{1}{2}\int \sqrt{2x^2 - 6x + 5}dx$

Let $2x^2 - 6x + 5 = t$

$(4x - 6)dx = dt$

$\therefore \quad I = \frac{1}{4}\int \sqrt{t}dt - \frac{\sqrt{2}}{2}\int \sqrt{x^2 - 3x + \frac{5}{2}}dx$

$\Rightarrow \quad I = \frac{1}{4}\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{\sqrt{2}}\int \sqrt{\left(x - \frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2}dx$

$= \frac{1}{6}(2x^2 - 6x + 5)^{\frac{3}{2}} - \frac{1}{\sqrt{2}}\left\{\frac{\left(x - \frac{3}{2}\right)}{2}\sqrt{x^2 - 3x + \frac{5}{2}} + \frac{1}{8}\log\left|\left(x - \frac{3}{2}\right) + \sqrt{x^2 - 3x + \frac{5}{2}}\right| + c\right.$

Hence,

$I = \frac{1}{6}(2x^2 - 6x + 5)^{\frac{3}{2}} - \frac{1}{8}(2x - 3)\sqrt{2x^2 - 6x + 5} - \frac{1}{8\sqrt{2}}\log\left|\left(x - \frac{3}{2}\right) + \sqrt{x^2 - 3x + \frac{5}{2}}\right| + c$

Indefinite Integrals Ex 19.29 Q7

Let $I = \int (x+1)\sqrt{x^2+x+1}\,dx$

Let $x+1 = \lambda \frac{d}{dx}\left(x^2+x+1\right)+\mu$

$= \lambda\left(2x+1\right)+\mu$

Equating similar terms, we get,

$2\lambda = 1 \Rightarrow \quad \lambda = \frac{1}{2}$

$\lambda+\mu = 1 \Rightarrow \quad \mu = \frac{1}{2}$

So,

$I = \int\left(\frac{1}{2}\left(2x+1\right)+\frac{1}{2}\right)\sqrt{x^2+x+1}\,dx$

$= \frac{1}{2}\int\left(2x+1\right)\sqrt{x^2+x+1}\,dx+\frac{1}{2}\int\sqrt{x^2+x+1}\,dx$

Let $x^2+x+1 = t$

$\Rightarrow \quad \left(2x+1\right)dx = dt$

$= \frac{1}{2}\int\sqrt{t}\,dt+\frac{1}{2}\int\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\,dx$

$= \frac{1}{2}\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}}+\frac{1}{2}\left\{\frac{\left(x+\frac{1}{2}\right)}{2}\sqrt{x^2+x+1}+\frac{3}{8}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x+1}\right|+c\right.$

$\Rightarrow \quad I = \frac{1}{3}\left(x^2+x+1\right)^{\frac{3}{2}}+\frac{1}{8}\left(2x+1\right)\sqrt{x^2+x+1}+\frac{3}{16}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x+1}\right|+c$

Hence,

$I = \frac{1}{3}\left(x^2+x+1\right)^{\frac{3}{2}}+\frac{1}{8}\left(2x+1\right)\sqrt{x^2+x+1}+\frac{3}{16}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x+1}\right|+c$

Indefinite Integrals Ex 19.29 Q8

Let $I = \int (2x+3)\sqrt{x^2+4x+3}dx$

Let $(2x+3) = \lambda \frac{d}{dx}(x^2+4x+3) + \mu$

$= \lambda(2x+4) + \mu$

Equating similar terms, we get,

$\lambda = 1$ and $4\lambda + \mu = 3$

$\Rightarrow \quad \mu = -1$

So,

$I = \int ((2x+4)+(-1))\sqrt{x^2+4x+3}dx$

$= \int (2x+4)\sqrt{x^2+4x+3}dx - \int \sqrt{x^2+4x+3}dx$

Let $x^2+4x+3 = t$

$\Rightarrow \quad (2x+4)dx = dt$

$\therefore \quad I = \int \sqrt{t}dt - \int \sqrt{(x+2)^2 - 1}dx$

$= \frac{3}{2}t^{\frac{3}{2}} - \frac{(x+2)}{2}\sqrt{x^2+4x+3} + \frac{1}{2}\log\left|(x+2)+\sqrt{x^2+4x+3}\right| + c$

Hence,

$$I = \frac{2}{3}(x^2+4x+3)^{\frac{3}{2}} - \left(\frac{x+2}{2}\right)\sqrt{x^2+4x+3} + \frac{1}{2}\log\left|(x+2)+\sqrt{x^2+4x+3}\right| + c$$

Indefinite Integrals Ex 19.29 Q9

Let $I = \int (2x-5)\sqrt{x^2-4x+3}dx$

Let $(2x-5) = \lambda \frac{d}{dx}(x^2-4x+3) + \mu$

$= \lambda(2x-4) + \mu$

Equating similar terms, we get,

$\lambda = 1$ and $-4\lambda + \mu = -5$

$\Rightarrow \quad \mu = -1$

So,

$I = \int ((2x-4)-1)\sqrt{x^2-4x+3}dx$

$= \int (2x-4)\sqrt{x^2-4x+3}dx - \int \sqrt{x^2-4x+3}dx$

Let $x^2-4x+3 = t$

$\Rightarrow \quad 2x-4dx = dt$

$I = \int \sqrt{t}dt - \int \sqrt{(x-2)^2 - 1}dx$

$= \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(x-2)}{2}\sqrt{x^2-4x+3} + \frac{1}{2}\log\left|(x-2)+\sqrt{x^2-4x+3}\right| + c$

Thus,

$$I = \frac{2}{3}(x^2-4x+3)^{\frac{3}{2}} - \frac{1}{2}(x-2)\sqrt{x^2-4x+3} + \frac{1}{2}\log\left|(x-2)+\sqrt{x^2-4x+3}\right| + c$$

Indefinite Integrals Ex 19.29 Q10

Let $I = \int x\sqrt{x^2+x}dx$

Let $x = \lambda \frac{d}{dx}\left(x^2+x\right)+\mu$

$= \lambda\left(2x+1\right)+\mu$

Equating similar terms, we get,

$2\lambda = 1 \qquad \Rightarrow \qquad \lambda = \frac{1}{2}$

$\lambda+\mu = 0 \qquad \Rightarrow \qquad \mu = -\frac{1}{2}$

So,

$I = \int\left(\frac{1}{2}\left(2x+1\right)-\frac{1}{2}\right)\sqrt{x^2+x}dx$

$= \frac{1}{2}\int\left(2x+1\right)\sqrt{x^2+x} - \frac{1}{2}\int\sqrt{x^2+x}dx$

Let $x^2+x = t$

$\Rightarrow \qquad \left(2x+1\right)dx = dt$

So,

$I = \frac{1}{2}\int\sqrt{t}dt - \frac{1}{2}\int\sqrt{\left(x+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}$

$I = \frac{1}{2}\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{2}\left\{\frac{\left(x+\frac{1}{2}\right)}{2}\sqrt{x^2+x} - \frac{1}{8}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x}\right|+c\right.$

Hence,

$I = \frac{1}{3}\left(x^2+x\right)^{\frac{3}{2}} - \frac{1}{8}\left(x+\frac{1}{2}\right)\sqrt{x^2+x} + \frac{1}{16}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x}\right|+c$

Indefinite Integrals Ex 19.29 Q11

Consider the integral $I = \int (x-3)\sqrt{x^2+3x-18}\,dx$

Let us express $x-3 = \lambda\frac{d}{dx}\left[x^2+3x-18\right]+\mu$

$\Rightarrow x-3 = \lambda\left[2x+3\right]+\mu$

$\Rightarrow x-3 = 2\lambda x + 3\lambda + \mu$

Comparing the coefficients, we have,

$2\lambda = 1$ and $3\lambda + \mu = -3$

$\Rightarrow \lambda = \frac{1}{2}$ and $3\times\frac{1}{2}+\mu = -3$

$\Rightarrow \lambda = \frac{1}{2}$ and $\mu = -3-\frac{3}{2}$

$\Rightarrow \lambda = \frac{1}{2}$ and $\mu = -\frac{9}{2}$

Then

$x-3 = \lambda\left[2x+3\right]+\mu$

Now the integral $I = \int (x-3)\sqrt{x^2+3x-18}\,dx$

$= \int\left[\frac{1}{2}[2x+3]-\frac{9}{2}\right]\sqrt{x^2+3x-18}\,dx$

$I = \frac{1}{2}\int[2x+3]\sqrt{x^2+3x-18}\,dx - \frac{9}{2}\int\sqrt{x^2+3x-18}\,dx$

$\Rightarrow I = I_1 + I_2$

where, $I_1 = \frac{1}{2}\int[2x+3]\sqrt{x^2+3x-18}\,dx$ and

$I_2 = -\frac{9}{2}\int\sqrt{x^2+3x-18}\,dx$

Let us consider the integral, I_1 :

$I_1 = \frac{1}{2}\int[2x+3]\sqrt{x^2+3x-18}\,dx$

Substituting, $x^2+3x-18 = t$

$\Rightarrow (2x+3)dx = dt$

Thus,

$I_1 = \frac{1}{2}\int\sqrt{t}\,dt$

$= \frac{1}{2}\times\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C$

$= \frac{1}{2} \times \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C$

$= \frac{1}{2} \times \frac{2}{3} \times t^{\frac{3}{2}} + C$

$= \frac{1}{3} \times t^{\frac{3}{2}} + C$

$= \frac{1}{3} \times \left(x^2 + 3x - 18\right)^{\frac{3}{2}} + C$

Now consider the integral

$I_2 = -\frac{9}{2}\int \sqrt{x^2 + 3x - 18}\, dx$

$= -\frac{9}{2}\int \sqrt{x^2 + 2 \times \frac{3}{2}x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 18}\, dx$

$= -\frac{9}{2}\int \sqrt{\left(x + \frac{3}{2}\right)^2 - \frac{9}{4} - 18}\, dx$

$= -\frac{9}{2}\int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9}{4} + 18\right)}\, dx$

$= -\frac{9}{2}\int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9+72}{4}\right)}\, dx$

$= -\frac{9}{2}\int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{81}{4}\right)}\, dx$

$= -\frac{9}{2}\int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2}\, dx$

$= -\frac{9}{2}\int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2}\, dx$

We know that $\int \sqrt{x^2 - a^2}dx = \frac{1}{2}x\sqrt{x^2 - a^2} - \frac{1}{2}a^2 \log \left|x + \sqrt{x^2 - a^2}\right| + C$

$\therefore I_2 = -\frac{9}{2}\left\{\frac{1}{2}\left(x + \frac{3}{2}\right)\sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2} - \frac{1}{2}\left(\frac{9}{2}\right)^2 \log \left|\left(x + \frac{3}{2}\right) + \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2}\right|\right\} + C$

$= -\frac{9}{4}\left\{\left(\frac{2x+3}{2}\right)\sqrt{x^2 + 3x - 18} - \left(\frac{729}{4}\right)\log \left|\left(x + \frac{3}{2}\right) + \sqrt{x^2 + 3x - 18}\right|\right\} + C$

$= -\frac{9}{8}(2x + 3)\sqrt{x^2 + 3x - 18} + \frac{729}{16}\log \left|\left(x + \frac{3}{2}\right) + \sqrt{x^2 + 3x - 18}\right| + C$

Thus, $I = = \frac{1}{3}\left(x^2 + 3x - 18\right)^{\frac{3}{2}} - \frac{9}{8}(2x + 3)\sqrt{x^2 + 3x - 18} + \frac{729}{16}\log \left|\left(x + \frac{3}{2}\right) + \sqrt{x^2 + 3x - 18}\right| + C$

Indefinite Integrals Ex 19.29 Q12

$$\int (x+3)\sqrt{3-4x-x^2}dx$$

$$\text{Let } x+3 = A\frac{d}{dx}(3-4x-x^2)+B$$
$$x+3 = A(-4-2x)+B$$
$$x+3 = -2Ax+B-4A$$
$$-2A = 1, B-4A = 3$$
$$A = -\frac{1}{2},$$
$$B = 4\times\left(-\frac{1}{2}\right)+3 = 1$$

$$\int (x+3)\sqrt{3-4x-x^2}dx$$
$$x+3 = -\frac{1}{2}(-4-2x)+1$$
$$\int\left[-\frac{1}{2}(-4-2x)+1\right]\sqrt{3-4x-x^2}dx$$
$$= -\frac{1}{2}\int(-4-2x)\sqrt{3-4x-x^2}dx + \int\sqrt{3-4x-x^2}dx$$
$$= I_1 + I_2 ....(i)$$
$$I_1 = -\frac{1}{2}\int(-4-2x)\sqrt{3-4x-x^2}dx$$
$$\text{Let } z = 3-4x-x^2$$
$$dz = -4-2x$$
$$I_1 = -\frac{1}{2}\int\sqrt{z}dz$$
$$= -\frac{1}{2}\left[\frac{z^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]$$
$$= -\frac{1}{2}\left[\frac{z^{\frac{3}{2}}}{\frac{3}{2}}\right]$$
$$= -\left[\frac{\left(3-4x-x^2\right)^{\frac{3}{2}}}{3}\right]$$
$$I_2 = \int\sqrt{3-4x-x^2}dx$$
$$= \int\sqrt{3-(x^2+4x+4)+4}dx$$
$$= \int\sqrt{7-(x^2+4x+4)}dx$$
$$= \int\sqrt{\left(\sqrt{7}\right)^2-(x+2)^2}dx$$
$$= \frac{(x+2)\sqrt{\left(\sqrt{7}\right)^2-(x+2)^2}}{2} + \frac{1}{2}\left(\sqrt{7}\right)^2\tan^{-1}\left(\frac{x+2}{\sqrt{7}}\right)+C$$
$$= \frac{(x+2)\sqrt{3-4x-x^2}}{2} + \frac{7}{2}\tan^{-1}\left(\frac{x+2}{\sqrt{7}}\right)$$

$$\text{From}(i),$$
$$= I_1 + I_2$$
$$= -\frac{1}{3}\left(3-4x-x^2\right)^{\frac{3}{2}} + \frac{1}{2}(x+2)\sqrt{3-4x-x^2} + \frac{7}{2}\tan^{-1}\left(\frac{x+2}{\sqrt{7}}\right)+C$$

Ex 19.30

Indefinite Integrals Ex 19.30 Q1

Let $\int \frac{2x+1}{(x+1)(x-2)} = \frac{A}{(x+1)} + \frac{B}{(x-2)}$

$\Rightarrow \quad 2x+1 = A(x-2) + B(x+1)$

Put $x = 2$

$\Rightarrow \quad 5 = 3B \Rightarrow \quad B = \frac{5}{3}$

Put $x = -1$

$\Rightarrow \quad -1 = -3A \quad \Rightarrow \quad A = \frac{1}{3}$

So,

$$\int \frac{2x+1}{(x+1)(x-2)} dx = \frac{1}{3}\int \frac{dx}{x+1} + \frac{5}{3}\int \frac{dx}{x-2}$$
$$= \frac{1}{3}\log|x+1| + \frac{5}{3}\log|x-2| + c$$

Thus,

$$I = \frac{1}{3}\log|x+1| + \frac{5}{3}\log|x-2| + c$$

Indefinite Integrals Ex 19.30 Q2

Let $\int \frac{1}{x(x-2)(x-4)}dx = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x-4}$

$\Rightarrow \quad 1 = A(x-2)(x-4) + B(x)(x-4) + Cx(x-2)$

Put $x = 0$

$\Rightarrow \quad 1 = 8A \Rightarrow \quad A = \frac{1}{8}$

Put $x = 2$

$\Rightarrow \quad 1 = -4B \quad \Rightarrow \quad B = -\frac{1}{4}$

Put $x = 4$

$\Rightarrow \quad 1 = 8C \Rightarrow \quad C = \frac{1}{8}$

So,

$$\int \frac{1}{x(x-2)(x-4)}dx = \frac{1}{8}\int \frac{dx}{x} + \left(-\frac{1}{4}\right)\int \frac{dx}{x-2} + \frac{1}{8}\int \frac{dx}{x-4}$$
$$= \frac{1}{8}\log|x| - \frac{1}{4}\log|x-2| + \frac{1}{8}\log|x-4| + c$$
$$= \frac{1}{8}\log\left|\frac{x(x-4)}{(x-2)^2}\right| + c$$

Thus,

$$I = \frac{1}{8}\log\left|\frac{x(x-4)}{(x-2)^2}\right| + c$$

Indefinite Integrals Ex 19.30 Q3

Let $I = \int \frac{x^2+x-1}{x^2+x-6}dx$

$= \int 1 + \frac{5}{x^2+x-6}dx$

$\Rightarrow \quad I = \int dx + \int \frac{5dx}{(x+3)(x-2)} \qquad ---(1)$

Let $\frac{5}{(x+3)(x-2)} = \frac{A}{x+3} + \frac{B}{x-2}$

$\Rightarrow \quad 5 = A(x-2) + B(x+3)$

Put $x = 2$

$\Rightarrow \quad 5 = 5B \Rightarrow \quad B = 1$

Put $x = -3$

$\Rightarrow \quad 5 = -5A \quad \Rightarrow \quad A = -1$

$\therefore \quad I = \int dx + \int \frac{-dx}{x+3} + \int \frac{dx}{x-2}$

$= x - \log|x+3| + \log|x-2| + c$

Hence,

$I = x - \log|x+3| + \log|x-2| + c$

Indefinite Integrals Ex 19.30 Q4

Let $I = \int \frac{3 + 4x - x^2}{(x+2)(x-1)} dx$

$= \int -1 + \frac{5x+1}{(x+2)(x-1)} dx$

$\Rightarrow \quad I = -\int dx + \int \frac{5x+1}{(x+2)(x-1)} dx \qquad ---(1)$

Let $\frac{5x+1}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$

$\Rightarrow \quad 5x + 1 = A(x-1) + B(x+2)$

Put $x = 1$

$\Rightarrow \quad 6 = 3B \Rightarrow \quad B = 2$

Put $x = -2$

$\Rightarrow \quad -9 = -3A \quad \Rightarrow \quad A = 3$

So,

$I = -\int dx + 3\int \frac{dx}{x+2} + 2\int \frac{dx}{x-1}$

$I = -x + 3\log|x+2| + 2\log|x-1| + c$

Indefinite Integrals Ex 19.30 Q5

Let $I = \int \frac{x^2+1}{x^2-1} dx$

$= \int 1 + \frac{2}{x^2-1} dx$

$= \int dx + \int \frac{2dx}{(x+1)(x-1)}$

$= \int dx + \int \frac{-1}{x+1} + \frac{1}{x-1} dx$

$= x - \log|x+1| + \log|x-1| + c$

$I = x + \log\left|\frac{x-1}{x+1}\right| + c$

Indefinite Integrals Ex 19.30 Q6

Let $I = \int \frac{x^2}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$

$\Rightarrow \quad x^2 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$

Put $x = 1$

$\Rightarrow \quad 1 = 2A \Rightarrow \quad A = \frac{1}{2}$

Put $x = 2$

$\Rightarrow \quad 4 = -B \Rightarrow \quad B = -4$

Put $x = 3$

$\Rightarrow \quad 9 = 2C \Rightarrow \quad C = \frac{9}{2}$

Thus,

$$I = \int \frac{x^2}{(x-1)(x-2)(x-3)} dx = \frac{1}{2}\int \frac{dx}{x-1} - 4\int \frac{dx}{x-2} + \frac{9}{2}\int \frac{dx}{x-3}$$

$$= \frac{1}{2}\log|x-1| - 4\log|x-2| + \frac{9}{2}\log|x-3| + c$$

Hence,

$$I = \frac{1}{2}\log|x-1| - 4\log|x-2| + \frac{9}{2}\log|x-3| + c$$

Indefinite Integrals Ex 19.30 Q7

$$\frac{5x}{(x+1)(x^2-4)} = \frac{5x}{(x+1)(x+2)(x-2)}$$

Let $\frac{5x}{(x+1)(x+2)(x-2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)} + \frac{C}{(x-2)}$

$$5x = A(x+2)(x-2) + B(x+1)(x-2) + C(x+1)(x+2) \qquad \ldots(1)$$

Substituting $x = -1$, -2, and 2 respectively in equation (1), we obtain

$A = \frac{5}{3}$, $B = -\frac{5}{2}$, and $C = \frac{5}{6}$

$$\therefore \frac{5x}{(x+1)(x+2)(x-2)} = \frac{5}{3(x+1)} - \frac{5}{2(x+2)} + \frac{5}{6(x-2)}$$

$$\Rightarrow \int \frac{5x}{(x+1)(x^2-4)} dx = \frac{5}{3}\int \frac{1}{(x+1)} dx - \frac{5}{2}\int \frac{1}{(x+2)} dx + \frac{5}{6}\int \frac{1}{(x-2)} dx$$

$$= \frac{5}{3}\log|x+1| - \frac{5}{2}\log|x+2| + \frac{5}{6}\log|x-2| + C$$

Indefinite Integrals Ex 19.30 Q8

Let $I = \int \frac{x^2+1}{x\left(x^2-1\right)} dx = \int \frac{x^2+1}{x\left(x+1\right)\left(x-1\right)} dx$

Let $\frac{x^2+1}{x\left(x+1\right)\left(x-1\right)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}$

$\Rightarrow \quad x^2+1 = A\left(x+1\right)\left(x-1\right) + B.x\left(x-1\right) + Cx\left(x+1\right)$

Put $x = 0$

$\Rightarrow \quad 1 = -A \Rightarrow \quad A = -1$

Put $x = -1$

$\Rightarrow \quad 2 = 2B \quad \Rightarrow \quad B = 1$

Put $x = 1$

$\Rightarrow \quad 2 = 2C \Rightarrow \quad C = 1$

Thus,

$I = -\int \frac{dx}{x} + \int \frac{dx}{x+1} + \int \frac{dx}{x-1}$

$= -\log|x| + \log|x+1| + \log|x-1| + c$

$I = \log\left|\frac{x^2-1}{x}\right| + c$

Indefinite Integrals Ex 19.30 Q9

Let $I = \int \frac{2x-3}{\left(x^2-1\right)\left(2x+3\right)} dx = \int \frac{2x-3}{\left(x+1\right)\left(x-1\right)\left(2x+3\right)} dx$

Let $\frac{2x-3}{\left(x+1\right)\left(x-1\right)\left(2x+3\right)} = \frac{A}{\left(x+1\right)} + \frac{B}{\left(x-1\right)} + \frac{C}{\left(2x+3\right)}$

$\Rightarrow \quad 2x-3 = A\left(x-1\right)\left(2x+3\right) + B\left(x+1\right)\left(2x+3\right) + C\left(x^2-1\right)$

Put $x = -1$

$\Rightarrow \quad -5 = -2A \quad \Rightarrow \quad A = \frac{5}{2}$

Put $x = 1$

$\Rightarrow \quad -1 = 10B \quad \Rightarrow \quad B = -\frac{1}{10}$

Put $x = -\frac{3}{2}$

$\Rightarrow \quad -6 = \frac{5}{4}C \quad \Rightarrow \quad C = -\frac{24}{5}$

Thus,

$I = \frac{5}{2}\int \frac{dx}{x+1} - \frac{1}{10}\int \frac{dx}{x-1} - \frac{24}{5}\int \frac{dx}{2x+3}$

$= \frac{5}{2}\log|x+1| - \frac{1}{10}\log|x-1| - \frac{24}{5}.\frac{1}{2}\log|2x+3| + c$

Hence,

$I = \frac{5}{2}\log|x+1| - \frac{1}{10}\log|x-1| - \frac{12}{5}\log|2x+3| + c$

Indefinite Integrals Ex 19.30 Q10

Let $I = \int \frac{x^3}{(x-1)(x-2)(x-3)} dx$

$= \int 1 + \frac{6x^2 - 9x + 6}{(x-1)(x-2)(x-3)} dx$

Let $\frac{6x^2 - 11x + 6}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$

$\Rightarrow \quad 6x^2 - 11x + 6 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$

Put $x = 1$

$\Rightarrow \quad 1 = 2A \Rightarrow \quad A = \frac{1}{2}$

Put $x = 2$

$\Rightarrow \quad 8 = -B \quad \Rightarrow \quad B = -8$

Put $x = 3$

$\Rightarrow \quad 27 = 2C \quad \Rightarrow \quad C = \frac{27}{2}$

Thus,

$I = \int dx + \frac{1}{2}\int \frac{dx}{x-1} - 8\int \frac{dx}{x-2} + \frac{27}{2}\int \frac{dx}{x-3}$

$= x + \frac{1}{2}\log|x-1| - 8\log|x-2| + \frac{27}{2}\log|x-3| + c$

Hence,

$I = x + \frac{1}{2}\log|x-1| - 8\log|x-2| + \frac{27}{2}\log|x-3| + c$

Indefinite Integrals Ex 19.30 Q11

Let $\int \frac{\sin 2x}{(1+\sin x)(2+\sin x)} dx = \frac{A}{1+\sin x} + \frac{B}{2+\sin x}$

$\Rightarrow \quad \sin 2x = A(2+\sin x) + B(1+\sin x)$

$\Rightarrow \quad 2\sin x\cos x = (2A+B) + (A+B)\sin x$

Equating similar terms, we get,

$2A + B = 0 \quad \Rightarrow \quad B = -2A$ and

$A + B = 2\cos x \Rightarrow \quad -A = 2\cos x$

$\Rightarrow \quad A = -2\cos x$

and $B = +4\cos x$

Thus,

$I = \int -\frac{2\cos x}{1+\sin x}dx + \int \frac{4\cos x}{2+\sin x}dx$

$= -2\log|1+\sin x| + 4\log|2+\sin x| + c$

$I = \log\left|\frac{(2+\sin x)^4}{(1+\sin x)^2}\right| + c$

Indefinite Integrals Ex 19.30 Q12

Let $\int \frac{2x}{\left(x^2+1\right)\left(x^2+3\right)} dx = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+3}$

$$\Rightarrow \quad 2x = (Ax+B)\left(x^2+3\right) + (Cx+D)\left(x^2+1\right)$$
$$= (A+C)x^3 + (B+D)x^2 + (3A+C)x + (3B+D)$$

Equating similar terms, we get,

$A+C=0,\ B+D=0,\ 3A+C=2$ and $3B+D=0$

$\Rightarrow \quad A=-C,\ B=D=0\ 2A=2 \quad \Rightarrow \quad A=1\ \&\ C=-1$

Thus,

$$I = \int \frac{xdx}{x^2+1} - \int \frac{xdx}{x^2+3}$$
$$= \frac{1}{2}\log\left|x^2+1\right| - \frac{1}{2}\log\left|x^2+3\right| + c$$

$$I = \frac{1}{2}\log\left|\frac{x^2+1}{x^2+3}\right| + c$$

Indefinite Integrals Ex 19.30 Q13

Let $\int \frac{1}{x\log x\,(2+\log x)} = \frac{A}{x\log x} + \frac{B}{x\,(2+\log x)}$

$\Rightarrow \quad 1 = A(2+\log x) + B\log x$

Put $x=1$

$\Rightarrow \quad 1 = 2A \Rightarrow \quad A = \frac{1}{2}$

Put $x = 10^{-2}$

$\Rightarrow \quad 1 = -2B \quad \Rightarrow \quad B = -\frac{1}{2}$

Thus,

$$I = \frac{1}{2}\int \frac{dx}{x\log x} + \left(-\frac{1}{2}\right)\int \frac{dx}{x\,(2+\log x)}$$
$$= \frac{1}{2}\log\left|\log x\right| - \frac{1}{2}\log\left|2+\log x\right| + c$$

$$I = \frac{1}{2}\log\left|\frac{\log x}{2+\log x}\right| + c$$

Indefinite Integrals Ex 19.30 Q15

Let $\frac{ax^2+bx+c}{(x-a)(x-b)(x-c)} = \frac{A}{x-a}+\frac{B}{x-b}+\frac{C}{x-c}$

$\Rightarrow \quad ax^2+bx+c = A(x-b)(x-c)+B(x-a)(x-c)+C(x-a)(x-b)$

Put $x = a$

$\Rightarrow \quad a^3+ba+c = (a-b)(a-c)A \Rightarrow \qquad A = \frac{a^3+ba+c}{(a-b)(a-c)}$

Put $x = b$

$\Rightarrow \quad ab^2+b^2+c = (b-a)(b-c)B \Rightarrow \qquad B = \frac{ab^2+b^2+c}{(b-a)(b-c)}$

Put $x = c$

$\Rightarrow \quad ac^2+bc+c = (c-a)(c-b)C \Rightarrow \qquad C = \frac{ac^2+bc+c}{(c-a)(c-b)}$

Thus,

$$I = \frac{a^3+ba+c}{(a-b)(a-c)}\int\frac{dx}{x-a}+\frac{ab^2+b^2+c}{(b-a)(b-c)}\int\frac{dx}{x-b}+\frac{ac^2+bc+c}{(c-a)(c-b)}\int\frac{dx}{x-c}$$

Hence,

$$I = \frac{a^3+ba+c}{(a-b)(a-c)}\log|x-a|+\frac{ab^2+b^2+c}{(b-a)(b-c)}\log|x-b|+\frac{ac^2+bc+c}{(c-a)(c-b)}\log|x-c|+c$$

Indefinite Integrals Ex 19.30 Q16

Consider the integral

$$I=\int \frac{x}{\left(x^2+1\right)(x-1)}dx$$

Now let us separate the fraction $\frac{x}{\left(x^2+1\right)(x-1)}$

through partial fractions.

$$\frac{x}{\left(x^2+1\right)(x-1)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+1}$$

$$\Rightarrow \frac{x}{\left(x^2+1\right)(x-1)}=\frac{A\left(x^2+1\right)+(Bx+C)(x-1)}{\left(x^2+1\right)(x-1)}$$

$$\Rightarrow x=A\left(x^2+1\right)+(Bx+C)(x-1)$$

$$\Rightarrow x=Ax^2+A+Bx^2-Bx+Cx-C$$

Comparing the coefficients, we have,

A+B=0, $-B+C=1$ and $A-C=0$

Solving the equations, we get,

$\Rightarrow A=\frac{1}{2}$, $B=-\frac{1}{2}$ and $C=\frac{1}{2}$

$$\Rightarrow \frac{x}{\left(x^2+1\right)(x-1)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+1}$$

$$\Rightarrow \frac{x}{\left(x^2+1\right)(x-1)}=\frac{1}{2}\times\frac{1}{x-1}-\frac{1}{2}\times\frac{x-1}{x^2+1}$$

$$\Rightarrow \frac{x}{\left(x^2+1\right)(x-1)}=\frac{1}{2(x-1)}-\frac{x}{2\left(x^2+1\right)}+\frac{1}{2\left(x^2+1\right)}$$

Thus, we have,

$$I=\int \frac{x}{\left(x^2+1\right)(x-1)}dx$$

$$=\int\left[\frac{1}{2(x-1)}-\frac{x}{2\left(x^2+1\right)}+\frac{1}{2\left(x^2+1\right)}\right]dx$$

$$=\int\frac{dx}{2(x-1)}-\int\frac{xdx}{2\left(x^2+1\right)}+\int\frac{dx}{2\left(x^2+1\right)}$$

$$=\frac{1}{2}\int\frac{dx}{(x-1)}-\frac{1}{2}\int\frac{xdx}{\left(x^2+1\right)}+\frac{1}{2}\int\frac{dx}{\left(x^2+1\right)}$$

$$=\frac{1}{2}\int\frac{dx}{(x-1)}-\frac{1}{2}\times\frac{1}{2}\int\frac{2xdx}{\left(x^2+1\right)}+\frac{1}{2}\int\frac{dx}{\left(x^2+1\right)}$$

$$=\frac{1}{2}\log|x-1|-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\tan^{-1}x+C$$

Indefinite Integrals Ex 19.30 Q17

Let $I = \int \frac{1}{(x-1)(x+1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+2}$

$\Rightarrow \quad 1 = A(x+1)(x+2) + B(x-1)(x+2) + C(x^2-1)$

Put $x = 1$

$\Rightarrow \quad 1 = 6A \Rightarrow \quad A = \frac{1}{6}$

Put $x = -1$

$\Rightarrow \quad 1 = -2B \quad \Rightarrow \quad B = -\frac{1}{2}$

Put $x = -2$

$\Rightarrow \quad 1 = 3C \Rightarrow \quad C = \frac{1}{3}$

So,

$I = \frac{1}{6}\int \frac{dx}{x-1} - \frac{1}{2}\int \frac{dx}{x+1} + \frac{1}{3}\int \frac{dx}{x+2}$

$I = \frac{1}{6}\log|x-1| - \frac{1}{2}\log|x+1| + \frac{1}{3}\log|x+2| + c$

Indefinite Integrals Ex 19.30 Q18

Consider the integral

$$I=\int \frac{x^2}{(x^2+4)(x^2+9)}dx$$

Now let us separate the fraction $\frac{x^2}{(x^2+4)(x^2+9)}$

through partial fractions.

Substitute $x^2 = t$

$$\frac{x^2}{(x^2+4)(x^2+9)} = \frac{t}{(t+4)(t+9)}$$

$$\Rightarrow \frac{t}{(t+4)(t+9)} = \frac{A}{t+4} + \frac{B}{t+9}$$

$$\Rightarrow \frac{t}{(t+4)(t+9)} = \frac{A(t+9)+B(t+4)}{(t+4)(t+9)}$$

$$\Rightarrow t = A(t+9)+B(t+4)$$

$$\Rightarrow t = At + 9A + Bt + 4B$$

Comparing the coefficients, we have,

A+B=1 and 9A+4B=0

$$\Rightarrow A=-\frac{4}{5} \text{ and } B=\frac{9}{5}$$

$$\Rightarrow \frac{x^2}{(x^2+4)(x^2+9)} = -\frac{4}{5(t+4)} + \frac{9}{5(t+9)}$$

$$\Rightarrow \frac{x^2}{(x^2+4)(x^2+9)} = -\frac{4}{5(x^2+4)} + \frac{9}{5(x^2+9)}$$

Thus, we have,

$$I=\int \frac{x^2}{(x^2+4)(x^2+9)}dx$$

$$= \int\left[-\frac{4}{5(x^2+4)} + \frac{9}{5(x^2+9)}\right]dx$$

$$= -\int \frac{4dx}{5(x^2+4)} + \int \frac{9dx}{5(x^2+9)}$$

$$= -\frac{4}{5}\int \frac{dx}{(x^2+4)} + \frac{9}{5}\int \frac{dx}{(x^2+9)}$$

$$= -\frac{4}{5} \times \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + \frac{9}{5} \times \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$$

$$= -\frac{2}{5}\tan^{-1}\left(\frac{x}{2}\right) + \frac{3}{5}\tan^{-1}\left(\frac{x}{3}\right) + C$$

Indefinite Integrals Ex 19.30 Q19

Let $\int \frac{5x^2-1}{x(x-1)(x+1)}dx = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$

$\Rightarrow \quad 5x^2 - 1 = A(x^2-1) + B(x+1)x + C(x-1)x$

Put $x = 0$

$\Rightarrow \quad -1 = -A \Rightarrow \quad A = 1$

Put $x = +1$

$\Rightarrow \quad 4 = 2B \Rightarrow \quad B = 2$

Put $x = -1$

$\Rightarrow \quad 4 = 2C \quad \Rightarrow \quad C = 2$

So,

$$I = \int \frac{dx}{x} + \int \frac{2dx}{x-1} + \int \frac{2dx}{x+1}$$

$$= \log|x| + 2\log|x-1| + 2\log|x+1| + c$$

$$I = \log\left|x\left(x^2-1\right)^2\right|$$

Indefinite Integrals Ex 19.30 Q20

Let $I = \int \frac{x^2 + 6x - 8}{x^3 - 4x} dx$

$\Rightarrow \quad I = \int \frac{x^2 + 6x - 8}{x(x+2)(x-2)} dx$

Now,

Let $\frac{x^2 + 6x - 8}{x(x+2)(x-2)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-2}$

$\Rightarrow \quad x^2 + 6x - 8 = A(x^2 - 4) + B(x-2)x + C(x+2)x$

Put $x = 0$

$\Rightarrow \quad -8 = -4A \Rightarrow \quad A = 2$

Put $x = -2$

$\Rightarrow \quad -16 = 8B \quad \Rightarrow \quad B = -2$

Put $x = 2$

$\Rightarrow \quad 8 = 8C \Rightarrow \quad C = 1$

Thus,

$I = \int \frac{2dx}{x} - \int \frac{2dx}{x+2} + \int \frac{dx}{x-2}$

$= 2\log|x| - 2\log|x+2| + \log|x-2| + c$

$\therefore \quad I = \log\left|\frac{x^2(x-2)}{(x+2)^2}\right| + c$

Indefinite Integrals Ex 19.30 Q21

Let $\int \frac{x^2 + 1}{(2x+1)(x^2 - 1)} = \frac{A}{2x+1} + \frac{Bx + C}{x^2 - 1}$

$\Rightarrow \quad x^2 + 1 = A(x^2 - 1) + (Bx + C)(2x + 1)$

$= (A + 2B)x^2 + (B + 2C)x + (-A + C)$

Equating similar terms, we get,

$A + 2B = 1$, $B + 2C = 0$ and $-A + C = 1$

Solving we get,

$A = -\frac{5}{3} \qquad B = \frac{4}{3} \qquad C = -\frac{2}{3}$

Thus,

$I = -\frac{5}{3}\int \frac{dx}{2x+1} + \int \frac{\frac{4}{3}x - \frac{2}{3}}{x^2 - 1} dx$

$= -\frac{5}{3}\int \frac{dx}{2x+1} + \frac{2}{3}\int \frac{2xdx}{x^2 - 1} - \frac{2}{3}\int \frac{dx}{x^2 - 1}$

$= -\frac{5}{3}\int \frac{dx}{2x+1} + \frac{2}{3}\int \frac{2x - 1}{(x+1)(x-1)} dx$

$= -\frac{5}{3}\int \frac{dx}{2x+1} + \frac{2}{3}\left(\int \left(\frac{\frac{3}{2}}{(x+1)} + \frac{\frac{1}{2}}{(x-1)}\right) dx\right)$

$I = -\frac{5}{6}\log|2x+1| + \log|x+1| + \frac{1}{3}\log|x-1| + c$

Indefinite Integrals Ex 19.30 Q22

Let $I = \int \frac{dx}{x\left[6(\log x)^2 + 7\log x + 2\right]}$

$= \int \frac{1}{x(2\log x + 1)(3\log x + 2)}dx$

Now,

Let $\frac{1}{x(2\log x + 1)(3\log x + 2)} = \frac{A}{x(2\log x + 1)} + \frac{B}{x(3\log x + 2)}$

$\Rightarrow \quad 1 = A(3\log x + 2) + B(2\log x + 1)$

Put $x = 10^{-\frac{1}{2}}$

$\Rightarrow \quad 1 = \frac{1}{2}A \Rightarrow \quad A = 2$

Put $x = 10^{-\frac{2}{3}}$

$\Rightarrow \quad 1 = -\frac{1}{3}B \quad \Rightarrow \quad B = -3$

$\therefore \quad I = \int \frac{2dx}{x(2\log x + 1)} - \int \frac{3dx}{x(3\log x + 2)}$

$= \log|2\log x + 1| - \log|3\log x + 2| + c$

$\therefore \quad I = \log\left|\frac{2\log x + 1}{3\log x + 2}\right| + c$

Indefinite Integrals Ex 19.30 Q23

$\frac{1}{x(x^n + 1)}$

Multiplying numerator and denominator by x^{n-1}, we obtain

$\frac{1}{x(x^n+1)} = \frac{x^{n-1}}{x^{n-1}x(x^n+1)} = \frac{x^{n-1}}{x^n(x^n+1)}$

Let $x^n = t \Rightarrow x^{n-1}dx = dt$

$\therefore \int \frac{1}{x(x^n+1)}dx = \int \frac{x^{n-1}}{x^n(x^n+1)}dx = \frac{1}{n}\int \frac{1}{t(t+1)}dt$

Let $\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{(t+1)}$

$1 = A(1+t) + Bt \qquad \ldots(1)$

Substituting $t = 0, -1$ in equation (1), we obtain

$A = 1$ and $B = -1$

$\therefore \frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{(1+t)}$

$\Rightarrow \int \frac{1}{x(x^n+1)}dx = \frac{1}{n}\int\left\{\frac{1}{t} - \frac{1}{(t+1)}\right\}dx$

$= \frac{1}{n}\left[\log|t| - \log|t+1|\right] + C$

$= -\frac{1}{n}\left[\log|x^n| - \log|x^n+1|\right] + C$

$= \frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right| + C$

Indefinite Integrals Ex 19.30 Q24

Let $\int \frac{x}{\left(x^2-a^2\right)\left(x^2-b^2\right)} = \frac{Ax+B}{\left(x^2-a^2\right)} + \frac{Cx+D}{\left(x^2-b^2\right)}$

$\Rightarrow \quad x = \left(Ax+B\right)\left(x^2-b^2\right) + \left(Cx+D\right)\left(x^2-a^2\right)$

$\quad x = \left(A+C\right)x^3 + \left(B+D\right)x^2 + \left(-Ab^2-Ca^2\right)x + \left(-Bb^2-Da^2\right)$

$\Rightarrow \quad A+C=0,\ B+D=0,\ -Ab^2-Ca^2=1,\ -Bb^2-Da^2=0$

We get $B=0,\ D=0,\ C=\frac{1}{b^2-a^2},\ A=-\frac{1}{b^2-a^2}$

Thus,

$$I = -\frac{1}{b^2-a^2}\int \frac{xdx}{x^2-a^2} + \frac{1}{b^2-a^2}\int \frac{xdx}{x^2-b^2}$$

$$I = -\frac{1}{2\left(b^2-a^2\right)}\log\left|x^2-a^2\right| + \frac{1}{2\left(b^2-a^2\right)}\log\left|x^2-b^2\right| + c$$

Indefinite Integrals Ex 19.30 Q25

Consider the integral

$$I = \int \frac{x^2+1}{\left(x^2+4\right)\left(x^2+25\right)}dx$$

Let $y=x^2$

Thus,

$$\frac{x^2+1}{\left(x^2+4\right)\left(x^2+25\right)} = \frac{y+1}{\left(y+4\right)\left(y+25\right)}$$

$$\Rightarrow \frac{y+1}{\left(y+4\right)\left(y+25\right)} = \frac{A}{y+4} + \frac{B}{y+25}$$

$$\Rightarrow \frac{y+1}{\left(y+4\right)\left(y+25\right)} = \frac{A\left(y+25\right)+B\left(y+4\right)}{\left(y+4\right)\left(y+25\right)}$$

$\Rightarrow y+1 = Ay+25A+By+4B$

Comparing the coefficients, we have,

$A+B=1$ and $25A+4B=1$

Solving the above equations, we have,

$A=\frac{-1}{7}$ and $B=\frac{8}{7}$

Thus, $\int \frac{x^2+1}{\left(x^2+4\right)\left(x^2+25\right)}dx$

$$= \int \frac{\frac{-1}{7}}{x^2+4}dx + \int \frac{\frac{8}{7}}{x^2+25}dx$$

$$= \frac{-1}{7}\int \frac{1}{x^2+4}dx + \frac{8}{7}\int \frac{1}{x^2+25}dx$$

$$= \frac{-1}{7} \times \frac{1}{2}\tan^{-1}\frac{x}{2} + \frac{8}{7} \times \frac{1}{5}\tan^{-1}\frac{x}{5} + C$$

$$= \frac{-1}{14}\tan^{-1}\frac{x}{2} + \frac{8}{35}\tan^{-1}\frac{x}{5} + C$$

Indefinite Integrals Ex 19.30 Q26

Let $I = \int \frac{x^3 + x + 1}{x^2 - 1} dx$

$= \int \left(x + \frac{2x + 1}{x^2 - 1} \right) dx$

Now,

Let $\frac{2x + 1}{x^2 - 1} = \frac{A}{x + 1} + \frac{B}{x - 1}$

$\Rightarrow \quad 2x + 1 = A(x - 1) + B(x + 1)$

Put $x = 1$

$\Rightarrow \quad 3 = 2B \Rightarrow \quad B = \frac{3}{2}$

Put $x = -1$

$\Rightarrow \quad -1 = -2A \Rightarrow \quad A = \frac{1}{2}$

$\therefore \quad I = \int x dx + \frac{1}{2} \int \frac{dx}{x + 1} + \frac{3}{2} \int \frac{dx}{x - 1}$

$I = \frac{x^2}{2} + \frac{1}{2} \log|x + 1| + \frac{3}{2} \log|x - 1| + c$

Indefinite Integrals Ex 19.30 Q27

Let $\frac{3x - 2}{(x + 1)^2 (x + 3)} = \frac{A}{(x + 1)} + \frac{B}{(x + 1)^2} + \frac{C}{(x + 3)}$

$\Rightarrow \quad 3x - 2 = A(x + 1)(x + 3) + B(x + 3) + C(x + 1)^2$

$= (A + C)x^2 + (4A + B + 2C)x + (3A + 3B + C)$

Equating similar terms, we get,

$A + C = 0 \quad \Rightarrow \quad A = -C$

$4A + B + 2C = 3 \quad \Rightarrow \quad B = -2C = 3$

$3A + 3B + C = -2 \Rightarrow \quad 3B - 2C = -2$

Solving, we get, $B = -\frac{5}{2}$, $C = -\frac{11}{4}$ & $A = \frac{11}{4}$

Thus,

$I = \frac{11}{4} \int \frac{dx}{x + 1} - \frac{5}{2} \int \frac{dx}{(x + 1)^2} - \frac{11}{4} \int \frac{dx}{x + 3}$

$I = \frac{11}{4} \log|x + 1| + \frac{5}{2(x + 1)} - \frac{11}{4} \log|x + 3| + c$

Indefinite Integrals Ex 19.30 Q28

Let $\frac{2x+1}{(x+2)(x-3)^2} = \frac{A}{x+2} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$

$$\Rightarrow \quad 2x+1 = A(x-3)^2 + B(x+2)(x-3) + C(x+2)$$
$$= (A+B)x^2 + (-6A - B + C)x + (9A - 6B + 2C)$$

Equating similar terms, we get,

$A + B = 0 \quad \Rightarrow \quad A = -B$
$-6A - B + C = 2 \quad \Rightarrow \quad 5B + C = 2$
$9A - 6B + 2C = 1 \Rightarrow \quad -15B + 2C = 1$

Solving, we get, $B = \frac{3}{25}$, $C = \frac{7}{5}$, $A = -\frac{3}{25}$

Thus,
$$I = -\frac{3}{25}\int \frac{dx}{x+2} + \frac{3}{25}\int \frac{dx}{x-3} + \frac{7}{5}\int \frac{dx}{(x-3)^2}$$

$$I = -\frac{3}{25}\log|x+2| + \frac{3}{25}\log|x-3| - \frac{7}{5(x-3)} + c$$

Indefinite Integrals Ex 19.30 Q29

Let $\frac{x^2+1}{(x-2)^2(x+3)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+3}$

$$\Rightarrow \quad x^2+1 = A(x-2)(x+3) + B(x+3) + C(x-2)^2$$
$$= (A+C)x^2 + (A+B-4C)x + (-6A+3B+4C)$$

Equating similar terms, we get,

$A + C = 1$, $A + B - 4C = 0$, $-6A + 3B + 4C = 1$

Solving, we get, $A = \frac{3}{5}$, $B = 1$, $C = \frac{2}{5}$

Thus,
$$I = \frac{3}{5}\int \frac{dx}{x-2} + \int \frac{dx}{(x-2)^2} + \frac{2}{5}\int \frac{dx}{x+3}$$

$$I = \frac{3}{5}\log|x-2| - \frac{1}{(x-2)} + \frac{2}{5}\log|x+3| + c$$

Indefinite Integrals Ex 19.30 Q30

Let $\frac{x}{(x-1)^2(x+2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$

$x = A(x-1)(x+2) + B(x+2) + C(x-1)^2$

Substituting x = 1, we obtain

$B = \frac{1}{3}$

Equating the coefficients of x^2 and constant term, we obtain

$A + C = 0$

$-2A + 2B + C = 0$

On solving, we obtain

$A = \frac{2}{9}$ and $C = \frac{-2}{9}$

$$\therefore \frac{x}{(x-1)^2(x+2)} = \frac{2}{9(x-1)} + \frac{1}{3(x-1)^2} - \frac{2}{9(x+2)}$$

$$\Rightarrow \int \frac{x}{(x-1)^2(x+2)}dx = \frac{2}{9}\int \frac{1}{(x-1)}dx + \frac{1}{3}\int \frac{1}{(x-1)^2}dx - \frac{2}{9}\int \frac{1}{(x+2)}dx$$

$$= \frac{2}{9}\log|x-1| + \frac{1}{3}\left(\frac{-1}{x-1}\right) - \frac{2}{9}\log|x+2| + C$$

$$= \frac{2}{9}\log\left|\frac{x-1}{x+2}\right| - \frac{1}{3(x-1)} + C$$

Indefinite Integrals Ex 19.30 Q31

Let $\frac{x^2}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$

$$\Rightarrow \quad x^2 = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$$
$$= (A+B)x^2 + (2A+C)x + (A-B-C)$$

Equating similar terms,

$A + B = 1,\ 2A + C = 0,\ A - B - C = 0$

Solving, we get, $A = \frac{1}{4},\ B = \frac{3}{4},\ C = -\frac{1}{2}$

Thus,

$$I = \frac{1}{4}\int \frac{dx}{x-1} + \frac{3}{4}\int \frac{dx}{x+1} - \frac{1}{2}\int \frac{dx}{(x+1)^2}$$
$$= \frac{1}{4}\log|x-1| + \frac{3}{4}\log|x+1| + \frac{1}{2(x+1)} + c$$

$$I = \frac{1}{4}\log|x-1| + \frac{3}{4}\log|x+1| + \frac{1}{2(x+1)} + c$$

Indefinite Integrals Ex 19.30 Q32

Let $\frac{x^2+x-1}{(x+1)^2(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+2)}$

$$\Rightarrow \quad x^2+x-1 = A(x+1)(x+2) + B(x+2) + C(x+1)^2$$
$$= (A+C)x^2 + (3A+B+2C)x + (2A+2B+C)$$

Equating similar terms

$A+C=1,\ 3A+B+2C=1,\ 2A+2B+C=-1$

Solving, we get, $A=0,\ B=-1,\ C=1$

Thus,

$$I = 0.\int\frac{dx}{x+1} + (-1)\int\frac{dx}{(x+1)^2} + 1.\int\frac{dx}{(x+2)}$$
$$= +\frac{1}{x+1} + \log|x+2| + c$$

$$I = \frac{1}{x+1} + \log|x+2| + c$$

Indefinite Integrals Ex 19.30 Q33

Let $\frac{2x^2+7x-3}{x^2(2x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(2x+1)}$

$$\Rightarrow \quad 2x^2+7x-3 = Ax(2x+1) + B(2x+1) + Cx^2$$

Equating similar terms, we get,

$2A+C=2,\ A+2B=7,\ B=-3$

Solving, we get, $A=13,\ C=-24$

Thus,

$$I = \int\frac{13dx}{x} - \int\frac{3dx}{x^2} - 24\int\frac{dx}{2x+1}$$

$$I = 13\log|x| + \frac{3}{x} - 12\log|2x+1| + c$$

Indefinite Integrals Ex 19.30 Q34

Let $I = \int \frac{5x^2 + 20x + 6}{x^3 + 2x^2 + x} dx$

$= \int \frac{5x^2 + 20x + 6}{x(x+1)^2} dx$

Now,

Let $\frac{5x^2 + 20x + 6}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

$\Rightarrow \quad 5x^2 + 20x + 6 = A(x+1)^2 + Bx(x+1) + Cx$

Equating similar terms, we get,

$A + B = 5,\ 2A + B + C = 20,\ A = 6$

Solving, we get, $B = -1,\ C = 9$

Thus,

$I = \int \frac{6dx}{x} - 1\int \frac{dx}{x+1} + 9\int \frac{dx}{(x+1)^2}$

$\therefore \quad I = 6\log|x| - \log|x+1| - \frac{9}{x+1} + c$

Indefinite Integrals Ex 19.30 Q35

Let $\frac{18}{(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{Bx + C}{x^2+4}$

$\Rightarrow \quad 18 = A(x^2+4) + (Bx+C)(x+2)$

$18 = (A+B)x^2 + (2B+C)x + (4A+2C)$

Equating similar terms, we get,

$A + B = 0,\ 2B + C = 0,\ 4A + 2C = 18$

Solving, we get, $A = \frac{9}{4},\ B = -\frac{9}{4},\ C = \frac{9}{2}$

Thus,

$I = \frac{9}{4}\int \frac{dx}{x+2} + \left(-\frac{9}{4}\right)\int \frac{x}{x^2+4} dx + \frac{9}{2}\int \frac{dx}{x^2+4}$

$I = \frac{9}{4}\log|x+2| - \frac{9}{8}\log|x^2+4| + \frac{9}{4}\tan^{-1}\left(\frac{x}{2}\right) + c \qquad \left[\because \int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{9}\right]$

Indefinite Integrals Ex 19.30 Q36

Let $\frac{5}{\left(x^2+1\right)\left(x+2\right)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+2}$

$\Rightarrow \quad 5 = \left(Ax+B\right)\left(x+2\right) + C\left(x^2+1\right)$

Equating similar terms, we get,

$A + C = 0,\ 2A + B = 0,\ 2B + C = 5$

Solving, we get, $A = -1,\ B = 2,\ C = 1$

Thus,

$I = \int \frac{-x+2}{x^2+1}dx + \int \frac{dx}{x+2}$

$= \int \frac{-xdx}{x^2+1} + 2\int \frac{dx}{x^2+1} + \int \frac{dx}{x+2}$

$I = -\frac{1}{2}\log\left|x^2+1\right| + 2\tan^{-1}x + \log\left|x+2\right| + c$

Indefinite Integrals Ex 19.30 Q37

Let $\frac{x}{\left(x+1\right)\left(x^2+1\right)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$

$\Rightarrow \quad x = A\left(x^2+1\right) + \left(Bx+C\right)\left(x+1\right)$

Equating similar terms, we get,

$A + B = 0,\ B + C = 1,\ A + C = 0$

Solving, we get, $A = -\frac{1}{2},\ B = \frac{1}{2},\ C = \frac{1}{2}$

Thus,

$I = -\frac{1}{2}\int \frac{dx}{x+1} + \frac{1}{2}\int \frac{xdx}{x^2+1} + \frac{1}{2}\int \frac{dx}{x^2+1}$

$I = -\frac{1}{2}\log\left|x+1\right| + \frac{1}{4}\log\left|x^2+1\right| + \frac{1}{2}\tan^{-1}x + c$

Indefinite Integrals Ex 19.30 Q38

Let $I = \int \frac{dx}{1+x+x^2+x^3}$

$\Rightarrow \quad I = \int \frac{dx}{\left(x^2+1\right)(x+1)}$

Now,

Let $\frac{1}{\left(x^2+1\right)(x+1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+1}$

$\Rightarrow \quad 1 = (Ax+B)(x+1) + C\left(x^2+1\right)$

Equating similar terms, we get,

$A+C=0,\ A+B=0,\ B+C=1$

Solving, we get, $A = -\frac{1}{2},\ B = \frac{1}{2},\ C = \frac{1}{2}$

Thus,

$I = -\frac{1}{2}\int \frac{xdx}{x^2+1} + \frac{1}{2}\int \frac{dx}{x^2+1} + \frac{1}{2}\int \frac{dx}{x+1}$

$I = -\frac{1}{4}\log\left|x^2+1\right| + \frac{1}{2}\tan^{-1}x + \frac{1}{2}\log\left|x+1\right| + c$

Indefinite Integrals Ex 19.30 Q39

Let $\frac{1}{(x+1)^2\left(x^2+1\right)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{Cx+D}{x^2+1}$

$$\Rightarrow \quad 1 = A(x+1)\left(x^2+1\right) + B\left(x^2+1\right) + (Cx+D)(x+1)^2$$
$$= (A+C)x^3 + (A+B+2C+D)x^2 + (A+C+2D)x + (A+B+D)$$

Equating similar terms, we get,

$A+C=0,\ A+B+2C+D=0,\ A+C+2D=0,\ A+B+D=1$

Solving, we get, $A = \frac{1}{2},\ B = \frac{1}{2},\ C = -\frac{1}{2},\ D = 0$

Thus,

$I = \frac{1}{2}\int \frac{dx}{x+1} + \frac{1}{2}\int \frac{dx}{(x+1)^2} - \frac{1}{2}\int \frac{xdx}{x^2+1}$

$I = \frac{1}{2}\log\left|x+1\right| - \frac{1}{2(x+1)} - \frac{1}{4}\log\left|x^2+1\right| + c$

Indefinite Integrals Ex 19.30 Q40

Let $I = \int \frac{2x}{x^3 - 1} dx = \int \frac{2x}{(x-1)(x^2+x+1)} dx$

Now,

Let $\frac{2x}{(x-1)(x^2+x+1)} = \frac{A}{(x-1)} + \frac{Bx+C}{x^2+x+1}$

$$\Rightarrow \quad 2x = A(x^2+x+1) + (Bx+C)(x-1)$$
$$= (A+B)x^2 + (A-B+C)x + (A-C)$$

Equating similar terms,

$A + B = 0,\ A - B + C = 2,\ \ A - C = 0,$

Solving, we get, $A = \frac{2}{3},\ B = -\frac{2}{3},\ C = \frac{2}{3}$

Thus,

$$I = \frac{2}{3}\int \frac{dx}{x-1} - \frac{2}{3}\int \frac{(x-1)dx}{x^2+x+1}$$

$$= \frac{2}{3}\int \frac{dx}{x-1} - \frac{2}{3}.\frac{1}{2}\int \frac{(2x-2)dx}{x^2+x+1}$$

$$\Rightarrow \quad I = \frac{2}{3}\int \frac{dx}{x-1} - \frac{1}{3}\int \frac{2x+1}{x^2+x+1}dx + \int \frac{dx}{x^2+x+1}$$

$$= \frac{2}{3}\int \frac{dx}{x-1} - \frac{1}{3}\int \frac{2x+1}{x^2+x+1}dx + \int \frac{dx}{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{2}{3}\log|x-1| - \frac{1}{3}\log\left|x^2+x+1\right| + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + c$$

Hence,

$$I = \frac{2}{3}\log|x-1| - \frac{1}{3}\log\left|x^2+x+1\right| + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + c$$

Indefinite Integrals Ex 19.30 Q41

Let $\frac{1}{\left(x^2+1\right)\left(x^2+4\right)} = \frac{Ax+B}{\left(x^2+1\right)} + \frac{Cx+D}{x^2+4}$

$$\Rightarrow \quad 1 = \left(Ax+B\right)\left(x^2+4\right) + \left(Cx+D\right)\left(x^2+1\right)$$
$$= \left(A+C\right)x^3 + \left(B+D\right)x^2 + \left(4A+C\right)x + 4B + D$$

Equating similar terms, we get,

$A + C = 0,\ B + D = 0,\ 4A + C = 0,\ 4B + D = 1$

Solving, we get, $A = 0,\ B = \frac{1}{3},\ C = 0,\ D = -\frac{1}{3}$

Thus,

$$I = \int \frac{\frac{1}{3}dx}{\left(x^2+1\right)} - \int \frac{\frac{1}{3}dx}{\left(x^2+4\right)}$$

$$= \frac{1}{3}\tan^{-1}x - \frac{1}{6}\tan^{-1}\left(\frac{x}{2}\right) + c \qquad \left[\because \int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}\right]$$

$$\therefore \quad I = \frac{1}{3}\tan^{-1}x - \frac{1}{6}\tan^{-1}\left(\frac{x}{2}\right) + c$$

Indefinite Integrals Ex 19.30 Q42

Let $\frac{x^2}{\left(x^2+1\right)\left(3x^2+4\right)} = \frac{Ax+B}{\left(x^2+1\right)} + \frac{Cx+D}{\left(3x^2+4\right)}$

$$\Rightarrow \quad x^2 = \left(Ax+B\right)\left(3x^2+4\right) + \left(Cx+D\right)\left(x^2+1\right)$$
$$= \left(3A+C\right)x^3 + \left(3B+D\right)x^2 + \left(4A+C\right)x + 4B + D$$

Equating similar terms, we get,

$3A + C = 0,\ 3B + D = 1,\ 4A + C = 0,\ 4B + D = 0$

Solving, we get, $A = 0,\ B = -1,\ C = 0,\ D = 4$

Thus,

$$I = \int \frac{-dx}{\left(x^2+1\right)} + \int \frac{4dx}{\left(3x^2+4\right)}$$

$$= -\tan^{-1}x + \frac{4}{3}\int \frac{dx}{x^2 + \left(\frac{2}{\sqrt{3}}\right)^2}$$

$$= -\tan^{-1}x + \frac{4}{3}.\frac{\sqrt{3}}{2}\tan^{-1}\left(\frac{\sqrt{3}x}{2}\right) + c$$

$$I = \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}x}{2}\right) - \tan^{-1}x + c$$

Indefinite Integrals Ex 19.30 Q43

To evaluate the integral follow the steps:

$$\int \frac{3x+5}{x^3-x^2-x+1}dx$$

Let $\frac{3x+5}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1}$

$$3x+5 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$$

For $x=1$ $\quad B=4$

For $x=-1$ $C=\frac{1}{2}$

For $x=0$ $\quad A=-\frac{1}{2}$,

Therefore

$$\int \frac{3x+5}{(x-1)^2(x+1)}dx = -\frac{1}{2}\int\frac{dx}{x-1} + 4\int\frac{dx}{(x-1)^2} + \frac{1}{2}\int\frac{dx}{x+1}$$

$$= -\frac{1}{2}\ln|(x-1)| - \frac{4}{(x-1)} + \frac{1}{2}\ln|(x+1)| + c$$

$$= \frac{1}{2}\ln\left|\frac{x+1}{x-1}\right| - \frac{4}{(x-1)} + c$$

Indefinite Integrals Ex 19.30 Q44

Let $I = \int \frac{x^3-1}{x^3+x}dx$

$$= \int 1 - \frac{(x+1)}{x^3+x}dx$$

$$= \int dx - \int \frac{x+1}{x^3+x}dx$$

Let $\frac{x+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

$$\Rightarrow \quad x+1 = A(x^2+1) + (Bx+C)(x)$$

$$= (A+B)x^2 + (B+C)x + A$$

Equating similar terms, we get,

$A+B=0$, $C=1$, $A=1$

Solving, we get, $A=1$, $B=-1$, $C=1$

Thus,

$$I = -\int\frac{dx}{x} - \int\frac{-x+1}{x^2+1}dx + \int dx$$

$$= -\log|x| + \int\frac{xdx}{x^2+1} - \int\frac{dx}{x^2+1} + \int dx$$

$$\therefore \quad I = x - \log|x| + \frac{1}{2}\log|x^2+1| - \tan^{-1}x + c$$

$$\therefore \quad I = x - \log|x| + \frac{1}{2}\log|x^2+1| - \tan^{-1}x + c$$

Indefinite Integrals Ex 19.30 Q45

To evaluate the integral follow the steps:

$$\int \frac{x^2+x+1}{(x+1)^2(x+2)}dx$$

Let $\frac{x^2+x+1}{(x+1)^2(x+2)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+2}$

$x^2+x+1 = A(x+1)(x+2)+B(x+2)+C(x+1)^2$

For $x=-1$ $B=1$

For $x=-2$ $C=3$

For $x=0$ $A=-2$,

Therefore

$$\int \frac{x^2+x+1}{(x+1)^2(x+2)}dx = -2\int\frac{dx}{x+1} + \int\frac{dx}{(x+1)^2} + 3\int\frac{dx}{x+2}$$

$$= -2\ln|x+1| - \frac{1}{x+1} + 3\ln|x+2| + c$$

Indefinite Integrals Ex 19.30 Q46

Let $\frac{1}{x(x^4+1)} = \frac{A}{x} + \frac{Bx^3+Cx^2+Dx+E}{x^4+1}$

$$\Rightarrow \quad 1 = A(x^4+1) + (Bx^3+Cx^2+Dx+E)x$$

$$= (A+B)x^4 + Cx^3 + Dx^2 + Ex + A$$

Equating similar terms, we get,

$A+B=0,\ C=0,\ D=0,\ E=0,\ A=1$

$\therefore \quad B=-1$

Thus,

$$I = \int\frac{dx}{x} + \int -\frac{x^3dx}{x^4+1}$$

$$= \log|x| - \frac{1}{4}\log|x^4+1| + c$$

$$I = \frac{1}{4}\log\left|\frac{x^4}{x^4+1}\right| + c$$

Indefinite Integrals Ex 19.30 Q47

Consider the integral

$$I=\int \frac{1}{x\left(x^3+8\right)}dx$$

Rewriting the above integrand, we have,

$$I=\int \frac{x^2}{x^3\left(x^3+8\right)}dx$$

$$=\frac{1}{3}\int \frac{3x^2}{x^3\left(x^3+8\right)}dx$$

Now substituting $x^3=t$, we have,

$3x^2dx=dt$

$$\Rightarrow I=\frac{1}{3}\int \frac{dt}{t\left(t+8\right)}$$

Let us separata the integrand by partial fractions.

Thus,

$$\frac{1}{t\left(t+8\right)}=\frac{A}{t}+\frac{B}{t+8}$$

$$\Rightarrow \frac{1}{t\left(t+8\right)}=\frac{A\left(t+8\right)+Bt}{t\left(t+8\right)}$$

$$\Rightarrow 1=A\left(t+8\right)+Bt$$

$$\Rightarrow 1=At+8A+Bt$$

Comparing the coefficients, we have,

$A+B=0$ and $8A=1$

$$\Rightarrow A=\frac{1}{8} \text{ and } B=-\frac{1}{8}$$

Therefore,

$$I=\frac{1}{3}\int \frac{dt}{t\left(t+8\right)}$$

$$=\frac{1}{3}\int \left[\frac{\frac{1}{8}}{t}-\frac{\frac{1}{8}}{t+8}\right]dt$$

$$=\frac{1}{3}\times\frac{1}{8}\int \frac{dt}{t}dt-\frac{1}{3}\times\frac{1}{8}\int \frac{dt}{t+8}$$

$$=\frac{1}{24}\log t-\frac{1}{24}\times\log\left(t+8\right)+C$$

$$=\frac{1}{24}\log x^3-\frac{1}{24}\times\log\left(x^3+8\right)+C$$

$$=\frac{3}{24}\log x-\frac{1}{24}\times\log\left(x^3+8\right)+C$$

$$=\frac{1}{8}\log x-\frac{1}{24}\times\log\left(x^3+8\right)+C$$

Indefinite Integrals Ex 19.30 Q48

Let $\frac{3}{(1-x)(1+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{1+x^2}$

$\Rightarrow \quad 3 = A(1+x^2) + (Bx+C)(1-x)$

$= (A-B)x^2 + (B-C)x + (A+C)$

Equating similar terms, we get,

$A - B = 0,\ B - C = 0,\ A + C = 3$

Solving we get,

$A = C = \frac{3}{2}$ and $B = \frac{3}{2}$

Thus,

$I = \frac{3}{2}\int \frac{dx}{1-x} + \frac{3}{2}\int \frac{xdx}{1+x^2} + \frac{3}{2}\int \frac{dx}{1+x^2}$

$= -\frac{3}{2}\log|1-x| + \frac{3}{2}\log|1+x^2| + \frac{3}{2}\tan^{-1}x + c$

$I = \frac{3}{4}\left[\log\left|\frac{1+x^2}{(1-x)^2}\right| + 2\tan^{-1}x + c\right]$

Indefinite Integrals Ex 19.30 Q49

Let

$\sin x = t$

$\Rightarrow \quad \cos x = dt$

$\therefore \quad \int \frac{\cos x}{(1-\sin x)^3(2+\sin x)} = \int \frac{1}{(1-t)^3(2+t)}dt$

Let $\quad f(t) = \frac{1}{(1-t)^3(2+t)}$

Then, suppose

$\frac{1}{(1-t)^3(2+t)} = \frac{A}{1-t} + \frac{B}{(1-t)^2} + \frac{c}{(1-t)^3} + \frac{D}{(2+t)}$

$\Rightarrow \quad 1 = A(1-t)^2(2+t) + B(1-t)(2+t) + C(2+t) + D(1-t)^3$

Put $t = 1$

$1 = 3C$

$\Rightarrow \quad C = \frac{1}{3}$

Put $t = -2$

$1 = 27D$

$\Rightarrow \quad D = \frac{1}{27}$

Similarly, we can find that $A = \frac{-1}{27}$ and $B = \frac{+1}{9}$

$\therefore \quad \int \frac{1}{(1-t)^3(2+t)}dt = \frac{-1}{27}\int \frac{1}{1-t}dt + \frac{1}{9}\int \frac{dt}{(1-t)^2} + \frac{1}{3}\int \frac{dt}{(1-t)^3} + \frac{1}{27}\int \frac{dt}{2+t}$

$= \frac{-1}{27}\log|1-t| + \frac{1}{9(1-t)} + \frac{1}{6(1-t)^2} + \frac{1}{27}\log|2+t| + c$

Putting $t = \sin x$, we get

$\int \frac{\cos x}{(1-\sin x)^3(2+\sin x)}dx$

$= \frac{-1}{27}\log|1-\sin x| + \frac{1}{9(1-\sin x)} + \frac{1}{6(1-\sin x)^2} + \frac{1}{27}\log|2+\sin x| + C$

Indefinite Integrals Ex 19.30 Q50

Consider the integral

$$I=\int \frac{2x^2+1}{x^2\left(x^2+4\right)}dx$$

Now let us separate the fraction $\frac{2x^2+1}{x^2\left(x^2+4\right)}$ through partial fractions.

Substitute $x^2=t$, then

$$\frac{2x^2+1}{x^2\left(x^2+4\right)}=\frac{2t+1}{t\left(t+4\right)}$$

$$\Rightarrow \frac{2t+1}{t\left(t+4\right)}=\frac{A}{t}+\frac{B}{t+4}$$

$$\Rightarrow \frac{2t+1}{t\left(t+4\right)}=\frac{A\left(t+4\right)+Bt}{t\left(t+4\right)}$$

$$\Rightarrow 2t+1=A\left(t+4\right)+Bt$$

$$\Rightarrow 2t+1=At+4A+Bt$$

Comparing the coefficients, we have,

$A+B=2$ and $4A=1$

$\Rightarrow A=\frac{1}{4}$ and $B=\frac{7}{4}$

$$\Rightarrow \frac{2x^2+1}{x^2\left(x^2+4\right)}=\frac{1}{4x^2}+\frac{7}{4\left(x^2+4\right)}$$

Thus, we have,

$$I=\int \frac{2x^2+1}{x^2\left(x^2+4\right)}dx$$

$$=\frac{1}{4}\int \frac{dx}{x^2}+\frac{7}{4}\int \frac{dx}{\left(x^2+4\right)}$$

$$=-\frac{1}{4x}+\frac{7}{4}\times\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+C$$

$$=-\frac{1}{4x}+\frac{7}{8}\tan^{-1}\left(\frac{x}{2}\right)+C$$

Indefinite Integrals Ex 19.30 Q51

To evaluate the integral follow the steps:

$$\int \frac{\cos x}{(1-\sin x)(2-\sin x)}dx$$

Let $1-\sin x=t$ and

$-\cos x dx=dt$

Therefore

$$-\int \frac{dt}{t(1+t)}=-\int\left(\frac{1}{t}-\frac{1}{t+1}\right)dt$$

$$=\ln\left|(t+1)\right|-\ln|t|+c$$

$$=\ln\left|\frac{t+1}{t}\right|+c$$

$$=\ln\left|\frac{2-\sin x}{1-\sin x}\right|+c$$

Indefinite Integrals Ex 19.30 Q52

Let $\frac{2x+1}{(x-2)(x-3)} = \frac{A}{(x-2)} + \frac{B}{(x-3)}$

$$\Rightarrow \quad 2x+1 = A(x-3) + B(x-2)$$
$$= (A+B)x + (-3A-2B)$$

Equating similar terms, we get,

$A + B = 2$, and $-3A - 2B = 1$

Thus,

$$I = -5\int \frac{dx}{x-2} + 7\int \frac{dx}{x-3}$$

$$= -5\log|x-2| + 7\log|x-3| + c$$

$$I = \log\left|\frac{(x-3)^7}{(x-2)^5}\right| + c$$

Indefinite Integrals Ex 19.30 Q53

Let $x^2 = y$

Then $\frac{1}{(y+1)(y+2)} = \frac{A}{y+1} + \frac{B}{y+2}$

$$\Rightarrow \quad 1 = A(y+2) + B(y+1)$$
$$= (A+B)y + (2A+B)$$

Equating similar terms, we get,

$A + B = 0$, and $2A + B = 1$

Solving, we get,

Thus,

$$I = \int \frac{dx}{x^2+1} - \int \frac{dx}{x^2+2}$$

$$I = \tan^{-1} x - \frac{1}{\sqrt{2}} \tan^{-1} \frac{x}{\sqrt{2}} + c$$

Indefinite Integrals Ex 19.30 Q54

To evaluate the integral follow the steps:

$$\int \frac{1}{x\left(x^4-1\right)}dx$$

Let $\frac{1}{x\left(x^4-1\right)}=\frac{A}{x}+\frac{B}{x+1}+\frac{C}{x-1}+\frac{D}{x^2+1}$

$$1=A(x+1)(x-1)\left(x^2+1\right)+Bx(x-1)\left(x^2+1\right)+Cx(x+1)\left(x^2+1\right)+Dx(x+1)(x-1)$$

For $x=0 \quad A=-1$,

For $x=1 \quad C=\frac{1}{4}$

For $x=-1\; B=\frac{1}{4}$

For $x=2 \quad D=\frac{1}{4}$

Therefore

$$\int \frac{1}{x\left(x^4-1\right)}dx=-\int \frac{1}{x}dx+\frac{1}{4}\int \frac{dx}{x+1}+\frac{1}{4}\int \frac{dx}{x-1}+\frac{1}{4}\int \frac{dx}{x^2+1}$$

$$=-\ln|x|+\frac{1}{4}\ln|(x+1)|+\frac{1}{4}\ln|(x-1)|+\frac{1}{4}\ln\left|\left(x^2+1\right)\right|+c$$

$$=\frac{1}{4}\ln\left|\frac{x^4-1}{x^4}\right|+c$$

Indefinite Integrals Ex 19.30 Q55

To evaluate the integral follow the steps:

$$\int \frac{1}{\left(x^4-1\right)}dx$$

Let $\frac{1}{\left(x^4-1\right)}=\frac{A}{x+1}+\frac{B}{x-1}+\frac{C}{x^2+1}$

$$1=A(x-1)\left(x^2+1\right)+B(x+1)\left(x^2+1\right)+C(x+1)(x-1)$$

For $x=1 \quad B=\frac{1}{4}$

For $x=-1\; A=-\frac{1}{4}$

For $x=0 \quad C=-\frac{1}{2}$,

Therefore

$$\int \frac{1}{\left(x^4-1\right)}dx=-\frac{1}{4}\int \frac{dx}{x+1}+\frac{1}{4}\int \frac{dx}{x-1}-\frac{1}{2}\int \frac{dx}{x^2+1}$$

$$=-\frac{1}{4}\ln|(x+1)|+\frac{1}{4}\ln|(x-1)|-\frac{1}{2}\tan^{-1}x+c$$

$$=\frac{1}{4}\ln\left|\frac{x-1}{x+1}\right|-\frac{1}{2}\tan^{-1}x+c$$

Indefinite Integrals Ex 19.30 Q57

Let $I = \int \frac{dx}{\cos x\left(5 - 4\sin x\right)}$

$$= \int \frac{\cos x dx}{\cos^2 x\left(5 - 4\sin x\right)}$$

$$= \int \frac{\cos x dx}{\left(1 - \sin^2 x\right)\left(5 - 4\sin x\right)}$$

Let $\sin x = t \quad \Rightarrow \quad \cos x dx = dt$

$\therefore \quad I = \int \frac{dt}{\left(1 - t^2\right)\left(5 - 4t\right)}$

Now,

Let $\frac{1}{\left(1 - t^2\right)\left(5 - 4t\right)} = \frac{A}{1 - t} + \frac{B}{1 + t} + \frac{C}{5 - 4t}$

$\Rightarrow \quad 1 = A\left(1 + t\right)\left(5 - 4t\right) + B\left(1 - t\right)\left(5 - 4t\right) + C\left(1 - t^2\right)$

Put $t = 1$

$\Rightarrow \quad 1 = 2A \quad \Rightarrow \quad A = \frac{1}{2}$

Put $t = -1$

$\Rightarrow \quad 1 = 18B \quad \Rightarrow \quad B = \frac{1}{18}$

Put $t = \frac{5}{4}$

$\Rightarrow \quad 1 = -\frac{9C}{16} \Rightarrow \quad C = -\frac{16}{9}$

Thus,

$I = \frac{1}{2}\int \frac{dt}{1 - t} + \frac{1}{18}\int \frac{dt}{1 + t} - \frac{16}{9}\int \frac{dt}{5 - 4t}$

$= -\frac{1}{2}\log\left|1 - t\right| + \frac{1}{18}\log\left|1 + t\right| + \frac{4}{9}\log\left|5 - 4t\right| + c$

Hence,

$I = -\frac{1}{2}\log\left|1 - \sin x\right| + \frac{1}{18}\log\left|1 + \sin x\right| + \frac{4}{9}\log\left|5 - 4\sin x\right| + c$

Indefinite Integrals Ex 19.30 Q58

$$\text{Let } I = \int \frac{1}{\sin x\,(3+2\cos x)}dx$$
$$= \int \frac{\sin x dx}{\sin^2 x\,(3+2\cos x)}$$
$$= \int \frac{\sin x dx}{(1-\cos^2 x)(3+2\cos x)}$$

Let $\cos x = t$

$\Rightarrow \quad -\sin x dx = dt$

$$\therefore \quad I = \int \frac{dt}{(t^2-1)(3+2t)}$$

Now,

$$\text{Let } \frac{1}{(t^2-1)(3+2t)} = \frac{A}{t-1} + \frac{B}{t+1} + \frac{C}{3+2t}$$

$$\Rightarrow \quad 1 = A(t+1)(3+2t) + B(t-1)(3+2t) + C(t^2-1)$$

Put $t = 1$

$$\Rightarrow \quad 1 = 10A \Rightarrow \quad A = \frac{1}{10}$$

Put $t = -1$

$$\Rightarrow \quad 1 = -2B \quad \Rightarrow \quad B = -\frac{1}{2}$$

Put $t = -\frac{3}{2}$

$$\Rightarrow \quad 1 = \frac{5}{4}.C \Rightarrow \quad C = \frac{4}{5}$$

Thus,

$$I = \frac{1}{10}\int \frac{dt}{t-1} - \frac{1}{2}\int \frac{dt}{t+1} + \frac{5}{4}\int \frac{dt}{3+2t}$$
$$= \frac{1}{10}\log|t-1| - \frac{1}{2}\log|t+1| + \frac{2}{5}\log|3+2t| + c$$

Hence,

$$I = \frac{1}{10}\log|\cos x - 1| - \frac{1}{2}\log|\cos x + 1| + \frac{2}{5}\log|3+2\cos x| + c$$

Indefinite Integrals Ex 19.30 Q59

$$\text{Let } I = \int \frac{1}{\sin x + \sin 2x} dx$$
$$= \int \frac{dx}{\sin x + 2\sin x \cos x}$$

$$= \int \frac{\sin x dx}{\left(1 - \cos^2 x\right) + 2\left(1 - \cos^2 x\right)\cos x}$$

Let $\cos x = t \quad \Rightarrow \qquad -\sin x dx = dt$

$$\therefore \qquad I = \int \frac{dt}{\left(t^2 - 1\right) + 2\left(t^2 - 1\right)t}$$
$$= \int \frac{dt}{\left(t^2 - 1\right)\left(1 + 2t\right)}$$

$$\text{Let } \int \frac{1}{\left(t^2 - 1\right)\left(1 + 2t\right)} = \frac{A}{t-1} + \frac{B}{t+1} + \frac{C}{1+2t}$$

$$\Rightarrow \qquad 1 = A\left(t+1\right)\left(1+2t\right) + B\left(t-1\right)\left(1+2t\right) + C\left(t^2 - 1\right)$$

Put $t = 1$

$$\Rightarrow \qquad 1 = 6A \Rightarrow \qquad A = \frac{1}{6}$$

Put $t = -1$

$$\Rightarrow \qquad 1 = 2B \Rightarrow \qquad B = \frac{1}{2}$$

Put $t = -\frac{1}{2}$

$$\Rightarrow \qquad 1 = -\frac{3}{4}C \Rightarrow \qquad C = -\frac{4}{3}$$

Thus,

$$I = \frac{1}{6}\int \frac{dt}{t-1} + \frac{1}{2}\int \frac{dt}{t+1} - \frac{4}{3}\int \frac{dt}{1+2t}$$
$$= \frac{1}{6}\log\left|t-1\right| + \frac{1}{2}\log\left|t+1\right| - \frac{2}{3}\log\left|1+2t\right| + c$$

Hence,

$$I = \frac{1}{6}\log\left|\cos x - 1\right| + \frac{1}{2}\log\left|\cos x + 1\right| - \frac{2}{3}\log\left|1 + 2\cos x\right| + c$$

Indefinite Integrals Ex 19.30 Q60

Let $I = \int \frac{x+1}{x\left(1+xe^x\right)}dx$

$$= \int \frac{(x+1)\left(1+xe^x - xe^x\right)}{x\left(1+xe^x\right)}dx$$

$$= \int \frac{(x+1)\left(1+xe^x\right)}{x\left(1+xe^x\right)}dx - \int \frac{(x+1)\left(xe^x\right)}{x\left(1+xe^x\right)}dx$$

$$= \int \frac{(x+1)}{x}dx - \int \frac{e^x(x+1)}{1+xe^x}dx$$

$$= \int \frac{(x+1)e^x}{xe^x}dx - \int \frac{e^x(x+1)}{1+xe^x}dx$$

$$= \log\left|xe^x\right| - \log\left|1+xe^x\right| + c$$

$$\therefore \quad I = \log\left|\frac{xe^x}{1+xe^x}\right| + c$$

Indefinite Integrals Ex 19.30 Q61

$$f(x) = \frac{\left(x^2+1\right)\left(x^2+2\right)}{\left(x^2+3\right)\left(x^2+4\right)}$$

Now,

$$\frac{\left(x^2+1\right)\left(x^2+2\right)}{\left(x^2+3\right)\left(x^2+4\right)}$$

$$= \frac{x^4+3x^2+2}{x^4+7x^2+12}$$

$$= \frac{\left(x^4+7x^2+12\right)-4x^2-10}{x^4+7x^2+12}$$

$$= 1 - \frac{4x^2+10}{x^4+7x^2+12}$$

Now,

$$\frac{4x^2+10}{x^4+7x^2+12} = \frac{4x^2+10}{\left(x^2+3\right)\left(x^2+4\right)}$$

Let $\frac{4x^2+10}{\left(x^2+3\right)\left(x^2+4\right)} = \frac{Ax+B}{x^2+3} + \frac{Cx+D}{x^2+4}$

$\Rightarrow \quad 4x^2+10 = (Ax+B)\left(x^2+4\right) + (Cx+D)\left(x^2+3\right)$

Let $x = 0$, we get

$10 = 4B + 3D$ ---(i)

If $x = 1$, we get

$14 = 5(A+B) + 4(C+D) = 5A + 5B + 4C + 4D$ ---(ii)

if $x = -1$, we get

$14 = 5(-A+B) + 4(-C+D) = -5A + 5B - 4C + 4D$ ---(iii)

Applying (ii) and (iii), we get

$28 = 10B + 8D$

$\Rightarrow \quad 14 = 5B + 4D$ ---(iv)

From (i), we get

$10 = 4B + 3D$ ---(i)

Multiplying equation (iv) by 3 and (i) by 4 and subtracting, we get

$42 - 40 = 15B - 16B$

$\Rightarrow \quad 2 = -B$

or $\quad B = -2$ ---(v)

Putting value of B in (i), we get

$10 = 4(-2) + 3D$

$\frac{10+8}{3} = D$

$\Rightarrow \quad D = 6$ ---(vi)

Comparing coefficients of x^3 in

$$4x^2 + 10 = (Ax + B)(x^2 + 4) + (Cx + 4)(x^2 + 3)$$, we get,

$$0 = A + C \qquad \text{---(vii)}$$

Comparing coefficients of x, we get

$$0 = 4A + 3C \qquad \text{---(viii)}$$

$\Rightarrow \quad A = C = 0$

$$\therefore \quad f(x) = 1 - \frac{(-2)}{x^2+3} - \frac{6}{x^2+4}$$

$$= 1 + \frac{2}{x^2+3} - \frac{6}{x^2+4}$$

$$\therefore \quad \int f(x)\,dx = \int 1 + \frac{2}{x^2+3} - \frac{6}{x^2+4}\,dx$$

$$= x + \frac{2}{\sqrt{3}}\tan^{-1}\times\frac{x}{\sqrt{3}} - 3\tan^{-1}\frac{x}{2} + c$$

Indefinite Integrals Ex 19.30 Q62

Let $x^2 = y$

$$\therefore \quad \frac{4x^4+3}{(x^2+2)(x^2+3)(x^2+4)} = \frac{4y^2+3}{(y+2)(y+3)(y+4)}$$

Now,

Let $\dfrac{4y^2+3}{(y+2)(y+3)(y+4)} = \dfrac{A}{y+2} + \dfrac{B}{y+3} + \dfrac{C}{y+4}$

$$\Rightarrow \quad 4y^2 + 3 = A(y+3)(y+4) + B = (y+2)(y+4) + C(y+2)(y+3)$$

$$= (A+B+C)y^2 + (7A+6A+5C)y + 12A + 8B + 6C$$

Equating similar terms,

$A + B + C = 4,\ 7A + 6A + 5C = 0,\ 12A + 8B + 6C = 3$

Solving, we get

$A = \dfrac{19}{2},\ B = -39,\ C = \dfrac{67}{2}$

Thus,

$$I = \frac{19}{2}\int\frac{dx}{x^2+2} + (-39)\int\frac{dx}{x^2+3} + \frac{67}{2}\int\frac{dx}{x^2+4}$$

$$\Rightarrow \quad I = \frac{19}{2\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) - \frac{39}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + \frac{67}{4}\tan^{-1}\left(\frac{x}{2}\right) + c$$

Hence,

$$I = \frac{19}{2\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) - \frac{39}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + \frac{67}{4}\tan^{-1}\left(\frac{x}{2}\right) + c$$

Indefinite Integrals Ex 19.30 Q63

$$\frac{x^4}{(x-1)(x^2+1)} = \frac{x^4}{x^3-x^2+x-1}$$

$$= \frac{x(x^3-x^2+x-1)+1(x^3-x^2+x-1)+1}{(x^3-x^2+x-1)}$$

$$= x+1+\frac{1}{(x-1)(x^2+1)}$$

Now, suppose

$$\frac{1}{(x-1)(x^2+1)} = \frac{A}{x-1}+\frac{Bx+C}{x^2+1}$$

$\Rightarrow \quad 1 = A(x^2+1)+(Bx+C)(x-1)$

Put $x = 1$

$1 = 2A$

$\Rightarrow \quad A = \frac{1}{2}$

Put $x = 0$

$1 = A - C$

$\Rightarrow \quad C = A - 1 = -\frac{1}{2}$

Put $x = -1$

$1 = 2A + 2B - 2C = 2(A - C) + 2B$

$\Rightarrow \quad 1 = 2 + 2B$

$\Rightarrow \quad 2B = -1$

$\Rightarrow \quad B = -\frac{1}{2}$

$$\therefore \quad \int \frac{x^4}{(x-1)(x^2+1)}dx = \int x\,dx + \int 1\,dx + \frac{1}{2}\int \frac{1}{x-1}dx - \frac{1}{2}\int \frac{x+1}{x^2+1}dx$$

$$= \frac{x^2}{2}+x+\frac{1}{2}\log|x-1|-\frac{1}{4}\log(x^2+1)-\frac{1}{2}\tan^{-1}x+C$$

Ex 19.31

Indefinite Integrals Ex 19.31 Q1

$$I = \int \frac{x^2+1}{x^4+x^2+1} dx$$

$$= \int \frac{1+\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}} dx$$

Dividing numerator and denominator by x^2

$$= \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x-\frac{1}{x}\right)^2+3} dx$$

Let $x - \frac{1}{x} = t \Rightarrow \quad \left(1+\frac{1}{x^2}\right)dx = dt$

$$\therefore \quad I = \int \frac{dt}{t^2+3}$$

$$= \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{t}{\sqrt{3}}\right) + c$$

$$\therefore \quad I = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right) + c$$

Indefinite Integrals Ex 19.31 Q2

$$\int \sqrt{\cot\theta}\,d\theta$$

Let $\cot\theta = x^2$

$\Rightarrow \quad -\operatorname{cosec}^2\theta\,d\theta = 2x\,dx$

$$\Rightarrow \quad d\theta = \frac{-2x}{\operatorname{cosec}^2\theta}dx$$

$$= \frac{-2x}{1+\cot^2\theta}dx$$

$$= \frac{-2x}{1+x^4}dx$$

$$\therefore \quad I = -\int \frac{2x^2}{1+x^4}dx$$

$$= -\int \frac{2}{\frac{1}{x^2}+x^2}dx$$

Dividing numerator and denominator by x^2

$$= -\int \frac{1+\frac{1}{x^2}+1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx$$

$$= -\int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+2} - \int \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x+\frac{1}{x}\right)^2-2}$$

Let $x-\frac{1}{x} = t \Rightarrow \quad \left(1+\frac{1}{x^2}\right)dx = dt$

and $\quad x+\frac{1}{x} = z \Rightarrow \quad \left(1-\frac{1}{x^2}\right)dx = dz$

$$\Rightarrow \quad I = -\int \frac{dt}{t^2+2} - \int \frac{dz}{z^2-2}$$

$$= -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right) - \frac{1}{2\sqrt{2}}\log\left|\frac{z-\sqrt{2}}{z+\sqrt{2}}\right| + c$$

$$= -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) - \frac{1}{2\sqrt{2}}\log\left|\frac{x^2+1-\sqrt{2}x}{x^2+1+\sqrt{2}x}\right| + c$$

$$I = -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\cot\theta-1}{\sqrt{2\cot\theta}}\right) - \frac{1}{2\sqrt{2}}\log\left|\frac{\cot\theta+1-\sqrt{2\cot\theta}}{\cot\theta+1-\sqrt{2\cot\theta}}\right| + c$$

Indefinite Integrals Ex 19.31 Q3

Let $I = \int \frac{x^2+9}{x^4+81}dx$

Dividing numerator and denominator by x^2

$$I = \int \frac{1+\frac{9}{x^2}}{x^2+\frac{81}{x^2}}dx$$

$$= \int \frac{1+\frac{9}{x^2}}{\left(x-\frac{9}{x}\right)^2+18}dx$$

Let $\left(x-\frac{9}{x}\right) = t \Rightarrow \quad \left(1+\frac{9}{x^2}\right)dx = dt$

$$\therefore \quad I = \int \frac{dt}{t^2+18}$$

$$\Rightarrow \quad I = \frac{1}{3\sqrt{2}}\tan^{-1}\left(\frac{t}{3\sqrt{2}}\right)+c$$

Thus,

$$I = \frac{1}{3\sqrt{2}}\tan^{-1}\left(\frac{x^2-9}{3\sqrt{2}x}\right)+c$$

Indefinite Integrals Ex 19.31 Q4

Let $I = \int \frac{1}{x^4+x^2+1}dx$

Dividing numerator and denominator by x^2

$$I = \int \frac{\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}}dx$$

$$= \frac{1}{2}\int \frac{1+\frac{1}{x^2}-1+\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}}dx$$

$$= \frac{1}{2}\left\{\int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+3} - \int \frac{\left(1-\frac{1}{x^2}\right)}{\left(x+\frac{1}{x}\right)^2-1}dx\right\}$$

Let $\left(x-\frac{1}{x}\right) = t$

$\Rightarrow \quad \left(1+\frac{1}{x^2}\right)dx = dt$

and $\quad \left(x+\frac{1}{x}\right) = z$

$\Rightarrow \quad \left(1-\frac{1}{x^2}\right)dx = dz$

$\therefore \quad I = \frac{1}{2}\int \frac{dt}{t^2+3} - \frac{1}{2}\int \frac{dz}{z^2-1}$

$\Rightarrow \quad I = \frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{t}{\sqrt{3}}\right) - \frac{1}{4}\log\left|\frac{z-1}{z+1}\right| + c$

$\Rightarrow \quad I = \frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right) - \frac{1}{4}\log\left|\frac{x^2+1-x}{x^2+1+x}\right| + c$

Indefinite Integrals Ex 19.31 Q5

Let $I = \int \frac{x^2-3x+1}{x^4+x^2+1}dx$

Dividing numerator and denominator by x^2

$\therefore \quad I = \int \frac{1-\frac{3}{x}+\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}}dx$

$$= \int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+3} - \int \frac{3x}{x^4+x^2+1}dx$$

Let $\left(x-\frac{1}{x}\right) = t$

$\Rightarrow \quad \left(1+\frac{1}{x^2}\right)dx = dt \qquad$ [For Ist part]

Let $\quad x^2 = z$

$\Rightarrow \quad 2xdx = dz \qquad$ [For IInd part]

$\therefore \quad I = \int \frac{dt}{t^2+3} - \frac{3}{2}\int \frac{dz}{z^2+z+1}$

$\Rightarrow \quad = \int \frac{dt}{t^2+3} - \frac{3}{2}\int \frac{dz}{\left(z+\frac{1}{2}\right)^2+\frac{3}{4}}$

$\Rightarrow \quad = \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{t}{\sqrt{3}}\right) - \frac{3}{2}\times\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{z+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + c$

$\Rightarrow \quad I = \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right) - \sqrt{3}\tan^{-1}\left(\frac{2z+1}{\sqrt{3}}\right) + c$

Hence,

$$I = \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right) - \sqrt{3}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right) + c$$

Indefinite Integrals Ex 19.31 Q6

Let $I = \int \frac{x^2+1}{x^4-x^2+1}dx$

Dividing numerator and denominator by x^2

$$\therefore \quad I = \int \frac{\left(1+\frac{1}{x^2}\right)}{x^2-1+\frac{1}{x^2}}dx$$

$$= \int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+1}$$

Let $\left(x-\frac{1}{x}\right) = t$

$$\Rightarrow \quad \left(1+\frac{1}{x^2}\right)dx = dt$$

$$\Rightarrow \quad I = \int \frac{dt}{t^2+1}$$

$$= tan^{-1} t + c$$

$$\therefore \quad I = tan^{-1}\left(\frac{x^2-1}{x}\right)+c$$

Indefinite Integrals Ex 19.31 Q7

Let $I = \int \frac{x^2-1}{x^4+1}dx$

Dividing numerator and denominator by x^2

$$\therefore \quad I = \int \frac{\left(1-\frac{1}{x^2}\right)}{x^2+\frac{1}{x^2}}dx$$

$$= \int \frac{\left(1-\frac{1}{x^2}\right)}{\left(x+\frac{1}{x}\right)^2-2}dx$$

Let $\left(x+\frac{1}{x}\right) = t$

$$\Rightarrow \quad \left(1-\frac{1}{x^2}\right)dx = dt$$

$$\therefore \quad I = \int \frac{dt}{t^2-2}$$

$$= \frac{1}{2\sqrt{2}} log\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right|+c$$

So,

$$I = \frac{1}{2\sqrt{2}} log\left|\frac{x^2+1-\sqrt{2}x}{x^2+1+\sqrt{2}x}\right|+c$$

Indefinite Integrals Ex 19.31 Q8

Let $I = \int \frac{x^2+1}{x^4+7x^2+1} dx$

Dividing numerator and denominator by x^2

$$\therefore \quad I = \int \frac{\left(1+\frac{1}{x^2}\right)}{x^2+7+\frac{1}{x^2}} dx$$

$$= \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x-\frac{1}{x}\right)^2+9} dx$$

Let $\left(x-\frac{1}{x}\right) = t$

$$\Rightarrow \quad \left(1+\frac{1}{x^2}\right)dx = dt$$

$$\therefore \quad I = \int \frac{dt}{t^2+9}$$

$$= \frac{1}{3}tan^{-1}\left|\frac{t}{3}\right| + c$$

Hence,

$$I = \frac{1}{3}tan^{-1}\left(\frac{x^2-1}{3x}\right) + c$$

Indefinite Integrals Ex 19.31 Q9

$$\text{Let} \quad I = \int \frac{(x-1)^2}{x^4+x^2+1} dx$$

$$= \int \frac{x^2-2x+1}{x^4+x^2+1} dx$$

Dividing numerator and denominator by x^2

$$\therefore \quad I = \int \frac{1-\frac{2}{x}+\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}} dx$$

$$= \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x-\frac{1}{x}\right)^2+3} dx - \int \frac{2x}{x^4+x^2+1} dx$$

Let $\left(x-\frac{1}{x}\right) = t$

$$\Rightarrow \quad \left(1+\frac{1}{x^2}\right)dx = dt \qquad \text{[For Ist part]}$$

Let $\quad x^2 = z$

$$\Rightarrow \quad 2xdx = dz \qquad \text{[For IInd part]}$$

$$\therefore \quad I = \int \frac{dt}{t^2+3} - \int \frac{dz}{z^2+z+1}$$

$$= \int \frac{dt}{t^3+3} - \int \frac{dz}{\left(z+\frac{1}{2}\right)^2+\frac{3}{4}}$$

$$\Rightarrow \quad I = \frac{1}{\sqrt{3}}tan^{-1}\left(\frac{t}{\sqrt{3}}\right) - \frac{2}{\sqrt{3}}tan^{-1}\left(\frac{z+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + c$$

$$\Rightarrow \quad I = \frac{1}{\sqrt{3}}tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right) - \frac{2}{\sqrt{3}}tan^{-1}\left(\frac{2z+1}{\sqrt{3}}\right) + c$$

Hence,

$$I = \frac{1}{\sqrt{3}}tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right) - \frac{2}{\sqrt{3}}tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right) + c$$

Indefinite Integrals Ex 19.31 Q10

Let $I = \int \frac{1}{x^4 + 3x^2 + 1} dx$

Dividing numerator and denominator by x^2

$$\therefore \quad I = \int \frac{\frac{1}{x^2}}{x^2 + 3 + \frac{1}{x^2}} dx$$

$$= \frac{1}{2} \int \frac{\left(1 + \frac{1}{x^2}\right) - \left(1 - \frac{1}{x^2}\right)}{x^2 + 3 + \frac{1}{x^2}} dx$$

$$= \frac{1}{2} \int \frac{\left(1 + \frac{1}{x^2}\right)}{\left(x - \frac{1}{x}\right)^2 + 5} dx - \frac{1}{2} \int \frac{\left(1 - \frac{1}{x^2}\right)}{\left(x + \frac{1}{x}\right)^2 + 1} dx$$

Let $\left(x - \frac{1}{x}\right) = t$

$$\Rightarrow \quad \left(1 + \frac{1}{x^2}\right) dx = dt$$

and $\quad x + \frac{1}{x} = z$

$$\Rightarrow \quad \left(1 - \frac{1}{x^2}\right) dx = dz$$

$$\therefore \quad I = \frac{1}{2} \int \frac{dt}{t^2 + 5} - \frac{1}{2} \int \frac{dz}{z^2 + 1}$$

$$= \frac{1}{2\sqrt{5}} \tan^{-1}\left(\frac{t}{\sqrt{5}}\right) - \frac{1}{2} \tan^{-1}(z) + c$$

Hence,

$$I = \frac{1}{2\sqrt{5}} \tan^{-1}\left(\frac{x^2 - 1}{\sqrt{5}x}\right) - \frac{1}{2} \tan^{-1}\left(\frac{x^2 + 1}{x}\right) + c$$

Indefinite Integrals Ex 19.31 Q11

Consider the integral

$$I=\int \frac{1}{\sin^4 x+\sin^2 x\cos^2 x+\cos^4 x}dx$$

Divide both the numerator and the denominator by $\cos^4 x$, we have,

$$I=\int \frac{\frac{1}{\cos^4 x}}{\frac{\sin^4 x+\sin^2 x\cos^2 x+\cos^4 x}{\cos^4 x}}dx$$

$$=\int \frac{\sec^4 x}{\tan^4 x+\tan^2 x+1}dx$$

$$=\int \frac{\sec^2 x \times \sec^2 x}{\tan^4 x+\tan^2 x+1}dx$$

$$=\int \frac{\left(\tan^2 x+1\right)\times \sec^2 x}{\tan^4 x+\tan^2 x+1}dx$$

Substituting $\tan x = t$; $\sec^2 x dx = dt$

Thus,

$$I=\int \frac{\left(1+t^2\right)dt}{t^4+t^2+1}$$

$$=\int \frac{\left(1+\frac{1}{t^2}\right)dt}{\left(t^2+\frac{1}{t^2}+1\right)}$$

$$=\int \frac{\left(1+\frac{1}{t^2}\right)dt}{\left(t^2+\frac{1}{t^2}-2+2+1\right)}$$

$$=\int \frac{\left(1+\frac{1}{t^2}\right)dt}{\left(t-\frac{1}{t}\right)^2+3}$$

$$=\int \frac{\left(1+\frac{1}{t^2}\right)dt}{\left(t-\frac{1}{t}\right)^2+3}$$

Substituting $z=t-\frac{1}{t}$; $dz=\left(1+\frac{1}{t^2}\right)dt$

$$I=\int \frac{dz}{z^2+3}$$

$$\Rightarrow I=\int \frac{dz}{z^2+\left(\sqrt{3}\right)^2}$$

$$I=\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{z}{\sqrt{3}}\right)+C$$

$$=\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{t-\frac{1}{t}}{\sqrt{3}}\right)+C$$

$$=\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{\tan x-\frac{1}{\tan x}}{\sqrt{3}}\right)+C$$

$$I=\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{\tan x-\cot x}{\sqrt{3}}\right)+C$$

Ex 19.32

Indefinite Integrals Ex 19.32 Q1

Let $\quad I = \int \frac{1}{(x-1)\sqrt{x+2}}\,dx$

Let $x + 2 = t^2$

$$\therefore \quad I = \int \frac{2t\,dt}{\left(t^2-3\right)t}$$

$$= 2\int \frac{dt}{t^2-3}$$

$$= \frac{2}{2\sqrt{3}} \log\left|\frac{t-\sqrt{3}}{t+\sqrt{3}}\right| + c$$

Thus,

$$I = \frac{1}{\sqrt{3}} \log\left|\frac{\sqrt{x-2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right| + c$$

Indefinite Integrals Ex 19.32 Q2

Let $\quad I = \int \frac{1}{(x-1)\sqrt{2x+3}}\,dx$

Let $\quad I = \int \frac{1}{(x-1)\sqrt{2x+3}}\,dx$

Let $2x + 3 = t^2$

$\Rightarrow \quad 2dx = 2t\,dt$

$$\therefore \quad I = \int \frac{t\,dt}{\left(\frac{t^2-3}{2}-1\right)t}$$

$$= 2\int \frac{dt}{t^2-5}$$

$$= \frac{2}{2\sqrt{5}} \log\left|\frac{t-\sqrt{5}}{t+\sqrt{5}}\right| + c$$

Thus,

$$I = \frac{1}{\sqrt{5}} \log\left|\frac{\sqrt{2x+3}-\sqrt{5}}{\sqrt{2x+3}+\sqrt{5}}\right| + c$$

Indefinite Integrals Ex 19.32 Q3

Let $\quad I = \int \frac{1}{(x-1)\sqrt{x+2}}dx$

$$I = \int \frac{(x-1)+2}{(x-1)\sqrt{x+2}}dx$$

$$I = \int \frac{dx}{\sqrt{x+2}} + 2\int \frac{dx}{(x-1)\sqrt{x+2}} \qquad \text{---(A)}$$

Now,

$$\int \frac{dx}{\sqrt{x+2}} = 2\sqrt{x+2} + c_1$$

and,

$$\int \frac{dx}{(x-1)\sqrt{x+2}}$$

Let $x+2 = t^2$

$\Rightarrow \qquad dx = 2tdt$

$$\therefore \qquad \int \frac{dx}{(x-1)\sqrt{x+2}} = 2\int \frac{tdt}{(t^2-3)t} = 2\int \frac{dt}{t^2-3}$$

$$= \frac{2\times 1}{2\sqrt{3}} log \left|\frac{t-\sqrt{3}}{t+\sqrt{3}}\right| + c_2$$

$$= \frac{1}{\sqrt{3}} log \left|\frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right| +_2$$

Thus, form (A),

$$I = 2\sqrt{x+2} + c_1 + \frac{2}{\sqrt{3}} log \left|\frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right| + c_2$$

Hence,

$$I = 2\sqrt{x+2} + \frac{2}{\sqrt{3}} log \left|\frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right| + c$$

Indefinite Integrals Ex 19.32 Q4

Let $\quad I = \int \frac{x^2}{(x-1)\sqrt{x+2}}dx$

$$= \int \frac{(x^2-1+1)}{(x-1)\sqrt{x+2}}dx$$

$$= \int \frac{(x+1)(x-1)}{(x-1)\sqrt{x+2}}dx + \int \frac{dx}{(x-1)\sqrt{x+2}}$$

$$= \int \frac{(x+1)}{\sqrt{x+2}}dx + \int \frac{dx}{(x-1)\sqrt{x+2}}$$

$$= \int \frac{(x+2)-1}{\sqrt{x+2}}dx + \int \frac{dx}{(x-1)\sqrt{x+2}}$$

$$I = \int \sqrt{x+2}dx - \int \frac{dx}{\sqrt{x+2}} + \int \frac{dx}{(x-1)\sqrt{x+2}} \qquad \text{---(A)}$$

Now,

$$\int \sqrt{x+2}dx = \frac{2}{3}(x+2)^{\frac{3}{2}} + c_1$$

and,

$$\int \frac{dx}{\sqrt{x+2}} = 2\sqrt{x+2} + c_2$$

$$\int \frac{dx}{(x-1)\sqrt{x+2}}$$

Let $x+2 = t^2$

$\Rightarrow \qquad dx = 2tdt$

$$\therefore \qquad 2\int \frac{tdt}{(t^2-3)t} = 2\int \frac{dt}{t^2-3}$$

$$= \frac{2}{2\sqrt{3}} log \left|\frac{t-\sqrt{3}}{t+\sqrt{3}}\right| + c_3$$

$$\therefore \qquad \int \frac{dx}{(x-1)\sqrt{x+2}} = \frac{1}{\sqrt{3}} log \left|\frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right| + c_3$$

Thus, form (A)

$$I = \frac{2}{3}(x+2)^{\frac{3}{2}} - 2\sqrt{x+2} + \frac{1}{\sqrt{3}} log \left|\frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right| + c \qquad [\text{when } c = c_1 + c_2 + c_3]$$

Indefinite Integrals Ex 19.32 Q5

Let $\quad I = \int \frac{x}{(x-3)\sqrt{x+1}}\, dx$

$$= \int \frac{(x-3)+3}{(x-3)\sqrt{x+1}}\, dx$$

$$I = \int \frac{dx}{\sqrt{x+1}} + 3\int \frac{dx}{(x-3)\sqrt{x+1}} \qquad \text{---(A)}$$

Now,

$$\int \frac{dx}{\sqrt{x+1}} = 2\sqrt{x+1} + c_1$$

and,

$$\int \frac{dx}{\sqrt{x+2}} = 2\sqrt{x+2} + c_2$$

$$\int \frac{dx}{(x-3)\sqrt{x+1}}$$

Let $x + 1 = t^2$

$\Rightarrow \quad dx = 2tdt$

$$\therefore \quad \int \frac{dx}{(x-3)\sqrt{x+1}} = 2\int \frac{tdt}{(t^2-4)t}$$

$$= 2\left|\frac{dt}{t^2-4}\right|$$

$$= \frac{2}{2\times 2} log\left|\frac{t-2}{t+2}\right| + c_2$$

$$\therefore \quad \int \frac{dx}{(x-3)\sqrt{x+1}} = \frac{1}{2} log\left|\frac{\sqrt{x+1}-2}{\sqrt{x+1}+2}\right| + c_2$$

Thus, form (A)

$$I = 2\sqrt{x+1} + \frac{3}{2} log\left|\frac{\sqrt{x+1}-2}{\sqrt{x+1}+2}\right| + c \qquad [\text{when } c = c_1 + c_2]$$

Indefinite Integrals Ex 19.32 Q6

Let $I = \int \frac{x}{\left(x^2+1\right)\sqrt{x}}dx$

Let $x = t^2$

$\Rightarrow \quad dx = 2tdt$

$\therefore \quad 2\int \frac{tdt}{\left(t^2+1\right)t}$

$= 2\left|\frac{dt}{t^4+1}\right|$

Dividing numerator and denominator by t^2

$$I = 2\int \frac{\frac{t}{t^2}}{\left(t^2+\frac{1}{t^2}\right)}dt$$

$$= \int \frac{\left(1+\frac{1}{t^2}\right)-\left(1-\frac{1}{t^2}\right)}{\left(t^2+\frac{1}{t^2}\right)}dt$$

$$= \int \frac{\left(1+\frac{1}{t^2}\right)}{\left(t-\frac{1}{t}\right)^2+2}dt - \int \frac{\left(1-\frac{1}{t^2}\right)}{\left(1+\frac{1}{t}\right)^2-2}dt$$

Let $t - \frac{1}{t} = z$

$\Rightarrow \quad \left(1+\frac{1}{t^2}\right)dt = dz \qquad$ [For Ist part]

and,

$t + \frac{1}{t} = y$

$\Rightarrow \quad \left(1-\frac{1}{t^2}\right)dt = dy \qquad$ [For IInd part]

$\therefore \quad I = \int \frac{dz}{z^2+2} - \int \frac{dy}{y^2-2}$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{z}{\sqrt{2}}\right) - \frac{1}{2\sqrt{2}}\log\left|\frac{y-\sqrt{2}}{y+\sqrt{2}}\right| + c$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t^2-1}{\sqrt{2}t}\right) - \frac{1}{2\sqrt{2}}\log\left|\frac{x+1-\sqrt{2x}}{x+1+\sqrt{2x}}\right| + c$$

Indefinite Integrals Ex 19.32 Q7

Let $I = \int \frac{x}{\left(x^2 + 2X + 2\right)\sqrt{x+1}}dx$

Let $x + 1 = t^2$

$\Rightarrow \quad dx = 2tdt$

$$= 2\int \frac{\left(t^2 - 1\right)tdt}{\left(t^4 + 1\right)t}$$

$$= 2\int \frac{\left(t^2 - 1\right)dt}{\left(t^4 + 1\right)}$$

$$= 2\int \frac{\left(1 - \frac{1}{t^2}\right)dt}{t^2 + \frac{1}{t^2}}$$

$$= 2\int \frac{\left(1 - \frac{1}{t^2}\right)dt}{\left(t + \frac{1}{t}\right)^2 - 2}$$

Let $t + \frac{1}{t} = y$

$\Rightarrow \quad \left(1 - \frac{1}{t^2}\right)dt = dy$

$\therefore \quad I = 2\int \frac{dy}{y^2 - 2}$

$$= \frac{2}{2\sqrt{2}} \log\left|\frac{y - \sqrt{2}}{y + \sqrt{2}}\right| + c$$

Thus,

$$I = \frac{1}{\sqrt{2}} \log\left|\frac{t^2 + 1 - \sqrt{2}t}{t^2 + 1 + \sqrt{2}t}\right| + c$$

Hence,

$$I = \frac{1}{\sqrt{2}} \log\left|\frac{x + 2 - \sqrt{2(x+1)}}{x + 2 + \sqrt{2(x+1)}}\right| + c$$

Indefinite Integrals Ex 19.32 Q8

Let $I = \int \frac{1}{(x-1)\sqrt{x^2 + 1}}dx$

Let $x - 1 = \frac{1}{t}$

$\Rightarrow \quad dx = -\frac{1}{t^2}dt$

$$\therefore \quad I = -\int \frac{\frac{1}{t^2}dt}{\frac{1}{t}\sqrt{\left(\frac{1}{t} + 1\right)^2 + 1}}$$

$$= -\int \frac{dt}{\sqrt{2t^2 + 2t + 1}}$$

$$= -\frac{1}{\sqrt{2}}\int \frac{dt}{\sqrt{t^2 + t + \frac{1}{2}}}$$

$$= -\frac{1}{\sqrt{2}}\int \frac{dt}{\sqrt{\left(t + \frac{1}{2}\right)^2 + \frac{1}{4}}}$$

$$\therefore \quad I = -\frac{1}{\sqrt{2}} \log\left|\left(t + \frac{1}{2}\right) + \sqrt{\left(t + \frac{1}{2}\right)^2 + \frac{1}{4}}\right| + c \quad \left[\text{When } t = \frac{1}{x-1}\right]$$

Indefinite Integrals Ex 19.32 Q9

Let $I = \int \frac{1}{(x+1)\sqrt{x^2+x+1}}dx$

Let $x+1 = \frac{1}{t}$

$\Rightarrow \quad dx = -\frac{1}{t^2}dt$

$\therefore \quad I = -\int \frac{\frac{1}{t^2}dt}{\frac{1}{t}\sqrt{\left(\frac{1}{t^2}+\frac{1}{t}-1\right)}}$

$= -\int \frac{dt}{\sqrt{1+t-t^2}}$

$= -\int \frac{dt}{\sqrt{\frac{5}{4}-\left(\frac{1}{4}-t+t^2\right)}}$

$= -\int \frac{dt}{\sqrt{\frac{5}{4}-\left(t-\frac{1}{2}\right)^2}}$

$= -\sin^{-1}\left(\frac{t-\frac{1}{2}}{\frac{\sqrt{5}}{2}}\right)+c$

$\therefore \quad I = -\sin^{-1}\left(\frac{2t-1}{\sqrt{5}}\right)+c \qquad \left[\text{When } t = \frac{1}{x+1}\right]$

Indefinite Integrals Ex 19.32 Q10

Let $I = \int \frac{1}{\left(x^2-1\right)\sqrt{x^2+1}}dx$

Let $x = \frac{1}{t}$

$\Rightarrow \quad dx = -\frac{1}{t^2}dt$

$\therefore \quad I = -\int \frac{\frac{1}{t^2}dt}{\left(\frac{1}{t^2}-1\right)\sqrt{\left(\frac{1}{t^2}+1\right)}}$

$= -\int \frac{tdt}{\left(1-t^2\right)\sqrt{1+t^2}}$

Let $1+t^2 = u^2$

$\Rightarrow \quad 2tdt = 2udu$

$I = \int \frac{u\,du}{\left(u^2-2\right)u}$

$= \int \frac{du}{u^2-2}$

$\therefore \quad I = \frac{1}{2\sqrt{2}}\log\left|\frac{u-\sqrt{2}}{u+\sqrt{2}}\right|+c$

$= \frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{1+t^2}-\sqrt{2}}{\sqrt{1+t^2}+\sqrt{2}}\right|+c$

Hence,

$I = -\frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{2}x+\sqrt{x^2+1}}{\sqrt{2}x-\sqrt{x^2+1}}\right|+c$

Indefinite Integrals Ex 19.32 Q11

$$I = \int \frac{x}{\left(x^2+4\right)\sqrt{x^2+1}}dx$$

Let $x^2+1 = u^2$

$\Rightarrow \quad 2xdx = 2u\,du$

$$\therefore \quad I = \int \frac{u}{\left(u^2+3\right)u}du$$

$$= \int \frac{1}{u^2+3}du$$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{u}{\sqrt{3}}\right)+C$$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\left(\sqrt{\frac{x^2+1}{3}}\right)+C$$

Indefinite Integrals Ex 19.32 Q12

Let $\quad I = \int \frac{1}{\left(1+x^2\right)\sqrt{1-x^2}}dx$

Let $x = \frac{1}{t}$

$\Rightarrow \quad dx = -\frac{1}{t^2}dt$

$$\therefore \quad I = \int \frac{-\frac{1}{t^2}dt}{\left(\frac{1}{t^2}+1\right)\sqrt{\left(1-\frac{1}{t^2}\right)}}$$

$$= -\int \frac{tdt}{\left(t^2+1\right)\sqrt{t^2-1}}$$

Let $t^2-1 = u^2$

$\Rightarrow \quad 2tdt = 2udu$

$$I = -\int \frac{udu}{\left(u^2+2\right)u}$$

$$= -\int \frac{du}{u^2+2}$$

$$= -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right)+c$$

$$= -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{t^2-1}}{\sqrt{2}}\right)+c$$

Thus,

$$I = -\frac{1}{\sqrt{2}}\tan^{-1}\left|\sqrt{\frac{1-x^2}{2x^2}}\right|+c$$

Indefinite Integrals Ex 19.32 Q13

Let $\quad I = \int \frac{1}{\left(2x^2+3\right)\sqrt{x^2-4}}\,dx$

Let $x = \frac{1}{t}$

$\Rightarrow \qquad dx = -\frac{1}{t^2}dt$

$$\therefore \qquad I = \int \frac{-\frac{1}{t^2}dt}{\left(\frac{2}{t^2}+3\right)\sqrt{\left(\frac{1}{t^2}-4\right)}}$$

$$= -\int \frac{tdt}{\left(2+3t^2\right)\sqrt{1-4t^2}}$$

Let $1-4t^2 = u^2$

$\Rightarrow \qquad -8tdt = 2udu$

$$\therefore \qquad I = \frac{1}{4}\int \frac{udu}{\frac{\left(11-3u^2\right)}{4}u}$$

$$= \frac{1}{3}\int \frac{du}{\frac{11}{3}-u^2}$$

$$= \frac{1}{2\sqrt{33}}\log\left|\frac{u-\sqrt{\frac{11}{3}}}{u+\sqrt{\frac{11}{3}}}\right|+c$$

$$= \frac{1}{2\sqrt{33}}\log\left|\frac{\sqrt{1-4t^2}-\sqrt{\frac{11}{3}}}{\sqrt{1-4t^2}+\sqrt{\frac{11}{3}}}+c\right|$$

Hence,

$$I = \frac{1}{2\sqrt{33}}\log\left|\frac{\sqrt{11}x+\sqrt{3x^2-12}}{\sqrt{11}x-\sqrt{3x^2-12}}\right|+c$$

Indefinite Integrals Ex 19.32 Q14

$$I = \int \frac{x}{\left(x^2+4\right)\sqrt{x^2+9}}\,dx$$

Let $x^2+9 = u^2$

$\Rightarrow \qquad 2xdx = 2u\,du$

$$\therefore \qquad I = \int \frac{u}{\left(u^2-5\right)u}du$$

$$= \int \frac{du}{u^2-5}$$

$$= \frac{1}{2\sqrt{5}}\log\left(\frac{u-\sqrt{5}}{u+\sqrt{5}}\right)+C$$

$$= \frac{1}{2\sqrt{5}}\log\left(\frac{\sqrt{x^2+9}-\sqrt{5}}{\sqrt{x^2+9}+\sqrt{5}}\right)+C$$

Ex 20.1

Definite Integrals Ex 20.1 Q1

We know that $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

Now,

$$\int_4^9 \frac{1}{\sqrt{x}} dx$$

$$= \left[\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right]_4^9$$

$$= \left[\frac{\sqrt{x}}{\frac{1}{2}}\right]_4^9$$

$$= 2[\sqrt{9} - \sqrt{4}]$$

$$= 2[3 - 2]$$

$$= 2$$

$$\therefore \int_4^9 \frac{1}{\sqrt{x}} dx = 2$$

Definite Integrals Ex 20.1 Q2

We know that $\int \frac{dx}{x} = \log x + C$

Now,

$$\int_{-2}^3 \frac{1}{x+7} dx$$

$$= [\log(x+7)]_{-2}^3$$

$$= [\log 10 - \log 5]_{-2}^3$$

$$= \log \frac{10}{5} \qquad \left[\because \log a - \log b = \log \frac{a}{b}\right]$$

$$= \log 2$$

$$\therefore \int_{-2}^3 \frac{1}{x+7} dx = \log 2$$

Definite Integrals Ex 20.1 Q3

Let $x = \sin\theta$

$\Rightarrow dx = \cos\theta d\theta$

Now,

$x = 0 \Rightarrow \theta = 0$

$x = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

$$\therefore \int_0^{\frac{\pi}{6}} \frac{1}{\sqrt{1-\sin^2\theta}} \cos\theta d\theta$$

$$= \int_0^{\frac{\pi}{6}} \frac{\cos\theta d\theta}{\cos\theta}$$

$$= \int_0^{\frac{\pi}{6}} d\theta$$

$$= [\theta]_0^{\frac{\pi}{6}}$$

$$= \left[\frac{\pi}{6} - 0\right]$$

$$= \frac{\pi}{6}$$

$$\therefore \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} = \frac{\pi}{6}$$

Definite Integrals Ex 20.1 Q4

We have,

$$I = \int_0^1 \frac{1}{1+x^2} dx$$

$$= \left[\tan^{-1} x\right]_0^1$$

$$= \left[\tan^{-1} 1 - \tan^{-1} 0\right]$$

$$= \left[\frac{\pi}{4} - 0\right] \qquad \left[\because \tan^{-1} 1 = \frac{\pi}{4}\right]$$

$$= \frac{\pi}{4}$$

$$\therefore \int_0^1 \frac{1}{1+x^2} dx = \frac{\pi}{4}$$

Definite Integrals Ex 20.1 Q5

Let $x^2+1=t$

$\Rightarrow 2x\,dx = dt$

$\Rightarrow x\,dx = \dfrac{dt}{2}$

Now,

$x = 2 \Rightarrow t = 5$

$x = 3 \Rightarrow t = 10$

$$\therefore \int_2^3 \frac{x}{x^2+1}dx = \frac{1}{2}\int_5^{10}\frac{dt}{t} \quad = \frac{1}{2}\left[\log t\right]_5^{10}$$

$$= \frac{1}{2}\left[\log 10 - \log 5\right]$$

$$= \frac{1}{2}\left[\log\frac{10}{5}\right]$$

$$= \frac{1}{2}\left[\log 2\right]$$

$$= \log\sqrt{2}$$

$$\therefore \int_2^3 \frac{x}{x^2+1} = \log\sqrt{2}$$

Definite Integrals Ex 20.1 Q6

We have,

$$\int_0^\infty \frac{1}{a^2+b^2x^2}dx = \frac{1}{b^2}\int_0^\infty \frac{1}{\left(\frac{a}{b}\right)^2+x^2}dx$$

We know that $\int \dfrac{1}{a^2+x^2} = \dfrac{1}{a}\tan^{-1}\dfrac{x}{a}$

$$\therefore \frac{1}{b^2}\int_0^\infty \frac{1}{\left(\frac{a}{b}\right)^2+x^2}dx = \frac{1}{b^2}\left[\frac{b}{a}\tan^{-1}\left(\frac{bx}{a}\right)\right]_0^\infty$$

$$= \frac{1}{ab}\left[\tan^{-1}\left(\frac{bx}{a}\right)\right]_0^\infty$$

$$= \frac{1}{ab}\left[\tan^{-1}\infty - \tan^{-1}0\right]$$

$$= \frac{1}{ab}\left[\frac{\pi}{2} - 0\right]$$

$$= \frac{\pi}{2ab}$$

$$\Rightarrow \int_0^\infty \frac{1}{a^2+b^2x^2}dx = \frac{\pi}{2ab}$$

Definite Integrals Ex 20.1 Q7

We have,

$$\int_{-1}^{1} \frac{1}{1+x^2}\,dx$$

We know that $\int \frac{1}{1+x^2}\,dx = \tan^{-1} x$

Now,

$$\int_{-1}^{1} \frac{1}{1+x^2}\,dx$$

$$= \left[\tan^{-1} x\right]_{-1}^{1}$$

$$= \left[\tan^{-1} 1 - \tan^{-1}(-1)\right]$$

$$= \left[\frac{\pi}{4} - \left(\frac{-\pi}{4}\right)\right] \qquad \left[\because \tan^{-1}(-1) = \frac{-\pi}{4}\right]$$

$$= \left[\frac{\pi}{4} + \frac{\pi}{4}\right]$$

$$= \frac{2\pi}{4}$$

$$\therefore \int_{-1}^{1} \frac{1}{1+x^2}\,dx = \frac{\pi}{2}$$

Definite Integrals Ex 20.1 Q8

We have,

$$\int_{0}^{\infty} e^{-x}\,dx$$

We know that $\int e^{-x}\,dx = -e^{-x}$

Now,

$$\int_{0}^{\infty} e^{-x}\,dx$$

$$= \left[-e^{-x}\right]_{0}^{\infty}$$

$$= \left[-e^{-\infty} + e^{-0}\right] \qquad \left[\because e^{\infty} = 0, \quad e^{0} = 1\right]$$

$$= \left[-0 + 1\right]$$

$$\therefore \int_{0}^{\infty} e^{-x}\,dx = 1$$

Definite Integrals Ex 20.1 Q9

We have,

$$\int_{0}^{1} \frac{x}{x+1}\,dx \qquad \left[\text{Add and subtract 1 in numerator}\right]$$

$$= \int_{0}^{1} \frac{(x+1)-1}{x+1}\,dx$$

$$= \int_{0}^{1} 1\,dx - \int_{0}^{1} \frac{1}{x+1}\,dx$$

$$= \left[x\right]_{0}^{1} - \left[\log(x+1)\right]_{0}^{1}$$

$$= 1 - \left[\log 2 - \log 1\right]$$

$$= 1 - \log\frac{2}{1}$$

$$= 1 - \log 2$$

$$= \log e - \log 2 \qquad \left[\because \log e = 1\right]$$

$$= \log\frac{e}{2}$$

$$\therefore \int_{0}^{1} \frac{x}{x+1}\,dx = \log\frac{e}{2}$$

Definite Integrals Ex 20.1 Q10

We have,

$$\int_0^{\frac{\pi}{2}} (\sin x + \cos x) dx$$

$$= \int_0^{\frac{\pi}{2}} \sin x dx + \int_0^{\frac{\pi}{2}} \cos x dx$$

$$= \left[-cosx\right]_0^{\frac{\pi}{2}} + \left[\sin x\right]_0^{\frac{\pi}{2}}$$

$$= \left[\cos\frac{\pi}{2} + \cos 0\right] + \left[\sin\frac{\pi}{2} - \sin 0\right]$$

$$= \left[-0 + 1\right] + 1$$

$$= 1 + 1$$

$$= 2$$

$$\therefore \int_0^{\frac{\pi}{2}} (\sin x + \cos x) dx = 2$$

Definite Integrals Ex 20.1 Q11

We have,

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x dx$$

We know that $\int \cot x dx = \log(\sin x)$

Now,

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x dx$$

$$= \left[\log(\sin x)\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \left[\log\left(\sin\frac{\pi}{2}\right) - \log\left(\sin\frac{\pi}{4}\right)\right]$$

$$= \left[\log 1 - \log\frac{1}{\sqrt{2}}\right]$$

$$= \left[0 - (\log 1 - \log\sqrt{2})\right]$$

$$= \log\sqrt{2} \qquad \left[\because \log a^n = n\log a\right]$$

$$= \frac{1}{2}\log 2$$

$$\therefore \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x dx = \frac{1}{2}\log 2$$

Definite Integrals Ex 20.1 Q12

We have,

$$\int_0^{\frac{\pi}{4}} \sec x dx$$

We know that $\int \sec x dx = \log(\sec x + \tan x)$

$$\therefore \int_0^{\frac{\pi}{4}} \sec x dx$$

$$= \left[\log(\sec x + \tan x)\right]_0^{\frac{\pi}{4}}$$

$$= \left[\log(\sqrt{2} + 1) - \log(1 + 0)\right]$$

$$= \log(\sqrt{2} + 1) \qquad \left[\because \log 1 = 0\right]$$

$$\therefore \int_0^{\frac{\pi}{4}} \sec x dx = \log(\sqrt{2} + 1)$$

Definite Integrals Ex 20.1 Q13

Let $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x \, dx$

$\int \operatorname{cosec} x \, dx = \log\left|\operatorname{cosec} x - \cot x\right| = F(x)$

By second fundamental theorem of calculus, we obtain

$$I = F\left(\frac{\pi}{4}\right) - F\left(\frac{\pi}{6}\right)$$
$$= \log\left|\operatorname{cosec}\frac{\pi}{4} - \cot\frac{\pi}{4}\right| - \log\left|\operatorname{cosec}\frac{\pi}{6} - \cot\frac{\pi}{6}\right|$$
$$= \log\left|\sqrt{2} - 1\right| - \log\left|2 - \sqrt{3}\right|$$
$$= \log\left(\frac{\sqrt{2} - 1}{2 - \sqrt{3}}\right)$$

Definite Integrals Ex 20.1 Q14

We have,

$$\int_0^1 \frac{1-x}{1+x} dx$$

Let $x = \cos 2\theta \Rightarrow dx = -2\sin 2\theta d\theta$

Now,

$x = 0 \Rightarrow \theta = \frac{\pi}{4}$

$x = 1 \Rightarrow \theta = 0$

Now,

$$\int_0^1 \frac{1-x}{1+x} dx$$
$$= \int_{\frac{\pi}{4}}^{0} \frac{1-\cos 2\theta}{1+\cos 2\theta} \times (-2\sin 2\theta) d\theta$$
$$= \int_0^{\frac{\pi}{4}} \frac{2\sin^2\theta}{2\cos^2\theta} \times 2\sin 2\theta d\theta \qquad \left[\because -\int_a^b f(x)dx = \int_b^a f(x)dx\right]$$
$$= \int_0^{\frac{\pi}{4}} \frac{4\sin^3\theta}{\cos\theta} d\theta$$

Let $\cos\theta = t$

$\Rightarrow -\sin\theta d\theta = dt$

Now,

$\theta = 0 \Rightarrow t = 1$

$\theta = \frac{\pi}{4} \Rightarrow t = \frac{1}{\sqrt{2}}$

$$\therefore \int_0^{\frac{\pi}{4}} \frac{4\sin^3\theta}{\cos\theta} d\theta$$
$$= -4\int_1^{\frac{1}{\sqrt{2}}} \frac{(1-t^2)}{t} dt$$
$$= -4\left[\log t - \frac{t^2}{2}\right]_1^{\frac{1}{\sqrt{2}}}$$
$$= -4\left[\log\left(\frac{1}{\sqrt{2}}\right) - \frac{1}{4} - 0 + \frac{1}{2}\right]$$
$$= -4\left[-\log\sqrt{2} + \frac{1}{4}\right]$$

$$\therefore \int_0^1 \frac{1-x}{1+x} dx = 2\log 2 - 1$$

Definite Integrals Ex 20.1 Q15

$$I=\int_0^{\pi}\frac{1}{1+\sin x}dx$$

Multiplying Numerator and Denominator by $(1-\sin x)$

$$I=\int_0^{\pi}\frac{1}{1+\sin x}\times\frac{1-\sin x}{1-\sin x}dx$$
$$=\int_0^{\pi}\frac{(1-\sin x)}{(1^2-\sin^2 x)}dx$$
$$=\int_0^{\pi}\frac{1-\sin x}{(\cos^2 x)}dx$$
$$=\int_0^{\pi}\frac{1}{\cos^2 x}dx-\int_0^{\pi}\frac{\sin x}{\cos^2 x}dx$$
$$=\int_0^{\pi}\sec^2 x dx-\int_0^{\pi}\tan x.\sec x dx$$
$$=[\tan x]_0^{\pi}-[\sec x]_0^{\pi}$$
$$=[\tan\pi-\tan 0]-[\sec\pi-\sec 0]$$
$$=[0-0]-[-1-1]$$
$$=2$$
$$I=2$$

$$\therefore\int_0^{\pi}\frac{1}{1+\sin x}dx=2$$

Definite Integrals Ex 20.1 Q16

We have,

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{1}{1+\sin x}dx$$

We know,

$$\sin x=\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

$$\therefore\frac{1}{1+\sin x}=\frac{1}{1+\left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}=\frac{1+\tan^2\frac{x}{2}}{\left(1+\tan\frac{x}{2}\right)^2}=\frac{\sec^2\frac{x}{2}}{\left(1+\tan\frac{x}{2}\right)^2}$$

$$\Rightarrow\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{1}{1+\sin x}dx=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{\sec^2\frac{x}{2}}{\left(1+\tan\frac{x}{2}\right)^2}dx$$

If $f(x)$ is an even function $\int_{-a}^{a}f(x)dx=2\int_0^{a}f(x)dx$

So,

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{\sec^2\frac{x}{2}}{\left(1+\tan\frac{x}{2}\right)^2}dx=2\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{\sec^2\frac{x}{2}}{\left(1+\tan\frac{x}{2}\right)^2}dx$$

let $1+\tan\frac{x}{2}=t$

$\Rightarrow \frac{1}{2}\sec^2\frac{x}{2}dx = dt$

Now,

$x=-\frac{\pi}{4} \Rightarrow t = 1-\tan\frac{\pi}{8}$

$x=\frac{\pi}{4} \Rightarrow t = 1+\tan\frac{\pi}{8}$

$$\therefore 2\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sec^2\frac{x}{2}}{\left(1+\tan\frac{x}{2}\right)^2}dx = 2\int_{1-\tan\frac{\pi}{8}}^{1+\tan\frac{\pi}{8}} \frac{8dt}{t^2}$$

$$= 2\left[\frac{-1}{t}\right]_{1-\tan\frac{\pi}{8}}^{1+\tan\frac{\pi}{8}}$$

$$= 2\left[\frac{1}{1-\tan\frac{\pi}{8}} - \frac{1}{1+\tan\frac{\pi}{8}}\right]$$

$$= 2\left[\frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}\right]$$

$$= 2\tan\frac{\pi}{4} \qquad \left[\because \tan 2A = \frac{2\tan A}{1-\tan^2 A}\right]$$

$$= 2$$

$$\therefore \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1+\sin x}dx = 2$$

Definite Integrals Ex 20.1 Q17

Let $I=\int_0^{\frac{\pi}{2}}\cos^2 x\,dx$

$$\int \cos^2 x\,dx = \int\left(\frac{1+\cos 2x}{2}\right)dx = \frac{x}{2}+\frac{\sin 2x}{4} = \frac{1}{2}\left(x+\frac{\sin 2x}{2}\right) = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \left[\mathrm{F}\left(\frac{\pi}{2}\right) - \mathrm{F}(0)\right]$$

$$= \frac{1}{2}\left[\left(\frac{\pi}{2}-\frac{\sin\pi}{2}\right)-\left(0+\frac{\sin 0}{2}\right)\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{2}+0-0-0\right]$$

$$= \frac{\pi}{4}$$

Definite Integrals Ex 20.1 Q18

We have,

$$\int_0^{\frac{\pi}{2}} \cos^3 x dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos 3x + 3\cos x}{4} dx \qquad \left[\because \cos 3x = 4\cos^3 x - 3\cos x\right]$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{2}} \left(\cos 3x + 3\cos x\right) dx$$

$$= \frac{1}{4}\left[\frac{\sin 3x}{3} + 3\sin x\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{4}\left[\left(\frac{\sin 3\frac{\pi}{2}}{3} + 3\sin\frac{\pi}{2}\right) - \left(\frac{\sin 0}{3} + 3\sin 0\right)\right]$$

$$= \frac{1}{4}\left[\left(\frac{-1}{3} + 3\right) - \left(0 + 0\right)\right] = \frac{2}{3}$$

$$= \frac{1}{4}\left[\frac{8}{3}\right]$$

$$= \frac{2}{3}$$

$$\therefore \int_0^{\frac{\pi}{2}} \cos^3 x dx = \frac{2}{3}$$

Definite Integrals Ex 20.1 Q19

We have,

$$\int_0^{\frac{\pi}{6}} \cos x \cos 2x dx \qquad \left[\because 2\cos C \cos D = \cos(C + D) - \cos(C - D)\right]$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{6}} 2\cos x \cos 2x dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{6}} \left(\cos 3x + \cos x\right) dx$$

$$= \frac{1}{2}\int\left[\frac{\sin 3x}{3} + \sin x\right]_0^{\frac{\pi}{6}}$$

$$= \frac{1}{2}\left[\left(\frac{\sin 3\frac{\pi}{6}}{3} + \sin\frac{\pi}{6}\right) - \left(\sin 0 - \sin 0\right)\right]$$

$$= \frac{1}{2}\left[\frac{\sin\frac{\pi}{2}}{3} + \sin\frac{\pi}{6}\right]$$

$$= \frac{1}{2}\left(\frac{1}{3} + \frac{1}{2}\right)$$

$$= \frac{1}{2}\left(\frac{5}{6}\right)$$

$$= \frac{5}{12}$$

$$\therefore \int_0^{\frac{\pi}{6}} \cos x \cos 2x dx = \frac{5}{12}$$

Definite Integrals Ex 20.1 Q20

We have,

$$\int_0^{\frac{\pi}{2}} \sin x \sin 2x dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} 2\sin x \sin 2x dx \qquad [\because 2\sin C \times \sin D = \cos(D-C) - \cos(D+C)]$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} (\cos x - \cos 3x) dx$$

$$= \frac{1}{2}\left[\sin x - \frac{\sin 3x}{3}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2}\left[\left(\sin\frac{\pi}{2} - \sin 0\right) - \left(\frac{\sin 3\frac{\pi}{2}}{3} - \frac{\sin 0}{3}\right)\right]$$

$$= \frac{1}{2}\left[(1-0) - \left(\frac{-1}{3} - 0\right)\right] \qquad \left[\because \sin 3\frac{\pi}{2} = -1\right]$$

$$= \frac{1}{2} \times \frac{4}{3}$$

$$= \frac{2}{3}$$

$$\therefore \int_0^{\frac{\pi}{2}} \sin x \sin 2x dx = \frac{2}{3}$$

Definite Integrals Ex 20.1 Q21

We have,

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} (\tan x + \cot x)^2 dx$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\frac{\sin^2 x + \cot^2 x}{\sin x \cos x}\right)^2 dx$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\frac{1}{\sin x \cos x}\right)^2 dx$$

Multiplying numerator and denominator by 2

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\frac{2}{2\sin x \cos x}\right)^2 dx$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\frac{2}{\sin 2x}\right)^2 dx \qquad [\because 2\sin x \cos x = \sin 2x]$$

$$= 4\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} cosec^2 x dx$$

$$= 4\left[-\frac{\cot 2x}{2}\right]_{\frac{\pi}{3}}^{\frac{\pi}{4}}$$

$$= 2\left[-\cot\frac{\pi}{2} + \cot 2\frac{\pi}{3}\right]$$

$$= 2\left[\frac{-1}{\sqrt{3}} - 0\right]$$

$$= \frac{-2}{\sqrt{3}}$$

$$\therefore \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} (\tan x + \cot x)^2 dx = \frac{-2}{\sqrt{3}}$$

Definite Integrals Ex 20.1 Q22

We have,

$$\int_0^{\frac{\pi}{2}} \cos^4 x dx$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{2}} \left(1+\cos 2x\right)^2 dx \qquad \left[\because 2\cos^2 x = 1+\cos 2x\right]$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{2}} \left(1+\cos^2 2x + 2\cos 2x\right) dx$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{2}} \left(1+\frac{1+\cos 4x}{2} + 2\cos 2x\right) dx$$

$$= \frac{1}{4}\left[x+\frac{1}{2}x+\frac{\sin 4x}{8}+\sin 2x\right]_0^{\frac{\pi}{2}} \qquad \left[\because \int \cos 4x dx = \frac{\sin 4x}{4}\right]$$

$$= \frac{1}{4}\left[\frac{\pi}{2}+\frac{\pi}{4}+0+0-0-0-0-0\right]$$

$$= \frac{1}{4}\times\frac{3\pi}{4}$$

$$= \frac{3\pi}{16}$$

$$\therefore \int_0^{\frac{\pi}{2}} \cos^4 x dx = \frac{3\pi}{16}$$

Definite Integrals Ex 20.1 Q23

We have,

$$\int_0^{\frac{\pi}{2}} \left(a^2\cos^2 x + b^2\left(1-\cos^2 x\right)\right) dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\left(a^2-b^2\right)\cos^2 x + b^2\right) dx$$

$$= \frac{a^2-b^2}{2}\int_0^{\frac{\pi}{2}} \left(1+\cos 2x\right) dx + b^2 \int_0^{\frac{\pi}{2}} dx$$

$$= \frac{a^2-b^2}{2}\left[x+\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}} + b^2[x]_0^{\frac{\pi}{2}}$$

$$= \frac{a^2-b^2}{2}\left[\frac{\pi}{2}+0-0-0\right] + b^2\left[\frac{\pi}{2}-0\right]$$

$$= \frac{a^2-b^2}{2}\left[\frac{\pi}{2}\right] + b^2\left[\frac{\pi}{2}\right]$$

$$= a^2\frac{\pi}{4} + b^2\left[\frac{\pi}{2}-\frac{\pi}{4}\right]$$

$$= \frac{\pi a^2}{4} + \frac{\pi b^2}{4}$$

$$= \frac{\pi}{4}\left(a^2+b^2\right)$$

$$\therefore \int_0^{\frac{\pi}{2}} (a^2\cos^2 x + b^2\sin^2 x) dx = \frac{\pi}{4}\left(a^2+b^2\right)$$

Definite Integrals Ex 20.1 Q24

We have,

$$\int_0^{\frac{\pi}{2}} \sqrt{1+\sin x}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{1+\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}\, dx \qquad \text{We use } \sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

$$\int_0^{\frac{\pi}{2}} \sqrt{1+\sin x}\, dx = \int_0^{\frac{\pi}{2}} \sqrt{\frac{\left(1+\tan\frac{x}{2}\right)^2}{1+\tan^2\frac{x}{2}}}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{\frac{\left(1+\tan\frac{x}{2}\right)^2}{\sec^2\frac{x}{2}}}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{1+\tan\frac{x}{2}}{\sec\frac{x}{2}}\right) dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\cos\frac{x}{2}+\sin\frac{x}{2}\right) dx$$

$$= \left[2\sin\frac{x}{2} - 2\cos\frac{x}{2}\right]_0^{\frac{\pi}{2}}$$

$$= 2\left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - 0 + 1\right]$$

$$\therefore \int_0^{\frac{\pi}{2}} \sqrt{1+\sin x}\, dx = 2$$

Definite Integrals Ex 20.1 Q25

We have,

$$\int_0^{\frac{\pi}{2}} \sqrt{1+\cos x}$$

We use $1+\cos x = 2\cos^2\frac{x}{2}$

$$= \int_0^{\frac{\pi}{2}} \sqrt{2\cos^2\frac{x}{2}}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{2}\cos\frac{x}{2} dx$$

$$= \sqrt{2}\left[2\sin\frac{x}{2}\right]_0^{\frac{\pi}{2}}$$

$$= 2\sqrt{2}\left[\frac{1}{\sqrt{2}}\right]$$

$$= 2$$

$$\therefore \int_0^{\frac{\pi}{2}} \sqrt{1+\cos x} = 2$$

Definite Integrals Ex 20.1 Q26

We have,

$$\int x\sin x\, dx = x\int \sin x dx - \int\left(\int \sin x dx\right)\left(\frac{dx}{dx}\right) dx$$

$$= -x\cos x + \int \cos x dx$$

$$\therefore \int_0^{\frac{\pi}{2}} x\sin x\, dx = \left[-x\cos x + \sin x\right]_0^{\frac{\pi}{2}} = \left(-\frac{\pi}{2}\times 0\right) + 1 + 0 - 0 = 1$$

$$\therefore \int_0^{\frac{\pi}{2}} x\sin x dx = 1$$

Definite Integrals Ex 20.1 Q27

We have,

$$\int x\cos x\,dx = x\int \cos x dx - \int\left(\int \cos x dx\right)\frac{dx}{dx}dx = x\sin x - \int \sin x dx$$

$$\therefore \int_0^{\frac{\pi}{2}} x\cos x\,dx = \left[x\sin x + \cos x\right]_0^{\frac{\pi}{2}} = \left[\frac{\pi}{2} + 0 - 0 - 1\right] = \frac{\pi}{2} - 1$$

$$\therefore \int_0^{\frac{\pi}{2}} x\cos x dx = \frac{\pi}{2} - 1$$

Definite Integrals Ex 20.1 Q28

We have,

$$\int x^2\cos x\,dx = x^2\int \cos x dx - \int (2x)\left(\int \cos x dx\right)dx = x^2\sin x - \int \sin x.2x dx$$

$$= x^2\sin x - 2\left[x\int \sin x - \int\left(\int \sin x dx\right)dx\right]$$

$$= x^2\sin x - 2\left[-x\cos x + \int \cos x dx\right]$$

$$\therefore \int_0^{\frac{\pi}{2}} x^2\cos x\,dx = \left[x^2\sin x + 2x\cos x - 2\sin x\right]_0^{\frac{\pi}{2}}$$

$$= \left[\frac{\pi^2}{4} + 0 - 2 - 0 - 0 + 0\right]$$

$$= \frac{\pi^2}{4} - 2$$

$$\therefore \int_0^{\frac{\pi}{2}} x^2\cos x dx = \frac{\pi^2}{4} - 2$$

Definite Integrals Ex 20.1 Q29

We have,

$$\int x^2\sin x\,dx = x^2\int \sin x dx - \int 2x\left(\int \sin x dx\right)dx = x^2\cos x + \int 2x\cos x dx$$

$$= x^2\cos x + 2\left[x\int \cos x dx - \int\left(\int \cos x dx\right)dx\right]$$

$$= -x^2\cos x + 2\left[x\sin x - \int \sin x dx\right]$$

$$\therefore \int_0^{\frac{\pi}{4}} x^2\sin x\,dx = \left[-x^2\cos x + 2x\sin x + 2\cos x\right]_0^{\frac{\pi}{4}}$$

$$= \frac{-\pi^2}{16}.\frac{1}{\sqrt{2}} + \frac{\pi}{2}.\frac{1}{\sqrt{2}} + 2.\frac{1}{\sqrt{2}} + 0 - 0 - 2$$

$$= \frac{1}{\sqrt{2}}\left[\frac{-\pi^2}{16} + \frac{\pi}{2} + 2\right] - 2$$

$$= \sqrt{2} + \frac{\pi}{2\sqrt{2}} - \frac{\pi^2}{16\sqrt{2}} - 2$$

$$\therefore \int_0^{\frac{\pi}{4}} x^2\sin x dx = \sqrt{2} + \frac{\pi}{2\sqrt{2}} - \frac{\pi^2}{16\sqrt{2}} - 2$$

Definite Integrals Ex 20.1 Q30

We have,

$$\int x^2 \cos 2x\, dx = x^2 \int \cos 2x dx - \int 2x \left(\int \cos 2x dx\right) dx$$

$$= \frac{x^2 \sin 2x}{2} - \int 2x \times \frac{\sin 2x}{2} dx$$

$$= \frac{x^2 \sin 2x}{2} - \left[x \int \sin 2x dx - \int \left(\int \sin 2x dx\right) dx\right]$$

$$= \frac{x^2 \sin 2x}{2} + \left[\frac{x \cos 2x}{2} - \int \frac{x \cos 2x}{2}\right]$$

$$\therefore \quad \int_0^{\frac{\pi}{2}} x^2 \cos 2x\, dx = \left[\frac{x^2 \sin 2x}{2} + \frac{x \cos 2x}{2} - \frac{\sin 2x}{4}\right]_0^{\frac{\pi}{2}}$$

$$= \left[\frac{\pi^2}{8} \times 0 + \frac{\pi}{4}(-1) - 0 - 0 - 0 + 0\right]$$

$$= \frac{-\pi}{4}$$

$$\therefore \int_0^{\frac{\pi}{2}} x^2 \cos 2x dx = \frac{-\pi}{4}$$

Definite Integrals Ex 20.1 Q31

We have,

$$\int x^2 \cos^2 x dx = \int x^2 \left(\frac{1+\cos 2x}{2}\right) dx = \frac{1}{2} \int \left(x^2 + x^2 \cos 2x\right) dx = \frac{1}{2}\left[\int x^2 dx + \int x^2 \cos 2x dx\right] \quad ..(A)$$

Now,

$$\int_0^{\frac{\pi}{2}} x^2 dx = \left[\frac{x^3}{3}\right]_0^{\frac{\pi}{2}} = \frac{\pi^3}{24} \quad ...(B)$$

$$\int x^2 \cos 2x\, dx = x^2 \int \cos 2x dx - \int 2x \left(\int \cos 2x dx\right) dx = \frac{x^2 \sin 2x}{2} - \int \frac{\sin 2x}{2} .2x dx$$

$$= \frac{x^2 \sin 2x}{2} - \left[x \int \sin 2x - \int \left(\int \sin 2x dx\right) dx\right]$$

$$= \frac{x^2 \sin 2x}{2} + \frac{x \cos 2x}{2} - \int \frac{\cos 2x}{2} dx$$

$$\therefore \quad \int_0^{\frac{\pi}{2}} x^2 \cos 2x\, dx = \left[\frac{x^2 \sin 2x}{2} + \frac{x \cos 2x}{2} - \frac{\sin 2x}{4}\right]_0^{\frac{\pi}{2}} = \frac{-\pi}{4} \quad -(C)$$

Now, Put (B) & (C) in (A), we get,

$$\int_0^{\frac{\pi}{2}} x^2 \cos^2 x dx = \int_0^{\frac{\pi}{2}} x^2 dx + \int_0^{\frac{\pi}{2}} x^2 \cos 2x\, dx = \frac{1}{2}\left[\frac{\pi^3}{24} - \frac{\pi}{4}\right] = \frac{\pi^3}{48} - \frac{\pi}{8}$$

Definite Integrals Ex 20.1 Q32

We have,

$$\int \log x dx = \int 1.\log x dx = \log x \int dx - \int \left(\int dx\right) . \frac{1}{x} dx = x \log x - \int x . \frac{1}{x} dx = x \log x - \int dx$$

$$\therefore \quad \int_1^2 \log x dx = \left[x \log x - x\right]_1^2 = 2\log 2 - 2 - 0 + 1 = 2\log 2 - 1$$

Definite Integrals Ex 20.1 Q33

We have,

$$\int \frac{\log x}{(x+1)^2} dx = \int \frac{1}{(x+1)^2} \log x dx = \log x \int \frac{1}{(x+1)^2} dx - \int \left(\int \frac{1}{(x+1)^2} dx\right) \frac{1}{x} dx$$

$$= \frac{-\log x}{(x+1)} + \int \frac{1}{x(x+1)} dx$$

$$= \frac{-\log x}{(x+1)} + \int \left(\frac{1}{x} - \frac{1}{x+1}\right) dx$$

$$\therefore \quad \int_1^3 \frac{\log x}{(x+1)^2} dx = \left[\frac{-\log x}{x+1} + \log x - \log(x+1)\right]_1^3 = \frac{3}{4}\log 3 - \log 2$$

Definite Integrals Ex 20.1 Q34

Let $I = \int_1^e \frac{e^x}{x}\left(1 + x\log x\right)dx$

$I = \int_1^e \frac{e^x}{x}dx + \int_1^e e^x \log x dx$

$I = \left[e^x \log x\right]_1^e - \int_1^e e^x . \log x + \int_1^e e^x \log x$

$I = \left[e^x \log x\right]_1^e$

$I = \left[e^x \log e - e^1 . \log 1\right]$

$I = \left[e^e . 1 - 0\right]$

$I = e^e$

$\therefore \int_1^e \frac{e^x}{x}\left(1 + x\log x\right)dx = e^e$

Definite Integrals Ex 20.1 Q35

We have,

$\int_1^e \frac{\log x}{x}dx$

Let $\log x = t$

$\Rightarrow \frac{1}{x}dx = dt$

Now,

$x = 1 \Rightarrow t = 0$

$x = e \Rightarrow t = 1$

$\therefore \int_1^e \frac{\log x}{x}dx = \int_0^1 t\,dt$

$= \left[\frac{t^2}{2}\right]_0^1$

$= \frac{1}{2}$

$\therefore \int_1^e \frac{\log x}{x}dx = \frac{1}{2}$

Definite Integrals Ex 20.1 Q36

We have,

$\int_e^{e^2} \left\{\frac{1}{\log x} - \frac{1}{(\log x)^2}dx\right\}$

$I = \int \frac{1}{\log x}.1dx = \frac{1}{\log x}\int dx - \int\left(\int dx\right).\frac{d}{dx}\left(\frac{1}{\log x}\right)dx = \frac{x}{\log x} + \int \frac{1}{(\log x)^2}.x.\frac{1}{x}dx$

$= \frac{x}{\log x} + \int \frac{dx}{(\log x)^2}$

$\int_e^{e^2} \left\{\frac{1}{\log x} - \frac{1}{(\log x)^2}dx\right\} = \left[\frac{x}{\log x}\right]_e^{e^2} + \int_e^{e^2} \frac{dx}{(\log x)^2} - \int_e^{e^2} \frac{dx}{(\log x)^2}$

$= \left[\frac{x}{\log x}\right]_e^{e^2}$

$= \frac{e^2}{2} - e$

Definite Integrals Ex 20.1 Q37

We have,

$$\int_1^2 \frac{x+3}{x(x+2)}dx$$

$$= \int_1^2 \frac{x}{x(x+2)}dx + \int_1^2 \frac{3}{x(x+2)}dx$$

$$= \int_1^2 \frac{dx}{(x+2)} + \int_1^2 \frac{3}{x(x+2)}dx$$

$$= \left[\log(x+2)\right]_1^2 + \frac{3}{2}\int_1^2 \frac{1}{x} - \frac{1}{x+2}dx \qquad \left[\text{using partial fraction}\right]$$

$$= \left[\log(x+2)\right]_1^2 + \left[\frac{3}{2}\log x - \frac{3}{2}\log(x+2)\right]_1^2$$

$$= \left[\frac{3}{2}\log x - \frac{1}{2}\log(x+2)\right]_1^2$$

$$= \frac{1}{2}\left[3\log 2 - \log 4 + \log 3\right]$$

$$= \frac{1}{2}\left[3\log 2 - 2\log 2 + \log 3\right] \qquad \left[\because \log 4 = 2\log 2\right]$$

$$= \frac{1}{2}\left[\log 2 + \log 3\right]$$

$$= \frac{1}{2}\left[\log 6\right]$$

$$= \frac{1}{2}\log 6$$

$$\therefore \int_1^2 \frac{x+3}{x(x+2)}dx = \frac{1}{2}\log 6$$

Definite Integrals Ex 20.1 Q38

$$\text{Let } I = \int_0^1 \frac{2x+3}{5x^2+1}dx$$

$$\int \frac{2x+3}{5x^2+1}dx = \frac{1}{5}\int \frac{5(2x+3)}{5x^2+1}dx$$

$$= \frac{1}{5}\int \frac{10x+15}{5x^2+1}dx$$

$$= \frac{1}{5}\int \frac{10x}{5x^2+1}dx + 3\int \frac{1}{5x^2+1}dx$$

$$= \frac{1}{5}\int \frac{10x}{5x^2+1}dx + 3\int \frac{1}{5\left(x^2+\frac{1}{5}\right)}dx$$

$$= \frac{1}{5}\log\left(5x^2+1\right) + \frac{3}{5}\cdot\frac{1}{\frac{1}{\sqrt{5}}}\tan^{-1}\frac{x}{\frac{1}{\sqrt{5}}}$$

$$= \frac{1}{5}\log\left(5x^2+1\right) + \frac{3}{\sqrt{5}}\tan^{-1}\left(\sqrt{5}x\right)$$

$$= \mathrm{F}(x)$$

Definite Integrals Ex 20.1 Q39

$$\int_0^2 \frac{dx}{x+4-x^2} = \int_0^2 \frac{dx}{-\left(x^2-x-4\right)}$$

$$= \int_0^2 \frac{dx}{-\left(x^2-x+\frac{1}{4}-\frac{1}{4}-4\right)}$$

$$= \int_0^2 \frac{dx}{-\left[\left(x-\frac{1}{2}\right)^2-\frac{17}{4}\right]}$$

$$= \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}$$

Let $x-\frac{1}{2}=t \Rightarrow dx = dt$

When $x=0$, $t=-\frac{1}{2}$ and when $x=2$, $t=\frac{3}{2}$

$$\therefore \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2} = \int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{dt}{\left(\frac{\sqrt{17}}{2}\right)^2-t^2}$$

$$= \left[\frac{1}{2\left(\frac{\sqrt{17}}{2}\right)}\log\frac{\frac{\sqrt{17}}{2}+t}{\frac{\sqrt{17}}{2}-t}\right]_{-\frac{1}{2}}^{\frac{3}{2}}$$

$$= \frac{1}{\sqrt{17}}\left[\log\frac{\frac{\sqrt{17}}{2}+\frac{3}{2}}{\frac{\sqrt{17}}{2}-\frac{3}{2}} - \frac{\log\frac{\sqrt{17}}{2}-\frac{1}{2}}{\log\frac{\sqrt{17}}{2}+\frac{1}{2}}\right]$$

$$= \frac{1}{\sqrt{17}}\left[\log\frac{\sqrt{17}+3}{\sqrt{17}-3} - \log\frac{\sqrt{17}-1}{\sqrt{17}+1}\right]$$

$$= \frac{1}{\sqrt{17}}\log\frac{\sqrt{17}+3}{\sqrt{17}-3}\times\frac{\sqrt{17}+1}{\sqrt{17}-1}$$

$$= \frac{1}{\sqrt{17}}\log\left[\frac{17+3+4\sqrt{17}}{17+3-4\sqrt{17}}\right]$$

$$= \frac{1}{\sqrt{17}}\log\left[\frac{20+4\sqrt{17}}{20-4\sqrt{17}}\right]$$

$$= \frac{1}{\sqrt{17}}\log\left(\frac{5+\sqrt{17}}{5-\sqrt{17}}\right)$$

$$= \frac{1}{\sqrt{17}}\log\left[\frac{\left(5+\sqrt{17}\right)\left(5+\sqrt{17}\right)}{25-17}\right]$$

$$= \frac{1}{\sqrt{17}}\log\left[\frac{25+17+10\sqrt{17}}{8}\right]$$

$$= \frac{1}{\sqrt{17}}\log\left(\frac{42+10\sqrt{17}}{8}\right)$$

$$= \frac{1}{\sqrt{17}}\log\left(\frac{21+5\sqrt{17}}{4}\right)$$

Definite Integrals Ex 20.1 Q40

We have,

$$\int_0^1 \frac{1}{2x^2+x+1}dx$$

$$= \frac{1}{2}\int_0^1 \frac{1dx}{\left(x^2+\frac{1}{2}x+\frac{1}{2}\right)}$$

$$= \frac{1}{2}\int_0^1 \frac{dx}{\left(x+\frac{1}{4}\right)^2+\frac{1}{2}-\frac{1}{16}} \qquad \left[\text{Adding } \frac{1}{16} \text{ \& substracting } \frac{1}{16} \text{ in numerator}\right]$$

$$= \frac{1}{2}\int_0^1 \frac{dx}{\left(x+\frac{1}{4}\right)^2+\frac{7}{16}}$$

$$= \frac{1}{2}\int_0^1 \frac{dx}{\left(x+\frac{1}{4}\right)^2+\left(\frac{\sqrt{7}}{4}\right)^2}$$

$$= \frac{1}{2}\cdot\frac{4}{\sqrt{7}}\left[\tan^{-1}\left(\frac{x+\frac{1}{4}}{\frac{\sqrt{7}}{4}}\right)\right]_0^1$$

$$= \frac{2}{\sqrt{7}}\left\{\tan^{-1}\frac{5}{\sqrt{7}}-\tan^{-1}\left(\frac{1}{\sqrt{7}}\right)\right\}$$

$$\therefore \int_0^1 \frac{1}{2x^2+x+1}dx = \frac{2}{\sqrt{7}}\left\{\tan^{-1}\frac{5}{\sqrt{7}}-\tan^{-1}\left(\frac{1}{\sqrt{7}}\right)\right\}$$

Definite Integrals Ex 20.1 Q41

Let $I = \int_0^1 \sqrt{x(1-x)}\,dx$

let $x = \sin^2\theta$
$\Rightarrow dx = 2\sin\theta.\cos\theta d\theta$

Now,
$x = 0 \Rightarrow \theta = 0$
$x = 1 \Rightarrow \theta = \frac{\pi}{2}$

$\therefore\ I = \int_0^{\frac{\pi}{2}} \sqrt{\sin^2\theta\left(1-\sin^2\theta\right)}.2\sin\theta.\cos\theta d\theta$

$= \int_0^{\frac{\pi}{2}} 2\sin^2\theta.\cos^2\theta d\theta$

$= \frac{1}{2}\int_0^{\frac{\pi}{2}} 4\sin^2\theta.\cos^2\theta d\theta$

$= \frac{1}{2}\int_0^{\frac{\pi}{2}} \left(\sin^2 2\theta\right) d\theta$

$= \frac{1}{2}\int_0^{\frac{\pi}{2}} \left(\frac{1-\cos 4\theta}{2}\right) d\theta$

$= \frac{1}{4}\int_0^{\frac{\pi}{2}} \left(1-\cos 4\theta\right) d\theta$

$= \frac{1}{4}\int_0^{\frac{\pi}{2}} d\theta - \frac{1}{4}\int_0^{\frac{\pi}{2}} \cos 4\theta d\theta$

$= \frac{1}{4}\left[\theta\right]_0^{\frac{\pi}{2}} - \frac{1}{4}\left[\frac{\sin 4\theta}{4}\right]_0^{\frac{\pi}{2}}$

$= \frac{1}{4}\left[\frac{\pi}{2} - 0\right] - \frac{1}{16}\left[\sin\pi - \sin 0\right]$

$= \frac{\pi}{8} - \frac{1}{16}\left[0-0\right]$

$= \frac{\pi}{8}$

$I = \frac{\pi}{8}$

$\therefore\ \int_0^1 \sqrt{x(1-x)}\,dx = \frac{\pi}{8}$

Definite Integrals Ex 20.1 Q42

We have,

$\int_0^2 \frac{dx}{\sqrt{3+2x-x^2}}$

$\int_0^2 \frac{dx}{\sqrt{3+1-\left(x^2-2x+1\right)}}$ [Add and subtract 1 in denominator]

$= \int_0^2 \frac{dx}{\sqrt{(2)^2 (x-1)^2}}$ $\left[\because \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}\right]$

$= \left[\sin^{-1}\left(\frac{x-1}{2}\right)\right]_0^2$

$= \sin^{-1}\frac{1}{2} - \sin^{-1}\left(\frac{-1}{2}\right)$

$= \sin^{-1}\left(\sin\frac{\pi}{6}\right) - \sin^{-1}\left[\sin\left(\frac{-\pi}{6}\right)\right]$

$= \frac{\pi}{6} + \frac{\pi}{6}$

$= \frac{\pi}{3}$

$\therefore\ \int_0^2 \frac{dx}{\sqrt{3+2x-x^2}} = \frac{\pi}{3}$

Definite Integrals Ex 20.1 Q43

We have,

$$\int_0^4 \frac{dx}{\sqrt{4x - x^2}}$$

$$= \int_0^4 \frac{dx}{\sqrt{4 - 4 + 4x - x^2}} \qquad \left[\text{Add and subtract 4 in denominator}\right]$$

$$= \int_0^4 \frac{dx}{\sqrt{4 - \left(x^2 - 4x + 4\right)}}$$

$$= \int_0^4 \frac{dx}{\sqrt{\left(2\right)^2 - \left(x - 2\right)^2}}$$

$$= \left[\sin^{-1}\left(\frac{x-2}{2}\right)\right]_0^4 \qquad \left[\because \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a}\right]$$

$$= \sin^{-1}\left(1\right) - \sin^{-1}\left(-1\right)$$

$$= \frac{\pi}{2} - \left(-\frac{\pi}{2}\right)$$

$$= \frac{2\pi}{2} = \pi$$

$$\therefore \int_0^4 \frac{dx}{\sqrt{4x - x^2}} = \pi$$

Definite Integrals Ex 20.1 Q44

$$\int_{-1}^{1} \frac{dx}{x^2 + 2x + 5} = \int_{-1}^{1} \frac{dx}{\left(x^2 + 2x + 1\right) + 4} = \int_{-1}^{1} \frac{dx}{\left(x+1\right)^2 + \left(2\right)^2}$$

Let $x + 1 = t \Rightarrow dx = dt$

When $x = -1$, $t = 0$ and when $x = 1$, $t = 2$

$$\therefore \int_{-1}^{1} \frac{dx}{\left(x+1\right)^2 + \left(2\right)^2} = \int_0^2 \frac{dt}{t^2 + 2^2}$$

$$= \left[\frac{1}{2}\tan^{-1}\frac{t}{2}\right]_0^2$$

$$= \frac{1}{2}\tan^{-1}1 - \frac{1}{2}\tan^{-1}0$$

$$= \frac{1}{2}\left(\frac{\pi}{4}\right) = \frac{\pi}{8}$$

Definite Integrals Ex 20.1 Q45

We have,

$$\int_1^4 \frac{x^2+x}{\sqrt{2x+1}}\,dx$$

Let $2x+1=t^2$

$\Rightarrow 2dx = 2t\,dt$

Now,

$x=1 \Rightarrow t=\sqrt{3}$

$x=4 \Rightarrow t=3$

$$\therefore \int_1^4 \frac{x^2+x}{\sqrt{2x+1}}\,dx = \int_{\sqrt{3}}^{3} \frac{\left(\frac{t^2-1}{2}\right)^2+\left(\frac{t^2-1}{2}\right)}{t}\,t\,dt$$

$$= \frac{1}{4}\int_{\sqrt{3}}^{3}\left(t^4-2t^2+1+2t^2-2\right)dt$$

$$= \frac{1}{4}\int_{\sqrt{3}}^{3} t^4-1$$

$$= \frac{1}{4}\left[\frac{t^5}{5}-t\right]_{\sqrt{3}}^{3}$$

$$= \frac{1}{4}\left[\frac{243-9\sqrt{3}}{5}-3+\sqrt{3}\right]$$

$$= \frac{1}{4}\left[\frac{228}{5}-\sqrt{3}\,(4)\right]$$

$$= \frac{57-\sqrt{3}}{5}$$

$$\therefore \int_1^4 \frac{x^2+x}{\sqrt{2x+1}}\,dx = \frac{57-\sqrt{3}}{5}$$

Definite Integrals Ex 20.1 Q46

We have,

$$\int_0^1 x\,(1-x)^5\,dx$$

Expanding $(1-x)^5$ by Binomial theorem

$$\therefore (1-x)^5 = 1^5 + {}^5C_1(-x) + {}^5C_2(-x)^2 + {}^5C_3(-x)^3 + {}^5C_4(-x)^4 + {}^5C_5(-x)^5$$

$$= 1-5x+10x^2-10x^3+5x^4-x^5$$

$$= \int_0^1 x\left(1-5x+10x^2-10x^3+5x^4-x^5\right)dx$$

$$= \left[\frac{x^2}{2}-\frac{5x^3}{3}+\frac{10x^4}{4}-\frac{10x^5}{5}+\frac{5x^6}{6}-\frac{x^7}{7}\right]_0^1$$

$$= \frac{1}{2}-\frac{5}{3}+\frac{10}{4}-\frac{10}{5}+\frac{5}{6}-\frac{1}{7}$$

$$= \frac{1}{42}$$

$$\therefore \int_0^1 x\,(1-x)^5\,dx = \frac{1}{42}$$

Definite Integrals Ex 20.1 Q47

We have,

$$\int_1^2\left(\frac{x-1}{x^2}\right)e^x dx = \int_1^2 \frac{xe^x}{x^2} - \int_1^2 \frac{e^x}{x^2}dx = \int_1^2 \frac{e^x dx}{x} - \int_1^2 \frac{e^x}{x^2}dx$$

Expanding 1^{st} integral by by parts we get

$$= \frac{1}{x}\int_1^2 e^x dx - \int_1^2\left(\int e^x . \frac{d\left(1/x\right)}{dx}dx\right) - \int_1^2 \frac{e^x}{x^2}$$

$$= \left[\frac{e^x}{x}\right]_1^2 + \int_1^2 \frac{e^x}{x^2}dx - \int_1^2 \frac{e^x}{x^2}$$

$$= \left[\frac{e^x}{x}\right]_1^2$$

$$= \frac{e^2}{2} - e$$

$$\therefore \int_1^2\left(\frac{x-1}{x^2}\right)e^x dx = \frac{e^2}{2} - e$$

Definite Integrals Ex 20.1 Q48

We have,

$$\int_0^1\left(xe^{2x} + \sin\frac{\pi x}{2}\right)dx = \int_0^1 \underset{\text{I}}{x}\, \underset{\text{II}}{e^{2x}}\, dx + \int_0^1 \sin\frac{\pi x}{2}dx$$

Applying by parts in first integral

$$= x\int_0^1 e^{2x}dx - \int_0^1\left(\int e^{2x}dx.\right)\frac{dx}{dx}dx + \left[\frac{-\cos\frac{\pi x}{2}}{\frac{\pi}{2}}\right]_0^1$$

$$= \frac{xe^{2x}}{2} - \frac{1}{2}\int_0^1 e^{2x}dx + \frac{2}{\pi}[1-0]$$

$$= \frac{xe^{2x}}{2} - \frac{1}{2}\int_0^1 e^{2x}dx + \frac{2}{\pi}[1-0]$$

$$= \left[\frac{xe^{2x}}{2} - \frac{1}{4}e^{2x}\right]_0^1 + \frac{2}{\pi}[1-0]$$

$$= \frac{e^2}{2} - \frac{1}{4}e^2 + \frac{1}{4} + \frac{2}{\pi}[1-0]$$

$$= \frac{e^2}{4} + \frac{2}{\pi} + \frac{1}{4}$$

$$= \frac{e^2}{4} + \frac{1}{4} + \frac{2}{\pi}$$

$$\therefore \int_0^1\left(xe^{2x} + \sin\frac{\pi x}{2}\right)dx = \frac{e^2}{4} + \frac{1}{4} + \frac{2}{\pi}$$

Definite Integrals Ex 20.1 Q49

We have,

$$\int_0^1 \left(xe^x + \cos\frac{\pi x}{4} \right) dx$$

$$= \int_0^1 \underset{\text{I}}{x}\, \underset{\text{II}}{e^x}\, dx + \int_0^1 \cos\frac{\pi x}{4}\, dx$$

Applying by by parts in 1st integral we get,

$$= x\int_0^1 e^x dx - \int_0^1 \left(\int e^x dx \right) \frac{dx}{dx}.dx + \int_0^1 \cos\frac{\pi x}{4} dx$$

$$= \left[xe^x \right]_0^1 - \int_0^1 e^x dx + \left[\frac{\sin\frac{\pi x}{4}}{\frac{\pi}{4}} \right]_0^1$$

$$= \left[xe^x - e^x \right]_0^1 + \frac{4}{\pi}\left[\frac{1}{\sqrt{2}} - 0 \right]$$

$$= \left[e^x (x-1) \right]_0^1 + \frac{4}{\pi}\left[\frac{1}{\sqrt{2}} \right]$$

$$= 0 + 1 + \frac{4}{\pi\sqrt{2}}$$

$$= 1 + \frac{2\sqrt{2}}{\pi}$$

$$\therefore \int_0^1 \left(xe^x + \cos\frac{\pi x}{4} \right) dx = 1 + \frac{2\sqrt{2}}{\pi}$$

Definite Integrals Ex 20.1 Q50

$$\int_{\frac{\pi}{2}}^{\pi} e^x \frac{1-\sin x}{1-\cos x} dx = \int_{\frac{\pi}{2}}^{\pi} e^x \frac{1 - 2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}} dx \qquad \left[1-\cos x = 2\sin^2\frac{x}{2} \right]$$

$$= -\int_{\frac{\pi}{2}}^{\pi} e^x \left(-\frac{1}{2}\csc^2\frac{x}{2} + \cot\frac{x}{2} \right) dx$$

$$= -e^x \cot\frac{x}{2} \Big|_{\frac{\pi}{2}}^{\pi} \qquad \left[\int e^x \left(F(x) + F'(x) \right) dx = e^x F(x) \right]$$

$$= e^{\frac{\pi}{2}}$$

Definite Integrals Ex 20.1 Q51

We have,

$$\int_0^{2\pi} e^{x/2} \sin\left(\frac{x}{2} + \frac{\pi}{4} \right) dx = \int_0^{2\pi} e^{x/2} \left(\sin\frac{x}{2}\cos\frac{\pi}{4} + \cos\frac{x}{2}\sin\frac{\pi}{4} \right) dx$$

$$= \int_0^{2\pi} e^{x/2} \sin\frac{x}{2} . \frac{1}{\sqrt{2}} dx + \int_0^{2\pi} e^{x/2} \cos\frac{x}{2} . \frac{1}{\sqrt{2}} dx$$

Expanding 1st part by parts, we get,

$$\int_0^{2\pi} e^{x/2} \sin\left(\frac{x}{2} + \frac{\pi}{4} \right) dx = \frac{1}{\sqrt{2}} \left\{ \sin\frac{x}{2} \int_0^{2\pi} e^{x/2} dx - \int_0^{2\pi} \left(\int_0^{2\pi} e^{x/2} dx \right) . \frac{d\left(\sin\frac{x}{2} \right)}{dx} dx \right\} + \frac{1}{\sqrt{2}} \int_0^{2\pi} e^{x/2} . \cos\frac{x}{2} dx$$

$$= \frac{1}{\sqrt{2}} \left\{ \sin\frac{x}{2} . 2e^{x/2} \right\}_0^{2\pi} - \frac{1}{\sqrt{2}} \int_0^{2\pi} e^{x/2} 2 . \frac{1}{2} \cos\frac{x}{2} dx + \frac{1}{\sqrt{2}} \int_0^{2\pi} e^{x/2} \cos\frac{x}{2} dx$$

$$= \frac{1}{\sqrt{2}} \left\{ \sin\frac{x}{2} . 2e^{x/2} \right\}_0^{2\pi} = \frac{1}{\sqrt{2}} \{ 0 - 0 \} = 0$$

$$\therefore \int_0^{2\pi} e^{x/2} \sin\left(\frac{x}{2} + \frac{\pi}{4} \right) dx = 0$$

Definite Integrals Ex 20.1 Q52

$$\text{Let } I = \int_0^{2\pi} e^x \cos\left(\frac{\pi}{4}+\frac{x}{2}\right)dx = \left[\cos\left(\frac{\pi}{4}+\frac{x}{2}\right).e^x\right]_0^{2\pi} + \frac{1}{2}\int_0^{2\pi} e^x \sin\left(\frac{\pi}{4}+\frac{x}{2}\right)dx$$

$$\Rightarrow I = \left[\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)e^x\right]_0^{2\pi} + \frac{1}{2}\left[\left\{\sin\left(\frac{\pi}{4}+\frac{x}{2}\right).e^x\right\}_0^{2\pi} - \frac{1}{2}\int_0^{2\pi} e^x \cos\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]dx$$

$$I = \left[\cos\left(\pi+\frac{\pi}{4}\right)e^{2\pi} - \cos\frac{\pi}{4}\right] + \frac{1}{2}\left[\sin\left(\pi+\frac{\pi}{4}\right).e^{2\pi} - \sin\frac{\pi}{4} - \frac{1}{2}I\right]$$

$$I = \left[-\cos\frac{\pi}{4}.e^{2\pi} - \cos\frac{\pi}{4}\right] + \frac{1}{2}\left[-\sin\frac{\pi}{4}.e^{2\pi} - \sin\frac{\pi}{4}\right] - \frac{I}{4}$$

$$\frac{5I}{4} = -\frac{1}{\sqrt{2}}\left(e^{2\pi}+1\right) - \frac{1}{2}.\frac{1}{\sqrt{2}}\left(e^{2\pi}+1\right) = \frac{-3}{2\sqrt{2}}\left(e^{2\pi}+1\right)$$

$$I = \frac{-3\sqrt{2}}{5}\left(e^{2\pi}+1\right)$$

$$\therefore \int_0^{2\pi} e^x \cos\left(\frac{\pi}{4}+\frac{x}{2}\right)dx = \frac{-3\sqrt{2}}{5}\left(e^{2\pi}+1\right)$$

Definite Integrals Ex 20.1 Q53

$$\text{Let } I = \int_0^1 \frac{dx}{\sqrt{1+x}-\sqrt{x}}$$

$$I = \int_0^1 \frac{1}{\left(\sqrt{1+x}-\sqrt{x}\right)} \times \frac{\left(\sqrt{1+x}+\sqrt{x}\right)}{\left(\sqrt{1+x}+\sqrt{x}\right)}dx$$

$$= \int_0^1 \frac{\sqrt{1+x}+\sqrt{x}}{1+x-x}dx$$

$$= \int_0^1 \sqrt{1+x}\,dx + \int_0^1 \sqrt{x}\,dx$$

$$= \left[\frac{2}{3}(1+x)^{\frac{3}{2}}\right]_0^1 + \left[\frac{2}{3}(x)^{\frac{3}{2}}\right]_0^1$$

$$= \frac{2}{3}\left[(2)^{\frac{3}{2}}-1\right] + \frac{2}{3}[1]$$

$$= \frac{2}{3}(2)^{\frac{3}{2}}$$

$$= \frac{2\cdot 2\sqrt{2}}{3}$$

$$= \frac{4\sqrt{2}}{3}$$

Definite Integrals Ex 20.1 Q54

$$\int_1^2 \frac{x}{(x+1)(x+2)}dx = -\int_1^2 \frac{1}{x+1}dx + \int_1^2 \frac{2}{x+2}dx \qquad \text{[Using Partial Fraction]}$$

$$= -\log(x+1)\big|_1^2 + 2\log(x+2)\big|_1^2$$

$$= -(\log 3 - \log 2) + 2(\log 4 - \log 3)$$

$$= -3\log 3 + 5\log 2$$

$$= \log\frac{32}{27}$$

Definite Integrals Ex 20.1 Q55

Let $I = \int_0^{\frac{\pi}{2}} \sin^3 x\,dx$

$$I = \int_0^{\frac{\pi}{2}} \sin^2 x \cdot \sin x\,dx$$

$$= \int_0^{\frac{\pi}{2}} \left(1-\cos^2 x\right)\sin x\,dx$$

$$= \int_0^{\frac{\pi}{2}} \sin x\,dx - \int_0^{\frac{\pi}{2}} \cos^2 x \cdot \sin x\,dx$$

$$= \left[-\cos x\right]_0^{\frac{\pi}{2}} + \left[\frac{\cos^3 x}{3}\right]_0^{\frac{\pi}{2}}$$

$$= 1 + \frac{1}{3}[-1] = 1 - \frac{1}{3} = \frac{2}{3}$$

Hence, the given result is proved.

Definite Integrals Ex 20.1 Q56

Let $I = \int_0^{\pi} \left(\sin^2 \frac{x}{2} - \cos^2 \frac{x}{2}\right) dx$

$$= -\int_0^{\pi} \left(\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}\right) dx$$

$$= -\int_0^{\pi} \cos x\ dx$$

$$\int \cos x\ dx = \sin x = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}(\pi) - \mathrm{F}(0)$$

$$= \sin\pi - \sin 0$$

$$= 0$$

Definite Integrals Ex 20.1 Q57

$$\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x} dx$$

Let $2x = t \Rightarrow 2dx = dt$

When $x = 1$, $t = 2$ and when $x = 2$, $t = 4$

$$\therefore \int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x} dx = \frac{1}{2}\int_2^4 \left(\frac{2}{t} - \frac{2}{t^2}\right) e^t dt$$

$$= \int_2^4 \left(\frac{1}{t} - \frac{1}{t^2}\right) e^t dt$$

Let $\frac{1}{t} = f(t)$

Then, $f'(t) = -\frac{1}{t^2}$

$$\Rightarrow \int_2^4 \left(\frac{1}{t} - \frac{1}{t^2}\right) e^t dt = \int_2^4 e^t \left[f(t) + f'(t)\right] dt$$

$$= \left[e^t f(t)\right]_2^4$$

$$= \left[e^t \cdot \frac{2}{t}\right]_2^4$$

$$= \left[\frac{e^t}{t}\right]_2^4$$

$$= \frac{e^4}{4} - \frac{e^2}{2}$$

$$= \frac{e^4 - 2e^2}{4}$$

Definite Integrals Ex 20.1 Q58

$$\int_1^2 \frac{1}{\sqrt{(x-1)(2-x)}}\,dx$$

$$= \int_1^2 \frac{1}{\sqrt{-\left(x-\frac{3}{2}\right)^2+\left(\frac{1}{4}\right)}}\,dx$$

$$= \int_1^2 \frac{1}{\sqrt{\left(\frac{1}{2}\right)^2-\left(x-\frac{3}{2}\right)^2}}\,dx$$

$$= \left[\sin^{-1}(2x-3)\right]_1^2$$

$$= \sin^{-1}(1)-\sin^{-1}(-1)$$

$$= \pi$$

Definite Integrals Ex 20.1 Q59

We have,

$$\int_0^k \frac{dx}{2+8x^2} = \frac{\pi}{16}$$

$$\Rightarrow \frac{1}{8}\int_0^k \frac{dx}{\left(\frac{1}{2}\right)^2+x^2} = \frac{\pi}{16}$$

$$\Rightarrow \frac{1}{8}\left[2\tan^{-1}2x\right]_0^k = \frac{\pi}{16} \qquad \left[\because \int \frac{dx}{a^2-x^2} = 2\tan^{-1}\frac{x}{a}\right]$$

$$\Rightarrow \frac{1}{4}\left[\tan^{-1}2k-\tan^{-1}0\right] = \frac{\pi}{16}$$

$$\Rightarrow \tan^{-1}2k-0 = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}2k = \frac{\pi}{4}$$

$$\Rightarrow 2k = 1$$

$$k = \frac{1}{2}$$

Definite Integrals Ex 20.1 Q60

We have,

$$\int_0^a 3x^2 dx = 8$$

$$\Rightarrow \left[x^3\right]_0^a = 8$$

$$\Rightarrow a^3 = 8$$

$$\Rightarrow a = 2$$

Definite Integrals Ex 20.1 Q61

$$\int_{\pi}^{\frac{3\pi}{2}} \sqrt{1-(1-2\sin^2 x)}\,dx$$

$$\int_{\pi}^{\frac{3\pi}{2}} \sqrt{2\sin^2 x}\,dx$$

$$\sqrt{2}\int_{\pi}^{\frac{3\pi}{2}} \sin x\,dx$$

$$\sqrt{2}(-\cos x)_{\pi}^{\frac{3\pi}{2}}$$

$$= \sqrt{2}$$

Definite Integrals Ex 20.1 Q62

$$I = \int_0^{2\pi} \sqrt{1 + \sin\frac{x}{2}}\, dx$$

$$\Rightarrow I = \int_0^{2\pi} \sqrt{\sin^2\frac{x}{4} + \cos^2\frac{x}{4} + 2\sin\frac{x}{4}\cos\frac{x}{4}}\, dx$$

$$\Rightarrow I = \int_0^{2\pi} \sqrt{\left(\sin\frac{x}{4} + \cos\frac{x}{4}\right)^2}\, dx$$

$$\Rightarrow I = \int_0^{2\pi} \left(\sin\frac{x}{4} + \cos\frac{x}{4}\right) dx$$

$$\Rightarrow I = \left[\frac{-\cos\frac{x}{4}}{\frac{1}{4}} + \frac{\sin\frac{x}{4}}{\frac{1}{4}}\right]_0^{2\pi}$$

$$\Rightarrow I = 4(0 + 1 + 1 - 0)$$

$$\Rightarrow I = 8$$

Definite Integrals Ex 20.1 Q63

$$I = \int_0^{\pi/4} (\tan x + \cot x)^{-2}\, dx$$

$$I = \int_0^{\pi/4} \frac{1}{(\tan x + \cot x)^2}\, dx$$

$$I = \int_0^{\pi/4} \frac{1}{\left(\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}\right)^2}\, dx$$

$$I = \int_0^{\pi/4} (\sin x \cos x)^2\, dx$$

$$I = \int_0^{\pi/4} \sin^2 x (1 - \sin^2 x)\, dx$$

$$I = \int_0^{\pi/4} \sin^2 x\, dx - \int_0^{\pi/4} \sin^4 x\, dx$$

We know that by reduction formula,

$$\int \sin^n x\ dx = \frac{n-1}{n}\int \sin^{n-2} x\ dx - \frac{\cos x \sin^{n-1} x}{n}$$

For $n = 2$

$$\int \sin^2 x\ dx = \frac{2-1}{2}\int 1\ dx - \frac{\cos x \sin x}{2}$$

$$\int \sin^2 x\ dx = \frac{1}{2}x - \frac{\cos x \sin x}{2}$$

For $n = 4$

$$\int \sin^4 x\ dx = \frac{4-1}{4}\int \sin^2 x\ dx - \frac{\cos x \sin^3 x}{4}$$

$$\int \sin^4 x\ dx = \frac{3}{4}\left\{\frac{1}{2}x - \frac{\cos x \sin x}{2}\right\} - \frac{\cos x \sin^3 x}{4}$$

Hence,

$$I = \left\{\frac{1}{2}x - \frac{\cos x \sin x}{2}\right\}_0^{\pi/4} - \left\{\frac{3}{4}\left\{\frac{1}{2}x - \frac{\cos x \sin x}{2}\right\} - \frac{\cos x \sin^3 x}{4}\right\}_0^{\pi/4}$$

$$= \left\{\frac{\pi}{8} - \frac{1}{4}\right\} - \left\{\frac{3}{4}\left(\frac{\pi}{8} - \frac{1}{4}\right) - \frac{1}{16}\right\}$$

$$= \frac{\pi}{32}$$

$$\int_0^{\frac{\pi}{2}} (\sin x \cos x)^2 dx$$

$$\int_0^{\frac{\pi}{2}} \sin^2 x(1-\sin^2 x)dx$$

$$\int_0^{\frac{\pi}{2}} \sin^2 x - \sin^4 x dx$$

$$\int_0^{\frac{\pi}{2}} \sin^2 x dx - \int_0^{\frac{\pi}{2}} \sin^4 x dx$$

We know , By reduction formula

$$\int \sin^n x dx = \frac{n-1}{n}\int \sin^{n-2} x dx - \frac{\cos x \sin^{n-1} x}{n}$$

For n=2

$$\int \sin^2 x dx = \frac{2-1}{2}\int 1 dx - \frac{\cos x \sin x}{2}$$

$$\int \sin^2 x dx = \frac{1}{2}x - \frac{\cos x \sin x}{2}$$

For n=4

$$\int \sin^4 x dx = \frac{4-1}{4}\int \sin^2 x dx - \frac{\cos x \sin^3 x}{4}$$

$$\int \sin^4 x dx = \frac{3}{4}\left\{\frac{1}{2}x - \frac{\cos x \sin x}{2}\right\} - \frac{\cos x \sin^3 x}{4}$$

Hence

$$\left\{\frac{1}{2}x - \frac{\cos x \sin x}{2}\right\}_0^{\frac{\pi}{2}} - \left\{\frac{3}{4}\left\{\frac{1}{2}x - \frac{\cos x \sin x}{2}\right\} - \frac{\cos x \sin^3 x}{4}\right\}_0^{\frac{\pi}{2}}$$

$$\frac{\Pi}{4} - \frac{3}{4}\left\{\frac{\Pi}{4}\right\}$$

$$\frac{\Pi}{16}$$

Definite Integrals Ex 20.1 Q64

Using Integration By parts

$$\int f'g = fg - \int fg'$$

$$f' = x, g = \log(2x+1)$$

$$f = \frac{x^2}{2}, g' = \frac{2}{2x+1}$$

$$\int_0^1 x\log(1+2x)\,dx$$

$$= \left[\frac{x^2\log(1+2x)}{2}\right]_0^1 - \int_0^1 \frac{2x^2}{2(2x+1)}dx$$

$$= \frac{\log(3)}{2} - \int_0^1 \frac{x}{2} - \frac{1}{4} + \frac{1}{4(2x+1)}dx$$

$$= \frac{\log(3)}{2} - \left[\frac{x^2}{4} - \frac{x}{4} + \frac{1}{8}\log|2x+1|\right]_0^1$$

$$= \frac{\log(3)}{2} - \frac{1}{8}\log(3)$$

$$= \frac{3}{8}\log_e(3)$$

Definite Integrals Ex 20.1 Q65

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (\tan^2 x + 2\tan x \cot x + \cot^2 x)dx$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left\{\left(\sec^2 x - 1\right) + 2 + \left(\operatorname{cosec}^2 x - 1\right)\right\} dx$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left\{\sec^2 x + \operatorname{cosec}^2 x\right\} dx$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \operatorname{cosec}^2 x dx$$

$$\left(\tan x\right)_{\frac{\pi}{6}}^{\frac{\pi}{3}} + \left(-\cot x\right)_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$\left\{\sqrt{3} - \frac{1}{\sqrt{3}}\right\} - \left\{\frac{1}{\sqrt{3}} - \sqrt{3}\right\}$$

$$2\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$$

$$\frac{4}{\sqrt{3}}$$

Definite Integrals Ex 20.1 Q66

$$I = \int_0^{\pi/4} \left(a^2 \cos^2 x + b^2 \sin^2 x\right) dx$$

$$I = \int_0^{\pi/4} \left(a^2\left(1 - \sin^2 x\right) + b^2 \sin^2 x\right) dx$$

$$I = \int_0^{\pi/4} \left(a^2 - a^2 \sin^2 x + b^2 \sin^2 x\right) dx$$

$$I = \int_0^{\pi/4} a^2 + \left(b^2 - a^2\right) \sin^2 x \, dx$$

$$I = \int_0^{\pi/4} a^2 + \left(b^2 - a^2\right) \frac{\left(1 + \cos 2x\right)}{2} dx$$

$$I = \left[a^2 x + \frac{\left(b^2 - a^2\right)}{2}\left(x + \frac{\sin 2x}{2}\right)\right]_0^{\pi/4}$$

$$I = \left[\frac{a^2 \pi}{4} + \frac{\left(b^2 - a^2\right)}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right)\right]$$

$$I = \frac{\left(b^2 + a^2\right)\pi}{8} + \frac{\left(b^2 - a^2\right)}{4}$$

Definite Integrals Ex 20.1 Q67

$$\int_0^1 \frac{1}{x^4+2x^3+2x^2+2x+1}dx$$

$$\int_0^1 \frac{1}{(x+1)^2(x^2+1)}dx$$

$$\int_0^1 \left\{-\frac{x}{2(x^2+1)}+\frac{1}{2(x+1)}+\frac{1}{2(x+1)^2}\right\}dx$$

$$-\int_0^1 \frac{x}{2(x^2+1)}dx+\int_0^1 \frac{1}{2(x+1)}dx+\int_0^1 \frac{1}{2(x+1)^2}dx$$

$$-\left\{\frac{\log(x^2+1)}{4}\right\}_0^1+\left\{\frac{\log(x+1)}{2}\right\}_0^1-\left\{\frac{1}{2(x+1)}\right\}_0^1$$

$$-\frac{\log 2}{4}+\frac{\log 2}{2}-\frac{1}{4}+\frac{1}{2}$$

$$\frac{\log 2}{4}+\frac{1}{4}$$

=(1/4)log(2e)

Ex 20.2

Definite Integrals Ex 20.2 Q1

Let $I = \int_2^4 \frac{x}{x^2+1}dx$

$\int \frac{x}{x^2+1}dx = \frac{1}{2}\int \frac{2x}{x^2+1}dx = \frac{1}{2}\log\left(1+x^2\right) = F(x)$

By the second funamental theorem of calculus, we obtain

$I = F(4) - F(2)$

$= \frac{1}{2}\left[\log\left(1+4^2\right) - \log\left(1+2^2\right)\right]$

$= \frac{1}{2}[\log 17 - \log 5]$

$= \frac{1}{2}\log\left(\frac{17}{5}\right)$

Definite Integrals Ex 20.2 Q2

Let $1 + \log x = t$

Differentiating w.r.t. x, we get

$$\frac{1}{x}dx = dt$$

Now, $x = 1 \Rightarrow t = 1$

$x = 2 \Rightarrow t = 1 + \log 2$

$$\therefore \int_1^2 \frac{1}{x\left(1+\log x\right)^2}dx = \int_1^{1+\log 2} \frac{dt}{t^2}$$

$$= \left[\frac{-1}{t}\right]_1^{1+\log 2}$$

$$= \left[\frac{-1}{1+\log 2} + 1\right]$$

$$= \left[\frac{-1+1+\log 2}{1+\log 2}\right]$$

$$= \left[\frac{\log 2}{1+\log 2}\right] \qquad [\because \log e = 1]$$

$$= \frac{\log 2}{\log e + \log 2} \qquad [\log a + \log b = \log ab]$$

$$= \frac{\log 2}{\log 2e}$$

$$\therefore \int_1^2 \frac{1}{x\left(1+\log x\right)^2}dx = \frac{\log 2}{\log 2e}$$

Definite Integrals Ex 20.2 Q3

Let $9x^2 - 1 = t$

Differentiating w.r.t. x, we get

$$18x\,dx = dt$$

$$3x\,dx = \frac{dt}{6}$$

Now, $x = 1 \Rightarrow t = 8$

$x = 2 \Rightarrow t = 35$

$$\therefore \int_1^2 \frac{3x}{9x^2-1}dx = \int_8^{35} \frac{dt}{6t}$$

$$= \frac{1}{6}\left[\log t\right]_8^{35}$$

$$= \frac{1}{6}\left(\log 35 - \log 8\right)$$

$$\therefore \int_1^2 \frac{3x}{9x^2-1}dx = \frac{1}{6}\left(\log 35 - \log 8\right)$$

Definite Integrals Ex 20.2 Q4

Put $\sin x = \dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$\cos x = \dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$\int_0^{\frac{\pi}{2}} \frac{dx}{5\cos x + 3\sin x} = \int_0^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}\,dx}{5\left(1-\tan^2\frac{x}{2}\right)+6\tan\frac{x}{2}}$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}\,dx}{5-5\tan^2\frac{x}{2}+6\tan\frac{x}{2}}$$

Let $\tan\frac{x}{2} = t$

Differentiating w.r.t. x, we get

$$\frac{1}{2}\sec^2\frac{x}{2}\,dx = dt$$

Now, $x = 0 \Rightarrow t = 0$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$$\int_0^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}\,dx}{5-5\tan^2\frac{x}{2}+6\tan\frac{x}{2}} = \int_0^1 \frac{2dt}{5-5t^2+6t} = \frac{2}{5}\int \frac{dt}{1-t^2+\frac{6}{5}t}$$

Forming perfect square by adding and subtracting $\frac{9}{25}$

$$\frac{2}{5}\int_0^1 \frac{dt}{1-t^2+\frac{6}{5}t}$$

$$= \frac{2}{5}\int_0^1 \frac{dt}{\frac{34}{25}-\left(t-\frac{3}{5}\right)^2}$$

$$= \frac{2}{5}\cdot\frac{1}{2}\sqrt{\frac{25}{34}}\log\left(\frac{\sqrt{\frac{34}{25}}+t-\frac{3}{5}}{\sqrt{\frac{34}{25}}-t+\frac{3}{5}}\right)_0^1 \qquad \left[\because \int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left(\frac{x+a}{x-a}\right)\right]$$

$$= \frac{1}{\sqrt{34}}\left\{\log\left(\frac{\sqrt{34}+2}{\sqrt{34}-2}\right)-\log\left(\frac{\sqrt{34}-3}{\sqrt{34}+3}\right)\right\}$$

$$= \frac{1}{\sqrt{34}}\log\left(\frac{\left(\sqrt{34}+2\right)\left(\sqrt{34}-3\right)}{\left(\sqrt{34}-2\right)\left(\sqrt{34}-3\right)}\right)$$

$$= \frac{1}{\sqrt{34}}\log\left(\frac{40+5\sqrt{34}}{40-5\sqrt{34}}\right)$$

$$= \frac{1}{\sqrt{34}}\log\left(\frac{8+\sqrt{34}}{8-\sqrt{34}}\right)$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{dx}{5\cos x + 3\sin x} = \frac{1}{\sqrt{34}}\log\left(\frac{8+\sqrt{34}}{8-\sqrt{34}}\right)$$

Definite Integrals Ex 20.2 Q5

Let $a^2 + x^2 = t^2$

Differentiating w.r.t. x, we get

$2x\,dx = 2t\,dt$

$x\,dx = t\,dt$

Now, $x = 0 \Rightarrow t = 0$

$x = a \Rightarrow t = \sqrt{2}a$

$$\therefore \int_0^a \frac{x\,dx}{\sqrt{a^2 + x^2}}dx = \int_a^{\sqrt{2}a} \frac{t\,dt}{t}$$

$$= \int_a^{\sqrt{2}a} dt$$

$$= [t]_a^{\sqrt{2}a}$$

$$= \left[\sqrt{2}a - a\right]$$

$$= a\left(\sqrt{2} - 1\right)$$

$$\therefore \int_0^a \frac{x}{\sqrt{a^2 + x^2}}dx = a\left(\sqrt{2} - 1\right)$$

Definite Integrals Ex 20.2 Q6

Let $e^x = t$

Differentiating w.r.t. x, we get

$e^x\,dx = dt$

Now, $x = 0 \Rightarrow t = 1$

$x = 1 \Rightarrow t = e$

$$\therefore \int_0^1 \frac{e^x}{1 + e^{2x}}dx = \int_1^e \frac{dt}{1 + t^2}$$

$$= \left[\tan^{-1} t\right]_1^e \qquad \left[\because \int \frac{dt}{1 + t^2} = \tan^{-1} t\right]$$

$$= \left[\tan^{-1} e - \tan^{-1} 1\right] \qquad \left[\because \tan\frac{\pi}{4} = 1\right]$$

$$= \tan^{-1} e - \frac{\pi}{4}$$

$$\therefore \int_0^1 \frac{e^x}{1 + e^{2x}}dx = \tan^{-1} e - \frac{\pi}{4}$$

Definite Integrals Ex 20.2 Q7

Let $x^2 = t$

Differentiating w.r.t. x, we get

$2x\,dx = dt$

Now,

$x = 0 \Rightarrow t = 0$

$x = 1 \Rightarrow t = 1$

$$\therefore \int_0^1 xe^{x^2}dx = \int_0^1 \frac{e^t dt}{2}$$

$$= \frac{1}{2}\int_0^1 e^t dt$$

$$= \frac{1}{2}\left[e^t\right]_0^1$$

$$= \frac{1}{2}\left[e^1 - e^0\right] \qquad \left[\because e^0 = 1\right]$$

$$= \frac{1}{2}(e - 1)$$

$$\therefore \int_0^1 xe^{x^2}dx = \frac{1}{2}(e - 1)$$

Definite Integrals Ex 20.2 Q8

Let $\log x = t$

Differentiating w.r.t. x, we get

$$\frac{1}{x}dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = 3 \Rightarrow t = \log 3$

$$\int_1^3 \frac{\cos(\log x)}{x}dx$$

$$= \int_0^{\log 3} \cos t\, dt \qquad \left[\because \int \cos t = \sin t\right]$$

$$= \left[\sin t\right]_0^{\log 3}$$

$$= \sin(\log 3) - \sin 0$$

$$= \sin(\log 3)$$

$$\int_1^3 \frac{\cos(\log x)}{x}dx = \sin(\log 3)$$

Definite Integrals Ex 20.2 Q9

Let $x^2 = t$

Differentiating w.r.t. x, we get

$$2x\,dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = 1 \Rightarrow t = 1$

$$\therefore \int_0^1 \frac{2x}{1+x^4}dx$$

$$= \int_0^1 \frac{dt}{1+t^2}$$

$$= \left[\tan^{-1} t\right]_0^1$$

$$= \left[\tan^{-1} 1 - \tan^{-1} 0\right] \qquad \left[\because \tan\frac{\pi}{4} = 1\right]$$

$$= \frac{\pi}{4}$$

$$\therefore \int_0^1 \frac{2x}{1+x^4}dx = \frac{\pi}{4}$$

Definite Integrals Ex 20.2 Q10

Let $x = a\sin\theta$

Differentiating w.r.t. x, we get

$dx = a\cos\theta d\theta$

Now,

$x = 0 \Rightarrow \theta = 0$

$x = a \Rightarrow \theta = \frac{\pi}{2}$

$\therefore \int_0^a \sqrt{a^2 - x^2}\,dx$

$= \int_0^{\frac{\pi}{2}} \sqrt{a^2\left(1 - \sin^2\theta\right)}\,a\cos\theta d\theta$

$= a^2 \int_0^{\frac{\pi}{2}} \cos^2\theta d\theta \qquad \left[\because \left(1 - \sin^2\theta\right) = \cos^2\theta \text{ and } \frac{1 + \cos 2\theta}{2} = \cos 2\theta\right]$

$= \frac{a^2}{2} \int_0^{\frac{\pi}{2}} \left(1 + \cos 2\theta\right) d\theta$

$= \frac{a^2}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}$

$= \frac{a^2}{2}\left[\frac{\pi}{2} + 0 - 0 - 0\right]$

$= \frac{\pi a^2}{4}$

$\therefore \int_0^a \sqrt{a^2 - x^2}\,dx = \frac{\pi a^2}{4}$

Definite Integrals Ex 20.2 Q11

Let $I = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\cos^5\phi\, d\phi = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\cos^4\phi\cos\phi\, d\phi$

Also, let $\sin\phi = t \Rightarrow \cos\phi\, d\phi = dt$

When $\phi = 0$, $t = 0$ and when $\phi = \frac{\pi}{2}$, $t = 1$

$\therefore I = \int_0^1 \sqrt{t}\left(1 - t^2\right)^2 dt$

$= \int_0^1 t^{\frac{1}{2}}\left(1 + t^4 - 2t^2\right) dt$

$= \int_0^1 \left[t^{\frac{1}{2}} + t^{\frac{9}{2}} - 2t^{\frac{5}{2}}\right] dt$

$= \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{11}{2}}}{\frac{11}{2}} - \frac{2t^{\frac{7}{2}}}{\frac{7}{2}}\right]_0^1$

$= \frac{2}{3} + \frac{2}{11} - \frac{4}{7}$

$= \frac{154 + 42 - 132}{231}$

$= \frac{64}{231}$

Definite Integrals Ex 20.2 Q12

Let $\sin x = t$

Differentiating w.r.t. x, we get

$$\cos x\, dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$$\int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\sin^2 x} dx$$

$$= \int_0^1 \frac{dt}{1+t^2}$$

$$= \left[\tan^{-1} t\right]_0^1$$

$$= \left[\tan^{-1} 1 - \tan^{-1} 0\right] \qquad \left[\because \tan\frac{\pi}{4} = 1\right]$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\sin^2 x} dx = \frac{\pi}{4}$$

Definite Integrals Ex 20.2 Q13

Let $1+\cos\theta = t^2$

Differentiating w.r.t. x, we get

$$-\sin\theta\, d\theta = 2t\, dt$$

$$\sin\theta\, d\theta = -2t\, dt$$

Now,

$x = 0 \Rightarrow t = \sqrt{2}$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\sin\theta\, d\theta}{\sqrt{1+\cos\theta}}$$

$$= \int_{\sqrt{2}}^{1} \frac{-2t dt}{t}$$

$$= -2\int_{\sqrt{2}}^{1} dt$$

$$= -2\left[t\right]_{\sqrt{2}}^{1}$$

$$= -2\left[1-\sqrt{2}\right]$$

$$= 2\left[\sqrt{2}-1\right]$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\sin\theta\, d\theta}{\sqrt{1+\cos\theta}} = 2\left[\sqrt{2}-1\right]$$

Definite Integrals Ex 20.2 Q14

Let $3+4\sin x = t$

Differentiating w.r.t. x, we get

$$4\cos x dx = dt$$

$$\cos x dx = \frac{dt}{4}$$

Now,

$x = 0 \Rightarrow t = 3$

$x = \frac{\pi}{3} \Rightarrow t = 3+2\sqrt{3}$

$$\therefore \int_0^{\frac{\pi}{3}} \frac{\cos x}{3+4\sin x} dx$$

$$= \int_3^{3+2\sqrt{3}} \frac{dt}{4t}$$

$$= \frac{1}{4}\left[\log t\right]_3^{3+2\sqrt{3}}$$

$$= \frac{1}{4}\left[\log\left(3+2\sqrt{3}\right) - \log 3\right]$$

$$= \frac{1}{4}\log\left(\frac{3+2\sqrt{3}}{3}\right)$$

$$\therefore \int_0^{\frac{\pi}{3}} \frac{\cos x}{3+4\sin x} dx = \frac{1}{4}\log\left(\frac{3+2\sqrt{3}}{3}\right)$$

Definite Integrals Ex 20.2 Q15

Let $\tan^{-1} x = t$

Differentiating w.r.t. x, we get

$$\frac{1}{1+x^2} dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = 1 \Rightarrow t = \frac{\pi}{4}$

$$\therefore \int_0^1 \frac{\sqrt{\tan^{-1} x}}{1+x^2} dx$$

$$= \int_0^{\frac{\pi}{4}} t^{1/2} dt$$

$$= \left[\frac{t^{3/2}}{3/2}\right]_0^{\frac{\pi}{4}}$$

$$= \frac{2}{3}\left[t^{3/2}\right]_0^{\frac{\pi}{4}}$$

$$= \frac{2}{3}\left[\left(\frac{\pi}{4}\right)^{3/2} - 0\right]$$

$$= \frac{1}{12}\pi^{3/2}$$

$$\therefore \int_0^1 \frac{\sqrt{\tan^{-1} x}}{1+x^2} dx = \frac{1}{12}\pi^{3/2}$$

Definite Integrals Ex 20.2 Q16

$$\int_0^2 x\sqrt{x+2}\,dx$$

Let $x + 2 = t^2 \Rightarrow dx = 2tdt$

When $x = 0$, $t = \sqrt{2}$ and when $x = 2$, $t = 2$

$$\begin{aligned}
\therefore \int_0^2 x\sqrt{x+2}\,dx &= \int_{\sqrt{2}}^2 \left(t^2-2\right)\sqrt{t^2}\,2tdt \\
&= 2\int_{\sqrt{2}}^2 \left(t^2-2\right)t^2 dt \\
&= 2\int_{\sqrt{2}}^2 \left(t^4-2t^2\right)dt \\
&= 2\left[\frac{t^5}{5}-\frac{2t^3}{3}\right]_{\sqrt{2}}^2 \\
&= 2\left[\frac{32}{5}-\frac{16}{3}-\frac{4\sqrt{2}}{5}+\frac{4\sqrt{2}}{3}\right] \\
&= 2\left[\frac{96-80-12\sqrt{2}+20\sqrt{2}}{15}\right] \\
&= 2\left[\frac{16+8\sqrt{2}}{15}\right] \\
&= \frac{16\left(2+\sqrt{2}\right)}{15} \\
&= \frac{16\sqrt{2}\left(\sqrt{2}+1\right)}{15}
\end{aligned}$$

Definite Integrals Ex 20.2 Q17

Let $x = \tan\theta$

Differentiating w.r.t. x, we get

$dx = \sec^2\theta d\theta$

Now,

$x = 0 \Rightarrow \theta = 0$

$x = 1 \Rightarrow \theta = \frac{\pi}{4}$

$$\int_0^1 \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$$

$$= \int_0^{\frac{\pi}{4}} \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)\sec^2\theta d\theta \qquad \left[\because \tan^2\theta = \frac{2\tan\theta}{1-\tan^2\theta}\right]$$

$$= \int_0^{\frac{\pi}{4}} \tan^{-1}\left(\tan 2\theta\right)\sec^2\theta d\theta$$

$$= \int_0^{\frac{\pi}{4}} 2\theta\sec^2\theta d\theta$$

Applying by parts, we get

$$= 2\left[\theta\int_0^{\frac{\pi}{4}}\sec^2\theta d\theta - \int_0^{\frac{\pi}{4}}\left(\sec^2\theta d\theta\right)\frac{d\theta}{d\theta}d\theta\right]$$

$$=2\left[\theta\tan\theta\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}}\tan\theta d\theta]$$

$$= 2\left[\theta\tan\theta + \log(\cos\theta)\right]_0^{\frac{\pi}{4}}$$

$$= 2\left[\frac{\pi}{4} + \log\left(\frac{1}{\sqrt{2}}\right) - 0 - 0\right]$$

$$= 2\left[\frac{\pi}{4} + \frac{1}{2}\log 2\right]$$

$$= \frac{\pi}{2} - \log 2$$

$$\therefore \int_0^1 \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx = \frac{\pi}{2} - \log 2$$

Definite Integrals Ex 20.2 Q18

Let $\sin^2 x = t$

Differentiating w.r.t. x, we get

$$2\sin x\cos x\,dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$$\int_0^{\frac{\pi}{2}} \frac{\sin x\cos x}{1+\sin^4 x}dx$$

$$= \frac{1}{2}\int_0^1 \frac{dt}{1+t^2}$$

$$= \frac{1}{2}\left[\tan^{-1} t\right]_0^1$$

$$= \frac{1}{2}\left[\tan^{-1}(1) - \tan^{-1}(0)\right]$$

$$= \frac{1}{2}\left[\tan^{-1}\left(\tan\frac{\pi}{4}\right) - \tan^{-1}(\tan 0)\right]$$

$$= \frac{1}{2}\times\frac{\pi}{4}$$

$$= \frac{\pi}{8}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\sin x\cos x}{1+\sin^4 x}dx = \frac{\pi}{8}$$

Definite Integrals Ex 20.2 Q19

Putting $\cos x = \dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \dfrac{1-\tan^2\frac{x}{2}}{\sec^2\frac{x}{2}}$

$$\sin x = \frac{2\tan\frac{x}{2}}{\sec^2\frac{x}{2}}$$

$$\therefore \quad I = \int_0^{\frac{\pi}{2}} \frac{1}{a\cos x + b\sin x}dx = \int_0^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}}{a\left(1-\tan^2\frac{x}{2}\right)+2b\tan^2\frac{x}{2}}dx$$

Put $\tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2}dx = dt$$

If $x = 0,\ t = 0$ and if $x = \frac{\pi}{2},\ t = 1$

$$\Rightarrow \quad I = 2\int_0^1 \frac{dt}{a\left(1-t^2\right)+2bt}$$

$$= 2\int_0^1 \frac{dt}{-at^2+2bt+a}$$

$$= 2\int_0^1 \frac{dt}{-a\left[t^2-\frac{2b}{a}t-1\right]}$$

$$= \frac{2}{a}\int_0^1 \frac{dt}{-\left[\left(t-\frac{b}{a}\right)^2-1-\frac{b^2}{a^2}\right]}$$

$$= \frac{2}{a}\int_0^1 \frac{dt}{\left(\frac{b^2}{a^2}+1\right)-\left(t-\frac{b}{a}\right)^2}$$

$$= \frac{2}{a}\left[\frac{1}{2\sqrt{\frac{b^2+a^2}{a^2}}}\left(\log\left|\frac{\frac{\sqrt{b^2+a^2}}{a^2}+\left(t-\frac{b}{a}\right)}{\frac{\sqrt{b^2+a^2}}{a^2}-\left(t-\frac{b}{a}\right)}\right|\right)\right]_0^1$$

$$= \frac{1}{\sqrt{b^2+a^2}}\log\left(\frac{a+b+\sqrt{a^2+b^2}}{a+b-\sqrt{a^2+b^2}}\right)$$

Definite Integrals Ex 20.2 Q20

We know that $\sin x = \dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{1}{5+4\sin x}dx = \int_0^{\frac{\pi}{2}} \frac{1}{5+4\sin\left(\dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{\dfrac{5\left(1+\tan^2\frac{x}{2}\right)+4\left(2\tan\frac{x}{2}\right)}{1+\tan^2\frac{x}{2}}}dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{1+\tan^2\frac{x}{2}}{\left(5+5\tan^2\frac{x}{2}+8\tan\frac{x}{2}\right)}dx$$

$$\int_0^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}}{5+5\tan^2\frac{x}{2}+8\tan\frac{x}{2}}dx$$

Let $\tan\frac{x}{2} = t$

Differentiating w.r.t. x, we get

$$\frac{1}{2}\sec^2\frac{x}{2}dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}}{5+5\tan^2\frac{x}{2}+8\tan\frac{x}{2}}dx$$

$$= \int_0^1 \frac{2dt}{5+5t^2+8t}$$

$$= \frac{2}{5}\int_0^1 \frac{dt}{1+t^2+\frac{8}{5}t}$$

$$= \frac{2}{5}\int_0^1 \frac{dt}{1-\frac{16}{25}+\frac{16}{25}+t^2+\frac{8}{5}t} \quad \left[\text{Adding and subtracting } \frac{16}{25}\right]$$

$$= \frac{2}{5}\int_0^1 \frac{dt}{\left(\frac{3}{2}\right)^2+\left(t+\frac{4}{5}\right)^2}$$

$$= \frac{2}{5}\left[\frac{5}{3}\tan^{-1}\left(t+\frac{4}{5}\right)\times\frac{5}{3}\right]_0^1$$

$$= \frac{2}{3}\left[\tan^{-1}\left(1+\frac{4}{5}\right)\times\frac{5}{3}-\tan^{-1}\frac{4}{5}\times\frac{5}{3}\right]_0^1$$

$$= \frac{2}{3}\left[\tan^{-1}3-\tan^{-1}\frac{4}{3}\right]_0^1$$

$$= \frac{2}{3}\left[\tan^{-1}\left(\frac{3-\frac{4}{3}}{1+3\times\frac{4}{3}}\right)\right]_0^1 \qquad \left[\because \tan^{-1}A-\tan^{-1}B=\tan^{-1}\left(\frac{A-B}{1+AB}\right)\right]$$

$$= \frac{2}{3}\left[\tan^{-1}\frac{\frac{5}{3}}{5}\right]$$

$$= \frac{2}{3}\tan^{-1}\frac{1}{3}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{1}{5+4\sin x}dx = \frac{2}{3}\tan^{-1}\frac{1}{3}$$

Definite Integrals Ex 20.2 Q21

We have,

$$\int_0^{\pi} \frac{\sin x}{\sin x+\cos x}dx$$

Let $\sin x = K(\sin x+\cos x)+L\frac{d}{dx}(\sin x+\cos x)$

$= K(\sin x+\cos x)+L(\cos x-\sin x)$

$= \sin x(K-L)+\cos x(K+L)$

Equating similar terms

$K-L=1$

$K+L=0$

$\Rightarrow K=\frac{1}{2}$ and $L=-\frac{1}{2}$

$$\therefore \int_0^{\pi} \frac{\sin x}{\sin x+\cos x}dx = \frac{1}{2}\int_0^{\pi} dx+\left(\frac{-1}{2}\right)\int_0^{\pi}\frac{\cos x-\sin x}{\sin x+\cos x}dx$$

$$= \frac{1}{2}[x]_0^{\pi}-\frac{1}{2}\left(\log|\sin x+\cos x|\right)_0^{\pi} = \frac{\pi}{2}-\frac{1}{2}(0) = \frac{\pi}{2}$$

$$\therefore \int_0^{\pi} \frac{\sin x}{\sin x+\cos x}dx = \frac{\pi}{2}$$

Definite Integrals Ex 20.2 Q22

We know,

$$\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}},\ \cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

$$\therefore \frac{1}{3+2\sin x+\cos x}$$

$$= \frac{1}{3+2\left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)+\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}$$

$$= \frac{\left(1+\tan^2\frac{x}{2}\right)}{3\left(1+\tan^2\frac{x}{2}\right)+4\tan\frac{x}{2}+\left(1-\tan^2\frac{x}{2}\right)}$$

$$= \frac{\sec^2\frac{x}{2}dx}{3+3\tan^2\frac{x}{2}+4\tan\frac{x}{2}+1-\tan^2\frac{x}{2}}$$

$$\therefore \int_0^{\pi}\frac{1}{3+2\sin x+\cos x}dx = \int_0^{\pi}\frac{\sec^2\frac{x}{2}dx}{2\tan^2\frac{x}{2}+4\tan\frac{x}{2}+4}$$

Let $\tan\frac{x}{2} = t$

Differentiating w.r.t. x, we get

$$\frac{1}{2}\sec^2\frac{x}{2}dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = \pi \Rightarrow t = \infty$

$$\therefore \int_0^{\pi}\frac{\sec^2\frac{x}{2}dx}{2\tan^2\frac{x}{2}+4\tan\frac{x}{2}+4}$$

$$= \int_0^{\infty}\frac{dt}{t^2+2t+2}$$

$$= \int_0^{\infty}\frac{dt}{(t+1)^2+1}$$

$$= \left[\tan^{-1}(t+1)\right]_0^{\infty}$$

$$= \tan^{-1}(\infty)-\tan^{-1}(0+1)$$

$$= \tan^{-1}(\infty)-\tan^{-1}(1)$$

$$= \tan^{-1}\left(\tan\frac{\pi}{2}\right)-\tan^{-1}\left(\tan\frac{\pi}{4}\right)$$

$$= \frac{\pi}{2}-\frac{\pi}{4}$$

$$= \frac{2\pi-\pi}{4}$$

$$= \frac{\pi}{4}$$

$$\therefore \int_0^{\pi}\frac{1}{3+2\sin x+\cos x}dx = \frac{\pi}{4}$$

Definite Integrals Ex 20.2 Q23

We have,

$$\int_0^1 1\cdot \tan^{-1}x\,dx = \tan^{-1}x\int_0^1 dx - \int_0^1 \left(\int dx.\right)\frac{d}{dx}\left(\tan^{-1}x\right)dx$$

$$= \left[x\tan^{-1}x\right]_0^1 - \int_0^1 \frac{x}{1+x^2}dx$$

$$= \left[x\tan^{-1}x - \frac{1}{2}\log\left(1+x^2\right)\right]_0^1$$

$$= \frac{\pi}{4} - \frac{1}{2}\left(\log 2 - 0\right)$$

$$= \frac{\pi}{4} - \frac{1}{2}\log 2$$

$$\therefore \int_0^1 \tan^{-1}x\,dx = \frac{\pi}{4} - \frac{1}{2}\log 2$$

Definite Integrals Ex 20.2 Q24

Using Integration By parts

$$\int f'g = fg - \int fg'$$

$$f' = \frac{x}{\sqrt{1-x^2}},\ g = \sin^{-1}x$$

$$f = -\sqrt{1-x^2},\ g' = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx = -\sqrt{1-x^2}\,\sin^{-1}x - \int(-1)dx$$

$$\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx = -\sqrt{1-x^2}\,\sin^{-1}x + x$$

Hence

$$\int_0^{\frac{1}{2}} \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx = \left\{x - \sqrt{1-x^2}\,\sin^{-1}x\right\}_0^{\frac{1}{2}}$$

$$\int_0^{\frac{1}{2}} \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx = \left\{\frac{1}{2} - \sqrt{1-(\frac{1}{2})^2}\,\sin^{-1}\frac{1}{2}\right\}$$

$$\int_0^{\frac{1}{2}} \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx = \left\{\frac{1}{2} - \frac{\sqrt{3}}{2}\frac{\Pi}{6}\right\}$$

Definite Integrals Ex 20.2 Q25

$$I = \int_0^{\pi/4} \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$$

$$I = \int_0^{\pi/4} \left(\frac{\sqrt{\sin x}}{\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}}\right) dx$$

$$I = \int_0^{\pi/4} \left(\frac{\sin x + \cos x}{\sqrt{\sin x \cos x}}\right) dx$$

$$I = \sqrt{2}\int_0^{\pi/4} \left(\frac{\sin x + \cos x}{\sqrt{2 \sin x \cos x}}\right) dx$$

$$I = \sqrt{2}\int_0^{\pi/4} \left(\frac{\sin x + \cos x}{\sqrt{1 - (\sin x - \cos x)^2}}\right) dx$$

Let $\sin x - \cos x = t$

$(\cos x + \sin x)dx = dt$

$x = 0 \Rightarrow t = -1$ and $x = \frac{\pi}{4} \Rightarrow t = 0$

$$I = \sqrt{2}\int_{-1}^{0} \left(\frac{1}{\sqrt{1 - t^2}}\right) dt$$

$$I = \sqrt{2}\left[\sin^{-1} t\right]_{-1}^{0}$$

$$I = \sqrt{2}\left[\sin^{-1}(0) - \sin^{-1}(-1)\right]$$

$$I = \frac{\pi}{\sqrt{2}}$$

Definite Integrals Ex 20.2 Q26

We have,

$$\int_0^{\frac{\pi}{4}} \frac{\tan^3 x}{1 + \cos 2x} dx = \int_0^{\frac{\pi}{4}} \frac{\tan^3 x}{2\cos^2 x} dx = \frac{1}{2}\int_0^{\frac{\pi}{4}} \tan^3 x \sec^2 x \, dx$$

Let $\tan x = t \Rightarrow \sec^2 x \, dx = dt$

Now,

$x = 0 \Rightarrow t = 0$

$x = \frac{\pi}{4} \Rightarrow t = 1$

$$\therefore \frac{1}{2}\int_0^{\frac{\pi}{4}} \sec^2 x \tan^3 x \, dx = \frac{1}{2}\int_0^1 t^3 \, dt = \frac{1}{2}\left[\frac{t^4}{4}\right]_0^1 = \frac{1}{8}$$

$$\therefore \int_0^{\frac{\pi}{4}} \frac{\tan^3 x}{1 + \cos 2x} dx = \frac{1}{8}$$

Definite Integrals Ex 20.2 Q27

We know that,

$$\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$$

$$\frac{1}{5 + 3\cos x} = \frac{1}{5 + 3\left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right)} = \frac{1 + \tan^2 \frac{x}{2}}{5\left(1 + \tan^2 \frac{x}{2}\right) + 3\left(1 - \tan^2 \frac{x}{2}\right)} = \frac{\sec^2 \frac{x}{2} dx}{8 + 2\tan^2 \frac{x}{2}}$$

$$\therefore \int_0^{\pi} \frac{dx}{5 + 3\cos x} dx = \frac{1}{2}\int_0^{\pi} \frac{\sec^2 \frac{x}{2}}{2^2 + \tan^2 \frac{x}{2}} dx$$

Let $\tan\frac{x}{2} = t$

Differentiating w.r.t. x, we get

$$\frac{1}{2}\sec^2 \frac{x}{2} dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = \pi \Rightarrow t = \infty$

$$\therefore \frac{1}{2}\int_0^{\pi}\left(\frac{\sec^2 \frac{x}{2} dx}{2^2 + \tan^2 \frac{x}{2}}\right) dx$$

$$= \int_0^{\infty} \frac{dt}{2^2 + t^2}$$

$$= \left[\frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)\right]_0^{\infty}$$

$$= \frac{1}{2}\left[\tan^{-1}(\infty) - \tan^{-1}(0)\right]$$

$$= \frac{1}{2}\left[\tan^{-1}\left(\tan\frac{\pi}{2}\right) - \tan^{-1}(\tan 0)\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{2} - 0\right]$$

$$= \frac{\pi}{4}$$

$$\therefore \int_0^{\pi} \frac{dx}{5 + 3\cos x} dx = \frac{\pi}{4}$$

Definite Integrals Ex 20.2 Q28

We have,

$$\int_0^{\frac{\pi}{2}} \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x} dx$$

Dividing numerator and denominator by $\cos^2 x$

$$\int_0^{\frac{\pi}{2}} \left(\frac{\frac{1}{\cos^2 x}}{a^2 \frac{\sin^2 x}{\cos^2 x} + b^2 \frac{\cos^2 x}{\cos^2 x}} \right) dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{\sec^2 x}{a^2 \tan^2 x + b^2} \right) dx$$

$$= \frac{1}{a^2} \int_0^{\frac{\pi}{2}} \left(\frac{\sec^2 x}{\tan^2 x + \left(\frac{b}{a}\right)^2} \right) dx$$

Let $\tan x = t$

Differentiating w.r.t. x, we get

$$\sec^2 x \, dx = dt$$

When $x = 0 \Rightarrow t = 0$

$$x = \frac{\pi}{2} \Rightarrow t = \infty$$

$$\therefore \frac{1}{a^2} \int_0^{\frac{\pi}{2}} \left(\frac{\sec^2 x}{\tan^2 x + \left(\frac{b}{a}\right)^2} \right) dx$$

$$= \frac{1}{a^2} \int_0^{\infty} \frac{dt}{\left(\frac{b}{a}\right)^2 + t^2}$$

$$= \frac{1}{a^2} \left[\frac{a}{b} \tan^{-1} \frac{at}{b} \right]_0^{\infty}$$

$$= \frac{1}{a^2} \frac{a}{b} \left[\tan^{-1} \infty - \tan^{-1} 0 \right]$$

$$= \frac{1}{ab} \left[\tan^{-1} \tan \frac{\pi}{2} \right] \quad = \frac{\pi}{2ab}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x} dx = \frac{\pi}{2ab}$$

Definite Integrals Ex 20.2 Q29

$$I = \int_0^{\frac{\pi}{2}} \frac{x + \sin x}{1 + \cos x} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{x + 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{x \sec^2 \frac{x}{2}}{2} + \tan \frac{x}{2} \right) dx$$

$$= \left[x \tan\left(\frac{x}{2}\right) - \int_0^{\frac{\pi}{2}} \tan \frac{x}{2} dx + \int_0^{\frac{\pi}{2}} \tan \frac{x}{2} dx \right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2}$$

$$\therefore I = \int_0^{\frac{\pi}{2}} \frac{x + \sin x}{1 + \cos x} dx = \frac{\pi}{2}$$

Definite Integrals Ex 20.2 Q30

$$I=\int_0^1 \frac{\tan^{-1} x}{1+x^2}dx$$

Let $t=\tan^{-1} x$

$$dt=\frac{1}{1+x^2}dx$$

$$x=0, t=0$$

$$x=1, t=\frac{\pi}{4}$$

$$I=\int_0^{\frac{\pi}{4}} t\,dt$$

$$=\left[\frac{t^2}{2}\right]_0^{\frac{\pi}{4}}$$

$$=\frac{1}{2}\frac{\pi^2}{16}$$

$$=\frac{\pi^2}{32}$$

Definite Integrals Ex 20.2 Q31

$$I=\int_0^{\pi/4}\frac{\sin x+\cos x}{3+\sin 2x}\,dx$$

$$I=\int_0^{\pi/4}\left(\frac{\sin x+\cos x}{3+1-(\cos x-\sin x)^2}\right)dx$$

$$I=\int_0^{\pi/4}\left(\frac{\sin x+\cos x}{4-(\cos x-\sin x)^2}\right)dx$$

$$I=\frac{1}{4}\left[\log\left|\frac{2+\sin x-\cos x}{2-\sin x+\cos x}\right|\right]_0^{\pi/4}$$

$$I=-\frac{1}{4}\log\left(\frac{1}{3}\right)$$

$$I=\frac{1}{4}\log_e 3$$

Definite Integrals Ex 20.2 Q32

We have,

$$\int_0^1 x\tan^{-1}x\,dx=\tan^{-1}x\int_0^1 x\,dx-\int_0^1\left(\int x\,dx\right)\frac{d}{dx}\left(\tan^{-1}x\right)dx$$

$$=\left[\frac{x^2}{2}\tan^{-1}x\right]_0^1-\frac{1}{2}\int_0^1\frac{x^2}{1+x^2}\,dx$$

$$=\left[\frac{x^2}{2}\tan^{-1}x\right]_0^1-\frac{1}{2}\int_0^1\frac{1+x^2-1}{1+x^2}\,dx$$

$$=\frac{1}{2}\left(\frac{\pi}{4}\right)-\frac{1}{2}\left[\int_0^1 dx-\int_0^1\frac{dx}{1+x^2}\right]$$

$$=\frac{\pi}{8}-\frac{1}{2}\left[x-\tan^{-1}x\right]_0^1$$

$$=\frac{\pi}{8}-\frac{1}{2}\left[1-\frac{\pi}{4}\right]$$

$$=\frac{\pi}{8}-\frac{1}{2}+\frac{\pi}{8}$$

$$=\frac{\pi}{4}-\frac{1}{2}$$

$$\therefore\int_0^1 x\tan^{-1}x\,dx=\frac{\pi}{4}-\frac{1}{2}$$

Definite Integrals Ex 20.2 Q33

Let $I = \int \frac{1-x^2}{x^4+x^2+1}dx = -\int \frac{x^2-1}{x^4+x^2+1}dx.$

Then,

$$I = -\int \frac{1-\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}}dx \qquad \left[\begin{array}{l}\text{Dividing the numerator and}\\ \text{denominator by } x^2\end{array}\right]$$

$$\Rightarrow \quad I = -\int \frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)^2-1^2}dx$$

Let, $x+\frac{1}{x} = u$. Then, $d\left(x+\frac{1}{x}\right) = du \Rightarrow \left(1-\frac{1}{x^2}\right)dx = du$

$$\therefore \quad I = -\int \frac{du}{u^2-1^2}$$

$$\Rightarrow \quad I = -\frac{1}{2(1)}\log\left|\frac{u-1}{u+1}\right| + C$$

$$\Rightarrow \quad I = -\frac{1}{2}\log\left|\frac{x+\frac{1}{x}-1}{x+\frac{1}{x}+1}\right| + C = -\frac{1}{2}\log\left|\frac{x^2-x+1}{x^2+x+1}\right| + C$$

$$\therefore \quad \int_0^1 \frac{1-x^2}{x^4+x^2+1}dx = \left[-\frac{1}{2}\log\left|\frac{x^2-x+1}{x^2+x+1}\right|\right]_0^1 = \left(-\frac{1}{2}\log\left|\frac{1}{3}\right|\right) - \left(-\frac{1}{2}\log|1|\right) = \log\sqrt{3}$$

$$= \log 3^{\frac{1}{2}}$$

$$= \frac{1}{2}\log 3$$

Definite Integrals Ex 20.2 Q34

Let $1+x^2 = t$

Differentiating w.r.t. x, we get

$2x\,dx = dt$

Now, $x = 0 \Rightarrow t = 1$

$x = 1 \Rightarrow t = 2$

$$\int_0^1 \frac{24x^3}{\left(1+x^2\right)^4}dx = \int_1^2 \frac{12(t-1)}{t^4}dt$$

$$= 12\int_1^2 \left(\frac{1}{t^3} - \frac{1}{t^4}\right)dt$$

$$= 12\left[-\frac{1}{2t^2} - \frac{1}{3t^3}\right]_1^2$$

$$= 12\left[-\frac{1}{8} + \frac{1}{24} + \frac{1}{2} - \frac{1}{3}\right]$$

$$= 12\left[\frac{-3+1+12-8}{24}\right]$$

$$= \frac{12\times 2}{24} = 1$$

$$\therefore \int_0^1 \frac{24x^3}{\left(1+x^2\right)^4}dx = 1$$

Definite Integrals Ex 20.2 Q35

Let $x - 4 = t^3$

Differentiating w.r.t. x, we get

$$dx = 3t^2 dt$$

Now, $x = 4 \Rightarrow t = 0$

$x = 12 \Rightarrow t = 2$

$$\therefore \int_4^{12} x(x-4)^{\frac{1}{3}} dx = \int_0^2 (t^3+1) t.3t^2 dt$$

$$= 3\int_0^2 (t^6 + 4t^3) dt$$

$$= 3\left[\frac{t^7}{7} + t^4\right]_0^2$$

$$= 3\left[\frac{128}{7} + 16\right]$$

$$= \frac{720}{7}$$

$$\therefore \int_4^{12} x(x-4)^{\frac{1}{3}} dx = \frac{720}{7}$$

Definite Integrals Ex 20.2 Q36

We have,

$$\int_0^{\frac{\pi}{2}} \underset{\text{I}}{x^2} \underset{\text{II}}{\sin x}\, dx$$

Using by parts, we get

$$x^2 \int \sin x dx - \int \left(\int \sin x dx\right) \frac{dx^2}{dx} . dx$$

$$= x^2 \cos x + \int \cos x . 2x dx$$

Again applying by parts

$$= x^2 \cos x + 2\left[x \int \cos x dx - \int \left(\int \cos x dx\right) . \frac{dx}{dx} . dx\right]$$

$$= x^2 \cos x + 2\left[x \sin x - \int \sin x dx\right]$$

$$= \left[x^2 \cos x + 2x \sin x + 2\cos x\right]_0^{\frac{\pi}{2}}$$

$$= \pi + 0 - 0 - 0 - 2$$

$$= \pi - 2$$

$$\therefore \int_0^{\frac{\pi}{2}} x^2 \sin x dx = \pi - 2$$

Definite Integrals Ex 20.2 Q37

Let $x = \cos 2\theta$

Differentiating w.r.t. x, we get

$dx = -2\sin 2\theta d\theta$

Now, $x = 0 \Rightarrow \theta = \frac{\pi}{4}$

$x = 1 \Rightarrow \theta = 0$

$$\therefore \int_0^1 \sqrt{\frac{1-x}{1+x}}dx = \int_{\frac{\pi}{4}}^{0} \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}\left(-2\sin 2\theta\right)d\theta$$

$$= \int_0^{\frac{\pi}{4}} \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}\left(\ 2\sin 2\theta\right)d\theta \qquad \left[\because \sin 2\theta = 2\sin\theta\cos\theta;\ \text{and}\ \sin^2\theta = \frac{1-\cos 2\theta}{2}\right]$$

$$= 2\int_0^{\frac{\pi}{4}} \frac{\sin\theta}{\cos\theta}.\sin 2\theta d\theta$$

$$= 4\int_0^{\frac{\pi}{4}} \sin^2\theta d\theta$$

$$= 2\int_0^{\frac{\pi}{4}} \left(1-\cos 2\theta\right)d\theta$$

$$= 2\left[\theta - \frac{\sin^2\theta}{2}\right]_0^{\frac{\pi}{4}}$$

$$= 2\left[\frac{\pi}{4} - \frac{1}{2}\right]$$

$$= \frac{\pi}{2} - 1$$

$$\therefore \int_0^1 \sqrt{\frac{1-x}{1+x}}dx = \frac{\pi}{2} - 1$$

Definite Integrals Ex 20.2 Q38

We have,

$$\int_0^1 \frac{1-x^2}{\left(1+x^2\right)^2}dx = \int_0^1 \frac{-x^2\left(1-\frac{1}{x^2}\right)dx}{x^2\left(x+\frac{1}{x}\right)^2} = -\int_0^1 \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x+\frac{1}{x}\right)^2}$$

Let $x + \frac{1}{x} = t \Rightarrow 1 - \frac{1}{x^2}dx = dt$

When $x = 0 \Rightarrow t = \infty$

$x = 1 \Rightarrow t = 2$

$$\therefore \int_0^1 \frac{1-x^2}{\left(1+x^2\right)^2}dx = -\int_{\infty}^{2} \frac{dt}{t^2} = \int_2^{\infty} \frac{dt}{t^2} = \left[-\frac{1}{t}\right]_2^{\infty} = \left(\frac{1}{2} - 0\right) = \frac{1}{2}$$

Definite Integrals Ex 20.2 Q39

Put $t = x^5 + 1$, then $dt = 5x^4\ dx$.

Therefore, $\int 5x^4\sqrt{x^5+1}\ dx = \int \sqrt{t}\ dt = \frac{2}{3}dt = \frac{2}{3}t^{\frac{3}{2}} = \frac{2}{3}\left(x^5+1\right)^{\frac{3}{2}}$

Hence, $\int_{-1}^{1} 5x^4\sqrt{x^4+1}\ dx = \frac{2}{3}\left[\left(x^5+1\right)^{\frac{3}{2}}\right]_{-1}^{1}$

$$= \frac{2}{3}\left[\left(1^5+1\right)^{\frac{3}{2}} - \left(\left(-1\right)^5+1\right)^{\frac{3}{2}}\right]$$

$$= \frac{2}{3}\left[2^{\frac{3}{2}} - 0^{\frac{3}{2}}\right] = \frac{2}{3}\left(2\sqrt{2}\right) = \frac{4\sqrt{2}}{3}$$

Alternatively, first we transform the integral and then evaluate the transformed integral with new limits.

Let $t = x^5 + 1$. Then $dt = 5x^4\ dx$.

Note that, when $x = -1$, $t = 0$ and when $x = 1$, $t = 2$

Thus, as x varies from -1 to 1, t varies from 0 to 2

Therefore $\int_{-1}^{1} 5x^4\sqrt{x^5+1}\ dx = \int_0^2 \sqrt{t}\ dt$

$$= \frac{2}{3}\left[t^{\frac{3}{2}}\right]_0^2 = \frac{2}{3}\left[2^{\frac{3}{2}} - 0^{\frac{3}{2}}\right] = \frac{2}{3}\left(2\sqrt{2}\right) = \frac{4\sqrt{2}}{3}$$

Definite Integrals Ex 20.2 Q40

$$I = \int_0^{\pi/2} \frac{\cos^2 x}{1+3\sin^2 x}\, dx$$

$$I = \int_0^{\pi/2} \frac{\sec^2 x}{\sec^2 x\left(\sec^2 x + 3\tan^2 x\right)}\, dx$$

Put $\tan x = t$

$\sec^2 x\, dx = dt$

$x = 0 \Rightarrow t = 0$ and $x = \frac{\pi}{2} \Rightarrow t = \infty$

$$I = \int_0^{\infty} \frac{1}{\left(1+t^2\right)\left(1+4t^2\right)}\, dt$$

$$I = -\frac{1}{3}\int_0^{\infty}\left[\frac{1}{\left(1+t^2\right)} - \frac{1}{\left(1+4t^2\right)}\right] dt$$

$$I = -\frac{1}{3}\left[\tan^{-1} t - 2\tan^{-1} 2t\right]_0^{\infty}$$

$$I = \frac{\pi}{6}$$

Definite Integrals Ex 20.2 Q41

Let $I = \int_0^{\frac{\pi}{4}} \sin^3 2t \cos 2t\, dt$. consider $\int \sin^3 2t \cos 2t\, dt$

Put $\sin 2t = u$ so that $2\cos 2t\, dt = du$ or $\cos 2t\, dt = \frac{1}{2}\, du$

So $\int \sin^3 2t \cos 2t\, dt = \frac{1}{2}\int u^3\, du$

$$= \frac{1}{8}\left[u^4\right] = \frac{1}{8}\sin^4 2t = F(t) \text{ say}$$

Therefore, by the second fundamental theorem of integrals calculus

$$I = F\left(\frac{\pi}{4}\right) - F(o) = \frac{1}{8}\left[\sin^4 \frac{\pi}{2} - \sin^4 0\right] = \frac{1}{8}$$

Definite Integrals Ex 20.2 Q42

Let $5 - 4\cos\theta = t$

Differentiating w.r.t. x, we get

$4\sin\theta d\theta = dt$

Now, $\theta = 0 \Rightarrow t = 1$

$\theta = \pi \Rightarrow t = 9$

$$\therefore \int_0^{\pi} 5\left(5 - 4\cos\theta\right)^{\frac{1}{4}} \sin\theta d\theta$$

$$= \frac{5}{4}\int_1^9 t^{\frac{1}{4}} dt$$

$$= \frac{5}{4}\left[\frac{4}{5}.t^{\frac{5}{4}}\right]_1^9$$

$$= 3^{\frac{5}{2}} - 1$$

$$= 9\sqrt{3} - 1$$

$$\therefore \int_0^{\pi} 5\left(5 - 4\cos\theta\right)^{\frac{1}{4}} \sin\theta d\theta = 9\sqrt{3} - 1$$

Definite Integrals Ex 20.2 Q43

We have,

$$\int_0^{\frac{\pi}{6}} \cos^{-3} 2\theta \sin 2\theta d\theta$$

$$= \int_0^{\frac{\pi}{6}} \frac{\sin 2\theta}{\cos^3 2\theta} d\theta$$

$$= \int_0^{\frac{\pi}{6}} \tan 2\theta . \sec^2 2\theta d\theta$$

Let $\tan 2\theta = t$

Differentiating w.r.t. x, we get

$$2 \sec^2 2\theta d\theta = dt$$

Now, $\theta = 0 \Rightarrow t = 0$

$$\theta = \frac{\pi}{6} \Rightarrow t = \sqrt{3}$$

$$\therefore \int_0^{\frac{\pi}{6}} \tan 2\theta . \sec^2 2\theta d\theta = \frac{1}{2} \int_0^{\sqrt{3}} t dt = \frac{1}{2} \left[\frac{t^2}{2} \right]_0^{\sqrt{3}}$$

$$= \frac{3}{4}$$

$$\therefore \int_0^{\frac{\pi}{6}} \cos^{-3} 2\theta \sin 2\theta d\theta = \frac{3}{4}$$

Definite Integrals Ex 20.2 Q44

Let $x^{\frac{2}{3}} = t$

Differentiating w.r.t. x, we get

$$\frac{3}{2} \sqrt{x} dx = dt$$

Now, $x = 0 \Rightarrow t = 0$

$$x = \pi^{\frac{2}{3}} \Rightarrow t = \pi$$

$$\therefore \int_0^{\pi^{\frac{2}{3}}} \sqrt{x} \cos^2 x^{\frac{3}{2}} dx$$

$$= \frac{2}{3} \int_0^{\pi} \cos^2 t \, dt$$

$$= \frac{1}{3} \int_0^{\pi} 1 + \cos 2t \, dt \qquad \left[\because 2\cos^2 t = t + \cos 2t \right]$$

$$= \frac{1}{3} \left[t + \frac{\sin 2t}{t} \right]_0^{\pi}$$

$$= \frac{1}{3} [\pi + 0 - 0 - 0] = \frac{\pi}{3}$$

$$\therefore \int_0^{\pi^{\frac{2}{3}}} \sqrt{x} \cos^2 x^{\frac{3}{2}} dx = \frac{\pi}{3}$$

Definite Integrals Ex 20.2 Q45

Let $1+\log x = t$
Differentiating w.r.t. x, we get

$$\frac{1}{x}dx = dt$$

When $x = 1 \Rightarrow t = 1$

$x = 2 \Rightarrow t = 1+\log 2$

$$\therefore \int_1^2 \frac{dx}{x(1+\log x)^2}$$
$$= \int_1^{1+\log 2} \frac{dt}{t^2}$$
$$= \left[-\frac{1}{t}\right]_1^{1+\log 2}$$
$$= 1 - \frac{1}{1+\log 2}$$
$$= \frac{\log 2}{1+\log 2}$$

$$\therefore \int_1^2 \frac{dx}{x(1+\log x)^2} = \frac{\log 2}{1+\log 2}$$

Definite Integrals Ex 20.2 Q46

We have,

$$\int_0^{\frac{\pi}{2}} \cos^5 x\,dx = \int_0^{\frac{\pi}{2}} \left(1-\sin^2 x\right)^2 \cos x\,dx$$

Let $\sin x = t$
Differentiating w.r.t. x, we get
$\cos x dx = dt$

When $x = 0 \Rightarrow t = 0$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$$\int_0^{\frac{\pi}{2}} \left(1-\sin^2 x\right)^2 \cos x\,dx$$
$$= \int_0^1 \left(1-t^2\right)^2 dt$$
$$= \int_0^1 \left(1-2t^2+t^4\right)dt$$
$$= \left[t-\frac{2}{3}t^3+\frac{t^5}{5}\right]_0^1$$
$$= 1-\frac{2}{3}+\frac{1}{5}$$
$$= \frac{8}{15}$$

$$\therefore \int_0^{\frac{\pi}{2}} \cos^5 x\,dx = \frac{8}{15}$$

Definite Integrals Ex 20.2 Q47

Let $I = \int \frac{\sqrt{x}}{\left(30 - x^{\frac{3}{2}}\right)} dx$. We first find the anti derivative of the integrand.

Pu $30 - x^{\frac{3}{2}} = t$. Then $-\frac{3}{2}\sqrt{x}\, dx = dt$ or $\sqrt{x}\, dx = -\frac{2}{3} dt$

Thus, $\int \frac{\sqrt{x}}{\left(30 - x^{\frac{3}{2}}\right)^2} dx = -\frac{2}{3}\int \frac{dt}{t^2} = \frac{2}{3}\left[\frac{1}{t}\right] = \frac{2}{3}\left[\frac{1}{\left(30 - x^{\frac{3}{2}}\right)}\right] = f(x)$

Therefore, by the second fundamental theorem of calculus, we have

$$I = F(9) - F(4) = \frac{2}{3}\left[\frac{1}{\left(30 - x^{\frac{3}{2}}\right)}\right]_4^9$$

$$= \frac{2}{3}\left[\frac{1}{(30 - 27)} - \frac{1}{30 - 8}\right] = \frac{2}{3}\left[\frac{1}{3} - \frac{1}{22}\right] = \frac{19}{99}$$

Definite Integrals Ex 20.2 Q48

Let $\cos x = t$

Differentiating w.r.t. x, we get

$-\sin x dx = dt$

When $x = 0 \Rightarrow t = 1$

$x = \pi \Rightarrow t = -1$

Now,

$$\int_0^{\pi} \sin^3 x \left(1 + 2\cos x\right)\left(1 + \cos x\right)^2 dx$$

$$= \int_0^{\pi} \sin^2 x \left(1 + 2\cos x\right)\left(1 + \cos x\right)^2 . \sin x dx$$

$$= -\int_{-1}^{1} \left(1 - t^2\right)\left(1 + 2t\right)\left(1 + t\right)^2 dt \qquad \left[\sin^2 x = 1 - \cos^2 x\right]$$

$$= \int_{-1}^{1} \left(1 + 2t - t^2 - 2t^3\right)\left(1 + t^2 + 2t\right) dt$$

$$= \int_{-1}^{1} \left(1 - t^2 + 2t + 2t + 2t^3 + 4t^2 - t^2 - t^4 - 2t^3 - 2t^5 - 4t^4\right) dt$$

$$= \int_{-1}^{1} \left(1 + 4t + 4t^2 - 2t^3 - 5t^4 - 2t^5\right) dt$$

$$= \left[t + 2t^2 + \frac{4}{3}t^3 - \frac{t^4}{2} - t^5 - \frac{t6}{3}\right]_{-1}^{1}$$

$$= \left[2 + 0 + \frac{8}{3} - 0 - 2 - 0\right] = \frac{8}{3}$$

$$\therefore \int_0^{\pi} \sin^3 x \left(1 + 2\cos x\right)\left(1 + \cos x\right)^2 dx = \frac{8}{3}$$

Definite Integrals Ex 20.2 Q49

$$I = \int_0^{\frac{\pi}{2}} 2\sin x\cos x\tan^{-1}(\sin x)dx$$

Let $t = \sin x$

$dt = \cos x dx$

$x = 0, t = 0$

$x = \frac{\pi}{2}, t = 1$

$$I = \int_0^1 2t\tan^{-1}(t)dt$$

$$= 2\left[\frac{1}{2}t^2\tan^{-1}t - \frac{t}{2} + \frac{1}{2}\tan^{-1}t\right]_0^1$$

$$= 2\left[\frac{\pi}{4} - \frac{1}{2}\right]$$

$$= \frac{\pi}{2} - 1$$

$$\therefore I = \int_0^{\frac{\pi}{2}} 2\sin x\cos x\tan^{-1}(\sin x)dx = \frac{\pi}{2} - 1$$

Definite Integrals Ex 20.2 Q50

Let $\sin x = t$

Differentiating w.r.t. x, we get

$\cos x\, dx = dt$

Now,

$x = 0 \Rightarrow t = 0$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$$\therefore \int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x)dx = 2\int_0^1 t\tan^{-1} t\, dt \qquad [\because \sin 2x = 2\sin x\cos x]$$

Using by parts

$$= 2\left\{\tan^{-1} t\int t dt - \int\left(\int t dt\right)\frac{d\tan^{-1}t}{dt}dt\right\}$$

$$= 2\left\{\frac{t^2}{2}\tan^{-1} - \frac{1}{2}\int\frac{t^2}{1+t^2}dt\right\}$$

$$= 2\left\{\frac{t^2}{2}\tan^{-1}t - \frac{1}{2}\left(\int dt - \int\frac{dt}{1+t^2}\right)\right\}$$

$$= 2\left[\frac{t^2}{2}\tan^{-1}t - \frac{1}{2}\left(t - \tan^{-1}t\right)\right]_0^1$$

$$= 2\left\{\frac{1}{2}\frac{\pi}{4} - \frac{1}{2}\left(1 - \frac{\pi}{4}\right)\right\}$$

$$= 2\left\{\frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8}\right\}$$

$$= 2\left(\frac{\pi}{4} - \frac{1}{2}\right) = \frac{\pi}{2} - 1$$

$$\therefore \int_0^{\frac{\pi}{2}} \sin 2x\tan^{-1}(\sin x)dx = \frac{\pi}{2} - 1$$

Definite Integrals Ex 20.2 Q51

We have,

$$\int_0^1 \left(\cos^{-1} x\right)^2 dx = \left(\cos^{-1} x\right)^2 \int_0^1 dx - \int_0^1 \left(\int dx\right) \frac{d\left(\cos^{-1} x\right)^2}{dx} dx$$

$$= \left[x\left(\cos^{-1} x\right)^2\right]_0^1 + \int_0^1 \frac{x.2\cos^{-1}}{\sqrt{1-x^2}} dx$$

Now,

Let $\cos^{-1} x = t \Rightarrow -\frac{1}{\sqrt{1-x^2}} dx = dt$

When $x = 0 \Rightarrow t = \frac{\pi}{2}$

$x = 1 \Rightarrow t = 0$

$$\therefore \int_0^1 \frac{2x\cos^{-1} x}{\sqrt{1-x^2}} dx = -2\int_{\frac{\pi}{2}}^0 t\cos t\,dt = 2\int_0^{\frac{\pi}{2}} t\cos t\,dt$$

$$= 2\left[t\int \cos t\,dt - \int \left(\cos t\,dt\right)\frac{dt}{dt}.dt\right]_0^{\frac{\pi}{2}}$$

$$= 2\left[t\sin t - \int \sin t\,dt\right]_0^{\frac{\pi}{2}}$$

$$= 2\left[t\sin t + \cos t\right]_0^{\frac{\pi}{2}}$$

$$= 2\left[\frac{\pi}{2} - 1\right]$$

$$\int_0^1 \left(\cos^{-1} x\right)^2 dx = \left[x\left(\cos^{-1} x\right)^2\right]_0^1 + \int_0^1 \frac{x.2\cos^{-1}}{\sqrt{1-x^2}} dx = \left[x\left(\cos^{-1} x\right)^2\right]_0^1 + 2\left(\frac{\pi}{2} - 1\right)$$

$$= 0 - 0 + 2\left(\frac{\pi}{2} - 1\right)$$

$$= \left(\pi - 2\right)$$

$$\therefore \int_0^1 \left(\cos^{-1} x\right)^2 dx = \left(\pi - 2\right)$$

Definite Integrals Ex 20.2 Q53

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{1+\cos x}}{(1-\cos x)^{\frac{3}{2}}}dx$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{2\cos^2\frac{x}{2}}}{\left(2\sin^2\frac{x}{2}\right)^{\frac{3}{2}}}dx \qquad \left[\begin{array}{l} \because 1+\cos x = 2\cos^2\frac{x}{2} \\ 1-\cos x = 2\sin^2\frac{x}{2} \end{array}\right]$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{2}\cos\frac{x}{2}}{2\sqrt{2}\sin^3\frac{x}{2}}dx$$

$$= \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cot\frac{x}{2}\,cosec^2\frac{x}{2}\,dx \qquad \left[\begin{array}{l} \because cosec^2\frac{x}{2} = \dfrac{1}{\sin^2\frac{x}{2}} \\ \cot\frac{x}{2} = \dfrac{\cos\frac{x}{2}}{\sin\frac{x}{2}} \end{array}\right]$$

Let $\cot\frac{x}{2} = t$

Differentiating w.r.t. x, we get

$$\frac{-1}{2}cosec^2\frac{x}{2} = dt$$

Now, $x = \frac{\pi}{3} \Rightarrow t = \sqrt{3}$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$$\therefore \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cot\frac{x}{2}\,cosec^2\frac{x}{2}\,dx = -\int_{\sqrt{3}}^{1} t\,dt = -\left[\frac{t^2}{2}\right]_{\sqrt{3}}^{1} = \frac{-1}{2}[1-3]$$

$= 1$

$$\therefore \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{1+\cos x}}{(1-\cos x)^{\frac{3}{2}}}dx = 1$$

Definite Integrals Ex 20.2 Q54

Substitute $x^2 = a^2\cos 2\theta$

Differentiating w.r.t. x, we get

$$2x\,dx = -2a^2\sin 2\theta\,d\theta$$

Now, $x = 0 \Rightarrow \theta = \frac{\pi}{4}$

$x = a \Rightarrow \theta = 0$

$$\therefore \int_0^a x\sqrt{\frac{a^2-x^2}{a^2+x^2}}dx = \int_{\frac{\pi}{4}}^{0} \sqrt{\frac{a^2(1-\cos 2\theta)}{a^2-(1-\cos 2\theta)}}\left(-a^2\sin 2\theta\right)d\theta$$

$$= -a^2\int_{\frac{\pi}{4}}^{0} \frac{\sin\theta}{\cos\theta}\sin 2\theta\,d\theta$$

$$= a^2\int_0^{\frac{\pi}{4}} 2\sin^2\theta\,d\theta$$

$$= a^2\int_0^{\frac{\pi}{4}} (1-\cos 2\theta)\,d\theta$$

$$= a^2\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{4}}$$

$$= a^2\left[\frac{\pi}{4} - \frac{1}{2}\right]$$

$$\therefore \int_0^a x\sqrt{\frac{a^2-x^2}{a^2+x^2}}dx = a^2\left[\frac{\pi}{4} - \frac{1}{2}\right]$$

Definite Integrals Ex 20.2 Q55

Let $x = a\cos 2\theta$

Differentiating w.r.t. x, we get

$dx = -2a\sin 2\theta$

Now, $x = -a \Rightarrow \theta = \frac{\pi}{2}$

$x = a \Rightarrow \theta = 0$

$$\therefore \int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}dx = \int_{\frac{\pi}{2}}^{0} \sqrt{\frac{a(1-\cos 2\theta)}{a(1+\cos 2\theta)}}(-2\sin 2\theta)d\theta$$

$$= 2a\int_{0}^{\frac{\pi}{2}} \frac{\sin\theta}{\cos\theta}.\sin 2\theta d\theta \qquad \left[\begin{array}{l} \because 1-\cos 2\theta = 2\sin^2\theta \\ 1+\cos 2\theta = 2\cos^2\theta \\ -\int_a^b f(x)dx = \int_b^a f(x)dx \end{array}\right]$$

$$= 2a\int_{0}^{\frac{\pi}{2}} \frac{\sin\theta.2\sin\theta\cos\theta}{\cos\theta}$$

$$= 4a\int_{0}^{\frac{\pi}{2}} s\,in^2\theta d\theta$$

$$= 2a\int_{0}^{\frac{\pi}{2}} (1-\cos 2\theta)d\theta$$

$$= 2a\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}$$

$$= 2a\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}$$

$$= 2a\left[\frac{\pi}{2} - 0 - 0 + 0\right] = \pi a$$

$$\therefore \int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}dx = \pi a$$

Definite Integrals Ex 20.2 Q56

Let $\cos x = t$

Differentiating w.r.t. x, we get

$-\sin x dx = dt$

Now, $x = 0 \Rightarrow t = 1$

$x = \frac{\pi}{2} \Rightarrow t = 0$

$$\therefore \int_{0}^{\frac{\pi}{2}} \frac{\sin x\cos x dx}{\cos^2 x + 3\cos x + 2}$$

$$= -\int_{1}^{0} \frac{tdt}{t^2 + 3t + 2}$$

$$= \int_{0}^{1} \frac{tdt}{(t+2)(t+1)} \qquad \left[\because -\int_a^b f(x) = \int_b^a f(x)\right]$$

$$= \int_{0}^{1} \left(-\frac{1}{t+1} + \frac{2}{t+2}\right)dt \qquad [\text{Applying partial fraction}]$$

$$= \left[-\log|1+t| + 2\log|t+2|\right]_0^1$$

$$= -\log 2 + 2\log 3 + 0 - 2\log 2$$

$$= 2\log 3 - 3\log 2$$

$$= \log\frac{9}{8}$$

$$\therefore \int_{0}^{\frac{\pi}{2}} \frac{\sin x\cos x dx}{\cos^2 x + 3\cos x + 2} = \log\frac{9}{8}$$

Definite Integrals Ex 20.2 Q57

$$I = \int_0^{\pi/2} \frac{\tan x}{1+m^2\tan^2 x}\,dx$$

$$I = \int_0^{\pi/2} \frac{\sin x\cos x}{\cos^2 x + m^2\sin^2 x}\,dx$$

Put $\sin^2 x = t$ then $2\sin x\cos x\, dx = dt$

$x = 0 \Rightarrow t = 0$ and $x = \frac{\pi}{2} \Rightarrow t = 1$

$$I = \frac{1}{2}\int_0^1 \frac{1}{(1-t)+m^2 t}\,dt$$

$$I = \frac{1}{2}\int_0^1 \frac{1}{(m^2-1)t+1}\,dt$$

$$I = \frac{1}{2}\left[\frac{1}{m^2-1}\log\left|(m^2-1)t+1\right|\right]_0^1$$

$$I = \frac{1}{2}\left[\frac{1}{m^2-1}\log\left|m^2\right| - \frac{1}{m^2-1}\ln|1|\right]$$

$$I = \frac{1}{2}\left[\frac{\log\left|m^2\right|}{m^2-1}\right]$$

$$I = \frac{1}{2}\left[\frac{2\log|m|}{m^2-1}\right]$$

$$I = \frac{\log|m|}{m^2-1}$$

Definite Integrals Ex 20.2 Q58

$$I = \int_0^{1/2} \frac{1}{(1+x^2)\sqrt{1-x^2}}\,dx$$

Let $x = \sin u$

$dx = \cos u\ du$

$$I = \int_0^{\pi/6} \frac{1}{(1+\sin^2 u)}\,du$$

$$I = \int_0^{\pi/6} \frac{\sec^2 u}{(1+2\tan^2 u)}\,du$$

Let $\tan u = v$

$dv = \sec^2 u\, du$

$$I = \int_0^{1/\sqrt{3}} \frac{1}{(1+2v^2)}\,dv$$

$$I = \frac{1}{\sqrt{2}}\left[\tan^{-1}\left(\sqrt{2}v\right)\right]_0^{1/\sqrt{3}}$$

$$I = \frac{1}{\sqrt{2}}\left[\tan^{-1}\left(\sqrt{\frac{2}{3}}\right)\right]$$

Definite Integrals Ex 20.2 Q59

$$I = \int_{1/3}^{1} \frac{(x - x^3)^{1/3}}{x^4} dx$$

$$I = \int_{1/3}^{1} \frac{\left(\frac{1}{x^2} - 1\right)^{1/3}}{x^3} dx$$

Let $\frac{1}{x^2} - 1 = t$

$\frac{-2}{x^3} dx = dt$

$x = \frac{1}{3} \Rightarrow t = 8$ and $x = 1 \Rightarrow t = 0$

$$I = -\frac{1}{2}\int_{8}^{0} (t)^{1/3} dt$$

$$I = -\frac{1}{2}\left[\frac{t^{4/3}}{4/3}\right]_8^0$$

$$I = -\frac{1}{2}[0 - 12]$$

$I = 6$

Definite Integrals Ex 20.2 Q60

$$\int \sec^2 x \frac{\tan^2 x}{\tan^6 x + 2\tan^3 x + 1} dx$$

$$u = \tan x \to \frac{du}{dx} = \sec^2 x$$

$$\int \frac{u^2}{u^6 + 2u^3 + 1} du$$

$$v = u^3 \to \frac{dv}{du} = 3u^2$$

$$\frac{1}{3}\int \frac{1}{v^2 + 2v + 1} dv$$

$$\frac{1}{3}\int \frac{1}{(v+1)^2} dv$$

$$-\frac{1}{3(v+1)}$$

$$-\frac{1}{3(u^3+1)}$$

$$-\frac{1}{3(\tan^3 x + 1)}$$

$$\left\{-\frac{1}{3(\tan^3 x + 1)}\right\}_0^{\frac{\pi}{4}}$$

$$\left\{-\frac{1}{6} + \frac{1}{3}\right\}$$

$$\frac{1}{6}$$

Definite Integrals Ex 20.2 Q61

$$\int_0^{\frac{\pi}{2}} \sqrt{\cos x(1-\cos^2 x)}\tan^2 x\cos^2 x dx$$

$$\int_0^{\frac{\pi}{2}} \sqrt{\cos x\sin^2 x}\sin^2 x dx$$

$$\int_0^{\frac{\pi}{2}} \sqrt{\cos x}\sin^3 x dx$$

$$\cos x = t \to -\sin x = \frac{dt}{dx}$$

$$-\int_1^0 \sqrt{t}(1-t^2)dt$$

$$\int_0^1 (\sqrt{t}-t^{\frac{5}{2}})dt$$

$$\left\{\frac{2t^{\frac{3}{2}}}{3}-\frac{2t^{\frac{7}{2}}}{7}\right\}_0^1$$

$$\frac{2}{3}-\frac{2}{7}$$

$$\frac{8}{21}$$

Definite Integrals Ex 20.2 Q62

$$I = \int_0^{\pi/2} \frac{\cos x}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^n} dx$$

$$I = \int_0^{\pi/2} \frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^n} dx$$

$$I = \int_0^{\pi/2} \frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^{n-1}} dx$$

Let $\cos\frac{x}{2}+\sin\frac{x}{2} = t$

$$\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)dx = 2dt$$

$x = 0 \Rightarrow t = 1$ and $x = \frac{\pi}{2} \Rightarrow t = \sqrt{2}$

$$I = \int_1^{\sqrt{2}} \frac{2}{(t)^{n-1}} dt$$

$$I = \left[\frac{2t^{-n+2}}{-n+2}\right]_1^{\sqrt{2}}$$

$$I = \frac{2}{2-n}\left[\left(\sqrt{2}\right)^{2-n}-1\right]$$

$$I = \frac{2}{2-n}\left[2^{1-\frac{n}{2}}-1\right]$$

Ex 20.3

Definite Integrals Ex 20.3 Q1(i)

We have,

$$\int_1^4 f(x)\,dx$$

$$= \int_1^2 (4x+3)\,dx + \int_2^4 (3x+5)\,dx$$

$$= \left[\frac{4x^2}{2}+3x\right]_1^2 + \left[\frac{3x^2}{2}+5x\right]_2^4$$

$$= \left[\left(\frac{16}{2}+6\right)-\left(\frac{4}{2}+3\right)\right]+\left[\left(\frac{48}{2}+20\right)-\left(\frac{12}{2}+10\right)\right]$$

$$= \left[(14-5)\right]+\left[(44-16)\right]$$

$$= 9+28$$

$$= 37$$

Definite Integrals Ex 20.3 Q1(ii)

We have,

$$\int_0^9 f(x)\,dx$$

$$= \int_0^{\frac{\pi}{2}} \sin x\,dx + \int_{\frac{\pi}{2}}^3 1\,dx + \int_3^9 e^{x-3}dx$$

$$= \left[-\cos x\right]_0^{\frac{\pi}{2}} + \left[x\right]_{\frac{\pi}{2}}^3 + \left[e^{x-3}\right]_3^9$$

$$= \left[-\cos\frac{\pi}{2}+\cos 0\right]+\left[3-\frac{\pi}{2}\right]+\left[e^{9-2}-e^{3-3}\right]$$

$$= \left[0+1\right]+\left[3-\frac{\pi}{2}\right]+\left[e^6-e^0\right]$$

$$= 0+1+3-\frac{\pi}{2}+e^6-e^0$$

$$= 1+3-\frac{\pi}{2}+e^6-1$$

$$= 3-\frac{\pi}{2}+e^6$$

Definite Integrals Ex 20.3 Q1(iii)

We have,

$$\int_1^4 f(x)\,dx$$

$$= \int_1^3 (7x+3)\,dx + \int_3^4 8x\,dx$$

$$= \left[\frac{7x^2}{2}+3x\right]_1^3 + \left[\frac{8x^2}{2}\right]_3^4$$

$$= \left[\left(\frac{7\times 9}{2}+3\times 3\right)-\left(\frac{7\times 1}{2}+3\times 1\right)\right]+\left[\left(\frac{8\times 16}{2}-\frac{8\times 9}{2}\right)\right]$$

$$= \left[\frac{63}{2}+9-\frac{7}{2}-3\right]+\left[64-36\right]$$

$$= 34+28$$

$$= 62$$

Definite Integrals Ex 20.3 Q2

We have,

$$\int_{-4}^4 |x+2|\,dx$$

$$= \int_{-4}^{-2} -(x+2)\,dx + \int_{-2}^4 (x+2)\,dx$$

$$= -\left[\frac{x^2}{2}+2x\right]_{-4}^{-2} + \left[\frac{x^2}{2}+2x\right]_{-2}^{4}$$

$$= -\left[\left(\frac{4}{2}-4\right)-\left(\frac{16}{2}-8\right)\right]+\left[\left(\frac{16}{2}+8\right)-\left(\frac{4}{2}-4\right)\right]$$

$$= -\left[(-2)-(0)\right]+\left[(16)-(-2)\right]$$

$$= -\left[-2\right]+\left[16+2\right]$$

$$= 2-18$$

$$= 20$$

$$\therefore \int_{-4}^4 |x+2|\,dx = 20$$

Definite Integrals Ex 20.3 Q3

We have,

$$\int_{-3}^{3} |x+1|dx$$

$$= \int_{-3}^{-1} -(x+1)dx + \int_{-1}^{3}(x+1)dx$$

$$= -\left[\frac{x^2}{2}+x\right]_{-3}^{-1} + \left[\frac{x^2}{2}+x\right]_{-1}^{3}$$

$$= -\left[\left(\frac{1}{2}-1\right)-\left(\frac{9}{2}-3\right)\right]+\left[\left(\frac{9}{2}+3\right)-\left(\frac{1}{2}-1\right)\right]$$

$$= -\left[\left(-\frac{1}{2}\right)-\left(1\frac{1}{2}\right)\right]+\left[\left(7\frac{1}{2}\right)-\left(-\frac{1}{2}\right)\right]$$

$$= -\left[-\frac{1}{2}-1\frac{1}{2}\right]+\left[7\frac{1}{2}+\frac{1}{2}\right]$$

$$= [-2]+[8]$$

$$= 2+8$$

$$= 10$$

$$\therefore \int_{-3}^{3} |x+1|dx = 10$$

Definite Integrals Ex 20.3 Q4

We have,

$$\int_{-1}^{1} |2x+1|dx$$

$$= \int_{-1}^{\frac{1}{2}} -(2x+1)\,dx + \int_{-\frac{1}{2}}^{1}(2x+1)\,dx$$

$$= -\left[\frac{2x^2}{2}+x\right]_{-1}^{-\frac{1}{2}} + \left[\frac{2x^2}{2}+x\right]_{-\frac{1}{2}}^{1}$$

$$= -\left[\left(\frac{2}{8}-\frac{1}{2}\right)-\left(\frac{2}{2}-1\right)\right]+\left[\left(\frac{2}{2}+1\right)-\left(\frac{2}{8}-\frac{1}{2}\right)\right]$$

$$= -\left[\left(\frac{1}{4}-\frac{1}{2}\right)-(1-1)\right]+\left[(1+1)-\left(\frac{1}{4}-\frac{1}{2}\right)\right]$$

$$= -\left[-\frac{1}{4}\right]+\left[2+\frac{1}{4}\right]$$

$$= \frac{1}{4}+2+\frac{1}{4}$$

$$= 2\frac{1}{2}$$

$$\therefore \int_{-1}^{1} |2x+1|dx = \frac{5}{2}$$

Definite Integrals Ex 20.3 Q5

(i)

$$\int_{-2}^{2}|2x+3|\,dx$$

$$= \int_{-2}^{-\frac{3}{2}} -(2x+3)\,dx + \int_{-\frac{3}{2}}^{2} (2x+3)\,dx$$

$$= -\left[\frac{2x^2}{2}+3x\right]_{-2}^{-\frac{3}{2}} + \left[\frac{2x^2}{2}+3x\right]_{-\frac{3}{2}}^{2}$$

$$= -\left[\left(\frac{2\times 9}{2\times 4}-\frac{9}{2}\right)-\left(\frac{2\times 4}{2}-6\right)\right]+\left[\left(\frac{2\times 4}{2}+6\right)-\left(\frac{2\times 9}{2\times 4}-\frac{9}{2}\right)\right]$$

$$= -\left[\left(\frac{18}{8}-\frac{9}{2}\right)-\left(\frac{8}{2}-6\right)\right]+\left[\left(\frac{8}{2}+6\right)-\left(\frac{18}{8}-\frac{9}{2}\right)\right]$$

$$= -\left[\left(\frac{9}{4}-\frac{9}{2}\right)-(-2)\right]+\left[(10)-\left(\frac{9}{4}-\frac{9}{2}\right)\right]$$

$$= \left[-\frac{9}{4}+2\right]+\left[10+\frac{9}{4}\right]$$

$$= \frac{9}{4}-2+10+\frac{9}{4}$$

$$\Rightarrow 8\frac{9}{2}$$

$$= 12\frac{1}{2}$$

$$\therefore \int_{-2}^{2}|2x+3|\,dx = \frac{25}{2}$$

Definite Integrals Ex 20.3 Q6

(ii)

We have,

$$f(x) = |x^2-3x+2|$$

$$= |(x-1)(x-2)|$$

$$= \begin{cases} x^2-3x+2 & 0 \le x \le 1 \\ -(x^2-3x+2) & 1 \le x \le 2 \end{cases}$$

Hence,

$$\int_{0}^{2}|x^2-3x+2|\,dx$$

$$= \int_{0}^{1}(x^2-3x+2)\,dx + \int_{1}^{2} -(x^2-3x+2)\,dx$$

$$= \left[\frac{x^3}{3}-\frac{3x^2}{2}+2x\right]_0^1 - \left[\frac{x^3}{3}-\frac{3x^2}{2}+2x\right]_1^2$$

$$= \left[\frac{1}{3}-\frac{3}{2}+2-0\right]-\left[\frac{8}{3}-\frac{12}{2}+4-\frac{1}{3}+\frac{3}{2}+2\right]$$

$$= \left[\frac{1}{6}\right]-\left[-\frac{5}{6}\right]$$

$$= \frac{1}{6}+\frac{5}{6}$$

$$= 1$$

$$\therefore \int_{0}^{2}|x^2-3x+2|\,dx = 1$$

Definite Integrals Ex 20.3 Q7

$$\int_0^3 |3x-1|\,dx = \int_0^{\frac{1}{3}} -(3x-1)\,dx + \int_{\frac{1}{3}}^{3} (3x-1)\,dx$$

$$= -\left[\frac{3x^2}{2} - x\right]_0^{\frac{1}{3}} + \left[\frac{3x^2}{2} - x\right]_{\frac{1}{3}}^{3}$$

$$= -\left[\left(\frac{3}{9\times 2} - \frac{1}{3}\right) - (0)\right] + \left[\left(\frac{3\times 9}{2} - 3\right) - \left(\frac{3}{9\times 2} - \frac{1}{3}\right)\right]$$

$$= -\left[\left(\frac{1}{6} - \frac{1}{3}\right)\right] + \left[\left(\frac{27}{2} - 3\right) - \left(\frac{1}{6} - \frac{1}{3}\right)\right]$$

$$= -\left[\left(-\frac{1}{6}\right)\right] + \left[\left(10\frac{1}{2}\right) - \left(\frac{-1}{6}\right)\right]$$

$$= -\left[\left(-\frac{1}{6}\right)\right] + \left[10\frac{1}{2} + \frac{1}{6}\right]$$

$$= \frac{1}{6} + 10\frac{1}{2} + \frac{1}{6}$$

$$= \frac{1}{3} + \frac{21}{2} = \frac{2+63}{6} = \frac{65}{6}$$

$$= \frac{65}{6}$$

$$\therefore \int_0^3 |3x-1|\,dx = \frac{65}{6}$$

Definite Integrals Ex 20.3 Q8

$$\int_{-6}^{6} |x+2|\,dx$$

$$= \int_{-6}^{-2} -(x+2)\,dx + \int_{-2}^{6} (x+2)\,dx$$

$$= -\left[\frac{x^2}{2} + 2x\right]_{-6}^{-2} + \left[\frac{x^2}{2} + 2x\right]_{-2}^{6}$$

$$= -\left[\left(\frac{4}{2} + 2(\cdot 2)\right) - \left(\frac{36}{2} - 12\right)\right] + \left[\left(\frac{36}{2} + 12\right) - \left(\frac{4}{2} - 4\right)\right]$$

$$= -\left[(2-4) - (18-12)\right] + \left[(18+12) - (2-4)\right]$$

$$= -[-8] + [30+2]$$

$$= 8 + 32$$

$$= 40$$

$$\therefore \int_{-6}^{6} |x+2|\,dx = 40$$

Definite Integrals Ex 20.3 Q9

$$\int_{-2}^{2} |x+1|\,dx = \int_{-2}^{-1} -(x+1)\,dx + \int_{-1}^{2} (x+1)\,dx$$

$$= -\left[\frac{x^2}{2} + x\right]_{-2}^{-1} + \left[\frac{x^2}{2} + x\right]_{-1}^{2}$$

$$= -\left[\left(\frac{1}{2} - 1\right) - \left(\frac{4}{2} - 2\right)\right] + \left[\left(\frac{4}{2} + 2\right) - \left(\frac{1}{2} - 1\right)\right]$$

$$= -\left[\left(-\frac{1}{2}\right) - 0\right] + \left[4 + \frac{1}{2}\right]$$

$$= \frac{1}{2} + 4\frac{1}{2}$$

$$= 5$$

$$\therefore \int_{-2}^{2} |x+1|\,dx = 5$$

Definite Integrals Ex 20.3 Q10

$$\int_1^2 |x-3|dx = \int_1^2 -(x-3)\,dx \qquad [x-3<0 \text{ for } 1>x>2]$$

$$= -\left[\frac{x^2}{2}-3x\right]_1^2$$

$$= -\left[\left(\frac{4}{2}-6\right)-\left(\frac{1}{2}-3\right)\right]$$

$$= -\left[(-4)-\left(-2\frac{1}{2}\right)\right]$$

$$= -\left[-4+2\frac{1}{2}\right]$$

$$= -\left[-\frac{3}{2}\right]$$

$$= \frac{3}{2}$$

$$\therefore \int_1^2 |x-3|dx = \frac{3}{2}$$

Definite Integrals Ex 20.3 Q11

$$\int_0^{\frac{\pi}{2}} |\cos 2x|\,dx$$

$$= \int_0^{\frac{\pi}{4}} -\cos 2x\,dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} +\cos 2x\,dx$$

$$= \left[\frac{+\sin 2x}{2}\right]_0^{\frac{\pi}{4}} + \left[\frac{-\sin 2x}{2}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \frac{1}{2}\left[\sin\frac{\pi}{2}-\sin 0\right]+\frac{1}{2}\left[\sin\pi+\sin\frac{\pi}{2}\right]$$

$$= \frac{1}{2}[1]+\frac{1}{2}[1]$$

$$= \frac{1}{2}+\frac{1}{2}$$

$$= 1$$

$$\therefore \int_0^{\frac{\pi}{2}} |\cos 2x|\,dx = 1$$

Definite Integrals Ex 20.3 Q12

$$\int_0^{2\pi} |\sin x|dx = \int_0^{\pi} \sin x\,dx + \int_{\pi}^{2\pi} -\sin x\,dx$$

$$= [-\cos x]_0^{\pi} + [\cos x]_{\pi}^{2\pi}$$

$$= [1+1]+[1+1]$$

$$\int_0^{2\pi} |\sin x|dx = 4$$

Definite Integrals Ex 20.3 Q13

$$\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} |\sin x|\,dx$$

$$= \int_{\frac{-\pi}{4}}^{0} -\sin x\,dx + \int_0^{\frac{\pi}{4}} \sin x\,dx$$

$$= [\cos x]_{\frac{-\pi}{4}}^{0} + [-\cos x]_0^{\frac{\pi}{4}}$$

$$= \left(1-\frac{1}{\sqrt{2}}\right)-\left(\frac{1}{\sqrt{2}}-1\right)$$

$$= (2-\sqrt{2})$$

$$\therefore \int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} |\sin x|\,dx = 2-\sqrt{2}$$

Definite Integrals Ex 20.3 Q14

We have,

$$I = \int_2^8 |x - 5| dx$$

We have,

$$|x - 5| = \begin{cases} x - 5 & \text{if } x \in (5, 8) \\ -(x - 5) & \text{if } x \in (2, 5) \end{cases}$$

Hence,

$$I = \int_2^5 -(x - 5) dx + \int_5^8 (x - 5) dx$$

$$= -\left[\frac{x^2}{2} - 5x\right]_2^5 + \left[\frac{x^2}{2} - 5x\right]_5^8$$

$$= -\left[\left(\frac{25}{2} - 25\right) - \left(\frac{4}{2} - 10\right)\right] + \left[\left(\frac{64}{2} - 40\right) - \left(\frac{25}{2} - 25\right)\right]$$

$$= -\left[-\frac{25}{2} + 8\right] + \left[(-8) + \left(\frac{25}{2}\right)\right]$$

$$= \frac{25}{2} - 8 - 8 + \frac{25}{2}$$

$$= 25 - 16 = 9$$

$$\therefore \int_2^8 |x - 5| dx = 9$$

Definite Integrals Ex 20.3 Q15

We have,

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \{\sin|x| + \cos|x|\} dx$$

Let $f(x) = \sin|x| + \cos|x|$

Then, $f(x) = f(-x)$

$\therefore$ f(x) is an even function.

So, $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \{\sin|x| + \cos|x|\} dx = 2\int_0^{\frac{\pi}{2}} (\sin x + \cos x) dx = 2[-\cos x + \sin x]_0^{\frac{\pi}{2}} = 4$

$$\therefore \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \{\sin|x| + \cos|x|\} dx = 4$$

Definite Integrals Ex 20.3 Q16

$$I = \int_0^4 |x-1| dx$$

It can be seen that, $(x - 1) \le 0$ when $0 \le x \le 1$ and $(x - 1) \ge 0$ when $1 \le x \le 4$

$$I = \int_0^1 |x-1| dx + \int_1^4 |x-1| dx \qquad \left(\int_a^b f(x) = \int_a^c f(x) + \int_c^b f(x)\right)$$

$$= \int_0^1 -(x-1) dx + \int_1^4 (x-1) dx$$

$$= \left[x - \frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2} - x\right]_1^4$$

$$= 1 - \frac{1}{2} + \frac{(4)^2}{2} - 4 - \frac{1}{2} + 1$$

$$= 1 - \frac{1}{2} + 8 - 4 - \frac{1}{2} + 1$$

$$= 5$$

Definite Integrals Ex 20.3 Q17

$$Let\ I=\int_1^4\{|x-1|+|x-2|+|x-4|\}dx$$

$$=\int_1^2\{(x-1)-(x-2)-(x-4)\}dx+\int_2^4\{(x-1)+(x-2)-(x-4)\}dx$$

$$=\int_1^2\{(x-1-x+2-x+4)\}dx+\int_2^4\{(x-1+x-2-x+4)\}dx$$

$$=\int_1^2(5-x)dx+\int_2^4(x+1)dx$$

$$=\left[5x-\frac{x^2}{2}\right]_1^2+\left[\frac{x^2}{2}+x\right]_2^4$$

$$=\left[10-2-5+\frac{1}{2}\right]+[8+4-2-2]$$

$$=\frac{7}{2}+8$$

$$I=\frac{23}{2}$$

Definite Integrals Ex 20.3 Q18

We have,

$$I=\int_{-5}^{0}\left(|x|+|x+2|+|x+5|\right)dx=\int_{-5}^{0}|x|dx+\int_{-5}^{0}|x+2|dx+\int_{-5}^{0}|x+5|dx$$

$$\Rightarrow I=\int_{-5}^{0}-x\,dx+\int_{-5}^{-2}-(x+2)dx+\int_{-2}^{0}(x+2)dx+\int_{-5}^{0}(x+5)dx$$

$$=\left[\frac{-x^2}{2}\right]_{-5}^{0}+\left[\frac{-x^2}{2}-2x\right]_{-5}^{-2}+\left[\frac{x^2}{2}+2x\right]_{-2}^{0}+\left[\frac{x^2}{2}+5x\right]_{-5}^{0}$$

$$=\left[+\frac{25}{2}\right]-\left[\frac{4}{2}-4-\frac{25}{2}+10\right]+\left[0+0-\frac{4}{2}+4\right]+\left[0+0-\frac{25}{2}+25\right]$$

$$=\frac{25}{2}-\left[8-\frac{25}{2}\right]+[2]+\left[25-\frac{25}{2}\right]$$

$$=\frac{25}{2}-8+\frac{25}{2}+2+25-\frac{25}{2}$$

$$=19+\frac{25}{2}=31\frac{1}{2}$$

$$I=\frac{63}{2}$$

Definite Integrals Ex 20.3 Q19

$$|x|=\begin{cases}x, & x\geq 0\\ -x, & x<0\end{cases}$$

$$|x-2|=\begin{cases}x-2, & x\geq 2\\ 2-x, & x<2\end{cases}$$

$$|x-4|=\begin{cases}x-4, & x\geq 4\\ 4-x, & x<4\end{cases}$$

Splitting the limits of the integral, we get

$$\int_0^4\left(|x|+|x-2|+|x-4|\right)dx$$

$$=\int_0^2\left(|x|+|x-2|+|x-4|\right)dx+\int_2^4\left(|x|+|x-2|+|x-4|\right)dx$$

$$=\int_0^2(x+2-x+4-x)dx+\int_2^4(x+x-2+4-x)dx$$

$$=\int_0^2(6-x)dx+\int_2^4(2+x)dx$$

$$=\left[6x-\frac{x^2}{2}\right]_0^2+\left[2x+\frac{x^2}{2}\right]_2^4$$

$$=[12-2]+[16-6]$$

$$=10+10$$

$$=20$$

Definite Integrals Ex 20.3 Q20

$$\int_{-1}^{2}|x+1|\,dx+\int_{-1}^{2}|x|\,dx+\int_{-1}^{2}|x-1|\,dx$$
$$\int_{-1}^{2}(x+1)dx-\int_{-1}^{0}xdx+\int_{0}^{2}xdx-\int_{-1}^{1}(x-1)dx+\int_{1}^{2}(x-1)dx$$
$$\left\{\frac{x^2}{2}+x\right\}_{-1}^{2}-\left\{\frac{x^2}{2}\right\}_{-1}^{0}+\left\{\frac{x^2}{2}\right\}_{0}^{2}-\left\{\frac{x^2}{2}-x\right\}_{-1}^{1}+\left\{\frac{x^2}{2}-x\right\}_{1}^{2}$$
$$\left\{(4)-(-\frac{1}{2})\right\}-\left\{-\frac{1}{2}\right\}+\{2\}-\left\{(-\frac{1}{2})-(\frac{3}{2})\right\}+\left\{(0)-(-\frac{1}{2})\right\}$$
$$\left\{4+\frac{1}{2}\right\}+\left\{\frac{1}{2}\right\}+\{2\}+\{2\}+\left\{\frac{1}{2}\right\}$$
$$\frac{19}{2}$$

Definite Integrals Ex 20.3 Q21

$$\int_{-2}^{0}xe^{-x}dx+\int_{0}^{2}xe^{x}dx$$

For

$$\int_{-2}^{0}xe^{-x}dx$$

Using Integration By parts

$$\int f'g=fg-\int fg'$$
$$f'=e^{-x},g=x$$
$$f=-e^{-x},g'=1$$
$$\int_{-2}^{0}xe^{-x}dx=\left\{-xe^{-x}\right\}_{-2}^{0}+\int_{-2}^{0}e^{-x}dx$$
$$\int_{-2}^{0}xe^{-x}dx=\left\{-xe^{-x}-e^{-x}\right\}_{-2}^{0}$$
$$\int_{-2}^{0}xe^{-x}dx=\left\{(-1)-(2e^2-e^2)\right\}$$
$$\int_{-2}^{0}xe^{-x}dx=\left\{-1-e^2\right\}$$

For

$$\int_{0}^{2}xe^{x}dx$$

Using Integration By parts

$$\int f'g=fg-\int fg'$$
$$f'=e^{x},g=x$$
$$f=e^{x},g'=1$$
$$\int_0^2 xe^x dx=\left\{xe^x\right\}_0^2-\int_0^2 e^x dx$$
$$\int_0^2 xe^x dx=\left\{xe^x-e^x\right\}_0^2$$
$$\int_0^2 xe^x dx=2e^2-e^2+1$$
$$\int_0^2 xe^x dx=e^2+1$$

Hence answer is,

$$\int_{-2}^{2}xe^{|x|}dx=-1-e^2+e^2+1=0$$

Definite Integrals Ex 20.3 Q22

$$-\int_{-\frac{\Pi}{4}}^{0} \sin^2 x dx + \int_{0}^{\frac{\Pi}{2}} \sin^2 x dx$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$-\int_{-\frac{\Pi}{4}}^{0} \frac{1-\cos 2x}{2} dx + \int_{0}^{\frac{\Pi}{2}} \frac{1-\cos 2x}{2} dx$$

$$-\frac{1}{2}\left\{x - \frac{\sin 2x}{2}\right\}_{-\frac{\Pi}{4}}^{0} + \frac{1}{2}\left\{x - \frac{\sin 2x}{2}\right\}_{0}^{\frac{\Pi}{2}}$$

$$-\frac{1}{2}\left\{-(-\frac{\Pi}{4} + \frac{1}{2})\right\} + \frac{1}{2}\left\{\frac{\Pi}{2}\right\}$$

$$\left\{-\frac{\Pi}{8} + \frac{1}{4}\right\} + \left\{\frac{\Pi}{4}\right\}$$

$$\frac{\Pi}{8} + \frac{1}{4}$$

$$\frac{\Pi + 2}{8}$$

Definite Integrals Ex 20.3 Q23

$$\int_{0}^{\frac{\Pi}{2}} \cos^2 x dx - \int_{\frac{\Pi}{2}}^{\Pi} \cos^2 x dx$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\int_{0}^{\frac{\Pi}{2}} \frac{1+\cos 2x}{2} dx - \int_{\frac{\Pi}{2}}^{\Pi} \frac{1+\cos 2x}{2} dx$$

$$\frac{1}{2}\left\{x + \frac{\sin 2x}{2}\right\}_{0}^{\frac{\Pi}{2}} - \frac{1}{2}\left\{x + \frac{\sin 2x}{2}\right\}_{\frac{\Pi}{2}}^{\Pi}$$

$$\frac{\Pi}{4} - \frac{\Pi}{4}$$

$$0$$

Definite Integrals Ex 20.3 Q24

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} (2\sin|x| + \cos|x|) dx$$

$$= \int_{-\frac{\pi}{4}}^{0} (-2\sin x + \cos x) dx + \int_{0}^{\frac{\pi}{2}} (2\sin x + \cos x) dx$$

$$= [2\cos x + \sin x]_{-\frac{\pi}{4}}^{0} + [-2\cos x + \sin x]_{0}^{\frac{\pi}{2}}$$

$$= 2 + 0 - 0 + 1 + 0 + 1 + 2 - 0$$

$$= 6$$

Definite Integrals Ex 20.3 Q25

$$\int_{\frac{\pi}{2}}^{\pi} \sin^{-1}(\sin x)dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} xdx + \int_{\frac{\pi}{2}}^{\pi} (\Pi - x)dx$$

$$\Rightarrow \left\{\frac{x^2}{2}\right\}_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \left\{\Pi x - \frac{x^2}{2}\right\}_{\frac{\pi}{2}}^{\pi}$$

$$\Rightarrow \left\{(\Pi^2 - \frac{\Pi^2}{2}) - \left(\frac{\Pi^2}{2} - \frac{\Pi^2}{8}\right)\right\}$$

$$\Rightarrow \left\{\frac{\Pi^2}{2} - \frac{3\Pi^2}{8}\right\}$$

$$\Rightarrow \frac{\Pi^2}{8}$$

Definite Integrals Ex 20.3 Q27

[x]=0 for 0
and [x]=1 for 1
Hence

$$\int_0^1 0 + \int_1^2 2xdx$$

$$\left\{x^2\right\}_1^2$$

$$3$$

Definite Integrals Ex 20.3 Q18

$$\int_0^{2\pi} \cos^{-1}(\cos x)dx$$

$$= -\int_0^{\pi} \cos^{-1}(\cos x)dx + \int_{\pi}^{2\pi} \cos^{-1}(\cos x)dx$$

$$= -\int_0^{\pi} xdx + \int_{\pi}^{2\pi} xdx$$

$$= -\left[\frac{x^2}{2}\right]_0^{\pi} + \left[\frac{x^2}{2}\right]_{\pi}^{2\pi}$$

$$= -\frac{\pi^2}{2} + \frac{4\pi^2}{2} - \frac{\pi^2}{2}$$

$$= \pi^2$$

Definite Integrals Ex 20.3 Q33

$$\text{Let } I = \int_a^b \frac{f(x)}{f(x) + f(a+b-x)}dx \qquad --(i)$$

We know that $\int_a^b f(x) = \int_a^b f(a+b-x)dx$

Then

$$I = \int_a^b \frac{f(a+b-x)}{f(a+b-x) + f\{a+b-(a+b-x)\}}dx$$

$$I = \int_a^b \frac{f(a+b-x)}{f(a+b-x)f(x)}dx \qquad --(ii)$$

Adding (i) & (ii)

$$2I = \int_a^b \frac{f(x) + f(a+b-x)}{f(x) + f(a+b-x)}dx$$

$$2I = \int_a^b dx$$

$$I = [x]_a^b$$

$$I = \frac{1}{2}[b-a]$$

$$I = \frac{b-a}{2}$$

Ex 20.4A

Definite Integrals Ex 20.4A Q1

We know

$$\int_0^{2\Pi} f(x)dx = \int_0^{2\Pi} f(2\Pi - x)dx$$

Hence

$$\int_0^{2\Pi} \frac{e^{\sin x}}{e^{\sin x}+e^{-\sin x}}dx = \int_0^{2\Pi} \frac{e^{\sin(2\Pi-x)}}{e^{\sin(2\Pi-x)}+e^{-\sin(2\Pi-x)}}dx$$

We know

$$\sin(2\Pi - x) = -\sin x$$

$$\int_0^{2\Pi} \frac{e^{\sin x}}{e^{\sin x}+e^{-\sin x}}dx = \int_0^{2\Pi} \frac{e^{-\sin x}}{e^{-\sin x}+e^{\sin x}}dx$$

If

$$I = \int_0^{2\Pi} \frac{e^{\sin x}}{e^{\sin x}+e^{-\sin x}}dx$$

Then also

$$I = \int_0^{2\Pi} \frac{e^{-\sin x}}{e^{-\sin x}+e^{\sin x}}dx$$

Hence

$$2I = \int_0^{2\Pi} \frac{e^{-\sin x}}{e^{-\sin x}+e^{\sin x}}dx + \int_0^{2\Pi} \frac{e^{\sin x}}{e^{-\sin x}+e^{\sin x}}dx$$

$$2I = \int_0^{2\Pi} \frac{e^{-\sin x}}{e^{-\sin x}+e^{\sin x}} + \frac{e^{\sin x}}{e^{-\sin x}+e^{\sin x}}dx$$

$$2I = \int_0^{2\Pi} dx$$

$$2I = 2\Pi$$

$$I = \Pi$$

Definite Integrals Ex 20.4A Q2

We know

$$\int_0^{2\Pi} f(x)dx = \int_0^{2\Pi} f(2\Pi - x)dx$$

Hence

$$\int_0^{2\Pi} \log(\sec x + \tan x)dx = \int_0^{2\Pi} \log(\sec(2\Pi - x) + \tan(2\Pi - x))dx$$

$$\int_0^{2\Pi} \log(\sec x + \tan x)dx = \int_0^{2\Pi} \log(\sec x - \tan x)dx$$

If

$$I = \int_0^{2\Pi} \log(\sec x + \tan x)dx$$

Then

$$I = \int_0^{2\Pi} \log(\sec x - \tan x)dx$$

$$2I = \int_0^{2\Pi} \log(\sec x + \tan x)dx + \int_0^{2\Pi} \log(\sec x - \tan x)dx$$

$$2I = \int_0^{2\Pi} \log(\sec x + \tan x) + \log(\sec x - \tan x)dx$$

$$2I = \int_0^{2\Pi} \log(\sec^2 x - \tan^2 x)dx$$

$$2I = \int_0^{2\Pi} \log(1)dx$$

$$2I = 0$$

$$I = 0$$

Definite Integrals Ex 20.4A Q3

We know

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

Hence

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan(\frac{\Pi}{2}-x)}}{\sqrt{\tan(\frac{\Pi}{2}-x)}+\sqrt{\cot(\frac{\Pi}{2}-x)}}dx$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cot x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$$

If

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$$

Then

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cot x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$$

So

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cot x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}} + \frac{\sqrt{\cot x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1dx$$

$$2I = \frac{\Pi}{6}$$

$$I = \frac{\Pi}{12}$$

Definite Integrals Ex 20.4A Q4

We know

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

Hence

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin(\frac{\Pi}{2}-x)}}{\sqrt{\sin(\frac{\Pi}{2}-x)}+\sqrt{\cos(\frac{\Pi}{2}-x)}}dx$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

If

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

Then

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

Hence

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}+\frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1dx$$

$$2I = \frac{\Pi}{6}$$

$$I = \frac{\Pi}{12}$$

Definite Integrals Ex 20.4A Q5

We know

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

Hence

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x}dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2(-x)}{1+e^{-x}}dx$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x}dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^{-x}}dx$$

If

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x}dx$$

Then

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^{-x}}dx$$

So

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x} + \frac{\tan^2 x}{1+e^{-x}}dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x} + \frac{\tan^2 x}{1+e^{-x}}dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x} + \frac{e^x \tan^2 x}{1+e^x}dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x + e^x \tan^2 x}{1+e^x}dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{(1+e^x)\tan^2 x}{1+e^x}dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x + e^x \tan^2 x}{1+e^x} dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{(1+e^x)\tan^2 x}{1+e^x} dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2 x dx$$

$$I = \frac{1}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2 x dx$$

We know

If f(x) is even

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$$

If f(x) is odd

$$\int_{-a}^{a} f(x)dx = 0$$

Here

$$f(x) = \tan^2 x$$

f(x) is even, hence

$$I = \int_{0}^{\frac{\pi}{4}} \tan^2 x dx$$

$$I = \int_{0}^{\frac{\pi}{4}} \sec^2 x - 1 dx$$

$$I = \{\tan x - x\}_0^{\frac{\pi}{4}}$$

$$I = 1 - \frac{\pi}{4}$$

Note: Answer given in the book is incorrect.

Definite Integrals Ex 20.4A Q6

We know

$$\int_{a}^{b} f(x)dx = \int_{a}^{b} f(a+b-x)dx$$

Hence

$$\int_{-a}^{a} \frac{1}{1+a^x} dx = \int_{-a}^{a} \frac{1}{1+a^{-x}} dx$$

If

$$I = \int_{-a}^{a} \frac{1}{1+a^x} dx$$

Then

$$I = \int_{-a}^{a} \frac{1}{1+a^{-x}} dx$$

So

$$2I = \int_{-a}^{a} \frac{1}{1+a^x} + \frac{1}{1+a^{-x}} dx$$

$$2I = \int_{-a}^{a} \frac{1}{1+a^x} + \frac{a^x}{1+a^x} dx$$

$$2I = \int_{-a}^{a} 1 dx$$

$$2I = 2a$$

$$I = a$$

Definite Integrals Ex 20.4A Q7

We know

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

Hence

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{1+e^{\tan x}}dx = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{1+e^{-\tan x}}dx$$

If

$$I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{1+e^{\tan x}}dx$$

Then

$$I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{1+e^{-\tan x}}dx$$

So

$$2I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{1+e^{\tan x}} + \frac{1}{1+e^{-\tan x}}dx$$

$$2I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{1+e^{\tan x}} + \frac{e^{\tan x}}{1+e^{\tan x}}dx$$

$$2I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 1dx$$

$$2I = \frac{2\Pi}{3}$$

$$I = \frac{\Pi}{3}$$

Definite Integrals Ex 20.4A Q8

We know

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

Hence

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^{x}}dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2(-x)}{1+e^{-x}}dx$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^{x}}dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^{-x}}dx$$

If

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^{x}}dx$$

Then

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^{-x}}dx$$

So

$$2I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\cos^2 x}{1+e^x}+\frac{\cos^2 x}{1+e^{-x}}dx$$

$$2I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\cos^2 x}{1+e^x}+\frac{e^x\cos^2 x}{1+e^x}dx$$

$$2I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{(1+e^x)\cos^2 x}{1+e^x}dx$$

$$2I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos^2 xdx$$

$$2I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{1+\cos 2x}{2}dx$$

$$I=\frac{1}{4}\left\{x+\frac{\sin 2x}{2}\right\}_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$I=\frac{1}{4}\left\{\left(\frac{\Pi}{2}\right)-\left(-\frac{\Pi}{2}\right)\right\}$$

$$I=\frac{\Pi}{4}$$

Note: Answer given in the book is incorrect.

Definite Integrals Ex 20.4A Q9

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{x^{11}-3x^9+5x^7-x^5+1}{\cos^2 x}dx$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{x^{11}-3x^9+5x^7-x^5}{\cos^2 x}+\frac{1}{\cos^2 x}dx$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{x^{11}-3x^9+5x^7-x^5}{\cos^2 x}dx+\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\sec^2 xdx$$

If f(x) is even

$$\int_{-a}^{a}f(x)dx=2\int_{0}^{a}f(x)dx$$

If f(x) is odd

$$\int_{-a}^{a}f(x)dx=0$$

Here

$\frac{x^{11}-3x^9+5x^7-x^5}{\cos^2 x}$ is odd and

$\sec^2 x$ is even. Hence

$$0+2\int_{0}^{\frac{\pi}{4}}\sec^2 xdx$$

$$2\{\tan x\}_0^{\frac{\pi}{4}}$$

$$2$$

Definite Integrals Ex 20.4A Q10

$$I=\int_a^b \frac{x^{1/n}}{x^{1/n}+(a+b-x)^{1/n}}dx$$

$$I=\int_a^b \frac{(a+b-x)^{1/n}}{(a+b-x)^{1/n}+x^{1/n}}dx$$

$$2I=\int_a^b \frac{x^{1/n}}{x^{1/n}+(a+b-x)^{1/n}}dx+\int_a^b \frac{(a+b-x)^{1/n}}{(a+b-x)^{1/n}+x^{1/n}}dx$$

$$2I=\int_a^b \frac{x^{1/n}+(a+b-x)^{1/n}}{x^{1/n}+(a+b-x)^{1/n}}dx$$

$$I=\frac{1}{2}\int_a^b dx$$

$$I=\frac{b-a}{2}$$

Definite Integrals Ex 20.4A Q11

We have,

$$I=\int_0^{\frac{\pi}{2}}\left(2\log\cos x-\log\sin 2x\right)dx$$

$$=\int_0^{\frac{\pi}{2}}\left(\log\cos^2 x-\log\sin 2x\right)dx$$

$$=\int_0^{\frac{\pi}{2}}\log\frac{\cos^2 x}{\sin x}dx$$

$$=\int_0^{\frac{\pi}{2}}\log\frac{\cos^2 x}{2\sin x.\cos x}dx$$

$$=\int_0^{\frac{\pi}{2}}\log\frac{\cos x}{2\sin x}dx$$

$$=\int_0^{\frac{\pi}{2}}\left(\log\cos x-\log\sin x-\log 2\right)dx$$

$$=\int_0^{\frac{\pi}{2}}\log\cos x\,dx-\int_0^{\frac{\pi}{2}}\log\sin x\,dx-\int_0^{\frac{\pi}{2}}\log 2$$

We know that $\int_0^{\frac{\pi}{2}}\log\cos x\,dx=\int_0^{\frac{\pi}{2}}\log\sin x\,dx$ $\qquad -(i)$

Hence from equation (i)

$$I=-\int_0^{\frac{\pi}{2}}\log 2=-\frac{\pi}{2}\log 2$$

Definite Integrals Ex 20.4A Q12

Let $I = \int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx \qquad ...(1)$

It is known that, $\left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$

$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}dx \qquad ...(2)$

Adding (1) and (2), we obtain

$$2I = \int_0^a \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}dx$$
$$\Rightarrow 2I = \int_0^a 1\,dx$$
$$\Rightarrow 2I = [x]_0^a$$
$$\Rightarrow 2I = a$$
$$\Rightarrow I = \frac{a}{2}$$

Definite Integrals Ex 20.4A Q13

Let $I = \int_0^5 \frac{\sqrt[4]{x+4}}{\sqrt[4]{x+4}+\sqrt[4]{9-x}}dx \qquad --(i)$

We know that $\int_0^a f(x) = \int_0^a f(a-x)$

So,

$$I = \int_0^5 \frac{\sqrt[4]{(5-x)+4}}{\sqrt[4]{(5-x)+4}+\sqrt[4]{9-(5-x)}}dx$$
$$I = \int_0^5 \frac{\sqrt[4]{9-x}}{\sqrt[4]{9-x}+\sqrt[4]{4+x}}dx \qquad --(ii)$$

Adding (i) & (ii)

$$2I = \int_0^5 \frac{\sqrt[4]{x+4}}{\sqrt[4]{x+4}+\sqrt[4]{9-x}}dx + \int_0^5 \frac{\sqrt[4]{9-x}}{\sqrt[4]{9-x}+\sqrt[4]{4+x}}dx$$
$$2I = \int_0^5 \frac{\sqrt[4]{x+4}+\sqrt[4]{9-x}}{\sqrt[4]{x+4}+\sqrt[4]{9-x}}dx$$
$$2I = \int_0^5 dx$$
$$2I = [x]_0^5$$
$$I = \frac{1}{2}[5-0] = \frac{5}{2}$$

$$\therefore \int_0^5 \frac{\sqrt[4]{x+4}}{\sqrt[4]{x+4}+\sqrt[4]{9-x}}dx = \frac{5}{2}$$

Definite Integrals Ex 20.4A Q14

Let $I = \int_0^7 \frac{\sqrt[3]{x}}{\sqrt[3]{x} + \sqrt[3]{7-x}} dx$ --(i)

We know that $\int_0^a f(x) = \int_0^a f(a-x)$

Hence,

$I = \int_0^7 \frac{\sqrt[3]{7-x}}{\sqrt[3]{7-x} + \sqrt[3]{x}} dx$ --(ii)

Adding (i) & (ii)

$$2I = \int_0^7 \frac{\sqrt[3]{x}}{\sqrt[3]{x} + \sqrt[3]{7-x}} dx + \int_0^7 \frac{\sqrt[3]{7-x}}{\sqrt[3]{7-x} + \sqrt[3]{x}} dx$$

$$2I = \int_0^7 \frac{\sqrt[3]{x} + \sqrt[3]{7-x}}{\sqrt[3]{x} + \sqrt[3]{7-x}} dx$$

$$2I = \int_0^7 dx$$

$$2I = [x]_0^7$$

$$I = \frac{7}{2}$$

Definite Integrals Ex 20.4A Q15

Let $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1 + \sqrt{\tan x}} dx$

$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ --(i)

We know that $\int_a^b f(x) = \int_a^b f(a+b-x) dx$

Hence,

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos\left(\frac{\pi}{2} - x\right)}}{\sqrt{\cos\left(\frac{\pi}{2} - x\right)} + \sqrt{\sin\left(\frac{\pi}{2} - x\right)}} dx$$

$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ --(ii)

Adding (i) & (ii)

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} dx$$

$$2I = [x]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$I = \frac{\pi}{12}$$

Definite Integrals Ex 20.4A Q16

$$I = \int_a^b x f(x)dx$$

$$I = \int_a^b (a+b-x) f(a+b-x)\,dx$$

$$I = \int_a^b (a+b-x) f(x)\,dx ....[\because f(a+b-x) = f(x)]$$

$$I = \int_a^b (a+b) f(x)\,dx - \int_a^b f(x)dx$$

$$I = (a+b)\int_a^b f(x)\,dx - I$$

$$2I = (a+b)\int_a^b f(x)dx$$

$$I = \frac{(a+b)}{2}\int_a^b f(x)dx$$

$$\therefore \int_a^b x f(x)\,dx = \frac{(a+b)}{2}\int_a^b f(x)\,dx$$

Ex 20.4B

Definite Integrals Ex 20.4B Q1

We have,

$$\frac{1}{1+\tan x} = \frac{1}{1+\frac{\sin x}{\cos x}} = \frac{\cos x}{\cos x + \sin x}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{dx}{1+\tan x} = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x} dx$$

Let

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x} dx \qquad --(\text{I})$$

So,

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos\left(\frac{\pi}{2} - x\right)}{\cos\left(\frac{\pi}{2} - x\right) + \sin\left(\frac{\pi}{2} - x\right)} dx \qquad \left[\because \int_0^a f(x)dx = \int_0^a f(a-x)\,dx\right]$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x}{\cos x + \sin x} dx \qquad --(\text{II})$$

Hence, adding (I) & (II)

$$2I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x} dx + \int_0^{\frac{\pi}{2}} \frac{\sin x}{\cos x + \sin x} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos x + \sin x}{\cos x + \sin x} dx$$

$$= \int_0^{\frac{\pi}{2}} dx$$

$$2I = [x]_0^{\frac{\pi}{2}}$$

$$2I = \left[\frac{\pi}{2} - 0\right] \quad \Rightarrow \quad I = \frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q2

We have,

$$\frac{1}{1+\cot x} = \frac{1}{1+\frac{\cos x}{\sin x}} = \frac{\sin x}{\sin x + \cos x}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{1}{1+\cot x} dx = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx$$

Let

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx \qquad --(\mathrm{I})$$

So,

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx \qquad \left[\left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx\right]\right]$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + \cos x} dx \qquad --(\mathrm{II})$$

Adding (I) & (II)

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx + \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + \cos x} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$2I = \int_0^{\frac{\pi}{2}} dx$$

$$= [x]_0^{\frac{\pi}{2}}$$

$$2I = \left[\frac{\pi}{2} - 0\right]$$

$$I = \frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q3

We have,

$$\frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}}=\frac{\sqrt{\frac{\cos x}{\sin x}}}{\sqrt{\frac{\cos x}{\sin x}}+\sqrt{\frac{\sin x}{\cos x}}}=\frac{\sqrt{\frac{\cos x}{\sin x}}}{\frac{\cos x+\sin x}{\sqrt{\sin x}.\sqrt{\cos x}}}=\sqrt{\frac{\cos x}{\sin x}}\times\frac{\sqrt{\sin x}.\sqrt{\cos x}}{\cos x+\sin x}$$

$$=\frac{\cos x}{\cos x+\sin x}$$

$$\therefore \int_0^{\frac{\pi}{2}}\frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}}dx=\int_0^{\frac{\pi}{2}}\frac{\cos x}{\cos x+\sin x}dx$$

Let

$$I=\int_0^{\frac{\pi}{2}}\frac{\cos x}{\cos x+\sin x}dx \qquad --(\mathrm{I})$$

So,

B $$I=\int_0^{\frac{\pi}{2}}\frac{\cos\left(\frac{\pi}{2}-x\right)}{\cos\left(\frac{\pi}{2}-x\right)+\sin\left(\frac{\pi}{2}-x\right)}dx \qquad \left[\because \int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$$

$$=\int_0^{\frac{\pi}{2}}\frac{\sin x}{\cos x+\sin x}dx \qquad --(\mathrm{II})$$

Adding (I) & (II)

$$2I=\int_0^{\frac{\pi}{2}}\frac{\cos x}{\cos x+\sin x}dx+\int_0^{\frac{\pi}{2}}\frac{\sin x}{\cos x+\sin x}dx$$

$$2I=\int_0^{\frac{\pi}{2}}\frac{\cos x+\sin x}{\cos x+\sin x}dx$$

$$2I=\int_0^{\frac{\pi}{2}}dx$$

$$2I=[x]_0^{\frac{\pi}{2}}$$

$$2I=\left[\frac{\pi}{2}-0\right]$$

$$I=\frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q4

$$\text{Let } I=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx \qquad \ldots(1)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)+\cos^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)}dx \qquad \left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\cos^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx$$

$$\Rightarrow 2I=\int_0^{\frac{\pi}{2}}1\cdot dx$$

$$\Rightarrow 2I=[x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I=\frac{\pi}{2}$$

$$\Rightarrow I=\frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q5

$$\int_0^{\frac{\pi}{2}} \frac{\sin^n x}{\sin^n x + \cos^n x} dx$$

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin^n x}{\sin^n x + \cos^n x} dx \qquad --(i)$$

So,

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin^n\left(\frac{\pi}{2} - x\right)}{\sin^n\left(\frac{\pi}{2} - x\right) + \cos^n\left(\frac{\pi}{2} - x\right)} dx \qquad \left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos^n x}{\sin^n x + \cos^n x} \qquad --(II)$$

Adding (I) & (II)

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^n x}{\sin^n x + \cos^n x} dx + \int_0^{\frac{\pi}{2}} \frac{\cos^n x}{\sin^n x + \cos^n x}$$

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^n + \cos^n x}{\sin^n + \cos^n x} dx$$

$$2I = \int_0^{\frac{\pi}{2}} dx$$

$$2I = [x]_0^{\frac{\pi}{2}}$$

$$2I = \left[\frac{\pi}{2} - 0\right]$$

$$I = \frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q6

We have,

$$\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\tan x}} dx = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

Let

$$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \qquad --(i)$$

So

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos\left(\frac{\pi}{2} - x\right)}}{\sqrt{\cos\left(\frac{\pi}{2} - x\right)} + \sqrt{\sin\left(\frac{\pi}{2} - x\right)}} dx \qquad \left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} \qquad --(ii)$$

Adding (i) & (ii)

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx + \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$2I = \int_0^{\frac{\pi}{2}} dx$$

$$2I = [x]_0^{\frac{\pi}{2}}$$

$$I = \frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q7

$$Let\ I = \int_0^a \frac{dx}{x+\sqrt{a^2-x^2}}$$

Let $x = a\sin\theta$

$dx = a\cos\theta d\theta$

Now, $x = 0 \Rightarrow \theta = 0$

$x = a \Rightarrow \theta = \frac{\pi}{2}$

$$I = \int_0^{\frac{\pi}{2}} \frac{a\cos\theta d\theta}{a\sin\theta + a\cos\theta}$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sin\theta+\cos\theta} \quad --(i)$$

So,

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos\left(\frac{\pi}{2}-\theta\right)}{\sin\left(\frac{\pi}{2}-\theta\right)+\cos\left(\frac{\pi}{2}-\theta\right)} d\theta \qquad \left[\because \int_0^a f(x)dx = \int_0^a f(a-x)\,dx\right]$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin\theta}{\cos\theta+\sin\theta} \quad --(ii)$$

Adding (i) & (ii) we get

$$2I = \int_0^{\frac{\pi}{2}} \frac{\cos\theta+\sin\theta}{\cos\theta+\sin\theta} d\theta$$

$$2I = \int_0^{\frac{\pi}{2}} d\theta$$

$$2I = \frac{1}{2}[\theta]_0^{\frac{\pi}{2}}$$

$$I = \frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q8

Put $x = \tan\theta$

$\Rightarrow$ $dx = \sec^2\theta d\theta$

If $x = 0,\ \theta = 0$

If $x = \infty,\ \theta = \frac{\pi}{2}$

$\therefore$ $I = \int_0^{\infty} \frac{\log x}{1+x^2} dx$

$$= \int_0^{\frac{\pi}{2}} \frac{\log(\tan\theta)\sec^2\theta d\theta}{1+\tan^2\theta}$$

$$\Rightarrow \quad I = \int_0^{\frac{\pi}{2}} \log(\tan\theta) d\theta \quad ---(i)$$

$$\Rightarrow \quad I = \int_0^{\frac{\pi}{2}} \log\tan\left(\frac{\pi}{2}-\theta\right) d\theta$$

$$\Rightarrow \quad I = \int_0^{\frac{\pi}{2}} \log\cot(\theta) d\theta \quad ---(ii)$$

Adding (i) and (ii), we get

$$2I = \int_0^{\frac{\pi}{2}} (\log\tan\theta + \log\cot\theta) d\theta$$

$$\Rightarrow \quad 2I = \int_0^{\frac{\pi}{2}} \log 1 \times dx = \int_0^{\frac{\pi}{2}} 0 \times dx = 0$$

$$\Rightarrow \quad I = 0$$

Definite Integrals Ex 20.4B Q9

Let $x = \tan\theta$

$\Rightarrow \quad dx = \sec^2\theta d\theta$

If $\quad x = 0, \ \theta = 0$

If $\quad x = 1, \ \theta = \frac{\pi}{4}$

$\therefore \quad \int_0^1 \frac{\log(1+x)}{1+x^2}dx$

$\Rightarrow \quad I = \int_0^{\frac{\pi}{4}} \log(1+\tan\theta)d\theta$

$\Rightarrow \quad I = \int_0^{\frac{\pi}{4}} \log\left\{1+\tan\left(\frac{\pi}{4}-\theta\right)\right\}d\theta$

$\Rightarrow \quad I = \int_0^{\frac{\pi}{4}} \log\left\{1+\frac{1-\tan\theta}{1+\tan\theta}\right\}d\theta$

$\Rightarrow \quad I = \int_0^{\frac{\pi}{4}} \log\left(\frac{2}{1+\tan\theta}\right)d\theta$

$\Rightarrow \quad I = \int_0^{\frac{\pi}{4}} (\log 2 - \log(1+\tan\theta))d\theta$

$\Rightarrow \quad 2I = \int_0^{\frac{\pi}{4}} \log 2 \times d\theta = \frac{\pi}{4}\log 2$

$\Rightarrow \quad I = \frac{\pi}{8}\log 2$

Definite Integrals Ex 20.4B Q10

$$I = \int_0^{\infty} \frac{x}{(1+x)(1+x^2)}dx$$

Let,

$$\frac{x}{(1+x)(1+x^2)} = \frac{A}{1+x} + \frac{Bx+C}{1+x^2}$$

$$\Rightarrow x = A(1+x^2) + (Bx+C)(1+x)$$

Equating coeffcients, we get

$A + B = 0 \Rightarrow A = -B$

$B + C = 1 \Rightarrow -2A = 1$

$A + C = 0 \Rightarrow A = -C$

$\therefore \ A = -\frac{1}{2}, \ B = \frac{1}{2}, \ C = \frac{1}{2}$

So,

$$I = \int_0^{\infty} \left(\frac{-\frac{1}{2}}{1+x} + \frac{1}{2}\frac{x+1}{x^2+1}\right)dx$$

$$= \int_0^{\infty} -\frac{1}{2}\frac{dx}{1+x} + \frac{1}{2}\int_0^{\infty}\frac{x}{x^2+1}dx + \frac{1}{2}\int_0^{\infty}\frac{dx}{1+x^2}$$

$$= \left[-\frac{1}{2}\log|1+x| + \frac{1}{4}\log|x^2+1| + \frac{1}{2}\tan^{-1}x\right]_0^{\infty}$$

$$= 0 + 0 + \frac{\pi}{4} + 0 - 0 - 0$$

$$= \frac{\pi}{4}$$

$$\because \int_0^{\infty} \frac{x}{(1+x)(1+x^2)}dx = \frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q11

We have,

$$I = \int_0^{\pi} \frac{x \tan x}{\sec x \operatorname{cosec} x} dx$$

$$I = \int_0^{\pi} \frac{x\left(\frac{\sin x}{\cos x}\right)}{\left(\frac{1}{\cos x}\right)\left(\frac{1}{\sin x}\right)} dx$$

$$I = \int_0^{\pi} x \sin^2 x \, dx \qquad \ldots(i)$$

$$I = \int_0^{\pi} (\pi - x) \sin^2 (\pi - x) dx \qquad \left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx\right]$$

$$I = \int_0^{\pi} (\pi - x) \sin^2 x \, dx \qquad --(ii)$$

Add (i) and (ii), we get

$$2I = \int_0^{\pi} (\pi) \sin^2 x \, dx = \pi \int_0^{\pi} \frac{1 - \cos 2x}{2} dx = \frac{\pi}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi} = \frac{\pi}{2}\left[\pi - 0 - 0 + 0\right] = \frac{\pi^2}{2}$$

$$\therefore \int_0^{\pi} \frac{x \tan x}{\sec x \operatorname{cosec} x} dx = \frac{\pi^2}{4}$$

Definite Integrals Ex 20.4B Q12

Let $I = \int_0^{\pi} x \sin x . \cos^4 x \, dx \qquad --(i)$

So,

$$I = \int_0^{\pi} (\pi - x) \sin(\pi - x) . \cos^4 (\pi - x) dx \qquad \left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx\right]$$

$$= \int_0^{\pi} (\pi - x) \sin x . \cos^4 x \, dx$$

$$= \int_0^{\pi} \pi \sin x . \cos^4 x \, dx - \int_0^{\pi} x \sin x . \cos^4 x \, dx$$

So from equation (i)

$$I = \int_0^{\pi} \pi \sin x . \cos^4 x \, dx - I$$

$$2I = \pi \int_0^{\pi} \sin x . \cos^4 x \, dx$$

Let $t = \cos x \, dx$

$dt = - \sin \; x \, dx$

As,

$x = 0 \quad t = 1$

$x = \pi \quad t = -1$

Hence

$$2I = \pi \int_{-1}^{+1} t^4 dt = \pi \left[\frac{t^5}{5}\right]_{-1}^{1} = \pi \left[\frac{1}{5} + \frac{1}{5}\right]$$

$$I = \frac{\pi}{5}$$

Definite Integrals Ex 20.4B Q13

Let $I = \int_0^{\pi} x \sin^3 x \, dx$

$= \int_0^{\pi} (\pi - x) \sin^3 (\pi - x) dx \qquad \left[\because \int_0^{a} f(x) dx = \int_0^{a} f(a - x) dx\right]$

$= \int_0^{\pi} \pi \sin^3 x \, dx - \int_0^{\pi} x \sin^3 x \, dx$

$\therefore \ I = \int_0^{\pi} \pi \sin^3 x \, dx - I$

$\Rightarrow 2I = \pi \int_0^{\pi} \sin^3 x \, dx$

$\Rightarrow 2I = \pi \int_0^{\pi} \frac{3 \sin x - \sin 3x}{4} dx$

$= \frac{\pi}{4} \int_0^{\pi} (3 \sin x - \sin 3x) dx$

$= \frac{\pi}{4} \left[-3 \cos x + \frac{\cos 3x}{3}\right]_0^{\pi}$

$= \frac{\pi}{4} \left[\left(-3 \cos \pi + \frac{\cos 3\pi}{3}\right) - \left(-3 \cos 0 + \frac{\cos 0}{3}\right)\right]$

$= \frac{\pi}{4} \left[\left(3 - \frac{1}{3}\right) - \left(-3 + \frac{1}{3}\right)\right]$

$= \frac{\pi}{4} \left[3 - \frac{1}{3} + 3 - \frac{1}{3}\right]$

$\frac{\pi}{4} \left[6 - \frac{2}{3}\right]$

$= \frac{\pi}{4} \times \frac{16}{3} \quad = \quad \frac{4\pi}{3}$

$\therefore \ I = \frac{2\pi}{3}$

Definite Integrals Ex 20.4B Q14

We have,

$$I = \int_0^{\pi} x \log \sin x \, dx = \int_0^{\pi} (\pi - x) \log \sin(\pi - x) \, dx$$

$$I = \pi \int_0^{\pi} \log \sin(x) \, dx - \int_0^{\pi} x \log \sin x \, dx$$

$$2I = \pi \int_0^{\pi} \log \sin x \, dx$$

Since $f(x) = f(-x)$, f(x) is an even function.

$$\therefore \; 2I = 2\pi \int_0^{\frac{\pi}{2}} \log \sin x \, dx$$

$$I = \pi \int_0^{\frac{\pi}{2}} \log \sin x \, dx \qquad \ldots(i)$$

$$\Rightarrow I = \pi \int_0^{\frac{\pi}{2}} \log \sin\left(\frac{\pi}{2} - x\right) dx = \pi \int_0^{\frac{\pi}{2}} \log \cos x \, dx \quad \ldots(ii)$$

Now adding (i) & (ii) we get

$$2I = \pi \int_0^{\frac{\pi}{2}} \log \sin x \, dx + \pi \int_0^{\frac{\pi}{2}} \log \cos x \, dx = \pi \int_0^{\frac{\pi}{2}} (\log \sin x + \log \cos x) \, dx = \pi \int_0^{\frac{\pi}{2}} \log \sin x . \cos x \, dx$$

$$\Rightarrow 2I = \pi \int_0^{\frac{\pi}{2}} \log\left(\frac{2 \sin x . \cos x}{2}\right) dx = \pi \int_0^{\frac{\pi}{2}} \log\left(\frac{\sin 2x}{2}\right) dx = \pi \int_0^{\frac{\pi}{2}} \log \sin 2x \, dx - \pi \int_0^{\frac{\pi}{2}} \log 2 \, dx \qquad \ldots(iii)$$

Now let $I = \int_0^{\frac{\pi}{2}} \log \sin 2x \, dx$

Putting $2x = t$ we get

$$I_1 = \int_0^{\pi} \log \sin t \frac{dt}{2} = \frac{1}{2} \int_0^{\pi} \log \sin t . dt = \frac{1}{2} \times 2\pi \int_0^{\frac{\pi}{2}} \log \sin t \, dt = \pi \int_0^{\frac{\pi}{2}} \log \sin x \, dx = I$$

So from (iii) we get

$$2I = I - \pi \frac{\pi}{2} \log 2$$

$$I = -\frac{\pi^2}{2} \log 2$$

Definite Integrals Ex 20.4B Q15

$$\text{Let } I = \int_0^{\pi} \frac{x \sin x}{1+\sin x} dx$$

$$= \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\sin x} dx \qquad \left[\because \int_0^a f(x)dx = \int_0^a f(a-x)\,dx\right]$$

$$I = \int_0^{\pi} \frac{\pi \sin x}{1+\sin x} dx - \int_0^{\pi} \frac{x \sin x}{1+\sin x} dx$$

$$2I = \pi \int_0^{\pi} \frac{\sin x}{1+\sin x} dx$$

$$2I = \pi \int_0^{\pi} \frac{\sin x}{1+\sin x} \times \frac{(1-\sin x)}{(1-\sin x)} dx$$

$$2I = \pi \int_0^{\pi} \frac{\sin x - \sin^2 x}{1+\sin^2 x} dx$$

$$2I = \pi \int_0^{\pi} \frac{(\sin x - \sin^2 x)}{\cos^2 x} dx$$

$$2I = \pi \int_0^{\pi} \left(\tan x . \sec x - \tan^2 x\right) dx$$

$$2I = \pi \int_0^{\pi} \left[\tan x . \sec x - \left(\sec^2 x - 1\right)\right] dx$$

$$2I = \pi \int_0^{\pi} \left(\sec x . \tan x - \sec^2 x + 1\right) dx$$

$$2I = \pi \int_0^{\pi} \left(\sec x . \tan x - \sec^2 x + 1\right) dx$$

$$2I = \pi \left[\sec x - \tan x + x\right]_0^{\pi}$$

$$2I = \pi \left[\left(\sec \pi - \tan \pi + \pi\right) - \left(\sec 0 - \tan 0 + 0\right)\right]$$

$$2I = \pi \left[\left(-1 - 0 + \pi\right) - \left(1 - 0 + 0\right)\right]$$

$$2I = \pi \left(\pi - 1 - 1\right)$$

$$I = \frac{\pi}{2}\left(\pi - 2\right)$$

$$\therefore \int_0^{\pi} \frac{x \sin x}{1+\sin x} dx = \pi \left(\frac{\pi}{2} - 1\right)$$

Definite Integrals Ex 20.4B Q16

We have

$$I=\int_0^{\pi}\frac{x\,dx}{1+\cos\alpha\sin x}\qquad --(i)$$

$$\therefore \int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$$

$$I=\int_0^{\pi}\frac{(\pi-x)\,dx}{1+\cos\alpha\sin(\pi-x)}=\int_0^{\pi}\frac{(\pi-x)\,dx}{1+\cos\alpha\sin x}\qquad --(ii)$$

Adding (i) & (ii) we get

$$2I=\pi\int_0^{\pi}\frac{\pi}{1+\cos\alpha.\sin x}dx$$

Substituting $\sin x=\dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$2I=\pi\int_0^{\pi}\frac{\sec^2\frac{x}{2}}{1+\tan^2\frac{x}{2}\,2\cos\alpha.\tan\frac{x}{2}}dx=\pi\int_0^{\pi}\frac{\sec^2\frac{x}{2}dx}{1-\cos^2\alpha+\left(\cos\alpha.\tan\frac{x}{2}\right)^2}dx$$

Let $\tan\frac{x}{2}=t\Rightarrow\frac{1}{2}\sec^2\frac{x}{2}dx=dt$

When $x=0\quad t=0$

$\qquad\pi\ \Rightarrow t=\alpha$

$$2I=\int_0^{\alpha}\frac{dt}{\left(1+\cos^2\alpha\right)+\left(\cos\alpha+t\right)^2}dx=2\pi.\frac{1}{\sqrt{1+\cos^2\alpha}}.\left[\tan^{-1}\left(\frac{\cos\alpha+1}{\sqrt{1+\cos^2\alpha}}\right)\right]_0^{\alpha}$$

$$=\frac{2\pi}{\sin\alpha}\left[\frac{\pi}{2}-\tan^{-1}\cot\alpha\right]$$

$$=\frac{2\pi}{\sin\alpha}\left[\cot^{-1}\left(\cot\alpha\right)\right]$$

$$=\frac{2\pi}{\sin\alpha}.\alpha$$

$$\Rightarrow I=\frac{\pi\alpha}{\sin\alpha}$$

Definite Integrals Ex 20.4B Q17

Let $I=\int_0^{\pi}x\cos^2x\,dx$

$$I=\int_0^{\pi}(\pi-x)\cos^2(\pi-x)\,dx\qquad\left[\because\int_0^a f(x)dx=\int_0^a f(a-x)\,dx\right]$$

$$I=\pi\int_0^{\pi}\cos^2x\,dx-\int_0^{\pi}x\cos^2x\,dx$$

$$2I=\pi\int_0^{\pi}\cos^2x\,dx$$

$$=\pi\int_0^{\pi}\left(\frac{1+\cos2x}{2}\right)dx\qquad\text{Since }\cos^2x=\frac{1+\cos2x}{2}$$

$$=\frac{\pi}{2}\int_0^{\pi}(1+\cos2x)dx$$

$$=\frac{\pi}{2}\left[x+\left(-\frac{\sin2x}{2}\right)\right]_0^{\pi}$$

$$\therefore 2I=\frac{\pi}{2}\left[\pi-\frac{\sin2.\pi}{2}-0+\frac{\sin0}{2}\right]$$

$$\Rightarrow 2I=\frac{\pi}{2}\left[\pi-0-0+0\right]$$

$$I=\frac{\pi^2}{4}$$

Definite Integrals Ex 20.4B Q18

$$I = \int_{\pi/6}^{\pi/3} \frac{1}{1+\cot^{3/2} x} dx$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sin^{3/2} x}{\sin^{3/2} x+\cos^{3/2} x} dx$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sin^{3/2}\left(\frac{\pi}{2}-x\right)}{\sin^{3/2}\left(\frac{\pi}{2}-x\right)+\cos^{3/2}\left(\frac{\pi}{2}-x\right)} dx = \int_{\pi/6}^{\pi/3} \frac{\cos^{3/2}(x)}{\cos^{3/2}(x)+\sin^{3/2}(x)} dx$$

$$\therefore 2I = \int_{\pi/6}^{\pi/3} \frac{\sin^{3/2} x}{\sin^{3/2} x+\cos^{3/2} x} dx + \int_{\pi/6}^{\pi/3} \frac{\cos^{3/2}(x)}{\cos^{3/2}(x)+\sin^{3/2}(x)} dx$$

$$2I = \int_{\pi/6}^{\pi/3} \frac{\sin^{3/2} x+\cos^{3/2} x}{\sin^{3/2} x+\cos^{3/2} x} dx$$

$$I = \frac{1}{2}\int_{\pi/6}^{\pi/3} dx$$

$$I = \frac{\pi}{12}$$

Definite Integrals Ex 20.4B Q19

$$I = \int_0^{\frac{\pi}{2}} \frac{\tan^7 x}{\tan^7 x+\cot^7 x} dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\tan^7\left(\frac{\Pi}{2}-x\right)}{\tan^7\left(\frac{\Pi}{2}-x\right)+\cot^7\left(\frac{\Pi}{2}-x\right)} dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\cot^7 x}{\tan^7 x+\cot^7 x} dx$$

Hence

$$2I = \int_0^{\frac{\pi}{2}} \frac{\tan^7 x}{\tan^7 x+\cot^7 x} + \frac{\cot^7 x}{\tan^7 x+\cot^7 x} dx$$

$$2I = \int_0^{\frac{\pi}{2}} 1 dx$$

$$2I = \frac{\Pi}{2}$$

$$I = \frac{\Pi}{4}$$

Definite Integrals Ex 20.4B Q20

$$I = \int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}} dx$$

$$I = \int_2^8 \frac{\sqrt{10-(8+2-x)}}{\sqrt{(8+2-x)}+\sqrt{10-(8+2-x)}} dx$$

$$I = \int_2^8 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{10-x}} dx$$

$$2I = \int_2^8 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{10-x}} + \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}} dx$$

$$2I = \int_2^8 1 dx$$

$$2I = 6$$

$$I = 3$$

Definite Integrals Ex 20.4B Q21

$$\int_0^{\pi} x\sin x\cos^2 x dx = \int_0^{\pi} (\Pi - x)\sin(\Pi - x)\cos^2(\Pi - x)dx$$

$$\int_0^{\pi} x\sin x\cos^2 x dx = \int_0^{\pi} (\Pi - x)\sin x\cos^2 x dx$$

$$\int_0^{\pi} x\sin x\cos^2 x dx = \int_0^{\pi} \Pi\sin x\cos^2 x dx - \int_0^{\pi} x\sin x\cos^2 x dx$$

$$2\int_0^{\pi} x\sin x\cos^2 x dx = \int_0^{\pi} \Pi\sin x\cos^2 x dx$$

$$\int_0^{\pi} x\sin x\cos^2 x dx = \frac{\Pi}{2}\int_0^{\pi} \sin x\cos^2 x dx$$

Now

$$\int_0^{\pi} \sin x\cos^2 x dx$$

Let cosx=t

$$\sin x dx = -dt$$

$$-\int_1^{-1} t^2 dt$$

$$\int_{-1}^{1} t^2 dt$$

$$\left\{\frac{t^3}{3}\right\}_{-1}^{1}$$

$$\frac{2}{3}$$

$$\therefore \int_0^{\pi} x\sin x\cos^2 x dx = \frac{\pi}{2}\times\frac{2}{3} = \frac{\pi}{3}$$

Definite Integrals Ex 20.4B Q22

We have,

$$I = \int_0^{\frac{\pi}{2}} \frac{x\sin x.\cos x}{\sin^4 x + \cos^4 x}dx \qquad --(i)$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2} - x\right)\cos x.\sin x}{\cos^4 x + \sin^4 x}dx \qquad --(ii)$$

Adding (i) & (ii)

$$2I = \frac{\pi}{2}\int_0^{\frac{\pi}{2}} \frac{\sin x.\cos x}{\cos^4 x + \sin^4 x}dx$$

$$2I = \frac{\pi}{4}\int_0^{\frac{\pi}{2}} \frac{2\sin x.\cos x}{\cos^4 x + \sin^4 x}dx$$

Let $t = \sin^2 x$

$$\Rightarrow 2I = \frac{\pi}{4}\int_0^1 \frac{1}{(1-t)^2 + t^2}dt$$

$$\Rightarrow 2I = \frac{\pi}{8}\int_0^1 \frac{1}{\left(t - \frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}dt$$

$$\Rightarrow 2I = \frac{\pi}{8}\times 2\left[\tan^{-1}(2t-1)\right]_0^1$$

$$\Rightarrow I = \frac{\pi}{8}\left[\frac{\pi}{4} + \frac{\pi}{4}\right] \quad = \quad \frac{\pi^2}{16}$$

Definite Integrals Ex 20.4B Q23

Let $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 x\, dx$

$f(-x) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3(-x)\, dx$

$= -\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 x\, dx$

Here $f(x) = -f(+x)$

Hence $f(x)$ is odd function.

So,
$I = 0$

Definite Integrals Ex 20.4B Q24

We have,

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x\, dx = 2\int_0^{\frac{\pi}{2}} \sin^4 x\, dx \qquad [\because \sin^4 x \text{ is an even function}]$$

$$= 2\int_0^{\frac{\pi}{2}} (\sin^2 x)^2\, dx$$

$$= 2\int_0^{\frac{\pi}{2}} \left(\frac{1-\cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} (1-\cos 2x)^2\, dx$$

$$= \frac{1}{2}\left[\int_0^{\frac{\pi}{2}} (1+\cos^2 2x - 2\cos 2x)\, dx\right]$$

$$= \frac{1}{2}\left[\int_0^{\frac{\pi}{2}} \left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right) dx\right]$$

$$= \frac{1}{4}\left[\int_0^{\frac{\pi}{2}} (3 - 4\cos 2x + \cos 4x)\, dx\right]$$

$$= \frac{1}{4}\left[3x - \frac{4\sin 2x}{2} + \frac{\sin 4x}{4}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{4}\left[\left\{\frac{3\pi}{2} - 2\sin \pi + \frac{1}{4}\sin 2\pi\right\} - \{0 - 0 + 0\}\right]$$

$$= \frac{1}{4}\left[\frac{3\pi}{2} - 0 + 0\right] = \frac{1}{4} \times \frac{3\pi}{2}$$

$$= \frac{3\pi}{8}$$

$$\therefore \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x\, dx = \frac{3\pi}{8}$$

Definite Integrals Ex 20.4B Q25

We have,

$$I = \int_{-1}^{1} \log\left(\frac{2-x}{2+x}\right) dx$$

Since, $\log\left\{\frac{2-(-x)}{2+(-x)}\right\} = -\log\left(\frac{2-x}{2+x}\right)$ $\therefore$ This is an odd function.

Hence,
$I = 0$

Definite Integrals Ex 20.4B Q26

We have,

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x \, dx$$

$\sin^2 x$ is even function.

Hence,

$$I = 2\int_0^{\frac{\pi}{4}} \sin^2 x \, dx = 2\int_0^{\frac{\pi}{4}} \left(\frac{1-\cos 2x}{2}\right) dx = \frac{2}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{4}} = \frac{1}{2}\left[\frac{2\pi}{4} - \sin\frac{\pi}{2} - 0 + \sin 0\right]$$

$$= \frac{1}{2}\left[\frac{2\pi}{4} - 1\right]$$

$$= \frac{\pi}{4} - \frac{1}{2}$$

$$\therefore \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x \, dx = \frac{\pi}{4} - \frac{1}{2}$$

Definite Integrals Ex 20.4B Q27

$$I = \int_0^{\pi} \log(1-\cos x)\, dx$$

$$= \int_0^{\pi} \log\left(2\sin^2\frac{x}{2}\right) dx$$

$$= \int_0^{\pi} \log 2 \, dx + \int_0^{\pi} \log \sin^2\frac{x}{2} \, dx$$

$$= \int_0^{\pi} \log 2 \, dx + 2\int_0^{\pi} \log \sin\frac{x}{2} \, dx$$

$$I = \log 2[x]_0^{\pi} + 4\int_0^{\frac{\pi}{2}} \log \sin t \, dt \qquad \left[\text{Put} \quad t = \frac{x}{2} \Rightarrow dt = \frac{1}{2}dx\right]$$

$$I = \pi \log 2 + 4I_1 \qquad \ldots(i)$$

$$I_1 = \int_0^{\frac{\pi}{2}} \log \sin t \, dt \qquad \ldots(ii)$$

$$= \int_0^{\frac{\pi}{2}} \log \cos t \, dt \qquad \ldots(iii)$$

Adding (ii) & (iii) we get

$$2I_1 = \int_0^{\frac{\pi}{2}} \log \sin t.\cos t \, dt = \int_0^{\frac{\pi}{2}} \log\left(\frac{\sin 2t}{2}\right) dt = \int_0^{\frac{\pi}{2}} \log \sin 2t \, dt - \frac{\pi}{2}\log 2$$

We know the property $\int_a^b f(x) = \int_a^b f(t)$

$$2I_1 = I_1 - \frac{\pi}{2}\log 2$$

$$\Rightarrow I_1 = -\frac{\pi}{2}\log 2 \qquad \ldots(iv)$$

Putting the value from (iv) to (i)

$$I = \pi \log 2 + 4\left(-\frac{\pi}{2}\log 2\right) = \pi \log 2 - 2\pi \log 2 = -\pi \log 2$$

$$I = -\pi \log 2$$

Definite Integrals Ex 20.4B Q28

We have,

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \log\left(\frac{2-\sin x}{2+\sin x}\right)dx$$

Let $f(x) = \log\left(\frac{2-\sin x}{2+\sin x}\right)$

Then,

$$f(-x) = \log\left(\frac{2-\sin(-x)}{2+\sin(-x)}\right) = -\log\left(\frac{2-\sin x}{2+\sin x}\right) = -f(x)$$

Thus, $f(x)$ is an odd function.

$$\therefore I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \log\left(\frac{2-\sin x}{2+\sin x}\right)dx = 0$$

Definite Integrals Ex 20.4B Q29

$$I = \int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}\, dx$$

$$I = \int_{-\pi}^{\pi} \frac{2x}{1+\cos^2 x}\, dx + \int_{-\pi}^{\pi} \frac{2x\sin x}{1+\cos^2 x}\, dx$$

$$I = 0 + \int_{-\pi}^{\pi} \frac{2x\sin x}{1+\cos^2 x}\, dx \ldots\ldots\ldots \left[\because \frac{2x}{1+\cos^2 x} \text{ is an odd function}\right]$$

$$I = 2\int_{0}^{\pi} \frac{2x\sin x}{1+\cos^2 x}\, dx \ldots\ldots\ldots \left[\because \frac{2x\sin x}{1+\cos^2 x} \text{ is an even function}\right]$$

$$I = 4\int_{0}^{\pi} \frac{x\sin x}{1+\cos^2 x}\, dx$$

$$I = 2\pi\int_{0}^{\pi} \frac{\sin x}{1+\cos^2 x}\, dx \ldots\ldots\ldots \left[\because \int_0^a xf(x)dx = \frac{a}{2}\int_0^a f(x)dx\right]$$

Put $\cos x = t$ then $-\sin x\, dx = dt$

$$I = -2\pi\int_{1}^{-1} \frac{1}{1+t^2}\, dt$$

$$I = -2\pi\left[\tan^{-1} t\right]_1^{-1}$$

$$I = \pi^2$$

Definite Integrals Ex 20.4B Q30

$$I = \int_{-a}^{a} \log\left(\frac{a-\sin\theta}{a+\sin\theta}\right) d\theta$$

Let $f(\theta) = \log\left(\frac{a-\sin\theta}{a+\sin\theta}\right)$

$$f(-\theta) = \log\left(\frac{a-\sin(-\theta)}{a+\sin(-\theta)}\right) = -\log\left(\frac{a-\sin\theta}{a+\sin\theta}\right) = -f(\theta)$$

$\therefore$ $f(\theta) = \log\left(\frac{a-\sin\theta}{a+\sin\theta}\right)$ is an odd function.

$$\therefore I = \int_{-a}^{a} \log\left(\frac{a-\sin\theta}{a+\sin\theta}\right) d\theta = 0$$

Definite Integrals Ex 20.4B Q31

$$I = \int_{-2}^{2} \frac{3x^3 + 2|x|+1}{x^2+|x|+1}\, dx$$

$$I = \int_{-2}^{2} \frac{3x^3}{x^2+|x|+1}\, dx + \int_{-2}^{2} \frac{2|x|+1}{x^2+|x|+1}\, dx$$

$$I = 0 + \int_{-2}^{2} \frac{2|x|+1}{x^2+|x|+1}\, dx \ldots\ldots\ldots \left[\because \frac{3x^3}{x^2+|x|+1} \text{ is an odd function}\right]$$

$$I = 2\int_{0}^{2} \frac{2|x|+1}{x^2+|x|+1}\, dx \ldots\ldots\ldots \left[\because \frac{2|x|+1}{x^2+|x|+1} \text{ is an even function}\right]$$

$$I = 2\left[\log\left(x^2+|x|+1\right)\right]_0^2$$

$$I = 2\left[\log(4+2+1) - \log(1)\right]$$

$$I = 2\log_e(7)$$

Definite Integrals Ex 20.4B Q32

$$I = \int_{-3\pi/2}^{-\pi/2} \left\{\sin^2(3\pi + x) + (\pi + x)^3\right\}\, dx$$

Substitute $\pi + x = u$ then $dx = du$

$$I = \int_{-\pi/2}^{\pi/2} \left\{\sin^2(2\pi + u) + (u)^3\right\}\, du$$

$$I = \int_{-\pi/2}^{\pi/2} \left\{\sin^2(u) + (u)^3\right\}\, du$$

$$I = \left[\frac{1}{2}\left(u - \frac{1}{2}\sin(2u)\right) + \frac{u^4}{4}\right]_{-\pi/2}^{\pi/2}$$

$$I = \frac{\pi}{2}$$

Definite Integrals Ex 20.4B Q33

$$\text{Let } I = \int_0^2 x\sqrt{2-x}\,dx$$

$$I = \int_0^2 (2-x)\sqrt{x}\,dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$= \int_0^2 \left\{2x^{\frac{1}{2}} - x^{\frac{3}{2}}\right\}dx$$

$$= \left[2\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right) - \frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^2$$

$$= \left[\frac{4}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}\right]_0^2$$

$$= \frac{4}{3}(2)^{\frac{3}{2}} - \frac{2}{5}(2)^{\frac{5}{2}}$$

$$= \frac{4\times 2\sqrt{2}}{3} - \frac{2}{5}\times 4\sqrt{2}$$

$$= \frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5}$$

$$= \frac{40\sqrt{2} - 24\sqrt{2}}{15}$$

$$= \frac{16\sqrt{2}}{15}$$

Definite Integrals Ex 20.4B Q34

$$Let\ I = \int_0^1 \log\left(\frac{1}{x} - 1\right)dx$$
$$= \int_0^1 \log\left(\frac{1-x}{x}\right)dx$$
$$= \int_0^1 \log(1-x)dx - \int_0^1 \log(x)dx$$
$$Applying\ the\ property,\ \int_0^a f(x)dx = \int_0^a f(a-x)dx$$
$$Thus,\ I = \int_0^1 \log(1-(1-x))dx - \int_0^1 \log(x)dx$$
$$= \int_0^1 \log(1-1+x)dx - \int_0^1 \log(x)dx$$
$$= \int_0^1 \log(x)dx - \int_0^1 \log(x)dx$$
$$= 0$$

Definite Integrals Ex 20.4B Q35

$$I = \int_{-1}^{1} |x \cos \pi x| dx$$
$$\text{Let } f(x) = |x \cos \pi x|$$
$$f(-x) = |-x \cos(-\pi x)| = |-x \cos(\pi x)| = |x \cos \pi x| = f(x)$$
$$\therefore I = \int_{-1}^{1} |x \cos \pi x| dx = 2\int_0^1 |x \cos \pi x| dx$$

Now,

$$f(x) = |x \cos \pi x| = \begin{cases} x \cos \pi x, \text{ if } 0 \le x \le \frac{1}{2} \\ -x \cos \pi x, \text{ if } \frac{1}{2} < x < 1 \end{cases}$$
$$\therefore I = 2\int_0^1 |x \cos \pi x| dx$$
$$\Rightarrow I = 2\left[\int_0^{\frac{1}{2}} x \cos \pi x\ dx + \int_{\frac{1}{2}}^{1} -x \cos \pi x\, dx\right]$$
$$\Rightarrow I = 2\left\{\left[\frac{x}{\pi}\sin \pi x + \frac{1}{\pi^2}\cos \pi x\right]_0^{\frac{1}{2}} - \left[\frac{x}{\pi}\sin \pi x + \frac{1}{\pi^2}\cos \pi x\right]_{\frac{1}{2}}^{1}\right\}$$
$$\Rightarrow I = 2\left\{\left[\frac{1}{2\pi} - \frac{1}{\pi^2}\right] - \left[-\frac{1}{\pi^2} - \frac{1}{2\pi}\right]\right\}$$
$$\Rightarrow I = \frac{2}{\pi}$$

Definite Integrals Ex 20.4B Q36

$$I=\int_0^{\Pi}\left(\frac{x}{1+\sin^2 x}+\cos^7 x\right)dx$$

$$I=\int_0^{\Pi}\left(\frac{\Pi-x}{1+\sin^2(\Pi-x)}+\cos^7(\Pi-x)\right)dx$$

$$I=\int_0^{\Pi}\left(\frac{\Pi-x}{1+\sin^2 x}-\cos^7 x\right)dx$$

$$2I=\int_0^{\Pi}\left(\frac{\Pi}{1+\sin^2 x}\right)dx$$

$$2I=\Pi\int_0^{\Pi}\frac{1}{1+\sin^2 x}dx$$

$$2I=\Pi\int_0^{\Pi}\frac{1}{1+2\tan^2 x}\sec^2 x dx$$

$$\mathrm{I}=\pi\int_0^{\frac{\pi}{2}}\frac{1}{1+2\tan^2 x}\sec^2 x dx..........\left[\because\int_0^{2a}f(x)dx=2\int_0^{a}f(x)dx,\text{ if } f(2a-x)=f(x)\right]$$

Let $\tan x = v$

$dv = \sec^2 x\ dx$

$$\Rightarrow \mathrm{I}=\pi\int_0^{\infty}\frac{1}{1+2v^2}dv$$

$$\Rightarrow \mathrm{I}=\pi\left[\frac{\tan^{-1}\left(\sqrt{2}v\right)}{\sqrt{2}}\right]_0^{\infty}$$

$$\Rightarrow \mathrm{I}=\pi\left[\frac{\pi}{2\sqrt{2}}\right]$$

$$\Rightarrow \mathrm{I}=\frac{\pi^2}{2\sqrt{2}}$$

Definite Integrals Ex 20.4B Q37

$$I=\int_0^{\pi}\frac{x}{1+\cos\alpha\sin x}dx$$

Then,

$$I=\int_0^{\pi}\frac{(\pi-x)}{1+\cos\alpha\sin(\pi-x)}dx$$

$$I=\int_0^{\pi}\frac{(\pi-x)}{1+\cos\alpha\sin x}dx$$

$$2I=\pi\int_0^{\pi}\frac{1}{1+\cos\alpha\sin x}dx$$

$$2I=\pi\int_0^{\pi}\frac{1+\tan^2\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)+2\cos\alpha\tan\left(\frac{x}{2}\right)}dx$$

$$I=\frac{\pi}{2}\int_0^{\pi}\frac{\sec^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+2\cos\alpha\tan\left(\frac{x}{2}\right)+1}dx$$

Put $\tan\left(\frac{x}{2}\right)=t$ then $\sec^2\left(\frac{x}{2}\right)dx=2dt$

$x=0\Rightarrow t=0$ and $x=\pi\Rightarrow t=\infty$

$$I=\frac{\pi}{2}\int_0^{\infty}\frac{2}{t^2+2t\cos\alpha+1}dt$$

$$I=\pi\int_0^{\infty}\frac{1}{(t+\cos\alpha)^2+(1-\cos^2\alpha)}dt$$

$$I=\pi\int_0^{\infty}\frac{1}{(t+\cos\alpha)^2+\sin^2\alpha}dt$$

$$I=\frac{\pi}{\sin\alpha}\left[\tan^{-1}\left(\frac{t+\cos\alpha}{\sin\alpha}\right)\right]_0^{\infty}$$

$$I=\frac{\pi\alpha}{\sin\alpha}$$

Definite Integrals Ex 20.4B Q38

We know

$$\int_0^{2a}f(x)dx=\int_0^{a}f(x)+\int_0^{a}f(2a-x)dx$$

Also here

$$f(x)=f(2\Pi-x)$$

So

$$I=\int_0^{2\Pi}\sin^{100}x\cos^{101}xdx=2\int_0^{\Pi}\sin^{100}x\cos^{101}xdx$$

$$I=2\int_0^{\Pi}\sin^{100}(\Pi-x)\cos^{101}(\Pi-x)dx$$

$$I=-2\int_0^{\Pi}\sin^{100}x\cos^{101}xdx$$

Hence

$$2I=0$$

$$I=0$$

Definite Integrals Ex 20.4B Q39

$$I = \int_0^{\pi/2} \frac{a\sin x + b\cos x}{\sin x + \cos x} dx$$

Then,

$$I = \int_0^{\pi/2} \frac{a\sin\left(\frac{\pi}{2} - x\right) + b\cos\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$$

$$I = \int_0^{\pi/2} \frac{a\cos x + b\sin x}{\cos x + \sin x} dx$$

$$2I = \int_0^{\pi/2} \frac{a\sin x + b\cos x}{\sin x + \cos x} dx + \int_0^{\pi/2} \frac{a\cos x + b\sin x}{\cos x + \sin x} dx$$

$$2I = (a+b)\int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$I = \frac{(a+b)}{2}\int_0^{\pi/2} 1 dx$$

$$I = \frac{(a+b)\pi}{4}$$

Definite Integrals Ex 20.4B Q40

We have,

$$I = \int_0^{2a} f(x)\,dx$$

Then

$$I = \int_0^{a} f(x)\,dx + \int_a^{2a} f(x)\,dx$$

$$I = \int_0^{a} f(x)\,dx + I_1$$

where, $I_1 = \int_a^{2a} f(x)\,dx$

Let $2a - t = x$ then $dx = -dt$

If $t = a \Rightarrow x = a$

If $t = 2a \Rightarrow x = 0$

$$I_1 = \int_0^{2a} f(x)\,dx = \int_a^{0} f(2a-t)(-dt) = -\int_a^{0} f(2a-t)\,dt$$

$$I_1 = \int_0^{a} f(2a-t)\,dt = \int_0^{a} f(2a-x)\,dx$$

$$\therefore I = \int_0^{a} f(x)\,dx + \int_0^{a} f(2a-x)\,dx$$

$$I = \int_0^{a} f(x)\,dx + \int_0^{a} f(x)\,dx = 2\int_0^{a} f(x)\,dx \quad [f(2a-x) = f(x)]$$

Hence Proved.

Definite Integrals Ex 20.4B Q41

We have,

$$I = \int_0^{2a} f(x)dx = \int_0^{a} f(x)dx + \int_a^{2a} f(x)dx$$

$$I = \int_0^{a} f(x)dx + I_1$$

Let $2a - t = x$ then $dx = -dt$

$t = a,\ x = a$

$t = 2a \quad x = 0$

$$I_1 = \int_0^{2a} f(x) = \int_a^{0} f(2a - t)(-dt)$$

$$= -\int_a^{0} f(2a - t)dt$$

$$I_1 = \int_0^{a} f(2a - t)dt = \int_0^{a} f(2a - x)dx$$

$$I = \int_0^{a} f(x)dx + \int_0^{a} f(2a - x)dx$$

$$I = \int_0^{a} f(x)dx - \int_0^{a} f(x) \qquad [\because f(2a - x) = -f(x)]$$

$$I = 0$$

Hence,

$$\int_0^{2a} f(x)dx = 0$$

Definite Integrals Ex 20.4B Q42

(i) We have,

$$I = \int_{-a}^{a} f(x^2)dx$$

Clearly $f(x^2)$ is an even function.

So,

$$\int_{-a}^{a} f(t) = 2\int_0^{a} f(t)$$

$$I = 2\int_0^{a} f(x^2)dx$$

(ii) We have,

$$I = \int_{-a}^{a} xf(x^2)dx$$

Clearly, $xf(x^2)$ is odd function.

So, $I = 0$

$$\therefore \int_{-a}^{a} xf(x^2)dx = 0$$

Definite Integrals Ex 20.4B Q43

We have from LHS,

$$I = \int_0^{2a} f(x)\,dx = \int_0^{a} f(x)\,dx + \int_a^{2a} f(x)\,dx \qquad \ldots(i)$$

Let $x = 2a - t$, then $dx = -dt$

$x = a \Rightarrow t = a$, and $x = 2a \Rightarrow t = 0$

$$\therefore \int_0^{2a} f(x)\,dx = -\int_a^{0} f(2a-t)\,dt$$

$$\Rightarrow \int_0^{2a} f(x)\,dx = \int_0^{a} f(2a-t)\,dt$$

$$\Rightarrow \int_0^{2a} f(x)\,dx = \int_0^{a} f(2a-x)\,dx$$

Substituting $\int_0^{2a} f(x)\,dx = \int_0^{a} f(2a-x)\,dx$ in (i)

we get,

$$\int_0^{2a} f(x)\,dx = \int_0^{a} f(x)\,dx + \int_0^{a} f(2a-x)\,dx$$

$$\Rightarrow \int_0^{2a} f(x)\,dx = \int_0^{a} \{f(x) + f(2a-x)\}\,dx$$

Definite Integrals Ex 20.4B Q44

$$I = \int_a^b x f(x)\,dx$$

$$\Rightarrow I = \int_a^b (a+b-x)\, f(a+b-x)\,dx$$

$$\Rightarrow I = \int_a^b (a+b-x)\, f(x)\,dx \ldots\ldots\ldots\ldots \left[\text{Given that } f(a+b-x) = f(x)\right]$$

$$\Rightarrow I = \int_a^b (a+b)\, f(x)\,dx - \int_a^b x f(x)\,dx$$

$$\Rightarrow I = \int_a^b (a+b)\, f(x)\,dx - I$$

$$\Rightarrow 2I = \int_a^b (a+b)\, f(x)\,dx$$

$$\Rightarrow I = \frac{a+b}{2} \int_a^b f(x)\,dx$$

Definite Integrals Ex 20.4B Q45

We have,

$$I = \int_{-a}^{a} f(x)\,dx = \int_{-a}^{0} f(x)\,dx + \int_0^{a} f(x)\,dx$$

Let $x = -t$ then $dx = -dt$

$x = -a \Rightarrow t = a$

$x = 0 \Rightarrow t = 0$

$$\therefore \int_{-a}^{a} f(x)\,dx = \int_a^{0} f(-t)(-dt) = -\int_a^{0} f(-t)\,dt$$

$$\Rightarrow \int_{-a}^{a} f(x)\,dx = \int_0^{a} f(-t)\,dt$$

$$\Rightarrow \int_{-a}^{0} f(x)\,dx = \int_0^{a} f(-x)\,dx$$

$$\therefore \int_{-a}^{a} f(x)\,dx = \int_0^{a} f(-x)\,dx + \int_0^{a} f(x)\,dx$$

Hence,

$$\int_{-a}^{a} f(x)\,dx = \int_0^{a} \{f(-x) + f(x)\}\,dx$$

Proved

Definite Integrals Ex 20.4B Q46

$$I = \int_0^{\pi} x f(\sin x)\,dx$$

$$I = \int_0^{\pi} (\Pi - x) f(\sin(\Pi - x))\,dx$$

$$I = \int_0^{\pi} (\Pi - x) f(\sin x)\,dx$$

$$2I = \int_0^{\pi} \Pi f(\sin x)\,dx$$

$$I = \frac{\Pi}{2}\int_0^{\pi} f(\sin x)\,dx$$

Ex 20.5

Definite Integrals Ex 20.5 Q1

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----+f(a+(n-1)h)\right], \text{ where } h=\frac{b-a}{n}$$

Here, $a=0,\ b=3$ and $f(x)=(x+4)$

$$h=\frac{3}{n}\Rightarrow nh=3$$

Thus, we have,

$$\Rightarrow I=\int_0^3 (x+4)\,dx$$

$$\Rightarrow I=\lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+----f((n-1)h)\right]$$

$$\Rightarrow I=\lim_{h\to 0} h\left[4+(h+4)+(2h+4)+----+((n-1)h+4)\right]$$

$$\Rightarrow I=\lim_{h\to 0} h\left[4n+h(1+2+3+-----+(n-1))\right]$$

$$\Rightarrow I=\lim_{h\to 0} h\left[4n+h\left(\frac{n(n-1)}{2}\right)\right] \qquad \left[\because h\to 0\ \&\ h=\frac{3}{n}\Rightarrow n\to\infty\right]$$

$$\Rightarrow I=\lim_{n\to\infty}\frac{3}{n}\left[4n+\frac{3}{n}\frac{(n^2-1)}{2}\right]$$

$$\Rightarrow I=\lim_{n\to\infty} 12+\frac{9}{2}\left(1-\frac{1}{n}\right)$$

$$=12+\frac{9}{2}=\frac{33}{2}$$

$$\therefore \int_0^3 (x+4)\,dx=\frac{33}{2}$$

Definite Integrals Ex 20.5 Q2

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----+f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here $a = 0,\ b = 2$

$$\Rightarrow h = \frac{2}{n}\ \&\ f(x) = x+3$$

Thus, we have,

$$I = \int_0^2 (x+3)\,dx$$

$$\Rightarrow I = \lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+----f((n-1)h)\right]$$

$$\Rightarrow I = \lim_{h\to 0} h\left[3+(h+3)+(2h+3)+(3h+3)----+(n-1)h+3\right]$$

$$= \lim_{h\to 0} h\left[3n+h(1+2+3+----(n-1))\right]$$

$$= \lim_{h\to 0} h\left[3n+h\frac{n(n-1)}{2}\right]$$

$$\because h = \frac{2}{n}\ \&\text{if } h\to 0 \Rightarrow n\to\infty$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[3n+\frac{2}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} \left[6+\frac{2}{n}n^2\left(1-\frac{1}{n}\right)\right]$$

$$= 6+2 = 8$$

$$\therefore \int_0^2 (x+3)\,dx = 8$$

Definite Integrals Ex 20.5 Q3

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----+f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here $a = 1,\ b = 3$ and $f(x) = 3x-2$

$$h = \frac{2}{n} \Rightarrow nh = 2$$

Thus, we have,

$$I = \int_1^3 (3x-2)\,dx$$

$$\Rightarrow I = \lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+----f(1+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[1+\{3(1+h)-2\}+\{3(1+2h)-2\}+-----+\ \{3(1+(n-1)h)-2\}\right]$$

$$= \lim_{h\to 0} h\left[n+3h(1+2+3+----(n-1))\right]$$

$$= \lim_{h\to 0} h\left[n+3h\frac{n(n-1)}{2}\right]$$

$$\because h = \frac{2}{n} \qquad \therefore \text{if } h\to 0 \Rightarrow n\to\infty$$

$$\therefore \lim_{n\to\infty} \frac{2}{n}\left[n+\frac{6}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} 2+\frac{6}{n^2}n^2\left(1-\frac{1}{n}\right)$$

$$= \lim_{n\to\infty} 2+6 = 8$$

$$\therefore \int_1^3 (3x-2)\,dx = 8$$

Definite Integrals Ex 20.5 Q4

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----+f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here $a = -1,\ b = 1$ and $f(x) = x+3$

$\therefore h = \dfrac{2}{n} \Rightarrow nh = 2$

Thus, we have,

$$I = \int_{-1}^{1} (x+3)\,dx$$

$$I = \lim_{h\to 0} h\left[f(-1)+f(-1+h)+f(-1+2h)+----+f(-1+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[2+(2+h)+(2+2h)+----+\{(n-1)h+2\}\right]$$

$$= \lim_{h\to 0} h\left[2n+h(1+2+3+----)\right]$$

$$= \lim_{h\to 0} h\left[2n+h\frac{n(n-1)}{2}\right] \qquad \left[\because h = \frac{2}{n}\ \&\text{ if } h\to 0 \Rightarrow n\to\infty\right]$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[2n+\frac{2}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} 4+\frac{2n^2}{n^2}\left(1-\frac{1}{n}\right)$$

$$= 4+2 = 6$$

$$\therefore \int_{-1}^{1} (x+3)\,dx = 6$$

Definite Integrals Ex 20.5 Q5

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here $a = 0,\ b = 5$

and $f(x) = (x+1)$

$\therefore h = \dfrac{5}{n} \Rightarrow nh = 5$

Thus, we have,

$$I = \int_0^5 (x+1)\,dx$$

$$I = \lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+----f\{(n-1)h\}\right]$$

$$= \lim_{h\to 0} h\left[1+(h+1)+(2h+1)+-----+((n-1)h+1)\right]$$

$$= \lim_{h\to 0} h\left[n+h(1+2+3+----(n-1))\right]$$

$\because h = \dfrac{5}{n}$ and if $h\to 0,\ n\to\infty$

$$= \lim_{n\to\infty} \frac{5}{n}\left[n+\frac{5}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} 5+\frac{25}{2n^2}n^2\left(1-\frac{1}{n}\right)$$

$$= 5+\frac{25}{2}$$

$$\therefore \int_0^5 (x+1)\,dx = \frac{35}{2}$$

Definite Integrals Ex 20.5 Q6

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 1$, $b = 3$

and $f(x) = (2x+3)$

$\therefore\ h = \dfrac{2}{n} \Rightarrow nh = 2$

Thus, we have,

$$I = \int_1^3 (2x+3)\,dx$$

$$= \lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+----f(1+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[2+3+\{2(1+h)+3\}+\{2(1+2h)+3\}----+2\{1+(n-1)+3\}\right]$$

$$= \lim_{h\to 0} h\left[5+(5+2h)+(5+4h)+----5+2(n-1)h\right]$$

$$= \lim_{h\to 0} h\left[5n+2h(1+2+3+----(n-1))\right]$$

$\because h = \dfrac{2}{n}$ and if $h\to 0 \Rightarrow n\to\infty$

$$\therefore \lim_{n\to\infty} \frac{2}{n}\left[5n+\frac{4}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} \left[10+\frac{4}{n^2}\frac{n(n-1)}{2}\right] = 14$$

$$\therefore \int_1^3 (2x+3)\,dx = 14$$

Definite Integrals Ex 20.5 Q7

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 3$, $b = 5$

and $f(x) = (2-x)$

$\therefore\ h = \dfrac{2}{n} \Rightarrow nh = 2$

Thus, we have,

$$I = \int_3^5 (2-x)\,dx$$

$$= \lim_{h\to 0} h\left[f(3)+f(3+h)+f(3+2h)+----f(3+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[(2-3)+\{2-(3+h)\}+\{2-(3+2h)\}+----\{2-(3+(n-1)h\}\right]$$

$$= \lim_{h\to 0} h\left[-1+(-1-h)+(-1-2h)+----\{-1-(n-1)h\}\right]$$

$$= \lim_{h\to 0} h\left[-n-h(1+2+----(n-1)h\right]$$

$\because h = \dfrac{2}{n}$ & if $h\to 0 \Rightarrow n\to\infty$

$$\therefore \lim_{n\to\infty} \frac{2}{n}\left[-n-\frac{2}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} -2-\frac{2}{n^2}n^2\left(1-\frac{1}{n}\right) = -2-2 = -4$$

$$\therefore \int_3^5 (2-x)\,dx = -4$$

Definite Integrals Ex 20.5 Q8

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here $a = 0,\ b = 2$ and $f(x) = (x^2+1)$

$\therefore\ h = \dfrac{2}{n} \Rightarrow nh = 2$

Thus, we have,

$$I = \int_0^2 (x^2+1)dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+----f\{(n-1)h\}\right]$$

$$= \lim_{h\to 0} h\left[1+(h^2+1)+\{(2h)^2+1\}+----\{(n-1)h)^2+1\}\right]$$

$$= \lim_{h\to 0} h\left[n+h^2\left(1+2^2+3^2+----+(n-1)^2\right)\right]$$

$\because h = \dfrac{2}{n}$ & if $h \Rightarrow 0 \Rightarrow n \to \infty$

$$\therefore \lim_{n\to\infty} \frac{2}{n}\left[n+\frac{4}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty} 2+\frac{4}{3n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= 2+\frac{4}{3}\times 2 = \frac{14}{3}$$

$$\therefore \int_0^2 (x^2+1)dx = \frac{14}{3}$$

Definite Integrals Ex 20.5 Q9

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here $a = 1,\ b = 2$ and $f(x) = x^2$

$\therefore\ h = \dfrac{1}{n} \Rightarrow nh = 1$

Thus, we have,

$$I = \int_1^2 x^2 dx$$

$$= \lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+----f(1+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[1+(1+h)^2+(1+2h)^2+-----+(1+(n-1)h\ )^2\right]$$

$$= \lim_{h\to 0} h\left[1+(1+2h+h^2)+(1+2\times 2h+2\times 2h^2)+----(1+2\times(n-1)h+(1-n)^2h^2)\right]$$

$$= \lim_{h\to 0} h\left[n+2h\{1+2+3---(n-1)\}+h^2\{1^2+2^2+3^2+---(n-1)^2\}\right]$$

$\because h = \dfrac{1}{n}$ & if $h \to 0 \Rightarrow n \to \infty$

$$= \lim_{n\to\infty} \frac{1}{n}\left[n+\frac{2}{n}\frac{n(n-1)}{2}+\frac{1}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty} 1+\frac{n^2}{n^2}\left(1-\frac{1}{n}\right)+\frac{1}{6n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= 1+1+\frac{2}{6} = \frac{7}{3}$$

$$\therefore \int_1^2 x^2 dx = \frac{7}{3}$$

Definite Integrals Ex 20.5 Q10

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h\right]$$

where $h = \dfrac{b-a}{n}$

Here $a=2,\ b=3$ and $f(x)=2x^2+1$

$\therefore\ h=\dfrac{1}{n}\ \Rightarrow\ nh=1$

Thus, we have,

$$I=\int_2^3\left(2x^2+1\right)dx$$

$$=\lim_{h\to 0} h\left[f(2)+f(2+h)+f(2+2h)+----f(2+(n-1)h)\right]$$

$$=\lim_{h\to 0} h\left[\left(2\times 2^2+1\right)\left\{2(2+h)^2+1\right\}+\left\{2(2+2h)^2+1\right\}+-----+\left\{2\left(2+(n-1)h\right)^2+1\right\}\right]$$

$$=\lim_{h\to 0} h\left[9n+8h(1+2+3---)+2h^2\left(1^2+2^2+3^2+--\right)\right]$$

$\because h=\dfrac{1}{n}$ & if $h\to 0\Rightarrow n\to\infty$

$$=\lim_{n\to\infty}\frac{1}{n}\left[9n+\frac{8}{n}\frac{n(n-1)}{2}+\frac{2}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$=\lim_{n\to\infty} 9+\frac{4}{n^2}n^2\left(1-\frac{1}{n}\right)+\frac{1}{3n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$=9+4+\frac{2}{3}=\frac{41}{3}$$

$$\therefore\ \int_2^3\left(2x^2+1\right)dx=\frac{41}{3}$$

Definite Integrals Ex 20.5 Q11

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h\right]$$

where $h = \dfrac{b-a}{n}$

Here $a=1,\ b=2$ and $f(x)=x^2-1$

$\therefore\ h=\dfrac{1}{n}\ \Rightarrow\ nh=1$

Thus, we have,

$$I=\int_1^2\left(x^2-1\right)dx$$

$$=\lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+----f(1+(n-1)h)\right]$$

$$=\lim_{h\to 0} h\left[\left(1^2-1\right)\left\{(1+h)^2-1\right\}+\left\{(1+2h)^2-1\right\}+-----+\left\{\left(1+(n-1)h\right)^2-1\right\}\right]$$

$$=\lim_{h\to 0} h\left[0+2h(1+2+3+---)+h^2\left(1+2^2+3^2+--\right)\right]$$

$\because h=\dfrac{1}{n}$ & if $h\to 0\Rightarrow n\to\infty$

$$=\lim_{n\to\infty}\frac{1}{n}\left[\frac{2}{n}\frac{n(n-1)}{2}+\frac{1}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n^2}n^2\left(1-\frac{1}{n}\right)+\frac{1}{6n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$=1+\frac{2}{6}=\frac{4}{3}$$

$$\therefore\ \int_1^2\left(x^2-1\right)dx=\frac{4}{3}$$

Definite Integrals Ex 20.5 Q12

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$
where $h = \dfrac{b-a}{n}$

Here $a=0$, $b=2$ and $f(x)=x^2+4$

$\therefore h = \dfrac{2}{n} \Rightarrow nh = 2$

Thus, we have,

$$I = \int_0^2 (x^2+4)dx$$
$$= \lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+----f(0+(n-1)h)\right]$$
$$= \lim_{h\to 0} h\left[4(h^2+4)+\{(2h)^2+4\}+----\{(n-1)h^2+4\}\right]$$
$\because h = \dfrac{2}{n}$ & if $h\to 0 \Rightarrow n\to\infty$
$$= \lim_{n\to\infty} \frac{2}{n}\left[4n+\frac{4}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$
$$= \lim_{n\to\infty} 8+\frac{4}{3n^2}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$
$$= 8+\frac{4\times 2}{3} = \frac{32}{3}$$

$$\therefore \int_0^2 (x^2+4)dx = \frac{32}{3}$$

Definite Integrals Ex 20.5 Q13

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$
where $h = \dfrac{b-a}{n}$

Here $a=1$, $b=4$ and $f(x)=x^2-x$

$h = \dfrac{3}{n} \Rightarrow nh = 3$

Thus, we have,

$$I = \int_1^4 (x^2-x)dx$$
$$= \lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+----f(1+(n-1)h)\right]$$
$$= \lim_{h\to 0} h\left[(1^2-1)+\{(1+h)^2-(1+h)\}+\{(1+2h)^2-(1+h)\}+----\right]$$
$$= \lim_{h\to 0} h\left[0+(h+h^2)+\{2h+(2h)^2\}+----\right]$$
$$= \lim_{h\to 0} h\left[h+(1+2+3+---(n-1))+h^2\{(1+2^2+3^2+---(n-1)^2\}\right]$$
$\because h = \dfrac{3}{n}$ & if $h\to 0 \Rightarrow n\to\infty$
$$= \lim_{n\to\infty} \frac{3}{n}\left[\frac{3}{n}\frac{n(n-1)}{2}+\frac{9}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$
$$= \lim_{n\to\infty} \frac{9}{n^2}n^2\left(1-\frac{1}{n}\right)+\frac{3}{2n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$
$$= \frac{9}{2}+3 = \frac{27}{2}$$

$$\therefore \int_1^4 (x^2-x)dx = \frac{27}{2}$$

Definite Integrals Ex 20.5 Q14

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here, $a = 0$, $b = 1$ and $f(x) = 3x^2+5x$

$$h = \frac{1}{n} \Rightarrow nh = 1$$

Thus, we have,

$$I = \int_0^1 \left(3x^2+5x\right)dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(0+h)+f(0+2h)+----f(0+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[\left\{0+\left(3h^2+5h\right)+\left\{3(2h)^2+5(2h)\right\}+----\right]$$

$$= \lim_{h\to 0} h\left[\left\{3h^2\left(1+2^2+3^2+---(n-1)^2\right)\right\}+5h\left\{1+2+3+---(n-1)\right\}\right]$$

$$\because h = \frac{1}{n} \text{ if } h \to 0 \Rightarrow n \to \infty$$

$$= \lim_{n\to\infty} \frac{1}{n}\left[\frac{3}{n^2}\frac{n(n-1)(2n-1)}{6}+\frac{5}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} \frac{3}{n^3}\frac{n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)}{6}+\frac{5}{2n^2}n^2\left(1-\frac{1}{n}\right)$$

$$= \frac{3\times 2}{6}+\frac{5}{2} = \frac{7}{2}$$

$$\therefore \int_0^1 \left(3x^2+5x\right)dx = \frac{7}{2}$$

Definite Integrals Ex 20.5 Q15

We have

$$\int_a^b f(x) = dx \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+...+f(a+(n-1)h)\right]$$

Where $h = \dfrac{b-a}{n}$

Here

$a = 0, b = 2$ and $f(x) = e^x$

Now

$$h = \frac{2}{n}$$

$$nh = 2$$

Thus, we have

$$I = \int_0^2 e^x dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+...+f((n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[1+e^h+e^{2h}+...+e^{(n-1)h}\right]$$

$$= \lim_{h\to 0} h\left\{\frac{\left(e^h\right)^n-1}{e^h-1}\right\}$$

$$= \lim_{h\to 0} h\left\{\frac{e^{nh}-1}{e^h-1}\right\}$$

$$= \lim_{h\to 0} h\left\{\frac{e^2-1}{e^h-1}\right\} \quad [nh = 2]$$

$$= \lim_{h\to 0}\left\{\frac{e^2-1}{\frac{e^h-1}{h}}\right\}$$

$$= e^2-1$$

Definite Integrals Ex 20.5 Q16

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here, $a = a,\ b = b$ and $f(x) = e^x$

$\therefore\ h = \dfrac{b-a}{n} \Rightarrow nh = b-a$

Thus, we have,

$$I = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[e^a + e^{a+h} + e^{a+2h} + ----e^{a+(n-1)h}\right]$$

$$= \lim_{h\to 0} he^a\left[1+e^h+e^{2h}+e^{3h}+----e^{(n-1)h}\right]$$

$$= \lim_{h\to 0} he^a\left[1+e^h+\left(e^h\right)^2+\left(e^h\right)^3+----\left(e^h\right)^{n-1}\right]$$

$$= \lim_{h\to 0} he^a\left\{\frac{\left(e^h\right)^n-1}{e^h-1}\right\} \qquad \left[\because a+ar+ar^2+----ar^{n-1} = a\left\{\frac{r^n-1}{r-1}\right\} \text{ if } r>1\right]$$

$$= \lim_{h\to 0} he^a n\left\{\frac{e^{nh}-1}{nh}\right\}\times\left(\frac{h}{e^{h-1}}\right) \qquad \left[\because \lim_{\theta\to 0}\frac{e^\theta-1}{\theta} = 1 \quad \& \quad nh = b-a\right]$$

$$\therefore \lim_{h\to 0}\left(e^{b-a}-1\right) = e^b - e^a$$

$$\therefore \int_a^b e^x dx = e^b - e^a$$

Definite Integrals Ex 20.5 Q17

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+\ldots+f(a+(n-1)h)\right], \text{ where } h = \frac{b-a}{n}.$$

Since we have to find $\int_a^b \cos x dx$

We have, $\quad f(x) = \cos x$

$\therefore \quad I = \int_a^b \cos x dx$

$\Rightarrow \quad I = \lim_{h\to 0} h\left[\cos a + \cos(a+h)+\cos(a+2h)+\ldots+\cos(a+(n-1)h)\right]$

$\Rightarrow \quad I = \lim_{h\to 0} h\left[\dfrac{\cos\left(a+(n-1)\frac{h}{2}\right)\sin\frac{nh}{2}}{\sin\frac{h}{2}}\right] = \lim_{h\to 0} h\left[\dfrac{\cos\left(a+\frac{nh}{2}-\frac{h}{2}\right)\sin\frac{nh}{2}}{\sin\frac{h}{2}}\right]$

$\Rightarrow \quad I = \lim_{h\to 0} h\left[\dfrac{\cos\left(a+\frac{b-a}{2}-\frac{h}{2}\right)\sin\left(\frac{b-a}{2}\right)}{\sin\frac{h}{2}}\right] \qquad [\because nh = b-a]$

$\Rightarrow \quad I = \lim_{h\to 0}\left[\dfrac{\frac{h}{2}}{\sin\frac{h}{2}}\times 2\cos\left(\frac{a+b}{2}-\frac{h}{2}\right)\sin\left(\frac{b-a}{2}\right)\right]$

$\Rightarrow \quad I = \lim_{h\to 0}\left(\dfrac{\frac{h}{2}}{\sin\frac{h}{2}}\right)\times\lim_{h\to 0} 2\cos\left(\frac{a+b}{2}-\frac{h}{2}\right)\sin\left(\frac{b-a}{2}\right) = 2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{b-a}{2}\right)$

$\Rightarrow \quad I = \sin b - \sin a \qquad [\because 2\cos A\sin B = \sin(A-B)-\sin(A+B)]$

Definite Integrals Ex 20.5 Q18

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 0$, $b = \frac{\pi}{2}$ and $f(x) = \sin x$

$$\therefore \quad h = \frac{\frac{\pi}{2}-0}{n} = \frac{\pi}{2n} \qquad nh = \frac{2}{\pi}$$

Thus, we have,

$$I = \int_0^{\frac{\pi}{2}} \sin x\, dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(0+h)+f(0+2h)+----f\{0+(n-1)h\}\right]$$

$$= \lim_{h\to 0} h\left[\sin 0+\sin h+\sin 2h+----\sin(n-1)h\right]$$

$$= \lim_{h\to 0} h\left[\frac{\sin\left(\frac{nh}{2}-\frac{h}{2}\right)\times \sin\frac{nh}{2}}{\sin\frac{h}{2}}\right]$$

$$= \lim_{h\to 0} h\left[\frac{\sin\left(\frac{\pi}{4}-\frac{h}{2}\right)\times \sin\frac{\pi}{4}}{\sin\frac{h}{2}}\right]$$

$$\left[\therefore \lim_{h\to 0}\frac{\sin\theta}{\theta}=1\right] \qquad \therefore \lim_{h\to 0}\frac{h}{\sin\frac{h}{2}}\left[\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\right]$$

$$= 2\times\frac{1}{2} = 1$$

$$\therefore \int_0^{\frac{\pi}{2}} \sin x\, dx = 1$$

Definite Integrals Ex 20.5 Q19

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here, $a = 0,\ b = \dfrac{\pi}{2}$ and $f(x) = \cos x$

$$\therefore\quad h = \frac{\frac{\pi}{2}-0}{n} = \frac{\pi}{2n} \qquad nh = \frac{2}{\pi}$$

Thus, we have,

$$I = \int_0^{\frac{\pi}{2}} \cos x\,dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(0+h)+f(0+2h)+----f\{0+(n-1)h\}\right]$$

$$= \lim_{h\to 0} h\left[\cos 0+\cos h+\cos 2h+----\cos(n-1)h\right]$$

$$= \lim_{h\to 0} h\left[\frac{\cos\left(\frac{nh}{2}-\frac{h}{2}\right)\times\cos\frac{nh}{2}}{\cos\frac{h}{2}}\right]$$

$$= \lim_{h\to 0} h\left[\frac{\cos\left(\frac{\pi}{4}-\frac{h}{2}\right)\times\cos\frac{\pi}{4}}{\cos\frac{h}{2}}\right]$$

$$\left[\because \lim_{h\to 0}\frac{\cos\theta}{\theta} = 1\right] \qquad \therefore \lim_{h\to 0}\frac{h}{\cos\frac{h}{2}}\left[\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\right]$$

$$= 2\times\frac{1}{2} = 1$$

$$\therefore \int_0^{\frac{\pi}{2}} \cos x\,dx = 1$$

Definite Integrals Ex 20.5 Q20

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h\right]$$

where $h = \dfrac{b-a}{n}$

Here, $a = 1,\ b = 4$ and $f(x) = 3x^2+2x$

$$I = \lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+----f(a+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[(3+2)+\left\{3(1+h)^2+2(1-+h)+\left\{3(1+2h)^2+2(1+2h)\right\}+----\right.\right]$$

$$= \lim_{h\to 0} h\left[5+8h(1+2+3+--)+3h^2\left(1+2^2+3^2+---\right)\right]$$

$$\because h = \frac{3}{n}\ \&\ \text{if } h\to 0 \Rightarrow n\to\infty$$

$$= \lim_{n\to\infty}\frac{3}{n}\left[5n+\frac{24}{n}\frac{n(n-1)}{2}+\frac{27}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty} 15+\frac{36}{n^2}n^2\left(1-\frac{1}{n}\right)+\frac{27}{2n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= 15+36+27 \quad = 78$$

$$\therefore \int_1^4 \left(3x^2+2x\right)dx = 78$$

Definite Integrals Ex 20.5 Q21

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here, $a = 0$, $b = 2$ and $f(x) = 3x^2 - 2$

$\therefore \quad h = \dfrac{2}{n} \Rightarrow nh = 2$

Thus, we have,

$$I = \int_0^2 \left(3x^2 - 2\right)dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(0+h)+f(0+2h)+----f\{0+(n-1)h\}\right]$$

$$= \lim_{h\to 0} h\left[-2+\left(3h^2-2\right)+\left\{3(2h)^2-2\right\}+----\right]$$

$$= \lim_{h\to 0} h\left[-2h+3h^2\left(1+2^2+3^2+----\right)\right]$$

$\because h = \dfrac{2}{n}$ if $h\to 0 \Rightarrow n\to\infty$

$$= \lim_{n\to\infty} \frac{2}{n}\left[-2n+\frac{12}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty} -4+\frac{4}{n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right) = -4+8 = 4$$

$$\therefore \int_0^2 \left(3x^2-2\right)dx = 4$$

Definite Integrals Ex 20.5 Q22

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here, $a = 0$, $b = 2$ and $f(x) = x^2 + 2$

$\therefore \quad h = \dfrac{2}{n} \Rightarrow nh = 2$

Thus, we have,

$$I = \int_0^2 \left(x^2 + 2\right)dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(0+h)+f(2h)+----f(0+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[2+\left(h^2+2\right)+\left\{(2h)^2+2\right\}+----\right]$$

$$= \lim_{h\to 0} h\left[2h+h^2\left(1+2^2+3^2+----(n-1)^2\right]\right.$$

$\because h = \dfrac{2}{n}$ & if $h\to 0 \Rightarrow n\to\infty$

$$= \lim_{n\to\infty} \frac{2}{n}\left[2n+\frac{4}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty} 4+\frac{4}{3n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= 4+\frac{8}{3} = \frac{20}{3}$$

$$\therefore \int_0^2 \left(x^2+2\right)dx = \frac{20}{3}$$

Definite Integrals Ex 20.5 Q23

It is known that,

$$\int_a^b f(x)\,dx=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+\ldots+f(a+(n-1)h)\right],\text{ where } h=\frac{b-a}{n}$$

Here, $a=0,\ b=4,$ and $f(x)=x+e^{2x}$

$$\therefore h=\frac{4-0}{n}=\frac{4}{n}$$

$$\Rightarrow \int_0^4\left(x+e^{2x}\right)dx=(4-0)\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f(h)+f(2h)+\ldots f((n-1)h)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\left(0+e^0\right)+\left(h+e^{2h}\right)+\left(2h+e^{2\cdot 2h}\right)+\ldots+\left\{(n-1)h+e^{2(n-1)h}\right\}\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[1+\left(h+e^{2h}\right)+\left(2h+e^{4h}\right)+\ldots+\left\{(n-1)h+e^{2(n-1)h}\right\}\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\left\{h+2h+3h+\ldots+(n-1)h\right\}+\left(1+e^{2h}+e^{4h}+\ldots+e^{2(n-1)h}\right)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[h\left\{1+2+\ldots(n-1)\right\}+\left(\frac{e^{2hn}-1}{e^{2h}-1}\right)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\frac{\left(h(n-1)n\right)}{2}+\left(\frac{e^{2hn}-1}{e^{2h}-1}\right)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\frac{4}{n}\cdot\frac{(n-1)n}{2}+\left(\frac{e^8-1}{e^{\frac{8}{n}}-1}\right)\right]$$

$$=4(2)+4\lim_{n\to\infty}\frac{\left(e^8-1\right)}{\left(\frac{e^{\frac{8}{n}}-1}{\frac{8}{n}}\right)8}$$

$$=8+\frac{4\cdot\left(e^8-1\right)}{8}\qquad\qquad\left(\lim_{x\to 0}\frac{e^x-1}{x}=1\right)$$

$$=8+\frac{e^8-1}{2}$$

$$=\frac{15+e^8}{2}$$

Definite Integrals Ex 20.5 Q24

We have,

$$\int_a^b f(x)dx=\lim_{h\to 0}h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

where $h=\dfrac{b-a}{n}$

Thus, we have,

$$I=\int_0^2\left(x^2+x\right)dx$$

$$=\lim_{h\to 0}h\left[f(0)+f(0+h)+f(0+2h)+----f\{(n-1)h\}\right]$$

$$=\lim_{h\to 0}h\left[0+\left(h^2+h\right)+\left\{(2h)^2+2h\right\}+----\right]$$

$$=\lim_{h\to 0}h\left[\left\{h^2\left(1+2^2+3^2+----(n-1)^2\right\}+h\left\{1+2+3----(n-1)\right\}\right]$$

$$\because h=\frac{2}{n}\ \ \&\text{ if } h\to 0\Rightarrow n\to\infty$$

$$=\lim_{n\to\infty}\frac{2}{n}\left[\frac{4}{n^2}\frac{n(n-1)(2n-1)}{6}+\frac{2}{n}\frac{n(n-1)}{2}\right]$$

$$=\lim_{n\to\infty}\frac{4}{3n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)+\frac{2}{n^2}n^2\left(1-\frac{1}{n}\right)$$

$$=\frac{8}{3}+2\ \ =\frac{14}{3}$$

$$\therefore\int_0^2\left(x^2+x\right)dx=\frac{14}{3}$$

Definite Integrals Ex 20.5 Q25

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here, $a = 0,\ b = 2$ and $f(x) = x^2+2x+1$

$\therefore\ h = \dfrac{2}{n} \Rightarrow nh = 2$

Thus, we have,

$$I = \int_0^2 \left(x^2+2x+1\right)dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+----f(0+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[1+\left(h^2+2h+1\right)+\left\{(2h)^2+2\times 2h+1\right\}+----\right]$$

$$= \lim_{h\to 0} h\left[n+h^2\left(1+2^2+3^2+----(n-1)^2+2h\left(1+2+3----(n-1)\right)\right]$$

$\because h = \dfrac{2}{n}$ if $h\to 0 \Rightarrow n\to\infty$

$$= \lim_{n\to\infty} \frac{2}{n}\left[n+\frac{4}{n^2}\frac{n(n-1)(2n-1)}{6}+\frac{4}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} 2+\frac{4}{3n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)+\frac{4}{n^2}n^2\left(1-\frac{1}{n}\right)$$

$$= 2+\frac{8}{3}+4 = \frac{26}{3}$$

$$\therefore \int_0^2 \left(x^2+2x+1\right)dx = \frac{26}{3}$$

Definite Integrals Ex 20.5 Q26

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here, $a = 0,\ b = 3$ and $f(x) = 2x^2+3x+5$

$\therefore\ h = \dfrac{3}{n} \Rightarrow nh = 3$

Thus, we have,

$$I = \int_0^3 \left(2x^2+3x+5\right)dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+----f((n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[5+\left(2h^2+3h+5\right)+\left\{2(2h)^2+3\times 2h+5\right\}+----\right]$$

$$= \lim_{h\to 0} h\left[5n+2h^2\left(1+2^2+3^2+----(n-1)^2+3h\left(1+2+3----(n-1)\right)\right)\right]$$

$\because h = \dfrac{3}{n}$ & if $h\to 0 \Rightarrow n\to\infty$

$$= \lim_{n\to\infty} \frac{2}{n}\left[5n+\frac{18}{n^2}\frac{n(n-1)(2n-1)}{6}+\frac{9}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} 15+\frac{9}{n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)+\frac{27}{2n^2}n^2\left(1-\frac{1}{n}\right)$$

$$= 15+18+\frac{27}{2} = \frac{93}{2}$$

$$\therefore \int_0^3 \left(2x^2+3x+5\right)dx = \frac{93}{2}$$

Definite Integrals Ex 20.5 Q27

It is known that,

$$\int_a^b f(x)\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+\ldots+f(a+(n-1)h)\right],\ \text{where } h=\frac{b-a}{n}$$

Here, $a=a$, $b=b$, and $f(x)=x$

$$\therefore \int_a^b x\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[a+(a+h)\ldots(a+2h)\ldots a+(n-1)h\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{a+a+a+\ldots+a}_{n\text{ times}}\right)+\left(h+2h+3h+\ldots+(n-1)h\right)\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[na+h\left(1+2+3+\ldots+(n-1)\right)\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[na+h\left\{\frac{(n-1)(n)}{2}\right\}\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[na+\frac{n(n-1)h}{2}\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{n}{n}\left[a+\frac{(n-1)h}{2}\right]$$

$$=(b-a)\lim_{n\to\infty}\left[a+\frac{(n-1)h}{2}\right]$$

$$=(b-a)\lim_{n\to\infty}\left[a+\frac{(n-1)(b-a)}{2n}\right]$$

$$=(b-a)\lim_{n\to\infty}\left[a+\frac{\left(1-\frac{1}{n}\right)(b-a)}{2}\right]$$

$$=(b-a)\left[a+\frac{(b-a)}{2}\right]$$

$$=(b-a)\left[\frac{2a+b-a}{2}\right]$$

$$=\frac{(b-a)(b+a)}{2}$$

$$=\frac{1}{2}\left(b^2-a^2\right)$$

Definite Integrals Ex 20.5 Q28

Let $I = \int_0^5 (x+1)\,dx$

It is known that,

$$\int_a^b f(x)\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)\ldots f(a+(n-1)h)\right], \text{ where } h=\frac{b-a}{n}$$

Here, $a=0$, $b=5$, and $f(x)=(x+1)$

$$\Rightarrow h=\frac{5-0}{n}=\frac{5}{n}$$

$$\begin{aligned}
\therefore \int_0^5 (x+1)\,dx &= (5-0)\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f\left(\frac{5}{n}\right)+\ldots+f\left((n-1)\frac{5}{n}\right)\right]\\
&= 5\lim_{n\to\infty}\frac{1}{n}\left[1+\left(\frac{5}{n}+1\right)+\ldots\left\{1+\left(\frac{5(n-1)}{n}\right)\right\}\right]\\
&= 5\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{1+1+1\ldots1}_{n\text{ times}}\right)+\left[\frac{5}{n}+2\cdot\frac{5}{n}+3\cdot\frac{5}{n}+\ldots(n-1)\frac{5}{n}\right]\right]\\
&= 5\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{5}{n}\{1+2+3\ldots(n-1)\}\right]\\
&= 5\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{5}{n}\cdot\frac{(n-1)n}{2}\right]\\
&= 5\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{5(n-1)}{2}\right]\\
&= 5\lim_{n\to\infty}\left[1+\frac{5}{2}\left(1-\frac{1}{n}\right)\right]\\
&= 5\left[1+\frac{5}{2}\right]\\
&= 5\left[\frac{7}{2}\right]\\
&= \frac{35}{2}
\end{aligned}$$

Definite Integrals Ex 20.5 Q29

It is known that,

$$\int_a^b f(x)\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+f(a+2h)\ldots f\{a+(n-1)h\}\right], \text{ where } h=\frac{b-a}{n}$$

Here, $a=2$, $b=3$, and $f(x)=x^2$

$$\Rightarrow h=\frac{3-2}{n}=\frac{1}{n}$$

$$\therefore \int_2^3 x^2 dx = (3-2)\lim_{n\to\infty}\frac{1}{n}\left[f(2)+f\left(2+\frac{1}{n}\right)+f\left(2+\frac{2}{n}\right)\ldots f\left\{2+(n-1)\frac{1}{n}\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[(2)^2+\left(2+\frac{1}{n}\right)^2+\left(2+\frac{2}{n}\right)^2+\ldots\left(2+\frac{(n-1)}{n}\right)^2\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[2^2+\left\{2^2+\left(\frac{1}{n}\right)^2+2\cdot2\cdot\frac{1}{n}\right\}+\ldots+\left\{(2)^2+\frac{(n-1)^2}{n^2}+2\cdot2\cdot\frac{(n-1)}{n}\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{2^2+\ldots+2^2}_{n\text{ times}}\right)+\left\{\left(\frac{1}{n}\right)^2+\left(\frac{2}{n}\right)^2+\ldots+\left(\frac{n-1}{n}\right)^2\right\}+2\cdot2\cdot\left\{\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+\ldots+\frac{(n-1)}{n}\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[4n+\frac{1}{n^2}\left\{1^2+2^2+3^2\ldots+(n-1)^2\right\}+\frac{4}{n}\left\{1+2+\ldots+(n-1)\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[4n+\frac{1}{n^2}\left\{\frac{n(n-1)(2n-1)}{6}\right\}+\frac{4}{n}\left\{\frac{n(n-1)}{2}\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[4n+\frac{n\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)}{6}+\frac{4n-4}{2}\right]$$

$$=\lim_{n\to\infty}\left[4+\frac{1}{6}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)+2-\frac{2}{n}\right]$$

$$=4+\frac{2}{6}+2$$

$$=\frac{19}{3}$$

Definite Integrals Ex 20.5 Q30

We have

$$\int_a^b f(x)=dx\lim_{h\to0}h\left[f(a)+f(a+h)+f(a+2h)+\ldots+f(a+(n-1)h)\right]$$

Where $h=\dfrac{b-a}{n}$

Here

$$a=1, b=3 \text{ and } f(x)=x^2+x$$

Now

$$h=\frac{2}{n}$$

$$nh=2$$

Thus, we have

$$I=\int_1^3\left(x^2+x\right)dx$$

$$=\lim_{h\to0}h\left[f(1)+f(1+h)+f(1+2h)+\ldots+f(1+(n-1)h)\right]$$

$$=\lim_{h\to0}h\left[\left(1^2+1\right)+\left\{(1+h)^2+(1+h)\right\}+\left\{(1+2h)^2+(1+2h)\right\}+\ldots\right]$$

$$=\lim_{h\to0}h\left[\left(1^2+(1+h)^2+(1+2h)^2+\ldots\right)+\left\{1+(1+h)+(1+2h)+\ldots\right\}\right]$$

$$=\lim_{h\to0}h\left[\left(n+2h(1+2+3+\ldots)+h^2\left(1+2^2+3^3+\ldots\right)\right)+\left(n+h(1+2+3+\ldots)\right)\right]$$

$$=\lim_{h\to0}h\left[\left(2n+3h(1+2+3+\ldots+(n-1))+h^2\left(1+2^2+3^3+\ldots+(n-1)^2\right)\right)\right]$$

$$\because h=\frac{2}{n} \text{ \& if } h\to0\Rightarrow n\to\infty$$

$$=\lim_{h\to\infty}\frac{2}{n}\left[2n+\frac{9}{n}\frac{n(n-1)}{2}+\frac{9}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$=\frac{38}{3}$$

Definite Integrals Ex 20.5 Q31

We have

$$\int_a^b f(x) = dx \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+...+f(a+(n-1)h)\right]$$

Where $h=\frac{b-a}{n}$

Here

$$a=0, b=2 \text{ and } f(x)=x^2-x$$

Now

$$h=\frac{2}{n}$$

$$nh=2$$

Thus, we have

$$I=\int_0^2 (x^2-x)dx$$

$$=\lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+...+f((n-1)h)\right]$$

$$=\lim_{h\to 0} h\left[\{(0)^2-(0)\}+\{(h)^2-(h)\}+\{(2h)^2-(2h)\}+...\right]$$

$$=\lim_{h\to 0} h\left[\left((h)^2+(2h)^2+...\right)-\{(h)+(2h)+...\}\right]$$

$$=\lim_{h\to 0} h\left[h^2\left(1+2^2+3^3+...+(n-1)^2\right)-h\{1+2+3+...+(n-1)\}\right]$$

$$\because h=\frac{2}{n} \text{ \& if } h\to 0 \Rightarrow n\to\infty$$

$$=\lim_{h\to\infty} \frac{2}{n}\left[\frac{9}{n^2}\frac{n(n-1)(2n-1)}{6}-\frac{9}{n}\frac{n(n-1)}{2}\right]$$

$$=\frac{2}{3}$$

Definite Integrals Ex 20.5 Q32

We have

$$\int_a^b f(x) = dx \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+...+f(a+(n-1)h)\right]$$

Where $h=\frac{b-a}{n}$

Here

$$a=1, b=3 \text{ and } f(x)=2x^2+5x$$

Now

$$h=\frac{2}{n}$$

$$nh=2$$

Thus, we have

$$I=\int_1^3 (2x^2+5x)dx$$

$$=\lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+...+f(1+(n-1)h)\right]$$

$$=\lim_{h\to 0} h\left[(2+5)+\{2(1+h)^2+5(1+h)\}+\{2(1+2h)^2+5(1+2h)\}+...\right]$$

$$=\lim_{h\to 0} h\left[\left(7n+9h(1+2+3+...)+2h^2(1+2^2+3^3+...)\right)\right]$$

$$\because h=\frac{2}{n} \text{ \& if } h\to 0 \Rightarrow n\to\infty$$

$$=\lim_{h\to\infty} \frac{2}{n}\left[7n+\frac{18}{n}\frac{n(n-1)}{2}+\frac{8}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$=\frac{112}{3}$$

Definite Integrals Ex 20.5 Q33

Given,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a) + f(a+h) + f(a+2h) + \ldots + f\left(a+(n-1)h\right)\right],$$

where $h = \dfrac{b-a}{n}$

Here, $f(x) = 3x^2 + 1,\ a = 1,\ b = 3$. Therefore, $h = \dfrac{3-1}{n} = \dfrac{2}{n}$

$$\therefore I = \int_1^3 \left(3x^2 + 1\right)dx$$

$$\Rightarrow I = \lim_{h\to 0} h\left[f(1) + f(1+h) + f(1+2h) + \ldots + f\left(1+(n-1)h\right)\right]$$

$$\Rightarrow I = \lim_{h\to 0} h\left[3(1)^2 + 1 + 3(1+h)^2 + 1 + 3(1+2h)^2 + 1 + \ldots + 3\left(1+(n-1)h\right)^2 + 1\right]$$

$$\Rightarrow I = \lim_{h\to 0} h\left[3n + n + 6h\left(1+2+3+\ldots+(n-1)\right) + 3h^2\left(1^2 + 2^2 + \ldots + (n-1)^2\right)\right]$$

$$\Rightarrow I = \lim_{n\to\infty} \frac{2}{n}\left[4n + \frac{12}{n}\left(1+2+3+\ldots+(n-1)\right) + 3\times\frac{4}{n^2}\left(1^2 + 2^2 + \ldots + (n-1)^2\right)\right]$$

$$\Rightarrow I = \lim_{n\to\infty}\left[8 + \frac{24}{n^2}\times\frac{n(n-1)}{2} + \frac{24}{n^3}\times\frac{(n-1)(n)(2n-1)}{6}\right]$$

$$\Rightarrow I = \lim_{n\to\infty}\left[8 + 12\left(1-\frac{1}{n}\right) + 4\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)\right]$$

$$\Rightarrow I = 8 + 12 + 4\times 2 = 28$$

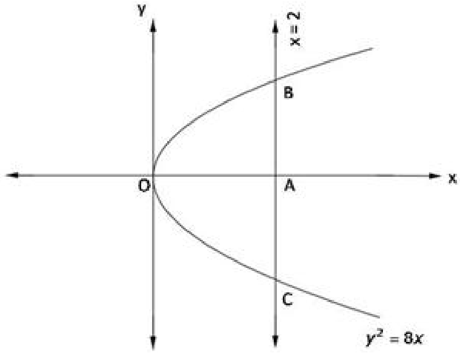
Ex 21.1

Areas of Bounded Regions Ex 21.1 Q1

Given equations are

$x = 2$ ---(1)
and $y^2 = 8x$ ---(2)

Equation (1) represents a line parallel to y-axis and equation (2) represents a parabola with vertex at origin and x-axis as its axis, A rough sketch is given as below:-



We have to find the area of shaded region . We sliced it in vertical rectangle width of rectangle = Δx,
Length = $(y - 0) = y$
Area of rectangle = $y \Delta x$

This rectangle can move horizontal from $x = 0$ to $x = 2$

Required area = Shaded region $OCBO$

$$= 2(\text{Shaded region } OABO)$$
$$= 2\int_0^2 y\,dx$$
$$= 2\int_0^2 \sqrt{8x}\,dx$$
$$= 2.2\sqrt{2}\int_0^2 \sqrt{x}\,dx$$
$$= 4\sqrt{2}\left[\frac{2}{3}x\sqrt{x}\right]_0^2$$
$$= 4\sqrt{2}\left[\left(\frac{2}{3}.2\sqrt{2}\right) - \left(\frac{2}{3}.0.\sqrt{0}\right)\right]$$
$$= 4\sqrt{2}\left(\frac{4\sqrt{2}}{3}\right)$$

Required area = $\frac{32}{3}$ square units

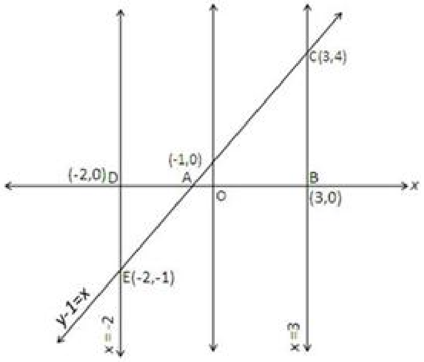
Areas of Bounded Regions Ex 21.1 Q2

To find area of region bounded by x-axis the ordinates $x = -2$ and $x = 3$ and

$$y - 1 = x \qquad ---(1)$$

Equation (1) is a line that meets at axes at $(0,1)$ and $(-1,0)$.

A rough sketch of the curve is as under:-



Shaded region is required area.

Required area = Region $ABCA$ + Region $ADEA$

$$A = \int_{-1}^{3} y\,dx + \left|\int_{-2}^{-1} y\,dx\right|$$

$$= \int_{-1}^{3} (x+1)\,dx + \left|\int_{-2}^{-1} (x+1)\,dx\right|$$

$$= \left(\frac{x^2}{2} + x\right)_{-1}^{3} + \left|\left(\frac{x^2}{2} + x\right)_{-2}^{-1}\right|$$

$$= \left[\left(\frac{9}{2} + 3\right) - \left(\frac{1}{2} - 1\right)\right] + \left|\left(\frac{1}{2} - 1\right) - (2 - 2)\right|$$

$$= \left[\frac{15}{2} + \frac{1}{2}\right] + \left|-\frac{1}{2}\right|$$

$$= 8 + \frac{1}{2}$$

$$A = \frac{17}{2} \text{ sq. units}$$

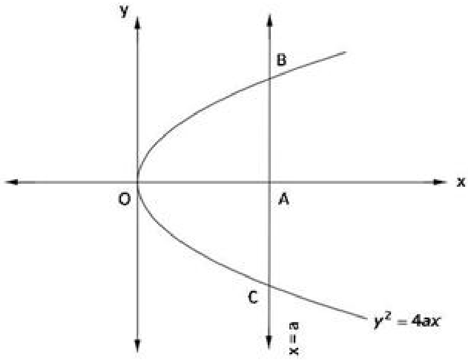
Areas of Bounded Regions Ex 21.1 Q3

We have to find the area of the region bounded by

$x = a$ $\quad ---(1)$

and $y^2 = 4ax$ $\quad ---(2)$

Equation (1) represents a line parallel to y-axis and equation (2) represents a parabola with vertex at origin and axis as x-axis. A rough sketch of the two curves is as below:-



We have to find the area of the shaded region. Now, we slice it in rectangles.
Width $= \Delta x$, Length $= y - 0 = y$

Area rectangle $= y\Delta x$

This approximating rectangle can move from $x = 0$ to $x = a$.

$$
\begin{aligned}
\text{Required area} &= \text{Region } OCBO \\
&= 2\left(\text{Region } OABO\right) \\
&= 2\int_0^a \sqrt{4ax}\, dx \\
&= 2.2\sqrt{a}\int_0^a \sqrt{x}dx \\
&= 4\sqrt{a}.\left(\frac{2}{3}x\sqrt{x}\right)_0^a \\
&= 4\sqrt{a}.\left(\frac{2}{3}a\sqrt{a}\right)
\end{aligned}
$$

Required area $= \dfrac{8}{3}a^2$ square units

Areas of Bounded Regions Ex 21.1 Q4

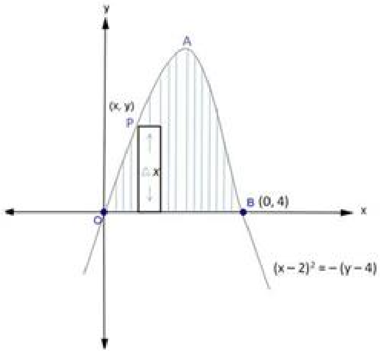
We have to find area bounded by x-axis and parabola

$y = 4x - x^2$

$\Rightarrow \quad x^2 - 4x + 4 = -y + 4$

$\Rightarrow \quad (x-2)^2 = -(y-4) \qquad ---(1)$

Equation (1) represents a downward parabola with vertex $(2,4)$ and passing through $(0,0)$ and $(0,4)$. A rough sketch is as below:-



the shaded region represents the required area. We slice the region in approximation rectangles with width $= \Delta x$, length $= y - 0 = y$

Area of rectangle $= y \Delta x$.

This approximation rectangle slide from $x = 0$ to $x = a$, so

Required area = Region $OABO$

$$= \int_0^4 \left(4x - x^2\right) dx$$

$$= \left(4\frac{x^2}{2} - \frac{x^3}{3}\right)_0^4$$

$$= \left(\frac{4 \times 16}{2} - \frac{64}{3}\right) - (0 - 0)$$

$$= \frac{64}{6}$$

Required area $= \dfrac{32}{3}$ square units

Areas of Bounded Regions Ex 21.1 Q5

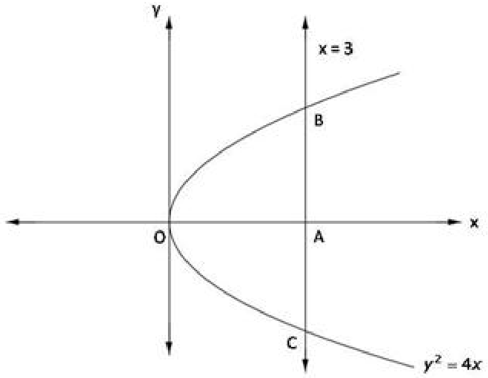
To find area bounded by

$y^2 = 4x$ ---(1)

and $x = 3$ ---(2)

Equation (1) represents a parabola with vertex at origin and axis as x-axis and equation (2) represents a line parallel to y-axis.

A rough sketch of the equations is as below:-



Shaded region represents the required area we slice this area with approximation rectangles with Width $=\Delta x$, length $= y - 0 = y$

Area of rectangle $= y\Delta x$.

This approximation rectangle can slide from $x = 0$ to $x = 3$, so

Required area = Region $OCBO$

$= 2(\text{Region } OABO)$

$= 2\int_0^3 y dx$

$= 2\int_0^3 \sqrt{4x} dx$

$= 4\int_0^3 \sqrt{x} dx$

$= 4\left(\frac{2}{3}x\sqrt{x}\right)_0^3$

$= \frac{8}{3}.3\sqrt{3}$

Required area $= 8\sqrt{3}$ square units

Areas of Bounded Regions Ex 21.1 Q6

We have to find the area enclosed by

$$y = 4 - x^2$$

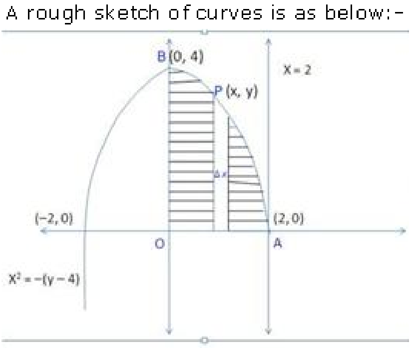
$$\Rightarrow \quad x^2 = -(y-4) \qquad ---(1)$$

$$x = 0 \qquad ---(2)$$

$$x = 2 \qquad ---(3)$$

Equation (1) represent a downward parabola with vertex at $(0,4)$ and passing through $(2,0),(-2,0)$. Equation (2) represents y-axis and equation (3) represents a line parallel to y-axis.

A rough sketch of curves is as below:-



Shaded region represents the required area. We slice this region into approximation rectangles with Width $= \Delta x$, length $= y - 0 = y$

Area of rectangle $= y\Delta x$.

This approximation rectangle move from $x = 0$ to $x = 2$, so

Required area = (Region $OABO$)

$$= \int_0^2 \left(4 - x^2\right) dx$$

$$= \left(4x - \frac{x^3}{3}\right)_0^2$$

$$= \left[4(2) - \frac{(2)^3}{3}\right] - [0]$$

$$= \left[\frac{24-8}{3}\right]$$

Required area $= \frac{16}{3}$ square units

Areas of Bounded Regions Ex 21.1 Q7

We have to find area enclosed by x-axis and

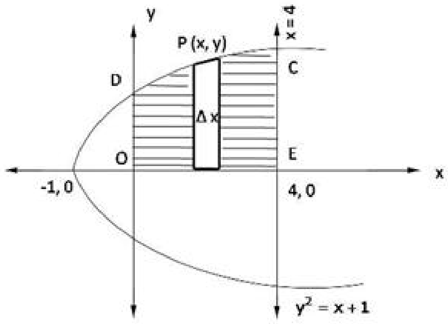
$$y = \sqrt{x+1}$$

$$\Rightarrow \quad y^2 = x + 1 \quad ---(1)$$

$$\text{and } x = 0 \quad ---(2)$$

$$x = 4 \quad ---(3)$$

Equation (1) represent a parabola with vertex at $(-1, 0)$ and passing through $(0,1)$ and $(0,-1)$. Equation (2) is y-axis and equation (3) is a line parallel to y-axis passing through $(4,0)$. So rough sketch of the curve is as below:-



We slice the required region in approximation rectangle with its Width $= \Delta x$, and length $= y - 0 = y$

Area of rectangle $= y\Delta x$.

Approximation rectangle moves from $x = 0$ to $x = 4$. So

$$\begin{aligned}
\text{Required area} &= \text{Shaded region} \\
&= (\text{Region } OECDO) \\
&= \int_0^4 y\,dx \\
&= \int_0^4 \sqrt{x+1}\,dx \\
&= \left(\frac{2}{3}(x+1)\sqrt{x+1}\right)_0^4 \\
&= \frac{2}{3}\left[\left((4+1)\sqrt{4+1}\right) - \left((0+1)\sqrt{0+1}\right)\right]
\end{aligned}$$

Required area $= \frac{2}{3}\left[5\sqrt{5} - 1\right]$ square units

Thus, Required area $= \frac{2}{3}\left(5^{\frac{3}{2}} - 1\right)$ square units

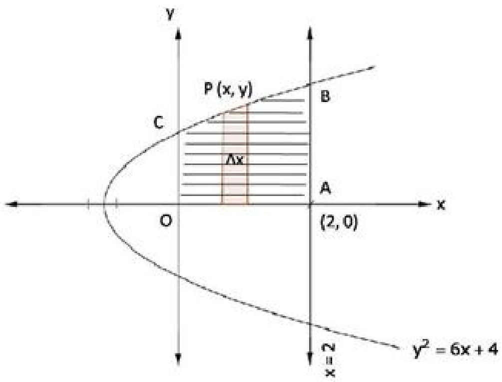
Areas of Bounded Regions Ex 21.1 Q8

We have to find area enclosed by x-axis

$x = 0,\ x = 2$ ---(1)

and $y^2 = 6x + 4$ ---(2)

Equation (1) represents y-axis and a line parallel to y-axis passing through (2,0) respectively. Equation (2) represents a parabola with vertex at $\left(-\frac{2}{3},0\right)$ and passes through the points $(0,2),(0,-2)$, so rough sketch of the curves is as below:-



Shaded region represents the required area. It is sliced in approximation rectangle with its Width $= \Delta x$, and length $= (y - 0) = y$

Area of rectangle $= y\Delta x$.

This approximation rectangle slide from $x = 0$ to $x = 2$, so

Required area = Region $OABCO$

$$= \int_0^2 \sqrt{6x+4}\,dx$$

$$= \left\{\frac{2}{3}\frac{(6x+4)\sqrt{6x+4}}{6}\right\}_0^2$$

$$= \frac{1}{9}\left[\left((12+4)\sqrt{12+4}\right) - \left((0+4)\sqrt{0+4}\right)\right]$$

$$= \frac{1}{9}\left[16\sqrt{16} - 4\sqrt{4}\right]$$

$$= \frac{1}{9}(64-8)$$

Required area $= \frac{56}{9}$ square units

Areas of Bounded Regions Ex 21.1 Q9

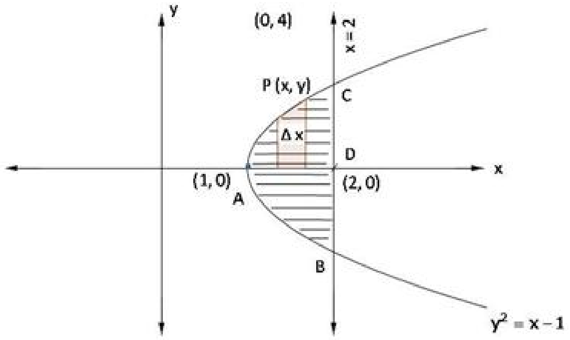
We have to find area enclosed by

$y^2 = x - 1$ --- (1)

and $x = 2$ --- (2)

Equation (1) is a parabola with vertex at (1,0) and axis as x-axis. Equation (2) represents a line parallel to y-axis passing through (2,0).

A rough sketch of curves is as below:-



Shaded region shows the required area. We slice it in approximation rectangle with its Width $= \Delta x$ and length $= y - 0 = y$

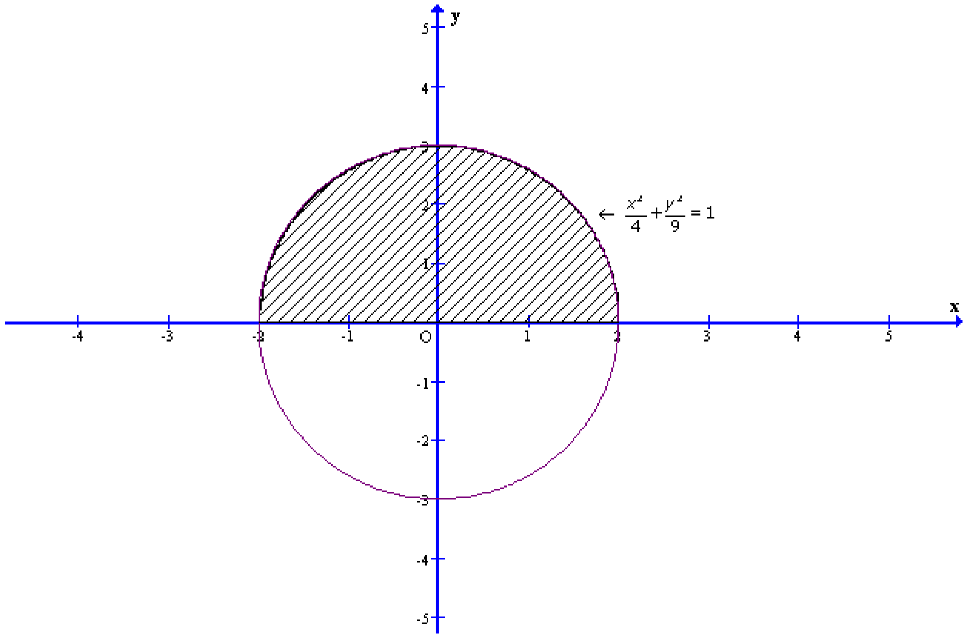
Area of the rectangle $= y\Delta x$.

This rectangle can slide from $x = 1$ to $x = 2$, so

$$\begin{aligned}\text{Required area} &= \text{Region } ABCA \\ &= 2\left(\text{Region } AOCA\right) \\ &= 2\int_1^2 y\,dx \\ &= 2\int_1^2 \sqrt{x-1}\,dx \\ &= 2\left(\frac{2}{3}(x-1)\sqrt{x-1}\right)_1^2 \\ &= \frac{4}{3}\left[\left((2-1)\sqrt{2-1}\right) - \left((1-1)\sqrt{1-1}\right)\right] \\ &= \frac{4}{3}(1-0)\end{aligned}$$

Required area $= \frac{4}{3}$ square units

Areas of Bounded Regions Ex 21.1 Q10



It can be observed that ellipse is symmetrical about x-axis.

Area bounded by ellipse $= 2\int_0^2 y\,dx$

$= 2\int_0^2 3\sqrt{1-\frac{x^2}{4}}\,dx$

$= 3\int_0^2 \sqrt{4-x^2}\,dx$

$= 3\left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$

$= 3\left[1(0)+2\sin^{-1}(1)-0-2\sin^{-1}(0)\right]$

$= 3[\pi]$

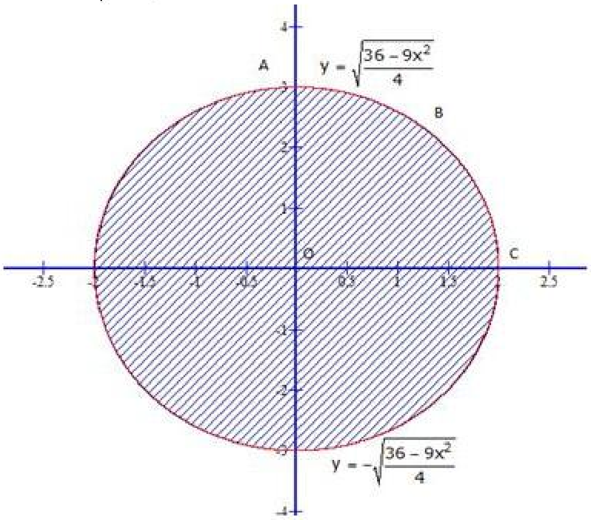
$= 3\pi$ sq. units

Areas of Bounded Regions Ex 21.1 Q11

$9x^2 + 4y^2 = 36$

$\Rightarrow \dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$

$\Rightarrow y = \pm\sqrt{\dfrac{36 - 9x^2}{4}}$



Area of Sector OABCO =

$\displaystyle\int_0^2 \sqrt{\frac{36 - 9x^2}{4}}\,dx$

$= \dfrac{3}{2}\displaystyle\int_0^2 \sqrt{4 - x^2}\,dx$

$= \dfrac{3}{2}\left[\dfrac{x\sqrt{4 - x^2}}{2} + \dfrac{2^2}{2}\sin^{-1}\left(\dfrac{x}{2}\right)\right]_0^2$

$= \dfrac{3}{2}\left[\dfrac{2\sqrt{4 - 2^2}}{2} + \dfrac{2^2}{2}\sin^{-1}\left(\dfrac{2}{2}\right)\right] - \dfrac{3}{2}\left[\dfrac{0\sqrt{4 - 0^2}}{2} + \dfrac{2^2}{2}\sin^{-1}\left(\dfrac{0}{2}\right)\right]$

$= \dfrac{3}{2}.2.\dfrac{\pi}{2} - 0$

$= \dfrac{3\pi}{2}$ sq. units

Area of the whole figure = 4 × Ar. D OABCO

$= 4 \times \dfrac{3\pi}{2}$

= 6p sq. units

Areas of Bounded Regions Ex 21.1 Q12

We have to find area enclosed between the curve and x-axis.

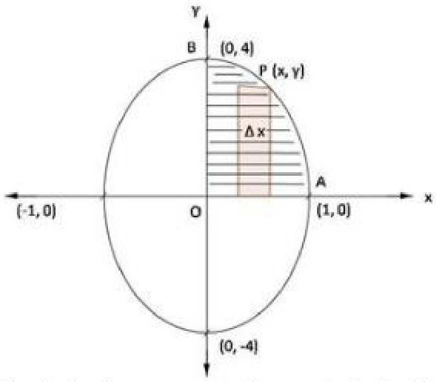
$y = 2\sqrt{1-x^2},\ x \in [0,1]$

$\Rightarrow \quad y^2 + 4x^2 = 4,\ x \in [0,1]$

$\Rightarrow \quad \frac{x^2}{1} + \frac{y^2}{4} = 1,\ x \in [0,1] \qquad\qquad ---(1)$

Equation (1) represents an ellipse with centre at origin and passes through $(\pm1,0)$ and $(0,\pm2)$ and $x \in [0,1]$ as represented by region between y-axis and line $x = 1$.

A rough sketch of curves is as below:-



Shaded region represents the required. We slice it into approximation rectangles of Width $= \Delta x$ and length $= y$

Area of the rectangle $= y\Delta x$.

The approximation rectangle slides from $x = 0$ to $x = 1$, so

Required area = Region $OAPBO$

$$= \int_0^1 y\,dx$$

$$= \int_0^1 2\sqrt{1-x^2}\,dx$$

$$= 2\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}(x)\right]_0^1$$

$$= 2\left[\left(\frac{1}{2}\sqrt{1-1} + \frac{1}{2}\sin^{-1}(1)\right) - (0+0)\right]$$

$$= 2\left[0 + \frac{1}{2}\cdot\frac{\pi}{2}\right]$$

Required area $= \frac{\pi}{2}$ square units

Areas of Bounded Regions Ex 21.1 Q13

To find area under the curves

$y = \sqrt{a^2 - x^2}$

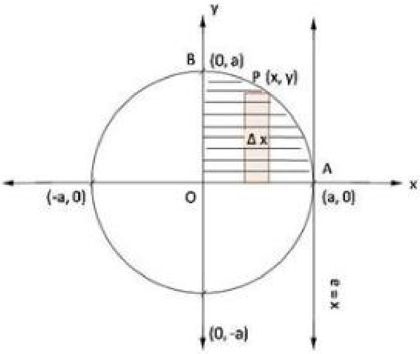
$\Rightarrow \quad x^2 + y^2 = a^2 \qquad ---(1)$

Between $x = 0 \qquad ---(2)$

$x = a \qquad ---(3)$

Equation (1) represents a circle with centre $(0,0)$ and passes axes at $(0,\pm a)$ $(\pm a,0)$ equation (2) represents y-axis and equation $x = a$ represent a line parallel to y-axis passing through $(a,0)$.

A rough sketch of the curves is as below:-



Shaded region represents the required area. We slice it into approximation rectangles of Width $= \Delta x$ and length $= y - 0 = y$

Area of the rectangle $= y\Delta x$.

The approximation rectangle can slide from $x = 0$ to $x = a$, so

Required area = Region $OAPBO$

$$= \int_0^a y\,dx$$

$$= \int_0^a \sqrt{a^2 - x^2}\,dx$$

$$= \left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$$

$$= \left[\left(\frac{a}{2}\sqrt{a^2 - a^2} + \frac{a^2}{2}\sin^{-1}(1)\right) - (0)\right]$$

$$= \left[0 + \frac{a^2}{2}\cdot\frac{\pi}{2}\right]$$

Required area $= \frac{\pi}{4}a^2$ square units

Areas of Bounded Regions Ex 21.1 Q14

To find area bounded by x-axis and

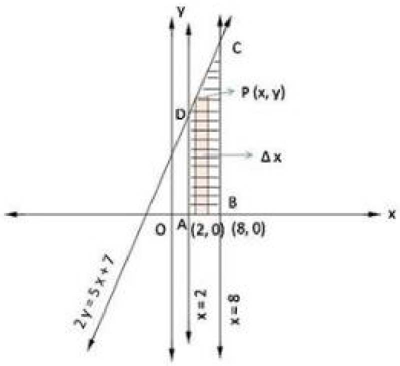
$2y = 5x + 7$ ---(1)

$x = 2$ ---(2)

$x = 8$ ---(3)

Equation (1) represents line passing through $\left(-\frac{7}{5}, 0\right)$ and $\left(0, \frac{7}{2}\right)$ equation (2), (3) shows line parallel to y-axis passing through $(2,0)$, $(8,0)$ respectively.

A rough sketch of curves is as below:-



Shaded region represents the required area. We slice the region into approximation rectangles of Width $= \Delta x$ and length $= y$

Area of the rectangle $= y\Delta x$.

This approximation rectangle slides from $x = 2$ to $x = 8$, so

Required area $=$ (Region $ABCDA$)

$$= \int_2^8 \left(\frac{5x+7}{2}\right) dx$$

$$= \frac{1}{2}\left(\frac{5x^2}{2} + 7x\right)_2^8$$

$$= \frac{1}{2}\left[\left(\frac{5(8)^2}{2} + 7(8)\right) - \left(\frac{5(2)^2}{2} + 7(2)\right)\right]$$

$$= \frac{1}{2}\left[(160 + 56) - (10 + 14)\right]$$

$$= \frac{192}{2}$$

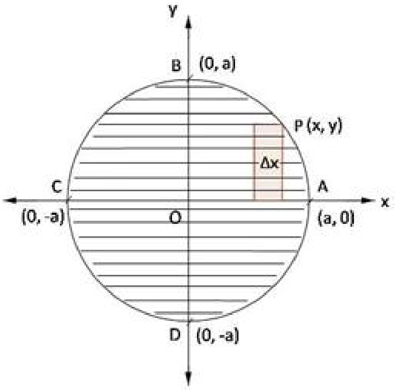
Required area $=$ 96 square units

Areas of Bounded Regions Ex 21.1 Q15

We have to find the area of circle

$$x^2 + y^2 = a^2 \qquad ---(1)$$

Equation (1) represents a circle with centre $(0,0)$ and radius a, so it meets the axes at $(\pm a, 0), (0, \pm a)$. A rough sketch of the curve is given below:-



Shaded region is the required area. We slice the region $AOBA$ in rectangles of width Δx and length $= y - 0 = y$

Area of rectangle $= y\Delta x$.

This approximation rectangle can slide from $x = 0$ to $x = a$, so

Required area = Region $ABCDA$

$= 4(\text{Region } ABOA)$

$= 4\left(\int_0^a y\,dx\right)$

$= 4\int_0^a \sqrt{a^2 - x^2}\,dx$

$= 4\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$

$= 4\left[\left(\frac{a}{2}\sqrt{a^2 - a^2} + \frac{a^2}{2}\sin^{-1}\frac{a}{a}\right) - (0 + 0)\right]$

$= 4\left[0 + \frac{a^2}{2}.\frac{\pi}{2}\right]$

$= 4\left(\frac{a^2\pi}{4}\right)$

Required area $= \pi a^2$ sq.units

Areas of Bounded Regions Ex 21.1 Q16

To find area enclosed by

$x = -2$, $x = 3$, $y = 0$ and $y = 1 + |x + 1|$

$\Rightarrow \quad y = 1 + x + 1$, if $x + 1 \geq 0$

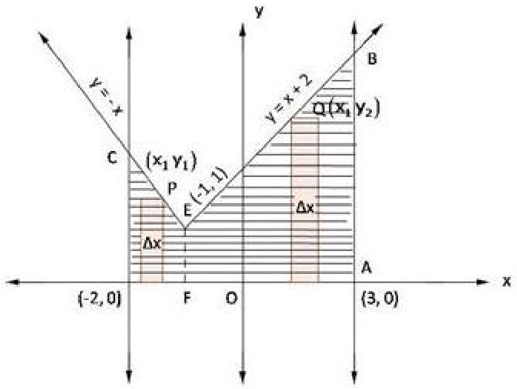
$\Rightarrow \quad y = 2 + x \qquad\qquad ---(1)$, if $x \geq -1$

And $\quad y = 1 - (x + 1)$, if $x + 1 < 0$

$\Rightarrow \quad y = 1 - x - 1$, if $x < -1$

$\Rightarrow \quad y = -x \qquad\qquad ---(2)$, if $x < -1$

So, equation (1) is a straight line that passes through $(0,2)$ and $(-1,1)$. Equation (2) is a line passing through $(-1,1)$ and $(-2,2)$ and it is enclosed by line $x = 2$ and $x = 3$ which are lines parallel to y-axis and pass through $(2,0)$ and $(3,0)$ respectively $y = 0$ is x-axis. So, a rough sketch of the curves is given as:-



Shaded region represents the required area.

So, required area = Region $(ABECDFA)$

Required area = $(\text{region } ABEFA + \text{region } ECDFE) \qquad ---(1)$

region $ECDFE$ is sliced into approximation rectangle with width Δx and length y_1.

Area of those approximation rectangle is $y_1 \Delta x$ and these slids from $x = -2$ to $x = -1$.

Region $ABEFA$ is sliced into approximation rectangle with width Δx and length y_2.

Area of those rectangle is $y_2 \Delta x$ which slides from $x = -1$ to $x = 3$. So, using equation (1),

$$\text{Required area} = \int_{-2}^{-1} y_1 dx + \int_{-1}^{3} y_2 dx$$

$$= \int_{-2}^{-1} (-x) dx + \int_{-1}^{3} (x + 2) dx$$

$$= -\left[\frac{x^2}{2}\right]_{-2}^{-1} + \left[\frac{x^2}{2} + 2x\right]_{-1}^{3}$$

$$= -\left[\frac{1}{2} - \frac{4}{2}\right] + \left[\left(\frac{9}{2} + 6\right) - \left(\frac{1}{2} - 2\right)\right]$$

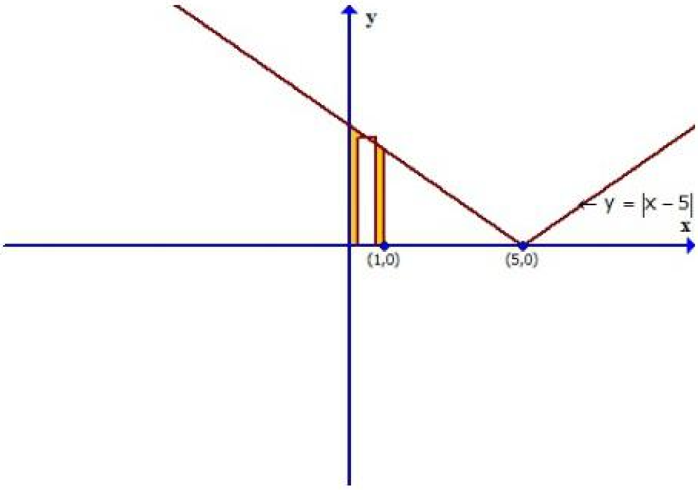
$$= \frac{3}{2} + \left(\frac{21}{2} + \frac{3}{2}\right)$$

$$= \frac{27}{2}$$

Required area = $\frac{27}{2}$ sq.units

Areas of Bounded Regions Ex 21.1 Q17

Consider the sketch of the given graph: $y = |x-5|$



Therefore,

Required area $= \int_0^1 y\,dx$

$= \int_0^1 |x-5|\,dx$

$= \int_0^1 -(x-5)\,dx$

$= \left[\frac{-x^2}{2} + 5x\right]_0^1$

$= \left[-\frac{1}{2} + 5\right]$

$= \frac{9}{2}$ *sq. units*

Therefore, the given integral represents the area bounded by the curves, $x = 0, y = 0$, $x = 1$ *and* $y = -(x-5)$.

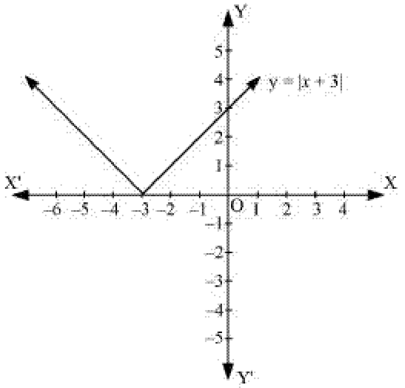
Areas of Bounded Regions Ex 21.1 Q18

The given equation is $y = |x+3|$

The corresponding values of x and y are given in the following table.

x	-6	-5	-4	-3	-2	-1	0
y	3	2	1	0	1	2	3

On plotting these points, we obtain the graph of $y = |x+3|$ as follows.



It is known that, $(x+3) \le 0$ for $-6 \le x \le -3$ and $(x+3) \ge 0$ for $-3 \le x \le 0$

$$\therefore \int_{-6}^{0} |(x+3)| dx = -\int_{-6}^{-3} (x+3) dx + \int_{-3}^{0} (x+3) dx$$

$$= -\left[\frac{x^2}{2} + 3x\right]_{-6}^{-3} + \left[\frac{x^2}{2} + 3x\right]_{-3}^{0}$$

$$= -\left[\left(\frac{(-3)^2}{2} + 3(-3)\right) - \left(\frac{(-6)^2}{2} + 3(-6)\right)\right] + \left[0 - \left(\frac{(-3)^2}{2} + 3(-3)\right)\right]$$

$$= -\left[-\frac{9}{2}\right] - \left[-\frac{9}{2}\right]$$

$$= 9$$

Areas of Bounded Regions Ex 21.1 Q19

We have,

$$y = |x+1| = \begin{cases} x+1, & \text{if } x+1 \geq 0 \\ -(x+1), & \text{if } x+1 < 0 \end{cases}$$

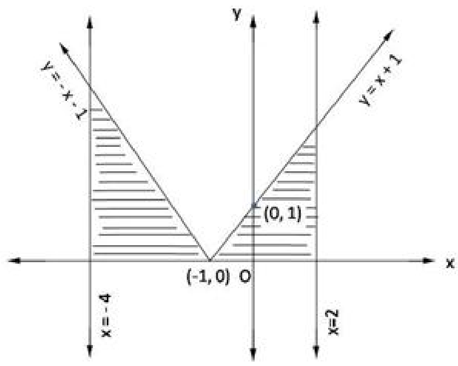
$$y = \begin{cases} (x+1), & \text{if } x \geq -1 \\ -x-1, & \text{if } x < -1 \end{cases}$$

$\Rightarrow \quad y = x+1 \qquad (1)$

and $y = -x-1 \qquad (2)$

Equation (1) represents a line which meets axes at $(0,1)$ and $(-1,0)$. Equation (2) represents a line passing through $(0,-1)$ and $(-1,0)$

A rough sketch is given below:-



$$\int_{-4}^{2} |x+1|\,dx = \int_{-4}^{-1} -(x+1)\,dx + \int_{-1}^{2} (x+1)\,dx$$

$$= -\left[\frac{x^2}{2}+x\right]_{-4}^{-1} + \left[\frac{x^2}{2}+x\right]_{-1}^{2}$$

$$= -\left[\left(\frac{1}{2}-1\right)-\left(\frac{16}{2}-4\right)\right]+\left[\left(\frac{4}{2}+2\right)-\left(\frac{1}{2}-1\right)\right]$$

$$= -\left[\left(-\frac{1}{2}-4\right)\right]+\left[4+\frac{1}{2}\right]$$

$$= \frac{9}{2}+\frac{9}{2}$$

$$= \frac{18}{2}$$

Required area = 9 sq. unit

Areas of Bounded Regions Ex 21.1 Q20

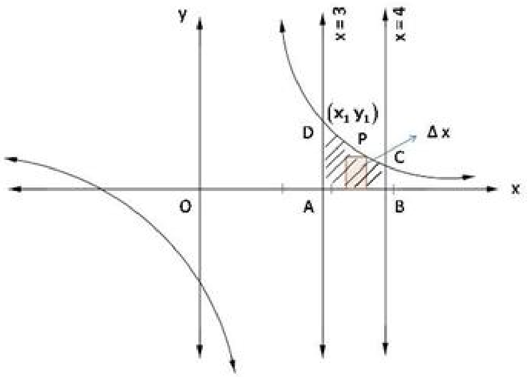
To find the area bounded by

x axis, $x = 3$, $x = 4$ and $xy - 3x - 2y - 10 = 0$

$\Rightarrow \quad y(x-2) = 3x + 10$

$\Rightarrow \quad y = \dfrac{3x+10}{x-2}$

A rough sketch of the curves is given below:-



Shaded region is required region.

It is sliced in rectangle with width $= \Delta x$ and length$= y$

Area of rectangle $= y\Delta x$

This approximation rectangle slide from $x = 3$ to $x = 4$. So,

Required area = Region $ABCDA$

$$= \int_3^4 y\,dx$$

$$= \int_3^4 \left(\frac{3x+10}{x-2}\right)dx$$

$$= \int_3^4 \left(3 + \frac{16}{x-2}\right)dx$$

$$= (3x)_3^4 + 16\{\log|x-2|\}_3^4$$

$$= (12 - 9) + 16(\log 2 - \log 1)$$

Required area $= (3 + 16\log 2)$ sq. units

Areas of Bounded Regions Ex 21.1 Q21

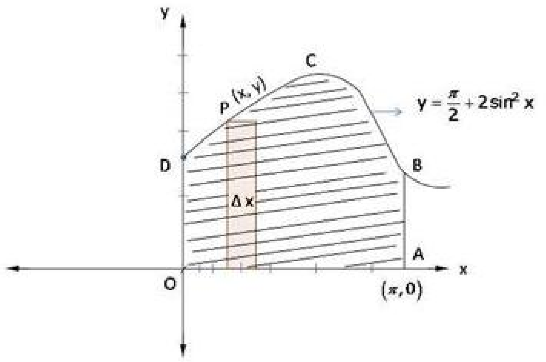
To find area bounded by $y = \frac{\pi}{2} + 2\sin^2 x$,

x-axis, $x = 0$ and $x = \pi$

A table for values of $y = \frac{\pi}{2} + 2\sin^2 x$ is:-

X	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\frac{\pi}{2} + 2\sin^2 x$	1.57	2.07	2.57	3.07	3.57	3.07	2.57	2.07	1.57

A rough sketch of the curves is given below:-



Shaded region represents required area. We slice it into rectangles of width $= \Delta x$ and length $= y$

Area of rectangle $= y\Delta x$

The approximation rectangle slides from $x = 0$ to $x = \pi$. So,

Required area = (Region $ABCDO$)

$$= \int_0^\pi y\,dx$$

$$= \int_0^\pi \left(\frac{\pi}{2} + 2\sin^2 x\right)dx$$

$$= \int_0^\pi \left(\frac{\pi}{2} + 1 - \cos 2x\right)dx$$

$$= \left[\frac{\pi}{2}x + x - \frac{\sin 2x}{2}\right]_0^\pi$$

$$= \left\{\left(\frac{\pi^2}{2} + \pi - \frac{\sin 2x}{2}\right) - (0)\right\}$$

$$= \frac{\pi^2}{2} + \pi$$

Required area $= \frac{\pi}{2}(\pi + 2)$ sq. units

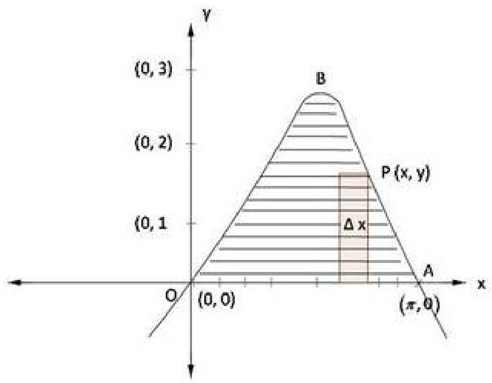
Areas of Bounded Regions Ex 21.1 Q22

To find area between by x-axis, $x = 0$, $x = \pi$ and

$$y = \frac{x}{\pi} + 2\sin^2 x \qquad ---(1)$$

The table for equation (1) is:-

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	0	0.66	1.25	1.88	2.5	1.88	1.25	0.66	0



Shaded region is the required area. We slice the area into rectangles with width $= \Delta x$, length $= y$

Area of rectangle $= y\Delta x$

The approximation rectangle slides from $x = 0$ to $x = \pi$. So,

Required area $=$ (Region $ABOA$)

$$= \int_0^\pi y\,dx$$

$$= \int_0^\pi \left(\frac{x}{\pi} + 2\sin^2 x\right)dx$$

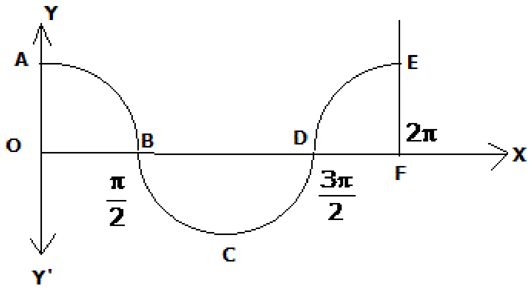
$$= \int_0^\pi \left(\frac{x}{\pi} + 1 - \cos 2x\right)dx$$

$$= \left[\frac{x^2}{2\pi} + x - \frac{\sin 2x}{2}\right]_0^\pi$$

$$= \left(\frac{\pi^2}{2\pi} + \pi - 0\right) - (0)$$

Required area $= \frac{3\pi}{2}$ sq. units

Areas of Bounded Regions Ex 21.1 Q23



From the figure, we notice that

The required area= area of the region OABO + area of the region BCDB + area of the region DEFD

Thus, the reqd. area $= \int_0^{\pi/2} \cos x \, dx + \left| \int_{\pi/2}^{3\pi/2} \cos x \, dx \right| + \int_{3\pi/2}^{2\pi} \cos x \, dx$

$= [\sin x]_0^{\pi/2} + \left| [\sin x]_{\pi/2}^{3\pi/2} \right| + [\sin x]_{3\pi/2}^{2\pi}$

$= \left[\sin \frac{\pi}{2} - \sin 0\right] + \left| \sin \frac{3\pi}{2} - \sin \frac{\pi}{2} \right| + \left[\sin 2\pi - \sin \frac{3\pi}{2}\right]$

$= 1 + 2 + 1 = 4$ square units

Areas of Bounded Regions Ex 21.1 Q24

To find area under the curve

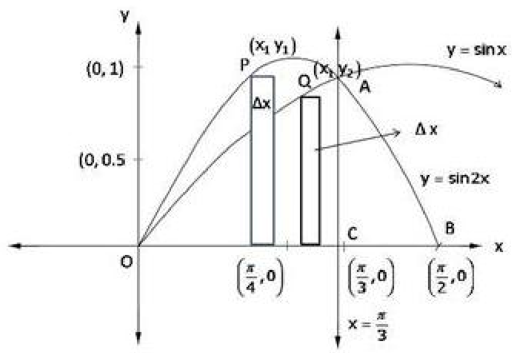
$y = \sin x$ ---(1)

and $y = \sin 2x$ ---(2)

between $x = 0$ and $x = \frac{\pi}{3}$.

X	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$Y = \sin x$	0	0.5	0.7	0.8	1
$Y = \sin 2x$	0	0.8	1	0.8	0

A rough sketch of the curve is given below:-



Area under curve $y = \sin 2x$

It is sliced in rectangles with width $= \Delta x$ and length $= y_1$

Area of rectangle $= y_1 \Delta x$

This approximation rectangle slides from $x = 0$ to $x = \frac{\pi}{3}$. So, Required area = Region $OPACO$

$A_1 = \int_0^{\frac{\pi}{3}} y_1 dx$

$= \int_0^{\frac{\pi}{3}} \sin 2x \, dx$

$= \left[\frac{-\cos 2x}{2}\right]_0^{\frac{\pi}{3}}$

$= -\left[-\frac{1}{4} - \frac{1}{2}\right]$

$A_1 = \frac{3}{4}$ sq.units

Area under curve $y = \sin x$:

It is sliced in rectangles with width Δx and langth y_2
Area of rectangle = $y_2 \Delta x$

This approximation rectangle slides from $x = 0$ to $x = \frac{\pi}{3}$. So,

Required area = Region $OQACO$

$= \int_0^{\frac{\pi}{3}} y_2 dx$

$= \int_0^{\frac{\pi}{3}} \sin x \, dx$

$= \left[-\cos x\right]_0^{\frac{\pi}{3}}$

$= -\left[\cos\frac{\pi}{3} - \cos 0\right]$

$= -\left(\frac{1}{2} - 1\right)$

$A_2 = \frac{1}{2}$ sq.units

So,

$A_2 : A_1 = \frac{1}{2} : \frac{3}{4}$

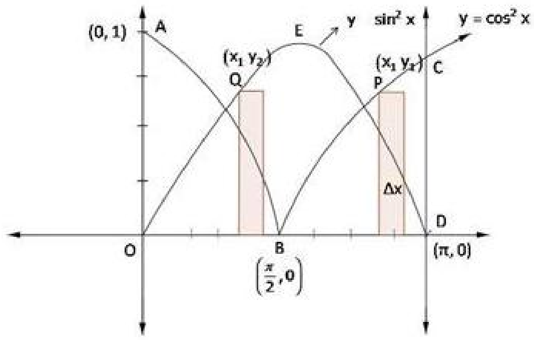
$A_2 : A_1 = 2 : 3$

Areas of Bounded Regions Ex 21.1 Q25

To compare area under curves
$y = \cos^2 x$ and $y = \sin^2 x$ between $x = 0$ and $x = \pi$.

Table for $y = \cos^2 x$ and $y = \sin^2 x$ is

X	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$Y=\cos^2 x$	1	0.75	0.5	0.25	0	0.25	0.5	0.75	1
$Y=\sin^2 x$	0	0.25	0.5	0.75	1	0.75	0.5	0.25	0



Area of region enclosed by $y = \cos^2 x$ and axis

A_1 = Region $OABO$ + Region $BCDB$

$= 2\,(\text{Region } BCDB)$

$= 2\int_{\frac{\pi}{2}}^{\pi} \cos^2 x \, dx$

$= 2\int_{\frac{\pi}{2}}^{\pi} \left(\frac{1-\cos 2x}{2}\right) dx$

$= \left[x - \frac{\sin 2x}{2}\right]_{\frac{\pi}{2}}^{\pi}$

$= \left[(\pi - 0) - \left(\frac{\pi}{2} - 0\right)\right]$

$= \pi - \frac{\pi}{2}$

$A_1 = \frac{\pi}{2}$ sq.units $\qquad$ ---(1)

Area of region enclosed by $y = \sin^2 x$ and axis

A_2 = Region $OEDO$

$= \int_0^{\pi} \sin^2 x \, dx$

$= \int_0^{\pi} \left(\frac{1-\cos 2x}{2}\right) dx$

$= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi}$

$= \frac{1}{2}\left[(\pi - 0) - (0)\right]$

$A_2 = \frac{\pi}{2}$ sq. units $\qquad$ ---(2)

From equation (1) and (2),
$A_1 = A_2$

So,
Area enclosed by $y = \cos^2 x$ = Area enclosed by $y = \sin^2 x$

Areas of Bounded Regions Ex 21.1 Q26

The required area fig., of the region BOB'RFSB is enclosed by the ellipse and the lines $x = 0$ and $x = ae$.

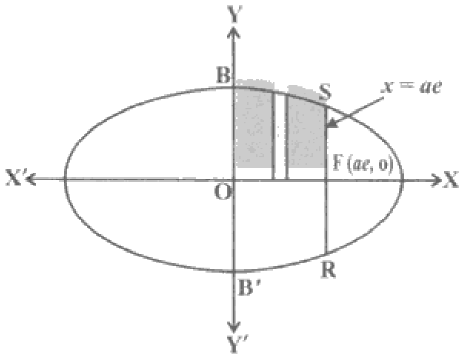
Note that the area of the region BOB'RFSB

$$= 2\int_0^{ae} y\,dx = 2\frac{b}{a}\int_0^{ae} \sqrt{a^2 - x^2}\,dx$$

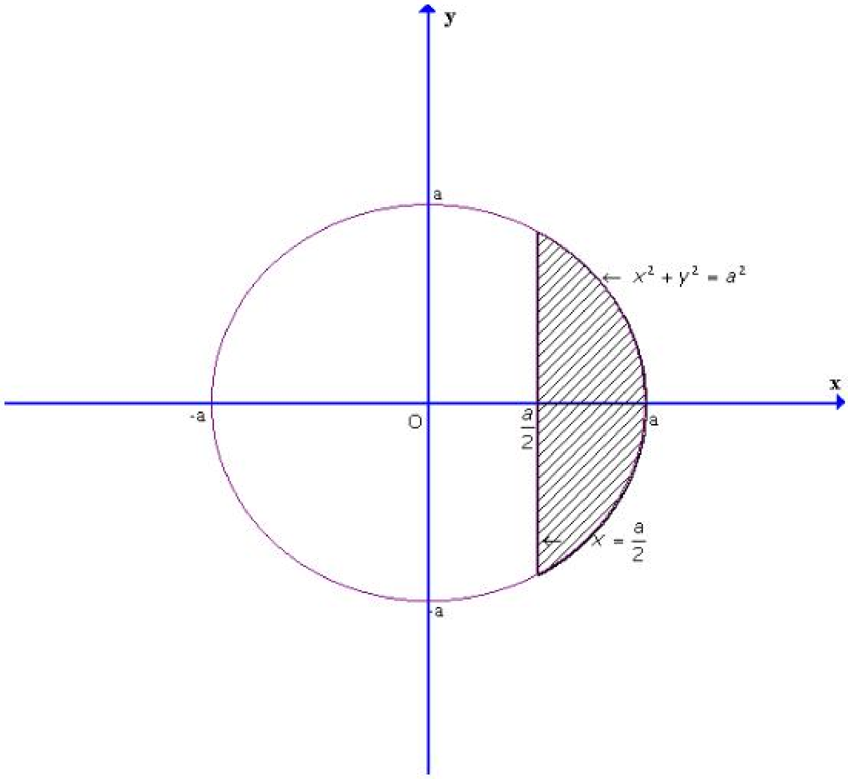
$$= \frac{2b}{a}\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^{ae}$$

$$= \frac{2b}{2a}\left[ae\sqrt{a^2 - a^2e^2} + a^2\sin^{-1}e\right]$$

$$= ab\left[e\sqrt{1 - e^2} + \sin^{-1}e\right]$$



Areas of Bounded Regions Ex 21.1 Q27



Area of the minor segment of the circle

$$=2\int_{\frac{a}{2}}^{a} \sqrt{a^2 - x^2}dx$$

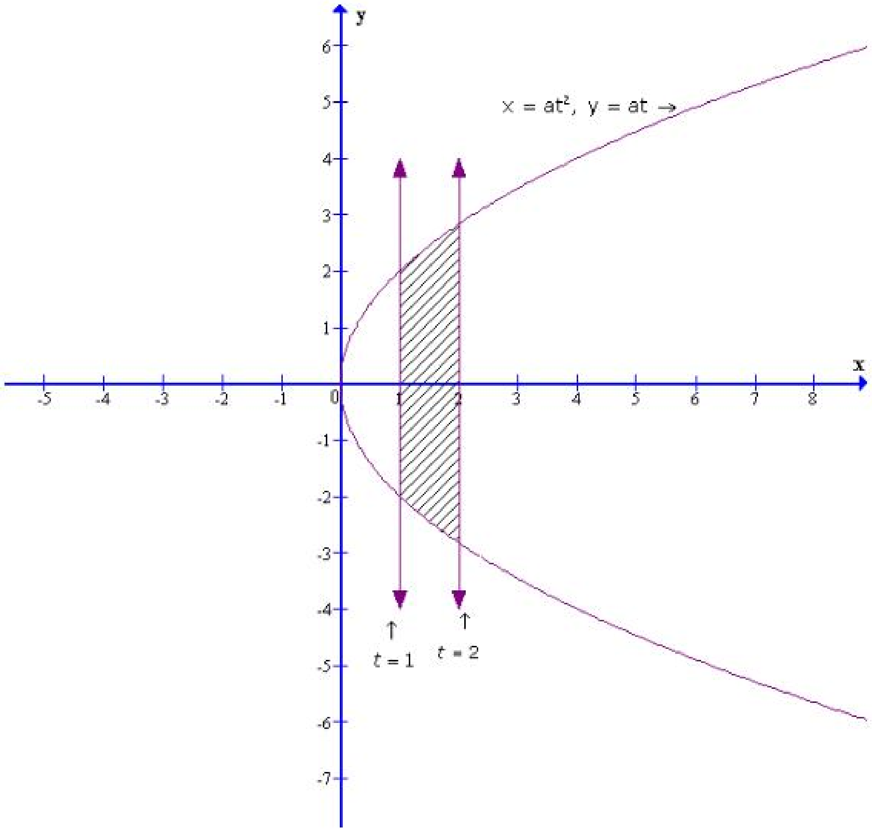
$$= 2\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{2}\right]_{\frac{a}{2}}^{a}$$

$$= 2\left[\frac{a}{2}(0) + \frac{a^2}{2}\sin^{-1}\left(\frac{a}{2}\right) - \frac{a}{4}\sqrt{a^2 - \frac{a^2}{4}} - \frac{a^2}{2}\sin^{-1}\frac{a}{4}\right]$$

$$= 2\left[\frac{a^2}{2}\sin^{-1}\left(\frac{a}{2}\right) - \frac{a}{4}\sqrt{a^2 - \frac{a^2}{4}} - \frac{a^2}{2}\sin^{-1}\frac{a}{4}\right]$$

$$= \frac{a^2}{12}\left(4\pi - 3\sqrt{3}\right)\text{sq. units}$$

Areas of Bounded Regions Ex 21.1 Q28



Area of the bounded region

$$= 2\int_1^2 y\,\frac{dx}{dt}\,dt$$

$$= 2\int_1^2 (2at)(2at)\,dt$$

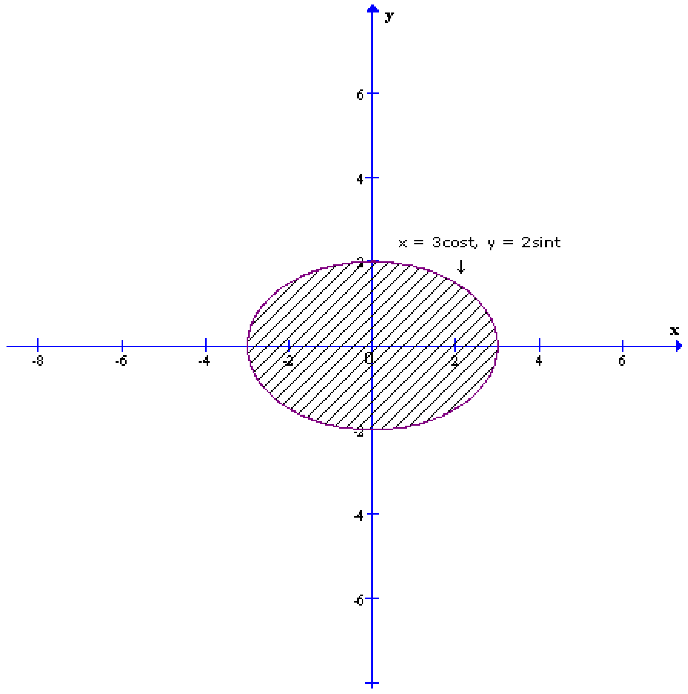
$$= 8a^2\int_1^2 t^2\,dt$$

$$= 8a^2\left[\frac{t^3}{3}\right]_1^2$$

$$= 8a^2\left[\frac{8}{3}-\frac{1}{3}\right]$$

$$= \frac{56a^2}{3} \text{ sq. units}$$

Areas of Bounded Regions Ex 21.1 Q29



Area of the bounded region

$= 4\int_0^{\frac{\pi}{2}} 2\sin t \, dt$

$= -8[\cos t]_0^{\frac{\pi}{2}}$

$= -8[0-1]$

$= 8$sq units

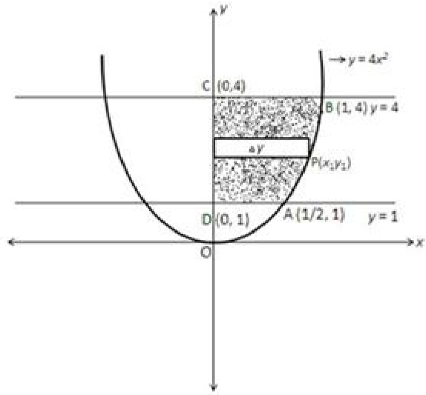
Ex 21.2

Areas of Bounded Regions Ex-21-2 Q1

To find the area enclosed in first quadrant by

$x = 0$, $y = 1$, $y = 4$ and
$y = 4x^2$ $\qquad\qquad ---(1)$

Equation (1) represents a parabola with vertex $(0,0)$ and axis as y-axis. $x = 0$ is y-axis and $y = 1$, $y = 4$ are lines parallel to x-axis passing through $(0,1)$ and $(0,4)$ respectively. A rough sketch of the curves is given as:-



Shaded region is required area and it is sliced into rectangles with area $x\Delta y$ it slides from $y = 1$ to $y = 4$, so

Required area = Region $ABCDA$

$$= \int_1^4 x\,dy$$

$$= \int_1^4 \sqrt{\frac{y}{4}}\,dy$$

$$= \frac{1}{2}\int_1^4 \sqrt{y}\,dy$$

$$= \frac{1}{2}\left[\frac{2}{3}y\sqrt{y}\right]_1^4$$

$$= \frac{1}{2}\left[\left(\frac{2}{3}.4.\sqrt{4}\right) - \left(\frac{2}{3}.1.\sqrt{1}\right)\right]$$

$$= \frac{1}{2}\left[\frac{16}{3} - \frac{2}{3}\right]$$

Required area $= \frac{7}{3}$ sq. units

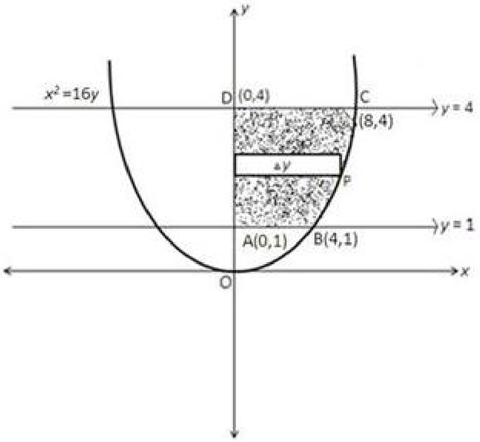
Areas of Bounded Regions Ex-21-2 Q2

To find region in first quadrant bounded by $y = 1$, $y = 4$ and y-axis and

$x^2 = 16y \qquad ---(1)$

Equation (1) represents a parabola with vertex $(0,0)$ and axes as y-axis.

A rough sketch of the curves is as under:-



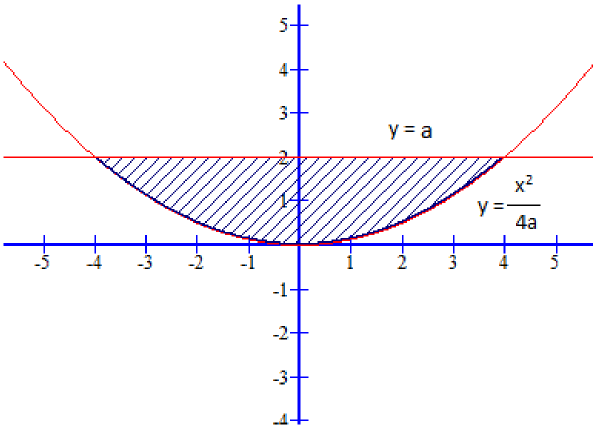
Shaded region is required area it is sliced in rectangles of area $x\Delta y$ which slides from $y = 1$ to $y = 4$, so

Required area = Region $ABCDA$

$$A = \int_1^4 x dy$$
$$= \int_1^4 4\sqrt{y}\, dy$$
$$= 4.\left[\frac{2}{3}y\sqrt{y}\right]_1^4$$
$$= 4.\left[\left(\frac{2}{3}.4\sqrt{4}\right) - \left(\frac{2}{3}.1.\sqrt{1}\right)\right]$$
$$= 4\left[\frac{16}{3} - \frac{2}{3}\right]$$

$A = \frac{56}{3}$ sq. units

Areas of Bounded Regions Ex-21-2 Q3



$$\text{Area of the region} = 2 \times \int_0^{2a} \left(a - \frac{x^2}{4a}\right) dx$$

$$= 2 \times \left[ax - \frac{x^3}{12a}\right]_0^{2a}$$

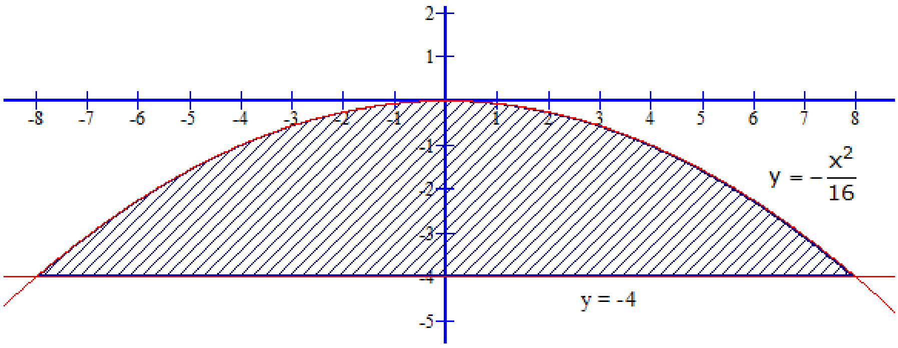
$$= 2\left[a(2a - 0) - \frac{(2a)^3 - 0^3}{12a}\right]$$

$$= 2\left[2a^2 - \frac{8a^3}{12a}\right]$$

$$= 2\left[\frac{16a^3}{12a}\right]$$

$$= \frac{8}{3}a^2 \text{sq. units}$$

Areas of Bounded Regions Ex-21-2 Q4



$$\text{Area of the region} = 2 \times \int_0^8 \left[-\frac{x^2}{16} - (-4)\right] dx$$

$$= 2 \times \left[-\frac{x^3}{48} + 4x\right]_0^8$$

$$= 2 \times \left[4x - \frac{x^3}{48}\right]_0^8$$

$$= 2 \times \left[4(8 - 0) - \frac{(8)^3 - 0^3}{48}\right]$$

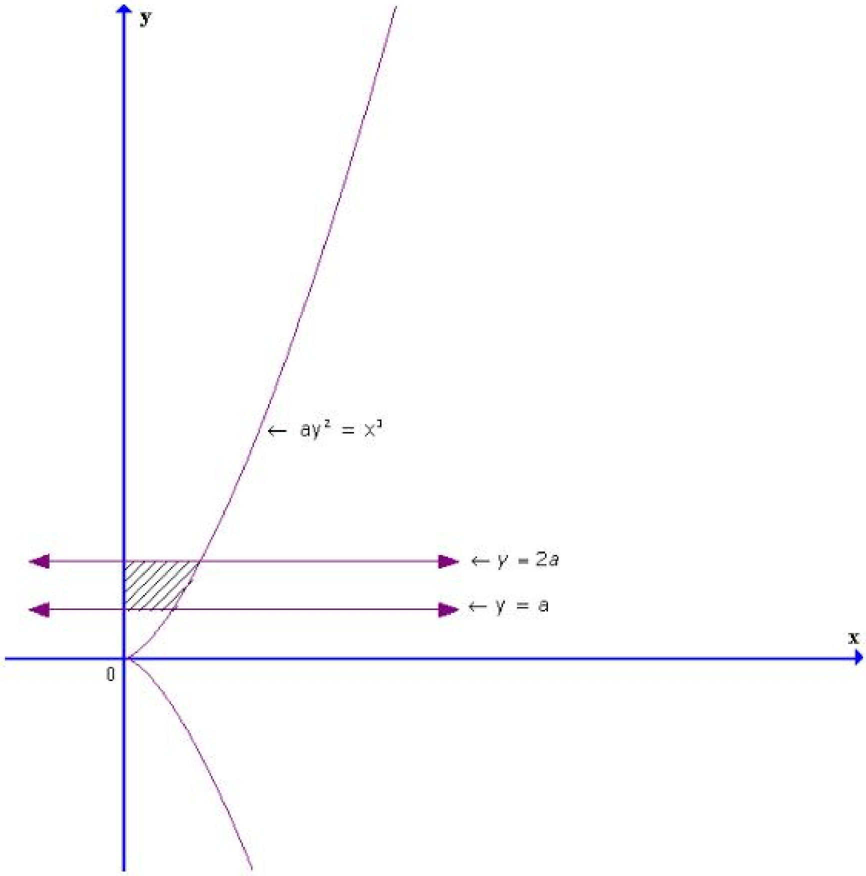
$$= 2 \times \left[32 - \frac{512}{48}\right]$$

$$= 2 \times \left[32 - \frac{32}{3}\right]$$

$$= 2 \times \left[\frac{96 - 32}{3}\right]$$

$$= 2 \times \frac{64}{3} = \frac{128}{3} \text{sq. units}$$

Areas of Bounded Regions Ex-21-2 Q5



Area of the bounded region

$$= \int_a^{2a} \left(ay^2\right)^{\frac{1}{3}} dy$$

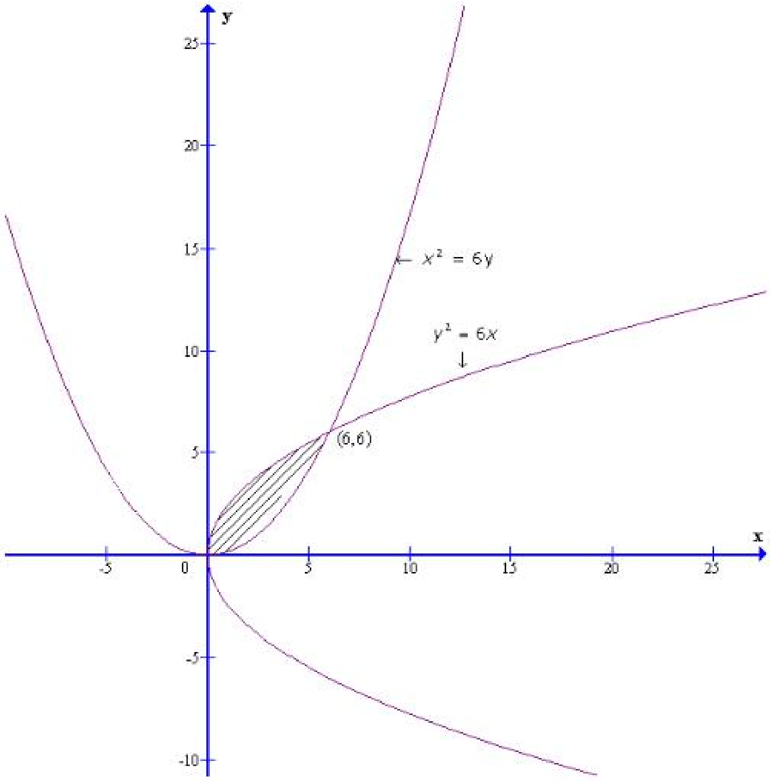
$$= a^{\frac{1}{3}} \int_a^{2a} y^{\frac{2}{3}} dy$$

$$= a^{\frac{1}{3}} \left[\frac{3}{5} y^{\frac{5}{3}}\right]_o^{2a}$$

$$= \frac{3}{5}\left(2^{\frac{5}{3}} - 1\right) a^2 \text{ sq units}$$

Ex 21.3

Areas of Bounded Regions Ex-21-3 Q1



Area of the bounded region

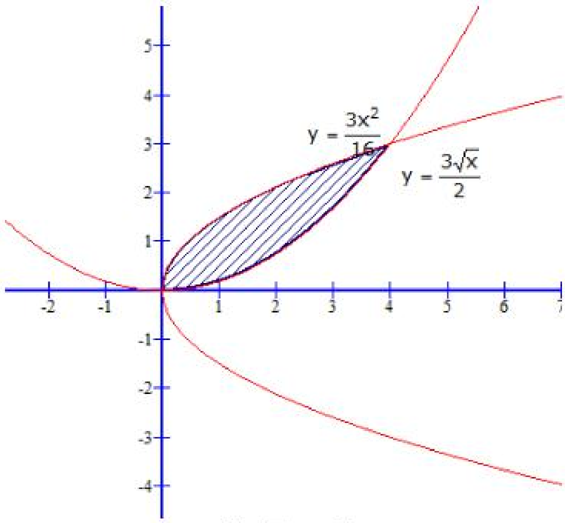
$$= \int_0^6 \sqrt{6x} - \frac{x^2}{6}\, dx$$

$$= \left[\sqrt{6}\,\frac{x^{3/2}}{3/2} - \frac{x^3}{18}\right]_0^6$$

$$= \left[\sqrt{6}\,\frac{(6)^{3/2}}{3/2} - \frac{(6)^3}{18} - 0\right]$$

= 12 sq. units

Areas of Bounded Regions Ex-21-3 Q2



$$\text{Area of the region} = \int_0^4 \left[\frac{3\sqrt{x}}{2} - \frac{3x^2}{16}\right] dx$$

$$= \left[x^{3/2} - \frac{x^3}{16}\right]_0^4$$

$$= \left[(4)^{3/2} - \frac{(4)^3}{16}\right]$$

$$= \left[8 - \frac{64}{16}\right]$$

$$= [8 - 4] = 4 \text{ sq.units}$$

Areas of Bounded Regions Ex-21-3 Q3

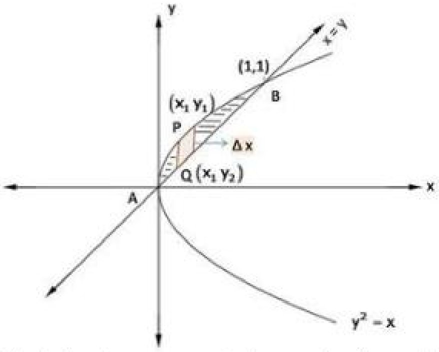
We have to find area of region bounded by

$y^2 = x$ ---(1)

and $y = x$ ---(2)

Equation (1) represents parabola with vertex (0,0) and axis as x-axis and equation (2) represents a line passing through origin and intersecting parabola at (0,0) and (1,1).

A rough sketch of curves is as below:-



Shaded region represents the required area. We slice it in rectangle with Width $= \Delta x$, length $= y_1 - y_2$

Area of rectangle $= (y_1 - y_2)\Delta x$

The approximation triangle can slide from $x = 0$ to $x = 1$.

Required area = region $AOBPA$

$$= \int_0^1 (y_1 - y_2)\,dx$$

$$= \int_0^1 \left(\sqrt{x} - x\right)dx$$

$$= \left[\frac{2}{3}x\sqrt{x} - \frac{x^2}{2}\right]_0^1$$

$$= \left[\frac{2}{3}.1.\sqrt{1} - \frac{(1)^2}{2}\right] - [0]$$

$$= \left[\frac{2}{3} - \frac{1}{2}\right]$$

Required area $= \frac{1}{6}$ square units

Areas of Bounded Regions Ex-21-3 Q4

We have to find area bounded by the curves

$$y = 4 - x^2$$

$$\Rightarrow \quad x^2 = -(y-4) \qquad ---(1)$$

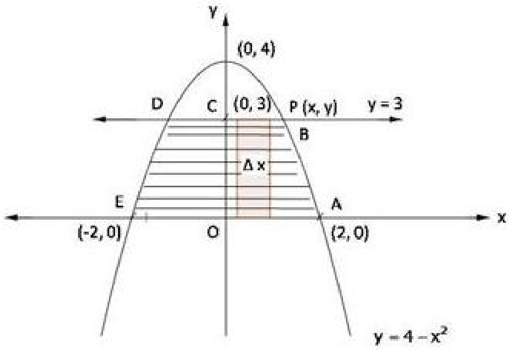
$$\text{and } y = 0 \qquad ---(2)$$

$$y = 3 \qquad ---(3)$$

Equation (1) represents a parabola with vertex $(0,4)$ and passes through $(0,2)$, $(0,-2)$

Equation (1) is x-axis and equation (3) is a line parallel to x-axis passing through $(0,3)$.

A rough sketch of curves is below:-



Shaded region represents the required area. We slice it in approximation rectangle with its Width $= \Delta x$ and length $= y - 0 = y$

Area of the rectangle $= y\Delta x$.

This approximation rectangle can slide from $x = 0$ to $x = 2$ for region $OABCO$.

Required area = Region $ABDEA$

$$= 2(\text{Region } OABCO)$$

$$= 2\int_0^2 y\,dx$$

$$= 2\int_0^2 (4 - x^2)\,dx$$

$$= 2\left(4x - \frac{x^3}{3}\right)_0^2$$

$$= 2\left[\left(8 - \frac{8}{3}\right) - (0)\right]$$

Required area $= \frac{32}{3}$ square units

Areas of Bounded Regions Ex-21-3 Q5

Here to find area $\left\{(x, y) : \frac{x^2}{a^2} + \frac{y^2}{b^2} \le 1 \le \frac{x}{a} + \frac{y}{b}\right\}$

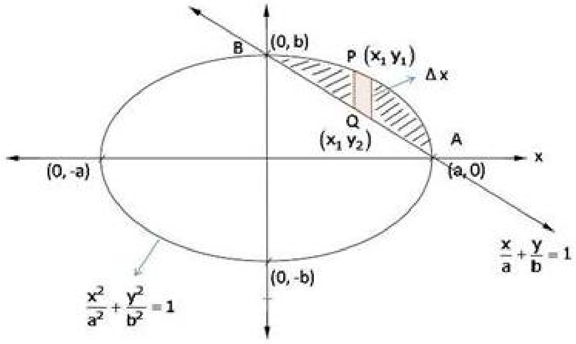
So,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad ---(1)$$

$$\frac{x}{a} + \frac{y}{b} = 1 \qquad ---(2)$$

Equation (1) represents ellipse with centre at origin and passing through $(\pm a, 0)$, $(0, \pm b)$ equation (2) represents a line passing through $(a,0)$ and $(0,b)$.

A rough sketch of curves is below:- let $a > b$



Shaded region is the required region as by substituting $(0,0)$ in $\frac{x^2}{a^2} + \frac{y^2}{b^2} \le 1$ gives a true statement and by substituting $(0,0)$ in $1 \le \frac{x}{a} + \frac{y}{b}$ gives a false statement.

We slice the shaded region into approximation rectangles with Width $= \Delta x$, length $= (y_1 - y_2)$

Area of the rectangle $= (y_1 - y_2)$

The approximation rectangle can slide from $x = 0$ to $x = a$, so

$$\text{Required area} = \int_0^a \left[\frac{b}{a}\sqrt{a^2 - x^2} - \frac{b}{a}(a - x)\right] dx$$

$$= \frac{b}{a}\int_0^a \left[\sqrt{a^2 - x^2} - (a - x)\right] dx$$

$$= \frac{b}{a}\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) - ax + \frac{x^2}{2}\right]_0^a$$

$$= \frac{b}{a}\left[\left(\frac{a}{2}\sqrt{a^2 - a^2} + \frac{a^2}{2}\sin^{-1}(1) - a^2 + \frac{a^2}{2}\right) - (0 + 0 + 0 + 0)\right]$$

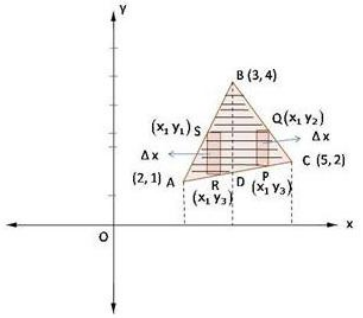
$$= \frac{b}{a}\left[\frac{a^2}{2} \cdot \frac{\pi}{2} - \frac{a^2}{2}\right]$$

$$= \frac{b}{a}\frac{a^2}{2}\left(\frac{\pi - 2}{2}\right)$$

Required area $= \frac{ab}{4}(\pi - 2)$ square units

Areas of Bounded Regions Ex-21-3 Q6

Here we have find area of the triangle whose vertices are $A(2,1)$, $B(3,4)$ and $C(5,2)$



Equation of AB,

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

$$y - 1 = \left(\frac{4-1}{3-2}\right)(x-2)$$

$$y - 1 = \frac{3}{1}(x-2)$$

$$y = 3x - 6 + 1$$

$$y = 3x - 5 \qquad ---(1)$$

Equation of BC,

$$y - 4 = \left(\frac{2-4}{5-3}\right)(x-3)$$

$$= \frac{-2}{2}(x-3)$$

$$y - 4 = -x + 3$$

$$y = -x + 7 \qquad ---(2)$$

Equation of AC,

$$y - 1 = \left(\frac{2-1}{5-2}\right)(x-2)$$

$$y - 1 = \frac{1}{3}(x-2)$$

$$y = \frac{1}{3}x - \frac{2}{3} + 1$$

$$y = \frac{1}{3}x + \frac{1}{3} \qquad ---(3)$$

Shaded area $\triangle ABC$ is the required area.

$ar(\triangle \mathrm{ABC}) = ar(\triangle ABD) + ar(\triangle BDC)$

For $ar(\triangle ABD)$: we slice the region into approximation rectangle with width $= \Delta x$ and length $(y_1 - y_3)$ area of rectangle $= (y_1 - y_3)\Delta x$

This approximation rectangle slides from $x = 2$ to $x = 3$

$$ar(\triangle ABD) = \int_2^3 (y_1 - y_3)dx$$

$$= \int_2^3 \left[(3x-5) - \left(\frac{1}{3}x + \frac{1}{3}\right)\right]dx$$

$$= \int_2^3 \left(3x - 5 - \frac{1}{3}x - \frac{1}{3}\right)dx$$

$$= \int_2^3 \left(\frac{8x}{3} - \frac{16}{3}\right)dx$$

$$= \frac{8}{3}\left(\frac{x^2}{2} - 12x\right)_2^3$$

$$= \frac{8}{3}\left[\left(\frac{9}{2} - 6\right) - (2-4)\right]$$

$$= \frac{8}{3}\left[-\frac{3}{2} + 2\right]$$

$$= \frac{8}{3} \times \frac{1}{2}$$

$$ar(\triangle ABD) = \frac{4}{3} \text{ sq. unit}$$

For $ar(\triangle BDC)$: we slice the region into rectangle with width $= \Delta x$ and length $(y_2 - y_3)$. Area of rectangle $= (y_2 - y_3)\Delta x$

The approximation rectangle slides from $x = 3$ to $x = 5$.

$$\text{Area}(\triangle BDC) = \int_3^5 (y_2 - y_3)dx$$
$$= \int_3^5 \left[(-x+7) - \left(\frac{1}{3}x + \frac{1}{3}\right)\right]dx$$
$$= \int_3^5 \left(-x + 7 - \frac{1}{3}x - \frac{1}{3}\right)dx$$
$$= \int_3^5 \left(-\frac{4}{3}x + \frac{20}{3}\right)dx$$
$$= -\left(\frac{4x^2}{6} - \frac{20}{3}x\right)_3^5$$
$$= -\left[\left(\frac{4(5)^2}{6} + \frac{20(5)}{3}\right) - \left(\frac{4(3)^2}{6} - \frac{20}{3}(3)\right)\right]$$
$$= -\left[\left(\frac{50}{3} - \frac{100}{3}\right) - (6 - 20)\right]$$
$$= -\left[-\frac{50}{3} + 14\right]$$
$$= -\left[-\frac{8}{3}\right]$$

$ar(\triangle BDC) = \frac{8}{3}$ sq. units

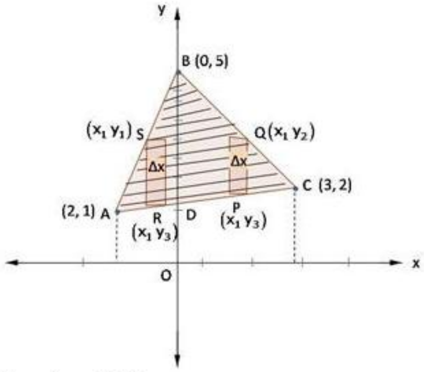
So, $ar(\triangle ABC) = ar(\triangle ABD) + ar(\triangle BDC)$

$$= \frac{4}{3} + \frac{8}{3}$$
$$= \frac{12}{3}$$

$ar(\triangle ABC) = 4$ sq. units

Areas of Bounded Regions Ex-21-3 Q7

We have to find area of the triangle whose vertices are $A(-1,1)$, $B(0,5)$, $C(3,2)$



Equation of AB,

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

$$y - 1 = \left(\frac{5-1}{0+1}\right)(x+1)$$

$$y - 1 = \frac{4}{1}(x+1)$$

$$y = 4x + 4 + 1$$

$$y = 4x + 5 \qquad ---(1)$$

Equation of BC,

$$y - 5 = \left(\frac{2-5}{3-0}\right)(x-0)$$

$$= \frac{-3}{3}(x-0)$$

$$y - 5 = -x$$

$$y = 5 - x \qquad ---(2)$$

Equation of AC,

$$y - 5 = \left(\frac{2-5}{3-0}\right)(x-0)$$

$$= \frac{-3}{3}(x-0)$$

$$y - 5 = -x$$

$$y = 5 - x \qquad ---(2)$$

Equation of AC,

$$y - 1 = \left(\frac{2-1}{3+1}\right)(x+1)$$

$$y - 1 = \frac{1}{4}(x+1)$$

$$y = \frac{1}{4}x + \frac{1}{4} + 1$$

$$y = \frac{1}{4}(x+5) \qquad ---(3)$$

Shaded area $\triangle ABC$ is the required area.
$ar(\triangle ABC) = ar(\triangle ABD) + ar(\triangle BDC)$
For $ar(\triangle ABD)$: we slice the region into approximation rectangle with width $= \Delta x$ and length $(y_1 - y_3)$ area of rectangle $= (y_1 - y_3)\Delta x$

This approximation rectangle slides from $x = -1$ to $x = 0$, so

$$ar(\triangle ABD) = \int_{-1}^{0}(y_1 - y_3)dx$$

$$= \int_{-1}^{0}\left[(4x+5) - \frac{1}{4}(x+5)\right]dx$$

$$= \int_{-1}^{0}\left(4x + 5 - \frac{x}{4} - \frac{5}{4}\right)dx$$

$$= \int_{-1}^{0}\left(\frac{15}{4}x + \frac{15}{4}\right)dx$$

$$= \frac{15}{4}\left(\frac{x^2}{2} + x\right)_{-1}^{0}$$

$$= \frac{15}{4}\left[(0) - \left(\frac{1}{2} - 1\right)\right]$$

$$= \frac{15}{4} \times \frac{1}{2}$$

$ar(\Delta ABD) = \frac{15}{8}$ sq. units

For $ar(\Delta BDC)$: we slice the region into rectangle with width $= \Delta x$ and length $(y_2 - y_3)$. Area of rectangle $= (y_2 - y_3)\Delta x$

The approximation rectangle slides from $x = 0$ to $x = 3$.

$$\text{Area}(\Delta BDC) = \int_0^3 (y_2 - y_3)dx$$
$$= \int_0^3 \left[(5 - x) - \left(\frac{1}{4}x + \frac{5}{4}\right)\right]dx$$
$$= \int_0^3 \left(5 - x - \frac{1}{4}x - \frac{5}{4}\right)dx$$
$$= \int_0^3 \left(-\frac{5}{4}x + \frac{15}{4}\right)dx$$
$$= \frac{5}{4}\left(3x - \frac{x^2}{2}\right)_0^3$$
$$= \frac{5}{4}\left[9 - \frac{9}{2}\right]$$

$ar(\Delta BDC) = \frac{45}{8}$ sq. units

So, $ar(\Delta ABC) = ar(\Delta ABD) + ar(\Delta BDC)$

$$= \frac{15}{8} + \frac{45}{8}$$
$$= \frac{60}{8}$$

$ar(\Delta ABC) = \frac{15}{2}$ sq. units

Areas of Bounded Regions Ex-21-3 Q8

To find area of triangular region bounded by

$y = 2x + 1$ (Say, line AB) ---(1)

$y = 3x + 1$ (Say, line BC) ---(2)

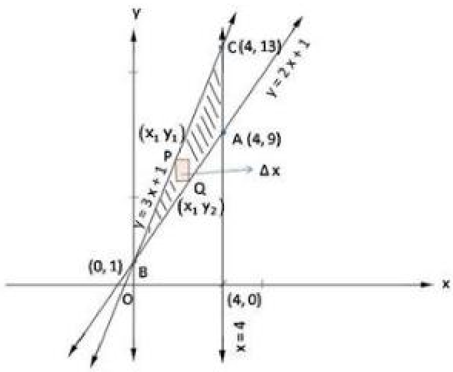
$y = 4$ (Say, line AC) ---(3)

equation (1) represents a line passing through points $(0,1)$ and $\left(-\frac{1}{2}, 0\right)$, equation (2) represents a line passing through points $(0,1)$ and $\left(-\frac{1}{3}, 0\right)$. Equation (3) represents a line parallel to y-axis passing through $(4,0)$.

Solving equation (1) and (2) gives point $B(0,1)$

Solving equation (2) and (3) gives point $C(4,13)$

Solving equation (1) and (3) gives point $A(4,9)$



Shaded region $ABCA$ gives required triangular region. We slice this region into approximation rectangle with width $= \Delta x$, length $= (y_1 - y_2)$.

Area of rectangle $= (y_1 - y_2)\Delta x$

This qpproximation rectangle slides from $x = 0$ to $x = 4$, so

Required area = (Region $ABCA$)

$= \int_0^4 (y_1 - y_2)dx$

$= \int_0^4 [(3x + 1) - (2x + 1)]dx$

$= \int_0^4 x dx$

$= \left[\frac{x^2}{2}\right]_0^4$

Required area = 8 sq. units

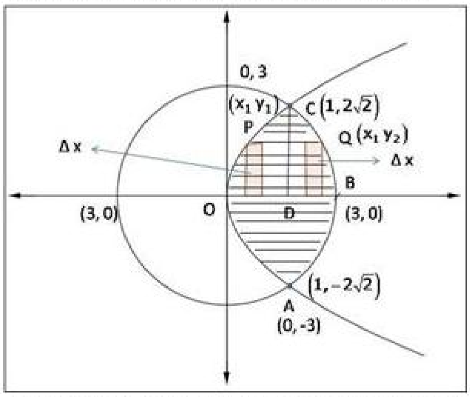
Areas of Bounded Regions Ex-21-3 Q9

To find area $\{(x,y) : y^2 \le 8x, x^2 + y^2 \le 9\}$ given equation is

$$y^2 = 8x \qquad ---(1)$$
$$x^2 + y^2 = 9 \qquad ---(2)$$

Equation (1) represents a parabola with vertex $(0,0)$ and axis as x-axis, equation (2) represents a circle with centre $(0,0)$ and radius $\sqrt{9} = 3$, so it meets area at $(\pm 3,0)$, $(0,\pm 3)$. point of intersection of parabola and circle is $(1,2\sqrt{2})$ and $(1,-2\sqrt{2})$.

A rough sketch of the curves is as below:-



Shaded region is the required region.

$$\text{Required area} = \text{Region } OABCO$$
$$= 2(\text{Region } OBCO)$$
$$\text{Required area} = 2(\text{region } ODCO + \text{region } DBCD)$$
$$= 2\left[\int_0^1 \sqrt{8x}\,dx + \int_1^3 \sqrt{9-x^2}\,dx\right]$$

$$= 2\left[\left(2\sqrt{2}.\frac{2}{3}x\sqrt{x}\right)_0^1 + \left(\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3}\right)_1^3\right]$$
$$= 2\left[\left(\frac{4\sqrt{2}}{3}.1.\sqrt{1}\right) + \left\{\left(\frac{3}{2}.\sqrt{9-9} + \frac{9}{2}\sin^{-1}(1)\right) - \left(\frac{1}{2}\sqrt{9-1} + \frac{9}{2}\sin^{-1}\frac{1}{3}\right)\right\}\right]$$
$$= 2\left[\frac{4\sqrt{2}}{3} + \left\{\left(\frac{9}{2}.\frac{\pi}{2}\right) - \left(\frac{2\sqrt{2}}{2} - \frac{9}{2}\sin^{-1}\left(\frac{1}{3}\right)\right)\right\}\right]$$

$$= 2\left[\frac{4\sqrt{2}}{3} + \frac{9\pi}{4} - \sqrt{2} - \frac{9}{2}\sin^{-1}\left(\frac{1}{3}\right)\right]$$

$$\text{Required area} = 2\left[\frac{\sqrt{2}}{3} + \frac{9\pi}{4} - \frac{9}{2}\sin^{-1}\left(\frac{1}{3}\right)\right] \text{ square units}$$

Areas of Bounded Regions Ex-21-3 Q10

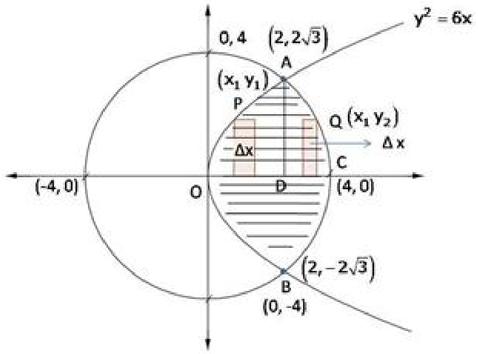
To find the area of common to

$x^2 + y^2 = 16 \quad ---(1)$

$y^2 = 6x \quad ---(2)$

Equation (1) represents a parabola with vertex (0,0) and axis as x-axis, equation (2) represents a circle with centre (0,0) and radius $\sqrt{16} = 4$, so it meets areas at $(\pm 4,0)$, $(0,\pm 4,0)$. points of intersection of parabola and circle are $(2,2\sqrt{3})$ and $(2,-2\sqrt{3})$.

A rough sketch of the curves is as below:-



Shaded region represents the required area.

Required area = Region $OBCAO$

Required area = 2 (region $ODAO$ + region $DCAD$) $\quad ---(1)$

Region $ODAO$ is divided into approximation rectangle with area $y_1 \Delta x$ and slides from $x = 0$ to $x = 2$. And region $DCAD$ is divided into approximation rectangle with area $y_2 \Delta x$ and slides from $x = 2$ and $x = 4$. So using equation (1),

Required area $= 2\left(\int_0^2 y_1 dx + \int_2^4 y_2 dx\right)$

$= 2\left[\int_0^2 \sqrt{6x}\,dx + \int_2^4 \sqrt{16 - x^2}\,dx\right]$

$= 2\left[\left\{\sqrt{6}.\frac{2}{3}x\sqrt{x}\right\}_0^2 + \left\{\frac{x}{2}\sqrt{16-x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right\}_2^4\right]$

$= 2\left[\left\{\sqrt{6}.\frac{2}{3}2.\sqrt{2}\right\} + \left\{\left(\frac{4}{2}\sqrt{16-16} + \frac{16}{2}\sin^{-1}\frac{4}{4}\right) - \left(\frac{2}{2}\sqrt{16-4} + \frac{16}{2}\sin^{-1}\frac{2}{4}\right)\right\}\right]$

$= 2\left[\frac{4}{3}\sqrt{12} + \left\{\left(0 + 8\sin^{-1}(1)\right) - \left(1.\sqrt{12} + 8\sin^{-1}\left(\frac{1}{2}\right)\right)\right\}\right]$

$= 2\left[\frac{8\sqrt{3}}{3} + \left\{\left(8.\frac{\pi}{2}\right) - \left(2\sqrt{3} + 8.\frac{\pi}{6}\right)\right\}\right]$

$= 2\left\{\frac{8\sqrt{3}}{3} + 4\pi - 2\sqrt{3} - \frac{4\pi}{3}\right\}$

$= 2\left\{\frac{2\sqrt{3}}{3} + \frac{8\pi}{3}\right\}$

Required area $= \frac{4}{3}\left(4\pi + \sqrt{3}\right)$ sq.units

Areas of Bounded Regions Ex-21-3 Q11

Equation of the given circles are

$x^2 + y^2 = 4$ ...(1)

And $(x - 2)^2 + y^2 = 4$ ...(2)

Equation (1) is a circle with centre O at eh origin and radius 2. Equation (2) is a circle with centre C (2,0) and radius 2. Solving equations (1) and (2), we have

$(x - 2)^2 + y^2 = x^2 + y^2$

Or $x^2 - 4x + 4 + y^2 = x^2 + y^2$

Or $x = 1$ which gives $y \pm\sqrt{3}$

Thus, the points of intersection of the given circles are A $(1, \sqrt{3})$ and A' $(1, -\sqrt{3})$ as shown in the fig.,

Required area of the enclosed region OACA'O between circle

$= 2$ [area of the region ODCAO] (Why ?)

$= 2$ [area of the region ODAO + areaof the region DCAD]

$$= 2\left[\int_0^1 y\,dx + \int_1^2 y\,dx\right]$$

$$= 2\left[\int_0^1 \sqrt{4 - (x-2)^2}\,dx + \int_1^2 \sqrt{4 - x^2}\,dx\right] \quad \text{(Why ?)}$$

$$= 2\left[\frac{1}{2}(x-2)\sqrt{4-(x-2)^2} + \frac{1}{2}\times 4\sin^{-1}\left(\frac{x-2}{2}\right)\right]_0^1 + 2\left[\frac{1}{2}x\sqrt{4-x^2} + \frac{1}{2}\times 4\sin^{-1}\frac{x}{2}\right]_1^2$$

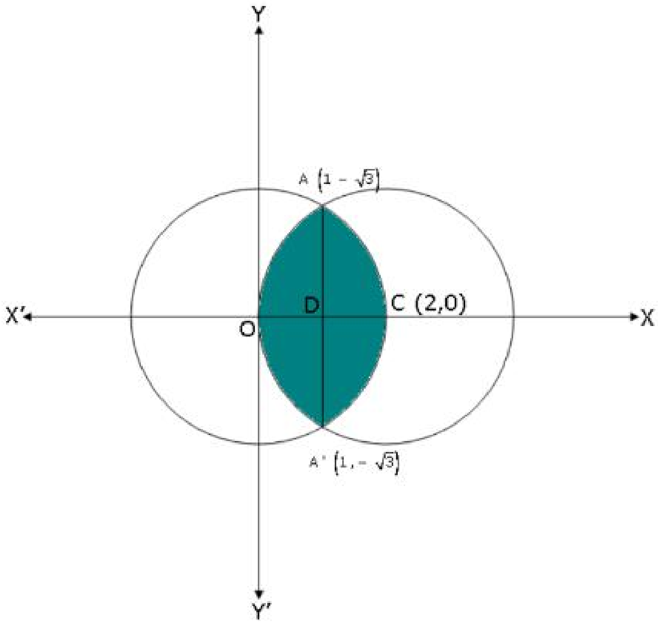
$$= \left[(x-2)\sqrt{4-(x-2)^2} + 4\sin^{-1}\left(\frac{x-2}{2}\right)\right]_0^1 + \left[x\sqrt{4-x^2} + 4\sin^{-1}\frac{x}{2}\right]_1^2$$

$$= \left[\left(-\sqrt{3} + 4\sin^{-1}\left(\frac{-1}{2}\right)\right) - 4\sin^{-1}(-1)\right] + \left[4\sin^{-1}1 - \sqrt{3} - 4\sin^{-1}\frac{1}{2}\right]$$

$$= \left[\left(-\sqrt{3} - 4\times\frac{\pi}{6}\right) + 4\times\frac{\pi}{2}\right] + \left[4\times\frac{\pi}{2} - \sqrt{3} - 4\times\frac{\pi}{6}\right]$$

$$= \left(-\sqrt{3} - \frac{2\pi}{3} + 2\pi\right) + \left(2\pi - \sqrt{3} - \frac{2\pi}{3}\right)$$

$$= \frac{8\pi}{3} - 2\sqrt{3} \text{ square units}$$



Areas of Bounded Regions Ex-21-3 Q12

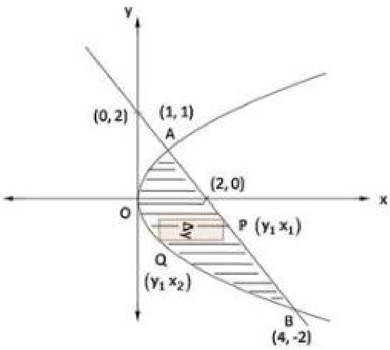
To find region enclosed by

$y^2 = x$ --- (1)

$x + y = 2$ --- (2)

Equation (1) represents a parabola with vertex at origin and its axis as x-axis, equation (2) represents a line passing through (2,0) and (0,2). points of intersection of line and parabola are (1,1) and (4,-2).

A rough sketch of curves is as below:-



Shaded region represents the required area. We slice it in rectangles of width Δy and length = $(x_1 - x_2)$.

Area of rectangle = $(x_1 - x_2)\Delta y$.

This approximation rectangle slides from $y = -2$ to $y = 1$, so

Required area = Region $AOBA$

$$= \int_{-2}^{1}(x_1 - x_2)\,dy$$

$$= \int_{-2}^{1}\left(2 - y - y^2\right)dy$$

$$= \left[2y - \frac{y^2}{2} - \frac{y^3}{3}\right]_{-2}^{1}$$

$$= \left[\left(2 - \frac{1}{2} - \frac{1}{3}\right) - \left(-4 - 2 + \frac{8}{3}\right)\right]$$

$$= \left[\left(\frac{12 - 3 - 2}{6}\right) - \left(\frac{-12 - 6 + 8}{3}\right)\right]$$

$$= \frac{7}{6} + \frac{10}{3}$$

Required area = $\frac{9}{2}$ sq.units

Areas of Bounded Regions Ex-21-3 Q13

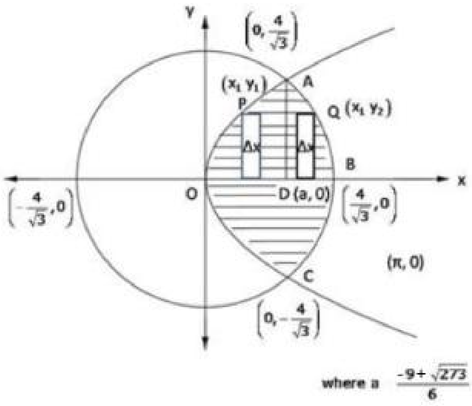
To find area $\{(x,y) : y^2 \le 3x, 3x^2 + 3y^2 \le 16\}$

$$\Rightarrow \quad y^2 = 3x \qquad ---(1)$$
$$3x^2 + 3y^2 = 16$$
$$x^2 + y^2 = \frac{16}{3} \qquad ---(2)$$

Equation (1) represents a parabola with vertex $(0,0)$ and axis as x-axis,

equation (2) represents a circle with centre $(0,0)$ and radius $\frac{4}{\sqrt{3}}$ and meets axes at

$\left(\pm\frac{4}{\sqrt{3}}, 0\right)$ and $\left(0, \pm\frac{4}{\sqrt{3}}\right)$. A rough sketch of the curves is given below:-



$$\text{Required area} = \text{Region } OCBAO$$
$$= 2(\text{Region } OBAO)$$
$$= 2(\text{Region } ODAO + \text{Region } DBAD)$$
$$= 2\left[\int_0^a \sqrt{3x}\,dx + \int_a^{\frac{4}{\sqrt{3}}} \sqrt{\left(\frac{4}{\sqrt{3}}\right)^2 - x^2}\,dx\right]$$

$$A = 2\left[\left(\sqrt{3}.\frac{2}{3}x\sqrt{x}\right)_0^a + \left(\frac{x}{2}\sqrt{\left(\frac{4}{\sqrt{3}}\right)^2 - x^2} + \frac{16}{6}\sin^{-1}\frac{x\sqrt{3}}{4}\right)_a^{\frac{4}{\sqrt{3}}}\right]$$

$$= 2\left[\left(\frac{2}{\sqrt{3}}a\sqrt{a}\right) + \left\{\left(0 + \frac{8}{3}\sin^{-1}(1)\right) - \left(\frac{a}{2}\sqrt{\left(\frac{4}{\sqrt{3}}\right)^2 - a^2} + \frac{8}{3}\sin^{-1}\frac{a\sqrt{3}}{4}\right)\right\}\right]$$

Thus, $A = \frac{4}{\sqrt{3}}a^{\frac{3}{2}} + \frac{8\pi}{3} - a\sqrt{\frac{16}{3} - a^2} - \frac{16}{3}\sin^{-1}\left(\frac{\sqrt{3}a}{4}\right)$

Where, $a = \frac{-9+\sqrt{273}}{6}$

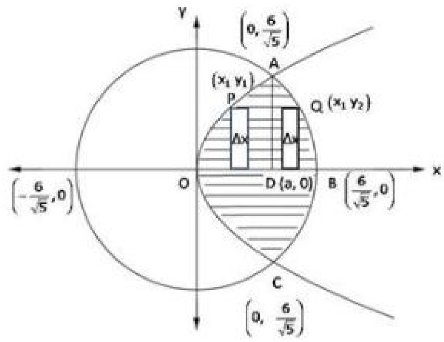
Areas of Bounded Regions Ex-21-3 Q14

To find area $\left\{(x,y) : y^2 \le 5x, 5x^2 + 5y^2 \le 36\right\}$

$$\Rightarrow \quad y^2 = 5x \qquad ---(1)$$
$$5x^2 + 5y^2 = 36$$
$$x^2 + y^2 = \frac{36}{5} \qquad ---(2)$$

Equation (1) represents a parabola with vertex $(0,0)$ and axis as x-axis.

Equation (2) represents a circle with centre $(0,0)$ and radius $\frac{6}{\sqrt{5}}$ and meets axes at $\left(\pm\frac{6}{\sqrt{5}},0\right)$ and $\left(0,\pm\frac{6}{\sqrt{5}}\right)$. x ordinate of point of intersection of circle and parabola is a where $a = \frac{-25+\sqrt{1345}}{10}$. A rough sketch of curves is:-



Required area = Region $OCBAO$

$$A = 2\left(\text{Region } OBAO\right)$$
$$= 2\left(\text{Region } ODAO + \text{Region } DBAD\right)$$
$$= 2\left[\int_0^a \sqrt{5x}\,dx + \int_a^{\frac{6}{\sqrt5}} \sqrt{\left(\frac{6}{\sqrt5}\right)^2 - x^2}\,dx\right]$$
$$= 2\left[\left(\sqrt5 . \frac{2}{3}x\sqrt{x}\right)_0^a + \left(\frac{x}{2}\sqrt{\left(\frac{6}{\sqrt5}\right)^2 - x^2} + \frac{36}{10}\sin^{-1}\left(\frac{x\sqrt5}{6}\right)\right)_a^{\frac{6}{\sqrt5}}\right]$$
$$= \frac{4\sqrt5}{3}a\sqrt{a} + 2\left\{\left(0 + \frac{18}{5}.\frac{\pi}{2}\right) - \left(\frac{a}{2}\sqrt{\left(\frac{6}{\sqrt5}\right)^2 - a^2} + \frac{18}{5}\sin^{-1}\left(\frac{a\sqrt5}{6}\right)\right)\right\}$$

Thus, $A = \frac{4\sqrt5}{a}a^{\frac{3}{2}} + \frac{18\pi}{5} - a\sqrt{\frac{36}{5} - a^2} - \frac{36}{5}\sin^{-1}\left(\frac{a\sqrt5}{6}\right)$

Where, $a = \frac{-25+\sqrt{1345}}{10}$

Areas of Bounded Regions Ex-21-3 Q15

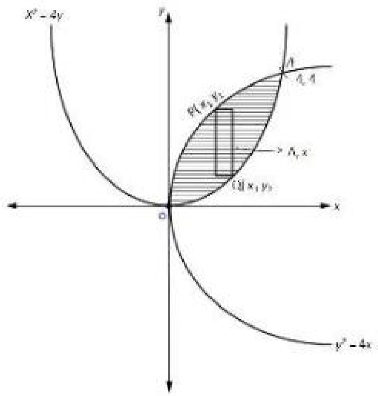
To find area bounded by

$$y^2 = 4x \qquad ---(1)$$

$$x^2 = 4y \qquad ---(2)$$

Equation (1) represents a parabola with vertex (0,0) and axis as x-axis.
Equation (2) represents a parabola with vertex (0,0) and axis as y-axis.
Points of intersection of parabolas are (0,0) and (4,4).

A rough sketch is given as:-



The shaded region is required area and it is sliced into rectangles with width Δx and length $(y_1 - y_2)$. Area of rectangle = $(y_1 - y_2)\Delta x$.

This approximation rectangle slide from $x = 0$ to $x = 4$, so

Required area = Region $OQAPO$

$$A = \int_0^4 (y_1 - y_2)dx$$
$$= \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right)dx$$
$$= \left[2.\frac{2}{3}x\sqrt{x} - \frac{x^3}{12}\right]_0^4$$
$$= \left[\left(\frac{4}{3}.4\sqrt{4} - \frac{64}{12}\right) - (0)\right]$$

$$A = \frac{32}{3} - \frac{16}{3}$$

$$A = \frac{16}{3} \text{ sq.units}$$

Areas of Bounded Regions Ex-21-3 Q16

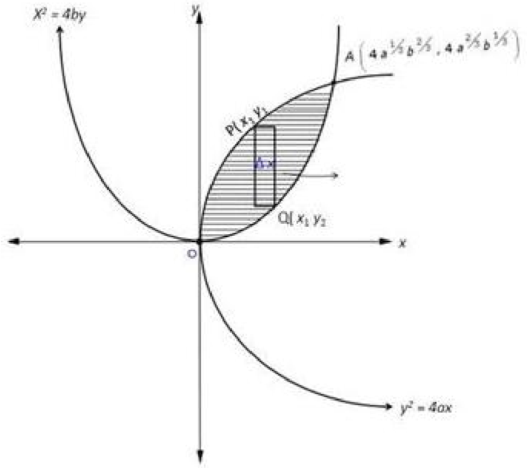
To find area enclosed by

$y^2 = 4ax \qquad ---(1)$

$x^2 = 4by \qquad ---(2)$

Equation (1) represents a parabola with vertex $(0,0)$ and axis as x-axis,
equation (2) represents a parabola with vertex $(0,0)$ and axis as y-axis,
points of intersection of parabolas are $(0,0)$ and $\left(4a\frac{1}{3}\, b\frac{2}{3},\ 4a\frac{2}{3}\, b\frac{1}{3}\right)$

A rough sketch is given as:-



The shaded region is required area and it is sliced into rectangles of width = Δx and length $(y_1 - y_2)$.

Area of rectangle = $(y_1 - y_2)\Delta x$.

This approximation rectangle slides from $x = 0$ to $x = 4a\frac{1}{3}\, b\frac{2}{3}$, so

Required area = Region $OQAPO$

$$= \int_0^{4a\frac{1}{3} b\frac{2}{3}} (y_1 - y_2)\, dx$$

$$= \int_0^{4a\frac{1}{3} b\frac{2}{3}} \left(2\sqrt{a}.\sqrt{x} - \frac{x^2}{4b}\right) dx$$

$$= \left[2\sqrt{a}.\frac{2}{3} x\sqrt{x} - \frac{x^3}{12b}\right]_0^{4a\frac{1}{3} b\frac{2}{3}}$$

$$= \frac{32\sqrt{a}}{3}.a\frac{1}{3}\, b\frac{2}{3}\, a\frac{1}{6}\, b\frac{1}{3} - \frac{64ab^2}{12b}$$

$$= \frac{32}{3}ab - \frac{16}{3}ab$$

$A = \frac{16}{3}ab$ sq.units

Areas of Bounded Regions Ex-21-3 Q17

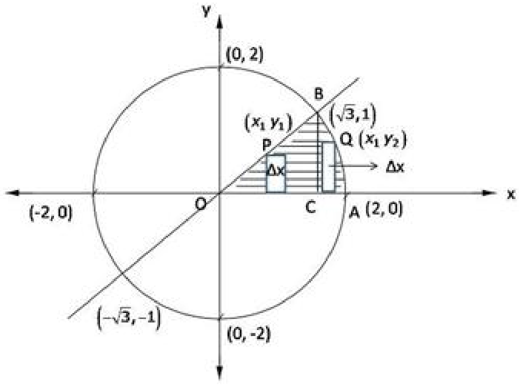
To find area in first quadrant enclosed by x-axis.

$x = \sqrt{3}y$ --- (1)

$x^2 + y^2 = 4$ --- (2)

Equation (1) represents a line passing through $(0,0), (-\sqrt{3},-1), (\sqrt{3},1)$. Equation (2) represents a circle with centre $(0,0)$ and passing through $(\pm 2,0), (0,\pm 2)$. Points of intersection of line and circle are $(-\sqrt{3},-1)$ and $(\sqrt{3},1)$.

A rough sketch of curves is given below:-



Required area = Region $OABO$

A = Region $OCBO$ + Region $ABCA$

$= \int_0^{\sqrt{3}} y_1 dx + \int_{\sqrt{3}}^{2} y_2 dx$

$= \int_0^{\sqrt{3}} \frac{x}{\sqrt{3}} dx + \int_{\sqrt{3}}^{2} \sqrt{4-x^2} dx$

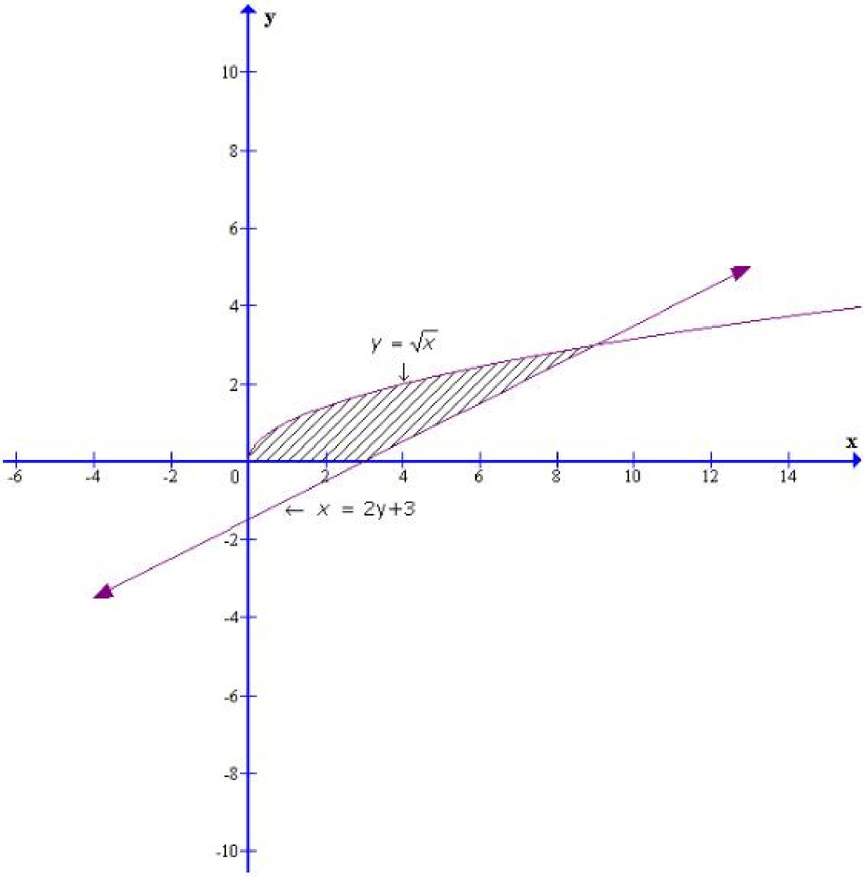
$= \left(\frac{x^2}{2\sqrt{3}}\right)_0^{\sqrt{3}} + \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_{\sqrt{3}}^{2}$

$= \left(\frac{3}{2\sqrt{3}} - 0\right) + \left[\left(0 + 2\sin^{-1}(1)\right) - \left(\frac{\sqrt{3}}{2}.1 + 2\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right]$

$= \frac{\sqrt{3}}{2} + 2.\frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2.\frac{\pi}{3}$

$A = \frac{\pi}{3}$ sq.units

Areas of Bounded Regions Ex-21-3 Q18



Area of the bounded region

$$= \int_0^3 \sqrt{x}\, dx + \int_3^9 \sqrt{x} - \left(\frac{x-3}{2}\right) dx$$

$$= \left[\frac{x^{3/2}}{3/2}\right]_0^3 + \left[\frac{x^{3/2}}{3/2} - \frac{x^2}{4} + \frac{3x}{2}\right]_3^9$$

$$= \left[\frac{(3)^{3/2}}{3/2} - 0\right] + \left[\frac{(9)^{3/2}}{3/2} - \frac{(9)^2}{4} + \frac{3(9)}{2} - \frac{(3)^{3/2}}{3/2} + \frac{(3)^2}{4} - \frac{3(3)}{2}\right]$$

= 9 sq. units

Areas of Bounded Regions Ex-21-3 Q19

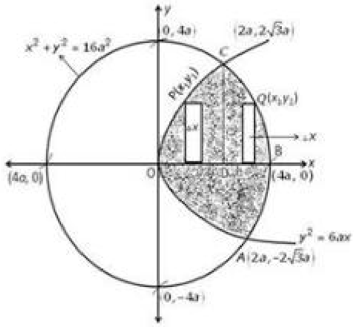
To find area in enclosed by

$x^2 + y^2 = 16a^2 \qquad ---(1)$

and $y^2 = 6ax \qquad ---(2)$

Equation (1) represents a circle with centre $(0,0)$ and meets axes $(\pm 4a, 0), (0, \pm 4a)$. Equation (2) represents a parabola with vertex $(0,0)$ and axis as x-axis. Points of intersection of circle and parabola are $(2a, 2\sqrt{3}a), (2a, -2\sqrt{3}a)$.

A rough sketch of curves is given as:-



Region $ODCO$ is sliced into rectangles of area $= y_1 \Delta x$ and it slides from $x = 0$ to $x = 2a$.

Region $BCDB$ is sliced into rectangles of area $= y_2 \Delta x$ it slides from $x = 2a$ to $x = 4a$. So,

Required area $= 2[\text{Region } ODCO + \text{Region } BCDB]$

$$= 2\left[\int_0^{2a} y_1 dx + \int_{2a}^{4a} y_2 dx\right]$$

$$= 2\left[\int_0^{2a} \sqrt{6ax}\, dx + \int_{2a}^{4a} \sqrt{16a^2 - x^2}\, dx\right]$$

$$= 2\left[\sqrt{6a}.\left(\frac{2}{3}x\sqrt{x}\right)_0^{2a} + \left[\frac{x}{2}\sqrt{16a^2 - x^2} + \frac{16a^2}{2}\sin^{-1}\left(\frac{x}{4a}\right)\right]_{2a}^{4a}\right]$$

$$= 2\left[\left(\sqrt{6a}.\frac{2}{3}2a\sqrt{2a}\right) + \left[\left(0 + 8a^2.\frac{\pi}{2}\right) - \left(a\sqrt{12a^2} + 8a^2.\frac{\pi}{6}\right)\right]\right]$$

$$= 2\left[\frac{8\sqrt{3}a^2}{3} + 4a^2\pi - 2\sqrt{3}a^2 - \frac{4}{3}a^2\pi\right]$$

$$= 2\left[\frac{2\sqrt{3}a^2}{3} + \frac{8a^2\pi}{3}\right]$$

$$A = \frac{4a^2}{3}\left(4\pi + \sqrt{3}\right) \text{ sq.units}$$

Areas of Bounded Regions Ex-21-3 Q20

To find area lying above x-axis and included in the circle

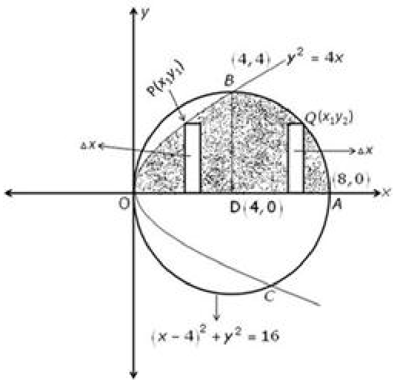
$x^2 + y^2 = 8x$

$(x-4)^2 + y^2 = 16$ --- (1)

and $y^2 = 4x$ --- (2)

Equation (1) represents a circle with centre (4,0) and meets axes at (0,0) and (8, 0).
Equation(2) represent a parabola with vertex (0,0) and axis as x-axis. They intersect at (4,−4) and (4,4).

A rough sketch of the curves is as under:-



Shaded region is the required region

Required area = Region $OABO$

Required area = Region $ODBO$ + Region $DABD$ --- (1)

Region $ODBO$ is sliced into rectangles of area $y_1 \Delta x$. This approximation rectangle can slide from $x = 0$ to $x = 4$. So,

$$\text{Region } ODBO = \int_0^4 y_1 dx$$
$$= \int_0^4 2\sqrt{x}dx$$
$$= 2\left(\frac{2}{3}x\sqrt{x}\right)_0^4$$

$$\text{Region } ODBO = \frac{32}{3} \text{ sq. units} \quad \text{--- (2)}$$

Region $DABD$ is sliced into rectangles of area $y_2 \Delta x$. Which moves from $x = 4$ to $x = 8$. So,

$$\text{Region } DABD = \int_4^8 y_2 dx$$
$$= \int_4^8 \sqrt{16-(x-4)^2}dx$$
$$= \left[\frac{(x-4)}{2}\sqrt{16-(x-4)^2} + \frac{16}{2}\sin^{-1}\left(\frac{x-4}{4}\right)\right]_4^8$$
$$= \left[\left(0 + 8.\frac{\pi}{2}\right) - (0+0)\right]$$

$$\text{Region } DABD = 4\pi \text{ sq. units} \quad \text{--- (3)}$$

Using (1), (2) and (3), we get

$$\text{Required area} = \left(\frac{32}{3} + 4\pi\right)$$

$$A = 4\left(\pi + \frac{8}{3}\right) \text{ sq.units}$$

Areas of Bounded Regions Ex-21-3 Q21

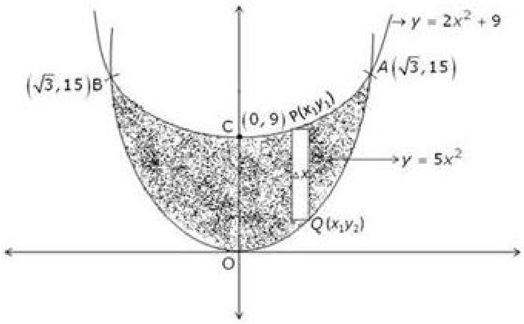
To find area enclosed by

$y = 5x^2 \qquad\qquad ---(1)$

$y = 2x^2 + 9 \qquad\qquad ---(2)$

Equation (1) represents a parabola with vertex $(0,0)$ and axis as y-axis. Equation (2) represents a parabola with vertex $(0,9)$ and axis as y-axis. Points of intersection of parabolas are $(\sqrt{3},15)$ and $(-\sqrt{3},15)$.

A rough sketch of curves is given as:-



Region $AOCA$ is sliced into rectangles with area $(y_1 - y_2)\Delta x$. It slides from $x = 0$ to $x = \sqrt{3}$, so

$$\begin{aligned}
\text{Required area} &= \text{Region } AOBCA \\
&= 2(\text{Region } AOCA) \\
&= 2\int_0^{\sqrt{3}} (y_1 - y_2)\,dx \\
&= 2\int_0^{\sqrt{3}} \left(2x^2 + 9 - 5x^2\right)dx \\
&= 2\int_0^{\sqrt{3}} \left(9 - 3x^2\right)dx \\
&= 2\left[9x - x^3\right]_0^{\sqrt{3}} \\
&= 2\left[\left(9\sqrt{3} - 3\sqrt{3}\right) - (0)\right]
\end{aligned}$$

Required area $= 12\sqrt{3}$ sq.units

Areas of Bounded Regions Ex-21-3 Q22

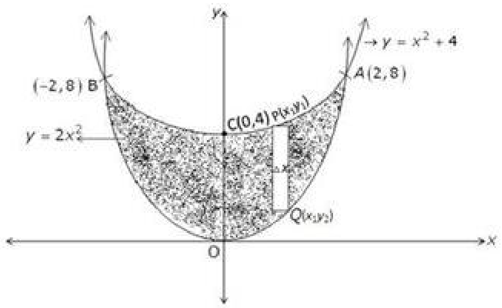
To find area enclosed by

$y = 2x^2$ ---(1)

$y = x^2 + 4$ ---(2)

Equation (1) represents a parabola with vertex (0,0) and axis as y-axis. Equation (2) represents a parabola with vertex (0,4) and axis as y-axis. Points of intersection of parabolas are (2,8) and (−2,8).

A rough sketch of curves is given as:-



Region $AOCA$ is sliced into rectangles with area $(y_1 - y_2)\Delta x$. And it slides from $x = 0$ to $x = 2$

Required area = Region $AOBCA$

$A = 2$ (Region $AOCA$)

$$= 2\int_0^2 (y_1 - y_2)\,dx$$

$$= 2\int_0^2 \left(x^2 + 4 - 2x^2\right)dx$$

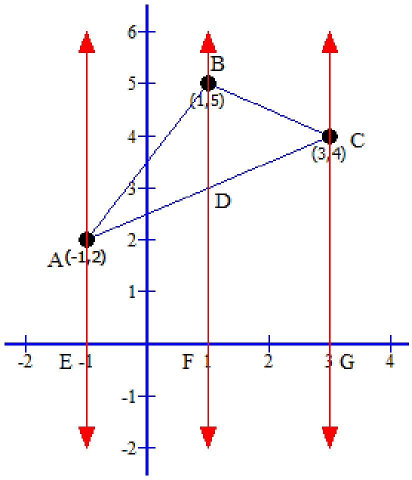
$$= 2\int_0^2 \left(4 - x^2\right)dx$$

$$= 2\left[4x - \frac{x^3}{3}\right]_0^2$$

$$= 2\left[\left(8 - \frac{8}{3}\right) - (0)\right]$$

$$A = \frac{32}{3} \text{ sq.units}$$

Areas of Bounded Regions Ex-21-3 Q23



Equation of side AB,

$$\frac{x+1}{1+1}=\frac{y-2}{5-2}$$

$$\Rightarrow \frac{x+1}{2}=\frac{y-2}{3}$$

$$\Rightarrow 3x+3=2y-4$$

$$\Rightarrow 2y-3x=7$$

$$\therefore\ y=\frac{3x+7}{2}\ldots\ldots.(i)$$

Equation of side BC,

$$\frac{x-1}{3-1}=\frac{y-5}{4-5}$$

$$\Rightarrow \frac{x-1}{2}=\frac{y-5}{-1}$$

$$\Rightarrow -x+1=2y-10$$

$$\Rightarrow 2y=11-x$$

$$\therefore\ y=\frac{11-x}{2}\ldots\ldots(ii)$$

Equation of side AC,

$$\frac{x+1}{3+1}=\frac{y-2}{4-2}$$

$$\Rightarrow \frac{x+1}{4}=\frac{y-2}{2}$$

$$\Rightarrow \frac{x+1}{2}=\frac{y-2}{1}$$

$$\Rightarrow x+1=2y-4$$

$$\Rightarrow 2y=5+x$$

$$\therefore\ y=\frac{5+x}{2}$$

Area of required region

= Area of EABFE + Area of BFGCB - Area of AEGCA

$$=\int_{-1}^{1} y_{AB}dx+\int_{1}^{3} y_{BC}dx-\int_{-1}^{3} y_{AC}dx$$

$$=\int_{-1}^{1}\frac{3x+7}{2}dx+\int_{1}^{3}\frac{11-x}{2}dx-\int_{-1}^{3}\frac{5+x}{2}dx$$

$$=\frac{1}{2}\left[\frac{3x^2}{2}+7x\right]_{-1}^{1}+\frac{1}{2}\left[11x-\frac{x^2}{2}\right]_{1}^{3}-\frac{1}{2}\left[5x+\frac{x^2}{2}\right]_{-1}^{3}$$

$$=\frac{1}{2}\left[\frac{3(1^2-1^2)}{2}+7(1-(-1))\right]+\frac{1}{2}\left[11(3-1)-\frac{(3)^2-1^2}{2}\right]$$

$$-\frac{1}{2}\left[5(3-(-1))+\frac{(3)^2-1^2}{2}\right]$$

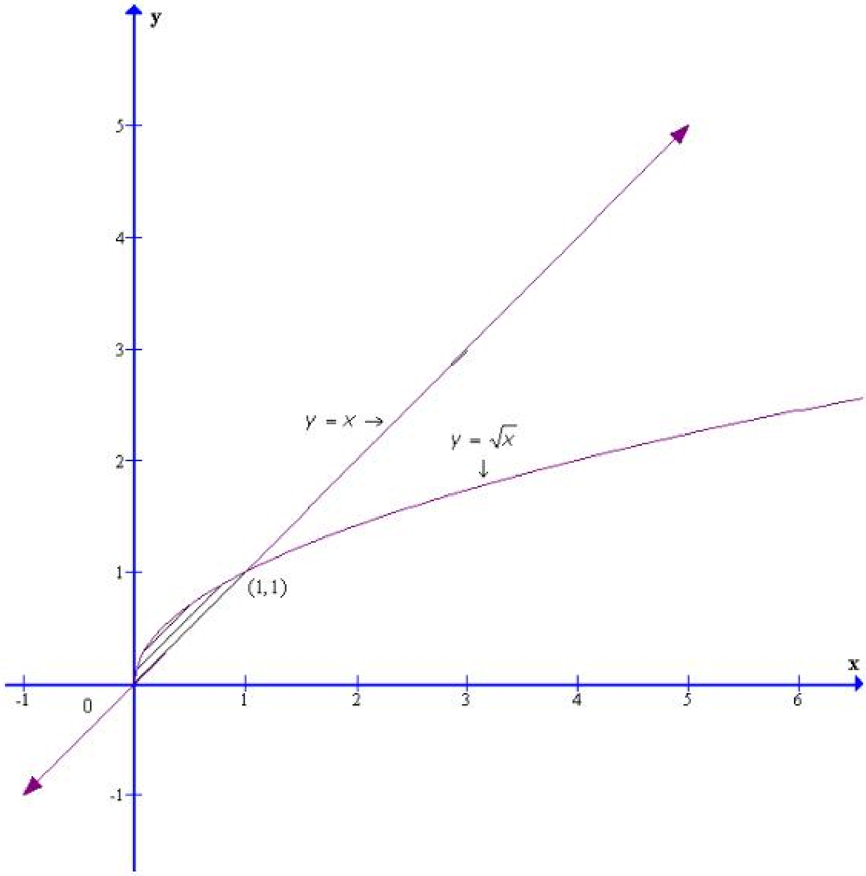
$$=\frac{1}{2}[0+14]+\frac{1}{2}[22-4]-\frac{1}{2}[20+4]$$

$$=7+\frac{1}{2}\times 18-\frac{1}{2}\times 24$$

$$=7+9-12$$

$$=4 \text{ sq units}$$

Areas of Bounded Regions Ex-21-3 Q24



Area of the bounded region

$= \int_0^1 \sqrt{x} - x \, dx$

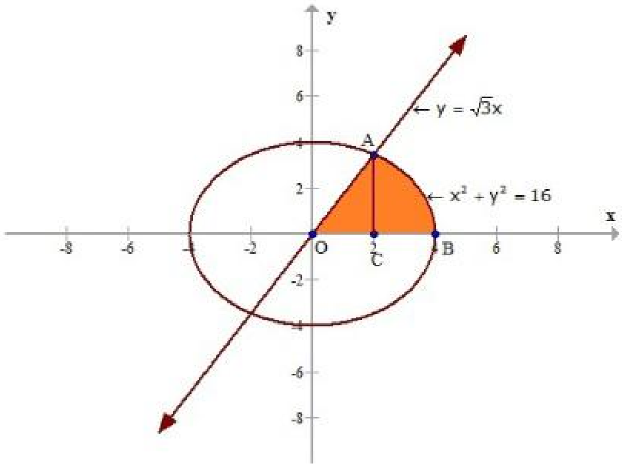
$= \left[\frac{x^{3/2}}{3/2} - \frac{x^2}{2} \right]_0^1$

$= \left[\frac{2}{3} - \frac{1}{2} \right]$

$= \frac{1}{6}$ sq. units

Areas of Bounded Regions Ex-21-3 Q25

Consider the following graph.



We have, $y=\sqrt{3}x$

Substituting this value in $x^2+y^2=16$,

$x^2+(\sqrt{3}x)^2=16$

$\Rightarrow x^2+3x^2=16$

$\Rightarrow 4x^2=16$

$\Rightarrow x^2=4$

$\Rightarrow x=\pm 2$

Since the shaded region is in the first quadrant, let us take the positive value of x.

Therefore, $x=2$ and $y=2\sqrt{3}$ are the coordinates of the intersection point A.

Thus, area of the shaded region $OAB=$ Area $OAC+$ Area ACB

$\Rightarrow \text{Area } OAB=\int_0^2 \sqrt{3}x\,dx+\int_2^4 \sqrt{16-x^2}\,dx$

$\Rightarrow \text{Area } OAB=\left(\frac{\sqrt{3}x^2}{2}\right)_0^2+\frac{1}{2}\left[x\sqrt{16-x^2}+16\sin^{-1}\left(\frac{x}{4}\right)\right]_2^4$

$\Rightarrow \text{Area } OAB=\left(\frac{\sqrt{3}\times 4}{2}\right)+\frac{1}{2}\left[16\sin^{-1}\left(\frac{4}{4}\right)\right]-\frac{1}{2}\left[4\sqrt{16-12}+16\sin^{-1}\left(\frac{2}{4}\right)\right]$

$\Rightarrow \text{Area } OAB=2\sqrt{3}+\frac{1}{2}\left[16\times\frac{\pi}{2}\right]-\frac{1}{2}\left[4\sqrt{3}+16\sin^{-1}\left(\frac{1}{2}\right)\right]$

$\Rightarrow \text{Area } OAB=2\sqrt{3}+4\pi-2\sqrt{3}-\frac{4\pi}{3}$

$\Rightarrow \text{Area } OAB=4\pi-\frac{4\pi}{3}$

$\Rightarrow \text{Area } OAB=\frac{8\pi}{3}$ sq. units.

Areas of Bounded Regions Ex-21-3 Q26

To find area bounded by

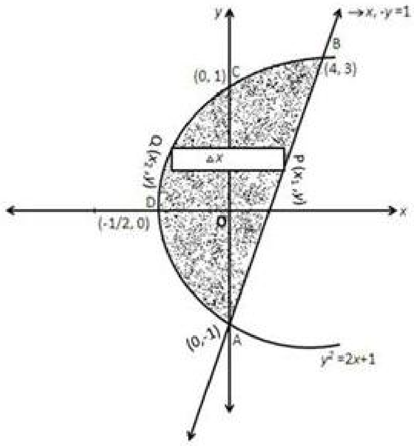
$y^2 = 2x + 1$ --- (1)

and $x - y = 1$ --- (2)

Equation (1) is a parabola with vertex $\left(-\frac{1}{2}, 0\right)$ and passes through $(0,1), (0,-1)$.

Equation (2) is a line passing through $(1,0)$ and $(0,-1)$. Points of intersection of parabola and line are $(3,2)$ and $(0,-1)$.

A rough sketch of the curves is given as:-



Shaded region represents the required area. It is sliced in rectangles of area $(x_1 - x_2)\Delta y$. It slides from $y = -1$ to $y = 3$, so

Required area = Region $ABCDA$

$$= \int_{-1}^{3} (x_1 - x_2)\, dy$$

$$= \int_{-1}^{3} \left(1 + y - \frac{y^2 - 1}{2}\right) dy$$

$$= \frac{1}{2} \int_{-1}^{3} \left(2 + 2y - y^2 + 1\right) dy$$

$$= \frac{1}{2} \int_{-1}^{3} \left(3 + 2y - y^2\right) dy$$

$$= \frac{1}{2} \left[3y + y^2 - \frac{y^3}{3}\right]_{-1}^{3}$$

$$= \frac{1}{2} \left[(9 + 9 - 9) - \left(-3 + 1 + \frac{1}{3}\right)\right]$$

$$= \frac{1}{2} \left[9 + \frac{5}{3}\right]$$

$$= \frac{32}{6}$$

Required area $= \frac{16}{3}$ sq. units

Areas of Bounded Regions Ex-21-3 Q27

To find region bounded by curves

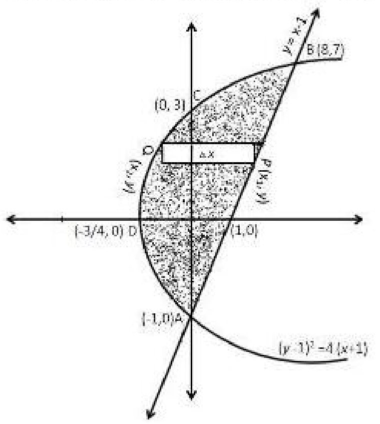
$y = x - 1$ --- (1)

and $(y-1)^2 = 4(x+1)$ --- (2)

Equation (1) represents a line passing through $(1,0)$ and $(0,-1)$ equation (2) represents a parabola with vertex $(-1,1)$ passes through $(0,3), (0,-1), \left(-\frac{3}{4}, 0\right)$.

Their points of intersection $(0,-1)$ and $(8,7)$.

A rough sketch of curves is given as:-



Shaded region is required area. It is sliced in rectangles of area $(x_1 - x_2)\Delta y$.
It slides from $y = -1$ to $y = 7$, so

Required area = Region $ABCDA$

$$A = \int_{-1}^{7} (x_1 - x_2)\, dy$$

$$= \int_{-1}^{7} \left(y + 1 - \frac{(y-1)^2}{4} + 1 \right) dy$$

$$= \frac{1}{4}\int_{-1}^{7} \left(4y + 4 - y^2 - 1 + 2y + 4\right) dy$$

$$= \frac{1}{4}\int_{-1}^{7} \left(6y + 7 - y^2\right) dy$$

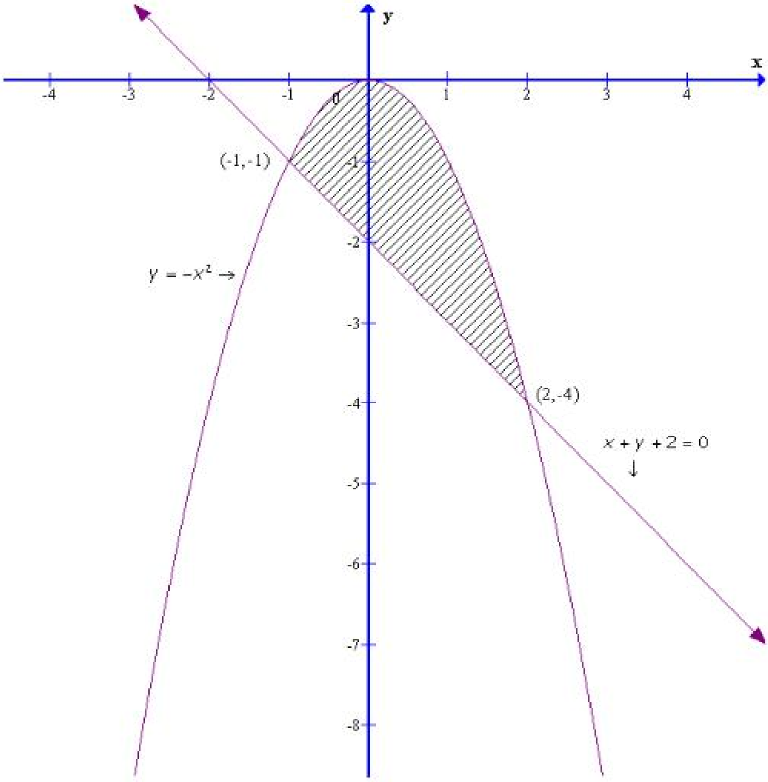
$$= \frac{1}{4}\left[3y^2 + 7y - \frac{y^3}{3}\right]_{-1}^{7}$$

$$= \frac{1}{4}\left[\left(147 + 49 - \frac{343}{3}\right) - \left(3 - 7 + \frac{1}{3}\right)\right]$$

$$= \frac{1}{4}\left[\frac{245}{3} + \frac{11}{3}\right]$$

$$A = \frac{64}{3} \text{ sq. units}$$

Areas of Bounded Regions Ex-21-3 Q28



Area of the bounded region

$= \int_{-1}^{2} -x^2 - (-2-x)\, dx$

$= \left[-\frac{x^3}{3} + 2x + \frac{x^2}{2} \right]_{-1}^{2}$

$= \left[-\frac{8}{3} + 6 \right] - \left(\frac{1}{3} + \frac{1}{2} - 2 \right)$

$= \frac{9}{2}$ sq.units

Areas of Bounded Regions Ex-21-3 Q29

To find area bounded by

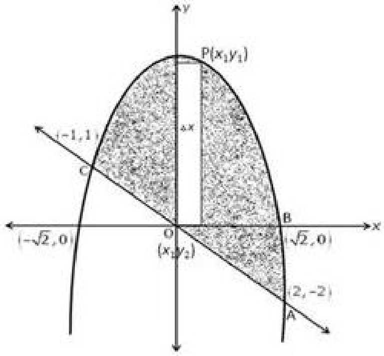
$y = 2 - x^2$ ---(1)

and $y + x = 0$ ---(2)

Equation (1) represents a parabola with vertex $(0,2)$ and downward, meets axes at $(\pm\sqrt{2},0)$.

Equation (2) represents a line passing through $(0,0)$ and $(2,-2)$. The points of intersection of line and parabola are $(2,-2)$ and $(-1,1)$.

A rough sketch of curves is as follows:-



Shaded region is sliced into rectangles with area $= (y_1 - y_2)\Delta x$. It slides from $x = -1$ to $x = 2$, so

Required area = Region $ABPCOA$

$$A = \int_{-1}^{2}(y_1 - y_2)dx$$

$$= \int_{-1}^{2}(2 - x^2 + x)dx$$

$$= \left[2x - \frac{x^3}{3} + \frac{x^2}{2}\right]_{-1}^{2}$$

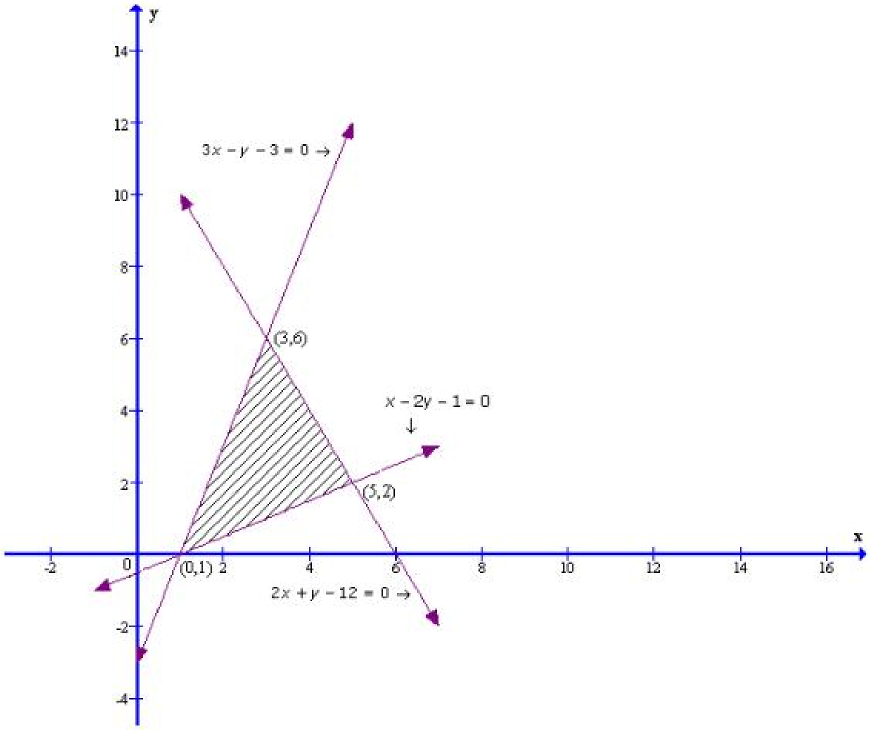
$$= \left[\left(4 - \frac{8}{3} + 2\right) - \left(-2 + \frac{1}{3} + \frac{1}{2}\right)\right]$$

$$= \left[\frac{10}{3} + \frac{7}{6}\right]$$

$$= \frac{27}{6}$$

$$A = \frac{9}{2} \text{ sq. units}$$

Areas of Bounded Regions Ex-21-3 Q30



Area of the bounded region

$$= \int_0^3 3x - 3 - \left(\frac{x-1}{2}\right)dx + \int_3^5 12 - 2x - \left(\frac{x-1}{2}\right)dx$$

$$= \left[\frac{3x^2}{2} - 3x - \frac{x^2}{4} + \frac{1}{2}x\right]_0^3 + \left[12x - 2\frac{x^2}{2} - \frac{x^2}{4} + \frac{1}{2}x\right]_3^5$$

$$= \left[\frac{27}{2} - 9 - \frac{9}{4} + \frac{3}{2}\right] + \left[60 - 25 - \frac{25}{4} + \frac{5}{2} - 36 + 9 + \frac{9}{4} - \frac{3}{2}\right]$$

= 11 sq.units

Areas of Bounded Regions Ex-21-3 Q31

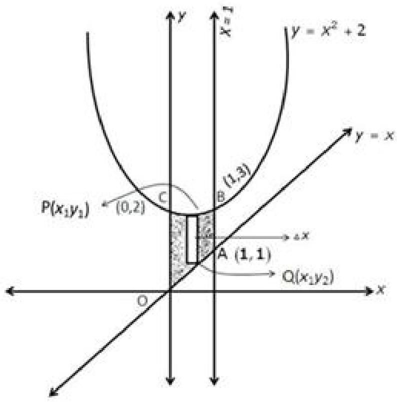
To find area bounded by $x = 0$, $x = 1$

and

$y = x \qquad ---(1)$

$y = x^2 + 2 \qquad ---(2)$

Equation (1) is a line passing through $(2,2)$ and $(0,0)$. Equation (2) is a parabola upward with vertex at $(0,2)$. A rough sketch of curves is as under:-



Shaded region is sliced into rectangles of area = $(y_1 - y_2)\Delta x$. It slides from $x = 0$ to $x = 1$, so

Required area = Region $OABCO$

$A = \int_0^1 (y_1 - y_2)dx$

$= \int_0^1 (x^2 + 2 - x)dx$

$= \left[\frac{x^3}{3} + 2x - \frac{x^2}{2}\right]_0^1$

$= \left[\left(\frac{1}{3} + 2 - \frac{1}{2}\right) - (0)\right]$

$= \left(\frac{2 + 12 - 3}{6}\right)$

$A = \frac{11}{6}$ sq. units

Areas of Bounded Regions Ex-21-3 Q32

To find area bounded by

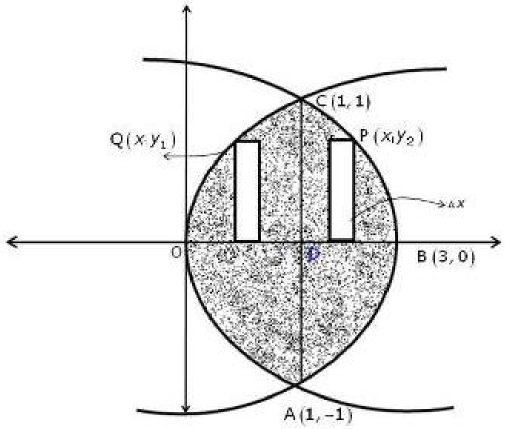
$x = y^2 \qquad ---(1)$

and

$x = 3 - 2y^2$

$2y^2 = -(x-3) \qquad ---(2)$

Equation (1) represents an upward parabola with vertex $(0,0)$ and axis -y. Equation (2) represents a parabola with vertex $(3,0)$ and axis as x-axis. They intersect at $(1,-1)$ and $(1,1)$. A rough sketch of the curves is as under:-



Required area = Region $OABCO$

$A = 2$ Region $OBCO$

$= 2[\text{Region } ODCO + \text{Region } BDCB]$

$= 2\left[\int_0^1 y_1 dx + \int_1^3 y_2 dx\right]$

$= 2\left[\int_0^1 \sqrt{x}dx + \int_1^3 \sqrt{\frac{3-x}{2}}dx\right]$

$= 2\left[\left(\frac{2}{3}x\sqrt{x}\right)_0^1 + \left(\frac{2}{3}.\left(\frac{3-x}{2}\right)\sqrt{\frac{3-x}{2}}.(-2)\right)_1^3\right]$

$= 2\left[\left(\frac{2}{3} - 0\right) + \left\{(0) - \left(\frac{2}{3}.1.1.(-2)\right)\right\}\right]$

$= 2\left[\frac{2}{3} + \frac{4}{3}\right]$

$A = 4$ sq. units

Areas of Bounded Regions Ex-21-3 Q33

To find area of $\triangle ABC$ with $A(4,1)$, $B(6,6)$ and $C(8,4)$.

Equation of AB,

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

$$y - 1 = \left(\frac{6 - 1}{6 - 4}\right)(x - 4)$$

$$y - 1 = \frac{5}{2}x - 10$$

$$y = \frac{5}{2}x - 9 \qquad ---(1)$$

Equation of BC,

$$y - 6 = \left(\frac{4 - 6}{8 - 6}\right)(x - 6)$$

$$= -1(x - 6)$$

$$y = -x + 12 \qquad ---(2)$$

Equation of AC,

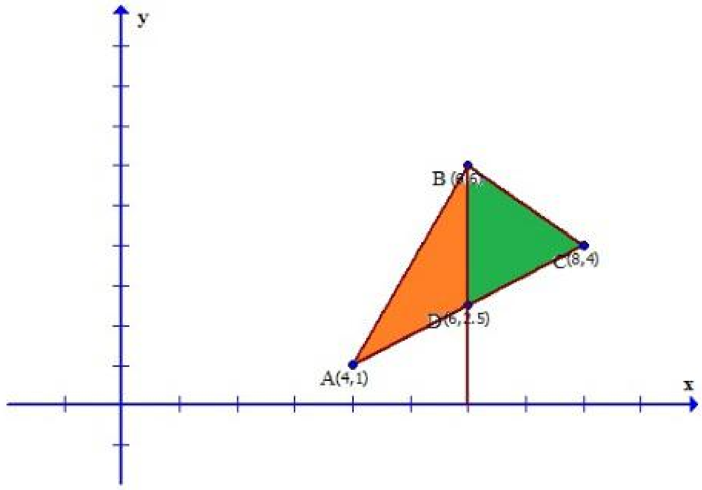
$$y - 1 = \left(\frac{4 - 1}{8 - 4}\right)(x - 4)$$

$$y - 1 = \frac{3}{4}(x - 4)$$

$$\Rightarrow \quad y = \frac{3}{4}x - 3 + 1$$

$$y = \frac{3}{4}x - 2 \qquad ---(3)$$

A rough sketch is as under:-



Clearly, Area of $\Delta ABC = Area\ ADB + Area\ BDC$

Area ADB: To find the area ADB, we slice it into vertical strips.

We observe that each vertical strip has its lower end on side AC and the upper end on AB. So the approximating rectangle has

$Length = y_2 - y_1$

$Width = \Delta x$

$Area = (y_2 - y_1)\Delta x$

Since the approximating rectangle can move from $x = 4$ *to* 6,

the area of the triangle ADB $= \int_4^6 (y_2 - y_1)dx$

$\Rightarrow$ *area of the triangle ADB* $= \int_4^6 \left[\left(\frac{5x}{2} - 9\right) - \left(\frac{3}{4}x - 2\right)\right]dx$

$\Rightarrow$ *area of the triangle ADB* $= \int_4^6 \left(\frac{5x}{2} - 9 - \frac{3}{4}x + 2\right)dx$

$\Rightarrow$ *area of the triangle ADB* $= \int_4^6 \left(\frac{7x}{4} - 7\right)dx$

$\Rightarrow$ *area of the triangle ADB* $= \left(\frac{7x^2}{4 \times 2} - 7x\right)_4^6$

$\Rightarrow$ *area of the triangle ADB* $= \left(\frac{7 \times 36}{8} - 7 \times 6\right) - \left(\frac{7 \times 16}{8} - 7 \times 4\right)$

$\Rightarrow$ *area of the triangle ADB* $=\left(\frac{63}{2}-42-14+28\right)$

$\Rightarrow$ *area of the triangle ADB* $=\left(\frac{63}{2}-28\right)$

Similarly, Area BDC $=\int_6^8 (y_4-y_3)dx$

$\Rightarrow Area\ BDC=\int_6^8 (y_4-y_3)dx$

$\Rightarrow Area\ BDC=\int_6^8 \left[(-x+12)-\left(\frac{3}{4}x-2\right)\right]dx$

$\Rightarrow Area\ BDC=\int_6^8 \left[\frac{-7x}{4}+14\right]dx$

$\Rightarrow Area\ BDC=\left[-\frac{7x^2}{8}+14x\right]_6^8$

$\Rightarrow Area\ BDC=\left[-\frac{7\times 64}{8}+14\times 8\right]-\left[-\frac{7\times 36}{8}+14\times 6\right]$

$\Rightarrow Area\ BDC=\left[-56+112+\frac{63}{2}-84\right]$

$\Rightarrow Area\ BDC=\left(\frac{63}{2}-28\right)$

Thus, Area ABC = Area ADB + Area BDC

$\Rightarrow Area\ ABC=\left(\frac{63}{2}-28\right)+\left(\frac{63}{2}-28\right)$

$\Rightarrow Area\ ABC=63-56$

$\Rightarrow Area\ ABC=7\ sq.\ units$

Areas of Bounded Regions Ex-21-3 Q34

To find area of region

$\left\{(x,y): |x-1| \le y \le \sqrt{5-x^2}\right\}$

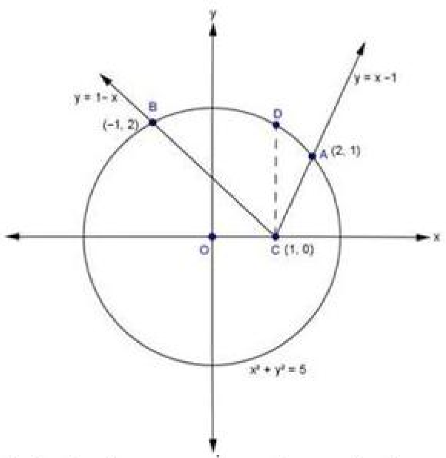
$\Rightarrow \quad |x-1| = y$

$\Rightarrow \quad y = \begin{cases} 1-x, \text{ if } x < 1 & ---(1) \\ x-1, \text{ if } x \ge 1 & ---(2) \end{cases}$

And $\quad x^2+y^2=5 \qquad ---(3)$

Equation (1) and (2) represent straight lines and equation (3) is a circle with centre $(0,0)$, meets axes at $(\pm\sqrt{5},0)$ and $(0,\pm\sqrt{5})$.

A rough sketch of the curves is as under:



Shaded region represents the required area.

Required area = Region $BCDB$ + Region $CADC$

$$A = \int_{-1}^{1}(y_1 - y_2)\,dx + \int_1^2 (y_1 - y_2)\,dx$$
$$= \int_{-1}^{1}\left[\sqrt{5-x^2} - 1 + x\right]dx + \int_1^2\left(\sqrt{5-x^2} - x + 1\right)dx$$
$$= \left[\frac{x}{2}\sqrt{5-x^2} + \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}} - x + \frac{x^2}{2}\right]_{-1}^{1} + \left[\frac{x}{2}\sqrt{5-x^2} + \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}} - \frac{x^2}{2} + x\right]_1^2$$
$$= \left[\left(\frac{1}{2}.2 + \frac{5}{2}\sin^{-1}\left(\frac{1}{\sqrt{5}}\right) - 1 + \frac{1}{2}\right) - \left(-\frac{1}{2}.2 - \frac{5}{2}\sin^{-1}\left(\frac{1}{\sqrt{5}}\right) + 1 + \frac{1}{2}\right)\right]$$
$$+\left[\left(1.1. + \frac{5}{2}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right) - 2 + 2\right) - \left(\frac{1}{2}.2 + \frac{5}{2}\sin^{-1}\frac{1}{\sqrt{5}} - \frac{1}{2} + 1\right)\right]$$

$$= \left[1 + \frac{5}{2}\sin^{-1}\frac{1}{\sqrt{5}} - \frac{1}{2} + 1 + \frac{5}{2}\sin^{-1}\frac{1}{\sqrt{5}} - \frac{3}{2}\right] + \left[1 + \frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}} - 1 - \frac{5}{2}\sin^{-1}\frac{1}{\sqrt{5}} - \frac{1}{2}\right]$$

$$= 5\sin^{-1}\frac{1}{\sqrt{5}} + \frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}} - \frac{5}{2}\sin^{-1}\frac{1}{\sqrt{5}} - \frac{1}{2}$$

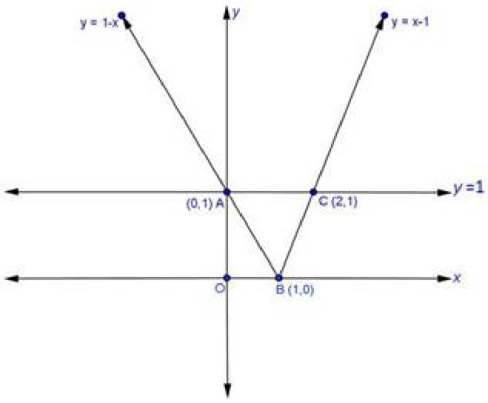
$$A = \left[\frac{5}{2}\left(\sin^{-1}\frac{2}{\sqrt{5}} + \sin^{-1}\frac{1}{\sqrt{5}}\right) - \frac{1}{2}\right] \text{ sq. units.}$$

Areas of Bounded Regions Ex-21-3 Q35

To find area bounded by $y = 1$ and

$$y = |x - 1|$$
$$y = \begin{cases} x - 1, \text{ if } x \ge 0 & ---(1) \\ 1 - x, \text{ if } x < 0 & ---(2) \end{cases}$$

A rough sketch of the curve is as under:-



Shaded region is the required area. So

Required area = Region $ABCA$

A = Region $ABDA$ + Region $BCDB$
$$= \int_0^1 (y_1 - y_2)\,dx + \int_1^2 (y_1 - y_3)\,dx$$
$$= \int_0^1 (1 - 1 + x)\,dx + \int_1^2 (1 - x + 1)\,dx$$
$$= \int_0^1 x\,dx + \int_1^2 (2 - x)\,dx$$
$$= \left(\frac{x^2}{2}\right)_0^1 + \left(2x - \frac{x^2}{2}\right)_1^2$$
$$= \left(\frac{1}{2} - 0\right) + \left[(4-2) - \left(2 - \frac{1}{2}\right)\right]$$

$$= \frac{1}{2} + \left(2 - 2 + \frac{1}{2}\right)$$

A = 1 sq. unit

Areas of Bounded Regions Ex-21-3 Q36

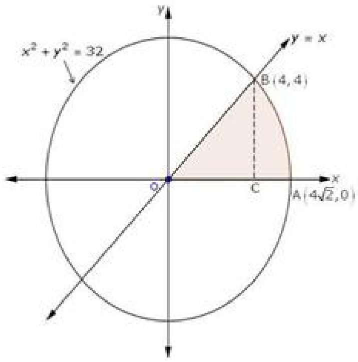
To find area of in first quadrant enclosed by x-axis, the line $y = x$ and circle

$$x^2 + y^2 = 32 \qquad ---(1)$$

Equation (1) is a circle with centre $(0,0)$ and meets axes at $\left(\pm 4\sqrt{2}, 0\right), \left(0, \pm 4\sqrt{2}\right)$.

And $y = x$ is a line passes through $(0,0)$ and intersect circle at $(4,4)$.

A rough sketch of curve is as under:-



Required area is shaded region $OABO$

Region $OABO$ = Region $OCBO$ + Region $CABC$

$$= \int_0^4 y_1 dx + \int_4^{4\sqrt{2}} y_2 dx$$
$$= \int_0^4 x dx + \int_4^{4\sqrt{2}} \sqrt{32 - x^2} dx$$
$$= \left(\frac{x^2}{2}\right)_0^4 + \left[\frac{x}{2}\sqrt{32 - x^2} + \frac{32}{2} \sin^{-1} \frac{x}{4\sqrt{2}}\right]_4^{4\sqrt{2}}$$
$$= (8 - 0) + \left[\left(0 + 16.\frac{\pi}{2}\right) - \left(8 + 16.\frac{\pi}{4}\right)\right]$$

$$= 8 + 8\pi - 8 - 4\pi$$

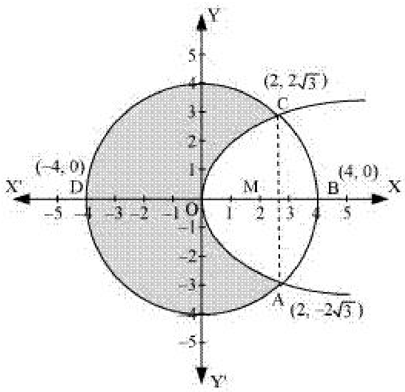
$A = 4\pi$ sq. units

Areas of Bounded Regions Ex-21-3 Q37

The given equations are

$x^2 + y^2 = 16 \ldots (1)$

$y^2 = 6x \ldots (2)$



Area bounded by the circle and parabola

$$= 2\left[\text{Area}(OADO) + \text{Area}(ADBA)\right]$$

$$= 2\left[\int_0^2 \sqrt{16x}\,dx + \int_2^4 \sqrt{16 - x^2}\,dx\right]$$

$$= 2\left[\sqrt{6}\left\{\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right\}_0^2\right] + 2\left[\frac{x}{2}\sqrt{16-x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_2^4$$

$$= 2\sqrt{6} \times \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^2 + 2\left[8 \cdot \frac{\pi}{2} - \sqrt{16-4} - 8\sin^{-1}\left(\frac{1}{2}\right)\right]$$

$$= \frac{4\sqrt{6}}{3}\left(2\sqrt{2}\right) + 2\left[4\pi - \sqrt{12} - 8\frac{\pi}{6}\right]$$

$$= \frac{16\sqrt{3}}{3} + 8\pi - 4\sqrt{3} - \frac{8}{3}\pi$$

$$= \frac{4}{3}\left[4\sqrt{3} + 6\pi - 3\sqrt{3} - 2\pi\right]$$

$$= \frac{4}{3}\left[\sqrt{3} + 4\pi\right]$$

$$= \frac{4}{3}\left[4\pi + \sqrt{3}\right] \text{ square units}$$

Area of circle $= \pi\,(r)^2$

$= \pi\,(4)^2 = 16\pi$ square units

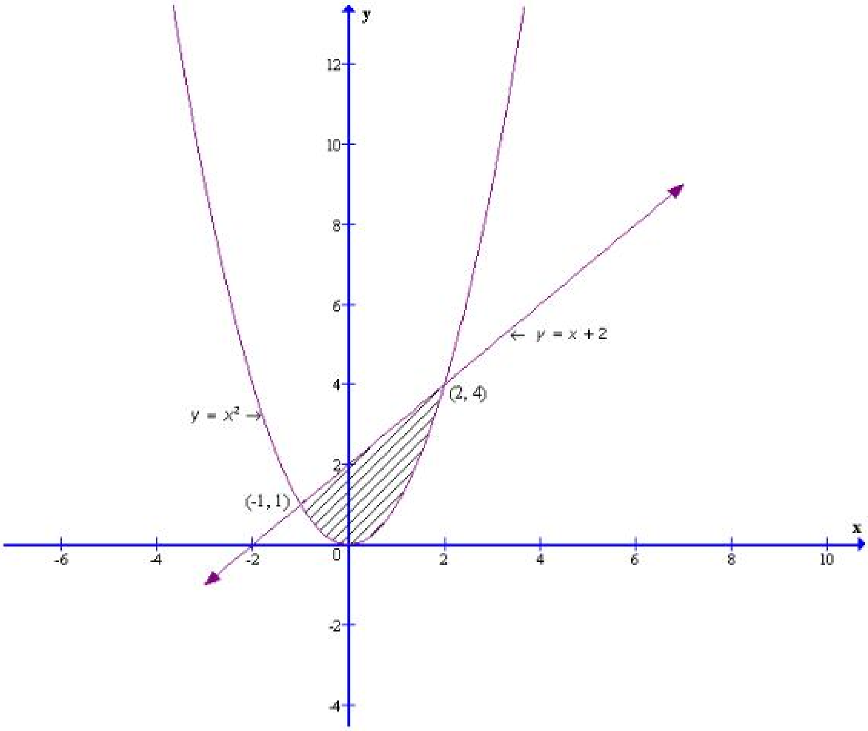
Thus, Required area $= 16\pi - \frac{4}{3}\left[4\pi + \sqrt{3}\right]$

$$= \frac{4}{3}\left[4 \times 3\pi - 4\pi - \sqrt{3}\right]$$

$$= \frac{4}{3}\left(8\pi - \sqrt{3}\right)$$

$$= \left(\frac{32}{3}\pi - \frac{4\sqrt{3}}{3}\right) \text{sq. units}$$

Areas of Bounded Regions Ex-21-3 Q38



Area of the bounded region

$= \int_{-1}^{2} x+2-x^2 \, dx$

$= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$

$= \frac{4}{2} + 4 - \frac{8}{3} - \frac{1}{2} + 2 - \frac{1}{3}$

$= \frac{9}{2}$ sq.units

Areas of Bounded Regions Ex-21-3 Q39

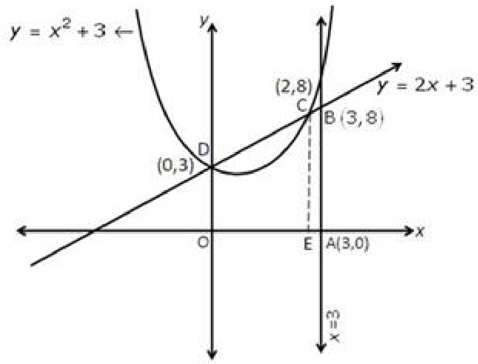
To find area of region

$$\{(x,y) : 0 \le y \le x^2 + 3,\ 0 \le y \le 2x + 3,\ 0 \le x \le 3\}$$

$$\Rightarrow \quad y = x^2 + 3 \qquad ---(1)$$
$$y = 2x + 3 \qquad ---(2)$$

and $\quad x = 0, x = 3$

Equation (1) represents a parabola with vertex (3,0) and axis as y-axis. Equation (2) represents a line a passing through (0,3) and $\left(-\frac{3}{2}, 0\right)$, a rough sketch of curve is as under:-



Required area = Region $ABCDOA$

A = Region $ABCEA$ + Region $ECDOE$

$$= \int_2^3 y_1 dx + \int_0^2 y_2 dx$$
$$= \int_2^3 (2x + 3)\, dx + \int_0^2 \left(x^2 + 3\right) dx$$
$$= \left(x^2 + 3x\right)_2^3 + \left(\frac{x^3}{3} + x\right)_0^2$$
$$= \left[(9 + 9) - (4 + 6)\right] + \left[\left(\frac{8}{3} + 2\right) - (0)\right]$$
$$= \left[18 - 10\right] + \left[\frac{14}{3}\right]$$
$$= 8 + \frac{14}{3}$$

$A = \frac{38}{3}$ sq. units

Areas of Bounded Regions Ex-21-3 Q40

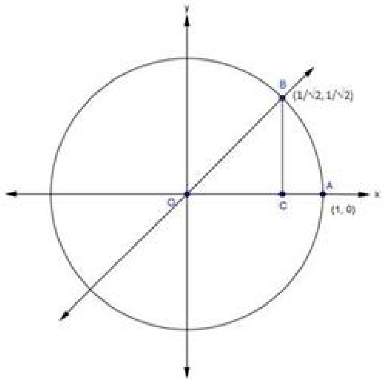
To find area bounded by positive x-axis and curve

$y = \sqrt{1 - x^2}$

$x^2 + y^2 = 1 \qquad ---(1)$

$x = y \qquad ---(2)$

Equation (1) represents a circle with centre $(0,0)$ and meets axes at $(\pm 1, 0), (0, \pm 1)$.
Equation (2) represents a line passing through $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ and they are also points of intersection. A rough sketch of the curve is as under:-



Required area = Region $OABO$

A = Region $OCBO$ + Region $CABC$

$$= \int_0^{\frac{1}{\sqrt{2}}} y_1 dx + \int_{\frac{1}{\sqrt{2}}}^1 y_2 dx$$

$$= \int_0^{\frac{1}{\sqrt{2}}} x dx + \int_{\frac{1}{\sqrt{2}}}^1 \sqrt{1 - x^2} dx$$

$$= \left[\frac{x^2}{2}\right]_0^{\frac{1}{\sqrt{2}}} + \left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1} x\right]_{\frac{1}{\sqrt{2}}}^1$$

$$= \left[\frac{1}{4} - 0\right] + \left[\left(0 + \frac{1}{2}.\frac{\pi}{2}\right) - \left(\frac{1}{2\sqrt{2}}.\frac{1}{\sqrt{2}} + \frac{1}{2}.\frac{\pi}{4}\right)\right]$$

$$= \frac{1}{4} + \frac{\pi}{4} - \frac{1}{4} - \frac{\pi}{8}$$

$A = \frac{\pi}{8}$ sq. units

Areas of Bounded Regions Ex-21-3 Q41

To find area bounded by lines

$y = 4x + 5$ (Say AB) $\quad ---(1)$

$y = 5 - x$ (Say BC) $\quad ---(2)$

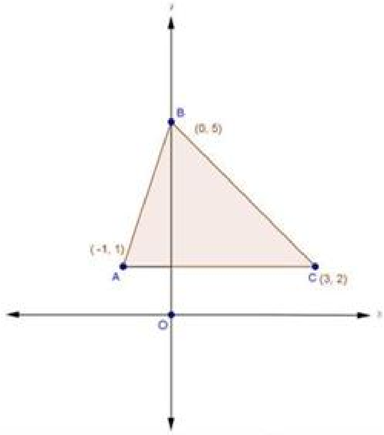
$4y = x + 5$ (Say AC) $\quad ---(3)$

By solving equation (1) and (2), we get $B(0,5)$

By solving equation (2) and (3), we get $C(3,2)$

By solving equation (1) and (3), we get $A(-1,1)$

A rough sketch of the curve is as under:-



Shaded area $\triangle ABC$ is the required area.

Required area $= ar(\triangle ABD) + ar(\triangle BDC) \quad ---(1)$

$$ar(\triangle ABD) = \int_{-1}^{0}(y_1 - y_3)\,dx$$

$$= \int_{-1}^{0}\left(4x + 5 - \frac{x}{4} - \frac{5}{4}\right)dx$$

$$= \int_{-1}^{0}\left(\frac{15x}{4} + \frac{15}{4}\right)dx$$

$$= \frac{15}{4}\left(\frac{x^2}{2}+x\right)_{-1}^{0}$$

$$= \frac{15}{4}\left[(0) - \left(\frac{1}{2} - 1\right)\right]$$

$$= \frac{15}{4} \times \frac{1}{2}$$

$$ar(\triangle ABD) = \frac{15}{8} \text{ sq. units} \qquad ---(2)$$

$$ar(\triangle BDC) = \int_0^3 (y_2 - y_3)\,dx$$

$$= \int_0^3 \left[(5-x) - \left(\frac{x}{4} + \frac{5}{4}\right)\right] dx$$

$$= \int_0^3 \left[5 - x - \frac{x}{4} - \frac{5}{4}\right] dx$$

$$= \int_0^3 \left(\frac{-5x}{4} + \frac{15}{4}\right) dx$$

$$= \frac{5}{4}\left(3x - \frac{x^2}{2}\right)$$

$$= \frac{5}{4}\left(9 - \frac{9}{2}\right)$$

$$ar(\triangle BDC) = \frac{45}{8} \text{ sq. units} \qquad ---(3)$$

Using equation (1), (2) and (3),

$$ar(\triangle ABC) = \frac{15}{8} + \frac{45}{8}$$

$$= \frac{60}{8}$$

$$ar(\triangle ABC) = \frac{15}{2} \text{ sq. units}$$

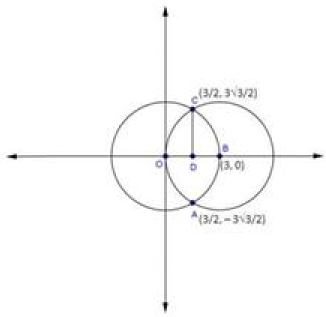
Areas of Bounded Regions Ex-21-3 Q42

To find area enclosed by

$$x^2 + y^2 = 9 \qquad ---(1)$$

$$(x-3)^2 + y^2 = 9 \qquad ---(2)$$

Equation (1) represents a circle with centre $(0,0)$ and meets axes at $(\pm 3, 0), (0, \pm 3)$.
Equation (2) is a circle with centre $(3,0)$ and meets axes at $(0,0), (6,0)$.
they intersect each other at $\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$ and $\left(\frac{3}{2}, -\frac{3\sqrt{3}}{2}\right)$. A rough sketch of the curves is as under:



Shaded region is the required area.

Required area = Region $OABCO$

$$A = 2\left(\text{Region } OBCO\right)$$

$$= 2\left(\text{Region } ODCO + \text{Region } DBCD\right)$$

$$= 2\left[\int_0^{\frac{3}{2}} \sqrt{9-\left(x-3\right)^2}dx + \int_{\frac{3}{2}}^{3} \sqrt{9-x^2}dx\right]$$

$$= 2\left[\left\{\frac{\left(x-3\right)}{2}\sqrt{9-\left(x-3\right)^2} + \frac{9}{2}\sin^{-1}\frac{\left(x-3\right)}{3}\right\}_0^{\frac{3}{2}} + \left\{\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right\}_{\frac{3}{2}}^{3}\right]$$

$$= 2\left[\left\{\left(-\frac{3}{4}\sqrt{9-\frac{9}{4}} + \frac{9}{2}\sin^{-1}\left(-\frac{3}{6}\right)\right) - \left(0 + \frac{9}{2}\sin^{-1}\left(-1\right)\right)\right\} + \left\{\left(0 + \frac{9}{2}\sin^{-1}\left(1\right)\right) - \left(\frac{3}{4}\sqrt{9-\frac{9}{4}} + \frac{9}{2}\sin^{-1}\left(\frac{1}{2}\right)\right)\right\}\right]$$

$$= 2\left[\left\{-\frac{9\sqrt{3}}{8} - \frac{9}{2}.\frac{\pi}{6} + \frac{9}{2}.\frac{\pi}{2}\right\} + \left\{\frac{9}{2}.\frac{\pi}{2} - \frac{9\sqrt{3}}{8} - \frac{9}{2}.\frac{\pi}{6}\right\}\right]$$

$$= 2\left[-\frac{9\sqrt{3}}{8} - \frac{3\pi}{4} + \frac{9\pi}{4} + \frac{9\pi}{4} - \frac{9\sqrt{3}}{8} - \frac{3\pi}{4}\right]$$

$$= 2\left[\frac{12\pi}{4} - \frac{18\sqrt{3}}{8}\right]$$

$$A = \left(6\pi - \frac{9\sqrt{3}}{2}\right) \text{ sq. units}$$

Areas of Bounded Regions Ex-21-3 Q43

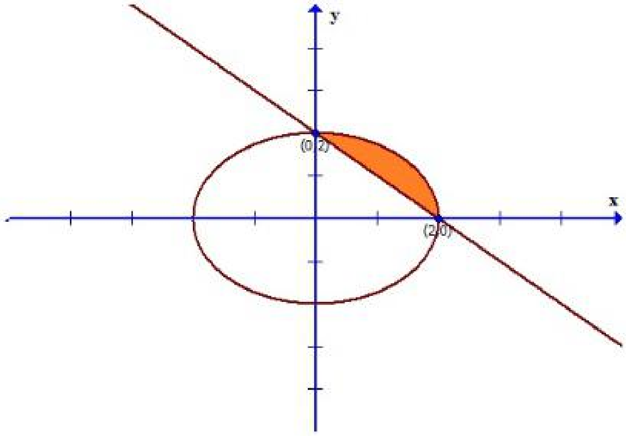
The equation of the given curves are

$x^2+y^2=4....(1)$

$x+y=2.......(2)$

Clearly $x^2+y^2=4$ represents a circle and $x+y=2$ is the equation of a straight line cutting x and y axes at (0,2) and (2,0) respectively.

The smaller region bounded by these two curves is shaded in the following figure.



Length $=y_2-y_1$

Width $=\Delta x$ and

Area $=(y_2-y_1)\Delta x$

Since the approximating rectangle can move from $x=0$ to $x=2$, the required area is given by

$$A=\int_0^2 (y_2-y_1)dx$$

We have $y_1=2-x$ and $y_2=\sqrt{4-x^2}$

Thus,

$$A=\int_0^2\left(\sqrt{4-x^2}-2+x\right)dx$$

$$\Rightarrow A=\int_0^2\left(\sqrt{4-x^2}\right)dx-2\int_0^2 dx+\int_0^2 xdx$$

$$\Rightarrow A=\left[\frac{x\sqrt{4-x^2}}{2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2-2(x)_0^2+\left(\frac{x^2}{2}\right)_0^2$$

$$\Rightarrow A=\frac{4}{2}\sin^{-1}\left(\frac{2}{2}\right)-4+2$$

$$\Rightarrow A=2\sin^{-1}(1)-2$$

$$\Rightarrow A=2\times\frac{\pi}{2}-2$$

$$\Rightarrow A=\pi-2 \text{ sq.units}$$

Areas of Bounded Regions Ex-21-3 Q44

To find area of region

$$\left\{(x,y): \frac{x^2}{9}+\frac{y^2}{4} \le 1 \le \frac{x}{3}+\frac{y}{2}\right\}$$

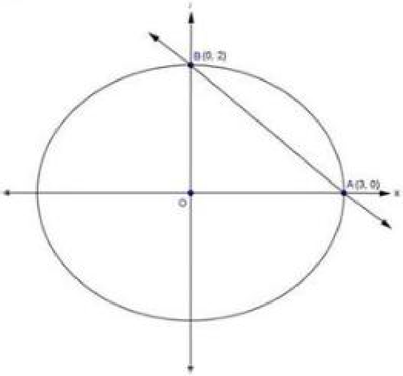
Here

$$\frac{x^2}{9}+\frac{y^2}{4}=1 \qquad ---(1)$$

$$\frac{x}{3}+\frac{y}{2}=1 \qquad ---(2)$$

Equation (1) represents an ellipse with centre at origin and meets axes at $(\pm 3,0)$, $(0,\pm 2)$. Equation (2) is a line that meets axes at $(3,0),(0,2)$.

A rough sketch is as under:



Shaded region represents required area. This is sliced into rectangles with area $(y_1 - y_2)\Delta x$ which slides from $x = 0$ to $x = 3$, so

Required area = Region $APBQA$

$$A = \int_0^3 (y_1 - y_2)\,dx$$

$$= \int_0^3\left[\frac{2}{3}\sqrt{9-x^2}\,dx - \frac{2}{3}(3-x)\,dx\right]$$

$$= \frac{2}{3}\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)-3x+\frac{x^2}{2}\right]_0^3$$

$$= \frac{2}{3}\left[\left\{0+\frac{9}{2}.\frac{\pi}{2}-9+\frac{9}{2}\right\}-\{0\}\right]$$

$$= \frac{2}{3}\left[\frac{9\pi}{4}-\frac{9}{2}\right]$$

$$A = \left(\frac{3\pi}{2}-3\right) \text{ sq. units}$$

Areas of Bounded Regions Ex-21-3 Q45

To find area enclosed by

$$y = |x-1|$$

$$\Rightarrow \quad y = \begin{cases} -(x-1), & \text{if } x-1<0 \\ (x-1), & \text{if } x-1 \ge 0 \end{cases}$$

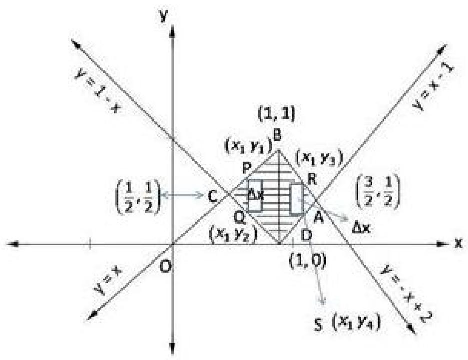
$$\Rightarrow \quad y = \begin{cases} 1-x, & \text{if } x<1 & ---(1) \\ x-1, & \text{if } x \ge 1 & ---(2) \end{cases}$$

And $\quad y = -|x-1|+1$

$$\Rightarrow \quad y = \begin{cases} +(x-1)+1, & \text{if } x-1<0 \\ -(x-1)+1, & \text{if } x-1 \ge 0 \end{cases}$$

$$y = \begin{cases} x, & \text{if } x<1 & ---(3) \\ -x+2, & \text{if } x \ge 1 & ---(4) \end{cases}$$

A rough sketch of equation of lines $(1),(2),(3),(4)$ is given as:



Shaded region is the required area.

Required area = Region $ABCDA$

Required area = Region $BDCB$ + Region $ABDA$ $\quad ---(1)$

Region $BDCB$ is sliced into rectangles of area $= (y_1 - y_2)\Delta x$ and it slides from $x = \frac{1}{2}$ to $x = 1$

Region $ABDA$ is sliced into rectangle of area $= (y_3 - y_4)\Delta x$ and it slides from $x = 1$ to $x = \frac{3}{2}$. So, using equation (1),

Required area = Region $BDCB$ + Region $ABDA$

$$= \int_{\frac{1}{2}}^{1}(y_1 - y_2)dx + \int_{1}^{\frac{3}{2}}(y_3 - y_4)dx$$

$$= \int_{\frac{1}{2}}^{1}(x-1+x)dx + \int_{1}^{\frac{3}{2}}(-x+2-x+1)dx$$

$$= \int_{\frac{1}{2}}^{1}(2x-1)dx + \int_{1}^{\frac{3}{2}}(3-2x)dx$$

$$= \left[x^2 - x\right]_{\frac{1}{2}}^{1} + \left[3x - x^2\right]_{1}^{\frac{3}{2}}$$

$$= \left[(1-1) - \left(\frac{1}{4} - \frac{1}{2}\right)\right] + \left[\left(\frac{9}{2} - \frac{9}{4}\right) - (3-1)\right]$$

$$= \frac{1}{4} + \frac{9}{4} - 2$$

$A = \frac{1}{2}$ sq.units

Areas of Bounded Regions Ex-21-3 Q46

To find area enclosed by

$$3x^2 + 5y = 32$$

$$3x^2 = -5\left(y - \frac{32}{5}\right) \qquad ---(1)$$

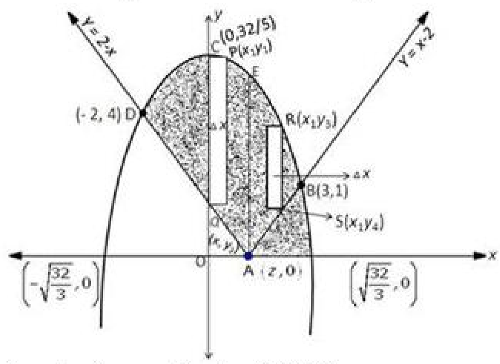
And

$$y = |x - 2|$$

$$\Rightarrow \qquad y = \begin{cases} -(x-2), \text{ if } x - 2 < 1 \\ (x-2), \text{ if } x - 2 \geq 1 \end{cases}$$

$$\Rightarrow \qquad y = \begin{cases} 2 - x, \text{ if } x < 2 \\ x - 2, \text{ if } x \geq 2 \end{cases} \qquad ---(2)$$

Equation (1) represents a downward parabola with vertex $\left(0, \frac{32}{5}\right)$ and equation (2) represents lines. A rough sketch of curves is given as:-



Required area = Region $ABECDA$

$$A = \text{Region } ABEA + \text{Region } AECDA$$

$$= \int_2^3 (y_3 - y_4)\,dx + \int_{-2}^2 (y_1 - y_2)\,dx$$

$$= \int_2^3 \left(\frac{32 - 3x^2}{5} - x + 2\right)dx + \int_{-2}^2 \left(\frac{32 - 3x^2}{5} - 2 + x\right)dx$$

$$= \int_2^3 \left(\frac{32 - 3x^2 - 5x + 10}{5}\right)dx + \int_{-2}^2 \left(\frac{32 - 3x^2 - 10 + 5x}{5}\right)dx$$

$$= \frac{1}{5}\left[\int_2^3 (42 - 3x^2 - 5x)\,dx + \int_{-2}^2 (22 - 3x^2 + 5x)\,dx\right]$$

$$A = \frac{1}{5}\left[\left(42x - x^3 - \frac{5x^2}{2}\right)_2^3 + \left(22x - x^3 + \frac{5x^2}{2}\right)_{-2}^2\right]$$

$$= \frac{1}{5}\left[\left\{\left(126 - 27 - \frac{45}{2}\right) - (84 - 8 - 10)\right\} + \left\{(44 - 8 + 10) - (-44 + 8 + 10)\right\}\right]$$

$$= \frac{1}{5}\left[\left\{\frac{153}{2} - 66\right\} + \{46 + 26\}\right]$$

$$= \frac{1}{5}\left[\frac{21}{2} + 72\right]$$

$$A = \frac{33}{2} \text{ sq. units}$$

Areas of Bounded Regions Ex-21-3 Q47

To area enclosed by

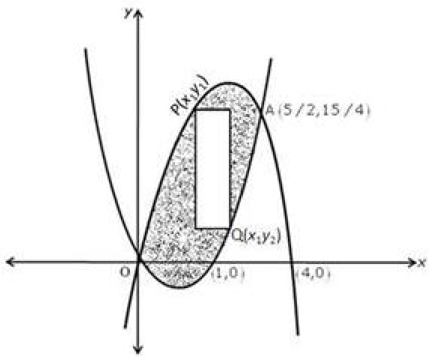
$$y = 4x - x^2$$
$$\Rightarrow \quad -y = x^2 - 4x + 4 - 4$$
$$\Rightarrow \quad -y + 4 = (x-2)^2$$
$$\Rightarrow \quad -(y-4) = (x-2)^2 \quad ---(1)$$

and $y = x^2 - x$

$$\left(y + \frac{1}{4}\right) = \left(x - \frac{1}{2}\right)^2 \quad ---(2)$$

Equation (1) represents a parabola downward with vertex at $(2,4)$ and meets axes at $(4,0),(0,0)$. Equation (2) represents a parabola upword whose vertex is $\left(\frac{1}{2}, -\frac{1}{4}\right)$ and meets axes at $(1,0),(0,0)$. Points of intersection of parabolas are $(0,0)$ and $\left(\frac{5}{2}, \frac{15}{4}\right)$.

A rough sketch of the curves is as under:-



Shaded region is required area it is sliced into rectangles with area $= (y_1 - y_2)\Delta x$. It slides from $x = 0$ to $x = \frac{5}{2}$, so

Required area = Region $OQAP$

$$A = \int_0^{\frac{5}{2}} (y_1 - y_2)dx$$
$$= \int_0^{\frac{5}{2}} \left[4x - x^2 - x^2 + x\right]dx$$
$$= \int_0^{\frac{5}{2}} \left[5x - 2x^2\right]dx$$
$$= \left[\frac{5x^2}{2} - \frac{2}{3}x^3\right]_0^{\frac{5}{2}}$$
$$= \left[\left(\frac{125}{8} - \frac{250}{24}\right) - (0)\right]$$

$$A = \frac{125}{24} \text{ sq. units}$$

Areas of Bounded Regions Ex-21-3 Q48

Given curves are

$y = 4x - x^2$

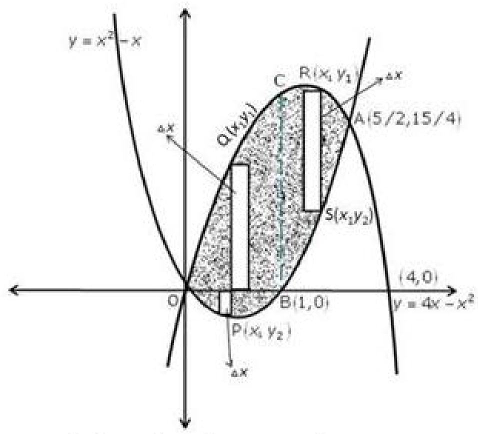
$\Rightarrow \quad -(y - 4) = (x - 2)^2 \qquad ---(1)$

and

$y = x^2 - x$

$\Rightarrow \quad \left(y + \frac{1}{4}\right)^2 = \left(x - \frac{1}{2}\right)^2 \qquad ---(2)$

Equation (1) represents a parabola downward with vertex at $(2,4)$ and meets axes at $(4,0),(0,0)$. Equation (2) represents a parabola upward whose vertex is $\left(\frac{1}{2},-\frac{1}{4}\right)$ and meets axes at $(1,0),(0,0)$ and $\left(\frac{5}{2},\frac{15}{4}\right)$. A rough sketch of the curves is as under:-



Area of the region above x-axis

A_1 = Area of region $OBACO$

$= \text{Region } OBCO + \text{Region } BACB$

$= \int_0^1 y_1 dx + \int_1^{\frac{5}{2}} (y_1 - y_2)\, dx$

$= \int_0^1 (4x - x^2)\, dx + \int_1^{\frac{5}{2}} (4x - x^2 - x^2 + x)\, dx$

$= \left(\frac{4x^2}{2} - \frac{x^3}{3}\right)_0^1 + \left[\frac{5x^2}{2} - \frac{2x^3}{3}\right]_1^{\frac{5}{2}}$

$= \left(2 - \frac{1}{3}\right) + \left[\left(\frac{125}{8} - \frac{250}{24}\right) - \left(\frac{5}{2} - \frac{2}{3}\right)\right]$

$= \frac{5}{3} + \frac{125}{24} - \frac{11}{6}$

$= \frac{121}{24}$ sq. units

Area of the region below x-axis

A_2 = Area of region $OPBO$

$= \text{Region } OBCO + \text{Region } BACB$

$= \left|\int_0^1 y_2 dx\right|$

$= \left|\int_0^1 (x^2 - x)\, dx\right|$

$= \left|\left(\frac{x^3}{3} - \frac{x^2}{2}\right)_0^1\right|$

$= \left|\left(\frac{1}{3} - \frac{1}{2}\right) - (0)\right|$

$= \left|-\frac{1}{6}\right|$

$A_2 = \frac{1}{6}$ sq. units

$A_1 : A_2 = \frac{121}{24} : \frac{1}{6}$

$\Rightarrow \quad A_1 : A_2 = \frac{121}{24} : \frac{4}{24}$

$\Rightarrow \quad A_1 : A_2 = 121 : 4$

Areas of Bounded Regions Ex-21-3 Q49

To find area bounded by the curve

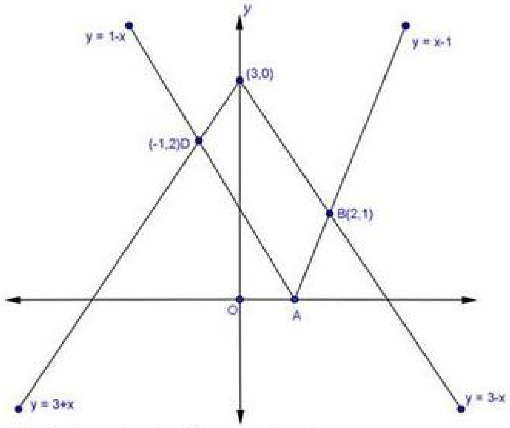
$$y = |x - 1|$$

$$\Rightarrow \quad y = \begin{cases} 1 - x, \text{ if } x < 1 & ---(1) \\ x - 1, \text{ if } x \geq 1 & ---(2) \end{cases}$$

and $\quad y = 3 - |x|$

$$\Rightarrow \quad y = \begin{cases} 3 + x, \text{ if } x < 0 & ---(3) \\ 3 - x, \text{ if } x \geq 0 & ---(4) \end{cases}$$

Drawing the rough sketch of lines $(1),(2),(3)$ and (4) as under:-



Shaded region is the required area

Required area = Region $ABCDA$

$$A = \text{Region } ABFA + \text{Region } AFCEA + \text{Region } CDEC$$
$$= \int_1^2 (y_1 - y_2)\,dx + \int_0^1 (y_1 - y_3)\,dx + \int_{-1}^0 (y_4 - y_3)\,dx$$
$$= \int_1^2 (3 - x - x + 1)\,dx + \int_0^1 (3 - x - 1 + x)\,dx + \int_{-1}^0 (3 + x - 1 + x)\,dx$$

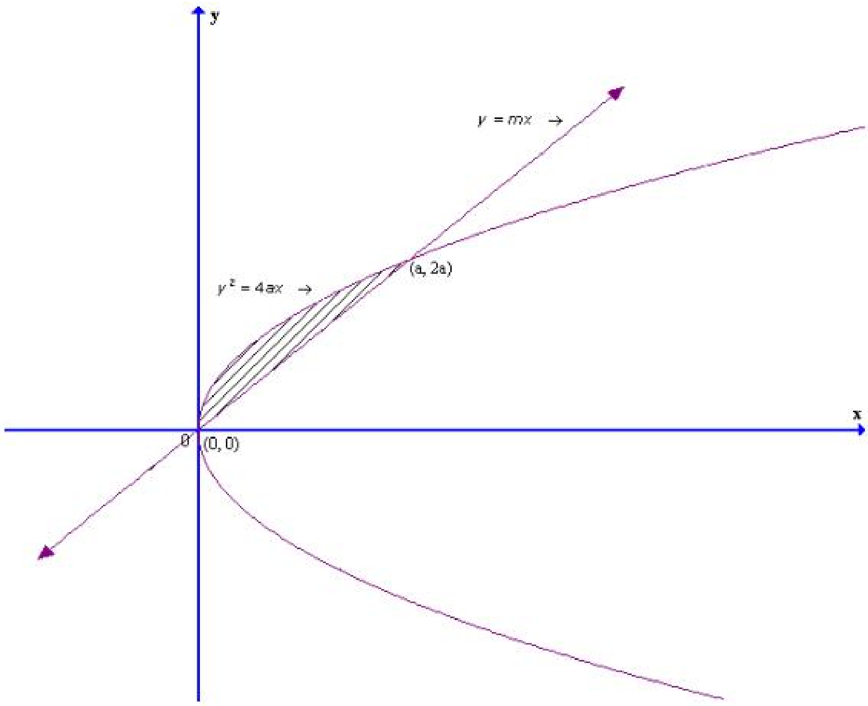
$$= \int_1^2 (4 - 2x)\,dx + \int_0^1 2dx + \int_{-1}^0 (2 + 2x)\,dx$$

$$= \left[4x - x^2\right]_1^2 + \left[2x\right]_0^1 + \left[2x + x^2\right]_{-1}^0$$
$$= \left[(8 - 4) - (4 - 1)\right] + \left[2 - 0\right] + \left[(0) - (-2 + 1)\right]$$

$$= (4 - 3) + 2 + 1$$

A = 4 sq. unit

Areas of Bounded Regions Ex-21-3 Q50



Area of the bounded region $= \dfrac{a^2}{12}$

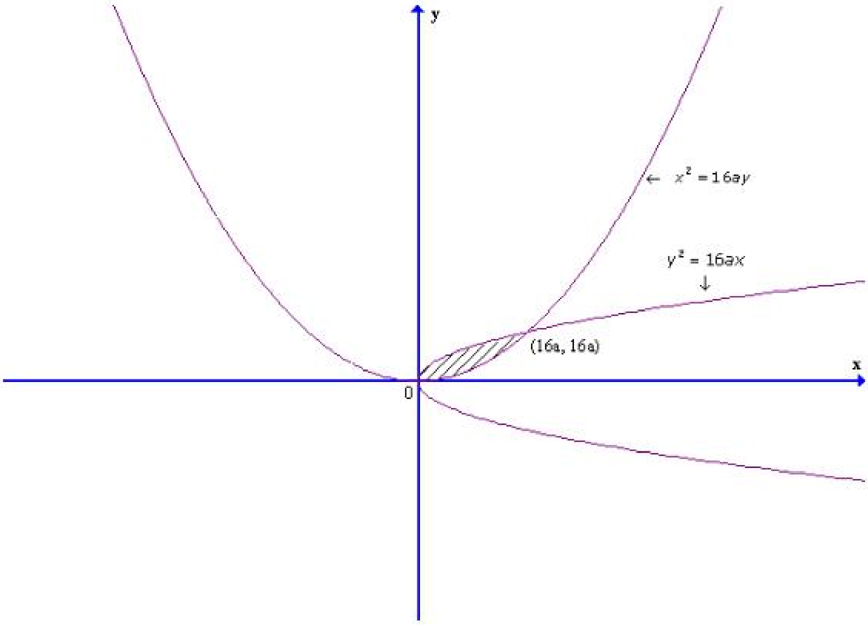
$$\frac{a^2}{12} = \int_0^a \sqrt{4ax} - mx\,dx$$

$$\frac{a^2}{12} = \left[2\sqrt{a}\,\frac{x^{3/2}}{3/2} - m\frac{x^2}{2}\right]_0^a$$

$$\frac{a^2}{12} = \frac{4a^2}{3} - m\frac{a^2}{2}$$

$m = 2$

Areas of Bounded Regions Ex-21-3 Q 51



Area of the bounded region $= \frac{1024}{3}$

$$\frac{1024}{3} = \int_0^{16a} \sqrt{16ax} - \frac{x^2}{16a}\, dx$$

$$\frac{1024}{3} = \left[4\sqrt{a}\,\frac{x^{3/2}}{3/2} - \frac{x^3}{48a}\right]_0^{16a}$$

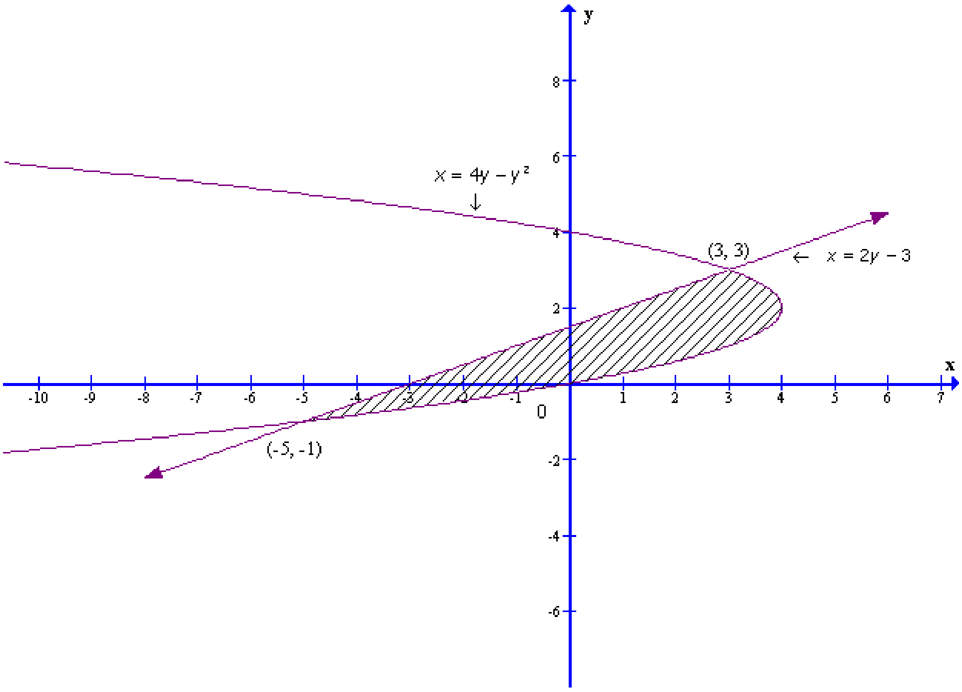
$$\frac{1024}{3} = \frac{(16a)^2 \times 2}{3} - \frac{(16a)^3}{48a}$$

$a = 2$

Note: Answer given in the book is incorrect.

Ex 21.4

Areas of Bounded Regions Ex-21-4 Q1



Area of the bounded region

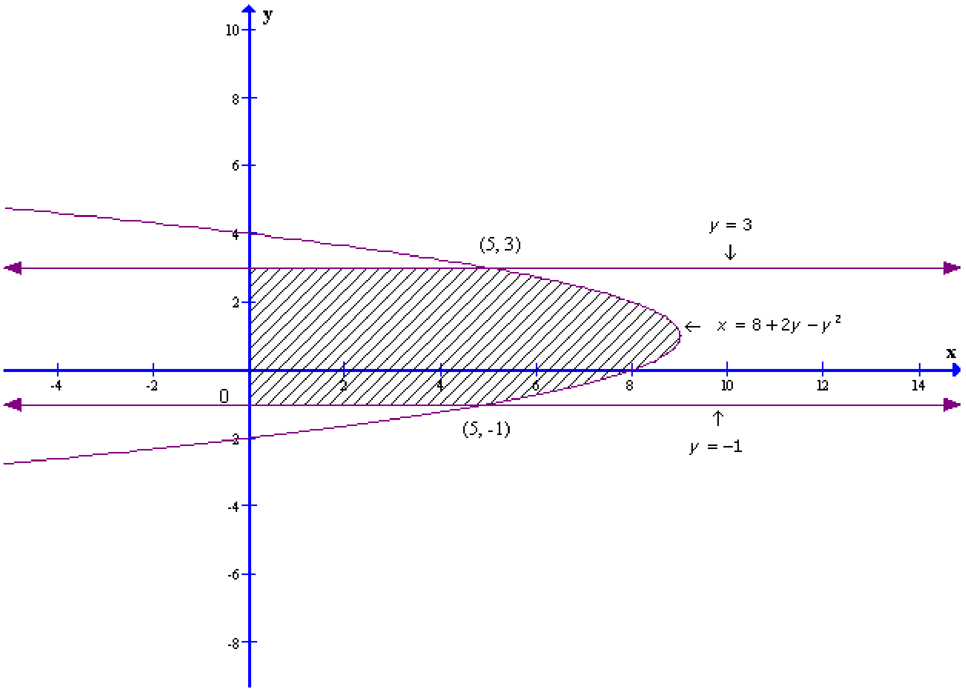
$$= \int_{-1}^{3} (4y - y^2 - 2y + 3)\, dy$$

$$= \left[2\frac{y^2}{2} - \frac{y^3}{3} + 3y\right]_{-1}^{3}$$

$$= 9 - 9 + 9 - 1 - \frac{1}{3} + 3 - \frac{(16a)^3}{48a}$$

$$= \frac{32}{3} sq. \text{ units}$$

Areas of Bounded Regions Ex-21-4 Q2



Area of the bounded region

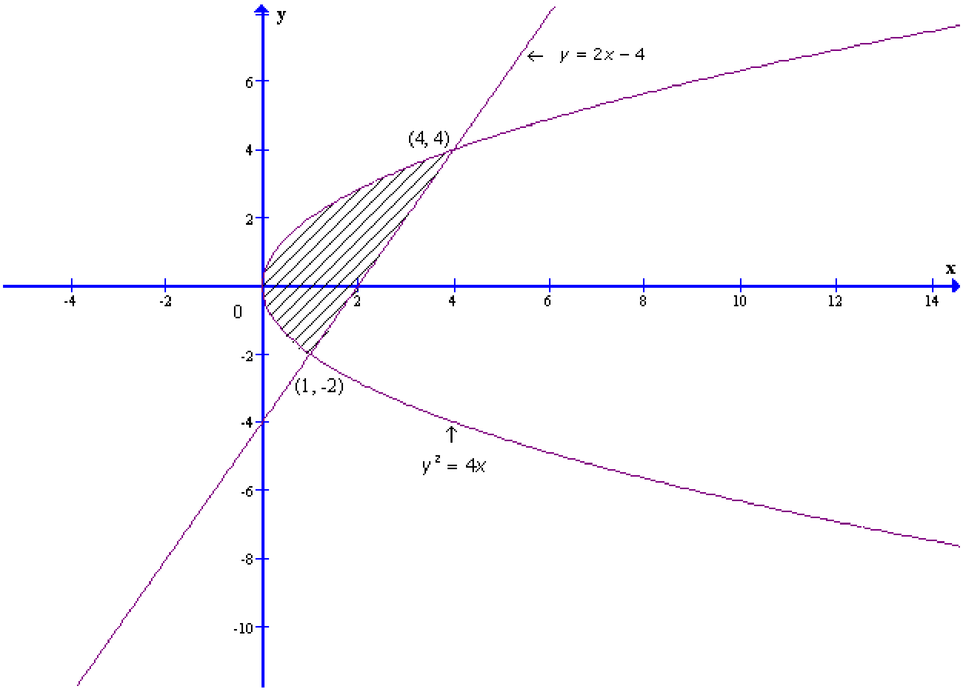
$$= \int_{-1}^{3} (5-0)\ dy + \int_{-1}^{3} 8 + 2y - y^2 - 5\ dy$$

$$= [5y]_{-1}^{3} + \left[3y + y^2 - \frac{y^3}{3}\right]_{-1}^{3}$$

$$= 15 + 5 + 9 + 9 - \frac{27}{3} + 3 - 1 - \frac{1}{3}$$

$$= \frac{92}{3} sq.\ units$$

Areas of Bounded Regions Ex-21-4 Q3



Area of the bounded region

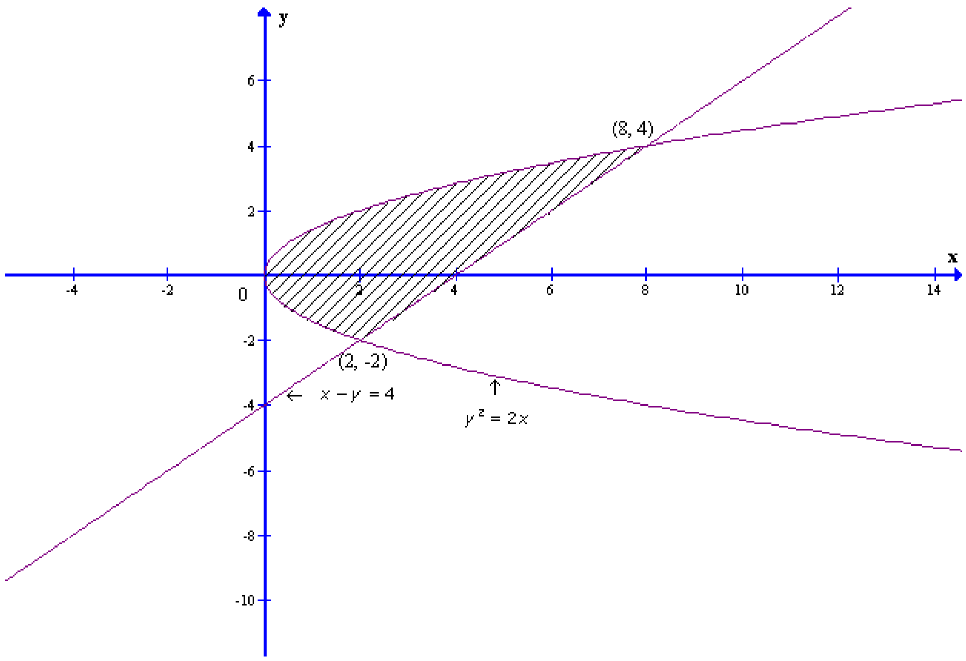
$$= \int_{-2}^{4} \left(\frac{y+4}{2} - \frac{y^2}{4} \right) dy$$

$$= \left[\frac{y^2}{4} + 2y - \frac{y^3}{12} \right]_{-2}^{4}$$

$$= 4 + 8 - \frac{16}{3} - 1 + 4 - \frac{2}{3}$$

= 9 sq. units

Areas of Bounded Regions Ex-21-4 Q4



Area of the bounded region

$$= \int_{-2}^{4} \left(y + 4 - \frac{y^2}{2} \right) dy$$

$$= \left[\frac{y^2}{2} + 4y - \frac{y^3}{6} \right]_{-2}^{4}$$

$$= 8 + 16 - \frac{32}{3} - 2 + 8 - \frac{4}{3}$$

= 18 sq. units

Ex 22.1

Differential Equations Ex 22.1 Q1

$$\frac{d^3x}{dt^3}+\frac{d^{2x}}{dt^2}+\left(\frac{dx}{dt}\right)^2=e^t$$

The highest order differential coefficient is $\frac{d^3x}{dt^3}$ and its power is 1.

So, it is a non-linear differential equation with order 3 and degree 1.

Differential Equations Ex 22.1 Q2

$$\frac{d^2y}{dx^2}+4y=0$$

It is a linear differential equation.

The highest order differential coefficient is $\frac{d^2y}{dx^2}$ and its power is 1.

So, it is a linear differential equation with order 2 and degree 1.

Differential Equations Ex 22.1 Q3

$$\left(\frac{dy}{dx}\right)^2+\frac{1}{\left(\frac{dy}{dx}\right)}=2$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)^3+1=2\left(\frac{dy}{dx}\right)$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)^3-2\left(\frac{dy}{dx}\right)+1=0$$

This is a polynomial in $\frac{dy}{dx}$.

The highest order differential coefficient is $\frac{dy}{dx}$ and its power is 3.

So, it is a non-linear differential equation with order 1 and degree 3.

Differential Equations Ex 22.1 Q4

Consider the given differential equation, $\sqrt{1+\left(\frac{dy}{dx}\right)^2}=\left(c\frac{d^2y}{dx^2}\right)^{\frac{1}{3}}$

Squaring on both the sides, we have

$$1+\left(\frac{dy}{dx}\right)^2=\left(c\frac{d^2y}{dx^2}\right)^{\frac{2}{3}}$$

Cubing on both the sides, we have

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3=\left\{\left(c\frac{d^2y}{dx^2}\right)^{\frac{2}{3}}\right\}^3$$

$$\Rightarrow 1+\left(\frac{dy}{dx}\right)^6+3\left(\frac{dy}{dx}\right)^2+3\left(\frac{dy}{dx}\right)^4=c^2\left(\frac{d^2y}{dx^2}\right)^2$$

$$\Rightarrow c^2\left(\frac{d^2y}{dx^2}\right)^2-\left(\frac{dy}{dx}\right)^6-3\left(\frac{dy}{dx}\right)^4-3\left(\frac{dy}{dx}\right)^2-1=0$$

The highest order differential coefficient in this equation is $\frac{d^2y}{dx^2}$ *and its power is* 2.

Therefore, the given differential equation is a non – linear differential equation of second order and second degree.

Differential Equations Ex 22.1 Q5

$$\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+xy=0$$

The highest order differential coefficient is $\frac{d^2y}{dx^2}$ and its power is 1.

So, it is a non-linear differential equation with order 2 and degree 1.

Differential Equations Ex 22.1 Q6

Consider the given differential equation,

$$\sqrt[3]{\frac{d^2y}{dx^2}} = \sqrt{\frac{dy}{dx}}$$

Cubing on both the sides of the above equation, we have

$$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^{\frac{3}{2}}$$

Squaring on both the sides of the above equation, we have

$$\left(\frac{d^2y}{dx^2}\right)^2 = \left[\left(\frac{dy}{dx}\right)^{\frac{3}{2}}\right]^2$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)^2 = \left[\left(\frac{dy}{dx}\right)\right]^3$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)^2 - \left[\left(\frac{dy}{dx}\right)\right]^3 = 0$$

The highest order differential coefficient in this equation is $\frac{d^2y}{dx^2}$

and its power is 2.

Therefore, the given differential equation is a non – linear differential equation of second order and second degree.

Differential Equations Ex 22.1 Q7

$$\frac{d^4y}{dx^4} = \left[c + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}$$

$$\Rightarrow \quad \left(\frac{d^4y}{dx^4}\right)^2 = \left[c + \left(\frac{dy}{dx}\right)^2\right]^3$$

$$\Rightarrow \quad \left(\frac{d^4y}{dx^4}\right)^2 = c^3 + \left(\frac{dy}{dx}\right)^6 + 3c\left(\frac{dy}{dx}\right)^2 + 3c^2\left(\frac{dy}{dx}\right)$$

$$\Rightarrow \quad \left(\frac{d^4y}{dx^4}\right)^2 - \left(\frac{dy}{dx}\right)^6 - 3c\left(\frac{dy}{dx}\right)^2 - 3c^2\left(\frac{dy}{dx}\right) - c^3 = 0$$

The highest order differential coefficient is $\left(\frac{d^4y}{dx^4}\right)$ and its power is 2.

It is a non-linear differential equation with order 4 and degree 2.

Differential Equations Ex 22.1 Q8

$$x + \left(\frac{dy}{dx}\right) = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$\Rightarrow \quad \left(x + \frac{dy}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow \quad x^2 + \left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) = 1 + \left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow \quad 2x\left(\frac{dy}{dx}\right) + x^2 - 1 = 0$$

$$\Rightarrow \quad \frac{dy}{dx} + \frac{x}{2} - \frac{1}{2x} = 0$$

The highest order differential coefficient is $\frac{dy}{dx}$ and power is 1.

So, it is a linear differential equation with order 1 and degree 1.

Differential Equations Ex 22.1 Q9

$$y\frac{d^2x}{dy^2} = y^2 + 1$$

$$\frac{d^2x}{dy^2} - y - \frac{1}{y} = 0$$

The differential coefficient is $\frac{d^2x}{dy^2}$ and its power is 1.

So, it is a linear differential equation with order 2 and degree 1.

Differential Equations Ex 22.1 Q10

$$s^2\frac{d^2t}{ds^2}+st\frac{dt}{ds}=s$$

The differential coefficient of highest order is $\frac{d^2t}{ds^2}$ and power is 1.

So, it is a non-linear differnetial equation with order 2 and degree 1.

Differential Equations Ex 22.1 Q11

$$x^2\left(\frac{d^2y}{dx^2}\right)^3+y\left(\frac{dy}{dx}\right)^4+y^4=0$$

The highest order differential coefficient is $\frac{d^2y}{dx^2}$ and its power is 3.

So, it is a non-linear differnetial equation with order 2 and degree 3.

Differential Equations Ex 22.1 Q12

$$\frac{d^3y}{dx^3}+\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{dy}{dx}\right)+4y=\sin x$$

The highest order differential coefficient is $\frac{d^3y}{dx^3}$ and its power is 1.

So, it is a non-linear differnetial equation with order 3 and degree 1.

Differential Equations Ex 22.1 Q13

$$\left(xy^2+x\right)dx+\left(y-x^2y\right)dy=0$$

$$\left(y-x^2y\right)\frac{dy}{dx}+xy^2+x=0$$

$$y\left(1-x^2\right)\frac{dy}{dx}+x\left(y^2+1\right)=0$$

The highest order differential coefficient is $\frac{dy}{dx}$ and its power is 1.

So, it is a non-linear differnetial equation with order 1 and degree 1.

Differential Equations Ex 22.1 Q14

$$\sqrt{1-y^2}dx+\sqrt{1-x^2}dy=0$$

$$\sqrt{1-x^2}\frac{dy}{dx}+\sqrt{1-y^2}=0$$

$$\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0$$

The highest order differential coefficient is $\frac{dy}{dx}$ and its power is 1.

So, it is a non-linear differnetial equation with order 1 and degree 1.

Differential Equations Ex 22.1 Q15

$$\frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^{\frac{2}{3}}$$

$$\left(\frac{d^2y}{dx^2}\right)^3=\left(\frac{dy}{dx}\right)^2$$

$$\left(\frac{d^2y}{dx^2}\right)^3=\left(\frac{dy}{dx}\right)^2=0$$

The highest order differential coefficient is $\frac{d^2y}{dx^2}$ and its power is 3.

So, it is a non-linear differnetial equation with order 2 and degree 3.

Differential Equations Ex 22.1 Q16

$$2\frac{d^2y}{dx^2}+3\sqrt{1-\left(\frac{dy}{dx}\right)^2}-y=0$$

$$2\frac{d^2y}{dx^2}=-3\sqrt{1-\left(\frac{dy}{dx}\right)^2}-y$$

Squaring both the sides,

$$4\left(\frac{d^2y}{dx^2}\right)^2=9\left(1-\left(\frac{dy}{dx}\right)^2-y\right)$$

$$4\left(\frac{d^2y}{dx^2}\right)^2+9\left(\frac{dy}{dx}\right)^2+9y-9=0$$

The highest order differential coefficient is $\frac{d^2y}{dx^2}$ and its power is 2.

So, it is a non-linear differential equation with order 2 and degree 2.

Differential Equations Ex 22.1 Q17

$$5\frac{d^2y}{dx^2}=\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}$$

$$\left\{5\left(\frac{d^2y}{dx^2}\right)\right\}^2=\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^3$$

$$25\left(\frac{d^2y}{dx^2}\right)^2=1+\left(\frac{dy}{dx}\right)^6+3\left(\frac{dy}{dx}\right)^2+3\left(\frac{dy}{dx}\right)^4$$

$$25\left(\frac{d^2y}{dx^2}\right)^2-\left(\frac{dy}{dx}\right)^6-3\left(\frac{dy}{dx}\right)^4-3\left(\frac{dy}{dx}\right)^2-1=0$$

The highest order differential coefficient is $\frac{d^2y}{dx^2}$ and its power is 2.

So, it is a non-linear differential equation with order 2 and degree 2

Differential Equations Ex 22.1 Q18

$$y=x\frac{dy}{dx}+a\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$\left(y-x\frac{dy}{dx}\right)^2=\left(a\sqrt{1-\left(\frac{dy}{dx}\right)^2}\right)^2$$

$$y^2+x^2\left(\frac{dy}{dx}\right)^2-2xy\frac{dy}{dx}=a\left(1-\left(\frac{dy}{dx}\right)^2\right)$$

$$x^2\left(\frac{dy}{dx}\right)^2-2xy\frac{dy}{dx}+y+a\left(\frac{dy}{dx}\right)^2-a=0$$

$$\left(x^2+a\right)\left(\frac{dy}{dx}\right)^2-2xy\frac{dy}{dx}+y-a=0$$

The highest order differential coefficient is $\frac{dy}{dx}$ and poer is 2.

So, it is a non-linear differential equation with order 1 and degree 2.

Differential Equations Ex 22.1 Q19

$$y=px+\sqrt{a^2p^2+b^2},\ p=\frac{dy}{dx}$$

$$y-px=\sqrt{a^2p^2+b^2}$$

$$(y-px)^2=\left(a^2p^2+b^2\right)$$

$$y^2+p^2x^2-2xyp=a^2p^2+b^2$$

$$x^2p^2-a^2p^2-2xyp+y^2-b^2=0$$

$$\left(x^2-a^2\right)p^2-2xyp+\left(y^2-b^2\right)=0$$

$$\left(x^2-a^2\right)\left(\frac{dy}{dx}\right)^2-2xy\left(\frac{dy}{dx}\right)+\left(y^2-b^2\right)=0$$

The highest order differential coefficient is $\frac{dy}{dx}$ and its power is 2.

So, it is a non-linear differential equation of order 1 and degree 2

Differential Equations Ex 22.1 Q20

$$\frac{dy}{dx}+e^y=0$$

The highest order differential coefficient is $\frac{dy}{dx}$ and its power is 1.
So, it is a non-linear differential equation of order 1 and degree 1.

Differential Equations Ex 22.1 Q21

$$\left(\frac{d^2y}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)^2=x\sin\left(\frac{d^2y}{dx^2}\right)$$
$$\left(\frac{d^2y}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)^2-x\sin\left(\frac{d^2y}{dx^2}\right)=0$$

The highest order differential coefficient is $\left(\frac{d^2y}{dx^2}\right)$ and it is not a polynomial of derviative,

So, it is a non-linear differential equation of order 2 but degree is not defined.

Differential Equations Ex 22.1 Q22

$$(y'')^2+(y')^3+\sin y=0$$

The highest order of differential coefficient is y'' and its power is 2,
So, it is a non-linear differential equation of order 2 and degree 2.

Differential Equations Ex 22.1 Q23

$$\frac{d^2y}{dx^2}+5x\left(\frac{dy}{dx}\right)^2-6y=\log x$$

The highest order differential coefficient is $\frac{d^2y}{dx^2}$ and its power is 1.

So, it is a non-linear differential equation with order 2 and degree 1.

Differential Equations Ex 22.1 Q24

$$\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}+\frac{dy}{dx}+y\sin y=0$$

The highest order differential coefficient is $\frac{d^3y}{dx^3}$ and its power is 1.

So, it is a linear differential equation of order 3 and degree 1.

Differential Equations Ex 22.1 Q25

$$\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^2=x^2\log\left(\frac{d^2y}{dx^2}\right)$$
$$\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^2-x^2\log\left(\frac{d^2y}{dx^2}\right)=0$$

The highest order derivative is $\frac{d^2y}{dx^2}$ but it is not a polynomial in $\frac{dy}{dx}$.

So, it is a non-linear differential equation of order 2 but degree is not defined.

Differential Equations Ex 22.1 Q26

The order of a differential equation is the order of the highest order derivative appearing in the equation.
The degree of a differential equation is the degree of the highest order derivative.
Consider the given differential equation

$$\left(\frac{dy}{dx}\right)^3-4\left(\frac{dy}{dx}\right)^2+7y=\sin x$$

In the above equation, the order of the highest order derivative is 1.
So the differential equation is of order 1.
In the above differential equation, the power of the highest order derivative is 3.
Hence, it is a differential equation of degree 3.
Since the degree of the above differential equation is 3, more than one, it is a non-linear differential equation.

Ex 22.2

Differential Equations Ex 22.2 Q1

$$y^2=(x-c)^3 \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$2y\frac{dy}{dx}=3(x-c)^2$$

$$(x-c)^2=\frac{2y}{3}\frac{dy}{dx}$$

$$(x-c)^2=\left(\frac{2y}{3}\frac{dy}{dx}\right)^{\frac{1}{2}}$$

Put the value of $(x-c)$ in equation (i),

$$y^2=\left\{\left(\frac{2y}{3}\frac{dy}{dx}\right)^{\frac{1}{2}}\right\}^3$$

$$y^2=\left(\frac{2y}{3}\frac{dy}{dx}\right)^{\frac{3}{2}}$$

Squaring both the sides,

$$y^4=\left(\frac{2y}{3}\frac{dy}{dx}\right)^3$$

$$y^4=\frac{8y^3}{27}\left(\frac{dy}{dx}\right)^3$$

$$27y=8\left(\frac{dy}{dx}\right)^3.$$

Differential Equations Ex 22.2 Q2

$$y=e^{mx} \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx}=me^{mx} \qquad \text{—(ii)}$$

From equation (i),

$$y=e^{mx}$$

$$\log y=mx$$

$$m=\frac{\log y}{x}$$

Put the value of m and e^{mx} in equation (i),

$$\frac{dy}{dx}=\frac{\log y}{x}y$$

$$x\frac{dy}{dx}=y\log y$$

Differential Equations Ex 22.2 Q3(i)

$$y^2=4ax \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$2y\frac{dy}{dx}=4a \qquad \text{—(ii)}$$

Put the value of a from equation (i) in (ii),

$$2y\frac{dy}{dx}=4\left(\frac{y^2}{4x}\right)$$

$$2y\frac{dy}{dx}=\frac{y^2}{x}$$

$$2x\frac{dy}{dx}=y$$

Differential Equations Ex 22.2 Q3(ii)

$$y=cx+2c^2+c^3 \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx}=c \qquad \text{—(ii)}$$

Put the value of c from equation (ii) in (i),

$$y=\left(\frac{dy}{dx}\right)x+2\left(\frac{dy}{dx}\right)^2+\left(\frac{dy}{dx}\right)^3$$

Differential Equations Ex 22.2 Q3(iii)

$$xy = a^2 \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$x\frac{dy}{dx} + y(1) = 0$$

$$x\frac{dy}{dx} + y = 0$$

Differential Equations Ex 22.2 Q3(iv)

$$y = ax^2 + bx + c$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 2ax + b$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = 2a$$

Again, differentiating it with respect to x,

$$\frac{d^3y}{dx^3} = 0$$

Differential Equations Ex 22.2 Q4

$$y = Ae^{2x} + Be^{-2x} \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 2Ae^{2x} - 2Be^{-2x}$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = 4Ae^{2x} + 4Be^{-2x}$$

$$= 4\left(Ae^{2x} + Be^{-2x}\right)$$

$$\frac{d^2y}{dx^2} = 4y \qquad \text{[Using equation (i)]}$$

Differential Equations Ex 22.2 Q5

$$x = A\cos nt + B\sin nt$$

Differentiating with respect to t,

$$\frac{dx}{dt} = -An\sin nt + nB\cos nt$$

Again, differentiating with respect to t,

$$\frac{d^2x}{dt^2} = -An^2\cos nt - n^2B\sin t$$

$$= -n^2(A\cos nt + B\sin nt)$$

$$\frac{d^2x}{dt^2} = -n^2x \qquad \text{[Using equation (i)]}$$

$$\frac{d^2x}{dt^2} + n^2x = 0$$

Differential Equations Ex 22.2 Q6

$$y^2 = a\left(b - x^2\right)$$

Differentiating it with respect to x,

$$2y\frac{dy}{dx} = a(-2x) \qquad \text{—(i)}$$

Again, differentiating it with respect to x,

$$2\left[y\frac{d^2y}{dx^2} + \frac{dy}{dx} \times \frac{dy}{dx}\right] = -2a$$

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -\left(\frac{2y}{-2x}\frac{dy}{dx}\right)$$

Using equatoin (i)

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = \frac{y}{x}\frac{dy}{dx}$$

$$x\left\{y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right\} = y\frac{dy}{dx}$$

Differential Equations Ex 22.2 Q7

$$y^2 - 2ay + x^2 = a^2 \qquad \text{---(i)}$$

Differentiating it with respect to x,

$$2y\frac{dy}{dx} - 2a\frac{dy}{dx} + 2x = 0$$

$$y\frac{dy}{dx} + x = a\frac{dy}{dx}$$

$$a = \frac{y\frac{dy}{dx} + x}{\frac{dy}{dx}}$$

Put the value of a in equation (i),

$$y^2 - 2\left[\frac{y\frac{dy}{dx} + x}{\frac{dy}{dx}}\right]y + x^2 = \left[\frac{y\frac{dy}{dx} + x}{\frac{dy}{dx}}\right]^2$$

Put $\frac{dy}{dx} = y'$

$$y^2 - 2\left(\frac{yy' + x}{y'}\right)y + x^2 = \left(\frac{yy' + x}{y'}\right)^2$$

$$\frac{y'y^2 - 2y'y^2 - 2xy + y'x^2}{y'} = \frac{y'^2y^2 + x^2 + 2xyy'}{y'^2}$$

$$y'^2y^2 - 2y'^2y^2 - 2xyy' + y'^2x^2 - y'^2y^2 - x^2 - 2xyy' = 0$$

$$-4xyy' + y'^2x^2 - x^2 - 2y'^2y^2 = 0$$

$$y'^2\left(x^2 - 2y^2\right) - 4xyy' - x^2 = 0$$

Differential Equations Ex 22.2 Q8

$$(x-a)^2 + (y-b)^2 = r^2 \qquad \text{---(i)}$$

Differentiating with respect to x,

$$2(x-a) + 2(y-b)\frac{dy}{dx} = 0$$

$$(x-a) + (y-b)\frac{dy}{dx} = 0 \qquad \text{---(ii)}$$

Differentiating with respect to x,

$$1 + (y-b)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)\left(\frac{dy}{dx}\right) = 0$$

$$1 + (y-b)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$

$$(y-b) = -\left\{\frac{\left(\frac{dy}{dx}\right)^2 + 1}{\frac{d^2y}{dx^2}}\right\} \qquad \text{---(iii)}$$

Put $(y-b)$ in equation (ii),

$$(x-a) - \left\{\frac{\left(\frac{dy}{dx}\right)^2 + 1}{\frac{d^2y}{dx^2}}\right\}\frac{dy}{dx} =$$

$$(x-a)\left(\frac{d^2y}{dx^2}\right) - \left(\frac{dy}{dx}\right)^3 - \left(\frac{dy}{dx}\right) = 0$$

$$(x-a)\frac{d^2y}{dx^2} = \frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3$$

$$(x-a) = \frac{\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3}{\frac{d^2y}{dx^2}} \qquad \text{---(iv)}$$

Put the value of $(x-a)$ and $(y-b)$ from equation (iii) and (iv) in equation (i),

$$\left\{\frac{\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3}{\frac{d^2y}{dx^2}}\right\}^2 + \left\{\frac{\left(\frac{dy}{dx}\right)^2 + 1}{\frac{d^2y}{dx^2}}\right\}^2 = r^2$$

Put $\frac{dy}{dx} = y'$ **and** $\frac{d^2y}{dx^2} = y''$

$$\left(y' + y'^3\right)^2 + \left(y'^2 + 1\right)^2 = r^2y''^2$$

$$y'^2\left(1 + y'^2\right)^2 + \left(1 + y'^2\right)^2 = r^2y''^2$$

Differential Equations Ex 22.2 Q9

We know that, equation of a circle with centre at (h,k) and radius r is given by,

$$\left(x-h^2\right)+(y-k)^2=r^2 \qquad \text{---(i)}$$

Here, centre lies, on y-axis, so $h=0$

$$\Rightarrow \quad x^2+(y-k)^2=r^2 \qquad \text{---(ii)}$$

Also, given that, circle is passing through origin, so

$$0+k^2=r^2$$
$$k^2=r^2$$

So, equation (ii) becomes,

$$x^2+(y-k)^2=k^2$$
$$x^2+y^2-2yk=0$$
$$2yk=x^2+y^2$$
$$k=\frac{x^2+y^2}{2y}$$

Differentiating with respect to x,

$$0=\frac{2y\left(2x+2y\frac{dy}{dx}\right)-\left(x^2+y^2\right)2\frac{dy}{dx}}{(2y)^2}$$
$$0=4xy+4y^2\frac{dy}{dx}-2x^2\frac{dy}{dx}-2y^2\frac{dy}{dx}$$
$$0=2y^2\frac{dy}{dx}-2x^2\frac{dy}{dx}+4xy$$
$$x^2\frac{dy}{dx}-y^2\frac{dy}{dx}=2xy$$
$$\left(x^2-y^2\right)\frac{dy}{dx}=2xy$$

Differential Equations Ex 22.2 Q10

Equation of circle with centre (h,k) and radius r is given by

$$(x-h)^2+(y-k)^2=r^2 \qquad \text{---(i)}$$

Here, centre lie on x-axis, so

$$k=0$$
$$\Rightarrow \quad (x-h)^2+y^2=r^2 \qquad \text{---(ii)}$$

Also, given that, circle is passing through $(0,0)$, so,

$$h^2=r^2$$

So, equation (ii) becomes,

$$(x-h)^2+y^2=h^2$$
$$x^2+h^2-2xh+y^2=h^2$$
$$x^2-2xh+y^2=0$$
$$2xh=x^2+y^2$$
$$h=\frac{x^2+y^2}{2x}$$

Differentiating it with respect to x,

$$0=\frac{\left(2x+2y\frac{dy}{dx}\right)2x-\left(x^2+y^2\right)2}{(2x)^2}$$
$$\left(2x+2y\frac{dy}{dx}\right)2x-\left(x^2+y^2\right)^2=0$$
$$2x^2+2yx\frac{dy}{dx}-x^2-y^2=0$$
$$\left(x^2-y^2\right)+2xy\frac{dy}{dx}=0$$

Differential Equations Ex 22.2 Q11

Let A be the surface area of rain drain, V be its volume, and r be the radius of rain drop.
Given,

$$\frac{dV}{dt} \propto A$$

$$\frac{dV}{dt} = -kA \qquad \text{[negative because } V \text{ decreases with increase in } t\text{]}$$

where k is the constant of proportionality.

So,

$$\frac{d}{dt}\left(\frac{4\pi}{3}r^3\right) = -k\left(4\pi r^2\right)$$

$$4\pi r^2 \frac{dr}{dt} = -k\left(4\pi r^2\right)$$

$$\frac{dr}{dt} = -k$$

Differential Equations Ex 22.2 Q12

Equation of parabolas with lotus rectum '$(4a)$' and whose area is parallel to x axes and vertex at (h,k) is given by,

$$(y-k)^2 = 4a(x-h)$$

Differentiating with respect to x,

$$2(y-k)y_1 = 4a(1)$$

$$(y-k)y_1 = 2a \qquad \text{---(i)}$$

Differentiating with respect to x,

$$(y-k)y_2 + (y_1)(y_1) = 0$$

$$(y-k)y_2 + (y_1)^2 = 0$$

$$\left(\frac{2a}{y_1}\right)^{y_2} + (y_1)^2 = 0$$

Using equation (i)

$$2ay_2 + (y_1)^3 = 0$$

Differential Equations Ex 22.2 Q13

$$y = 2\left(x^2-1\right) + ce^{-x^2} \qquad \text{---(i)}$$

Differentiating it in equation (i),

$$\frac{dy}{dx} = 4x - 2cxe^{-x^2} \qquad \text{---(ii)}$$

Now,

$$\frac{dy}{dx} + 2xy$$

$$= 4x - 2cxe^{-x^2} + 2x\left[2\left(x^2-1\right) + ce^{-x^2}\right]$$

$$= 4x - 2cxe^{-x^2} + 4x^3 - 4x + 2xce^{-x^2}$$

$$= 4x^3$$

So,

$$\frac{dy}{dx} + 2xy = 4x^3$$

Which is given equation, so

$y = 2\left(x^2+1\right) + ce^{-x^2}$ is the solution of the equation.

Differential Equations Ex 22.2 Q14

$$y = \left(\sin^{-1}x\right)^2 + A\cos^{-1}x + B$$

$$\frac{dy}{dx} = 2\sin^{-1}x \times \left(\frac{1}{\sqrt{1-x^2}}\right) + A \times \left(\frac{-1}{\sqrt{1-x^2}}\right) + 0$$

$$\sqrt{1-x^2}\frac{dy}{dx} = 2\sin^{-1}x - A$$

$$\sqrt{1-x^2}\frac{d^2y}{dx^2} + \frac{dy}{dx}\left(\frac{1}{\sqrt{1-x^2}}\right)(-2x) = 2 \times \left(\frac{1}{\sqrt{1-x^2}}\right) - 0$$

$$\left(1-x^2\right)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 2 = 0$$

Note: Answer given in the book is incorrect.

Differential Equations Ex 22.2 Q15(i)

Consider the given equation.,

$(2x+a)^2+y^2=a^2....(1)$

Differentiating the above equation with respect to x, we have,

$$2(2x+a)+2y\frac{dy}{dx}=0$$

$$\Rightarrow (2x+a)+y\frac{dy}{dx}=0$$

$$\Rightarrow 2x+a=-y\frac{dy}{dx}$$

$$\Rightarrow a=-2x-y\frac{dy}{dx}$$

Substituting the value of a in equation (1), we have

$$\left(2x-2x-y\frac{dy}{dx}\right)^2+y^2=\left(-2x-y\frac{dy}{dx}\right)^2$$

$$\Rightarrow \left(y\frac{dy}{dx}\right)^2+y^2=\left(4x^2+y^2\left(\frac{dy}{dx}\right)^2+4xy\frac{dy}{dx}\right)$$

$$\Rightarrow y^2=4x^2+4xy\frac{dy}{dx}$$

$$\Rightarrow y^2-4x^2-4xy\frac{dy}{dx}=0$$

Differential Equations Ex 22.2 Q15(ii)

$$(2x-a)^2-y^2=a^2$$
$$4x^2+a^2-ax-y^2=a^2$$
$$4x^2-4ax-y^2=0$$
$$4ax=4x^2-y^2$$
$$a=\frac{4x^2-y^2}{4x}$$

Differentiating it with respect to x,

$$0=\left[\frac{4x\left(8x-2y\frac{dy}{dx}\right)-4\left(4x^2-y^2\right)}{(4x)^2}\right]$$

$$32x^2-8xy\frac{dy}{dx}-16x^2+4y^2=0$$

$$16x^2-8xy\frac{dy}{dx}+4y^2=0$$

$$4x^2+y^2=2xy\frac{dy}{dx}$$

Differential Equations Ex 22.2 Q15(iii)

Consider the given equation,

$(x-a)^2+2y^2=a^2....(1)$

Differentiating the above equation with respect to x, we have

$$2(x-a)+4y\frac{dy}{dx}=0$$

$$\Rightarrow (x-a)+2y\frac{dy}{dx}=0$$

$$\Rightarrow (x-a)=-2y\frac{dy}{dx}$$

$$\Rightarrow a=x+2y\frac{dy}{dx}$$

Substituting the value of a in equation (1), we have

$$\left(x-x+2y\frac{dy}{dx}\right)^2+2y^2=\left(x+2y\frac{dy}{dx}\right)^2$$

$$\Rightarrow 4y^2\left(\frac{dy}{dx}\right)^2+2y^2=x^2+4y^2\left(\frac{dy}{dx}\right)^2+4xy\frac{dy}{dx}$$

$$\Rightarrow 2y^2-x^2=4xy\frac{dy}{dx}$$

Differential Equations Ex 22.2 Q16(i)

$$x^2+y^2=a^2$$

Differentiating it with respect to x,

$$2x+2y\frac{dy}{dx}=0$$

$$x+y\frac{dy}{dx}=0$$

Differential Equations Ex 22.2 Q16(ii)

$$x^2-y^2=a^2$$

Differentiating it with respect to x,

$$2x-2y\frac{dy}{dx}=0$$

$$x-y\frac{dy}{dx}=0$$

Differential Equations Ex 22.2 Q16(iii)

$$y^2=4ax$$

$$\frac{y^2}{x}=4a$$

Differentiating it with respect to x,

$$\left[\frac{x\times 2y\frac{dy}{dx}-y^2(1)}{x^2}\right]=0$$

$$2xy\frac{dy}{dx}-y^2=0$$

$$2x\frac{dy}{dx}-y=0$$

Differential Equations Ex 22.2 Q16(iv)

$$x^2+(y-b)^2=1 \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$2x+2(y-b)\frac{dy}{dx}=0$$

$$x+(y-b)\frac{dy}{dx}=0$$

$$(y-b)\frac{dy}{dx}=-x$$

$$(y-b)=\frac{-x}{\frac{dy}{dx}}$$

Put the value of $(y-b)$ is equation (i)

$$x^2\left(\frac{-x}{\frac{dy}{dx}}\right)^2=1$$

$$x^2\left(\frac{dy}{dx}\right)^2+x^2=\left(\frac{dy}{dx}\right)^2$$

$$x^2\left\{\left(\frac{dy}{dx}\right)^2+1\right\}=\left(\frac{dy}{dx}\right)^2$$

Differential Equations Ex 22.2 Q16(v)

$$(x-a)^2-y^2=1 \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$2(x-a)-2y\frac{dy}{dx}=0$$

$$(x-a)-y\frac{dy}{dx}=0$$

$$(x-a)=y\frac{dy}{dx}$$

Put the value of $(x-a)$ is equation (i)

$$\left(y\frac{dy}{dx}\right)^2-y^2=1$$

$$y^2\left(\frac{dy}{dx}\right)^2-y^2=1$$

Differential Equations Ex 22.2 Q16(vi)

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$$\frac{b^2x^2-a^2y^2}{a^2b^2}=1$$

$$b^2x^2-a^2y^2=a^2b^2$$

Differentiating it with respect to x,

$$2xb^2-2a^2y\frac{dy}{dx}=0$$

$$xb^2-ya^2\frac{dy}{dx}=0 \qquad \text{---(i)}$$

Again, differentiating it with respect to x,

$$b^2-a^2\left(y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)\left(\frac{dy}{dx}\right)\right)=0$$

$$b^2=a^2\left(y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right)$$

Put the value of b^2 in equation (i)

$$xb^2-ya^2\frac{dy}{dx}=0$$

$$xa^2\left(y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right)-ya^2\frac{dy}{dx}=0$$

$$xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=0$$

$$x\left\{y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right\}=y\frac{dy}{dx}$$

Differential Equations Ex 22.2 Q16(vii)

$$y^2=4a(x-b)$$

Differentiating it with respect to x,

$$2y\frac{dy}{dx}=4a$$

Again, differentiating it with respect to x,

$$2\left[y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)\left(\frac{dy}{dx}\right)\right]=0$$

$$y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$$

Differential Equations Ex 22.2 Q16(viii)

$$y=ax^3$$

Differentiating it with respect to x,

$$\frac{dy}{dx}=3ax^2$$

$$=3\left(\frac{y}{x^3}\right)x^2$$

Using equation (i)

$$\frac{dy}{dx}=\frac{3y}{x}$$

$$x\frac{dy}{dx}=3y$$

Differential Equations Ex 22.2 Q16(ix)

$$x^2+y^2=ax^3$$

$$\frac{x^2+y^2}{x^3}=a$$

Differentiating it with respect to x,

$$\left[\frac{(x^3)\left(2x+2y\frac{dy}{dx}\right)-(x^2+y^2)(3x^2)}{(x^3)^2}\right]=0$$

$$2x^4+2x^3y\frac{dy}{dx}-3x^4-3x^2y^2=0$$

$$2x^3y\frac{dy}{dx}-x^4-3x^2y^2=0$$

$$2x^3y\frac{dy}{dx}=x^4+3x^2y^2$$

$$2x^3y\frac{dy}{dx}=x^2(x^2+3y^2)$$

$$2xy\frac{dy}{dx}=(x^2+3y^2)$$

Differential Equations Ex 22.2 Q16(x)

$$y = e^{ax} \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = ae^{ax}$$

$$\frac{dy}{dx} = ay \qquad \text{—(ii)}$$

From equation (i),

$$y = e^{ax}$$

$$\log y = ax$$

$$a = \frac{\log y}{x}$$

Put the value of a in equation (ii),

$$\frac{dy}{dx} = \left(\frac{\log y}{x}\right) y$$

$$x\frac{dy}{dx} = y \log y$$

Differential Equations Ex 22.2 Q17

We know that the equation of said family of ellipses is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \text{--------(i)}$$

DIfferentiating (i) w.r.t. x , we get

$$\frac{2x}{a^2} + \frac{2y}{b^2} \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow \qquad \frac{y}{x}\left(\frac{dy}{dx}\right) = \frac{-b^2}{a^2} \qquad \text{--------(ii)}$$

DIfferentiating (ii) w.r.t. x , we get

$$\frac{y}{x}\left(\frac{d^2y}{dx^2}\right) + \left(\frac{x\frac{dy}{dx} - y}{x^2}\right)\frac{dy}{dx} = 0$$

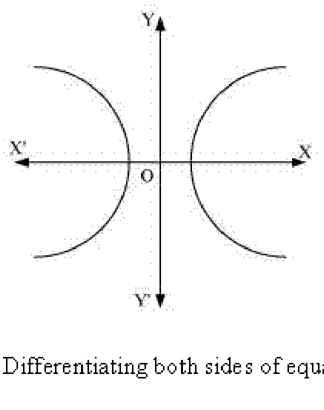
$$\Rightarrow \qquad xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$$

which is the required differential equation.

Differential Equations Ex 22.2 Q18

The equation of the family of hyperbolas with the centre at origin and foci along the x-axis is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad ...(1)$$



Differentiating both sides of equation (1) with respect to x, we get:

$$\frac{2x}{a^2} - \frac{2yy'}{b^2} = 0$$

$$\Rightarrow \frac{x}{a^2} - \frac{yy'}{b^2} = 0 \qquad ...(2)$$

Again, differentiating both sides with respect to x, we get:

$$\frac{1}{a^2} - \frac{y' \cdot y' + yy''}{b^2} = 0$$
$$\Rightarrow \frac{1}{a^2} = \frac{1}{b^2}\left((y')^2 + yy''\right)$$

Substituting the value of $\frac{1}{a^2}$ in equation (2), we get:

$$\frac{x}{b^2}\left((y')^2 + yy''\right) - \frac{yy'}{b^2} = 0$$
$$\Rightarrow x(y')^2 + xyy'' - yy' = 0$$
$$\Rightarrow xyy'' + x(y')^2 - yy' = 0$$

This is the required differential equation.

Differential Equations Ex 22.2 Q19

Let C denote the family of circles in the second quadrant and touching the coordinate axes.

Let $(-a,a)$ be the coordinate of the centre of any member of this family.

Equation representing the family C is

$$(x+a)^2 + (y-a)^2 = a^2 \qquad --------(i)$$

$$\text{or } x^2 + y^2 + 2ax - 2ay + a^2 = 0 \qquad --------(ii)$$

Differentiating eqn (ii) w.r.t. x, we get

$$2x + 2y\frac{dy}{dx} + 2a - 2a\frac{dy}{dx} = 0$$
$$\Rightarrow x + y\frac{dy}{dx} = a\left(\frac{dy}{dx} - 1\right)$$
$$\Rightarrow \qquad a = \frac{x + yy'}{y' - 1}$$

Substituting the value of a in (ii), we get

$$\left[x + \frac{x+yy'}{y'-1}\right]^2 + \left[y - \frac{x+yy'}{y'-1}\right]^2 = \left[\frac{x+yy'}{y'-1}\right]^2$$
$$\Rightarrow \quad \left[xy' - x + x + yy'\right]^2 + \left[yy' - y - x - yy'\right]^2 = \left[x + yy'\right]^2$$
$$\Rightarrow \quad (x+y)^2 y'' + (x+y)^2 = \left[x + yy'\right]^2$$
$$\Rightarrow \quad (x+y)^2\left[(y')^2 + 1\right] = \left[x + yy'\right]^2$$

which is the differential equation representing the given family of circles.

Ex 22.3

Differential Equations Ex 22.3 Q1

$$y = be^x + ce^{2x} \quad \text{—(i)}$$

Differentiating both sides with respect to x,

$$\frac{dy}{dx} = be^x + 2ce^{2x} \quad \text{—(ii)}$$

Differentiating both sides with respect to x,

$$\frac{d^2y}{dx^2} = be^x + 4ce^{2x} \quad \text{—(iii)}$$

Now,

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y$$
$$= be^x + 4ce^{2x} - 3\left(be^x + 2ce^{2x}\right) + 2\left(be^x + ce^{2x}\right)$$
$$= be^x + 4ce^{2x} - 3be^x - 6ce^{2x} + 2be^x + 2ce^{2x}$$
$$= 3be^x - 3be^x + 6ce^{2x} - 6ce^{2x}$$
$$= 0$$

So,

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$$

Differential Equations Ex 22.3 Q2

$$y = 4\sin 3x \quad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 4(3)\cos 3x$$
$$\frac{dy}{dx} = 12\cos 3x \quad \text{—(ii)}$$

Differentiating it wtih respect to x,

$$\frac{d^2y}{dx^2} = -12(3)\sin 3x$$
$$\frac{d^2y}{dx^2} = -36\sin 3x \quad \text{—(iii)}$$

Now,

$$\frac{d^2y}{dx^2} + 9y$$
$$= -36\sin 3x + 9(4\sin 3x)$$
$$= -36\sin 3x + 36\sin 3x$$
$$= 0$$

So, $y = 4\sin 3x$ is a solution of

$$\frac{d^2y}{dx^2} + 9y = 0$$

Differential Equations Ex 22.3 Q3

$$y = ae^{2x} + be^{-x} \quad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 2ae^{2x} - be^{-x} \quad \text{—(ii)}$$

Differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = 4ae^{2x} + be^{-x} \quad \text{—(ii)}$$

Now,

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y$$
$$= \left(4ae^{2x} + be^{-x}\right) - \left(2ae^{2x} - be^{-x}\right) - 2\left(ae^{2x} + be^{-x}\right)$$
$$= 4ae^{2x} + be^{-x} - 2ae^{2x} + be^{-x} - 2ae^{2x} - 2be^{-x}$$
$$= 4ae^{2x} - 4ae^{2x} + 2be^{-x} - 2be^{-x}$$
$$= 0$$

Differential Equations Ex 22.3 Q4

The given function is $y = A\cos x + B\sin x$ ---------(i)

Differentiating both sides of eqn (i) w.r.t x, successively, we get

$$\frac{dy}{dx} = -A\sin x + B\cos x$$

$$\frac{d^2y}{dx^2} = -A\cos x - B\sin x$$

Substituting these values of $\frac{d^2y}{dx^2}$ and y in the given differential equation,

L.H.S = $(-A\cos x - B\sin x) + (A\cos x + B\sin x) = 0 =$ R.HS

Therefore, the given function is a solution of the given differential equation.

Differential Equations Ex 22.3 Q5

$$y = A\cos 2x - B\sin 2x \qquad \text{---(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = -2A\sin 2x - 2B\cos 2x$$

$$\frac{dy}{dx} = -2(A\sin 2x + B\cos 2x) \qquad \text{---(ii)}$$

Differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = -2[2A\cos 2x - 2B\sin 2x]$$

$$= -4[A\cos 2x - B\sin 2x]$$

$$\frac{d^2y}{dx^2} = -4y$$

$$\frac{d^2y}{dx^2} + 4y = 0$$

Differential Equations Ex 22.3 Q6

$$y = Ae^{Bx} \qquad \text{---(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = ABe^{Bx} \qquad \text{---(ii)}$$

Differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = AB^2e^{Bx}$$

$$= \frac{(ABe^{Bx})^2}{(Ae^{Bx})}$$

$$\frac{d^2y}{dx^2} = \frac{1}{y}\left(\frac{dy}{dx}\right)^2$$

Differential Equations Ex 22.3 Q7

$$y = \frac{a}{x} + b \qquad \text{---(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = -\frac{a}{x^2} \qquad \text{---(ii)}$$

Differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = \frac{2a}{x^3}$$

$$= -\frac{2}{x}\left(-\frac{a}{x^2}\right)$$

$$\frac{d^2y}{dx^2} = -\frac{2}{x}\left(\frac{dy}{dx}\right)$$

$$\frac{d^2y}{dx^2} + \frac{2}{x}\left(\frac{dy}{dx}\right) = 0$$

Differential Equations Ex 22.3 Q8

$$y^2 = 4ax \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$2y\frac{dy}{dx} = 4a$$

$$\frac{dy}{dx} = \frac{4a}{2y}$$

$$\frac{dy}{dx} = \frac{2a}{y} \qquad \text{—(ii)}$$

Now,

$$x\frac{dy}{dx} + a\frac{dy}{dx}$$

$$= 2\frac{xa}{y} + a\left(\frac{y}{2a}\right)$$

$$= \frac{4a^2x + ay^2}{2ay}$$

$$= \frac{ay^2 + ay^2}{2ay}$$

$$= y$$

So,

$$x\frac{dy}{dx} + a\frac{dx}{dy} = y$$

Differential Equations Ex 22.3 Q9

$$Ax^2 + By^2 = 1$$

Differentiating it with respect to x,

$$2Ax + 2By\frac{dy}{dx} = 0$$

$$y\frac{dy}{dx} = \frac{-2Ax}{2B}$$

$$y\frac{dy}{dx} = -\frac{Ax}{B} \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -\frac{A}{B}$$

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = \frac{y}{x}\frac{dy}{dx}$$

Using equation (i)

$$x\left\{y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right\} = y\frac{dy}{dx}$$

Differential Equations Ex 22.3 Q10

$$y = ax^3 + bx^2 + c$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 3ax^2 + 2bx$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = 6ax + 2b$$

Differentiating it with respect to x

$$\frac{d^3y}{dx^3} = 6a$$

Differential Equations Ex 22.3 Q11

$$y = \frac{c-x}{1+cx} \qquad \text{---(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \left[\frac{(1+cx)(-1)-(c-x)(c)}{(1+cx)}\right]$$

$$\frac{dy}{dx} = \left[\frac{-1-cx-c^2+cx}{(1+cx)^2}\right]$$

$$= \frac{-1-c^2}{(1+cx)^2}$$

$$\frac{dy}{dx} = \frac{-(1+c^2)}{(1+cx)^2} \qquad \text{---(ii)}$$

Now,

$$(1+x^2)\frac{dy}{dx}+(1+y^2)$$

$$= (1+x^2)\left[\frac{-(1+c^2)}{(1+cx)^2}\right]+\left[1+\left(\frac{c-x}{1+cx}\right)^2\right]$$

$$= \frac{-(1+x^2)(1+c^2)}{(1+cx)^2}+\left[\frac{(1+cx)^2+(c-x)^2}{(1+cx)^2}\right]$$

$$= \frac{-1-x^2-c^2-x^2c^2+1+c^2x^2+2cx+c^2+x^2-2cx}{(1+cx)^2}$$

$$= \frac{0}{(1+cx)^2}$$

$$= 0$$

So,

$$(1+x^2)\frac{dy}{dx}+(1+y^2) = 0$$

Differential Equations Ex 22.3 Q12

$$y = e^x(A\cos x + B\sin x) \ldots\ldots(i)$$

$$\frac{dy}{dx} = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x)$$

$$\frac{dy}{dx} = e^x\left[(A+B)\cos x - (A-B)\sin x\right] \ldots\ldots\ldots(ii)$$

$$\frac{d^2y}{dx^2} = e^x\left[(A+B)\cos x - (A-B)\sin x\right] + e^x\left[-(A+B)\sin x - (A-B)\cos x\right]$$

$$\frac{d^2y}{dx^2} = 2e^x(B\cos x - A\sin x) \ldots\ldots\ldots\ldots(iii)$$

Adding (i) and (iii) we get

$$y + \frac{1}{2}\frac{d^2y}{dx^2} = e^x\left[(A+B)\cos x - (A-B)\sin x\right]$$

$$2y + \frac{d^2y}{dx^2} = 2\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

Hence $y = e^x(A\cos x + B\sin x)$ is the solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0.$$

Differential Equations Ex 22.3 Q13

$$y = cx + 2c^2 \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{y}{dx} = c \qquad \text{—(ii)}$$

Now,

$$2\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} - y$$
$$= 2c^2 + xc - cx + 2c^2 \qquad \left[\text{Using equation (i) and (ii)}\right]$$
$$= 0$$

So,

$$2\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} - y = 0$$

Differential Equations Ex 22.3 Q14

$$y = -x - 1 \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = -1 \qquad \text{—(ii)}$$

So,

$$(y - x)dy - \left(y^2 - x^2\right)dx$$
$$= \left[(y-2)\frac{dy}{dx} - \left(y^2 - x^2\right)\right]dx$$
$$= \left[(-x-1-x)(-1) - \left\{(-x-1)^2 - x^2\right\}\right]$$

Using equation (i) and (ii),

$$= \left[x + 1 + x - \left(x^2 + 1 + 2x - x^2\right)\right]dx$$
$$= \left[2x + 1 - 2x - 1\right]dx$$
$$= 0$$

So,

$$(y - x)dy - \left(y^2 - x^2\right)dx = 0$$

Differential Equations Ex 22.3 Q15

$$y^2 = 4a(x + a) \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$2y\frac{dy}{dx} = 4a(1)$$
$$\frac{dy}{dx} = \frac{2a}{y} \qquad \text{—(ii)}$$

Now,

$$y\left\{1 - \left(\frac{dy}{dx}\right)^2\right\}$$
$$= \left[y^2\left\{1 - \left(\frac{dy}{dx}\right)^2\right\}\right]\frac{1}{y}$$
$$= \left[4a(x+a) - 4a(x+a)\left(\frac{2a}{y}\right)^2\right]\frac{1}{y}$$

Using equation (i) and (ii)

$$= \left[4ax + 4a^2 - \frac{16a3x}{y^2} - \frac{16a^4}{y^2}\right]\frac{1}{y}$$
$$= \frac{4a}{y^3}\left[xy^2 + ay^2 - 4a^2x - 4a^3\right]$$
$$= \frac{4a}{y^3}\left[y^2(a+x) - 4a^2(x+a)\right]$$
$$= \frac{4a}{y^3}(a+x)\left(y^2 - 4a^2\right)$$
$$= \frac{4a}{y^3}\left(\frac{y^2}{4a}\right)\left(y^2 - 4a^2\right)$$

Using equation (i) and (ii)

$$= \frac{1}{y}\left(y^2 - 4a^2\right)$$
$$= \frac{1}{y}\left[4ax + 4a^2 - 4a\right]$$
$$= \frac{1}{y}(4ax)$$
$$= 2x\left(\frac{2a}{y}\right)$$
$$= 2x\frac{dy}{dx}$$

So,

$$y\left\{1 - \left(\frac{dy}{dx}\right)^2\right\} = 2x\frac{dy}{dx}$$

Differential Equations Ex 22.3 Q16

$$y = ce^{\tan^{-1}x}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = ce^{\tan^{-1}x} \times \left(\frac{1}{1+x^2}\right)$$

$$\left(1+x^2\right)\frac{dy}{dx} = ce^{\tan^{-1}x}$$

$$\left(1+x^2\right)\frac{dy}{dx} = y$$

Again, differentiating it with respecet to x,

$$2x\frac{dy}{dx} + \left(1+x^2\right)\frac{d^2y}{dx^2} = \frac{dy}{dx}$$

$$2x\frac{dy}{dx} - \frac{dy}{dx} + \left(1+x^2\right)\frac{d^2y}{dx^2} = 0$$

$$(2x-1)\frac{dy}{dx} + \left(1+x^2\right)\frac{d^2y}{dx^2} = 0$$

Differential Equations Ex 22.3 Q17

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0$$

$$y = e^{m\cos^{-1}x}$$

$$\frac{dy}{dx} = \frac{me^{m\cos^{-1}x}}{-\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = \frac{-my}{\sqrt{1-x^2}} \text{........(i)}$$

$$\frac{d^2y}{dx^2} = \frac{\sqrt{(1-x^2)}.\left(-m\frac{dy}{dx}\right) - (-my)\frac{(-2x)}{2\sqrt{(1-x^2)}}}{(1-x^2)} \text{[From (i)]}$$

$$\frac{d^2y}{dx^2} = \frac{(-m)(-my) - x\frac{dy}{dx}}{(1-x^2)} \text{[From (i)]}$$

$$(1-x^2)\frac{d^2y}{dx^2} = m^2y - x\frac{dy}{dx}$$

$$(1-x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2y = 0$$

Hence Proved

Differential Equations Ex 22.3 Q18

$$y = \log\left(x + \sqrt{x^2+a^2}\right)^2$$

Differentiating it with respect to x,

$$\frac{dy}{dx}\frac{1}{\left(x+\sqrt{a^2+x^2}\right)^2} \times 2\left(x+\sqrt{x^2+a^2}\right)\frac{d}{dx}\left(x+\sqrt{x^2+a^2}\right)$$

$$= \frac{2}{\left(x+\sqrt{a^2+x^2}\right)} \times \left(1 + \frac{1}{2\sqrt{x^2+a^2}}(2x)\right)$$

$$= \frac{2}{\left(x+\sqrt{a^2+x^2}\right)}\left(\frac{\sqrt{x^2+a^2}+x}{2\sqrt{x^2+a^2}}\right)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{a^2+x^2}}$$

$$\sqrt{a^2+x^2}\frac{dy}{dx} = 1 \qquad \text{---(i)}$$

Again, differentiating it with respecet to x,

$$\sqrt{1-x^2}\frac{d^2y}{dx^2} + \frac{1}{2\sqrt{1-x^2}}(-2x)\frac{dy}{dx} = -m\frac{dy}{dx}$$

$$\sqrt{1-x^2}\frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}}\frac{dy}{dx} - m\left(\frac{-e^{m\cos^{-1}x}m}{\sqrt{1-x^2}}\right) = 0$$

Using equation (i),

$$\sqrt{a^2+x^2}\frac{d^2y}{dx^2} + \frac{2x}{2\sqrt{a^2+x^2}}\frac{dy}{dx} = 0$$

$$\left(a^2+x^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$$

Differential Equations Ex 22.3 Q19

$$y = 2\left(x^2 - 1\right) + ce^{-x^2} \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 2(2x) + ce^{-x^2}(-2x)$$

$$\frac{dy}{dx} = 4x - 2cxe^{-x^2} \qquad \text{—(ii)}$$

Now,

$$\frac{dy}{dx} + 2xy$$

$$= 4x - 2cxe^{-x^2} + 2x\left[2\left(x^2 - 1\right) + ce^{-x^2}\right]$$

Using equation (i) and (ii),

$$= 4x - 2cxe^{-x^2} + 2x\left(2x^2 - 2 + ce^{-x^2}\right)$$

$$= 4x - 2cxe^{-x^2} + 4x^3 - 4x + 2xce^{-x^2}$$

$$= 4x^3$$

So,

$$\frac{dy}{dx} + 2xy = 4x^3$$

Differential Equations Ex 22.3 Q20

$$y = e^{-x} + ax + b$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = -e^{-x} + a$$

Differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = e^{-x}$$

$$\frac{1}{e^{-x}}\frac{d^2y}{dx^2} = 1$$

$$e^{x}\frac{d^2y}{dx^2} = 1$$

Differential Equations Ex 22.3 Q21(i)

$$y = ax \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = a$$

$$= \frac{ax}{x} \qquad \left[\because x \in R - \{0\}\right]$$

$$\frac{dy}{dx} = \frac{y}{x} \qquad \left[\text{Using equation (i)}\right]$$

$$x\frac{dy}{dx} = y$$

So, $y = ax$ is the solution of the given equation.

Differential Equations Ex 22.3 Q21(ii)

$$y = \pm\sqrt{a^2 - x^2}$$

Squaring both the sides,

$$y^2 = \left(a^2 - x^2\right)$$

Differentiating it with respect to x,

$$2y\frac{dy}{dx} = -2x$$

$$y\frac{dy}{dx} = -x$$

$$x + y\frac{dy}{dx} = 0$$

So,

$y = \pm\sqrt{a^2 - x^2}$ **is the solution of the given equation.**

Differential Equations Ex 22.3 Q21(iii)

$$y = \frac{a}{x+a}$$

$$\frac{dy}{dx} = \frac{a}{(x+a)^2} \times (-1) = -\frac{a}{(x+a)^2}$$

Consider,

$$x\frac{dy}{dx} + y = -\frac{ax}{(x+a)^2} + \frac{a}{x+a} = \frac{-ax + ax + a^2}{(x+a)^2} = \frac{a^2}{(x+a)^2} = y^2$$

$$x\frac{dy}{dx} + y = y^2$$

Hence $y = \frac{a}{x+a}$ is the solution of the differential equation $x\frac{dy}{dx} + y = y^2$.

Differential Equations Ex 22.3 Q21(iv)

$$y = ax + b + \frac{1}{2x}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = a - \frac{1}{2x^2}$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = 0 - \frac{(-2)}{2x^3}$$

$$\frac{d^2y}{dx^2} = \frac{1}{x^3}$$

$$x^3\frac{d^2y}{dx^2} = 1$$

So,

$y = ax + b + \frac{1}{2x}$ **is the solution of the given equation.**

Differential Equations Ex 22.3 Q21(v)

$$y = \frac{1}{4}(x \pm a)^2$$

Case I:

$$y = \frac{1}{4}(x+a)^2$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{1}{4}2(x+a)$$

$$\frac{dy}{dx} = \frac{1}{2}(x+a)$$

Squaring both sides,

$$\left(\frac{dy}{dx}\right)^2 = \frac{1}{4}(x+a)^2$$

$$\left(\frac{dy}{dx}\right)^2 = y \qquad \text{[Using equation (i)]}$$

So,

$y = \frac{1}{4}(x+a)$ **is the solution of the given equation.**

Case II:

$$y = \frac{1}{4}(x-a)^2 \qquad \text{---(ii)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{1}{4}2(x-a)$$

$$\frac{dy}{dx} = \frac{1}{2}(x-a)$$

Squaring both the sides,

$$\left(\frac{dy}{dx}\right)^2 = \frac{1}{4}(x-a)^2$$

$$\left(\frac{dy}{dx}\right)^2 = y^2 \qquad \text{[Using equation (ii)]}$$

So,

$y = \frac{1}{4}(x-a)$ **is the solution of the given equation.**

Ex 22.4

Differential Equations Ex 22.4 Q1

Here, $y = \log x$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{1}{x}$$

$$x\frac{dy}{dx} = 1$$

So, $y = \log x$ is a solutionof the equation

If $x = 1$, $y = \log 1 = 0$

So,

$$y(1) = 0$$

Differential Equations Ex 22.4 Q2

Here, $y = e^x$

Differentiating it with respect to x,

$$\frac{dy}{dx} = e^x$$

$$\frac{dy}{dx} = y$$

So, $y = e^x$ is a solution of the equation

If $x = 0$, $y = e^0 = 1$

So,

$$y(0) = 1$$

Differential Equations Ex 22.4 Q3

Here, $y = \sin x$ —(i)

Differentiating it with respect to x,

$$\frac{dy}{dx} = \cos x \quad \text{—(ii)}$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = -\sin x$$

$$\frac{d^2y}{dx^2} = -y$$

$$\frac{d^2y}{dx^2} + y = 0$$

So, $y = \sin x$ is a solution of the equation.

Put $x = 0$ in equation (i),

$\Rightarrow$ $y = \sin 0$

$\Rightarrow$ $y = 0$

$\Rightarrow$ $y(0) = 0$

Put $x = 0$ in equation (ii),

$y' = \cos 0$

$y' = 1$

$\Rightarrow$ $y'(0) = 1$

Differential Equations Ex 22.4 Q4

Here, $y = e^x + 1$ —(i)

Differentiating it with respect to x,

$$\frac{dy}{dx} = e^x$$

$$\frac{dy}{dx} = y - 1 \qquad \text{—(ii)}$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = \frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$$

It is given differential equation. So,

$y = e^x + 1$ is a solutionof the equation

Put $x = 0$ in equation (i),

$\Rightarrow \quad y = e^0 + 1 = 2$

$y(0) = 2$

Put $x = 0$ in equation (ii),

$y' = e^0 = 1$

$y'(0) = 1$

Differential Equations Ex 22.4 Q5

Here, $y = e^{-x} + 2$ —(i)

Differentiating it with respect to x,

$$\frac{dy}{dx} = -e^{-x}$$

$$\frac{dy}{dx} = -(y - 2) \qquad \text{[Using equaton (i)]}$$

$$\frac{dy}{dx} + y = 2$$

It is given differential equation. So,

$y = e^{-x} + 2$ is a solutionof the equation

Put $x = 0$ in equation (i),

$y = e^0 + 2$

$= 1 + 2$

$y = 3$

So,

$y(0) = 3$

Differential Equations Ex 22.4 Q6

$y = \sin x + \cos x$ —(i)

Differentiating it with respect to x,

$$\frac{dy}{dx} = \cos x - \sin x \qquad \text{—(ii)}$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = -\sin x - \cos x$$

$$\frac{d^2y}{dx^2} = -(\sin x + \cos x)$$

$$\frac{d^2y}{dx^2} = -y \qquad \text{[Using equation (i)]}$$

$$\frac{d^2y}{dx^2} + y = 0$$

It is the given equation, so

$y = \sin x + \cos x$ is the solution of the given equation

Put $x = 0$ in equation (i),

$y = \sin 0 + \cos 0$

$y = 0 + 1$

$y = 1$

So,

$y(0) = 1$

Put $x = 0$ in equation (i),

$\frac{dy}{dx} = \cos 0 - \sin 0$

$y' = 1$

So,

$y'(0) = 1$

Differential Equations Ex 22.4 Q7

$$y = e^x + e^{-x} \qquad \text{---(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = e^x - e^{-x} \qquad \text{---(ii)}$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = e^x + e^{-x}$$

$$\frac{d^2y}{dx^2} = y \qquad [\text{Using equation (i)}]$$

$$\frac{d^2y}{dx^2} - y = 0$$

It is the given equation, so

$y = e^x + e^{-x}$ is the solution of the given equation.

Put $x = 0$ in equation (i),

$$y = e^0 + e^0$$
$$y = 2$$

So,

$$y(0) = 2$$

Put $x = 0$ in equation (ii),

$$y' = e^0 - e^0$$
$$y' = 0$$

So,

$$y'(0) = 0$$

Differential Equations Ex 22.4 Q8

$$y = e^x + e^{2x} \qquad \text{---(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = e^x + 2e^{2x} \qquad \text{---(ii)}$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = e^x + 4e^{2x}$$
$$= (3-2)e^x + (6-2)e^{2x}$$
$$= 3e^x - 2e^x + 6e^{2x} - 2e^{2x}$$
$$= 3e^x + 6e^{2x} - 2e^x - 2e^{2x}$$
$$= 3\left(e^x + 2e^{2x}\right) - 2\left(e^x + e^{2x}\right)$$

$$\frac{d^2y}{dx^2} = 3\frac{dy}{dx} - 2y \qquad [\text{Using equation(i) and (ii)}]$$

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$$

It is the given equation, so

$y = e^x + 2e^{2x}$ is the solution of the given equation.

Put $x = 0$ in equation (i),

$$y = e^0 + e^0$$
$$y = 1 + 1$$
$$y = 2$$

So,

$$y(0) = 2$$

Put $x = 0$ in equation (ii),

$$\frac{dy}{dx} = e^0 + 2e^0$$
$$y' = 1 + 2$$
$$y' = 3$$

So,

$$y'(0) = 3$$

Differential Equations Ex 22.4 Q9

$$y = xe^x + e^x \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \left[x \times \frac{d}{dx}\left(e^x\right) + e^x \frac{d}{dx}(x)\right] + e^x$$

$$= xe^x + e^x(1) + e^x$$

$$\frac{dy}{dx} = xe^x + 2e^x \qquad \text{—(ii)}$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = x\frac{d}{dx}\left(e^x\right) + e^x \frac{d}{dx}(x) + 2e^x$$

$$= (2-1)xe^x + (4-1)e^x$$

$$= 2xe^x - xe^x + 4e^x - e^x$$

$$= 2xe^x + 4e^x - xe^x - e^x$$

$$= 2\left(xe^x + 2e^x\right) - \left(xe^x + 1\right)$$

$$\frac{d^2y}{dx^2} = 2\frac{dy}{dx} - y \qquad \text{[Using equation(i) and (ii)]}$$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$$

It is the given equation, so

$y = xe^x + e^x$ is the solution of the given equation.

Put $y = 0$ in equation (i),

$$y = 0 + e^0$$

$$y = 1$$

So,

$$y(0) = 1$$

Put $y = 0$ in equation (ii),

$$\frac{dy}{dx} = 0 + 2e^0$$

$$y' = 2$$

So,

$$y'(0) = 2$$

Ex 22.5

Differential Equations Ex 22.5 Q1

$$\frac{dy}{dx} = x^2 + x - \frac{1}{x},\ x \neq 0$$

$$\int dy = \int \left(x^2 + x - \frac{1}{x}\right) dx$$

$$y = \frac{x^3}{3} + \frac{x^2}{2} - \log|x| + c,\ x \neq 0$$

Differential Equations Ex 22.5 Q2

$$\frac{dy}{dx} = x^5 + x^2 - \frac{2}{x},\ x \neq 0$$

$$\int dy = \int \left(x^5 + x^2 - \frac{2}{x}\right) dx$$

$$y = \frac{x^6}{6} + \frac{x^3}{3} - 2\log|x| + c,\ x \neq 0$$

Differential Equations Ex 22.5 Q3

$$\frac{dy}{dx} + 2x = e^{3x}$$

$$\frac{dy}{dx} = e^{3x} - 2x$$

$$\int dy = \int \left(e^{3x} - 2x\right) dx$$

$$y = \frac{e^{3x}}{3} - \frac{2x^2}{2} + c$$

$$y = \frac{e^{3x}}{3} - x^2 + c$$

$$y + x^2 = \frac{1}{3}e^{3x} + c$$

Differential Equations Ex 22.5 Q4

$$\left(x^2 + 1\right)\frac{dy}{dx} = 1$$

$$\int dy = \int \frac{dx}{x^2 + 1}$$

$$y = \tan^{-1} x + c$$

Differential Equations Ex 22.5 Q6

$$(x + 2)\frac{dy}{dx} = x^2 + 3x + 7$$

$$dy = \left(\frac{x^2 + 3x + 7}{x + 2}\right) dx$$

$$dy = \left(x + 1 + \frac{5}{x + 2}\right) dx$$

$$\int dy = \int \left(x + 1 + \frac{5}{x + 2}\right) dx$$

$$y = \frac{x^2}{2} + x + 5\log|x + 2| + c$$

$$x \neq -2$$

Differential Equations Ex 22.5 Q7

$$\frac{dy}{dx} = \tan^{-1} x$$

$$dy = \tan^{-1} x dx$$

$$\int dy = \int \tan^{-1} x dx$$

$$y = \tan^{-1} x \times \int 1 dx - \int \left(\frac{1}{1 + x^2} \int dx\right) dx + c$$

Using integration by parts

$$y = x\tan^{-1} x - \int \frac{x}{1 + x^2} dx + c$$

$$y = x\tan^{-1} x - \frac{1}{2}\int \frac{2x}{1 + x^2} dx + c$$

$$y = x\tan^{-1} x - \frac{1}{2}\log\left|1 + x^2\right| + c$$

Differential Equations Ex 22.5 Q8

$$\frac{dy}{dx} = \log x$$

$$\Rightarrow dy = \log x \times dx$$

$$\Rightarrow \int dy = \int \log x dx$$

$$\Rightarrow y = \log x \times \int 1 dx - \int\left(\frac{1}{x}\int 1 dx\right)dx + C \quad [\text{Using integration by parts}]$$

$$\Rightarrow y = x\log x - \int dx + C$$

$$\Rightarrow y = x\log x - x + C$$

$$\Rightarrow y = x(\log x - 1) + C, \text{ where } x \in (0, \infty)$$

Differential Equations Ex 22.5 Q9

$$\frac{1}{x}\frac{dy}{dx} = \tan^{-1} x$$

$$dy = x\tan^{-1} x dx$$

$$\int dy = \int x\tan^{-1} x dx$$

$$y = \tan^{-1} x\int x dx - \int\left(\frac{1}{1+x^2}\int x dx\right)dx + c$$

Using integration by parts

$$y = \frac{x^2}{2}\tan^{-1} x - \int \frac{x^2}{2\left(1+x^2\right)}dx + c$$

$$= \frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\int \frac{x^2}{1+x^2}dx + c$$

$$= \frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\int\left(1 - \frac{1}{x^2+1}\right)dx + c$$

$$y = \frac{x^2}{2}\tan^{-1} x - \frac{1}{2}x + \frac{1}{2}\tan^{-1} x + c$$

$$y = \frac{1}{2}\left(x^2+1\right)\tan^{-1} x - \frac{1}{2}x + c$$

Differential Equations Ex 22.5 Q10

$$\frac{dy}{dx} = \cos^3 x\sin^2 x + x\sqrt{2x+1}$$

$$dy = \left(\cos^3 x\sin^2 x + x\sqrt{2x+1}\right)dx$$

$$\int dy = \int \cos^3 x\sin^2 x dx + \int x\sqrt{2x+1}dx$$

$$y = I_1 + I_2 \qquad \text{---(i)}$$

$$I_1 = \int \cos^3 x\sin^2 x dx$$

$$= \int \cos^2 x \times \cos x \times \sin^2 x dx$$

$$I_1 = \int\left(1 - \sin^2 x\right)\sin^2 x\cos x dx$$

Put $\sin x = t$

$$\cos x dx = dt$$

$$I_1 = \int\left(1 - t^2\right)t^2 dt$$

$$= \int\left(t^2 - t^4\right)dt$$

$$= \frac{t^3}{3} - \frac{t^5}{5} + c_1$$

$$I_1 = \frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + c_1$$

And,

$$I_2 = \int x\sqrt{2x+1}dx$$

Put $2x + 1 = v^2$

$$2dx = 2vdv$$

$$I_2 = \int\left(\frac{v^2-1}{2}\right)v \times vdv$$

$$= \frac{1}{2}\int\left(v^4 - v^2\right)dv$$

$$= \frac{1}{2}\left(\frac{v^5}{5} - \frac{v^3}{3}\right) + c_2$$

$$I_2 = \frac{1}{10}(2x+1)^{\frac{5}{2}} - \frac{1}{6}(2x+1)^{\frac{3}{2}} + c_2$$

Put the I_1 and I_2 in equation (i),

$$y = I_1 + I_2$$

$$y = \frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + \frac{1}{10}(2x+1)^{\frac{5}{2}} - \frac{1}{6}(2x+1)^{\frac{3}{2}} + c$$

As $\quad c = c_1 + c_2$

Differential Equations Ex 22.5 Q11

$$(\sin x+\cos x)\,dy+(\cos x-\sin x)\,dx=0$$
$$(\sin x+\cos x)\,dy=(\sin x-\cos x)\,dx$$
$$dy=\frac{(\sin x-\cos x)}{\sin x+\cos x}dx$$
$$\int dy=-\int\left(\frac{\cos x-\sin x}{\sin x+\cos x}\right)dx$$

Put $\sin x+\cos x=t$

$$(\cos x-\sin x)\,dx=dt$$
$$\int dy=-\int\frac{1}{t}dt$$
$$y=-\log|t|+c$$
$$y+\log|\sin x+\cos x|=c$$

Differential Equations Ex 22.5 Q12

$$\frac{dy}{dx}-x\sin^2 x=\frac{1}{x\log x}$$
$$\frac{dy}{dx}=\frac{1}{x\times\log x}+x\sin^2 x$$
$$dy=\left(\frac{1}{x\log x}+x\sin^2 x\right)dx$$
$$\int dy=\int\frac{1}{x\log x}dx+\int x\sin^2 x dx$$
$$y=I_1+I_2 \qquad \text{---(i)}$$
$$I_1=\int\frac{1}{x\log x}dx$$

Let $\log x=t$

$$\frac{1}{x}dx=dt$$
$$I_1=\int\frac{dt}{t}$$
$$=\log|t|+c_1$$
$$I_1=\log|\log x|+c_1$$
$$I_2=\int x\sin^2 x dx$$
$$=\int x\frac{(1-\cos 2x)}{2}dx$$
$$=\frac{1}{2}\int(x-x\cos 2x)dx$$
$$=\frac{1}{2}\int xdx-\frac{1}{2}\int x\cos 2xdx$$
$$=\frac{1}{2}\left(\frac{x^2}{2}\right)-\frac{1}{2}\left[x\int\cos 2xdx-\int(1\times\int\cos 2xdx)\,dx\right]+c_2$$
$$=\frac{x^2}{4}-\frac{1}{2}\left[\frac{x\sin 2x}{2}-\int\frac{\sin 2x}{2}dx\right]+c_2$$
$$=\frac{x^2}{4}-\frac{1}{2}\left[\frac{x\sin 2x}{2}+\frac{\cos 2x}{4}\right]+c_2$$
$$I_2=\frac{x^2}{4}-\frac{x\sin 2x}{4}-\frac{\cos 2x}{8}+c_2$$

Put the value of I_1 and I_2 in equation (i),

$$y=I_1+I_2$$
$$y=\log|\log x|+\frac{x^2}{4}-\frac{x\sin 2x}{4}-\frac{\cos 2x}{8}+c \text{ as } c_1+c_2=c$$

Differential Equations Ex 22.5 Q13

$$\frac{dy}{dx}=x^5\tan^{-1}(x^3)$$
$$dy=x^5\tan^{-1}(x^3)dx$$
$$\int dy=\int x^5\tan^{-1}(x^3)dx$$

Put $x^3=t$

$\Rightarrow$ $3x^2dx=dt$

$\Rightarrow$ $x^2dx=\dfrac{dt}{3}$

So,

$$\int dy=\frac{1}{3}\left[\tan^{-1}t\int tdt-\int\left(\frac{1}{1+t^2}\times\int tdt\right)\right]dt+c$$

Using integration by parts

$$y=\frac{1}{3}\left[\frac{t^2}{2}+\tan^{-1}t-\int\frac{t^2}{2(t^2+1)}dt\right]+c$$
$$=\frac{1}{6}t^2\tan^{-1}t-\frac{1}{6}\int\left(\frac{t^2}{t^2+1}\right)dt+c$$
$$y=\frac{1}{6}t^2\tan^{-1}t-\frac{1}{6}\int\left(1-\frac{1}{t^2+1}\right)dt+c$$
$$=\frac{1}{6}t^2\tan^{-1}t-\frac{1}{6}t+\frac{1}{6}\tan^{-1}t+c$$
$$y=\frac{1}{6}(t^2+1)\tan^{-1}t-\frac{1}{6}t+c$$
$$y=\frac{1}{6}\left[(t^2+1)\tan^{-1}t-t\right]+c$$

So,

$$y=\frac{1}{6}\left[(x^6+1)\tan^{-1}(x^3)-x^3\right]+c$$

Differential Equations Ex 22.5 Q14

$$\sin^4 x \frac{dy}{dx} = \cos x$$
$$dy = \frac{\cos x}{\sin^4 x} dx$$
$$\int dy = \int \frac{\cos x}{\sin^4 x} dx$$
Put $\sin x = t$
$$\cos x dx = dt$$
$$\int dy = \int \frac{dt}{t^4}$$
$$y = \frac{1}{-3t^3} + c$$
$$y = -\frac{1}{3\sin^3 x} + c$$
$$y = -\frac{1}{3}\cos ec^3 x + c$$

Differential Equations Ex 22.5 Q15

$$\cos x \frac{dy}{dx} - \cos 2x = \cos 3x$$
$$\cos x \frac{dy}{dx} = \cos 3x + \cos 2x$$
$$\frac{dy}{dx} = \frac{4\cos^3 x - 3\cos x + 2\cos^2 x - 1}{\cos x}$$
$$\frac{dy}{dx} = \frac{4\cos^3 x}{\cos x} - \frac{3\cos x}{\cos x} + \frac{2\cos^2 x}{\cos x} - \frac{1}{\cos x}$$
$$\frac{dy}{dx} = 4\cos^2 x - 3 + 2\cos x - \sec x$$
$$\frac{dy}{dx} = 4\left(\frac{\cos 2x + 1}{2}\right) - 3 + 2\cos x - \sec x$$
$$dy = (2\cos 2x + 2 - 3 + 2\cos x - \sec x) dx$$
$$\int dy = \int (2\cos 2x - 1 + 2\cos x - \sec x) dx$$
$$y = \sin 2x - x + 2\sin x - \log|\sec x + \tan x| + c$$

Differential Equations Ex 22.5 Q16

$$\sqrt{1 - x^4} dy = x dx$$
$$dy = \frac{x dx}{\sqrt{1 - x^4}}$$
$$\int dy = \int \frac{x dx}{\sqrt{1 - x^4}}$$
Let $x^2 = t$
$$2x dx = dt$$
$$\Rightarrow \quad x dx = \frac{dt}{2}$$
$$\int dy = \int \frac{dt}{2\sqrt{1 - t^2}}$$
$$y = \frac{1}{2}\sin^{-1}(t) + c$$
$$y = \frac{1}{2}\sin^{-1}(x^2) + c$$

Differential Equations Ex 22.5 Q17

$$\sqrt{a + x} dy + x dx = 0$$
$$\sqrt{a + x} dy = -x dx$$
$$dy = \frac{-x}{\sqrt{a + x}} dx$$
$$\int dy = -\int \frac{x}{\sqrt{a + x}} dx$$
Put $a + x = t^2$
$$dx = 2t dt$$
$$\int dy = -\int \left(\frac{t^2 - a}{t}\right) 2t dt$$
$$\int dy = 2\int (a - t^2) dt$$
$$y = 2\left(at - \frac{t^3}{3}\right) + c$$
$$y + \frac{2}{3}t^3 - 2at = c$$
$$y + \frac{2}{3}(a + x)^{\frac{3}{2}} - 2a\sqrt{a + x} = c$$

Differential Equations Ex 22.5 Q18

$$\left(1+x^2\right)\frac{dy}{dx}-x=2\tan^{-1}x$$
$$\left(1+x^2\right)\frac{dy}{dx}=2\tan^{-1}x+x$$
$$dy=\left(\frac{2\tan^{-1}x+x}{1+x^2}\right)dx$$
$$\int dy=\int\left(\frac{2\tan^{-1}x+x}{1+x^2}\right)dx$$
$$y=\int(2t+\tan t)dt \qquad \left[\tan^{-1}x=t\right]$$
$$=\frac{1}{2}\log\left|1+x^2\right|+\left(\tan^{-1}x\right)^2+c$$

Differential Equations Ex 22.5 Q19

$$\frac{dy}{dx}=x\log x$$
$$dy=x\log x dx$$
$$\int dy=\int x\log x dx$$
$$y=\log|x|\int x dx-\int\left(\frac{1}{x}\int x dx\right)dx+c$$

Using integration by parts

$$=\frac{x^2}{2}\log|x|-\int\frac{x^2}{2x}dx+c$$
$$=\frac{x^2}{2}\log|x|-\frac{1}{2}\int x dx+c$$
$$y=\frac{x^2}{2}\log|x|-\frac{x^2}{4}+c$$

Differential Equations Ex 22.5 Q20

$$\frac{dy}{dx}=xe^x-\frac{5}{2}+\cos^2 x$$
$$dy=\left(xe^x-\frac{5}{2}+\cos^2 x\right)dx$$
$$\int dy=\int xe^x dx-\frac{5}{2}\int dx+\int\cos^2 x dx$$
$$\int dy=\int xe^x dx-\frac{5}{2}\int dx+\int\left(\frac{1+\cos 2x}{2}\right)dx$$
$$=\int xe^x-\frac{5}{2}\int dx+\frac{1}{2}\int dx+\frac{1}{2}\int\cos 2x dx$$
$$\int dy=\int xe^x-2\int dx+\frac{1}{2}\int\cos 2x dx$$
$$y=\left[x\times\int e^x dx-\int\left(1\times\int e^x dx\right)dx\right]-2x+\frac{1}{2}\frac{\sin 2x}{2}+c$$

Using integration by parts

$$y=xe^x-e^x-2x+\frac{1}{4}\sin 2x+c$$
$$y=xe^x-e^x-2x+\frac{1}{4}\sin 2x+c$$

Differential Equations Ex 22.5 Q21

The given differential equation is:

$$\left(x^3+x^2+x+1\right)\frac{dy}{dx}=2x^2+x$$
$$\Rightarrow \frac{dy}{dx}=\frac{2x^2+x}{\left(x^3+x^2+x+1\right)}$$
$$\Rightarrow dy=\frac{2x^2+x}{(x+1)\left(x^2+1\right)}dx$$

Integrating both sides, we get:

$$\int dy=\int\frac{2x^2+x}{(x+1)\left(x^2+1\right)}dx \qquad ...(1)$$

$$\text{Let } \frac{2x^2+x}{(x+1)\left(x^2+1\right)}=\frac{A}{x+1}+\frac{Bx+C}{x^2+1}. \qquad ...(2)$$
$$\Rightarrow \frac{2x^2+x}{(x+1)\left(x^2+1\right)}=\frac{Ax^2+A+(Bx+C)(x+1)}{(x+1)\left(x^2+1\right)}$$
$$\Rightarrow 2x^2+x=Ax^2+A+Bx^2+Bx+Cx+C$$
$$\Rightarrow 2x^2+x=(A+B)x^2+(B+C)x+(A+C)$$

Comparing the coefficients of x^2 and x, we get:

$A+B=2$

$B+C=1$

$A+C=0$

Solving these equations, we get:

$$A=\frac{1}{2},\ B=\frac{3}{2} \text{ and } C=\frac{-1}{2}$$

Substituting the values of A, B, and C in equation (2), we get:

$$\frac{2x^2+x}{(x+1)\left(x^2+1\right)}=\frac{1}{2}\cdot\frac{1}{(x+1)}+\frac{1}{2}\frac{(3x-1)}{\left(x^2+1\right)}$$

Therefore, equation (1) becomes:

$$\int dy=\frac{1}{2}\int\frac{1}{x+1}dx+\frac{1}{2}\int\frac{3x-1}{x^2+1}dx$$
$$\Rightarrow y=\frac{1}{2}\log(x+1)+\frac{3}{2}\int\frac{x}{x^2+1}dx-\frac{1}{2}\int\frac{1}{x^2+1}dx$$
$$\Rightarrow y=\frac{1}{2}\log(x+1)+\frac{3}{4}\cdot\int\frac{2x}{x^2+1}dx-\frac{1}{2}\tan^{-1}x+\mathrm{C}$$
$$\Rightarrow y=\frac{1}{2}\log(x+1)+\frac{3}{4}\log\left(x^2+1\right)-\frac{1}{2}\tan^{-1}x+\mathrm{C}$$
$$\Rightarrow y=\frac{1}{4}\left[2\log(x+1)+3\log\left(x^2+1\right)\right]-\frac{1}{2}\tan^{-1}x+\mathrm{C}$$
$$\Rightarrow y=\frac{1}{4}\left[(x+1)^2\left(x^2+1\right)^3\right]-\frac{1}{2}\tan^{-1}x+\mathrm{C} \qquad ...(3)$$

Now, $y=1$ when $x=0$.

$$\Rightarrow 1=\frac{1}{4}\log(1)-\frac{1}{2}\tan^{-1}0+\mathrm{C}$$
$$\Rightarrow 1=\frac{1}{4}\times 0-\frac{1}{2}\times 0+\mathrm{C}$$
$$\Rightarrow \mathrm{C}=1$$

Substituting C = 1 in equation (3), we get:

$$y=\frac{1}{4}\left[\log(x+1)^2\left(x^2+1\right)^3\right]-\frac{1}{2}\tan^{-1}x+1$$

Differential Equations Ex 22.5 Q22

$$\sin\left(\frac{dy}{dx}\right) = k, \; y(0) = 1$$

$$\frac{dy}{dx} = \sin^{-1} k$$

$$dy = \sin^{-1} k dx$$

$$\int dy = \int \sin^{-1} k dx$$

$$y = x \sin^{-1} k + c \qquad \text{---(i)}$$

Put $x = 0, y = 1$

$$1 = 0 + c$$

$$1 = c$$

Put $c = 1$ in equation (i),

$$y = x \sin^{-1} k + 1$$

$$y - 1 = x \sin^{-1} k$$

Differential Equations Ex 22.5 Q23

$$e^{\frac{dy}{dx}} = x + 1, \quad y(0) = 3$$

$$\frac{dy}{dx} = \log(x+1)$$

$$dy = \log(x+1) dx$$

$$\int dy = \int \log(x+1) dx$$

$$y = \log(x+1) \int 1 \times dx - \int\left(\frac{1}{x+1} \times \int 1 \times dx\right) dx + c$$

Using integration by parts

$$y = x \log(x+1) - \int\left(\frac{x}{x+1}\right) dx + c$$

$$= x \log(x+1) - \int\left(1 - \frac{1}{x+1}\right) dx + c$$

$$= x \log(x+1) - x + \log(x+1) + c$$

$$y = (x+1) \log(x+1) - x + c \qquad \text{---(i)}$$

Put $y = 3, x = 0$

$$3 = 0 + c$$

$\Rightarrow$ $c = 3$

Using equation (i),

$$y = (x+1) \log(x+1) - x + 3$$

Differential Equations Ex 22.5 Q24

$$c'(x) = 2 + 0.15x, \quad c(0) = 100$$

$$c'(x) dx = (2 + 0.15x) dx$$

$$\int c'(x) dx = \int 2dx + 0.15 \int x dx$$

$$c(x) = 2x + 0.15 \frac{x^2}{2} + c \qquad \text{---(i)}$$

Put $x = 0, c(x) = 100$

$$100 = 2(0) + 0 + c$$

$$100 = c$$

Put $c = 100$ in equation (i),

$$c(x) = 2x + (0.15) \frac{x^2}{2} + 100$$

Differential Equations Ex 22.5 Q25

$$x \frac{dy}{dx} + 1 = 0, \quad y(-1) = 0$$

$$x \frac{dy}{dx} = -1$$

$$dy = -\frac{dx}{x}$$

$$\int dy = -\int \frac{dx}{x}$$

$$y = -\log|x| + c \qquad \text{---(i)}$$

Put $x = -1$ and $y = 0$

$$0 = 0 + c$$

$$c = 0$$

Put $c = 0$ in equation (i),

$$y = -\log|x|, x < 0$$

Differential Equations Ex 22.5 Q26

$$x\left(x^2-1\right)\frac{dy}{dx}=1, y(2)=0$$

$$\frac{dy}{dx}=\frac{1}{x\left(x^2-1\right)}$$

$$dy=\frac{1}{x\left(x^2-1\right)}dx$$

$$\int dy=\int\left(\frac{1}{x\left(x^2-1\right)}\right)dx$$

$$y=\frac{1}{2}\int\frac{1}{x-1}dx-\int\frac{1}{x}dx+\frac{1}{2}\int\frac{1}{x+1}dx$$

$$=\frac{1}{2}\log|x-1|-\log|x|+\frac{1}{2}\log|x+1|+c$$

Putting $x=2, y=0$, we have

$$y=\frac{1}{2}\log|x-1|-1\log|x|+\frac{1}{2}\log|x+1|+c$$

$$0=\frac{1}{2}\log|2-1|-\log|2|+\frac{1}{2}\log|2+1|+c$$

$$c=\log|2|-\frac{1}{2}\log|3|$$

Putting the value of c, we have

$$y=\frac{1}{2}\log|x-1|-1\log|x|+\frac{1}{2}\log|x+1|+c$$

$$=\log\frac{4}{3}\left(\frac{x^2-1}{x^2}\right)$$

Ex 22.6

Differential Equations Ex 22.6 Q1

$$\frac{dy}{dx} + \frac{1+y^2}{y} = 0, \qquad y \neq 0$$

$$\frac{dy}{dx} = -\frac{1+y^2}{y}$$

$$\int \frac{y}{1+y^2} dy = -\int dx$$

$$\int \frac{2y}{1+y^2} dy = -2\int dx$$

$$\log\left|1+y^2\right| = -2x + c_1$$

$$\frac{1}{2}\log\left|1+y^2\right| + x = c$$

Differential Equations Ex 22.6 Q2

$$\frac{dy}{dx} = \frac{1+y^2}{y^3}, \quad y \neq 0$$

$$\frac{y^3}{1+y^2} dy = dx$$

$$\int\left(y - \frac{y}{y^2+1}\right) dy = \int dx$$

$$\int y dy - \int \frac{y}{y^2+1} dy = \int dx$$

$$\int y dy - \frac{1}{2}\int \frac{2y}{y^2+1} dy = \int dx$$

$$\frac{y^2}{2} - \frac{1}{2}\log\left|y^2+1\right| = x + c$$

Differential Equations Ex 22.6 Q3

$$\frac{dy}{dx} = \sin^2 y$$

$$\frac{dy}{\sin^2 y} = dx$$

$$\int \operatorname{cosec}^2 y dy = \int dx$$

$$-\cot x = x + c_1$$

$$x + \cot x = c$$

Differential Equations Ex 22.6 Q4

$$\frac{dy}{dx} = \frac{1-\cos 2y}{1+\cos 2y}$$

$$= \frac{2\sin^2 y}{2\cos^2 y}$$

$$\frac{dy}{dx} = \tan^2 y$$

$$\frac{dy}{\tan^2 y} = dx$$

$$\int \cot^2 y dy = \int dx$$

$$\int\left(\operatorname{cosec}^2 y - 1\right) dy = \int dx$$

$$-\cot y - y + c = x$$

$$c = x + y + \cot y$$

Ex 22.7

Differential Equations Ex 22.7 Q1

$$(x-1)\frac{dy}{dx} = 2xy$$

Separating the variables,

$$\int \frac{dy}{y} = \int \frac{2x}{x-1}dx$$

$$\int \frac{dy}{y} = \int \left(2 + \frac{2}{x-1}\right)dx$$

$$\log y = 2x + 2\log|x-1| + c$$

Differential Equations Ex 22.7 Q2

$$\left(x^2+1\right)dy = xydx$$

$$\int \frac{1}{y}dy = \int \frac{x}{x^2+1}dx$$

$$\int \frac{1}{y}dy = \frac{1}{2}\int \frac{2x}{x^2+1}dx$$

$$\log y = \frac{1}{2}\log\left|x^2+1\right| + \log c$$

$$y = \sqrt{x^2+1} \times c$$

Differential Equations Ex 22.7 Q3

$$\frac{dy}{dx} = \left(e^x+1\right)y$$

$$\int \frac{1}{y}dy = \int \left(e^x+1\right)dx$$

$$\log|y| = e^x + x + c$$

Differential Equations Ex 22.7 Q4

$$(x-1)\frac{dy}{dx} = 2x^3y$$

$$\frac{dy}{y} = \frac{2x^3}{x-1}dx$$

$$\int \frac{dy}{y} = 2\int \left(x^2+x+1+\frac{1}{x-1}\right)dx$$

$$\log|y| = 2\left(\frac{x^3}{3}+\frac{x^2}{2}+x+\log|x-1|\right)+c$$

$$\log|y| = \frac{2}{3}x^3 + x^2 + 2x + 2\log|x-1| + c$$

Differential Equations Ex 22.7 Q5

$$xy(y+1)dy = \left(x^2+1\right)dx$$

$$y(y+1)dy = \frac{x^2+1}{x}dx$$

$$\int \left(y^2+y\right)dy = \int \left(x+\frac{1}{x}\right)dx$$

$$\frac{y^3}{3}+\frac{y^2}{2} = \frac{x^2}{2} + \log|x| + c$$

Differential Equations Ex 22.7 Q6

$$5\frac{dy}{dx} = e^x y^4$$

$$5\int \frac{dy}{y^4} = \int e^x dx$$

$$5\left(\frac{y^{-4+1}}{-4+1}\right) = e^x + c$$

$$-\frac{5}{3y^3} = e^x + c$$

Differential Equations Ex 22.7 Q7

$$x \cos y dy = \left(xe^x \log x + e^x\right) dx$$

$$\int \cos y dy = \int e^x \left(\log x + \frac{1}{x}\right) dx$$

$$\sin y = e^x \log x + c$$

Since, $\int \left(f(x) + f'(x)\right) e^x dx = e^x f(x) + c$

Differential Equations Ex 22.7 Q8

$$\frac{dy}{dx} = e^{x+y} + x^2 e^y$$

$$= e^x e^y + x^2 e^y$$

$$\frac{dy}{dx} = e^y \left(e^x + x^2\right)$$

$$\int e^{-y} dy = \int \left(e^x + x^2\right) dx$$

$$-e^{-y} = e^x + \frac{x^3}{3} + c$$

Differential Equations Ex 22.7 Q9

$$x \frac{dy}{dx} + y = y^2$$

$$x \frac{dy}{dx} = \left(y^2 - y\right)$$

$$\frac{1}{y^2 - y} dy = \frac{dx}{x}$$

$$\int \left(\frac{1}{y-1} - \frac{1}{y}\right) dy = \int \frac{dx}{x}$$

$$\log|y-1| - \log|y| = \log|x| + \log|c|$$

$$\log\left|\frac{y-1}{y}\right| = |xc|$$

$$y - 1 = xyc$$

Differential Equations Ex 22.7 Q10

$$\left(e^y + 1\right) \cos x dx + e^y \sin x dy = 0$$

$$\left(e^y + 1\right) \cos x dx = -e^y \sin x dy$$

$$\int \frac{\cos x}{\sin x} dx = -\int \frac{e^y}{e^y + 1} dy$$

$$\int \cot x dx = -\int \frac{e^y}{e^y + 1} dy$$

$$\log|\sin x| = -\log\left|e^y + 1\right| + \log|c|$$

$$\sin x = \frac{c}{e^y + 1}$$

$$\sin x \left(e^y + 1\right) = c$$

Differential Equations Ex 22.7 Q11

$$x\cos^2 y\,dx = y\cos^2 x\,dy$$
$$\frac{x}{\cos^2 x}dx = \frac{y}{\cos^2 y}dy$$
$$\int x\sec^2 x\,dx = \int y\sec^2 y\,dy$$
$$x\times\int\sec^2 x - \int\left(1\times\int\sec^2 x\,dx\right)dx = y\int\sec^2 y\,dy - \int\left(1\times\int\sec^2 y\,dy\right)dy$$
$$x\tan x - \int\tan x\,dx = y\tan y - \int\tan y\,dy + c$$
$$x\tan x - \log|\sec x| = y\tan y - \log|\sec y| + c$$

Differential Equations Ex 22.7 Q12

$$xy\,dy = (y-1)(x+1)dx$$
$$\frac{y}{y-1}dy = \frac{x+1}{x}dx$$
$$\int\left(1+\frac{1}{y-1}\right)dy = \int\left(1+\frac{1}{x}\right)dx$$
$$y + \log|y-1| = x + \log|x| + c$$
$$y - x = \log|x| - \log|y-1| + c$$

Differential Equations Ex 22.7 Q13

$$x\frac{dy}{dx} + \cot y = 0$$
$$x\frac{dy}{dx} = -\cot y$$
$$\int\tan y\,dy = -\int\frac{dx}{x}$$
$$\log|\sec y| = -\log|x| + \log|c|$$
$$\sec y = \frac{c}{x}$$
$$x\sec y = c$$
$$x = c\cos y$$

Differential Equations Ex 22.7 Q14

$$\frac{dy}{dx} = \frac{xe^x\log x + e^x}{x\cos y}$$
$$\int\cos y\,dy = \int e^x\left(\log x + \frac{1}{x}\right)dx$$
$$\sin y = e^x\log x + c$$

Since, $\int e^x\left(f(x) + f'(x)\right)dx = e^x f(x) + c$

Differential Equations Ex 22.7 Q15

$$\frac{dy}{dx} = e^{x+y} + e^y x^3$$
$$\frac{dy}{dx} = e^y\left(e^x + x^3\right)$$
$$\int e^{-y}dy = \int\left(e^x + x^3\right)dx$$
$$-e^{-y} = e^x + \frac{x^4}{4} + c_1$$
$$e^x + \frac{x^4}{4} + e^{-y} = c$$

Differential Equations Ex 22.7 Q16

$$y\sqrt{1+x^2} + x\sqrt{1+y^2}\frac{dy}{dx} = 0$$
$$x\sqrt{1+y^2}\frac{dy}{dx} = -y\sqrt{1+x^2}$$
$$\int\frac{\sqrt{1+y^2}}{y}dy = -\int\frac{\sqrt{1+x^2}}{x}dx$$
$$\int\frac{y\sqrt{1+y^2}}{y^2}dy = -\int\frac{x\sqrt{1+x^2}}{x^2}dx$$

Let $1+y^2 = t^2$

$\Rightarrow$ $2y\,dy = 2t\,dt$

$1+x^2 = v^2$

$\Rightarrow$ $2x\,dx = 2v\,dv$

$$\int\frac{t\times t\,dt}{t^2-1} = -\int\frac{v\times v\,dv}{v^2-1}$$
$$\int\frac{t^2dt}{t^2-1} = -\int\frac{v^2dv}{v^2-1}$$
$$\int\left(1+\frac{1}{t^2-1}\right)dt = \int\left(1+\frac{1}{v^2-1}\right)dv$$
$$t + \frac{1}{2}\log\left|\frac{t-1}{t+1}\right| = -v - \log\left|\frac{v-1}{v+1}\right| + c$$
$$\sqrt{1+y^2} + \frac{1}{2}\log\left|\frac{\sqrt{y^2+1}-1}{\sqrt{y^2+1}+1}\right| = -\sqrt{1+x^2} - \frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right| + c$$
$$\sqrt{1+y^2} + \sqrt{1+x^2} + \frac{1}{2}\log\left|\frac{\sqrt{y^2+1}-1}{\sqrt{y^2+1}+1}\right| + \frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right| = c$$

Differential Equations Ex 22.7 Q17

$$\sqrt{1+x^2}dy + \sqrt{1+y^2}dx = 0$$
$$\sqrt{1+x^2}dy = -\sqrt{1+y^2}dx$$
$$\int \frac{dy}{\sqrt{1+y^2}} = -\int \frac{dx}{\sqrt{1+x^2}}$$
$$log\left|y+\sqrt{1+y^2}\right| = -log\left|x+\sqrt{1+x^2}\right| = log\,|c|$$
$$\left(y+\sqrt{1+y^2}\right)\left(x+\sqrt{1+x^2}\right) = c$$

Differential Equations Ex 22.7 Q18

$$\sqrt{1+x^2+y^2+x^2y^2} + xy\frac{dy}{dx} = 0$$
$$\sqrt{\left(1+x^2\right)+y^2\left(1+x^2\right)} = -xy\frac{dy}{dx}$$
$$\sqrt{\left(1+x^2\right)\left(1+y^2\right)} = -xy\frac{dy}{dx}$$
$$\frac{ydy}{\sqrt{1+y^2}} = -\frac{\sqrt{1+x^2}}{x}dx$$
$$\int \frac{ydy}{\sqrt{1+y^2}} = -\int \frac{x\sqrt{1+x^2}}{x^2}dx$$

Let $1+y^2 = t^2$

$\Rightarrow$ $2ydy = 2tdt$

$1+x^2 = v^2$

$\Rightarrow$ $2xdx = 2vdv$

$$\int \frac{tdt}{t} = -\int \frac{v \times vdv}{v^2-1}$$
$$\int dt = -\int \frac{v^2}{v^2-1}dv$$
$$-\int dt = \int\left(1+\frac{1}{v^2-1}\right)dv$$
$$-t = v + \frac{1}{2}\log\left|\frac{v-1}{v+1}\right| + c_1$$
$$-\sqrt{1+y^2} = \sqrt{1+x^2} + \frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right| + c_1$$
$$\sqrt{1+x^2} + \sqrt{1+y^2} + \frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right| = c$$

Differential Equations Ex 22.7 Q19

$$\frac{dy}{dx} = \frac{e^x\left(\sin^2 x + \sin x 2x\right)}{y\left(2\log y + 1\right)}$$
$$y\left(2\log y + 1\right)dy = e^x\left(\sin^2 x + \sin 2x\right)dx$$
$$\int\left(2y\log y + y\right)dy = \int e^x\left(\sin^2 x + \sin 2x\right)dx$$
$$2\left[\log y \times \int ydy - \int\left(\frac{1}{2}\int ydy\right)dy\right] + \frac{y^2}{2} = e^x \sin^2 x + c$$

Using integration by parts and

$$\int\left(f(x)+f'(x)\right)e^x dxdy + \frac{y^2}{2} = e^x \sin^2 x + c$$
$$y^2\log y - \frac{y^2}{2} + \frac{y^2}{2} = e^x \sin^2 x + c$$
$$y^2\log y = e^x \sin^2 x + c$$

Differential Equations Ex 22.7 Q20

$$\frac{dy}{dx} = \frac{x\left(2\log x + 1\right)}{\sin y + y\cos y}$$
$$\int\left(\sin y + y\cos y\right)dy = \int\left(2x\log x + x\right)dx$$
$$\int \sin ydy + \int y\cos ydy = 2\int x\log xdx + \int xdx$$
$$\int \sin ydy + \left\{y \times \left(\int \cos ydy\right) - \int\left(1 \times \int \cos ydy\right)dy\right\} = 2\left\{\log x\int xdx - \int\left(\frac{1}{x}\int xdx\right)dx\right\} + \int xdx + c$$
$$\int \sin ydy + y\sin y - \int \sin ydy = x^2\log x - 2\int\frac{x}{2}dx + \int xdx + c$$
$$y\sin y = x^2\log x + c$$

Differential Equations Ex 22.7 Q21

$$\left(1-x^2\right)dy + xydx = xy^2dx$$
$$\left(1-x^2\right)dy = dx\left(xy^2 - xy\right)$$
$$\left(1-x^2\right)dy = xy\left(y-1\right)dx$$
$$\int\frac{dy}{y\left(y-1\right)} = \int\frac{xdx}{1-x^2}$$
$$\int\left(\frac{1}{y-1}-\frac{1}{y}\right)dy = \frac{1}{2}\int\frac{2x}{1-x^2}dx$$
$$\int\left(\frac{1}{y-1}-\frac{1}{y}\right)dy = -\frac{1}{2}\int\frac{-2x}{1-x^2}dx$$
$$\log\left|y-1\right| - \log\left|y\right| = -\frac{1}{2}\log\left|1-x^2\right| + c$$

Differential Equations Ex 22.7 Q22

$$\tan ydx + \sec^2 y\tan xdy = 0$$
$$\tan ydx = -\sec^2 y\tan xdy$$
$$-\frac{dx}{\tan x} = \frac{\sec^2 ydy}{\tan y}$$
$$-\int\cot xdx = \int\frac{\sec^2 ydy}{\tan y}$$
$$-\log\left|\sin x\right| = \log\left|\tan y\right| + \log\left|c\right|$$
$$\frac{1}{\sin x} = c\tan y$$
$$\sin x\tan y = c_1$$

Differential Equations Ex 22.7 Q23

$$\left(1+x\right)\left(1+y^2\right)dx + \left(1+y\right)\left(1+x^2\right)dy = 0$$
$$\left(1+x\right)\left(1+y^2\right)dx = -\left(1+y\right)\left(1+x^2\right)dy$$
$$\frac{\left(1+y\right)dy}{\left(1+y^2\right)} = -\frac{\left(1+x\right)}{\left(1+x^2\right)}dx$$
$$\int\left(\frac{1}{1+y^2}+\frac{y}{1+y^2}\right)dy = -\int\left[\frac{1}{1+x^2}+\frac{x}{1+x^2}\right]dx$$
$$\int\frac{1}{1+y^2}dy + \frac{1}{2}\int\frac{2y}{1+y^2}dy = -\int\frac{1}{1+x^2}dx - \frac{1}{2}\int\frac{2x}{1+x^2}dx$$
$$\tan^{-1}\left(y\right) + \frac{1}{2}\log\left|1+y^2\right| = -\tan^{-1}x - \frac{1}{2}\log\left|1+x^2\right| + c$$
$$\tan^{-1}x + \tan^{-1}y + \frac{1}{2}\log\left|\left(1+y^2\right)\left(1+x^2\right)\right| = c$$

Differential Equations Ex 22.7 Q24

$$\tan y\frac{dy}{dx} = \sin\left(x+y\right) + \sin\left(x-y\right)$$
$$\tan y\frac{dy}{dx} = 2\sin\left\{\frac{\left(x+y\right)+\left(x-y\right)}{2}\right\}\cos\left\{\frac{\left(x+y\right)-\left(x-y\right)}{2}\right\}$$
$$= 2\sin\left(\frac{x+y+x-y}{2}\right)\cos\left(\frac{x+y-x+y}{2}\right)$$
$$\tan y\frac{dy}{dx} = 2\sin x\cos y$$
$$\frac{\tan y}{\cos y}dy = 2\sin xdx$$
$$\int\sec y\tan ydy = 2\int\sin xdx$$
$$\sec y = -2\cos x + c$$
$$\sec y + 2\cos x = c$$

Differential Equations Ex 22.7 Q25

$$\cos x\cos y\frac{dy}{dx} = -\sin x\sin y$$
$$\frac{\cos y}{\sin y}dy = -\frac{\sin x}{\cos x}dx$$
$$\int\cot ydy = -\int\tan xdx$$
$$\log\sin y = \log\cos x + \log c$$
$$\sin y = c\cos x$$

Differential Equations Ex 22.7 Q26

$$\frac{dy}{dx}+\frac{\cos x\sin y}{\cos y}=0$$
$$\frac{dy}{dx}=-\cos x\tan y$$
$$\frac{dy}{\tan y}=-\cos x dx$$
$$\int\cot y dy=-\int\cos x dx$$
$$\log|\sin y|=-\sin x+c$$
$$\sin x+\log|\sin y|=c$$

Differential Equations Ex 22.7 Q27

$$x\sqrt{1-y^2}dx+y\sqrt{1-x^2}dy=0$$
$$x\sqrt{1-y^2}dx=-y\sqrt{1-x^2}dy$$
$$\frac{ydy}{\sqrt{1-y^2}}=-\frac{xdx}{\sqrt{1-x^2}}$$
$$\frac{1}{-2}\int\frac{-2y}{\sqrt{1-y^2}}dy=\frac{1}{2}\int\frac{-2x}{\sqrt{1-x^2}}dx$$
$$-\frac{1}{2}2\times\sqrt{1-y^2}=\frac{1}{2}\times 2\sqrt{1-x^2}+c_1$$
$$\sqrt{1-y^2}+\sqrt{1-x^2}=c$$

Differential Equations Ex 22.7 Q28

$$y\left(1+e^x\right)dy=(y+1)e^x dx$$
$$\frac{ydy}{y+1}=\frac{e^x dx}{1+e^x}$$
$$\int\left(1-\frac{1}{y+1}\right)dy=\int\left(\frac{e^x}{1+e^x}\right)dx$$
$$y-\log|y+1|=\log\left|1+e^x\right|+c$$

Differential Equations Ex 22.7 Q29

$$(y+xy)dx+\left(x-xy^2\right)dy=0$$
$$y(1+x)dx=\left(xy^2-x\right)dy$$
$$y(1+x)dx=x\left(y^2-1\right)dy$$
$$\frac{\left(y^2-1\right)dy}{y}=\frac{1+x}{x}dx$$
$$\int\left(y-\frac{1}{y}\right)dy=\int\left(\frac{1}{x}+1\right)dx$$
$$\frac{y^2}{2}-\log|y|=\log|x|+x+c_1$$
$$\frac{y^2}{2}-x-\log|y|-\log|x|=c_1$$
$$\log|x|+x+\log|y|-\frac{y^2}{2}=c$$

Differential Equations Ex 22.7 Q30

$$\frac{dy}{dx}=1-x+y-xy$$
$$=(1-x)+y(1-x)$$
$$\frac{dy}{dx}=(1-x)(1+y)$$
$$\int\frac{dy}{1+y}=\int(1-x)dx$$
$$\log|y+1|=x-\frac{x^2}{2}+c$$

Differential Equations Ex 22.7 Q31

$$\left(y^2+1\right)dx-\left(x^2+1\right)dy=0$$
$$\left(y^2+1\right)dx=\left(x^2+1\right)dy$$
$$\int\frac{dy}{y^2+1}=\int\frac{dx}{x^2+1}$$
$$\tan^{-1}y=\tan^{-1}x+c$$

Differential Equations Ex 22.7 Q32

$$dy+(x+1)(y+1)dx=0$$
$$dy=-(x+1)(y+1)dx$$
$$\int\frac{dy}{y+1}=-\int(x+1)dx$$
$$\log|y+1|=-\frac{x^2}{2}-x+c$$
$$\log|y+1|+\frac{x^2}{2}+x=c$$

Differential Equations Ex 22.7 Q33

$$\frac{dy}{dx}=\left(1+x^2\right)\left(1+y^2\right)$$
$$\int\frac{dy}{1+y^2}=\int\left(1+x^2\right)dx$$
$$\tan^{-1}y=x+\frac{x^3}{3}+c$$
$$\tan^{-1}y-x-\frac{x^3}{3}=c$$

Differential Equations Ex 22.7 Q34

$$(x-1)\frac{dy}{dx}=2x^3y$$
$$\frac{dy}{y}=\frac{2x^3dx}{x-1}$$
$$\int\frac{dy}{y}=2\int\left(x^2+x+1+\frac{1}{x-1}\right)dx$$
$$\log|y|=\log e^{\left(\frac{2}{3}x^3+x^2+2x\right)}+\log|x-1|^2+\log|c|$$
$$y=c|x-1|^2e^{\left(\frac{2}{3}x^3+x^2+2x\right)}$$

Differential Equations Ex 22.7 Q35

$$\frac{dy}{dx}=e^{x+y}+e^{-x+y}$$
$$=e^x\times e^y+e^{-x}\times e^y$$
$$\frac{dy}{dx}=e^y\left(e^x+e^{-x}\right)$$
$$\frac{dy}{e^y}=\left(e^x+e^{-x}\right)dx$$
$$\int e^{-y}dy=\int\left(e^x+e^{-x}\right)dx$$
$$-e^{-y}=e^x-e^{-x}+c$$
$$e^{-x}-e^{-y}=e^x+c$$

Differential Equations Ex 22.7 Q36

$$\frac{dy}{dx} = \left(\cos^2 x - \sin^2 x\right)\cos^2 y$$

$$\frac{dy}{\cos^2 y} = \left(\cos^2 x - \sin^2 x\right)dx$$

$$\int \sec^2 y dy = \int \cos 2x dx$$

$$\tan y = \frac{\sin 2x}{2} + c$$

Differential Equations Ex 22.7 Q37(i)

$$\left(xy^2 + 2x\right)dx + \left(x^2 y + 2y\right)dy = 0$$

$$\left(x^2 y + 2y\right)dy = -\left(xy^2 + 2x\right)dx$$

$$y\left(x^2 + 2\right)dy = -x\left(y^2 + 2\right)dx$$

$$\frac{y}{y^2 + 2}dy = -\frac{x}{x^2 + 2}dx$$

$$\int \frac{2y}{y^2 + 2}dy = -\int \frac{2x}{x^2 + 2}dx$$

$$\log\left|y^2 + 2\right| = -\log\left|x^2 + 2\right| + \log|c|$$

$$\left|y^2 + 2\right| = \left|\frac{c}{x^2 + 2}\right|$$

$$y^2 + 2 = \frac{c}{x^2 + 2}$$

Differential Equations Ex 22.7 Q37(ii)

Consider the given equation

$$\operatorname{cosec} x \log y \frac{dy}{dx} + x^2 y^2 = 0$$

$$\Rightarrow \frac{\log y dy}{y^2} = \frac{-x^2 dx}{\operatorname{cosec} x}$$

$$\Rightarrow -\frac{\log y dy}{y^2} = x^2 \sin x dx$$

Integrating on both the sides,

$$\Rightarrow -\int \frac{\log y dy}{y^2} = \int x^2 \sin x dx$$

Using integration by parts on both sides

$$\Rightarrow \frac{\log y + 1}{y} = -x^2 \cos x + 2\left(x \sin x + \cos x\right) + C$$

$$\Rightarrow \frac{\log y + 1}{y} + x^2 \cos x - 2\left(x \sin x + \cos x\right) = C$$

Differential Equations Ex 22.7 Q38(i)

$$xy\frac{dy}{dx} = 1 + x + y + xy$$

$$= (1 + x) + y(1 + x)$$

$$xy\frac{dy}{dx} = (1 + x)(1 + y)$$

$$\int \frac{y dy}{y + 1} = \int \frac{1 + x}{x}dx$$

$$\int \left(1 - \frac{1}{y + 1}\right)dy = \int \left(\frac{1}{x} + 1\right)dx$$

$$y - \log|y + 1| = \log|x| + x + \log|c|$$

$$y = \log|cx(y + 1)| + x$$

Differential Equations Ex 22.7 Q38(ii)

$$y\left(1-x^2\right)\frac{dy}{dx}=x\left(1+y^2\right)$$

$$\frac{ydy}{\left(1+y^2\right)}=\frac{xdx}{1-x^2}$$

$$-\int\frac{2ydy}{\left(1+y^2\right)}=\int\frac{-2x}{\left(1-x^2\right)}dx$$

$$-\log\left|1+y^2\right|=\log\left|1-x^2\right|+\log\left|c_1\right|$$

$$\log|c|=\log\left|1-x^2\right|+\log\left|1+y^2\right|$$

$$c=\left(1-x^2\right)\left(1+y^2\right)$$

Differential Equations Ex 22.7 Q38(iii)

$$ye^{x/y}\,dx=\left(xe^{x/y}+y^2\right)dy$$

$$ye^{x/y}\,dx-xe^{x/y}dy=y^2dy$$

$$\left(ydx-xdy\right)e^{x/y}=y^2dy$$

$$\left(\frac{ydx-xdy}{y^2}\right)e^{x/y}=dy$$

$$e^{x/y}\,d\left(\frac{x}{y}\right)=dy$$

Integrating on both the sides we get,

$e^{x/y}=y+C$, which is the required solution.

Differential Equations Ex 22.7 Q38(iv)

$$(1+y^2)\tan^{-1}x\,dx+2y\,(1+x^2)dy=0$$

$$\left(1+y^2\right)\tan^{-1}x\,dx=-2y\left(1+x^2\right)dy$$

$$-\frac{\tan^{-1}x}{2\left(1+x^2\right)}dx=\frac{y}{\left(1+y^2\right)}dy$$

Integrating on both the sides

$$\int-\frac{\tan^{-1}x}{2\left(1+x^2\right)}dx=\int\frac{y}{\left(1+y^2\right)}dy$$

$$-\left(\tan^{-1}x\left(\frac{1}{2}\tan^{-1}x\right)-\int\frac{1}{\left(1+x^2\right)}\left(\frac{1}{2}\tan^{-1}x\right)dx\right)=\frac{1}{2}\ln\left(y^2+1\right)+C$$

$$-\frac{1}{4}\left(\tan^{-1}x\right)^2=\frac{1}{2}\ln\left(y^2+1\right)+C_1$$

$$\frac{1}{2}\left(\tan^{-1}x\right)^2+\ln\left(y^2+1\right)=C$$

Differential Equations Ex 22.7 Q39

$$\frac{dy}{dx}=y\tan 2x,\ y(0)=2$$

$$\int\frac{dy}{y}=\int\tan 2x\,dx$$

$$\log|y|=\frac{1}{2}\log|\sec 2x|+\log c$$

$$y=\sqrt{\sec 2x}\,c \qquad \text{---(i)}$$

Put $x=0, y=2$

$$2=\sqrt{\sec 0}\times c$$

$$2=c$$

Put $c=2$ in equation (i),

$$y=2\sqrt{\sec 2x}$$

$$y=\frac{2}{\sqrt{\cos 2x}}$$

Differential Equations Ex 22.7 Q40

$$2x\frac{dy}{dx} = 3y,\ y(1) = 2$$

$$\int\frac{2dy}{y} = \int\frac{3dx}{x}$$

$$2\log|y| = 3\log|x| + \log c$$

$$y^2 = x^3c \qquad ---(i)$$

Put $x = 1,\ y = 2$

$$4 = c$$

Put $c = 4$ in equation (i),

$$y^2 = 4x^3$$

Differential Equations Ex 22.7 Q41

$$xy\frac{dy}{dx} = y + 2,\ y(2) = 0$$

$$\frac{ydy}{y+2} = \frac{dx}{x}$$

$$\int\left(1 - \frac{2}{y+2}\right)dy = \int\frac{dx}{x}$$

$$y - 2\log|y+2| = \log|x| + \log|c| \qquad ---(i)$$

Put $y = 0,\ x = 2$

$$0 - 2\log 2 = \log 2 + \log c$$

$$-3\log 2 = \log c$$

$$\log\left(\frac{1}{8}\right) = \log c$$

$$c = \frac{1}{8}$$

Put $c = \frac{1}{8}$ in equation (i),

$$y - 2\log|y+2| = \log\left|\frac{x}{8}\right|$$

Differential Equations Ex 22.7 Q42

$$\frac{dy}{dx} = 2e^x y^3,\ y(0) = \frac{1}{2}$$

$$\int\frac{dy}{y^3} = \int 2e^x dx$$

$$-\frac{1}{2y^2} = 2e^x + c \qquad ---(i)$$

Put $x = 0,\ y = \frac{1}{2}$

$$-\frac{4}{2} = 2e^0 + c$$

$$-2 = 2 + c$$

$$c = -4$$

Put $c = -4$ in equation (i),

$$-\frac{1}{2y^2} = 2e^x - 4$$

$$-1 = 4e^x y^2 - 8y^2$$

$$-1 = -y^2\left(8 - 4e^x\right)$$

$$y^2\left(8 - 4e^x\right) = 1$$

Differential Equations Ex 22.7 Q43

$$\frac{dr}{dt} = -rt,\ r(0) = r_0$$

$$\int\frac{dr}{r} = -\int tdt$$

$$\log|r| = -\frac{t^2}{2} + c \qquad ---(i)$$

Put $t = 0,\ r = r_0$ inequation (i),

$$\log|r_0| = 0 + c$$

$$\log|r_0| = c$$

Now,

$$\log|r| = -\frac{t^2}{2} + \log|r_0|$$

$$\frac{r}{r_0} = e^{-\frac{t^2}{2}}$$

$$r = r_0 e^{-\frac{t^2}{2}}$$

Differential Equations Ex 22.7 Q44

$$\frac{dy}{dx} = y\sin 2x,\ y(0) = 1$$

$$\int \frac{dy}{y} = \int \sin 2x dx$$

$$\log|y| = -\frac{\cos 2x}{2} + c \qquad \text{---(i)}$$

Put $y = 1,\ x = 0$

$$\log|1| = -\frac{\cos 0}{2} + c$$

$$0 = -\frac{1}{2} + c$$

$$c = \frac{1}{2}$$

So,

$$\log|y| = -\frac{\cos 2x}{2} + \frac{1}{2}$$

$$= \frac{1 - \cos 2x}{2}$$

$$\log|y| = \sin^2 x$$

$$y = e^{\sin^2 x}$$

Differential Equations Ex 22.7 Q45(i)

$$\frac{dy}{dx} = y\tan x,\ y(0) = 1$$

$$\int \frac{dy}{y} = \int \tan x dx$$

$$\log|y| = \log|\sec x| + c \qquad \text{---(i)}$$

Put $y = 1,\ x = 0$

$$0 = \log(1) + c$$

$$c = 0$$

Put $c = 0$ in equation (i),

$$\log y = \log|\sec x|$$

$$y = \sec x \qquad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

Differential Equations Ex 22.7 Q45(ii)

$$2x\frac{dy}{dx} = 5y,\ y(1) = 1$$

$$\int \frac{2dy}{y} = \int \frac{5dx}{x}$$

$$2\log|y| = 5\log|x| + c \qquad \text{---(i)}$$

Put $x = 1,\ y = 1$

$$2\log(1) = 5\log(1) + c$$

$$0 = c$$

Put $c = 0$ in equation (i),

$$2\log|y| = 5\log|x|$$

$$y^2 = |x|^5$$

$$y = |x|^{\frac{5}{2}}$$

Differential Equations Ex 22.7 Q45(iii)

$$\frac{dy}{dx} = 2e^{2x}y^2,\ y(0) = -1$$
$$\int \frac{dy}{y^2} = \int 2e^{2x}dx$$
$$-\frac{1}{y} = \frac{2e^{2x}}{2} + c$$
$$-\frac{1}{y} = e^{2x} + c \qquad ---(i)$$

Put $y = -1,\ x = 0$

$$1 = e^0 + c$$
$$1 = 1 + c$$
$$c = 0$$

Put $c = 0$ in equation (i),

$$-\frac{1}{y} = e^{2x}$$
$$y = -e^{-2x}$$

Differential Equations Ex 22.7 Q45(iv)

$$\cos y\frac{dy}{dx} = e^x,\ y(0) = \frac{\pi}{2}$$
$$\int \cos y\,dy = \int e^x dx$$
$$\sin y = e^x + c \qquad ---(i)$$

Put $x = 0,\ y = \frac{\pi}{2}$

$$\sin\left(\frac{\pi}{2}\right) = e^0 + c$$
$$1 = 1 + c$$
$$c = 0$$

Put $c = 0$ in equation (i),

$$\sin y = e^x$$
$$y = \sin^{-1}\left(e^x\right)$$

Differential Equations Ex 22.7 Q45(v)

$$\frac{dy}{dx} = 2xy,\ y(0) = 1$$
$$\int \frac{dy}{y} = \int 2x dx$$
$$\log|y| = 2\frac{x^2}{2} + c$$
$$\log|y| = x^2 + c \qquad ---(i)$$

Put $x = 0,\ y = 1$

$$\log(1) = 0 + c$$
$$0 = 0 + c$$
$$c = 0$$

Put $c = 0$ in equation (i),

$$\log y = x^2$$
$$y = e^{x^2}$$

Differential Equations Ex 22.7 Q45(vi)

$$\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2, y(0) = 1$$
$$= (1+x^2)(1+y^2)$$
$$\int \frac{dy}{1+y^2} = \int (1+x^2)dx$$
$$\tan^{-1} y = x + \frac{x^3}{3} + c \qquad ---(i)$$

Put $x = 0, y = 1$

$$\tan^{-1} y = x + \frac{x^3}{3} + c$$
$$c = \frac{\pi}{4}$$

Put $c = \frac{\pi}{4}$ in equation (i)

$$\tan^{-1} y = x + \frac{x^3}{3} + \frac{\pi}{4}$$

Differential Equations Ex 22.7 Q45(vii)

$$xy\frac{dy}{dx} = (x+2)(y+2),\ y(1) = -1$$
$$\frac{ydy}{(y+2)} = \frac{(x+2)}{x}dx$$
$$\int\left(1-\frac{2}{y+2}\right)dy = \int\left(1+\frac{2}{x}\right)dx$$
$$y - x - 2\log(y+2) - 2\log x = c$$
$$\text{Put } x = 1,\ y = -1$$
$$-1-1-2\log(-1+2) - 2\log 1 = c$$
$$\Rightarrow -2 = c$$
$$\text{Thus, we have}$$
$$y - x - 2\log(y+2) - 2\log x = -2$$

Differential Equations Ex 22.7 Q45(viii)

$$\frac{dy}{dx} = 1 + x + y^2 + xy^2$$
$$\frac{dy}{dx} = (1+x)(1+y^2)$$
$$\frac{1}{(1+y^2)}dy = (1+x)dx$$

Integrating on both the sides we get

$$\int\frac{1}{(1+y^2)}dy = \int(1+x)dx$$
$$\tan^{-1}y = x + \frac{x^2}{2} + C \ldots(i)$$

Put $y = 0,\ x = 0$ then

$$\tan^{-1}0 = 0 + 0 + C$$
$$C = 0$$

From(i) we have

$$\tan^{-1}y = x + \frac{x^2}{2}$$
$$y = \tan\left(x + \frac{x^2}{2}\right)$$

Differential Equations Ex 22.7 Q45(ix)

$$2(y+3) - xy\frac{dy}{dx} = 0$$
$$2(y+3) = xy\frac{dy}{dx}$$
$$\frac{2}{x}dx = \frac{y}{y+3}dy$$

Integrating on both the sides we get

$$\int\frac{2}{x}dx = \int\frac{y}{y+3}dy$$
$$2\ln|x| = y + 3 - 3\ln|y+3| + C \ldots..(i)$$

Put $x = 1$ and $y = -2$ in eq (i)

$$2\ln|1| = -2 + 3 - 3\ln|-2+3| + C$$
$$0 = 1 - 0 + C$$
$$C = -1$$

From eq (i) we have

$$2\ln|x| = y + 3 - 3\ln|y+3| - 1$$
$$\ln(|x|)^2 = y + 2 - \ln(|y+3|)^3$$
$$\ln(|x|)^2 + -\ln(|y+3|)^3 = y + 2$$
$$x^2(y+3)^3 = e^{y+2}$$

Differential Equations Ex 22.7 Q46

$$x\frac{dy}{dx}+\cot y = 0,\ y=\frac{\pi}{4}\ \text{at}\ x=\sqrt{2}$$
$$x\frac{dy}{dx} = -\cot y$$
$$\frac{dy}{\cot y} = -\frac{dx}{x}$$
$$\int \tan y\,dy = -\int\frac{dx}{x}+c$$
$$\log|\sec y| = -\log|x|+c \qquad \text{---(i)}$$

Put $x=\sqrt{2},\ y=\frac{\pi}{4}$

$$\log\left|\sec\frac{\pi}{4}\right| = -\log\left|\sqrt{2}\right|+c$$
$$\log\left|\sqrt{2}\right| = -\frac{1}{2}\log 2+c$$
$$\frac{1}{2}\log 2 = -\frac{1}{2}\log 2+c$$
$$\log 2 = c$$

Put c in equation (i),

$$\log|\sec y| = -\log|x|+\log 2$$
$$\sec y = \frac{2}{x}$$
$$x = \frac{2}{\sec y}$$
$$x = 2\cos y$$

Differential Equations Ex 22.7 Q47

$$\left(1+x^2\right)\frac{dy}{dx}+\left(1+y^2\right) = 0,\ y=1\ \text{at}\ x=0$$
$$\left(1+x^2\right)\frac{dy}{dx} = -\left(1+y^2\right)$$
$$\int\frac{dy}{\left(1+y^2\right)} = -\int\frac{dx}{1+x^2}$$
$$\tan^{-1}y = -\tan^{-1}x+c \qquad \text{---(i)}$$

Put $x=0,\ y=1$

$$\tan^{-1}(1) = -\tan^{-1}0+c$$

Put c in equation (1),

$$\tan^{-1}y = -\tan^{-1}x+\frac{\pi}{4}$$
$$\tan^{-1}y = \left(\frac{\pi}{4}-\tan^{-1}x\right)$$
$$y = \tan\left(\frac{\pi}{4}-\tan^{-1}x\right)$$
$$y = \frac{\tan\frac{\pi}{4}-\tan\left(\tan^{-1}x\right)}{1+\tan\frac{\pi}{4}\tan\left(\tan^{-1}x\right)}$$
$$y = \frac{1-x}{1+x}$$
$$y+xy = 1-x$$
$$x+y = 1-xy$$

Differential Equations Ex 22.7 Q48

$$\frac{dy}{dx} = \frac{2x(\log x + 1)}{\sin y + y\cos y}, \; y = 0 \text{ at } x = 1$$

$$\int(\sin y + y\cos y)dy = \int 2x(\log x + 1)dx$$

$$\Rightarrow \quad \int \sin y dy + \int y\cos y dy = \int 2x\log x dx + 2\int x dx$$

$$\Rightarrow \quad -\cos y + \left[y \times \int \cos y dy - \int(1 \times \int \cos y dy)dy\right] = 2\left[\log x \int x dx - \int\left(\frac{1}{x}\int x dx\right)dx\right] + x^2 + c$$

$$\Rightarrow \quad -\cos y + y\sin y - \int \sin y dy = 2\frac{x^2}{2}\log x - 2\int \frac{x}{2}dx + x^2 + c$$

$$\Rightarrow \quad -\cos y + y\sin y + \cos y = x^2\log x - \frac{x^2}{2} + x^2 + c$$

$$y\sin y = x^2\log x + \frac{x^2}{2} + c \qquad \text{---(i)}$$

Put $y = 0,\ x = 1$

$$0 = 0 + \frac{1}{2} + c$$

$$c = -\frac{1}{2}$$

Put $c = -\frac{1}{2}$ in equation (i),

$$y\sin y = x^2\log x + \frac{x^2}{2} - \frac{1}{2}$$

$$2y\sin y = 2x^2\log x + x^2 - 1$$

Differential Equations Ex 22.7 Q49

$$e^{\frac{dy}{dx}} = x + 1$$

$$\frac{dy}{dx} = \log(x+1), \; y = 3 \text{ at } x = 0$$

$$\int dy = \int \log(x+1)dx$$

$$y = \log|x+1| \times \int 1 \times dx - \int\left(\frac{1}{x+1} \times \int 1 dx\right)dx + c$$

Using integration by parts

$$y = x\log|x+1| - \int \frac{x}{x+1}dx + c$$

$$y = x\log|x+1| - \left(\int\left(1 - \frac{1}{x+1}\right)dx\right) + c$$

$$= x\log|x+1| - (x - \log|x+1|) + c$$

$$y = x\log|x+1| - x + \log|x+1| + c$$

$$y = (x+1)\log|x+1| - x + c \qquad \text{---(i)}$$

Put $y = 3$ and $x = 0$

$$3 = 0 - 0 + c$$

$$c = 3$$

Put $c = 3$ in equation (i),

$$y = (x+1)\log|x+1| - x + 3$$

Differential Equations Ex 22.7 Q50

$$\cos y dy + \cos x \sin y dx = 0$$

$$\cos y dy = -\cos x \sin y dx$$

$$\frac{\cos y}{\sin y}dy = -\cos x dx$$

$$\int \cot y dy = -\int \cos x dx$$

$$\log|\sin y| = -\sin x + c \qquad \text{---(i)}$$

Put $y = \frac{\pi}{2}$ and $x = \frac{\pi}{2}$

$$\log\left|\sin\frac{\pi}{2}\right| = -\sin\frac{\pi}{2} + c$$

$$0 = -1 + c$$

$$c = 1$$

Put $c = 1$ in equation (1),

$$\log|\sin y| = 1 - \sin x$$

$$\log|\sin y| + \sin x = 1$$

Differential Equations Ex 22.7 Q51

$$\frac{dy}{dx} = -4xy^2,\ y = 1 \text{ when } x = 0$$

$$\int \frac{dy}{y^2} = -4\int x dx$$

$$-\frac{1}{y} = -4\frac{x^2}{2} + c \qquad \text{---(i)}$$

Put $y = 1$ and $x = 0$

$$-1 = 0 + c$$

$$c = -1$$

Plut $c = -1$ in equation (i),

$$-\frac{1}{y} = -2x^2 - 1$$

$$\frac{1}{y} = 2x^2 + 1$$

$$y = \frac{1}{2x^2 + 1}$$

Differential Equations Ex 22.7 Q52

The differential equation of the curve is:

$$y' = e^x \sin x$$

$$\Rightarrow \frac{dy}{dx} = e^x \sin x$$

$$\Rightarrow dy = e^x \sin x$$

Integrating both sides, we get:

$$\int dy = \int e^x \sin x \, dx \qquad \ldots(1)$$

Let $I = \int e^x \sin x \, dx.$

$$\Rightarrow I = \sin x \int e^x dx - \int\left(\frac{d}{dx}(\sin x) \cdot \int e^x dx\right) dx$$

$$\Rightarrow I = \sin x \cdot e^x - \int \cos x \cdot e^x dx$$

$$\Rightarrow I = \sin x \cdot e^x - \left[\cos x \cdot \int e^x dx - \int\left(\frac{d}{dx}(\cos x) \cdot \int e^x dx\right) dx\right]$$

$$\Rightarrow I = \sin x \cdot e^x - \left[\cos x \cdot e^x - \int(-\sin x) \cdot e^x dx\right]$$

$$\Rightarrow I = e^x \sin x - e^x \cos x - I$$

$$\Rightarrow 2I = e^x(\sin x - \cos x)$$

$$\Rightarrow I = \frac{e^x(\sin x - \cos x)}{2}$$

Differential Equations Ex 22.7 Q53

The differential equation of the given curve is:

$$xy\frac{dy}{dx}=(x+2)(y+2)$$
$$\Rightarrow\left(\frac{y}{y+2}\right)dy=\left(\frac{x+2}{x}\right)dx$$
$$\Rightarrow\left(1-\frac{2}{y+2}\right)dy=\left(1+\frac{2}{x}\right)dx$$

Integrating both sides, we get:

$$\int\left(1-\frac{2}{y+2}\right)dy=\int\left(1+\frac{2}{x}\right)dx$$
$$\Rightarrow\int dy-2\int\frac{1}{y+2}dy=\int dx+2\int\frac{1}{x}dx$$
$$\Rightarrow y-2\log(y+2)=x+2\log x+C$$
$$\Rightarrow y-x-C=\log x^2+\log(y+2)^2$$
$$\Rightarrow y-x-C=\log\left[x^2(y+2)^2\right] \qquad ...(1)$$

Differential Equations Ex 22.7 Q54

Let the rate of change of the volume of the balloon be k (where k is a constant)

$$\Rightarrow\frac{dv}{dt}=k$$
$$\Rightarrow\frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)=k \qquad \left[\text{Volume of sphere}=\frac{4}{3}\pi r^3\right]$$
$$\Rightarrow\frac{4}{3}\pi\cdot 3r^2\cdot\frac{dr}{dt}=k$$
$$\Rightarrow 4\pi r^2\,dr=k\,dt$$

Integrating both sides, we get:

$$4\pi\int r^2dr=k\int dt$$
$$\Rightarrow 4\pi\cdot\frac{r^3}{3}=kt+C$$
$$\Rightarrow 4\pi r^3=3(kt+C) \qquad ...(1)$$

Now, at $t=0$, $r=3$:

$4\pi\times 3^3=3\ (k\times 0+C)$

$108\pi=3C$

$C=36\pi$

At $t=3$, $r=6$:

$4\pi\times 6^3=3\ (k\times 3+C)$

$864\pi=3\ (3k+36\pi)$

$3k=-288\pi-36\pi=252\pi$

$k=84\pi$

Substituting the values of k and C in equation (1), we get:

$$4\pi r^3=3\left[84\pi t+36\pi\right]$$
$$\Rightarrow 4\pi r^3=4\pi(63t+27)$$
$$\Rightarrow r^3=63t+27$$
$$\Rightarrow r=(63t+27)^{\frac{1}{3}}$$

Thus, the radius of the balloon after t seconds is $(63t+27)^{\frac{1}{3}}$.

Differential Equations Ex 22.7 Q55

Let p, t, and r represent the principal, time, and rate of interest respectively.

It is given that the principal increases continuously at the rate of $r\%$ per year.

$$\Rightarrow \frac{dp}{dt}=\left(\frac{r}{100}\right)p$$
$$\Rightarrow \frac{dp}{p}=\left(\frac{r}{100}\right)dt$$

Integrating both sides, we get:

$$\int\frac{dp}{p}=\frac{r}{100}\int dt$$
$$\Rightarrow \log p=\frac{rt}{100}+k$$
$$\Rightarrow p=e^{\frac{rt}{100}+k} \qquad ...(1)$$

It is given that when $t=0$, $p=100$.

$\Rightarrow 100=e^{k}$ ... (2)

Now, if $t=10$, then $p=2\times 100=200$.

$$200=e^{\frac{r}{10}+k}$$
$$\Rightarrow 200=e^{\frac{r}{10}}\cdot e^{k}$$
$$\Rightarrow 200=e^{\frac{r}{10}}\cdot 100 \qquad \text{(From (2))}$$
$$\Rightarrow e^{\frac{r}{10}}=2$$
$$\Rightarrow \frac{r}{10}=\log_e 2$$
$$\Rightarrow \frac{r}{10}=0.6931$$
$$\Rightarrow r=6.931$$

Hence, the value of r is 6.93%.

Differential Equations Ex 22.7 Q56

Let p and t be the principal and time respectively.

It is given that the principal increases continuously at the rate of 5% per year.

$$\Rightarrow \frac{dp}{dt}=\left(\frac{5}{100}\right)p$$
$$\Rightarrow \frac{dp}{dt}=\frac{p}{20}$$
$$\Rightarrow \frac{dp}{p}=\frac{dt}{20}$$

Integrating both sides, we get:

$$\int\frac{dp}{p}=\frac{1}{20}\int dt$$
$$\Rightarrow \log p=\frac{t}{20}+C$$
$$\Rightarrow p=e^{\frac{t}{20}+C} \qquad ...(1)$$

Now, when $t=0$, $p=1000$.

$1000=e^{C}$ ... (2)

Differential Equations Ex 22.7 Q57

Let y be the number of bacteria at any instant t.

It is given that the rate of growth of the bacteria is proportional to the number present.

$$\therefore \frac{dy}{dt} \propto y$$
$$\Rightarrow \frac{dy}{dt} = ky \text{ (where } k \text{ is a constant)}$$
$$\Rightarrow \frac{dy}{y} = kdt$$

Integrating both sides, we get:

$$\int \frac{dy}{y} = k\int dt$$
$$\Rightarrow \log y = kt + C \qquad ...(1)$$

Let y_0 be the number of bacteria at $t = 0$.

$\log y_0 = C$

Substituting the value of C in equation (1), we get:

$$\log y = kt + \log y_0$$
$$\Rightarrow \log y - \log y_0 = kt$$
$$\Rightarrow \log\left(\frac{y}{y_0}\right) = kt$$
$$\Rightarrow kt = \log\left(\frac{y}{y_0}\right) \qquad ...(2)$$

Also, it is given that the number of bacteria increases by 10% in 2 hours.

$$\Rightarrow y = \frac{110}{100} y_0$$
$$\Rightarrow \frac{y}{y_0} = \frac{11}{10} \qquad ...(3)$$

Substituting this value in equation (2), we get:

$$k \cdot 2 = \log\left(\frac{11}{10}\right)$$
$$\Rightarrow k = \frac{1}{2}\log\left(\frac{11}{10}\right)$$

Therefore, equation (2) becomes:

$$\frac{1}{2}\log\left(\frac{11}{10}\right) \cdot t = \log\left(\frac{y}{y_0}\right)$$
$$\Rightarrow t = \frac{2\log\left(\frac{y}{y_0}\right)}{\log\left(\frac{11}{10}\right)} \qquad ...(4)$$

Now, let the time when the number of bacteria increases from 100000 to 200000 be t_1.

$y = 2y_0$ at $t = t_1$

From equation (4), we get:

$$t_1 = \frac{2\log\left(\frac{y}{y_0}\right)}{\log\left(\frac{11}{10}\right)} = \frac{2\log 2}{\log\left(\frac{11}{10}\right)}$$

Hence, in $\frac{2\log 2}{\log\left(\frac{11}{10}\right)}$ hours the number of bacteria increases from 100000 to 200000.

Differential Equations Ex 22.7 Q58

Consider the given equation

$$\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx} = -\cos x$$

$$\Rightarrow \frac{dy}{(1+y)} = \frac{-\cos x dx}{(2+\sin x)}$$

Integrating both the sides,

$$\Rightarrow \int \frac{dy}{(1+y)} = \int \frac{-\cos x dx}{(2+\sin x)}$$

$$\Rightarrow \log(1+y) = -\log(2+\sin x) + \log\ C$$

$$\Rightarrow \log(1+y) + \log(2+\sin x) = \log\ C$$

$$\Rightarrow \log(1+y)(2+\sin x) = \log\ C$$

$$\Rightarrow (1+y)(2+\sin x) = C \ldots (1)$$

Given that $y(0) = 1$

$$\Rightarrow (1+1)(2+\sin 0) = C$$

$$\Rightarrow C = 4$$

Substituting the value of C in equation (1), we have,

$$\Rightarrow (1+y)(2+\sin x) = 4$$

$$\Rightarrow (1+y) = \frac{4}{(2+\sin x)}$$

$$\Rightarrow y = \frac{4}{(2+\sin x)} - 1 \ldots (2)$$

We need to find the value of $y\left(\frac{\pi}{2}\right)$

Substituting the value of $x=\frac{\pi}{2}$ in equation (2), we get,

$$y = \frac{4}{\left(2+\sin\ \frac{\pi}{2}\right)} - 1$$

$$\Rightarrow y = \frac{4}{(2+1)} - 1$$

$$\Rightarrow y = \frac{4}{3} - 1$$

$$\Rightarrow y = \frac{1}{3}$$

Ex 22.8

Differential Equations Ex 22.8 Q1

$$\frac{dy}{dx} = (x+y+1)^2$$

Let $x+y+1 = v$

$$1+\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{dv}{dx} - 1$$

So,

$$\frac{dv}{dx} - 1 = v^2$$

$$\frac{dv}{dx} = v^2 + 1$$

$$\int \frac{1}{v^2+1} = \int dx$$

$$\tan^{-1}(v) = x + c$$

$$\tan^{-1}(x+y+1) = x + c$$

Differential Equations Ex 22.8 Q2

$$\frac{dy}{dx} \times \cos(x-y) = 1$$

Let $x - y = v$

$$1 - \frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = 1 - \frac{dv}{dx}$$

So,

$$\left(1 - \frac{dv}{dx}\right)\cos v = 1$$

$$1 - \frac{dv}{dx} = \sec v$$

$$1 - \sec v = \frac{dv}{dx}$$

$$dx = \frac{dv}{1 - \sec v}$$

$$dx = \frac{\cos v}{1 - \cos v}dv$$

$$\int dx = \int \frac{\cos^2\frac{v}{2} - \sin^2\frac{v}{2}}{2\sin^2\frac{v}{2}}dv$$

$$\int dx = \int \frac{1}{2}\cot\left(\frac{v}{2}\right)dv - \frac{1}{2}dv$$

$$2\int dx = \int \cot^2\left(\frac{v}{2}\right)dv - \int dv$$

$$2\int dx = \int\left(\operatorname{cosec}^2\frac{v}{2} - 1\right)dv - \int dv$$

$$2x = -2\cot\left(\frac{v}{2}\right)dv - v - v + c_1$$

$$2(x+v) = -2\cot\frac{v}{2} + c_1$$

$$x + x - y = -\cot\left(\frac{x-y}{2}\right) + c$$

$$c + y = \cot\left(\frac{x-y}{2}\right)$$

Differential Equations Ex 22.8 Q3

$$\frac{dy}{dx} = \frac{(x-y)+3}{2(x-y)+5}$$

Let $x - y = v$

$$1 - \frac{dy}{dx} = \frac{dv}{dx}$$

So,

$$1 - \frac{dv}{dx} = \frac{v+3}{2v+5}$$

$$\frac{dv}{dx} = 1 - \frac{v+3}{2v+5}$$

$$= \frac{2v+5-v-3}{2v+5}$$

$$\frac{dv}{dx} = \frac{v+2}{2v+5}$$

$$\frac{2v+5}{v+2}dv = dx$$

$$\frac{(2v+4)+1}{v+2}dv = dx$$

$$\int\left(2 + \frac{1}{v+2}\right)dv = \int dx$$

$$2v + \log|v+2| = x + c$$

$$2(x-y) + \log|x-y+2| = x + c$$

Differential Equations Ex 22.8 Q4

$$\frac{dy}{dx} = (x+y)^2$$

Let $x + y = v$

$$1 + \frac{dy}{dx} = \frac{dv}{dx}$$

So,

$$\frac{dv}{dx} - 1 = v^2$$

$$\frac{dv}{dx} = 1 + v^2$$

$$\int \frac{1}{1+v^2} dv = \int dx$$

$$\tan^{-1} v = x + c$$

$$\tan^{-1}(x+y) = x + c$$

$$x + y = \tan(x+c)$$

Differential Equations Ex 22.8 Q5

$$(x+y)^2 \frac{dy}{dx} = 1$$

Let $x + y = v$

$$1 + \frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{dv}{dx} - 1$$

So,

$$v^2\left(\frac{dv}{dx} - 1\right) = 1$$

$$\frac{dv}{dx} = \frac{1}{v^2} + 1$$

$$\frac{dv}{dx} = \frac{v^2+1}{v^2}$$

$$\frac{v^2}{v^2+1} dv = dx$$

$$\int \frac{v^2+1-1}{v^2+1} dv = \int dx$$

$$\int \left(1 - \frac{1}{v^2+1}\right) dv = \int dx$$

$$v - \tan^{-1}(v) = x + c$$

$$x + y - \tan^{-1}(x+y) = x + c$$

$$y - \tan^{-1}(x+y) = c$$

Differential Equations Ex 22.8 Q6

$$\cos^2(x-2y) = 1 - \frac{2dy}{dx}$$

Let $x - 2y = v$

$$1 - \frac{2dy}{dx} = \frac{dv}{dx}$$

So,

$$\cos^2 v = \frac{dv}{dx}$$

$$\int dx = \int \sec^2 v dv$$

$$x = \tan v + c$$

$$x = \tan(x-2y) + c$$

Differential Equations Ex 22.8 Q7

The given differential equation can be written as

$$\frac{dy}{dx} = \frac{1}{\cos(x+y)}$$

Let $x + y = \upsilon$. Then,

$$1 + \frac{dy}{dx} = \frac{d\upsilon}{dx} \Rightarrow \frac{dy}{dx} = \frac{d\upsilon}{dx} - 1$$

Putting $x + y = \upsilon$ and $\frac{dy}{dx} = \frac{d\upsilon}{dx} - 1$ the given differential equation, we get

$$\Rightarrow \quad \frac{d\upsilon}{dx} - 1 = \frac{1}{\cos\upsilon}$$

$$\Rightarrow \quad \frac{d\upsilon}{dx} = \frac{1+\cos\upsilon}{\cos\upsilon}$$

$$\Rightarrow \quad \frac{\cos\upsilon}{1+\cos\upsilon} d\upsilon = dx$$

$$\Rightarrow \quad \frac{\cos\upsilon(1-\cos\upsilon)}{1-\cos^2\upsilon} d\upsilon = dx$$

$$\Rightarrow \quad \left(\cot\upsilon \operatorname{cosec}\upsilon - \cot^2\upsilon\right) d\upsilon = dx$$

$$\Rightarrow \quad \left(\cot\upsilon \operatorname{cosec}\upsilon - \operatorname{cosec}^2\upsilon + 1\right) d\upsilon = dx$$

$$\Rightarrow \quad -\operatorname{cosec}\upsilon + \cot\upsilon + \upsilon = x + C$$

$$\Rightarrow \quad -\operatorname{cosec}(x+y) + \cot(x+y) + x + y = x + C$$

$$\Rightarrow \quad -\operatorname{cosec}(x+y) + \cot(x+y) + y = C$$

$$\Rightarrow \quad -\frac{1-\cos(x+y)}{\sin(x+y)} + y = C$$

$$\Rightarrow \quad -\tan\left(\frac{x+y}{2}\right) + y = C$$

We have,

$y(0) = 0$ i.e. $y = 0$ when $x = 0$

Putting $x = 0$ and $y = 0$ in (i), we get $C = 0$.

Putting $C = 0$ in (i), we get

$$-\tan\left(\frac{x+y}{2}\right) + y = 0 \Rightarrow y = \tan\left(\frac{x+y}{2}\right),$$ which is the required solution.

Differential Equations Ex 22.8 Q8

$$\frac{dy}{dx} = \tan(x+y)$$

Let $x + y = v$

$$1 + \frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dv}{dx} - 1 = \tan v$$

$$\frac{dv}{dx} = 1 + \tan v$$

$$\frac{1}{1+\tan v}dv = dx$$

$$\frac{\cos v}{\cos v + \sin v}dv = dx$$

$$\left(\frac{2\cos v}{\cos v + \sin v}\right)dv = 2dx$$

$$\left(\frac{\cos v + \sin v + \cos v - \sin v}{\cos v + \sin v}\right)dv = 2dx$$

$$\int dv + \int\left(\frac{\cos v - \sin v}{\cos v + \sin v}\right)dv = 2\int dx$$

$$v + \log\left|\cos v + \sin v\right| = 2x + c$$

$$x + y + \log\left|\cos(x+y) + \sin(x+y)\right| = 2x + c$$

$$y - x + \log\left|\cos(x+y) + \sin(x+y)\right| = c$$

Differential Equations Ex 22.8 Q9

$$2v - v\frac{dv}{dx} = \frac{dv}{dx}$$

$$\Rightarrow 2v = v\frac{dv}{dx} + \frac{dv}{dx}$$

$$\Rightarrow 2v = (v+1)\frac{dv}{dx}$$

$$\Rightarrow \frac{(v+1)}{v}dv = 2dx$$

$$(x+y)(dx - dy) = dx + dy$$

$$(x+y)\left(1 - \frac{dy}{dx}\right) = 1 + \frac{dy}{dx}$$

Let $x + y = v$

$$1 + \frac{dy}{dx} = \frac{dv}{dx}$$

So,

$$v\left(1 - \left(\frac{dv}{dx} - 1\right)\right) = \frac{dv}{dx}$$

$$v\left(2 - \frac{dv}{dx}\right) = \frac{dv}{dx}$$

$$2v - v\frac{dv}{dx} = \frac{dv}{dx}$$

$$\Rightarrow 2v = v\frac{dv}{dx} + \frac{dv}{dx}$$

$$\Rightarrow 2v = (v+1)\frac{dv}{dx}$$

$$\Rightarrow \frac{(v+1)}{v}dv = 2dx$$

$$\int\left(1 + \frac{1}{v}\right)dv = 2\int dx$$

$$v + \log|v| = 2x + c$$

$$x + y + \log|x+y| = 2x + c$$

$$y - x + \log|x+y| = c$$

Differential Equations Ex 22.8 Q10

$$(x+y+1)\frac{dy}{dx} = 1$$

Let $x + y = v$

$$1 + \frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{dv}{dx} - 1$$

So,

$$(v+1)\left(\frac{dv}{dx} - 1\right) = 1$$

$$(v+1)\frac{dv}{dx} - (v+1) = 1$$

$$(1+v)\frac{dv}{dx} = 1 + 1 + v$$

$$\frac{v+1dv}{2+v} = dx$$

$$\int\left(1 - \frac{1}{v+2}\right)dv = \int dx$$

$$v - \log|v+2| = x + \log c$$

$$x + y - \log|x+y+2| = x + \log c$$

$$y = \log c|x+y+2|$$

$$e^y = c(x+y+2)$$

$$ke^y = x + y + 2 \qquad [k = 1/c]$$

$$x = ke^y - y - 2$$

Differential Equations Ex 22.8 Q11

$$\frac{dy}{dx} + 1 = e^{x+y}$$

Let $x+y = v$

$$1 + \frac{dy}{dx} = \frac{dv}{dx}$$

$\therefore$ Given differential equation becomes,

$$\frac{dv}{dx} = e^v$$

$$\frac{1}{e^v}dv = dx$$

Integrating on both the sides we get

$$-e^{-v} = x + C$$

$$\therefore -e^{-(x+y)} = x + C$$

Ex 22.9

Differential Equations Ex 22.9 Q1

Here, $x^2dy + y(x+y)dx = 0$

$$\frac{dy}{dx} = -\frac{y(x+y)}{x^2}$$

It is homogeneous equation

Put $y = vx$

and,

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So,

$$v + x\frac{dv}{dx} = -\frac{vx(x+vx)}{x^2}$$

$$v + x\frac{dv}{dx} = -v - v^2$$

$$x\frac{dv}{dx} = -2v - v^2$$

$$\int \frac{1}{v^2+2v}dv = -\int \frac{dx}{x}$$

$$\int \frac{1}{v^2+2v+1-1}dv = -\int \frac{dx}{x}$$

$$\int \frac{1}{(v+1)^2-(1)^2}dv = -\int \frac{dx}{x}$$

$$\frac{1}{2}\log\left|\frac{v+1-1}{v+1+1}\right| = -\log|x| + \log|c|$$

$$\log\left|\frac{v}{v+2}\right|^{\frac{1}{2}} = -\log\left|\frac{c}{x}\right|$$

$$\frac{v}{v+2} = \frac{c^2}{x^2}$$

$$\frac{\frac{y}{x}}{\frac{y}{x}+2} = \frac{c^2}{x^2}$$

$$\frac{y}{y+2x} = \frac{c^2}{x^2}$$

$$yx^2 = (y+2x)c^2$$

Differential Equations Ex 22.9 Q2

$$\frac{dy}{dx} = \frac{y-x}{y+x}$$

It is homogeneous equation

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So,

$$v + x\frac{dv}{dx} = \frac{vx-x}{vx+x}$$

$$v + x\frac{dv}{dx} = \frac{v-1}{v+1}$$

$$x\frac{dv}{dx} = \frac{v-1}{v+1} - v$$

$$= \frac{v-1-v^2-v}{v+1}$$

$$x\frac{dv}{dx} = -\frac{(1+v^2)}{v+1}$$

$$\int \frac{v+1}{v^2+1}dv = -\int \frac{dx}{x}$$

$$\int \frac{v}{v^2+1}dv + \int \frac{1}{v^2+1}dv = -\int \frac{dx}{x}$$

$$\frac{1}{2}\int \frac{2v}{v^2+1}dv + \int \frac{1}{v^2+1}dv = -\int \frac{dx}{x}$$

$$\frac{1}{2}\text{og}|v^2+1| + \tan^{-1}v = -\log|x| + \log|c|$$

$$\log\left|\frac{y^2+x^2}{x^2}\right| + 2\tan^{-1}\left(\frac{y}{x}\right) = 2\log\left|\frac{c}{x}\right|$$

$$\log|x^2+y^2| - 2\log|x| + 2\tan^{-1}\left(\frac{y}{x}\right) = 2\log\left|\frac{c}{x}\right|$$

$$\log|x^2+y^2| + 2\tan^{-1}\left(\frac{y}{x}\right) = 2\log(c)$$

$$\log|x^2+y^2| + 2\tan^{-1}\left(\frac{y}{x}\right) = k$$

Differential Equations Ex 22.9 Q3

Here, $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$.

It is a homogeneous equation

Put $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{v^2x^2 - x^2}{2xvx}$$

$$x\frac{dv}{dx} = \frac{v^2 - 1}{2v} - \frac{v}{1}$$

$$= \frac{v^2 - 1 - 2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{-1 - v^2}{2v}$$

$$\int \frac{2v}{1+v^2}dv = -\int \frac{dx}{x}$$

$$\log\left|1 + v^2\right| = -\log|x| + \log|c|$$

$$1 + v^2 = \frac{c}{x}$$

$$1 + \frac{y^2}{x^2} = \frac{c}{x}$$

$$x^2 + y^2 = cx$$

Differential Equations Ex 22.9 Q4

Here, $\frac{xdy}{dx} = x + y,\ x \neq 0$

$\frac{dy}{dx} = \frac{x + y}{x}$

It is a homogeneous equation

Put $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{x + vx}{x}$$

$$v + x\frac{dv}{dx} = 1 + v$$

$$\int dv = \int \frac{dx}{x}$$

$$v = \log|x| + c$$

$$\frac{y}{x} = \log|x| + c$$

$$y = x\log|x| + cx$$

Differential Equations Ex 22.9 Q5

Here, $\left(x^2 - y^2\right)dx - 2xydy = 0$

$\frac{dy}{dx} = \frac{x^2 - y^2}{2xy}$

It is a homogeneous equation

Put $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{x^2 - v^2x^2}{2xvx}$$

$$x\frac{dv}{dx} = \frac{1 - v^2}{2v} - v$$

$$x\frac{dv}{dx} = \frac{1 - v^2 - 2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1 - 3v^2}{2v}$$

$$\int \frac{2v}{1 - 3v^2}dv = \int \frac{dx}{x}$$

$$\frac{1}{-3}\int \frac{-6v}{1 - 3v^2}dv = \int \frac{dx}{x}$$

$$\int \frac{-6v}{1 - 3v^2} = -3\int \frac{dx}{x}$$

$$\log\left|1 - 3v^2\right| = -3\log|x| + \log|c|$$

$$1 - 3v^2 = \frac{c}{x^3}$$

$$x^3\left(1 - \frac{3y^2}{x^2}\right) = c$$

$$\frac{x^3\left(x^2 - 3y^2\right)}{x^2} = c$$

$$x\left(x^2 - 3y^2\right) = c$$

Differential Equations Ex 22.9 Q6

$$\frac{dy}{dx}=\frac{x+y}{x-y}$$

Here it is a homogeneous equation

Put $y=vx$

And

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

So,

$$v+x\frac{dv}{dx}=\frac{1+v}{1-v}$$

$$x\frac{dv}{dx}=\frac{1+v}{1-v}-v$$

$$x\frac{dv}{dx}=\frac{1+v^2}{1-v}$$

$$\frac{1-v}{1+v^2}dv=\frac{dx}{x}$$

$$\int\frac{1-v}{1+v^2}dv=\int\frac{dx}{x}$$

$$\int\frac{1}{1+v^2}dv-\frac{1}{2}\int\frac{2v}{1+v^2}dv=\int\frac{dx}{x}$$

$$\tan^{-1}v-\frac{1}{2}\log\left(1+v^2\right)=\log x+c$$

$$\tan^{-1}\frac{y}{x}=\frac{1}{2}\log\left(x^2+y^2\right)+c$$

Differential Equations Ex 22.9 Q7

Here, $2xy\frac{dy}{dx}=x^2+y^2$

$$\frac{dy}{dx}=\frac{x^2+y^2}{2xy}$$

It is a homogeneous equation

Put $y=vx$

and, $\frac{dy}{dx}=v+x\frac{dv}{dx}$

So,

$$v+x\frac{dv}{dx}=\frac{x^2+v^2x^2}{2xvx}$$

$$x\frac{dv}{dx}=\frac{1+v^2}{2v}-v$$

$$x\frac{dv}{dx}=\frac{1+v^2-2v^2}{2v}$$

$$x\frac{dv}{dx}=\frac{1-v^2}{2v}$$

$$\frac{2v}{1-v^2}dv=\frac{dx}{x}$$

$$\int\frac{-2v}{1-v2}dv=-\int\frac{dx}{x}$$

$$\log\left|1-v^2\right|=-\log|x|+\log c$$

$$\left(1-v^2\right)=\frac{c}{x}$$

$$x\left(1-\frac{y^2}{x^2}\right)=c$$

$$\frac{x\left(x^2-y^2\right)}{x^2}=c$$

$$x^2-y^2=cx$$

Differential Equations Ex 22.9 Q8

Consider the given differential equation

$$x^2\frac{dy}{dx}=x^2-2y^2+xy$$

$$\Rightarrow \frac{dy}{dx}=\frac{x^2-2y^2+xy}{x^2}$$

This is a homogeneous differential equation.

Substituting $y=vx$ *and* $\frac{dy}{dx}=v+x\frac{dv}{dx}$, *we have*

$$v+x\frac{dv}{dx}=\frac{x^2-2v^2\times x^2+x\times v\times x}{x^2}$$

$$\Rightarrow v+x\frac{dv}{dx}=1-2v^2+v$$

$$\Rightarrow x\frac{dv}{dx}=1-2v^2$$

$$\Rightarrow \frac{dv}{1-2v^2}=\frac{dx}{x}$$

$$\Rightarrow \frac{dv}{v^2-\frac{1}{2}}=-2\frac{dx}{x}$$

$$\Rightarrow \int\frac{dv}{\left(\frac{1}{\sqrt{2}}\right)^2-v^2}=2\int\frac{dx}{x}$$

$$\Rightarrow \int\frac{dv}{\left(\frac{1}{\sqrt{2}}\right)^2-v^2}=2\int\frac{dx}{x}$$

$$\Rightarrow \frac{\sqrt{2}}{2}\log\left(\frac{\frac{1}{\sqrt{2}}+v}{\frac{1}{\sqrt{2}}-v}\right)=2\log x+\log C$$

$$\Rightarrow \frac{1}{\sqrt{2}}\log\left(\frac{\frac{1}{\sqrt{2}}+\frac{y}{x}}{\frac{1}{\sqrt{2}}-\frac{y}{x}}\right)=2\log x+\log C$$

$$\Rightarrow \frac{1}{\sqrt{2}}\log\left(\frac{x+y\sqrt{2}}{x-y\sqrt{2}}\right)=2\log x+\log C$$

$$\Rightarrow \frac{1}{\sqrt{2}}\log\left(\frac{x+y\sqrt{2}}{x-y\sqrt{2}}\right)=\log x^2+\log C$$

$$\Rightarrow \log\left(\frac{x+y\sqrt{2}}{x-y\sqrt{2}}\right)^{\frac{1}{\sqrt{2}}}=\log Cx^2$$

$$\Rightarrow \left(\frac{x+y\sqrt{2}}{x-y\sqrt{2}}\right)^{\frac{1}{\sqrt{2}}}=Cx^2$$

$$\Rightarrow \left(\frac{x+y\sqrt{2}}{x-y\sqrt{2}}\right)=\left(Cx^2\right)^{\sqrt{2}}$$

Differential Equations Ex 22.9 Q9

Here, $xy\frac{dy}{dx} = x^2 - y^2$

$$\frac{dy}{dx} = \frac{x^2 - y^2}{xy}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{x^2 - v^2x^2}{xvx}$$

$$x\frac{dv}{dx} = \frac{1 - v^2}{v} - v$$

$$x\frac{dv}{dx} = \frac{1 - v^2 - v^2}{v}$$

$$x\frac{dv}{dx} = \frac{1 - 2v^2}{v}$$

$$\frac{v}{1 - 2v^2}dv = \frac{dx}{x}$$

$$\int\frac{-4v}{1 - 2v^2}dv = -4\int\frac{dx}{x}$$

$$\log\left|1 - 2v^2\right| = -4\log|x| + \log c$$

$$\left(1 - 2\frac{y^2}{x^2}\right) = \frac{c}{x^4}$$

$$\left(\frac{x^2 - 2y^2}{x^2}\right) = \frac{c}{x^4}$$

$$x^2\left(x^2 - 2y^2\right) = c$$

Differential Equations Ex 22.9 Q10

Here, $ye^{\frac{x}{y}}dx = \left(xe^{\frac{x}{y}} + y\right)dy$

$$\frac{dx}{dy} = \frac{xe^{\frac{x}{y}} + y}{ye^{\frac{x}{y}}}$$

It is a homogeneous equation

Put $x = vy$

and $\frac{dx}{dy} = v + y\frac{dv}{dy}$

So,

$$v + y\frac{dv}{dy} = \frac{vye^{\frac{vy}{y}} + y}{ye^{\frac{vy}{y}}}$$

$$v + y\frac{dv}{dy} = \frac{ve^v + 1}{e^v}$$

$$y\frac{dv}{dy} = \frac{ve^v + 1}{e^v} - v$$

$$y\frac{dv}{dy} = \frac{ve^v + 1 - ve^v}{e^v}$$

$$y\frac{dv}{dy} = \frac{1}{e^v}$$

$$\int evdv = \int\frac{dy}{y}$$

$$e^v = \log|y| + c$$

$$e^{\frac{x}{y}} = \log y + c$$

Differential Equations Ex 22.9 Q11

Here, $x^2\frac{dy}{dx} = x^2 + xy + y^2$

$$\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$$

It is a homogeneous equaton

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{x^2 + xvx + v^2x^2}{x^2}$$

$$x\frac{dv}{dx} = 1 + v + v^2 - v^2$$

$$x\frac{dv}{dx} = 1 + v^2$$

$$\int\frac{dv}{1+v^2} = \int\frac{dx}{x}$$

$$\tan^{-1} v = \log|x| + c$$

$$\tan^{-1}\frac{y}{x} = \log|x| + c$$

Differential Equations Ex 22.9 Q12

Here, $\left(y^2 - 2xy\right)dx = \left(x^2 - 2xy\right)dy$

$$\frac{dy}{dx} = \frac{y^2 - 2xy}{x^2 - 2xy}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{v^2x^2 - 2xvx}{x^2 - 2xvx}$$

$$v + x\frac{dv}{dx} = \frac{v^2 - 2v}{1 - 2v}$$

$$x\frac{dv}{dx} = \frac{v^2 - 2v}{1 - 2v} - v$$

$$= \frac{v^2 - 2v - v + 2v^2}{1 - 2v}$$

$$x\frac{dv}{dx} = \frac{3v^2 - 3v}{1 - 2v}$$

$$\frac{1 - 2v}{3\left(v^2 - v\right)}dv = \frac{dx}{x}$$

$$\frac{-}{3}\frac{(2v - 1)}{v^2 - v}\,dv = \frac{dx}{x}$$

$$\int\frac{2v - 1}{v^2 - v}dv = -3\int\frac{dx}{x}$$

$$\log\left|v^2 - v\right| = -3\log|x| + \log c$$

$$v^2 - v = \frac{c}{x^3}$$

$$\frac{y^2}{x^2} - \frac{y}{x} = \frac{c}{x^3}$$

$$y^2 - xy = \frac{c}{x}$$

$$x\left(y^2 - xy\right) = c$$

Differential Equations Ex 22.9 Q13

Here, $2xydx + \left(x^2 + 2y^2\right)dy = 0$

$$\frac{dy}{dx} = \frac{2xy}{x^2 + 2y^2}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{2xvx}{x^2 + 2v^2x^2}$$

$$v + x\frac{dv}{dx} = \frac{2v}{1 + 2v^2}$$

$$x\frac{dv}{dx} = \frac{2v}{1 + 2v^2} - v$$

$$= \frac{2v - v - 2v^3}{1 + 2v^2}$$

$$x\frac{dv}{dx} = \frac{v - 2v^3}{1 + 2v^2}$$

$$\int \frac{1 + 2v^2}{v - 2v^3}dv = \int \frac{dx}{x} \qquad \text{---(i)}$$

$$\frac{1 + 2v^2}{v - 2v^3} = \frac{1 + 2v^2}{v\left(1 - 2v^2\right)}$$

$$\frac{1 + 2v^2}{v\left(1 - 2v^2\right)} = \frac{A}{v} + \frac{Bv + c}{1 - 2v^2}$$

$$\frac{1 + 2v^2}{v\left(1 - 2v^2\right)} = \frac{A\left(1 - 2v^2\right) + (Bv + c)v}{v\left(1 - 2v^2\right)}$$

$$1 + 2v^2 = A - 2Av^2 + Bv^2 + cv$$

$$1 + 2v^2 = v^2(-2A + B) + cv + A$$

Comparing the coefficients of like powers of v,

$$A = 1$$

$$c = 0$$

$$-2A + B = 2$$

$$-2 + B = 0$$

$$B = 4$$

$$\frac{1 + 2v^2}{v - 2v^3} = \frac{1}{v} + \frac{4v}{1 - 2v^2}$$

$$\frac{1 + 2v^2}{v - 2v^3} = \frac{1}{v} - \frac{(-4v)}{\left(1 - 2v^2\right)}$$

Differential Equations Ex 22.9 Q14

Here, $3x^2dy = \left(3xy + y^2\right)dx$

$$\frac{dy}{dx} = \frac{3xy + y^2}{3x^2}$$

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{3xvx + v^2x^2}{3x^2}$$

$$v + x\frac{dv}{dx} = \frac{3v + v^2}{3}$$

$$x\frac{dv}{dx} = \frac{3v + v^2}{3} - v$$

$$x\frac{dv}{dx} = \frac{3v + v^2 - 3v}{3}$$

$$x\frac{dv}{dx} = \frac{v^2}{3}$$

$$3\int \frac{1}{v^2}dv = \int \frac{dx}{x}$$

$$3\left(-\frac{1}{v}\right) = \log|x| + c$$

$$-\frac{3x}{y} = \log|x| + c$$

Differential Equations Ex 22.9 Q15

Here, $\frac{dy}{dx} = \frac{x}{2y + x}$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{x}{2vx + x}$$
$$v + x\frac{dv}{dx} = \frac{1}{2v + 1}$$
$$x\frac{dv}{dx} = \frac{1}{2v + 1} - v$$
$$x\frac{dv}{dx} = \frac{1 - 2v^2 - v}{2v + 1}$$
$$\int\frac{2v + 1}{1 - v - 2v^2}dv = \int\frac{dx}{x}$$
$$-\int\frac{2v + 1}{2v^2 + v - 1}dv = \int\frac{dx}{x}$$
$$\frac{1}{2}\int\frac{4v + 2}{2v^2 + v - 1}dv = -\int\frac{dx}{x}$$
$$\int\frac{4v + 1 + 1}{2v^2 + v - 1}dv = -2\int\frac{dx}{x}$$
$$\int\frac{4v + 1}{2v^2 + v - 1}dv + \int\frac{1}{2v^2 + v - 1}dv = -2\int\frac{dx}{x}$$
$$\int\frac{4v + 1}{2v^2 + v - 1}dv + \frac{1}{2}\int\frac{1}{v^2 + \frac{v}{2} - \frac{1}{2}}dv = -2\int\frac{dx}{x}$$
$$\int\frac{4v + 1}{2v^2 + v - 1}dv + \frac{1}{2}\int\frac{dv}{v^2 + 2v\left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 - \frac{1}{2}} = -2\int\frac{dx}{x}$$
$$\int\frac{4v + 1}{2v^2 + v - 1}dv + \frac{1}{2}\int\frac{dv}{\left(v + \frac{1}{4}\right)^2 - \left(\frac{3}{4}\right)^2} = -2\int\frac{dx}{x}$$
$$\log\left|2v^2 + v - 1\right| + \frac{1}{2} \times \frac{1}{2\left(\frac{3}{4}\right)}\log\left|\frac{v + \frac{1}{4} - \frac{3}{4}}{v + \frac{1}{4} + \frac{3}{4}}\right| = -2\log|x| + \log c$$

Differential Equations Ex 22.9 Q16

Here, $(x + 2y)dx - (2x - y)dy = 0$

$$\frac{dy}{dx} = \frac{(x + 2y)}{(2x - y)}$$

It is a homogeneous equation

Put $y = vx$

$\therefore$ $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{x + 2vx}{2x - vx}$$
$$v + x\frac{dv}{dx} = \frac{1 + 2v}{2 - v}$$
$$x\frac{dv}{dx} = \frac{1 + 2v}{2 - v} - \frac{v}{1}$$
$$x\frac{dv}{dx} = \frac{1 + 2v - 2v + v^2}{2 - v}$$
$$x\frac{dv}{dx} = \frac{1 + v^2}{2 - v}$$
$$\frac{2 - v}{1 + v^2} = \frac{dx}{x}$$
$$\int\frac{2 - v}{1 + v^2}dv = \int\frac{dx}{x}$$
$$\int\frac{2}{1 + v^2}dv - \int\frac{v}{1 + v^2}dv = \int\frac{dx}{x}$$
$$2\tan^{-1}v - \frac{1}{2}\log\left|1 + v^2\right| = \log|x| + \log c$$
$$2\tan^{-1}v = \log xc + \log\left|1 + v^2\right|^{\frac{1}{2}}$$
$$e^{2\tan^{-1}v} = \left(1 + v^2\right)^{\frac{1}{2}}xc$$
$$e^{2\tan^{-1}\frac{y}{x}} = \left\{\frac{\left(y^2 + x^2\right)^{\frac{1}{2}}}{x}\right\}xc$$
$$e^{2\tan^{-1}\frac{y}{x}} = \left(y^2 + x^2\right)^{\frac{1}{2}}c$$

Differential Equations Ex 22.9 Q17

Here, $\frac{dy}{dx} = \frac{y}{x} - \sqrt{\frac{y^2}{x^2} - 1}$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{vx}{x} - \sqrt{\frac{v^2x^2}{x^2} - 1}$$

$$v + x\frac{dv}{dx} = v - \sqrt{v^2 - 1}$$

$$x\frac{dv}{dx} = v - \sqrt{v^2 - 1} - v$$

$$x\frac{dv}{dx} = -\sqrt{v^2 - 1}$$

$$\int \frac{dv}{\sqrt{v^2 - 1}} - \int \frac{dx}{x}$$

$$\log\left|v + \sqrt{v^2 - 1}\right| = -\log|x| + \log c$$

$$\left(\frac{y}{x} + \sqrt{v^2 - 1}\right) = \frac{c}{x}$$

$$y + \sqrt{y^2 - x^2} = c$$

Differential Equations Ex 22.9 Q18

$$\frac{dy}{dx} = \frac{y}{x}\left\{\log\left(\frac{y}{x}\right) + 1\right\}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{vx}{x}\left\{\log\left(\frac{vx}{x}\right) + 1\right\}$$

$$v + x\frac{dv}{dx} = v\log v + v$$

$$x\frac{dv}{dx} = v\log v$$

$$\int \frac{1}{v\log v}dv = \int \frac{dx}{x}$$

$$\log\log v = \log|x| + \log c$$

$$\log v = xc$$

$$\log\frac{y}{x} = xc$$

$$\frac{y}{x} = e^{xc}$$

$$y = xe^{xc}$$

Differential Equations Ex 22.9 Q19

$$\frac{dy}{dx} = \frac{y}{x} + \sin\left(\frac{y}{x}\right)$$

Here it is a homogeneous equation

Put $y = vx$

And

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So,

$$v + x\frac{dv}{dx} = v + \sin v$$

$$x\frac{dv}{dx} = \sin v$$

$$\operatorname{cosec} v\,dv = \frac{dx}{x}$$

$$\int \operatorname{cosec} v\,dv = \int \frac{dx}{x}$$

$$\log\tan\frac{v}{2} = \log x + \log c$$

$$\tan\frac{v}{2} = Cx$$

$$\tan\frac{y}{2x} = Cx$$

Differential Equations Ex 22.9 Q20

Here, $y^2dx + \left(x^2 - xy + y^2\right)dy = 0$

$$\frac{dy}{dx} = \frac{-y^2}{x^2 - xy + y^2}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{-v^2x^2}{x^2 - xvx + v^2x^2}$$

$$v + x\frac{dv}{dx} = \frac{-v^2}{1 - v + v^2}$$

$$x\frac{dv}{dx} = \frac{-v^2}{1 - v + v^2} - \frac{v}{1}$$

$$= \frac{-v^2 - v + v^2 - v^3}{1 - v + v^2}$$

$$x\frac{dv}{dx} = \frac{-v - v^3}{v^2 - v + 1}$$

$$\frac{v^2 - v + 1}{-v\left(1 + v^2\right)}dv = \frac{dx}{x}$$

$$\left(\frac{1}{1 + v^2} - \frac{1}{v}\right)dv = \frac{dx}{x}$$

$$-\int\frac{1}{v}dv + \int\frac{1}{1 + v^2}dv = \int\frac{dx}{x}$$

$$-\log|v| + \tan^{-1}v = \log|x| + \log c$$

$$\log\left|\frac{x}{y}\right| + \tan^{-1}\left(\frac{y}{x}\right) = \log c$$

$$\tan^{-1}\left(\frac{y}{x}\right) = \log xc - \log\frac{x}{y}$$

$$\tan^{-1}\left(\frac{y}{x}\right) = \log\left(\frac{xcy}{x}\right)$$

$$\tan^{-1}\left(\frac{y}{x}\right) = \log(cy)$$

$$e^{\tan^{-1}\left(\frac{y}{x}\right)} = cy$$

Differential Equations Ex 22.9 Q21

Here, $\left[x\sqrt{x^2 + y^2} - y^2\right]dx + xydy = 0$

$$\frac{dy}{dx} = \frac{\left[y^2 - x\sqrt{x^2 + y^2}\right]}{xy}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{\left[v^2x^2 - x\sqrt{x^2 + v^2x^2}\right]}{xvx}$$

$$v + x\frac{dv}{dx} = \frac{\left[v^2 - \sqrt{1 + v^2}\right]}{v}$$

$$x\frac{dv}{dx} = \frac{v^2 - \sqrt{1 + v^2}}{v} - v$$

$$= \frac{v^2 - \sqrt{1 + v^2} - v^2}{v}$$

$$x\frac{dv}{dx} = \frac{-\sqrt{1 + v^2}}{v}$$

$$\int\frac{v}{\sqrt{1 + v^2}}dv = -\int\frac{dx}{x}$$

$$\frac{1}{2}\int\frac{2v}{\sqrt{1 + v^2}}dv = -\int\frac{dx}{x}$$

Let $1 + v^2 = t$

$$2vdv = dt$$

$$\frac{1}{2}\int\frac{1}{\sqrt{t}}dt = -\int\frac{dx}{x}$$

$$\frac{1}{2}\times 2\sqrt{t} = -\log|x| + \log c$$

$$\sqrt{1 + v^2} = \log\left|\frac{c}{x}\right|$$

$$\frac{\sqrt{x^2 + y^2}}{x} = \log\left|\frac{c}{x}\right|$$

$$\sqrt{x^2 + y^2} = x\log\left|\frac{c}{x}\right|$$

Differential Equations Ex 22.9 Q22

Here, $x\dfrac{dy}{dx} = y - x\cos^2\left(\dfrac{y}{x}\right)$

$$\frac{dy}{dx} = \frac{y - x\cos^2\left(\frac{y}{x}\right)}{x}$$

It is a homogeneous equation

Put $y = vx$

and $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{vx - x\cos^2\left(\frac{vx}{x}\right)}{x}$$

$$v + x\frac{dv}{dx} = v - \cos^2 v$$

$$x\frac{dv}{dx} = v - \cos^2 v - v$$

$$x\frac{dv}{dx} = -\cos^2 v$$

$$\frac{dv}{\cos^2 v} = -\frac{dx}{x}$$

$$\int \sec^2 v\,dv = -\int\frac{dx}{x}$$

$$\tan v = -\log|x| + \log c$$

$$\tan\frac{y}{x} = \log\left|\frac{c}{x}\right|$$

Differential Equations Ex 22.9 Q23

Here, $\dfrac{y}{x}\cos\left(\dfrac{y}{x}\right)dx - \left\{\dfrac{x}{y}\sin\left(\dfrac{y}{x}\right) + \cos\left(\dfrac{y}{x}\right)\right\}dy = 0$

$$\frac{dy}{dx} = \frac{\frac{y}{x}\cos\left(\frac{y}{x}\right)}{\frac{x}{y}\sin\left(\frac{y}{x}\right) + \cos\left(\frac{y}{x}\right)}$$

It is a homogeneous equation

Put $y = vx$

and $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{\frac{vx}{x}\cos\left(\frac{vx}{x}\right)}{\frac{x}{vx}\sin\left(\frac{vx}{x}\right) + \cos\left(\frac{vx}{x}\right)}$$

$$= \frac{v\cos v}{\frac{1}{v}\sin v + \cos v}$$

$$v + x\frac{dv}{dx} = \frac{v^2\cos v}{\sin v + v\cos v}$$

$$x\frac{dv}{dx} = \frac{v^2\cos v}{\sin v + v\cos v} - v$$

$$x\frac{dv}{dx} = \frac{v^2\cos v - v\sin v - v^2\cos v}{\sin v + v\cos v}$$

$$x\frac{dv}{dx} = \frac{-v\sin v}{\sin v + v\cos v}$$

$$\frac{\sin v + v\cos v}{v\sin v}dv = -\frac{dx}{x}$$

$$\int\left(\frac{1}{v} + \cot v\right)dv = -\log|x| + \log c$$

$$\log|v| + \log|\sin v| = \log\left|\frac{c}{x}\right|$$

$$\log|v\sin v| = \log\left|\frac{c}{x}\right|$$

$$|v\sin v| = \left|\frac{c}{x}\right|$$

$$\left|x\left(\frac{y}{x}\right)\sin\left(\frac{y}{x}\right)\right| = |c|$$

$$\left|y\sin\frac{y}{x}\right| = c$$

Differential Equations Ex 22.9 Q24

Here, $xy\log\left(\frac{x}{y}\right)dx + \left\{y^2 - x^2\log\left(\frac{x}{y}\right)\right\}dy = 0$

$$\frac{dy}{dx} = \frac{x^2\log\left(\frac{x}{y}\right) - y^2}{xy\log\left(\frac{x}{y}\right)}$$

It is a homogeneous equation

Put $\quad x = vy$

and $\quad \frac{dx}{dy} = v + y\frac{dv}{dy}$

So,

$$v + y\frac{dv}{dy} = \frac{v^2y^2\log\left(\frac{vy}{y}\right) - y^2}{vy\log\left(\frac{vy}{y}\right)}$$

$$v + y\frac{dv}{dy} = \frac{v^2\log v - 1}{v\log v}$$

$$y\frac{dv}{dy} = \frac{v^2\log v - 1}{v\log v} - v$$

$$y\frac{dv}{dy} = \frac{v^2\log v - 1 - v^2\log v}{v\log v}$$

$$y\frac{dv}{dy} = \frac{-1}{v\log v}$$

$$\int v\log v\,dv = -\int\frac{dy}{y}$$

$$\log v \times \int v\,dv - \int\frac{1}{v}\times\int v\,dv\,dv = -\log|y| + \log c$$

Itegrating it by parts

$$\frac{v^2}{2}\log v\int\frac{1}{v}\times\frac{v^2}{2}dv = \log\left|\frac{c}{y}\right|$$

$$\frac{v^2}{2}\log v - \frac{1}{2}\int v\,dv = \log\left|\frac{c}{y}\right|$$

$$\frac{v^2}{2}\log v - \frac{v^2}{4} = \log\left|\frac{c}{y}\right|$$

$$\frac{v^2}{2}\left[\log v - \frac{1}{2}\right] = \log\left|\frac{c}{y}\right|$$

Differential Equations Ex 22.9 Q25

$$\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$$

Here it is a homogeneous equation

Put $\quad x = vy$

And

$$\frac{dx}{dy} = v + y\frac{dv}{dy}$$

So,

$$\frac{dx}{dy} = -\frac{e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)}{\left(1 + e^{\frac{x}{y}}\right)}$$

$$v + y\frac{dv}{dy} = -\frac{e^{\frac{vy}{y}}\left(1 - \frac{vy}{y}\right)}{\left(1 + e^{\frac{vy}{y}}\right)}$$

$$= -\frac{e^v(1 - v)}{(1 + e^v)}$$

$$y\frac{dv}{dy} = -\frac{e^v(1 - v)}{(1 + e^v)} - v$$

$$= \frac{-e^v(1 - v) - v(1 + e^v)}{(1 + e^v)}$$

$$\frac{(1 + e^v)}{-e^v(1 - v) - v(1 + e^v)}dv = \frac{dy}{y}$$

$$x + ye^{x/y} = c$$

Differential Equations Ex 22.9 Q26

Here, $\left(x^2+y^2\right)\frac{dy}{dx} = 8x^2 - 3xy + 2y^2$

$$\frac{dy}{dx} = \frac{8x^2 - 3xy + 2y^2}{x^2+y^2}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{8x^2 - 3xvx + 2v^2x^2}{x^2+v^2x^2}$$

$$v + x\frac{dv}{dx} = \frac{8 - 3v + 2v^2}{1+v^2}$$

$$x\frac{dv}{dx} = \frac{8 - 3v + 2v^2}{1+v^2} - v$$

$$= \frac{8 - 3v + 2v^2 - v - v^3}{1+v^2}$$

$$x\frac{dv}{dx} = \frac{8 - 4v + 2v^2 - v^3}{1+v^2}$$

$$\frac{1+v^2}{8 - 4v + 2v^2 - v^3}dv = \frac{dx}{x}$$

$$\frac{1+v^2}{4(2-v)+v^2(2-v)}dv = \frac{dx}{x}$$

$$\frac{1+v^2}{4(2-v)+v^2(2-v)}dv = \frac{dx}{x}$$

$$\int\frac{1+v^2}{\left(4+v^2\right)(2-v)}dv = \int\frac{dx}{x} \qquad \text{---(A)}$$

$$\frac{1+v^2}{\left(4+v^2\right)(2-v)} = \frac{Av+B}{4+v^2} + \frac{c}{2-v}$$

$$\frac{1+v^2}{\left(4+v^2\right)(2-v)} = \frac{(Av+B)(2-v)+c\left(4+v^2\right)}{\left(4+v^2\right)(2-v)}$$

$$1+v^2 = 2Av - Av^2 + 2B - Bv + 4c + cv^2$$

$$1+v^2 = v^2(-A+c) + v(2A-B) + 2B + 4c$$

Comparing the coefficients of like powers of v

$$-A + c = 1 \qquad \text{---(i)}$$

$$2A - B = 0$$

$$\Rightarrow \quad B = 2A \qquad \text{---(ii)}$$

$$2B + 4c = 1 \qquad \text{---(iii)}$$

Solving equation (i), (ii) and (iii)

$$A = -\frac{3}{8}, B = -\frac{3}{4}, C = \frac{5}{8}$$

Using equation (A)

$$\int\frac{\left(-\frac{3}{8}v - \frac{3}{4}\right)}{4+v^2}dv + \frac{5}{8}\int\frac{1}{2-v}dv = \int\frac{dx}{x}$$

$$-\frac{3}{8}\int\frac{v+2}{4+v^2}dv + \frac{5}{8}\int\frac{1}{2-v}dv = \int\frac{dx}{x}$$

$$-\frac{3}{8}\int\frac{v}{4+v^2}dv - \frac{3}{8}\int\frac{1}{4+v^2}dv + \frac{5}{8}\int\frac{1}{2-v}dv = \int\frac{dx}{x}$$

$$-\frac{3}{16}\log\left|4+v^2\right| - \frac{3}{8}\tan^{-1}\frac{v}{2} - \frac{5}{8}\log|2-v| = \log|x| + loc$$

$$-\left[\log\left|4+v^2\right|^{\frac{3}{16}} + \log e^{\frac{3}{8}\tan^{-1}\left(\frac{v}{2}\right)} + \log(2-v)^{\frac{5}{8}}\right] = \log|xc|$$

$$\left(4+v^2\right)^{\frac{3}{16}} \times e^{\frac{3}{8}\tan^{-1}\left(\frac{v}{2}\right)} \times (2-v)^{\frac{5}{8}} = \frac{c}{x}$$

$$\frac{\left(4x^2+y^2\right)^{\frac{3}{16}}}{x^{\frac{3}{8}}} \times e^{\frac{3}{8}\tan^{-1}\left(\frac{y}{2x}\right)} \frac{(2x-y)^{\frac{5}{8}}}{x^{\frac{5}{8}}} = \frac{c}{x}$$

$$\left(4x^2+y^2\right)^{\frac{3}{16}} \times (2x-y)^{\frac{5}{8}} = c\,e^{\frac{-3}{8}\tan^{-1}\left(\frac{y}{2x}\right)}$$

Differential Equations Ex 22.9 Q27

Here, $\left(x^2 - 2xy\right)dy + \left(x^2 - 3xy + 2y^2\right)dx = 0$

$$\frac{dy}{dx} = \frac{x^2 - 3xy + 2y^2}{2xy - x^2}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{x^2 - 3xvx + 2v^2x^2}{2xvx - x^2}$$

$$x\frac{dv}{dx} = \frac{1 - 3v + 2v^2}{2v - 1} - v$$

$$x\frac{dv}{dx} = \frac{1 - 3v + 2v^2 - 2v^2 + v}{2v - 1}$$

$$x\frac{dv}{dx} = \frac{1 - 2v}{2v - 1}$$

$$\frac{2v - 1}{1 - 2v}dv = \frac{dx}{x}$$

$$\frac{1 - 2v}{1 - 2v}dv = -\int\frac{dx}{x}$$

$$\int dv = -\int\frac{dx}{x}$$

$$v = -\log|x| + C$$

$$y/x + \log x = C$$

Differential Equations Ex 22.9 Q28

Here, $x\frac{dy}{dx} = y - x\cos^2\left(\frac{y}{x}\right)$

$$\frac{dy}{dx} = \frac{y - x\cos^2\left(\frac{y}{x}\right)}{x}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{vx - x\cos^2\left(\frac{vx}{x}\right)}{x}$$

$$v + x\frac{dv}{dx} = v - \cos^2 v$$

$$x\frac{dv}{dx} = v - \cos^2 v - v$$

$$x\frac{dv}{dx} = -\cos^2 v$$

$$\frac{dv}{\cos^2 v} = -\frac{dx}{x}$$

$$\int \sec^2 v\,dv = -\int\frac{dx}{x}$$

$$\tan v = -\log|x| + \log c$$

$$\tan\frac{y}{x} = \log\left|\frac{c}{x}\right|$$

Differential Equations Ex 22.9 Q29

Here, $x\frac{dy}{dx} - y = 2\sqrt{y^2 - x^2}$

$$\frac{dy}{dx} = \frac{2\sqrt{y^2 - x^2} + y}{x}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{2\sqrt{v^2x^2 - x^2} + vx}{x}$$

$$v + x\frac{dv}{dx} = 2\sqrt{v^2 - 1} + v$$

$$x\frac{dv}{dx} = 2\sqrt{v^2 - 1}$$

$$\int\frac{dv}{\sqrt{v^2 - 1}} = 2\int\frac{dx}{x}$$

$$\log\left|v + \sqrt{v^2 - 1}\right| = 2\log|x| + \log|c|$$

$$\log\left|v + \sqrt{v^2 - 1}\right| = \log\left|cx^2\right|$$

$$v + \sqrt{v^2 - 1} = \left|cx^2\right|$$

$$\frac{y}{x} + \sqrt{\frac{y^2}{x^2} - 1} = \left|cx^2\right|$$

$$\left(y + \sqrt{y^2 - x^2}\right) = cx^3$$

Differential Equations Ex 22.9 Q30

Here, $x\cos\left(\frac{y}{x}\right)(ydx+xdy)=y\sin\left(\frac{y}{x}\right)(xdy-ydx)$

$$yx\cos\left(\frac{y}{x}\right)+x^2\cos\left(\frac{y}{x}\right)\frac{dy}{dx}=xy\sin\left(\frac{y}{x}\right)-y^2\sin\left(\frac{y}{x}\right)$$

$$\frac{dy}{dx}=\left(\frac{-y^2\sin\left(\frac{y}{x}\right)-xy\cos\left(\frac{y}{x}\right)}{x^2\cos\left(\frac{y}{x}\right)-xy\sin\left(\frac{y}{x}\right)}\right)$$

$$\frac{dy}{dx}=\frac{-xy\cos\left(\frac{y}{x}\right)-y^2\sin\left(\frac{y}{x}\right)}{x^2\cos\left(\frac{y}{x}\right)-xy\sin\left(\frac{y}{x}\right)}$$

It is a homogeneous equation

Put $y=vx$

and $\frac{dy}{dx}=v+x\frac{dv}{dx}$

So,

$$v+x\frac{dv}{dx}=\frac{-xvx\cos\left(\frac{vx}{x}\right)-v^2x^2\sin\left(\frac{vx}{x}\right)}{x^2\cos\left(\frac{vx}{x}\right)-xvx\sin\left(\frac{vx}{x}\right)}$$

$$x\frac{dv}{dx}=\frac{-v\cos v-v^2\sin v}{\cos v-v\sin v}-v$$

$$x\frac{dv}{dx}=\frac{-v\cos v-v^2\sin v-v\cos v+v^2\sin v}{\cos v-v\sin v}$$

$$x\frac{dv}{dx}=\frac{-2v\cos v}{\cos v-v\sin v}$$

$$\int\frac{\cos v-v\sin v}{v\cos v}dv=-2\int\frac{dx}{x}$$

$$\int\left(\frac{1}{v}-\tan v\right)dv=-2\int\frac{dx}{x}$$

Differential Equations Ex 22.9 Q31

Here, $\left(x^2+3xy+y^2\right)dx-x^2dy=0$

$$\frac{dy}{dx}=\frac{x^2+3xy+y^2}{x^2}$$

It is a homogeneous equation

Put $y=vx$

and $\frac{dy}{dx}=v+x\frac{dv}{dx}$

So,

$$v+x\frac{dv}{dx}=\frac{x^2+3xvx+v^2x^2}{x^2}$$

$$v+x\frac{dv}{dx}=1+3v+v^2$$

$$x\frac{dv}{dx}=1+2v+v^2$$

$$x\frac{dv}{dx}=(v+1)^2$$

$$\int\frac{1}{(v+1)^2}dv=\int\frac{dx}{x}$$

$$-\frac{1}{v+1}=\log|x|-c$$

$$\frac{x}{x+y}+\log|x|=c$$

Differential Equations Ex 22.9 Q32

Here, $(x-y)\frac{dy}{dx}=x+2y$

$$\frac{dy}{dx}=\frac{x+2y}{x-y}$$

It is a homogeneous equation

Put $y=vx$

and $\frac{dy}{dx}=v+x\frac{dv}{dx}$

So,

$$v+x\frac{dv}{dx}=\frac{x+2vx}{x-vx}$$

$$x\frac{dv}{dx}=\frac{1+2v}{1-v}-v$$

$$x\frac{dv}{dx}=\frac{1+2v-v+v^2}{1-v}$$

$$x\frac{dv}{dx}=\frac{1+v+v^2}{1-v}$$

$$\frac{1-v}{v^2+v+1}dv=\frac{dx}{x}$$

$$-\frac{v-1}{v^2+v+1}dv=\frac{dx}{x}$$

$$\frac{1}{2}\times\frac{2v-2}{v^2+v+1}dv=\frac{-dx}{x}$$

$$\int\frac{(2v+1)-3}{v^2+v+1}dv=-\int\frac{2dx}{x}$$

$$\int\frac{2v+1}{v^2+v+1}dv-\int\frac{3}{v^2+2v\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1}=-2\int\frac{dx}{x}$$

$$\int\frac{2v+1}{v^2+v+1}dv-\int\frac{3}{\left(v+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dv=-2\int\frac{dx}{x}$$

$$\log\left|v^2+v+1\right|-3\left(\frac{2}{\sqrt{3}}\right)\tan^{-1}\left(\frac{v+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)=-2\log|x|+c$$

$$\log\left|y^2+xy+x^2\right|=2\sqrt{3}\tan^{-1}\left(\frac{2y+x}{x\sqrt{3}}\right)+c$$

Differential Equations Ex 22.9 Q33

$$\left(2x^2y + y^3\right)dx + \left(xy^2 + 3x^3\right)dy = 0$$

$$\frac{dy}{dx} = \frac{2x^2y + y^3}{3x^3 - xy^2}$$

It is a homogeneous equation

Put $\quad y = vx$

and $\quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{2x^2vx + v^3x^3}{3x^3 - xv^2x^2}$$

$$x\frac{dv}{dx} = \frac{2v + v^3}{3 - v^2} - v$$

$$= \frac{2v + v^3 - 3v + v^3}{3 - v^2}$$

$$x\frac{dv}{dx} = \frac{2v^3 - v}{3 - v^2}$$

$$\int \frac{3 - v^2}{2v^3 - v}dv = \int \frac{dx}{x} \qquad \text{---(i)}$$

$$\frac{3 - v^2}{v\left(2v^2 - 1\right)} = \frac{A}{(v)} + \frac{Bv + c}{\left(2v^2 - 1\right)}$$

$$3 - v^2 = A\left(2v^2 - 1\right) + (Bv + c)(v)$$

$$= 2Av^2 - A + Bv^2 + cv$$

$$3 - v^2 = (2A + B)v^2cv - A$$

Comparing the coefficient of like powers of v

$A = -3$

$C = 0$

and $\quad 2A + B = -1$

$\Rightarrow \quad 2(-3) + B = -1$

$\Rightarrow \quad B = 5$

So,

$$\int \frac{-3}{v}dv + \int \frac{5v}{2v^2 - 1}dv = \int \frac{dx}{x}$$

$$-3\int \frac{1}{v}dv + \frac{5}{4}\int \frac{4v}{2v^2 - 1}dv = \int \frac{dx}{x}$$

$$-3\log|v| + \frac{5}{4}\log\left|2v^2 - 1\right| = \log|x| + \log|c|$$

$$-12\log|v| + 5\log\left|2v^2 - 1\right| = 4\log|x| + 4\log|c|$$

$$\frac{\left|2v^2 - 1\right|^5}{v^{12}} = x^4c^4$$

$$\frac{\left|2y^2 - x^2\right|^5}{x^{10}} = x^4c^4\left(\frac{y}{x}\right)^{12}$$

$$\left|2y^2 - x^2\right|^5 = x^{14}c^4\frac{y^{12}}{x^{12}}$$

$$x^2c^4y^{12} = \left|2y^2 - x^2\right|^5$$

Differential Equations Ex 22.9 Q34

$$x\frac{dy}{dx} - y + x\sin\left(\frac{y}{x}\right) = 0$$

$$\frac{dy}{dx} = \frac{y - x\sin\left(\frac{y}{x}\right)}{x}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{vx - x\sin\left(\frac{vx}{x}\right)}{x}$$

$$x\frac{dv}{dx} = v - \sin v - v$$

$$\int \operatorname{cosec} v\,dv = -\int\frac{dx}{x}$$

$$\log|\operatorname{cosec} v + \cot v| = -\log\frac{c}{x}$$

$$\log|\operatorname{cosec} v + \cot v| = \log\frac{x}{c}$$

$$\operatorname{cossec}\left(\frac{y}{x}\right) + \frac{\cos\left(\frac{y}{x}\right)}{\sin\left(\frac{y}{x}\right)} = \frac{x}{c}$$

$$\frac{\left(1 + \cos\frac{y}{x}\right)}{\sin\left(\frac{y}{x}\right)} = \frac{x}{c}$$

$$x\sin\left(\frac{y}{x}\right) = c\left(1 + \cos\frac{y}{x}\right)$$

Differential Equations Ex 22.9 Q35

$$ydx + \left\{x\log\left(\frac{y}{x}\right)\right\}dy - 2xdy = 0$$

$$y + x\log\left(\frac{y}{x}\right)\frac{dy}{dx} - 2x\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{y}{2x - x\log\left(\frac{y}{x}\right)}$$

It is a homogeneous equation

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So,

$$v + x\frac{dv}{dx} = \frac{vx}{2x - x\log\left(\frac{vx}{x}\right)}$$

$$x\frac{dv}{dx} = \frac{v}{2 - \log v} - v$$

$$x\frac{dv}{dx} = \frac{v - 2v + v\log v}{2 - \log v}$$

$$\int\frac{\log v - 2}{v(\log v - 1)}dv = -\int\frac{dx}{x}$$

Let $\log v - 1 = t$

$$\frac{1}{v}dv = dt$$

$$\int\left(\frac{t-1}{t}\right)dt = -\int\frac{dx}{x}$$

$$t - \log|t| = \log\left|\frac{c}{x}\right|$$

$$\log v - 1\log(\log v - 1) = \log\left|\frac{c}{x}\right|$$

$$\log e^{\log v - 1} - \log|\log v - 1| = \log\left|\frac{c}{x}\right|$$

$$e^{\log\left(\frac{v}{e}\right)} = \frac{c}{x}|\log v - 1|$$

$$\frac{v}{e} = \frac{c}{x}|\log v - 1|$$

$$y = c_1\left\{\log\left|\frac{y}{x}\right| - 1\right\}$$

Differential Equations Ex 22.9 Q36(i)

$$\left(x^2 + y^2\right)dx = 2xydy,\ y(1) = 0$$

$$\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$$

It is a homogenues equation

Put $\quad y = vx$

and $\quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{x^2 + v^2x^2}{2xvx}$$

$$x\frac{dv}{dx} = \frac{1 + v^2}{2v} - v$$

$$x\frac{dv}{dx} = \frac{1 + v^2 - 2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1 - v^2}{2v}$$

$$\int\frac{2v}{1 - v^2} = \int\frac{dx}{x}$$

$$\log\left|1 - v^2\right| = -\log|x| + \log|c|$$

$$\log\left|1 - v^2\right| = \log\left|\frac{c}{x}\right|$$

$$\left|\frac{x^2 - y^2}{x^2}\right| = \left|\frac{c}{x}\right|$$

$$\left|x^2 - y^2\right| = |cx| \qquad \text{---(i)}$$

Put $y = 0,\ x = 1$

$$1 - 0 = c$$

$$c = 1$$

Put the value of c in equation (i),

$$\left|x^2 - y^2\right| = |x|$$

$$\left(x^2 - y^2\right)^2 = x^2$$

Differential Equations Ex 22.9 Q36(ii)

Here, $xe^{\frac{y}{x}} - y + x\frac{dy}{dx} = 0,\ y(e) = 0$

$$\frac{dy}{dx} = \frac{y - xe^{\frac{y}{x}}}{x}$$

It is a homogeneous equation

Put $\quad y = vx$

and $\quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{vx - xe^{\frac{vx}{x}}}{x}$$

$$x\frac{dv}{dx} = v - e^{v} - v$$

$$x\frac{dv}{dx} = -e^{v}$$

$$\int -e^{-v}dv = \int \frac{dx}{x}$$

$$e^{v} = \log|xc|$$

$$v = \log(\log|xc|)$$

$$\frac{y}{x} = \log\log|x| + k$$

$$y = x\log(\log|x|) + k \qquad ---(i)$$

Put $\quad y = 0,\ x = e$

$$0 = e\log(\log e) + k$$

$$0 = e \times 0 + k$$

$$0 = k$$

Using equation (i),

$$y = x\log(\log|x|)$$

Differential Equations Ex 22.9 Q36(iii)

$\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\frac{y}{x} = 0, y(1) = 0$

Here it is a homogeneous equation

Put $\quad y = vx$

And

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So,

$$v + x\frac{dv}{dx} = \frac{vx}{x} - \operatorname{cosec}\frac{vx}{x}$$

$$x\frac{dv}{dx} = v - \operatorname{cosec} v - v$$

$$= -\operatorname{cosec} v$$

$$\frac{dv}{\operatorname{cosec} v} = -\frac{dx}{x}$$

$$\sin v\,dv = -\frac{dx}{x}$$

$$-\cos v = -\log|x| + c$$

$$-\cos\frac{y}{x} = -\log|x| + c$$

Now putting $y = 0, x = 1$, we have

$$c = -1$$

Now

$$-\cos\frac{y}{x} + 1 = -\log|x|$$

$$\log|x| = \cos\frac{y}{x} - 1$$

Differential Equations Ex 22.9 Q36(iv)

$$\left(xy - y^2\right)dx - x^2dy = 0,\ y(1) = 1$$

$$\frac{dy}{dx} = \frac{xy - y^2}{x^2}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{xvx - v^2x^2}{x^2}$$

$$x\frac{dv}{dx} = v - v^2 - v$$

$$x\frac{dv}{dx} = -v^2$$

$$-\int\frac{1}{v^2}dv = \int\frac{dx}{x}$$

$$-\left(-\frac{1}{v}\right) = \log|x| + c$$

$$\frac{x}{y} = \log|x| + c \qquad ---(i)$$

Put $y = 1,\ x = 1$

$1 = c$

Using equation (1),

$$x = y\left[\log|x| + 1\right]$$

$$y = \frac{x}{\left[\log|x| + 1\right]}$$

Differential Equations Ex 22.9 Q36(v)

$$\frac{dy}{dx} = \frac{y(x + 2y)}{x(2x + y)},\ y(1) = 2$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{vx(x + 2vx)}{x(2x + vx)}$$

$$x\frac{dv}{dx} = \frac{v(1 + 2v)}{(2 + v)} - v$$

$$x\frac{dv}{dx} = \frac{v + 2v^2 - 2v - v^2}{2 + v}$$

$$x\frac{dv}{dx} = \frac{v^2 - v}{2 + v}$$

$$\frac{2 + v}{v^2 - v}dv = \frac{dx}{x}$$

$$\int\frac{2 + v}{v^2 - v}dv = \int\frac{dx}{x} \qquad ---(i)$$

$$\frac{2 + v}{v(v - 1)} = \frac{A}{v} + \frac{B}{v - 1}$$

$$\frac{2 + v}{v(v - 1)} = \frac{A(v - 1) + Bv}{v(v - 1)}$$

$$2 + v = (A + B)v - A$$

Comparing the coefficients of like powers of v,

$A = -2$

$A + B = 1$

$\Rightarrow \quad -2 + B = 1$

$\Rightarrow \quad B = 3$

Using equation (i),

$$\int\frac{-2}{v}dv + 3\int\frac{1}{v - 1}dv = \int\frac{dx}{x}$$

$$-2\log|v| + 3\log|v - 1| = \log|cx|$$

$$|v - 1|^3 = v^2cx$$

$$\frac{|y - x|^3}{x^3} = \frac{y^2}{x^2}cx$$

Differential Equations Ex 22.9 Q36(vi)

$$\left(y^4 - 2x^3y\right)dx + \left(x^4 - 2xy^3\right)dy = 0$$

$$\frac{dy}{dx} = \frac{2x^3y - y^4}{x^4 - 2xy^3}$$

It is a homogeneous equation

Put $\quad y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So,

$$v + x\frac{dv}{dx} = \frac{2x^3vx - x^4v^4}{x^4 - 2xv^3x^3}$$

$$x\frac{dv}{dx} = \frac{2v - v^4}{1 - 2v^3} - v$$

$$x\frac{dv}{dx} = \frac{2v - v^4 - v + 2v^4}{1 - 2v^3}$$

$$x\frac{dv}{dx} = \frac{v^4 + v}{1 - 2v^3}$$

$$\int \frac{1 - 2v^3}{v\left(v^3 + 1\right)}dv = \int \frac{dx}{x} \qquad \text{---(i)}$$

$$\frac{1 - 2v^3}{v\left(v+1\right)\left(v^2 - v + 1\right)} = \frac{A}{v} + \frac{B}{V + 1} + \frac{cv + D}{v^2 - v + 1}$$

$$1 - 2v^3 = A\left(v^3 + 1\right) + Bv\left(v^2 - v + 1\right) + \left(cv + D\right)\left(v^2 + v\right)$$

$$= Av^3 + A + cv^3 - Bv^2 + cv + cv^3 + cv^2 + Dv^2 + Dv$$

$$1 - 2v^3 = v^3\left(A + B + C\right) + v^2\left(-B + C + D\right) + v\left(B + D\right) + A$$

Comparing the coefficients of like powers of v

$$A = 1 \qquad \text{---(ii)}$$

$$B + D = 0 \qquad \text{---(iii)}$$

$$-B + C + D = 0 \qquad \text{---(iv)}$$

$$A + B + C = -2 \qquad \text{---(v)}$$

Solution of equation (ii), (iii), (iv), (v) gives

$$A = 1,\ b = -1,\ c = -2,\ x = 1$$

Using equation (i),

$$\int \frac{1}{v}dv - \int \frac{1}{v+1}dv - \int \frac{2v - 1}{v^2 - v + 1}dv = \int \frac{dx}{x}$$

$$\log|v| - \log|v + 1| - \log\left|v^2 - v + 1\right| = \log|xc|$$

$$\log\left|\frac{v}{v^3 + 1}\right| = \log|xc|$$

Differential Equations Ex 22.9 Q36(vii)

Here, $x\left(x^2+3y^2\right)dx+y\left(y^2+3x^2\right)dy=0,\ y(1)=1$

$$\frac{dy}{dx}=-\frac{x\left(x^2+3y^2\right)}{y\left(y^2+3x^2\right)}$$

It is a homogeneous equation

Put $\quad y=vx$

and $\quad \frac{dy}{dx}=v+x\frac{dv}{dx}$

So,

$$v+x\frac{dv}{dx}=-\frac{x\left(x^2+3v^2x^2\right)}{vx\left(v^2x^2+3x^2\right)}$$

$$x\frac{dv}{dx}=-\frac{\left(1+3v^2\right)}{v\left(v^2+3\right)}-v$$

$$x\frac{dv}{dx}=\frac{-1-3v^2-v^4-3v^2}{v\left(v^2+3\right)}$$

$$=\frac{-v^4-6v^2-1}{v\left(v^2+3\right)}$$

$$\frac{v\left(v^2+3\right)}{v^4+6v^2+1}dc=-\frac{dx}{x}$$

$$\int\frac{4v^3+12v}{v^4+6v^2+1}dv=-4\int\frac{dx}{x}$$

$$\log\left|v^4+6v^2+1\right|=\log\left|\frac{c}{x^4}\right|$$

$$\left|v^4+6v^2+1\right|=\left|\frac{c}{x^4}\right|$$

$$\left|y^4+6y^2x^2+x^4\right|=|c| \qquad \text{---(i)}$$

Put $\quad y=1,\ x=1$

$$(1+6+1)=c$$

$\Rightarrow \quad c=8$

Put $c=8$ in equation (i),

$$\left(y^4+x^4+6x^2y^2\right)=8$$

Differential Equations Ex 22.9 Q36(viii)

$$\left\{x\sin^2\left(\frac{y}{x}\right)-y\right\}dx+xdy=0$$

$$\left\{x\sin^2\left(\frac{y}{x}\right)-y\right\}dx=-xdy$$

$$\sin^2\left(\frac{y}{x}\right)+\frac{y}{x}=\frac{dy}{dx}\ldots\ldots..(i)$$

Let $v=\frac{y}{x}$

$$v+x\frac{dv}{dx}=\frac{dy}{dx}$$

From eq (i)

$$\sin^2 v+v=v+x\frac{dv}{dx}$$

$$\frac{1}{\sin^2 v}dv=\frac{1}{x}dx$$

Integrating on both the sides we have,

$$\int\frac{1}{\sin^2 v}dv=\int\frac{1}{x}dx$$

$$-\cot v=\log(x)+C$$

$$-\cot\left(\frac{y}{x}\right)=\log(x)+C\ldots\ldots\ldots(ii)$$

Put x= 1 y = $\frac{\pi}{4}$ in eq (ii)

$$-\cot\left(\frac{\pi}{4}\right) = \log(1) + C$$

C = −1

From eq (ii) we have

$$-\cot\left(\frac{y}{x}\right) = \log(x) - 1$$

Differential Equations Ex 22.9 Q36(ix)

$$\left\{x\sin^2\left(\frac{y}{x}\right) - y\right\}dx + xdy = 0$$

Here it is a homogeneous equation

Put $y = vx$

And

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So,

$$v + x\frac{dv}{dx} = -\sin^2\left(\frac{vx}{x}\right) + \frac{vx}{x}$$

$$x\frac{dv}{dx} = -\sin^2 v$$

$$\frac{dv}{\sin^2 v} = -\frac{dx}{x}$$

$$\cot\left(\frac{y}{x}\right) = \log|cx|$$

$$x\frac{dy}{dx} - y + x\sin\left(\frac{y}{x}\right) = 0, y(2) = \pi$$

Here it is a homogeneous equation

Put $y = vx$

And

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So,

$$v + x\frac{dv}{dx} = \frac{vx}{x} - \sin\left(\frac{vx}{x}\right)$$

$$x\frac{dv}{dx} = -\sin v$$

$$\frac{dv}{\sin v} = -\frac{dx}{x}$$

$$\operatorname{cosec} v dv = -\frac{dx}{x}$$

$$-\log(\operatorname{cosec} v + \cot v) = -\log x + c$$

Now putting $y = \pi, x = 2$, we have

$c = 0.301$

Now

$$-\log\left(\operatorname{cosec}\left(\frac{y}{x}\right) + \cot\left(\frac{y}{x}\right)\right) = -\log x + 0.301$$

Differential Equations Ex 22.9 Q37

Consider the given equation

$$x\cos\left(\frac{y}{x}\right)\frac{dy}{dx}=y\cos\left(\frac{y}{x}\right)+x$$

This is a homogeneous differential equation.

Thus, substiuting $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

in the above equation, we get,

$$x\cos\left(\frac{vx}{x}\right)\left(v+x\frac{dv}{dx}\right) = vx\cos\left(\frac{vx}{x}\right)+x$$

$$\Rightarrow \cos v\left(v+x\frac{dv}{dx}\right)=v\cos\left(\frac{vx}{x}\right)+1$$

$$\Rightarrow v\cos v+x\cos v\frac{dv}{dx}=v\cos v+1$$

$$\Rightarrow x\cos v\frac{dv}{dx}=1$$

$$\Rightarrow \cos v\,dv=\frac{dx}{x}$$

Integrating both the sides,

$$\Rightarrow \int\cos v\,dv=\int\frac{dx}{x}$$

$$\Rightarrow \sin v = \log x + C$$

$$\Rightarrow \sin\left(\frac{y}{x}\right) = \log x + C \ldots(1)$$

Given that when $x = 1$, $y = \frac{\pi}{4}$

Substituting the values, $x = 1$ and $y = \frac{\pi}{4}$

in equation (1), we get,

$$\Rightarrow \sin\left(\frac{\frac{\pi}{4}}{1}\right) = \log 1 + C$$

$$\Rightarrow \sin\left(\frac{\pi}{4}\right) = 0 + C$$

$$\Rightarrow \frac{1}{\sqrt{2}} = C$$

Subsituting the value of C, in equation (1) we get,

$$\sin\left(\frac{y}{x}\right) = \log x + \frac{1}{\sqrt{2}}$$

Differential Equations Ex 22.9 Q38

consider the given equation

$$(x-y)\frac{dy}{dx} = x + 2y$$

This is a homogeneous equation.

Substiuting y=vx and $\frac{dy}{dx} = \left(v + x\frac{dv}{dx}\right)$ in

the above equation, we have,

$$(x-vx)\left(v + x\frac{dv}{dx}\right) = x + 2vx$$

$$\Rightarrow (1-v)\left(v + x\frac{dv}{dx}\right) = 1 + 2v$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{1+2v}{1-v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+2v}{1-v} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+2v-v(1-v)}{1-v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+2v-v+v^2}{1-v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+v+v^2}{1-v}$$

$$\Rightarrow \frac{(1-v)dv}{(1+v+v^2)} = \frac{dx}{x}$$

Integrating on both the sides, we have,

Integrating on both the sides, we have,

$$\Rightarrow \int\frac{(1-v)dv}{(1+v+v^2)} = \int\frac{dx}{x}$$

$$\Rightarrow \frac{3}{2}\int\frac{dv}{(1+v+v^2)} - \int\frac{1}{2}\frac{(2v+1)dv}{(1+v+v^2)} = \int\frac{dx}{x}$$

$$\Rightarrow \frac{3}{2}\int\frac{dv}{v^2+\frac{1}{4}+v+\frac{3}{4}} - \frac{1}{2}\int\frac{(2v+1)dv}{(1+v+v^2)} = \int\frac{dx}{x}$$

$$\Rightarrow \frac{3}{2}\int\frac{dv}{\left(v+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} - \frac{1}{2}\int\frac{(2v+1)dv}{(1+v+v^2)} = \int\frac{dx}{x}$$

$$\Rightarrow \frac{3}{2}\times\frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}\frac{v+\frac{1}{2}}{\frac{\sqrt{3}}{2}} - \frac{1}{2}\log\left(1+v+v^2\right) = \log x + C$$

$$\Rightarrow \sqrt{3}\tan^{-1}\frac{2v+1}{\sqrt{3}} - \frac{1}{2}\log\left(1+v+v^2\right) = \log x + C$$

$$\Rightarrow \sqrt{3}\tan^{-1}\frac{2\left(\frac{y}{x}\right)+1}{\sqrt{3}} - \frac{1}{2}\log\left(1+\left(\frac{y}{x}\right)+\left(\frac{y}{x}\right)^2\right) = \log x + C\ldots(1)$$

Given that when x = 1, y = 0

Substituting the values, in the above equation, we get,

$$\Rightarrow \sqrt{3}\tan^{-1}\frac{2\times 0+1}{\sqrt{3}}-\frac{1}{2}\log\left(1+0+0^2\right)=\log 1+C$$

$$\Rightarrow \sqrt{3}\tan^{-1}\frac{1}{\sqrt{3}}-\frac{1}{2}\times 0=0+C$$

$$\Rightarrow C=\sqrt{3}\times\frac{\pi}{6}$$

$$\Rightarrow C=\frac{\pi}{2\sqrt{3}}$$

Thus, equation (1) becomes,

$$\sqrt{3}\tan^{-1}\frac{2\left(\frac{y}{x}\right)+1}{\sqrt{3}}-\frac{1}{2}\log\left(1+\left(\frac{y}{x}\right)+\left(\frac{y}{x}\right)^2\right)=\log x+\frac{\pi}{2\sqrt{3}}$$

$$\Rightarrow \sqrt{3}\tan^{-1}\frac{2y+x}{x\sqrt{3}}-\frac{\pi}{2\sqrt{3}}=\log x+\frac{1}{2}\log\left(1+\left(\frac{y}{x}\right)+\left(\frac{y}{x}\right)^2\right)$$

$$\Rightarrow 2\sqrt{3}\tan^{-1}\frac{2y+x}{x\sqrt{3}}-\frac{\pi}{\sqrt{3}}=\log x^2+\log\left(\frac{x^2+xy+y^2}{x^2}\right)$$

$$\Rightarrow 2\sqrt{3}\tan^{-1}\frac{2y+x}{x\sqrt{3}}-\frac{\pi}{\sqrt{3}}=\log\left(x^2+xy+y^2\right)$$

Differential Equations Ex 22.9 Q39

$$\frac{dy}{dx}=\frac{xy}{x^2+y^2}$$

$$\frac{dy}{dx}=\left(\frac{1}{\frac{x}{y}+\frac{y}{x}}\right)\ldots\ldots.(i)$$

Let $v=\frac{y}{x}$

$$x\frac{dv}{dx}+v=\frac{dy}{dx}$$

From (i) we have,

$$x\frac{dv}{dx}+v=\left(\frac{1}{\frac{1}{v}+v}\right)$$

$$\left(-\frac{1}{v^3}-\frac{1}{v}\right)dv=\frac{1}{x}dx$$

Integrating on both the sides we have

$$\frac{1}{2v^2}-\log v=\log x+C$$

$$\Rightarrow \frac{x^2}{2y^2}=\log\left(\frac{y}{x}\times x\right)+C\ldots\ldots..(ii)$$

Put x =0 , y = 1

$$0=\log(1)+C$$

$$C=0$$

From eq (ii) we have

$$\frac{x^2}{2y^2}=\log(y)$$

Ex 22.10

Differential Equations Ex 22.10 Q1

Here, $\frac{dy}{dx}+2y = e^{3x}$

This is a linear differential equation, comparing it with

$$\frac{dy}{dx}+Py = Q$$

$$P = 2, Q = e^{3x}$$

I.F. $= e^{\int Pdx}$

$= e^{\int 2dx}$

$= e^{2x}$

Multiplying both the sides by I.F.

$$e^{2x}\frac{dy}{dx}+e^{2x}2y = e^{2x}\times e^{3x}$$

$$e^{2x}\frac{dy}{dx}+e^{2x}2y = e^{5x}$$

Integrating it with respect to x,

$$ye^{2x} = \int e^{5x}dx + c$$

$$ye^{2x} = \frac{e^{5x}}{5}+c$$

$$y = \frac{e^{3x}}{5}+ce^{-2x}$$

Differential Equations Ex 22.10 Q2

Here, $4\frac{dy}{dx}+8y = 5e^{-3x}$

$$\frac{dy}{dx}+2y = \frac{5}{4}e^{-3x}$$

This is a linear differential equation, comparing it with

$$\frac{dy}{dx}+Py = Q$$

$$P = 2, Q = \frac{5}{4}e^{-3x}$$

I.F. $= e^{\int Pdx}$

$= e^{\int 2dx}$

$= e^{2x}$

Solution of the equation is given by

$$y\times(\text{I.F}) = \int Q\times(\text{I.F})dx + c$$

$$ye^{2x} = \int \frac{5}{4}e^{-3x}\times e^{2x}dx + c$$

$$ye^{2x} = \int \frac{5}{4}e^{-x}dx + c$$

$$ye^{2x} = \frac{-5}{4}e^{-x} + c$$

$$y = \frac{-5}{4}e^{-3x} + ce^{-2x}$$

Differential Equations Ex 22.10 Q3

Here, $\frac{dy}{dx}+2y = 6e^{x}$

It is a linear differential equation, comparing it with

$$\frac{dy}{dx}+Py = Q$$

$$P = 2, Q = 6e^{x}$$

I.F. $= e^{\int Pdx}$

$= e^{\int 2dx}$

$= e^{2x}$

Solution of the equation is given by,

$$y\times(\text{I.F}) = \int Q\times(\text{I.F})dx + c$$

$$y\times\left(e^{2x}\right) = \int 6e^{x}\times e^{2x}dx + c$$

$$= \int 6e^{3x}dx + c$$

$$ye^{2x} = \frac{6}{3}e^{3x} + c$$

$$ye^{2x} = 2e^{3x} + c$$

$$y = 2e^{x} + ce^{-2x}$$

Differential Equations Ex 22.10 Q4

Here, $\frac{dy}{dx} + y = e^{-2x}$

This is a linear differential equation, comparing it with

$$\frac{dy}{dx} + Py = Q$$

$$P = 1, Q = e^{-2x}$$

I.F. $= e^{\int P dx}$

$$= e^{\int 2dx}$$

$$= e^{x}$$

Solution of the equation is given by,

$$y \times (\text{I.F}) = \int Q \times (\text{I.F}) dx + c$$

$$y \times e^{x} = \int e^{-2x} \times e^{x} dx + c$$

$$= \int e^{-x} + c$$

$$ye^{x} = \frac{e^{-x}}{-1} + c$$

$$y = -e^{-2x} + ce^{-x}$$

Differential Equations Ex 22.10 Q6

Here, $\frac{dy}{dx} + 2y = 4x$

It is a linear differential equation, comparing it with

$$\frac{dy}{dx} + Py = Q$$

$$P = 2, Q = 4x$$

I.F. $= e^{\int P dx}$

$$= e^{\int 2dx}$$

$$= e^{2x}$$

Solution of the equation is given by,

$$y \times (\text{I.F}) = \int Q \times (\text{I.F}) dx + c$$

$$y \times e^{2x} = \int 4x \times e^{2x} dx + c$$

$$= 4\left[x \times \int e^{2x} dx - \int \left(1 \times \int e^{2x} dx\right) dx\right] + c$$

Using integration by parts

$$y \times e^{2x} = 4\left[x \times \frac{e^{2x}}{2} - \int \frac{e^{2x}}{2} dx\right] + c$$

$$= 2xe^{2x} - 2\frac{e^{2x}}{2} + c$$

$$ye^{2x} = 2xe^{2x} - e^{2x} + c$$

$$ye^{2x} = (2x - 1)e^{2x} + c$$

$$y = (2x - 1) + ce^{-2x}$$

Differential Equations Ex 22.10 Q7

Here, $x\frac{dy}{dx} + y = xe^{x}$

$$\frac{dy}{dx} + \frac{y}{x} = e^{x}$$

It is a linear differential equation, comparing it with

$$\frac{dy}{dx} + Py = Q$$

$$P = \frac{1}{x}, Q = e^{x}$$

I.F. $= e^{\int P dx}$

$$= e^{\int \frac{1}{x} dx}$$

$$= e^{\log x}$$

$$= x$$

Solution of the equation is given by,

$$y \times (\text{I.F}) = \int Q \times (\text{I.F}) dx + c$$

$$y \times (x) = \int e^{x} \times x dx + c$$

$$xy = x\int e^{x} dx - \int \left(1 \times \int e^{x} dx\right) dx + c$$

Using integration by parts

$$= xe^{x} - \int e^{x} dx + c$$

$$= xe^{x} - e^{x} + c$$

$$xy = (x - 1)e^{x} + c$$

$$y = \left(\frac{x - 1}{x}\right)e^{x} + \frac{c}{x},\ x > 0$$

Differential Equations Ex 22.10 Q8

Here, $\frac{dy}{dx}+\frac{4x}{x^2+1}y=-\frac{1}{\left(x^2+1\right)^2}$

It is a linear differential equation, comparing it with

$$\frac{dy}{dx}+Py=Q$$

$$P=\frac{4x}{x^2+1},\ Q=-\frac{1}{\left(x^2+1\right)^2}$$

I.F. $=e^{\int Pdx}$

$$=e^{\int\frac{4x}{x^2+1}dx}$$

$$=e^{2\int\frac{2x}{x^2+1}dx}$$

$$=e^{2\log\left|x^2+1\right|}$$

$$=\left(x^2+1\right)^2$$

Solution of the equation is given by,

$$y\times(\text{I.F})=\int Q\times(\text{I.F})dx+c$$

$$y\left(x^2+1\right)^2=\int-\frac{1}{\left(x^2+1\right)^2}\left(x^2+1\right)^2xdx+c$$

$$y\left(x^2+1\right)^2=\int-dx+c$$

$$y\left(x^2+1\right)^2=-x+c$$

$$y=-\frac{x}{\left(x^2+1\right)^2}+\frac{c}{\left(x^2+1\right)^2}$$

Differential Equations Ex 22.10 Q9

Here, $x\frac{dy}{dx}+y=x\log x$

$$\frac{dy}{dx}+\frac{y}{x}=\log x$$

It is a linear differential equation, comparing it with

$$\frac{dy}{dx}+Py=Q$$

$$P=\frac{1}{x},\ Q=\log x$$

I.F. $=e^{\int Pdx}$

$$=e^{\int\frac{1}{x}dx}$$

$$=e^{\log|x|}$$

$$=x,\ x>0$$

Solution of the equation is given by,

$$y\times(\text{I.F})=\int Q\times(\text{I.F})dx+c$$

$$y\times x=\int(\log x)(x)dx+c$$

$$yx=\log x\times\int xdx-\int\left(\frac{1}{x}\times\int xdx\right)dx+c$$

$$=\frac{x^2}{2}\log x-\int\frac{x^2}{2x}dx+c$$

$$=\frac{x^2}{2}\log x-\int\frac{x}{2}dx+c$$

$$yx=\frac{x^2}{2}\log x-\frac{x^2}{4}+c$$

$$y=\frac{x}{2}\log x-\frac{x}{4}+\frac{c}{x},\ x>0$$

Differential Equations Ex 22.10 Q10

Here, $x\frac{dy}{dx} - y = (x-1)e^x$

$$\frac{dy}{dx} - \frac{y}{x} = \left(\frac{x-1}{x}\right)e^x$$

It is a linear differential equation. Comparing the equation by,

$$\frac{dy}{dx} + Py = Q$$

$$P = -\frac{1}{x}, Q = \left(\frac{x-1}{x}\right)e^x$$

I.F. $= e^{\int P dx}$

$$= e^{-\int \frac{1}{x}dx}$$

$$= e^{-\log|x|}$$

$$= \frac{1}{x}, \ x > 0$$

Solution of the equation is given by,

$$y \times (\text{I.F}) = \int Q \times (\text{I.F}) dx + c$$

$$y\left(\frac{1}{x}\right) = \int \left(\frac{x-1}{x}\right)e^x \left(\frac{1}{x}\right)dx + c$$

$$\frac{y}{x} = \int \left(\frac{1}{x} - \frac{1}{x^2}\right)e^x dx + c$$

$$\frac{y}{x} = \frac{1}{x}e^x + c$$

Since $\int [f(x) + f'(x)]e^x dx = f(x)e^x + c$

$$y = e^x + \mathrm{Cx}, \ x > 0$$

Differential Equations Ex 22.10 Q11

Here, $\frac{dy}{dx} + \frac{y}{x} = x^3$

It is a linear differential equation. Comparing the equation by,

$$\frac{dy}{dx} + Py = Q$$

$$P = \frac{1}{x}, Q = x^3$$

I.F. $= e^{\int P dx}$

$$= e^{\int \frac{1}{x}dx}$$

$$= e^{\log|x|}$$

$$= x, \ x > 0$$

Solution of the equation is given by,

$$y \times (\text{I.F}) = \int Q \times (\text{I.F}) dx + c$$

$$y \times x = \int x^3 \times (x) dx + c$$

$$xy = \frac{x^5}{5} + c$$

$$y = \frac{x^4}{5} + \frac{c}{x}, \ x > 0$$

Differential Equations Ex 22.10 Q12

$\frac{dy}{dx}+y=\sin x$

It is a linear differential equation. Comparing it with,

$$\frac{dy}{dx}+Py=Q$$

$$p=1, Q=\sin x$$

I.F.

$$=e^{\int pdx}$$

$$=e^{\int dx}$$

$$=e^{x}$$

Solution of the equation is given by,

$$y\times(\text{I.F})=\int Q\times(\text{I.F})dx+c$$

$$y(e^{x})=\int \sin x\times(e^{x})dx+c$$

$$ye^{x}=\frac{e^{x}}{2}(\sin x-\cos x)+c$$

Differential Equations Ex 22.10 Q13

Here, $\frac{dy}{dx}+y=\cos x$

It is a linear differential equation. Comparing the equation by,

$$\frac{dy}{dx}+Py=Q$$

$$P=1, Q=\cos x$$

I.F. $=e^{\int Pdx}$

$$=e^{\int dx}$$

$$=e^{x}$$

Solution of the equation is given by,

$$y\times(\text{I.F})=\int Q\times(\text{I.F})dx+c$$

$$y\left(e^{x}\right)=\int(\cos x)\left(e^{x}\right)+c_1 \qquad \text{---(i)}$$

Let $I=\int e^{x}\cos x dx$

$$=\cos x\times\int e^{x}dx\int\left(\sin x\int e^{x}dx\right)dx+c_2$$

Using integration by parts

$$I=e^{x}\cos x+\int \sin x e^{x}dx+c$$

$$=e^{x}\cos x+\left[\sin x\int e^{x}dx-\int\left(\cos x\int e^{x}dx\right)dx\right]+c_2$$

$$I=e^{x}\cos x+\sin e^{x}-I+c_2$$

$$2I=e^{x}\left(\cos x+\sin x\right)+c_2$$

$$I=\frac{e^{x}}{2}(\cos x+\sin x)+\frac{c_2}{2}$$

$$I=\frac{e^{x}}{2}(\cos x+\sin x)+c_3$$

Putting I in equation (i),

$$ye^{x}=\frac{e^{x}}{2}(\cos x+\sin x)+c_1+c_3$$

$$ye^{x}=\frac{e^{x}}{2}(\cos x+snx)+c$$

$$y=\frac{1}{2}(\cos x+\sin x)+ce^{-x}$$

Differential Equations Ex 22.10 Q14

$$\frac{dy}{dx}+2y=\sin x$$

It is a linear differential equation. Comparing it with,

$$\frac{dy}{dx}+Py=Q$$

$$p=2, Q=\sin x$$

I.F.

$$=e^{\int p dx}$$

$$=e^{\int dx}$$

$$=e^{2x}$$

Solution of the equation is given by,

$$y\times(\text{I.F})=\int Q\times(\text{I.F})dx+c$$

$$y\left(e^{2x}\right)=\int \sin x\times\left(e^{2x}\right)dx+c$$

$$ye^{2x}=\frac{e^{2x}}{5}(2\sin x-\cos x)+c$$

Differential Equations Ex 22.10 Q15

Here, $\frac{dy}{dx}-y\tan x=-2\sin x$

It is a linear differential equation. Comparing the equation by,

$$\frac{dy}{dx}+Py=Q$$

$$P=-\tan x, Q=-2\sin x$$

I.F. $=e^{\int P dx}$

$$=e^{-\int \tan x dx}$$

$$=e^{-\log \sec x}$$

$$=\frac{1}{\sec x}$$

Solution of the equation is given by,

$$y\times(\text{I.F})=\int Q\times(\text{I.F})dx+c$$

$$\frac{y}{\sec x}=\int -\frac{2\sin x}{\sec x}dx+c$$

$$y\cos x=-\int 2\sin x\cos x dx+c$$

$$y\cos x=-\int \sin 2x dx+c$$

$$y\cos x=\frac{\cos 2x}{2}+c$$

$$y=\frac{\cos 2x}{2\cos x}+\frac{c}{\cos x}$$

Differential Equations Ex 22.10 Q16

Here, $\left(1+x^2\right)\frac{dy}{dx}+y = tan^{-1}x$

$$\frac{dy}{dx}+\frac{y}{1+x^2} = \frac{tan^{-1}x}{1+x^2}$$

It is a linear differential equation. Comparing the equation by,

$$\frac{dy}{dx}+Py = Q$$

$$P = \frac{1}{1+x^2}, Q = \frac{tan^{-1}x}{1+x^2}$$

I.F. $= e^{\int P dx}$

$$= e^{\int \frac{1}{1+x^2}dx}$$

$$= e^{tan^{-1}x}$$

Solution of the equation is given by,

$$y \times (\text{I.F}) = \int Q \times (\text{I.F}) dx + c$$

$$y\left(e^{tan^{-1}x}\right) = \int \frac{tan^{-1}x}{1+x^2}e^{tan^{-1}x}dx + c$$

Let $tan^{-1}x = t$

$$\frac{1}{1+t^2}dx = dt$$

So,

$$ye^t = \int t \times e^t dt + c$$

$$= t \times \int e^t dt - \int \left(1 \times e^t dt\right) dt + c$$

Using integration by parts

$$ye^t = te^t - e^t + c$$

$$y = (t-1) ce^{-t}$$

$$y = \left(tan^{-1}x - 1\right) + ce^{-tan^{-1}x}$$

Differential Equations Ex 22.10 Q17

Here, $\frac{dy}{dx}+y\tan x = \cos x$

It is a linear differential equation. Comparing the equation by,

$$\frac{dy}{dx}+Py = Q$$

$$P = \tan x, Q = \cos x$$

I.F. $= e^{\int P dx}$

$$= e^{\int \tan x dx}$$

$$= e^{\log|\sec x|}$$

Solution of the equation is given by,

$$y \times (\text{I.F}) = \int Q \times (\text{I.F}) dx + c$$

$$y \sec x = \int \cos x (\sec x) dx + c$$

$$\frac{y}{\cos x} = \int dx + c$$

$$\frac{y}{\cos x} = x + c$$

$$y = x\cos x + c\cos x$$

Differential Equations Ex 22.10 Q18

$\frac{dy}{dx}+y\cot x=x^2\cot x+2x$

It is a linear differential equation. Comparing it with,

$$\frac{dy}{dx}+Py=Q$$

$$p=\cot x, Q=x^2\cot x+2x$$

I.F.

$=e^{\int pdx}$

$=e^{\int \cot dx}$

$=e^{\log \sin x}$

$=\sin x$

Solution of the equation is given by,

$$y\times(\text{I.F})=\int Q\times(\text{I.F})dx+c$$

$$y(\sin x)=\int\left(x^2\cos x+2x\sin x\right)dx+c$$

$$y\sin x=\int x^2\cos x dx+\int 2x\sin x dx+C$$

$$=x^2\sin x+C$$

Differential Equations Ex 22.10 Q19

Here, $\frac{dy}{dx}+y\tan x=x^2\cos^2 x$

It is a linear differential equation. Comparing the equation by,

$$\frac{dy}{dx}+Py=Q$$

$$P=\tan x, Q=x^2\cos^2 x$$

I.F. $=e^{\int Pdx}$

$=e^{\int \tan xdx}$

$=e^{\log|\sec x|}$

$=\sec x$

Solution of the equation is given by,

$$y\times(\text{I.F})=\int Q\times(\text{I.F})dx+c$$

$$y\sec x=\int x^2\cos^2 x\left(\sec x\right)dx+c$$

$$=\int x^2\cos xdx+c$$

$$=x^2\int\cos xdx-\int\left(2x\int\cos xdx\right)dx+c$$

Using integration by parts

$$y\left(\sec x\right)=x^2\sin x-2\int x\sin xdx+c$$

$$=x^2\sin x-2\left[x\times\int\sin xdx-\int\left(1\times\int\sin xdx\right)dx\right]+c$$

$$y\sec x=x^2\sin x+2x\cos x-2\sin x+c$$

$$y=x^2\sin x\cos x+2x\cos^2 x-2\sin x\cos x+c\cos x$$

Differential Equations Ex 22.10 Q20

Here, $\left(1+x^2\right)\frac{dy}{dx}+y=e^{\tan^{-1}x}$

$$\frac{dy}{dx}+\frac{y}{1+x^2}=\frac{e^{\tan^{-1}x}}{1+x^2}$$

It is a linear differential equation. Comparing the equation by,

$$\frac{dy}{dx}+Py=Q$$

$$P=\frac{1}{1+x^2},\ Q=\frac{e^{\tan^{-1}x}}{1+x^2}$$

I.F. $=e^{\int P dx}$

$$=e^{\int\frac{1}{1+x^2}dx}$$

$$=e^{\tan^{-1}x}$$

Solution of the equation is given by,

$$y\times(\text{I.F})=\int Q\times(\text{I.F})\,dx+c$$

$$y\left(e^{\tan^{-1}x}\right)=\int\frac{e^{\tan^{-1}x}}{1+x^2}\times e^{\tan^{-1}x}dx+c$$

Let $e^{\tan^{-1}x}=t$

$$e^{\tan^{-1}x}*\frac{1}{1+x^2}dx=dt$$

$$y(t)=\int t\,dt+c$$

$$yt=\frac{t^2}{2}+c$$

$$y=\frac{t}{2}+\frac{c}{t}$$

$$y=\left(\frac{1}{2}e^{\tan^{-1}x}+ce^{-\tan^{-1}x}\right)$$

Differential Equations Ex 22.10 Q21

Here, $x\,dy=\left(2y+2x^4+x^2\right)dx$

$$x\frac{dy}{dx}=2y+2x^4+x^2$$

$$\frac{dy}{dx}-\frac{2}{x}y=2x^3+x$$

It is a linear differential equation. Comparing it with equation,

$$\frac{dy}{dx}+Py=Q$$

$$P=-\frac{2}{x},\ Q=2x^3+x$$

I.F. $=e^{\int P dx}$

$$=e^{-2\int\frac{1}{x}dx}$$

$$=e^{-2\log|x|}$$

$$=e^{\log\left(\frac{1}{x^2}\right)}$$

$$=\frac{1}{x^2}$$

Solution of the equation is given by,

$$y\times(\text{I.F})=\int Q\times(\text{I.F})\,dx+c$$

$$y\left(\frac{1}{x^2}\right)=\int\left(2x^3+x\right)\left(\frac{1}{x^2}\right)dx+c$$

$$\frac{y}{x^2}=\int\left(2x+\frac{1}{x}\right)dx+c$$

$$\frac{y}{x^2}=2\frac{x^2}{2}+\log|x|+c$$

$$y=x^4+x^2\log|x|+cx^2$$

Differential Equations Ex 22.10 Q22

Here, $\left(1+y^2\right)+\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$

$$\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=-\left(1+y^2\right)$$

$$e^{\tan^{-1}y}-x=\left(1+y^2\right)\frac{dy}{dx}$$

$$\left(1+y^2\right)\frac{dx}{dy}+x=e^{\tan^{-1}y}$$

$$\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{e^{\tan^{-1}y}}{1+y^2}$$

It is a linear differential equation. Comparing the equation with,

$$\frac{dx}{dy}+Px=Q$$

$$P=\frac{1}{1+y^2},\ Q=\frac{e^{\tan^{-1}y}}{1+y^2}$$

I.F. $=e^{\int P dy}$

$$=e^{\int\frac{1}{1+y^2}dy}$$

$$=e^{\tan^{-1}y}$$

Solution of the equation is given by,

$$x\times(\text{I.F})=\int Q\times(\text{I.F})\,dy+c$$

$$x\left(e^{\tan^{-1}y}\right)=\int\frac{e^{\tan^{-1}y}}{1+y^2}\left(e^{\tan^{-1}y}\right)dy+c$$

Let $e^{\tan^{-1}y}=t$

$$e^{\tan^{-1}y}\left(\frac{1}{1+y^2}\right)dy=dt$$

$$xt=\int t\,dt+c$$

$$xt=\frac{t^2}{t}+c$$

$$x=\frac{1}{2}t+\frac{c}{t}$$

$$x=\frac{1}{2}e^{\tan^{-1}y}+ce^{-\tan^{-1}y}$$

Differential Equations Ex 22.10 Q23

Here, $y^2\frac{dx}{dy}+x-\frac{1}{y}=0$

$$\frac{dx}{dy}+\frac{x}{y^2}=\frac{1}{y^3}$$

It is a linear differential equation. Comparing the equation with,

$$\frac{dx}{dy}+Px=Q$$

$$P=\frac{1}{y^2},\ Q=\frac{1}{y^3}$$

I.F. $=e^{\int P dy}$

$$=e^{\int \frac{1}{y^2}dy}$$

$$=e^{-\frac{1}{y}}$$

Solution of the equation is given by,

$$x\times(\text{I.F})=\int Q\times(\text{I.F})\,dy+c$$

$$x\left(e^{-\frac{1}{y}}\right)=\int\frac{1}{y^3}\left(e^{-\frac{1}{y}}\right)dy+c$$

Let $e^{-\frac{1}{y}}=t$

$\Rightarrow$ $\frac{1}{y}=-\log t$

$$e^{-\frac{1}{y}}\times\frac{1}{y^2}dy=dt$$

$$x(t)=\int\frac{1}{y}dt+c$$

$$=-\int \log+dt+c$$

$$=-\left[\log t\times\int 1\times dt-\int\left(\frac{1}{t}\int 1\times dt\right)dt\right]+c$$

$$=-\left[t\log t-\int\frac{t}{t}dt\right]+c$$

$$x(t)=-t\log t+t+c$$

$$x(t)=-t[\log t-1]+c$$

$$x=-\left[-\frac{1}{y}-1\right]+ce^{\frac{1}{y}}$$

$$x=\frac{1}{y}+1+ce^{\frac{1}{y}}$$

$$x=\left(\frac{1+y}{y}\right)+ce^{\frac{1}{y}}$$

Differential Equations Ex 22.10 Q24

Here, $\left(2x-10y^3\right)\frac{dy}{dx}+y=0$

$$y\frac{dx}{dy}+2x-10y^3=0$$

$$\frac{dx}{dy}=\frac{2}{y}x=10y^2$$

It is a linear differential equation. Comparing the equation with,

$$\frac{dx}{dy}+Px=Q$$

$$P=\frac{2}{y},\ Q=10y^2$$

I.F. $=e^{\int P dy}$

$$=e^{\int\frac{2}{y}dy}$$

$$=e^{2\log|y|}$$

$$=y^2$$

Solution of the equation is given by,

$$x\times(\text{I.F})=\int Q\times(\text{I.F})\,dy+c$$

$$x\left(y^2\right)=\int 10y^2\left(y^2\right)dy+c$$

$$xy^2=10\frac{y^5}{5}+c$$

$$xy^2=2y^5+c$$

$$x=2y^3+\frac{c}{y^2}$$

$$x=2y^3+cy^{-2}$$

Differential Equations Ex 22.10 Q25

Here, $(x+\tan y)dy = \sin 2y dx$

$$x+\tan y = \sin 2y\frac{dx}{dy}$$

$$\sin 2y\frac{dx}{dy} - x = \tan y$$

$$\frac{dx}{dy} - \operatorname{cosec} 2y x = \frac{\tan y}{\sin 2y}$$

It is a linear differential equation. Comparing it with,

$$\frac{dx}{dy} + Px = Q$$

$$P = -\operatorname{cosec} 2y,\ Q = \frac{\tan y}{\sin 2y}$$

I.F. $= e^{-\int \operatorname{cosec} 2y dy}$

$= e^{-\int \operatorname{cosec} 2y dy}$

$= e^{-\frac{1}{2}\log \tan y}$

$= e^{\log\sqrt{\cot y}}$

$= \sqrt{\cot y}$

Solution of the equation is given by,

$$x\times(\text{I.F}) = \int Q\times(\text{I.F})dy + c$$

$$x\sqrt{\cot y} = \int \frac{\tan y}{\sin 2y}\sqrt{\cot y}dy + c$$

$$= \int \frac{\sqrt{\tan y}}{\left(\frac{2\tan y}{1+\tan^2 y}\right)}dy + c$$

$$x\sqrt{\cot y} = \int \frac{1+\tan^2 y}{2\sqrt{\tan y}}dy + c$$

$$\frac{x}{\sqrt{\tan y}} = \frac{1}{2}\int \frac{\sec^2 y}{\sqrt{\tan y}}dy + c$$

Put $\tan y = t$

$$\sec^2 y\times dy = dt$$

$$\frac{x}{\sqrt{\tan y}} = \frac{1}{2}\int \frac{dt}{\sqrt{t}} + c$$

$$= \frac{1}{2}\times 2\sqrt{t} + c$$

$$\frac{x}{\sqrt{\tan y}} = \sqrt{\tan y} + c$$

$$x = \tan y + c\sqrt{\tan y}$$

Differential Equations Ex 22.10 Q26

Here, $dx + xdy = e^{-y}\sec^2 y dy$

$$\frac{dx}{dy} + x = e^{-y}\sec^2 y$$

It is a linear differential equation. Comparing it with,

$$\frac{dx}{dy} + Px = Q$$

$$P = 1,\ Q = e^{-y}\sec^2 y$$

I.F. $= e^{\int P dy}$

$= e^{\int dy}$

$= e^{y}$

Solution of the equation is given by,

$$x\times(\text{I.F}) = \int Q\times(\text{I.F})dy + c$$

$$xe^{y} = \int e^{-y}\sec^2 y e^{y} dy + c$$

$$= \int \sec^2 y dy + c$$

$$xe^{y} = \int \tan y + c$$

$$x = e^{-y}(\tan y + c)$$

Differential Equations Ex 22.10 Q27

Here, $\frac{dy}{dx} = y \tan x - 2 \sin x$

$\frac{dy}{dx} - y \tan x = -2 \sin x$

It is a linear differential equation. Comparing it with,

$\frac{dy}{dx} + Py = Q$

$P = -\tan x, Q = -2 \sin x$

I.F. $= e^{\int P dx}$

$= e^{-\int \tan x dx}$

$= e^{-\log \sec x}$

$= \frac{1}{\sec x}$

$= \cos x$

Solution of the equation is given by,

$y \times (\text{I.F}) = \int Q \times (\text{I.F}) dx + c$

$y \cos x = -\int 2 \sin x \cos x dx + c$

Let $\sin x = t$

$\cos x dx = dt$

$y(\cos x) = -\int 2t dt + c$

$= -t^2 + c$

$y \cos x = -\sin^2 x + c$

$y = \sec x \left(-\sin^2 x + c\right)$

Differential Equations Ex 22.10 Q28

Here, $\frac{dy}{dx} + y \cos x = \sin x \cos x$

It is a linear differential equation. Comparing it with,

$\frac{dy}{dx} + Py = Q$

$P = \cos x, Q = \sin x \cos x$

I.F. $= e^{\int P dx}$

$= e^{\int \cos x dx}$

$= e^{\sin x}$

Solution of the equation is given by,

$y \times (\text{I.F}) = \int Q \times (\text{I.F}) dx + c$

$y\left(e^{\sin x}\right) = \int \sin x \cos x e^{\sin x} dx + c$

Let $\sin x = t$

$\cos x dx = dt$

$ye^t = \int t \times e^t dt + c$

$= t \times \int e^t dt - \int \left(1 \int e^t dt\right) dt + c$

$ye^t = te^t - e^t + c$

$ye^t = e^t (t-1) + c$

$y = t - 1 + ce^{-t}$

$y = \sin x - 1 + ce^{-\sin x}$

Differential Equations Ex 22.10 Q29

Here, $\left(1+x^2\right)\frac{dy}{dx}-2xy=\left(x^2+2\right)\left(x^2+1\right)$

$$\frac{dy}{dx}-\frac{2x}{x^2+1}y=\left(x^2+2\right)$$

It is a linear differential equation. Comparing it with,

$$\frac{dy}{dx}+Py=Q$$

$$P=-\frac{2x}{x^2+1},\ Q=x^2+2$$

I.F. $=e^{\int P dx}$

$$=e^{-\int\frac{2x}{x^2+1}dx}$$

$$=e^{-\log\left|x^2+1\right|}$$

$$=\frac{1}{\left(x^2+1\right)}$$

Solution of the equation is given by,

$$y\times(\text{I.F})=\int Q\times(\text{I.F})dx+c$$

$$y\left(\frac{1}{x^2+1}\right)=\int\left(\frac{x^2+2}{x^2+1}\right)dx+c$$

$$=\int\left(1+\frac{1}{x^2+1}\right)dx+c$$

$$\frac{y}{\left(x^2+1\right)}=x+\tan^{-1}x+c$$

$$y=\left(x^2+1\right)\left(x+\tan^{-1}x+c\right)$$

Differential Equations Ex 22.10 Q30

Here, $(\sin x)\frac{dy}{dx}+y\cos x=2\sin^2 x\cos x$

$$\frac{dy}{dx}+y\cot x=2\sin x\cos x$$

It is a linear differential equation. Comparing it with,

$$\frac{dy}{dx}+Py=Q$$

$$P=\cot x,\ Q=2\sin x\cos x$$

I.F. $=e^{\int P dx}$

$$=e^{\int \cot x dx}$$

$$=e^{\log \sin x}$$

$$=\sin x$$

Solution of the equation is given by,

$$y\times(\text{I.F})=\int Q\times(\text{I.F})dx+c$$

$$y(\sin x)=\int 2\sin x\cos x(\sin x)dx+c$$

ysin x = (2/3)sin^3x+C

Differential Equations Ex 22.10 Q32

Here, $\frac{dy}{dx} + \frac{2y}{x} = \cos x$

It is a linear differential equation. Comparing it with,

$$\frac{dy}{dx} + Py = Q$$

$$P = \frac{2}{x}, Q = \cos x$$

I.F. $= e^{\int P dx}$

$$= e^{2\int \frac{1}{x} dx}$$

$$= e^{2\log|x|}$$

$$= x^2$$

Solution of the equation is given by,

$$y \times (\text{I.F}) = \int Q \times (\text{I.F.}) dx + c$$

$$y\left(x^2\right) = \int \cos x \left(x^2\right) dx + c$$

$$yx^2 = \int x^2 \cos x dx + c$$

$$= x^2 \int \cos x - \int \left(2x \times \int \cos x dx\right) dx + c$$

Using integration by parts

$$yx^2 = x^2 \sin x - \int 2x \sin x dx + c$$

$$= x^2 \sin x - 2\left[x \times \int \sin x dx - \int \left(1 \times \int \sin x dx\right) dx\right] + c$$

$$= x^2 \sin x + 2x \cos x - 2\int \cos x dx + c$$

$$yx^2 = x^2 \sin x + 2x \cos x - 2 \sin x + c$$

$$y = \sin x + \frac{2}{x} \cos x - \frac{2}{x^2} \sin x + \frac{c}{x^2}$$

Differential Equations Ex 22.10 Q33

Here, $\frac{dy}{dx} - y = xe^x$

It is a linear differential equation. Comparing it with,

$$\frac{dy}{dx} + Py = Q$$

$$P = -1, Q = xe^x$$

I.F. $= e^{\int P dx}$

$$= e^{-\int dx}$$

$$= e^{-x}$$

Solution of the equation is given by,

$$y \times (\text{I.F}) = \int Q \times (\text{I.F.}) dx + c$$

$$ye^{-x} = \int xe^x \times e^{-x} dx + c$$

$$= \int x dx + c$$

$$ye^{-x} = \frac{x^2}{2} + c$$

$$y = e^x \left(\frac{x^2}{2} + c\right)$$

Differential Equations Ex 22.10 Q34

Here, $\frac{dy}{dx} + 2y = xe^{4x}$

It is a linear differential equation. Comparing it with,

$$\frac{dy}{dx} + Py = Q$$

$$P = 2, Q = xe^{4x}$$

I.F. $= e^{\int P dx}$

$$= e^{\int 2 dx}$$

$$= e^{2x}$$

Solution of the equation is given by,

$$y \times (\text{I.F}) = \int Q \times (\text{I.F.}) dx + c$$

$$y\left(e^{2x}\right) = \int xe^{4x}\left(e^{2x}\right) dx + c$$

$$= \int xe^{6x} dx + c$$

$$= x \times \int e^{6x} dx - \int\left(1 \int e^{6x} \times dx\right) + c$$

Using integration by parts

$$ye^{2x} = x \times \frac{e^{6x}}{6} - \int \frac{e^{6x}}{6} dx + c$$

$$ye^{2x} = \frac{x}{6}e^{6x} - \frac{e^{6x}}{36} + c$$

$$y = \frac{x}{6}e^{4x} - \frac{e^{4x}}{36} + ce^{-2x}$$

Differential Equations Ex 22.10 Q35

Here, $\left(x + 2y^2\right) \frac{dy}{dx} = y$

$$y\frac{dx}{dy} - x = 2y^2$$

$$\frac{dx}{dy} - \frac{x}{y} = 2y$$

It is a linear differential equation. Comparing it with,

$$\frac{dx}{dy} + Px = Q$$

$$P = -\frac{1}{y}, Q = 2y$$

I.F. $= e^{\int P dy}$

$$= e^{-\int \frac{1}{y} dy}$$

$$= e^{-\log|y|}$$

$$= \frac{1}{y}, \ y > 0$$

Solution of the equation is given by,

$$x \times (\text{I.F}) = \int Q \times (\text{I.F.}) dx + c$$

$$x\left(\frac{1}{y}\right) = \int 2y\left(\frac{1}{y}\right) dy + c$$

$$= \int 2 dy + c$$

$$x\left(\frac{1}{y}\right) = 2y + c \qquad \text{---(i)}$$

Given, when $x = 2$, $y = 1$

So,

$$2 = 2 + c$$

$$c = 0$$

Put the value of c in equation (i),

$$x = 2y^2$$

Differential Equations Ex 22.10 Q36(ii)

Here, $\frac{dy}{dx} - y = \cos 2x$

It is a linear differential equation. Comparing it with,

$$\frac{dy}{dx} + Py = Q$$

$$P = -1, Q = \cos 2x$$

I.F. $= e^{\int P dx}$

$= e^{-\int dx}$

$= e^{-x}$

Solution of the equation is given by,

$$y \times (\text{I.F}) = \int Q \times (\text{I.F.})\, dx + c$$

$$y \times e^{-x} = \int \cos 2x \times e^{-x} dx + c \qquad \text{---(i)}$$

$$I = \int \cos 2x e^{-x} dx = \cos 2x \times \left(-e^{-x}\right) - \int \left(\frac{\sin 2x}{2}\right) e^{-x} dx \qquad \text{[Using integration by parts]}$$

$$I = -e^{-x} \cos 2x - \frac{1}{2}\left[\left(-\sin 2x e^{-x}\right) + \int \frac{\cos 2x}{2} e^{-x} dx\right]$$

$$I = -e^{-x} \cos 2x + \frac{1}{2} \sin 2x e^{-x} - \frac{1}{4} I$$

$$\frac{5}{4} I = \frac{e^{-x}}{2} (\sin 2x - 2\cos 2x)$$

$$I = \frac{2}{5} e^{-x} (\sin 2x - 2\cos 2x)$$

So, solution of the equation is given by

$$y = \frac{2}{5}(\sin 2x - 2\cos 2x) + ce^{x}$$

Differential Equations Ex 22.10 Q36(iii)

Here, $x\frac{dy}{dx} - y = (x+1)e^{-x}$

$$\frac{dy}{dx} - \frac{y}{x} = \left(\frac{x+1}{x}\right)e^{-x}$$

It is a linear differential equation. Comparing it with,

$$\frac{dy}{dx} + Py = Q$$

$$P = -\frac{1}{x}, Q = \left(\frac{x+1}{x}\right)e^{-x}$$

I.F. $= e^{\int P dx}$

$= e^{-\int \frac{1}{x} dx}$

$= e^{-\log|x|}$

$= e^{\log\left(\frac{1}{x}\right)}$

$= \frac{1}{x}, \quad x > 0$

Solution of the equation is given by,

$$y \times (\text{I.F}) = \int Q \times (\text{I.F.})\, dx + c$$

$$y \times \left(\frac{1}{x}\right) = \int \left(\frac{x+1}{x}\right) e^{-x} \times \left(\frac{1}{x}\right) dx + c$$

$$\frac{y}{x} = \int \left(\frac{1}{x} + \frac{1}{x^2}\right) e^{-x} dx + c$$

Let $-x = t$

$$-dx = dt$$

$$y\left(-\frac{1}{x}\right) = \int \left(-\frac{1}{t} + \frac{1}{t^2}\right) e^{t} dt + c$$

$$y\left(-\frac{1}{x}\right) = -\frac{1}{t} e^{t} + c$$

$\left[\text{Since } \int \{f(x) + f'(x)\} e^{x} dx = f(x) e^{x} + c\right]$

$$-\frac{y}{x} = \frac{1}{x} e^{-x} + c$$

$$y = -\left(e^{-x} + cx\right)$$

$$y = -e^{-x} + c_1 x$$

Differential Equations Ex 22.10 Q36(iv)

Here, $x\frac{dy}{dx}+y=x^4$

$$\frac{dy}{dx}+\frac{y}{x}=x^3$$

It is a linear differential equation. Comparing it with,

$$\frac{dy}{dx}+Py=Q$$

$$P=\frac{1}{x},\ Q=x^3$$

I.F. $=e^{\int P dx}$

$$=e^{\int\frac{1}{x}dx}$$

$$=e^{\log|x|}$$

$$=x$$

Solution of the equation is given by,

$$y\times(\text{I.F})=\int Q\times(\text{I.F.})\,dx+c$$

$$yx=\int x^3(x)dx+c$$

$$xy=\frac{x^5}{5}+c$$

$$y=\frac{x^4}{5}+\frac{c}{x},\ x>0$$

Differential Equations Ex 22.10 Q36(v)

Here, $(x\log x)\frac{dy}{dx}+y=\log x$

$$\frac{dy}{dx}+\frac{y}{x\log x}=\frac{1}{x}$$

It is a linear differential equation. Comparing it with,

$$\frac{dy}{dx}+Py=Q$$

$$P=\frac{1}{x\log x},\ Q=\frac{1}{x}$$

I.F. $=e^{\int P dx}$

$$=e^{\int\frac{1}{x\log x}dx}$$

$$=e^{\log|\log x|}$$

$$=\log x$$

Solution of the equation is given by,

$$y\times(\text{I.F})=\int Q\times(\text{I.F.})\,dx+c$$

$$y(\log x)=\int\frac{1}{x}(\log x)dx+c$$

$$y(\log x)=\frac{(\log x)^2}{2}+c$$

$$y=\frac{1}{2}\log x+\frac{c}{\log x},\ x>0,\ x\neq 1$$

Ex 22.11

Differential Equations Ex 22.11 Q1

Let A be the surface area of balloon, so

$$\frac{dA}{dt} \propto t$$

$$\Rightarrow \quad \frac{dA}{dt} = \lambda t$$

$$\Rightarrow \quad \frac{d}{dt}\left(4\pi r^2\right) = \lambda t$$

$$\Rightarrow \quad 8\pi r\frac{dr}{dt} = \lambda t$$

$$\Rightarrow \quad 8\pi r\, dr = \lambda t$$

$$\Rightarrow \quad 8\pi\int r\, dr = \lambda\int t\, dt$$

$$\Rightarrow \quad 8\pi\frac{r^2}{2} = \frac{\lambda t^2}{2} + c$$

$$\Rightarrow \quad 4\pi r^2 = \frac{\lambda t^2}{2} + c ----(1)$$

Given $r = 1$ unit when $t = 0$, so

$$4\pi(1)^2 = 0 + c$$

$$\Rightarrow \quad 4\pi = c$$

Using it is equation (i),

$$4\pi r^2 = \frac{\lambda t^2}{2} + 4\pi ----(2)$$

Also, given $r = 2$ units when $t = 3$ sec.

$$4\pi(2)^2 = \frac{\lambda(3)^2}{2} + 4\pi$$

$$\Rightarrow \quad 16\pi = \frac{9}{2}\lambda + 4\pi$$

$$\Rightarrow \quad \frac{9}{2}\lambda = 12\pi$$

$$\Rightarrow \quad \lambda = \frac{24}{9}\pi$$

$$\Rightarrow \quad \lambda = \frac{8}{3}\pi$$

Now,equation (2) becomes

$$4\pi r^2 = \frac{8\pi}{6}t^2 + 4\pi$$

$$\Rightarrow \quad 4\pi\left(r^2 - 1\right) = \frac{4}{3}\pi t^2$$

$$\Rightarrow \quad r^2 - 1 = \frac{1}{3}t^2$$

$$\Rightarrow \quad r^2 = 1 + \frac{1}{3}t^2$$

$$\therefore \quad r = \sqrt{\left(1 + \frac{1}{3}t^2\right)}$$

Differential Equations Ex 22.11 Q2

Let the population after time t be P and initial population be P_0.

So,

$$\frac{dP}{dt} = 5\% \times P$$

$$\Rightarrow \quad \frac{dP}{dt} = \frac{P}{20}$$

$$\Rightarrow \quad 20\frac{dP}{P} = dt$$

$$\Rightarrow \quad 20\int\frac{dP}{P} = \int dt$$

$$\Rightarrow \quad 20\log|P| = t + c \text{ ----} (1)$$

Given $P = P_0$ when $t = 0$

$$20\log(P_0) = 0 + c$$

$$\Rightarrow \quad 20\log(P_0) = c$$

Now, equation (1) becomes

$$20\log(P) = t + 20\log(P_0)$$

$$\Rightarrow \quad 20\log\left(\frac{P}{P_0}\right) = t$$

Let time is t, when $P = 2P_0$, so,

$$20\log\left(\frac{2P}{P_0}\right) = t_1$$

$$\Rightarrow \quad 20\log 2 = t_1$$

Required time period $= 20\log 2$ years

Differential Equations Ex 22.11 Q3

Let P be the population at any time t and P_0 be the initial population.

So

$$\frac{dP}{dt} \propto P$$

$$\Rightarrow \quad \frac{dP}{dt} = \lambda P$$

$$\Rightarrow \quad \frac{dP}{dt} = \lambda\, dt$$

$$\Rightarrow \quad \int\frac{dP}{dt} = \lambda\int dt +$$

$$\Rightarrow \quad \log P = \lambda t + c \text{ ---} (1)$$

Here, $P = P_0$ t when $t = 0$,

$$\log(P_0) = 0 + c$$

$$\Rightarrow \quad c = \log(P_0)$$

Now, equation (1) becomes

$$\log(P) = \lambda t + \log(P_0)$$

$$\Rightarrow \quad \log\left(\frac{P}{P_0}\right) = \lambda t \text{ ---} (2)$$

Given $P = 2P_0$ when $t = 25$

$$\log\left(\frac{2P_0}{P_0}\right) = 25\lambda$$

$$\Rightarrow \quad \log 2 = 25\lambda$$

$$\Rightarrow \quad \lambda = \frac{\log 2}{25}$$

Now equation (2) becomes

$$\log\left(\frac{P}{P_0}\right) = \left(\frac{\log 2}{25}\right)t$$

let t_1 be the time to become population 500000 from 100000, so,

$$\log\left(\frac{500000}{100000}\right) = \frac{\log 2}{25}t_1$$

$$\Rightarrow \quad t_1 = \frac{25\log 5}{\log 2}$$

$$\Rightarrow \quad = \frac{25(1.609)}{(0.6931)} = 58$$

Required time $= 58$ years

Differential Equations Ex 22.11 Q4

Let C be the count of bacteria at any time t.
It is given that

$$\frac{dC}{dt} \propto C$$

$$\Rightarrow \frac{dC}{dt} = \lambda C, \textit{ where } \lambda \textit{ is a constant of proportionality}$$

$$\Rightarrow \frac{dC}{C} = \lambda dt$$

$$\Rightarrow \int \frac{dC}{C} = \lambda \int dt$$

$$\Rightarrow \log C = \lambda t + \log K \ldots.(1)$$

Initially, at $t = 0$, $C = 100000$
Thus, we have,

$$\log 100000 = \lambda \times 0 + \log K \ldots.(2)$$

$$\Rightarrow \log 100000 = \log K \ldots.(3)$$

At $t = 2$, $C = 100000 + 100000 \times \frac{10}{100} = 110000$

Thus, from (1), *we have,*

$$\log 110000 = \lambda \times 2 + \log K \ldots.(4)$$

Subtracting equation (2) *from* (4), *we have,*

$$\log 110000 - \log 100000 = 2\lambda$$

$$\Rightarrow \log 11 \times 10000 - \log 10 \times 10000 = 2\lambda$$

$$\Rightarrow \log \frac{11 \times 10000}{10 \times 10000} = 2\lambda$$

$$\Rightarrow \log \frac{11}{10} = 2\lambda$$

$$\Rightarrow \lambda = \frac{1}{2} \log \frac{11}{10} \ldots.(5)$$

We need to find the time 't' in which the count reaches 200000.
Substituting the values of λ *and* K *from equations* (3) *and* (5) *in equation* (1), *we have*

$$\log 200000 = \frac{1}{2} \log \frac{11}{10} t + \log 100000$$

$$\Rightarrow \frac{1}{2} \log \frac{11}{10} t = \log 200000 - \log 100000$$

$$\Rightarrow \frac{1}{2} \log \frac{11}{10} t = \log \frac{200000}{100000}$$

$$\Rightarrow \frac{1}{2} \log \frac{11}{10} t = \log 2$$

$$\Rightarrow t = \frac{2 \log 2}{\log \frac{11}{10}} \textit{ hours}$$

Differential Equations Ex 22.11 Q5

Given that, interest is compounded 6% per annum. Let P be principal

$$\frac{dP}{dt} = \frac{\Pr}{100}$$
$$\frac{dP}{dt} = \frac{r}{100}dt$$
$$\int \frac{dP}{P} = \int \frac{r}{100}dt$$
$$\log P = \frac{rt}{100} + c - - - (1)$$

Let P_0 be the initial principal at $t = 0$,

$$\log(P_0) = 0 + c$$
$$c = \log(P_0)$$

Put value of C is equation (1)

$$\log(P) = \frac{rt}{100} + \log(P_0)$$
$$\log\left(\frac{P}{P_0}\right) = \frac{rt}{100}$$

Case I:

Here, $P_0 = 1000, t = 10$ years and $r = 6$

$$\log\left(\frac{P}{1000}\right) = \frac{6 \times 10}{100}$$
$$\log P - \log 1000 = 0.6$$
$$\log P = \log e^{0.6} + \log 1000$$
$$= \log\left(e^{0.6} + 1000\right)$$
$$= \log(1.822 + 1000)$$
$$\log P = \log 1822$$

so,

$$P = \text{Rs}\,1822$$

Rs 1000 will be Rs 1822 after 10 years

Differential Equations Ex 22.11 Q6

Let A be the amount of bacteria present at time t and A_0 be the initial amount of bacteria. Here,

$$\frac{dA}{dt} \propto A$$

$$\frac{dA}{dt} = \lambda A$$

$$\int \frac{dA}{A} = \int \lambda dt$$

$$\log A = \lambda t + c - - - (1)$$

When $t = 0$, $A = A_0$

$$\log(A_0) = 0 + c$$

$$c = \log A_0$$

Using equation (1),

$$\log A = \lambda t + \log A_0$$

$$\log\left(\frac{A}{A_0}\right) = \lambda t - - - - - (2)$$

Given, bacteria triples is 5 hours, so $A = 3A_0$, when $t = 5$

so, $$\log\left(\frac{3A_0}{A_0}\right) = 5\lambda$$

$$\log 3 = 5\lambda$$

$$\lambda = \frac{\log 3}{5}$$

Putting the value of λ in equation (2)

$$\log\left(\frac{A}{A_0}\right) = \frac{\log 3}{5}t$$

Case I: let A_1 be the number of bacteria present 10 hours, os

$$\log\left(\frac{A_1}{A_0}\right) = \frac{\log 3}{5} \times 10$$

$$\log\left(\frac{A_1}{A_0}\right) = 2\log 3$$

$$\log\left(\frac{A_1}{A_0}\right) = 2(1.0986)$$

$$\log\left(\frac{A_1}{A_0}\right) = 2.1972$$

$$A_1 = A_0 e^{2.1972}$$

$$A_1 = A_0 9$$

thus

There will be 9 times the bateria present is 10 hours.

Case II: let t_1 be the time necessary for the bacteria to be 10 times, os

$$\log\left(\frac{A}{A_0}\right) = \frac{\log 3}{5} \times t$$

$$\log\left(\frac{10A_0}{A_0}\right) = \frac{\log 3}{5} \times t_1$$

$$5\ \log 10 = \log 3 t_1$$

$$5\frac{\log 10}{\log 3} = t_1$$

Required time is $\frac{5\log 10}{\log 3}$ hours

Differential Equations Ex 22.11 Q7

Let P be the population of the city at any time t.
It is given that

$$\frac{dP}{dt} \propto P$$

$$\Rightarrow \frac{dP}{dt} = \lambda P, \text{ where } \lambda \text{ is a constant of proportionality}$$

$$\Rightarrow \frac{dP}{P} = \lambda dt$$

$$\Rightarrow \int \frac{dP}{P} = \lambda \int dt$$

$$\Rightarrow \log P = \lambda t + \log K \ldots.(1)$$

Initially, at $t = 1990,\ P = 200000$

Thus, we have,

$$\log 200000 = \lambda \times 1990 + \log K \ldots.(2)$$

At $t = 2000,\ P = 250000$

Thus, from (1), *we have,*

$$\log 250000 = \lambda \times 2000 + \log K \ldots.(3)$$

Subtracting equation (2) *from* (3), *we have,*

$$\log 250000 - \log 200000 = 10\lambda$$

$$\Rightarrow \log \frac{4}{5} = 10\lambda$$

$$\Rightarrow \lambda = \frac{1}{10} \log \frac{4}{5} \ldots.(4)$$

Substituting the value of λ *from equation* (4) *in equation* (1), *we have*

$$\log 200000 = 1990 \times \frac{1}{10} \log \frac{4}{5} + \log K$$

$$\Rightarrow \log K = \log 200000 - 199 \times \log \frac{4}{5} \ldots.(5)$$

Substituting the value of λ, $\log K$ *and* $t = 2010$ *in equation* (1), *we have*

$$\log P = \left\{\frac{1}{10} \log \frac{4}{5}\right\} 2010 + \log 200000 - 199 \times \log \frac{4}{5}$$

$$\Rightarrow \log P = \log \left\{\frac{4}{5}\right\}^{201} + \log\left(200000 \times \left(\frac{5}{4}\right)^{199}\right)$$

$$\Rightarrow P = \left\{\frac{4}{5}\right\}^{201} \times 200000 \times \left(\frac{5}{4}\right)^{199}$$

$$\Rightarrow P = \left(\frac{5}{4}\right)^{2} \times 200000 = \frac{25}{16} \times 200000 = 312500$$

Differential Equations Ex 22.11 Q8

Given,

$$C'(x) = \frac{dC}{dx} = 2 + 0.15x$$

$$dC = (2 + 0.15x)\, dx$$

$$\int dC = \int (2 + 0.15x) dx$$

$$C = 2x + \frac{0.15x^2}{2} + \lambda \text{ - - - - } (1)$$

Given $C = 100$ when $x = 0$, so

$$100 = 0 + 0 + \lambda$$

$$\lambda = 100$$

Put the value of λ in equation (1) total cost function is

$$C(x) = 2x + \frac{0.15x^2}{2} + 100$$

$$C(x) = 2x + 0.075x^2 + 100$$

Differential Equations Ex 22.11 Q9

Let P be principal at any time t at the rate of $r\%$ per annum, so

$$\frac{dP}{dt} = \frac{\text{Pr}}{100}$$

$$\frac{dP}{P} = \frac{r}{100}dt$$

$$\int\frac{dP}{P} = \frac{r}{100}\int dt$$

$$\log P = \frac{rt}{100} + c - - - (1)$$

Let P_o be the initial amount, so

$$\log(P_o) = 0 + c$$

$$c = \log(P_o)$$

Put the value of C in equation (1),

$$\log P = \frac{rt}{100} + \log P_o$$

$$\log P - \log P_o = \frac{rt}{100}$$

$$\log\left(\frac{P}{P_o}\right) = \frac{rt}{100}$$

For $t = 1, r = 8\%$

$$\log\left(\frac{P}{P_o}\right) = \frac{8 \times 1}{100}$$

$$\log\frac{P}{P_o} = 0.08$$

$$\frac{P}{P_o} = e^{0.08}$$

$$\frac{P}{P_o} = 1.0833$$

$$\frac{P}{P_o} - 1 = 1.0833 - 1$$

$$\frac{P - P_o}{P_o} = 0.0833$$

percentage increase in amount in one year

$$= 0.0833 \times 100$$

$$= 8.33\%$$

Required percentage = 8.33%

Differential Equations Ex 22.11 Q10

Here,

$$L\frac{di}{dt}+Ri=E$$

$$\frac{di}{dt}+\frac{R}{L}i=\frac{E}{L}$$

It is a linear differential equation. Compound it with $\frac{\text{dy}}{\text{dx}}+Py=Q$

$$P=\frac{R}{L},\ Q=\frac{E}{L}$$

$$I.F.=e^{\int Pdt}$$

$$=e^{\int\frac{P}{L}dt}$$

$$I.F.==e^{\left(\frac{R}{L}\right)t}$$

Solution of the equation is given by

$$i\left(I.F.\right)=\int Q\left(I.F.\right)dt+c$$

$$i\left(e^{\left(\frac{R}{L}\right)t}\right)=\int\frac{E}{L}\left(e^{\left(\frac{R}{L}\right)t}\right)dt+c$$

$$i\left(e^{\left(\frac{R}{L}\right)t}\right)=\frac{E}{L}\times\frac{L}{R}\left(e^{\left(\frac{R}{L}\right)t}\right)+c$$

$$i\left(e^{\left(\frac{R}{L}\right)t}\right)=\frac{E}{L}\left(e^{\left(\frac{R}{L}\right)t}\right)+c$$

$$i=\left(\frac{E}{L}\right)+c\left(e^{\left(\frac{R}{L}\right)t}\right)------(1)$$

Initiatially there was no current, so put $i=0, t=0$

$$0=\frac{F}{R}+ce^{0}$$

$$0=\frac{F}{R}+c$$

$$c=-\frac{F}{R}$$

Using Equation (1)

$$i=\frac{F}{R}-\frac{F}{R}e^{\left(-\frac{R}{L}\right)t}$$

$$i=\frac{F}{R}\left(1-e^{\left(-\frac{R}{L}\right)t}\right)$$

Differential Equations Ex 22.11 Q11

Let A be the quantity of mass at any time t, so

$$\frac{dA}{dt} \propto A$$
$$\frac{dA}{dP} = -\lambda A$$
$$\frac{dA}{A} = -\lambda dt$$
$$\int \frac{dA}{A} = -\lambda \int dt$$
$$\log A = -\lambda t + c \text{ - - - } (1)$$

Let initial quantity of mass be A_0, so

$$\log A_0 = -\lambda (0) + c$$
$$\log(A_0) = c$$

Now, equation (1) becames,

$$\log\ A = -\lambda t + \log A_0$$
$$\log\left(\frac{A}{A_0}\right) = -\lambda t$$

Let t_1 be the required time to half the mass , so $A = \frac{1}{2}A_0$,

Now, $\log\left(\frac{A}{A_0}\right) = -\lambda t$

$$\log\left(\frac{A}{2A}\right) = -\lambda t$$
$$-\log 2 = -\lambda t$$
$$\frac{1}{\lambda}\log 2 = t$$

Required time is $\frac{1}{\lambda}$ $\log 2$ units where λ is constant of proportionality.

Differential Equations Ex 22.11 Q12

Let A be the quantity of radius at any time t, so

$$\frac{dA}{dt} \propto A$$
$$\frac{dA}{dP} = -\lambda A$$
$$\frac{dA}{A} = -\lambda t$$
$$\int \frac{dA}{A} = -\lambda \int dt$$
$$\log A = -\lambda t + c \text{ - - - } (1)$$

Let A_0 be the initial amount of radius percentage , so

$$\log A_0 = -\lambda (0) + c$$
$$c = \log(A_0)$$

Using, equation (1),

$$\log\ A = -\lambda t + \log A_0$$
$$\log\left(\frac{A}{A_0}\right) = -\lambda t \text{ - - - - - } (2)$$

Given,its half-life is 1590 years, so

$$\log\left(\frac{\frac{1}{2}A_0}{A_0}\right) = -\lambda(1590)$$
$$\log\left(\frac{1}{2}\right) = -\lambda(1590)$$
$$-\log 2 = -\lambda(1590)$$
$$\log 2 = \lambda(1590)$$
$$\frac{\log 2}{1590} = \lambda$$

Now, equation (1) becomes

$$\log\left(\frac{A}{A_0}\right) = -\frac{\log 2}{1590}t$$

Differential Equations Ex 22.11 Q13

Slope of tangent at point $(x, y) = -\frac{x}{y}$

$$\frac{dy}{dx} = -\frac{x}{y}$$
$$y\,dy = -x dx$$
$$\int y\,dt = -\int x\,dx$$
$$\frac{y^2}{2} + \frac{x^2}{2} = c_1$$
$$x^2 + y^2 = c - - - - - (1)$$

Given, curve is passing through $(3, -4)$, so

$$(3)^2 + (-4)^2 = c$$
$$9 + 16 = c$$
$$c = 25$$

So, using equation (1),

$$x^2 + y^2 = 25$$

$$x^2 + y^2 = 25$$

Differential Equations Ex 22.11 Q14

$$y - x\frac{dy}{dx} = y^2 + \frac{dy}{dx}$$
$$\frac{dy}{dx} + x\frac{dy}{dx} = y - y^2$$
$$(1+x)\frac{dy}{dx} = y - y^2$$
$$\frac{dy}{y - y^2} = \frac{dx}{1+x}$$
$$\frac{dy}{y(1-y)} = \frac{dx}{1+x}$$
$$\int\left(\frac{1}{y} + \frac{1}{1-y}\right)dx = \int\frac{dx}{1+x}$$
$$\log|y| - \log|1 - y = \log|1+x|| + \log|c|$$
$$\frac{y}{1-y} = c(1+x)$$
$$y = (1-y)c(1+x) - - - - (1)$$

It is passing through $(2,2)$ so,

$$2 = (1-2)c(1+2)$$
$$2 = -3c$$
$$c = -\frac{2}{3}$$

Now,equation (1) becomes,

$$y = -\frac{2}{3}(1-y)(1+x)$$
$$3y = -2(1+x-y-xy)$$
$$3y + 2 + 2x - 2y - 2xy = 0$$
$$y + 2x - 2xy + 2 = 0$$

$$2xy - 2x - 2 - y = 0$$

Chapter 22 Differential Equations Ex 22.11 Q15

It is passing through $\left(1, \frac{\pi}{4}\right)$, so,

$$\tan\left(\frac{\pi}{4}\right) = -\log|1| + c$$
$$1 = 0 + c$$
$$c = 1$$

Now, equation (1) becomes

$$\tan\left(\frac{y}{x}\right) = -\log|x| + 1$$

Therefore,

$$\tan\left(\frac{y}{x}\right) = \log\left|\frac{e}{x}\right|$$

Differential Equations Ex 22.11 Q16

Let $P(x, y)$ be the point of contact of tangent and curve $y = f(x)$. and It cuts axes at A and B so, equatin of tangent at $P(x, y)$

$$Y - y = \frac{dy}{dx}(X - x)$$

Put $X = 0$

$$Y - y = \frac{dy}{dx}(-x)$$

$$Y = y - x\frac{dy}{dx}$$

So, coordinate of $A = \left(0, y - x\frac{dy}{dx}\right)$

Put $Y = 0$,

$$0 - y = \frac{dy}{dx}(X - x)$$

$$-y\frac{dx}{dy} = X - x$$

$$X = x - y\frac{dx}{dy}$$

Coordinate of $B = \left(x - y\frac{dx}{dy}, 0\right)$

Given, (intercept on x -axis) = 4(ordinate)

$$x - y\frac{dx}{dy} = 4y$$

$$y\frac{dx}{dy} + 4y = x$$

$$\frac{dx}{dy} + 4 = \frac{x}{y}$$

$$\frac{dx}{dy} - \frac{x}{y} = -4$$

It is a linear different equation. Comparing it with $\frac{dx}{dy} + Px = Q$

$P = -\frac{1}{y}$, $Q = -4$

$$I.F. = e^{\int P dy}$$

$$= e^{-\int \frac{1}{y} dy}$$

$$= e^{-\log y}$$

$$= \frac{1}{y}$$

Solution of the equation is given by,

$$x(I.F.) = \int Q(I.F.)dy + \log c$$

$$x\left(\frac{1}{y}\right) = \int(-4)\left(\frac{1}{y}\right)dy + \log c$$

$$\frac{x}{y} = -4\log y + \log c$$

$$e^{\frac{x}{y}} = \frac{c}{y^4}$$

Differential Equations Ex 22.11 Q17

Slope at any point $= y + 2x$

$$\frac{dy}{dx} = y + 2x$$

$$\frac{dy}{dx} - y = 2x$$

It is a linear differential equation. comparing it with $\frac{dy}{dx} + Py = Q$

$P = -1, Q = 2x$

$$I.F. = e^{\int P dx}$$

$$= e^{\int(-1)dx}$$

$$= e^{-x}$$

Solution of the equation is given by

$$(I.F.) = \int Q\,(I.F.)\,dx + c$$

$$y\left(e^{-x}\right) = \int (2x)\left(e^{-x}\right)dx + c$$

$$y\left(e^{-x}\right) = 2\int xe^{-x}dx + c$$

$$y\left(e^{-x}\right) = 2\left[x\left(-e^{-x}\right) + \int 1e^{-x}dx\right] + c$$

$$y\left(e^{-x}\right) = -2xe^{-x} - 2e^{-x} + c$$

$$y = -2x - 2 + ce^{x}$$

$$y + 2(x+1) = ce^{x} \text{ --- } (1)$$

It is passing through origin,

$$0 + 2(0+1) = ce^{0}$$

$$2 = c$$

Now, equation (1) becomes,

$$y + 2(x+1) = 2e^{x}$$

Differential Equations Ex 22.11 Q18

Given, tangent makes on angle $\tan^{-1}(2x + 3y)$ with x-axis,

Slope of tangent $= \tan\theta$

$$\frac{dy}{dx} = \tan\left(\tan^{-1}(2x + 3y)\right)$$

$$\frac{dy}{dx} = 2x + 3y$$

$$\frac{dy}{dx} - 3y = 2x$$

It is a linear differetial equation comparing it with $\frac{dy}{dx} + Py = Q$

$P = -3, Q = 2x$

$$I.F. = e^{\int P dx}$$

$$= e^{-\int 3dx}$$

$$= e^{-3x}$$

Solution of the equation on given by

$$y\,(\text{I.F.}) = \int Q\,(I.F.)\,dx + c$$

$$y\left(e^{-3x}\right) = \int 2xe^{-3x}dx + c$$

$$= 2\left[x\left(\frac{-e^{-3x}}{3}\right) - \int 1.\left(\frac{-e^{-3x}}{3}\right)dx\right] + c$$

$$= -\frac{2}{3}xe^{-3x} + \frac{2}{3}\int e^{-3x}dx + c$$

$$y\left(e^{-3x}\right) = -\frac{2}{3}xe^{-3x} + \frac{2}{9}e^{-3x} + c$$

$$y = -\frac{2}{3}x - \frac{2}{9} + ce^{3x} \text{ ----- } (1)$$

It is passing through (1,2),

$$2 = -\frac{2}{3} - \frac{2}{9} + ce^{3}$$

$$2 = -\frac{8}{9} + ce^{3}$$

$$\frac{26}{9} = ce^{3}$$

$$c = \frac{26}{9}e^{-3}$$

Now equation (1) becomes,

$$ye^{-3x} = \left(-\frac{2}{3}x - \frac{2}{9}\right)e^{-3x} + \frac{26}{9}e^{-3}$$

Differential Equations Ex 22.11 Q19

Let $P(x,y)$ be the point of contact of tangent whit curve $y = f(x)$ equatin of tangent at $P(x,y)$ is

$$Y - y = \frac{dy}{dx}(X - x)$$

Put Y = 0

$$-y = \frac{dy}{dx}(X - x)$$

$$X = X - \frac{ydx}{dx}$$

Coordinate of $B = \left(x - y\frac{dx}{dy}, 0\right)$

Given, (intercept on x - axis) = $4x$

$$x - y\frac{dx}{dy} = 2x$$

$$-y\frac{dx}{dy} = 2x - x$$

$$-y\frac{dx}{dy} = x$$

$$-\frac{dx}{x} = \frac{dy}{y}$$

$$-\int\frac{dx}{x} = \int\frac{dy}{y}$$

$$-\log x = \log y + c \text{ - - - } (1)$$

It is passing through $(1,2)$

$$-\log 1 = \log 2 + c$$

$$c = -\log 2$$

Put c in equation (1)

$$-\log x = \log y - \log 2$$

$$\frac{1}{x} = \frac{y}{2}$$

$$xy = 2$$

Differential Equations Ex 22.11 Q20

$$x(x+1)\frac{dy}{dx} - y = x(x+1)$$

$$\frac{dy}{dx} - \frac{y}{x(x+1)} = \frac{x(x+1)}{x(x+1)}$$

$$\frac{dy}{dx} - \frac{y}{x(x+1)} = 1$$

It is linear differential equation coparing it with $\frac{dy}{dx} + Py = Q$

$$P = -\frac{1}{x(x+1)}, \qquad Q = 1$$

$$I.F. = e^{\int\frac{1}{x(x+1)}dx}$$

$$= e^{\int\left(\frac{1}{x} - \frac{1}{(x+1)}\right)dx}$$

$$= e^{-\log|x| + \log|x+1|}$$

$$= e^{\log\left(\frac{x+1}{x}\right)}$$

$$= \frac{x+1}{x}$$

Solution of the equation is given by

$$y(I.F.) = \int Q(I.F.)dx + c$$

$$y\left(\frac{x+1}{x}\right) = \int\left(\frac{x+1}{x}\right)dx + c$$

$$y\left(\frac{x+1}{x}\right) = \int\left(1 + \frac{1}{x}\right)dx + c$$

$$y\left(\frac{x+1}{x}\right) = x + \log|x| + c \text{ - - - } (1)$$

It is passing through $(1,0)$, so

$$0 = 1 + \log(1) + c$$

$$-1 = c$$

Now,equation (1) becomes,

$$y\left(\frac{x+1}{x}\right) = x + \log|x| - 1$$

$$y(x+1) = x(x + \log x - 1)$$

Differential Equations Ex 22.11 Q21

Slope of the curve $= \frac{2y}{x}$

$$\frac{dy}{dx} = \frac{2y}{x}$$
$$\frac{dy}{y} = \frac{2}{x}dx$$
$$\int \frac{dy}{y} = 2\int \frac{1}{x}dx$$
$$\log|y| = 2\log|x| + \log|c|$$
$$y = x^2c \text{ ---} (1)$$

It is passing through $(3, -4)$ so,

$$-4 = (3)^2 c$$
$$-4 = 9c$$
$$c = -\frac{4}{9}$$

Now, equation (1) becomes,

$$y = -\frac{4}{9}x^2$$
$$9y = -4x^2$$

$$9y + 4x^2 = 0$$

Differential Equations Ex 22.11 Q22

Given,

Slope of the equation $= x + 3y - 1$

$$\frac{dy}{dx} = x + 3y - 1$$
$$\frac{dy}{dx} - 3y = x - 1$$

It is a linear differential equation. Camparing it with $\frac{dy}{dx} + Py = Q$

$P = -3, Q = x - 1$

$$I.F. = e^{\int P dx}$$
$$= e^{\int -3dx}$$
$$= e^{-3x}$$

Solution of the equation is given by,

$$y(I.F.) = \int Q(I.F.)dx + c$$
$$y\left(e^{-3x}\right) = \int (x-1)\left(e^{-3x}\right)dx + c$$
$$y\left(e^{-3x}\right) = (x-1)\left(-\frac{1}{3}e^{-3x}\right) - \int (1)\left(\frac{-e^{-3x}}{3}\right)dx + c$$
$$y\left(e^{-3x}\right) = -\frac{(x-1)}{3}e^{-3x} + \left(-\frac{e^{-3x}}{9}\right) + c$$
$$y = -\frac{x}{3} + \frac{1}{3} - \frac{1}{9} + ce^{3x}$$
$$y = -\frac{x}{3} + \frac{2}{9} + ce^{3x}$$

It is passing through origin, so

$$0 = 0 + \frac{2}{9} + ce^{3(0)}$$
$$0 = \frac{2}{9} + c$$
$$c = -\frac{2}{9}$$

Now, equation (1) becomes,

$$y = -\frac{x}{3} + \frac{2}{9} - \frac{2}{9}e^{3x}$$
$$9y = -3x + 2 - 2e^{3x}$$

$$3(3y + x) = 2\left(1 - e^{3x}\right)$$

Differential Equations Ex 22.11 Q23

Given,

Slope at point $(x, y) = x + xy$

$$\frac{dy}{dx} = x(y+1)$$

$$\frac{dy}{y+1} = x\,dx$$

$$\int \frac{dy}{y+1} = \int x\,dx$$

$$\log|y+1| = \frac{x^2}{2} + c - - - - - (1)$$

It is passing through $(0,1)$, so,

$$\log z = 0 + c$$

$$c = \log 2$$

Now,equation (2) becomes,

$$\log|y+1| = \frac{x^2}{2} + \log 2$$

$$y + 1 = 2e^{\frac{x^2}{2}}$$

Differential Equations Ex 22.11 Q24

$$y^2 - 2xy\frac{dy}{dx} - x^2 = 0$$

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$$

It is a homeganeous equation.

put, $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Now,

$$x\frac{dv}{dx} + v = \frac{v^2x^2 - x^2}{2xvx}$$

$$x\frac{dv}{dx} = \frac{v^2 - 1}{2v} - v$$

$$x\frac{dv}{dx} = \frac{v^2 - 1 - 2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{-v^2 - 1}{2v}$$

$$\int \frac{2v}{v^2+1}dv = -\int \frac{dx}{x}$$

$$\log|v^2 + 1| = -\log|x| + \log|c|$$

$$v^2 + 1 = \frac{c}{x}$$

$$\frac{y^2 + x^2}{x^2} = \frac{c}{x}$$

$$y^2 + x^2 = cx$$

$$y^2 + x^2 - cx = 0$$

Differentiating it with respect to x,

$$2x + 2y\frac{dy}{dx} - c = 0$$

$$\frac{dy}{dx} = \frac{c - 2x}{2y}$$

Let (h,k) be the point where tangent passes through origin and length is equal to h. so, equation of tangent at (h,k) is

$$(y-k)=\left(\frac{dy}{dx}\right)_{(h,k)}(x-h)$$

$$(y-k)=\left(\frac{c-2h}{2k}\right)(x-h)$$

$$2ky-2k^2=xc-2hx-hc+2h^2$$

$$x(c-2h)-2ky+2k^2-hc+2h^2=0$$

$$x(c-2h)-2ky+2\left(k^2+h^2\right)-hc=0$$

$$x(c-2h)-2ky+2(ch)-hc=0$$

$\left[\text{Since } h^2+k^2=ch \text{ as } (h,k) \text{ is on the curve}\right]$

$$x(c-2h)-2ky+hc=0$$

length of perpendicular as tangent from origin is

$$L=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

$$=\left|\frac{(0)(c-2h)+(0)(-2k)+hc}{\sqrt{(c-2h)^2+(-2k)^2}}\right|$$

$$=\frac{hc}{\sqrt{c^2+4h^2+4k^2-4ch}}$$

$$L=\frac{hc}{\sqrt{c^2+4\left(h^2+k^2-ch\right)}}$$

$$=\frac{hc}{\sqrt{c^2+4(0)}}$$

$$=\frac{hc}{c}$$

$$=c$$

Hence,

$x^2+y^2=cx$ is the required curve

Differential Equations Ex 22.11 Q25

Let $P(x,y)$ be the point of contact of tangent and curve $y=f(x)$. Equation tangent at $P(x,y)$ is

$$Y-y=\frac{dx}{dy}(X-x)$$

put $Y=0$

$$-y=\frac{dx}{dy}(X-x)$$

$$-y=\frac{dx}{dy}(X-x)$$

$$X=x-y\frac{dx}{dy}$$

coordinate of $B=\left(x-y\frac{dx}{dy},0\right)$

Given,

Distance between foot of ordinate of the point of contact and the point of intersection of tangent and x – axis $=2x$

$$BC=2x$$

$$\sqrt{\left(x-y\frac{dx}{dy}-x\right)^2+(0)^2}=2x$$

$$y\frac{dx}{dy}=2x$$

$$y\frac{dx}{x}=2\frac{dy}{y}$$

$$\int\frac{dx}{x}=2\int\frac{dy}{y}$$

$$\log x=2\log y+\log c\ ---(1)$$

It is passing through $(1,2)$,

$$\log 1=2\log 2+\log c$$

$$-2\log 2=\log c$$

$$\log\left(\frac{1}{4}\right)=\log c$$

$$c=\frac{1}{4}$$

Put value of c in equation (1),

$$\log x=2\log y+\log\left(\frac{1}{4}\right)$$

$$x=\frac{y^2}{4}$$

$$y^2=4x$$

Differential Equations Ex 22.11 Q26

Equation of normal on point (x, y) on the curve

$$Y - y = \frac{-dx}{dy}(X - x)$$

Itis passing through $(3,0)$

$$0 - y = \frac{-dx}{dy}(3 - x)$$

$$y = \frac{dx}{dy}(3 - x)$$

$$ydy = (3 - x)dx$$

$$\int y\,dy = \int (3 - x)dx$$

$$\frac{y^2}{2} = 3x - \frac{x^2}{2} + c \,---\,(1)$$

It passing through $(3,4)$, so,

$$\frac{16}{2} = 9 - \frac{9}{2} + c$$

$$\frac{16}{2} = \frac{9}{2} + c$$

$$c = 7$$

Put $c = 7$ is equation (1)

$$\frac{y^2}{2} = 3x - \frac{x^2}{2} + \frac{7}{2}$$

$$y^2 = 6x - x^2 + 7$$

Differential Equations Ex 22.11 Q27

Let A be the quantity of bacteria present in culture at any time t and initial quantity of bacteria is A_0.

$$\frac{dA}{dt} \propto A$$

$$\frac{dA}{dt} = \lambda A$$

$$\frac{dA}{A} = \lambda dt$$

$$\int \frac{dA}{A} = \lambda \int dt$$

$$\log A = \lambda t + c \,----\,(1)$$

Initially, $A = A_0, t = 0$

$$\log A_0 = 0 + c$$

$$\log A_0 = c$$

Now eqution (1) becomes,

$$\log A = \lambda t + \log A_0$$

$$\log\left(\frac{A}{A_0}\right) = \lambda t \,-----\,(2)$$

Given $A = 2A_0$ when $t = 6$ hours

$$\log\left(\frac{A}{A_0}\right) = 6\lambda$$

$$\frac{\log 2}{6} = \lambda$$

Now equation (2) becomes,

$$\log\left(\frac{A}{A_0}\right) = \frac{\log 2}{6}t$$

Now, $A = 8A_0$

so, $$\log\left(\frac{8A_0}{A_0}\right) = \frac{\log 2}{6}t$$

$$\log 2^3 = \frac{\log 2}{6}t$$

$$3\log 2 = \frac{\log 2}{6}t$$

$$18 = t$$

Therefore,

Bacteria becomes 8 times in 18 hours

Differential Equations Ex 22.11 Q28

Let A be the quantity of radium present at time t and A_0 be the initial quantity of radium.

$$\frac{dA}{dt} \propto A$$
$$\frac{dA}{dt} = -\lambda A$$
$$\frac{dA}{A} = -\lambda dt$$
$$\int \frac{dA}{A} = -\lambda \int dt$$
$$\log A = -\lambda t + c \text{ - - - - } (2)$$

Now, $A = A_0$ when $t = 0$

$$\log A_0 = 0 + c$$
$$c = \log A_0$$

Put value of c in equation

$$\log A = -\lambda t + \log A_0$$
$$\log\left(\frac{A}{A_0}\right) = -\lambda t \text{ - - - } (2)$$

Given that,

In 25 years bacteria decomposes 1.1% , so

$$A = (100 - 1.1)\% = 98.9\% = 0.989 A_0, t = 5$$
$$\log\left(\frac{0.989 A_0}{A_0}\right) = -\lambda 25$$
$$\log(0.989) = -25\lambda$$
$$\lambda = -\frac{1}{25}\log(0.989)$$

Now, equation (2) becomes,

$$\log\left(\frac{A}{A_0}\right) = \left\{\frac{1}{25}\log(0.989)\right\}t$$

Now $A = \frac{1}{2}A_0$

$$\log\left(\frac{A}{2A}\right) = \frac{1}{25}\log(0.989)t$$

$$\frac{-\log 2 \times 25}{\log(0.989)} = t$$
$$-\frac{0.6931 \times 25}{0.01106} = t$$

$$t = 1567 \text{ years.}$$

Required time = 1567 years

Differential Equations Ex 22.11 Q29

Given,

$$\text{Slope of tangent} = \frac{x^2 + y^2}{2xy}$$
$$\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$$

It is a homeganeous equation.

put, $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Now,

$$v + x\frac{dv}{dx} = \frac{x^2 + v^2x^2}{2xvx}$$
$$x\frac{dv}{dx} = \frac{1 + v^2}{2v} - v$$
$$x\frac{dv}{dx} = \frac{1 + v^2 - 2v^2}{v}$$
$$x\frac{dv}{dx} = \frac{1 - v^2}{v}$$
$$\frac{v}{1 - v^2}dv = \frac{dx}{x}$$
$$\int \frac{v}{1 - v^2}dv = \int \frac{dx}{x}$$
$$\int \frac{-2v}{1 - v^2}dv = \int \frac{-2dx}{x}$$
$$\log\left|1 - v^2\right| = -2\log x + \log c$$
$$1 - \frac{y^2}{x^2} = \frac{c}{x^2}$$
$$\frac{x^2 - y^2}{x^2} = \frac{c}{x^2}$$

$$x^2 - y^2 = c$$

It is equation of rectangular hyperbola.

Differential Equations Ex 22.11 Q30

Given,

Slope of tangent at $(x, y) = x + y$

$$\frac{dy}{dx} = x + y$$

$$\frac{dy}{dx} - y = x$$

It is a linear differential equation.Comparing it with $\frac{dy}{dx} + Py = Q$

$P = -1, Q = x$

$$I.F. = e^{\int P dx}$$
$$= e^{\int(-1)dx}$$
$$= e^{-x}$$

Solution of equation is given by,

$$y\left(I.F.\right) = \int Q\left(I.F.\right)dx + c$$

$$y\left(e^{-x}\right) = \int xe^{-x}dx + c$$

$$ye^{-x} = x\left(e^{-x}\right) + \int\left(1 \times e^{-x}\right)dx + c$$

[Using integration by parts]

$$ye^{-x} = -xe^{-x} - e^{-x} + c$$

$$y = -x - 1 + ce^{x} - - - - - (1)$$

It is passing through origin

$$0 = 0 - 1 + ce^{0}$$
$$1 = c$$

Put $c = 1$ is equation

$$y = -x - 1 + e^{x}$$

$$y + x + 1 = e^{x}$$

Differential Equations Ex 22.11 Q31

We know that the slope of the tangent to the curve is $\frac{dy}{dx}$.

$\therefore \quad \frac{dy}{dx} = x + xy$

$\Rightarrow \quad \frac{dy}{dx} - xy = y \quad ------(i)$

This is a linear differential equation of the type $\frac{dy}{dx} + Py = Q$

where $P = -x$ and $Q = x$.

So, $\quad I.F. = e^{\int -x dx} = e^{\frac{-x^2}{2}}$

$\therefore$ Solution of the given equation is given by

$$y.\, e^{\frac{-x^2}{2}} = \int x.e^{\frac{-x^2}{2}}\, dx + C \quad ------(ii)$$

Let $\quad I = \int x.e^{\frac{-x^2}{2}}\, dx$

Let $\frac{-x^2}{2} = t$, then $-x\,dx = dt$ or $x\,dx = -dt$

$\therefore \quad I = \int x.e^{\frac{-x^2}{2}}\, dx = \int -e^{t}\, dt = -e^{t} = -e^{\frac{-x^2}{2}}$

Substituting the value of I in (ii), we get

$$y.\, e^{\frac{-x^2}{2}} = -e^{\frac{-x^2}{2}} + C$$

or $\quad y = -1 + Ce^{\frac{x^2}{2}} \quad ------(iii)$

This equation (iii) passes through (0,1)

$\therefore \quad 1 = -1 + Ce^{0} \quad \Rightarrow \quad C = 2$

Substituting the value of C in (iii), we get

$$y = -1 + 2e^{\frac{x^2}{2}}$$

which is the equation of the required curve.

Differential Equations Ex 22.11 Q32

Given,

Slope of tangent at $(x,y) = x^2$

$\frac{dy}{dx} = x^2$

$dy = x^2 dx$

$\int dy = \int x^2 dx$

$y = \frac{x^3}{3} + c - - - - (1)$

It is passing through $(-1,1)$

$1 = \frac{(-1)}{3} + c$

$1 = -\frac{1}{3} + c$

$c = 1 + \frac{1}{3}$

$c = \frac{4}{3}$

Put is equation

$y = \frac{x^3}{3} + \frac{4}{3}$

$3y = x^3 + 4$

Differential Equations Ex 22.11 Q33

Given,

y (Slope of tangent) $= x$

$y\frac{dy}{dx} = x$

$ydy = xdx$

$\int ydy = \int xdx$

$\frac{y^2}{2} = \frac{x^2}{2} + c - - - - (1)$

It is passing through $(0,a)$

$\frac{a^2}{2} = 0 + c$

$c = \frac{a^2}{2}$

Put $c = \frac{a^2}{2}$ is equation (1)

$\frac{y^2}{2} = \frac{x^2}{2} + \frac{a^2}{2}$

$y^2 = x^2 + a^2$

Differential Equations Ex 22.11 Q34

Let $P(x,y)$ be the point on the curve $y = f(x)$ such that tangent at P cuts the coordinate axes at A and B.

The equation of tangent is,

$Y - y = \frac{dy}{dx}(X - x)$

Put $Y = 0$

$-y = \frac{dy}{dx}(X - x)$

$-y\frac{dy}{dx} + x = X$

Coordinate of $B = \left(-y\frac{dy}{dx} + x, 0\right)$

Here, x intercept of tangent $= y$

$-y\frac{dx}{dy} + x = y$

$\frac{dx}{dy} - \frac{x}{y} = -1$

It is a linear differential equation on comparing it with $\frac{dx}{dy} + py = Q$

$P = \frac{1}{y}, Q = -1$

$I.F. = e^{\int\left(\frac{1}{y}\right)dy}$

$= e^{\log y}$

$= \frac{1}{y}$

Solution of the equation is given by,

$x(I.F.) = \int Q(I.F)dy + c$

$x\left(\frac{1}{y}\right) = \int(-1)\left(\frac{1}{y}\right)dy + c$

$x\left(\frac{1}{y}\right) = -\log y + c - - - - (1)$

It is passing through $(1,1)$

$\frac{1}{1} = -\log 1 + c$

$c = 1$

put $c = 1$ is equation (1),

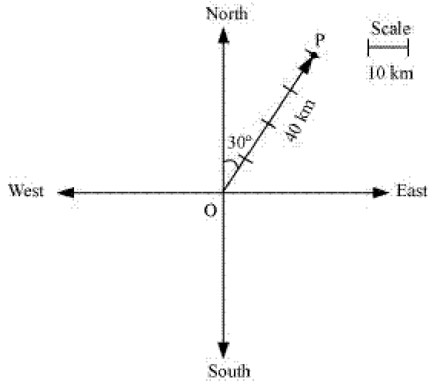
$\frac{x}{y} = -\log y + 1$

$x = y - y\log y$

$x + y\log y = y$

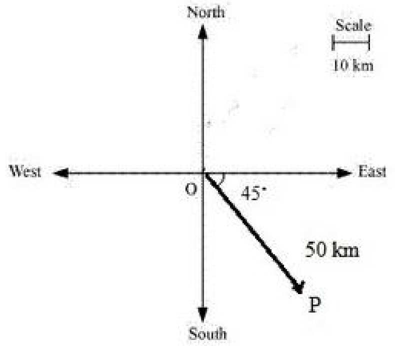
Ex 23.1

Algebra of Vectors Ex 23.1 Q1(i)



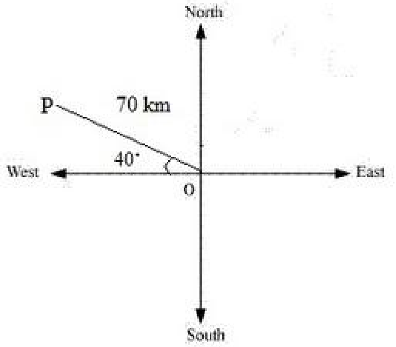
Here, vector $\overrightarrow{OP}$ represents the displacement of 40 km, 30° East of North.

Algebra of Vectors Ex 23.1 Q1(ii)



Here, vector $\overrightarrow{OP}$ represents the displacement of 50 km, south-east

Algebra of Vectors Ex 23.1 Q1(iii)



Here, vector $\overrightarrow{OP}$ represents the displacement of 70 km, 40° north of west

Algebra of Vectors Ex 23.1 Q2

(i) 15 kg is a scalar quantity because it involves only mass

(ii) 20 kg weight is a vector quantity as it involves both magnitude and direction.

(iii) 45° is a scalar quantity as it involves only magnitude.

(iv) 10 meters south-east is a vector quantity as it involve direction.

(v) 50 m/s² is a scalar quantity as it involves magnitude of acceleration.

Algebra of Vectors Ex 23.1 Q3

(i) Time period is a scalar quantity as it involves only magnitude.

(ii) Distance is a scalar quantity as it involves only magnitude.

(iii) Displacement is vector quantity as it involves both magnitude and direction.

(iv) Force is a vector quantity as it involves both magnitude and direction.

(v) Work done is a scalar quantity as it involves only magnitude.

(vi) Velocity is a vector quantity as it involves both magnitude as well as direction.

(vii) Acceleration is a vector quantity because it involves both magnitude as well as direction.

Algebra of Vectors Ex 23.1 Q4

(i)
Collinear vectors are
$\vec{x}, \vec{z}$ and $\vec{b}$
$\vec{y}, \vec{c}$
$\vec{a}, \vec{d}$

(ii)
Equal vectors are
$\vec{y}$ and $\vec{c}$
$\vec{x}$ and $\vec{b}$
$\vec{a}$ and $\vec{d}$

(iii)
Coinitial vector are
$\vec{a}$, $\vec{y}$ and $\vec{z}$

(iv)
Collinear but not equal
$\vec{b}$ and $\vec{z}$
$\vec{x}$ and $\vec{z}$

Algebra of Vectors Ex 23.1 Q5

(i) a and b are collinear, it is true.

(ii) Two collinear vectors are may not be equal in magnitude, so it is false.

(iii) Zero vector may not be unique, so it is false.

(iv) Two vectors having same magnitude are may not be collinear so it is false.

(v) Two collinear vectors having the same magnitude are may not be equal, so it is false.

Ex 23.2

Algebra of Vectors Ex 23.2 Q1

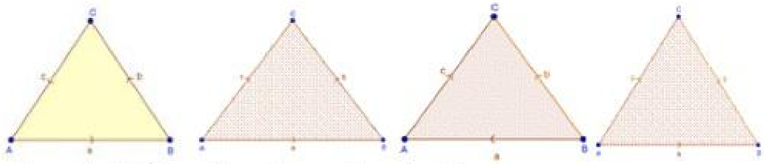
Given that, P, Q, R are collinear.

It also given that, $\overrightarrow{PQ} = \vec{a}$ and $\overrightarrow{QR} = \vec{b}$

$$\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$$
$$= \vec{a} + \vec{b}$$

$$\overrightarrow{PR} = \vec{a} + \vec{b}$$

Algebra of Vectors Ex 23.2 Q2



Given that, $\vec{a}, \vec{b}$, and $\vec{c}$ are three sides of a triangle.

$\vec{a} + \vec{b} + \vec{c}$

$= \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$

$= \overrightarrow{AC} + \overrightarrow{CA}$ $\quad\left[\text{Since } \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}\right]$

$= \overrightarrow{AC} - \overrightarrow{AC}$ $\quad\left[\text{Since } \overrightarrow{CA} = \overrightarrow{AC}\right]$

$= \vec{0}$

So, $\vec{a} + \vec{b} + \vec{c} = \vec{0}$

Triangle law says that, if vectors are represented in magnitude and direction by the two sides of triangle taken is same order, then their sum is represented by the third side taken in reverse order.

Thus,
$\vec{a} + \vec{b} = \vec{c}$
or
$\vec{a} + \vec{c} = \vec{b}$
$\vec{b} + \vec{c} = \vec{a}$

Algebra of Vectors Ex 23.2 Q3

Here, it is given that $\vec{a}$ and $\vec{b}$ are two non-collinear vectors having the same initial point.

Let $\vec{a} = \overrightarrow{AB}$ and $\vec{b} = \overrightarrow{AD}$, So we can draw a parallelogram ABCD as above.

By the properties of parallelogram
$\overrightarrow{BC} = \vec{b}$ and $\overrightarrow{DC} = \vec{a}$

In ΔABC,
Using triangle law,
$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$
$$\vec{a} + \vec{b} = \overrightarrow{AC} \qquad --\text{(i)}$$

In ΔABD,
Using triangle law,
$$\overrightarrow{AD} + \overrightarrow{DB} = \overrightarrow{AB}$$
$$\vec{b} + \overrightarrow{DB} = \vec{a}$$
$$\overrightarrow{DB} = \vec{a} - \vec{b} \qquad --\text{(ii)}$$

From equation (i) and (ii), we get that

$\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are diagonals of a parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$

Algebra of Vectors Ex 23.2 Q4

Given that m is a scalar and $\vec{a}$ is a vector such that

$m\vec{a} = \vec{0}$

$m\left(a_1\hat{i} + b_1\hat{j} + c_1\hat{k}\right) = 0 \times \hat{i} + 0 \times \hat{j} + 0 \times \hat{k}$ $\left[\text{since let } \vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}\right]$

$ma_1\hat{i} + mb_1\hat{j} + mc_1\hat{k} = 0 \times \hat{i} + 0 \times \hat{j} + 0 \times \hat{k}$

Comparing the coefficients of $\hat{i}, \hat{j}, \hat{k}$ of LHS and RHS,

$ma_1 = 0 \Rightarrow m = 0$ or $a_1 = 0$ (i)

$mb_1 = 0 \Rightarrow m = 0$ or $b_1 = 0$ (ii)

$mc_1 = 0 \Rightarrow m = 0$ or $c_1 = 0$ (iii)

From (i), (ii) and (iii)

$m = 0$ or $a_1 = b_1 = c_1 = 0$

$\Rightarrow m = 0$ or $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{j} = 0$

$\Rightarrow m = 0$ or $\vec{a} = 0$

Algebra of Vectors Ex 23.2 Q5

(i)

Let $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$

$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$

Given that, $a = -b$

$a_1\hat{i} + b_1\hat{j} + c_1\hat{k} = -a_2\hat{i} - b_2\hat{j} - c_2\hat{k}$

Comparing the coefficients of i, j, k in LHS and RHS,

$a_1 = -a_2$ (1)

$b_1 = -b_2$ (2)

$c_1 = -c_2$ (3)

$\left|\vec{a}\right| = \sqrt{a_1^2 + b_1^2 + c_1^2}$

Using (1), (2)and (3),

$\left|\vec{a}\right| = \sqrt{(-a_2)^2 + (-b_2)^2 + (-c_2)^2}$

$\left|\vec{a}\right| = \sqrt{a_2^2 + b_2^2 + c_2^2}$

$\therefore \left|\vec{a}\right| = \left|\vec{b}\right|$

(ii)

Given a and b are two vectors such that $\left|\vec{a}\right| = \left|\vec{b}\right|$

It means magnitude of vector $\vec{a}$ is equal to the magnitude of vector $\vec{b}$, but we cannot conclude anything about the direction of the vector.

So,it is false that

$\left|\vec{a}\right| = \left|\vec{b}\right| \Rightarrow \vec{a} = \pm\vec{b}$

(iii)

Given for any vector $\vec{a}$ and $\vec{b}$

$\left|\vec{a}\right| = \left|\vec{b}\right|$

It means magnitude of the vector $\vec{a}$ and $\vec{b}$ are equal but we cannot say any thing about the direction of the vector $\vec{a}$ and $\vec{b}$. And we know that $\vec{a} = \vec{b}$ means magnitude and same direction. So, it is false.

Algebra of Vectors Ex 23.2 Q6

Here it is given that $ABCD$ is a quadrilateral.

In ΔADC, using triangle law, we get

$\overrightarrow{CD} + \overrightarrow{DA} = \overrightarrow{CA}$ --(i)

In ΔABC, using triangle law, we get

$\overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{BA}$ --(ii)

Put value of $\overrightarrow{CA}$ in equation (ii),

$\overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA} = \overrightarrow{BA}$

Adding $\overrightarrow{BA}$ on both the sides,

$\overrightarrow{BA} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA} = \overrightarrow{BA} + \overrightarrow{BA}$

$\therefore \overrightarrow{BA} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA} = 2\overrightarrow{BA}$

Algebra of Vectors Ex 23.2 Q7

(i)

Given that $ABCDE$ is a pentagon.

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA}$

$= \left(\overrightarrow{AB} + \overrightarrow{BC}\right) + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA}$

$= \overrightarrow{AC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA}$ $\left[\text{Using triangle law in } \Delta ABC,\ \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}\right]$

$= \left(\overrightarrow{AC} + \overrightarrow{CD}\right) + \overrightarrow{DE} + \overrightarrow{EA}$

$= \left(\overrightarrow{AD}\right) + \overrightarrow{DE} + \overrightarrow{EA}$ $\left[\text{Using triangle law in } \Delta ACD,\ \overrightarrow{AC} + \overrightarrow{CD} = \overrightarrow{AD}\right]$

$= \overrightarrow{AD} + \overrightarrow{DA}$

$= \overrightarrow{AD} - (-\overrightarrow{AD})$

$= \vec{0}$

$\therefore \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA} = \vec{0}$

(ii)

It is given that ABCDE is a pentagon, So

$\overrightarrow{AB} + \overrightarrow{AE} + \overrightarrow{BC} + \overrightarrow{DC} + \overrightarrow{ED} + \overrightarrow{AC}$

$= \left(\overrightarrow{AB} + \overrightarrow{BC}\right) + \overrightarrow{AE} + \overrightarrow{DC} + \overrightarrow{ED} + \overrightarrow{AC}$

$= \overrightarrow{AC} + \overrightarrow{DC} + \left(\overrightarrow{AE} + \overrightarrow{ED}\right) + \overrightarrow{AC}$ $\left[\text{Using triangle law in } \Delta ABC,\ \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}\right]$

$= \overrightarrow{AC} + \overrightarrow{DC} + \left(\overrightarrow{AD}\right) + \overrightarrow{AC}$

$= \overrightarrow{AC} + \overrightarrow{DC} - \overrightarrow{DA} + \overrightarrow{AC}$

$= \overrightarrow{AC} + \overrightarrow{DC} + \overrightarrow{AD} + \overrightarrow{AC}$

$= \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{DC} + \overrightarrow{AC}$

$= \overrightarrow{AC} + \overrightarrow{AC} + \overrightarrow{AC}$

$= 3\overrightarrow{AC}$

So,

$\overrightarrow{AB} + \overrightarrow{AE} + \overrightarrow{BC} + \overrightarrow{DC} + \overrightarrow{ED} + \overrightarrow{AC} = 3\overrightarrow{AC}$

Algebra of Vectors Ex 23.2 Q8

Let O be the centre of a regular octagon, we know that the centre of a regular octagon bisects all the diagonals passing through it.

Thus,

$\overrightarrow{OA} = -\overrightarrow{OE}$ (i)

$\overrightarrow{OB} = -\overrightarrow{OF}$ (ii)

$\overrightarrow{OC} = -\overrightarrow{OG}$ (iii)

$\overrightarrow{OD} = -\overrightarrow{OH}$ (iv)

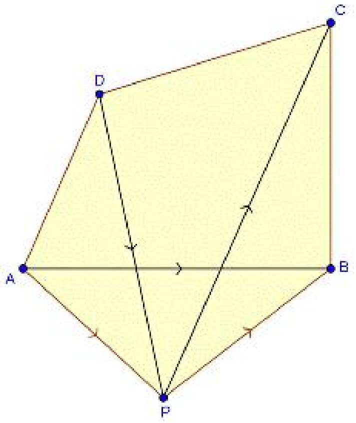
Adding equation (i), (ii), and (iv),

$\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} = -\overrightarrow{OE} - \overrightarrow{OF} - \overrightarrow{OG} - \overrightarrow{OH}$

$\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} + = -\left(\overrightarrow{OE} + \overrightarrow{OF} + \overrightarrow{OG} + \overrightarrow{OH}\right)$

$\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} + \overrightarrow{OE} + \overrightarrow{OF} + \overrightarrow{OG} + \overrightarrow{OH} = \vec{0}$

Algebra of Vectors Ex 23.2 Q9



Given, $\overrightarrow{AP} + \overrightarrow{PB} + \overrightarrow{PD} = \overrightarrow{PC}$

$\overrightarrow{AP} + \overrightarrow{PB} = \overrightarrow{PC} - \overrightarrow{PD}$

$\overrightarrow{AP} + \overrightarrow{PB} = \overrightarrow{PC} + \overrightarrow{DP}$ $\quad\left[\text{Since } \overrightarrow{DP} = -\overrightarrow{PD}\right]$

$\overrightarrow{AP} + \overrightarrow{PB} = \overrightarrow{DP} + \overrightarrow{PC}$

$\overrightarrow{AB} = \overrightarrow{DC}$ $\quad\left[\begin{array}{l}\text{Using triangle law in } \Delta APB,\ \overrightarrow{AP} + \overrightarrow{PB} = \overrightarrow{AB} \\ \text{Using triangle law in } \Delta DPC,\ \overrightarrow{DP} + \overrightarrow{PC} = \overrightarrow{DC}\end{array}\right]$

Therefore, AB is parallel to DC and equal is magnitude.
Hence, $ABCD$ is a parallelogram.

Algebra of Vectors Ex 23.2 Q10

We need to show that

$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} = 6\overrightarrow{AO}$

We know that centre O of the hexagon bisects the diagonal $\overrightarrow{AD}$

$\therefore \quad \overrightarrow{AO} = \frac{1}{2}\overrightarrow{AD};\ \overrightarrow{BO} = -\overrightarrow{EO};\ \overrightarrow{CO} = -\overrightarrow{FO}$

Now

$\overrightarrow{AB} + \overrightarrow{BO} = \overrightarrow{AO}$

$\overrightarrow{AC} + \overrightarrow{CO} = \overrightarrow{AO}$

$\overrightarrow{AD} + \overrightarrow{DO} = \overrightarrow{AO}$

$\overrightarrow{AE} + \overrightarrow{EO} = \overrightarrow{AO}$

$\overrightarrow{AF} + \overrightarrow{FO} = \overrightarrow{AO}$

Adding these equations we get

$$\left(\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF}\right) + \left(\overrightarrow{BO} + \overrightarrow{CO} + \overrightarrow{DO} + \overrightarrow{EO} + \overrightarrow{FO}\right) = 5\ \overrightarrow{AO}$$

$\Rightarrow \quad \left(\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF}\right) + \overrightarrow{DO} = 5\ \overrightarrow{AO}$

But $\quad \overrightarrow{DO} = -\overrightarrow{AO}$

$\therefore \quad \overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} = 6\ \overrightarrow{AO}.$

Ex 23.3

Algebra of Vectors Ex 23.3 Q1

Point R divides the line joining the two points P and Q in the ratio 1:2 internally.

Position vector of point R $=\dfrac{1\left(\vec{a}-2\vec{b}\right)+2\left(2\vec{a}+\vec{b}\right)}{1+2}=\dfrac{5\vec{a}}{3}$

Point R divides the line joining the two points P and Q in the ratio 1:2 externally.

Position vector of point R $=\dfrac{1\left(\vec{a}-2\vec{b}\right)-2\left(2\vec{a}+\vec{b}\right)}{1-2}=\dfrac{-3\vec{a}-4\vec{b}}{-1}=3\vec{a}+4\vec{b}$

Algebra of Vectors Ex 23.3 Q2

Here it is given that $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$ be the position vectors of the four distinct points A,B,C,D such that
$\vec{b}-\vec{a}=\vec{c}-\vec{d}$

Given that,
$\vec{b}-\vec{a}=\vec{c}-\vec{d}$
$\overrightarrow{AB}=\overrightarrow{DC}$

So, AB is parallel and equal to DC (in magnitude).

Hence,
$ABCD$ *is* a parallelogram.

Algebra of Vectors Ex 23.3 Q3

Here, it is given that $\vec{a},\vec{b}$ are position vector of A and B.

Let C be a point in AB produced such that $AC=3AB$.

It is clear that point C divides the line AB in ratio $3:2$ externally.
So position vector point C is given by

$$\vec{c}=\frac{m\vec{b}-n\vec{a}}{m-n}$$
$$=\frac{3\vec{b}-2\vec{a}}{3-2}$$
$$\vec{c}=3\vec{b}-2\vec{a}$$

Again, let D be a point in BA produced such that $BD=2BA$.

Let $\vec{d}$ be the position vector of D. It is clear that point D divides the line AB in $1:2$ externally. So position vector of D is given by

$$\vec{d}=\frac{m\vec{a}-n\vec{b}}{m-n}$$
$$=\frac{2\vec{a}-\vec{b}}{2-1}$$
$$\vec{d}=2\vec{a}-\vec{b}$$

$$\vec{c}=3\vec{b}-2\vec{a}$$
$$\vec{d}=2\vec{a}-\vec{b}$$

Algebra of Vectors Ex 23.3 Q4

We have given that

$3\vec{a} - 2\vec{b} + 5\vec{c} - 6\vec{d} = \vec{0}$

$3\vec{a} + 5\vec{c} = 2\vec{b} + 6\vec{d}$ (i)

Sum of the coefficients on both the sides of the equation (i) is 8, so divide equation (i) by 8 on both the sides,

$$\frac{3\vec{a} + 5\vec{c}}{8} = \frac{2\vec{b} + 6\vec{d}}{8}$$

$$\frac{3\vec{a} + 5\vec{c}}{3+5} = \frac{2\vec{b} + 6\vec{d}}{2+6}$$

It shows that position vector of a point p dividing AC in the ratio $3:5$, is same as that of a point dividing BD in the ratio of $2:6$.

Hence, point P is common to AC and BD. Therefore, P is the point of intersection of AC and BD.

So, A, B, C and D are coplanar.

Position vector of point P is given by

$$\frac{3\vec{a} + 5\vec{c}}{8} \quad \text{or} \quad \frac{2\vec{b} + 6\vec{d}}{8}$$

Algebra of Vectors Ex 23.3 Q5

We have given that

$5\vec{p} - 2\vec{q} + 6\vec{r} - 9\vec{s} = \vec{0}$

Where $\vec{p}, \vec{q}, \vec{r}$ and $\vec{s}$ are the position vectors of point P, Q, R and S.

$5\vec{p} + 6\vec{r} = 2\vec{q} + 9\vec{s}$ (i)

Sum of the coefficients on both the sides of the equation (i) is 11. So divide equation (i) by 11 on both the sides.

$$\frac{5\vec{p} + 6\vec{r}}{11} = \frac{2\vec{q} + 9\vec{s}}{11}$$

$$\frac{5\vec{p} + 6\vec{r}}{5+6} = \frac{2\vec{q} + 9\vec{s}}{2+9}$$

It shows that position vector of a point A dividing PR in the ratio of $6:5$ and QS in the ratio of $9:2$. Thus, A is the common point to PR and QS and it is also point of intersection of PQ and QS.

So,

P, Q, R and S are coplanar

Position vector of point A is given by

$$\frac{5p + 6q}{11} \quad \text{or} \quad \frac{2\vec{q} + 9\vec{s}}{11}$$

Algebra of Vectors Ex 23.3 Q6

Let ABC be a triangle.

Let the position vectors of A, B and C with respect to some origin, O be $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively.

Let D be the point on BC where the bisector of $\angle A$ meets.

Let $\vec{d}$ position vector of D which divides BC internally in the ratio β and γ, where $\beta = |\overrightarrow{AC}|$ and $\gamma = |\overrightarrow{AB}|$

Thus, $\beta = |\vec{c} - \vec{a}|$ and $\gamma = |\vec{b} - \vec{a}|$

Thus, by section formula, the position vector of D is given by

$$\overrightarrow{OD} = \frac{\beta\vec{b} + \gamma\vec{c}}{\beta + \gamma}$$

Let $\alpha = |\vec{b} - \vec{c}|$

Incentre is the concurrent point of angle bisectors.

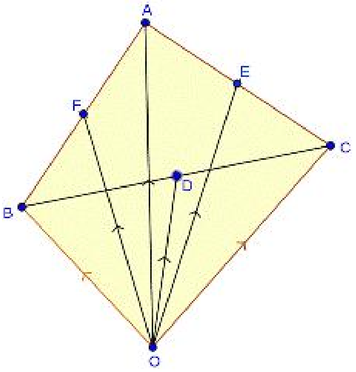
Thus, Incentre divides the line AD in the ratio $\alpha : \beta + \gamma$

Thus, the position vector of incentre is

$$\text{equal to } \frac{\alpha\vec{a} + \frac{\beta\vec{b} + \gamma\vec{c}}{(\beta + \gamma)} \times (\beta + \gamma)}{\alpha + \beta + \gamma} = \frac{\alpha\vec{a} + \beta\vec{b} + \gamma\vec{c}}{\alpha + \beta + \gamma}$$

Ex 23.4

Algebra of Vectors Ex 23.4 Q1



Here, in ΔABC, D, E, F are the mid points of the sides of BC, CA and AB respectively. And O is any point in space.

Let $\vec{a}, \vec{b}, \vec{c}, \vec{d}, \vec{e}, \vec{f}$ be the position vector of point A, B, C, D, E, F with respect to O.

So, $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OC} = \vec{c}$
$\overrightarrow{OD} = \vec{d}$, $\overrightarrow{OE} = \vec{e}$, $\overrightarrow{OF} = \vec{f}$

$$\vec{d} = \frac{\vec{b} + \vec{c}}{2}$$

$$\vec{e} = \frac{\vec{a} + \vec{c}}{2} \qquad \text{[Using mid point formula]}$$

$$\vec{f} = \frac{\vec{a} + \vec{b}}{2}$$

$$\begin{aligned}
\overrightarrow{OD} + \overrightarrow{OE} + \overrightarrow{OF} &= \vec{d} + \vec{e} + \vec{f} \\
&= \frac{\vec{b} + \vec{c}}{2} + \frac{\vec{a} + \vec{c}}{2} + \frac{\vec{a} + \vec{b}}{2} \\
&= \frac{\vec{b} + \vec{c} + \vec{a} + \vec{c} + \vec{a} + \vec{b}}{2} \\
&= \frac{2\left(\vec{a} + \vec{b} + \vec{c}\right)}{2} \\
&= \vec{a} + \vec{b} + \vec{c} \\
&= \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}
\end{aligned}$$

So,
$$\overrightarrow{OD} + \overrightarrow{OE} + \overrightarrow{OF} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}$$

Algebra of Vectors Ex 23.4 Q2

Here, we have to show that the sum of the three vectors ditermined by medians of a triangle directed from the vertices is zero.

Let ABC is triangle such that position vector of A, B and C are $\vec{a}, \vec{b}$ and $\vec{c}$ respectively.

As AD, BE, CF are medians, D, E and F are mid points.

Position vector of $D = \dfrac{\vec{b}+\vec{c}}{2}$ [Using mid point formula]

Position vector of $E = \dfrac{\vec{c}+\vec{a}}{2}$ [Using mid point formula]

Position vector of $F = \dfrac{\vec{a}+\vec{b}}{2}$ [Using mid point formula]

Now,

$$\overrightarrow{AD}+\overrightarrow{BE}+\overrightarrow{CF}$$
$$= \left(\frac{\vec{b}+\vec{c}}{2}-\vec{a}\right)+\left(\frac{\vec{c}+\vec{a}}{2}-\vec{b}\right)+\left(\frac{\vec{a}+\vec{b}}{2}-\vec{c}\right)$$
$$= \frac{\vec{b}+\vec{c}-2\vec{a}}{2}+\frac{\vec{c}+\vec{a}-2\vec{b}}{2}+\frac{\vec{a}+\vec{b}-2\vec{c}}{2}$$
$$= \frac{\vec{b}+\vec{c}-2\vec{a}+\vec{c}+\vec{a}-2\vec{b}+\vec{a}+\vec{b}-2\vec{c}}{2}$$
$$= \frac{2\vec{b}+2\vec{c}+2\vec{a}-2\vec{b}-2\vec{a}-2\vec{c}}{2}$$
$$= \frac{\vec{0}}{2}$$
$$= \vec{0}$$

$$\therefore\ \overrightarrow{AD}+\overrightarrow{BE}+\overrightarrow{CF}=\vec{0}$$

Algebra of Vectors Ex 23.4 Q3

Here, it is given that $ABCD$ is a parallelogram, P is the point of intersection of diagonals and O be the point of reference.

Using triangle law in ΔAOP,

$\overrightarrow{OP}+\overrightarrow{PA}=\overrightarrow{OA}$ (i)

Using triangle law in ΔOBP,

$\overrightarrow{OP}+\overrightarrow{PB}=\overrightarrow{OB}$ (ii)

Using triangle law in ΔOPC,

$\overrightarrow{OP}+\overrightarrow{PC}=\overrightarrow{OC}$ (iii)

Using triangle law in ΔOPD,

$\overrightarrow{OP}+\overrightarrow{PD}=\overrightarrow{OD}$ (iv)

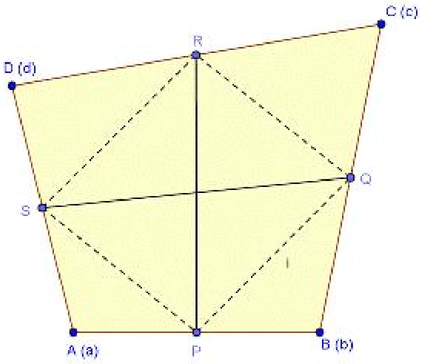
Adding equation (i), (ii), (iii), and (iv),

$$\overrightarrow{OP}+\overrightarrow{PA}+\overrightarrow{OP}+\overrightarrow{PB}+\overrightarrow{OP}+\overrightarrow{PC}+\overrightarrow{OP}+\overrightarrow{PD}=\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}$$
$$4\overrightarrow{OP}+\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}+\overrightarrow{PD}=\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}$$
$$4\overrightarrow{OP}+\overrightarrow{PA}+\overrightarrow{PB}-\overrightarrow{PA}-\overrightarrow{PB}=\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}$$

$\left[\begin{array}{l}\text{Since } \overrightarrow{PC}=-\overrightarrow{PA} \text{ and } \overrightarrow{PD}=-\overrightarrow{PB} \\ \text{as } P \text{ is mid point of } AC, BD\end{array}\right]$

$$4\overrightarrow{OP}=\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}$$

Algebra of Vectors Ex 23.4 Q4



Let $ABCD$ be a quadrilateral and P,Q,R,S be the mid points of sides AB, BC, CD and DA respectively.

Let position vector of A,B,C and D be $\vec{a},\vec{b},\vec{c}$, and $\vec{d}$.

So position vector of P,Q,R and S are $\left(\frac{\vec{a}+\vec{b}}{2}\right)$, $\left(\frac{\vec{b}+\vec{c}}{2}\right)$, $\left(\frac{\vec{c}+\vec{d}}{2}\right)$ and $\left(\frac{\vec{d}+\vec{a}}{2}\right)$ respectively.

Position vector of $\overrightarrow{PQ}$

= Position vector of Q − Position vector of P

$= \left(\frac{\vec{b}+\vec{c}}{2}\right)-\left(\frac{\vec{a}+\vec{b}}{2}\right)$

$= \frac{\vec{b}+\vec{c}-\vec{a}-\vec{b}}{2}$

$= \frac{\vec{c}-\vec{a}}{2}$ (i)

Position vector of $\overrightarrow{SR}$

= Position vector of R − Position vector of S

$= \left(\frac{\vec{c}+\vec{d}}{2}\right)-\left(\frac{\vec{a}+\vec{d}}{2}\right)$

$= \frac{\vec{c}+\vec{d}-\vec{a}-\vec{d}}{2}$

$= \frac{\vec{c}-\vec{a}}{2}$ (ii)

Using (i) and (ii) ,

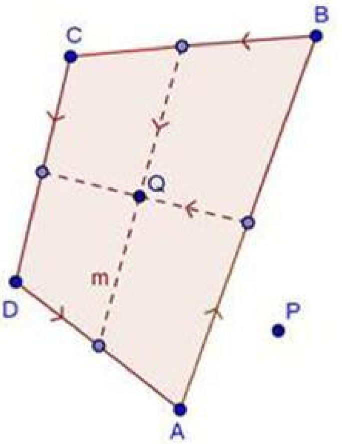
$\overrightarrow{PQ} = \overrightarrow{SR}$

So, $PQRS$ is a parallelogram.

Therefore, PR bisects QS [as diagonals of parallelogram]

Line segment joining the mid point of opposite sides of a quadrilateral bisects each other.

Algebra of Vectors Ex 23.4 Q5



Let $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ be the position vectors of the points $A, B, C,$ and D respectively.

Then, position vector of

$$\text{mid point of } AB = \frac{\vec{a}+\vec{b}}{2}$$

$$\text{mid point of } BC = \frac{\vec{b}+\vec{c}}{2}$$

$$\text{mid point of } CD = \frac{\vec{c}+\vec{d}}{2}$$

$$\text{mid point of } DA = \frac{\vec{a}+\vec{d}}{2}$$

Q is the mid point of the line joining the mid points of AB and CD

$$\therefore \quad p.r. \text{ or } Q = \frac{\frac{\vec{a}+\vec{a}}{2}+\frac{\vec{c}+\vec{d}}{2}}{2}$$

$$= \frac{\vec{a}+\vec{b}+\vec{c}+\vec{d}}{4}$$

Let $\vec{p}$ be the position vector of P.

Then,

$$\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}+\overrightarrow{PD}$$

$$= \vec{a}-\vec{p}+\vec{b}-\vec{p}+\vec{c}-\vec{p}+\vec{d}-\vec{p}$$

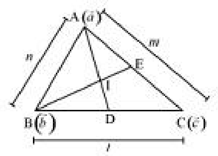
$$= \left(\vec{a}+\vec{b}+\vec{c}+\vec{d}\right)-4\vec{p}$$

$$= 4\left(\frac{\vec{a}+\vec{b}+\vec{c}+\vec{d}}{4}-\vec{p}\right)$$

$$= 4\overrightarrow{PQ}$$

Algebra of Vectors Ex 23.4 Q6

Let $A\left(\vec{a}\right), B\left(\vec{b}\right)$ and $C\left(\vec{c}\right)$ be the position vectors of the vertices of the triangle ΔABC and the length of the sides BC, CA and AB be l, m and n respectively.



The internal bisector of a triangle divides the opposite side in the ratio of the sides containing the angles.

Since AD is the internal bisector of the $\angle ABC$,

$$\frac{BD}{DC} = \frac{AB}{AC} = \frac{n}{m} \qquad (1)$$

Therefore position vector of $D = \dfrac{n\vec{c}+m\vec{b}}{m+n}$

Let the internal bisector intersect at I.

$$\frac{ID}{AI} = \frac{BD}{AB} \qquad (2)$$

$$\frac{BD}{DC} = \frac{n}{m}$$

Therefore,

$$\frac{CD}{BD} = \frac{m}{n}$$

$$\frac{CD+BD}{BD} = \frac{m+n}{n}$$

$$\frac{BC}{BD} = \frac{m+n}{n}$$

$$BD = \frac{\ln}{m+n} \qquad (3)$$

From (2) and (3), we get

$$\frac{ID}{AI} = \frac{\ln}{m+n}$$

Therefore,

$$\text{Position vector of } I = \frac{\left(\dfrac{nc+mb}{m+n}\right)(m+n)+la}{l+m+n} = \frac{la+mb+nc}{l+m+n}$$

Similarly, we can prove that I lie on the internal bisectors of angles B and C.

Hence the three bisectors are concurrent.

Ex 23.5

Algebra of Vectors Ex 23.5 Q1

Here $\vec{a} = -4\hat{i} - 3\hat{j}$

$$|\vec{a}| = \sqrt{(-4)^2 + (-3)^2}$$
$$= \sqrt{16+9}$$
$$= \sqrt{25}$$
$$= 5$$

$\therefore |\vec{a}| = 5$

Algebra of Vectors Ex 23.5 Q2

Here $\vec{a} = 12\hat{i} + n\hat{j}$

$$|\vec{a}| = \sqrt{(12)^2 + (n)^2}$$
$$13 = \sqrt{144+n^2} \qquad \left[\text{Since } |\vec{a}| = 13\right]$$

Squaring both sides,

$$(13)^2 = \left(\sqrt{144+n^2}\right)^2$$
$$169 = 144 + n^2$$
$$n^2 = 169 - 144$$
$$n^2 = 25$$
$$n = \pm\sqrt{25}$$
$$n = \pm 5$$

Algebra of Vectors Ex 23.5 Q3

Here, $\vec{a} = \sqrt{3}\hat{i} + \hat{j}$

Let $\vec{b}$ is any vector parallel to $\vec{a}$

So, $\vec{b} = \lambda\vec{a}$ (where λ is any scalar)

$$= \lambda\left(\sqrt{3}\hat{i} + \hat{j}\right)$$
$$\vec{b} = \lambda\sqrt{3}\hat{i} + \lambda\hat{j}$$
$$|\vec{b}| = \sqrt{(\lambda\sqrt{3})^2 + (\lambda)^2}$$
$$= \sqrt{3\lambda^2 + \lambda^2}$$
$$= \sqrt{4\lambda^2}$$
$$|\vec{b}| = 2\lambda$$
$$4 = 2\lambda$$
$$\lambda = \frac{4}{2}$$
$$\lambda = 2$$

$$\therefore \vec{b} = \lambda\sqrt{3}\hat{i} + \lambda\hat{j}$$
$$\vec{b} = 2\sqrt{3}\hat{i} + 2\hat{j}$$

Algebra of Vectors Ex 23.5 Q4

(i) Here, $A = (4, -1)$

$B = (1, 3)$

Position vector of $A = 4\hat{i} - \hat{j}$

Position vector of $B = \hat{i} + 3\hat{j}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$= (\hat{i} + 3\hat{j}) - (4\hat{i} - \hat{j})$

$= \hat{i} + 3\hat{j} - 4\hat{i} + \hat{j}$

$\overrightarrow{AB} = -3\hat{i} + 4\hat{j}$

$\left|\overrightarrow{AB}\right| = \sqrt{(-3)^2 + (4)^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

$\left|\overrightarrow{AB}\right| = 5$

$\overrightarrow{AB} = -3\hat{i} + 4\hat{j}$

(ii) Here, $A = (-6, 3)$

$B = (-2, -5)$

Position vector of $A = -6\hat{i} + 3\hat{j}$

Position vector of $B = -2\hat{i} - 5\hat{j}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$= (-2\hat{i} - 5\hat{j}) - (-6\hat{i} + 3\hat{j})$

$= -2\hat{i} - 5\hat{j} + 6\hat{i} - 3\hat{j}$

$\overrightarrow{AB} = 4\hat{i} - 8\hat{j}$

$\left|\overrightarrow{AB}\right| = \sqrt{(4)^2 + (-8)^2}$

$= \sqrt{16 + 64}$

$= \sqrt{80}$

$= \sqrt{16 \times 5}$

$= 4\sqrt{5}$

$\left|\overrightarrow{AB}\right| = 4\sqrt{5}$

$\overrightarrow{AB} = 4\hat{i} - 8\hat{j}$

Algebra of Vectors Ex 23.5 Q5

Here, $A = (-1, 3)$

$B = (-2, 1)$

Position vector of $A = -\hat{i} + 3\hat{j}$

Position vector of $B = -2\hat{i} + 1\hat{j}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$= (-2\hat{i} + \hat{j}) - (-\hat{i} + 3\hat{j})$

$= -2\hat{i} + \hat{j} + \hat{i} - 3\hat{j}$

$= -\hat{i} - 2\hat{j}$

So,

Coordinate of the position vector equivalent to $\overrightarrow{AB} = (-1, -2)$

Algebra of Vectors Ex 23.5 Q6

Here, $A = (-2, -1)$

$B = (3, 0)$

$C = (1, -2)$

Let $D = (x, y)$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$$= \left(3\hat{i} - 0 \times \hat{j}\right) - \left(-2\hat{i} - \hat{j}\right)$$
$$= 3\hat{i} - 0 \times \hat{j} + 2\hat{i} + \hat{j}$$
$$\overrightarrow{AB} = 5\hat{i} + \hat{j}$$

$\overrightarrow{DC}$ = Position vector of C – Position vector of D

$$= \left(\hat{i} - 2\hat{j}\right) - \left(x\hat{i} + y\hat{j}\right)$$
$$= \hat{i} - 2\hat{j} - x\hat{i} - y\hat{j}$$
$$\overrightarrow{DC} = (1 - x)\hat{i} + (-2 - y)\hat{j}$$

Since $ABCD$ is a parallelogram, which have equal and parallel opposite sides.

So, $\overrightarrow{AB} = \overrightarrow{DC}$

$$5\hat{i} + \hat{j} = (1 - x)\hat{i} + (-2 - y)\hat{j}$$

Comparing components of LHS and RHS

$$5 = 1 - x$$
$$x = 1 - 5$$
$$x = -4$$
$$1 = -2 - y$$
$$y = -2 - 1$$
$$y = -3$$

So, coordinate of D is $(-4, -3)$

Algebra of Vectors Ex 23.5 Q7

Here, $A(3, 4), B(5, -6), C(4, -1)$

$$\vec{a} = 3\hat{i} + 4\hat{j}$$
$$\vec{b} = 5\hat{i} - 6\hat{j}$$
$$\vec{c} = 4\hat{i} - \hat{j}$$

Now,

$$\vec{a} + 2\vec{b} - 3\vec{c} = \left(3\hat{i} + 4\hat{j}\right) + 2\left(5\hat{i} - 6\hat{j}\right) - 3\left(4\hat{i} - \hat{j}\right)$$
$$= 3\hat{i} + 4\hat{j} + 10\hat{i} - 12\hat{j} - 12\hat{i} + 3\hat{j}$$
$$= \hat{i} - 5\hat{j}$$

$$\therefore \vec{a} + 2\vec{b} - 3\vec{c} = \hat{i} - 5\hat{j}$$

Algebra of Vectors Ex 23.5 Q9

$$\left|\overrightarrow{AB}\right| = 5 \text{ units}$$
$$\left|\overrightarrow{BC}\right| = \sqrt{(8)^2}$$
$$\left|\overrightarrow{BC}\right| = 8 \text{ units}$$
$$\left|\overrightarrow{AC}\right| = \sqrt{(-3)^2 + (8)^2}$$
$$= \sqrt{9 + 16}$$
$$= \sqrt{25}$$
$$\left|\overrightarrow{AC}\right| = 5 \text{ units}$$

Here, $\left|\overrightarrow{AB}\right| = \left|\overrightarrow{AC}\right|$

$$5 = 5$$

So, there are two sides AB,and BC of ΔABC have same length.

ΔABC is an isosceles triangle.

Algebra of Vectors Ex 23.5 Q10

Let $\vec{a} = \hat{i} + \sqrt{3}\hat{j}$

Suppose $\vec{b}$ is any vector parallel to $\vec{a}$

$\vec{b} = \lambda\vec{a}$ where λ is a scalar

$$= \lambda\left(\hat{i} + \sqrt{3}\hat{j}\right)$$

$$\vec{b} = \lambda\hat{i} + \sqrt{3}\lambda\hat{j}$$

$$\left|\vec{b}\right| = \sqrt{(\lambda)^2 + \left(\sqrt{3}\lambda\right)^2}$$
$$= \sqrt{\lambda^2 + 3\lambda^2}$$
$$= \sqrt{4\lambda^2}$$
$$= 2\lambda$$

Unit vector of $\vec{b} = \dfrac{\vec{b}}{\left|\vec{b}\right|}$

$$\hat{b} = \frac{\lambda\hat{i} + \sqrt{3}\lambda\hat{j}}{2\lambda}$$

$$\hat{b} = \frac{\left(\hat{i} + \sqrt{3}\ \hat{j}\right)}{2}$$

$$\hat{b} = \frac{1}{2}\left(\hat{i} + \sqrt{3}\ \hat{j}\right)$$

Algebra of Vectors Ex 23.5 Q11

(i) Here, $P = (3, 2)$

Position vector of $P = 3\hat{i} + 2\hat{j}$

Component of P along x-axis $= 3\hat{i}$

Component of P along y-axis $= 2\hat{j}$

(ii) Here, $Q = (-5, 1)$

Position vector of $Q = -5\hat{i} + \hat{j}$

Component of Q along x-axis $= -5\hat{i}$

Component of Q along y-axis $= \hat{j}$

(ii) Here, $R = (-11, -9)$

Position vector of $R = -11\hat{i} - 9\hat{j}$

Component of R along x-axis $= -11\hat{i}$

Component of R along y-axis $= -9\hat{j}$

(iv) Here, $S = (4, -3)$

Position vector of $S = 4\hat{i} - 3\hat{j}$

Component of S along x-axis $= 4\hat{i}$

Component of S along y-axis $= -3\hat{j}$

Ex 23.6

Algebra of Vectors Ex 23.6 Q1

Magnitude of a vector $x\hat{i}+y\hat{j}+z\hat{k}$ is given by $\sqrt{(x)^2+y^2+z^2}$.

So,

$$|\vec{a}| = \sqrt{(2)^2+(3)^2+(-6)^2}$$
$$= \sqrt{4+9+36}$$
$$= \sqrt{49}$$
$$= 7$$

$$\therefore |\vec{a}| = 7$$

Algebra of Vectors Ex 23.6 Q2

Unit vector of $\vec{a} = \dfrac{\vec{a}}{|\vec{a}|}$

$$\text{Unit vector of } 3\hat{i}+4\hat{j}-12\hat{k} = \frac{3\hat{i}+4\hat{j}-12\hat{k}}{\sqrt{(3)^2+(4)^2+(-12)^2}}$$
$$= \frac{3\hat{i}+4\hat{j}-12\hat{k}}{\sqrt{9+16+144}}$$
$$= \frac{3\hat{i}+4\hat{j}-12\hat{k}}{\sqrt{169}}$$

Unit vector of $\left(3\hat{i}+4\hat{j}-12\hat{k}\right) = \dfrac{1}{13}\left(3\hat{i}+4\hat{j}-12\hat{k}\right)$

Algebra of Vectors Ex 23.6 Q3

Let $\vec{a} = \hat{i}-\hat{j}+3\hat{k}$
$\vec{b} = 2\hat{i}+\hat{j}-2\hat{k}$
$\vec{b} = \hat{i}+2\hat{j}-2\hat{k}$

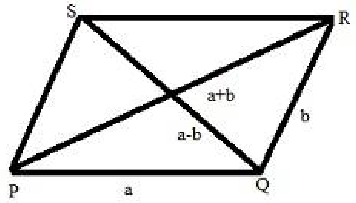
Let d be the resultant of $\vec{a}, \vec{b},$ and $\vec{c}$,

$$\vec{d} = \vec{a}+\vec{b}+\vec{c}$$
$$= \left(\hat{i}-\hat{j}+3\hat{k}\right)+\left(2\hat{i}+\hat{j}-2\hat{k}\right)+\left(\hat{i}+2\hat{j}-2\hat{k}\right)$$
$$\vec{d} = 4\hat{i}+2\hat{j}-\hat{k}$$

$$\text{Unit vector } \vec{d} = \frac{\vec{d}}{|\vec{d}|}$$
$$= \frac{4\hat{i}+2\hat{j}-\hat{k}}{\sqrt{(4)^2+(2)^2+(-1)^2}}$$
$$\vec{d} = \frac{4\hat{i}+2\hat{j}-\hat{k}}{\sqrt{16+4+1}}$$

Algebra of Vectors Ex 23.6 Q4

Let $PQRS$ be a parallelogram such that $PQ = \vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $QR = \vec{b} = -2\hat{i} + \hat{j} + 2\hat{k}$



In $\triangle PQR$

$$\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$$

$$\overrightarrow{PR} = \vec{a} + \vec{b} = \hat{i} + \hat{j} - \hat{k} + \left(-2\hat{i} + \hat{j} + 2\hat{k}\right)$$

$$\overrightarrow{PR} = -\hat{i} + 2\hat{j} + \hat{k}$$

In $\triangle PSQ$

$$\overrightarrow{PS} + \overrightarrow{SQ} = \overrightarrow{PQ}$$

$$\overrightarrow{SQ} = \vec{a} - \vec{b} = \hat{i} + \hat{j} - \hat{k} - \left(-2\hat{i} + \hat{j} + 2\hat{k}\right)$$

$$\overrightarrow{SQ} = 3\hat{i} + 0\hat{j} - 3\hat{k}$$

The unit vector along $\overrightarrow{PR} = \dfrac{\overrightarrow{PR}}{\left|\overrightarrow{PR}\right|} = \dfrac{-\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{1+4+1}} = \dfrac{1}{\sqrt{6}}\left(-\hat{i} + 2\hat{j} + \hat{k}\right)$

The unit vector along $\overrightarrow{SQ} = \dfrac{\overrightarrow{SQ}}{\left|\overrightarrow{SQ}\right|} = \dfrac{3\hat{i} + 0\hat{j} - 3\hat{k}}{\sqrt{9+0+9}} = \dfrac{1}{\sqrt{2}}\left(\hat{i} - \hat{k}\right)$

Algebra of Vectors Ex 23.6 Q5

$$3\vec{a} - 2\vec{b} + 4\vec{c} = 3\left(3\hat{i} - \hat{j} - 4\hat{k}\right) - 2\left(-2\hat{i} + 4\hat{j} - 3\hat{k}\right) + 4\left(\hat{i} + 2\hat{j} - \hat{k}\right)$$

$$= 9\hat{i} - 3\hat{j} - 12\hat{k} + 4\hat{i} - 8\hat{j} + 6\hat{k} + 4\hat{i} + 8\hat{j} - 4\hat{k}$$

$$= 17\hat{i} - 3\hat{j} - 10\hat{k}$$

$$\left|3\vec{a} - 2\vec{b} + 4\vec{c}\right| = \sqrt{(17)^2 + (-3)^2 + (10)^2}$$

$$= \sqrt{289 + 9 + 100}$$

$$= \sqrt{398}$$

$$\left|3\vec{a} - 2\vec{b} + 4\vec{c}\right| = \sqrt{398}$$

Algebra of Vectors Ex 23.6 Q6

Here, $\overrightarrow{PQ} = 3\hat{i} + 2\hat{j} - \hat{k}$

Position vector of $P = \hat{i} - \hat{j} + 2\hat{k}$

$\overrightarrow{PQ}$ = Position vector of Q – Position vector of P

$3\hat{i} + 2\hat{j} - \hat{k}$ = Position vector of $Q - \left(\hat{i} - \hat{j} + 2\hat{k}\right)$

Position vector of $Q = \left(3\hat{i} + 2\hat{j} - \hat{k}\right) + \left(\hat{i} - \hat{j} + 2\hat{k}\right)$

$= 4\hat{i} + \hat{j} + \hat{k}$

Coordinates of $Q = (4, 1, 1)$

Algebra of Vectors Ex 23.6 Q7

Let $\vec{A} = \hat{i} - \hat{j}$

$\vec{B} = 4\hat{i} + 3\hat{j} + \hat{k}$

$\vec{C} = 2\hat{i} - 4\hat{j} + 5\hat{k}$

$$\overrightarrow{AB} = \vec{B} - \vec{A}$$
$$= \left(4\hat{i} + 3\hat{j} + \hat{k}\right) - \left(\hat{i} - \hat{j}\right)$$
$$= 4\hat{i} + 3\hat{j} + \hat{k} - \hat{i} + \hat{j}$$
$$= 3\hat{i} + 4\hat{j} + \hat{k}$$

$$\left|\overrightarrow{AB}\right| = \sqrt{(3)^2 + (4)^2 + (1)^2} = \sqrt{9+16+1} = \sqrt{26}$$

$$\overrightarrow{BC} = \vec{C} - \vec{B}$$
$$= \left(2\hat{i} - 4\hat{j} + 5\hat{k}\right) - \left(4\hat{i} + 3\hat{j} + \hat{k}\right)$$
$$= 2\hat{i} - 4\hat{j} + 5\hat{k} - 4\hat{i} - 3\hat{j} - \hat{k}$$
$$= -2\hat{i} - 7\hat{j} + 4\hat{k}$$

$$\left|\overrightarrow{BC}\right| = \sqrt{(2)^2 + (-7)^2 + (4)^2} = \sqrt{4+49+16} = \sqrt{69}$$

$$\overrightarrow{CA} = \vec{A} - \vec{C}$$
$$= \hat{i} - \hat{j} - \left(2\hat{i} - 4\hat{j} + 5\hat{k}\right)$$
$$= \hat{i} - \hat{j} - 2\hat{i} + 4\hat{j} - 5\hat{k}$$
$$= -\hat{i} + 3\hat{j} - 5\hat{k}$$

$$\left|\overrightarrow{CA}\right| = \sqrt{(-1)^2 + (3)^2 + (-5)^2} = \sqrt{1+9+25} = \sqrt{35}$$

Here, $\left|AB\right|^2 + \left|\overrightarrow{CA}\right|^2 = \left|\overrightarrow{BC}\right|^2$

$$26 + 35 = 69$$
$$61 \neq 69$$
$$\text{LHS} \neq \text{RHS}$$

Since sum of square of two sides is not equal to the square of third sides. So,
ΔABC is not a right triangle

Algebra of Vectors Ex 23.6 Q8

Here,

Let vertex $\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

vertex $\vec{B} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

vertex $\vec{C} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

Side $\overrightarrow{AB} = \vec{B} - \vec{A}$
$$= \left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right) - \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right)$$
$$\overrightarrow{AB} = \left(b_1 - a_1\right)\hat{i} + \left(b_2 - a_2\right)\hat{j} + \left(b_3 - a_3\right)\hat{k}$$

$$\overrightarrow{BC} = \vec{C} - \vec{B}$$
$$= \left(c_1\hat{i} + c_2\hat{j} + c_3\hat{k}\right) - \left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right)$$
$$= c_1\hat{i} + c_2\hat{j} + c_3\hat{k} - b_1\hat{i} - b_2\hat{j} - b_3\hat{k}$$
$$\overrightarrow{BC} = \left(c_1 - b_1\right)\hat{i} + \left(c_2 - b_2\right)\hat{j} + \left(c_3 - b_3\right)\hat{k}$$

$$\overrightarrow{AC} = \left(c_1\hat{i} + c_2\hat{j} + c_3\hat{k}\right) - \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right)$$
$$= c_1\hat{i} + c_2\hat{j} + c_3\hat{k} - a_1\hat{i} - a_2\hat{j} - a_3\hat{k}$$
$$\overrightarrow{AC} = \left(c_1 - a_1\right)\hat{i} + \left(c_2 - a_2\right)\hat{j} + \left(c_3 - a_3\right)\hat{k}$$

$$\left|\overrightarrow{AB}\right| = \sqrt{\left(b_1 - a_1\right)^2 + \left(b_2 - a_2\right)^2 + \left(b_3 - a_3\right)^2}$$
$$\left|\overrightarrow{BC}\right| = \sqrt{\left(c_1 - b_1\right)^2 + \left(c_2 - b_2\right)^2 + \left(c_3 - b_3\right)^2}$$
$$\left|\overrightarrow{AC}\right| = \sqrt{\left(c_1 - a_1\right)^2 + \left(c_2 - a_2\right)^2 + \left(c_3 - a_3\right)^2}$$

Algebra of Vectors Ex 23.6 Q9

Here, given vertex $A = (1, -1, 2)$

$$\vec{A} = \hat{i} - \hat{j} + 2\hat{k}$$

vertex $B = (2, 1, 3)$

$$\vec{B} = 2\hat{i} + \hat{j} + 3\hat{k}$$

vertex $C = (-1, 2, -1)$

$$\vec{C} = -\hat{i} + 2\hat{j} - \hat{k}$$

$$\text{Centroid } \vec{O} = \frac{\vec{A} + \vec{B} + \vec{C}}{3}$$

$$= \frac{\left(\hat{i} - \hat{j} + 2\hat{k}\right) + \left(2\hat{i} + \hat{j} + 3\hat{k}\right) + \left(-\hat{i} + 2\hat{j} - \hat{k}\right)}{3}$$

$$= \frac{2\hat{i} + 2\hat{j} + 4\hat{k}}{3}$$

$$\text{Centroid } \vec{O} = \frac{2\hat{i}}{3} + \frac{2\hat{j}}{3} + \frac{4\hat{k}}{3}$$

Algebra of Vectors Ex 23.6 Q10

The position vector of point R dividing the line segment joining two points

P and Q in the ratio $m: n$ is given by:

i. Internally:

$$\frac{m\vec{b} + n\vec{a}}{m + n}$$

ii. Externally:

$$\frac{m\vec{b} - n\vec{a}}{m - n}$$

Position vectors of P and Q are given as:

$$\overrightarrow{OP} = \hat{i} + 2\hat{j} - \hat{k} \text{ and } \overrightarrow{OQ} = -\hat{i} + \hat{j} + \hat{k}$$

(i) The position vector of point R which divides the line joining two points P and Q internally in the ratio 2:1 is given by,

$$\overrightarrow{OR} = \frac{2\left(-\hat{i} + \hat{j} + \hat{k}\right) + 1\left(\hat{i} + 2\hat{j} - \hat{k}\right)}{2+1} = \frac{\left(-2\hat{i} + 2\hat{j} + 2\hat{k}\right) + \left(\hat{i} + 2\hat{j} - \hat{k}\right)}{3}$$

$$= \frac{-\hat{i} + 4\hat{j} + \hat{k}}{3} = -\frac{1}{3}\hat{i} + \frac{4}{3}\hat{j} + \frac{1}{3}\hat{k}$$

Algebra of Vectors Ex 23.6 Q11

Here, $P\left(2\hat{i} - 3\hat{j} + 4\hat{k}\right)$ and

$Q\left(4\hat{i} + \hat{j} - 2\hat{k}\right)$

We know that,

If A and B are two points with position vector $\vec{a}$ and $\vec{b}$ then the position vector of mid point C is given by

$$\frac{\vec{a} + \vec{b}}{2}$$

Let R is the mid point of PQ.

$$\text{Position vector of } R = \frac{\vec{P} + \vec{Q}}{2}$$

$$\vec{R} = \frac{2\hat{i} - 3\hat{j} + 4\hat{k} + 4\hat{i} + \hat{j} - 2\hat{k}}{2}$$

$$= \frac{6\hat{i} - 2\hat{j} + 2\hat{k}}{2}$$

$$= \frac{2\left(3\hat{i} - \hat{j} + \hat{k}\right)}{2}$$

Position vector of mid point $= 3\hat{i} - \hat{j} + \hat{k}$

Algebra of Vectors Ex 23.6 Q12

Here, point $P = (1,2,3)$

$$\vec{P} = \hat{i} + 2\hat{j} + 3\hat{k}$$

Point $Q = (4,5,6)$

$$\vec{Q} = 4\hat{i} + 5\hat{j} + 6\hat{k}$$

$$\begin{aligned}\overrightarrow{PQ} &= \text{Position vector of } Q - \text{Position vector of } P\\ &= \left(4\hat{i} + 5\hat{j} + 6\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)\\ &= 4\hat{i} + 5\hat{j} + 6\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k}\\ &= 3\hat{i} + 3\hat{j} + 3\hat{k}\\ &= 3\left(\hat{i} + \hat{j} + k\right)\end{aligned}$$

$$\begin{aligned}\left|\overrightarrow{PQ}\right| &= 3\sqrt{(1)^2 + (1)^2 + (1)^2}\\ &= 3\sqrt{1+1+1}\\ \left|\overrightarrow{PQ}\right| &= 3\sqrt{3}\end{aligned}$$

Unit vector in the direction of $\overrightarrow{PQ} = \dfrac{\overrightarrow{PQ}}{\left|\overrightarrow{PQ}\right|}$

$$= \frac{3\left(\hat{i} + \hat{j} + \hat{k}\right)}{3\sqrt{3}}$$

Unit vector is the direction of $\overrightarrow{PQ} = \dfrac{1}{\sqrt{3}}\left(\hat{i} + \hat{j} + \hat{k}\right)$

Algebra of Vectors Ex 23.6 Q13

The position vectors of A, B and C are $2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ respectively.

Therefore,

$\overrightarrow{AB} = -\hat{i} - 2\hat{j} - 6\hat{k}, \overrightarrow{BC} = 2\hat{i} - \hat{j} + \hat{k}$ and $\overrightarrow{CA} = -\hat{i} + 3\hat{j} + 5\hat{k}$

Clearly, $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0$.

So, $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CA}$ form a triangle.

Now

$$\begin{aligned}\left|\overrightarrow{AB}\right| &= \sqrt{1+4+36} = \sqrt{41}\\ \left|\overrightarrow{BC}\right| &= \sqrt{4+1+1} = \sqrt{6}\\ \left|\overrightarrow{CA}\right| &= \sqrt{1+9+25} = \sqrt{35}\end{aligned}$$

Clearly,

$$\left|\overrightarrow{AB}\right|^2 = \left|\overrightarrow{BC}\right|^2 + \left|\overrightarrow{CA}\right|^2$$

$$AB^2 = BC^2 + CA^2$$

Hence ΔABC is a right triangle right angle at C.

Algebra of Vectors Ex 23.6 Q14

Find the position vector of the mid point of the vector joining the points P (2, 3, 4) and Q (4, 1, − 2).

Solution 16:

The position vector of mid-point R of the vector joining points P (2, 3, 4) and Q (4, 1, − 2) is given by,

$$\overline{\text{OR}} = \frac{\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) + \left(4\hat{i} + \hat{j} - 2\hat{k}\right)}{2} = \frac{(2+4)\hat{i} + (3+1)\hat{j} + (4-2)\hat{k}}{2}$$

$$= \frac{6\hat{i} + 4\hat{j} + 2\hat{k}}{2} = 3\hat{i} + 2\hat{j} + \hat{k}$$

Algebra of Vectors Ex 23.6 Q15

$x\left(\hat{i}+\hat{j}+\hat{k}\right)$ is a unit vector if $\left|x\left(\hat{i}+\hat{j}+\hat{k}\right)\right|=1$.

Now,

$$\left|x\left(\hat{i}+\hat{j}+\hat{k}\right)\right|=1$$
$$\Rightarrow \sqrt{x^2+x^2+x^2}=1$$
$$\Rightarrow \sqrt{3x^2}=1$$
$$\Rightarrow \sqrt{3}\,x=1$$
$$\Rightarrow x=\pm\frac{1}{\sqrt{3}}$$

Hence, the required value of x is $\pm\frac{1}{\sqrt{3}}$.

Algebra of Vectors Ex 23.6 Q16

We have,

$\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+3\hat{k}$ and $\vec{c}=\hat{i}-2\hat{j}+\hat{k}$

$$\begin{aligned}2\vec{a}-\vec{b}+3\vec{c}&=2\left(\hat{i}+\hat{j}+\hat{k}\right)-\left(2\hat{i}-\hat{j}+3\hat{k}\right)+3\left(\hat{i}-2\hat{j}+\hat{k}\right)\\&=2\hat{i}+2\hat{j}+2\hat{k}-2\hat{i}+\hat{j}-3\hat{k}+3\hat{i}-6\hat{j}+3\hat{k}\\&=3\hat{i}-3\hat{j}+2\hat{k}\end{aligned}$$
$$\left|2\vec{a}-\vec{b}+3\vec{c}\right|=\sqrt{3^2+(-3)^2+2^2}=\sqrt{9+9+4}=\sqrt{22}$$

Hence, the unit vector along $2\vec{a}-\vec{b}+3\vec{c}$ is

$$\frac{2\vec{a}-\vec{b}+3\vec{c}}{\left|2\vec{a}-\vec{b}+3\vec{c}\right|}=\frac{3\hat{i}-3\hat{j}+2\hat{k}}{\sqrt{22}}=\frac{3}{\sqrt{22}}\hat{i}-\frac{3}{\sqrt{22}}\hat{j}+\frac{2}{\sqrt{22}}\hat{k}.$$

Algebra of Vectors Ex 23.6 Q17

Here, $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$

$\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$

$2\vec{a} - \vec{b} + 3\vec{c}$

$$= 2\left(\hat{i} + \hat{j} + \hat{k}\right) - \left(4\hat{i} - 2\hat{j} + 3\hat{k}\right) + 3\left(\hat{i} - 2\hat{j} + \hat{k}\right)$$

$$= 2\hat{i} + 2\hat{j} + 2\hat{k} - 4\hat{i} + 2\hat{j} - 3\hat{k} + 3\hat{i} - 6\hat{j} + 3\hat{k}$$

$$= \hat{i} - 2\hat{j} + 2\hat{k}$$

Let $\vec{d}$ is a vector parallel to $2\vec{a} - \vec{b} + 3\vec{c}$

So, $\vec{d} = \lambda\left(2\vec{a} - \vec{b} + 3\vec{c}\right)$

Where λ is any scalar

$$= \lambda\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$$

$$\vec{d} = \lambda\hat{i} - \lambda 2\hat{j} + \lambda 2\hat{k} \qquad \text{(i)}$$

Given that $\left|\vec{d}\right| = 6$

$$\sqrt{(\lambda)^2 + (-2\lambda)^2 + (2\lambda)^2} = 6$$

$$\sqrt{\lambda^2 + 4\lambda^2 + 4\lambda^2} = 6$$

$$\sqrt{9\lambda^2} = 6$$

$$3\lambda = 6$$

$$\lambda = \frac{6}{3}$$

$$\lambda = 2$$

Put the value of λ in equation (i)

$\vec{d} = 2\hat{i} - 2(2)\hat{j} + 2(2)\hat{k}$

$= 2\hat{i} - 4\hat{j} + 4\hat{k}$

A vector of magnitude 6 which is parallel to $2\vec{a} - \vec{b} + 3\vec{c}$ is given by

$2\hat{i} - 4\hat{j} + 4\hat{k}$

Algebra of Vectors Ex 23.6 Q18

Given that

$\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$

and

$\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

Thus, Find a vector of magnitude of 5 *units parallel to the resultant of the vectors*

$\vec{a} + \vec{b} = 2\hat{i} + 3\hat{j} - \hat{k} + \hat{i} - 2\hat{j} + \hat{k}$

$\Rightarrow \vec{a} + \vec{b} = 3\hat{i} + \hat{j}$

$\Rightarrow \left|\vec{a} + \vec{b}\right| = \sqrt{9 + 1} = \sqrt{10}$

Thus, the unit vector along the resultant vector $\vec{a} + \vec{b}$ *is*

$$\frac{3\hat{i} + \hat{j}}{\sqrt{10}}$$

The vector of magnitude of 5 *units parallel to the resultant*

$$vector = \frac{3\hat{i} + \hat{j}}{\sqrt{10}} \times 5 = \sqrt{\frac{5}{2}}\left(3\hat{i} + \hat{j}\right)$$

Algebra of Vectors Ex 23.6 Q19

Let D be the point at which median drawn from A touches side BC.

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be the position vectors of the vertices A, B and C.

Position vector of D = $\frac{\vec{b}+\vec{c}}{2}$.........[Since D is midpoint of B and C]

$\overrightarrow{AD} = \frac{\vec{b}+\vec{c}}{2} - \vec{a} = \frac{\vec{b}+\vec{c}-2\vec{a}}{2} = \frac{\vec{b}-\vec{a}+\vec{c}-\vec{a}}{2} = \frac{\overrightarrow{AB}+\overrightarrow{AC}}{2} = \frac{\hat{j}+\hat{i}+3\hat{i}-\hat{j}+4\hat{k}}{2}$

$\overrightarrow{AD} = 2\hat{i}+2\hat{k}$

$\left|\overrightarrow{AD}\right| = \sqrt{4+4} = 4\sqrt{2}$ units

Note : Answer given in the book is incorrect.

Ex 23.7

Algebra of Vectors Ex 23.7 Q1

Here, position vector of A = Position vector of $A = \vec{a} - 2\vec{b} + 3\vec{c}$

position vector of B = Position vector of $B = 2\vec{a} + 3\vec{b} - 4\vec{c}$

position vector of C = Position vector of $C = -7\vec{b} + 10\vec{c}$

$\overrightarrow{AB}$ = position vector of B – position vector of A

$$= \left(2\vec{a} + 3\vec{b} - 4\vec{c}\right) - \left(\vec{a} - 2\vec{b} + 3\vec{c}\right)$$
$$= 2\vec{a} + 3\vec{b} - 4\vec{c} - \vec{a} + 2\vec{b} - 3\vec{c}$$
$$\overrightarrow{AB} = \vec{a} + 5\vec{b} - 7\vec{c}$$

$\overrightarrow{BC}$ = position vector of C – position vector of B

$$= \left(-7\vec{b} + 10\vec{c}\right) - \left(2\vec{a} + 3\vec{b} - 4\vec{c}\right)$$
$$= -7\vec{b} + 10\vec{c} - 2\vec{a} - 3\vec{b} + 4\vec{c}$$
$$\overrightarrow{BC} = -2\vec{a} - 10\vec{b} + 14\vec{c}$$

From $\overrightarrow{AB}$ and $\overrightarrow{BC}$, we get

$$\overrightarrow{BC} = -2\left(\overrightarrow{AB}\right)$$

So, $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel but $\vec{B}$ is a common vector. Hence, A, B, C are collinear.

Algebra of Vectors Ex 23.7 Q2(i)

Let the points be A, B, C

Position vector of $A = \vec{a}$

Position vector of $B = \vec{b}$

Position vector of $C = 3\vec{a} - 2\vec{b}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$$= \vec{b} - \vec{a}$$

$\overrightarrow{BC}$ = Position vector of C – Position vector of B

$$= 3\vec{a} - 2\vec{b} - \vec{b}$$
$$= 3\vec{a} - 3\vec{b}$$

Using $\overrightarrow{AB}$ and $\overrightarrow{BC}$

Let $\overrightarrow{BC} = \lambda\left(\overrightarrow{AB}\right)$ [where λ is and scalar]

$$3\vec{a} - 3\vec{b} = \lambda\left(\vec{b} - \vec{a}\right)$$
$$3\vec{a} - 3\vec{b} = \lambda\vec{b} - \lambda\vec{a}$$
$$3\vec{a} - 3\vec{b} = \lambda\vec{a} + \lambda\vec{b}$$

Comparing the coefficients of LHS and RHS, we get

$$-\lambda = 3$$
$$\lambda = 3$$
$$\lambda = -3$$

Since the value of λ are different.

Therefore,

A, B, C are not collinear.

Algebra of Vectors Ex 23.7 Q2(ii)

Let the points be A, B, C

Position vector of $A = \vec{a} + \vec{b} + \vec{c}$
Position vector of $B = 4\vec{a} + 3\vec{b}$
Position vector of $C = 10\vec{a} + 7\vec{b} - 2\vec{c}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$= \left(4\vec{a} + 3\vec{b}\right) - \left(\vec{a} + \vec{b} + \vec{c}\right)$

$= 4\vec{a} + 3\vec{b} - \vec{a} - \vec{b} - \vec{c}$

$\overrightarrow{AB} = 3\vec{a} + 2\vec{b} - \vec{c}$

$\overrightarrow{BC}$ = Position vector of C – Position vector of B

$= \left(10\vec{a} + 7\vec{b} - 2\vec{c}\right) - \left(4\vec{a} + 3\vec{b}\right)$

$= 10\vec{a} + 7\vec{b} - 2\vec{c} - 4\vec{a} - 3\vec{b}$

$\overrightarrow{BC} = 6\vec{a} + 4\vec{b} - 2\vec{c}$

Using $\overrightarrow{AB}$ and $\overrightarrow{BC}$

$\overrightarrow{BC} = 2\left(\overrightarrow{AB}\right)$

So, $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but $\vec{B}$ is a common vector. Hence, A,B,C are collinear.

Algebra of Vectors Ex 23.7 Q3

Let the points be A, B, C

Position vector of $A = \hat{i} + 2\hat{j} + 3\hat{k}$
Position vector of $B = 3\hat{i} + 4\hat{j} + 7\hat{k}$
Position vector of $C = -3\hat{i} - 2\hat{j} - 5\hat{k}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$= \left(3\hat{i} + 4\hat{j} + 7\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

$= 3\hat{i} + 4\hat{j} + 7\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k}$

$\overrightarrow{AB} = 2\hat{i} + 2\hat{j} + 4\hat{k}$

$\overrightarrow{BC}$ = Position vector of C – Position vector of B

$= \left(-3\hat{i} - 2\hat{j} - 5\hat{k}\right) - \left(3\hat{i} + 4\hat{j} + 7\hat{k}\right)$

$= -3\hat{i} - 2\hat{j} - 5\hat{k} - 3\hat{i} - 4\hat{j} - 7\hat{k}$

$\overrightarrow{BC} = -6\hat{i} - 6\hat{j} - 12\hat{k}$

Using $\overrightarrow{AB}$ and $\overrightarrow{BC}$ we get

$\overrightarrow{BC} = -3\left(\overrightarrow{AB}\right)$

So, $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but $\vec{B}$ is a common vector. Hence, A, B, C are collinear.

Algebra of Vectors Ex 23.7 Q4

Let the points be A, B, C

Position vector of $A = 10\hat{i} + 3\hat{j}$

Position vector of $B = 12\hat{i} - 5\hat{j}$

Position vector of $C = a\hat{i} + 11\hat{j}$

Given that, A,B,C are collinear

$\Rightarrow \overrightarrow{AB}$ and $\overrightarrow{BC}$ are collinear

$\Rightarrow \overrightarrow{AB} = \lambda\left(\overrightarrow{BC}\right)$ (Where λ is same scalar)

$\Rightarrow$ Position vector of B – Position vector of $A = \lambda$ – (Position vector of C – Position vector of B)

$$\Rightarrow \quad \left(12\hat{i} - 5\hat{j}\right) - \left(10\hat{i} + 3\hat{j}\right) = \lambda\left[\left(a\hat{i} + 11\hat{j}\right) - \left(12\hat{i} - 5\hat{j}\right)\right]$$

$$\Rightarrow \quad 12\hat{i} - 5\hat{j} - 10\hat{i} - 3\hat{j} = \lambda\left(a\hat{i} + 11\hat{j} - 12\hat{i} + 5\hat{j}\right)$$

$$\Rightarrow \quad 2\hat{i} - 8\hat{j} = \left(\lambda a - 12\lambda\right)\hat{i} = \left(11\lambda + 5\lambda\right)\hat{j}$$

Comparing the coefficients of LHS and RHS, we get

$\lambda a - 12\lambda = 2$ (i)

$-8 = 11\lambda + 5\lambda$ (ii)

$-8 = 16\lambda$

$\lambda = \frac{-8}{16}$

$\lambda = -\frac{1}{2}$

Put the value of λ in equation (i),

$$\lambda a - 12\lambda = 2$$

$$\left(-\frac{1}{2}\right)a - 12\left(-\frac{1}{2}\right) = 2$$

$$-\frac{1}{2}a + \frac{12}{2} = 2$$

$$-\frac{1}{2}a + 6 = 2$$

$$-\frac{1}{2}a = 2 - 6$$

$$-\frac{1}{2}a = -4$$

$$a = (-4) \times (-2)$$

$$a = 8$$

Algebra of Vectors Ex 23.7 Q5

Let A, B, C be the points then

Position vector of $A = \vec{a} + \vec{b}$
Position vector of $B = \vec{a} - \vec{b}$
Position vector of $C = \vec{a} + \lambda\vec{b}$

$\overrightarrow{AB}$ = Position vector of B − Position vector of A

$$= \left(\vec{a} - \vec{b}\right) - \left(\vec{a} + \vec{b}\right)$$
$$= \vec{a} - \vec{b} - \vec{a} - \vec{b}$$
$$\overrightarrow{AB} = -2\vec{b}$$

$\overrightarrow{BC}$ = Position vector of C − Position vector of B

$$= \left(\vec{a} + \lambda\vec{b}\right) - \left(\vec{a} - \vec{b}\right)$$
$$= \vec{a} + \lambda\vec{b} - \vec{a} + \vec{b}$$
$$= \lambda\vec{b} + \vec{b}$$
$$\overrightarrow{BC} = \left(\lambda + 1\right)\vec{b}$$

Using $\overrightarrow{AB}$ and $\overrightarrow{BC}$, we get

$$\overrightarrow{AB} = \left[\frac{(\lambda+1)}{-2}\right]\left(\overrightarrow{BC}\right)$$

Let $\left(\frac{\lambda+1}{-2}\right) = \mu$

Since λ is a real number. So,
μ is also a real no.

So, $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$, but $\vec{B}$ is a common vector. Hence,
A, B, C are collinear.

Algebra of Vectors Ex 23.7 Q6

Here, $\overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OB} + \overrightarrow{OC}$

$$\overrightarrow{OA} - \overrightarrow{BO} = \overrightarrow{BO} - \overrightarrow{CO}$$
$$\overrightarrow{AB} = \overrightarrow{BC}$$

So, $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but $\vec{B}$ is a common vector. Hence,
A, B, C are collinear.

Algebra of Vectors Ex 23.7 Q7

Let the given points be A and B

Position vector of $A = 2\hat{i} - 3\hat{j} + 4\hat{k}$
Position vector of $B = -4\hat{i} + 6\hat{j} - 8\hat{k}$

Let O be the initial point having postion vector

$$0 \times \hat{i} + 0 \times \hat{j} + 0 \times \hat{k}$$

$\overrightarrow{OA}$ = Position vector of A − Position vector of O

$$= \left(2\hat{i} - 3\hat{j} + 4\hat{k}\right) - \left(0 \times \hat{i} + 0 \times \hat{j} + 0 \times \hat{k}\right)$$
$$= 2\hat{i} - 3\hat{j} + 4\hat{k}$$

$\overrightarrow{OB}$ = Position vector of B − Position vector of O

$$= \left(-4\hat{i} + 6\hat{j} - 8\hat{k}\right) - \left(0 \times \hat{i} + 0 \times \hat{j} + 0 \times \hat{k}\right)$$
$$\overrightarrow{OB} = -4\hat{i} + 6\hat{j} - 8\hat{k}$$

Using OA and OB, we get

$$\overrightarrow{OB} = -2\left(\overrightarrow{OA}\right)$$

Therefore, $\overrightarrow{OA}$ is parallel to $\overrightarrow{OB}$ but O is the common point to them. Hence,
A and B are collinear.

Algebra of Vectors Ex 23.7 Q8

Here, $A = (m, -1)$

$B = (2, 1)$

$C = (4, 5)$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$= \left(2\hat{i} + \hat{j}\right) - \left(m\hat{i} - \hat{j}\right)$

$= 2\hat{i} + \hat{j} - m\hat{i} + \hat{j}$

$= (2 - m)\hat{i} + 2\hat{j}$

$\overrightarrow{BC}$ = Position vector of C – Position vector of B

$= \left(4\hat{i} + 5\hat{j}\right) - \left(2\hat{i} + \hat{j}\right)$

$= 4\hat{i} + 5\hat{j} - 2\hat{i} - \hat{j}$

$\overrightarrow{BC} = 2\hat{i} + 4\hat{j}$

A, B, C are collinear. So, $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are collinear.

So, $\overrightarrow{AB} = \lambda\left(\overrightarrow{BC}\right)$

$(2 - m)\hat{i} + 2\hat{j} = \lambda\left(2\hat{i} + 4\hat{j}\right)$, for λ scalar

$(2 - m)\hat{i} + 2\hat{j} = 2\lambda i + 4\lambda\hat{j}$

Comparing the coefficient of LHS and RHS.

$2 - m = 2\lambda$

$\frac{2 - m}{2} = \lambda \qquad$ (i)

$2 = 4\lambda$

$\frac{2}{4} = \lambda$

$\frac{1}{2} = \lambda \qquad$ (ii)

Using (i) and (ii)

$\frac{2 - m}{2} = \frac{1}{2}$

$4 - 2m = 2$

$-2m = 2$

$-2m = 2 - 4$

$-2m = -2$

$m = \frac{-2}{-2}$

$m = 1$

$\therefore m = 1$

Algebra of Vectors Ex 23.7 Q9

Here, let $A = (3, 4)$

$B = (-5, 16)$

$C = (5, 1)$

$\overrightarrow{AB}$ = Position vector of B − Position vector of A

$= \left(-5\hat{i} + 16\hat{j}\right) - \left(3\hat{i} + 4\hat{j}\right)$

$= -5\hat{i} + 16\hat{j} - 3\hat{i} - 4\hat{j}$

$\overrightarrow{AB} = -8\hat{i} + 12\hat{j}$

$\overrightarrow{BC}$ = Position vector of C − Position vector of B

$= \left(5\hat{i} + \hat{j}\right) - \left(-5\hat{i} + 16\hat{j}\right)$

$= 5\hat{i} + \hat{j} + 5\hat{i} - 16\hat{j}$

$\overrightarrow{BC} = 10\hat{i} - 15\hat{j}$

So, $4\left(\overrightarrow{AB}\right) = -5\left(\overrightarrow{BC}\right)$

$\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but B is a common point.

Hence, A, B, C are collinear.

Algebra of Vectors Ex 23.7 Q10

Here, it is given that vectors

$a = 2\hat{i} - 3\hat{j}$ and $b = -6\hat{i} + m\hat{j}$ are collinear.

So, $a = \lambda b$, for a scalar λ

$2\hat{i} - 3\hat{j} = \lambda\left(-6\hat{i} + m\hat{j}\right)$

$2\hat{i} - 3\hat{j} = -6\lambda\hat{i} + \lambda m\hat{j}$

Comparing the coefficients of LHS and RHS,

$2 = -6\lambda$

$\lambda = \frac{2}{-6}$

$\lambda = \frac{-1}{3}$ (i)

$-3 = \lambda m$

$\lambda = \frac{-3}{m}$ (ii)

From (i) and (ii),

$\frac{-1}{3} = \frac{-3}{m}$

$m = 3 \times 3$

$= 9$

$\therefore m = 9$

Algebra of Vectors Ex 23.7 Q11

The given points are A (1, −2, −8), B (5, 0, −2), and C (11, 3, 7).

$$\therefore \overrightarrow{AB} = (5-1)\hat{i} + (0+2)\hat{j} + (-2+8)\hat{k} = 4\hat{i} + 2\hat{j} + 6\hat{k}$$
$$\overrightarrow{BC} = (11-5)\hat{i} + (3-0)\hat{j} + (7+2)\hat{k} = 6\hat{i} + 3\hat{j} + 9\hat{k}$$
$$\overrightarrow{AC} = (11-1)\hat{i} + (3+2)\hat{j} + (7+8)\hat{k} = 10\hat{i} + 5\hat{j} + 15\hat{k}$$
$$\left|\overrightarrow{AB}\right| = \sqrt{4^2+2^2+6^2} = \sqrt{16+4+36} = \sqrt{56} = 2\sqrt{14}$$
$$\left|\overrightarrow{BC}\right| = \sqrt{6^2+3^2+9^2} = \sqrt{36+9+81} = \sqrt{126} = 3\sqrt{14}$$
$$\left|\overrightarrow{AC}\right| = \sqrt{10^2+5^2+15^2} = \sqrt{100+25+225} = \sqrt{350} = 5\sqrt{14}$$
$$\therefore \left|\overrightarrow{AC}\right| = \left|\overrightarrow{AB}\right| + \left|\overrightarrow{BC}\right|$$

Thus, the given points A, B, and C are collinear.

Now, let point B divide AC in the ratio $\lambda : 1$. Then, we have:

$$\overrightarrow{OB} = \frac{\lambda\overrightarrow{OC} + \overrightarrow{OA}}{(\lambda+1)}$$
$$\Rightarrow 5\hat{i} - 2\hat{k} = \frac{\lambda\left(11\hat{i}+3\hat{j}+7\hat{k}\right)+\left(\hat{i}-2\hat{j}-8\hat{k}\right)}{\lambda+1}$$
$$\Rightarrow (\lambda+1)\left(5\hat{i}-2\hat{k}\right) = 11\lambda\hat{i} + 3\lambda\hat{j} + 7\lambda\hat{k} + \hat{i} - 2\hat{j} - 8\hat{k}$$
$$\Rightarrow 5(\lambda+1)\hat{i} - 2(\lambda+1)\hat{k} = (11\lambda+1)\hat{i} + (3\lambda-2)\hat{j} + (7\lambda-8)\hat{k}$$

On equating the corresponding components, we get:

$$5(\lambda+1) = 11\lambda+1$$
$$\Rightarrow 5\lambda+5 = 11\lambda+1$$
$$\Rightarrow 6\lambda = 4$$
$$\Rightarrow \lambda = \frac{4}{6} = \frac{2}{3}$$

Hence, point B divides AC in the ratio $2:3$.

Algebra of Vectors Ex 23.7 Q12

We have

$\overrightarrow{AP}$ = *Position vector of P − Position vector of A*

$\Rightarrow \overrightarrow{AP} = \hat{i} + 2\hat{j} + 3\hat{k} - \left(-2\hat{i} + 3\hat{j} + 5\hat{k}\right) = 3\hat{i} - \hat{j} - 2\hat{k}$

$\overrightarrow{PB}$ = *Position vector of B − Position vector of P*

$\Rightarrow \overrightarrow{PB} = 7\hat{i} - \hat{k} - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) = 6\hat{i} - 2\hat{j} - 4\hat{k}$

Clearly, $\overrightarrow{PB} = 2\overrightarrow{AP}$

so vectors $\overrightarrow{AP}$ *and* $\overrightarrow{PB}$ *are collinear.*

But P is a point common to $\overrightarrow{AP}$ *and* $\overrightarrow{PB}$.

Hence P, A, B are collinear points.

Similarly, $\overrightarrow{CP} = \hat{i} + 2\hat{j} + 3\hat{k} - \left(-3\hat{i} - 2\hat{j} - 5\hat{k}\right) = 4\hat{i} + 4\hat{j} + 8\hat{k}$

and $\overrightarrow{PD} = 3\hat{i} + 4\hat{j} + 7\hat{k} - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) = 2\hat{i} + 2\hat{j} + 4\hat{k}$

So vectors $\overrightarrow{CP}$ *and* $\overrightarrow{PD}$ *are collinear.*

But P is a common point to $\overrightarrow{CP}$ *and* $\overrightarrow{CD}$.

Hence, C, P, D are collinear points.

Thus, A, B, C, D and P are points such that A, P, B and C, P, D are two sets of collinear points. Hence AB and CD intersect at the point P

Algebra of Vectors Ex 23.7 Q13

Points (λ, - 10, 3), (1 -1, 3) and (3, 5, 3) are collinear.

$\therefore$ (λ, - 10, 3) = x(1 -1, 3) + y(3, 5, 3) for some scalars x and y.

$\Rightarrow \lambda = x + 3y,\ \ -10 = -x + 5y$ and $3 = 3x + 3y$

Solving $-10 = -x + 5y$ and $3 = 3x + 3y$ for x and y we get,

$x = \frac{5}{2}$ and $y = -\frac{3}{2}$

Now,

$\lambda = x + 3y$

$\Rightarrow \lambda = \frac{5}{2} + 3\left(-\frac{3}{2}\right) = -2$

Ex 23.8

Algebra of Vectors Ex 23.8 Q1

(i) Let P, Q, R be the points whose position vectors are $2\hat{i}+\hat{j}-\hat{k}$, $3\hat{i}-2\hat{j}+\hat{k}$ and $\hat{i}+4\hat{j}-3\hat{k}$ respectively.

$$\overrightarrow{PQ} = \text{Position vector of } Q - \text{Position vector of } P$$
$$= \left(3\hat{i}-2\hat{j}+\hat{k}\right)-\left(2\hat{i}+\hat{j}-\hat{k}\right)$$
$$= 3\hat{i}-2\hat{j}+\hat{k}-2\hat{i}-\hat{j}+\hat{k}$$
$$\overrightarrow{PQ} = \hat{i}-3\hat{j}+2\hat{k}$$

$$\overrightarrow{QR} = \text{Position vector of } R - \text{Position vector of } Q$$
$$= \left(\hat{i}+4\hat{j}-3\hat{k}\right)-\left(3\hat{i}-2\hat{j}+\hat{k}\right)$$
$$= \hat{i}+4\hat{j}-3\hat{k}-3\hat{i}+2\hat{j}-\hat{k}$$
$$= -2\hat{i}+6\hat{j}-4\hat{k}$$
$$\overrightarrow{QR} = -2\overrightarrow{PQ}$$

Therefore, $\overrightarrow{QR}$ is parallel to $\overrightarrow{PQ}$ but there is a common point Q. So, P, Q, R are collinear.

(ii) Let P, Q, R be the points represented be the vectors are $3\hat{i}-2\hat{j}+4\hat{k}$, $\hat{i}+\hat{j}+\hat{k}$ and $-\hat{i}+4\hat{j}-2\hat{k}$ respectively.

$$\overrightarrow{PQ} = \text{Position vector of } Q - \text{Position vector of } P$$
$$= \left(-\hat{i}+4\hat{j}-2\hat{k}\right)-\left(3\hat{i}-2\hat{j}+4\hat{k}\right)$$
$$= \hat{i}+\hat{j}+\hat{k}-3\hat{i}+2\hat{j}-4\hat{k}$$
$$= -2\hat{i}-3\hat{j}-3\hat{k}$$

$$\overrightarrow{QR} = \text{Position vector of } R - \text{Position vector of } Q$$
$$= \left(-\hat{i}+4\hat{j}-2\hat{k}\right)-\left(\hat{i}+\hat{j}+\hat{k}\right)$$
$$= -\hat{i}+4\hat{j}-2\hat{k}-\hat{i}-\hat{j}-\hat{k}$$
$$= -2\hat{i}+3\hat{j}-3\hat{k}$$
$$\overrightarrow{PQ} = \overrightarrow{QR}$$

So, $\overrightarrow{PQ}$ is parallel to $\overrightarrow{QR}$ but Q is the common point Q. So, P, Q, R are collinear.

Algebra of Vectors Ex 23.8 Q2(i)

Here, $\vec{A} = 6\hat{i}-7\hat{j}-\hat{k}$
$\vec{B} = 2\hat{i}-3\hat{j}+\hat{k}$
$\vec{C} = 4\hat{i}-5\hat{j}-0\times\hat{k}$

$$\overrightarrow{AB} = \vec{B}-\vec{A}$$
$$= \left(2\hat{i}-3\hat{j}+\hat{k}\right)-\left(6\hat{i}-7\hat{j}-\hat{k}\right)$$
$$= 2\hat{i}-3\hat{j}+\hat{k}-6\hat{i}+7\hat{j}+\hat{k}$$
$$\overrightarrow{AB} = -4\hat{i}+4\hat{j}+2\hat{k}$$

$$\overrightarrow{BC} = \vec{C}-\vec{B}$$
$$= \left(4\hat{i}-5\hat{j}-0\times\hat{k}\right)-\left(2\hat{i}-3\hat{j}+\hat{k}\right)$$
$$= 4\hat{i}-5\hat{j}-0\times\hat{k}-2\hat{i}+3\hat{j}-\hat{k}$$
$$\overrightarrow{BC} = 2\hat{i}-2\hat{j}-\hat{k}$$

$$\overrightarrow{AB} = -2\left(\overrightarrow{BC}\right)$$

So, $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but B is the common point. So, A, B, C are collinear.

Algebra of Vectors Ex 23.8 Q2(ii)

Here, $\vec{A} = 2\hat{i} - \hat{j} + 3\hat{k}$

$\vec{B} = 4\hat{i} + 3\hat{j} + \hat{k}$

$\vec{C} = 3\hat{i} + \hat{j} + 2\hat{k}$

$\overrightarrow{AB} = \vec{B} - \vec{A}$

$= \left(4\hat{i} + 3\hat{j} + \hat{k}\right) - \left(2\hat{i} - \hat{j} + 3\hat{k}\right)$

$= 4\hat{i} + 3\hat{j} + \hat{k} - 2\hat{i} + \hat{j} - 3\hat{k}$

$\overrightarrow{AB} = 2\hat{i} + 4\hat{j} - 2\hat{k}$

$\overrightarrow{BC} = \vec{C} - \vec{B}$

$= \left(3\hat{i} + \hat{j} + 2\hat{k}\right) - \left(4\hat{i} + 3\hat{j} + \hat{k}\right)$

$= 3\hat{i} + \hat{j} + 2\hat{k} - 4\hat{i} - 3\hat{j} - \hat{k}$

$\overrightarrow{BC} = -\hat{i} - 2\hat{j} + \hat{k}$

So, $\overrightarrow{AB} = -2\left(\overrightarrow{BC}\right)$

$\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but $\vec{B}$ is a common vector.

Therefore, A, B, C are collinear.

Algebra of Vectors Ex 23.8 Q2(iii)

Here, $\vec{A} = \hat{i} + 2\hat{j} + 7\hat{k}$

$\vec{B} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

$\vec{C} = 3\hat{i} + 10\hat{j} - \hat{k}$

$\overrightarrow{AB} = \vec{B} - \vec{A}$

$= \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 7\hat{k}\right)$

$= 2\hat{i} + 6\hat{j} + 3\hat{k} - \hat{i} - 2\hat{j} - 7\hat{k}$

$\overrightarrow{AB} = \hat{i} + 4\hat{j} - 4\hat{k}$

$\overrightarrow{BC} = \vec{C} - \vec{B}$

$= \left(3\hat{i} + 10\hat{j} - \hat{k}\right) - \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right)$

$= 3\hat{i} + \hat{j} + 2\hat{k} - 2\hat{i} - 6\hat{j} - 3\hat{k}$

$\overrightarrow{BC} = \hat{i} + 4\hat{j} - 4\hat{k}$

$\overrightarrow{AB} = \overrightarrow{BC}$

So, $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but $\vec{B}$ is a common vector.

So, A, B, C are collinear.

Algebra of Vectors Ex 23.8 Q2(iv)

Here, $\vec{A} = -3\hat{i} - 2\hat{j} - 5\hat{k}$

$\vec{B} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{C} = 3\hat{i} + 4\hat{j} + 7\hat{k}$

$\overrightarrow{AB} = \vec{B} - \vec{A}$

$= \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - \left(-3\hat{i} - 2\hat{j} - 5\hat{k}\right)$

$= \hat{i} + 2\hat{j} + 3\hat{k} + 3\hat{i} + 2\hat{j} + 5\hat{k}$

$\overrightarrow{AB} = 4\hat{i} + 4\hat{j} + 8\hat{k}$

$\overrightarrow{BC} = \vec{C} - \vec{B}$

$= \left(3\hat{i} + 4\hat{j} + 7\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

$= 3\hat{i} + 4\hat{j} + 7\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k}$

$\overrightarrow{BC} = 2\hat{i} + 2\hat{j} + 4\hat{k}$

So, $\overrightarrow{AB} = 2\overrightarrow{BC}$

Hence, $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but $\vec{B}$ is a common vector.

Therefore, A, B, C are collinear.

Algebra of Vectors Ex 23.8 Q2(v)

Here, $\vec{A} = 2\hat{i} - \hat{j} + 3\hat{k}$

$\vec{B} = 3\hat{i} - 5\hat{j} + \hat{k}$

$\vec{C} = -\hat{i} + 11\hat{j} + 9\hat{k}$

$\overrightarrow{AB} = \vec{B} - \vec{A}$

$= \left(3\hat{i} - 5\hat{j} + \hat{k}\right) - \left(2\hat{i} - \hat{j} + 3\hat{k}\right)$

$= 3\hat{i} - 5\hat{j} + \hat{k} - 2\hat{i} + \hat{j} - 3\hat{k}$

$\overrightarrow{AB} = \hat{i} - 4\hat{j} - 2\hat{k}$

$\overrightarrow{BC} = \vec{C} - \vec{B}$

$= \left(-\hat{i} + 11\hat{j} + 9\hat{k}\right) - \left(3\hat{i} - 5\hat{j} + \hat{k}\right)$

$= -\hat{i} + 11\hat{j} + 9\hat{k} - 3\hat{i} + 5\hat{j} - \hat{k}$

$= -4\hat{i} + 16\hat{j} + 8\hat{k}$

So, $\overrightarrow{AB} = -4\left(\overrightarrow{BC}\right)$

$\overrightarrow{AB}$ is parallel to vector $\overrightarrow{BC}$ but $\vec{B}$ is a common vector.

So, A, B, C are collinear

Algebra of Vectors Ex 23.8 Q3(i)

We know that,

Three vectors are coplanar if one of the vector can be expressed as the linear combination of other two.

Let,

$5\vec{a} + 6\vec{b} + 7\vec{c} = x\left(7\vec{a} - 8\vec{b} + 9\vec{c}\right) + y\left(3\vec{a} + 20\vec{b} + 5\vec{c}\right)$

$5\vec{a} + 6\vec{b} + 7\vec{c} = 7\vec{a}x - 8\vec{b}x + 9\vec{c}x + 3\vec{a}y + 20\vec{b}y + 5\vec{c}y$

$5\vec{a} + 6\vec{b} + 7\vec{c} = \left(7x + 3y\right)\vec{a} + \left(-8x + 20y\right)\vec{b} + \left(9x + 5y\right)\vec{c}$

Comparing the LHS and RHS,

$7x + 3y = 5 \quad \text{(i)}$

$-8x + 20y = 6 \quad \text{(ii)}$

$9x + 5y = 7 \quad \text{(iii)}$

For solving (i) and (ii),

Subtract $-8 \times$ (i) from $7 \times$ (ii),

$$\begin{array}{r} -56x + 140y = 42 \\ \underset{(+)}{-}\,56x \underset{(+)}{-} 24y = \underset{(+)}{-}\,40 \\ \hline 164y = 82 \end{array}$$

$y = \dfrac{82}{164}$

$y = \dfrac{1}{2}$

Put $y = \frac{1}{2}$ in equation (i),

$$7x + 3y = 5$$
$$7x + 3\left(\frac{1}{2}\right) = 5$$
$$7x + \frac{3}{2} = 5$$
$$7x = \frac{5}{1} - \frac{3}{2}$$
$$7x = \frac{10-3}{2}$$
$$7x = \frac{7}{2}$$
$$x = \frac{7}{14}$$
$$x = \frac{1}{2}$$

Now, put $x = \frac{1}{2}$ and $y = \frac{1}{2}$ in equation (iii),

$$9x + 5y = 7$$
$$9\left(\frac{1}{2}\right) + 5\left(\frac{1}{2}\right) = 7$$
$$\frac{9}{2} + \frac{5}{2} = 7$$
$$\frac{14}{2} = 7$$
$$7 = 7$$
$$\text{LHS} = \text{RHS}$$

$\therefore$ The value of x,y satisfy equation (iii).

So,
$5\vec{a} + 6\vec{b} + 7\vec{c},\ 7\vec{a} - 8\vec{b} + 9\vec{c},\ 3\vec{a} + 20\vec{b} + 5\vec{c}$ are coplanar.

Algebra of Vectors Ex 23.8 Q3(ii)

We know that,
Three vectors are coplanar if one of them can be expressed as the linear combination of other two.

Let
$$\vec{a} - 2\vec{b} + 3\vec{c} = x\left(-3\vec{b} + 5\vec{c}\right) + y\left(-2\vec{a} + 3\vec{b} - 4\vec{c}\right)$$
$$\vec{a} - 2\vec{b} + 3\vec{c} = -3\vec{b}x + 5\vec{c}x + 2\vec{a}y + 3\vec{b}y - 4\vec{c}y$$
$$\vec{a} - 2\vec{b} + 3\vec{c} = (-2y)\vec{a} + (-3x + 3y)\vec{b} + (5x - 4y)\vec{c}$$

Comparing the LHS and RHS,
$$-2y = 1 \qquad \text{(i)}$$
$$-3x + 3y = -2 \qquad \text{(ii)}$$
$$5x - 4y = 3 \qquad \text{(iii)}$$

From solving (i) and $y = -\frac{1}{2}$

Put value of y in equation (ii),

$$-3x + 3y = -2$$
$$-3x + 3\left(-\frac{1}{2}\right) = -2$$
$$-3x - \frac{3}{2} = -2$$
$$-3x = \frac{-2}{1} + \frac{3}{2}$$
$$-3x = \frac{-4+3}{2}$$
$$-3x = \frac{-1}{2}$$
$$x = \frac{-1}{-6}$$
$$x = \frac{1}{6}$$

Put value of x and y in equation (iii)

$$5x - 4y = 3$$
$$5\left(\frac{1}{6}\right) - 4\left(-\frac{1}{2}\right) = 3$$
$$\frac{5}{6} + \frac{4}{2} = 3$$
$$\frac{5+12}{6} = 3$$
$$\frac{17}{6} = 3$$
$$\text{LHS} \neq \text{RHS}$$

So, value of x and y do not satisfy the equation (iii).

So,
vectors $\vec{a} - 2\vec{b} + 3\vec{c},\ -3\vec{b} + 5\vec{c},$ and $-2\vec{a} + 3\vec{b} - 4\vec{c}$ are not coplanar.

Algebra of Vectors Ex 23.8 Q4

Here,

Position vector of $P = 6\hat{i} - 7\hat{j}$

Position vector of $Q = 16\hat{i} - 19\hat{j} - 4\hat{k}$

Position vector of $R = 3\hat{j} - 6\hat{k}$

Position vector of $S = 2\hat{i} - 5\hat{j} + 10\hat{k}$

$\overrightarrow{PQ}$ = Position vector of Q – Position vector of P

$$= \left(16\hat{i} - 19\hat{j} - 4\hat{k}\right) - \left(6\hat{i} - 7\hat{j}\right)$$
$$= 16\hat{i} - 19\hat{j} - 4\hat{k} - 6\hat{i} + 7\hat{j}$$
$$= 10\hat{i} - 12\hat{j} - 4\hat{k}$$

$\overrightarrow{PR}$ = Position vector of R – Position vector of P

$$= \left(3\hat{j} - 6\hat{k}\right) - \left(6\hat{i} - 7\hat{j}\right)$$
$$= 3\hat{j} - 6\hat{k} - 6\hat{i} + 7\hat{j}$$
$$= -6\hat{i} + 10\hat{j} - 6\hat{k}$$

$\overrightarrow{PS}$ = Position vector of S – Position vector of P

$$= \left(2\hat{i} - 5\hat{j} + 10\hat{k}\right) - \left(6\hat{i} - 7\hat{j}\right)$$
$$= 2\hat{i} - 5\hat{j} + 10\hat{k} - 6\hat{i} + 7\hat{j}$$
$$= -4\hat{i} + 2\hat{j} + 10\hat{k}$$

Let $\overrightarrow{PQ} = x\overrightarrow{PR} + y\overrightarrow{PS}$

$$10\hat{i} - 12\hat{j} - 4\hat{k} = x\left(-6\hat{i} + 10\hat{j} - 6\hat{k}\right) + \left(-4\hat{i} + 2\hat{j} + 10\hat{k}\right)$$
$$= -6x\hat{i} + x10\hat{j} - 6x\hat{k} - 4y\hat{i} + 2y\hat{j} + 10y\hat{k}$$
$$10\hat{i} - 12\hat{j} - 4\hat{k} = \left(-6x - 4y\right)\hat{i} + \left(10x + 2y\right)\hat{j} + \left(-6x + 10y\right)\hat{k}$$

Comparing the coefficients of $\hat{i}, \hat{j}$ and $\hat{k}$ of LHS and RHS,

$-6x - 4y = 10$

$3x + 2y = -5 \qquad (i)$

$10x + 2y = -12 \qquad (ii)$

$-6x + 10y = -4 \qquad (iii)$

Substracting (i) from (ii),

$$\begin{array}{rcl} 10x + 2y &=& -12 \\ 3x + 2y &=& -5 \\ (-)\quad (-) & & (+) \\ \hline 7x \qquad &=& -7 \end{array}$$

$$x = \frac{-7}{7}$$
$$x = -1$$

Put $x = -1$ in equation (i)

$$3x + 2y = -5$$
$$3(-1) + 2y = -5$$
$$-3 + 2y = -5$$
$$2y = -5 + 3$$
$$2y = -2$$
$$y = \frac{-2}{2}$$
$$y = -1$$

Put $x = -1$ and $y = -1$ in equation (iii),

$$-6x + 10y = -4$$
$$-6(-1) + 10(-1) = -4$$
$$6 - 10 = -4$$
$$-4 = -4$$
$$\text{LHS} = \text{RHS}$$

Therefore,

P, Q, R, S are coplanar.

Algebra of Vectors Ex 23.8 Q5(i)

We know that, three vectors are coplanar if one of the vector can be expressed as linear combination of other two.

Let,

$$2\hat{i} - \hat{j} + \hat{k} = x\left(\hat{i} - 3\hat{j} - 5\hat{k}\right) + y\left(3\hat{i} - 4\hat{j} - 4\hat{k}\right)$$

$$2\hat{i} - \hat{j} + \hat{k} = x\hat{i} - 3x\hat{j} - 5x\hat{k} + 3y\hat{i} - 4y\hat{j} - 4y\hat{k}$$

$$2\hat{i} - \hat{j} + \hat{k} = (x + 3y)\hat{i} + (-3x - 4y)\hat{j} + (-5x - 4y)\hat{k}$$

Comparing the coefficients of LHS and RHS,

$$x + 3y = 2 \qquad \text{(i)}$$

$$-3x - 4y = -1 \qquad \text{(ii)}$$

$$-5x - 4y = 1 \qquad \text{(iii)}$$

For solving equation (i) and (ii),

Add $3 \times$(i) with equation (ii),

$$\begin{array}{r} 3x + 9y = 6 \\ \underline{-3x - 4y = -1} \\ 5y = 5 \end{array}$$

$$y = \frac{5}{5}$$

$$y = 1$$

Put y in equation (i),

$$x + 3y = 2$$

$$x + 3(1) = 2$$

$$x + 3 = 2$$

$$x = 2 - 3$$

$$x = -1$$

Put the value of x and y in equation (iii),

$$-5x - 4y = 1$$

$$-5(-1) - 4(1) = 1$$

$$5 - 4 = 1$$

$$1 = 1$$

$$\text{LHS} = \text{RHS}$$

So, the value of x and y satisfy equation (iii).

Hence, vectors are coplanar.

Algebra of Vectors Ex 23.8 Q5(ii)

We know that,

Three vectors are coplanar if one of the vector can be expressed as the linear combination of other two vectors.

Let,

$\hat{i}+\hat{j}+\hat{k}=x\left(2\hat{i}+3\hat{j}-\hat{k}\right)+y\left(-\hat{i}-2\hat{j}+2\hat{k}\right)$

$\hat{i}+\hat{j}+\hat{k}=2x\hat{i}+3x\hat{j}-x\hat{k}+-y\hat{i}-2y\hat{j}+2y\hat{k}$

$\hat{i}+\hat{j}+\hat{k}=\left(2x-y\right)\hat{i}+\left(3x-2y\right)\hat{j}+\left(-x+2y\right)\hat{k}$

Comparing the coefficients of LHS and RHS,

$2x-y=1$ (i)

$3x-2y=1$ (ii)

$-x+2y=1$ (iii)

For solving (i) and (ii),

Subtracting $2\times$(i) from (ii),

$$\begin{array}{rcl} 3x - 2y &=& 1 \\ 4x - 2y &=& 2 \\ (-) \quad (+) & & (-) \\ \hline -x &=& -1 \end{array}$$

$x=1$

Put the value of x in equation (i),

$2x-y=1$

$2(1)-y=1$

$2-y=1$

$-y=1-2$

$-y=-1$

$y=1$

Put the value of x and y in equation (iii),

$-x+2y=1$

$-(1)+2(1)=1$

$-1+2=1$

$1=1$

$\text{LHS}=\text{RHS}$

The value of x and y satisfy equation (iii).

Hence, vectors are coplanar.

Algebra of Vectors Ex 23.8 Q6(i)

We know that,

Three vectors are coplanar if one of them vector can be expressed as the linear combination of the other two.

Let,

$$\left(3\hat{i}+\hat{j}-\hat{k}\right) = x\left(2\hat{i}-\hat{j}+7\hat{k}\right)+y\left(7\hat{i}-\hat{j}+23\hat{k}\right)$$

$$= 2x\hat{i}-x\hat{j}+7x\hat{k}+7y\hat{i}-y\hat{j}+23y\hat{k}$$

$$\left(3\hat{i}+\hat{j}-\hat{k}\right) = \left(2x+7y\right)\hat{i}+\left(-x-y\right)\hat{j}+\left(7x+23y\right)\hat{k}$$

Equating the coefficients of LHS and RHS,

$2x+7y = 3$ (i)

$-x-y = 1$ (ii)

$7x+23y = -1$ (iii)

For solving (i) and (ii),

Add (i) and $2\times$(ii),

$$\begin{array}{r} 2x+7y = 3 \\ \underline{-2x-2y = 2} \\ 5y = 5 \end{array}$$

$y = \frac{5}{5}$

$y = 1$

Put the value of y in equation (i),

$2x+7y = 3$

$2x+7(1) = 3$

$2x+7 = 3$

$2x = 3-7$

$2x = -4$

$x = \frac{-4}{2}$

$x = -2$

Put the value of x and y in equation (iii),

$7x+23y = -1$

$7(2)+23(1) = -1$

$14+23 = -1$

$37 = -1$

$\text{LHS} \neq \text{RHS}$

The value of x and y do not satisfy the equation (iii).

Hence, vectors are non-coplanar.

Algebra of Vectors Ex 23.8 Q6(ii)

We know that,

Three vectors are coplanar if any one of the vector can be expressed as the linear combination of other two vectors.

Let,

$$\hat{i}+2\hat{j}+3\hat{k}=x\left(2\hat{i}+\hat{j}+3\hat{k}\right)+y\left(\hat{i}+\hat{j}+\hat{k}\right)=2x\hat{i}+x\hat{j}+3x\hat{k}+y\hat{i}+y\hat{j}+y\hat{k}$$

$$\therefore\ \hat{i}+2\hat{j}+3\hat{k}=(2x+y)\hat{i}+(x+2y)\hat{j}+(3x+y)\hat{k}$$

Comparing the coefficients of LHS and RHS,

$2x+y=1 \quad$ (i)

$x+2y=2 \quad$ (ii)

$3x+y=3 \quad$ (iii)

Subtracting $2\times$(ii) from equation (i),

$$\begin{array}{r} 2x+4y=4 \\ 2x+y=1 \\ (-)\ \ (-)\ \ (-) \\ \hline 3y=3 \end{array}$$

$y=\frac{3}{3}$

$y=1$

Put the value of y in equation (i),

$2x+y=1$

$2x+1=1$

$2x=1-1$

$2x=0$

$x=\frac{0}{2}$

$x=0$

Put the value of x and y in equation (iii),

$3x+y=3$

$3(0)+1=3$

$0+1=3$

$1=3$

LHS $\neq$ RHS

The value of x and y do not satisfy the equation (iii).

Hence, vectors are non-coplanar.

Algebra of Vectors Ex 23.8 Q7(i)

We know that,
Three vectors are coplanar if any one of them can be expressed as the linear combination of other two vectors.

Let,

$$\left(2\vec{a} - \vec{b} + 3\vec{c}\right) = x\left(\vec{a} + \vec{b} - 2\vec{c}\right) + y\left(\vec{a} + \vec{b} - 3\vec{c}\right)$$

$$= \vec{a}x + \vec{b}x - 2\vec{c}x + \vec{a}y + \vec{b}y - 3\vec{c}y$$

$$\left(2\vec{a} - \vec{b} + 3\vec{c}\right) = (x+y)\vec{a} + (x+y)\vec{b} + (-2x - 3y)\vec{c}$$

Comparing the coefficients of LHS and RHS,

$x + y = 2 \quad$ (i)

$x + y = -1 \quad$ (ii)

$-2x - 3y = 3 \quad$ (iii)

For solving the equation (i) and (ii),
Subtracting (ii) from (i),

$$\begin{array}{rcl} x + y &=& 2 \\ x + y &=& -1 \\ (-)\;(-) & & (+) \\ \hline 0 &=& 3 \end{array}$$

There is no value of x and y that can satisfy the equation (iii).
Hence, vectors are non-coplanar.

Algebra of Vectors Ex 23.8 Q7(ii)

We know that,
Three vectors are coplanar if any one of them can be expressed as the linear combination of other two vectors.

Let

$$\vec{a} + 2\vec{b} + 3\vec{c} = x\left(2\vec{a} + \vec{b} + 3\vec{c}\right) + y\left(\vec{a} + \vec{b} + \vec{c}\right)$$

$$= 2\vec{a}x + \vec{b}x + 3\vec{c}x + \vec{a}y + \vec{b}y + \vec{c}y$$

$$\vec{a} + 2\vec{b} + 3\vec{c} = (2x+y)\vec{a} + (x+y)\vec{b} + (3x+y)\vec{c}$$

Comparing the coefficients of LHS and RHS,

$2x + y = 1 \quad$ (i)

$x + y = 2 \quad$ (ii)

$3x + y = 3 \quad$ (iii)

For solving the equation (i) and (ii),
Subtracting equation (i) from equation (ii),

$$\begin{array}{rcl} x + y &=& 2 \\ 2x + y &=& 1 \\ (-)\;\;(-) & & (-) \\ \hline -x &=& 1 \end{array}$$

$x = -1$

Put the value of x in equation (i)

$x + y = 2$

$-1 + y = 2$

$y = 2 + 1$

$y = 3$

Put the x and y in equation (iii),

$3x + y = 3$

$3(-1) + 3 = 3$

$-3 + 3 = 3$

$0 = 3$

LHS $\neq$ RHS

The value of x and y do not satisfy the equation (iii).

Hence, vectors are non-coplanar.

Algebra of Vectors Ex 23.8 Q8

We know that,

Three vectors are coplanar if any one of them can be expressed as the linear combination of other two vectors.

Let

$\vec{a} = x\vec{b} + y\vec{c}$

$= x\left(2\hat{i} + \hat{j} + 3\hat{k}\right) + y\left(\hat{i} + \hat{j} + \hat{k}\right)$

$= 2\hat{i}x + \hat{j}x + 3\hat{k}x + \hat{i}y + \hat{j}y + \hat{k}y$

$\hat{i} + 2\hat{j} + 3\hat{k} = (2x + y)\hat{i} + (x + y)\hat{j} + (3x + y)\hat{k}$

Comparing the coefficient of LHS and RHS,

$2x + y = 1$ (i)

$x + y = 2$ (ii)

$3x + y = 3$ (iii)

For solving (i) and (ii),

Subtracting (i) from (ii),

$$\begin{array}{r} x + y = 2 \\ 2x + y = 1 \\ (-)\ \ (-)\ \ (-) \\ \hline -x \quad = 1 \\ x = -1 \end{array}$$

Put the value of x in equation (i),

$x + y = 2$

$-1 + y = 2$

$y = 2 + 1$

$y = 3$

Put the values of x and y in equation (iii)

$3x + y = 3$

$3(-1) + 3 = 3$

$-3 + 3 = 3$

$0 = 3$

LHS $\neq$ RHS

The values of x and y do not satisfy equation (iii).

Hence,

$\vec{a}, \vec{b}, \vec{c}$ are non coplanar.

Let,

$$\vec{d} = x\vec{b} + y\hat{j} + z\hat{k}$$

$$= x\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + y\left(2\hat{i} + \hat{j} + 3\hat{k}\right) + z\left(\hat{i} + \hat{j} + \hat{k}\right)$$

$$= x\hat{i} + 2x\hat{j} + 3x\hat{k} + 2y\hat{i} + \hat{j}y + 3y\hat{k} + z\hat{i} + z\hat{j} + z\hat{k}$$

$$2\hat{i} - \hat{j} - 3\hat{k} = \left(x + 2y + z\right)\hat{i} + \left(2x + y + z\right)\hat{j} + \left(3x + 3y + z\right)\hat{k}$$

Comparing the coefficient of LHS and RHS,

$x + 2y + z = 2 \qquad (i)$

$2x + y + z = -1 \qquad (ii)$

$3x + 3y + z = -3 \qquad (iii)$

Subtracting equation (i) from (ii),

$$\begin{array}{l} 2x + y + z = -1 \\ \underset{(-)}{x} \underset{(-)}{+ 2y} \underset{(-)}{+ z} = \underset{(-)}{2} \\ \hline x - y \quad = -3 \qquad (iv) \end{array}$$

Subtracting equation (ii) from (iii),

$$\begin{array}{l} 3x + 3y + z = -3 \\ \underset{(-)}{2x} \underset{(-)}{+ y} \underset{(-)}{+ z} = \underset{(+)}{-1} \\ \hline x + 2y \quad = -2 \qquad (v) \end{array}$$

Subtracting (iv) from (v),

$$\begin{array}{l} x + 2y = -2 \\ \underset{(-)}{x} \underset{(+)}{- y} = \underset{(+)}{-3} \\ \hline 3y = 1 \\ y = \frac{1}{3} \end{array}$$

Put y in equation (v),

$x + 2y = -2$

$x + 2\left(\frac{1}{3}\right) = -2$

$2 + \frac{2}{3} = -2$

$x = \frac{-2}{1} - \frac{2}{3}$

$= \frac{-6 - 2}{3}$

$x = \frac{-8}{3}$

Put value of x and y in equation (i),

$x + 2y + z = 2$

$\frac{-8}{3} + 2\left(\frac{1}{3}\right) + z = 2$

$\frac{-8}{3} + \frac{2}{3} + z = 2$

$z = \frac{2}{1} + \frac{8}{3} - \frac{2}{3}$

$z = \frac{6 + 8 - 2}{3}$

$z = \frac{14 - 2}{3}$

$z = \frac{12}{3}$

$z = 4$

So,

$\vec{d} = x\vec{a} + y\vec{b} + z\vec{c}$

$\vec{d} = \left(\frac{-8}{3}\right)\vec{a} + \left(\frac{1}{3}\right)\vec{b} + (4)\vec{c}$

Algebra of Vectors Ex 23.8 Q9

Necessary Condition: Let $\vec{a},\vec{b},\vec{c}$ are three coplanar vectors. Then one of them can be expressed as the linear combination of other two vectors.

Let, $\vec{c} = x\vec{a} + y\vec{b}$
$x\vec{a} + y\vec{b} - \vec{c} = 0$

Put $x = l,\ y = m,\ (-1) = n$
$l\vec{a} + m\vec{b} + n\vec{c} = 0$

Thus, if $\vec{a},\vec{b},\vec{c}$ are coplanar vectors,
then there exist scalars l,m,n
$l\vec{a} + m\vec{b} + n\vec{c} = 0$
Such that l,m,n are not all zero simultaneously.

Sufficient Condition: Let $\vec{a},\vec{b},\vec{c}$ be three vectors such that there exist scalars l,m,n not all zero simultaneously satisfying $l\vec{a} + m\vec{b} + n\vec{c} = 0$

$l\vec{a} + m\vec{b} + n\vec{c} = 0$
$n\vec{c} = -l\vec{a} - m\vec{b}$

Dividing by n, both the sides

$$\frac{n\vec{c}}{n} = \frac{-l\vec{a}}{n} - \frac{m\vec{b}}{n}$$

$$\vec{c} = \left(-\frac{l}{n}\right)\vec{a} + \left(-\frac{m}{n}\right)\vec{b}$$

$\vec{c}$ is a linear combination of $\vec{a}$ and $\vec{b}$

Hence, $\vec{a},\vec{b},\vec{c}$ are coplanar vectors.

Algebra of Vectors Ex 23.8 Q10

Given that, A, B, C and D are four points with position vector $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ respectively.

Let A, B, C, D are coplanar.

If so, there exists x, y, z, u not all zero such that

$x\vec{a} + y\vec{b} + z\vec{c} + u\vec{d} = 0$

$x + y + z + u = 0$

Let, $x = 3,\ y = -2,\ z = 1,\ u = -2$

$3\vec{a} - 2\vec{b} + \vec{c} - 2\vec{d} = 0$

and, $x + y + z + u = 3 + (-2) + 1 + (-2)$

$= 4 - 4$

$= 0$

Thus, A, B, C, D are coplanar.

if $3\vec{a} - 2\vec{b} + \vec{c} - 2\vec{d} = 0$

Let $3\vec{a} - 2\vec{b} + \vec{c} - 2\vec{d} = 0$

$3\vec{a} + \vec{c} = 2\vec{b} + 2\vec{d}$

Divide by sum of the coefficients that is by 4 on both sides,

$$\frac{3\vec{a} + \vec{c}}{4} = \frac{2\vec{b} + 2\vec{d}}{4}$$

$$\frac{3\vec{a} + \vec{c}}{3 + 1} = \frac{2\vec{b} + 2\vec{d}}{2 + 2}$$

It shows that P is the point which divides AC in ratio $1:3$ internally as well as BD in ratio $2:2$ internally.

Thus, P is the point of intersection of AC and BD.

Hence,

A, B, C, D are coplanar.

We can say that,

A, B, C, D are coplanar if and only if Let $3\vec{a} - 2\vec{b} + \vec{c} - 2\vec{d} = \vec{0}$

Ex 23.9

Algebra of Vectors Ex 23.9 Q1

We know that, If l, m, n are the direction cosine of a vector and α, β, γ can the direction angle, then

$$l = \cos\alpha,\ m = \cos\beta\ n = \cos\gamma$$

and, $l^2 + m^2 + n^2 = 1 \qquad \text{(i)}$

$\therefore \quad l = \cos 45°,\ m = \cos 60°,\ n = \cos 120°$

$$l = \frac{1}{\sqrt{2}},\ m = \frac{1}{2}, n = -\frac{1}{2}$$

Put l, m, n in equation (i)

$$l^2 + m^2 + n^2 = 1$$
$$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{-1}{2}\right)^2 = 1$$
$$\frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1$$
$$\frac{2+1+1}{4} = 1$$
$$\frac{4}{4} = 1$$
$$1 = 1$$
$$\text{LHS} = \text{RHS}$$

Therefore,a vector can have direction angle $45°, 60°, 120°$.

Algebra of Vectors Ex 23.9 Q2

Here, $l = 1,\ m = 1,\ n = 1$

Put it in

$$l^2 + m^2 + n^2 = 1$$
$$(1)^2 + (1)^2 + (1)^2 = 1$$
$$1 + 1 + 1 = 1$$
$$3 = 1$$
$$\text{LHS} \neq \text{RHS}$$

Therefore,
1,1,1 can not be direction cosines of a straight line.

Algebra of Vectors Ex 23.9 Q3

Here, $\alpha = \frac{\pi}{4},\ \beta = \frac{\pi}{4},\ \gamma = ?$

$$l = \cos\alpha = \cos\frac{\pi}{4}$$
$$l = \frac{1}{\sqrt{2}}$$

$$m = \cos\beta = \cos\frac{\pi}{4}$$
$$m = \frac{1}{\sqrt{2}}$$

$$n = \cos\gamma$$

Put value of $l, m,$ and n in

$$l^2 + m^2 + n^2 = 1$$
$$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2\gamma = 1$$
$$\frac{1}{2} + \frac{1}{2} + \cos^2\gamma = 1$$
$$1 + \cos^2\gamma = 1$$
$$\cos^2\gamma = 1 - 1$$
$$\cos^2\gamma = 0$$
$$\cos\gamma = 0$$
$$\gamma = \cos^{-1}(0)$$
$$\gamma = \frac{\pi}{2}$$

The angle made by the vector with the z-axis $= \frac{\pi}{2}$

Algebra of Vectors Ex 23.9 Q4

Here, $\alpha = \beta = \gamma$

$\Rightarrow \cos\alpha = \cos\beta = \cos\gamma$

$\Rightarrow l = m = n = x$ (say)

We know that,

$$l^2 + m^2 + n^2 = 1$$
$$x^2 + x^2 + x^2 = 1$$
$$3x^2 = 1$$
$$x^2 = \frac{1}{3}$$
$$x = \pm\frac{1}{\sqrt{3}}$$

$$l = \pm\frac{1}{\sqrt{3}},\ m = \pm\frac{1}{\sqrt{3}},\ n = \pm\frac{1}{\sqrt{3}}$$

Hence, direction cosiner of $\vec{r}$ are,

$$\pm\frac{1}{\sqrt{3}},\ \pm\frac{1}{\sqrt{3}},\ \pm\frac{1}{\sqrt{3}}$$

Vector $\vec{r} = \left|\vec{r}\right|\left(l\hat{i} + m\hat{j} + n\hat{k}\right)$

$$= 6\left(\pm\frac{1}{\sqrt{3}}\hat{i} + \pm\frac{1}{\sqrt{3}}\hat{j} + \pm\frac{1}{\sqrt{3}}\hat{k}\right)$$

$$= \frac{\pm 6 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}\left(\hat{i} + \hat{j} + \hat{k}\right) \quad \text{[Rationalizing the denominator]}$$

$$= \frac{\pm 6\sqrt{3}}{3}\left(\hat{i} + \hat{j} + \hat{k}\right)$$

$$\vec{r} = \pm 2\sqrt{3}\left(\hat{i} + \hat{j} + \hat{k}\right)$$

Algebra of Vectors Ex 23.9 Q5

Here, $\alpha = 45°,\ \beta = 60°,\ \gamma = \theta$ (say)

$$l = \cos\alpha$$
$$= \cos 45°$$
$$l = \frac{1}{\sqrt{2}}$$

$$m = \cos\beta$$
$$= \cos 60°$$
$$m = \frac{1}{2}$$

$$n = \cos\theta$$

Put l, m, and n in

$$l^2 + m^2 + n^2 = 1$$
$$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2\theta = 1$$
$$\frac{2+1}{4} + \cos^2\theta = 1$$
$$\frac{3}{4} + \cos^2\theta = 1$$
$$\cos^2\theta = \frac{1}{1} - \frac{3}{4}$$
$$= \frac{4-3}{4}$$
$$\cos^2\theta = \frac{1}{2}$$

So, $l = \frac{1}{\sqrt{2}},\ m = \frac{1}{2},\ n = \pm\frac{1}{2}$

The required,

vector $\vec{r} = \left|\vec{r}\right|\left(l\hat{i} + m\hat{j} + n\hat{k}\right)$

$$= 8\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{2}\hat{j} \pm \frac{1}{2}\hat{k}\right)$$

$$= 8\,\frac{\sqrt{2}\hat{i} + \hat{j} \pm \hat{k}}{2}$$

$$\vec{r} = 4\left(\sqrt{2}\hat{i} + \hat{j} \pm \hat{k}\right)$$

Algebra of Vectors Ex 23.9 Q6

(i)

Here, the direction ratios of the vector

$2\hat{i} + 2\hat{j} - \hat{k} = 2,\ 2,\ -1$

The direction cosines of the vector

$= \frac{2}{|\vec{r}|}, \frac{2}{|\vec{r}|}, \frac{-1}{|\vec{r}|}$

$= \frac{2}{\sqrt{(2)^2 + (2)^2 + (-1)^2}}, \frac{2}{\sqrt{(2)^2 + (2)^2 + (-1)^2}}, \frac{-1}{\sqrt{(2)^2 + (2)^2 + (-1)^2}}$

$= \frac{2}{\sqrt{4+4+1}}, \frac{2}{\sqrt{4+4+1}}, \frac{-1}{\sqrt{4+4+1}}$

$= \frac{2}{\sqrt{9}}, \frac{2}{\sqrt{9}}, \frac{-1}{\sqrt{9}}$

$= \frac{2}{3}, \frac{2}{3}, \frac{-1}{3}$

(ii)

Here, let $\vec{r} = 6\hat{i} - 2\hat{j} - 3\hat{k}$

and, $|\vec{r}| = \sqrt{(6)^2 + (-2)^2 + (-3)^2}$

$= \sqrt{36 + 4 + 9}$

$= \sqrt{49}$

$|\vec{r}| = 7$

The direction cosines of $\vec{r}$ are given by

$= \frac{6}{|\vec{r}|}, \frac{-2}{|\vec{r}|}, \frac{-3}{|\vec{r}|}$

$= \frac{6}{7}, \frac{-2}{7}, \frac{-3}{7}$

Algebra of Vectors Ex 23.9 Q7(i)

Let, $\vec{r} = \hat{i} - \hat{j} + \hat{k}$

The direction ratios of the vector $\vec{r} = 1, -1, 1$

And, $|\vec{r}| = \sqrt{(1)^2 + (-1)^2 + (1)^2}$

$= \sqrt{1+1+1}$

$= \sqrt{3}$

The direction cosines of the vector $\vec{r}$

$= \frac{1}{|\vec{r}|}, \frac{-1}{|\vec{r}|}, \frac{1}{|\vec{r}|}$

$= \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

So, $l = \cos\alpha = \frac{1}{\sqrt{3}}$

$\alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$m = \cos\beta = \frac{-1}{\sqrt{3}}$

$\beta = \cos^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

$n = \cos\gamma = \frac{1}{\sqrt{3}}$

$\gamma = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Thus, angles made by $\vec{r}$ with the coordinate axes are given by

$\cos^{-1}\left(\frac{1}{\sqrt{3}}\right), \cos^{-1}\left(-\frac{1}{\sqrt{3}}\right), \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Algebra of Vectors Ex 23.9 Q7(ii)

Let, $\vec{r} = \hat{j} - \hat{k}$

$\vec{r} = 0 \times \hat{i} + \hat{j} - \hat{k}$

The direction ratios of $\vec{r} = 0, 1, -1$

and, $|\vec{r}| = \sqrt{(0)^2 + (1)^2 + (-1)^2}$

$= \sqrt{0+1+1}$

$|\vec{r}| = \sqrt{2}$

The direction cosines of the $\vec{r}$ are given by

$= \frac{0}{|\vec{r}|}, \frac{1}{|\vec{r}|}, \frac{-1}{|\vec{r}|}$

$= \frac{0}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$

So, $l = \cos\alpha = 0$

$\alpha = \cos^{-1}(0)$

$\alpha = \frac{\pi}{2}$

$m = \cos\beta = \frac{1}{\sqrt{2}}$

$\beta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\beta = \frac{\pi}{4}$

$n = \cos\gamma = -\frac{1}{\sqrt{2}}$

$\gamma = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

$\lambda = \pi - \frac{\pi}{7}$

$\gamma = \frac{3\pi}{4}$

So, angles made by the vector $\vec{r}$ with coordinate axes are given by

$\frac{\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}$

Algebra of Vectors Ex 23.9 Q7(iii)

Let, $4\hat{i} + 8\hat{j} + \hat{k} = \vec{r}$

The direction ratios of $\vec{r} = 4, 8, 1$

And, $|\vec{r}| = \sqrt{(4)^2 + (8)^2 + (1)^2}$

$= \sqrt{16 + 64 + 1}$

$= \sqrt{81}$

$|\vec{r}| = 9$

The direction cosines of the $\vec{r}$ are given by

$= \frac{4}{|\vec{r}|}, \frac{8}{|\vec{r}|}, \frac{1}{|\vec{r}|}$

$= \frac{4}{9}, \frac{8}{9}, \frac{1}{9}$

Now, $l = \cos\alpha = \frac{4}{9}$

$\alpha = \cos^{-1}\left(\frac{4}{9}\right)$

$m = \cos\beta = \frac{8}{9}$

$\beta = \cos^{-1}\left(\frac{8}{9}\right)$

$n = \cos\gamma = \frac{1}{9}$

$\gamma = \cos^{-1}\left(\frac{1}{9}\right)$

The angles made by the vector $\vec{r}$ with the coordinate axes are given by

$\cos^{-1}\left(\frac{4}{9}\right), \cos^{-1}\left(\frac{8}{9}\right), \cos^{-1}\left(\frac{1}{9}\right)$

Algebra of Vectors Ex 23.9 Q8

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$.

Then,

$|\vec{a}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

Therefore, the direction cosines of $\vec{a}$ are $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$.

Now, let α, β, and γ be the angles formed by $\vec{a}$ with the positive directions of x, y, and z axes.

Then, we have $\cos\alpha = \frac{1}{\sqrt{3}}, \cos\beta = \frac{1}{\sqrt{3}}, \cos\gamma = \frac{1}{\sqrt{3}}$.

Hence, the given vector is equally inclined to axes OX, OY, and OZ.

Algebra of Vectors Ex 23.9 Q9

Let a vector be equally inclined to axes OX, OY, and OZ at angle α.

Then, the direction cosines of the vector are cos α, cos α, and cos α.

Now,

$\cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$

$\Rightarrow 3\cos^2\alpha = 1$

$\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$

Hence, the direction cosines of the vector which are equally inclined to the axes are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$.

Algebra of Vectors Ex 23.9 Q10

Let unit vector $\vec{a}$ have (a_1, a_2, a_3) components.

$\Rightarrow \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

Since $\vec{a}$ is a unit vector, $|\vec{a}| = 1$.

Also, it is given that $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$, and an acute angle θ with $\hat{k}$.

Then, we have:

$\cos\frac{\pi}{3} = \frac{a_1}{|\vec{a}|}$

$\Rightarrow \frac{1}{2} = a_1 \qquad [|\vec{a}| = 1]$

$\cos\frac{\pi}{4} = \frac{a_2}{|\vec{a}|}$

$\Rightarrow \frac{1}{\sqrt{2}} = a_2 \qquad [|\vec{a}| = 1]$

Also, $\cos\theta = \frac{a_3}{|\vec{a}|}$.

$\Rightarrow a_3 = \cos\theta$

Now,

$|a| = 1$

$\Rightarrow \sqrt{a_1^2 + a_2^2 + a_3^2} = 1$

$$\Rightarrow \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2\theta = 1$$

$$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2\theta = 1$$

$$\Rightarrow \frac{3}{4} + \cos^2\theta = 1$$

$$\Rightarrow \cos^2\theta = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

$$\therefore a_3 = \cos\frac{\pi}{3} = \frac{1}{2}$$

Hence, $\theta = \frac{\pi}{3}$ and the components of $\vec{a}$ are $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}\right)$.

The Plane Ex 23.9 Q11

Let l, m, n be the direction cosines of the vector $\vec{r}$.

$l = \cos\alpha$, $m = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ and $n = \cos\left(\frac{\pi}{2}\right) = 0$

$$l^2 + m^2 + n^2 = 1$$

$$l^2 + \frac{1}{2} + 0 = 1$$

$$l = \pm\frac{1}{\sqrt{2}}$$

$$\vec{r} = |\vec{r}|\left(l\hat{i} + m\hat{j} + n\hat{k}\right)$$

$$\vec{r} = 3\sqrt{2}\left(\pm\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + 0\right)$$

$$\vec{r} = \pm 3\hat{i} + 3\hat{j}$$

The Plane Ex 23.9 Q12

Let l, m, n be the direction cosines of the vector $\vec{r}$.

Vector $\vec{r}$ is indined at equal angles to the three axes.

$l = \cos\alpha$, $m = \cos\alpha$ and $n = \cos\alpha$

$\Rightarrow l = m = n$

$$l^2 + m^2 + n^2 = 1$$

$$\Rightarrow l = m = n = \pm\frac{1}{\sqrt{3}}$$

$$\vec{r} = |\vec{r}|\left(l\hat{i} + m\hat{j} + n\hat{k}\right)$$

$$\vec{r} = 2\sqrt{3}\left(\pm\frac{1}{\sqrt{3}}\hat{i} \pm \frac{1}{\sqrt{3}}\hat{j} \pm \frac{1}{\sqrt{3}}\hat{k}\right)$$

$$\vec{r} = \pm 2\hat{i} \pm 2\hat{j} \pm 2\hat{k}$$

Ex 24.1

Scalar or Dot Product Ex 24.1 Q1

(i)

$\vec{a} \cdot \vec{b}$

$= \left(\hat{i} - 2\hat{j} + \hat{k}\right) \cdot \left(4\hat{i} - 4\hat{j} + 7\hat{k}\right)$

$= (1)(4) + (-2) \cdot (-4) + (1)(7)$

$= 4 + 8 + 7$

$= 19$

$\vec{a} \cdot \vec{b} = 19$

(ii)

$\vec{a} \cdot \vec{b} = \left(\hat{j} + 2\hat{k}\right) \cdot \left(2\hat{i} + \hat{k}\right)$

$= \left(0 \times \hat{i} + \hat{j} + 2\hat{k}\right)\left(2\hat{i} + 0 \times \hat{j} + \hat{k}\right)$

$= (0)(2) + (1)(0) + (2)(1)$

$= 0 + 0 + 2$

$\vec{a} \cdot \vec{b} = 2$

(iii)

$\vec{a} \cdot \vec{b} = \left(\hat{j} - \hat{k}\right) \cdot \left(2\hat{i} + 3\hat{j} - 2\hat{k}\right)$

$= \left(0 \times \hat{i} + \hat{j} - \hat{k}\right)\left(2\hat{i} + 3\hat{j} - 2\hat{k}\right)$

$= (0)(2) + (1)(3) + (-1)(-2)$

$= 0 + 3 + 2$

$\vec{a} \cdot \vec{b} = 5$

Scalar or Dot Product Ex 24.1 Q2

(i)

$\vec{a}$ and $\vec{b}$ are prependicular

$\Rightarrow \vec{a}\vec{b} = 0$

$\Rightarrow \left(\lambda\hat{i} + 2\hat{j} + \hat{k}\right) \cdot \left(4\hat{i} - 9\hat{j} + 2\hat{k}\right) = 0$

$\Rightarrow (\lambda)(4) + (2)(-9) + (1)(2) = 0$

$\Rightarrow 4\lambda - 18 + 2 = 0$

$\Rightarrow 4\lambda - 16 = 0$

$\Rightarrow 4\lambda = 16$

$\Rightarrow \lambda = \frac{16}{4}$

$\Rightarrow \lambda = 4$

(ii)

$\vec{a}$ and $\vec{b}$ are prependicular

$\Rightarrow \vec{a}\vec{b} = 0$

$\Rightarrow \left(\lambda\hat{i} + 2\hat{j} + \hat{k}\right) \cdot \left(5\hat{i} - 9\hat{j} + 2\hat{k}\right) = 0$

$\Rightarrow (\lambda)(5) + (2)(-9) + (1)(2) = 0$

$\Rightarrow 5\lambda - 18 + 2 = 0$

$\Rightarrow 5\lambda - 16 = 0$

$\Rightarrow 5\lambda = 16$

$\Rightarrow \lambda = \frac{16}{5}$

(iii)

$\vec{a}$ and $\vec{b}$ are prependicular

$\Rightarrow \vec{a}.\vec{b} = 0$

$\Rightarrow \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)\left(3\hat{i} + 2\hat{j} - \lambda\hat{k}\right) = 0$

$\Rightarrow (2)(3) + (3)(2) + (4)(-\lambda) = 0$

$\Rightarrow 6 + 6 - 4\lambda = 0$

$\Rightarrow 12 - 4\lambda = 0$

$\Rightarrow -4\lambda = -12$

$\Rightarrow \lambda = \frac{-12}{-4}$

$\Rightarrow \lambda = 3$

(iv)

$\vec{a}$ and $\vec{b}$ are prependicular

$\Rightarrow \vec{a}.\vec{b} = 0$

$\Rightarrow \left(\lambda\hat{i} + 3\hat{j} + 2\hat{k}\right)\left(\hat{i} - \hat{j} + 3\hat{k}\right) = 0$

$\Rightarrow (\lambda)(1) + (3)(-1) + (2)(3) = 0$

$\Rightarrow \lambda - 3 + 6 = 0$

$\Rightarrow \lambda + 3 = 0$

$\Rightarrow \lambda = -3$

Scalar or Dot Product Ex 24.1 Q3

We know that, if θ is the angle between $\vec{a}$ and $\vec{b}$, then

$$\cos\theta = \frac{\vec{a}\,.\,\vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{6}{4\times 3}$$

$$= \frac{6}{12}$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\theta = \frac{\pi}{3}$$

Angle between $\vec{a}$ and $\vec{b}$ = $\frac{\pi}{3}$

Scalar or Dot Product Ex 24.1 Q4

$$\left(\vec{a} - 2\vec{b}\right) = \left(\hat{i} - \hat{j}\right) - 2\left(-\hat{j} + 2\hat{k}\right)$$

$$= \left(\hat{i} - \hat{j}\right) + 2\hat{j} - 4\hat{k}$$

$$= \left(\hat{i} + \hat{j} - 4\hat{k}\right)$$

$$\left(\vec{a} + \vec{b}\right) = \left(\hat{i} - \hat{j}\right) + \left(-\hat{j} + 2\hat{k}\right)$$

$$= \hat{i} - \hat{j} - \hat{j} + 2\hat{k}$$

$$= \left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$$

Now,

$$\left(\vec{a} - 2\vec{b}\right).\left(\vec{a} + \vec{b}\right)$$

$$= \left(\hat{i} + \hat{j} - 4\hat{k}\right)\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$$

$$= (1)(1) + (1)(-2) + (-4)(2)$$

$$= 1 - 2 - 8$$

$$= -9$$

$$\left(\vec{a} - 2\vec{b}\right).\left(\vec{a} + \vec{b}\right) = -9$$

Scalar or Dot Product Ex 24.1 Q5(i)

Let θ be the angle between $\vec{a}$ and $\vec{b}$, then

$$\cos\theta = \frac{\vec{a}\,.\,\vec{b}}{|\vec{a}||\vec{b}|} \qquad ---(i)$$

$$\vec{a}\,.\,\vec{b} = \left(\hat{i}-\hat{j}\right)\left(\hat{j}+\hat{k}\right)$$
$$= \left(\hat{i}-\hat{j}+0\times\hat{k}\right)\left(0\times\hat{i}+\hat{j}+\hat{k}\right)$$
$$= (1)(0)+(-1)(1)+(0)(1)$$
$$= 0-1+0$$
$$\vec{a}\,.\,\vec{b} = -1$$

$$|\vec{a}| = \left|\hat{i}-\hat{j}\right|$$
$$= \left|\hat{i}-\hat{j}+0\times\hat{k}\right|$$
$$= \sqrt{(1)^2+(-1)^2+(0)^2}$$
$$= \sqrt{1+1+0}$$
$$|\vec{a}| = \sqrt{2}$$

$$|\vec{b}| = \left|\hat{j}+\hat{k}\right|$$
$$= \left|0\times\hat{i}+\hat{j}+\hat{k}\right|$$
$$= \sqrt{(0)^2+(1)^2+(1)^2}$$
$$= \sqrt{0+1+1}$$
$$|\vec{b}| = \sqrt{2}$$

Put $\vec{a}\times\vec{b}, |\vec{a}|$ and $|\vec{b}|$ in equation (i)

$$\cos\theta = \frac{\vec{a}\,.\,\vec{b}}{|\vec{a}||\vec{b}|}$$
$$= \frac{-1}{\sqrt{2}\times\sqrt{2}}$$
$$\cos\theta = \frac{-1}{2}$$
$$\theta = \cos^{-1}\left(-\frac{1}{2}\right)$$
$$\theta = \pi-\frac{\pi}{3}$$
$$\theta = \frac{2\pi}{3}$$

Angle between $\vec{a}$ and $\vec{b} = \frac{2\pi}{3}$

Scalar or Dot Product Ex 24.1 Q5(ii)

Let θ be the angle between two vactor $\overline{a} = 3\hat{i}-2\hat{j}-6\hat{k}$ and $\vec{b} = 4\hat{i}-\hat{j}+8\hat{k}$

$$\cos\theta = \frac{\overline{a}.\vec{b}}{|\overline{a}||\vec{b}|} \cdots(1)$$

$$\overline{a}.\vec{b} = \left(3\hat{i}-2\hat{j}-6\hat{k}\right)\left(4\hat{i}-\hat{j}+8\hat{k}\right)$$
$$= 3*4+(-2)(-1)+(-6)8$$
$$= 12+2-48$$
$$= -34$$

$$|\overline{a}| = \sqrt{3^2+(-2)^2+(-6)^2}$$
$$= \sqrt{49}$$
$$= 7$$

$$|\vec{b}| = \sqrt{4^2+(-1)^2+8^2}$$
$$= \sqrt{81}$$
$$= 9$$

Putting value of $|\overline{a}|, |\vec{b}|$ and $\overline{a}.\vec{b}$ in equation (1)

$$\cos\theta = \frac{\overline{a}.\vec{b}}{|\overline{a}||\vec{b}|}$$
$$= \frac{-34}{7*9}$$
$$= \frac{-34}{63}$$
$$\theta = \cos^{-1}\left(\frac{-34}{63}\right)$$
$$= 122.66^0$$

Scalar or Dot Product Ex 24.1 Q5(iii)

Let the angle between $\vec{a}$ and $\vec{b}$ be θ, then

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} \qquad ---(i)$$

$$\vec{a}.\ \vec{b} = \left(2\hat{i} - \hat{j} + 2\hat{k}\right)\left(4\hat{i} + 4\hat{j} - 2\hat{k}\right)$$
$$= (2)(4) + (-1)(4) + (2)(-2)$$
$$= 8 - 4 - 4$$
$$\vec{a}.\ \vec{b} = 0$$

$$|\vec{a}| = \left|2\hat{i} - \hat{j} + 2\hat{k}\right|$$
$$= \sqrt{(2)^2 + (-1)^2 + (2)^2}$$
$$= \sqrt{4 + 1 + 4}$$
$$= \sqrt{9}$$
$$|\vec{a}| = 3$$

$$|\vec{b}| = \left|4\hat{i} + 4\hat{j} - 2\hat{k}\right|$$
$$= \sqrt{(4)^2 + (4)^2 + (-2)^2}$$
$$= \sqrt{16 + 16 + 4}$$
$$= \sqrt{36}$$
$$|\vec{b}| = 6$$

Put $\vec{a}.\ \vec{b}$, $|\vec{a}|$ and $|\vec{b}|$ in equation (i)

$$\cos\theta = \frac{\vec{a}.\ \vec{b}}{|\vec{a}||\vec{b}|}$$
$$= \frac{0}{3\times 6}$$
$$= \frac{0}{18}$$
$$\cos\theta = 0$$
$$\theta = \cos^{-1}(0)$$

Angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{2}$

Scalar or Dot Product Ex 24.1 Q5(iv)

Let θ be the angle between vector $\vec{a}$ and $\vec{b}$, then

$$\cos\theta = \frac{\vec{a}:\vec{b}}{|\vec{a}||\vec{b}|} \qquad ---(i)$$

$$\vec{a}\ \ \vec{b} = \left(2\hat{i} - 3\hat{j} + \hat{k}\right)\left(\hat{i} + \hat{j} - 2\hat{k}\right)$$
$$= (2)(1) + (-3)(1) + (1)(-2)$$
$$= 2 - 3 - 2$$
$$\vec{a}\cdot\vec{b} = -3$$

$$|\vec{a}| = \left|2\hat{i} - 3\hat{j} + \hat{k}\right|$$
$$= \sqrt{(2)^2 + (-3)^2 + (-1)^2}$$
$$= \sqrt{4 + 9 + 1}$$
$$= \sqrt{14}$$

$$|\vec{b}| = \left|\hat{i} + \hat{j} - 2\hat{k}\right|$$
$$|\vec{b}| = \sqrt{(1)^2 + (1)^2 + (-2)^2}$$
$$= \sqrt{1 + 1 + 4}$$
$$|\vec{b}| = \sqrt{6}$$

Put $\vec{a}.\ \vec{b}$, $|\vec{a}|$ and $|\vec{b}|$ in equation (i),

$$\cos\theta = \frac{\vec{a}.\ \vec{b}}{|\vec{a}||\vec{b}|}$$
$$= \frac{-3}{\sqrt{14}\times\sqrt{6}}$$
$$\cos\theta = \frac{-3}{\sqrt{84}}$$
$$\theta = \cos^{-1}\left(\frac{-3}{\sqrt{84}}\right)$$

Angle between vector $\vec{a}$ and $\vec{b}$ $= \cos^{-1}\left(\frac{-3}{\sqrt{84}}\right)$

Scalar or Dot Product Ex 24.1 Q5(v)

Let θ be the angle between vector $\vec{a}$ and $\vec{b}$, then

$$\cos\theta = \frac{\vec{a}\,.\,\vec{b}}{|\vec{a}||\vec{b}|} \qquad ---(i)$$

$$\vec{a}\,.\,\vec{b} = \left(\hat{i}+2\hat{j}-\hat{k}\right)\left(\hat{i}-\hat{j}+\hat{k}\right)$$
$$= (1)(1)+(2)(-1)+(-1)(1)$$
$$= 1-2-1$$
$$\vec{a}\,.\,\vec{b} = -2$$

$$|\vec{a}| = \left|\hat{i}+2\hat{j}-\hat{k}\right|$$
$$= \sqrt{(1)^2+(2)^2+(-1)^2}$$
$$= \sqrt{1+4+1}$$
$$= \sqrt{6}$$

$$|\vec{b}| = \left|\hat{i}-\hat{j}+\hat{k}\right|$$
$$|\vec{b}| = \sqrt{(1)^2+(-1)^2+(1)^2}$$
$$= \sqrt{1+1+1}$$
$$|\vec{b}| = \sqrt{3}$$

Put $\vec{a}\,.\,\vec{b}$, $|\vec{a}|$, $|\vec{b}|$ in equation (i),

$$\cos\theta = \frac{\vec{a}\,.\,\vec{b}}{|\vec{a}||\vec{b}|}$$
$$= \frac{-2}{\sqrt{6}\sqrt{3}}$$
$$= \frac{-2}{\sqrt{18}}$$
$$= \frac{-2\times\sqrt{2}}{3\sqrt{2}\times\sqrt{2}}$$
$$= \frac{-2\sqrt{2}}{3\times 2}$$
$$\cos\theta = \frac{-\sqrt{2}}{3}$$
$$\theta = \cos^{-1}\left(\frac{-\sqrt{2}}{3}\right)$$

Angle between vector $\vec{a}$ and $\vec{b}$ $= \cos^{-1}\left(\frac{-\sqrt{2}}{3}\right)$

Scalar or Dot Product Ex 24.1 Q6

Component along x-, y- and z-axis are $\hat{i}, \hat{j}$ and $\hat{k}$ respectively.

Let θ_1 be the angle between $\vec{a}$ and $\hat{i}$.

$$\cos\theta_1 = \frac{\vec{a}.\hat{i}}{|\vec{a}||\hat{i}|}$$
$$= \frac{\left(\hat{i}-\hat{j}+\sqrt{2}\hat{k}\right)\left(\hat{i}+0.\hat{j}+0.\hat{k}\right)}{\left|\hat{i}-\hat{j}+\sqrt{2}\hat{k}\right|\left|\hat{i}+0.\hat{j}+0.\hat{k}\right|}$$
$$= \frac{(1)(1)+(-1)(0)+\left(\sqrt{2}\right)(0)}{\sqrt{(1)^2+(-1)^2+\left(\sqrt{2}\right)^2}\cdot\sqrt{(1)^2+(0)^2+(0)^2}}$$
$$= \frac{1+0+0}{\sqrt{4}\sqrt{1}}$$
$$\cos\theta_1 = \frac{1}{2}$$
$$\theta_1 = \frac{\pi}{3}$$

Let θ_2 be the angle between $\vec{a}$ and $\hat{j}$.

$$\cos\theta_2 = \frac{\vec{a}.\hat{j}}{|\vec{a}||\hat{j}|}$$
$$= \frac{\left(\hat{i}-\hat{j}+\sqrt{2}\hat{k}\right)\left(0.\hat{i}+\hat{j}+0\hat{k}\right)}{\sqrt{(1)^2+(-1)^2+\left(\sqrt{2}\right)^2}\cdot\sqrt{(0)^2+(1)^2+(0)^2}}$$
$$= \frac{(1)(0)+(-1)(1)+\left(\sqrt{2}\right)(0)}{\sqrt{1+1+2}\ \sqrt{1}}$$
$$= \frac{-1}{\sqrt{4}\sqrt{1}}$$
$$= \frac{-1}{2}$$
$$\cos\theta_2 = -\frac{1}{2}$$
$$\theta_2 = \pi - \frac{\pi}{3}$$
$$\theta_2 = \frac{2\pi}{3}$$

Let θ_3 be the angle between $\vec{a}$ and $\hat{k}$, then

$$\cos\theta_3 = \frac{\vec{a}.\hat{k}}{|\vec{a}||\hat{k}|}$$
$$= \frac{\left(\hat{i}-\hat{j}+\sqrt{2}\hat{k}\right)\left(0.\hat{i}+0.\hat{j}+\hat{k}\right)}{\sqrt{(1)^2+(-1)^2+\left(\sqrt{2}\right)^2}\ \sqrt{(0)^2+(0)^2+(1)^2}}$$
$$= \frac{(1)(0)+(-1)(0)+\left(\sqrt{2}\right)(1)}{\sqrt{1+1+2}\ \sqrt{1}}$$
$$= \frac{\sqrt{2}}{\sqrt{4}\ \sqrt{1}}$$
$$\cos\theta_3 = \frac{1}{\sqrt{2}}$$
$$\theta_3 = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$
$$\theta_3 = \frac{\pi}{4}$$

So, the angle between vector $\vec{a}$ and x-axis is $\frac{\pi}{3}$, vector $\vec{a}$ and y-axis is $\frac{2\pi}{3}$, vector $\vec{a}$ and z-axis is $\frac{\pi}{4}$.

Scalar or Dot Product Ex 24.1 Q7(i)

Let the requird vector be $x\hat{i}+y\hat{j}+z\hat{k}$

According to question,

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(\hat{i}+\hat{j}-3\hat{k}\right)=0$$
$$(x)(1)+(y)(1)+(z)(-3)=0$$
$$x+y-3z=0 \qquad ---(i)$$

And,

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(\hat{i}+3\hat{j}-2\hat{k}\right)=5$$
$$(x)(1)+(y)(3)+(z)(-2)=5$$
$$x+3y-2z=5 \qquad ---(ii)$$

And,

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(2\hat{i}+\hat{j}+4\hat{k}\right)=8$$
$$(x)(2)+(y)(1)+(z)(4)=8$$
$$2x+y+4z=8 \qquad ---(iii)$$

Subtracting (i) from (ii),

$$\begin{array}{l} x+3y-2z=5 \\ \underset{(-)}{x}\underset{(-)}{+y}\underset{(+)}{-3z}=0 \\ \hline \quad 2y+z=5 \qquad ---(iv) \end{array}$$

Subtracting $2\times$(ii) from (iii),

$$\begin{array}{l} 2x+y+4z=8 \\ \underset{(-)}{2x}\underset{(-)}{+6y}\underset{(+)}{-4z}=\underset{(-)}{10} \\ \hline \quad -5y+8z=-2 \qquad ---(v) \end{array}$$

Subtracting $8\times$(iv) from (v),

$$\begin{array}{l} -5y+8z=-2 \\ \underset{(-)}{18y}\underset{(-)}{+8z}=\underset{(-)}{40} \\ \hline -21y \quad =-42 \qquad ---(vi) \end{array}$$

$$y=\frac{-42}{-21}$$
$$y=2$$

Put $y=2$ in equation (iv),

$$2y+z=5$$
$$2(2)+z=5$$
$$4+z=5$$
$$z=5-4$$
$$z=1$$

Put $y=2$ and $z=1$ in equation (i),

$$x+y-3z=0$$
$$x+(2)-3(1)=0$$
$$x+2-3=0$$
$$x-1=0$$
$$x=1$$

The required vector $=x\hat{i}+y\hat{j}+z\hat{k}$

The required vector $=\hat{i}+2\hat{j}+\hat{k}$

Scalar or Dot Product Ex 24.1 Q8(i)

Here, $\hat{a}$ and $\hat{b}$ are unit vectors, then

$$|\hat{a}| = |\hat{b}| = 1$$

$$|\hat{a}+\hat{b}|^2 = (\hat{a}+\hat{b})^2$$

$$= (\hat{a})^2 + (\hat{b})^2 + 2\hat{a}.\hat{b}$$

$$= |\hat{a}|^2 + |\hat{b}|^2 + 2\hat{a}.\hat{b}$$

$$= (1)^2 + (1)^2 + 2\hat{a}.\hat{b}$$

$$|\hat{a}+\hat{b}|^2 = 2 + 2\hat{a}\times\hat{b}$$

$$|\hat{a}+\hat{b}|^2 = 2 + 2\times|\hat{a}||\hat{b}|\cos\theta \qquad \left[\text{Since } \vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta\right]$$

$$|\hat{a}+\hat{b}|^2 = 2 + 2\times1\times1\times\cos\theta$$

$$= 2 + 2\cos\theta$$

$$|\hat{a}+\hat{b}|^2 = 2(1+\cos\theta)$$

$$|\hat{a}+\hat{b}|^2 = 2\left(2\cos^2\frac{\theta}{2}\right) \qquad \left[\text{Since } 1+\cos\theta = 2\cos^2\frac{\theta}{2}\right]$$

$$|\hat{a}+\hat{b}|^2 = 4\cos^2\frac{\theta}{2}$$

$$|\hat{a}+\hat{b}| = \sqrt{4\cos^2\frac{\theta}{2}}$$

$$|\hat{a}+\hat{b}| = 2\cos\frac{\theta}{2}$$

$$\cos\frac{\theta}{2} = \frac{1}{2}|\hat{a}+\hat{b}|$$

Scalar or Dot Product Ex 24.1 Q8(ii)

Here, $\hat{a}$ and $\hat{b}$ are unit vectors

$$|\hat{a}| = |\hat{b}| = 1$$

$$\frac{|\hat{a}-\hat{b}|^2}{|\hat{a}+\hat{b}|^2} = \frac{(\hat{a}-\hat{b})^2}{(\hat{a}+\hat{b})^2}$$

$$= \frac{(\hat{a})^2 + (\hat{b})^2 - 2\hat{a}.\hat{b}}{(\hat{a})^2 + (\hat{b})^2 + 2\hat{a}.\hat{b}}$$

$$= \frac{|\hat{a}|^2 + |\hat{b}|^2 - 2\hat{a}.\hat{b}}{|\hat{a}|^2 + |\hat{b}|^2 + 2\hat{a}.\hat{b}}$$

$$\frac{|\hat{a}-\hat{b}|^2}{|\hat{a}+\hat{b}|^2} = \frac{(1)^2 + (1)^2 - 2|\vec{a}||\vec{b}|\cos\theta}{(1)^2 + (1)^2 + 2|\vec{a}||\vec{b}|\cos\theta} \qquad \left[\text{Since } \vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta\right]$$

$$\frac{|\hat{a}-\hat{b}|^2}{|\hat{a}+\hat{b}|^2} = \frac{1+1-2(1)(1)\cos\theta}{1+1+2(1)(1)\cos\theta}$$

$$= \frac{2-2\cos\theta}{2+2\cos\theta}$$

$$= \frac{2(1-\cos\theta)}{2(1+\cos\theta)}$$

$$= \frac{2\times\sin^2\frac{\theta}{2}}{2\times\cos^2\frac{\theta}{2}} \qquad \left[\text{Since } 1-\cos\theta = 2\sin^2\frac{\theta}{2},\ 1+\cos\theta = 2\cos^2\frac{\theta}{2}\right]$$

$$\frac{|\hat{a}-\hat{b}|^2}{|\hat{a}+\hat{b}|^2} = \tan^2\frac{\theta}{2}$$

$$\tan\frac{\theta}{2} = \frac{|\hat{a}-\hat{b}|}{|\hat{a}+\hat{b}|}$$

Scalar or Dot Product Ex 24.1 Q9

Let $\hat{a}$ and $\hat{b}$ are two unit vectors

Then, $|\hat{a}| = |\hat{b}| = 1$

And sum of $\hat{a}$ and $\hat{b}$ is a unit vector, then

$$|\hat{a}+\hat{b}| = 1$$

Taking square of both the sides,

$$|\hat{a}+\hat{b}|^2 = (1)^2$$

$$(\hat{a}+\hat{b})^2 = 1$$

$$(\hat{a})^2 + (\hat{b})^2 + 2\hat{a}.\ \hat{b} = 1$$

$$|\hat{a}|^2 + |\hat{b}|^2 + 2\hat{a}.\ \hat{b} = 1$$

$$(1)^2 + (1)^2 + 2\hat{a}.\ \hat{b} = 1$$

$$2 + 2\hat{a}\ \ \hat{b} = 1$$

$$2\hat{a}.\ \hat{b} = 1 - 2$$

$$2\hat{a}.:\hat{b} = -1$$

$$\hat{a}.\ \hat{b} = \frac{-1}{2} \qquad ---(i)$$

$$|\hat{a}-\hat{b}|^2 = (\hat{a}-\hat{b})^2$$

$$= (\hat{a})^2 + (\hat{b})^2 - 2\hat{a}.\ \hat{b}$$

$$= |\hat{a}|^2 + |\hat{b}|^2 - 2\times\hat{a}.\ \hat{b}$$

$$= (1)^2 + (1)^2 - 2\times\left(\frac{-1}{2}\right)$$

Using equation (i)

$$= 1 + 1 + \frac{2}{2}$$

$$= 1 + 1 + 1$$

$$|\hat{a}-\hat{b}|^2 = 3$$

$$|\hat{a}-\hat{b}| = \sqrt{3}$$

Scalar or Dot Product Ex 24.1 Q10

Given that $\vec{a},\vec{b},\vec{c}$ are mutually prependicular, so,

$$\vec{a}.\ \vec{b} = \vec{b}.\ \vec{c} = \vec{c}.\ \vec{a} = 0$$

and $\vec{a},\vec{b}$ and $\vec{c}$ are unit vectors, so

$$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$$

Now,

$$|\vec{a}+\vec{b}+\vec{c}|^2 = (\vec{a}+\vec{b}+\vec{c})^2$$

$$= (\vec{a})^2 + (\vec{b})^2 + (\vec{c})^2 + 2\vec{a}\vec{b} + 2\vec{b}\vec{c} + 2\vec{c}\vec{a}$$

$$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(0) + 2(0) + 2(0)$$

$$= (1)^2 + (1)^2 + (1)^2 + 0$$

$$|\vec{a}+\vec{b}+\vec{c}|^2 = 1+1+1$$

$$|\vec{a}+\vec{b}+\vec{c}|^2 = 3$$

$$|\vec{a}+\vec{b}+\vec{c}| = \sqrt{3}$$

Scalar or Dot Product Ex 24.1 Q11

Here, $\left|\vec{a}+\vec{b}\right| = 60$

Squaring both the sides,

$$\left|\vec{a}+\vec{b}\right|^2 = (60)^2$$

$$\left(\vec{a}+\vec{b}\right) = (60)^2$$

$$\left(\vec{a}\right)^2 + \left(\vec{b}\right)^2 + 2\vec{a}\vec{b} = 3600$$

$$\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + 2\vec{a}\vec{b} = 3600 \qquad ---(i)$$

Now, $\left|\vec{a}-\vec{b}\right| = 40$

Squaring both the sides,

$$\left|\vec{a}-\vec{b}\right|^2 = (40)^2$$

$$\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 - 2\vec{a}\vec{b} = 1600 \qquad ---(ii)$$

Adding (i) and (ii),

$$2\left|\vec{a}\right|^2 + 2\left|\vec{b}\right|^2 + 2\vec{a}\vec{b} - 2\vec{a}\vec{b} = 3600 - 1600$$

$$2\left|\vec{a}\right|^2 + 2(46)^2 = 5200$$

$$2\left|\vec{a}\right|^2 = 5200 - 4232$$

$$2\left|\vec{a}\right|^2 = 968$$

$$\left|\vec{a}\right|^2 = \frac{968}{2}$$

$$\left|\vec{a}\right|^2 = 484$$

$$\left|\vec{a}\right| = \sqrt{484}$$

$$\left|\vec{a}\right| = 22$$

Scalar or Dot Product Ex 24.1 Q12

Let θ be the angle between $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}$

Then,

$$\cos\theta = \frac{\left(\hat{i}+\hat{j}+\hat{k}\right).\left(\hat{i}\right)}{\left|\hat{i}+\hat{j}+\hat{k}\right|\left|\hat{i}\right|}$$

$$= \frac{1}{\frac{1}{\sqrt{3}}}$$

$$= \sqrt{3}$$

Similarly, if α and γ are angles that $\hat{i}+\hat{j}+\hat{k}$ make with $\hat{j}$ and $\hat{k}$

Then,

$$\cos\alpha = \sqrt{3}$$

and $$\cos\gamma = \sqrt{3}$$

Therefore, $\hat{i}+\hat{j}+\hat{k}$ is equally inclined the three axes.

Scalar or Dot Product Ex 24.1 Q13

We have,

$$\vec{a} = \frac{1}{7}\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$$

$$\vec{b} = \frac{1}{7}\left(3\hat{i}-6\hat{j}+2\hat{k}\right)$$

$$\vec{c} = \frac{1}{7}\left(6\hat{i}+2\hat{j}-3\hat{k}\right)$$

Then,

$$\vec{a}.\vec{b} = \frac{1}{7}\left(2\hat{i}+3\hat{j}+6\hat{k}\right)\times\frac{1}{7}\left(3\hat{i}-6\hat{j}+2\hat{k}\right)$$

$$= \frac{1}{49}(6-18+12) = 0$$

Similarly,

$$\vec{b}.\vec{c} = \vec{a}.\vec{c} = 0$$

$\therefore$ $\vec{a},\vec{b},\vec{c}$ are mutually perpendicular

Scalar or Dot Product Ex 24.1 Q14

Let $\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right)=0$

$$\left|\vec{a}\right|^2-\left|\vec{b}\right|^2=0$$

$$\left|\vec{a}\right|^2=\left|\vec{b}\right|^2$$

$$\left|\vec{a}\right|=\left|\vec{b}\right|$$

Let $\left|\vec{a}\right|=\left|\vec{b}\right|$

Squaring both the sides.

$$\left|\vec{a}\right|^2=\left|\vec{b}\right|^2$$

$$\left|\vec{a}\right|^2-\left|\vec{b}\right|^2=0$$

$$\left(\vec{a}\right)^2-\left(\vec{b}\right)^2=0$$

$$\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right)=0$$

Thus,

$$\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right)=0 \Leftrightarrow \left|\vec{a}\right|=\left|\vec{b}\right|$$

Scalar or Dot Product Ex 24.1 Q15

If $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+\hat{j}-2\hat{k}$ and $\vec{c}=\hat{i}+3\hat{j}-\hat{k}$,

find λ

Given that $\vec{a}$ is perpendicular to $\lambda\vec{b}+\vec{c}$

$\therefore\ \vec{a}\bullet\left(\lambda\vec{b}+\vec{c}\right)=0$

$\lambda\vec{a}\bullet\vec{b}+\vec{a}\bullet\vec{c}=0$

$\lambda\left(2\hat{i}-\hat{j}+\hat{k}\right)\bullet\left(\hat{i}+\hat{j}-2\hat{k}\right)+\left(2\hat{i}-\hat{j}+\hat{k}\right)\bullet\left(\hat{i}+3\hat{j}-\hat{k}\right)=0$

$\lambda\left(2-1-2\right)+\left(2-3-1\right)=0$

$-\lambda-2=0$

$\lambda=2$

Scalar or Dot Product Ex 24.1 Q16

$\vec{p}=5\hat{i}+\lambda\hat{j}-3\hat{k}$ and $\vec{q}=\hat{i}+3\hat{j}-5\hat{k}$

$\vec{p}+\vec{q}$

$=5\hat{i}+\lambda\hat{j}-3\hat{k}+\hat{i}+3\hat{j}-5\hat{k}$

$=6\hat{i}+(\lambda+3)\hat{j}-8\hat{k}$

$\vec{p}-\vec{q}$

$=5\hat{i}+\lambda\hat{j}-3\hat{k}-\hat{i}-3\hat{j}+5\hat{k}$

$=4\hat{i}+(\lambda-3)\hat{j}+2\hat{k}$

$(\vec{p}+\vec{q}).(\vec{p}-\vec{q})=0$

$\Rightarrow [6\hat{i}+(\lambda+3)\hat{j}-8\hat{k}].[4\hat{i}+(\lambda-3)\hat{j}+2\hat{k}]=0$

$\Rightarrow 24+(\lambda^2-9)-16=0$

$\Rightarrow \lambda^2-9+8=0$

$\Rightarrow \lambda^2-1=0$

$\therefore \lambda=\pm1$

Scalar or Dot Product Ex 24.1 Q17

According to question $\overline{\beta}_1$ is parallel to $\overline{\alpha}$. So

$$\overline{\beta}_1 = \gamma\overline{\alpha}$$
$$= \gamma\left(3\hat{i}+4\hat{j}+5\hat{k}\right)$$

$$\overline{\beta} = \overline{\beta_1} + \overline{\beta_2}$$
$$2\hat{i}+\hat{j}-4\hat{k} = \gamma\left(3\hat{i}+4\hat{j}+5\hat{k}\right)+\overline{\beta_2} \qquad \left(\textit{putting } \overline{\beta} \textit{ and } \overline{\beta_1}\right)$$
$$\overline{\beta_2} = (2-3\gamma)\hat{i}+(1-4\gamma)\hat{j}-(4+5\gamma)\hat{k}$$

Again $\overline{\beta}_2$ is perpendicular to $\overline{\alpha}$. So

$$\overline{\beta}_2.\overline{\alpha} = 0$$
$$\left[(2-3\gamma)\hat{i}+(1-4\gamma)\hat{j}-(4+5\gamma)\hat{k}\right].\left(3\hat{i}+4\hat{j}+5\hat{k}\right) = 0$$
$$6-9\gamma+4-16\gamma-20-25\gamma = 0$$
$$-50\gamma = 10$$
$$\gamma = -\frac{1}{5}$$

$$\overline{\beta}_1 = -\frac{1}{5}\left(3\hat{i}+4\hat{j}+5\hat{k}\right)$$
$$\overline{\beta} = \overline{\beta_1} + \overline{\beta_2}$$
$$2\hat{i}+\hat{j}-4\hat{k} = -\frac{1}{5}\left(3\hat{i}+4\hat{j}+5\hat{k}\right)+\overline{\beta_2} \qquad \left(\textit{putting } \overline{\beta} \textit{ and } \overline{\beta_1}\right)$$
$$\overline{\beta_2} = \frac{1}{5}\left(13\hat{i}+9\hat{j}-15\hat{k}\right)$$
$$\overline{\beta} = -\frac{1}{5}\left(3\hat{i}+4\hat{j}+5\hat{k}\right)+\frac{1}{5}\left(13\hat{i}+9\hat{j}-15\hat{k}\right)$$

Scalar or Dot Product Ex 24.1 Q18

Consider $\vec{a} = 2\hat{i}+4\hat{j}+3\hat{k}$ and $\vec{b} = 3\hat{i}+3\hat{j}-6\hat{k}$.

Then,

$\vec{a}\cdot\vec{b} = 2.3+4.3+3(-6) = 6+12-18 = 0$

We now observe that:

$|\vec{a}| = \sqrt{2^2+4^2+3^2} = \sqrt{29}$

$\therefore \vec{a} \neq \vec{0}$

$|\vec{b}| = \sqrt{3^2+3^2+(-6)^2} = \sqrt{54}$

$\therefore \vec{b} \neq \vec{0}$

Hence, the converse of the given statement need not be true.

Scalar or Dot Product Ex 24.1 Q19

Here,

$$\vec{b}+\vec{c}=\left(\hat{i}-3\hat{j}+5\hat{k}\right)\left(2\hat{i}+\hat{j}-4\hat{k}\right)$$
$$=3\hat{i}+2\hat{j}+\hat{k}$$
$$\vec{b}+\vec{c}=\vec{a}$$

$\therefore$ $\vec{a},\vec{b},\vec{c}$ are represents the sides of a triangle.

$$\left|\vec{a}\right|=\sqrt{(3)^2+(-2)^2+(1)^2}$$
$$=\sqrt{9+4+1}$$
$$=\sqrt{14}$$

$$\left|\vec{b}\right|=\sqrt{(1)^2+(-3)^2+(5)^2}$$
$$=\sqrt{1+9+25}$$
$$\left|\vec{b}\right|=\sqrt{35}$$

$$\left|\vec{c}\right|=\sqrt{(2)^2+(1)^2+(-4)^2}$$
$$=\sqrt{4+1+16}$$
$$=\sqrt{21}$$

$$\left(\sqrt{21}\right)^2+\left(\sqrt{14}\right)^2=\left(\sqrt{35}\right)^2$$
$$21+14=35$$
$$35=35$$

$$\left|\vec{c}\right|^2+\left|\vec{a}\right|^2=\left|\vec{b}\right|^2$$

$\therefore$ By the pythagorous theorem,

Triangle formed by $\vec{a},\vec{b},\vec{c}$ is a right angled triangled.

Scalar or Dot Product Ex 24.1 Q20

The given vectors are $\vec{a}=2\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=-\hat{i}+2\hat{j}+\hat{k}$, and $\vec{c}=3\hat{i}+\hat{j}$.

Now,

$$\vec{a}+\lambda\vec{b}=\left(2\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(-\hat{i}+2\hat{j}+\hat{k}\right)=(2-\lambda)\hat{i}+(2+2\lambda)\hat{j}+(3+\lambda)\hat{k}$$

If $\left(\vec{a}+\lambda\vec{b}\right)$ is perpendicular to $\vec{c}$, then

$$\left(\vec{a}+\lambda\vec{b}\right)\cdot\vec{c}=0.$$
$$\Rightarrow\left[(2-\lambda)\hat{i}+(2+2\lambda)\hat{j}+(3+\lambda)\hat{k}\right]\cdot\left(3\hat{i}+\hat{j}\right)=0$$
$$\Rightarrow(2-\lambda)3+(2+2\lambda)1+(3+\lambda)0=0$$
$$\Rightarrow 6-3\lambda+2+2\lambda=0$$
$$\Rightarrow -\lambda+8=0$$
$$\Rightarrow \lambda=8$$

Hence, the required value of λ is 8.

Scalar or Dot Product Ex 24.1 Q21

$$\vec{A} = 0.\hat{i} - \hat{j} - 2\hat{k}$$
$$\vec{B} = 3\hat{i} + \hat{j} + 4\hat{k}$$
$$\vec{C} = 5\hat{i} + 7\hat{j} + \hat{k}$$

$$\overrightarrow{AB} = \vec{B} - \vec{A}$$
$$= \left(3\hat{i} + \hat{j} + 4\hat{k}\right) - \left(0.\hat{i} - \hat{j} - 2\hat{k}\right)$$
$$= 3\hat{i} + \hat{j} + 4\hat{k} - 0.\hat{i} + \hat{j} + 2\hat{k}$$
$$\overrightarrow{AB} = 3\hat{i} + 2\hat{j} + 6\hat{k}$$

$$\overrightarrow{BC} = \vec{C} - \vec{B}$$
$$= \left(5\hat{i} + 7\hat{j} + \hat{k}\right) - \left(3\hat{i} + \hat{j} + 4\hat{k}\right)$$
$$= 5\hat{i} + 7\hat{j} + \hat{k} - 3\hat{i} - \hat{j} - 4\hat{k}$$
$$\overrightarrow{BC} = 2\hat{i} + 6\hat{j} - 3\hat{k}$$

$$\overrightarrow{AC} = \vec{C} - \vec{A}$$
$$= \left(5\hat{i} + 7\hat{j} + \hat{k}\right) - \left(-\hat{j} - 2\hat{k}\right)$$
$$= 5\hat{i} + 7\hat{j} + \hat{k} + \hat{j} + 2\hat{k}$$
$$\overrightarrow{AC} = 5\hat{i} + 8\hat{j} + 3\hat{k}$$

Angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$,

$$\cos A = \frac{\overrightarrow{AB}.\overrightarrow{AC}}{\left|\overrightarrow{AB}\right|\left|\overrightarrow{AC}\right|}$$
$$= \frac{\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)\left(5\hat{i} + 8\hat{j} + 3\hat{k}\right)}{\sqrt{(3)^2 + (2)^2 + (6)^2}\sqrt{(5)^2 + (8)^2 + (3)^2}}$$
$$= \frac{(3)(5) + (2)(8) + (6)(3)}{\sqrt{9 + 4 + 36}\sqrt{25 + 64 + 9}}$$
$$= \frac{15 + 16 + 18}{\sqrt{49}\sqrt{98}}$$
$$= \frac{49}{\sqrt{49}\sqrt{49 \times 2}}$$
$$\cos A = \frac{49}{49\sqrt{2}}$$
$$\cos A = \frac{1}{\sqrt{2}}$$
$$A = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$
$$= \frac{\pi}{4}$$
$$\angle A = \frac{\pi}{4}$$

Angle between $\overrightarrow{BC}$ and $\overrightarrow{BA}$

$$\cos B = \frac{\overrightarrow{BC}.\overrightarrow{BA}}{\left|\overrightarrow{BC}\right|\left|\overrightarrow{BA}\right|}$$
$$= \frac{\left(2\hat{i} + 6\hat{j} - 3\hat{k}\right)\left(-3\hat{i} - 2\hat{j} - 6\hat{k}\right)}{\sqrt{(2)^2 + (6)^2 + (-3)^2}\sqrt{(-3)^2 + (-2)^2 + (-6)^2}}$$
$$= \frac{(2)(-3) + (6)(-2) + (-3)(-6)}{\sqrt{4 + 36 + 9}\sqrt{9 + 4 + 36}}$$
$$= \frac{-6 - 12 + 18}{\sqrt{49}\sqrt{98}}$$
$$\cos B = \frac{-18 + 18}{49}$$
$$= \frac{0}{49}$$
$$\cos B = 0$$
$$B = \cos^{-1}(0)$$
$$\angle B = \frac{\pi}{2}$$

We know that,

$$\angle A + \angle B + \angle C = \pi$$
$$\frac{\pi}{4} + \frac{\pi}{2} + \angle C = \pi$$
$$\frac{3\pi}{4} + \angle C = \pi$$
$$\angle C = \frac{\pi}{1} - \frac{3\pi}{4}$$
$$\angle C = \frac{4\pi - 3\pi}{4}$$
$$\angle C = \frac{\pi}{4}$$
$$\angle A = \frac{\pi}{4}$$
$$\angle B = \frac{\pi}{2}$$

Scalar or Dot Product Ex 24.1 Q22

Let θ be the angle between the vectors $\vec{a}$ and $\vec{b}$.

It is given that $|\vec{a}| = |\vec{b}|$, $\vec{a} \cdot \vec{b} = \frac{1}{2}$, and $\theta = 60°$. ...(1)

We know that $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$.

$$\therefore \frac{1}{2} = |\vec{a}||\vec{a}|\cos 60° \qquad [\text{Using (1)}]$$
$$\Rightarrow \frac{1}{2} = |\vec{a}|^2 \times \frac{1}{2}$$
$$\Rightarrow |\vec{a}|^2 = 1$$
$$\Rightarrow |\vec{a}| = |\vec{b}| = 1$$

Scalar or Dot Product Ex 24.1 Q23

Given

$$\vec{a} = 4\hat{i} - 3\hat{j} + \hat{k}$$
$$\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$$
$$\vec{c} = \hat{i} - \hat{j}$$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$$= \left(2\hat{i} - 4\hat{j} + 5\hat{k}\right) - \left(4\hat{i} - 3\hat{j} + \hat{k}\right)$$
$$= 2\hat{i} - 4\hat{j} + 5\hat{k} - 4\hat{i} + 3\hat{j} - \hat{k}$$
$$\overrightarrow{AB} = -2\hat{i} - \hat{j} + 4\hat{k}$$

$\overrightarrow{BC}$ = Position vector of C – Position vector of B

$$= \left(\hat{i} - \hat{j}\right) - \left(2\hat{i} - 4\hat{j} + 5\hat{k}\right)$$
$$= \hat{i} - \hat{j} - 2\hat{i} + 4\hat{j} - 5\hat{k}$$
$$= -\hat{i} + 3\hat{j} - 5\hat{k}$$

$\overrightarrow{CA}$ = Position vector of A – Position vector of C

$$= \left(4\hat{i} - 3\hat{j} + \hat{k}\right) - \left(\hat{i} - \hat{j}\right)$$
$$= 4\hat{i} - 3\hat{j} + \hat{k} - \hat{i} + \hat{j}$$
$$= 3\hat{i} - 2\hat{j} + \hat{k}$$

Now, $\overrightarrow{AB}.\overrightarrow{CA}$

$$= \left(-2\hat{i} - \hat{j} + 4\hat{k}\right).\left(3\hat{i} - 2\hat{j} + \hat{k}\right)$$
$$= (-2)(3) + (-1)(-2) + (4)(1)$$
$$= -6 + 2 + 4$$
$$= -6 + 6$$
$$= 0$$

So, $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{CA}$

$\angle A$ is right angle.

Hence, ABC is a right triangle

Scalar or Dot Product Ex 24.1 Q24

Given,

$$A=(1,2,3)$$
$$B=(-1,0,0)$$
$$C=(0,1,2)$$

Position vector of $A=\hat{i}+2\hat{j}+3\hat{k}$

Position vector of $B=-\hat{i}+0\hat{j}+0\hat{k}$

Position vector of $C=0\hat{i}+\hat{j}+2\hat{k}$

$\overline{AB}$ = Position vector of B - Position vector of A

$$=\left(-\hat{i}+0\hat{j}+0\hat{k}\right)-\left(\hat{i}+2\hat{j}+3\hat{k}\right)$$
$$=-2\hat{i}-2\hat{j}-3\hat{k}$$

$\overline{BC}$ = Position vector of C - Position vector of B

$$=\left(0\hat{i}+\hat{j}+2\hat{k}\right)-\left(-\hat{i}+0\hat{j}+0\hat{k}\right)$$
$$=\hat{i}+\hat{j}+2\hat{k}$$

$\overline{AC}$ = Position vector of C - Position vector of A

$$=\left(0\hat{i}+\hat{j}+2\hat{k}\right)-\left(1\hat{i}+2\hat{j}+3\hat{k}\right)$$
$$=-\hat{i}-\hat{j}-\hat{k}$$

$$\overline{AB}.\overline{BC}=\left(-2\hat{i}-2\hat{j}-3\hat{k}\right).\left(\hat{i}+\hat{j}+2\hat{k}\right)$$
$$=-2-2-6$$
$$=-10$$

$$\angle ABC=\frac{\overrightarrow{AB}.\overrightarrow{BC}}{\left|\overrightarrow{AB}\right|\left|\overrightarrow{BC}\right|}$$
$$=\frac{-10}{\sqrt{(-2)^2+(-2)^2+(-3)^2}\sqrt{1^2+1^2+2^2}}$$
$$=\frac{-10}{\sqrt{17}\sqrt{6}}$$
$$=\frac{-10}{\sqrt{102}}$$
$$\angle ABC=\cos^{-1}\left(\frac{-10}{\sqrt{102}}\right)$$

Scalar or Dot Product Ex 24.1 Q25

Given

$$A=(0,1,1)$$
$$B=(\ \ 3,1,5)$$
$$C=(0,3,3)$$

Position vector of $A = 0\hat{i}+\hat{j}+\hat{k}$
Position vector of $B = 3\hat{i}+\hat{j}+5\hat{k}$
Position vector of $C = 0.\hat{i}+3\hat{j}+3\hat{k}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$$=\left(3\hat{i}+\hat{j}+5\hat{k}\right)-\left(0.\hat{i}+\hat{j}+\hat{k}\right)$$
$$=3\hat{i}+\hat{j}+5\hat{k}-\hat{j}-\hat{k}$$
$$\overrightarrow{AB}=3\hat{i}+4\hat{k}$$

$\overrightarrow{BC}$ = Position vector of C – Position vector of B

$$=\left(0.\hat{i}+3\hat{j}+3\hat{k}\right)-\left(3\hat{i}+\hat{j}+5\hat{k}\right)$$
$$\overrightarrow{BC}=3\hat{j}+3\hat{k}-3\hat{i}-\hat{j}-5\hat{k}$$
$$=-3\hat{i}+2\hat{j}-2\hat{k}$$

$\overrightarrow{AC}$ = Position vector of C – Position vector of A

$$=\left(-3\hat{j}+3\hat{k}\right)-\left(\hat{j}+\hat{k}\right)$$
$$=3\hat{j}+3\hat{k}-\hat{j}-\hat{k}$$
$$=2\hat{j}+2\hat{k}$$

$\overrightarrow{BC}.\overrightarrow{AC}$

$= \left(-3\hat{i} + 2\hat{j} - 2\hat{k}\right)\left(2\hat{j} + 2\hat{k}\right)$

$= (-3)(0) + (2)(2) + (-2)(+2)$

$= 0 + 4 - 4$

$= 0$

So, $\overrightarrow{BC}$ and $\overrightarrow{AC}$ is perpendicular

$\Rightarrow$ $\angle C$ is right angle.

Scalar or Dot Product Ex 24.1 Q26

Projection of $\left(\vec{b} + \vec{c}\right)$ on $\vec{a}$

$$= \frac{\left(\vec{b} + \vec{c}\right).\vec{a}}{|\vec{a}|}$$

$$= \frac{\vec{b}.\vec{a} + \vec{c}.\vec{a}}{\sqrt{(2)^2 + (-2)^2 + (1)^2}}$$

$$= \frac{\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)\left(2\hat{i} - 2\hat{j} + \hat{k}\right)\left(2\hat{i} - \hat{j} + 4\hat{k}\right)\left(2\hat{i} - 2\hat{j} + \hat{k}\right)}{\sqrt{4 + 4 + 1}}$$

$$= \frac{(1)(2) + (2)(-2) + (-2)(1) + (2)(2) + (-1)(-2) + (4)(1)}{\sqrt{9}}$$

$$= \frac{2 - 4 - 2 + 4 + 2 + 4}{3}$$

$$= \frac{12 - 6}{3} = \frac{6}{3} = 2$$

Projection of $\left(\vec{b} + \vec{c}\right) = 2$

Scalar or Dot Product Ex 24.1 Q27

$\vec{a} + \vec{b} = \left(5\hat{i} - \hat{j} - 3\hat{k}\right) + \left(\hat{i} + 3\hat{j} - 5\hat{k}\right)$

$= 5\hat{i} - \hat{j} - 3\hat{k} + \hat{i} + 3\hat{j} - 5\hat{k}$

$\vec{a} + \vec{b} = 6\hat{i} + 2\hat{j} - 8\hat{k}$ ---(i)

$\vec{a} - \vec{b} = \left(5\hat{i} - \hat{j} - 3\hat{k}\right) - \left(\hat{i} + 3\hat{j} - 5\hat{k}\right)$

$= 5\hat{i} - \hat{j} - 3\hat{k} - \hat{i} - 3\hat{j} + 5\hat{k}$

$\vec{a} - \vec{b} = 4\hat{i} - 4\hat{j} + 2\hat{k}$ ---(ii)

Now, $\left(\vec{a} + \vec{b}\right).\left(\vec{a} - \vec{b}\right)$

$= \left(6\hat{i} + 2\hat{j} - 8\hat{k}\right).\left(4\hat{i} - 4\hat{j} + 2\hat{k}\right)$

$= (6)(4) + (2)(-4) + (-8)(2)$

$= 24 - 8 - 16$

$= 0$

So, $\left(\vec{a} + \vec{b}\right)$ and $\left(\vec{a} - \vec{b}\right)$ are perpendicular.

Scalar or Dot Product Ex 24.1 Q28

Let unit vector $\vec{a}$ have (a_1, a_2, a_3) components.

$\Rightarrow \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

Since $\vec{a}$ is a unit vector, $|\vec{a}| = 1$.

Also, it is given that $\vec{a}$ makes angles $\frac{\pi}{4}$ with $\hat{i}$, $\frac{\pi}{3}$ with $\hat{j}$, and an acute angle θ with $\hat{k}$.

Then, we have:

$$\cos\frac{\pi}{4} = \frac{a_1}{|\vec{a}|}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = a_1 \qquad \left[|\vec{a}| = 1\right]$$

$$\cos\frac{\pi}{3} = \frac{a_2}{|\vec{a}|}$$

$$\Rightarrow \frac{1}{2} = a_2 \qquad \left[|\vec{a}| = 1\right]$$

Also, $\cos\theta = \frac{a_3}{|\vec{a}|}$.

$\Rightarrow a_3 = \cos\theta$

Now,

$|a| = 1$

$\Rightarrow \sqrt{a_1^2 + a_2^2 + a_3^2} = 1$

$\Rightarrow \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2\theta = 1$

$\Rightarrow \frac{1}{2} + \frac{1}{4} + \cos^2\theta = 1$

$\Rightarrow \frac{3}{4} + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = 1 - \frac{3}{4} = \frac{1}{4}$

$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

$\therefore a_3 = \cos\frac{\pi}{3} = \frac{1}{2}$

Hence, $\theta = \frac{\pi}{3}$ and the components of $\vec{a}$ are $\left(\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}\right)$.

Scalar or Dot Product Ex 24.1 Q29

$\left(3\vec{a} - 5\vec{b}\right) \cdot \left(2\vec{a} + 7\vec{b}\right)$

$= 3\vec{a} \cdot 2\vec{a} + 3\vec{a} \cdot 7\vec{b} - 5\vec{b} \cdot 2\vec{a} - 5\vec{b} \cdot 7\vec{b}$

$= 6\vec{a} \cdot \vec{a} + 21\vec{a} \cdot \vec{b} - 10\vec{a} \cdot \vec{b} - 35\vec{b} \cdot \vec{b}$

$= 6|\vec{a}|^2 + 11\vec{a} \cdot \vec{b} - 35|\vec{b}|^2$

$= \mathbf{6*2^2 + 11*1 - 35*1^2}$

$= \mathbf{35 - 35}$

$= \mathbf{0}$

Scalar or Dot Product Ex 24.1 Q30(i)

We have,

$(\bar{x} - \bar{a}).(\bar{x} + \bar{a}) = 8$

$\Rightarrow |\bar{x}|^2 - |\bar{a}|^2 = 8$

$\Rightarrow |\bar{x}|^2 - 1^2 = 8 \qquad \sin ce\, |\bar{a}| = 1$

$\Rightarrow |\bar{x}|^2 = 8 + 1$

$\Rightarrow |\bar{x}|^2 = 9$

$\Rightarrow |\bar{x}| = 3$

Scalar or Dot Product Ex 24.1 Q30(ii)

We have,

$(\bar{x} - \bar{a}).(\bar{x} + \bar{a}) = 12$

$\Rightarrow |\bar{x}|^2 - |\bar{a}|^2 = 12$

$\Rightarrow |\bar{x}|^2 - 1^2 = 12 \qquad \sin ce\, |\bar{a}| = 1$

$\Rightarrow |\bar{x}|^2 = 12 + 1$

$\Rightarrow |\bar{x}|^2 = 13$

$\Rightarrow |\bar{x}| = \sqrt{13}$

Scalar or Dot Product Ex 24.1 Q31(i)

Here, $\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right) = 12$

$|\vec{a}|^2 - |\vec{b}|^2 = 12$

$\left(2|\vec{b}|\right)^2 - |\vec{b}|^2 = 12$ $\quad\left[\text{Using } |\vec{a}| = 2|\vec{b}|\right]$

$4|\vec{b}|^2 - |\vec{b}|^2 = 12$

$3|\vec{b}|^2 = 12$

$|\vec{b}|^2 = \frac{12}{3}$

$|\vec{b}|^2 = 4$

$|\vec{b}| = 2$

$|\vec{a}| = 2|\vec{b}| = 2(2)$

$|\vec{a}| = 4$

$|\vec{b}| = 2$

Scalar or Dot Product Ex 24.1 Q31(ii)

$\left(\vec{a}\cdot\vec{b}\right).\left(\vec{a}-\vec{b}\right) = 8$

$\Rightarrow \vec{a}\cdot\vec{a} - \vec{a}.\vec{b} + \vec{b}\cdot\vec{a} - \vec{b}\cdot\vec{b} = 8$

$\Rightarrow |\vec{a}|^2 - |\vec{b}|^2 = 8$

$\Rightarrow \left(8|\vec{b}|\right)^2 - |\vec{b}|^2 = 8$ $\quad\left[|\vec{a}| = 8|\vec{b}|\right]$

$\Rightarrow 64|\vec{b}|^2 - |\vec{b}|^2 = 8$

$\Rightarrow 63|\vec{b}|^2 = 8$

$\Rightarrow |\vec{b}|^2 = \frac{8}{63}$

$\Rightarrow |\vec{b}| = \sqrt{\frac{8}{63}}$ $\quad$[Magnitude of a vector is non-negative]

$\Rightarrow |\vec{b}| = \frac{2\sqrt{2}}{3\sqrt{7}}$

$|\vec{a}| = 8|\vec{b}| = \frac{8\times 2\sqrt{2}}{3\sqrt{7}} = \frac{16\sqrt{2}}{3\sqrt{7}}$

Scalar or Dot Product Ex 24.1 Q31(iii)

Here, $\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right) = 3$

$|\vec{a}|^2 - |\vec{b}|^2 = 3$

$\left(2|\vec{b}|\right)^2 - |\vec{b}|^2 = 3$ $\quad\left[\text{Using } |\vec{a}| = 2|\vec{b}|\right]$

$4|\vec{b}|^2 - |\vec{b}|^2 = 3$

$3|\vec{b}|^2 = 3$

$|\vec{b}|^2 = \frac{3}{3}$

$|\vec{b}|^2 = 1$

$|\vec{b}| = 1$

$|\vec{a}| = 2|\vec{b}|$

$= 2(1)$

$|\vec{a}| = 2$

$|\vec{b}| = 1$

Scalar or Dot Product Ex 24.1 Q32(i)

$$\left|\vec{a}-\vec{b}\right|^2 = \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 - 2\vec{a}.\vec{b}$$
$$= (2)^2 + (5)^2 - 2(8)$$
$$= 4 + 25 - 16$$
$$\left|\vec{a}-\vec{b}\right|^2 = 13$$

$$\left|\vec{a}-\vec{b}\right| = \sqrt{13}$$

Scalar or Dot Product Ex 24.1 Q32(ii)

$$\left|\vec{a}-\vec{b}\right|^2 = \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 - 2\vec{a}.\vec{b}$$
$$= (3)^2 + (4)^2 - 2.(1)$$
$$= 9 + 16 - 2$$
$$\left|\vec{a}-\vec{b}\right|^2 = 23$$

$$\left|\vec{a}-\vec{b}\right| = \sqrt{23}$$

Scalar or Dot Product Ex 24.1 Q32(iii)

We have

$$\left|\vec{a}-\vec{b}\right|^2 = \left(\vec{a}-\vec{b}\right).\left(\vec{a}-\vec{b}\right)$$
$$= \vec{a}.\vec{a} - \vec{a}.\vec{b} - \vec{b}.\vec{a} + \vec{b}.\vec{b}$$
$$= \left|\vec{a}\right|^2 - 2\left(\vec{a}.\vec{b}\right) + \left|\vec{b}\right|^2 = (2)^2 - 2(4) + (3)^2 = 5$$
$$\therefore \quad \left|\vec{a}-\vec{b}\right| = \sqrt{5}$$

Scalar or Dot Product Ex 24.1 Q33(i)

We have,

$\left|\vec{a}\right| = \sqrt{3}, \left|\vec{b}\right| = 2$ and $\vec{a}.\vec{b} = \sqrt{6}$

Let θ be the angle between $\vec{a}$ and $\vec{b}$. Then

$$\cos\theta = \frac{\vec{a}.\vec{b}}{\left|\vec{a}\right|\left|\vec{b}\right|}$$
$$= \frac{\sqrt{6}}{\sqrt{3}\times 2}$$
$$= \frac{1}{\sqrt{2}}$$
$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$
$$= \frac{\pi}{4}$$

Scalar or Dot Product Ex 24.1 Q33(ii)

Let the angle between $\vec{a}$ and $\vec{b}$ is θ, then

$$\cos\theta = \frac{\vec{a}.\vec{b}}{\left|\vec{a}\right|\left|\vec{b}\right|}$$
$$= \frac{1}{3.3}$$
$$\cos\theta = \frac{1}{9}$$

$$\theta = \cos^{-1}\left(\frac{1}{9}\right)$$

Scalar or Dot Product Ex 24.1 Q34

Let $\vec{a} = \vec{u} + \vec{v}$

$5\hat{i} - 2\hat{j} + 5\hat{k} = \vec{u} + \vec{v}$ ---(i)

Such that $\vec{u}$ is parallel to $\vec{b}$ and $\vec{v}$ is perpendicular to $\vec{b}$.

Now, $\vec{u}$ is parallel to $\vec{b}$

$\vec{u} = \lambda\vec{b}$

$= \lambda\left(3\hat{i} + \hat{k}\right)$

$\vec{u} = 3\lambda\hat{i} + \lambda\hat{k}$ ---(ii)

Put value of $\vec{u}$ in equation (i),

$5\hat{i} - 2\hat{j} + 5\hat{k} = \left(3\lambda\hat{i} + \lambda\hat{k}\right) + \vec{v}$

$\vec{v} = 5\hat{i} - 2\hat{j} + 5\hat{k} - 3\lambda\hat{i} - \lambda\hat{k}$

$\vec{v} = (5 - 3\lambda)\hat{i} + (-2)\hat{j} + (5 - \lambda)\hat{k}$

$\vec{v}$ is perpendicular to $\vec{b}$

Then, $\vec{v}.\vec{b} = 0$

$\left[(5 - 3\lambda)\hat{i} + (-2)\hat{j} + (5 - \lambda)\hat{k}\right].\left(3\hat{i} + 0 \times \hat{j} + \hat{k}\right) = 0$

$(5 - 3\lambda)(3) + (-2)(0) + (5 - \lambda)(1) = 0$

$15 - 9\lambda + 0 + 5 - \lambda = 0$

$20 - 10\lambda = 0$

$-10\lambda = -20$

$\lambda = \frac{-20}{-10}$

$\lambda = 2$

Put λ in equation (ii)

$\vec{u} = 3\lambda\hat{i} + \lambda\hat{k}$

$= 3(2)\hat{i} + (2)\hat{k}$

$\vec{u} = 6\hat{i} + 2\hat{k}$

Put the value of $\vec{u}$ in equation (i)

$5\hat{i} - 2\hat{j} + 5\hat{k} = \vec{u} + \vec{v}$

$5\hat{i} - 2\hat{j} + 5\hat{k} = \left(6\hat{i} + 2\hat{k}\right) + \vec{v}$

$\vec{v} = 5\hat{i} - 2\hat{j} + 5\hat{k} - 6\hat{i} - 2\hat{k}$

$\vec{v} = -\hat{i} - 2\hat{j} + 3\hat{k}$

$\vec{a} = \left(6\hat{i} + 2\hat{k}\right) + \left(-\hat{i} - 2\hat{j} + 3\hat{k}\right)$

Scalar or Dot Product Ex 24.1 Q35

Vectors $\vec{a}$ and $\vec{b}$ have same magnitude, then

$|\vec{a}| = |\vec{b}| = x$ (Say)

Let θ be the angle between $\vec{a}$ and $\vec{b}$, then

$\cos\theta = \frac{\vec{a}\vec{b}}{|\vec{a}||\vec{b}|}$

$\cos 30° = \frac{3}{x.x}$

$\frac{\sqrt{3}}{2} = \frac{3}{x^2}$

$\sqrt{3}x^2 = 6$

$x^2 = \frac{6}{\sqrt{3}}$

Rationalizing the denominator,

$x^2 = \frac{6 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

$x^2 = \frac{6\sqrt{3}}{3}$

$x^2 = 2\sqrt{3}$

$x = \sqrt{2\sqrt{3}}$

$|\vec{a}| = |\vec{b}| = \sqrt{2\sqrt{3}}$

Scalar or Dot Product Ex 24.1 Q36

Let $\left(2\hat{i}-\hat{j}+3\hat{k}\right)=\vec{a}+\vec{b}$ ---(i)

Such that $\vec{a}$ is a vector parallel to vector $\left(2\hat{i}+4\hat{j}-2\hat{k}\right)$ and $\vec{b}$ is a vector perpendicular to the vector $\left(2\hat{i}+4\hat{j}-2\hat{k}\right)$.

Since, $\vec{a}$ is parallel to $\left(2\hat{i}+4\hat{j}-2\hat{k}\right)$

$$\vec{a}=\lambda\left(2\hat{i}+4\hat{j}-2\hat{k}\right)$$
$$\vec{a}=2\lambda\hat{i}+4\lambda\hat{j}-2\lambda\hat{k} \qquad ---\text{(ii)}$$

Put value of $\vec{a}$ in equation (i),

$$\left(2\hat{i}-\hat{j}+3\hat{k}\right)=\left(2\lambda\hat{i}+4\lambda\hat{j}-2\lambda\hat{k}\right)+\vec{b}$$
$$\vec{b}=2\hat{i}-\hat{j}+3\hat{k}-2\lambda\hat{i}-4\lambda\hat{j}+2\lambda\hat{k}$$
$$\vec{b}=(2-2\lambda)\hat{i}+(-1-4\lambda)\hat{j}+(3+2\lambda)\hat{k}$$

$\vec{b}$ is a vector perpendicular to the vector $\left(2\hat{i}+4\hat{j}-2\hat{k}\right)$, then

$$\vec{b}.\left(2\hat{i}+4\hat{j}-2\hat{k}\right)=0$$
$$\left[(2-2\lambda)\hat{i}+(-1-4\lambda)\hat{j}+(3+2\lambda)\hat{k}\right]\left(2\hat{i}+4\hat{j}-2\hat{k}\right)=0$$
$$(2-2\lambda)(2)+(-1-4\lambda)(4)+(3+2\lambda)(-2)=0$$
$$4-4\lambda-4-16\lambda-6-4\lambda=0$$
$$-6-24\lambda=0$$
$$-24\lambda=6$$
$$\lambda=-\frac{1}{4}$$

Put λ in equation (ii),

$$\vec{a}=2\lambda\hat{i}+4\lambda\hat{j}-2\lambda\hat{k}$$
$$=2\left(-\frac{1}{4}\right)\hat{i}+4\left(-\frac{1}{4}\right)\hat{j}-2\left(-\frac{1}{4}\right)\hat{k}$$
$$\vec{a}=-\frac{1}{2}\hat{i}-\hat{j}+\frac{1}{2}\hat{k}$$

Put the value of $\vec{a}$ in equation (i),

$$\left(2\hat{i}-\hat{j}+3\hat{k}\right)=\left(-\frac{1}{2}\hat{i}-\hat{j}+\frac{1}{2}\hat{k}\right)+\vec{b}$$

$$\vec{b}=2\hat{i}-\hat{j}+3\hat{k}+\frac{1}{2}\hat{i}+\hat{j}-\frac{1}{2}\hat{k}$$
$$=\frac{4\hat{i}-2\hat{j}+6\hat{k}+\hat{i}+2\hat{j}-\hat{k}}{2}$$
$$=\frac{5\hat{i}+5\hat{k}}{2}$$
$$\vec{b}=\frac{5}{2}\left(\hat{i}+\hat{k}\right)$$

$$\left(2\hat{i}-\hat{j}+3\hat{k}\right)=\left(-\frac{1}{2}\hat{i}-\hat{j}+\frac{1}{2}\hat{k}\right)+\frac{5}{2}\left(\hat{i}+\hat{k}\right)$$

Scalar or Dot Product Ex 24.1 Q37

Let $\left(6\hat{i} - 3\hat{j} - 6\hat{k}\right) = \vec{a} + \vec{b}$ $\qquad\qquad ---(i)$

Such that $\vec{a}$ is parallel to $\left(\hat{i} + \hat{j} + \hat{k}\right)$ and $\vec{b}$ is perpendicular to $\left(\hat{i} + \hat{j} + \hat{k}\right)$.

Since, $\vec{a}$ is parallel to $\left(\hat{i} + \hat{j} + \hat{k}\right)$

$$\vec{a} = \lambda\left(\hat{i} + \hat{j} + \hat{k}\right) \qquad ---(ii)$$

Put $\vec{a}$ in equation (i),

$$\left(6\hat{i} - 3\hat{j} - 6\hat{k}\right) = \left(\lambda\hat{i} + \lambda\hat{j} + \lambda\hat{k}\right) + \vec{b}$$
$$\vec{b} = 6\hat{i} - \lambda\hat{i} - 3\hat{j} - \lambda\hat{j} - 6\hat{k} - \lambda\hat{k}$$
$$\vec{b} = (6 - \lambda)\hat{i} + (-3 - \lambda)\hat{j} + (-6 - \lambda)\hat{k}$$

$\vec{b}$ is a vector perpendicular to the vector $\left(\hat{i} + \hat{j} + \hat{k}\right)$, then

$$\vec{b}.\left(\hat{i} + \hat{j} + \hat{k}\right) = 0$$
$$\left[(6 - \lambda)\hat{i} + (-3 - \lambda)\hat{j} + (-6 - \lambda)\hat{k}\right]\left(\hat{i} + \hat{j} + \hat{k}\right) = 0$$
$$(6 - \lambda)(1) + (-3 - \lambda)(1) + (-6 - \lambda)(1) = 0$$
$$6 - \lambda - 3 - \lambda - 6 - \lambda = 0$$
$$-3 - 3\lambda = 0$$
$$-3 = 3\lambda$$
$$\lambda = \frac{-3}{3}$$
$$\lambda = -1$$

Put value of λ in (ii),

$$\vec{a} = -1.\left(\hat{i} + \hat{j} + \hat{k}\right)$$
$$\vec{a} = -\hat{i} - \hat{j} - \hat{k}$$

Using $\vec{a}$ in equation (i),

$$\left(6\hat{i} - 3\hat{j} - 6\hat{k}\right) = \left(-\hat{i} - \hat{j} - \hat{k}\right) + \vec{b}$$
$$\vec{b} = 6\hat{i} + \hat{i} - 3\hat{j} + \hat{j} - 6\hat{k} + \hat{k}$$
$$\vec{b} = 7\hat{i} - 2\hat{j} - 5\hat{k}$$

Thus,

Vector $\vec{a} = -\hat{i} - \hat{j} - \hat{k}$ and

$\vec{b} = 7\hat{i} - 2\hat{j} - 5\hat{k}$

are required vectors.

Scalar or Dot Product Ex 24.1 Q38

Here, $\left(\vec{a} + \vec{b}\right)$ is orthogonal to $\left(\vec{a} - \vec{b}\right)$

Then, $\left(\vec{a} + \vec{b}\right).\left(\vec{a} - \vec{b}\right) = 0$

$$\left|\vec{a}\right|^2 = \left|\vec{b}\right|^2 = 0$$
$$\left\{\sqrt{(5)^2 + (-1)^2 + (7)^2}\right\}^2 - \left\{\sqrt{(1)^2 + (-1)^2 + (\lambda)^2}\right\}^2 = 0$$
$$(25 + 1 + 49) - \left(1 + 1 + \lambda^2\right) = 0$$
$$75 - \left(2 + \lambda^2\right) = 0$$
$$75 - 2 - \lambda^2 = 0$$
$$-\lambda^2 = -73$$
$$\lambda = \sqrt{73}$$

Scalar or Dot Product Ex 24.1 Q39

It is given that $\vec{a}\cdot\vec{a}=0$ and $\vec{a}\cdot\vec{b}=0$.

Now,

$\vec{a}\cdot\vec{a}=0 \Rightarrow |\vec{a}|^2=0 \Rightarrow |\vec{a}|=0$

$\therefore \vec{a}$ is a zero vector.

Hence, vector $\vec{b}$ satisfying $\vec{a}\cdot\vec{b}=0$ can be any vector

Scalar or Dot Product Ex 24.1 Q40

Given that $\vec{c}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, so,

$$\vec{a}\cdot\vec{c}=0 \text{ and } \vec{b}\cdot\vec{c}=0$$

Now, $\vec{c}\cdot(\vec{a}+\vec{b})$

$=\vec{c}\cdot\vec{a}+\vec{c}\cdot\vec{b}$

$=0+0$

$=0$

$\therefore \vec{c}$ is perpendicular to $(\vec{a}+\vec{b})$

$\vec{c}\cdot(\vec{a}-\vec{b})$

$=\vec{c}\cdot\vec{a}-\vec{c}\cdot\vec{b}$

$=0-0$

$=0$

$\therefore \vec{c}$ is perpendicular to $(\vec{a}-\vec{b})$

Scalar or Dot Product Ex 24.1 Q41

Here $|\vec{a}|=a$, $|\vec{b}|=b$

LHS $=\left(\frac{\vec{a}}{a^2}-\frac{\vec{b}}{b^2}\right)^2$

$=\left(\frac{\vec{a}}{a^2}\right)^2+\left(\frac{\vec{b}}{b^2}\right)^2-2\frac{\vec{a}}{a^2}\cdot\frac{\vec{b}}{b^2}$

$=\frac{|\vec{a}|^2}{a^4}+\frac{|\vec{b}|^2}{b^4}-\frac{2\vec{a}\vec{b}}{a^2b^2}$

$=\frac{a^2}{a^4}+\frac{b^2}{b^4}-\frac{2\vec{a}\vec{b}}{a^2b^2}$ $\quad$ [Since $|\vec{a}|=a$, $|\vec{b}|=b$]

$=\frac{1}{a^2}+\frac{1}{b^2}-\frac{2\vec{a}\vec{b}}{a^2b^2}$

$=\frac{b^2+a^2-2\vec{a}\vec{b}}{a^2b^2}$

$=\frac{|\vec{a}|^2+|\vec{b}|^2-2\vec{a}\vec{b}}{a^2b^2}$

$=\frac{(\vec{a}-\vec{b})^2}{a^2b^2}$

$=\left(\frac{\vec{a}-\vec{b}}{ab}\right)^2$

$=$ RHS

Hence proved

$$\therefore \quad \left(\frac{\vec{a}}{a^2}-\frac{\vec{b}}{b^2}\right)^2=\left(\frac{\vec{a}-\vec{b}}{ab}\right)^2$$

Scalar or Dot Product Ex 24.1 Q42

Given that

$\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors such that

$\vec{d}.\vec{a} = \vec{d}.\vec{b} = \vec{d}.\vec{c} = 0$

Given that

$\vec{d}.\vec{a} = 0$

$\Rightarrow$ $\vec{d}$ perpendicular to $\vec{a}$

or $\vec{d} = 0$ $\quad ---(i)$

$\vec{d}.\vec{b} = 0$

$\Rightarrow$ $\vec{d}$ is perpendicular to $\vec{b}$ or $\vec{d} = 0$ $\quad ---(ii)$

$\vec{d}.\vec{c} = 0$

$\Rightarrow$ $\vec{d}$ is perpendicular to $\vec{c}$ or $\vec{d} = 0$ $\quad ---(iii)$

From (i), (ii), (iii), we get

$\vec{d}$ is perpendicular to $\vec{a}, \vec{b}, \vec{c}$ or $\vec{d} = 0$, but $\vec{d}$ can not be perpendicular to $\vec{a}, \vec{b}$ and $\vec{c}$ because $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors, so

$\vec{d} = 0$

Scalar or Dot Product Ex 24.1 Q43

Given that

$\vec{a}$ is perpendicular to $\vec{b}$ and $\vec{c}$

It means,

$\vec{a}.\vec{b} = 0$ and $\vec{a}.\vec{c} = 0$ $\quad ---(i)$

Let $\vec{r}$ be some vector in the plane of $\vec{b}$ and $\vec{c}$

Then, $\vec{r}, \vec{b}, \vec{c}$ are coplanar

We know that,

Three vectors are coplanar if one of them is expressible as linear combination of other two vectors.

Let $\vec{r} = x\vec{b} + y\vec{c}$

where x and y are same scalar

$\vec{r}.\vec{a} = \left(x\vec{b} + y\vec{c}\right).\vec{a}$ $\quad$ [Taking dot product with $\vec{a}$ on both the side]

$\vec{r}.\vec{a} = x\vec{b}.\vec{a} + y\vec{c}.\vec{a}$

$= x.0 + y.0$ $\quad$ [Using (i)]

$\vec{r}.\vec{a} = 0 + 0$

$\vec{r}.\vec{a} = 0$

So, $\vec{r}$ is perpendicular to $\vec{a}$

Thus,

$\vec{a}$ is perpendicular to every vector in the plane of $\vec{b}$ and $\vec{c}$

Scalar or Dot Product Ex 24.1 Q44

We have,

$\vec{a} + \vec{b} + \vec{c} = \vec{0}$

$\vec{b} + \vec{c} = -\vec{a}$

Squaring both the sides.

$\left(\vec{b} + \vec{c}\right)^2 = \left(-\vec{a}\right)^2$

$\left|\vec{b}\right|^2 + \left|\vec{c}\right|^2 + 2\vec{b}.\vec{c} = \left|\vec{a}\right|^2$

$2\vec{b}.\vec{c} = \left|\vec{a}\right|^2 - \left|\vec{b}\right|^2 - \left|\vec{c}\right|^2$

$2\left|\vec{b}\right|\left|\vec{c}\right|\cos\theta = \left|\vec{a}\right|^2 - \left|\vec{b}\right|^2 - \left|\vec{c}\right|^2$ $\quad$ [Since $\vec{b}.\vec{c} = \left|\vec{b}\right|\left|\vec{c}\right|\cos\theta$]

$$\cos\theta = \frac{\left|\vec{a}\right|^2 - \left|\vec{b}\right|^2 - \left|\vec{c}\right|^2}{2\left|\vec{b}\right|\left|\vec{c}\right|}$$

Scalar or Dot Product Ex 24.1 Q45

Here, $\vec{u}+\vec{v}+\vec{w}=0$

Squaring both the sides,

$$\left(\vec{u}+\vec{v}+\vec{w}\right)^2=(0)^2$$

$$\left|\vec{u}\right|^2+\left|\vec{v}\right|^2+\left|\vec{w}\right|^2+2\vec{u}\vec{v}+2\vec{v}\vec{w}+2\vec{w}\vec{u}=0$$

$$(3)^2+(4)^2+(5)^2+2\left(\vec{u}.\vec{v}+\vec{v}.\vec{w}+\vec{w}.\vec{u}\right)=0$$

$$9+16+25+2\left(\vec{u}\vec{v}+\vec{v}\vec{w}+\vec{w}\vec{u}\right)=0$$

$$2\left(\vec{u}\vec{v}+\vec{v}\vec{w}+\vec{w}\vec{u}\right)=-50$$

$$\vec{u}\vec{v}+\vec{v}\vec{w}+\vec{w}\vec{u}=\frac{-50}{2}$$

$$\vec{u}\vec{v}+\vec{v}\vec{w}+\vec{w}\vec{u}=-25$$

Scalar or Dot Product Ex 24.1 Q46

Given

$$\vec{a}=x^2\hat{i}+2\hat{j}-2\hat{k}$$

$$\vec{b}=\hat{i}-\hat{j}+\hat{k}$$

$$\vec{c}=x^2\hat{i}+5\hat{j}-4k$$

Let θ be the angle between $\vec{a}$ and $\vec{b}$, then

$$\vec{a}.\vec{b}=\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta$$

$$\cos\theta=\frac{\vec{a}\vec{b}}{\left|\vec{a}\right|\left|\vec{b}\right|}$$

$$=\frac{\left(x^2\hat{i}+2\hat{j}-2\hat{k}\right)\left(\hat{i}-\hat{j}+\hat{k}\right)}{\sqrt{\left(x^2\right)^2+(2)^2+(-2)^2}\sqrt{(1)^2+(-1)^2+(1)^2}}$$

$$=\frac{\left(x^2\right)(1)+(2)(-1)+(-2)(1)}{\sqrt{x^4+4+4}\sqrt{1+1+1}}$$

$$=\frac{x^2-2-2}{\sqrt{8+x^2}\sqrt{3}}$$

$$\cos\theta=\frac{x^2-4}{\sqrt{3}\sqrt{8+x^4}}$$

Since θ is an acute angle, so

$$\cos\theta>0$$

$$\frac{x^2-4}{\sqrt{3}\sqrt{8+x^4}}>0$$

$$x^2-4>0$$

$$x^2>4$$

$$\Rightarrow \quad x<-2 \text{ or } x>2 \qquad ---(i)$$

Again, let ϕ be the angle between $\vec{b}$ and $\vec{c}$,

$$\cos\phi=\frac{\vec{b}\vec{c}}{\left|\vec{b}\right|\left|\vec{c}\right|}$$

$$=\frac{\left(\hat{i}-\hat{j}+\hat{k}\right)\left(x^2\hat{i}+5\hat{j}-4k\right)}{\sqrt{(1)^2+(-1)^2+(1)^2}\sqrt{\left(x^2\right)^2+(5)^2+(-4)^2}}$$

$$\cos\phi=\frac{(1)\left(x^2\right)+(-1)(5)+(1)(-4)}{\sqrt{3}\sqrt{x^4+25+16}}$$

$$\cos\phi=\frac{x^2-5-4}{\sqrt{3}\sqrt{x^2+41}}$$

$$\cos\phi=\frac{x^2-9}{\sqrt{3}\sqrt{x^2+41}}$$

Since ϕ is an obtuse angle, so

$$\cos\phi<0$$

$$\frac{x^2-9}{\sqrt{3}\sqrt{x^2+41}}<0$$

$$x^2-9<0$$

$$x^2<9$$

$$\Rightarrow \quad x>-3 \text{ and } x<3 \qquad ---(ii)$$

From

$$-3<x<-2 \text{ and } 2<x<3$$

$$x\in(-3,-2)\cup(2,3)$$

Scalar or Dot Product Ex 24.1 Q47

Here, $\vec{a}$ and $\vec{b}$ are mutually perpenducular, then

$$\vec{a}.\vec{b} = 0$$
$$\left(3\hat{i} + x\hat{j} - \hat{k}\right)\left(2\hat{i} + \hat{j} + y\hat{k}\right) = 0$$
$$(3)(2) + (x)(1) + (-1)(y) = 0$$
$$6 + x - y = 0$$
$$x - y = -6 \quad ---(i)$$

Also, $\vec{a}$ and $\vec{b}$ have equal magnitude,

$$|\vec{a}| = |\vec{b}|$$
$$\sqrt{(3)^2 + (x)^2 + (-1)^2} = \sqrt{(2)^2 + (1)^2 + (y)^2}$$
$$9 + x^2 + 1 = 4 + 1 + y^2$$
$$x^2 + 10 = 5y^2$$
$$x^2 - y^2 = 5 - 10$$
$$x^2 - y^2 = -5$$
$$(x + y)(x - y) = -5$$
$$(x + y)(-6) = -5 \quad [\text{Using } (i)]$$
$$-6x - 6y = -5$$
$$-(6x + 6y) = -5$$
$$6x + 6y = 5 \quad ---(ii)$$

Solving (i) and (ii),

$$6x + 6y = 5$$
$$\underline{6x - 6y = -36} \quad [(i) \times 6]$$
$$12x \quad = -31$$
$$x = \frac{-31}{12}$$

Put value of x in equation (i),

$$x - y = -6$$
$$\frac{-31}{12} - y = -6$$
$$-y = \frac{-6}{1} + \frac{31}{12}$$
$$-y = \frac{-72 + 31}{12}$$
$$y = \frac{41}{12}$$

Scalar or Dot Product Ex 24.1 Q48

Given

$\vec{a}$ and $\vec{b}$ are unit vectors

Then, $|\vec{a}| = |\vec{b}| = 1$

$$|\vec{a} + \vec{b}| = \sqrt{3}$$

Squaring both the sides,

$$|\vec{a} + \vec{b}|^2 = \left(\sqrt{3}\right)^2$$
$$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b} = 3$$
$$1 + 1 + 2\vec{a}.\vec{b} = 3$$
$$2 + 2\vec{a}.\vec{b} = 3$$
$$2\vec{a}.\vec{b} = 3 - 2$$
$$2\vec{a}.\vec{b} = 1$$
$$\vec{a}.\vec{b} = \frac{1}{2}$$

Now, $\left(2\vec{a} - 5\vec{b}\right).\left(3\vec{a} + \vec{b}\right)$

$$= 2\vec{a}.3\vec{a} + 2\vec{a}\vec{b} - 5\vec{b}.3\vec{a} - 5\vec{b}.\vec{b}$$
$$= 6\left(\vec{a}\right)^2 + 2\vec{a}.\vec{b} - 15\vec{a}.\vec{b} - 5\left(\vec{b}\right)^2$$
$$= 6|\vec{a}|^2 - 13\vec{a}\vec{b} - 5|\vec{b}|^2$$
$$= 6(1)^2 - 13\left(\frac{1}{2}\right) - 5(1)^2$$
$$= \frac{6}{1} - \frac{13}{2} - \frac{5}{1}$$
$$= \frac{12 - 13 - 10}{2}$$
$$= \frac{12 - 23}{2}$$
$$= -\frac{11}{2}$$

$$\left(2\vec{a} - 5\vec{b}\right).\left(3\vec{a} + \vec{b}\right) = -\frac{11}{2}$$

Scalar or Dot Product Ex 24.1 Q49

$|\bar{a}+\vec{b}|^2 = |\vec{b}|^2$

$\Rightarrow (\bar{a}+\vec{b}).(\bar{a}+\vec{b}) = \vec{b}.\vec{b}$

$\Rightarrow \bar{a}.\bar{a} + \bar{a}.\vec{b} + \vec{b}.\bar{a} + \vec{b}.\vec{b} = \vec{b}.\vec{b}$

$\Rightarrow \bar{a}.\bar{a} + \bar{a}.\vec{b} + \bar{a}.\vec{b} + \vec{b}.\vec{b} = \vec{b}.\vec{b}$

$\Rightarrow \bar{a}.\bar{a} + 2\bar{a}.\vec{b} = 0$

$\Rightarrow \bar{a}.(\bar{a} + 2\vec{b}) = 0$

$\therefore$ $\bar{a} + 2\vec{b}$ is perpendicular to $\bar{a}$.

Let $\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right) = 0$

$$\left|\vec{a}\right|^2 - \left|\vec{b}\right|^2 = 0$$

$$\left|\vec{a}\right|^2 = \left|\vec{b}\right|^2$$

$$\left|\vec{a}\right| = \left|\vec{b}\right|$$

Let $\left|\vec{a}\right| = \left|\vec{b}\right|$

Squaring both the sides.

$$\left|\vec{a}\right|^2 = \left|\vec{b}\right|^2$$

$$\left|\vec{a}\right|^2 - \left|\vec{b}\right|^2 = 0$$

$$\left(\vec{a}\right)^2 - \left(\vec{b}\right)^2 = 0$$

$$\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right) = 0$$

Thus,

$$\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right) = 0 \Leftrightarrow \left|\vec{a}\right| = \left|\vec{b}\right|$$

Ex 24.2

Scalar or Dot Product Ex 24.2 Q1

Let $\vec{o}$, $\vec{a}$ and $\vec{b}$ be the position vector of the O, A and B.

P and Q are points of trisection of AB.

Position vector of point P = $\dfrac{2\vec{a}+\vec{b}}{3}$

Position vector of point Q = $\dfrac{\vec{a}+2\vec{b}}{3}$

$$OP = \frac{2\vec{a}+\vec{b}}{3} - \vec{o} = \frac{2\vec{a}+\vec{b}-3\vec{o}}{3} = \frac{2OA+OB}{3}$$

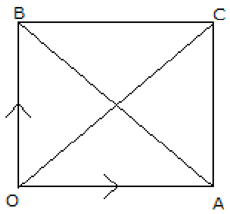
$$OQ = \frac{\vec{a}+2\vec{b}}{3} - \vec{o} = \frac{\vec{a}+2\vec{b}-3\vec{o}}{3} = \frac{OA+2OB}{3}$$

$$OP^2 + OQ^2 = \left(\frac{2OA+OB}{3}\right)^2 + \left(\frac{OA+2OB}{3}\right)^2$$

$$= \frac{5\left(OA^2+OB^2\right)+8(OA)(OB)\cos 90^\circ}{9}$$

$$= \frac{5}{9}AB^2 \ldots\ldots\ldots \left[\because OA^2+OB^2 = AB^2 \text{ and } \cos 90^\circ = 0\right]$$

Scalar or Dot Product Ex 24.2 Q2



Let OACB be a quadrilateral such that its diagonal bisect each other at right angles.
We know that if the diagonals of a quadrilateral bisect each other then its a parallelogram.
$\therefore$ OACB is a parallelogram.
$\Rightarrow$ OA = BC and OB = AC.

Taking O as origin let $\vec{a}$ and $\vec{b}$ be the position vector of the A and B.

AB and OC be the diagonals of quadrilateral which bisect each other at right angles.

$\therefore \overrightarrow{OC} \bullet \overrightarrow{AB} = 0$

$\Rightarrow \left(\vec{a}+\vec{b}\right) \bullet \left(\vec{b}-\vec{a}\right) = 0$

$\Rightarrow \left|\vec{b}\right|^2 = \left|\vec{a}\right|^2$

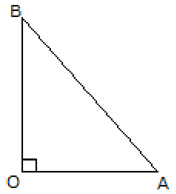
$\Rightarrow OB = OA$

Simillarly we can show that

OA = OB = BC = CA

Hence OACB is a rhombus.

Scalar or Dot Product Ex 24.2 Q3



Let OAC be a right triangle, right angled at O.

Taking O as origin let $\vec{a}$ and $\vec{b}$ be the position vector of the $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

$\overrightarrow{OA}$ is perpendicular to $\overrightarrow{OB}$

$\therefore \overrightarrow{OA} \bullet \overrightarrow{OB} = 0$

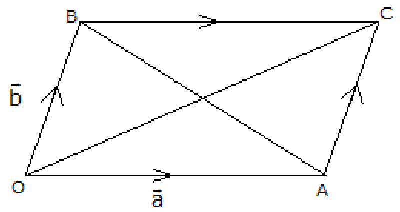
$\vec{a} \bullet \vec{b} = 0$

Now,

$\overrightarrow{AB}^2 = \left(\vec{b}-\vec{a}\right)^2 = (\vec{a})^2 + \left(\vec{b}\right)^2 - 2\vec{a}\bullet\vec{b} \;\; = (\vec{a})^2 + \left(\vec{b}\right)^2 - 0 = \left(\overrightarrow{OA}\right)^2 + \;\; \left(\overrightarrow{OB}\right)^2$

Hence proved.

Scalar or Dot Product Ex 24.2 Q4



Let OAC be a right triangle, right angled at O.

Taking O as origin let $\vec{a}$ and $\vec{b}$ be the position vector of the $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

$\overrightarrow{OA}$ is perpendicular to $\overrightarrow{OB}$

$\therefore \overrightarrow{OA} \bullet \overrightarrow{OB} = 0$

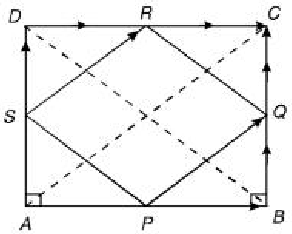
$\vec{a} \bullet \vec{b} = 0$

Now,

$\overrightarrow{AB}^2 = \left(\vec{b}-\vec{a}\right)^2 = (\vec{a})^2 + \left(\vec{b}\right)^2 - 2\vec{a}\bullet\vec{b} \;\; = (\vec{a})^2 + \left(\vec{b}\right)^2 - 0 = \left(\overrightarrow{OA}\right)^2 + \;\; \left(\overrightarrow{OB}\right)^2$

Hence proved.

Scalar or Dot Product Ex 24.2 Q5



ABCD be a rectangle.
Let P, Q, R and S be the midpoints of the sides AB, BC, CD and DA respectively.
Now,

$\overrightarrow{PQ} = \overrightarrow{PB} + \overrightarrow{BQ} = \frac{1}{2}\left(\overrightarrow{AB} + \overrightarrow{BC}\right) = \frac{1}{2}\overrightarrow{AC}$.........(i)

$\overrightarrow{SR} = \overrightarrow{SD} + \overrightarrow{DR} = \frac{1}{2}\left(\overrightarrow{AD} + \overrightarrow{DC}\right) = \frac{1}{2}\overrightarrow{AC}$.........(ii)

From (i) and (ii), we have

$\overrightarrow{PQ} = \overrightarrow{SR}$ i.e. sides PQ and SR are equal and parallel.
∴ PQRS is a parallelogram.

$(PQ)^2 = \overrightarrow{PQ} \bullet \overrightarrow{PQ} = \left(\overrightarrow{PB} + \overrightarrow{BQ}\right) \bullet \left(\overrightarrow{PB} + \overrightarrow{BQ}\right) = |PB|^2 + |BQ|^2$........(iii)

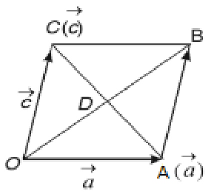
$(PS)^2 = \overrightarrow{PS} \bullet \overrightarrow{PS} = \left(\overrightarrow{PA} + \overrightarrow{PS}\right) \bullet \left(\overrightarrow{PA} + \overrightarrow{PS}\right) = |PA|^2 + |AS|^2 = |PB|^2 + |BQ|^2$........(iv)

From (iii) and (iv) we get,

$(PQ)^2 = (PQ)^2$ i. e. PQ = PS

⇒ The adjacent sides of PQRS are equal.
∴ PQRS is a rhombus.

Scalar or Dot Product Ex 24.2 Q6



Let OABC be a rhombus, whose diagonals OB and AC intersect at point D.
Let O be the origin.
Let the position vector of A and C be $\vec{a}$ and $\vec{c}$ respectively then,

$\overrightarrow{OA} = \vec{a}$ and $\overrightarrow{OC} = \vec{c}$

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OA} + \overrightarrow{OC} = \vec{a} + \vec{c}$............$\left[\because \overrightarrow{AB} = \overrightarrow{OC}\right]$

Position vector of mid-point of $\overrightarrow{OB} = \frac{1}{2}\left(\vec{a} + \vec{c}\right)$

Position vector of mid-point of $\overrightarrow{AC} = \frac{1}{2}\left(\vec{a} + \vec{c}\right)$

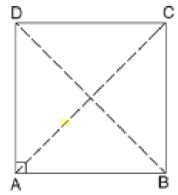
∴ Midpoints of OB and AC coincide.
∴ Diagonal OB and AC bisect each other.

$\overrightarrow{OB} \bullet \overrightarrow{AC} = \left(\vec{a} + \vec{c}\right) \bullet \left(\vec{c} - \vec{a}\right) = \left(\vec{c} + \vec{a}\right) \bullet \left(\vec{c} - \vec{a}\right) = \left|\vec{c}\right|^2 - \left|\vec{a}\right|^2 = \overrightarrow{OC} - \overrightarrow{OA} = 0$

$\left[\because \text{OC and OA are sides of the rhombus}\right]$

$\Rightarrow \overrightarrow{OB} \perp \overrightarrow{AC}$

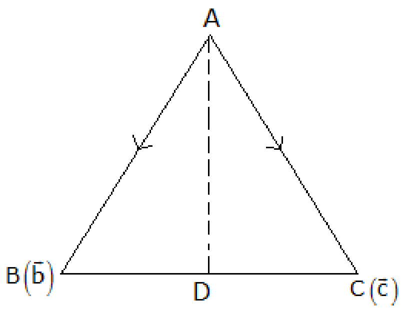
Scalar or Dot Product Ex 24.2 Q7



Let ABCD be a rectangle.
Take A as origin.
Let position vectors of point B, D be $\vec{a}$ and $\vec{b}$ respectively.
By parallelogram law,
$\overrightarrow{AC} = \vec{a} + \vec{b}$ and $\overrightarrow{BD} = \vec{a} - \vec{b}$
As ABCD is a rectangle, $AB \perp AD$
$\Rightarrow \vec{a} \bullet \vec{b} = 0$.........(i)

Now, diagonals AC and BD are perpendicular iff $\overrightarrow{AC} \bullet \overrightarrow{BD} = 0$
$\Rightarrow (\vec{a} + \vec{b})(\vec{a} - \vec{b}) = 0$
$\Rightarrow (\vec{a})^2 - (\vec{b})^2 = 0$
$\Rightarrow |\overrightarrow{AB}|^2 = |\overrightarrow{AD}|^2$
$\Rightarrow |AB| = |AD|$
Hence ABCD is a square.

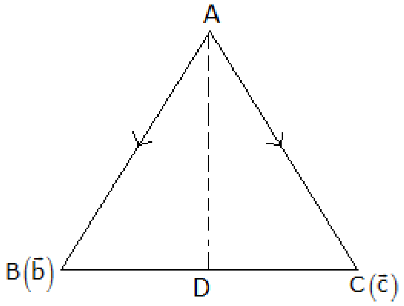
Scalar or Dot Product Ex 24.2 Q8



Take A as origin, let the position vectors of B and C are $\vec{b}$ and $\vec{c}$ respectively.
Position vector of D = $\frac{\vec{b} + \vec{c}}{2}$, $\overrightarrow{AB} = \vec{b}$ and $\overrightarrow{AC} = \vec{c}$.
$\overrightarrow{AD} = \frac{\vec{b} + \vec{c}}{2} - \vec{0} = \frac{\vec{b} + \vec{c}}{2}$

Consider, $2\left(AD^2 + CD^2\right)$
$= 2\left[\left(\frac{\vec{b} + \vec{c}}{2}\right)^2 + \left(\frac{\vec{b} + \vec{c}}{2} - \vec{c}\right)^2\right]$
$= 2\left[\left(\frac{\vec{b} + \vec{c}}{2}\right)^2 + \left(\frac{\vec{b} - \vec{c}}{2}\right)^2\right]$
$= \frac{1}{2}\left[(\vec{b} + \vec{c})^2 + (\vec{b} - \vec{c})^2\right]$
$= (\vec{b})^2 + (\vec{c})^2$
$= (\overrightarrow{AB})^2 + (\overrightarrow{AC})^2$
$= AB^2 + AC^2$
Hence proved.

Scalar or Dot Product Ex 24.2 Q9



Take A as origin, let the position vectors of B and C are $\vec{b}$ and $\vec{c}$ respectively.

Position vector of D = $\frac{\vec{b}+\vec{c}}{2}$, $\overrightarrow{AB} = \vec{b}$ and $\overrightarrow{AC} = \vec{c}$.

$\overrightarrow{AD} = \frac{\vec{b}+\vec{c}}{2} - \vec{0} = \frac{\vec{b}+\vec{c}}{2}$

AD is perpendicular to BC

$\Rightarrow \overrightarrow{AD} \bullet \overrightarrow{BC} = 0$

$\Rightarrow \left(\frac{\vec{b}+\vec{c}}{2}\right) \bullet (\vec{c} - \vec{b}) = 0$

$\Rightarrow (\vec{b}+\vec{c}) \bullet (\vec{c}-\vec{b}) = 0$

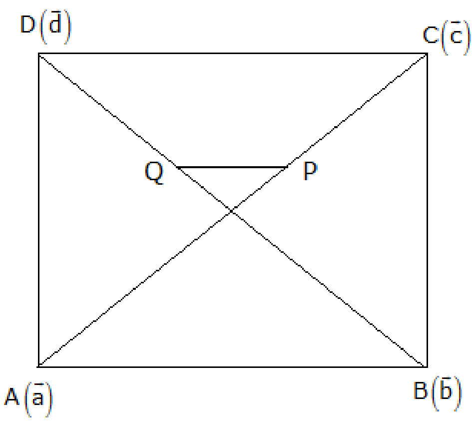
$\Rightarrow |\vec{c}|^2 = |\vec{b}|^2$

$\Rightarrow |\vec{c}| = |\vec{b}|$

$\Rightarrow AC = AB$

Hence ΔABC is an isoscales triangle.

Scalar or Dot Product Ex 24.2 Q10



Take O as origin, let the position vectors of A, B C and D are $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ respectively.

Position vector of P = $\frac{\vec{a}+\vec{c}}{2}$

Position vector of Q = $\frac{\vec{a}+\vec{d}}{2}$

LHS = $AB^2 + BC^2 + CD^2 + DA^2$

$= (\vec{b}-\vec{a})^2 + (\vec{c}-\vec{b})^2 + (\vec{d}-\vec{c})^2 + (\vec{d}-\vec{a})^2$

$= 2\left[(\vec{a})^2 + (\vec{b})^2 + (\vec{c})^2 + (\vec{d})^2 - \vec{a}\vec{b}\cos\theta_1 - \vec{b}\vec{c}\cos\theta_2 - \vec{d}\vec{c}\cos\theta_3 - \vec{c}\vec{a}\cos\theta_4\right]$

RHS = $AC^2 + BD^2 + 4PQ^2$

$= (\vec{c}-\vec{a})^2 + (\vec{d}-\vec{b})^2 + 4\left(\frac{\vec{a}+\vec{d}}{2} - \frac{\vec{a}+\vec{c}}{2}\right)^2$

$= 2\left[(\vec{a})^2 + (\vec{b})^2 + (\vec{c})^2 + (\vec{d})^2 - \vec{a}\vec{b}\cos\theta_1 - \vec{b}\vec{c}\cos\theta_2 - \vec{d}\vec{c}\cos\theta_3 - \vec{c}\vec{a}\cos\theta_4\right]$

= LHS

Hence proved.

Ex 25.1

Vector or Cross Product Ex 25.1 Q1

If $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and
$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$, then

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -2 \\ -1 & 0 & 3 \end{vmatrix}$$

$$= \hat{i}(9-0) - \hat{j}(3-2) + \hat{k}(0+3)$$

$$\vec{a} \times \vec{b} = 9\hat{i} - \hat{j} + 3\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{(9)^2 + (-1)^2 + (3)^2}$$
$$= \sqrt{81 + 1 + 9}$$

$$|\vec{a} \times \vec{b}| = \sqrt{91}$$

Vector or Cross Product Ex 25.1 Q2(i)

If $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and
$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$, then

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}(4-0) - \hat{j}(3-0) + \hat{k}(3-4)$$
$$= 4\hat{i} - 3\hat{j} - \hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{(4)^2 + (-3)^2 + (-1)^2}$$
$$= \sqrt{16 + 9 + 1}$$

$$|\vec{a} \times \vec{b}| = \sqrt{26}$$

Vector or Cross Product Ex 25.1 Q2(ii)

If $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and
$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$, then

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}(0-1) - \hat{j}(2-1) + \hat{k}(2-0)$$
$$= -\hat{i} - \hat{j} + 2\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{(-1)^2 + (-1)^2 + (2)^2}$$
$$= \sqrt{1 + 1 + 4}$$

$$|\vec{a} \times \vec{b}| = \sqrt{6}$$

Magnitude of $\vec{a} \times \vec{b} = \sqrt{6}$.

Vector or Cross Product Ex 25.1 Q3(i)

A vector perpendicular to both $\vec{a}$ and $\vec{b} = \vec{a} \times \vec{b}$.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$\vec{c}\,(\text{say}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 3 \\ -2 & 1 & -2 \end{vmatrix}$$

$$\vec{c} = \hat{i}\,(2-3) - \hat{j}\,(-8+6) + \hat{k}\,(4-2)$$
$$\vec{c} = -\hat{i} + 2\hat{j} + 2\hat{k}$$

$\vec{c}$ is a vector perpendicular to both $\vec{a}$ and $\vec{b}$.

$$\hat{c} = \frac{\vec{c}}{|\vec{c}|}$$

$$= \frac{-\hat{i} + 2\hat{j} + 2\hat{k}}{\sqrt{(-1)^2 + (2)^2 + (2)^2}}$$
$$= \frac{-\hat{i} + 2\hat{j} + 2\hat{k}}{\sqrt{1+4+4}}$$

$$= \frac{1}{3}\left(-\hat{i} + 2\hat{j} + 2\hat{k}\right)$$

So, unit vector perpendicular to both $\vec{a}$ and $\vec{b} = \frac{1}{3}\left(-\hat{i} + 2\hat{j} + 2\hat{k}\right)$.

Vector or Cross Product Ex 25.1 Q3(ii)

A vector perpendicular to the plane containing the vector $\vec{a}$ and $\vec{b}$ is given by
$\vec{a} \times \vec{b} = \pm\vec{c}$ (Say)

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$\vec{c} = \hat{i}\,(1-2) - \hat{j}\,(2-1) + \hat{k}\,(4-1)$$
$$\vec{c} = -\hat{i} - \hat{j} + 3\hat{k}$$

$$\hat{c} = \frac{\vec{c}}{|\vec{c}|}$$

$$= \frac{-\hat{i} - \hat{j} + 3\hat{k}}{\sqrt{(-1)^2 + (-1)^2 + (3)^2}}$$
$$= \frac{-\hat{i} - \hat{j} + 3\hat{k}}{\sqrt{1+1+9}}$$

$$= \frac{1}{\sqrt{11}}\left(-\hat{i} - \hat{j} + 3\hat{k}\right)$$

Unit vector perpendicular to the plane of $\vec{a}$ and $\vec{b} = \pm\frac{1}{\sqrt{11}}\left(-\hat{i} - \hat{j} + 3\hat{k}\right)$.

Vector or Cross Product Ex 25.1 Q4

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 4 & 3 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= \hat{i}\,(-4-3) - \hat{j}\,(0-3) + \hat{k}\,(0-4)$$
$$= -7\hat{i} + 3\hat{j} - 4\hat{k}$$

$$\left|\vec{a} \times \vec{b}\right| = \sqrt{(-7)^2 + (3)^2 + (-4)^2}$$
$$= \sqrt{49 + 9 + 16}$$

$$\left|\vec{a} \times \vec{b}\right| = \sqrt{74}$$

Vector or Cross Product Ex 25.1 Q5

$$\vec{b} = \hat{i} - 2\hat{k}$$
$$\hat{b} = \frac{\vec{b}}{\left|\vec{b}\right|}$$

$$= \frac{\hat{i} - 2\hat{k}}{\sqrt{(1)^2 + (-2)^2}}$$
$$= \frac{\hat{i} - 2\hat{k}}{\sqrt{1+4}}$$

$$= \frac{\hat{i} - 2\hat{k}}{\sqrt{5}}$$

$$2\hat{b} = \frac{2}{\sqrt{5}}\hat{i} - \frac{4}{\sqrt{5}}\hat{k}$$
And, $\vec{a} = 4\hat{i} + 3\hat{j} + \hat{k}$

If $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and
$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$2\hat{b} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{2}{\sqrt{5}} & 0 & -\frac{4}{\sqrt{5}} \\ 4 & 3 & 1 \end{vmatrix}$$

$$= \hat{i}\left(0 + \frac{12}{\sqrt{5}}\right) - \hat{j}\left(\frac{2}{\sqrt{5}} + \frac{16}{\sqrt{5}}\right) + \hat{k}\left(\frac{6}{\sqrt{5}} - 0\right)$$

$$2\hat{b} \times \vec{a} = \frac{12}{\sqrt{5}}\hat{i} - \frac{18}{\sqrt{5}}\hat{j} + \frac{6}{\sqrt{5}}\hat{k}$$

$$\left|2\hat{b} \times \vec{a}\right| = \sqrt{\left(\frac{12}{\sqrt{5}}\right)^2 + \left(-\frac{18}{\sqrt{5}}\right)^2 + \left(\frac{6}{\sqrt{5}}\right)^2}$$
$$= \sqrt{\frac{144}{5} + \frac{324}{5} + \frac{36}{5}}$$

$$\left|2\hat{b} \times \vec{a}\right| = \sqrt{\frac{504}{5}}$$

Vector or Cross Product Ex 25.1 Q6

$$\vec{a}+2\vec{b}=\left(3\hat{i}-\hat{j}-2\hat{k}\right)+2\left(2\hat{i}+3\hat{j}+\hat{k}\right)$$
$$=3\hat{i}-\hat{j}-2\hat{k}+4\hat{i}+6\hat{j}+2\hat{k}$$

$$\vec{a}+2\vec{b}=7\hat{i}+5\hat{j}$$

$$2\vec{a}-\vec{b}=2\left(3\hat{i}-\hat{j}-2\hat{k}\right)-\left(2\hat{i}+3\hat{j}+\hat{k}\right)$$
$$=6\hat{i}-2\hat{j}-4\hat{k}-2\hat{i}-3\hat{j}-\hat{k}$$

$$=4\hat{i}-5\hat{j}-5\hat{k}$$

We know that if $\vec{a}=a_1\hat{i}+b_1\hat{j}+c_1\hat{k}$ and $\vec{b}=a_2\hat{i}+b_2\hat{j}+c_2\hat{k}$ then,

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}$$

Therefore,

$$\left(\vec{a}+2\vec{b}\right)\times\left(2\vec{a}-\vec{b}\right)=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 7 & 5 & 0\\ 4 & -5 & -5\end{vmatrix}$$

$$=\hat{i}\left(-25-0\right)-\hat{j}\left(-35-0\right)+\hat{k}\left(-35-20\right)$$
$$=-25\hat{i}+35\hat{j}-55\hat{k}$$

$$\left(\vec{a}+2\vec{b}\right)\times\left(2\vec{a}-\vec{b}\right)=-25\hat{i}+35\hat{j}-55\hat{k}$$

Vector or Cross Product Ex 25.1 Q7(i)

Let, $\vec{a}=2\hat{i}+3\hat{j}+6\hat{k}$, $\vec{b}=3\hat{i}-6\hat{j}+2\hat{k}$

If $\vec{a}=a_1\hat{i}+b_1\hat{j}+c_1\hat{k}$ and
$\vec{b}=a_2\hat{i}+b_2\hat{j}+c_2\hat{k}$, then,

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}$$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 6\\ 3 & -6 & 2\end{vmatrix}$$

$$=\hat{i}\left(6+36\right)-\hat{j}\left(4-18\right)+\hat{k}\left(-12-9\right)$$
$$=42\hat{i}+14\hat{j}-21\hat{k}$$

$$=7\left(6\hat{i}+2\hat{j}-3\hat{k}\right)$$

$$\left|\vec{a}\times\vec{b}\right|=7\sqrt{(6)^2+(2)^2+(-3)^2}$$
$$=7\sqrt{36+4+9}$$

$$\left|\vec{a}\times\vec{b}\right|=7\sqrt{49}$$
$$\left|\vec{a}\times\vec{b}\right|=7\times7$$

$$\left|\vec{a}\times\vec{b}\right|=49$$

Vector or Cross Product Ex 25.1 Q7(ii)

Vector perpendicular to $\vec{a}$ and $\vec{b}$

with magnitude 1 = $\dfrac{\vec{a}\times\vec{b}}{\left|\vec{a}\times\vec{b}\right|}$

$$= \frac{1}{49}\left(7\left(6\hat{i}+2\hat{j}-3\hat{k}\right)\right)$$

$$= \frac{1}{7}\left(6\hat{i}+2\hat{j}-3\hat{k}\right)$$

vector of magnitude 49, which is perpendicular to $\vec{a}$ and $\vec{b}$

$$= 49\left(\frac{\vec{a}\times\vec{b}}{\left|\vec{a}\times\vec{b}\right|}\right)$$

$$= 49\left[\frac{1}{7}\left(6\hat{i}+2\hat{j}-3\hat{k}\right)\right]$$

$$= 42\hat{i}+14\hat{j}-21\hat{k}$$

The required vector = $42\hat{i}+14\hat{j}-21\hat{k}$

Vector or Cross Product Ex 25.1 Q7(iii)

If $\vec{a} = a_1\hat{i}+b_1\hat{j}+c_1\hat{k}$ and

$\vec{b} = a_2\hat{i}+b_2\hat{j}+c_2\hat{k}$, then,

$$\vec{a}\times\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}$$

$$= \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & -4\\ 6 & 5 & -2\end{vmatrix}$$

$$= \hat{i}\left(-2+20\right)-\hat{j}\left(-6+24\right)+\hat{k}\left(15-6\right)$$

$$= 18\hat{i}-18\hat{j}+9\hat{k}$$

$$= 9\left(2\hat{i}-2\hat{j}+\hat{k}\right)$$

$$\left|\vec{a}\times\vec{b}\right| = 9\sqrt{2^2+\left(-2\right)^2+\left(1\right)^2}$$

$$= 9\sqrt{4+4+1}$$

$$\left|\vec{a}\times\vec{b}\right| = 9\sqrt{9}$$

$$\left|\vec{a}\times\vec{b}\right| = 9\times 3$$

$$\left|\vec{a}\times\vec{b}\right| = 27$$

Vector or Cross Product Ex 25.1 Q7(iv)

Unit vector perpendicular to the vector

$$\vec{a} \text{ and } \vec{b} = \frac{\vec{a}\times\vec{b}}{\left|\vec{a}\times\vec{b}\right|}$$

$$= \frac{1}{27}\left(9\left(2\hat{i} - 2\hat{j} + \hat{k}\right)\right)$$

$$= \frac{1}{3}\left(2\hat{i} - 2\hat{j} + \hat{k}\right)$$

vector with length 3 and which is perpendicular to both $\vec{a}$ and $\vec{b}$

$$= 3\left(\frac{\vec{a}\times\vec{b}}{\left|\vec{a}\times\vec{b}\right|}\right)$$

$$= 3\left[\frac{1}{3}\left(2\hat{i} - 2\hat{j} + \hat{k}\right)\right]$$

$$= 2\hat{i} - 2\hat{j} + \hat{k}$$

Required vector $= 2\hat{i} - 2\hat{j} + \hat{k}$

Vector or Cross Product Ex 25.1 Q8(i)

Here, $\vec{a} = 2\hat{i} + 0.\hat{j} + 0.\hat{k}$

$\vec{b} = 0.\hat{i} + 3\hat{j} + 0.\hat{k}$,

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ 0 & 3 & 0 \end{vmatrix}$$

$$= \hat{i}\,(0 - 0) - \hat{j}\,(0 - 0) + \hat{k}\,(6 - 0)$$

$$= 6\hat{k}$$

Area of parallelogram $= \left|\vec{a}\times\vec{b}\right|$

$$= \left|0\hat{i} + 0.\hat{j} + 6\hat{k}\right|$$

$$= \sqrt{(0)^2 + (0)^2 + (6)^2}$$

Area of parallelogram = 6 sq.unit

Vector or Cross Product Ex 25.1 Q8(ii)

Let, $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$

$\vec{b} = \hat{i} - \hat{j}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{vmatrix}$$

$$= \hat{i}\,(0 + 3) - \hat{j}\,(0 - 3) + \hat{k}\,(-2 - 1)$$

$$= 3\hat{i} + 3\hat{j} - 3\hat{k}$$

$$= 3\left(\hat{i} + \hat{j} - \hat{k}\right)$$

Area of parallelogram $= \left|\vec{a}\times\vec{b}\right|$

$$= 3\sqrt{(1)^2 + (1)^2 + (-1)^2}$$

$$= 3\sqrt{3}$$

Area of parallelogram $= 3\sqrt{3}$ sq.unit

Vector or Cross Product Ex 25.1 Q8(iii)

Let, $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$

$\vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}$$

$$= \hat{i}\,(4 - 6) - \hat{j}\,(12 + 2) + \hat{k}\,(-9 - 1)$$

$$= -2\,\hat{i} - 14\hat{j} - 10\hat{k}$$

$$= -2\left(\hat{i} + 7\hat{j} + 5\hat{k}\right)$$

Area of parallelogram $= \left|\vec{a} \times \vec{b}\right|$

$$= 2\sqrt{(1)^2 + (7)^2 + (5)^2}$$

$$= 2\sqrt{1 + 49 + 25}$$

$$= 2\sqrt{75}$$

$$= 10\sqrt{3}$$

Area of parallelogram $= 10\sqrt{3}$ sq.unit

Vector or Cross Product Ex 25.1 Q8(iv)

Let, $\vec{a} = \hat{i} - 3\hat{j} + \hat{k}$

$\vec{b} = \hat{i} + \hat{j} + \hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}\,(-3 - 1) - \hat{j}\,(1 - 1) + \hat{k}\,(1 + 3)$$

$$= -4\hat{i} - 0.\hat{j} + 4\hat{k}$$

Area of parallelogram $= \left|\vec{a} \times \vec{b}\right|$

$$= \sqrt{(-4)^2 + (0)^2 + (4)^2}$$

$$= \sqrt{16 + 0 + 16}$$

$$= \sqrt{32}$$

$$= 4\sqrt{2}$$

Area of parallelogram $= 4\sqrt{2}$ sq.unit

Vector or Cross Product Ex 25.1 Q9(i)

Area of parallelogram = $\frac{1}{2}\left|\vec{d_1} \times \vec{d_2}\right|$

Here, $d_1 = 4\hat{i} - \hat{j} - 3\hat{k}$

$d_2 = -2\hat{i} + \hat{j} - 2\hat{k}$

$$\vec{d_1} \times \vec{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & -3 \\ -2 & 1 & -2 \end{vmatrix}$$

$$= \hat{i}(2+3) - \hat{j}(-8-6) + \hat{k}(4-2)$$

$$= 5\hat{i} + 14\hat{j} + 2\hat{k}$$

$$\left|\vec{d_1} \times \vec{d_2}\right| = \sqrt{(5)^2 + (14)^2 + (2)^2}$$

$$= \sqrt{25 + 196 + 4}$$

$$= \sqrt{225}$$

$$= 15$$

Area of parallelogram = $\frac{1}{2}\left|\vec{d_1} \times \vec{d_2}\right|$

Area of parallelogram = $\frac{15}{2}$ sq.unit

Vector or Cross Product Ex 25.1 Q9(ii)

Given, $d_1 = 2\hat{i} + \hat{k}$

$d_2 = \hat{i} + \hat{j} + \hat{k}$

$$\vec{d_1} \times \vec{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}(0-1) - \hat{j}(2-1) + \hat{k}(2-0)$$

$$= -\hat{i} - \hat{j} + 2\hat{k}$$

$$\left|\vec{d_1} \times \vec{d_2}\right| = \sqrt{(-1)^2 + (-1)^2 + (2)^2}$$

$$= \sqrt{1 + 1 + 4}$$

$$= \sqrt{6}$$

Area of parallelogram = $\frac{1}{2}\left|\vec{d_1} \times \vec{d_2}\right|$

Area of parallelogram = $\frac{1}{2}\sqrt{6}$ sq.unit

Vector or Cross Product Ex 25.1 Q9(iii)

Given, $d_1 = 3\hat{i} + 4\hat{j}$

$d_2 = \hat{i} + \hat{j} + \hat{k}$

$$\overrightarrow{d_1} \times \overrightarrow{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}\,(4-0) - \hat{j}\,(3-0) + \hat{k}\,(3-4)$$

$$= 4\hat{i} - 3\hat{j} - \hat{k}$$

$$\left|\overrightarrow{d_1} \times \overrightarrow{d_2}\right| = \sqrt{(4)^2 + (-3)^2 + (-1)^2}$$

$$= \sqrt{16 + 9 + 1}$$

$$= \sqrt{26}$$

Area of parallelogram $= \frac{1}{2}\left|\overrightarrow{d_1} \times \overrightarrow{d_2}\right|$

Area of parallelogram $= \frac{\sqrt{26}}{2}$ sq.unit

Vector or Cross Product Ex 25.1 Q9(iv)

Here, $d_1 = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$d_2 = 3\hat{i} - 6\hat{j} + 2\hat{k}$

$$\overrightarrow{d_1} \times \overrightarrow{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 3 & -6 & 2 \end{vmatrix}$$

$$= \hat{i}\,(6+36) - \hat{j}\,(4-18) + \hat{k}\,(-12-9)$$

$$= 42\hat{i} + 14\hat{j} - 21\hat{k}$$

$$= 7\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right)$$

$$\left|\overrightarrow{d_1} \times \overrightarrow{d_2}\right| = 7\sqrt{(6)^2 + (2)^2 + (-3)^2}$$

$$= 7\sqrt{36 + 4 + 9}$$

$$= 7\sqrt{49}$$

$$= 7 \times 7$$

$$= 49$$

Area of parallelogram $= \frac{1}{2}\left|\overrightarrow{d_1} \times \overrightarrow{d_2}\right|$

Area of parallelogram $= \frac{49}{2}$ sq.unit

Vector or Cross Product Ex 25.1 Q10

Given, $\vec{a} = 2\hat{i} + 5\hat{j} - 7\hat{k}$,
$\vec{b} = -3\hat{i} + 4\hat{j} + \hat{k}$,
$\vec{c} = \hat{i} - 2\hat{j} - 3\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 5 & -7 \\ -3 & 4 & 1 \end{vmatrix}$$

$$= \hat{i}(5 + 28) - \hat{j}(2 - 21) + \hat{k}(8 + 15)$$
$$= 33\hat{i} + 19\hat{j} + 23\hat{k}$$

$$(\vec{a} \times \vec{b}) \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 33 & 19 & 23 \\ 1 & -2 & -3 \end{vmatrix}$$

$$= \hat{i}(-57 + 46) - \hat{j}(-99 - 23) + \hat{k}(-66 - 19)$$
$$(\vec{a} \times \vec{b}) \times \vec{c} = -11\hat{i} + 122\hat{j} - 85\hat{k} \quad \text{---(i)}$$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 4 & 1 \\ 1 & -2 & -3 \end{vmatrix}$$

$$= \hat{i}(-12 + 2) - \hat{j}(9 - 1) + \hat{k}(6 - 4)$$
$$= -10\hat{i} - 8\hat{j} + 2\hat{k}$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 5 & -7 \\ -10 & -8 & 2 \end{vmatrix}$$

$$= \hat{i}(10 + 56) - \hat{j}(4 - 70) + \hat{k}(-16 + 50)$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = 66\hat{i} + 66\hat{j} + 36\hat{k} \quad \text{---(ii)}$$

From equation (i) and (ii)

$$\vec{a} \times (\vec{b} \times \vec{c}) \neq (\vec{a} \times \vec{b}) \times \vec{c}$$

Vector or Cross Product Ex 25.1 Q11

We know that, if θ be the angle between $\vec{a}$ and $\vec{b}$, then,

$$\vec{a} \times \vec{b} = |\vec{a}|.|\vec{b}|.\sin\theta.\hat{n}$$
$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|.|\sin\theta|.|\hat{n}|$$
$$8 = 2.5.\sin\theta.1 \qquad [\text{As } \hat{n} \text{ is a unit vector}]$$

$$\sin\theta = \frac{8}{10}$$
$$\sin\theta = \frac{4}{5}$$
$$\cos^2\theta = 1 - \sin^2\theta$$
$$= 1 - \left(\frac{4}{5}\right)^2$$
$$= 1 - \frac{16}{25}$$
$$= \frac{25 - 16}{25}$$
$$= \frac{9}{25}$$

$$\cos\ \theta = \frac{3}{5}$$

$$\vec{a}.\vec{b} = |\vec{a}|.|\vec{b}|.\cos\theta$$
$$= 2.5.\frac{3}{5}$$

$$\vec{a}.\vec{b} = 6$$

Vector or Cross Product Ex 25.1 Q12

Given, $\vec{a} = \frac{1}{7}\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$

$\vec{b} = \frac{1}{7}\left(3\hat{i} - 6\hat{j} + 2\hat{k}\right)$

$\vec{c} = \frac{1}{7}\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right)$,

$$\vec{a} \times \vec{b} = \frac{1}{7} \times \frac{1}{7}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 3 & -6 & 2 \end{vmatrix}$$

$$= \frac{1}{49}\left[\hat{i}(6+36) - \hat{j}(4-18) + \hat{k}(-12-9)\right]$$

$$= \frac{1}{49}\left[42\hat{i} + 14\hat{j} - 21\hat{k}\right]$$

$$= \frac{7\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right)}{49}$$

$$= \frac{1}{7}\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right)$$

$$\vec{a} \times \vec{b} = \vec{c} \qquad ---(i)$$

$$\vec{b} \times \vec{c} = \frac{1}{7} \times \frac{1}{7}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -6 & 2 \\ 6 & 2 & -3 \end{vmatrix}$$

$$= \frac{1}{49}\left[\hat{i}(18-4) - \hat{j}(-9-12) + \hat{k}(6+36)\right]$$

$$= \frac{1}{49}\left[14\hat{i} + 21\hat{j} + 42\hat{k}\right]$$

$$= \frac{7\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)}{49}$$

$$\vec{c} \times \vec{a} = \frac{1}{7} \times \frac{1}{7}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 2 & -3 \\ 2 & 3 & 6 \end{vmatrix}$$

$$\vec{c} \times \vec{a} = \frac{1}{49}\left[\hat{i}(12+9) - \hat{j}(36+6) + \hat{k}(18-4)\right]$$

$$= \frac{1}{49}\left[21\hat{i} - 42\hat{j} + 14\hat{k}\right]$$

$$= \frac{7\left(3\hat{i} - 6\hat{j} + 2\hat{k}\right)}{49}$$

$$= \frac{1}{7}\left(3\hat{i} - 6\hat{j} + 2\hat{k}\right)$$

$$\vec{c} \times \vec{a} = \vec{b} \qquad ---(iii)$$

From (i), (ii), and (iii),

$$\left.\begin{matrix} \vec{a} \times \vec{b} = \vec{c} \\ \vec{b} \times \vec{c} = \vec{a} \\ \vec{c} \times \vec{a} = \vec{b} \end{matrix}\right\} - (A)$$

$$\left|\vec{a}\right| = \frac{1}{7}\sqrt{(2)^2 + (3)^2 + (6)^2}$$

$$= \frac{1}{7}\sqrt{4+9+36}$$

$$= \frac{1}{7}\sqrt{49}$$

$$= \frac{1}{7} \times 7$$

$$\left|\vec{a}\right| = 1 \qquad ---(iv)$$

$$\left|\vec{b}\right| = \frac{1}{7}\sqrt{(3)^2 + (-6)^2 + (2)^2}$$

$$= \frac{1}{7}\sqrt{9+36+4}$$

$$= \frac{1}{7}\sqrt{49}$$

$$= \frac{7}{7}$$

$$\left|\vec{b}\right| = 1 \qquad ---(v)$$

$$\left|\vec{c}\right| = \frac{1}{7}\sqrt{(6)^2 + (2)^2 + (-3)^2}$$

$$= \frac{1}{7}36+4+9$$

$$= \frac{1}{7}\sqrt{49}$$

$$= \frac{7}{7}$$

$$\left|\vec{c}\right| = 1 \qquad ---(vi)$$

From equation (iv), (v), (vi),

$$\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right| = 1 \qquad ---(B)$$

From (A) and (B), We can say that
$\vec{a}, \vec{b}, \vec{c}$ is a right handed orthogonal system of unit vectors

Vector or Cross Product Ex 25.1 Q13

We know that, if θ is angle between $\vec{a}$ and $\vec{b}$,

$$\vec{a}\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$
$$60 = 13.5.\cos\theta$$
$$\cos\theta = \frac{60}{65}$$
$$\cos\theta = \frac{12}{13}$$
$$\sin^2\theta = 1 - \cos^2\theta$$
$$= 1 - \left(\frac{12}{13}\right)^2$$
$$= 1 - \frac{144}{169}$$
$$= \frac{169 - 144}{169}$$
$$= \frac{25}{169}$$
$$\sin\ \theta = \pm\sqrt{\frac{25}{169}}$$
$$= \pm\frac{5}{13}$$

$$|\sin\theta| = \frac{5}{13}$$

We know that,

$$\vec{a}\times\vec{b} = |\vec{a}|.|\vec{b}|.\sin\theta.\hat{n}$$
$$|\vec{a}\times\vec{b}| = |\vec{a}|.|\vec{b}|.|\sin\theta|.|\hat{n}|$$
$$= 13.5.\frac{5}{13}.1 \qquad \left[\text{Since, } \hat{n} \text{ is a unit vector}\right]$$

$$|\vec{a}\times\vec{b}| = 25$$

Vector or Cross Product Ex 25.1 Q14

We know that, if θ be the angle between $\vec{a}$ and $\vec{b}$, then

$$\vec{a}.\vec{b} = |\vec{a}|\ |\vec{b}|\cos\theta \qquad ---(i)$$

And, $\vec{a}\times\vec{b} = |\vec{a}|\ |\vec{b}|.\sin\theta.\hat{n}$

$$|\vec{a}\times\vec{b}| = |\vec{a}|.|\vec{b}|.|\sin\theta|.|\hat{n}|$$
$$= |\vec{a}|\ |\vec{b}||\sin\theta|.1 \qquad \left[\text{Since, } \hat{n} \text{ is a unit vector}\right]$$

$$|\vec{a}\times\vec{b}| = |\vec{a}|\ |\vec{b}|\sin\theta \qquad ---(ii)$$

Given that, $|\vec{a}\times\vec{b}| = \vec{a}.\vec{b}$

$$|\vec{a}|\ |\vec{b}|\sin\theta = |\vec{a}|.|\vec{b}|\cos\theta$$
$$\sin\theta = \cos\theta$$

$$\theta = \frac{\pi}{4}$$

Angle between $\vec{a}$ and $\vec{b}$ = $\frac{\pi}{4}$

Vector or Cross Product Ex 25.1 Q15

We have,

$\vec{a} \times \vec{b} = \vec{b} \times \vec{c}$

$\left(\vec{a} \times \vec{b}\right) - \left(\vec{b} \times \vec{c}\right) = \vec{0}$

$\left(\vec{a} \times \vec{b}\right) + \left(\vec{c} \times \vec{b}\right) = \vec{0}$ $\quad$ $\left[\text{Since, } \left(\vec{b} \times \vec{c}\right) = -\left(\vec{c} \times \vec{b}\right)\right]$

$\left(\vec{a} + \vec{c}\right) \times \vec{b} = \vec{0}$ $\quad$ [Using distributive property]

We know that, if $\vec{a} \times \vec{b} = \vec{0}$, then vector $\vec{a}$ is parallel to vector $\vec{b}$.

Thus, $\left(\vec{a} + \vec{c}\right)$ is parallel to $\vec{b}$

$\left(\vec{a} + \vec{c}\right) = \overrightarrow{mb}$

Where m is any scalar

Vector or Cross Product Ex 25.1 Q16

We know that,

$\vec{a} \times \vec{b} = |\vec{a}|\,|\vec{b}| \sin\theta\ \hat{n}$

$|\vec{a} \times \vec{b}| = |\vec{a}|\ \ |\vec{b}|\ \ |\sin\theta|\ |\hat{n}|$

$\qquad = |\vec{a}|\ |\vec{b}| |\sin\theta|.1$ $\quad$ [as $\hat{n}$ is a unit vector]

$|\vec{a} \times \vec{b}| = |\vec{a}|\ |\vec{b}| |\sin\theta|$

$\sqrt{(3)^2 + (2)^2 + (6)^2} = 2.7.|\sin\theta|$

$\sqrt{9 + 4 + 36} = 14.|\sin\theta|$

$\sqrt{49} = 14\,|\sin\theta|$

$\sin\theta = \frac{7}{14}$

$\sin\theta = \frac{1}{2}$

$\theta = \sin^{-1}\left(\frac{1}{2}\right)$

$\theta = \frac{\pi}{6}$

Angle between $\vec{a}$ and $\vec{b} = \frac{\pi}{6}$

Vector or Cross Product Ex 25.1 Q17

Given that $\vec{a} \times \vec{b} = \vec{0}$

This gives us four conclusions about $\vec{a}$ and $\vec{b}$

(i) $\vec{a} = \vec{0}$ or

(ii) $\vec{b} = \vec{0}$ or

(iii) $\vec{a} = \vec{b} = \vec{0}$ or

(iv) $\vec{a}$ is parallel to $\vec{b}$.

Also, it is given that $\vec{a}.\vec{b} = 0$

This also gives us four canclusions about $\vec{a}$ and $\vec{b}$.

(i) $\vec{a} = \vec{0}$ or

(ii) $\vec{b} = \vec{0}$ or

(iii) $\vec{a} = \vec{b} = \vec{0}$ or

(iv) $\vec{a}$ is perpendicular to $\vec{b}$.

Now,

$\vec{a}$ parallel $\vec{b}$ and $\vec{a}$ is perpendicular to $\vec{b}$ are not possible simultaneously.

So,

$\vec{a} = 0$ or $\vec{b} = 0$ or $\vec{a} = \vec{b} = \vec{0}$

Vector or Cross Product Ex 25.1 Q18

Given that $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors such that

$\vec{a} \times \vec{b} = \vec{c},\ \vec{b} \times \vec{c} = \vec{a},\ \vec{c} \times \vec{a} = \vec{b},$

$\vec{a} \times \vec{b} = \vec{c}$

$\Rightarrow \quad \vec{c}$ is a vector perpendicular to both $\vec{a}$ and $\vec{b}$ $\qquad ---(i)$

$\vec{b} \times \vec{c} = \vec{a}$

$\Rightarrow \quad \vec{a}$ is a vector perpendicular to $\vec{b}$ and $\vec{c}$ $\qquad ---(ii)$

$\vec{c} \times \vec{a} = \vec{b}$

$\Rightarrow \quad \vec{b}$ is a vector perpendicular to $\vec{a}$ and $\vec{c}$ $\qquad ---(iii)$

Using $(i), (ii)$ and (iii), we can see that $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular unit vectors.

Since, $\vec{a} \times \vec{b} = \vec{c}$
$\vec{b} \times \vec{c} = \vec{a}$
$\vec{c} \times \vec{a} = \vec{b}$

Therefore,
$\vec{a}, \vec{b}, \vec{c}$ form an orthonormal right handed traid of unit vectors.

Vector or Cross Product Ex 25.1 Q19

Here, Position vector of $A = \left(3\hat{i} - \hat{j} + 2\hat{k}\right)$

Position vector of $B = \left(\hat{i} - \hat{j} - 3\hat{k}\right)$

Position vector of $C = \left(4\hat{i} - 3\hat{j} + \hat{k}\right)$

$$\begin{aligned}\overrightarrow{AB} &= \vec{B} - \vec{A} \\ &= \left(\hat{i} - \hat{j} - 3\hat{k}\right) - \left(3\hat{i} - \hat{j} + 2\hat{k}\right) \\ &= \hat{i} - \hat{j} - 3\hat{k} - 3\hat{i} + \hat{j} - 2\hat{k} \\ &= -2\hat{i} - 5\hat{k}\end{aligned}$$

$$\begin{aligned}\overrightarrow{AC} &= \vec{C} - \vec{A} \\ &= \left(4\hat{i} - 3\hat{j} + \hat{k}\right) - \left(3\hat{i} - \hat{j} + 2\hat{k}\right) \\ &= 4\hat{i} - 3\hat{j} + \hat{k} - 3\hat{i} + \hat{j} - 2\hat{k} \\ &= \hat{i} - 2\hat{j} - \hat{k}\end{aligned}$$

Vector perpendicular to the plane ABC

$$= \overrightarrow{AC} \times \overrightarrow{AB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -1 \\ -2 & 0 & -5 \end{vmatrix}$$

$$\begin{aligned}\overrightarrow{AC} \times \overrightarrow{AB} &= \hat{i}\left(10 - 0\right) - \hat{j}\left(-5 - 2\right) + \hat{k}\left(0 - 4\right) \\ &= 10\hat{i} + 7\hat{j} - 4\hat{k}\end{aligned}$$

$$\begin{aligned}\left|\overrightarrow{AC} \times \overrightarrow{AB}\right| &= \sqrt{(10)^2 + (7)^2 + (-4)^2} \\ &= \sqrt{100 + 49 + 16} \\ &= \sqrt{165}\end{aligned}$$

Therefore, unit vector perpendicular to the plane $ABC = \dfrac{\overrightarrow{AC} \times \overrightarrow{AB}}{\left|\overrightarrow{AC} \times \overrightarrow{AB}\right|}$

$$= \frac{1}{\sqrt{165}}\left(10\hat{i} + 7\hat{j} - 4\hat{k}\right)$$

Unit vector perpendicular to the plane $ABC = \dfrac{1}{\sqrt{165}}\left(10\hat{i} + 7\hat{j} - 4\hat{k}\right)$

Vector or Cross Product Ex 25.1 Q20

Here, It is given that

In $\triangle ABC$

$\overrightarrow{BC} + \overrightarrow{CA} + \overrightarrow{AB}$
$= \overrightarrow{BA} + \overrightarrow{AB}$
$= \overrightarrow{BA} - \overrightarrow{BA}$ $\quad\left[\text{Since, } \overrightarrow{BA} = -\overrightarrow{AB}\right]$

$= \vec{0}$

Given that, $\left|\overrightarrow{BC}\right| = a$
$\left|\overrightarrow{CA}\right| = b$
$\left|\overrightarrow{AB}\right| = c$

Let, $\overrightarrow{BC} = \vec{a}$, $\overrightarrow{CA} = \vec{b}$ and $\overrightarrow{AB} = \vec{c}$

We have,

$$\overrightarrow{BC} + \overrightarrow{CA} + \overrightarrow{AB} = \vec{0}$$

$$\Rightarrow \overrightarrow{BC} + \overrightarrow{CA} = -\overrightarrow{AB}$$
$$\Rightarrow \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{BA}$$
$$\Rightarrow \vec{a} + \vec{b} = -\vec{c}$$
$$\Rightarrow \vec{a} + \vec{b} + \vec{c} = \vec{0}$$
$$\Rightarrow \vec{a} \times \left(\vec{a} + \vec{b} + \vec{c}\right) = \vec{a} \times \vec{0}$$
$$\Rightarrow \vec{a} \times \vec{a} + \vec{a} \times \vec{b} + \vec{a} \times \vec{c} = \vec{0}$$
$$\Rightarrow \vec{0} + \vec{a} \times \vec{b} + \vec{a} \times \vec{c} = \vec{0} \qquad \left[\text{Since, } \vec{a} \times \vec{a} = \vec{0}\right]$$
$$\Rightarrow \vec{a} \times \vec{b} = -\left(\vec{a} \times \vec{c}\right)$$

$$\Rightarrow \vec{a} \times \vec{b} = \vec{c} \times \vec{a} \qquad ---(i)$$

Again, $\vec{a} + \vec{b} + \vec{c} = \vec{0}$

$$\Rightarrow \vec{b} \times \left(\vec{a} + \vec{b} + \vec{c}\right) = \vec{b} \times \vec{0}$$
$$\Rightarrow \vec{b} \times \vec{a} + \vec{b} \times \vec{b} + \vec{b} \times \vec{c} = \vec{0}$$
$$\Rightarrow \vec{b} \times \vec{a} + \vec{0} + \vec{b} \times \vec{c} = \vec{0} \qquad \left[\text{Since, } \vec{b} \times \vec{b} = \vec{0}\right]$$
$$\Rightarrow \vec{b} \times \vec{a} + \vec{b} \times \vec{c} = \vec{0}$$
$$\Rightarrow \vec{b} \times \vec{c} = -\left(\vec{b} \times \vec{a}\right)$$

$$\Rightarrow \vec{b} \times \vec{c} = \vec{a} \times \vec{b} \qquad ---(ii)$$

From equation (i) and (ii), we get

$$\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$$

$$\Rightarrow \left|\vec{a} \times \vec{b}\right| = \left|\vec{b} \times \vec{c}\right| = \left|\vec{c} \times \vec{a}\right|$$
$$\Rightarrow \left|\vec{a}\right| \left|\vec{b}\right| \sin\left(\pi - C\right) = \left|\vec{b}\right| \left|\vec{c}\right| \sin\left(\pi - A\right) = \left|\vec{c}\right| \left|\vec{a}\right| \sin\left(\pi - B\right)$$
$$\Rightarrow ab \sin C = bc \sin A = ca \sin B$$

Dividing by abc

$$\Rightarrow \frac{ab \sin C}{abc} = \frac{bc \sin A}{abc} = \frac{ca \sin B}{abc}$$
$$\Rightarrow \frac{\sin C}{c} = \frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\Rightarrow \frac{c}{\sin C} = \frac{a}{\sin A} = \frac{b}{\sin B}$$

Vector or Cross Product Ex 25.1 Q21

Here, $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - 5\hat{k}$,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & 3 & -5 \end{vmatrix}$$

$$= \hat{i}(10-9) - \hat{j}(-5-6) + \hat{k}(3+4)$$

$$= \hat{i} + 11\hat{j} + 7\hat{k}$$

Now, $\vec{a}.(\vec{a} \times \vec{b}) = (\hat{i} - 2\hat{j} + 3\hat{k}).(\hat{i} + 11\hat{j} + 7\hat{k})$

$$\vec{a}.(\vec{a} \times \vec{b}) = (1)(1) + (-2)(11) + (3)(7)$$
$$= 1 - 22 + 21$$
$$= 22 - 22$$

$$\vec{a} \times (\vec{a} \times \vec{b}) = 0$$

Dot product of $\vec{a}$ and $\vec{a} \times \vec{b}$ is zero, then,
$\vec{a}$ is perpendicular to $(\vec{a} \times \vec{b})$

Vector or Cross Product Ex 25.1 Q22

Given $\vec{p}$ and $\vec{q}$ be unit vector with angle $30°$ between then

$$|\vec{p}| = |\vec{q}| = 1$$

$$\vec{p} \times \vec{q} = |\vec{p}|\,|\vec{q}| \sin 30° \hat{n}$$
$$= 1.1.\left(\frac{1}{2}\right)\hat{n}$$

$$|\vec{p} \times \vec{q}| = \left|\frac{\hat{n}}{2}\right|$$

$$|\vec{p} \times \vec{q}| = \frac{1}{2} \qquad ---(i) \qquad \left[\text{Since, } \hat{n} \text{ is a unit vector}\right]$$

$$\text{Area of parallelogram} = \frac{1}{2}|\vec{a} \times \vec{b}|$$
$$= \frac{1}{2}\left|(\vec{p} + 2\vec{q}) \times (2\vec{p} + \vec{q})\right|$$
$$= \frac{1}{2}\left|\vec{p} \times 2\vec{p} + \vec{p} \times \vec{q} + 2\vec{q} \times 2\vec{p} + 2\vec{q} \times \vec{q}\right|$$
$$= \frac{1}{2}\left|\vec{0} + \vec{p} \times \vec{q} + 2\vec{q} \times 2\vec{p} + \vec{0}\right| \qquad \left[\text{Since, } \vec{p} \times 2\vec{q} = \vec{0} \text{ and } 2\vec{q} \times \vec{q} = \vec{0}\right]$$
$$= \frac{1}{2}\left|\vec{p} \times \vec{q} + 4(\vec{q} \times \vec{p})\right|$$
$$= \frac{1}{2}\left|(\vec{p} \times \vec{q}) - 4(\vec{p} \times \vec{q})\right| \qquad \left[\text{Since, } \vec{q} \times \vec{p} = -\vec{p} \times \vec{q}\right]$$
$$= \frac{1}{2}\left|-3(\vec{p} \times \vec{q})\right|$$
$$= \frac{3}{2}|\vec{p} \times \vec{q}|$$
$$= \frac{3}{2} \times \frac{1}{2} \qquad \left[\text{Using (i)}\right]$$
$$= \frac{3}{4}$$

$$\text{Area of parallelogram} = \frac{3}{4} \text{ sq. unit}$$

Vector or Cross Product Ex 25.1 Q23

We know that

$$\vec{a}\times\vec{b} = \left|\vec{a}\right|.\left|\vec{b}\right|.\sin\theta.\hat{n}$$

$$\left|\vec{a}\times\vec{b}\right| = \left|\vec{a}\right|\left|\vec{b}\right|\sin\theta.\left|\hat{n}\right|$$

$$= \left|\vec{a}\right|\left|\vec{b}\right|\sin\theta.1 \qquad \left[\text{Since, } \hat{n} \text{ is unit vector}\right]$$

$$\left|\vec{a}\times\vec{b}\right| = \left|\vec{a}\right|\left|\vec{b}\right|\sin\theta$$

Squaring both the sides,

$$\left|\vec{a}\times\vec{b}\right|^2 = \left|\vec{a}\right|^2\left|\vec{b}\right|^2\sin^2\theta$$

$$= \left|\vec{a}\right|^2\left|\vec{b}\right|^2\left(1-\cos^2\theta\right)$$

$$= \left|\vec{a}\right|^2\left|\vec{b}\right|^2 - \left|\vec{a}\right|^2\left|\vec{b}\right|^2\cos^2\theta$$

$$= \left|\vec{a}\right|^2\left|\vec{b}\right|^2 - \left(\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta\right)^2$$

$$= \left(\vec{a}\right)^2\left(\vec{b}\right)^2 - \left(\vec{a}.\vec{b}\right)^2 \qquad \left[\text{Since, } \left|\vec{a}\right|\left|\vec{b}\right|\cos\theta = \vec{a}.\vec{b}\right]$$

$$\left|\vec{a}\times\vec{b}\right|^2 = \left(\vec{a}.\vec{a}\right)\left(\vec{b}.\vec{b}\right) - \left(\vec{a}.\vec{b}\right)\left(\vec{b}.\vec{a}\right) \qquad \left[\text{Since, } \left(\vec{a}.\vec{b}\right) = \left(\vec{b}.\vec{a}\right)\right]$$

$$\left|\vec{a}\times\vec{b}\right|^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} \\ \vec{b}.\vec{a} & \vec{b}\vec{b} \end{vmatrix}$$

Vector or Cross Product Ex 25.1 Q24

Define of $\vec{a}\times\vec{b}$: – Let $\vec{a}$, $\vec{b}$ be two non-zero, non-parallel vectors. Then $\vec{a}\times\vec{b}$, in that order, is defined as a vector whose magnitude is $\left|\vec{a}\right|\ \left|\vec{b}\right|\sin\theta$, where θ is the angle between $\vec{a}$ and $\vec{b}$ and whose direction is perpendicular to the plane of $\vec{a}$ and $\vec{b}$ and this constitute a right handed system.

$$\vec{a}\times\vec{b} = \left|\vec{a}\right|\ \left|\vec{b}\right|\sin\theta\hat{n}$$

Where $\hat{n}$ is a unit vector perpendicular to the plane of $\vec{a}$ and $\vec{b}$ such that $\vec{a},\vec{b},\hat{n}$ form a right handed system.

Now,

$$\vec{a}\times\vec{b} = \left|\vec{a}\right|\ \left|\vec{b}\right|\sin\theta\ \hat{n}$$

$$\left|\vec{a}\times\vec{b}\right| = \left|\vec{a}\right|\ \left|\vec{b}\right|\left|\sin\theta\right|\ \left|\hat{n}\right|$$

$$= \left|\vec{a}\right|\ \left|\vec{b}\right|\left|\sin\theta\right|.1$$

$$\left|\vec{a}\times\vec{b}\right| = \left|\vec{a}\right|\ \left|\vec{b}\right|\left|\sin\theta\right|$$

$$= \frac{\vec{a}.\vec{b}}{\cos\theta}.\sin\theta \qquad \left[\text{Since, } \cos\theta = \frac{\vec{a}.\vec{b}}{\left|\vec{a}\right|\left|\vec{b}\right|}\right]$$

$$\left|\vec{a}\times\vec{b}\right| = \vec{a}.\vec{b}.\tan\theta$$

Vector or Cross Product Ex 25.1 Q25

$\vec{a} \times \vec{b} = |\vec{a}|\ |\vec{b}| \sin\theta . \hat{n}$

$|\vec{a} \times \vec{b}| = |\vec{a}|\ |\vec{b}|\ |\sin\theta|\ |\hat{n}|$

$35 = \sqrt{26}.7\ |\sin\theta|.1$

$\sin\theta = \dfrac{35}{\sqrt{26}.7}$

$\sin\theta = \dfrac{5}{\sqrt{26}}$

$\cos^2\theta = 1 - \sin^2\theta$

$= 1 - \left(\dfrac{5}{\sqrt{26}}\right)^2$

$= \dfrac{1}{1} - \dfrac{25}{26}$

$= \dfrac{26 - 25}{26}$

$= \dfrac{1}{26}$

$\cos\theta = \dfrac{1}{\sqrt{26}}$

$\vec{a}.\vec{b} = |\vec{a}|\ |\vec{b}| \cos\theta$

$= \sqrt{26}.7.\dfrac{1}{\sqrt{26}}$

$\vec{a}.\vec{b} = 7$

Vector or Cross Product Ex 25.1 Q26

Area of triangle $= \dfrac{1}{2}\left|\overrightarrow{OA} \times \overrightarrow{OB}\right|$

$\left|\overrightarrow{OA} \times \overrightarrow{OB}\right| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -3 & -2 & 1 \end{vmatrix}$

$= \hat{i}(2 + 6) - \hat{j}(1 + 9) + \hat{k}(-2 + 6)$

$= 8\hat{i} - 10\hat{j} + 4\hat{k}$

$= 2\left(4\hat{i} - 5\hat{j} + 2\hat{k}\right)$

Area of triangle $= \dfrac{1}{2}\left|\overrightarrow{OA} \times \overrightarrow{OB}\right|$

$= \dfrac{1}{2}\left[2\sqrt{(4)^2 + (-5)^2 + (2)^2}\right]$

$= \dfrac{1}{2}\left[2\sqrt{16 + 25 + 4}\right]$

$= \sqrt{45}$

$= 3\sqrt{5}$

Area of triangle $= 3\sqrt{5}$ Sq.unit

Vector or Cross Product Ex 25.1 Q27

Let $\vec{d} = d_1\hat{i} + d_2\hat{j} + d_3\hat{k}$.

Since $\vec{d}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, we have:

$$\vec{d} \cdot \vec{a} = 0$$
$$\Rightarrow d_1 + 4d_2 + 2d_3 = 0 \qquad \text{...(i)}$$

And,

$$\vec{d} \cdot \vec{b} = 0$$
$$\Rightarrow 3d_1 - 2d_2 + 7d_3 = 0 \qquad \text{...(ii)}$$

Also, it is given that:

$$\vec{c} \cdot \vec{d} = 15$$
$$\Rightarrow 2d_1 - d_2 + 4d_3 = 15 \qquad \text{...(iii)}$$

On solving (i), (ii), and (iii), we get:

$$d_1 = \frac{160}{3}, d_2 = -\frac{5}{3} \text{ and } d_3 = -\frac{70}{3}$$
$$\therefore \vec{d} = \frac{160}{3}\hat{i} - \frac{5}{3}\hat{j} - \frac{70}{3}\hat{k} = \frac{1}{3}\left(160\hat{i} - 5\hat{j} - 70\hat{k}\right)$$

Hence, the required vector is $\frac{1}{3}\left(160\hat{i} - 5\hat{j} - 70\hat{k}\right)$.

Vector or Cross Product Ex 25.1 Q28

Given, $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$
$\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

Let, $\vec{d} = \vec{a} + \vec{b}$

$$= \left(3\hat{i} + 2\hat{j} + 2\hat{k}\right) + \left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$$
$$\vec{d} = 4\hat{i} + 4\hat{j} - 0\,\hat{k}$$

And, $\vec{e} = \vec{a} - \vec{b}$

$$= \left(3\hat{i} + 2\hat{j} + 2\hat{k}\right) - \left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$$
$$\vec{e} = 2\hat{i} + 4\hat{k}$$

Let, $\vec{f}$ be any vector perpendicular to both $\vec{d}$ and $\vec{e}$

$$\vec{f} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix}$$

$$= \hat{i}\left(16 - 0\right) - \hat{j}\left(16 - 0\right) + \hat{k}\left(0 - 8\right)$$
$$\vec{f} = 16\hat{i} - 16\hat{j} - 8\hat{k}$$

$$= 8\left(2\hat{i} - 2\hat{j} - \hat{k}\right)$$

Let $\vec{g}$ be the required vector, then

$$\vec{g} = \lambda\vec{f} \qquad \text{and} \qquad \left|\vec{g}\right| = 1$$
$$\vec{g} = 8\lambda\left(2\hat{i} - 2\hat{j} - \hat{k}\right) \qquad ---(i)$$

$$\left|\vec{g}\right| = 1$$

$$8\lambda\sqrt{(2)^2 + (-2)^2 + (-1)^2} = 1$$
$$8\lambda\sqrt{4 + 4 + 1} = 1$$
$$8\lambda\sqrt{9} = 1$$
$$24\lambda = 1$$

$$\lambda = \frac{1}{24}$$

Put λ in (i)

$$\vec{g} = 8\left(\frac{1}{24}\right)\left(2\hat{i} - 2\hat{j} - \hat{k}\right)$$
$$\vec{g} = \frac{1}{3}\left(2\hat{i} - 2\hat{j} - \hat{k}\right)$$

Thus,

Unit vector perpendicular to $\left(\vec{a} + \vec{b}\right)$ and $\left(\vec{a} - \vec{b}\right) = \frac{1}{3}\left(2\hat{i} - 2\hat{j} - \hat{k}\right)$

Vector or Cross Product Ex 25.1 Q29

Given, $A = (2, 3, 5)$
$B = (3, 5, 8)$
$C = (2, 7, 8)$

Position vector of $A = 2\hat{i} + 3\hat{j} + 5\hat{k}$
Position vector of $B = 3\hat{i} + 5\hat{j} + 8\hat{k}$
Position vector of $C = 2\hat{i} + 7\hat{j} + 8\hat{k}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$$= \left(3\hat{i} + 5\hat{j} + 8\hat{k}\right) - \left(2\hat{i} + 3\hat{j} + 5\hat{k}\right)$$
$$= 3\hat{i} + 5\hat{j} + 8\hat{k} - 2\hat{i} - 3\hat{j} - 5\hat{k}$$

$$\overrightarrow{AB} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$\overrightarrow{AC}$ = Position vector of C – Position vector of A

$$= \left(2\hat{i} + 7\hat{j} + 8\hat{k}\right) - \left(2\hat{i} + 3\hat{j} + 5\hat{k}\right)$$
$$= 2\hat{i} + 7\hat{j} + 8\hat{k} - 2\hat{i} - 3\hat{j} - 5\hat{k}$$

$$\overrightarrow{AC} = 4\hat{j} + 3\hat{k}$$

Area of triangle $= \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 0 & 4 & 3 \end{vmatrix}$$

$$= \hat{i}(6 - 12) - \hat{j}(3 - 0) + \hat{k}(4 - 0)$$
$$\overrightarrow{AB} \times \overrightarrow{AC} = -6\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \sqrt{(-6)^2 + (-3)^2 + (4)^2}$$
$$= \sqrt{36 + 9 + 16}$$
$$= \sqrt{61}$$

Area of triangle $= \frac{1}{2}\sqrt{61}$ Sq. unit

Vector or Cross Product Ex 25.1 Q30

Let $\vec{d} = d_1\hat{i} + d_2\hat{j} + d_3\hat{k}$.

Since $\vec{d}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, we have:

$$\vec{d} \cdot \vec{a} = 0$$
$$\Rightarrow d_1 + 4d_2 + 2d_3 = 0 \qquad \text{...(i)}$$

And,

$$\vec{d} \cdot \vec{b} = 0$$
$$\Rightarrow 3d_1 - 2d_2 + 7d_3 = 0 \qquad \text{...(ii)}$$

Also, it is given that:

$$\vec{c} \cdot \vec{d} = 15$$
$$\Rightarrow 2d_1 - d_2 + 4d_3 = 15 \qquad \text{...(iii)}$$

On solving (i), (ii), and (iii), we get:

$$d_1 = \frac{160}{3}, d_2 = -\frac{5}{3} \text{ and } d_3 = -\frac{70}{3}$$
$$\therefore \vec{d} = \frac{160}{3}\hat{i} - \frac{5}{3}\hat{j} - \frac{70}{3}\hat{k} = \frac{1}{3}\left(160\hat{i} - 5\hat{j} - 70\hat{k}\right)$$

Hence, the required vector is $\frac{1}{3}\left(160\hat{i} - 5\hat{j} - 70\hat{k}\right)$.

Vector or Cross Product Ex 25.1 Q31

$$\vec{a} \cdot \vec{b} = 0$$

Then,

(i) Either $|\vec{a}| = 0$ or $|\vec{b}| = 0$, or $\vec{a} \perp \vec{b}$ $\left(\text{in case } \vec{a} \text{ and } \vec{b} \text{ are non-zero}\right)$

$$\vec{a} \times \vec{b} = 0$$

(ii) Either $|\vec{a}| = 0$ or $|\vec{b}| = 0$, or $\vec{a} \parallel \vec{b}$ $\left(\text{in case } \vec{a} \text{ and } \vec{b} \text{ are non-zero}\right)$

But, $\vec{a}$ and $\vec{b}$ cannot be perpendicular and parallel simultaneously.

Hence, $|\vec{a}| = 0$ or $|\vec{b}| = 0$.

Vector or Cross Product Ex 25.1 Q32

Take any parallel non-zero vectors so that $\vec{a} \times \vec{b} = \vec{0}$.

Let $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{b} = 4\hat{i} + 6\hat{j} + 8\hat{k}$.

Then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 4 & 6 & 8 \end{vmatrix} = \hat{i}(24-24) - \hat{j}(16-16) + \hat{k}(12-12) = 0\hat{i} + 0\hat{j} + 0\hat{k} = \vec{0}$$

It can now be observed that:

$$|\vec{a}| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$$
$$\therefore \vec{a} \neq \vec{0}$$
$$|\vec{b}| = \sqrt{4^2 + 6^2 + 8^2} = \sqrt{116}$$
$$\therefore \vec{b} \neq \vec{0}$$

Hence, the converse of the given statement need not be true.

Vector or Cross Product Ex 25.1 Q33

We have,

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k},\ \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k},\ \vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

$$(\vec{b}+\vec{c}) = (b_1+c_1)\hat{i} + (b_2+c_2)\hat{j} + (b_3+c_3)\hat{k}$$

$$\text{Now, } \vec{a}\times(\vec{b}+\vec{c})\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1+c_1 & b_2+c_2 & b_3+c_3 \end{vmatrix}$$

$$= \hat{i}\left[a_2(b_3+c_3) - a_3(b_2+c_2)\right] - \hat{j}\left[a_1(b_3+c_3) - a_3(b_1+c_1)\right] + \hat{k}\left[a_1(b_2+c_2) - a_2(b_1+c_1)\right]$$

$$= \hat{i}\left[a_2b_3 + a_2c_3 - a_3b_2 - a_3c_2\right] + \hat{j}\left[-a_1b_3 - a_1c_3 + a_3b_1 + a_3c_1\right] + \hat{k}\left[a_1b_2 + a_1c_2 - a_2b_1 - a_2c_1\right] \quad ...(1)$$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= \hat{i}\left[a_2b_3 - a_3b_2\right] + \hat{j}\left[b_1a_3 - a_1b_3\right] + \hat{k}\left[a_1b_2 - a_2b_1\right] \qquad (2)$$

$$\vec{a}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \hat{i}\left[a_2c_3 - a_3c_2\right] + \hat{j}\left[a_3c_1 - a_1c_3\right] + \hat{k}\left[a_1c_2 - a_2c_1\right] \qquad (3)$$

Vector or Cross Product Ex 25.1 Q34

Given that

$A = (1,1,2)$

$B = (2,3,5)$

$C = (1,5,5)$

Position vector of $A = \hat{i} + \hat{j} + 2\hat{k}$

Position vector of $B = 2\hat{i} + 3\hat{j} + 5\hat{k}$

Position vector of $C = \hat{i} + 5\hat{j} + 5\hat{k}$

$\overrightarrow{AB}$ = Position vector of B − Position vector of A

$= 2\hat{i} + 3\hat{j} + 5\hat{k} - (\hat{i} + \hat{j} + 2\hat{k})$

$= \hat{i} + 2\hat{j} + 3\hat{k}$

$\overrightarrow{AC}$ = Position vector of C − Position vector of A

$= \hat{i} + 5\hat{j} + 5\hat{k} - (\hat{i} + \hat{j} + 2\hat{k})$

$= 4\hat{j} + 3\hat{k}$

Area of triangle $= \frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|$

$$\overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 0 & 4 & 3 \end{vmatrix}$$

$= \hat{i}(6-12) - \hat{j}(3-0) + \hat{k}(4-0)$

$= -6\hat{i} - 3\hat{j} + 4\hat{k}$

$\left|\overrightarrow{AB}\times\overrightarrow{AC}\right| = \sqrt{(-6)^2 + (-3)^2 + 4^2}$

$= \sqrt{36+9+16}$

$= \sqrt{61}$

Area of the triangle $= \frac{1}{2}\sqrt{61}$ Sq.unit

Vector or Cross Product Ex 25.1 Q35

Let

$$\vec{a} = 2\hat{i} - 4\hat{j} + 5\hat{k}$$
$$\vec{b} = \hat{i} - 2\hat{j} - 3\hat{k}$$

The unit vector parallel to one of its diagonals is $\dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$.

Now

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 5 \\ 1 & -2 & -3 \end{vmatrix}$$
$$= \hat{i}(12+10) - \hat{j}(-6-5) + \hat{k}(-4+4)$$
$$= 22\hat{i} + 11\hat{j}$$
$$= 11(2\hat{i} + \hat{j})$$

$$|\vec{a}\times\vec{b}| = 11\sqrt{2^2+1^2}$$
$$= 11\sqrt{5}$$

Therefore

$$\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} = \frac{11(2\hat{i}+\hat{j})}{11\sqrt{5}}$$
$$= \frac{1}{\sqrt{5}}(2\hat{i}+\hat{j})$$

The unit vector parallel to one of its diagonals is $\frac{1}{\sqrt{5}}(2\hat{i}+\hat{j})$.

Again, the area of the parallelogram is $|\vec{a}\times\vec{b}| = 11\sqrt{5}$ Sq. unit

Ex 26.1

Scalar Triple Product Ex 26.1 Q1(i)

We have

$$\left[\hat{i}\,\hat{j}\,\hat{k}\right]+\left[\hat{j}\,\hat{k}\,\hat{i}\right]+\left[\hat{k}\,\hat{i}\,\hat{j}\right]=\left(\hat{i}\times\hat{j}\right)\cdot\hat{k}+\left(\hat{j}\times\hat{k}\right)\cdot\hat{i}+\left(\hat{k}\times\hat{i}\right)\cdot\hat{j}$$
$$=\hat{k}\cdot\hat{k}+\hat{i}\cdot\hat{i}+\hat{j}\cdot\hat{j}$$
$$=1+1+1$$
$$=3$$

Therefore, $\left[\hat{i}\,\hat{j}\,\hat{k}\right]+\left[\hat{j}\,\hat{k}\,\hat{i}\right]+\left[\hat{k}\,\hat{i}\,\hat{j}\right]=3$

Scalar Triple Product Ex 26.1 Q1(ii)

We have

$$\left[2\hat{i}\,\hat{j}\,\hat{k}\right]+\left[\hat{i}\,\hat{k}\,\hat{j}\right]+\left[\hat{k}\,\hat{j}\,2\hat{i}\right]=\left(2\hat{i}\times\hat{j}\right)\cdot\hat{k}+\left(\hat{i}\times\hat{k}\right)\cdot\hat{j}+\left(\hat{k}\times\hat{j}\right)\cdot 2\hat{i}$$
$$=2\hat{k}\cdot\hat{k}+\left(-\hat{j}\right)\cdot\hat{j}+\left(-\hat{i}\right)\cdot 2\hat{i}$$
$$=2-1-2$$
$$=-1$$

Therefore, $\left[2\hat{i}\,\hat{j}\,\hat{k}\right]+\left[\hat{i}\,\hat{k}\,\hat{j}\right]+\left[\hat{k}\,\hat{j}\,2\hat{i}\right]=-1$

Scalar Triple Product Ex 26.1 Q2(i)

We have

$$\left[\vec{a}\,\vec{b}\,\vec{c}\right]=\begin{vmatrix}2 & -3 & 0\\ 1 & 1 & -1\\ 3 & 0 & -1\end{vmatrix}$$
$$=2(-1-0)+3(-1+3)$$
$$=-2+6$$
$$=4$$

Therefore, $\left[\vec{a}\,\vec{b}\,\vec{c}\right]=4$

Scalar Triple Product Ex 26.1 Q2(ii)

We have

$$\left[\vec{a}\,\vec{b}\,\vec{c}\right]=\begin{vmatrix}1 & -2 & 3\\ 2 & 1 & -1\\ 0 & 1 & 1\end{vmatrix}$$
$$=1(1+1)+2(2+0)+3(2-0)$$
$$=2+4+6$$
$$=12$$

Therefore, $\left[\vec{a}\,\vec{b}\,\vec{c}\right]=12$

Scalar Triple Product Ex 26.1 Q3(i)

We know that the volume of a parallelepiped whose three adjacent edges are $\vec{a},\vec{b},\vec{c}$ is equal to $\left|\left[\vec{a}\,\vec{b}\,\vec{c}\right]\right|$.

We have

$$\left[\vec{a}\,\vec{b}\,\vec{c}\right]=\begin{vmatrix}2 & 3 & 4\\ 1 & 2 & -1\\ 3 & -1 & 2\end{vmatrix}$$
$$=2(4-1)-3(2+3)+4(-1-6)$$
$$=6-15-28$$
$$=-9-28$$
$$=-37$$

Therefore, the volume of the parallelepiped is $\left|\left[\vec{a}\,\vec{b}\,\vec{c}\right]\right|=|-37|=37$ cubic unit.

Scalar Triple Product Ex 26.1 Q3(ii)

Let $\vec{a}=2\hat{i}-3\hat{j}+4\hat{k},\vec{b}=\hat{i}+2\hat{j}-\hat{k},\vec{c}=3\hat{i}-\hat{j}-2\hat{k}$

We know that the volume of a parallelepiped whose three adjacent edges are $\vec{a},\vec{b},\vec{c}$ is equal to $\left|\left[\vec{a}\,\vec{b}\,\vec{c}\right]\right|$.

We have

$$\left[\vec{a}\,\vec{b}\,\vec{c}\right]=\begin{vmatrix}2 & -3 & 4\\ 1 & 2 & -1\\ 3 & -1 & -2\end{vmatrix}$$
$$=2(-4-1)+3(-2+3)+4(-1-6)$$
$$=-10+3-28$$
$$=-10-25$$
$$=-35$$

Therefore, the volume of the parallelepiped is $\left|\left[\vec{a}\,\vec{b}\,\vec{c}\right]\right|=|-35|=35$ cubic unit.

Scalar Triple Product Ex 26.1 Q3(iii)

Let $\vec{a}=11\hat{i}, \vec{b}=2\hat{j}, \vec{c}=13\hat{k}$

We know that the volume of a parallelepiped whose three adjacent edges are $\vec{a},\vec{b},\vec{c}$ is equal to $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|$.

We have

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\begin{vmatrix} 11 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 13 \end{vmatrix}$$

$$=11(26-0)+0+0$$

$$=286$$

Therefore, the volume of the parallelepiped is $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|=|286|=286$ cubic unit.

Scalar Triple Product Ex 26.1 Q1(iv)

Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}-\hat{j}+\hat{k}, \vec{c}=\hat{i}+2\hat{j}-\hat{k}$

We know that the volume of a parallelepiped whose three adjacent edges are $\vec{a},\vec{b},\vec{c}$ is equal to $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|$.

We have

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 2 & -1 \end{vmatrix}$$

$$=1(1-2)-1(-1-1)+1(2+1)$$

$$=-1+2+3$$

$$=4$$

Therefore, the volume of the parallelepiped is $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|=|4|=4$ cubic unit.

Scalar Triple Product Ex 26.1 Q4(i)

We know that three vectors $\vec{a},\vec{b},\vec{c}$ are coplanar iff their scalar triple product is zero i.e. $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$.

Here,

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\begin{vmatrix} 1 & 2 & -1 \\ 3 & 2 & 7 \\ 5 & 6 & 5 \end{vmatrix}$$

$$=1(10-42)-2(15-35)-1(18-10)$$

$$=-32+40-8$$

$$=0$$

Hence, the given vectors are coplanar.

Scalar Triple Product Ex 26.1 Q4(ii)

We know that three vectors $\vec{a},\vec{b},\vec{c}$ are coplanar iff their scalar triple product is zero i.e. $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$.

Here,

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix}$$

$$=-4(12+3)+6(-3+24)-2(1+32)$$

$$=-60+126-66$$

$$=0$$

Hence, the given vectors are coplanar.

Scalar Triple Product Ex 26.1 Q5(i)

We know that vectors $\vec{a},\vec{b},\vec{c}$ are coplanar iff $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$.

$\therefore \vec{a},\vec{b},\vec{c}$ are coplanar

$$\Rightarrow \left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$$

$$\begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ \lambda & -1 & \lambda \end{vmatrix}=0$$

$$=1(\lambda-1)+1(2\lambda+\lambda)+1(-2-\lambda)$$

$$=\lambda-1+3\lambda-2-\lambda$$

$$3=3\lambda$$

$$1=\lambda$$

Scalar Triple Product Ex 26.1 Q5(ii)

We know that vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar iff $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$.

$\therefore \vec{a}, \vec{b}, \vec{c}$ are coplanar

$$\Rightarrow \left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$$

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ \lambda & \lambda & 5 \end{vmatrix}=0$$

$$=2(10+3\lambda)+1(5+3\lambda)+1(\lambda-2\lambda)$$
$$=20+6\lambda+5+3\lambda-\lambda$$
$$-25=8\lambda$$
$$\lambda=-\frac{25}{8}$$

Scalar Triple Product Ex 26.1 Q5(iii)

We know that vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar iff $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$.

$\therefore \vec{a}, \vec{b}, \vec{c}$ are coplanar

$$\Rightarrow \left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$$

$$\begin{vmatrix} 1 & 2 & -3 \\ 3 & \lambda & 1 \\ 1 & 2 & 2 \end{vmatrix}=0$$

$$=1(2\lambda-2)-2(6-1)-3(6-\lambda)$$
$$=2\lambda-2-12+2-18+3\lambda$$
$$=5\lambda-30$$
$$30=5\lambda$$
$$\lambda=6$$

Scalar Triple Product Ex 26.1 Q5(iv)

We know that vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar iff $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$.

$\therefore \vec{a}, \vec{b}, \vec{c}$ are coplanar

$$\Rightarrow \left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$$

$$\begin{vmatrix} 1 & 3 & 0 \\ 0 & 0 & 5 \\ \lambda & -1 & 0 \end{vmatrix}=0$$

$$=1(0+5)-3(0-5\lambda)+0$$
$$=5+15\lambda$$
$$-5=15\lambda$$
$$\lambda=-\frac{1}{3}$$

Scalar Triple Product Ex 26.1 Q6

Let

$$OA=6\hat{i}-7\hat{j}, OB=16\hat{i}-19\hat{j}-4\hat{k}, OC=3\hat{j}-6\hat{k}, OD=2\hat{i}+5\hat{j}+10\hat{k}$$
$$AB=OB-OA=16\hat{i}-25\hat{j}-4\hat{k}$$
$$AC=OC-OA=-16\hat{i}-16\hat{j}+2\hat{k}$$
$$CD=OD-OC=2\hat{i}+2\hat{j}+16\hat{k}$$
$$AD=OD-OA=-4\hat{i}+12\hat{j}+10\hat{k}$$

The four points are co-planer if vectors $\overline{AB}, \overline{AC}, \overline{AD}$ are co-planer.

$$\begin{vmatrix} 16 & -25 & -4 \\ -16 & -16 & 2 \\ -4 & 12 & 10 \end{vmatrix}=16(-160-24)+25(-160+8)-4(-144+64)$$
$$\neq 0$$

Hence the points are not coplanar.

Scalar Triple Product Ex 26.1 Q7

$AB = position\ vector\ of\ B - position\ vector\ of\ A$

$$=4\hat{i}-2\hat{j}-2\hat{k}$$

$AC = position\ vector\ of\ C - position\ vector\ of\ A$

$$=-2\hat{i}+4\hat{j}-2\hat{k}$$

$AD = position\ vector\ of\ D - position\ vector\ of\ A$

$$=-2\hat{i}-2\hat{j}+4\hat{k}$$

The four points are co-planar if the vectors are co-planar.

$$Thus, \begin{vmatrix} 4 & -2 & -2 \\ -2 & 4 & -2 \\ -2 & -2 & 4 \end{vmatrix}=4[16-4]+2[-8-4]-2[4+8]=48-24-24=0$$

Hence proved.

Scalar Triple Product Ex 26.1 Q8

Let $OA = 6\hat{i} - 7\hat{j}$, $OB = 16\hat{i} - 19\hat{j} - 4\hat{k}$, $OC = 3\hat{i} - 6\hat{k}$, $OD = 2\hat{i} - 5\hat{j} + 10\hat{k}$

Thus,

$AB = OB - OA = 10\hat{i} - 12\hat{j} - 4\hat{k}$

$AC = OC - OA = -3\hat{i} + 7\hat{j} - 6\hat{k}$

$AD = OD - OA = -4\hat{i} + 2\hat{j} + 10\hat{k}$

The four points are co-planar if vectors AB, AC and AD are co-planar.

Thus, we have

$$\begin{vmatrix} 10 & -12 & -4 \\ -3 & 7 & -6 \\ -4 & 2 & 10 \end{vmatrix} = 10(70+12) + 12(-30-24) - 4(-6+28) = 820 - 648 - 88$$

Scalar Triple Product Ex 26.1 Q9

Let

Position vector of $A = -\hat{j} - \hat{k}$

Position vector of $B = 4\hat{i} + 5\hat{j} + \lambda\hat{k}$

Position vector of $C = 3\hat{i} + 9\hat{j} + 4\hat{k}$

Position vector of $D = -4\hat{i} + 4\hat{j} + 4\hat{k}$

The four points are coplanar if the vectors $\overline{AB}, \overline{AC}, \overline{AD}$ are coplanar.

$\overline{AB} = 4\hat{i} + 6\hat{j} + (\lambda + 1)\hat{k}$

$\overline{AC} = 3\hat{i} + 10\hat{j} + 5\hat{k}$

$\overline{AD} = -4\hat{i} + 5\hat{j} + 5\hat{k}$

$$\begin{vmatrix} 4 & 6 & (\lambda+1) \\ 3 & 10 & 5 \\ -4 & 5 & 5 \end{vmatrix} = 0$$

$4(50-25) - 6(15+20) + (\lambda+1)(15+40) = 0$

$100 - 210 + 55 + 55\lambda = 0$

$55\lambda = 55$

$\lambda = 1$

Scalar Triple Product Ex 26.1 Q10

$(\vec{a}-\vec{b}).\{(\vec{b}-\vec{c})\times(\vec{c}-\vec{a})\}$

$= [(\vec{a}-\vec{b}) \quad (\vec{b}-\vec{c}) \quad (\vec{c}-\vec{a})]$

$= [a \quad (\vec{b}-\vec{c}) \quad (\vec{c}-\vec{a})] + [-b \quad (\vec{b}-\vec{c}) \quad (\vec{c}-\vec{a})]$

$= 6[a \quad b \quad c] - 6[a \quad b \quad c]$

$= 0$

Scalar Triple Product Ex 26.1 Q11

If $\vec{a}$ represents the sides AB , If $\vec{b}$ represents the sides BC, If $\vec{c}$ represents the sides AC of the triangleABC.

$\vec{a}\times\vec{b}$ is perpendicular to the plane of the triangleABC.

$\vec{b}\times\vec{c}$ is perpendicular to the plane of the triangleABC.

$\vec{c}\times\vec{a}$ is perpendicular to the plane of the triangleABC.

Hence $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$ is a vector perpendicular to the plane of the triangleABC.

Scalar Triple Product Ex 26.1 Q12(i)

$\overline{a}, \overline{b}, \overline{c}$ are coplanar if

$[a \quad b \quad c] = 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 2 & c_3 \end{vmatrix} = 0$$

$0 - 1(c_3) + 1(2) = 0$

$c_3 = 2$

Scalar Triple Product Ex 26.1 Q12(ii)

$\overline{a}, \overline{b}, \overline{c}$ are coplanar if

$[a \quad b \quad c] = 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ -1 & c_2 & 1 \end{vmatrix} = 0$$

$0 - 1 + 1(c_2) = 0$

$c_2 = 1$

Scalar Triple Product Ex 26.1 Q13

Let

Position vector of $OA = 3\hat{i}+2\hat{j}+\hat{k}$

Position vector of $OB = 4\hat{i}+\lambda\hat{j}+5\hat{k}$

Position vector of $OC = 4\hat{i}+2\hat{j}-2\hat{k}$

Position vector of $OD = 6\hat{i}+5\hat{j}-\hat{k}$

The four points are coplanar if the vectors $\overline{AB}, \overline{AC}, \overline{AD}$ are coplanar.

$$\overline{AB} = \hat{i}+(\lambda-2)\hat{j}+4\hat{k}$$
$$\overline{AC} = \hat{i}+0\hat{j}-3\hat{k}$$
$$\overline{AD} = 3\hat{i}+3\hat{j}-2\hat{k}$$

$$\begin{vmatrix} 1 & (\lambda-2) & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0$$

$$1(9)-(\lambda-2)(-2+9)+4(3-0)=0$$
$$9-7\lambda+14+12=0$$
$$7\lambda=35$$
$$\lambda=5$$

Ex 27.1

Direction Cosines and Direction Ratios Ex 27.1 Q1

Let l, m and n be the direction cosines of a line.
$l = \cos 90° = 0$

$m = \cos 60° = \frac{1}{2}$

$n = \cos 30° = \frac{\sqrt{3}}{2}$

$\therefore$ The direction cosines of the line are $0, \frac{1}{2}, \frac{\sqrt{3}}{2}$.

Direction Cosines and Direction Ratios Ex 27.1 Q2

Let the direction cosines of the line be l, m, n.
Here,
$a = 2, b = -1, c = -2$ are the direction ratios of the line.

$$l = \pm\frac{a}{\sqrt{a^2+b^2+c^2}}, m = \pm\frac{b}{\sqrt{a^2+b^2+c^2}}, n = \pm\frac{c}{\sqrt{a^2+b^2+c^2}}$$

$$l = \frac{2}{\sqrt{2^2+(-1)^2+(-2)^2}}, m = \frac{-1}{\sqrt{2^2+(-1)^2+(-2)^2}}, n = \frac{-2}{\sqrt{2^2+(-1)^2+(-2)^2}}$$

$$l = \frac{2}{\sqrt{9}}, m = \frac{-1}{\sqrt{9}}, n = \frac{-2}{\sqrt{9}}$$

$$l = \frac{2}{3}, m = -\frac{1}{3}, n = -\frac{2}{3}$$

$\therefore$ The direction ratios of the line are $\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}$.

Direction Cosines and Direction Ratios Ex 27.1 Q3

The direction ratios of the line joining $(-2, 4, -5)$ and $(1, 2, 3)$ are,
$(1+2, 2-4, 3+5) = (3, -2, 8)$
Here, $a = 3, b = -2, c = 8$
Direction cosines are

$$\frac{3}{\sqrt{3^2+(-2)^2+8^2}}, \frac{-2}{\sqrt{3^2+(-2)^2+8^2}}, \frac{8}{\sqrt{3^2+(-2)^2+8^2}}$$

$$= \frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$$

Direction Cosines and Direction Ratios Ex 27.1 Q4

Here A $(2, 3, -4)$, B $(1, -2, 3)$ and C $(3, 8, -11)$.
Direction ratios of AB $= (1-2, -2-3, 3+4) = (-1, -5, 7)$
Direction ratios of BC $= (3-1, 8+2, -11-3) = (2, 10, -14)$

Here, the respective direction cosines of AB and AC,
$\frac{-1}{2} = \frac{-5}{10} = \frac{7}{-14}$ are proportional.

Also, B is the common point between the two lines,
$\therefore$ The points A $(2, 3, -4)$, B $(1, -2, 3)$ and C $(3, 8, -11)$ are collinear.

Direction Cosines and Direction Ratios Ex 27.1 Q5

$A(3,5,-4), B(-1,1,2)$ and $C(-5,-5,-2)$

The direction ratios of the side $AB = (-1-3, 1-5, 2+4)$

$= (-4,-4,6)$

Direction cosines of AB will be

$$\frac{-4}{\sqrt{(-4)^2+(-4)^2+6^2}}, \frac{-4}{\sqrt{(-4)^2+(-4)^2+6^2}}, \frac{6}{\sqrt{(-4)^2+(-4)^2+6^2}}$$

$$= \frac{-4}{\sqrt{68}}, \frac{-4}{\sqrt{68}}, \frac{6}{\sqrt{68}}$$

$$= \frac{-2}{\sqrt{17}}, \frac{-2}{\sqrt{17}}, \frac{3}{\sqrt{17}}$$

The direction ratios of the side $BC = (-5+1, -5-1, -2-2)$

$= (-4,-6,-4)$

Direction cosines of BC will be

$$\frac{-4}{\sqrt{(-4)^2+(-6)^2+(-4)^2}}, \frac{-6}{\sqrt{(-4)^2+(-6)^2+(-4)^2}}, \frac{-4}{\sqrt{(-4)^2+(-6)^2+(-4)^2}}$$

$$= \frac{-4}{\sqrt{68}}, \frac{-6}{\sqrt{68}}, \frac{-4}{\sqrt{68}}$$

$$= \frac{-2}{\sqrt{17}}, \frac{-3}{\sqrt{17}}, \frac{-2}{\sqrt{17}}$$

The direction ratios of the side $AC = (-5-3, -5-5, -2+4)$

$= (-8,-10,2)$

Direction cosines of AC will be

$$\frac{-8}{\sqrt{(-8)^2+(-10)^2+2^2}}, \frac{-10}{\sqrt{(-8)^2+(-10)^2+2^2}}, \frac{2}{\sqrt{(-8)^2+(-10)^2+2^2}}$$

$$= \frac{-8}{\sqrt{168}}, \frac{-10}{\sqrt{168}}, \frac{2}{\sqrt{168}}$$

$$= \frac{-4}{\sqrt{42}}, \frac{-5}{\sqrt{42}}, \frac{1}{\sqrt{42}}$$

Direction Cosines and Direction Ratios Ex 27.1 Q6

Let, θ be the angle between the vectors with direction ratios a, b, c and a_2, b_2, c_2 then.

$$\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$= \frac{(1)(4)+(-2)(3)+(1)(2)}{\sqrt{(1)^2+(-2)^2+(1)^2}\sqrt{(4)^2+(3)^2+(2)^2}}$$

$$= \frac{4-6+2}{\sqrt{1+4+1}\sqrt{16+9+4}}$$

$$= \frac{6-6}{\sqrt{6}\sqrt{29}}$$

$$= \frac{0}{\sqrt{174}}$$

$\cos\theta = 0$

$\theta = \cos^{-1}(0)$

$\theta = \frac{\pi}{2}$

Direction Cosines and Direction Ratios Ex 27.1 Q7

Here, given that the direction cosines of the vectors are proportional to 2, 3, − 6 and 3, − 4, 5.

Therefore, 2, 3, − 6 and 3, − 4, 5 are the direction ratios of two vectors.

Let, θ be the angle between two vectors having direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{(2)(3) + (3)(-4) + (-6)(5)}{\sqrt{(2)^2 + (3)^2 + (-6)^2}\sqrt{(3)^2 + (-4)^2 + (5)^2}}$$

$$\cos\theta = \frac{6 - 12 - 30}{\sqrt{4 + 9 + 36}\sqrt{9 + 16 + 25}}$$

$$= \frac{6 - 42}{\sqrt{49}\sqrt{50}}$$

$$= \frac{-36 \times \sqrt{2}}{7 \times 5 \times \sqrt{2} \times \sqrt{2}} \qquad \text{(Rationalizing the denominator)}$$

$$= \frac{-36\sqrt{2}}{70}$$

$$\cos\theta = \frac{-18\sqrt{2}}{35}$$

$$\theta = \cos^{-1}\left(\frac{-18\sqrt{2}}{35}\right)$$

Direction Cosines and Direction Ratios Ex 27.1 Q8

The vectors, represented by these are

$\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

and $\vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$

Let, θ be the angle between the lines,
then,

$$\cos\theta = \frac{\vec{a} \times \vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{(2\hat{i} + 3\hat{j} + 6\hat{k})(\hat{i} + 2\hat{j} + 2\hat{k})}{\sqrt{(2)^2 + (3)^2 + (6)^2}\sqrt{(1)^2 + (2)^2 + (2)^2}}$$

$$= \frac{(2)(1) + (3)(2) + (6)(2)}{\sqrt{4 + 9 + 36}\sqrt{1 + 4 + 4}}$$

$$= \frac{2 + 6 + 12}{\sqrt{49}\sqrt{9}}$$

$$= \frac{20}{7 \times 3}$$

$$\cos\theta = \frac{20}{21}$$

$$\theta = \cos^{-1}\left(\frac{20}{21}\right)$$

Angle between the lines = $\cos^{-1}\left(\frac{20}{21}\right)$

Direction Cosines and Direction Ratios Ex 27.1 Q9

We have, $(2,\ 3,\ 4), (-1, -2, 1)$ and $(5, 8, 7)$

Let the points are A, B, C respectively.

Position vector of $A = 2\hat{i} + 3\hat{j} + 4\hat{k}$
Position vector of $B = -\hat{i} - 2\hat{j} + \hat{k}$
Position vector of $C = 5\hat{i} + 8\hat{j} + 7\hat{k}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$= \left(-\hat{i} - 2\hat{j} + \hat{k}\right) - \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$
$= -\hat{i} - 2\hat{j} + \hat{k} - 2\hat{i} - 3\hat{j} - 4\hat{k}$

$\overrightarrow{AB} = -3\hat{i} - 5\hat{j} - 3\hat{k}$

$\overrightarrow{BC}$ = Position vector of C – Position vector of B

$= \left(5\hat{i} + 8\hat{j} + 7\hat{k}\right) - \left(-\hat{i} - 2\hat{j} + \hat{k}\right)$
$= 5\hat{i} + 8\hat{j} + 7\hat{k} + \hat{i} + 2\hat{j} - \hat{k}$

$\overrightarrow{BC} = 6\hat{i} + 10\hat{j} + 6\hat{k}$

Using $\overrightarrow{AB}$ and $\overrightarrow{BC}$, we get

$\overrightarrow{BC} = -2\ \overrightarrow{AB}$

So, $\overrightarrow{BC}$ is parallel to $\overrightarrow{AB}$ but $\vec{B}$ is the common vector,

Hence, A, B, C are collinear

Direction Cosines and Direction Ratios Ex 27.1 Q10

line throught points $(4,\ 7,\ 8)$ *and* $(2,\ 3,\ 4)$

$$\frac{x-4}{2} = \frac{y-7}{4} = \frac{z-8}{4} \rightarrow \frac{x-4}{1} = \frac{y-7}{2} = \frac{z-8}{2}$$

line through the points $(-1, -2, 1)$ and $(1, 2, 5)$

$$\frac{x+1}{-2} = \frac{y+2}{-4} = \frac{z-1}{-4} \rightarrow \frac{x+1}{1} = \frac{y+2}{2} = \frac{z-1}{2}$$

the directi on ratios are same for both the lines

$\therefore$ they are parallel to each other

Direction Cosines and Direction Ratios Ex 27.1 Q11

Given,

$A(1, -1, 2)$ and $B(3, 4, -2)$
$C(0, 3, 2)$ and $D(3, 5, 6)$

Direction ratios of line AB
$a_1 = 2,\quad b_1 = 5,\quad c_1 = -4$

Direction ratios of line CD
$a_2 = 3,\quad b_2 = 2,\quad c_2 = 4$

We know that, lines are perpendicular if

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$LHS = (2)(3) + (5)(2) + (-4)(4)$
$6 + 10 - 16$
$16 - 16$
0

$\therefore LHS = RHS$

Lines are perpendicular

Direction Cosines and Direction Ratios Ex 27.1 Q12

Here,

$A(0, 0, 0)$ and $B(2, 1, 1)$
$C(3, 5, -1)$ and $D(4, 3, -1)$

Direction ratios of line AB
$a_1 = 2,\quad b_1 = 1,\quad c_1 = 1$

Direction ratios of line CD
$a_2 = 1,\quad b_2 = -2,\quad c_2 = 0$

Now,

$a_1a_2 + b_1b_2 + c_1c_2$
$= (2)(1) + (1)(-2) + (1)(0)$
$= 2 - 2 + 0$

$= 0$

Since, $a_1a_2 + b_1b_2 + c_1c_2 = 0$, lines are perpendicular

Direction Cosines and Direction Ratios Ex 27.1 Q13

Given, that the direction ratios of lines are proportional to a, b, c and $b-c$, $c-a$, $a-b$.

Let, $\vec{x}$ and $\vec{y}$ be the vector parallel to these lines respectively, so

$\vec{x} = a\hat{i} + b\hat{j} + c\hat{k}$

And, $\vec{y} = (b-c)\hat{i} + (c-a)\hat{j} + (a-b)\hat{k}$

Let, θ be the angle between $\vec{x}$ and $\vec{y}$, so,

$$\cos\theta = \frac{\vec{x} \times \vec{y}}{|\vec{x}||\vec{y}|}$$

$$= \frac{\left(a\hat{i} + b\hat{j} + c\hat{k}\right)\left[(b-c)\hat{i} + (c-a)\hat{j} + (a-b)\hat{k}\right]}{\sqrt{a^2+b^2+c^2}\sqrt{(b-c)^2+(c-a)^2+(a-b)^2}}$$

$$= \frac{(a)(b-c) + b(c-a) + c(a-b)}{\sqrt{a^2+b^2+c^2}\sqrt{b^2+c^2-2bc+c^2+a^2-2ca+a^2+b^2-2ab}}$$

$$\cos\theta = \frac{ab-ac+bc-ba+ca-bc}{\sqrt{a^2+b^2+c^2}\sqrt{2a^2+2b^2+2c^2-2ab-2bc-2ca}}$$

$\cos\theta = 0$

$\theta = \cos^{-1}(0)$

$\theta = \frac{\pi}{2}$

Angle between the lines $= \frac{\pi}{2}$

Direction Cosines and Direction Ratios Ex 27.1 Q14

Here we have,

$A(1, 2, 3)$, $B(4, 5, 7)$, $C(-4, 3, -6)$ $D(2, 9, 2)$

Direction ratios of AB

$a_1 = 3,\quad b_1 = 3,\quad c_1 = 4$

Direction ratios of CD

$a_2 = 6,\quad b_2 = 6,\quad c_2 = 8$

Let, θ be the angle between AB and CD, so,

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\cos\theta = \frac{(3)(6)+(3)(6)+(4)(8)}{\sqrt{(3)^2+(3)^2+(4)^2}\sqrt{(6)^2+(6)^2+(8)^2}}$$

$$= \frac{18+18+32}{\sqrt{9+9+16}\sqrt{36+36+64}}$$

$$= \frac{68}{\sqrt{34}\sqrt{136}}$$

$$= \frac{68}{\sqrt{34}\ 2\sqrt{34}}$$

$$= \frac{68}{34 \times 2}$$

$\cos\theta = 1$

$\theta = \cos^{-1}(1)$

$\theta = 0°$

Therefore,

Angle between AB and CD = $0°$

Direction Cosines and Direction Ratios Ex 27.1 Q15

The given equations are

$2lm+2\text{ln}-mn=0$

$l+m+n=0$

$\rightarrow l=-(m+n)$...........(1)

$2l(m+n)=mn \rightarrow l=\dfrac{mn}{2(m+n)}$...............(2)

put $l=-(m+n)$ in (2)

$\rightarrow -(m+n)=\dfrac{mn}{2(m+n)} \rightarrow -2(m+n)^2=mn$

$\rightarrow -2\left(m^2+n^2+2mn\right)=mn \rightarrow \left(m^2+n^2+2mn\right)=-\dfrac{mn}{2}$

$\rightarrow \left(m^2+n^2+2mn+\dfrac{mn}{2}\right)=0 \rightarrow \left(m^2+n^2+\dfrac{5mn}{2}\right)=0$

$\rightarrow \left(2m^2+2n^2+5mn\right)=0 \rightarrow (2m+n)(m+2n)=0$

$\rightarrow m=-\dfrac{n}{2} \rightarrow l=-\left(n-\dfrac{n}{2}\right)=-\dfrac{n}{2}$

$\rightarrow m=-2n \rightarrow l=-(-2n+n)=n$

Thus the direction ratios of two lines are proportional to $-\dfrac{n}{2},-\dfrac{n}{2},n$

and $n,-2n,n$

i.e. $-\dfrac{1}{2},-\dfrac{1}{2},1$ and $1,-2,1$

Hence the direction cosines are

$-\dfrac{1}{\sqrt{3}},-\dfrac{1}{\sqrt{3}},\dfrac{2}{\sqrt{3}}$ and $1,-2,1$

Direction Cosines and Direction Ratios Ex 27.1 Q16(i)

Given that, $l + m + n = 0$ $\quad ---(i)$

$l^2 + m^2 - n^2 = 0$ $\quad ---(ii)$

From equation (i),

$l = -(m+n)$

Put the value of l in equation (ii),

$[-(m+n)]^2 + m^2 - n^2 = 0$

$(m+n)^2 + m^2 - n^2 = 0$

$m^2 + n^2 + 2mn + m^2 - n^2 = 0$

$2m^2 + 2mn = 0$

$2m(m+n) = 0$

$m = 0,\ m + n = 0$

$m = -n$ and $m = 0$

Put the value of $m = -n$ in equation (i)

$l = -(-n+n)$

$l = 0$

Again put the value of $m = 0$ in equation (i)

$l = -(m+n)$

$= -(0+n)$

$l = -n$

Thus the direction ratios are proportional to

$0, -n, n$ and $-n, 0, n$

$\Rightarrow \quad 0, -1, 1$ and $-1, 0, 1$

So, vectors parallel to these lines are

$\vec{a} = 0 \times \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = -\hat{i} + 0 \times \hat{j} + \hat{k}$ respectively.

Let, θ be the angle between the $\vec{a}$ and $\vec{b}$

So, $\cos\theta = \dfrac{\vec{a} \times \vec{b}}{|\vec{a}||\vec{b}|}$

$\vec{a} = 0 \times \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = -\hat{i} + 0 \times \hat{j} + \hat{k}$ respectively.

$$\cos\theta = \frac{\left(0 \times \hat{i} - \hat{j} + \hat{k}\right) \times \left(-\hat{i} + 0 \times \hat{j} + \hat{k}\right)}{\sqrt{0^2 + (-1)^2 + (1)^2}\sqrt{(-1)^2 + (0)^2 + (1)^2}}$$

$$= \frac{(0)(-1) + (-1)(0) + (1)(1)}{\sqrt{1+1}\sqrt{1+1}}$$

$$= \frac{0 + 0 + 1}{\sqrt{2} \times \sqrt{2}}$$

$$= \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\theta = \frac{\pi}{3}$$

So, angle between the lines $= \dfrac{\pi}{3}$

Direction Cosines and Direction Ratios Ex 27.1 Q16(ii)

Given that,

$2l - m + 2n = 0$ --- (i)

$mn + nl + lm = 0$ --- (ii)

From equation (i),

$2l - m + 2n = 0$

$m = 2l + 2n$

Put the value of m in equation (ii),

$mn + nl + lm = 0$

$(2l + 2n)n + nl + l(2l + 2n) = 0$

$2ln + 2n^2 + nl + 2l^2 + 2ln = 0$

$2l^2 + 5ln + 2n^2 = 0$

$2l^2 + 4ln + ln + 2n^2 = 0$

$2l(l + 2n) + n(l + 2n) = 0$

$(l + 2n)(2l + n) = 0$

$l + 2n = 0$ or $2l + n = 0$

$l = -2n$ or $l = -\frac{n}{2}$

Put the value of $l = -2n$ in equation (i)

$2l - m + 2n = 0$

$2(-2n) - m + 2n = 0$

$-4n - m + 2n = 0$

$-2n - m = 0$

$-2n = m$

$m = -2n$

Again, put the value of $l = -\frac{1}{2}n$ in equation (i)

$2l - m + 2n = 0$

$2\left(-\frac{1}{2}n\right) - m + 2n = 0$

$-n - m + 2n = 0$

$-m + n = 0$

$-m = -n$

$m = n$

So, direction cosines of the lines are given by,

$-2n, -2n, n$ or $-\frac{1}{2}n, n, n$

$-2, -2, 1$ or $-\frac{1}{2}, 1, 1$

So, vectors parallel to these lines

$\vec{a} = -2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = -\frac{1}{2}\hat{i} + \hat{j} + \hat{k}$ respectively.

Let, θ be the angle between $\vec{a}$ and $\vec{b}$,

$$\cos\theta = \frac{\vec{a} \times \vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{\left(-2\hat{i} - 2\hat{j} + \hat{k}\right) \times \left(-\frac{1}{2}\hat{i} + \hat{j} + \hat{k}\right)}{\sqrt{(-2)^2 + (-2)^2 + (1)^2}\sqrt{\left(-\frac{1}{2}\right)^2 + (1)^2 + (1)^2}}$$

$$= \frac{(-2)\left(-\frac{1}{2}\right) + (-2)(1) + (1)(1)}{\sqrt{4 + 4 + 1}\sqrt{\frac{1}{4} + 1 + 1}}$$

$$= \frac{1 - 2 + 1}{\sqrt{9}\sqrt{\frac{9}{4}}}$$

$$\cos\theta = \frac{0}{3 \times \frac{3}{2}}$$

$$\cos\theta = 0$$

$$\theta = \cos^{-1}(0)$$

$$\theta = \frac{\pi}{2}$$

Angle between the lines $= \frac{\pi}{2}$

Direction Cosines and Direction Ratios Ex 27.1 Q16(iii)

Here,

$l + 2m + 3n = 0$ ---(i)

$3lm - 4ln + mn = 0$ ---(ii)

From equation (i),

$l + 2m + 3n = 0$

$l = -2m - 3n$

Put the value of l in equation (ii),

$3lm - 4ln + mn = 0$

$3(-2m - 3n)m - 4(-2m - 3n)n + mn = 0$

$-6m^2 - 9nm + 8mn + 12n^2 + mn = 0$

$-6m^2 + 12n^2 = 0$

$-6m^2 = -12n^2$

$m^2 = 2n^2$

$m = \pm\sqrt{2n^2}$

$m = n\sqrt{2}$ or $m = -n\sqrt{2}$

Put $m = n\sqrt{2}$ in equation (i)

$l + 2m + 3n = 0$

$l + 2(n\sqrt{2}) + 3n = 0$

$l + n(2\sqrt{2} + 3) = 0$

$l = -(2\sqrt{2} + 3)n$

Again, m = $-\sqrt{2}n$ in equation (i)

$l + 2m + 3n = 0$

$l + 2(-\sqrt{2}n) + 3n = 0$

$l - 2\sqrt{2}n + 3n = 0$

$l + n(-2\sqrt{2} + 3) = 0$

$l = (2\sqrt{2} - 3)n$

Thus, direction cosines of the lines are given by,

$$-\left(2\sqrt{2}+3\right)n,\ \sqrt{2}n,\ n \quad or \quad \left(2\sqrt{2}-3\right)n,\ -\sqrt{2}n,\ n$$
$$-\left(2\sqrt{2}+3\right),\ \sqrt{2},\ 1 \quad or \quad \left(2\sqrt{2}-3\right),\ -\sqrt{2},\ 1$$

So, vectors parallel to these lines are

$$\vec{a} = -\left(2\sqrt{2}+3\right)\hat{i} + \sqrt{2}\hat{j} + \hat{k} \text{ and } \vec{b} = \left(2\sqrt{2}-3\right)\hat{i} - \sqrt{2}\hat{j} + \hat{k}$$

Let, θ be the angle between the lines,
then,

$$\cos\theta = \frac{\vec{a}\times\vec{b}}{\left|\vec{a}\right|\times\left|\vec{b}\right|}$$
$$= \frac{-\left(2\sqrt{2}+3\right)\times\left(2\sqrt{2}-3\right)+\left(\sqrt{2}\right)\times\left(-\sqrt{2}\right)+(1)(1)}{\sqrt{\left(2\sqrt{2}+3\right)^2+\left(\sqrt{2}\right)^2+(1)^2}\sqrt{\left(2\sqrt{2}-3\right)^2+\left(-\sqrt{2}\right)^2+(1)^2}}$$

$$= \frac{-(8-9)-2+1}{\sqrt{8+9+12\sqrt{2}+2+1}\sqrt{8+9-12\sqrt{2}+2+1}}$$

$$= \frac{-(-1)-2+1}{\sqrt{20+12\sqrt{2}}\sqrt{20-12\sqrt{2}}}$$
$$= \frac{1-2+1}{\sqrt{20+12\sqrt{2}}\sqrt{20-12\sqrt{2}}}$$

$\cos\theta = 0$
$\theta = \cos^{-1}(0)$

$\theta = \frac{\pi}{2}$

Angle between the lines $= \frac{\pi}{2}$

Direction Cosines and Direction Ratios Ex 27.1 Q16(iv)

The given equations are,
$2l + 2m - n = 0$.........(i)
$mn + ln + lm = 0$.........(ii)

From(i), we get $n = 2l + 2m$.
Putting $n = 2l + 2m$ in (ii), we get
$m(2l + 2m) + l(2l + 2m) + lm = 0$
$\Rightarrow 2lm + 2m^2 + 2l^2 + 2ml + lm = 0$
$\Rightarrow 2m^2 + 5lm + \ + 2l^2 = 0$
$\Rightarrow 2m^2 + 4lm + lm + 2l^2 = 0$
$\Rightarrow (2m + l)(m + 2l) = 0$
$\Rightarrow m = -\frac{l}{2}$ or $m = -2l$

By putting $m = -\frac{l}{2}$ in (i) we get $n = l$
By putting $m = -2l$ in (i) we get $n = -2l$

So direction ratios of two lines are proportional to
$l,\ -\frac{l}{2},\ l$ and $l,\ -2l,\ -2l$ or, $1,\ -\frac{1}{2}, 1$ and $1,\ -2,\ -2$

So, vectors parallel to these lines are

$\vec{a} = \hat{i} - \frac{1}{2}\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} - 2\hat{k}$

If θ is the angle between the lines, then θ is also the angle between $\vec{a}$ and $\vec{b}$.

$$\cos\theta = \frac{\vec{a}\bullet\vec{b}}{|\vec{a}||\vec{b}|} = \frac{1+1-2}{\sqrt{1+\frac{1}{4}+1}\sqrt{1+4+9}} = 0$$

$$\Rightarrow \theta = \cos^{-1}(0) = \frac{\pi}{2}$$

Ex 28.1

Straight Line in Space Ex 28.1 Q1

Vector equation of a line

is $\vec{r} = \vec{a} + \lambda\vec{b}$

The Cartesian equation of a line is

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{a_2} = \frac{x - x_3}{a_3}$$

Using the above formula,

Vector equation of the line,

$\vec{r} = (5\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 8\hat{k})$

The Cartesian equation of the line

$$\frac{x - 5}{3} = \frac{y - 2}{2} = \frac{z + 4}{-8}$$

Straight Line in Space Ex 28.1 Q2

The direction ratios of the line are

$(3 + 1, 4 - 0, 6 - 2) = (4, 4, 4)$

Since the line passes through $(-1, 0, 2)$

The vector equation of the line,

$\vec{r} = \vec{a} + \lambda\vec{b}$

$\Rightarrow \vec{r} = \vec{a} + \lambda\vec{b}$

$\Rightarrow \vec{r} = (-\vec{i} + 0\vec{j} + 2\vec{k}) + \lambda(4\vec{i} + 4\vec{j} + 4\vec{k})$

$\therefore$ The vector equation of the line,

$\vec{r} = (-\vec{i} + 0\vec{j} + 2\vec{k}) + \lambda(4\vec{i} + 4\vec{j} + 4\vec{k})$

Straight Line in Space Ex 28.1 Q3

We know that, vector equation of line passing through a fixed point $\vec{a}$ and parallel to vector $\vec{b}$ is

$$\vec{r} = \vec{a} + \lambda\vec{b}, \text{ where } \lambda \text{ is scalar}$$

Here, $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{a} = 5\hat{i} - 2\hat{j} + 4\hat{k}$

So, equation of required line is

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

$$\vec{r} = \left(5\hat{i} - 2\hat{j} + 4\hat{k}\right) + \lambda\left(2\hat{i} - \hat{j} + 3\hat{k}\right)$$

Put $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, so

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right) = \left(5 + 2\lambda\right)\hat{i} + \left(-2 - \lambda\right)\hat{j} + \left(4 + 3\lambda\right)\hat{k}$$

Comparing the coefficients of $\hat{i}, \hat{j}, \hat{k}$, so

$$x = 5 + 2\lambda,\ y = -2 - \lambda,\ z = 4 + 3\lambda$$

$$\Rightarrow \quad \frac{x - 5}{2} = \lambda,\ \frac{y + 2}{-0} = \lambda,\ \frac{z - 4}{3} = \lambda$$

Cortesian form of equation of the line is,

$$\frac{x - 5}{2} = \frac{y + 2}{-0} = \frac{z - 4}{3}$$

Straight Line in Space Ex 28.1 Q4

We know that, equation of line passing through a vector $\vec{a}$ and parallel to a vector $\vec{b}$ is given by,

$$\vec{r} = \vec{a} + \lambda\vec{b}, \text{ where } \lambda \text{ is scalar,}$$

Here, $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$

Required equation of line is,

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

$$\vec{r} = \left(2\hat{i} - 3\hat{j} + 4\hat{k}\right) + \lambda\left(3\hat{i} + 4\hat{j} - 5\hat{k}\right)$$

Put $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$x\hat{i} + y\hat{j} + z\hat{k} = (2 + 3\lambda)\hat{i} + (-3 + 4\lambda)\hat{j} + (4 - 5\lambda)\hat{k}$$

On equating coefficients of $\hat{i}, \hat{j}$ and $\hat{k}$,

$\Rightarrow \quad 2 + 3\lambda = x,\ -3 + 4\lambda = y,\ 4 - 5\lambda = z$

$\Rightarrow \quad \dfrac{x-2}{3} = \lambda,\ \dfrac{y+3}{4} = \lambda,\ \dfrac{z-4}{-5} = z$

So, cortesian form of equation of the line is

$$\frac{x-2}{3} = \frac{y+3}{4} = \frac{z-4}{-5}$$

Straight Line in Space Ex 28.1 Q5

$ABCD$ is a parallelogram.

$\Rightarrow \quad AC$ and BD bisect each other at point O (say).

Position vector of point $O = \dfrac{\vec{a} + \vec{c}}{2}$

$$= \frac{\left(4\hat{i} + 5\hat{j} - 10\hat{k}\right) + \left(-\hat{i} + 2\hat{j} + \hat{k}\right)}{2}$$

$$= \frac{3\hat{i} + 7\hat{j} - 9\hat{k}}{2}$$

Let position vector of point O and B are represented by $\vec{o}$ and $\vec{b}$.

Equation of the line BD is the line passing through O and B is given by

$$\vec{r} = \vec{a} + \lambda\left(\vec{b} - \vec{a}\right) \qquad \left[\begin{array}{l}\text{Since equation of the line passing through} \\ \text{two points } \vec{a} \text{ and } \vec{b}\end{array}\right]$$

$$\vec{r} = \vec{b} + \lambda\left(\vec{o} - \vec{b}\right)$$

$$= \left(2\hat{i} - 3\hat{j} + 4\hat{k}\right) + \lambda\left(\frac{3\hat{i} + 7\hat{j} - 9\hat{k}}{2} - 2\hat{i} - 3\hat{j} + 4\hat{k}\right)$$

$$\vec{r} = \left(2\hat{i} - 3\hat{j} + 4\hat{k}\right) + \lambda\left(3\hat{i} + 7\hat{j} - 9\hat{k} - 4\hat{i} + 6\hat{j} - 8\hat{k}\right)$$

$$\vec{r} = \left(2\hat{i} - 3\hat{j} + 4\hat{k}\right) + \lambda\left(-\hat{i} + 13\hat{j} - 17\hat{k}\right)$$

Put $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right) = (2 - \lambda)\hat{i} + (-3 + 13\lambda)\hat{j} + (4 - 17\lambda)\hat{k}$$

Equation the coefficients of $\hat{i}, \hat{j}, \hat{k}$, so

$\Rightarrow \quad x = 2 - \lambda,\ y = -3 - 13\lambda,\ z = 4 - 17\lambda$

$\Rightarrow \quad \dfrac{x-2}{-1} = \lambda,\ \dfrac{y+3}{13} = \lambda,\ \dfrac{z-4}{-17} = \lambda$

So equation of the line BD in cortesian form,

$$\frac{x-2}{-1} = \frac{y+3}{13} = \frac{z-4}{-17}$$

Straight Line in Space Ex 28.1 Q6

We know that, equation of line passing through two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1} \qquad ---(i)$$

Here, $(x_1, y_1, z_1) = A(1, 2, -1)$
$(x_2, y_2, z_2) = B(2, 1, 1)$

Using equation (i), equation of line AB,

$$\frac{x - 1}{2 - 1} = \frac{y - 2}{1 - 2} = \frac{z + 1}{1 + 1}$$

$$\frac{x - 1}{1} = \frac{y - 2}{-1} = \frac{z + 1}{2} = \lambda \text{ (say)}$$

$$x = \lambda + 1,\ y = -\lambda + 2,\ z = 2\lambda - 1$$

Vector form of equation of line AB is,

$$x\hat{i} + y\hat{j} + z\hat{k} = (\lambda + 1)\hat{i} + (-\lambda + 2)\hat{j} + (2\lambda - 1)\hat{k}$$

$$\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(\hat{i} - \hat{j} + 2\hat{k})$$

Straight Line in Space Ex 28.1 Q7

We know that vector equation of a line passing through $\vec{a}$ and parallel to vector $\vec{b}$ is given by,

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

Here, $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$

So, required vector equation of line is,

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$$

Now,

$$(x\hat{i} + y\hat{j} + z\hat{k}) = (1 + \lambda)\hat{i} + (2 - 2\lambda)\hat{j} + (3 + 3\lambda)\hat{k}$$

Equating the coefficients of $\hat{i}, \hat{j}, \hat{k}$,

$\Rightarrow \quad x = 1 + \lambda,\ y = 2 - 2\lambda,\ z = 3 + 3\lambda$

$\Rightarrow \quad x - 1 = \lambda,\ \dfrac{y - 2}{2} = \lambda,\ \dfrac{z - 3}{3} = \lambda$

So, required equation of line is cortesian form,

$$\frac{x - 1}{1} = \frac{y - 2}{-2} = \frac{z - 3}{3}$$

Straight Line in Space Ex 28.1 Q8

We know that, equation of a line passing through a point (x_1, y_1, z_1) and having direction ratios proportional to a, b, c is

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} \qquad ---(i)$$

Here, $(x_1, y_1, z_1) = (2, -1, 1)$ and

Given line $\frac{x-3}{2} = \frac{y+1}{7} = \frac{z-2}{-3}$ is prallel to required line.

$\Rightarrow \quad a = 2\mu,\ b = 7\mu,\ c = -3\mu$

So, equation of required line using equation (i),

$$\frac{x-2}{2\mu} = \frac{y+1}{7\mu} = \frac{z-1}{-3\mu}$$

$$\Rightarrow \quad \frac{x-2}{2} = \frac{y+1}{7} = \frac{z-1}{-3} = \lambda \text{ (say)}$$

$\Rightarrow \quad x = 2\lambda + 2,\ y = 7\lambda - 1,\ z = -3\lambda + 1$

So, $x\hat{i} + y\hat{j} + z\hat{k} = (2\lambda + 2)\hat{i} + (7\lambda - 1)\hat{j} + (-3\lambda + 1)\hat{k}$

$$\vec{r} = \left(2\hat{i} - \hat{j} + \hat{k}\right) + \lambda\left(2\hat{i} + 7\hat{j} - 3\hat{k}\right)$$

Straight Line in Space Ex 28.1 Q9

The Cartesian equation of the line is

$$\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2} \qquad \ldots(1)$$

The given line passes through the point (5, −4, 6). The position vector of this point is

$\vec{a} = 5\hat{i} - 4\hat{j} + 6\hat{k}$

Also, the direction ratios of the given line are 3, 7, and 2.

This means that the line is in the direction of vector, $\vec{b} = 3\hat{i} + 7\hat{j} + 2\hat{k}$

It is known that the line through position vector $\vec{a}$ and in the direction of the vector $\vec{b}$ is given by the equation, $\vec{r} = \vec{a} + \lambda\vec{b}, \lambda \in R$

$$\Rightarrow \vec{r} = \left(5\hat{i} - 4\hat{j} + 6\hat{k}\right) + \lambda\left(3\hat{i} + 7\hat{j} + 2\hat{k}\right)$$

This is the required equation of the given line in vector form.

Straight Line in Space Ex 28.1 Q10

We know that, equation of a line passing through a point (x_1, y_1, z_1) and having direction ratios proportional to a, b, c is

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} \qquad ---(i)$$

Here, $(x_1, y_1, z_1) = (1, -1, 2)$ and

Given line $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$ is prallel to required line, so

$\Rightarrow \quad a = \mu,\ b = 2\mu,\ c = -2\mu$

So, equation of required line using equation (i) is,

$$\frac{x-1}{\mu} = \frac{y+1}{2\mu} = \frac{z-2}{-2\mu}$$

$$\Rightarrow \quad \frac{x-1}{1} = \frac{y+1}{2} = \frac{z-2}{-2} = \lambda \text{ (say)}$$

$x = \lambda + 1,\ y = 2\lambda - 1,\ z = -2\lambda + 2$

So, $x\hat{i} + y\hat{j} + z\hat{k} = (\lambda + 1)\hat{i} + (2\lambda - 1)\hat{j} + (-2\lambda + 2)\hat{k}$

$$\vec{r} = \left(\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$$

Straight Line in Space Ex 28.1 Q11

Given, line is,

$$\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$$

$$\Rightarrow \quad \frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3} = \lambda \text{ (say)}$$

$$x = -2\lambda + 4,\ y = 6\lambda,\ z = -3\lambda + 1$$

So, $x\hat{i} + y\hat{j} + z\hat{k} = (-2\lambda + 4)\hat{i} + (6\lambda)\hat{j} + (-3\lambda + 1)\hat{k}$

$$\vec{r} = \left(4\hat{i} + \hat{k}\right) + \lambda\left(-2\hat{i} + 6\hat{j} - 3\hat{k}\right)$$

Direction ratios of the line are $= -2, 6, -3$

Direction cosines of the line are,

$$\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$$

$$\Rightarrow \quad \frac{-2}{\sqrt{(-2)^2+(6)^2+(-3)^2}}, \frac{6}{\sqrt{(-2)^2+(6)^2+(-3)^2}}, \frac{-3}{\sqrt{(-2)^2+(6)^2+(-3)^2}}$$

$$\Rightarrow \quad \frac{-2}{7}, \frac{6}{7}, \frac{-3}{7}$$

Straight Line in Space Ex 28.1 Q12

$$x = ay + b,$$
$$z = cy + d$$
$$\frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c} = \lambda(say)$$

So DR's of line are (a, 1, c)

From above equation, we can write

$$x = a\lambda + b$$
$$y = \lambda$$
$$z = c\lambda + d$$

So vector equation of line is

$$x\hat{i} + y\hat{j} + z\hat{k} = (b\hat{i} + d\hat{k}) + \lambda\left(a\hat{i} + \hat{j} + c\hat{k}\right)$$

Straight Line in Space Ex 28.1 Q13

We know that, equation of a line passing through $\vec{a}$ and parallel to vector $\vec{b}$ is,

$$\vec{r} = \vec{a} + \lambda\vec{b} \qquad ---(i)$$

Here, $\vec{a} = \hat{i} - 2\hat{j} - 3\hat{k}$

and, $\vec{b} =$ line joining $\left(\hat{i} - \hat{j} + 4\hat{k}\right)$ and $\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$

$$= \left(2\hat{i} + \hat{j} + 2\hat{k}\right) - \left(\hat{i} - \hat{j} + 4\hat{k}\right)$$
$$= 2\hat{i} - \hat{i} + \hat{j} + \hat{j} + 2\hat{k} - 4\hat{k}$$
$$= \hat{i} + 2\hat{j} - 2\hat{k}$$

Equation of the line is

$$\vec{r} = \left(\hat{i} - 2\hat{j} - 3\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$$

For cortesion form of equation put $x\hat{i} + y\hat{j} + z\hat{k}$,

$$x\hat{i} + y\hat{j} + z\hat{k} = (1+\lambda)\hat{i} + (-2+2\lambda)\hat{j} + (-3-2\lambda)\hat{k}$$

Equating coefficients of $\hat{i}, \hat{j}, \hat{k}$, so

$$x = 1 + \lambda,\ y = -2 + 2\lambda,\ z = -3 - 2\lambda$$

$$\Rightarrow \quad \frac{x-1}{1} = \lambda,\ \frac{y+2}{2} = \lambda,\ \frac{z+3}{-2} = \lambda$$

So, $\dfrac{x-1}{1} = \dfrac{y+2}{2} = \dfrac{z+3}{-2}$

Straight Line in Space Ex 28.1 Q14

Distance of point P from $Q = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

$$PQ = \sqrt{(3\lambda - 2 - 1)^2 + (2\lambda - 1 - 3)^2 + (2\lambda + 3 - 3)^2}$$

$$\Rightarrow \quad (5)^2 = (3\lambda - 3)^2 + (2\lambda - 4)^2 + (2\lambda)^2$$
$$\Rightarrow \quad 25 = 9\lambda^2 + 9 - 18\lambda + 4\lambda^2 + 16 - 16\lambda + 4\lambda^2$$
$$\Rightarrow \quad 17\lambda^2 - 34\lambda = 0$$
$$\Rightarrow \quad 17\lambda(\lambda - 2) = 0$$
$$\Rightarrow \quad \lambda = 0 \text{ or } 2$$

So, points on the line are $(3(0) - 2,\ 2(0) - 1,\ 2(0) + 3)$

$(3(2) - 2,\ 2(2) - 1,\ 2(2) + 3)$

$= (-2, -1, 3),\ (4, 3, 7)$

Straight Line in Space Ex 28.1 Q15

Let the given points are A, B, C with position vectors $\vec{a}, \vec{b}, \vec{c}$ respectively, so

$\vec{a} = -2\hat{i} + 3\hat{j},\ \vec{b} = \hat{i} + 2\hat{j} + 3\hat{k},\ \vec{c} = 7\hat{i} - \hat{k}$

We know that, equation of a line passing through $\vec{a}$ and $\vec{b}$ are,

$$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$$
$$= (-2\hat{i} + 3\hat{j}) + \lambda((\hat{i} + 2\hat{j} + 3\hat{k}) - (-2\hat{i} + 3\hat{j}))$$
$$= (-2\hat{i} + 3\hat{j}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k} + 2\hat{i} - 3\hat{j})$$
$$\vec{r} = (-2\hat{i} + 3\hat{j}) + \lambda(3\hat{i} - \hat{j} + 3\hat{k}) \qquad ---(i)$$

If A, B, C are collinear then $\vec{c}$ must satisfy equation (i),

$$7\hat{i} - \hat{k} = (-2 + 3\lambda)\hat{i} + (3 - \lambda)\hat{j} + (3\lambda)\hat{k}$$

Equation the coefficients of $\hat{i}, \hat{j}, \hat{k}$,

$-2 + 3\lambda = 7 \quad \Rightarrow \lambda = 3$

$3 - \lambda = 0 \quad \Rightarrow \lambda = 3$

$3\lambda = -1 \quad \Rightarrow \lambda = -\frac{1}{3}$

Since, value of λ are not equal, so,

Given points are not collinear.

Straight Line in Space Ex 28.1 Q16

We know that, equation of a line passing through a point (x_1, y_1, z_1) and having direction ratios proportional to a, b, c is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \qquad ---(i)$$

Here, $(x_1, y_1, z_1) = (1, 2, 3)$ and

Given line $\frac{-x - 2}{1} = \frac{y + 3}{7} = \frac{2z - 6}{3}$

$$\Rightarrow \quad \frac{x + 2}{-1} = \frac{y + 3}{7} = \frac{z - 3}{\frac{3}{2}}$$

It parallel to the required line, so

$$a = \mu,\ b = 7\mu,\ c = \frac{3}{2}\mu$$

So, equation of required line using equation (i) is,

$$\frac{x - 1}{-\mu} = \frac{y - 2}{7\mu} = \frac{z - 3}{\frac{3}{2}\mu}$$

$$\Rightarrow \quad \frac{x - 1}{-1} = \frac{y - 2}{7} = \frac{z - 3}{\frac{3}{2}}$$

Straight Line in Space Ex 28.1 Q17

Given equation of line is,

$$3x + 1 = 6y - 2 = 1 - z$$

Dividing all by 6,

$$\frac{3x+1}{6} = \frac{6y-2}{6} = \frac{1-z}{6}$$

$$\Rightarrow \quad \frac{3x}{6} + \frac{1}{6} = \frac{6y}{6} - \frac{2}{6} = \frac{1}{6} - \frac{z}{6}$$

$$\Rightarrow \quad \frac{1}{2}x + \frac{1}{6} = y - \frac{1}{3} = -\frac{z}{6} + \frac{1}{6}$$

$$\Rightarrow \quad \frac{1}{2}\left(x + \frac{1}{3}\right) = 1\left(y - \frac{1}{3}\right) = +\frac{1}{6}(z - 1)$$

$$\Rightarrow \quad \frac{x + \frac{1}{3}}{2} = \frac{y - \frac{1}{3}}{1} = \frac{z-1}{-6} = \lambda \ \text{(say)} \qquad ---(i)$$

Comparing it with equation of line passing through (x_1, y_1, z_1) and direction ratios a, b, c,

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

$$\Rightarrow \quad (x_1, y_1, z_1) = \left(-\frac{1}{3}, \frac{1}{3}, 1\right)$$

$$a = 2,\ b = 1,\ -6$$

So, direction ratios of the line are $= 2, 1, -6$

From equation (i),

$$x = \left(2\lambda - \frac{1}{3}\right),\ y = \left(\lambda + \frac{1}{3}\right),\ z = (-6\lambda + 1)$$

So, vector equation of the given line is,

$$x\hat{i} + y\hat{j} + z\hat{k} = \left(2\lambda - \frac{1}{3}\right)\hat{i} + \left(\lambda + \frac{1}{3}\right)\hat{j} + (-6\lambda + 1)\hat{k}$$

$$\vec{r} = \left(-\frac{1}{3}\hat{i} + \frac{1}{3}\hat{j} + \hat{k}\right) + \lambda\left(2\hat{i} + \hat{j} - 6\hat{k}\right)$$

Ex 28.2

Straight Line in Space Ex 28.2 Q1

Let $l_1 = \frac{12}{13}, m_1 = -\frac{3}{13}, n_1 = -\frac{4}{13}$

$l_2 = \frac{4}{13}, m_2 = \frac{12}{13}, n_1 = \frac{3}{13}$

$l_3 = \frac{3}{13}, m_3 = -\frac{4}{13}, n_3 = \frac{12}{13}$

$l_1 l_2 + m_1 m_2 + n_1 n_2$

$= \frac{12}{13} \times \frac{4}{13} + (-\frac{3}{13}) \times \frac{12}{13} + (-\frac{4}{13}) \times \frac{3}{13}$

$= \frac{48 - 36 - 12}{169} = 0$

$l_2 l_3 + m_2 m_3 + n_2 n_3$

$= \frac{4}{13} \times \frac{3}{13} + \frac{12}{13} \times (-\frac{4}{13}) + \frac{3}{13} \times \frac{12}{13}$

$= \frac{12 - 48 + 36}{169} = 0$

$l_1 l_3 + m_1 m_3 + n_1 n_3$

$= \frac{12}{13} \times \frac{3}{13} + (-\frac{3}{13}) \times (-\frac{4}{13}) + (-\frac{4}{13}) \times \frac{12}{13}$

$= \frac{36 + 12 - 48}{169} = 0$

$\therefore$ The lines are mutually perpendicular.

Straight Line in Space Ex 28.2 Q2

The direction ratios of a line passing through the points
$(1, -1, 2)$ and $(3, 4, -2)$ are
$(3 - 1, 4 + 1, -2 - 2)$
$= (2, 5, -4)$

The direction ratios of a line passing through the points
$(0, 3, 2)$ and $(3, 5, 6)$ are
$(3 - 0, 5 - 3, 6 - 2)$
$= (3, 2, 4)$

Angle between the lines

$$\cos\theta = \frac{(a_1 a_2 + b_1 b_2 + c_1 c_2)}{\sqrt{a_1{}^2 + b_1{}^2 + c_1{}^2}\sqrt{a_2{}^2 + b_2{}^2 + c_2{}^2}}$$

$$\cos\theta = \frac{[2 \times 3 + 5 \times 2 + (-4) \times 4]}{\sqrt{2^2 + 5^2 + (-4)^2}\sqrt{3^2 + 2^2 + 4^2}}$$

$$\cos\theta = \frac{0}{\sqrt{2^2 + 5^2 + (-4)^2}\sqrt{3^2 + 2^2 + 4^2}}$$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}$

The lines are mutually perpendicular.

Straight Line in Space Ex 28.2 Q3

The direction ratios of a line passing through the points
$(4, 7, 8)$ and $(2, 3, 4)$ are
$(4 - 2, 7 - 3, 8 - 4)$
$= (2, 4, 4)$
The direction ratios of a line passing through the points
$(-1, -2, 1)$ and $(1, 2, 5)$ are
$(-1 - 1, -2 - 2, 1 - 5)$
$= (-2, -4, -4)$

The direction ratios are proportional.

$\frac{2}{-2} = \frac{4}{-4} = \frac{4}{-4}$

Hence, the lines are mutually parallel.

Straight Line in Space Ex 28.2 Q4

The Cartesian equation of a line passing through (x_1, y_1, z_1)
and with direction ratios (a_1, b_1, c_1)

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$$

The Cartesian equation of a line passing through $(-2, 4, -5)$
and parallel to the line $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$ is

$$\frac{x+2}{3}=\frac{y-4}{5}=\frac{z+5}{6}$$

Straight Line in Space Ex 28.2 Q5

Given equations of lines are $\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z}{1}$ and $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$.

Clearly,

$7\times 1+(-5)\times 2+1\times 3$

$=7-10+3$

$=0$

$\therefore$ Lines $\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z}{1}$ and $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ are perpendicular to each other.

Straight Line in Space Ex 28.2 Q6

The direction ratios of a line joining the origin to the point $(2, 1, 1)$
are $(2-0, 1-0, 1-0)=(2, 1, 1)$
The direction ratios of a line joining $(3, 5, -1)$ and $(4, 3, -1)$
are $(4-3, 3-5, -1+1)=(1, -2, 0)$

Angle between the lines

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\cos\theta=\frac{2\times 1+1\times(-2)+1\times 0}{\sqrt{2^2+1^2+1^2}\sqrt{1^2+(-2)^2+0^2}}$$

$$\cos\theta=\frac{0}{\sqrt{6}\sqrt{5}}$$

$$\cos\theta=0$$

$$\therefore \theta=\frac{\pi}{2}$$

The lines are mutually perpendicular.

Straight Line in Space Ex 28.2 Q7

Vector equation of a line is

$\vec{r}=\vec{a}+\lambda\vec{b}$

The direction cosines of the x-axis are $(1, 0, 0)$. Equation of a line parallel
to the x-axis and passing through the origin is

$\vec{r}=(0\hat{i}+0\hat{j}+0\hat{k})+\lambda(1\hat{i}+0\hat{j}+0\hat{k})$

$\vec{r}=\lambda\hat{i}$

Straight Line in Space Ex 28.2 Q8(i)

We know that, If Q be the angle between two lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$, then

$$\cos\theta = \frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{\left|\overrightarrow{b_1}\right|.\left|\overrightarrow{b_2}\right|} \qquad ---(i)$$

Here, $\vec{r} = \left(4\hat{i} - \hat{j}\right) + \lambda\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$

and, $\vec{r} = \left(\hat{i} - \hat{j} + 2\hat{k}\right) - \mu\left(2\hat{i} + 4\hat{j} - 4\hat{k}\right)$

$$\Rightarrow \quad \overrightarrow{a_1} = 4\hat{i} - \hat{j}, \quad \overrightarrow{b_1} = \hat{i} + 2\hat{j} - 2\hat{k}$$
$$\overrightarrow{a_2} = \hat{i} - \hat{j} + 2\hat{k}, \overrightarrow{b_2} = 2\hat{i} + 4\hat{j} - 4\hat{k}$$

$$\left|\overrightarrow{b_1}\right| = \sqrt{(1)^2 + (2)^2 + (-2)^2} = 3$$
$$\left|\overrightarrow{b_2}\right| = \sqrt{(2)^2 + (4)^2 + (-4)^2} = 6$$

Let θ be the angle between given lines. So using equation (i),

$$\cos\theta = \frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{\left|\overrightarrow{b_1}\right|.\left|\overrightarrow{b_2}\right|}$$
$$= \frac{\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)\left(2\hat{i} + 4\hat{j} - 4\hat{k}\right)}{3.6}$$
$$= \frac{2 + 8 + 8}{18}$$
$$\cos\theta = 1$$

$$\theta = 0°$$

Straight Line in Space Ex 28.2 Q8(ii)

We know that, angle between two lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$, is given by

$$\cos\theta = \frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{\left|\overrightarrow{b_1}\right|.\left|\overrightarrow{b_2}\right|} \qquad ---(i)$$

Given lines are,

$$\vec{r} = \left(3\hat{i} + 2\hat{j} - 4\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$$
$$\vec{r} = \left(5\hat{j} - 2\hat{k}\right) + \mu\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$$

$$\Rightarrow \quad \overrightarrow{b_1} = \hat{i} + 2\hat{j} + 2\hat{k}, \ \overrightarrow{b_2} = 3\hat{i} + 2\hat{j} + 6\hat{k}$$

$$\left|\overrightarrow{b_1}\right| = \sqrt{(1)^2 + (2)^2 + (2)^2} = 3$$
$$\left|\overrightarrow{b_2}\right| = \sqrt{(3)^2 + (2)^2 + (6)^2} = 7$$

Let θ be the angle between given lines, so using equation (i),

$$\cos\theta = \frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{\left|\overrightarrow{b_1}\right|.\left|\overrightarrow{b_2}\right|}$$
$$= \frac{\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)}{3.7}$$
$$= \frac{3 + 4 + 12}{21}$$
$$= \frac{19}{21}$$

$$\theta = \cos^{-1}\left(\frac{19}{21}\right)$$

Straight Line in Space Ex 28.2 Q8(iii)

We know that, angle between two lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$, is given by

$$\cos\theta = \frac{\vec{b_1}.\vec{b_2}}{|\vec{b_1}||\vec{b_2}|} \qquad ---(i)$$

Equation of given lines are,

$$\vec{r} = \lambda\left(\hat{i} + \hat{j} + 2\hat{k}\right) \text{ and}$$

$$\vec{r} = 2\hat{j} + \mu\left[\left(\sqrt{3} - 1\right)\hat{i} - \left(\sqrt{3} + 1\right)\hat{j} + 4\hat{k}\right]$$

$$\Rightarrow \quad \vec{b_1} = \left(\hat{i} + \hat{j} + 2\hat{k}\right), \ \vec{b_2} = \left(\sqrt{3} - 1\right)\hat{i} - \left(\sqrt{3} + 1\right)\hat{j} + 4\hat{k}$$

Let θ be the angle between given lines, so using equation (i),

$$\cos\theta = \frac{\vec{b_1}.\vec{b_2}}{|\vec{b_1}||\vec{b_2}|}$$

$$= \frac{\left(\hat{i} + \hat{j} + 2\hat{k}\right)\left(\left(\sqrt{3} - 1\right)\hat{i} - \left(\sqrt{3} + 1\right)\hat{j} + 4\hat{k}\right)}{\sqrt{(1)^2 + (1)^2 + (2)^2}\sqrt{\left(\sqrt{3} - 1\right)^2 + \left(-\sqrt{3} - 1\right)^2 + (4)^2}}$$

$$= \frac{\sqrt{3} - 1 - \sqrt{3} - 1 + 8}{\sqrt{6}.\sqrt{3 + 1 - 2\sqrt{3} + 3 + 1 + 2\sqrt{3} + 16}}$$

$$= \frac{6}{\sqrt{6}.2\sqrt{6}}$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

Straight Line in Space Ex 28.2 Q9(i)

We know that, angle between two lines

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1} \text{ and } \qquad \frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$$

is given by,

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \qquad ---(i)$$

Here, given lines are,

$$\frac{x + 4}{3} = \frac{y - 1}{5} = \frac{z + 3}{4} \text{ and } \frac{x + 1}{1} = \frac{y - 4}{1} = \frac{z - 5}{2}$$

$$\Rightarrow \quad a_1 = 3, b_1 = 5, c_1 = 4, a_2 = 1, b_2 = 1, c_2 = 2$$

Let θ be the angle between given lines, so using equation (i),

$$\cos\theta = \frac{(3)(1) + (5)(1) + (4)(2)}{\sqrt{(3)^2 + (5)^2 + (4)^2}\sqrt{(1)^2 + (1)^2 + (2)^2}}$$

$$= \frac{3 + 5 + 8}{\sqrt{50}\sqrt{6}}$$

$$= \frac{16}{10\sqrt{3}}$$

$$\cos\theta = \frac{8}{5\sqrt{3}}$$

$$\theta = \cos^{-1}\left(\frac{8}{5\sqrt{3}}\right)$$

Straight Line in Space Ex 28.2 Q9(ii)

We know that, angle between two lines

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

is given by,

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \qquad ---(i)$$

Given, equation of lines are,

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{-3} \text{ and } \frac{x+3}{-1}=\frac{y-5}{8}=\frac{z-1}{4}$$

$\Rightarrow \quad a_1=2,\ b_1=3,\ c_1=-3,\ a_2=-1,\ b_2=8,\ c_2=4$

Let θ be the angle between two given lines, so using equation (i),

$$\cos\theta=\frac{(2)(-1)+(3)(8)+(-3)(4)}{\sqrt{(2)^2+(3)^2+(-3)^2}\sqrt{(-1)^2+(8)^2+(4)^2}}$$

$$=\frac{-2+24-12}{\sqrt{22}\sqrt{81}}$$

$$\cos\theta=\frac{10}{9\sqrt{22}}$$

$$\theta=\cos^{-1}\left(\frac{10}{9\sqrt{22}}\right)$$

Straight Line in Space Ex 28.2 Q9(iii)

We know that, angle between two lines

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

is given by,

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \qquad ---(i)$$

Given lines are,

$$\frac{5-x}{-2}=\frac{y+3}{1}=\frac{1-z}{3} \text{ and } \frac{x}{3}=\frac{1-y}{-2}=\frac{z+5}{-1}$$

$$\Rightarrow \quad \frac{x-5}{2}=\frac{y+3}{1}=\frac{z-1}{-3} \text{ and } \frac{x}{3}=\frac{y-1}{2}=\frac{z+5}{-1}$$

$\Rightarrow \quad a_1=2,\ b_1=1,\ c_1=-3,\ a_2=3,\ b_2=2,\ c_2=-1$

Let θ be the angle between given lines, so using equation (i),

$$\cos\theta=\frac{(2)(3)+(1)(2)+(-3)(-1)}{\sqrt{(2)^2+(1)^2+(-3)^2}\sqrt{(3)^2+(2)^2+(-1)^2}}$$

$$=\frac{6+2+3}{\sqrt{14}\sqrt{14}}$$

$$\cos\theta=\frac{11}{14}$$

$$\theta=\cos^{-1}\left(\frac{11}{14}\right)$$

Straight Line in Space Ex 28.2 Q9(iv)

We know that, angle between two lines

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

is given by,

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

Equation of given lines are,

$$\frac{x-2}{3}=\frac{y+3}{-2}, z=5 \text{ and } \frac{x+1}{1}=\frac{2y-3}{3}=\frac{z-5}{2}$$

$$\Rightarrow \quad \frac{x-2}{3}=\frac{y+3}{-2}, z=5 \text{ and } \frac{x+1}{1}=\frac{\frac{y-3}{3}}{\frac{3}{2}}=\frac{z-5}{2}$$

$$\Rightarrow \quad a_1=3,\ b_1=-2,\ c_1=0,\ a_2=1,\ b_2=\frac{3}{2},\ c_2=2$$

Let θ be the angle between given lines, so from equation (i),

$$\cos\theta=\frac{(3)(1)+(-2)\left(\frac{3}{2}\right)+(0)(2)}{\sqrt{(3)^2+(-2)^2+(0)^2}\sqrt{(1)^2+\left(\frac{3}{2}\right)^2+(2)^2}}$$

$$=\frac{3-3+0}{\sqrt{38}\sqrt{\frac{29}{4}}}$$

$$\cos\theta=0$$

$$\theta=\frac{\pi}{2}$$

Straight Line in Space Ex 28.2 Q9(v)

$$\frac{x-5}{1}=\frac{2y+6}{-2}=\frac{z-3}{1} \text{ and } \frac{x-2}{3}=\frac{y+1}{4}=\frac{z-6}{5}$$

$\hat{a}=\hat{i}-2\hat{j}+\hat{k}, \hat{b}=3\hat{i}+4\hat{j}+5\hat{k}$ are the vectors parallel to above lines

$\therefore$ angle between $\hat{a}$ and $\hat{b} \to \cos\theta=\dfrac{\hat{a}\cdot\hat{b}}{|\hat{a}||\hat{b}|}$

$$\cos\theta=\frac{(\hat{i}-2\hat{j}+\hat{k})\cdot(3\hat{i}+4\hat{j}+5\hat{k})}{|\hat{i}-2\hat{j}+\hat{k}||\hat{i}-2\hat{j}+\hat{k}|}=\frac{3-8+5}{|\hat{i}-2\hat{j}+\hat{k}||\hat{i}-2\hat{j}+\hat{k}|}=0$$

$$\cos\theta=0 \to \theta=90^\circ$$

Straight Line in Space Ex 28.2 Q9(vi)

$$\frac{x-2}{2}=\frac{y-1}{7}=\frac{z+3}{-3} \text{ and } \frac{x+2}{-1}=\frac{y-4}{2}=\frac{z-5}{4}$$

$\hat{a}=2\hat{i}+7\hat{j}-3\hat{k}, \hat{b}=-1\hat{i}+4\hat{j}+4\hat{k}$ are the vectors parallel to above lines

$\therefore$ angle between $\hat{a}$ and $\hat{b} \to \cos\theta=\dfrac{\hat{a}\cdot\hat{b}}{|\hat{a}||\hat{b}|}$

$$\cos\theta=\frac{(2\hat{i}+7\hat{j}-3\hat{k})\cdot(-1\hat{i}+2\hat{j}+4\hat{k})}{|(2\hat{i}+7\hat{j}-3\hat{k})||(-1\hat{i}+2\hat{j}+4\hat{k})|}=0$$

$$\cos\theta=0 \to \theta=90^\circ$$

Straight Line in Space Ex 28.2 Q10(i)

We know that, angle (θ) between two lines

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

is given by,

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \quad ---(i)$$

Here, $a_1=5,\ b_1=-12,\ c_1=13$
$a_2=-3,\ b_2=4,\ c_2=5$

Let θ be the required angle, so using equation (i),

$$\cos\theta=\frac{(5)(-3)+(-12)(4)+(13)(5)}{\sqrt{(5)^2+(-12)^2+(13)^2}\sqrt{(-3)^2+(4)^2+(5)^2}}$$

$$=\frac{-15-48+65}{\sqrt{169\times 2}\sqrt{25\times 2}}$$

$$=\frac{2}{65\times 2}$$

$$\cos\theta=\frac{1}{65}$$

$$\theta=\cos^{-1}\left(\frac{1}{65}\right)$$

Straight Line in Space Ex 28.2 Q10(ii)

We know that, angle (θ) between lines

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

is given by,

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \quad ---(i)$$

Here, $a_1=2,\ b_1=2,\ c_1=1$
$a_2=4,\ b_2=1,\ c_2=8$

Let θ be required angle, so using equation (i),

$$\cos\theta=\frac{(2)(4)+(2)(1)+(1)(8)}{\sqrt{(2)^2+(2)^2+(1)^2}\sqrt{(4)^2+(1)^2+(8)^2}}$$

$$=\frac{8+2+8}{3.9}$$

$$=\frac{18}{27}$$

$$\cos\theta=\frac{2}{3}$$

$$\theta=\cos^{-1}\left(\frac{2}{3}\right)$$

Straight Line in Space Ex 28.2 Q10(iii)

We know that, angle (θ) between two lines

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

is given by,

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \quad ---(i)$$

Here, $a_1=1,\ b_1=2,\ c_1=-2$
$a_2=-2,\ b_2=2,\ c_2=1$

Let θ be the required angle, so using equation (i),

$$\cos\theta=\frac{(1)(-2)+(2)(2)+(-2)(1)}{\sqrt{(1)^2+(2)^2+(-2)^2}\sqrt{(-2)^2+(2)^2+(1)^2}}$$

$$=\frac{-2+4-2}{3.3}$$

$$=\frac{0}{9}$$

$$\cos\theta=0$$

$$\theta=\frac{\pi}{2}$$

Straight Line in Space Ex 28.2 Q10(vi)

a, b, c and $b - c,\ c - a,\ a - b$ are direction ratios

these are the vectors with above direction ratios

$\hat{x} = a\hat{i} + b\hat{j} + c\hat{k},\ \hat{y} = (b - c)\hat{i} + (c - a)\hat{j} + (a - b)\hat{k}$

are the vectors parallel to two given lines

$\therefore$ angle between the lines with above

direction ratios are $\hat{x}$ and $\hat{y} \rightarrow \cos\theta = \dfrac{\hat{x}\cdot\hat{y}}{|\hat{x}||\hat{y}|}$

$$\cos\theta = \frac{(a\hat{i} + b\hat{j} + c\hat{k})\cdot((b-c)\hat{i} + (c-a)\hat{j} + (a-b)\hat{k})}{|a\hat{i} + b\hat{j} + c\hat{k}||(b-c)\hat{i} + (c-a)\hat{j} + (a-b)\hat{k}|}$$

$$= \frac{a(b-c) + b(c-a) + c(a-b)}{\sqrt{a^2+b^2+c^2}\sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2}}$$

$$= \frac{ab - ac + bc - ba + ca - cb}{\sqrt{a^2+b^2+c^2}\sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2}} = 0$$

$\cos\theta = 0 \rightarrow \theta = 90°$

Straight Line in Space Ex 28.2 Q11

We know that, angle (θ) between two lines

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1} \text{ and } \frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$$

is given by,

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Here, Direction ratios of first line is 2, 2, 1

$\Rightarrow \quad a_1 = 2,\ b_1 = 2,\ c_1 = 1$

Direction ratios of the line joining $(3, 1, 4)$ and $(7, 2, 12)$ is given by

$= (7 - 3), (2 - 1), (12 - 4)$

$= 4, 1, 8$

$\Rightarrow \quad a_2 = 4,\ b_2 = 1,\ c_2 = 8$

Let θ be the required angle, so using equation (i),

$$\cos\theta = \frac{(2)(4) + (2)(1) + (1)(8)}{\sqrt{(2)^2 + (2)^2 + (1)^2}\sqrt{(4)^2 + (1)^2 + (8)^2}}$$

$$= \frac{8 + 2 + 8}{3.9}$$

$$\cos\theta = \frac{2}{3}$$

$$\theta = \cos^{-1}\left(\frac{2}{3}\right)$$

Straight Line in Space Ex 28.2 Q12

We know that equation of a line passing through (x_1, y_1, z_1) and direction ratios are a, b, c is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \qquad ---(i)$$

Here, $(x_1, y_1, z_1) = (1, 2, -4)$

and required line is parallel to the given line

$$\frac{x-3}{4} = \frac{y-5}{2} = \frac{z+1}{3}$$

$\Rightarrow$ Direction ratios of the required line are proportional to $4, 2, 3$

$\Rightarrow$ $a = 4\lambda,\ b = 2\lambda,\ c = 3\lambda$

So, required equation of the line is

$$\Rightarrow \quad \frac{x-1}{4\lambda} = \frac{y-2}{2\lambda} = \frac{z+4}{3\lambda}$$

$$\Rightarrow \quad \frac{x-1}{4} = \frac{y-2}{2} = \frac{z+4}{3}$$

Straight Line in Space Ex 28.2 Q13

We know that, equation of a line passing through (x_1, y_1, z_1) and direction ratios are a, b, c is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \qquad ---(i)$$

Here, $(x_1, y_1, z_1) = (-1, 2, 1)$

and required line is parallel to the given line

$$\frac{2x-1}{4} = \frac{3y+5}{2} = \frac{2-z}{3}$$

$$\Rightarrow \quad \frac{x - \frac{1}{2}}{2} = \frac{y + \frac{5}{3}}{\frac{2}{3}} = \frac{z-2}{-3}$$

$\Rightarrow$ Direction ratios of the required line are proportional to $2, \frac{2}{3}, -3$

$\Rightarrow$ $a = 2\lambda,\ b = \frac{2}{3}\lambda,\ c = -3\lambda$

So, required equation of the line using equation (i),

$$\frac{x+1}{2\lambda} = \frac{y-2}{\frac{2}{3}\lambda} = \frac{z-1}{-3\lambda}$$

$$\Rightarrow \quad \frac{x+1}{2} = \frac{y-2}{\frac{2}{3}} = \frac{z-1}{-3}$$

Straight Line in Space Ex 28.2 Q14

We know that equation of *a* line passing through the point $\vec{a}$ and is the direction of vector $\vec{b}$ is

$$\vec{r} = \vec{a} + \lambda\vec{b} \qquad ---(i)$$

Here, $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$

and given that the required line is parallel to

$$\vec{r} = \left(\hat{i} - 2\hat{j} + \hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} - 5\hat{k}\right)$$

$$\Rightarrow \quad \vec{b} = \left(2\hat{i} + 3\hat{j} - 5\hat{k}\right).\mu$$

So, required equation of the line using equation (i) is

$$\vec{r} = \left(2\hat{i} - \hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} - 5\hat{k}\right).\mu$$

$$\vec{r} = \left(2\hat{i} - \hat{j} + 3\hat{k}\right) + \lambda'\left(2\hat{i} + 3\hat{j} - 5\hat{k}\right)$$

where λ' is a scalar such that $\lambda' = \lambda.\mu$

Straight Line in Space Ex 28.2 Q15

We know that, equation of a line passing through (x_1, y_1, z_1) with direction ratios a, b, c is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

So, equation of required line passing through $(2, 1, 3)$ is

$$\frac{x-2}{a} = \frac{y-1}{b} = \frac{z-3}{c} \qquad ---(1)$$

Given that line $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ is perpendicular to line (i), so

$$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$
$$(a)(1) + (b)(2) + (c)(3) = 0$$
$$a + 2b + 3c = 0 \qquad ---(2)$$

And line $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$ is perpendicular to line (i), so

$$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$
$$(a)(-3) + (b)(2) + (c)(5) = 0$$
$$-3a + 2b + 5c = 0 \qquad ---(3)$$

Solving equation (2) and (3) by cross multiplication,

$$\frac{a}{(2)(5) - (2)(3)} = \frac{b}{(-3)(3) - (1)(5)} = \frac{c}{(1)(2) - (-3)(2)}$$

$$\Rightarrow \quad \frac{a}{10-6} = \frac{b}{-9-5} = \frac{c}{2+6}$$

$$\Rightarrow \quad \frac{a}{4} = \frac{b}{-14} = \frac{c}{8}$$

$$\Rightarrow \quad \frac{a}{2} = \frac{b}{-7} = \frac{c}{4} = \lambda \text{ (Say)}$$

$$\Rightarrow \quad a = 2\lambda,\ b = -7\lambda,\ c = 4\lambda$$

Using a, b, c in equation (i),

$$\frac{x-2}{2\lambda} = \frac{y-1}{-7\lambda} = \frac{z-3}{4\lambda}$$

$$\Rightarrow \quad \frac{x-2}{2} = \frac{y-1}{-7} = \frac{z-3}{4}$$

Straight Line in Space Ex 28.2 Q16

We know that equation of a line passing through a point with position vector $\vec{\alpha}$ and perpendiculat to $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ is given by

$$\vec{r} = \vec{\alpha} + \lambda\left(\vec{b_1} \times \vec{b_2}\right) \qquad ---(i)$$

Here, $\vec{\alpha} = \left(\hat{i} + \hat{j} - 3\hat{k}\right)$

and required line is perpendicular to

$$\vec{r} = \hat{i} + \lambda\left(2\hat{i} + \hat{j} - 3\hat{k}\right) \text{ and}$$
$$\vec{r} = \left(2\hat{i} + \hat{j} - \hat{k}\right) + \mu\left(\hat{i} + \hat{j} + \hat{k}\right)$$

$$\Rightarrow \quad \vec{b_1} = \left(2\hat{i} + \hat{j} - 3\hat{k}\right),\ \vec{b_2} = \hat{i} + \hat{j} + \hat{k}$$

Now,

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix}$$
$$= \hat{i}(1+3) - \hat{j}(2+3) + \hat{k}(2-1)$$
$$\vec{b_1} \times \vec{b_2} = 4\hat{i} - 5\hat{j} + \hat{k}$$

Using equation, required equation of line is

$$\vec{r} = \vec{\alpha} + \lambda\left(\vec{b_1} \times \vec{b_2}\right)$$

$$\vec{r} = \left(\hat{i} + \hat{j} - 3\hat{k}\right) + \lambda\left(4\hat{i} - 5\hat{j} + \hat{k}\right)$$

Straight Line in Space Ex 28.2 Q17

We know that equation of a line passing through (x_1, y_1, z_1) and direction ratios as a, b, c is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \qquad ----(1)$$

So, equation of a line passing through $(1, -1, 1)$ is

$$\frac{x - 1}{a} = \frac{y + 1}{b} = \frac{z - 1}{c} \qquad ---(2)$$

Now, Directions ratios of the line joining $A(4, 3, 2)$ and $B(1, -1, 0)$

$= (1 - 4), (-1 - 3), (0 - 2)$

$\Rightarrow$ Direction ratios of line $AB = -3, -4, -2$

and, Directions ratios of the line joining $C(1, 2, -1)$ and $D(2, 1, 1)$

$= (2 - 1), (1 - 2), (1 + 1)$

$\Rightarrow$ Direction ratios of line $CD = 1, -1, 2$

Given that, line AB is perpendicular to line (2), so

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(a)(-3) + (b)(-4) + (c)(-2) = 0$

$-3a + 4b - 2c = 0$

$3a + 4b + 2c = 0 \qquad ---(3)$

and, line CD is also perpendicular to line (2), so

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(a)(1) + (b)(-1) + (c)(2) = 0$

$a - b + 2c = 0 \qquad ---(4)$

Solving equation (3) and (4) using cross multiplication,

$$\frac{a}{(4)(2) - (-1)(2)} = \frac{b}{(1)(2) - (3)(2)} = \frac{c}{(3)(-1) - (4)(1)}$$

$\Rightarrow \quad \dfrac{a}{8 + 2} = \dfrac{b}{2 - 6} = \dfrac{c}{-3 - 4}$

$\Rightarrow \quad \dfrac{a}{10} = \dfrac{b}{-4} = \dfrac{c}{-7} = \lambda$ (Say)

$\Rightarrow \quad a = 10\lambda,\ b = -4\lambda,\ c = -7\lambda$

Straight Line in Space Ex 28.2 Q18

We know that equation of a line passing through a point (x_1, y_1, z_1) and direction ratios a, b, c is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

So, equation of required line passing through $(1, 2, -4)$ is

$$\frac{x - 1}{a} = \frac{y - 2}{b} = \frac{z + 4}{c} \qquad ---(1)$$

Given that, line $\frac{x - 8}{8} = \frac{y + 9}{-16} = \frac{z - 10}{7}$ is perpendicular to line (1), so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$\Rightarrow \quad (a)(8) + (b)(-16) + (c)(7) = 0$$

$$\Rightarrow \quad 8a - 16b + 7c = 0 \qquad ----(2)$$

also, line $\frac{x - 15}{3} = \frac{y - 29}{8} = \frac{z - 5}{-5}$ is perpendicular to line (1), so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$\Rightarrow \quad (3)(a) + (8)(b) + (-5)(c) = 0$$

$$\Rightarrow \quad 3a + 8b - 5c = 0 \qquad ----(3)$$

Solving equation (2) and (3) by cross-multiplication,

$$\frac{a}{(-16)(-5) - (8)(7)} = \frac{b}{(3)(7) - (8)(-5)} = \frac{c}{(8)(8) - (3)(-16)}$$

$$\Rightarrow \quad \frac{a}{80 - 56} = \frac{b}{21 + 40} = \frac{c}{64 + 48}$$

$$\Rightarrow \quad \frac{a}{24} = \frac{b}{61} = \frac{c}{112} = \lambda \text{ (Say)}$$

$$\Rightarrow \quad a = 24\lambda,\ b = 61\lambda,\ c = 112\lambda$$

Put a, b, c in equation (1) to get required equation of the line, so

$$\frac{x - 1}{24\lambda} = \frac{y - 2}{61\lambda} = \frac{z + 4}{112\lambda}$$

$$\Rightarrow \quad \frac{x - 1}{24} = \frac{y - 2}{61} = \frac{z + 4}{112}$$

Straight Line in Space Ex 28.2 Q19

Equation of lines are,

$$\frac{x - 5}{7} = \frac{y + 2}{-5} = \frac{z}{1}$$

and, $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$

Now, $a_1a_2 + b_1b_2 + c_1c_2$

$$= (7)(1) + (-5)(2) + (1)(3)$$

$$= 7 - 10 + 3$$

$$= 0$$

So, given lines are perpendicular.

Straight Line in Space Ex 28.2 Q20

We know that, equation of a line passing through the point (x_1, y_1, z_1) and direction ratios a, b, c is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \qquad ---(1)$$

So, equation of line passing through $(2, -1, -1)$ is

$$\frac{x - 2}{a} = \frac{y + 1}{b} = \frac{z + 1}{c} \qquad ---(2)$$

Line (2) is parallel to given line,

$$6x - 2 = 3y + 1 = 2z - 2$$

$$\Rightarrow \quad \frac{6x - 2}{6} = \frac{3y + 1}{6} = \frac{2z - 2}{6}$$

$$\Rightarrow \quad \frac{x - \frac{1}{3}}{1} = \frac{y + \frac{1}{2}}{2} = \frac{z - \frac{1}{3}}{3}$$

So, $a = \lambda$, $b = 2\lambda$, $c = 3\lambda$

Using a, b, c in equation (2) to get required equation of line,

$$\frac{x - 2}{\lambda} = \frac{y + 1}{2\lambda} = \frac{z + 1}{3\lambda}$$

$$\Rightarrow \quad \frac{x - 2}{1} = \frac{y + 1}{2} = \frac{z + 1}{3} = \lambda \text{ (Say)}$$

$$\Rightarrow \quad x = \lambda + 2, \; y = 2\lambda - 1, \; z = 3\lambda - 1$$

So,

$$x\hat{i} + y\hat{j} + z\hat{k} = (\lambda + 2)\hat{i} + (2\lambda - 1)\hat{j} + (3\lambda - 1)\hat{k}$$

$$\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$$

Straight Line in Space Ex 28.2 Q21

The direction of ratios of the lines, $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$, are -3, $2k$, 2 and $3k$, 1, -5 respectively.

It is known that two lines with direction ratios, a_1, b_1, c_1 and a_2, b_2, c_2, are perpendicular, if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$\therefore -3(3k) + 2k \times 1 + 2(-5) = 0$$

$$\Rightarrow -9k + 2k - 10 = 0$$

$$\Rightarrow 7k = -10$$

$$\Rightarrow k = \frac{-10}{7}$$

Therefore, for $k = -\frac{10}{7}$, the given lines are perpendicular to each other.

Straight Line in Space Ex 28.2 Q22

The coordinates of A, B, C, and D are (1, 2, 3), (4, 5, 7), (−4, 3, −6), and (2, 9, 2) respectively.

The direction ratios of AB are (4 − 1) = 3, (5 − 2) = 3, and (7 − 3) = 4

The direction ratios of CD are (2 −(− 4)) = 6, (9 − 3) = 6, and (2 −(−6)) = 8

It can be seen that, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$

Therefore, AB is parallel to CD.

Thus, the angle between AB and CD is either 0° or 180°.

Straight Line in Space Ex 28.2 Q23

Given equation of line are,

$$\frac{x-5}{5\lambda+2}=\frac{2-y}{5}=\frac{1-z}{-1} \text{ and}$$
$$\frac{x}{1}=\frac{2y+1}{4\lambda}=\frac{1-z}{-3}$$

$$\Rightarrow \quad \frac{x-5}{5\lambda+2}=\frac{y-2}{-5}=\frac{z-1}{1} \qquad ---(1)$$

$$\text{and,} \quad \frac{x}{1}=\frac{y+\frac{1}{2}}{2\lambda}=\frac{z-1}{3} \qquad ---(2)$$

Given that line (1) and (2) are perpendicular,

So, $a_1a_2+b_1b_2+c_1c_2=0$

$$(5\lambda+2)(1)+(-5)(2\lambda)+(1)(3)=0$$
$$5\lambda+2-10\lambda+3=0$$
$$-5\lambda+5=0$$
$$\lambda=\frac{5}{5}$$

$$\lambda=1$$

Straight Line in Space Ex 28.2 Q24

The direction ratios of the line are

$$\frac{x+2}{2}=\frac{2y-7}{6}=\frac{5-z}{6}$$

2,6,6

The direction cosines of the line are

$$l=\frac{2}{\sqrt{2^2+6^2+6^2}}=\frac{2}{\sqrt{76}}$$
$$m=\frac{6}{\sqrt{2^2+6^2+6^2}}=\frac{6}{\sqrt{76}}$$
$$n=\frac{6}{\sqrt{2^2+6^2+6^2}}=\frac{6}{\sqrt{76}}$$
$$\left(\frac{2}{\sqrt{76}},\frac{6}{\sqrt{76}},\frac{6}{\sqrt{76}}\right)$$

$\therefore$ Vector equation of the line is

$$\vec{r}=\vec{a}+\lambda\vec{b}$$
$$\vec{r}=(-\vec{i}+2\vec{j}+3\vec{k})+\lambda(2\vec{i}+6\vec{j}+6\vec{k})$$

Ex 28.3

Straight Line in Space Ex 28.3 Q1

We have equation of first line,

$$\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3} = \lambda \text{ (Say)} \qquad ---(1)$$

General point on line (1) is

$(\lambda,\ 2\lambda + 2,\ 3\lambda - 3)$

Another line is,

$$\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4} = \mu \text{ (Say)} \qquad ---(2)$$

General point on line (2) is

$(2\mu + 2,\ 3\mu + 6,\ 4\mu + 3)$

If lines (1) and (2) intersect then they have a common point, so for same value of λ and μ, we must have,

$\lambda = 2\mu + 2 \quad \Rightarrow \lambda - 2\mu = 2 \qquad ---(3)$

$2\lambda + 2 = 3\mu + 6 \Rightarrow 2\lambda - 4\mu = 4 \qquad ---(4)$

$3\lambda - 3 = 4\mu + 3 \Rightarrow 3\lambda - 4\mu = 6 \qquad ---(5)$

Now, solving equation (3) and (4) to get λ and μ,

$$\begin{array}{r} 2\lambda - 4\mu = 4 \\ 2\lambda - 3\mu = 4 \\ (-) \quad (+) \quad (-) \\ \hline -\mu = 0 \end{array}$$

$\Rightarrow \quad \mu = 0$

Put $\mu = 0$ in equation (3),

$\lambda - 2\mu = 2$

$\lambda - 2(0) = 2$

$\lambda = 2$

Put λ and μ in equation (5),

$3\lambda - 4\mu = 6$

$3(2) - 4(0) = 6$

$6 = 6$

LHS = RHS

Straight Line in Space Ex 28.3 Q2

We have equation of first line,

$$\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5} = \lambda \ (\text{Say}) \qquad ---(1)$$

General point on line (1) is

$$(3\lambda+1,\ 2\lambda-1,\ 5\lambda+1)$$

Another line is,

$$\frac{x+2}{4} = \frac{y-1}{3} = \frac{z+1}{-2} = \mu \ (\text{Say}) \qquad ---(2)$$

General point on line (2) is,

$$(4\mu-2,\ 3\mu+1,\ -2\mu-1)$$

If lines (1) and (2) intersect, then they have a common point, so for same value of λ and μ, we must have,

$$3\lambda+1 = 4\mu-2 \Rightarrow 3\lambda-4\mu = -3 \qquad ---(3)$$

$$2\lambda-1 = 3\mu+1 \Rightarrow 2\lambda-3\mu = 2 \qquad ---(4)$$

$$5\lambda+1 = -2\mu-1 \Rightarrow 5\lambda+2\mu = -2 \qquad ---(5)$$

Solving equation (3) and (4) to get λ and μ,

$$\begin{array}{r} 6\lambda-8\mu = -6 \\ 6\lambda-9\mu = 6 \\ (-)\quad (+)\quad (-) \\ \hline \mu = -12 \end{array}$$

Put the value of μ in equation (3),

$$3\lambda-4(-12) = -3$$

$$3\lambda+48 = -3$$

$$3\lambda = -3-48$$

$$3\lambda = -51$$

$$\lambda = \frac{-51}{3}$$

$$\lambda = -17$$

Put the value of λ and μ in equation (5),

$$5\lambda+2\mu = -2$$

$$5(-17)+2(-12) = -2$$

$$-85-24 = -2$$

$$-109 \neq -2$$

$$\text{LHS} \neq \text{RHS}$$

Straight Line in Space Ex 28.3 Q3

Given equation of first line is

$$\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} = \lambda \ (\text{Say}) \qquad ---(1)$$

General point on line (1) is

$$(3\lambda - 1,\ 5\lambda - 3,\ 7\lambda - 5)$$

Another equation of line is

$$\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} = \mu \ (\text{Say}) \qquad ---(2)$$

General point on line (2) is,

$$(\mu + 2,\ 3\mu + 4,\ 5\mu + 6)$$

If lines (1) and (2) are intersecting then, they have a common point. So for same value of λ and μ, we must have,

$$3\lambda - 1 = \mu + 2 \ \Rightarrow 3\lambda - \mu = 3 \qquad ---(3)$$

$$5\lambda - 3 = 3\mu + 4 \Rightarrow 5\lambda - 3\mu = 7 \qquad ---(4)$$

$$7\lambda - 5 = 5\mu + 6 \ \Rightarrow 7\lambda - 5\mu = 11 \qquad ---(5)$$

Solving equation (3) and (4) to get λ and μ,

$$15\lambda - 5\mu = 15$$

$$15\lambda - 9\mu = 21$$

$$(-) \quad (+) \quad (-)$$

$$4\mu = -6$$

$$\mu = \frac{-3}{2}$$

Put the value of μ in equation (3),

$$3\lambda - \mu = 3$$

$$3\lambda - \left(-\frac{3}{2}\right) = 3$$

$$3\lambda = 3 - \frac{3}{2}$$

$$\lambda = \frac{1}{2}$$

Put the value of λ and μ in equation (5),

$$7\lambda - 5\mu = 11$$

$$7\left(\frac{1}{2}\right) - 5\left(-\frac{3}{2}\right) = 11$$

$$\frac{7}{2} + \frac{15}{2} = 11$$

$$\frac{22}{2} = 11$$

$$11 = 11$$

$$\text{LHS} \neq \text{RHS}$$

Since, the values of λ and μ obtained by solving (3) and (4) satisfy equation (5), Hence

Given lines intersect each other.

Point of intersection $= (3\lambda - 1,\ 5\lambda - 3,\ 7\lambda - 5)$

$$= \left\{\frac{3}{2} - 1,\ \left(\frac{5}{2} - 3\right),\ \left(\frac{7}{2} - 5\right)\right\}$$

$$= \left(\frac{1}{2},\ \frac{-1}{2},\ \frac{-3}{2}\right)$$

Point of intersection is $\left(\frac{1}{2},\ \frac{-1}{2},\ \frac{-3}{2}\right)$.

Straight Line in Space Ex 28.3 Q4

Equation of the line passing through $A(0,-1,-1)$ and $B(4,5,1)$ is given by

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

$$\frac{x-0}{4-0}=\frac{y+1}{5+1}=\frac{z+1}{1+1}$$

$$\frac{x}{4}=\frac{y+1}{6}=\frac{z+1}{2}=\lambda \text{ (say)}$$

So, general point on line AB is

$$(4\lambda,\ 4\lambda,\ 2\lambda-1)$$

Now, equation of the line passing through $C(3,9,4)$ and $D(-4,4,4)$ is

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

$$\frac{x-3}{-4-3}=\frac{y-9}{4-9}=\frac{z-4}{4-4}$$

$$\frac{x-3}{-7}=\frac{y-9}{-5}=\frac{z-4}{0}=\mu \text{ (say)}$$

So, general point on line CD is

$$(-7\mu+3,\ -5\mu+9,\ 0.\mu+4)$$

$$(-7\mu+3,\ -5\mu+9,\ 4)$$

If lines AB and CD intersect, there must be a common point to them. So we have to find λ and μ such that

$$4\lambda=-7\mu+3 \quad \Rightarrow 4\lambda+7\mu=3 \qquad ---(1)$$

$$6\lambda-1=-5\mu+9 \quad \Rightarrow 6\lambda+5\mu=10 \qquad ---(2)$$

$$2\lambda-1=4 \quad \Rightarrow 2\lambda-1=4 \qquad ---(3)$$

From equation (3),

$$2\lambda=4+1$$

$$\lambda=\frac{5}{2}$$

Put $\lambda=\frac{5}{2}$ in equation (2),

$$6\left(\frac{5}{2}\right)+5\mu=10$$

$$5\mu=10-15$$

$$5\mu=-5$$

$$\mu=-1$$

Now, put values of λ and μ in equation (1),

$$4\lambda+7(\mu)=3$$

$$4\left(\frac{5}{2}\right)+7(-1)=3$$

$$10-7=3$$

$$3=3$$

$$\text{LHS} \neq \text{RHS}$$

Since, the values of λ and μ by solving (2) and (3) satisfy equation (1), so

Line AB and CD are intersecting lines

Point of intersection of AB and CD

$$=(-7\mu+3,\ -5\mu+9,\ 4)$$

$$=(-7(-1)+3,\ -5(-1)+9,\ 4)$$

$$=(7+3,\ 5+9,\ 4)$$

$$=(10,14,4)$$

So, point of intersection of AB and CD $=(10,14,4)$.

Straight Line in Space Ex 28.3 Q5

Given equations of lines are

$$\vec{r} = \left(\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(3\hat{i} - \hat{j}\right)$$
$$\vec{r} = \left(4\hat{i} - \hat{k}\right) + \mu\left(2\hat{i} + 3\hat{k}\right)$$

If these lines intersect, they must have a common point, so, for some value of λ and μ we must have,

$$\left(\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(3\hat{i} - \hat{j}\right) = \left(4\hat{i} - \hat{k}\right) + \mu\left(2\hat{i} + 3\hat{k}\right)$$
$$(1 + 3\lambda)\hat{i} + (1 - \lambda)\hat{j} - \hat{k} = (4 + 2\mu)\hat{i} + (-1 + 3\mu)\hat{k}$$

Equation the coefficients of $\hat{i}, \hat{j}, \hat{k}$, we get

$$1 + 3\lambda = 4 + 2\mu \quad \Rightarrow 3\lambda - 2\mu = 3 \qquad ---(1)$$
$$1 - \lambda = 0 \quad \Rightarrow \lambda = 1 \qquad ---(2)$$
$$-1 = -1 + 3\mu \quad \Rightarrow \mu = 0 \qquad ---(3)$$

Put the value of λ and μ in equation (1),

$$3\lambda - 2\mu = 3$$
$$3(1) - 2(0) = 3$$
$$3 = 3$$
$$\text{LHS} = \text{RHS}$$

The value of λ and μ satisfy equation (1), so

Lines are intersecting.

Put value of λ in equation (1) to get point of intersection

$$\vec{r} = \left(\hat{i} + \hat{j} - \hat{k}\right) + (1)\left(3\hat{i} - \hat{j}\right)$$
$$= \hat{i} + \hat{j} - \hat{k} + 3\hat{i} - \hat{j}$$
$$= 4\hat{i} - \hat{k}$$

So, point of intersection is $(4, 0, -1)$.

Straight Line in Space Ex 28.3 Q6(i)

Given equations of lines are

$$\vec{r} = \left(\hat{i} - \hat{j}\right) + \lambda\left(2\hat{i} + \hat{k}\right) \quad \text{and}$$
$$\vec{r} = \left(2\hat{i} - \hat{j}\right) + \mu\left(\hat{i} + \hat{i} - \hat{k}\right)$$

If these lines intersect each other, there must be some common point, so, we must have λ and μ such that

$$\left(\hat{i} - \hat{j}\right) + \lambda\left(2\hat{i} + \hat{k}\right) = \left(2\hat{i} - \hat{j}\right) + \mu\left(\hat{i} + \hat{i} - \hat{k}\right)$$
$$(1 + 2\lambda)\hat{i} - \hat{j} + \lambda\hat{k} = (2 + \mu)\hat{i} + (-1 + \mu)\hat{j} - \mu\hat{k}$$

Equation the coefficients of $\hat{i}, \hat{j}$ and $\hat{k}$,

$$1 + 2\lambda = 2 + \mu \quad \Rightarrow 2\lambda - \mu = 1 \qquad ---(1)$$
$$-1 = -1 + \mu \quad \Rightarrow \mu = 0 \qquad ---(2)$$
$$\lambda = -\mu \quad \Rightarrow \lambda = 0 \qquad ---(3)$$

Put value of λ and μ in equation (1),

$$2\lambda - \mu = 1$$
$$2(0) - (0) = 1$$
$$0 = 1$$
$$\text{LHS} \neq \text{RHS}$$

Since, the values of λ and μ form equation (2) and (3) does not satisfy equation (1),

Hence, given lines do not intersect each other.

Straight Line in Space Ex 28.3 Q6(ii)

Given, equations of first line is

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{1} = \lambda \ (\text{say}) \qquad ---(1)$$

General point on line (1) is

$$(2\lambda+1,\ 3\lambda-1,\ \lambda)$$

Another equation of line is

$$\frac{x-1}{5} = \frac{y-2}{1},\ z = 3 \qquad ---(2)$$

$$\frac{x-1}{5} = \frac{y-2}{1} = \mu,\ (\text{say}),\ z = 3$$

General point on line (2) is

$$(5\mu+1,\ \mu+2,\ 3)$$

If line (1) and (2) intersect each other then, there is a common point to them, so, we must have value of λ and μ such that

$$2\lambda+1 = 5\mu+1 \quad \Rightarrow 2\lambda-5\mu = 0 \qquad ---(3)$$
$$3\lambda-1 = \mu+2 \quad \Rightarrow 3\lambda-\mu = 3 \qquad ---(4)$$
$$\lambda = 3 \quad \Rightarrow \lambda = 3 \qquad ---(5)$$

Put value of λ in equation (4),

$$3\lambda-\mu = 3$$
$$3(3)-\mu = 3$$
$$-\mu = 3-9$$
$$\mu = 6$$

Put the value of λ and μ in equation (3), so

$$2\lambda-5\mu = 0$$
$$2(3)-5(6) = 0$$
$$6-30 = 0$$
$$-24 \neq 0$$
$$\text{LHS} \neq \text{RHS}$$

Since the values of λ and μ obtained from equation (4) and (5) does not satisfy equation (3), so,

Given lines are not intersecting.

Straight Line in Space Ex 28.3 Q6(iii)

Given, equations of first line is,

$$\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0} = \lambda \text{ (say)} \qquad ---(1)$$

General point on line (1) is,

$(3\lambda + 1, -\lambda + 1, -1)$

Another equation of line is

$$\frac{x-4}{2} = \frac{y-0}{0} = \frac{z+1}{3} = \mu \text{ (say)} \qquad ---(2)$$

General point on line (2) is,

$(2\mu + 4, 0, 3\mu - 1)$

If line (1) and (2) intersecting then there must be a common point, so, we must have the value of λ and μ as

$$3\lambda + 1 = 2\mu + 4 \quad \Rightarrow 3\lambda - 2\mu = 3 \qquad ---(1)$$
$$-\lambda + 1 = 0 \quad \Rightarrow \lambda = 1 \qquad ---(2)$$
$$3\mu - 1 = -1 \quad \Rightarrow \mu = 0 \qquad ---(3)$$

Put the value of λ and μ in equation (1), so

$$3\lambda - 2\mu = 3$$
$$3(1) - 2(0) = 3$$
$$3 = 3$$
$$\text{LHS} \neq \text{RHS}$$

Since the values of λ and μ obtained by equation (2) and (3) satisfy equation (1), so,

Given lines are intersecting.

Straight Line in Space Ex 28.3 Q6(iv)

Given, equation of line is

$$\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5} = \lambda \text{ (say)} \qquad ---(1)$$

General point on line (1) is,

$$(4\lambda+5,\ 4\lambda+7,\ -5\lambda-3)$$

Another equation of line is,

$$\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3} = \mu \text{ (say)} \qquad ---(2)$$

General point on line (2) is

$$(7\mu+8,\ \mu+4,\ 3\mu+5)$$

If line (1) and (2) intersecting, then there must have some common point to them, so, we must have value of λ and μ such that

$$4\lambda+5 = 7\mu+8 \quad \Rightarrow 4\lambda - 7\mu = 3 \qquad ---(3)$$

$$4\lambda+5 = \mu+4 \quad \Rightarrow 4\lambda - \mu = -3 \qquad ---(4)$$

$$-5\lambda-3 = 3\mu+5 \quad \Rightarrow -5\lambda - 3\mu = 8 \qquad ---(5)$$

Solving equation (3) and (4) to find λ and μ,

$$4\lambda - 7\mu = 3$$

$$\underset{(-)}{4\lambda} \underset{(+)}{-} \mu = \underset{(-)}{-3}$$

$$-6\mu = 6$$

$$\mu = -1$$

Put value of λ in equation (3),

$$4\lambda - 7\mu = 3$$

$$4\lambda - 7(-1) = 3$$

$$4\lambda = 3-7$$

$$\lambda = -1$$

Put the value of λ and μ in equation (5),

$$-5\lambda - 3\mu = 8$$

$$-5(-1) - 3(-1) = 8$$

$$5+3 = 8$$

$$\text{LHS} = \text{RHS}$$

Straight Line in Space Ex 28.3 Q7

$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$

$\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

If the lines intersect eachother, then the shortest distance between the lines should be zero.

Here,

$\vec{a_1} = 3\hat{i} + 2\hat{j} - 4\hat{k}$

$\vec{a_2} = 5\hat{i} - 2\hat{j}$

$\vec{b_1} = \hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{b_2} = 3\hat{i} + 2\hat{j} + 6\hat{k}$

$$(\vec{b_1} \times \vec{b_2}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 3 & 2 & 6 \end{vmatrix}$$

$= \vec{i}(12-4) - \vec{j}(6-6) + \vec{k}(2-6)$

$= 8\vec{i} - 0\vec{j} - 4\vec{k}$

$(\vec{a_2} - \vec{a_1}) = (5\hat{i} - 2\hat{j} - 3\hat{i} - 2\hat{j} + 4\hat{k}) = (2\hat{i} - 4\hat{j} + 4\hat{k})$

$$\text{Shortest Distance, } d = \left|\frac{(\vec{b_1} \times \vec{b_2}).(\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|}\right|$$

$$= \left|\frac{(8\vec{i} - 0\vec{j} - 4\vec{k}).(2\hat{i} - 4\hat{j} + 4\hat{k})}{|8\vec{i} - 0\vec{j} - 4\vec{k}|}\right|$$

$$= \left|\frac{8 \times 2 - 0 \times 4 + (-4) \times 4}{|8\vec{i} - 0\vec{j} - 4\vec{k}|}\right|$$

$$= \left|\frac{0}{|8\vec{i} - 0\vec{j} - 4\vec{k}|}\right| = 0$$

Since the shortest distance is zero, the lines are intersect each other.

Point of intersection of the lines,

$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$

$\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

Lines in the Cartesian form,

$\frac{x-3}{1} = \frac{y-2}{2} = \frac{z+4}{2} = \lambda$

$x = \lambda + 3, y = 2\lambda + 2, z = 2\lambda - 4$

$\frac{x-5}{3} = \frac{y+2}{2} = \frac{z}{6} = \mu$

$x = 3\mu + 5, y = 2\mu - 2, z = 6\mu$

From coordinates of x,

$\lambda + 3 = 3\mu + 5$

$\lambda = 3\mu + 2$.....(i)

From coordinates of y,

$2\lambda + 2 = 2\mu - 2$

$\lambda = \mu - 2$.......(ii)

Solving (i) and (ii),

$\lambda = -4, \mu = -2$

Coordinates of the point of intersection,

$x = 3(-2) + 5, y = 2(-2) - 2, z = 6(-2)$

$x = -1, y = -6, z = -12$

$(-1, -6, -12)$

Ex 28.4

Straight Line in Space Ex 28.4 Q1

Let the foot of the perpendicular drawn from $P(3,-1,11)$ to the line

$\frac{x}{2}=\frac{y-2}{-3}=\frac{z-3}{4}$ is Q, so we have to find length of PQ.Q is general point on the line

$\frac{x}{2}=\frac{y-2}{-3}=\frac{z-3}{4}=\lambda \quad (say)$

Co-ordinate of $Q=(2\lambda,-3\lambda+2,4\lambda+3)$

Direction ratios of the given line $=2.-3,4$

Since PQ is perpendicular to the given line therefore

$$
\begin{aligned}
& a1a2+b1b2+c1c2=0 \\
& \Rightarrow 2(2\lambda-3)+(-3)(-3\lambda+3)+4(4\lambda-8)=0 \\
& \Rightarrow 4\lambda-6+9\lambda-9+16\lambda-32=0 \\
& \Rightarrow 29\lambda-47=0 \\
& \Rightarrow \lambda=\frac{47}{29}
\end{aligned}
$$

Therefore co-ordinates of Q

$=2\left(\frac{47}{29}\right),-3\left(\frac{47}{29}\right)+2,4\left(\frac{47}{29}\right)+3$

$=\frac{94}{29},\frac{-83}{29},\frac{275}{29}$

Distance between P and Q is

Straight Line in Space Ex 28.4 Q2

Let foot of the perpendicular drawn from the point $P(1,0,0)$ to the line $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ is Q. We have to find length of PQ.

Q is a general point on the line,

$$\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}=\lambda \quad (\text{say})$$

Coordinate of Q $=(2\lambda+1,-3\lambda-1,8\lambda-10)$

Direction ratios line PQ are

$$= (2\lambda+1-1),\ (-3\lambda-1-0),\ (8\lambda-10-0)$$

$$\Rightarrow \quad = (2\lambda),\ (-3\lambda-1),\ (8\lambda-10)$$

Since, line PQ is perpendicular to the given line, so

$$
\begin{aligned}
& a_1a_2+b_1b_2+c_1c_2=0 \\
& (2)(2\lambda)+(-3)(-3\lambda-1)+8(8\lambda-10)=0 \\
& 4\lambda+9\lambda+3+64\lambda-80=0 \\
& 77\lambda-77=0 \\
& \lambda=1
\end{aligned}
$$

Therefore, coordinate of Q is $(2\lambda+1,\ -3\lambda-1,\ 8\lambda-10)$

$$
\begin{aligned}
& = (2(1)+1,\ -3(1)-1,\ 8(1)-10) \\
& = (3,-4,-2)
\end{aligned}
$$

Therefore, coordinate of Q is $(2\lambda+1,\ -3\lambda-1,\ 8\lambda-10)$

$$
\begin{aligned}
& = (2(1)+1,\ -3(1)-1,\ 8(1)-10) \\
& = (3,-4,-2)
\end{aligned}
$$

$$
\begin{aligned}
PQ &= \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2} \\
&= \sqrt{(1-3)^2+(0+4)^2+(0+2)^2} \\
&= \sqrt{4+16+4} \\
&= \sqrt{24} \\
&= 2\sqrt{6}
\end{aligned}
$$

So, foot of perpendicular $=(3,-4,-2)$

length of perpendicular $=2\sqrt{6}$ units

Straight Line in Space Ex 28.4 Q3

Let the foot of the perpendicular drawn from $A(1,0,3)$ to the line joining the points $B(4,7,1)$

And $C(3,5,3)$ be D

Equation of line passing through $B(4,7,1)$ and $C(3,5,3)$ is

$$\frac{x-x1}{x2-x1}=\frac{y-y1}{y2-y1}=\frac{z-z1}{z2-z1}$$
$$\Rightarrow \frac{x-4}{3-4}=\frac{y-7}{5-7}=\frac{z-1}{3-1}$$
$$\Rightarrow \frac{x-4}{-1}=\frac{y-7}{-2}=\frac{z-1}{2}=\lambda\,(\text{say})$$

Direction ratio of AD are

$$(-\lambda+4-1),(-2\lambda+7-0),(2\lambda+1-3)$$
$$=(-\lambda+3),(-2\lambda+7),(2\lambda-2)$$

Line AD is perpendicular to BC so

$$a1a2+b1b2+c1c2=0$$
$$\Rightarrow(-1)(-\lambda+3)+(-2)(-2\lambda+7)+2(2\lambda-2)=0$$
$$\Rightarrow \lambda-3+4\lambda-14+4\lambda-4=0$$
$$\Rightarrow 9\lambda-21=0$$
$$\Rightarrow \lambda=\frac{21}{9}$$

Co-ordinates of D are

$$=\left(-\frac{21}{9}+4,(-2)\left(\frac{21}{9}+7\right),2\left(\frac{21}{9}+1\right)\right)$$
$$=\left(\frac{15}{9},\frac{21}{9},\frac{51}{9}\right)$$
$$=\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$$

Straight Line in Space Ex 28.4 Q4

Given that D is the foot of perpendicular from $A(1,0,4)$ on BC, so

Equation of line passing through B,C is

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

$$\Rightarrow \quad \frac{x-0}{2-0}=\frac{y+11}{-3+11}=\frac{z-3}{1-3}$$
$$\Rightarrow \quad \frac{x}{2}=\frac{y+11}{8}=\frac{z-3}{-2}=\lambda \ (\text{say})$$

Coordinate of $D = (2\lambda,\ 8\lambda-11,\ -2\lambda+3)$

Direction ratios of $AD = 2\lambda-1,\ 8\lambda-11-0,\ -2\lambda+3-4$

$$=(2\lambda-1),\ (8\lambda-11),\ (-2\lambda-1)$$

Since, line AD is perpendicular on BC, so

$$a_1a_2+b_1b_2+c_1c_2=0$$

$$\Rightarrow \quad (2)(2\lambda-1)+(8)(8\lambda-11)+(-2)(-2\lambda-1)=0$$
$$\Rightarrow \quad 4\lambda-2+64\lambda-88+4\lambda+2=0$$
$$\Rightarrow \quad 72\lambda-88=0$$
$$\Rightarrow \quad \lambda=\frac{88}{72}$$
$$\lambda=\frac{11}{9}$$

Coordinate of $D = (2\lambda,\ 8\lambda-11,\ -2\lambda+3)$

$$=\left(2\left(\frac{11}{9}\right),\ 8\left(\frac{11}{9}\right)-11,\ -2\left(\frac{11}{9}\right)+3\right)$$

Coordinate of $D=\left(\frac{22}{9},\ \frac{-11}{9},\ \frac{5}{9}\right)$

Straight Line in Space Ex 28.4 Q5

Let foot of the perpendicular from $P(2,3,4)$ is θ on the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$, so

Equation of given line is,

$$\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$$
$$\frac{x-4}{-2}=\frac{y}{6}=\frac{z-1}{-3}=\lambda \text{ (say)}$$

Coordinate of $Q=(-2\lambda+4,\ 6\lambda,\ -3\lambda+1)$

Direction ratios of $PQ=(-2\lambda+4-2),\ (6\lambda-3),\ (-3\lambda+1-4)$

$$=(-2\lambda+2),\ (6\lambda-3),\ (-3\lambda-3)$$

Line PQ is perpendicular to given line, so

$$a_1a_2+b_1b_2+c_1c_2=0$$
$$(-2)(-2\lambda+2)+(6)(6\lambda-3)+(-3)(-3\lambda-3)=0$$
$$4\lambda-4+36\lambda-18+9\lambda+9=0$$
$$49\lambda-13=0$$
$$\lambda=\frac{13}{49}$$

Coordinate of $Q=(-2\lambda+4,\ 6\lambda,\ -3\lambda+1)$

$$=\left(-2\left(\frac{13}{49}\right)+4,\ 6\left(\frac{13}{49}\right),\ -3\left(\frac{13}{49}\right)+1\right)$$
$$=\left(\frac{-26+196}{49},\ \frac{78}{49},\ \frac{-39+49}{49}\right)$$

Coordinate of $Q=\left(\frac{170}{49},\ \frac{78}{49},\ \frac{10}{49}\right)$

$$PQ=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$
$$=\sqrt{\left(\frac{170}{49}-2\right)^2+\left(\frac{78}{49}-3\right)^2+\left(\frac{10}{49}-4\right)^2}$$
$$=\sqrt{\left(\frac{72}{49}\right)^2+\left(\frac{69}{49}\right)^2+\left(-\frac{168}{49}\right)^2}$$
$$=\sqrt{\frac{5184+4761+34596}{2401}}$$
$$=\sqrt{\frac{44541}{2401}}$$
$$=\sqrt{\frac{909}{49}}$$
$$=\frac{3\sqrt{101}}{49}$$

Perpendicular distance from $(2,3,4)$ to given line is $\frac{3\sqrt{101}}{49}$ units.

Straight Line in Space Ex 28.4 Q6

Let θ be the foot of the perpendicular drawn from $P\left(2,4,-1\right)$ to the line

$$\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}$$

Given line is $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}=\lambda$ (say)

Coordinate of Q (General point on the line)

$$=\left(\lambda-5,\ 4\lambda-3,\ -9\lambda+6\right)$$

Direction ratios of PQ are

$$=\left(\lambda-5-2\right),\ \left(4\lambda-3-4\right),\ \left(-9\lambda+6+1\right)$$
$$=\lambda-7,\ 4\lambda-7,\ -9\lambda+7$$

Line PQ is perpendicular to the given line, so

$$a_1a_2+b_1b_2+c_1c_2=0$$
$$\left(1\right)\left(\lambda-7\right)+\left(4\right)\left(4\lambda-7\right)+\left(-9\right)\left(-9\lambda+7\right)=0$$
$$\lambda-7+16\lambda-28+81\lambda-63=0$$
$$98\lambda-98=0$$
$$\lambda=1$$

$\therefore$ Coordinate of Q $=\left(\lambda-5,\ 4\lambda-3,\ -9\lambda+6\right)$

$$=\left(1-5,\ 4\left(1\right)-3,\ -9\left(1\right)+6\right)$$

Coordinate of foot of perpendicular $=\left(-4,1,-3\right)$

So, equation of the perpendicular PQ is

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

$$\Rightarrow \quad \frac{x-2}{-4-2}=\frac{y-4}{1-4}=\frac{z+1}{-3+1}$$

$$\Rightarrow \quad \frac{x-2}{-6}=\frac{y-4}{-3}=\frac{z+1}{-2}$$

Straight Line in Space Ex 28.4 Q7

Let foot of the perpendicular drawn from $P\left(5,4,-1\right)$ to the given line is Q, so

Given equation of line is,

$$\vec{r}=\hat{i}+\lambda\left(2\hat{i}+9\hat{j}+5\hat{k}\right)$$
$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)=\left(1+2\lambda\right)\hat{i}+\left(9\lambda\right)\hat{j}+\left(5\lambda\right)\hat{k}$$

Equation the coefficients of $\hat{i},\hat{j}$ and $\hat{k}$

$$\Rightarrow \quad x=1+2\lambda,\ y=9\lambda,\ z=5\lambda$$
$$\Rightarrow \quad \frac{x-1}{2}=\lambda,\ \frac{y}{9}=\lambda,\ \frac{z}{5}=\lambda$$
$$\Rightarrow \quad \frac{x-1}{2}=\frac{y}{9}=\frac{z}{5}=\lambda \text{ (say)}$$

Coordinate of $Q=\left(2\lambda+1,\ 9\lambda,\ 5\lambda\right)$

Direction ratios of line PQ are

$$\left(2\lambda+1-5\right),\ 9\lambda-4,\ 5\lambda+1$$

$$\Rightarrow \quad 2\lambda-4,\ 9\lambda-4,\ 5\lambda+1$$

Straight Line in Space Ex 28.4 Q8

Let position vector of foot of perpendicular drown from $P\left(\hat{i}+6\hat{j}+3\hat{k}\right)$ on $\vec{r}=\left(\hat{j}+2\hat{k}\right)+\lambda\left(\hat{i}+2\hat{j}+3\hat{k}\right)$ be $Q\left(\vec{q}\right)$. So

Q is on the line $\vec{r}=\left(\hat{j}+2\hat{k}\right)+\lambda\left(\hat{i}+2\hat{j}+3\hat{k}\right)$

So, Position vector of Q = $(\lambda)\hat{i}+(1+2\lambda)\hat{j}+(2+3\lambda)\hat{k}$

$$\begin{aligned}\overrightarrow{PQ} &= \text{Position vector of Q} - \text{Position vector of } P\\ &= \left\{\lambda\hat{i}+(1+2\lambda)\hat{j}+(2+3\lambda)\hat{k}\right\}-\left(\hat{i}+6\hat{j}+3\hat{k}\right)\\ &= (\lambda-1)\hat{i}+(1+2\lambda-6)\hat{j}+(2+3\lambda-3)\hat{k}\\ \overrightarrow{PQ} &= (\lambda-1)\hat{i}+(2\lambda-5)\hat{j}+(3\lambda-1)\hat{k}\end{aligned}$$

Here, $\overrightarrow{PQ}$ is perpendicular to given line

So,

$$\left\{(\lambda-1)\hat{i}+(2\lambda-5)\hat{j}+(3\lambda-1)\hat{k}\right\}\left(\hat{i}+2\hat{j}+3\hat{k}\right)=0$$

$$\Rightarrow\quad (\lambda-1)(1)+(2\lambda-5)(2)+(3\lambda-1)(3)=0$$
$$\Rightarrow\quad \lambda-1+4\lambda-10+9\lambda-3=0$$
$$\Rightarrow\quad 14\lambda-14=0$$
$$\Rightarrow\quad \lambda=1$$

$$\begin{aligned}\text{Position vector of Q} &= \left(\hat{j}+2\hat{k}\right)+\lambda\left(\hat{i}+2\hat{j}+3\hat{k}\right)\\ &= \left(\hat{j}+2\hat{k}\right)+(1)\left(\hat{i}+2\hat{j}+3\hat{k}\right)\end{aligned}$$

Foot of perpendicular $=\hat{i}+3\hat{j}+5\hat{k}$

$$\begin{aligned}\overrightarrow{PQ} &= \text{Position vector of Q} - \text{Position vector of } P\\ &= \left(\hat{i}+3\hat{j}+5\hat{k}\right)-\left(\hat{i}+6\hat{j}+3\hat{k}\right)\\ &= \hat{i}+3\hat{j}+5\hat{k}-\hat{i}-6\hat{j}-3\hat{k}\\ &= -3\hat{i}+2\hat{k}\end{aligned}$$

$$\begin{aligned}\left|\overrightarrow{PQ}\right| &= \sqrt{(-3)^2+(2)^2}\\ &= \sqrt{13}\text{ units}\end{aligned}$$

Length of perpendicular $=\sqrt{13}$ units

Straight Line in Space Ex 28.4 Q9

Let Q be the perpendicular drown from $P\left(-\hat{i}+3\hat{j}+2\hat{k}\right)$ on the line

$$\vec{r}=\left(2\hat{j}+3\hat{k}\right)+\lambda\left(2\hat{i}+\hat{j}+3\hat{k}\right)$$

Let the position vector of Q be

$$\left(2\hat{j}+3\hat{k}\right)+\lambda\left(2\hat{i}+\hat{j}+3\hat{k}\right)$$
$$(2\lambda)\hat{i}+(2+\lambda)\hat{j}+(3+3\lambda)\hat{k}$$

$$\overrightarrow{PQ}=\text{Position vector of } Q-\text{Position vector of } P$$
$$=\left\{(2\lambda)\hat{i}+(2+\lambda)\hat{j}+(3+3\lambda)\hat{k}\right\}-\left(-\hat{i}+3\hat{j}+2\hat{k}\right)$$
$$=(2\lambda+1)\hat{i}+(2\lambda-3)\hat{j}+(3+3\lambda-2)\hat{k}$$
$$\overrightarrow{PQ}=(2\lambda+1)\hat{i}+(\lambda-1)\hat{j}+(3\lambda+1)\hat{k}$$

Since, $\overrightarrow{PQ}$ is perpendicular to the given line, so

$$\left\{(2\lambda+1)\hat{i}+(\lambda-1)\hat{j}+(3\lambda+1)\hat{k}\right\}\left(2\hat{i}+\hat{j}+3\hat{k}\right)=0$$
$$(2\lambda+1)(2)+(\lambda-1)(1)+(3\lambda+1)3=0$$
$$4\lambda+2+\lambda-1+9\lambda+3=0$$
$$14\lambda+4=0$$
$$\lambda=-\frac{4}{14}$$
$$\lambda=-\frac{2}{7}$$

Position vector of $Q=(2\lambda)\hat{i}+(2+\lambda)\hat{j}+(3+3\lambda)\hat{k}$

$$=2\left(-\frac{2}{7}\right)\hat{i}+\left(2-\frac{2}{7}\right)\hat{j}+\left(3+3\left(-\frac{2}{7}\right)\right)\hat{k}$$
$$=-\frac{4}{7}\hat{i}+\frac{12}{7}\hat{j}+\frac{15}{7}\hat{k}$$

Coordinates of foot of the perpendicular $=\left(-\frac{4}{7},\frac{12}{7},\frac{15}{7}\right)$

Equation of PQ is

$$\vec{r}=\vec{a}+\lambda\left(\vec{b}-\vec{a}\right)$$

$$\Rightarrow\quad \vec{r}=\left(-\hat{i}+3\hat{j}+2\hat{k}\right)+\lambda\left(\left(-\frac{4}{7}\hat{i}+\frac{12}{7}\hat{j}+\frac{15}{7}\hat{k}\right)-\left(-\hat{i}+3\hat{j}+2\hat{k}\right)\right)$$

Straight Line in Space Ex 28.4 Q10

Let foot of the perpendicular drawn from $(0,2,7)$ to the line $\frac{x+2}{-1}=\frac{y-1}{3}=\frac{z-3}{-2}$ be Q.

Given equation of the line is

$$\frac{x+2}{-1}=\frac{y-1}{3}=\frac{z-3}{-2}=\lambda\ \text{(say)}$$

Coordinate of Q is $(-\lambda-2,\ 3\lambda+1,\ -2\lambda+3)$

Direction ratios of PQ are $(-\lambda-2-0),\ (3\lambda+1-2),\ (-2\lambda+3-7)$

$$=(-\lambda-2),\ (3\lambda-1),\ (-2\lambda-4)$$

Since, PQ is perpendicular to given line, so

$$a_1a_2+b_1b_2+c_1c_2=0$$
$$(-1)(-\lambda-2)+(3)(3\lambda-1)+(-2)(-2\lambda-4)=0$$

$$\Rightarrow\quad \lambda+2+9\lambda-3+4\lambda+8=0$$
$$\Rightarrow\quad 14\lambda+7=0$$
$$\lambda=-\frac{1}{2}$$

Foot of the perpendicular $=(-\lambda-2,\ 3\lambda+1,\ -2\lambda+3)$

$$=\left(-\left(-\frac{1}{2}\right)-2,\ 3\left(-\frac{1}{2}\right)+1,\ -2\left(-\frac{1}{2}\right)+3\right)$$

Foot of the perpendicular $=\left(-\frac{3}{2},-\frac{1}{2},4\right)$

Straight Line in Space Ex 28.4 Q11

Let foot of the perpendicular from $P(1,2,-3)$ to the line $\frac{x+1}{2}=\frac{y-3}{-2}=\frac{z}{-1}$ be Q

Given equation of the line is

$$\frac{x+1}{2}=\frac{y-3}{-2}=\frac{z}{-1}=\lambda$$

$$\Rightarrow \quad x=2\lambda-1,\ y=-2\lambda+3,\ z=-\lambda$$

Coordinate of Q $(2\lambda-1,\ -2\lambda+3,\ -\lambda)$

Direction ratios of PQ are

$$(2\lambda-1-1),\ (-2\lambda+3-2),\ (-\lambda+3)$$

$$\Rightarrow \quad (2\lambda-2),\ (-2\lambda+1),\ (-\lambda+3)$$

Let PQ is perpendicular to given line, so

$$a_1a_2+b_1b_2+c_1c_2=0$$

$$\Rightarrow \quad (2)(2\lambda-2)+(-2)(-2\lambda+1)+(-1)(-\lambda+3)=0$$
$$\Rightarrow \quad 4\lambda-4+4\lambda-2+\lambda-3=0$$
$$\Rightarrow \quad 9\lambda-9=0$$
$$\Rightarrow \quad \lambda=1$$

Coordinate of foot of perpendicular

$$=(2\lambda-1,\ -2\lambda+3,\ -\lambda)$$
$$=(2(1)-1,\ -2(1)+3,\ -1)$$

$$=(1,1,-1)$$

Straight Line in Space Ex 28.4 Q12

Equation of line AB is

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

$$\Rightarrow \quad \frac{x-0}{-3-0}=\frac{y-6}{-6-6}=\frac{z+9}{3+9}$$
$$\Rightarrow \quad \frac{x}{-3}=\frac{y-6}{-12}=\frac{z+9}{12}=\lambda \ \text{(say)}$$

Coordinate of point $D=(-3\lambda,\ -12\lambda+6, 12\lambda-9)$

Direction ratios of $CD=(-3\lambda-7),\ (-12\lambda+6-4),\ (12\lambda-9+1)$

$$=(-3\lambda-7),\ (-12\lambda+2),\ (12\lambda-8)$$

Line CD is perpendicular to line AB, so

$$a_1a_2+b_1b_2+c_1c_2=0$$

$$\Rightarrow \quad (-3)(-3\lambda-7)+(-12)(-12\lambda+2)+(12)(12\lambda-8)=0$$
$$\Rightarrow \quad 9\lambda+21+144\lambda-24+144\lambda-96=0$$
$$\Rightarrow \quad 297\lambda-99=0$$
$$\Rightarrow \quad \lambda=\frac{1}{3}$$

Coordinate of $D=(-3\lambda,\ -12\lambda+6, 12\lambda-9)$

$$=\left(-3\left(\frac{1}{3}\right),\ -12\left(\frac{1}{3}\right)+6, 12\left(\frac{1}{3}\right)-9\right)$$

Coordinate of $D = (-1, 2, -5)$

Equation of CD is,

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$\Rightarrow \quad \frac{x - 7}{-1 - 7} = \frac{y - 4}{2 - 4} = \frac{z + 1}{-5 + 1}$$

$$\Rightarrow \quad \frac{x - 7}{-8} = \frac{y - 4}{-2} = \frac{z + 1}{-4}$$

$$\text{or} \quad \frac{x - 7}{4} = \frac{y - 4}{1} = \frac{z + 1}{2}$$

Straight Line in Space Ex 28.4 Q13

Let P ≡ (2, 4, -1).

In order to find the distance we need to find a point Q on the line.

We see that line is passing through the point Q(-5, -3, 6).

So,let take this point as required point.

Also line is parallel to the vector $\vec{b} = \hat{i} + 4\hat{j} - 9\hat{k}$.

Now, $\overrightarrow{PQ} = (-5\hat{i} - 3\hat{j} + 6\hat{k}) - (2\hat{i} + 4\hat{j} - \hat{k}) = -7\hat{i} - 7\hat{j} + 7\hat{k}$

$$\vec{b} \times \overrightarrow{PQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & -9 \\ -7 & -7 & 7 \end{vmatrix} = -35\hat{i} + 56\hat{j} + 21\hat{k}$$

$$|\vec{b} \times \overrightarrow{PQ}| = \sqrt{1225+3136+441} = \sqrt{4802}$$

$$|\vec{b}| = \sqrt{1 + 16 + 81} = \sqrt{98}$$

$$d = \frac{|\vec{b} \times \overrightarrow{PQ}|}{|\vec{b}|} = \frac{\sqrt{4802}}{\sqrt{98}} = 7$$

Straight Line in Space Ex 28.4 Q14

Let L be the foot of the perpendicular drawn from A(1, 8, 4) on the line joining the points B (0, -1, 3) and C (2, -3, -1).

Equation of the line passing through the points B (0, -1, 3) and C (2, -3, -1) is given by,

$$\vec{r} = \vec{b} + \lambda(\vec{c} - \vec{b})$$

$$\vec{r} = (0 + 2\lambda)\hat{i} + (-1 - 2\lambda)\hat{j} + (3 - 4\lambda)\hat{k}$$

Let positon vector of L be,

$$\vec{r} = (2\lambda)\hat{i} + (-1 - 2\lambda)\hat{j} + (3 - 4\lambda)\hat{k} \text{ ....(i)}$$

Then, $\overrightarrow{AL}$ = Position vector of L - position vector of A

$$\Rightarrow \overrightarrow{AL} = (2\lambda)\hat{i} + (-1 - 2\lambda)\hat{j} + (3 - 4\lambda)\hat{k} - (1\hat{i} + 8\hat{j} + 4\hat{k})$$

$$\Rightarrow \overrightarrow{AL} = (-1 + 2\lambda)\hat{i} + (-9 - 2\lambda)\hat{j} + (-1 - 4\lambda)\hat{k}$$

Since $\overrightarrow{AL}$ is perpendicular to the given line which is parallel to $\vec{b} = 2\hat{i} - 2\hat{j} - 4\hat{k}$

$\therefore \quad \overrightarrow{AL} \cdot \vec{b} = 0$

$\Rightarrow 2(-1 + 2\lambda) - 2(-9 - 2\lambda) - 4(-1 - 4\lambda) = 0$

$\Rightarrow -2 + 4\lambda + 18 + 4\lambda + 4 + 16\lambda = 0$

$\Rightarrow 24\lambda = -20$

$\Rightarrow \lambda = \frac{-5}{6}$

Putting value of $\lambda = \frac{-5}{6}$ in (i) we get

$$\vec{r} = -\frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{19}{3}\hat{k}$$

Coordinates of the foot of the perpendicular are $\left(-\frac{5}{3}, \frac{2}{3}, \frac{19}{3}\right)$.

Ex 28.5

Straight Line in Space Ex 28.5 Q1(i)

We know that, shortest distance between lines

$\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is given by

$$\text{S.D.} = \left|\frac{\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)}{\left|\vec{b_1} \times \vec{b_2}\right|}\right| \qquad ---(i)$$

Given equations of lines are.

$$\vec{r} = 3\hat{i} + 8\hat{j} + 3\hat{k} + \lambda\left(3\hat{i} - \hat{j} + \hat{k}\right)$$

$$\vec{r} = \left(-3\hat{i} - 7\hat{j} + 6\hat{k}\right) + \mu\left(-3\hat{i} + 2\hat{j} + 4\hat{k}\right)$$

$$\Rightarrow \quad \vec{a_1} = \left(3\hat{i} + 8\hat{j} + 3\hat{k}\right),\ \vec{b_1} = 3\hat{i} - \hat{j} + \hat{k}$$

$$\vec{a_2} = \left(-3\hat{i} - 7\hat{j} + 6\hat{k}\right),\ \vec{b_2} = \left(-3\hat{i} + 2\hat{j} + 4\hat{k}\right)$$

Now,

$$\vec{a_2} - \vec{a_1} = \left(-3\hat{i} - 7\hat{j} + 6\hat{k}\right) - \left(3\hat{i} + 8\hat{j} + 3\hat{k}\right)$$

$$= -3\hat{i} - 7\hat{j} + 6\hat{k} - 3\hat{i} - 8\hat{j} - 3\hat{k}$$

$$\left(\vec{a_2} - \vec{a_1}\right) = -6\hat{i} - 15\hat{j} + 3\hat{k}$$

$$\left(\vec{b_1} \times \vec{b_2}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix}$$

$$= \hat{i}\left(-4 - 2\right) - \hat{j}\left(12 + 3\right) + \hat{k}\left(6 - 3\right)$$

$$= \left(-6\hat{i} - 15\hat{j} + 3\hat{k}\right)$$

$$\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right) = \left(-6\hat{i} - 15\hat{j} + 3\hat{k}\right).\left(-6\hat{i} - 15\hat{j} + 3\hat{k}\right)$$

$$= \left(-6\right)\left(6\right) + \left(-15\right)\left(-15\right) + \left(3\right)\left(3\right)$$

$$= 36 + 225 + 9$$

$$= 270$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{\left(-6\right)^2 + \left(-15\right)^2 + \left(3\right)^2}$$

$$= \sqrt{36 + 225 + 9}$$

$$= \sqrt{270}$$

Substituting values of $\left|\vec{b_1} \times \vec{b_2}\right|$ and $\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)$ in equation (i) to get shortest distance between given lines, so

$$\text{S.D.} = \frac{270}{\sqrt{270}}$$

$$= \sqrt{270}$$

$$\text{S.D.} = 3\sqrt{30} \text{ units}$$

Straight Line in Space Ex 28.5 Q1(ii)

We know that, shortest distance between lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is given by

$$\text{S.D.} = \left|\frac{(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|}\right| \qquad ---(i)$$

Given equations of lines are,

$$\vec{r} = (3\hat{i} + 5\hat{j} + 7\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 7\hat{k}) \text{ and}$$
$$\vec{r} = (-\hat{i} - \hat{j} - \hat{k}) + \mu(7\hat{i} - 6\hat{j} + \hat{k})$$

$$\Rightarrow \quad \vec{a_1} = (3\hat{i} + 5\hat{j} + 7\hat{k}),\ \vec{b_1} = (\hat{i} - 2\hat{j} + 7\hat{k})$$
$$\vec{a_2} = (-\hat{i} - \hat{j} - \hat{k}),\ \vec{b_2} = (7\hat{i} - 6\hat{j} + \hat{k})$$

So, $\vec{a_2} - \vec{a_1} = (-\hat{i} - \hat{j} - \hat{k}) - (3\hat{i} + 5\hat{j} + 7\hat{k})$
$$= -\hat{i} - \hat{j} - \hat{k} - 3\hat{i} - 5\hat{j} - 7\hat{k}$$
$$= -4\hat{i} - 6\hat{j} - 8\hat{k} = -2(2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 7 \\ 7 & -6 & 1 \end{vmatrix}$$
$$= \hat{i}(-2 + 42) - \hat{j}(1 - 49) + \hat{k}(-6 + 14)$$
$$= 40\hat{i} + 48\hat{j} + 8\hat{k}$$
$$= 8(5\hat{i} + 6\hat{j} + \hat{k})$$

$$(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2}) = \{-2(2\hat{i} + 3\hat{j} + 4\hat{k})\}\{8(5\hat{i} + 6\hat{j} + \hat{k})\}$$
$$= -16[(2)(5) + (3)(6) + (4)(1)]$$
$$= -16[10 + 18 + 4]$$
$$= -16 \times 32$$
$$(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2}) = -512$$

$$|\vec{b_1} \times \vec{b_2}| = 8\sqrt{(5)^2 + (6)^2 + (1)^2}$$
$$= 8\sqrt{25 + 36 + 1}$$
$$= 8\sqrt{62}$$

Substituting values of $(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2})$ and $|\vec{b_1} \times \vec{b_2}|$ in equation (i) to get required shortest distance between given lines, so

$$\text{S.D.} = \left|\frac{-512}{8\sqrt{62}}\right|$$

$$\text{S.D.} = \frac{512}{\sqrt{3968}}$$

Straight Line in Space Ex 28.5 Q1(iii)

We know that, shortest distance between lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is given by

$$\text{S.D.} = \left|\frac{\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)}{\left|\vec{b_1} \times \vec{b_2}\right|}\right| \qquad ---(i)$$

Given equations of lines are,

$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) \text{ and}$$

$$\vec{r} = \left(2\hat{i} + 4\hat{j} + 5\hat{k}\right) + \mu\left(3\hat{i} + 4\hat{j} + 5\hat{k}\right)$$

Now, $\left(\vec{a_2} - \vec{a_1}\right) = \left(2\hat{i} + 4\hat{j} + 5\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

$$= 2\hat{i} + 4\hat{j} + 5\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k}$$

$$\left(\vec{a_2} - \vec{a_1}\right) = \hat{i} + 2\hat{j} + 2\hat{k}$$

$$\left(\vec{b_1} \times \vec{b_2}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$$= \hat{i}(15 - 16) - \hat{j}(10 - 12) + \hat{k}(8 - 9)$$

$$\left(\vec{b_1} \times \vec{b_2}\right) = -\hat{i} + 2\hat{j} - \hat{k}$$

$$\left(\vec{a_2} - \vec{a_1}\right)\left(\vec{b_1} \times \vec{b_2}\right) = \left(\hat{i} + 2\hat{j} + 2\hat{k}\right)\left(-\hat{i} + 2\hat{j} - \hat{k}\right)$$

$$= (1)(-1) + (2)(2) + (2)(-1)$$

$$= -1 + 4 - 2$$

$$\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right) = 1$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{(-1)^2 + (2)^2 + (-1)^2}$$

$$= \sqrt{1 + 4 + 1}$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{6}$$

Substituting values of $\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)$ and $\left|\vec{b_1} \times \vec{b_2}\right|$ in equation (i) to get the shortest distance between given lines, so

$$\text{S.D.} = \left|\frac{1}{\sqrt{6}}\right|$$

$$\text{S.D.} = \frac{1}{\sqrt{6}} \text{ units}$$

Straight Line in Space Ex 28.5 Q1(iv)

$$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-t)\hat{k}$$

$$\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$$

Above equations can be rewritten as

$$\vec{r} = (i - 2j + 3k) + t(-i + j - k)$$

$$\vec{r} = (i - j - k) + s(i + 2j - 2k)$$

Shortest distance is given by $\left|\dfrac{(a_2 - a_1)\cdot(b_1 \times b_2)}{|(b_1 \times b_2)|}\right|$

$$(b_1 \times b_2) = -3j - 3k$$

$$(a_2 - a_1) = j - 4k$$

$$(a_2 - a_1)\cdot(b_1 \times b_2) = 9$$

$$|(b_1 \times b_2)| = 3\sqrt{2}$$

Shortest distance is $\dfrac{9}{3\sqrt{2}} = \dfrac{3}{\sqrt{2}}$

Straight Line in Space Ex 28.5 Q1(v)

We know that, the shortest distance between lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is given by

$$\vec{r} = \left|\frac{(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|}\right| \qquad ---(i)$$

Given equations of lines are,

$$\vec{r} = (\lambda - 1)\hat{i} + (\lambda + 1)\hat{j} - (1 + \lambda)\hat{k}$$

$$\Rightarrow \quad \vec{r} = (-\hat{i} + \hat{j} - \hat{k}) + \lambda(\hat{i} + \hat{j} - \hat{k}) \text{ and}$$

$$\vec{r} = (1 - \mu)\hat{i} + (2\mu - 1)\hat{j} + (\mu + 2)\hat{k}$$

$$\Rightarrow \quad \vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-\hat{i} + 2\hat{j} + \hat{k})$$

So, $\vec{a_1} = (-\hat{i} + \hat{j} - \hat{k})$, $\vec{b_1} = (\hat{i} + \hat{j} - \hat{k})$ and

$\vec{a_2} = (\hat{i} - \hat{j} + 2\hat{k})$, $\vec{b_2} = (-\hat{i} + 2\hat{j} + \hat{k})$

$$(\vec{a_2} - \vec{a_1}) = (\hat{i} - \hat{j} + 2\hat{k}) - (-\hat{i} + \hat{j} - \hat{k})$$

$$= \hat{i} - \hat{j} + 2\hat{k} + \hat{i} - \hat{j} + \hat{k}$$

$$(\vec{a_2} - \vec{a_1}) = 2\hat{i} - 2\hat{j} + 3\hat{k}$$

$$(\vec{b_1} \times \vec{b_2}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ -1 & 2 & 1 \end{vmatrix}$$

$$= \hat{i}(1 + 2) - \hat{j}(1 - 1) + \hat{k}(2 + 1)$$

$$(\vec{b_1} \times \vec{b_2}) = 3\hat{i} + 3\hat{k}$$

$$(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2}) = (2\hat{i} - 2\hat{j} + 3\hat{k})(3\hat{i} + 3\hat{k})$$

$$= (2)(3) + (-2)(0) + (3)(3)$$

$$= 6 + 0 + 9$$

$$(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2}) = 15$$

$$|\vec{b_1} \times \vec{b_2}| = \sqrt{(3)^2 + (3)^2}$$

$$= \sqrt{18}$$

$$|\vec{b_1} \times \vec{b_2}| = 3\sqrt{2}$$

Substituting values of $(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2})$ and $|\vec{b_1} \times \vec{b_2}|$ in equation (i) to get the shortest distance between the given lines, so

$$\text{S.D.} = \left|\frac{15}{3\sqrt{2}}\right|$$

$$\text{S.D.} = \frac{5}{\sqrt{2}} \text{ units}$$

Straight Line in Space Ex 28.5 Q1(vi)

We know that, the shortest distance between lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is given by

$$\text{S.D.} = \left|\frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|}\right| \qquad ---(i)$$

Given equations of lines are,

$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \lambda(2\hat{i} - 5\hat{j} + 2\hat{k}) \text{ and}$$
$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \mu(\hat{i} - \hat{j} + \hat{k})$$

$$\Rightarrow \quad \vec{a_1} = (2\hat{i} - \hat{j} - \hat{k}),\ \vec{b_1} = (2\hat{i} - 5\hat{j} + 2\hat{k}) \text{ and}$$
$$\vec{a_2} = (\hat{i} + 2\hat{j} + \hat{k}),\ \vec{b_2} = (\hat{i} - \hat{j} + \hat{k})$$

$$(\vec{a_2} - \vec{a_1}) = (\hat{i} + 2\hat{j} + \hat{k}) - (2\hat{i} - \hat{j} - \hat{k})$$
$$= \hat{i} + 2\hat{j} + \hat{k} - 2\hat{i} + \hat{j} + \hat{k}$$
$$(\vec{a_2} - \vec{a_1}) = -\hat{i} + 3\hat{j} + 2\hat{k}$$

$$|\vec{b_1} \times \vec{b_2}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -5 & 2 \\ 1 & -1 & 1 \end{vmatrix}$$
$$= \hat{i}(-5 + 2) - \hat{j}(2 - 2) + \hat{k}(-2 + 5)$$
$$= -3\hat{i} + 3\hat{k}$$

$$(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = (-\hat{i} + 3\hat{j} + 2\hat{k})(-3\hat{i} + 3\hat{k})$$
$$= (-1)(-3) + (3)(0) + (2)(3)$$
$$= 3 + 0 + 6$$
$$(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2}) = 9$$

$$|\vec{b_1} \times \vec{b_2}| = \sqrt{(-3)^2 + (3)^2}$$
$$= \sqrt{9 + 9}$$
$$|\vec{b_1} \times \vec{b_2}| = 3\sqrt{2}$$

Substituting the values of $(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})$ and $|\vec{b_1} \times \vec{b_2}|$ in equation (i) to get the shortest distance between given lines, so

$$\text{S.D.} = \left|\frac{9}{3\sqrt{2}}\right|$$

$$\text{S.D.} = \frac{3}{\sqrt{2}}$$

Straight Line in Space Ex 28.5 Q1(vii)

Given,

$$\vec{r} = \hat{i} + \hat{j} + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right) \qquad ------(i)$$

and

$$\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu\left(3\hat{i} - 5\hat{j} + 2\hat{k}\right) \qquad ------(ii)$$

Comparing (i) and (ii) with $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ respectively, we get

$$\vec{a}_1 = \hat{i} + \hat{j}, \qquad \vec{b}_1 = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k}, \qquad \vec{b}_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$$

$\therefore \quad \vec{a}_2 - \vec{a}_1 = \hat{i} - \hat{k}$

and $\quad \vec{b}_1 \times \vec{b}_2 = \left(2\hat{i} - \hat{j} + \hat{k}\right) \times \left(3\hat{i} - 5\hat{j} + 2\hat{k}\right)$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = 3\hat{i} - \hat{j} - 7\hat{k}$$

So, $\quad \left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{9 + 1 + 49} = \sqrt{59}$

Hence, the shortest distance between the lines l_1 and l_2 is given by

$$d = \left|\frac{\left(\vec{b}_1 \times \vec{b}_2\right).\left(\vec{a}_2 - \vec{a}_1\right)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right| = \frac{|3 - 0 + 7|}{\sqrt{59}} = \frac{10}{\sqrt{59}}$$

Straight Line in Space Ex 28.5 Q1(viii)

The equation of lines are

$\vec{r} = (8 + 3\lambda)\hat{i} - (9 + 16\lambda)\hat{j} + (10 + 7\lambda)\hat{k}$ and $\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$

The lines pass through $\vec{a_1} = 8\hat{i} - 9\hat{j} + 10\hat{k}$ and $\vec{a_2} = 15\hat{i} + 29\hat{j} + 5\hat{k}$

and parallel to vectors, $\vec{b_1} = 3\lambda\hat{i} - 16\lambda\hat{j} + 7\lambda\hat{k}$ and $\vec{b_2} = 3\mu\hat{i} + 8\mu\hat{j} - 5\mu\hat{k}$

$$\vec{a_1} - \vec{a_2} = -7\hat{i} - 38\hat{j} + 5\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix} = 24\hat{i} + 36\hat{j} + 72\hat{k}$$

So, $\left(\vec{a_1} - \vec{a_2}\right)\cdot\left(\vec{b_1} \times \vec{b_2}\right) = -168 - 1368 + 360 = -1176$

$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{576 + 1296 + 5184} = 84$

$$S.D. = \left|\frac{\left(\vec{a_1} - \vec{a_2}\right)\cdot\left(\vec{b_1} \times \vec{b_2}\right)}{\left|\vec{b_1} \times \vec{b_2}\right|}\right| = \left|\frac{-1176}{84}\right| = 14$$

Straight Line in Space Ex 28.5 Q2(i)

Given lines are,

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda \ \text{(say)}$$

$$x = 2\lambda + 1,\ y = 3\lambda + 2,\ z = 4\lambda + 3$$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (2\lambda + 1)\hat{i} + (3\lambda + 2)\hat{j} + (4\lambda + 3)\hat{k}$$
$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$$

$$\Rightarrow \quad \vec{a_1} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right),\ \vec{b_1} = \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$$

and, $$\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-5}{5} = \mu \ \text{(say)}$$

$$x = 3\mu + 2,\ y = 4\mu + 3,\ z = 5\mu + 5$$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (3\mu + 2)\hat{i} + (4\mu + 3)\hat{j} + (5\mu + 5)\hat{k}$$
$$\vec{r} = \left(2\hat{i} + 3\hat{j} + 5\hat{k}\right) + \mu\left(3\hat{i} + 4\hat{j} + 5\hat{k}\right)$$

$$\Rightarrow \quad \vec{a_2} = \left(2\hat{i} + 3\hat{j} + 5\hat{k}\right),\ \vec{b_2} = \left(3\hat{i} + 4\hat{j} + 5\hat{k}\right)$$

We know that, the shortest distance between the lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ is given by,

$$\text{S.D.} = \left|\frac{\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)}{\left|\vec{b_1} \times \vec{b_2}\right|}\right| \qquad ---(i)$$

$$\left(\vec{a_2} - \vec{a_1}\right) = \left(2\hat{i} + 3\hat{j} + 5\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$$
$$= 2\hat{i} + 3\hat{j} + 5\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k}$$
$$\left(\vec{a_2} - \vec{a_1}\right) = \hat{i} + \hat{j} + 2\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$
$$= \hat{i}(15 - 16) - \hat{j}(10 - 12) + \hat{k}(8 - 9)$$

$$\left(\vec{b_1} \times \vec{b_2}\right) = -\hat{i} + 2\hat{j} - \hat{k}$$

$$\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right) = \left(\hat{i} + \hat{j} + 2\hat{k}\right)\left(-\hat{i} + 2\hat{j} - \hat{k}\right)$$
$$= (1)(-1) + (1)(2) + (2)(-1)$$
$$= -1 + 2 - 2$$
$$\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right) = -1$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{(-1)^2 + (2)^2 + (-1)^2}$$
$$= \sqrt{1 + 4 + 1}$$
$$= \sqrt{6}$$

Using the values of $\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)$ and $\left|\vec{b_1} \times \vec{b_2}\right|$ in equation (i) to get the shortest distance between given lines, so

$$\text{S.D.} = \left|\frac{-1}{\sqrt{3}}\right|$$

$$\text{S.D.} = \frac{1}{\sqrt{6}} \text{ units}$$

Straight Line in Space Ex 28.5 Q2(ii)

Given equations of line are,

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{1} = \lambda \ \text{(say)}$$

$\Rightarrow \quad x = 2\lambda + 1,\ y = 3\lambda - 1,\ z = \lambda$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (2\lambda + 1)\hat{i} + (3\lambda - 1)\hat{j} + \lambda\hat{k}$$
$$\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + 3\hat{j} + \hat{k})$$

$\Rightarrow \quad \vec{a_1} = \hat{i} - \hat{j},\ \vec{b_1} = 2\hat{i} + 3\hat{j} + \hat{k}$

and, $\quad \frac{x+1}{3} = \frac{y-2}{1} = \mu,\ z = 2$

$\Rightarrow \quad x = 3\mu - 1,\ y = \mu + 2,\ z = 2$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (3\mu - 1)\hat{i} + (\mu + 2)\hat{j} + 2\hat{k}$$
$$\vec{r} = (-\hat{i} + 2\hat{j} + 2\hat{k}) + \mu(3\hat{i} + \hat{j})$$

$\Rightarrow \quad \vec{a_2} = (-\hat{i} + 2\hat{j} + 2\hat{k}),\ \vec{b_2} = (3\hat{i} + \hat{j})$

We know that, the shortest distance between two lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ is given by,

$$\text{S.D.} = \left|\frac{(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|}\right| \qquad ---(i)$$

$$(\vec{a_2} - \vec{a_1}) = (\hat{i} - \hat{j}) - (-\hat{i} + 2\hat{j} + 2\hat{k})$$
$$= \hat{i} - \hat{j} + \hat{i} - 2\hat{j} - 2\hat{k}$$
$$(\vec{a_2} - \vec{a_1}) = 2\hat{i} - 3\hat{j} - 2\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 3 & 1 & 0 \end{vmatrix}$$
$$= \hat{i}(0-1) - \hat{j}(0-3) + \hat{k}(2-9)$$

$$(\vec{b_1} \times \vec{b_2}) = -\hat{i} + 3\hat{j} - 7\hat{k}$$

$$(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2}) = (2\hat{i} - 3\hat{j} - 2\hat{k})(-\hat{i} + 3\hat{j} - 7\hat{k})$$
$$= (2)(-1) + (-3)(3) + (-2)(-7)$$
$$= -2 - 9 + 14$$
$$(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2}) = 3$$

$$|\vec{b_1} \times \vec{b_2}| = \sqrt{(-1)^2 + (3)^2 + (-7)^2} = \sqrt{59}$$

Substitute the value of $(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2})$ and $|\vec{b_1} \times \vec{b_2}|$ in equation (i) to get the shortest distance between given lines, so

$$\text{S.D.} = \left|\frac{3}{\sqrt{59}}\right|$$

$$\text{S.D.} = \frac{3}{\sqrt{59}} \text{ units}$$

Straight Line in Space Ex 28.5 Q2(iii)

Given equation of lines are,

$$\frac{x-1}{-1} = \frac{y+2}{1} = \frac{z-3}{-2} = \lambda \ \text{(say)}$$

$\Rightarrow \quad x = -\lambda + 1,\ y = \lambda - 2,\ z = -2\lambda + 3$

$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$= (-\lambda + 1)\hat{i} + (\lambda - 2)\hat{j} + (-2\lambda + 3)\hat{k}$

$\vec{r} = \left(\hat{i} - 2\hat{j} + 3\hat{k}\right) + \lambda\left(-\hat{i} + \hat{j} - 2\hat{k}\right)$

$\Rightarrow \quad \vec{a_1} = \left(\hat{i} - 2\hat{j} + 3\hat{k}\right),\ \vec{b_1} = \left(-\hat{i} + \hat{j} - 2\hat{k}\right)$

and, $\quad \dfrac{x-1}{1} = \dfrac{y+1}{2} = \dfrac{z+1}{-2} = \mu \ \text{(say)}$

$\Rightarrow \quad x = \mu + 1,\ y = 2\mu - 1,\ z = -2\mu - 1$

$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$= (\mu + 1)\hat{i} + (2\mu - 1)\hat{j} + (-2\mu - 1)\hat{k}$

$\vec{r} = \left(\hat{i} - \hat{j} - \hat{k}\right) + \mu\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$

$\Rightarrow \quad \vec{a_2} = \left(\hat{i} - \hat{j} - \hat{k}\right),\ \vec{b_2} = \left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$

$\left(\vec{a_2} - \vec{a_1}\right) = \left(\hat{i} - \hat{j} - \hat{k}\right) - \left(\hat{i} - 2\hat{j} + 3\hat{k}\right)$

$= \hat{i} - \hat{j} - \hat{k} - \hat{i} + 2\hat{j} - 3\hat{k}$

$= \hat{j} - 4\hat{k}$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix}$$

$= \hat{i}(-2 + 4) - \hat{j}(2 + 2) + \hat{k}(-2 - 1)$

$\left(\vec{b_1} \times \vec{b_2}\right) = 2\hat{i} - 4\hat{j} - 3\hat{k}$

$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{(2)^2 + (-4)^2 + (-3)^2}$

$= \sqrt{29}$

$\left(\vec{a_2} - \vec{a_1}\right)\left(\vec{b_1} \times \vec{b_2}\right) = \left(\hat{j} - 4\hat{k}\right)\left(2\hat{i} - 4\hat{j} - 3\hat{k}\right)$

$= (0)(2) + (1)(-4) + (-4)(-3)$

$= 0 - 4 + 12$

$\left(\vec{a_2} - \vec{a_1}\right)\left(\vec{b_1} \times \vec{b_2}\right) = 8$

We know that, shortest distance between $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ is given by,

$$\text{S.D.} = \left|\frac{\left(\vec{a_2} - \vec{a_1}\right) \cdot \left(\vec{b_1} \times \vec{b_2}\right)}{\left|\vec{b_1} \times \vec{b_2}\right|}\right| \qquad ---(i)$$

So, shortest distance between given lines is

$$\text{S.D.} = \left|\frac{8}{\sqrt{29}}\right|$$

$$\text{S.D.} = \frac{8}{\sqrt{29}} \text{ units}$$

Straight Line in Space Ex 28.5 Q2(iv)

Given equation of lines are,

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} = \lambda \text{ (say)}$$

$\Rightarrow \quad x = \lambda + 3,\ y = -2\lambda + 5,\ z = \lambda + 7$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (\lambda + 3)\hat{i} + (-2\lambda + 5)\hat{j} + (\lambda + 7)\hat{k}$$
$$\vec{r} = \left(3\hat{i} + 5\hat{j} + 7\hat{k}\right) + \lambda\left(\hat{i} - 2\hat{j} + \hat{k}\right)$$

$$\Rightarrow \quad \vec{a_1} = \left(3\hat{i} + 5\hat{j} + 7\hat{k}\right),\ \vec{b_1} = \left(\hat{i} - 2\hat{j} + \hat{k}\right)$$

and, $\dfrac{x+1}{7} = \dfrac{y+1}{-6} = \dfrac{z+1}{1} = \mu$ (say)

$\Rightarrow \quad x = 7\mu - 1,\ y = -6\mu - 1,\ z = \mu - 1$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (7\mu - 1)\hat{i} + (-6\mu - 1)\hat{j} + (\mu - 1)\hat{k}$$
$$\vec{r} = \left(-\hat{i} - \hat{j} - \hat{k}\right) + \mu\left(7\hat{i} - 6\hat{j} + \hat{k}\right)$$

$$\Rightarrow \quad \vec{a_2} = \left(-\hat{i} - \hat{j} - \hat{k}\right),\ \vec{b_2} = 7\hat{i} - 6\hat{j} + \hat{k}$$

We know that, shortest distance between $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ is given by,

$$\text{S.D.} = \left|\frac{\left(\vec{a_2} - \vec{a_1}\right)\cdot\left(\vec{b_1} \times \vec{b_2}\right)}{\left|\vec{b_1} \times \vec{b_2}\right|}\right| \qquad ---(i)$$

$$\left(\vec{a_2} - \vec{a_1}\right) = \left(-\hat{i} - \hat{j} - \hat{k}\right) - \left(3\hat{i} + 5\hat{j} + 7\hat{k}\right)$$
$$= -\hat{i} - \hat{j} - \hat{k} - 3\hat{i} - 5\hat{j} - 7\hat{k}$$
$$\left(\vec{a_2} - \vec{a_1}\right) = -4\hat{i} - 6\hat{j} - 8\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 7 & -6 & 1 \end{vmatrix}$$
$$= \hat{i}(-2 + 6) - \hat{j}(1 - 7) + \hat{k}(-6 + 14)$$
$$\vec{b_1} \times \vec{b_2} = 4\hat{i} + 6\hat{j} + 8\hat{k}$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{(4)^2 + (6)^2 + (8)^2}$$
$$= \sqrt{16 + 36 + 64}$$
$$= \sqrt{116}$$
$$= 2\sqrt{29}$$

$$\left(\vec{a_2} - \vec{a_1}\right)\left(\vec{b_1} \times \vec{b_2}\right) = \left(-4\hat{i} - 6\hat{j} - 8\hat{k}\right)\left(4\hat{i} + 6\hat{j} + 8\hat{k}\right)$$
$$= (-4)(4) + (-6)(6) + (-8)(8)$$
$$= -16 - 36 - 64$$
$$= -116$$

Substituting the values of $\left(\vec{a_2} - \vec{a_1}\right)\cdot\left(\vec{b_1} \times \vec{b_2}\right)$ and $\left|\vec{b_1} \times \vec{b_2}\right|$ in equation (i) to get the shortest distance between the two given lines, so

$$\text{S.D.} = \left|\frac{-116}{2\sqrt{29}}\right|$$
$$= \frac{58}{\sqrt{29}}$$

Straight Line in Space Ex 28.5 Q3(i)

Given equations of lines are,

$$\vec{r} = \left(\hat{i} - \hat{j}\right) + \lambda\left(2\hat{i} + \hat{k}\right)$$

$$\Rightarrow \quad \vec{a_1} = \left(\hat{i} - \hat{j}\right),\ \vec{b_1} = \left(2\hat{i} + \hat{k}\right)$$

and, $\vec{r} = \left(2\hat{i} - \hat{j}\right) + \mu\left(\hat{i} + \hat{j} - \hat{k}\right)$

$$\Rightarrow \quad \vec{a_2} = \left(2\hat{i} - \hat{j}\right),\ \vec{b_2} = \left(\hat{i} + \hat{j} - \hat{k}\right)$$

We know that, shortest distance between lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ is given by

$$\text{S.D.} = \left|\frac{\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)}{\left|\vec{b_1} \times \vec{b_2}\right|}\right| \qquad ---(i)$$

$$\left(\vec{a_2} - \vec{a_1}\right) = \left(2\hat{i} - \hat{j}\right) - \left(\hat{i} - \hat{j}\right)$$
$$= 2\hat{i} - \hat{j} - \hat{i} + \hat{j}$$
$$= \hat{i}$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$
$$= \hat{i}\left(0 - 1\right) - \hat{j}\left(-2 - 1\right) + \hat{k}\left(2 - 0\right)$$
$$= -\hat{i} + 3\hat{j} + 2\hat{k}$$

$$\left(\vec{a_2} - \vec{a_1}\right)\left(\vec{b_1} \times \vec{b_2}\right) = \left(\hat{i}\right)\left(-\hat{i} + 3\hat{j} + 2\hat{k}\right)$$
$$= (1)(-1) + (0)(3) + (0)(2)$$
$$= -1 + 0 + 0$$
$$\left(\vec{a_2} - \vec{a_1}\right)\left(\vec{b_1} \times \vec{b_2}\right) = -1$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{(-1)^2 + (3)^2 + (2)^2}$$
$$= \sqrt{1 + 9 + 4}$$
$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{14}$$

So, shortest distance between the given lines using equation (1) is,

$$\text{S.D.} = \left|\frac{-1}{\sqrt{14}}\right|$$

$$= \frac{1}{\sqrt{14}} \text{ units}$$

$$\text{S.D} \neq 0$$

Since, shortest distance between lines is not zero, so
lines are not intersecting.

Straight Line in Space Ex 28.5 Q3(ii)

Given equations of lines are,

$$\vec{r} = \left(\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(3\hat{i} - \hat{j}\right)$$

$$\Rightarrow \quad \vec{a_1} = \left(\hat{i} + \hat{j} - \hat{k}\right), \vec{b_1} = \left(3\hat{i} - \hat{j}\right)$$

and, $\vec{r} = \left(4\hat{i} - \hat{k}\right) + \mu\left(2\hat{i} + 3\hat{k}\right)$

$$\Rightarrow \quad \vec{a_2} = \left(4\hat{i} - \hat{k}\right), \vec{b_2} = \left(2\hat{i} + 3\hat{k}\right)$$

We know that, shortest distance between two lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ is given by

$$\text{S.D.} = \left|\frac{\left(\vec{a_2} - \vec{a_1}\right)\cdot\left(\vec{b_1} \times \vec{b_2}\right)}{\left|\vec{b_1} \times \vec{b_2}\right|}\right| \qquad ---(i)$$

$$\left(\vec{a_2} - \vec{a_1}\right) = \left(4\hat{i} - \hat{k}\right) - \left(\hat{i} + \hat{j} - \hat{k}\right)$$
$$= 4\hat{i} - \hat{k} - \hat{i} - \hat{j} + \hat{k}$$
$$= 3\hat{i} - \hat{j}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix}$$
$$= \hat{i}\left(-3 - 0\right) - \hat{j}\left(9 - 0\right) + \hat{k}\left(0 + 2\right)$$
$$= -3\hat{i} - 9\hat{j} + 2\hat{k}$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{(-3)^2 + (-9)^2 + (2)^2}$$
$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{9 + 81 + 4}$$
$$= \sqrt{94}$$

$$\left(\vec{a_2} - \vec{a_1}\right)\cdot\left(\vec{b_1} \times \vec{b_2}\right) = \left(3\hat{i} - \hat{j}\right)\left(-3\hat{i} - 9\hat{j} + 2\hat{k}\right)$$
$$= (3)(-3) + (-1)(-9) + (0)(2)$$
$$= -9 + 9 + 0$$
$$= 0$$

Using $\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)$ and $\left|\vec{b_1} \times \vec{b_2}\right|$ in equation (i) to get shortest distance between given lines, so

$$\text{S.D.} = \left|\frac{0}{\sqrt{94}}\right|$$

$$\text{S.D} = 0$$

Since, shortest distance between the given lines is zero, so lines are intersecting.

Straight Line in Space Ex 28.5 Q3(iii)

Given equations of lines are,

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{1} = \lambda \ \text{(say)}$$

$\Rightarrow \quad x = 2\lambda + 1,\ y = 3\lambda - 1,\ z = \lambda$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (2\lambda + 1)\hat{i} + (3\lambda - 1)\hat{j} + (\lambda)\hat{k}$$
$$\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + 3\hat{j} + \hat{k})$$

$$\Rightarrow \quad \vec{a_1} = (\hat{i} - \hat{j}),\ \vec{b_1} = (2\hat{i} + 3\hat{j} + \hat{k})$$

and, $\quad \frac{x+1}{5} = \frac{y-2}{1} = \mu \ \text{(say)},\ z = 2$

$\Rightarrow \quad x = 5\mu - 1,\ y = \mu + 2,\ z = 2$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (5\mu - 1)\hat{i} + (\mu + 2)\hat{j} + 2\hat{k}$$
$$\vec{r} = (-\hat{i} + 2\hat{j} + 2\hat{k}) + \mu(5\hat{i} + \hat{j})$$

$$\Rightarrow \quad \vec{a_2} = (-\hat{i} + 2\hat{j} + 2\hat{k}),\ \vec{b_2} = (5\hat{i} + \hat{j})$$

We know that, the shortest distance between $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ is given by

$$\text{S.D.} = \left|\frac{(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|}\right| \qquad ---\text{(i)}$$

$$(\vec{a_2} - \vec{a_1}) = (-\hat{i} + 2\hat{j} + 2\hat{k}) - (\hat{i} - \hat{j})$$
$$= -\hat{i} + 2\hat{j} + 2\hat{k} - \hat{i} + \hat{j}$$
$$= -2\hat{i} + 3\hat{j} + 2\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 5 & 1 & 0 \end{vmatrix}$$
$$= \hat{i}(0-1) - \hat{j}(0-5) + \hat{k}(2-15)$$
$$\vec{b_1} \times \vec{b_2} = -\hat{i} + 5\hat{j} - 13\hat{k}$$

$$(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2}) = (-2\hat{i} + 3\hat{j} + 2\hat{k})(-\hat{i} + 5\hat{j} - 13\hat{k})$$
$$= (-2)(-1) + (3)(5) + (2)(-13)$$
$$= -9$$

$$|\vec{b_1} \times \vec{b_2}| = \sqrt{(-1)^2 + (5)^2 + (-13)^2}$$
$$= \sqrt{1 + 25 + 169}$$
$$= \sqrt{195}$$

Substituting the value of $(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2})$ and $|\vec{b_1} \times \vec{b_2}|$ in equation (i) to get shortest distance between given lines, so

$$\text{S.D.} = \left|\frac{-9}{\sqrt{195}}\right|$$
$$= \frac{9}{\sqrt{195}} \text{ units}$$

Since, shortest distance between given lines is not zero, so lines are not intersecting.

Straight Line in Space Ex 28.5 Q3(iv)

Given lines are,

$$\frac{x-5}{4} = \frac{y-7}{-5} = \frac{z+3}{-5} = \lambda \text{ (say)}$$

$$\Rightarrow \quad x = 4\lambda + 5,\ y = -5\lambda + 7,\ z = -5\lambda - 3$$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (4\lambda + 5)\hat{i} + (-5\lambda + 7)\hat{j} + (-5\lambda - 3)\hat{k}$$
$$\vec{r} = \left(5\hat{i} + 7\hat{j} - 3\hat{k}\right) + \lambda\left(4\hat{i} - 5\hat{j} - 5\hat{k}\right)$$

$$\Rightarrow \quad \vec{a_1} = \left(5\hat{i} + 7\hat{j} - 3\hat{k}\right),\ \vec{b_1} = \left(4\hat{i} - 5\hat{j} - 5\hat{k}\right)$$

and, $$\frac{x-8}{7} = \frac{y-7}{1} = \frac{z-5}{3} = \mu \text{ (say)}$$

$$\Rightarrow \quad x = 7\mu + 8,\ y = \mu + 7,\ 3\mu + 5$$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (7\mu + 8)\hat{i} + (\mu + 7)\hat{j} + (3\mu + 5)\hat{k}$$
$$\vec{r} = \left(8\hat{i} + 7\hat{j} + 5\hat{k}\right) + \mu\left(7\hat{i} + \hat{j} + 3\hat{k}\right)$$

$$\Rightarrow \quad \vec{a_2} = \left(8\hat{i} + 7\hat{j} + 5\hat{k}\right),\ \vec{b_2} = \left(7\hat{i} + \hat{j} + 3\hat{k}\right)$$

We know that, shortest distance between lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ is given by

$$\text{S.D.} = \left|\frac{\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)}{\left|\vec{b_1} \times \vec{b_2}\right|}\right| \qquad ---(i)$$

$$\left(\vec{a_2} - \vec{a_1}\right) = \left(8\hat{i} + 7\hat{j} + 5\hat{k}\right) - \left(5\hat{i} + 7\hat{j} - 3\hat{k}\right)$$
$$= 8\hat{i} + 7\hat{j} + 5\hat{k} - 5\hat{i} - 7\hat{j} + 3\hat{k}$$
$$= 3\hat{i} + 8\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -5 & -5 \\ 7 & 1 & 3 \end{vmatrix}$$
$$= \hat{i}(-15 + 5) - \hat{j}(12 + 35) + \hat{k}(4 + 35)$$
$$= -10\hat{i} - 47\hat{j} + 39\hat{k}$$

$$\left(\vec{a_2} - \vec{a_1}\right)\left(\vec{b_1} \times \vec{b_2}\right) = \left(3\hat{i} + 8\hat{k}\right)\left(-10\hat{i} - 47\hat{j} + 39\hat{k}\right)$$
$$= (3)(-10) + (0)(-4) + (8)(39)$$
$$= -30 + 312$$
$$= 282$$

Using equation (i) to get the shortest distance between the given lines, so

$$\text{S.D.} = \left|\frac{282}{\left|\vec{b_1} \times \vec{b_2}\right|}\right|$$

$$\text{S.D.} \neq 0$$

Since, the shortest distance between given lines is not equal to zero, so

Given lines are not intersecting.

Straight Line in Space Ex 28.5 Q4(i)

Given, equation of lines are,

$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right) \qquad ---(1)$$

$$\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \mu\left(-\hat{i} + \hat{j} - \hat{k}\right)$$

$$\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) - \mu\left(\hat{i} - \hat{j} + \hat{k}\right)$$

$$\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \mu'\left(\hat{i} - \hat{j} + \hat{k}\right) \qquad ---(2)$$

These two lines passes through the points having position vectors $\vec{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{a_2} = 2\hat{i} - \hat{j} - \hat{k}$ respectively and both are parallel to the vector $\vec{b} = \hat{i} - \hat{j} + \hat{k}$

We know that, shortest distance between parallel lines $\vec{r} = \vec{a_1} + \lambda\vec{b}$ and $\vec{r} = \vec{a_2} + \mu\vec{b}$ is given by

$$\text{S.D.} = \left|\frac{\left(\vec{a_2} - \vec{a_1}\right).\vec{b}}{\left|\vec{b}\right|}\right| \qquad ---(i)$$

$$\left(\vec{a_2} - \vec{a_1}\right) = \left(2\hat{i} - \hat{j} - \hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$$

$$= 2\hat{i} - \hat{j} - \hat{k} - \hat{i} - 2\hat{j} - 3\hat{k}$$

$$\left(\vec{a_2} - \vec{a_1}\right) = \hat{i} - 3\hat{j} - 4\hat{k}$$

$$\left(\vec{a_2} - \vec{a_1}\right) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & -4 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}\left(-3 - 4\right) - \hat{j}\left(1 + 4\right) + \hat{k}\left(-1 + 3\right)$$

$$\left(\vec{a_2} - \vec{a_1}\right) \times \vec{b} = -7\hat{i} - 5\hat{j} + 2\hat{k}$$

$$\left|\left(\vec{a_2} - \vec{a_1}\right) \times \vec{b}\right| = \sqrt{\left(-7\right)^2 + \left(-5\right)^2 + \left(2\right)^2}$$

$$= \sqrt{49 + 25 + 4}$$

$$= \sqrt{78}$$

$$\left|\vec{b}\right| = \sqrt{\hat{i} - \hat{j} + \hat{k}}$$

$$= \sqrt{\left(1\right)^2 + \left(-1\right)^2 + \left(1\right)^2}$$

$$\left|\vec{b}\right| = \sqrt{3}$$

Using $\left|\left(\vec{a_2} - \vec{a_1}\right) \times \vec{b}\right|$ and $\left|\vec{b}\right|$ in equation (1) to get the shortest distance between parallel lines, so

$$\text{S.D.} = \frac{\sqrt{78}}{\sqrt{3}}$$

$$\text{S.D.} = \sqrt{\frac{78}{3}}$$

$$\text{S.D.} = \sqrt{26} \text{ units}$$

Straight Line in Space Ex 28.5 Q4(ii)

Given, equation of lines are,

$$\vec{r} = \left(\hat{i} + \hat{j}\right) + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right) \qquad ---(1)$$

$$\vec{r} = \left(2\hat{i} + \hat{j} - \hat{k}\right) + \mu\left(4\hat{i} - 2\hat{j} + 2\hat{k}\right)$$

$$\vec{r} = \left(2\hat{i} + \hat{j} - \hat{k}\right) + 2\mu\left(2\hat{i} - \hat{j} + \hat{k}\right)$$

$$\vec{r} = \left(2\hat{i} + \hat{j} - \hat{k}\right) + \mu'\left(2\hat{i} - \hat{j} + \hat{k}\right) \qquad ---(2)$$

So, $\overrightarrow{a_1} = \left(\hat{i} + \hat{j}\right)$, $\overrightarrow{a_2} = 2\hat{i} + \hat{j} - \hat{k}$

$\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$

We know that, the shortest distance between the parallel lines $\vec{r} = \overrightarrow{a_1} + \lambda\vec{b}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\vec{b}$ is given by

$$\text{S.D.} = \frac{\left|\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right)\vec{b}\right|}{\left|\vec{b}\right|} \qquad ---(i)$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) = \left(2\hat{i} + \hat{j} - \hat{k}\right) - \left(\hat{i} + \hat{j}\right)$$

$$= 2\hat{i} + \hat{j} - \hat{k} - \hat{i} - \hat{j}$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) = \hat{i} - \hat{k}$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ 2 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}\left(0 - 1\right) - \hat{j}\left(1 + 2\right) + \hat{k}\left(-1 - 0\right)$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \times \vec{b} = -\hat{i} - 3\hat{j} - \hat{k}$$

$$\left|\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \times \vec{b}\right| = \sqrt{(-1)^2 + (-3)^2 + (-1)^2}$$

$$= \sqrt{1 + 9 + 1}$$

$$\left|\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \times \vec{b}\right| = \sqrt{11}$$

$$\left|\vec{b}\right| = \sqrt{(2)^2 + (-1)^2 + (1)^2}$$

$$= \sqrt{4 + 1 + 1}$$

$$\left|\vec{b}\right| = \sqrt{6}$$

Using $\left|\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \times \vec{b}\right|$ and $\left|\vec{b}\right|$ in equation (1) to get the shortest distance between the given lines, so

$$\text{S.D.} = \frac{\sqrt{11}}{\sqrt{6}}$$

$$\text{S.D.} = \sqrt{\frac{11}{6}} \text{ units}$$

Straight Line in Space Ex 28.5 Q5

Equation of line passing through $(0,0,0)$ and $(1,0,2)$ is given by $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

$$\vec{r} = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \lambda((1-0)\hat{i} + (0-0)\hat{j} + (2-0)\hat{k})$$

$$\vec{r} = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \lambda(\hat{i} + 2\hat{k}) \qquad ---(1)$$

Equation of another line passing through $(1,3,0)$ and $(0,3,0)$ is

$$\vec{r} = (\hat{i} + 3\hat{j} + 0\hat{k}) + \mu((0-1)\hat{i} + (3-3)\hat{j} + (0-0)\hat{k})$$

$$\vec{r} = (\hat{i} + 3\hat{j} + 0\hat{k}) + \mu(-\hat{i}) \qquad ---(2)$$

From equation (1) and (2)

$$\vec{a_1} = (0.\hat{i} + 0.\hat{j} + 0.\hat{k}),\ \vec{b_1} = (\hat{i} + 2\hat{k})$$

$$\vec{a_2} = (\hat{i} + 3\hat{j} + 0\hat{k}), \qquad \vec{b_2} = -\hat{i}$$

We know that, shortest distance between the lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ is given by

$$\text{S.D.} = \frac{\left|(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2})\right|}{\left|\vec{b_1} \times \vec{b_2}\right|} \qquad ---(3)$$

$$(\vec{a_2} - \vec{a_1}) = (\hat{i} + 3\hat{j} + 0.\hat{k}) - (0.\hat{i} + 0.\hat{j} + 0\hat{k})$$

$$(\vec{a_2} - \vec{a_1}) = (\hat{i} + 3\hat{j})$$

$$(\vec{b_1} \times \vec{b_2}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 2 \\ -1 & 0 & 0 \end{vmatrix}$$

$$= \hat{i}(0-0) - \hat{j}(0+2) + \hat{k}(-2)$$

$$(\vec{b_1} \times \vec{b_2}) = -2\hat{j}$$

$$(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2}) = (\hat{i} + 3\hat{j})(-2\hat{j})$$

$$= (1)(0) + (3)(-2)$$

$$(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2}) = -6$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{(-2)^2}$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = 2$$

Using $(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2})$ and $\left|\vec{b_1} \times \vec{b_2}\right|$ in equation (1) to get shortest distance between the lines, so

$$\text{S.D.} = \left|\frac{-6}{2}\right|$$

$$\text{S.D.} = 3 \text{ units}$$

Straight Line in Space Ex 28.5 Q6

Given equations of lines are,

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6} = \lambda \ \text{(say)}$$

$$\Rightarrow \quad x = 2\lambda + 1,\ y = 3\lambda + 2,\ z = 6\lambda - 4$$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (2\lambda + 1)\hat{i} + (3\lambda + 2)\hat{j} + (6\lambda - 4)\hat{k}$$
$$\vec{r} = \left(\hat{i} + 2\hat{j} - 4\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$$

$$\Rightarrow \quad \vec{a_1} = \hat{i} + 2\hat{j} - 4\hat{k},\ \vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

Another equation of line is,

$$\frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12} = \mu \ \text{(say)}$$

$$\Rightarrow \quad x = 4\mu + 3,\ y = 6\mu + 3,\ 12\mu - 5$$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (4\mu + 3)\hat{i} + (6\mu + 3)\hat{j} + (12\mu - 5)\hat{k}$$
$$= \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right) + \mu\left(4\hat{i} + 6\hat{j} + 12\hat{k}\right)$$
$$= \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right) + 2\mu\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$$
$$\vec{r} = \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right) + \mu'\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$$

$$\Rightarrow \quad \vec{a_2} = \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right),\ \vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

We know that, shortest distance between parallel lines $\vec{r} = \vec{a_1} + \lambda\vec{b}$ and $\vec{r} = \vec{a_2} + \mu\vec{b}$ is given by

$$\text{S.D.} = \frac{\left|\left(\vec{a_2} - \vec{a_1}\right) \times \vec{b}\right|}{\left|\vec{b}\right|} \qquad ---(i)$$

$$\left(\vec{a_2} - \vec{a_1}\right) = \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right) - \left(\hat{i} + 2\hat{j} - 4\hat{k}\right)$$
$$= 3\hat{i} + 3\hat{j} - 5\hat{k} - \hat{i} - 2\hat{j} + 4\hat{k}$$
$$\left(\vec{a_2} - \vec{a_1}\right) = 2\hat{i} + \hat{j} - \hat{k}$$

$$\left(\vec{a_2} - \vec{a_1}\right) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix}$$
$$= \hat{i}\,(6 + 3) - \hat{j}\,(12 + 2) + \hat{k}\,(6 - 2)$$
$$= 9\hat{i} - 14\hat{j} + 4\hat{k}$$

$$\left|\left(\vec{a_2} - \vec{a_1}\right) \times \vec{b}\right| = \sqrt{(9)^2 + (-14)^2 + (4)^2}$$
$$= \sqrt{81 + 196 + 16}$$
$$= \sqrt{293}$$

$$\left|\vec{b}\right| = \sqrt{(2)^2 + (3)^2 + (6)^2}$$
$$= \sqrt{4 + 9 + 36}$$
$$= \sqrt{49}$$
$$\left|\vec{b}\right| = 7$$

Using $\left|\left(\vec{a_2} - \vec{a_1}\right) \times \vec{b}\right|$ and $\left|\vec{b}\right|$ in equation (i) to get the shortest distance between given lines, so

$$\text{S.D.} = \frac{\sqrt{293}}{7} \text{ units}$$

Straight Line in Space Ex 28.5 Q7(i)

Here,

$a_1 = \hat{i} + 2\hat{j} + \hat{k}$

$b_1 = \hat{i} - \hat{j} + \hat{k}$

$a_2 = 2\hat{i} - \hat{j} - \hat{k}$

$b_2 = 2\hat{i} + \hat{j} + 2\hat{k}$

$\bar{a}_2 - \bar{a}_1 = 2\hat{i} - \hat{j} - \hat{k} - \hat{i} - 2\hat{j} - \hat{k} = \hat{i} - 3\hat{j} - 2\hat{k}$

$$b_1 \times b_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = \hat{i}(-2-1) - \hat{j}(2-2) + \hat{k}(1+2) = -3\hat{i} + 3\hat{k}$$

The shortest distance between the two lines,

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2).(\bar{a}_2 - \bar{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$d = \left| \frac{(-3\hat{i} + 3\hat{k}).(\hat{i} - 3\hat{j} - 2\hat{k})}{|-3\hat{i} - 3\hat{k}|} \right| = \left| \frac{-3-6}{\sqrt{(-3)^2 + (-3)^2}} \right| = \frac{9}{3\sqrt{2}}$$

The shortest distance between the two lines $= \dfrac{3}{\sqrt{2}}$ units

Straight Line in Space Ex 28.5 Q7(ii)

Here,

$\bar{a}_1 = -\hat{i} - \hat{j} - \hat{k}$

$\bar{a}_2 = 3\hat{i} + 5\hat{j} + 7\hat{k}$

$\vec{b}_1 = 7\hat{i} - 6\hat{j} + \hat{k}$

$\vec{b}_2 = \hat{i} - 2\hat{j} + \hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

$= \hat{i}(-6+2) - \hat{j}(7-1) + \hat{k}(-14+6)$

$= -4\hat{i} - 6\hat{j} - 8\hat{k}$

$\bar{a}_2 - \bar{a}_1 = \hat{i}(3+1) + \hat{j}(5+1) + \hat{k}(7+1)$

$= 4\hat{i} + 6\hat{j} + 8\hat{k}$

The shortest distance between two lines,

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2).(\bar{a}_2 - \bar{a}_1)}{|b_1 \times b_2|} \right|$$

$$= \left| \frac{(-4\hat{i} - 6\hat{j} - 8\hat{k}).(4\hat{i} + 6\hat{j} + 8\hat{k})}{\sqrt{(-4)^2 + (-6)^2 + (-8)^2}} \right|$$

$$= \left| \frac{-16 - 36 - 64}{\sqrt{116}} \right|$$

$$= \left| \frac{-116}{\sqrt{116}} \right|$$

$= 2\sqrt{29}$ units

Straight Line in Space Ex 28.5 Q7(iii)

Here,

$\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$,

$\vec{b}_1 = \hat{i} - 3\hat{j} + 2\hat{k}$

$\vec{a}_2 = 4\hat{i} + 5\hat{j} + 6\hat{k}$,

$\vec{b}_2 = 2\hat{i} + 3\hat{j} + \hat{k}$

$(\vec{a}_2 - \vec{a}_1) = 4\hat{i} + 5\hat{j} + 6\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k} = 3\hat{i} + 3\hat{j} + 3\hat{k}$

$\vec{b}_1 \times \vec{b}_2$

$= (\hat{i} - 3\hat{j} + 2\hat{k}) \times (2\hat{i} + 3\hat{j} + \hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$= \hat{i}(-3-6) - \hat{j}(1-4) + \hat{k}(3+6)$

$= -9\hat{i} + 3\hat{j} + 9\hat{k}$

Shortest distance between the two lines $= \left|\dfrac{(a_2 - a_1).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

$$= \left|\frac{(3\hat{i} + 3\hat{j} + 3\hat{k}).(-9\hat{i} + 3\hat{j} + 9\hat{k})}{|-9\hat{i} + 3\hat{j} + 9\hat{k}|}\right|$$

$$= \left|\frac{3 \times (-9) + 3 \times 3 + 3 \times 9}{\sqrt{(-9)^2 + 3^2 + 9^2}}\right|$$

$$= \left|\frac{-27 + 9 + 27}{\sqrt{(-9)^2 + 3^2 + 9^2}}\right|$$

$$= \left|\frac{9}{\sqrt{171}}\right| = \frac{3}{\sqrt{19}} \text{units}$$

Straight Line in Space Ex 28.5 Q7(iv)

Here,

$\vec{a}_1 = 6\hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{a}_2 = -4\hat{i} - \hat{k}$

$\vec{b}_1 = \hat{i} - 2\hat{j} + 2\hat{k}$

$\vec{b}_2 = 3\hat{i} - 2\hat{j} - 2\hat{k}$

$\vec{a}_2 - \vec{a}_1 = -4\hat{i} - \hat{k} - 6\hat{i} - 2\hat{j} - 2\hat{k}$

$= -10\hat{i} - 2\hat{j} - 3\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix}$$

$= \hat{i}(4+4) - \hat{j}(-2-6) + \hat{k}(-2+6)$

$= 8\hat{i} + 8\hat{j} + 4\hat{k}$

Shortest Distance $= \left|\dfrac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

$$= \left|\frac{(-10\hat{i} - 2\hat{j} - 3\hat{k}).(8\hat{i} + 8\hat{j} + 4\hat{k})}{|8\hat{i} - 8\hat{j} + 4\hat{k}|}\right|$$

$$= \left|\frac{(-10) \times 8 + (-2) \times 8 + (-3) \times 4}{\sqrt{8^2 + (-8)^2 + (-4)^2}}\right|$$

$$= \left|\frac{-80 - 16 - 12}{\sqrt{64 + 64 + 16}}\right| = \left|\frac{-108}{\sqrt{144}}\right| = 9 \text{ units}$$

Straight Line in Space Ex 28.5 Q8

$\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}$

$\vec{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k}$

$\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j} - 5\hat{k} - \hat{i} - 2\hat{j} + 4\hat{k}$

$= 2\hat{i} + \hat{j} - \hat{k}$

$\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix}$$

$= \hat{i}(6+3) - \hat{j}(12+2) + \hat{k}(6-2)$

$= 9\hat{i} - 14\hat{j} + 4\hat{k}$

Shortest distance betweeen 2lines

$$= \left| \frac{(\vec{a}_2 - \vec{a}_1) \times \vec{b}}{|\vec{b}|} \right|$$

$$= \left| \frac{9\hat{i} - 14\hat{j} + 4\hat{k}}{\left|\sqrt{2^2 + 3^2 + 6^2}\right|} \right|$$

$$= \left| \frac{9\hat{i} - 14\hat{j} + 4\hat{k}}{\sqrt{49}} \right|$$

$$= \left| \frac{\sqrt{9^2 + (-14)^2 + 4^2}}{\sqrt{49}} \right|$$

$$= \left| \frac{\sqrt{293}}{\sqrt{49}} \right| = \frac{\sqrt{293}}{7} \text{units}$$

Ex 29.1

The Plane 29.1 Q1(i)

Given three points are,
$(2,1,0),(3,-2,-2)$ and $(3,1,7)$

We know that, equation of plane passing through three points is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-2 & y-1 & z-0 \\ 3-2 & -2-1 & -2-0 \\ 3-2 & 1-1 & 7-0 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-2 & y-1 & z \\ 1 & -3 & -2 \\ 1 & 0 & 7 \end{vmatrix} = (x-2)(-21-0)-(y-1)(7+2)+z(0+3)=0$$

$$= -21x+42-9y+9+3z=0$$
$$= -21x-9y+3z+51=0$$

Dividing by -3, we get

Equation of plane, $7x+3y-z-17=0$

The Plane 29.1 Q1(ii)

Given points are,
$(-5,0,-6),(-3,10,-9)$ and $(-2,6,-6)$

We know that, equation of plane passing through three points is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x+5 & y-0 & z+6 \\ -3+5 & 10-0 & -9+6 \\ -2+5 & 6-0 & -6+6 \end{vmatrix} = 0$$

$$\begin{vmatrix} x+5 & y & z+6 \\ 2 & 10 & -3 \\ 3 & 6 & 0 \end{vmatrix} = 0$$

$(x+5)(0+18)-y(0+9)+(z+6)(12-30)=0$
$(x+5)(18)-y(9)+(z+6)(-18)=0$
$18x+90-9y-18z-108=0$

Dividing by 9, we get

Equation of plane, $2x-y-2z-2=0$

The Plane 29.1 Q1(iii)

Given three points are,

$(1,1,1), (1,-1,2)$ and $(-2,-2,2)$

We know that, equation of plane passing through three points is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 1-1 & -1-1 & 2-1 \\ -2-1 & -2-1 & 2-1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 0 & -2 & 1 \\ -3 & -3 & 1 \end{vmatrix} = 0$$

$(x-1)(-2+3) - (y-1)(0+3) + (z-1)(0-6) = 0$

$(x-1)(1) - (y-1)(3) + (z-1)(-6) = 0$

$x - 1 - 3y + 3 - 6z + 6 = 0$

$x - 3y - 6z + 8 = 0$

Equation of plane is, $x - 3y - 6z + 8 = 0$

The Plane 29.1 Q1(iv)

Given points are,

$(2,3,4), (-3,5,1)$ and $(4,-1,2)$

We know that, equation of plane passing through three points are given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-2 & y-3 & z-4 \\ -3-2 & 5-3 & 1-4 \\ 4-2 & -1-3 & 2-4 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-2 & y-3 & z-4 \\ -5 & 2 & -3 \\ 2 & -4 & -2 \end{vmatrix} = 0$$

$(x-2)(-4-12) - (y-3)(10+6) + (z-4)(20-4) = 0$

$(x-2)(-16) - (y-3)(16) + (z-4)(16) = 0$

$-16x + 32 - 16y + 48 + 16z - 64 = 0$

$-16x - 16y + 16z + 16 = 0$

Dividing by (-16), we get,

Equation of plane, $x + y - z - 1 = 0$

The Plane 29.1 Q1(v)

Given points are,
$(0,-1,0),(3,3,0)$ and $(1,1,1)$

We know that, equation of plane passing through three points is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-0 & y+1 & z-0 \\ 3-0 & 3+1 & 0-0 \\ 1-0 & 1+1 & 1-0 \end{vmatrix} = 0$$

$$\begin{vmatrix} x & y+1 & z \\ 3 & 4 & 0 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$x(4-0)-(y+1)(3-0)+z(6-4)=0$
$4x-(y+1)(3)+z(2)=0$
$4x-3y-3+2z=0$

$4x-3y+2z-3=0$

Equation of plane is, $4x-3y+2z-3=0$

The Plane 29.1 Q2

We have to prove that points
$(0,-1,-1),(4,5,1),(3,9,4)$ and $(-4,4,4)$ are coplanar.

First we shall find the equation of plane passing through three points:
$(0,-1,1),(4,5,1)$ and $(3,9,4)$

We know that equation of plane passing through three points is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-0 & y+1 & z+1 \\ 4-0 & 5+1 & 1+1 \\ 3-0 & 9+1 & 4+1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x & y+1 & z+1 \\ 4 & 6 & 2 \\ 3 & 10 & 5 \end{vmatrix} = 0$$

$x(30-20)-(y+1)(20-6)+(z+1)(40-18)=0$
$10x-(y+1)(14)+(z+1)(22)=0$
$10x-14y-14+22z+22=0$

$10x-14y+22z+8=0$

Dividing by 2, we get

$5x-7y+11z+4=0 \qquad ---(i)$

Now, for the fourth point $(-4,4,4)$ put $x=-4,\ y=4,\ z=4$ in equation (i),

$5(-4)-7(4)+11(4)+4=0$
$-20-28+44+4=0$
$-48+48=0$

$0=0$
$LHS=RHS$

Since, fourth point satisfies the equation of plane passing
through three points
So, all four points are collinear

Equation of common plane is, $5x-7y+11z+4=0$

The Plane 29.1 Q3(i)

Given, four points are
$(0,-1,0),(2,1,-1),(1,1,1)$ and $(3,3,0)$.

Now, first we find the equation of plane passing through three points:
$(0,-1,0),(2,1,-1),(1,1,1)$

We know that equation of plane passing through three points is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$$

$$\begin{vmatrix} x-0 & y+1 & z-0 \\ 2-0 & 1+1 & -1-0 \\ 1-0 & 1+1 & 1-0 \end{vmatrix}=0$$

$$\begin{vmatrix} x & y+1 & z \\ 2 & 2 & -1 \\ 1 & 2 & 1 \end{vmatrix}=0$$

$x(2+2)-(y+1)(2+1)+z(4-2)=0$
$x(4)-(y+1)(3)+z(2)=0$
$4x-3y-3+2z=0$

$4x-3y+2z-3=0 \qquad ---(i)$

Put, $x=3,\ y=3,\ z=0$ in equation (i), we get

$4x-3y+2z-3=0$
$4(3)-3(3)+2(0)-3=0$
$12-9+0-3=0$
$12-12=0$

$0=0$
$LHS=RHS$

Since, fourth point satisfies the equation of plane passing
through three points,
Hence, four points are coplanar

The Plane 29.1 Q3(ii)

Given, four points are
$(0,4,3),(-1,-5,-3),(-2,-2,1)$ and $(1,1,-1)$

First we shall find the equation of plane passing through three points:
$(0,4,3),(-1,-5,-3),(-2,-2,1)$

We know that, equation of plane passing through three given points is,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-0 & y-4 & z-3 \\ -1-0 & -5-4 & -3-3 \\ -2-0 & -2-4 & 1-3 \end{vmatrix} = 0$$

$$\begin{vmatrix} x & y-4 & z-3 \\ -1 & -9 & -6 \\ -2 & -6 & -2 \end{vmatrix} = 0$$

$x(18-36)-(y-4)(2-12)+(z-3)(6-18)=0$
$x(-18)-(y-4)(-10)+(z-3)(-12)=0$
$-18x+10y-40-12z+36=0$

$-18x+10y-12z-4=0 \quad\quad ---(i)$

Put, $x=1$, $y=1$, $z=-1$ in equation (i),

$-18(1)+10(1)-12(-1)-4=0$
$-18+10+12-4=0$
$-22+22=0$

$0=0$
$LHS=RHS$

So, fourth point $(1,1,-1)$ satisfies the equation of plane passing through three points,
Hence, four points are coplanar

Ex 29.2

The Plane 29.2 Q1

Given, intercepts on the coordinate axes are $2, -3$ and 4

We know that,
The equation of a plane whose intercepts on the coordinate axes are a, b and c respectively, is given by

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \qquad ---(i)$$

Here, $a = 2$, $b = -3$, $c = 4$

So,
Equation of required plane is

$$\frac{x}{2} + \frac{y}{-3} + \frac{z}{4} = 1$$
$$\frac{6x - 4y + 3z}{12} = 1$$

$$6x - 4y + 3z = 12$$

The Plane 29.2 Q2(i)

Reduce the equation $4x + 3y - 6z - 12 = 0$ in intercept form:

$$4x + 3y - 6z - 12 = 0$$
$$4x + 3y - 6z = 12$$

Divide by 12,

$$\frac{4x}{12} + \frac{3y}{12} - \frac{6z}{12} = \frac{12}{12}$$
$$\frac{x}{3} + \frac{y}{4} - \frac{z}{2} = 1$$

$$\frac{x}{3} + \frac{y}{4} + \frac{z}{(-2)} = 1 \qquad ---(i)$$

This is of the form,

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \qquad ---(ii)$$

Comparing equation (i) and (ii),

$a = 3$, $b = 4$, $c = -2$

Intercepts on the coordinate axes are $3, 4, -2$

The Plane 29.2 Q2(ii)

Reduce $2x+3y-z=6$ in the intercept form:

$2x+3y-z=6$

Divide by 6,

$$\frac{2x}{6}+\frac{3y}{6}-\frac{z}{6}=\frac{6}{6}$$
$$\frac{x}{3}+\frac{y}{2}-\frac{z}{6}=1$$

$$\frac{x}{3}+\frac{y}{2}+\frac{z}{(-6)}=1 \qquad ---(i)$$

We know intercept form of plane with a, b, c as intercepts on coordinate axes is,

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \qquad ---(ii)$$

Comparing equation (i) and (ii),

$a=3$, $b=2$, $c=-6$

So, intercepts on coordinate axes by the given plane are $3,2,-6$

The Plane 29.2 Q2(iii)

We have to find intercepts on coordinate axes by plane $2x-y+z=5$

$2x-y+z=5$

Divide by 5,

$$\frac{2x}{5}-\frac{y}{5}+\frac{z}{5}=\frac{5}{5}$$
$$\frac{x}{\left(\frac{5}{2}\right)}+\frac{y}{(-5)}+\frac{z}{5}=1 \qquad ---(i)$$

We know that if a,b,c are intercepts on coordinate axes by the plane,
then equation of such plane is given by,

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \qquad ---(ii)$$

Comparing the equation (i) and (ii),

$a=\frac{5}{2}$, $b=-5$, $c=5$

So, intercepts on coordinate axes by the plane are $\frac{5}{2},-5,5$.

The Plane 29.2 Q3

Here, it is given that the plane meets axes in A, B and C

Let, $A = (a,0,0)$, $B = (0,b,0)$, $C = (0,0,c)$

We have centroid of ΔABC is (α, β, γ) we know that, centroid of ΔABC is given by

$$\text{Centroid} = \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}$$

$$(\alpha, \beta, \gamma) = \left(\frac{a+0+0}{3}, \frac{0+b+0}{3}, \frac{0+0+c}{3}\right)$$

$$(\alpha, \beta, \gamma) = \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$$

So,

$$\frac{a}{3} = \alpha \Rightarrow a = 3\alpha \qquad ---(i)$$

$$\frac{b}{3} = \beta \Rightarrow b = 3\beta \qquad ---(ii)$$

$$\frac{c}{3} = \gamma \Rightarrow c = 3\gamma \qquad ---(iii)$$

We know that, if a, b, c are intercepts by plane on coordinate axes, then equation of the plane is given by

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Put a, b, c from equation (i), (ii) and (iii),

$$\frac{x}{3\alpha} + \frac{y}{3\beta} + \frac{z}{3\gamma} = 1$$

Multiplying by 3 on both the sides,

$$\frac{3x}{3\alpha} + \frac{3y}{3\beta} + \frac{3z}{3\gamma} = 3$$

$$\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$$

The Plane 29.2 Q4

Intercepts on the coordinate axes are equal.

We know that, if a, b, c are intercepts on coordinate axes by a plane, then equation of the plane is given by,

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Here, it is givin that $a = b = c = p$ (Say)

$$\frac{x}{p} + \frac{y}{p} + \frac{z}{p} = 1$$

$$\frac{x+y+z}{p} = 1$$

$$x + y + z = p \qquad ---(i)$$

It is given that plane is passing through the point $(2,4,6)$, so, using equation (i)

$$x + y + z = p$$

$$2 + 4 + 6 = p$$

$$12 = p$$

Put, value of p in equation (i)

$$x + y + z = 12$$

So, the required equation of the plane is given by,

$$x + y + z = 12$$

The Plane 29.2 Q5

Here, it is given that plane meets the coordinate axes at A, B and C with centroid of
$\triangle ABC$ is $(1, -2, 3)$

The equation of plane with intercepts a, b and c on the coordinate axes is given by,

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \qquad ---(\text{i})$$

We know that, centroid of a triangle is given by

$$\text{Centroid} = \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}$$

$$(1, -2, 3) = \left(\frac{a + 0 + 0}{3}, \frac{0 + b + 0}{3}, \frac{0 + 0 + c}{3}\right)$$

$$(1, -2, 3) = \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$$

Comparing LHS and RHS,

$$\frac{a}{3} = 1 \Rightarrow \quad a = 3 \qquad ---(\text{i})$$

$$\frac{b}{3} = -2 \Rightarrow \quad b = -6 \qquad ---(\text{ii})$$

$$\frac{c}{3} = 3 \Rightarrow \quad c = 9 \qquad ---(\text{iii})$$

Put, a, b, c is equation (i), we get the equation of required plane

$$\frac{x}{3} + \frac{y}{-6} + \frac{z}{9} = 1$$

$$\frac{6x - 3x + 2z}{18} = 1$$

$$6x - 3x + 2z = 18$$

Ex - 29.3

The Plane 29.3 Q1

We know that, vector equation of a plane passing through a point $\vec{a}$ and normal to $\vec{n}$ is given by,

$$\left(\vec{r}-\vec{a}\right).\vec{n}=0 \qquad ---(i)$$

Here,

$\vec{a}=2\hat{i}-\hat{j}+\hat{k}$
$\vec{n}=4\hat{i}+2\hat{j}-3\hat{k}$

Put, $\vec{a}$ and $\vec{n}$ in equation (i)

$$\left[\vec{r}-\left(2\hat{i}-\hat{j}+\hat{k}\right)\right].\left(4\hat{i}+2\hat{j}-3\hat{k}\right)=0$$
$$\vec{r}.\left(4\hat{i}+2\hat{j}-3\hat{k}\right)-\left(2\hat{i}-\hat{j}+\hat{k}\right).\left(4\hat{i}+2\hat{j}-3\hat{k}\right)=0$$
$$\vec{r}.\left(4\hat{i}+2\hat{j}-3\hat{k}\right)-\left[(2)(4)+(-1)(2)+(1)(-3)\right]=0$$
$$\vec{r}\left(4\hat{i}+2\hat{j}-3\hat{k}\right)-\left[8-2-3\right]=0$$

$$\vec{r}\left(4\hat{i}+2\hat{j}-3\hat{k}\right)-3=0$$

So, equation of required plane is given by,

$$\vec{r}.\left(4\hat{i}+2\hat{j}-3\hat{k}\right)=3$$

The Plane 29.3 Q2(i)

Given the vector equation of a plane,

$$\vec{r}.\left(12\hat{i}-3\hat{j}+4\hat{k}\right)+5=0$$

let, $\vec{r}=\left(x\hat{i}+y\hat{j}+z\hat{k}\right)$

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(12\hat{i}-3\hat{j}+4\hat{k}\right)+5=0$$
$$(x)(12)+(y)(-3)+(z)(4)+5=0$$

$$12x-3y+4z+5=0$$

Cartesian form of the equation of the plane is given by

$$12x-3y+4z+5=0$$

The Plane 29.3 Q2(ii)

Here, equation of the plane is,

$$\vec{r}.\left(-\hat{i}+\hat{j}+2\hat{k}\right)=9$$

let, $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, then

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(-\hat{i}+\hat{j}+2\hat{k}\right)=9$$
$$(x)(-1)+(y)(1)+(z)(2)=9$$

$$-x+y+2z=9$$

Cartesian form of the equation of plane is,

$$-x+y+2z=9$$

The Plane 29.3 Q3

We have to find vector equation of coordinate planes.
For xy-plane.

It passes through origin and is perpendicular to z-axis, so
Put $\vec{a} = 0.\hat{i} + 0.\hat{j} + 0\hat{k}$ and $\vec{n} = \hat{k}$ in the vector equation of plane passing through point $\vec{a}$ and perpendicular to vector $\vec{n}$

$$\left(\vec{r} - \vec{a}\right).\vec{n} = 0$$
$$\left(\vec{r} - 0.\hat{i} - 0.\hat{j} - 0\hat{k}\right).\hat{k} = 0$$

$$\vec{r}.\hat{k} = 0 \qquad\qquad ---(i)$$

For xz-plane,
It passes through origin and perpendicular to y-axis, so

$\vec{a} = 0.\hat{i} + 0.\hat{j} + 0\hat{k}$ and $\vec{n} = \hat{j}$

Equation of xz-plane is given by
$$\left(\vec{r} - \vec{a}\right).\vec{n} = 0$$
$$\left(\vec{r} - 0.\hat{i} - 0.\hat{j} - 0\hat{k}\right).\hat{j} = 0$$

$$\vec{r}.\hat{j} = 0$$

For yz-plane.

It passes through origin and is perpendicular to x-axis, so
$\vec{a} = 0.\hat{i} + 0.\hat{j} + 0\hat{k}$, $\vec{n} = \hat{i}$

$$\left(\vec{r} - \vec{a}\right).\vec{n} = 0$$
$$\left(\vec{r} - 0.\hat{i} - 0.\hat{j} - 0\hat{k}\right).\hat{i} = 0$$

$$\vec{r}.\hat{i} = 0$$

Hence, equation of xy, yz, zx-plane are given by

$$\vec{r}.\hat{k} = 0$$
$$\vec{r}.\hat{i} = 0$$
$$\vec{r}.\hat{j} = 0$$

The Plane 29.3 Q4(i)

Given, equation of plane is,

$$2x - y + 2z = 8$$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(2\hat{i} - \hat{j} + 2\hat{k}\right) = 8$$
$$\vec{r}.\left(2\hat{i} - \hat{j} + 2\hat{k}\right) = 8$$

So,
Vector equation of the plane is $\vec{r}.\left(2\hat{i} - \hat{j} + 2\hat{k}\right) = 8$

The Plane 29.3 Q4(ii)

Given, cartesian equation of the plane is,

$x + y - z = 5$

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\hat{i} + \hat{j} - \hat{k}\right) = 5$

$\vec{r}\left(\hat{i} + \hat{j} - \hat{k}\right) = 5$

So,

Vector equation of the plane is $\vec{r}\left(\hat{i} + \hat{j} - \hat{k}\right) = 5$

The Plane 29.3 Q4(iii)

Given, cartesian equation of plane is,

$x + y = 3$

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\hat{i} + \hat{j}\right) = 3$

$\vec{r}.\left(\hat{i} + \hat{j}\right) = 3$

So,

Vector equation of the plane is $\vec{r}.\left(\hat{i} + \hat{j}\right) = 3$

The Plane 29.3 Q5

We know that, vector equation of a plane passing through point $\vec{a}$ and perpendicular to the vector $\vec{n}$ is given by,

$\left(\vec{r} - \vec{a}\right).\vec{n} = 0 \qquad ---(i)$

The given plane is passing through the point $(1,-1,1)$ and normal to the line joining $A(1,2,5)$ and $B(-1,3,1)$. So,

$\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{n} = \overrightarrow{AB}$

= Position vector of $\vec{B}$ − Position vector of A

$= \left(-\hat{i} + 3\hat{j} + \hat{k}\right) - \left(\hat{i} + 2\hat{j} + 5\hat{k}\right)$

$= -\hat{i} + 3\hat{j} + \hat{k} - \hat{i} - 2\hat{j} - 5\hat{k}$

$= -2\hat{i} + \hat{j} - 4\hat{k}$

Put, $\vec{n}$ and $\vec{a}$ in equation (i),

$\left[\vec{r} - \left(\hat{i} - \hat{j} + \hat{k}\right)\right].\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) = 0$

$\vec{r}.\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) - \left(\hat{i} - \hat{j} + \hat{k}\right)\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) = 0$

$\vec{r}.\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) - \left[(1)(-2) + (-1)(1) + (1)(-4)\right] = 0$

$\vec{r}.\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) - \left[-2 - 1 - 4\right] = 0$

$\vec{r}\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) - \left[-7\right] = 0$

$\vec{r}.\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) + 7 = 0$

$\vec{r}.\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) = -7$

Multiplying by (-1) on both the sides

$\vec{r}\left(2\hat{i} - \hat{j} + 4\hat{k}\right) = 7$

Put, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(2\hat{i} - \hat{j} + 4\hat{k}\right) = 7$

$(x)(2) + (y)(-1) + (2)(4) = 7$

$2x - y + 4z = 7$

So, vector and cartesian equation the plane is

$\vec{r}.\left(2\hat{i} - \hat{j} + 4\hat{k}\right) = 7,\ 2x - y + 4z = 7$

The Plane 29.3 Q6

Here, it is given that $\bar{n} = \sqrt{3}$ and $\bar{n}$ makes equal angle with coordinate axes.

Let, $\bar{n}$ has direction cosine as l, m and n and it makes angle of α, β and γ with the coordinate axes. So

Here, $\alpha = \beta = \gamma$

$\Rightarrow \quad \cos\alpha = \cos\beta = \cos\gamma$

$\Rightarrow \quad l = m = n = p$ (Say)

We know that,

$l^2 + m^2 + n^2 = 1$

$p^2 + p^2 + p^2 = 1$

$3p^2 = 1$

$p^2 = \frac{1}{3}$

$p = \pm\frac{1}{\sqrt{3}}$

So,

$l = \pm\frac{1}{\sqrt{3}}$

$\cos\alpha = \pm\frac{1}{\sqrt{3}}$

Now, $\alpha = \cos^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

It gives, α is an obtuse angle so, neglect it.

Again, $\alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

It gives, α is an acute angle, so

$\cos\alpha = \frac{1}{\sqrt{3}}$

$\therefore \quad l = m = n = \frac{1}{\sqrt{3}}$

So,

$\vec{n} = |\vec{n}|\left(l\hat{i} + m\hat{j} + n\hat{k}\right)$

$= \sqrt{3}\left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right)$

$\vec{n} = \hat{i} + \hat{j} + \hat{k}$

And, $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$

We know that, vector equation of a plane passing through the point $\vec{a}$ and perpendicular to the vector $\vec{n}$ is given by,

$\left(\vec{r} - \vec{a}\right).\vec{n} = 0$

$\left[\vec{r} - \left(2\hat{i} + \hat{j} - \hat{k}\right)\right].\left(\hat{i} + \hat{j} + \hat{k}\right) = 0$

$\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) - \left(2\hat{i} + \hat{j} - \hat{k}\right)\left(\hat{i} + \hat{j} + \hat{k}\right) = 0$

$\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) - \left[(2)(1) + (1)(1) + (-1)(1)\right] = 0$

$\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) - \left[2 + 1 - 1\right] = 0$

$\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) - 2 = 0$

$\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) = 2$

Put, $\vec{r} = \left(x\hat{i} + y\hat{j} + z\hat{k}\right)$

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\hat{i} + \hat{j} + \hat{k}\right) = 2$

$(x)(1) + (y)(1) + (z)(1) = 2$

$x + y + z = 2$

So, vector and cartesian equation of the plane is,

$\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) = 2,\ x + y + z = 2$

The Plane 29.3 Q7

Here, it is given that foot of the perpendicular drawn from origin O to the plane is $P\left(12,-4,3\right)$

It means, the required plane is passing through $P\left(12,-4,3\right)$ and perpendicular to OP.

We know that, equation of a plane passing through $\vec{a}$ and perpendicular to $\vec{n}$ is given by,

$$\left(\vec{r}-\vec{a}\right).\vec{n}=0 \qquad ---\left(\text{i}\right)$$

Here, $\vec{a}=12\hat{i}-4\hat{j}+3\hat{k}$

And, $\vec{n}=\overrightarrow{OP}$

$=$ Position vector of P $-$ Position vector of O

$$=\left(12\hat{i}-4\hat{j}+3\hat{k}\right)-\left(0\hat{i}+0\hat{j}+0\hat{k}\right)$$

$$\vec{n}=12\hat{i}-4\hat{j}+3\hat{k}$$

Put, value of $\vec{a}$ and $\vec{n}$ in equation $\left(\text{i}\right)$,

$$\left[\vec{r}-\left(12\hat{i}-4\hat{j}+3\hat{k}\right)\right].\left(12\hat{i}-4\hat{j}+3\hat{k}\right)=0$$

$$\vec{r}.\left(12\hat{i}-4\hat{j}+3\hat{k}\right)-\left(12\hat{i}-4\hat{j}+3\hat{k}\right)\left(12\hat{i}-4\hat{j}+3\hat{k}\right)=0$$

$$\vec{r}.\left(12\hat{i}-4\hat{j}+3\hat{k}\right)-\left[\left(12\right)\left(12\right)+\left(-4\right)\left(-4\right)+\left(3\right)\left(3\right)\right]=0$$

$$\vec{r}.\left(12\hat{i}-4\hat{j}+3\hat{k}\right)-\left[144+16+9\right]=0$$

$$\vec{r}.\left(12\hat{i}-4\hat{j}+3\hat{k}\right)-169=0$$

Put, $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(12\hat{i}-4\hat{j}+3\hat{k}\right)-169=0$$

$$\left(x\right)\left(12\right)+\left(y\right)\left(-4\right)+\left(z\right)\left(3\right)=169$$

$$12x-4y+3z=169$$

So, the vector and cartesian equation of the required plane is,

$$\vec{r}.\left(12\hat{i}-4\hat{j}+3\hat{k}\right)=169,\ 12x-4y+3z=169$$

The Plane 29.3 Q8

Given that, the plane is passing through $P(2,3,1)$ having 5,3,2 as the direction ratios of the normal to the plane.

We know that,
Equation of a plane passing through a point $\vec{a}$ and $\vec{n}$ is a vector normal to the plane, is given by,

$$(\vec{r}-\vec{a}).\vec{n}=0 \qquad ---(i)$$

So, $\vec{a}=2\hat{i}+3\hat{j}+\hat{k}$
$\vec{n}=5\hat{i}+3\hat{j}+2\hat{k}$

Put, $\vec{a}$ and $\vec{n}$ in equation (i),

$$\left[\vec{r}-\left(2\hat{i}+3\hat{j}+\hat{k}\right)\right]\left(5\hat{i}+3\hat{j}+2\hat{k}\right)=0$$
$$\vec{r}\left(5\hat{i}+3\hat{j}+2\hat{k}\right)-\left(2\hat{i}+3\hat{j}+\hat{k}\right)\left(5\hat{i}+3\hat{j}+2\hat{k}\right)=0$$
$$\vec{r}\left(5\hat{i}+3\hat{j}+2\hat{k}\right)-\left[(2)(5)+(3)(3)+(1)(2)\right]=0$$
$$\vec{r}\left(5\hat{i}+3\hat{j}+2\hat{k}\right)-\left[10+9+2\right]=0$$
$$\vec{r}\left(5\hat{i}+3\hat{j}+2\hat{k}\right)-21=0$$

Put, $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(5\hat{i}+3\hat{j}+2\hat{k}\right)-21=0$$
$$(x)(5)+(y)(3)+(z)(2)=21$$

$$5x+3y+2z=21$$

The Plane 29.3 Q9

Here, given that P is the point $(2,3,-1)$ and required plane is passing through P at right angles to OP

It means, the plane is passing through P and OP is the vector normal to the plane.

We know that, equation of a plane, passing through a point $\vec{a}$ and $\vec{n}$ is vector normal to the plane, is given by,

$$(\vec{r}-\vec{a}).\vec{n}=0 \qquad ---(i)$$

Here, $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$
$\vec{n}=\overrightarrow{OP}$
$=$ Position vector of P $-$ Position vector of O

$$=\left(2\hat{i}+3\hat{j}-\hat{k}\right)-\left(0\hat{i}+0\hat{j}+0\hat{k}\right)$$
$$\vec{n}=2\hat{i}+3\hat{j}-\hat{k}$$

Put, the value of $\vec{a}$ and $\vec{n}$ in equation (i),

$$\left[\vec{r}-\left(2\hat{i}+3\hat{j}-\hat{k}\right)\right].\left(2\hat{i}+3\hat{j}-\hat{k}\right)=0$$
$$\vec{r}\left(2\hat{i}+3\hat{j}-\hat{k}\right)-\left[\left(2\hat{i}+3\hat{j}-\hat{k}\right)\left(2\hat{i}+3\hat{j}-\hat{k}\right)\right]=0$$
$$\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)-\left[(2)(2)+(3)(3)+(-1)(-1)\right]=0$$
$$\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)-\left[4+9+1\right]=0$$
$$\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)-14=0$$
$$\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)=14$$

Put, $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$
$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(2\hat{i}+3\hat{j}-\hat{k}\right)=14$$
$$(x)(2)+(y)(3)+(z)(-1)=14$$

$$2x+3y-z=14$$

Equation of required plane is,

$$2x+3y-z=14$$

The Plane 29.3 Q10

Here, given equation of plane is,

$2x + y - 2z = 3$

Dividing by 3 on both the sides,

$$\frac{2x}{3} + \frac{y}{3} - \frac{2z}{3} = \frac{3}{3}$$

$$\frac{x}{\frac{3}{2}} + \frac{y}{3} + \frac{z}{-\frac{3}{2}} = 1 \qquad ---(i)$$

We know that, if a,b,c are the intercepts by a plane on the coordinate axes, new equation of the plane is,

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \qquad ---(ii)$$

Comparing the equation (i) and (ii),

$a = \frac{3}{2}$, $b = 3$, $c = -\frac{3}{2}$

Again, given equation of plane is,

$$2x + y - 2z = 3$$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(2\hat{i} + \hat{j} - 2\hat{k}\right) = 3$$

$$\vec{r}\left(2\hat{i} + \hat{j} - 2\hat{k}\right) = 3$$

So, vector normal to the plane is given by

$$\vec{n} = 2\hat{i} + \hat{j} - 2\hat{k}$$

$$\left|\vec{n}\right| = \sqrt{(2)^2 + (1)^2 + (-2)^2}$$

$$= \sqrt{4 + 1 + 4}$$

$$= \sqrt{9}$$

$$\left|\vec{n}\right| = 3$$

Direction vector of $\vec{n} = 2,1,-2$

Direction vector of $\vec{n} = \frac{2}{\left|\vec{n}\right|}, \frac{1}{\left|\vec{n}\right|}, \frac{-2}{\left|\vec{n}\right|}$

$$= \frac{2}{3}, \frac{1}{3}, -\frac{2}{3}$$

So,

Intercepts by the plane on coordinate axes are $= \frac{3}{2}, 3, -\frac{3}{2}$

Direction cosine of normal to the plane are $= \frac{2}{3}, \frac{1}{3}, -\frac{2}{3}$

The Plane 29.3 Q11

Here, given that, the required plane passes through the point $(1,-2,5)$ and is perpendicular to the line joining origin O to the point $P\left(3\hat{i}+\hat{j}-\hat{k}\right)$.

We know that, equation of a plane passing through a point $\vec{a}$ and perpendicular to a vector $\vec{n}$ is given by,

$$\left(\vec{r}-\vec{a}\right).\vec{n}=0 \qquad ---(i)$$

Here, $\vec{a}=\hat{i}-2\hat{j}+5\hat{k}$

$\vec{n}=\overrightarrow{OP}$

$=$ Position vector of P − Position vector of O

$=\left(3\hat{i}+\hat{j}-\hat{k}\right)-\left(0\hat{i}+0\hat{j}+0\hat{k}\right)$

$\vec{n}=3\hat{i}+\hat{j}-\hat{k}$

Put, the value of $\vec{a}$ and $\vec{n}$ in equation (i), we get,

$$\left[\vec{r}-\left(\hat{i}-2\hat{j}+5\hat{k}\right)\right]\left(3\hat{i}+\hat{j}-\hat{k}\right)=0$$

$$\vec{r}.\left(3\hat{i}+\hat{j}-\hat{k}\right)-\left(\hat{i}-2\hat{j}+5\hat{k}\right)\left(3\hat{i}+\hat{j}-\hat{k}\right)=0$$

$$\vec{r}.\left(3\hat{i}+\hat{j}-\hat{k}\right)-\left[(1)(3)+(-2)(1)+(5)(-1)\right]=0$$

$$\vec{r}.\left(3\hat{i}+\hat{j}-\hat{k}\right)-\left[3-2-5\right]=0$$

$$\vec{r}.\left(3\hat{i}+\hat{j}-\hat{k}\right)-\left[-4\right]=0$$

$$\vec{r}.\left(3\hat{i}+\hat{j}-\hat{k}\right)+4=0$$

$$\vec{r}.\left(3\hat{i}+\hat{j}-\hat{k}\right)=-4$$

The Plane 29.3 Q12

We have to find the equation of plane that bisects $A(1,2,3)$ and $B(3,4,5)$ perpendicularly

We know that, equation of a plane passing through the point $\vec{a}$ and perpendicular to vector $\vec{n}$ is given by,

$$\left(\vec{r}-\vec{a}\right).\vec{n}=0 \qquad ---(i)$$

Here, $\vec{a}=$ mid-point of AB

$$=\frac{\text{Position vector of } A+\text{Position vector of } B}{2}$$

$$=\frac{\hat{i}+2\hat{j}+3\hat{k}+3\hat{i}+4\hat{j}+5\hat{k}}{2}$$

$$\vec{a}=\frac{4\hat{i}+6\hat{j}+8\hat{k}}{2}$$

$$\vec{a}=2\hat{i}+3\hat{j}+4\hat{k}$$

And, $\vec{n}=\overrightarrow{AB}$

$=$ Position vector of B − Position vector of A

$=\left(3\hat{i}+4\hat{j}+5\hat{k}\right)-\left(\hat{i}+2\hat{j}+3\hat{k}\right)$

$=3\hat{i}+4\hat{j}+5\hat{k}-\hat{i}-2\hat{j}-3\hat{k}$

$\vec{n}=2\hat{i}+2\hat{j}+2\hat{k}$

Put, the value of $\vec{a}$ and $\vec{n}$ in equation (i),

$$\vec{r}-\left(2\hat{i}+3\hat{j}+4\hat{k}\right)\left(2\hat{i}+2\hat{j}+2\hat{k}\right)=0$$

$$\vec{r}\left(2\hat{i}+2\hat{j}+2\hat{k}\right)-\left[\left(2\hat{i}+3\hat{j}+4\hat{k}\right)\left(2\hat{i}+2\hat{j}+2\hat{k}\right)\right]=0$$

$$\vec{r}\left(2\hat{i}+2\hat{j}+2\hat{k}\right)-\left[(2)(2)+(3)(2)+(4)(2)\right]=0$$

$$\vec{r}\left(2\hat{i}+2\hat{j}+2\hat{k}\right)-\left[4+6+8\right]=0$$

$$\vec{r}\left(2\hat{i}+2\hat{j}+2\hat{k}\right)-18=0$$

$$\vec{r}\left(2\hat{i}+2\hat{j}+2\hat{k}\right)=18$$

The Plane 29.3 Q13(i)

Given, two equation of plane are,

$x - y + z - 2 = 0$ and
$3x + 2y - z + 4 = 0$

$x - y + z = 2$
$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\hat{i} - \hat{j} + \hat{k}\right) = 2$
$\vec{r}.\vec{n_1} = 2$ --- (i)

$3x + 2y - z = -4$
$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(3\hat{i} + 2\hat{j} - \hat{k}\right) = -4$
$\vec{r}\left(3\hat{i} + 2\hat{j} - \hat{k}\right) = -4$
$\vec{r}.\vec{n_2} = -4$ --- (i)

From equation (i) and (ii), we get that
$\vec{n_1}$ is normal to equation (i) and
$\vec{n_2}$ is normal to equation (ii).

Now,
$\vec{n_1}.\vec{n_2} = \left(\hat{i} - \hat{j} + \hat{k}\right).\left(3\hat{i} + 2\hat{j} - \hat{k}\right)$
$= (1)(3) + (-1)(2) + (1)(-1)$
$= 3 - 2 - 1$
$= 3 - 3$

$= 0$

$\vec{n_1}.\vec{n_2} = 0$

So, $\vec{n_1}$ is perpendicular to $\vec{n_2}$

The Plane 29.3 Q13(ii)

Given, two vector equation of plane are,

$\vec{r}.\left(2\hat{i} - \hat{j} + 3\hat{k}\right) = 5$
$\vec{r}.\vec{n_1} = 5$

So, $\vec{n_1} = \left(2\hat{i} - \hat{j} + 3\hat{k}\right)$
And, $\vec{r}.\left(2\hat{i} - 2\hat{j} - 2\hat{k}\right) = 5$
$\vec{r}.\vec{n_2} = 5$

So, $\vec{n_2} = 2\hat{i} - 2\hat{j} - 2\hat{k}$

Now, $\vec{n_1}.\vec{n_2}$
$= \left(2\hat{i} - \hat{j} + 3\hat{k}\right).\left(2\hat{i} - 2\hat{j} - 2\hat{k}\right)$
$= (2)(2) + (-1)(-2) + (3)(-2)$
$= 4 + 2 - 6$
$= 6 - 6$

$= 0$

$\vec{n_1}.\vec{n_2} = 0$

Hence, normals to planes $\vec{n_1}$ and $\vec{n_2}$ are perpendicular.

The Plane 29.3 Q14

Given, equation of plane is,

$2x + 2y + 2z = 3$

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(2\hat{i} + 2\hat{j} + 2\hat{k}\right) = 3$

$\vec{r}.\left(2\hat{i} + 2\hat{j} + 2\hat{k}\right) = 3$

$\vec{r}.\vec{n} = d$

Normal to the plane $\vec{n} = 2\hat{i} + 2\hat{j} + 2\hat{k}$

Direction ratio of $\vec{n} = 2, 2, 2$

Direction cosine of $\vec{n} = \frac{2}{\left|\vec{n}\right|}, \frac{2}{\left|\vec{n}\right|}, \frac{2}{\left|\vec{n}\right|}$

$$\left|\vec{n}\right| = \sqrt{(2)^2 + (2)^2 + (2)^2}$$
$$= \sqrt{4 + 4 + 4}$$
$$= \sqrt{12}$$

$$\left|\vec{n}\right| = 2\sqrt{3}$$

Direction cosine of $\left|\vec{n}\right| = \frac{2}{2\sqrt{3}}, \frac{2}{2\sqrt{3}}, \frac{2}{2\sqrt{3}}$

$$= \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$$

So, $l = \frac{1}{\sqrt{3}}$, $m = \frac{1}{\sqrt{3}}$, $n = \frac{1}{\sqrt{3}}$

Let, α, β, γ be the angle that normal $\vec{n}$ makes with the coordinate axes respectively.

$l = \cos\alpha = \frac{1}{\sqrt{3}}$

$\alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ --- (i)

$m = \cos\beta = \frac{1}{\sqrt{3}}$

$\beta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ --- (ii)

$n = \cos\gamma = \frac{1}{\sqrt{3}}$

$\gamma = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ --- (iii)

From equation (i), (ii) and (iii),

$\alpha = \beta = \gamma$

So, normal to the plane, $\vec{n}$ is equally inclined with the coordinate axes.

The Plane 29.3 Q15

Given, equation of plane is,

$12x - 3y + 4z = 1$

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(12\hat{i} - 3\hat{j} + 4\hat{k}\right) = 1$

$\vec{r}.\vec{n} = 1$

So, normal to the plane is

$\vec{n} = 12\hat{i} - 3\hat{j} + 4\hat{k}$

$\left|\vec{n}\right| = \sqrt{(12)^2 + (-3)^2 + (4)^2}$

$= \sqrt{144 + 9 + 16}$

$= \sqrt{144 + 25}$

$= 169 = 13$

Unit vector $\hat{n} = \dfrac{12\hat{i} - 3\hat{j} + 4\hat{k}}{13}$

$= \dfrac{12\hat{i}}{13} - \dfrac{3}{13}\hat{j} + 4\hat{k}$

A vector normal to the plane with magnitude

$26 = 26\hat{n}$

$= 26\left(\dfrac{12\hat{i}}{13} - 3\hat{j} + 4\hat{k}\right)$

Required vector $= 24\hat{i} - 6\hat{j} + 8\hat{k}$

The Plane 29.3 Q16

Given that, line drawn from $A(4, -1, 2)$ meets a plane at right angle, at the point $B(-10, 5, 4)$.

We know that,

Equation of a plane passing through the point $\vec{a}$ and perpendicular to $\vec{n}$ is given by,

$\left(\vec{r} - \vec{a}\right).\vec{n} = 0 \qquad ---(i)$

Here, $\vec{a}$ = Position vector B

$\vec{a} = -10\hat{i} + 5\hat{j} + 4\hat{k}$

$\vec{n} = \overrightarrow{AB}$

= Position vector of B – Position vector of A

$= \left(-10\hat{i} + 5\hat{j} + 4\hat{k}\right) - \left(4\hat{i} - \hat{j} + 2\hat{k}\right)$

$= -10\hat{i} + 5\hat{j} + 4\hat{k} - 4\hat{i} + \hat{j} - 2\hat{k}$

$\vec{n} = -14\hat{i} + 6\hat{j} + 2\hat{k}$

Put, the value of $\vec{a}$ and $\vec{n}$ in equation (i),

$\left[\vec{r} - \left(-10\hat{i} + 5\hat{j} + 4\hat{k}\right)\right].\left(-14\hat{i} + 6\hat{j} + 2\hat{k}\right) = 0$

$\vec{r}.\left(-14\hat{i} + 6\hat{j} + 2\hat{k}\right) - \left(-10\hat{i} + 5\hat{j} + 4\hat{k}\right)\left(-14\hat{i} + 6\hat{j} + 2\hat{k}\right) = 0$

$\vec{r}\left(-14\hat{i} + 6\hat{j} + 2\hat{k}\right) - \left[(-10)(-14) + (5)(6) + (4)(2)\right] = 0$

$\vec{r}\left(-14\hat{i} + 6\hat{j} + 2\hat{k}\right) - \left[140 + 30 + 8\right] = 0$

$\vec{r}\left(-14\hat{i} + 6\hat{j} + 2\hat{k}\right) - 178 = 0$

$\vec{r}\left(-14\hat{i} + 6\hat{j} + 2\hat{k}\right) = 178$

The Plane 29.3 Q17

We have to find the equation of plane which bisects the line joining the points $A(-1,2,3)$ and $B(3,-5,6)$ at right angles.

Let, C be the mid-point of AB

We know that, equation of a plane passing through a point $\vec{a}$ and perpendicular to a vector $\vec{n}$ is given by,

$$(\vec{r}-\vec{a}).\vec{n} = 0 \qquad ---(i)$$

Here, $\vec{a}$ = Position vector of C

= Mid-point of A and B

$$= \frac{\text{Position vector of } A + \text{Position vector of } B}{2}$$

$$\vec{a} = \frac{-\hat{i}+2\hat{j}+3\hat{k} + 3\hat{i}-5\hat{j}+6\hat{k}}{2}$$

$$= \frac{2\hat{i}}{2} - \frac{3\hat{j}}{2} + \frac{9\hat{k}}{2}$$

$$\vec{a} = \hat{i} - \frac{3}{2}\hat{j} + \frac{9}{2}\hat{k}$$

$\vec{n} = \overrightarrow{AB}$

= Position vector of B – Position vector of A

$$= \frac{\left(3\hat{i}-5\hat{j}+6\hat{k}\right)-\left(-\hat{i}+2\hat{j}+3\hat{k}\right)}{2}$$

$$= \frac{3\hat{i}-5\hat{j}+6\hat{k}+\hat{i}-2\hat{j}-3\hat{k}}{2}$$

$$= \frac{4\hat{i}-7\hat{j}+3\hat{k}}{2}$$

$$= \frac{4}{2}\hat{i} - \frac{7}{2}\hat{j} + \frac{3}{2}\hat{k}$$

$$\vec{n} = 2\hat{i} - \frac{7}{2}\hat{j} + \frac{3}{2}\hat{k}$$

Put, the value of $\vec{a}$ and $\vec{n}$ in equation (i), we get,

$$\left[\vec{r} - \left(\hat{i}-\frac{3}{2}\hat{j}+\frac{9}{2}\hat{k}\right)\right]\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right) = 0$$

$$\vec{r}.\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right) - \left(\hat{i}-\frac{3}{2}\hat{j}+\frac{9}{2}\hat{k}\right)\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right) = 0$$

$$\vec{r}.\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right) - \left[(1)(2)+\left(-\frac{3}{2}\right)\left(-\frac{7}{2}\right)+\left(\frac{9}{2}\right)\left(+\frac{3}{2}\right)\right] = 0$$

$$\vec{r}.\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right) - \left[2+\frac{21}{4}+\frac{27}{4}\right] = 0$$

$$\vec{r}.\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right) - \left[\frac{29+27}{4}\right] = 0$$

$$\vec{r}.\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right) - \frac{56}{4} = 0$$

$$\vec{r}.\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right) - 14 = 0$$

Put, $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right) - 14 = 0$$

$$(x)(2)+(y)\left(-\frac{7}{2}\right)+(z)\left(+\frac{3}{2}\right) - 14 = 0$$

$$2x - \frac{7y}{2} + \frac{3z}{2} - 14 = 0$$

$$\frac{4x-7y+3z-28}{2} = 0$$

$$4x - 7y + 3z = 28$$

Equation of required plane is,

$$4x - 7y + 3z = 28$$

Vector equation of the plane:

Given that the required plane passes through the point $(5, 2, -4)$ having the position vector

$\vec{a} = 5\hat{i} + 2\hat{j} - 4\hat{k}$

Also given that the required plane is perpendicular to the line with direction ratios 2, 3 and -1.

Thus the vector equation of the normal vector to the plane is $\vec{n} = 2\hat{i} + 3\hat{j} - \hat{k}$.

We know that the vector equation of the plane passing through a point having position vector $\vec{a}$ and normal to vector $\vec{n}$ is given by $(\vec{r} - \vec{a}) \cdot \vec{n} = 0$ or, $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$.

Thus the required equation of the required plane is

$\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = (5\hat{i} + 2\hat{j} - 4\hat{k}) \cdot (2\hat{i} + 3\hat{j} - \hat{k})$

$\Rightarrow \vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = 10 + 6 + 4$

$\Rightarrow \vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = 20$

The Cartesian equation of the plane is

$\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = 20$

$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = 2$

$\Rightarrow 2x + 3y - z = 20$

The Plane Ex 29.3 Q19

Consider the point $P(1, 2, -3)$.

Thus the position vector of the point P is

$\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$

Direction ratios of the line OP, where O is the origin, are 1,2 and -3

Thus the vector equation of the normal vector, OP, to the plane is $\vec{n} = \hat{i} + 2\hat{j} - 3\hat{k}$.

We know that the vector equation of the plane passing through a point having position vector $\vec{a}$ and normal to vector $\vec{n}$ is given by $(\vec{r} - \vec{a}) \cdot \vec{n} = 0$ or, $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$.

Thus the required equation of the required plane is

$\vec{r} \cdot (\hat{i} + 2\hat{j} - 3\hat{k}) = (\hat{i} + 2\hat{j} - 3\hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k})$

$\Rightarrow \vec{r} \cdot (\hat{i} + 2\hat{j} - 3\hat{k}) = 1 + 4 + 9$

$\Rightarrow \vec{r} \cdot (\hat{i} + 2\hat{j} - 3\hat{k}) = 14$

$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k}) = 14$

$\Rightarrow x + 2y - 3z = 14$

The Plane Ex 29.3 Q20

O is the origin and the coordinates of A are (a, b, c).

$\overline{OA} = a\hat{i} + b\hat{j} + c\hat{k}$

$\therefore$ The direction the direction ratios of OA are proportional to, a, b, c.

$\therefore$ Direction cosines are,

$$\frac{a}{\sqrt{a^2 + b^2 + c^2}}, \frac{b}{\sqrt{a^2 + b^2 + c^2}}, \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

The equation of the line passing through A(a, b, c) and perpendicular to $\overline{OA}$ is,

$\left\{x\hat{i} + y\hat{j} + z\hat{k} - \left(a\hat{i} + b\hat{j} + c\hat{k}\right)\right\} \bullet a\hat{i} + b\hat{j} + c\hat{k} = 0$

$ax + by + cz = a^2 + b^2 + c^2$

Ex - 29.4

The Plane Ex 29.4 Q1

Here, it is given that, the required plane is at a distanceof 3 unit from origin and k is unit vector normal to it. We know that, vector equation of a plane normal to unit vector $\hat{n}$ and at distance d from origin, is

$$\vec{r}.\hat{n} = d$$

So, here $d = 3$ unit
$\hat{n} = \hat{k}$

The equation of the required plane is,

$$\vec{r}.\hat{k} = 3$$

The Plane Ex 29.4 Q2

We know that, vector equation of a plane which is at a distance d unit from origin and normal to unit vector $\hat{n}$ is given by

$$\vec{r}.\hat{n} = d \qquad ---(\text{i})$$

Here, $d = 5$ unit

$$\vec{n} = \hat{i} - 2\hat{j} - 2\hat{k}$$

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|}$$

$$= \frac{\hat{i} - 2\hat{j} - 2\hat{k}}{\sqrt{(1)^2 + (-2)^2 + (-2)^2}}$$

$$= \frac{\hat{i} - 2\hat{j} - 2\hat{k}}{\sqrt{9}}$$

$$\hat{n} = \frac{1}{3}\left(\hat{i} - 2\hat{j} - 2\hat{k}\right)$$

Put, value of d and $\hat{n}$ in equation (i),
The equation of required plane is,

$$\vec{r}.\frac{1}{3}\left(\hat{i} - 2\hat{j} - 2\hat{k}\right) = 5$$

The Plane Ex 29.4 Q3

Given equation of plane is,

$2x - 3y - 6z = 14$

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right).\left(2\hat{i} - 3\hat{j} - 6\hat{k}\right) = 14$

Dividing the equation by $\sqrt{(2)^2 + (-3)^2 + (-6)^2}$

$$\vec{r}.\frac{\left(2\hat{i} - 3\hat{j} - 6\hat{k}\right)}{\sqrt{4+9+36}} = \frac{14}{\sqrt{4+9+36}}$$

$$\vec{r}.\left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}\right) = \frac{14}{7}$$

$$\vec{r}.\left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}\right) = 2 \qquad ---(i)$$

We know that the vector equation of a plane with distance d from origin and normal to unit vector $\hat{n}$ is given by

$$\vec{r}.\hat{n} = d \qquad ---(ii)$$

Comparing (i) and (ii),

$d = 2$ and

$\hat{n} = \frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}$

So, distance of plane from origin = 2 unit

Direction cosine of normal to plane = $\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}$

The Plane Ex 29.4 Q4

Given equation of plane is,

$\vec{r}.\left(\hat{i} - 2\hat{j} + 2\hat{k}\right) + 6 = 0$

$\vec{r}.\left(\hat{i} - 2\hat{j} + 2\hat{k}\right) = -6$

Multiplying both the sides by (-1),

$\vec{r}.\left(-\hat{i} + 2\hat{j} - 2\hat{k}\right) = 6$

$$\vec{r}.\vec{n} = 6 \qquad ---(i)$$

Here, $\vec{n} = -\hat{i} + 2\hat{j} - 2\hat{k}$

$|\vec{n}| = \sqrt{(-1)^2 + (2)^2 + (-2)^2}$

$= \sqrt{1+4+4}$

$= \sqrt{9}$

$= 3$

Dividing equation (i) by $|\vec{n}| = 3$ both the sides,

$$\vec{r}.\frac{\vec{n}}{|\vec{n}|} = \frac{6}{|\vec{n}|}$$

$$\vec{r}.\frac{1}{3}\left(-\hat{i} + 2\hat{j} - 2\hat{k}\right) = \frac{6}{3}$$

$$\vec{r}\left(-\frac{\hat{i}}{3} + \frac{2}{3}\hat{j} - \frac{2}{3}\hat{k}\right) = 2 \qquad ---(ii)$$

We know that, equation of a plane at distance d from origin and normal to unit vector $\hat{n}$ is

$$\vec{r}.\hat{n} = d \qquad ---(iii)$$

Comparing equation (ii) and (iii),

$d = 2$

$\hat{n} = -\frac{\hat{i}}{3} + \frac{2}{3}\hat{j} - \frac{2}{3}\hat{k}$

The Plane Ex 29.4 Q5

Given equation of plane is,

$$2x - 3y + 6z + 14 = 0$$
$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right) = -14$$
$$\vec{r}.\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right) = -14$$

Multiplying by (-1) both the sides,

$$\vec{r}.\left(-2\hat{i} + 3\hat{j} - 6\hat{k}\right) = 14 \qquad ---(i)$$

So, $\vec{r}.\vec{n} = 14$

$$\vec{n} = -2\hat{i} + 3\hat{j} - 6\hat{k}$$
$$\left|\vec{n}\right| = \sqrt{(-2)^2 + (3)^2 + (-6)^2}$$
$$= \sqrt{4 + 9 + 36}$$
$$= \sqrt{49}$$

$$\left|\vec{n}\right| = 7$$

Dividing equation (i) by $\left|\vec{n}\right| \Rightarrow$ both the sides,

$$\vec{r}.\frac{\left(-2\hat{i} + 3\hat{j} - 6\hat{k}\right)}{7} = \frac{14}{7}$$

$$\vec{r}.\left(-\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}\right) = 2$$

$$-\frac{2}{7}x + \frac{3}{7}y - \frac{6}{7}z = 2$$

The Plane Ex 29.4 Q6

Given, direction ratios of perpendicular from origin to a plane is 12, – 3, 4

So,

Normal vector $= 12\hat{i} - 3\hat{j} + 4\hat{k}$

$$\vec{n} = 12\hat{i} - 3\hat{j} + 4\hat{k}$$
$$\left|\vec{n}\right| = \sqrt{(12)^2 + (-3)^2 + (4)^2}$$
$$= \sqrt{144 + 9 + 16}$$
$$= \sqrt{169}$$

$$\left|\vec{n}\right| = 13$$

Normal unit vector $\hat{n} = \dfrac{\vec{n}}{\left|\vec{n}\right|}$

$$= \frac{1}{13}\left(12\hat{i} - 3\hat{j} + 4\hat{k}\right)$$

Given that, perpendicular distance of plane from origin is 5 unit.

$\Rightarrow \qquad d = 5$ unit

We know that, equation of a plane at a distance d from origin and normal unit vector $\hat{n}$ is

$$\vec{r}.\hat{n} = d$$

So, vector equation of required plane is

$$\vec{r}.\left(\frac{12}{13}\hat{i} - \frac{3}{13}\hat{j} + \frac{4}{13}\hat{k}\right) = 5$$

Put, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\frac{12}{13}\hat{i} - \frac{3}{13}\hat{j} + \frac{4}{13}\hat{k}\right) = 5$$
$$(x)\left(\frac{12}{13}\right) + (y)\left(-\frac{3}{13}\right) + (z)\left(\frac{4}{13}\right) = 5$$

$$\frac{12}{13}x - \frac{3}{13}y + \frac{4}{13}z = 5$$

The Plane Ex 29.4 Q7

Given equation of plane is

$x + 2y + 3z - 6 = 0$

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - 6 = 0$

$\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) = 6 \qquad ---(i)$

$\vec{r}.\vec{n} = 6$

So, $\vec{n} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\left|\vec{n}\right| = \sqrt{(1)^2 + (2)^2 + (3)^2}$

$= \sqrt{1 + 4 + 9}$

$\left|\vec{n}\right| = \sqrt{14}$

Dividing equation (i) by $\sqrt{14}$, we get

$\vec{r}.\left(\frac{1}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\hat{k}\right) = \frac{6}{\sqrt{14}} \qquad ---(ii)$

We know that, vector equation of a plane at distance d unit from origin and normal to unit vector $\hat{n}$ is

$\vec{r}.\hat{n} = d \qquad ---(iii)$

Comparing (ii) and (iii), we get

Normal unit vector $= \hat{n} = \frac{1}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\hat{k}$

The Plane Ex 29.4 Q8

We know that, vector equation of a plane which is at a distance d from origin and normal to unit vector $\hat{n}$ is given by

$\vec{r}.\hat{n} = d \qquad ---(i)$

Here, given $\vec{d} = 3\sqrt{3}$ unit.

Let, $\vec{a} = \left(p\hat{i} + q\hat{j} + r\hat{k}\right)$

Where $\vec{a}$ is normal vector.

Given that, $\vec{a}$ is equally inclined to the coordinate axes

If l, m, n are direction cosines of $\vec{n}$,

Here, $l = m = n \qquad ---(ii)$

We know that,

$l^2 + m^2 + n^2 = 1$

$l^2 + l^2 + l^2 = 1 \qquad$ [Using (ii)]

$3l^2 = 1$

$l = \frac{1}{\sqrt{3}}$

So, $l = m = n = \frac{1}{\sqrt{3}}$

Here,

$$l = \frac{p}{|\vec{a}|} = \frac{1}{\sqrt{3}}$$

$$m = \frac{q}{|\vec{a}|} = \frac{1}{\sqrt{3}}$$

$$n = \frac{r}{|\vec{a}|} = \frac{1}{\sqrt{3}}$$

Now,

$$\vec{a} = p\hat{i} + q\hat{j} + r\hat{k}$$

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

$$= \frac{p}{|\vec{a}|}\hat{i} + \frac{q}{|\vec{a}|}\hat{j} + \frac{r}{|\vec{a}|}\hat{k}$$

$$\hat{a} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

Put the value of $d = 3\sqrt{3}$ unit and $\hat{n} = \hat{a} = \frac{\hat{i}}{\sqrt{3}} + \frac{\hat{j}}{\sqrt{3}} + \frac{\hat{k}}{\sqrt{3}}$ in equation (i), vector equation of the required plane is

$$\vec{r}.\frac{1}{\sqrt{3}}\left(\hat{i} + \hat{j} + \hat{k}\right) = 3\sqrt{3}$$

$$\vec{r}\left(\hat{i} + \hat{j} + \hat{k}\right) = 9$$

$$x + y + z = 9$$

The Plane Ex 29.4 Q9

Here, we have to find equating a plane passing through $A(1,2,1)$ and perpendicular to line joining $B(1,4,2)$ and $C(2,3,5)$.

We know that, the vector equation of a plane passing through a point $\vec{a}$ and perpendicular to vector $\vec{n}$ is given by,

$$(\vec{r}-\vec{a}).\vec{n} = 0 \qquad ---(i)$$

Here, $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$

$\vec{n} = \overrightarrow{BC}$

$= \text{Position vector of } C - \text{Position vector of } B$

$= \left(2\hat{i} + 3\hat{j} + 5\hat{k}\right) - \left(\hat{i} + 4\hat{j} + 2\hat{k}\right)$

$= 2\hat{i} + 3\hat{j} + 5\hat{k} - \hat{i} - 4\hat{j} - 2\hat{k}$

$\vec{n} = \hat{i} - \hat{j} + 3\hat{k}$

Put, $\vec{a}$ and $\vec{n}$ in equation (i),

Vector equation of plane is

$$\left[\vec{r} - \left(\hat{i} + 2\hat{j} + \hat{k}\right)\right].\left(\hat{i} - \hat{j} + 3\hat{k}\right) = 0$$

$$\vec{r}.\left(\hat{i} - \hat{j} + 3\hat{k}\right) - \left(\hat{i} + 2\hat{j} + \hat{k}\right)\left(\hat{i} - \hat{j} + 3\hat{k}\right) = 0$$

$$\vec{r}.\left(\hat{i} - \hat{j} + 3\hat{k}\right) - \left[(1)(1) + (2)(-1) + (1)(3)\right] = 0$$

$$\vec{r}.\left(\hat{i} - \hat{j} + 3\hat{k}\right) - \left[1 - 2 + 3\right] = 0$$

$$\vec{r}.\left(\hat{i} - \hat{j} + 3\hat{k}\right) - (4 - 2) = 0$$

$$\vec{r}.\left(\hat{i} - \hat{j} + 3\hat{k}\right) - 2 = 0$$

$$\vec{r}.\left(\hat{i} - \hat{j} + 3\hat{k}\right) = 2 \qquad ---(ii)$$

$$|\vec{n}| = \sqrt{(1)^2 + (-1)^2 + (3)^2}$$

$$= \sqrt{1+1+9}$$

$$= \sqrt{11}$$

Dividing equation (i) by $\sqrt{11}$,

$$\vec{r}.\left(\frac{1}{\sqrt{11}}\hat{i} - \frac{1}{\sqrt{11}}\hat{j} + \frac{3}{\sqrt{11}}\hat{k}\right) = \frac{2}{\sqrt{11}}$$

$$\vec{r}.\hat{n} = d$$

So, perpendicular distance of plane from origin = $\frac{2}{\sqrt{11}}$ units

Equation of plane, $\vec{r}.\left(\hat{i} - \hat{j} + 3\hat{k}\right) = 2$

Equation of plane, $x - y + 3z - 2 = 0$

The Plane 29.4 Q10

We know that the vector equation of a plane at a distance

'p' from the origin and normal to the unit vector $\hat{n}$ is $\vec{r}\cdot\hat{n}=p$

Vector normal to the plane is $\vec{n}=2\hat{i}-3\hat{j}+4\hat{k}$

The unit vector normal to the plane is

$$\hat{n}=\frac{2}{\sqrt{2^2+(-3)^2+4^2}}\hat{i}-\frac{3}{\sqrt{2^2+(-3)^2+4^2}}\hat{j}+\frac{4}{\sqrt{2^2+(-3)^2+4^2}}\hat{k}$$

$$\Rightarrow \hat{n}=\frac{2}{\sqrt{4+9+16}}\hat{i}-\frac{3}{\sqrt{4+9+16}}\hat{j}+\frac{4}{\sqrt{4+9+16}}\hat{k}$$

$$\Rightarrow \hat{n}=\frac{2}{\sqrt{29}}\hat{i}-\frac{3}{\sqrt{29}}\hat{j}+\frac{4}{\sqrt{29}}\hat{k}$$

Here, given that $p=\frac{6}{\sqrt{29}}$

Thus, the vector equation of the plane is

$$\vec{r}\cdot\left(\frac{2}{\sqrt{29}}\hat{i}-\frac{3}{\sqrt{29}}\hat{j}+\frac{4}{\sqrt{29}}\hat{k}\right)=\frac{6}{\sqrt{29}}$$

The Cartesian equation of the plane is

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\cdot\left(\frac{2}{\sqrt{29}}\hat{i}-\frac{3}{\sqrt{29}}\hat{j}+\frac{4}{\sqrt{29}}\hat{k}\right)=\frac{6}{\sqrt{29}}$$

$$\Rightarrow \left(\frac{2x}{\sqrt{29}}-\frac{3y}{\sqrt{29}}+\frac{4z}{\sqrt{29}}\right)=\frac{6}{\sqrt{29}}$$

$$\Rightarrow \left(\frac{2x-3y+4z}{\sqrt{29}}\right)=\frac{6}{\sqrt{29}}$$

$$\Rightarrow 2x-3y+4z=6$$

The Plane 29.4 Q11

The Cartesian equation of the given plane is

$2x-3y+4z-6=0.$

The above equation can be rewritten as

$2x-3y+4z=6$

Therefore, the vector equation of the plane is

$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\cdot\left(2\hat{i}-3\hat{j}+4\hat{k}\right)=6$

$\Rightarrow \vec{r}\cdot\left(2\hat{i}-3\hat{j}+4\hat{k}\right)=6 \ldots\text{(1)}$

We know that the vector equation of a plane at a distance

'p' from the origin and normal to unit vector $\hat{n}$ is $\vec{r}\cdot\hat{n}=p$

We have, $\vec{n}=2\hat{i}-3\hat{j}+4\hat{k}.$

Thus $|\vec{n}|=\sqrt{2^2+(-3)^2+4^2}=\sqrt{29}$

Dividing the equation (1) by $|\vec{n}|=\sqrt{29}$, we have,

$$\vec{r}\cdot\left(\frac{2\hat{i}-3\hat{j}+4\hat{k}}{\sqrt{29}}\right)=\frac{6}{\sqrt{29}}$$

Hence the normal form of the equation of the plane is

$$\vec{r}\cdot\left(\frac{2}{\sqrt{29}}\hat{i}-\frac{3}{\sqrt{29}}\hat{j}+\frac{4}{\sqrt{29}}\hat{k}\right)=\frac{6}{\sqrt{29}}$$

Hence the perpendicular distance of the

origin from the plane is $p=\frac{6}{\sqrt{29}}$.

Ex - 29.5

The Plane Ex 29.5 Q1

Given that, plane is passing through
$(1,1,1)$, $(1,-1,1)$ and $(-7,-3,-5)$

We know that, equation of plane passing through 3 points,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 1-1 & -1-1 & 1-1 \\ -7-1 & -3-1 & -5-1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 0 & -2 & 0 \\ -8 & -4 & -6 \end{vmatrix} = 0$$

$(x-1)(12-0)-(y-1)(0-0)+(z-1)(0-16)=0$
$(x-1)(12)-(y-1)(0)+(z-1)(-16)=0$
$12x-12-0-16z+16=0$

$12x-16z+4=0$

Dividing by 4,

Dividing by 4,

$3x-4z+1=0$
$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(3\hat{i}+0\hat{j}-4\hat{k}\right)+1=0$

$\vec{r}.\left(3\hat{i}-4\hat{k}\right)+1=0$

Equation of the required plane,

$\vec{r}.\left(3\hat{i}-4\hat{k}\right)+1=0$

The Plane Ex 29.5 Q2

Let $P(2,5,-3)$, $Q(-2,-3,5)$ and $R(5,3,-3)$ be the three points on a plane having position vectors $\vec{p}$, $\vec{q}$ and $\vec{s}$ respectively. Then the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are in the same plane.
Therefore, $\overrightarrow{PQ}\times\overrightarrow{PR}$ is a vector perpendicular to the plane.
Let $\vec{n}=\overrightarrow{PQ}\times\overrightarrow{PR}$

$\overrightarrow{PQ}=(-2-2)\hat{i}+(-3-5)\hat{j}+(5-(-3))\hat{k}$
$\Rightarrow \overrightarrow{PQ}=-4\hat{i}-8\hat{j}+8\hat{k}$
Similarly,
$\overrightarrow{PR}=(5-2)\hat{i}+(3-5)\hat{j}+(-3-(-3))\hat{k}$
$\Rightarrow \overrightarrow{PR}=3\hat{i}-2\hat{j}+0\hat{k}$
Thus
$\vec{n}=\overrightarrow{PQ}\times\overrightarrow{PR}$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -4 & -8 & 8\\ 3 & -2 & 0\end{vmatrix}$$

$=16\hat{i}+24\hat{j}+32\hat{k}$
The plane passes through the point P with position vector $\vec{p}=2\hat{i}+5\hat{j}-3\hat{k}$
Thus, its vector equation is
$\{\vec{r}-(2\hat{i}+5\hat{j}-3\hat{k})\}\cdot(16\hat{i}+24\hat{j}+32\hat{k})=0$
$\Rightarrow \vec{r}\cdot(16\hat{i}+24\hat{j}+32\hat{k})-(32+120-96)=0$
$\Rightarrow \vec{r}\cdot(16\hat{i}+24\hat{j}+32\hat{k})-56=0$
$\Rightarrow \vec{r}\cdot(16\hat{i}+24\hat{j}+32\hat{k})=56$
$\Rightarrow \vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k})=7$

The Plane Ex 29.5 Q3

Let $A(a,0,0)$, $B(0,b,0)$ and $C(0,0,c)$ be three points on a plane having their position vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively. Then vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are in the same plane.
Therefore, $\overrightarrow{AB}\times\overrightarrow{AC}$ is a vector perpendicular to the plane.
Let $\vec{n}=\overrightarrow{AB}\times\overrightarrow{AC}$

$\overrightarrow{AB}=(0-a)\hat{i}+(b-0)\hat{j}+(0-0)\hat{k}$
$\Rightarrow \overrightarrow{AB}=-a\hat{i}+b\hat{j}+0\hat{k}$
Similarly,
$\overrightarrow{AC}=(0-a)\hat{i}+(0-0)\hat{j}+(c-0)\hat{k}$
$\Rightarrow \overrightarrow{AC}=-a\hat{i}+0\hat{j}+c\hat{k}$
Thus
$\vec{n}=\overrightarrow{AB}\times\overrightarrow{AC}$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -a & b & 0\\ -a & 0 & c\end{vmatrix}$$

$\vec{n}=bc\hat{i}+ac\hat{j}+ab\hat{k}$
$\Rightarrow \hat{n}=\dfrac{bc\hat{i}+ac\hat{j}+ab\hat{k}}{\sqrt{b^2c^2+a^2c^2+a^2b^2}}$
The plane passes through the point P with position vector $\vec{a}=a\hat{i}+0\hat{j}+0\hat{k}$

Thus, the vector equation in the normal form is

$$\{\vec{r}-[(a\hat{i}+0\hat{j}+0\hat{k})]\}\cdot\left(\frac{bc\hat{i}+ac\hat{j}+ab\hat{k}}{\sqrt{b^2c^2+a^2c^2+a^2b^2}}\right)=0$$

$$\Rightarrow \vec{r}\cdot\frac{(bc\hat{i}+ac\hat{j}+ab\hat{k})}{\sqrt{b^2c^2+a^2c^2+a^2b^2}}=\frac{abc}{\sqrt{b^2c^2+a^2c^2+a^2b^2}}$$

$$\Rightarrow \vec{r}\cdot\frac{(bc\hat{i}+ac\hat{j}+ab\hat{k})}{\sqrt{b^2c^2+a^2c^2+a^2b^2}}=\frac{1}{\sqrt{\frac{b^2c^2+a^2c^2+a^2b^2}{a^2b^2c^2}}}$$

$$\Rightarrow \vec{r}\cdot\frac{(bc\hat{i}+ac\hat{j}+ab\hat{k})}{\sqrt{b^2c^2+a^2c^2+a^2b^2}}=\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}\ldots(1)$$

The vector equation of a plane normal to the unit vector $\hat{n}$ and at a distance 'd' from the origin is $\vec{r}\cdot\hat{n}=d\ldots(2)$
Given that the plane is at a distance 'p' from the origin.
Comparing equations (1) and (2), we have,

$$d=p=\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}$$

$$\Rightarrow \frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$$

The Plane Ex 29.5 Q4

Let $P(1,1,-1)$, $Q(6,4,-5)$ and $R(-4,-2,3)$ be three points on a plane having position vectors $\vec{p}$, $\vec{q}$ and $\vec{s}$ respectively. Then the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are in the same plane.
Therefore, $\overrightarrow{PQ}\times\overrightarrow{PR}$ is a vector perpendicular to the plane.
Let $\vec{n}=\overrightarrow{PQ}\times\overrightarrow{PR}$

$\overrightarrow{PQ}=(6-1)\hat{i}+(4-1)\hat{j}+(-5-(-1))\hat{k}$
$\Rightarrow\overrightarrow{PQ}=5\hat{i}+3\hat{j}-4\hat{k}$
Similarly,
$\overrightarrow{PR}=(-4-1)\hat{i}+(-2-1)\hat{j}+(3-(-1))\hat{k}$
$\Rightarrow\overrightarrow{PR}=-5\hat{i}-3\hat{j}+4\hat{k}$
Thus
Here, $\overrightarrow{PQ}=-\overrightarrow{PR}$
Therefore, the given points are collinear.
Thus, $\vec{n}=a\hat{i}+b\hat{j}+c\hat{k}$ where, $5a+3b-4c=0$
The plane passes through the point P with position vector $\vec{p}=\hat{i}+\hat{j}-\hat{k}$
Thus, its vector equation is
$\{\vec{r}-(\hat{i}+\hat{j}-\hat{k})\}\cdot(a\hat{i}+b\hat{j}+c\hat{k})=0$, where, $5a+3b-4c=0$

The Plane Ex 29.5 Q5

Let, A,B,C be the points with position vector $\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$, $\left(2\hat{i}-2\hat{j}-\hat{k}\right)$ and $\left(7\hat{i}+6\hat{k}\right)$ respectively. Then

$\overrightarrow{AB}$ = Position vector of B – Poosition vector of A

$$=\left(2\hat{i}-2\hat{j}-\hat{k}\right)-\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$$
$$=2\hat{i}-2\hat{j}-\hat{k}-3\hat{i}-4\hat{j}-2\hat{k}$$
$$=-\hat{i}-6\hat{j}-3\hat{k}$$

$\overrightarrow{BC}$ = Position vector of C – Poosition vector of B

$$=\left(7\hat{i}+6\hat{k}\right)-\left(2\hat{i}-2\hat{j}-\hat{k}\right)$$
$$=7\hat{i}+6\hat{k}-2\hat{i}+2\hat{j}+\hat{k}$$

$$\overrightarrow{BC}=5\hat{i}+2\hat{j}+7\hat{k}$$

A vector normal to A,B,C is a vector perpendicular to $\overrightarrow{AB}\times\overrightarrow{BC}$

$$\vec{n}=\overrightarrow{AB}\times\overrightarrow{BC}$$
$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & -6 & -3\\ 5 & 2 & 7\end{vmatrix}$$

$$\vec{n}=\hat{i}\left(-42+6\right)-\hat{j}\left(-7+15\right)+\hat{k}\left(-2+30\right)$$

$= -36\hat{i} - 8\hat{j} + 28\hat{k}$

We know that, equation of a plane passing through vector $\vec{a}$ and perpendicular to vector $\vec{n}$ is given by,

$$\vec{r}.\vec{n} = \vec{a}.\vec{n} \qquad ---(i)$$

Put $\vec{a}$ and $\vec{n}$ in equation (i),

$$\vec{r}.\left(-36\hat{i} - 8\hat{j} + 28\hat{k}\right) = \left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)\left(-36\hat{i} - 8\hat{j} + 28\hat{k}\right)$$

$$= (3)(-36) + (4)(-8) + (2)(28)$$
$$= -108 - 32 + 56$$
$$= -140 + 56$$

$$\vec{r}.\left(-36\hat{i} - 8\hat{j} + 28\hat{k}\right) = -84$$

Dividing by (-4), we get

$$\vec{r}.\left(9\hat{i} + 2\hat{j} - 7\hat{k}\right) = 21$$

Equation of required plane is,

$$\vec{r}.\left(9\hat{i} + 2\hat{j} - 7\hat{k}\right) = 21$$

Ex - 29.6

The Plane 29.6 Q1(i)

Given equation of two planes are

$$\vec{r}.\left(2\hat{i}-3\hat{j}+4\hat{k}\right)=1 \qquad ---(i)$$
$$\vec{r}.\left(-\hat{i}+\hat{j}\right)=4 \qquad ---(ii)$$

We know that, angle between two planes

$\vec{r}.\vec{n_1}=d_1$ and $\vec{r}.\vec{n_2}=d_2$ is given by,

$$\cos\theta=\frac{\vec{n_1}.\vec{n_2}}{\left|\vec{n_1}\right|\left|\vec{n_2}\right|} \qquad ---(iii)$$

Here, $\vec{n_1}=2\hat{i}-3\hat{j}+4\hat{k}$

$\vec{n_2}=-\hat{i}+\hat{j}$

$$\cos\theta=\frac{\left(2\hat{i}-3\hat{j}+4\hat{k}\right)\left(-\hat{i}+\hat{j}\right)}{\sqrt{(2)^2+(-3)^2+(4)^2}\sqrt{(-1)^2+(1)^2}}$$
$$=\frac{(2)(-1)+(-3)(1)+(4)(0)}{\sqrt{4+9+16}\sqrt{1+1}}$$
$$=\frac{-2-3+0}{\sqrt{29}\sqrt{2}}$$
$$\cos\theta=\frac{-5}{\sqrt{58}}$$

$$\theta=\cos^{-1}\left(\frac{-5}{\sqrt{58}}\right)$$

The Plane 29.6 Q1(ii)

Given equation of planes are

$$\vec{r}.\left(2\hat{i}-\hat{j}+2\hat{k}\right)=6 \qquad ---(i)$$
$$\vec{r}.\left(3\hat{i}+6\hat{j}-2\hat{k}\right)=9 \qquad ---(ii)$$

We know that, angle between the planes

$\vec{r}.\vec{n_1}=d_1$ and $\vec{r}.\vec{n_2}=d_2$ is given by,

$$\cos\theta=\frac{\vec{n_1}.\vec{n_2}}{\left|\vec{n_1}\right|\left|\vec{n_2}\right|} \qquad ---(iii)$$

Here, from equation (i) and (ii),

$\vec{n_1}=2\hat{i}-\hat{j}+2\hat{k}$

$\vec{n_2}=3\hat{i}+6\hat{j}-2\hat{k}$

Put $\vec{n_1}$ and $\vec{n_2}$ in equation (iii),

$$\cos\theta=\frac{\left(2\hat{i}-\hat{j}+2\hat{k}\right)\left(3\hat{i}+6\hat{j}-2\hat{k}\right)}{\sqrt{(2)^2+(-1)^2+(2)^2}\sqrt{(3)^2+(6)^2+(-2)^2}}$$
$$=\frac{(2)(3)+(-1)(6)+(2)(-2)}{\sqrt{4+1+4}\sqrt{9+36+4}}$$
$$\cos\theta=\frac{6-6+4}{\sqrt{9}\sqrt{49}}$$
$$=\frac{-4}{3.7}$$
$$=\frac{-4}{21}$$

$$\theta=\cos^{-1}\left(\frac{-4}{21}\right)$$

The Plane 29.6 Q1(iii)

Given equation of planes are

$$\vec{r}.\left(2\hat{i}+3\hat{j}-6\hat{k}\right)=5 \quad ---(i)$$
$$\vec{r}.\left(\hat{i}-2\hat{j}+2\hat{k}\right)=9 \quad ---(ii)$$

We know that, angle between equation of planes

$\vec{r}.\vec{n_1}=d_1$ and $\vec{r}.\vec{n_2}=d_2$ is given by,

$$\cos\theta=\frac{\vec{n_1}.\vec{n_2}}{|\vec{n_1}||\vec{n_2}|} \quad ---(iii)$$

From equation (i) and (ii),

$\vec{n_1}=2\hat{i}+3\hat{j}-6\hat{k}$

$\vec{n_2}=\hat{i}-2\hat{j}+2\hat{k}$

Put $\vec{n_1}$ and $\vec{n_2}$ in equation (iii),

$$\cos\theta=\frac{\left(2\hat{i}+3\hat{j}-6\hat{k}\right)\left(\hat{i}-2\hat{j}+2\hat{k}\right)}{\sqrt{(2)^2+(3)^2+(-6)^2}\sqrt{(1)^2+(-2)^2+(2)^2}}$$
$$=\frac{(2)(1)+(3)(-2)+(-6)(2)}{\sqrt{4+9+36}\sqrt{1+4+4}}$$
$$=\frac{2-6-12}{\sqrt{49}\sqrt{9}}$$
$$=\frac{-16}{7.3}$$
$$=\frac{-16}{21}$$

$$\theta=\cos^{-1}\left(\frac{-16}{21}\right)$$

The Plane 29.6 Q2(i)

Given, equation of planes are,

$$2x-y+z=4 \quad ---(i)$$
$$x+y+2z=3 \quad ---(ii)$$

We know that, angle between two planes

$a_1x+b_1y+c_1z+d_1=0$ and

$a_2x+b_2y+c_2z+d_2=0$ is given by

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \quad ---(iii)$$

Here, from equation (i) and (ii),

$a_1=2,\ b_1=-1,\ c_1=1$

$a_2=1,\ b_2=1,\ c_2=2$

Put then values in equation (iii),

$$\cos\theta=\frac{(2)(1)+(-1)(1)+(1)(2)}{\sqrt{(2)^2+(-1)^2+(1)^2}\sqrt{(1)^2+(1)^2+(2)^2}}$$
$$=\frac{2-1+2}{\sqrt{4+1+1}\sqrt{1+1+4}}$$
$$\cos\theta=\frac{4-1}{\sqrt{6}\sqrt{6}}$$
$$=\frac{3}{6}$$
$$=\frac{1}{2}$$

$$\theta=\cos^{-1}\left(\frac{1}{2}\right)$$

$$\theta=\frac{\pi}{3}$$

The Plane 29.6 Q2(ii)

Given, equation of two planes are,

$x + y - 2z = 3 \quad ---(i)$

$2x - 2y + z = 5 \quad ---(ii)$

We know that, angle between two planes

$a_1x + b_1y + c_1z + d_1 = 0$ and

$a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Here, from equation (i) and (ii),

$a_1 = 1,\ b_1 = 1,\ c_1 = -2$

$a_2 = 2,\ b_2 = -2,\ c_2 = 1$

Put these values in equation (iii),

$$\cos\theta = \frac{(1)(2) + (1)(-2) + (-2)(1)}{\sqrt{(1)^2 + (1)^2 + (-2)^2}\sqrt{(2)^2 + (-2)^2 + (1)^2}}$$

$$\cos\theta = \frac{2 - 2 - 2}{\sqrt{1 + 1 + 4}\sqrt{4 + 4 + 1}}$$

$$= \frac{-2}{\sqrt{6}\sqrt{9}}$$

$$= \frac{-2}{3\sqrt{6}}$$

$$\theta = \cos^{-1}\left(\frac{-2}{3\sqrt{6}}\right)$$

The Plane 29.6 Q2(iii)

Given, equation of planes are,

$x - y + z = 5 \quad ---(i)$

$x + 2y + z = 9 \quad ---(ii)$

We know that, angle between the planes

$a_1x + b_1y + c_1z + d_1 = 0$ and

$a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \quad ---(iii)$$

From equation (i) and (ii),

$a_1 = 1,\ b_1 = -1,\ c_1 = 1$

$a_2 = 1,\ b_2 = 2,\ c_2 = 1$

Put these values in equation (iii),

$$\cos\theta = \frac{(1)(1) + (-1)(2) + (1)(1)}{\sqrt{(1)^2 + (-1)^2 + (1)^2}\sqrt{(1)^2 + (2)^2 + (1)^2}}$$

$$\cos\theta = \frac{1 - 2 + 1}{\sqrt{1 + 1 + 1}\sqrt{1 + 4 + 1}}$$

$$= \frac{0}{\sqrt{3}\sqrt{6}}$$

$$\cos\theta = 0$$

$$\theta = \cos^{-1}(0)$$

$$\theta = \frac{\pi}{2}$$

The Plane 29.6 Q2(iv)

Given, equation of planes are,

$2x - 3y + 4z = 1$ --- (i)

$-x + y = 4$ --- (ii)

We know that, angle between two planes

$a_1x + b_1y + c_1z + d_1 = 0$ and
$a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Here, from equation (i) and (ii),
$a_1 = 2,\ b_1 = -3,\ c_1 = 4$
$a_2 = -1,\ b_2 = 1,\ c_2 = 0$

Put these values in equation (iii),

$$\cos\theta = \frac{(2)(-1) + (-3)(1) + (4)(0)}{\sqrt{(2)^2 + (-3)^2 + (4)^2}\sqrt{(-1)^2 + (1)^2 + (0)^2}}$$

$$\cos\theta = \frac{-2 - 3 + 0}{\sqrt{4 + 9 + 16}\sqrt{1 + 1 + 0}}$$

$$= \frac{-5}{\sqrt{29}\sqrt{2}}$$

$$\cos\theta = \frac{-5}{\sqrt{58}}$$

$$\theta = \cos^{-1}\left(\frac{-5}{\sqrt{58}}\right)$$

The Plane Ex 29.6 Q2(v)

We know that the angle between the planes
$a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \cdot \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Therefore, the angle between $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$

$$\cos\theta = \frac{2 \times 3 + 1 \times (-6) + (-2) \times (-2)}{\sqrt{2^2 + 1^2 + (-2)^2} \cdot \sqrt{3^2 + (-6)^2 + (-2)^2}}$$

$$\Rightarrow \cos\theta = \frac{6 - 6 + 4}{\sqrt{9} \cdot \sqrt{9 + 36 + 4}}$$

$$\Rightarrow \cos\theta = \frac{4}{3 \times 7}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{4}{21}\right)$$

The Plane 29.6 Q3(i)

Given equation of planes are

$\vec{r}.\left(2\hat{i} - \hat{j} + \hat{k}\right) = 5$ --- (i)

$\vec{r}.\left(-\hat{i} - \hat{j} + \hat{k}\right) = 3$ --- (ii)

We know that, planes

$\vec{r}.\vec{n_1} = d_1$ and $\vec{r}.\vec{n_2} = d_2$ are perpendicular

if $\vec{n_1}.\vec{n_2} = 0$ --- (iii)

From equation (i) and (ii),
$\vec{n_1} = 2\hat{i} - \hat{j} + \hat{k}$
$\vec{n_2} = -\hat{i} - \hat{j} + \hat{k}$

Put $\vec{n_1}$ and $\vec{n_2}$ in equation (iii),

$\vec{n_1}.\vec{n_2} = 0$
$\left(2\hat{i} - \hat{j} + \hat{k}\right)\left(-\hat{i} - \hat{j} + \hat{k}\right) = 0$
$(2)(-1) + (-1)(-1) + (1)(1) = 0$
$-2 + 1 + 1 = 0$

$0 = 0$
$LHS = RHS$

Hence, planes are at right angle

The Plane 29.6 Q3(ii)

Given, equation of planes are,

$x - 2y + 4z = 10$
$18x + 17y + 4z = 49$

$\Rightarrow \quad x - 2y + 4z - 10 = 0 \qquad ---(i)$
$\quad 18x + 17y + 4z - 49 = 0 \qquad ---(ii)$

We know that, planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are at right angles if $a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(iii)$

From (i) and (ii),
$a_1 = 1,\ b_1 = -2,\ c_1 = 4$
$a_2 = 18,\ b_2 = 17,\ c_2 = 4$

Put these in equation (iii),

$a_1a_2 + b_1b_2 + c_1c_2 = 0$
$(1)(18) + (-2)(17) + (4)(4) = 0$
$18 - 34 + 16 = 0$

$0 = 0$
$LHS = RHS$

Hence, planes are at right angles

The Plane 29.6 Q4(i)

Here, given equation of planes are

$\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) = 7 \qquad ---(i)$
$\vec{r}.\left(\lambda\hat{i} + 2\hat{j} - 7\hat{k}\right) = 26 \qquad ---(ii)$

We know that, planes $\vec{r}.\vec{n_1} = d_1$ and $\vec{r}.\vec{n_2} = d_2$ are perpendicular if

$\vec{n_1}.\vec{n_2} = 0 \qquad ---(iii)$

From equation (i) and (ii), we get
$\vec{n_1} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{n_2} = \lambda\hat{i} + 2\hat{j} - 7\hat{k}$

Since, (i) and (ii) are perpendicular, so from (iii),

$\vec{n_1}.\vec{n_2} = 0$
$\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)\left(\lambda\hat{i} + 2\hat{j} - 7\hat{k}\right) = 0$
$(1)(\lambda) + (2)(2) + (3)(-7) = 0$
$\lambda + 4 - 21 = 0$

$\lambda - 17 = 0$
$\lambda = 17$

The Plane 29.6 Q4(ii)

Given, that plane $2x - 4y + 3z - 5 = 0$ --- (i)
and $x + 2y + \lambda z - 5 = 0$ are --- (ii) perpendicular.

We know that, planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ --- (iii)

From equation (i) and (ii),
$a_1 = 2,\ b_1 = -4,\ c_1 = 3$
$a_2 = 1,\ b_2 = 2,\ c_2 = \lambda$

Put these in equation (iii),

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(2)(1) + (-4)(2) + (3)(\lambda) = 0$
$2 - 8 + 3\lambda = 0$
$-6 + 3\lambda = 0$
$3\lambda = 6$
$\lambda = \frac{6}{3}$

$\lambda = 2$

The Plane 29.6 Q4(iii)

Given, that planes
$3x - 6y - 2z - 7 = 0$ --- (i)
and $2x + y - \lambda z - 5 = 0$ --- (ii)
are perpendicular.

We know that, planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ --- (iii)

From (i) and (ii),
$a_1 = 3,\ b_1 = -6,\ c_1 = -2$
$a_2 = 2,\ b_2 = 1,\ c_2 = -\lambda$

Put these in equation (iii),

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(3)(2) + (-6)(1) + (-2)(-\lambda) = 0$
$6 - 6 + 2\lambda = 0$
$0 + 2\lambda = 0$
$2\lambda = 0$

$\lambda = 0$

The Plane 29.6 Q5

We know that equation of a plane passing through (x_1, y_1, z_1) is given by

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ --- (i)

Given, plane is passing through $(-1, -1, 2)$,

$a(x + 1) + b(y + 1) + c(z - 2) = 0$ --- (ii)

We know that plane $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ --- (iii)

Given, plane (ii) is perpendicular to plane
$3x + 2y - 3z = 1$ --- (iv)

So, using (ii),(iv) in (iii),
$a_1a_2 + b_1b_2 + c_1c_2 = 0$
$(a)(3) + (b)(2) + (c)(-3) = 0$

$3a + 2b - 3c = 0$ --- (v)

Also, plane (ii) is perpendicular to plane
$5x - 4y + z = 5$ --- (vi)

So, using (ii),(vi) in (iii),
$a_1a_2 + b_1b_2 + c_1c_2 = 0$
$(a)(5) + (b)(-4) + (c)(1) = 0$

$5a - 4b + c = 0$ --- (vii)

On solving (v) and (vii),

$$\frac{a}{(2)(1) - (-3)(-4)} = \frac{b}{(5)(-3) - (3)(1)} = \frac{c}{(3)(-4) - (2)(5)}$$
$$\frac{a}{2 - 12} = \frac{b}{-15 - 3} = \frac{c}{-12 - 10}$$
$$\frac{a}{-10} = \frac{b}{-18} = \frac{c}{-22} = \lambda \text{ (Say)}$$
$a = -10\lambda,\ b = -18\lambda,\ c = -22\lambda$

Put the value of a, b, c in equation (ii)

$a(x + 1) + b(y + 1) + c(z - 2) = 0$
$(-10\lambda)(x + 1) + (-18\lambda)(y + 1) + (-22\lambda)(z - 2) = 0$
$-10\lambda x - 10\lambda - 18\lambda y - 18\lambda - 22\lambda z + 44\lambda = 0$
$-10\lambda x - 18\lambda y - 22\lambda z + 16\lambda = 0$

Dividing by -2λ,
$5x + 9y + 11z - 8 = 0$

So, equation of required plane is,

$5x + 9y + 11z - 8 = 0$

The Plane 29.6 Q6

We know that equation of a plane passing through $(x_1, y_1 z_1)$ is given by

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ ---(i)

Now, equation of plane passing through $(1, -3, -2)$,

$a(x - 1) + b(y + 3) + c(z + 2) = 0$ ---(ii)

We know that planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ ---(iii)

Given, plane (ii) is perpendicular to plane
$x + 2y + 2z = 5$ ---(iv)

Using equation (ii),(iv) in (iii),
$(a)(1) + (b)(2) + (c)(2) = 0$

$a + 2b + 2c = 0$ ---(v)

Also, plane (ii) is perpendicular to plane
$3x + 3y + 2z = 8$ ---(vi)

Using equation (ii),(vi) in (iii),
$(a)(3) + (b)(3) + (c)(2) = 0$

$3a + 3b + 2c = 0$ ---(vii)

Solving (v) and (vii) by cross multiplication,

$$\frac{a}{(2)(2) - (3)(2)} = \frac{b}{(3)(2) - (1)(2)} = \frac{c}{(1)(3) - (2)(3)}$$

$$\frac{a}{4 - 6} = \frac{b}{6 - 2} = \frac{c}{3 - 6}$$

$$\frac{a}{-2} = \frac{b}{4} = \frac{c}{-3} = \lambda \text{ (Say)}$$

$a = -2\lambda,\ b = 4\lambda,\ c = -3\lambda$

Put a, b, c in equation (ii)

$a(x - 1) + b(y + 3) + c(z + 2) = 0$
$(-2\lambda)(x - 1) + (4\lambda)(y + 3) + (-3\lambda)(z + 2) = 0$
$-2\lambda x + 2\lambda + 4\lambda y + 12\lambda - 3\lambda z - 6\lambda = 0$
$-2\lambda x + 4\lambda y - 3\lambda z + 8\lambda = 0$

Dividing by $(-\lambda)$,
$2x - 4y + 3z - 8 = 0$

Equation of required plane is,

$2x - 4y + 3z - 8 = 0$

The Plane 29.6 Q7

We know that equation of a plane passing through a point (x_1, y_1, z_1) is

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0 \quad ---(i)$

Given that, plane is passing through origin, so

$a(x - 0) + b(y - 0) + c(z - 0) = 0$

$ax + by + cz = 0 \quad ---(ii)$

We know that planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0 \quad ---(iii)$

Given that, plane (ii) is perpendicular to plane

$x + 2y - z = 1 \quad ---(iv)$

Using $(ii),(iv)$ in equation (iii),

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(a)(1) + (b)(2) + (c)(-1) = 0$

$a + 2b - c = 0 \quad ---(v)$

Given, plane (ii) is perpendicular to plane

$3x - 4y + z = 5 \quad ---(vi)$

Using equation $(ii),(vi)$ in (iii),

$(a)(3) + (b)(-4) + (c)(1) = 0$

$3a - 4b + c = 0 \quad ---(vii)$

Solving (v) and (vii) by cross multiplication,

$$\frac{a}{(2)(1) - (-4)(-1)} = \frac{b}{(3)(-1) - (1)(1)} = \frac{c}{(1)(-4) - (2)(3)}$$

$$\frac{a}{2 - 4} = \frac{b}{-3 - 1} = \frac{c}{-4 - 6}$$

$$\frac{a}{-2} = \frac{b}{-4} = \frac{c}{-10} = \lambda \text{ (Say)}$$

$a = -2\lambda,\ b = -4\lambda,\ c = -10\lambda$

Put a,b,c in equation (ii)

$ax + by + cz = 0$

$-2\lambda x - 4\lambda y - 10\lambda z = 0$

Dividing by -2λ,

$x + 2y + 5z = 0$

Equation of required plane is,

$x + 2y + 5z = 0$

The Plane 29.6 Q8

We know that equation of a plane passing through (x_1, y_1, z_1) is given by

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

Given that, plane is passing through $(1, -1, 2)$, so

$a(x - 1) + b(y + 1) + c(z - 2) = 0$ $\quad\quad ---(i)$

Plane (i) is also passing through $(2, -2, 2)$, so $(2, -2, 2)$ must satisfy the equation (i),

$a(2 - 1) + b(-2 + 1) + c(2 - 2) = 0$

$a - b = 0$ $\quad\quad ---(ii)$

We know that planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ $\quad\quad ---(iii)$

Given that, plane (i) is perpendicular to plane

$6x - 2y + 2z - 9 = 0$ $\quad\quad ---(iv)$

Using plane (i), (iv) in equation (iii),

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(a)(6) + (b)(-2) + (c)(2) = 0$

$6a - 2b + 2c = 0$ $\quad\quad ---(v)$

Solving (ii) and (v) by cross-multiplication,

$$\frac{a}{(-1)(2) - (-2)(0)} = \frac{b}{(6)(0) - (1)(2)} = \frac{c}{(1)(-2) - (6)(-1)}$$

$$\frac{a}{-2 + 0} = \frac{b}{0 - 2} = \frac{c}{-2 + 6}$$

$$\frac{a}{-2} = \frac{b}{-2} = \frac{c}{4} = \lambda \text{ (Say)}$$

$\Rightarrow \quad a = -2\lambda,\ b = -2\lambda,\ c = 4\lambda$

Put a, b, c in equation (i)

$a(x - 1) + b(y + 1) + c(z - 2) = 0$

$(-2\lambda)(x - 1) + (-2\lambda)(y + 1) + (4\lambda)(z - 2) = 0$

$-2\lambda x + 2\lambda - 2\lambda y - 2\lambda + 4\lambda z - 8\lambda = 0$

$-2\lambda x - 2\lambda y + 4\lambda z - 8\lambda = 0$

Dividing by (-2λ),

$x + y - 2z + 4 = 0$

Equation of required plane is,

$x + y - 2z + 4 = 0$

The Plane 29.6 Q9

We know that, equation of plane passing through the point (x_1, y_1, z_1) is given by,

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

Here, the plane is passing through $(2, 2, 1)$

$a(x - 2) + b(y - 2) + c(z - 1) = 0$ ---(i)

It is also passing through $(9, 3, 6)$, so it must satisfy the equation (i),

$a(9 - 2) + b(3 - 2) + c(6 - 1) = 0$

$7a + b + 5c = 0$ ---(ii)

We know that, plane $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ ---(iii)

Given that, plane (i) is perpendicular to plane

$2x + 6y + 6z = 1$ ---(iv)

Using plane (i), (iv) in equation (iii),

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(a)(2) + (b)(6) + (c)(6) = 0$

$2a + 6b + 6c = 0$ ---(v)

Solving (ii) and (v) by cross-multiplication,

$$\frac{a}{(1)(6) - (5)(6)} = \frac{b}{(2)(5) - (7)(6)} = \frac{c}{(7)(6) - (2)(1)}$$

$$\frac{a}{6 - 30} = \frac{b}{10 - 42} = \frac{c}{42 - 2}$$

$$\frac{a}{-24} = \frac{b}{-32} = \frac{c}{40} = \lambda \text{ (Say)}$$

$\Rightarrow \quad a = -24\lambda,\ b = -32\lambda,\ c = 40\lambda$

The Plane 29.6 Q10

We know that, equation of plane passing through the point (x_1, y_1, z_1) is given by,

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

Given, the required plane is passing through $(-1, 1, 1)$,

$a(x + 1) + b(y - 1) + c(z - 1) = 0$ ---(i)

It is also passing through $(1, -1, 1)$, so it must satisfy the equation (i),

$a(1 + 1) + b(-1 - 1) + c(1 - 1) = 0$

$2a - 2b = 0$ ---(ii)

We know that, plane $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ ---(iii)

Given, plane (i) is perpendicular to plane

$x + 2y + 2z = 5$ ---(iv)

Using plane (i), (iv) in equation (iii),

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(a)(1) + (b)(2) + (c)(2) = 0$

$a + 2b + 2c = 0$ ---(v)

Solving (ii) and (v) by cross-multiplication,

$$\frac{a}{(-2)(2) - (2)(0)} = \frac{b}{(1)(0) - (2)(2)} = \frac{c}{(2)(2) - (1)(-2)}$$

$$\frac{a}{-4 - 0} = \frac{b}{0 - 4} = \frac{c}{4 + 2}$$

$$\frac{a}{-4} = \frac{b}{-4} = \frac{c}{6} = \lambda \text{ (Say)}$$

$\Rightarrow \quad a = -4\lambda,\ b = -4\lambda,\ c = 6\lambda$

Put the value of a, b, c in equation (i),

$$a(x+1)+b(y-1)+c(z-1)=0$$
$$(-4\lambda)(x+1)+(-4\lambda)(y-1)+(6\lambda)(z-1)=0$$
$$-4\lambda x+4\lambda-4\lambda y+4\lambda+6\lambda z-6\lambda=0$$

$$-4\lambda x-4\lambda y+6\lambda z-6\lambda=0$$

Dividing by (-2λ), we get
$$2x+2y-3z+3=0$$

The equation of required plane is,

$$2x+2y-3z+3=0$$

The Plane Ex 29.6 Q11

The equation of the plane parallel to ZOX is $y=$ constant.
Given that the $y-$intercept is 3.
Thus the equation of the plane is $y=3$.

The Plane Ex 29.6 Q12

The equation of any plane passing through $(1,-1,2)$
is $a(x-1)+b(y+1)+c(z-2)=0$....(1)
Given that, plane (1) is perpendicular to the planes
$2x+3y-2z=5$
and
$x+2y-3z=8$
Therefore, we have,
$2a+3b-2c=0$...(2)
and
$a+2b-3c=0$...(3)
Solving equations (2) and (3) by cross multiplication, we have,
$$\frac{a}{3\times(-3)-2\times(-2)}=\frac{b}{1\times(-2)-2\times(-3)}=\frac{c}{2\times2-1\times3}=\lambda(\text{say})$$
$$\Rightarrow\frac{a}{-9+4}=\frac{b}{-2+6}=\frac{c}{4-3}=\lambda$$
$$\Rightarrow\frac{a}{-5}=\frac{b}{4}=\frac{c}{1}=\lambda$$
Thus, we have,
$a=-5\lambda, b=4\lambda$ and $c=\lambda$
Substituting the above values in equation (1), we have,
$-5\lambda(x-1)+4\lambda(y+1)+\lambda(z-2)=0$
Since $\lambda\neq0$, we have,
$-5(x-1)+4(y+1)+(z-2)=0$
$\Rightarrow -5x+5+4y+4+z-2=0$
$\Rightarrow -5x+4y+z+7=0$
$\Rightarrow 5x-4y-z-7=0$
$\Rightarrow 5x-4y-z=7$
Thus the required equation of the plane is $5x-4y-z=7$

The Plane Ex 29.6 Q13

Given that the equation of the required
plane is parallel to the plane
$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=2$...(1)
$\therefore$ Vector equation of any plane parallel to (1) is
$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=k$...(2)
Since the given plane passes through (a, b, c), then
$(a\hat{i}+b\hat{j}+c\hat{k})\cdot(\hat{i}+\hat{j}+\hat{k})=k$
$\Rightarrow a+b+c=k$...(3)

Substituting the above value of k in equation (2), we have,
$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=a+b+c$
Thus the required equation of the plane is $x+y+z=a+b+c$

The Plane Ex 29.6 Q14

The equation of any plane passing through $(-1,3,2)$
is$(x+1)+b(y-3)+c(z-2)=0$....(1)
Given that, Plane (1) is perpendicular to the planes
$x+2y+3z=5$
and
$3x+3y+z=0$
Therefore, we have,
$a+2b+3c=0$...(2)
and
$3a+3b+c=0$...(3)
Solving equations (2) and (3) by cross multiplication, we have,

$$\frac{a}{2\times 1-3\times 3}=\frac{b}{3\times 3-1\times 1}=\frac{c}{1\times 3-3\times 2}=\lambda(\text{say})$$

$$\Rightarrow \frac{a}{2-9}=\frac{b}{9-1}=\frac{c}{3-6}=\lambda$$

$$\Rightarrow \frac{a}{-7}=\frac{b}{8}=\frac{c}{-3}=\lambda$$

Thus, we have,
$a=-7\lambda, b=8\lambda$ and $c=-3\lambda$
Substituting the above values in equation (1), we have,
$-7\lambda(x+1)+8\lambda(y-3)-3\lambda(z-2)=0$
Since $\lambda\neq 0$, we have,
$-7(x+1)+8(y-3)-3(z-2)=0$
$\Rightarrow -7x-7+8y-24-3z+6=0$
$\Rightarrow -7x+8y-3z-25=0$
$\Rightarrow 7x-8y+3z+25=0$

Thus the required equation of the plane is $7x-8y+3z+25=0$

The Plane Ex 29.6 Q15

The equation of any plane passing through $(2,1,-1)$
is $a(x-2)+b(y-1)+c(z+1)=0$....(1)
Also, the above plane passes through the point $(-1,3,4)$.
Thus, equation (1), becomes,
$a(-1-2)+b(3-1)+c(4+1)=0$
$\Rightarrow -3a+2b+5c=0$...(2)
Given that, Plane (1) is perpendicular to the plane
$x-2y+4z=10$
Therefore, we have,
$a-2b+4c=0$...(3)

Solving equations (2) and (3) by cross multiplication, we have,

$$\frac{a}{2\times 4-5\times(-2)}=\frac{b}{1\times 5-(-3)\times 4}=\frac{c}{(-3)\times(-2)-1\times 2}=\lambda(\text{say})$$

$$\Rightarrow \frac{a}{8+10}=\frac{b}{5+12}=\frac{c}{6-2}=\lambda$$

$$\Rightarrow \frac{a}{18}=\frac{b}{17}=\frac{c}{4}=\lambda$$

Thus, we have,
$a=18\lambda, b=17\lambda$ and $c=4\lambda$
Substituting the above values in equation (1), we have,
$18\lambda(x-2)+17\lambda(y-1)+4\lambda(z+1)=0$
Since $\lambda\neq 0$, we have,
$18(x-2)+17(y-1)+4(z+1)=0$
$\Rightarrow 18x-36+17y-17+4z+4=0$
$\Rightarrow 18x+17y+4z-49=0$

Thus the required equation of the plane is $18x+17y+4z-49=0$

Ex - 29.7

The Plane 29.7 Q1(i)

Here, $\vec{r} = \left(2\hat{i} - \hat{k}\right) + \lambda\hat{i} + \mu\left(\hat{i} - 2\hat{j} - \hat{k}\right)$

We know that, $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ represent a plane passing through a point having position vector $\vec{a}$ and parallel to vectors $\vec{b}$ and $\vec{c}$.

Here, $\vec{a} = 2\hat{i} - \hat{k}$, $\vec{b} = \hat{i}$, $\vec{c} = \hat{i} - 2\hat{j} - \hat{k}$

The given plane is perpendicular to a vector

$$\vec{n} = \vec{b} \times \vec{c}$$
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ 1 & -2 & -1 \end{vmatrix}$$

$$= \hat{i}\,(0 - 0) - \hat{j}\,(-1 - 0) + \hat{k}\,(-2 - 0)$$
$$= 0\hat{i} + \hat{j} - 2\hat{k}$$

$$\vec{n} = \hat{j} - 2\hat{k}.$$

We know that vector equation of plane in scalar product form is,

$$\vec{r}.\vec{n} = \vec{a}.\vec{n} \qquad ---(i)$$

Put $\vec{n}$ and $\vec{a}$ in equation (i),

$$\vec{r}.\left(\hat{j} - 2\hat{k}\right) = \left(2\hat{i} - \hat{k}\right)\left(\hat{j} - 2\hat{k}\right)$$
$$\vec{r}.\left(\hat{j} - 2\hat{k}\right) = (2)(0) + (0)(1) + (-1)(-2)$$
$$= 0 + 0 + 2$$
$$\vec{r}.\left(\hat{j} - 2\hat{k}\right) = 2$$

The equation in required form is,

$$\vec{r}.\left(\hat{j} - 2\hat{k}\right) = 2$$

The Plane 29.7 Q1(ii)

Here, $\vec{r} = (1 + s - t)\,\hat{i} + (2 - s)\,\hat{j} + (3 - 2s + 2t)\,\hat{k}$

$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + s\left(\hat{i} - \hat{j} - 2\hat{k}\right) + t\left(-\hat{i} + 2\hat{k}\right)$$

We know that, $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ represent a plane passing through a point having position vector $\vec{a}$ and parallel to vectors $\vec{b}$ and $\vec{c}$.

Here, $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} - \hat{j} - 2\hat{k}$, $\vec{c} = -\hat{i} + 2\hat{k}$

The given plane is perpendicular to a vector

$$\vec{n} = \vec{b} \times \vec{c}$$
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -2 \\ -1 & 0 & 2 \end{vmatrix}$$

$$= \hat{i}\,(-2 - 0) - \hat{j}\,(2 - 2) + \hat{k}\,(0 - 1)$$

$$\vec{n} = -2\hat{i} - \hat{k}$$

We know that, vector equation of a plane in scalar product form is,

$$\vec{r}.\vec{n} = \vec{a}.\vec{n} \qquad ---(i)$$

Put value of $\vec{a}$ and $\vec{n}$ in equation (i),

$\vec{r}.\left(-2\hat{i}-\hat{k}\right)=\left(-2\hat{i}-\hat{k}\right)\left(\hat{i}+2\hat{j}+3\hat{k}\right)$

$\vec{r}.\left(-2\hat{i}-\hat{k}\right)=(-2)(1)+(0)(2)+(-1)(3)$

$=-2+0-3$

$\vec{r}.\left(-2\hat{i}-\hat{k}\right)=-5$

Multiplying both the sides by (-1),

$\vec{r}.\left(2\hat{i}+\hat{k}\right)=5$

The equation in the required form,

$\vec{r}.\left(2\hat{i}+\hat{k}\right)=5$

The Plane 29.7 Q1(iii)

Given, equation of plane,

$\vec{r}=\left(\hat{i}+\hat{j}\right)+\lambda\left(\hat{i}+2\hat{j}-\hat{k}\right)+\mu\left(-\hat{i}+\hat{j}-2\hat{k}\right)$

We know that, $\vec{r}=\vec{a}+\lambda\vec{b}+\mu\vec{c}$ is the equation of a plane passing through point $\vec{a}$ and parallel to $\vec{b}$ and $\vec{c}$.

Here, $\vec{a}=\hat{i}+\hat{j}$, $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$, $\vec{c}=-\hat{i}+\hat{j}-2\hat{k}$

The given plane is perpendicular to a vector

$\vec{n}=\vec{b}\times\vec{c}$

$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -1\\ -1 & 1 & -2\end{vmatrix}$

$=\hat{i}(-4+1)-\hat{j}(-2-1)+\hat{k}(1+2)$

$=-3\hat{i}+3\hat{j}+3\hat{k}$

We know that, the equation of plane in scalar product form is given by,
$\vec{r}.\vec{n}=\vec{a}\vec{n}$

$\vec{r}.\left(-3\hat{i}+3\hat{j}+3\hat{k}\right)=\left(\hat{i}+\hat{j}\right)\left(-3\hat{i}+3\hat{j}+3\hat{k}\right)$

$=(1)(-3)+(1)(3)+(0)(3)$

$=-3+3$

$\vec{r}.\left(-3\hat{i}+3\hat{j}+3\hat{k}\right)=0$

Dividing by 3, we get

$\vec{r}.\left(-\hat{i}+\hat{j}+\hat{k}\right)=0$

Equation in required form is,

$\vec{r}.\left(-\hat{i}+\hat{j}+\hat{k}\right)=0$

The Plane 29.7 Q1(iv)

$\bar{r} = \hat{i} - \hat{j} + \lambda(\hat{i} + \hat{j} + \hat{k}) + \mu(4i - 2\hat{j} + 3k)$

Plane is passing through $(\hat{i} - \hat{j})$and parallel to

$b(\hat{i} + \hat{j} + \hat{k})$ *and* $c\,(4i - 2\hat{j} + 3k)$

$$n = b \times c = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 4 & -2 & 3 \end{vmatrix}$$

$n = 5i + j - 6k$

$\bar{r}.n = (\hat{i} - \hat{j}).(5i + j - 6k)$=5-1=4

$\bar{r}.(5i + j - 6k) = 4$

The Plane 29.7 Q2(i)

Here, given equation of plane is,

$$\vec{r} = \left(\hat{i} - \hat{j}\right) + s\left(-\hat{i} + \hat{j} + 2\hat{k}\right) + t\left(\hat{i} + 2\hat{j} + \hat{k}\right)$$

We know that, $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ represents the equation of a plane passing through a vector $\vec{a}$ and parallel to vector $\vec{b}$ and $\vec{c}$.

Here, $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = -\hat{i} + \hat{j} + 2\hat{k}$, $\vec{c} = \hat{i} + 2\hat{j} + \hat{k}$

Given plane is perpendicular to vector

$$\vec{n} = \vec{b} \times \vec{c}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix}$$

$$= \hat{i}\,(1 - 4) - \hat{j}\,(-1 - 2) + \hat{k}\,(-2 - 1)$$

$$\vec{n} = -3\hat{i} + 3\hat{j} - 3\hat{k}$$

We know that, equation of plane in the scalar product form,

$$\vec{r}.\vec{n} = \vec{a}.\vec{n} \qquad ---(i)$$

Put the value of $\vec{a}$ and $\vec{n}$ in equation (i),

$$\vec{r}.\left(\hat{i} - \hat{j}\right) = \left(\hat{i} - \hat{j}\right)\left(-3\hat{i} + 3\hat{j} - 3\hat{k}\right)$$

$$\vec{r}.\left(-3\hat{i}+3\hat{j}-3\hat{k}\right)=(1)(-3)+(-1)(3)+(0)(-3)$$
$$=-3-3+0$$

$$\vec{r}.\left(-3\hat{i}+3\hat{j}-3\hat{k}\right)=-6$$

Put $\vec{r}.=x\hat{i}+y\hat{j}+z\hat{k}$

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(-3\hat{i}+3\hat{j}-3\hat{k}\right)=-6$$
$$(x)(-3)+(y)(3)+(z)(-3)=-6$$

$$-3x+3y-3z=-6$$

Dividing by (-3), we get

$$x-y+z=2$$

Equation in required form is,

$$x-y+z=2$$

The Plane 29.7 Q2(ii)

Given, equation of plane,

$$\vec{r}=(1+s+t)\hat{i}+(2-s+t)\hat{j}+(3-2s+2t)\hat{k}$$
$$=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+s\left(\hat{i}-\hat{j}-2\hat{k}\right)+t\left(\hat{i}+\hat{j}+2\hat{k}\right)$$

We know that, $\vec{r}=\vec{a}+\lambda\vec{b}+\mu\vec{c}$ represents the equation of a plane passing through the vector $\vec{a}$ and parallel to vector $\vec{b}$ and $\vec{c}$.

Here, $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$
$\vec{b}=\hat{i}-\hat{j}-2\hat{k}$
$\vec{c}=\hat{i}+\hat{j}+2\hat{k}$

The given plane is perpendicular to vector

$$\vec{n}=\vec{b}\times\vec{c}$$
$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & -2\\ 1 & 1 & 2\end{vmatrix}$$

$$=\hat{i}(-2+2)-\hat{j}(2+2)+\hat{k}(1+1)$$
$$=0.\left(\hat{i}\right)-4\hat{j}+2\hat{k}$$
$$\vec{n}=-4\hat{j}+2\hat{k}$$

We know that, equation of plane in scalar product form is given by,
$$\vec{r}.\vec{n}=\vec{a}.\vec{n} \qquad ---(i)$$

Put, the value of $\vec{a}$ and $\vec{n}$ in (i),

$$\vec{r}.\left(-4\hat{j}+2\hat{k}\right) = \left(\hat{i}+2\hat{j}+3\hat{k}\right)\left(-4\hat{j}+2\hat{k}\right)$$

$$\vec{r}.\left(-4\hat{j}+2\hat{k}\right) = (1)(0)+(2)(-4)+(3)(2)$$
$$= 0-8+6$$

$$\vec{r}.\left(-4\hat{j}+2\hat{k}\right) = -2$$

Put $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(-4\hat{j}+2\hat{k}\right) = -2$$
$$(x)(0)+(y)(-4)+(z)(2) = -2$$

$$-4y+2z = -2$$

Dividing by (-2), we get

$$2y - z = 1$$

The equation in required form is,

$$2y - z = 1$$

The Plane 29.7 Q3(i)

Given, equation of plane is,

$$\vec{r} = (\lambda-2\mu)\hat{i}+(3-\mu)\hat{j}+(2\lambda+\mu)\hat{k}$$
$$\vec{r} = \left(3\hat{j}\right)+\lambda\left(\hat{i}+2\hat{k}\right)+\mu\left(-2\hat{i}-\hat{j}+\hat{k}\right)$$

We know that, $\vec{r} = \vec{a}+\lambda\vec{b}+\mu\vec{c}$ represents the equation of a plane passing through a point $\vec{a}$ and parallel to vector $\vec{b}$ and $\vec{c}$.

Given, $\vec{a} = 3\hat{j}$
$\vec{b} = \hat{i}+2\hat{k}$
$\vec{c} = -2\hat{i}-\hat{j}+\hat{k}$

The given plane is perpendicular to

$$\vec{n} = \vec{b}\times\vec{c}$$
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 2 \\ -2 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}(0+2)-\hat{j}(1+4)+\hat{k}(-1-0)$$

$$\vec{n} = 2\hat{i}-5\hat{j}-\hat{k}$$

Vector equation of plane in non-parametric form is,

$$\vec{r}.\vec{n} = \vec{a}.\vec{n}$$

$$\vec{r}.\left(2\hat{i} - 5\hat{j} - \hat{k}\right) = \left(3\hat{j}\right)\left(2\hat{i} - 5\hat{j} - \hat{k}\right)$$

$$= (0)(2) + (3)(-5) + (0)(-1)$$

$$= 0 - 15 + 0$$

$$\vec{r}.\left(2\hat{i} - 5\hat{j} - \hat{k}\right) = -15$$

$$\vec{r}.\left(2\hat{i} - 5\hat{j} - \hat{k}\right) + 15 = 0$$

The required form of equation is,

$$\vec{r}.\left(2\hat{i} - 5\hat{j} - \hat{k}\right) + 15 = 0$$

The Plane 29.7 Q3(ii)

Given, equation of plane is,

$$\vec{r} = \left(2\hat{i} + 2\hat{j} - \hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \mu\left(5\hat{i} - 2\hat{j} + 7\hat{k}\right)$$

We know that, $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ represents the equation of a plane passing through a vector $\vec{a}$ and parallel to vector $\vec{b}$ and $\vec{c}$.

Here, $\vec{a} = 2\hat{i} + 2\hat{j} - \hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{c} = 5\hat{i} - 2\hat{j} + 7\hat{k}$

The given plane is perpendicular to vector

$$\vec{n} = \vec{b} \times \vec{c}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 5 & -2 & 7 \end{vmatrix}$$

$$= \hat{i}\,(14 + 6) - \hat{j}\,(7 - 15) + \hat{k}\,(-2 - 10)$$

$$\vec{n} = 20\hat{i} + 8\hat{j} - 12\hat{k}$$

We know that, equation of a plane in non-parametric form is given by,

$$\vec{r}.\vec{n} = \vec{a}.\vec{n}$$

$$\vec{r}.\left(20\hat{i} + 8\hat{j} - 12\hat{k}\right) = \left(2\hat{i} + 2\hat{j} - \hat{k}\right)\left(20\hat{i} + 8\hat{j} - 12\hat{k}\right)$$

$$= (2)(20) + (2)(8) + (-1)(-12)$$

$$= 40 + 16 + 12$$

$$\vec{r}.\left(20\hat{i} + 8\hat{j} - 12\hat{k}\right) = 68$$

Dividing by 4,

$$\vec{r}.\left(5\hat{i} + 2\hat{j} - 3\hat{k}\right) = 17$$

Equation of plane in required form is,

$$\vec{r}.\left(5\hat{i} + 2\hat{j} - 3\hat{k}\right) = 17$$

Ex - 29.8

The Plane 29.8 Q1

Given, equation of plane is

$2x - 3y + z = 0 \qquad ---(i)$

We know that equation of a plane parallel the plane (i) is given by

$2x - 3y + z + \lambda = 0 \qquad ---(ii)$

Given that, plane (ii) is passing through the point $(1, -1, 2)$ so it must satisfy the equation (ii),

$2(1) - 3(-1) + (2) + \lambda = 0$
$2 + 3 + 2 + \lambda = 0$
$7 + \lambda = 0$

$\lambda = -7$

Put the value of λ in equation (ii),

$2x - 3y + z - 7 = 0$

So, equation of the required plane is,

$2x - 3y + z = 7$

The Plane 29.8 Q2

Given, equation of plane is

$\vec{r}.\left(2\hat{i} - 3\hat{j} + 5\hat{k}\right) + 2 = 0 \qquad ---(i)$

We know that equation of a plane parallel to the plane (i) is given by

$\vec{r}.\left(2\hat{i} - 3\hat{j} + 5\hat{k}\right) + \lambda = 0 \qquad ---(ii)$

Given that, plane (ii) is passing through vector $\left(3\hat{i} + 4\hat{j} - \hat{k}\right)$ so it must satisfy equation (ii),

$\left(3\hat{i} + 4\hat{j} - \hat{k}\right)\left(2\hat{i} - 3\hat{j} + 5\hat{k}\right) + \lambda = 0$
$(3)(2) + (4)(-3) + (-1)(5) + \lambda = 0$
$6 - 12 - 5 + \lambda = 0$
$-11 + \lambda = 0$

$\lambda = 11$

Put the value of λ in equation (ii),

$\vec{r}.\left(2\hat{i} - 3\hat{j} + 5\hat{k}\right) + 11 = 0$

Equation of required plane is,

$\vec{r}.\left(2\hat{i} - 3\hat{j} + 5\hat{k}\right) + 11 = 0$

The Plane 29.8 Q3

We know that, equation of a plane passing through the line of intersection of two planes

$a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$\left(a_1x + b_1y + c_1z + d_1\right) + \lambda\left(a_2x + b_2y + c_2z + d_2\right) = 0$

Given, equations of plane is,

$2x - 7y + 4z - 3 = 0$ and
$3x - 5y + 4z + 11 = 0$

So, equation of plane passing through the line of intersection of given two planes is

$\left(2x - 7y + 4z - 3\right) + \lambda\left(3x - 5y + 4z + 11\right) = 0$
$2x - 7y + 4z - 3 + 3\lambda x - 5\lambda y + 4\lambda z + 11\lambda = 0$
$x\left(2 + 3\lambda\right) + y\left(-7 - 5\lambda\right) + z\left(4 + 4\lambda\right) - 3 + 11\lambda = 0 \qquad ---(i)$

Plane (1) is passing through the points $(-2,1,3)$, so it satisfies the equation (i),

$(-2)\left(2 + 3\lambda\right) + (1)\left(-7 - 5\lambda\right) + (3)\left(4 + 4\lambda\right) - 3 + 11\lambda = 0$
$-4 - 6\lambda - 7 - 5\lambda + 12 + 12\lambda - 3 + 11\lambda = 0$
$-2 + 12\lambda = 0$
$12\lambda = 2$
$\lambda = \frac{2}{12}$

$\lambda = \frac{1}{6}$

Put λ in equation (i),

$x\left(2 + 3\lambda\right) + y\left(-7 - 5\lambda\right) + z\left(4 + 4\lambda\right) - 3 + 11\lambda = 0$
$x\left(2 + \frac{3}{6}\right) + y\left(-7 - \frac{5}{6}\right) + z\left(4 + \frac{4}{6}\right) - 3 + \frac{11}{6} = 0$
$x\left(\frac{12 + 3}{6}\right) + y\left(\frac{-42 - 5}{6}\right) + z\left(\frac{24 + 4}{6}\right) - \frac{18 + 11}{6} = 0$

$\frac{15}{6}x - \frac{47}{6}y + \frac{28}{6}z - \frac{7}{6} = 0$

Multiplying by 6, we get

$15x - 47y + 28z - 7 = 0$

Therefore, equation of required plane is,

$15x - 47y + 28z = 7$

The Plane 29.8 Q4

We know that, equation of a plane passing the line of intersection of planes

$\vec{r}.\vec{n_1} = d_1$ and $\vec{r}.\vec{n_2} = d_2$ is given by
$\vec{r}.\left(\vec{n_1} + \lambda \vec{n_2}\right) = d_1 + \lambda d_2$

So, equation of plane through the line of intersection of planes $\vec{r}.\left(\hat{i} + 3\hat{j} - \hat{k}\right) = 0$

and $\vec{r}.\left(\hat{j} + 2\hat{k}\right) = 0$ is given by

$$\vec{r}.\left[\left(\hat{i} + 3\hat{j} - \hat{k}\right) + \lambda\left(\hat{j} + 2\hat{k}\right)\right] = 0 \qquad ---(i)$$

Given that plane (i) is passing through the point $\left(2\hat{i} + \hat{j} - \hat{k}\right)$, so

$$\left(2\hat{i} + \hat{j} - \hat{k}\right)\left(\hat{i} + 3\hat{j} - \hat{k}\right) + \lambda\left(2\hat{i} + \hat{j} - \hat{k}\right)\left(\hat{j} + 2\hat{k}\right) = 0$$
$$(2)(1) + (1)(3) + (-1)(-1) + \lambda\left[(2)(0) + (1)(1) + (-1)(2)\right] = 0$$
$$(2 + 3 + 1) + \lambda(1 - 2) = 0$$
$$6 - \lambda = 0$$

$$\lambda = 6$$

Put λ in equation (i),

$$\vec{r}.\left[\left(\hat{i} + 3\hat{j} - \hat{k}\right) + \lambda\left(\hat{j} + 2\hat{k}\right)\right] = 0$$
$$\vec{r}.\left[\hat{i} + 3\hat{j} - \hat{k} + 6\left(\hat{j} + 2\hat{k}\right)\right] = 0$$
$$\vec{r}.\left[\hat{i} + 3\hat{j} - \hat{k} + 6\hat{j} + 12\hat{k}\right] = 0$$

$$\vec{r}.\left(\hat{i} + 9\hat{j} + 11\hat{k}\right) = 0$$

So, equation of required plane is,

$$\vec{r}.\left(\hat{i} + 9\hat{j} + 11\hat{k}\right) = 0$$

The Plane 29.8 Q5

We know that, equation of a plane passing through the line of intersection of $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by
$(a_1x + b_1y + c_1z + d_1) + \lambda(a_2x + b_2y + c_2z + d_2) = 0$

So, equation of plane passing through the line of intersection of plane $2x - y = 0$ and $3z - y = 0$ is

$$(2x - y) + \lambda(3z - y) = 0$$
$$2x - y + 3\lambda z - \lambda y = 0$$
$$x(2) + y(-1 - \lambda) + z(3\lambda) = 0 \qquad ---(i)$$

We know that, two planes are perpendicular if

$$a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(ii)$$

Given, plane (i) is perpendicular to plane

$$4x + 5y - 3z = 8 \qquad ---(iii)$$

Using (i) and (iii) in equation (ii),

$$(2)(4) + (-1 - \lambda)(5) + (3\lambda)(-3) = 0$$
$$8 - 5 - 5\lambda - 9\lambda = 0$$
$$3 - 14\lambda = 0$$
$$-14\lambda = -3$$

$$\lambda = \frac{3}{14}$$

Put the value of λ in equation (i),

$$2x + y(-1 - \lambda) + z(3\lambda) = 0$$
$$2x + y\left(-1 - \frac{3}{14}\right) + z.3\left(\frac{3}{14}\right) = 0$$
$$2x + y\left(\frac{-14 - 3}{14}\right) + \frac{9z}{14} = 0$$
$$2x + y\left(-\frac{17}{14}\right) + \frac{9z}{14} = 0$$

Multiplying with 14, we get

$$28x - 17y + 9z = 0$$

Equation of required plane is,

$$28x - 17y + 9z = 0$$

The Plane 29.8 Q6

We know that, the equation plane passing through the line of intersection of plane $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\left(a_1x + b_1y + c_1z + d_1\right) + \lambda\left(a_2x + b_2y + c_2z + d_2\right) = 0$$

Here, equation of plane passing through the intersection of plane $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$ is given by,

$$\left(x + 2y + 3z - 4\right) + \lambda\left(2x + y - z + 5\right) = 0$$
$$x + 2y + 3z - 4 + 2\lambda x + \lambda y - \lambda z + 5\lambda = 0$$
$$x\left(1 + 2\lambda\right) + y\left(2 + \lambda\right) + z\left(3 - \lambda\right) - 4 + 5\lambda = 0 \qquad ---(i)$$

We know, that two planes are perpendicular if

$$a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(ii)$$

Given that plane (i) is perpendicular to plane,

$$5x + 3y - 6z + 8 = 0 \qquad ---(iii)$$

Using plane (i) and (iii) in equation (ii),

$$(5)(1 + 2\lambda) + (3)(2 + \lambda) + (-6)(3 - \lambda) = 0$$
$$5 + 10\lambda + 6 + 3\lambda - 18 + 6\lambda = 0$$
$$-7 + 19\lambda = 0$$
$$19\lambda = 7$$

$$\lambda = \frac{7}{19}$$

Put value of λ in equation (i),

$$x\left(1 + 2\lambda\right) + y\left(2 + \lambda\right) + z\left(3 - \lambda\right) - 4 + 5\lambda = 0$$
$$x\left(1 + \frac{14}{19}\right) + y\left(2 + \frac{7}{19}\right) + z\left(3 - \frac{7}{19}\right) - 4 + \frac{35}{19} = 0$$
$$x\left(\frac{19 + 14}{19}\right) + y\left(\frac{38 + 7}{19}\right) + z\left(\frac{57 - 7}{19}\right)\frac{-76 + 35}{19} = 0$$

$$x\left(\frac{33}{19}\right) + y\left(\frac{45}{19}\right) + z\left(\frac{50}{19}\right) - \frac{41}{19} = 0$$

Multiplying by 19, we get

$$33x + 45y + 50z - 41 = 0$$

Equation of required plane is,

$$33x + 45y + 50z - 41 = 0$$

The Plane 29.8 Q7

We know that, equation of a plane passing through the line of intersection of planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by,

$$\left(a_1x + b_1y + c_1z + d_1\right) + \lambda\left(a_2x + b_2y + c_2z + d_2\right) = 0$$

So, equation of plane passing through the line of intersection of planes $x + 2y + 3z + 4 = 0$ and $x - y + z + 3 = 0$ is

$$\left(x + 2y + 3z + 4\right) + \lambda\left(x - y + z + 3\right) = 0$$
$$x\left(1 + \lambda\right) + y\left(2 - \lambda\right) + z\left(3 + \lambda\right) + 4 + 3\lambda = 0 \qquad ---(\text{i})$$

Equation (i) is passing through origin, so

$$\left(0\right)\left(1 + \lambda\right) + \left(0\right)\left(2 - \lambda\right) + \left(0\right)\left(3 + \lambda\right) + 4 + 3\left(\lambda\right) = 0$$
$$0 + 0 + 0 + 4 + 3\lambda = 0$$
$$3\lambda = -4$$

$$\lambda = -\frac{4}{3}$$

Put the value of λ in equation (i),

$$x\left(1 + \lambda\right) + y\left(2 - \lambda\right) + z\left(3 + \lambda\right) + 4 + 3\lambda = 0$$
$$x\left(1 - \frac{4}{3}\right) + y\left(2 + \frac{4}{3}\right) + z\left(3 - \frac{4}{3}\right) + 4 - \frac{12}{3} = 0$$
$$x\left(\frac{3 - 4}{3}\right) + y\left(\frac{6 + 4}{3}\right) + z\left(\frac{9 - 4}{3}\right) + 4 - 4 = 0$$

$$-\frac{x}{3} + \frac{10y}{3} + \frac{5z}{3} = 0$$

Multiplying by 3, we get

$$-x + 10y + 5z = 0$$
$$x - 10y - 5z = 0$$

The equation of required plane is,

$$x - 10y - 5z = 0$$

The Plane 29.8 Q8

We know that equation of plane passing through the line of intersection of planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by,

$$\left(a_1x + b_1y + c_1z + d_1\right) + \lambda\left(a_2x + b_2y + c_2z + d_2\right) = 0$$

So, equation of plane passing through the line of intersection of planes $x - 3y + 2z - 5 = 0$ and $2x - y + 3z - 1 = 0$ is given by

$$\left(x - 3y + 2z - 5\right) + \lambda\left(2x - y + 3z - 1\right) = 0$$
$$x\left(1 + 2\lambda\right) + y\left(-3 - \lambda\right) + z\left(2 + 3\lambda\right) - 5 - \lambda = 0 \qquad ---\text{(i)}$$

Plane (i) is passing through the point $(1,-2,3)$ so,

$$(1)(1 + 2\lambda) + (-2)(-3 - \lambda) + (3)(2 + 3\lambda) - 5 - \lambda = 0$$
$$1 + 2\lambda + 6 + 2\lambda + 6 + 9\lambda - 5 - \lambda = 0$$
$$8 + 12\lambda = 0$$
$$12\lambda = -8$$
$$\lambda = -\frac{8}{12}$$
$$\lambda = -\frac{2}{3}$$

Put the value of λ in equation (i),

$$x\left(1 + 2\lambda\right) + y\left(-3 - \lambda\right) + z\left(2 + 3\lambda\right) - 5 - \lambda = 0$$
$$x\left(1 - \frac{4}{3}\right) + y\left(-3 + \frac{2}{3}\right) + z\left(2 - \frac{6}{3}\right) - 5 + \frac{2}{3} = 0$$
$$x\left(\frac{3 - 4}{3}\right) + y\left(\frac{-9 + 2}{3}\right) + z\left(\frac{6 - 6}{3}\right)\frac{-15 + 2}{3} = 0$$

$$-\frac{1}{3}x - \frac{7}{3}y + z(0) - \frac{13}{3} = 0$$

Multiplying by (-3),

$$x + 7y + 13 = 0$$
$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\hat{i} + 7\hat{j}\right) + 13 = 0$$

$$\vec{r}\left(\hat{i} + 7\hat{j}\right) + 13 = 0$$

Equation of required plane is,

$$\vec{r}\left(\hat{i} + 7\hat{j}\right) + 13 = 0$$

The Plane 29.8 Q9

We know that, equation of plane passing through the line of intersection of planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\left(a_1x + b_1y + c_1z + d_1\right) + \lambda\left(a_2x + b_2y + c_2z + d_2\right) = 0$$

So, equation of plane passing through the line of intersection of planes is $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$ is given by,

$$\left(x + 2y + 3z - 4\right) + \lambda\left(2x + y - z + 5\right) = 0$$
$$x\left(1 + 2\lambda\right) + y\left(2 + \lambda\right) + z\left(3 - \lambda\right) - 4 + 5\lambda = 0 \qquad ---(i)$$

We know that two planes are perpendicular if
$$a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(ii)$$

Given that plane (i) is perpendicular to plane,

$$5x + 3y + 6z + 8 = 0 \qquad ---(iii)$$

Using (i) and (iii) in equation (ii),
$$(5)\left(1 + 2\lambda\right) + (3)\left(2 + \lambda\right) + (6)\left(3 - \lambda\right) = 0$$
$$5 + 10\lambda + 6 + 3\lambda + 18 - 6\lambda = 0$$
$$29 + 7\lambda = 0$$
$$7\lambda = -29$$

$$\lambda = -\frac{29}{7}$$

Put the value of λ in equation (i),
$$x\left(1 + 2\lambda\right) + y\left(2 + \lambda\right) + z\left(3 - \lambda\right) - 4 + 5\lambda = 0$$
$$x\left(1 - \frac{58}{7}\right) + y\left(2 - \frac{29}{7}\right) + z\left(3 + \frac{29}{7}\right) - 4 - \frac{145}{7} = 0$$
$$x\left(\frac{7 - 58}{7}\right) + y\left(\frac{14 - 29}{7}\right) + z\left(\frac{21 + 29}{7}\right)\frac{-28 - 145}{7} = 0$$

$$x\left(-\frac{51}{7}\right) + y\left(-\frac{15}{7}\right) + z\left(\frac{50}{7}\right) - \frac{173}{7} = 0$$

The Plane 29.8 Q10

$$\vec{r}.(\hat{i} + 3\hat{j}) + 6 = 0 \text{ and } \vec{r}.(3\hat{i} - \hat{j} - 4\hat{k}) = 0$$
$$x + 3y + 6 = 0;\ 3x - y - 4z = 0$$
$$x + 3y + 6 + \lambda(3x - y - 4z) = 0$$
$$x(1 + 3\lambda) + y(3 - \lambda) + -4z\lambda + 6 = 0$$
$$\text{Distance from origin to plane} = \left|\frac{6}{\sqrt{(1 + 3\lambda)^2 + (3 - \lambda)^2 + (4\lambda)^2}}\right| = 1$$
$$36 = (1 + 3\lambda)^2 + (3 - \lambda)^2 + (4\lambda)^2$$
$$36 = 1 + 6\lambda + 9\lambda^2 + 9 - 6\lambda + \lambda^2 + 16\lambda^2$$
$$26 = 26\lambda^2$$
$$\lambda^2 = 1$$
$$\lambda = \pm 1$$

$$Case: 1\ \lambda = 1$$
$$x + 3y + 6 + 1(3x - y - 4z) = 0$$
$$4x + 2y - 4z + 6 = 0$$

$$Case: 2\ \lambda = -1$$
$$x + 3y + 6 - 1(3x - y - 4z) = 0$$
$$2x - 4y - 4z - 6 = 0$$

The Plane 29.8 Q11

We know that equation of a plane passing through the line of intersection of two planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\left(a_1x + b_1y + c_1z + d_1\right) + \lambda\left(a_2x + b_2y + c_2z + d_2\right) = 0$$

So, equation of plane passing through the planes $2x + 3y - z + 1 = 0$ and $x + y - 2z + 3 = 0$ is

$$\left(2x + 3y - z + 1\right) + \lambda\left(x + y - 2z + 3\right) = 0$$
$$x\left(2 + \lambda\right) + y\left(3 + \lambda\right) + z\left(-1 - 2\lambda\right) + 1 + 3\lambda = 0 \qquad ---(i)$$

We know that two planes are perpendicular if
$$a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(ii)$$

Given, plane (i) is perpendicular to the plane,

$$3x - y - 2z - 4 = 0 \qquad ---(iii)$$

Using (i) and (iii) in equation (ii),
$$(3)(2 + \lambda) + (-1)(3 + \lambda) + (-2)(-1 - 2\lambda) = 0$$
$$6 + 3\lambda - 3 - \lambda + 2 + 4\lambda = 0$$
$$6\lambda + 5 = 0$$
$$6\lambda = -5$$

$$\lambda = -\frac{5}{6}$$

Put the value of λ in equation (i),
$$x\left(2 + \lambda\right) + y\left(3 + \lambda\right) + z\left(-1 - 2\lambda\right) + 1 + 3\lambda = 0$$
$$x\left(2 - \frac{5}{6}\right) + y\left(3 - \frac{5}{6}\right) + z\left(-1 + \frac{10}{6}\right) + 1 - \frac{15}{6} = 0$$
$$x\left(\frac{12 - 5}{6}\right) + y\left(\frac{18 - 5}{6}\right) + z\left(\frac{-6 + 10}{6}\right) + \frac{6 - 15}{6} = 0$$

$$\frac{7x}{6} + \frac{13y}{6} + \frac{4z}{6} - \frac{9}{6} = 0$$

The Plane 29.8 Q12

We know that, equation of a plane passing through the line of intersection of plane

$\vec{r}.\vec{n_1} - d_1 = 0$ and $\vec{r}.\vec{n_2} - d_2 = 0$ is

$\left(\vec{r}.\vec{n_1} - d_1\right) + \lambda\left(\vec{r}.\vec{n_2} - d_2\right) = 0$

So, equation of plane passing through the line of intersection of plane $\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - 4 = 0$ and $\vec{r}.\left(2\hat{i} + \hat{j} - \hat{k}\right) + 5 = 0$ is given by

$$\left[\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - 4\right] + \lambda\left[\vec{r}.\left(2\hat{i} + \hat{j} - \hat{k}\right) + 5\right] = 0$$

$$\vec{r}.\left[\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + \hat{j} - \hat{k}\right)\right] - 4 + 5\lambda = 0 \qquad ---(i)$$

We know that two planes are perpendicular if

$$\vec{n_1}.\vec{n_2} = 0 \qquad ---(ii)$$

Given that plane (i) is perpendicular to plane

$$\vec{r}.\left(5\hat{i} + 3\hat{j} - 6\hat{k}\right) + 8 = 0 \qquad ---(iii)$$

Using (i) and (iii) in equation (ii),

$$\left[\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + \hat{j} - \hat{k}\right)\right]\left(5\hat{i} + 3\hat{j} - 6\hat{k}\right) = 0$$

$$\left[\hat{i}\,(1 + 2\lambda) + \hat{j}\,(2 + \lambda) + \hat{k}\,(3 - \lambda)\right]\left(5\hat{i} + 3\hat{j} - 6\hat{k}\right) = 0$$

$$(1 + 2\lambda)(5) + (2 + \lambda)(3) + (3 - \lambda)(-6) = 0$$

$$5 + 10\lambda + 6 + 3\lambda - 18 + 6\lambda = 0$$

$$19\lambda - 7 = 0$$

$$\lambda = \frac{7}{19}$$

Put value of λ in equation (i),

$$\vec{r}.\left[\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + \hat{j} - \hat{k}\right)\right] - 4 + 5\lambda = 0$$

$$\vec{r}\left[\hat{i} + 2\hat{j} + 3\hat{k} + \frac{14}{19}\hat{i} + \frac{7}{19}\hat{j} - \frac{7}{19}\hat{k}\right] - 4 + 5\left(\frac{7}{19}\right) = 0$$

$$\vec{r}\left[\frac{33\hat{i}}{19} + \frac{45\hat{j}}{19} - \frac{50\hat{k}}{19}\right]\frac{-76 + 35}{19} = 0$$

$$\vec{r}\left(\frac{33\hat{i} + 45\hat{j} + 50\hat{k}}{19}\right) - \frac{41}{19} = 0$$

Multiplying by 19,

$$\vec{r}\left(33\hat{i} + 45\hat{j} + 50\hat{k}\right) - 41 = 0$$

Equation of required plane is,

$$\vec{r}\left(33\hat{i} + 45\hat{j} + 50\hat{k}\right) - 41 = 0$$

$$33x + 45y + 50z - 41 = 0$$

The Plane 29.8 Q13

The equation of a plane passing through the intersection of
$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k}) = 6$ and $\vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k}) = -5$ is
$[\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-6]+\lambda[\vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k})+5] = 0$
$\Rightarrow \vec{r}\cdot[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1+4\lambda)\hat{k}] = (6-5\lambda)$...(1)
$\Rightarrow [x\hat{i}+y\hat{j}+z\hat{k}]\cdot[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1+4\lambda)\hat{k}] = (6-5\lambda)$
$\Rightarrow [x(1+2\lambda)+y(1+3\lambda)+z(1+4\lambda)] = (6-5\lambda)$...(2)
The requried plane also passes through the point(1, 1, 1).
Substiuting $x = 1, y = 1, z = 1$ in equation (2), we have,
$1\times(1+2\lambda)+1\times(1+3\lambda)+1\times(1+4\lambda) = (6-5\lambda)$
$\Rightarrow 1+2\lambda+1+3\lambda+1+4\lambda = 6-5\lambda$
$\Rightarrow 3+9\lambda = 6-5\lambda$
$\Rightarrow 14\lambda = 6-3$
$\Rightarrow 14\lambda = 3$
$\Rightarrow \lambda = \frac{3}{14}$

Substituting the value $\lambda = \frac{3}{14}$ in equation (1), we have,
$$\vec{r}\cdot\left[\left(1+2\left(\frac{3}{14}\right)\right)\hat{i}+\left(1+3\left(\frac{3}{14}\right)\right)\hat{j}+\left(1+4\left(\frac{3}{14}\right)\right)\hat{k}\right] = \left(6-5\left(\frac{3}{14}\right)\right)$$
$$\Rightarrow \vec{r}\cdot\left[\frac{20}{14}\hat{i}+\frac{23}{14}\hat{j}+\frac{26}{14}\hat{k}\right] = \frac{69}{14}$$
$$\Rightarrow \vec{r}\cdot[20\hat{i}+23\hat{j}+26\hat{k}] = 69$$

The Plane 29.8 Q14

We know that, equation of the plane passing through the line of intersection of planes

$\vec{r}.\overrightarrow{n_1} - d_1 = 0$ and $\vec{r}.\overrightarrow{n_2} - d_2 = 0$ is

$\left(\vec{r}.\overrightarrow{n_1} - d_1\right) + \lambda\left(\vec{r}.\overrightarrow{n_2} - d_2\right) = 0$

So, equation of plane passing through the line of intersection of plane $\vec{r}.\left(2\hat{i} + \hat{j} + 3\hat{k}\right) - 7 = 0$ and $\vec{r}.\left(2\hat{i} + 5\hat{j} + 3\hat{k}\right) - 9 = 0$ is given by

$$\left[\vec{r}.\left(2\hat{i} + \hat{j} + 3\hat{k}\right) - 7\right] + \lambda\left[\vec{r}.\left(2\hat{i} + 5\hat{j} + 3\hat{k}\right) - 9\right] = 0$$

$$\vec{r}\left[\left(2\hat{i} + \hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 5\hat{j} + 3\hat{k}\right)\right] - 7 - 9\lambda = 0$$

$$\vec{r}\left[(2 + 2\lambda)\hat{i} + (1 + 5\lambda)\hat{j} + (3 + 3\lambda)\hat{k}\right] - 7 - 9\lambda = 0 \qquad ---(i)$$

Given that plane (i) is passing through

$\left(2\hat{i} + \hat{j} + 3\hat{k}\right)$, so

$$\left(2\hat{i} + \hat{j} + 3\hat{k}\right)\left[(2 + 2\lambda)\hat{i} + (1 + 5\lambda)\hat{j} + (3 + 3\lambda)\hat{k}\right] - 7 - 9\lambda = 0$$

$$(2)(2 + 2\lambda) + (1)(1 + 5\lambda) + (3)(3 + 3\lambda) - 7 - 9\lambda = 0$$

$$4 + 4\lambda + 1 + 5\lambda + 9 + 9\lambda - 7 - 9\lambda = 0$$

$$9\lambda + 7 = 0$$

$$9\lambda = -7$$

$$\lambda = -\frac{7}{9}$$

Put value of λ in equation (i),

$$\vec{r}.\left[(2 + 2\lambda)\hat{i} + (1 + 5\lambda)\hat{j} + (3 + 3\lambda)\hat{k}\right] - 7 - 9\lambda = 0$$

$$\vec{r}.\left[\left(2 + \frac{14}{9}\right)\hat{i} + \left(1 - \frac{35}{9}\right)\hat{j} + \left(3 - \frac{21}{9}\right)\hat{k}\right] - 7 + \frac{63}{9} = 0$$

$$\vec{r}.\left[\left(\frac{18 - 14}{9}\right)\hat{i} + \left(\frac{9 - 35}{9}\right)\hat{j} + \left(\frac{27 - 21}{9}\right)\hat{k}\right] - 7 + 7 = 0$$

$$\vec{r}.\left[\left(\frac{4}{9}\right)\hat{i} - \frac{26}{9}\hat{j} + \frac{6\hat{k}}{9}\right] + 0 = 0$$

$$\vec{r}.\left[\frac{4}{9}\hat{i} - \frac{26}{9}\hat{j} + \frac{6}{9}\hat{k}\right] = 0$$

Multiplying by $\left(\frac{9}{2}\right)$, we get

$$\vec{r}\left[2\hat{i} - 13\hat{j} + 3\hat{k}\right] = 0$$

Equation of required plane is,

$$\vec{r}.\left(2\hat{i} - 13\hat{j} + 3\hat{k}\right) = 0$$

The Plane 29.8 Q15

The equation of the family of planes through the intersection of planes
$3x - y + 2z = 4$ and $x + y + z = 2$ is,
$(3x - y + 2z - 4) + \lambda(x + y + z - 2) = 0$..........(i)

If it passes through $(2, 2, 1)$, then
$(6 - 2 + 2 - 4) + \lambda(2 + 2 + 1 - 2) = 0$
$\Rightarrow \lambda = -\frac{2}{3}$

Substituting $\lambda = -\frac{2}{3}$ in (i) we get, $7x - 5y + 4z = 0$ as the equation of the required plane.

The Plane 29.8 Q16

The equation of the family of planes through the line of intersection of planes
$x + y + z = 1$ and $2x + 3y + 4z = 5$ is,
$(x + y + z - 1) + \lambda(2x + 3y + 4z - 5) = 0$..........(i)
$(2\lambda + 1)x + (3\lambda + 1)y + (4\lambda + 1)z = 5\lambda + 1$

It is perpendicular to the plane $x - y + z = 0$.
$\therefore (2\lambda + 1)(1) + (3\lambda + 1)(-1) + (4\lambda + 1)(1) = 5\lambda + 1$
$\Rightarrow 2\lambda + 1 - 3\lambda - 1 + 4\lambda + 1 = 5\lambda + 1$
$\Rightarrow \lambda = -\frac{1}{3}$

Substituting $\lambda = -\frac{1}{3}$ in (i), we get, $x - z + 2 = 0$ as the equation of the required plane
and its vector equation is $\vec{r}\cdot(\hat{i} - \hat{k}) + 2 = 0$.

The equation of the family of planes parallel to $\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = 2$ is,
$\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = d$........(i)

If it passes through (a, b, c) then
$(a\hat{i} + b\hat{j} + c\hat{k})\cdot(\hat{i} + \hat{j} + \hat{k}) = d$
$\Rightarrow a + b + c = d$

Substituting $a + b + c = d$ in (i), we get,
$\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = a + b + c$

$x + y + z = a + b + c$ as the equation of the required plane.

Ex 29.9

The Plane Ex 29.9 Q1

We know that distance of a point $\vec{a}$ from a plane $\vec{r}.\vec{n} - d = 0$ is given by

$$D = \left|\frac{\vec{a}\vec{n} - d}{|\vec{n}|}\right| \text{ unit}$$

Here, $\vec{a} = 2\hat{i} - \hat{j} - 4\hat{k}$ and
plane $\vec{r}.\left(3\hat{i} - 4\hat{j} + 12\hat{k}\right) - 9 = 0$

$\vec{r}.\vec{n} - d = 0$

So, required distance

$$D = \left|\frac{\left(2\hat{i} - \hat{j} - 4\hat{k}\right)\left(3\hat{i} - 4\hat{j} + 12\hat{k}\right) - 9}{\sqrt{(3)^2 + (-4)^2 + (12)^2}}\right|$$
$$= \left|\frac{(2)(3) + (-1)(-4) + (-4)(12) - 9}{\sqrt{9 + 16 + 144}}\right|$$
$$= \left|\frac{6 + 4 - 48 - 9}{\sqrt{169}}\right|$$
$$= \left|-\frac{47}{13}\right|$$

$$= \frac{47}{13} \text{ units}$$

Required distance is $\frac{47}{13}$ units

The Plane Ex 29.9 Q2

We know that, distance of a point $\vec{a}$ to a plane $\vec{r}.\vec{n} - d = 0$ is given by

$$D = \left|\frac{\vec{a}\vec{n} - d}{|\vec{n}|}\right| \qquad ---(i)$$

Let D_1 be the distance of point $\left(\hat{i} - \hat{j} + 3\hat{k}\right)$

from the plane $\vec{r}.\left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) + 9 = 0$, then

$$D_1 = \left|\frac{\left(\hat{i} - \hat{j} + 3\hat{k}\right)\left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) + 9}{\sqrt{(5)^2 + (2)^2 + (-7)^2}}\right| \qquad \left[\text{Using equation (i)}\right]$$
$$= \left|\frac{(1)(5) + (-1)(2) + (3)(-7) + 9}{\sqrt{25 + 4 + 49}}\right|$$
$$= \left|\frac{5 - 2 - 21 + 9}{\sqrt{78}}\right|$$
$$= \left|-\frac{9}{\sqrt{78}}\right|$$

$$D_1 = \frac{9}{\sqrt{78}} \text{ units} \qquad ---(ii)$$

Again, let D_2 be the distance of point $\left(3\hat{i} + 3\hat{j} + 3\hat{k}\right)$ from the plane $\vec{r}.\left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) + 9 = 0$, then, using equation (i), we get

$$D_2 = \left|\frac{\left(3\hat{i} + 3\hat{j} + 3\hat{k}\right)\left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) + 9}{\sqrt{(5)^2 + (2)^2 + (-7)^2}}\right|$$
$$= \left|\frac{(3)(5) + (3)(2) + (3)(-7) + 9}{\sqrt{25 + 4 + 49}}\right|$$
$$= \left|\frac{15 + 6 - 21 + 9}{\sqrt{78}}\right|$$
$$= \left|\frac{9}{\sqrt{78}}\right|$$

$$= \frac{9}{\sqrt{78}} \text{ units} \qquad ---(iii)$$

From equation (ii) and (iii)
$D_1 = D_2$

Distance of point $\left(\hat{i} - \hat{j} + 3\hat{k}\right)$ from plane $\vec{r}.\left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) + 9 = 0$
= Distance of point $\left(3\hat{i} + 3\hat{j} + 3\hat{k}\right)$ from plane $\vec{r}.\left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) + 9 = 0$

The Plane Ex 29.9 Q3

We know that, distance of a point (x_1, y_1, z_1) from a plane $ax + by + cz + d = 0$ is given by

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right| \qquad ---(i)$$

So, distance of point $(2, 3, -5)$ from the plane $x + 2y - 2z - 9 = 0$ is given by

$$D = \left|\frac{2 + (2)(3) - 2(-5) - 9}{\sqrt{(1)^2 + (2)^2 + (-2)^2}}\right| \qquad \text{[Using equation (i)]}$$

$$= \left|\frac{2 + 6 + 10 - 9}{\sqrt{1 + 4 + 4}}\right|$$

$$= \left|\frac{9}{\sqrt{9}}\right|$$

$$= \left|\frac{9}{3}\right|$$

$D = 3$ units

The Plane Ex 29.9 Q4

Given equation of plane is

$$x + 2y - 2z + 8 = 0 \qquad ---(i)$$

We know that, equation of the plane parallel to plane (i) is given by

$$x + 2y - 2z + \lambda = 0 \qquad ---(ii)$$

We know that, distance (D) of a point (x_1, y_1, z_1) from a plane $ax + by + cz + d = 0$
is given by

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right| \qquad ---(iii)$$

Given, $D = 2$ unit is the distance of the plane (ii) from the point $(2, 1, 1)$, so
Using (i),

$$2 = \left|\frac{2 + (2)(1) - 2(1) + \lambda}{\sqrt{(1)^2 + (2)^2 + (-2)^2}}\right|$$

$$2 = \left|\frac{2 + 2 - 2 + \lambda}{\sqrt{1 + 4 + 4}}\right|$$

$$2 = \left|\frac{2 + \lambda}{\sqrt{9}}\right|$$

Squaring both the sides, we get

$$4 = \frac{(2 + \lambda)^2}{9}$$

$$36 = (2 + \lambda)^2$$

$$2 + \lambda = \pm 6$$

$$\Rightarrow \quad 2 + \lambda = 6 \quad or \quad 2 + \lambda = -6$$

$$\Rightarrow \quad \lambda = 4 \quad or \quad \lambda = -8$$

Put $\lambda = 4$ in equation (ii),
$x + 2y - 2z + 4 = 0$

Put $\lambda = -8$ in equation (ii),
$x + 2y - 2z - 8 = 0$

Hence, equation of the required plane are
$x + 2y - 2z + 4 = 0$
$x + 2y - 2z - 8 = 0$

The Plane Ex 29.9 Q5

We know that distance (D) of a point (x_1, y_1, z_1) from a plane $ax + by + cz + d = 0$ is given by

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{(a)^2 + (b)^2 + (c)^2}}\right| \qquad ---(i)$$

Let D_1 be the distance of the point $(1,1,1)$ from plane $3x + 4y - 12z + 13 = 0$, so using (i), we get

$$D_1 = \left|\frac{(3)(1) + (4)(1) - 12(1) + 13}{\sqrt{(3)^2 + (4)^2 + (-12)^2}}\right|$$

$$= \left|\frac{3 + 4 - 12 + 13}{\sqrt{9 + 16 + 144}}\right|$$

$$= \left|\frac{8}{\sqrt{169}}\right|$$

$$D = \frac{8}{13} \text{ units} \qquad ---(ii)$$

Let D_2 be the distane of a point $(-3,0,1)$ from the plane $3x + 4y - 12z + 13 = 0$, so using equation (i),

$$D_2 = \left|\frac{(3)(-3) + (4)(0) - 12(1) + 13}{\sqrt{(3)^2 + (4)^2 + (-12)^2}}\right|$$

$$= \left|\frac{-9 + 0 - 12 + 13}{\sqrt{9 + 4 + 144}}\right|$$

$$= \left|-\frac{8}{\sqrt{169}}\right|$$

$$D_2 = \frac{8}{13} \text{ units} \qquad ---(iii)$$

Hence, from equation (ii) and (iii)

$D_1 = D_2$

The Plane Ex 29.9 Q6

Given equation of plane is

$x - 2y + 2z - 3 = 0 \qquad ---(i)$

We know that, equation of a plane parallel to plane (i) is given by,

$x - 2y + 2z + \lambda = 0 \qquad ---(ii)$

We know that distance (D) of a point (x_1, y_1, z_1) from a plane $ax + by + cz + d = 0$ is given by,

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{(a)^2 + b^2 + c^2}}\right| \qquad ---(iii)$$

Given that, distance of plane (ii) from a point $(1, 1, 1)$ is one unit, so using (iii),

$$1 = \left|\frac{(1) - 2(1) + 2(1) + \lambda}{\sqrt{(1)^2 + (-2)^2 + (2)^2}}\right|$$

$$= \left|\frac{1 - 2 + 2 + \lambda}{\sqrt{1 + 4 + 4}}\right|$$

$$1 = \left|\frac{1 + \lambda}{\sqrt{9}}\right|$$

$$1 = \left|\frac{1 + \lambda}{3}\right|$$

Squaring both the sides,

$$1 = \frac{(1 + \lambda)^2}{9}$$

$$9 = (1 + \lambda)^2$$

$$1 + \lambda = \pm 3$$

$\Rightarrow \quad 1 + \lambda = 3 \quad \text{or} \quad 1 + \lambda = -3$

$\Rightarrow \quad \lambda = 2 \quad \text{or} \quad \lambda = -4$

Put the value of λ in equation (ii) to get the equations of required planes,

$x - 2y + 2z + 2 = 0$

$x - 2y + 2z - 4 = 0$

The Plane Ex 29.9 Q7

We know that, distance (D) of a point (x_1, y_1, z_1) from a plane $ax + by + cz + d = 0$ is given by,

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right| \qquad ---(i)$$

So, distance of point $(2, 3, 5)$ from xy-plane (we know that equation of xy-plane is $z = 0$) is

$$= \left|\frac{(2)(0) + (3)(0) + (5)(1) + 0}{\sqrt{(0)^2 + (0)^2 + (1)^2}}\right| \qquad [\text{Using } (i)]$$

$$= \left|\frac{0 + 0 + 5}{\sqrt{0 + 0 + 1}}\right|$$

$= 5$ unit

Distance of the point $(2, 3, 5)$ from xy-plane $= 5$ unit

The Plane Ex 29.9 Q8

We know that, distance (D) of a point $\vec{a}$ from a plane $\vec{r}.\vec{n} - d = 0$ is given by,

$$D = \left|\frac{\vec{a}.\vec{n} - d}{|\vec{n}|}\right| \qquad ---(i)$$

So, distance of point $\left(3\hat{i} + 3\hat{j} + 3\hat{k}\right)$ from plane $\vec{r}.\left(5\hat{i} + 2\hat{j} + 3\hat{k}\right) + 9 = 0$ is

$$D = \left|\frac{\left(3\hat{i} + 3\hat{j} + 3\hat{k}\right)\left(5\hat{i} + 2\hat{j} + 3\hat{k}\right) + 9}{\sqrt{(5)^2 + (2)^2 + (-7)^2}}\right|$$

$$= \left|\frac{(3)(5) + (3)(2) + (3)(-7) + 9}{\sqrt{25 + 4 + 49}}\right|$$

$$= \left|\frac{15 + 6 - 21 + 9}{\sqrt{78}}\right|$$

$$= \left|\frac{9}{\sqrt{78}}\right|$$

Therefore, required distance is

$$= \frac{9}{\sqrt{78}} \text{ units}$$

The Plane Ex 29.9 Q9

Distance of point $(1,1,1)$ from origin is $\sqrt{3}$

Distance of point $(1,1,1)$ from plane is $\left|\frac{1+\lambda}{\sqrt{3}}\right|$

$$\text{Product} = \left|\frac{1+\lambda}{\sqrt{3}}\right| \times \sqrt{3} = 5$$

$$|1 + \lambda| = 5$$

so λ=4 or -6

The Plane Ex 29.9 Q10

Consider

$$3x-4y+12z-6=0 \quad \ldots\ldots (1)$$

$$4x+3z-7=0 \quad \ldots\ldots (2)$$

The distance of a point (x_1, y_1, z_1) from the plane $3x-4y+12z-6=0$ is

$$D_1=\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$$

$$=\left|\frac{3x_1-4y_1+12z_1-6}{\sqrt{3^2+(-4)^2+12^2}}\right|$$

$$=\left|\frac{3x_1-4y_1+12z_1-6}{\sqrt{169}}\right|$$

$$=\left|\frac{3x_1-4y_1+12z_1-6}{13}\right|$$

The distance of the point (x_1, y_1, z_1) from the plane $4x+3z-7=0$ is

$$D_2=\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$$

$$=\left|\frac{4x_1+3z_1-7}{\sqrt{4^2+3^2}}\right|$$

$$=\left|\frac{4x_1+3z_1-7}{\sqrt{25}}\right|$$

$$=\left|\frac{4x_1+3z_1-7}{5}\right|$$

Since the point (x_1, y_1, z_1) are equidistant from the planes $3x-4y+12z-6=0$ and $4x+3z-7=0$

So

$$D_1=D_2$$

$$\left|\frac{3x_1-4y_1+12z_1-6}{13}\right|=\left|\frac{4x_1+3z_1-7}{5}\right|$$

$$\frac{3x_1-4y_1+12z_1-6}{13}=\pm\frac{4x_1+3z_1-7}{5}$$

Taking positive sign

$$\frac{3x_1-4y_1+12z_1-6}{13}=\frac{4x_1+3z_1-7}{5}$$

$$15x_1-20y_1+60z_1-30=52x_1+39z_1-91$$

$$37x_1+20y_1-21z_1-61=0$$

Taking negative sign

$$\frac{3x_1-4y_1+12z_1-6}{13}=-\frac{4x_1+3z_1-7}{5}$$

$$15x_1-20y_1+60z_1-30=-52x_1-39z_1+91$$

$$67x_1-20y_1+99z_1-121=0$$

The Plane Ex 29.9 Q11

The equation of any plane passing through A(2,5, − 3)

is $a(x-2)+b(y-5)+c(z+3)=0$....(1)

The above plane passes through the point B(− 2, − 3,5)

and hence, we have,

$a(-2-2)+b(-3-5)+c(5+3)=0$

$\Rightarrow -4a-8b+8c=0$...(2)

Again the required plane passes through the point C(5,3, − 3)

and hence, we have,

$a(5-2)+b(3-5)+c(-3+3)=0$

$\Rightarrow 3a-2b+0c=0$...(3)

Solving equations (2) and (3) by cross multiplication, we have,

$$\frac{a}{(-8)\times 0-(-2)\times 8}=\frac{b}{3\times 8-(-4)\times 0}=\frac{c}{(-4)\times(-2)-3\times(-8)}=\lambda(\text{say})$$

$$\Rightarrow \frac{a}{0+16}=\frac{b}{24+0}=\frac{c}{8+24}=\lambda$$

$$\Rightarrow \frac{a}{16}=\frac{b}{24}=\frac{c}{32}=\lambda$$

$$\Rightarrow \frac{a}{2}=\frac{b}{3}=\frac{c}{4}=\lambda$$

$\Rightarrow a=2\lambda, b=3\lambda$ and $c=4\lambda$

Substituting the above values in equation (1), we have,

$2\lambda(x-2)+3\lambda(y-5)+4\lambda(z+3)=0$

Since $\lambda \neq 0$, we have,

$2(x-2)+3(y-5)+4(z+3)=0$

$\Rightarrow 2x-4+3y-15+4z+12=0$

$\Rightarrow 2x+3y+4z-7=0$

Thus the equation of the plane is

$2x+3y+4z-7=0$

The distance from the point P(7,2,4) to the plane is

$$d=\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$$

$$\therefore \text{Distance, } d=\left|\frac{2x+3y+4z-7}{\sqrt{2^2+3^2+4^2}}\right|$$

$$\Rightarrow d_{(7,2,4)}=\left|\frac{2\times 7+3\times 2+4\times 4-7}{\sqrt{2^2+3^2+4^2}}\right|$$

$$\Rightarrow d_{(7,2,4)}=\left|\frac{29}{\sqrt{29}}\right|$$

$\Rightarrow d_{(7,2,4)}=\sqrt{29}$ units

The Plane Ex 29.9 Q12

Given that a plane is making intercepts − 6,3 and 4

respectively on the coordinate axes.

Thus the equation of the plane is

$$\frac{x}{-6}+\frac{y}{3}+\frac{z}{4}=1\text{...(1)}$$

We need to find the length of the perpendicular

from the origin on the plane.

If the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ is at a distance 'p', then

$$\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\text{...(2)}$$

Comparing equation (1) with the

general equation, we get,

$a=-6, b=3$ and $c=4$

Thus, equation (2) becomes,

$$\frac{1}{p^2}=\frac{1}{(-6)^2}+\frac{1}{3^2}+\frac{1}{4^2}$$

$$\Rightarrow \frac{1}{p^2}=\frac{4+16+9}{144}$$

$$\Rightarrow \frac{1}{p^2}=\frac{29}{144}$$

$$\Rightarrow p^2=\frac{144}{29}$$

$\Rightarrow p=\frac{12}{\sqrt{29}}$ units

Ex 29.10

The Plane Ex 29.10 Q1

Let $p\left(x_1, y_1, z_1\right)$ be any point as the plane $2x - y + 3z - 4 = 0$, then

$$2x_1 - y_1 + 3z_1 - 4 = 0 \qquad ---(i)$$

We know that distance (D) of a point $\left(x_1, y_1, z_1\right)$ from a plane $ax + by + cz + d = 0$ is given by

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{(a)^2 + (b)^2 + (c)^2}}\right| \qquad ---(ii)$$

Length of perpendicular from $P\left(x_1, y_1, z_1\right)$ to the plane $6x - 3y + 9z + 13 = 0$ is

$$= \left|\frac{6x_1 - 3y_1 + 9z_1 + 13}{\sqrt{(6)^2 + (-3)^2 + (9)^2}}\right| \qquad [\text{Using (ii)}]$$

$$= \left|\frac{3\left(2x_1 - y_1 + 3z_1\right) + 13}{\sqrt{36 + 9 + 81}}\right| \qquad [\text{Using (i)}]$$

$$= \left|\frac{3(4) + 13}{\sqrt{126}}\right|$$

$$= \left|\frac{12 + 13}{\sqrt{126}}\right|$$

$$= \left|\frac{25}{\sqrt{126}}\right|$$

$$= \frac{25}{3\sqrt{14}} \quad \text{units}$$

Since P is the point on plane (i) and $\frac{25}{3\sqrt{14}}$ is the distance of P from plane $6x - 3y + 9z + 13 = 0$, so the distance between the parallel planes is $\frac{25}{3\sqrt{14}}$ units

The Plane Ex 29.10 Q2

Equation of plane which is parallel to 2x-3y+5z+7=0
is of the form 2x-3y+5z=d
Above plane is passing through (3, 4, -1)
So, substitute above point in the equation, we get
6-12-5=d
d=-11
So plane equation is 2x-3y+5z=-11
Distance between planes is given by

$$\left|\frac{-7+11}{\sqrt{4+9+25}}\right| = \frac{4}{\sqrt{38}}$$

The Plane Ex 29.10 Q3

Given equations of planes are

$2x - 2y + z + 3 = 0$ ---(i)

$2x - 2y + z + 9 = 0$ ---(ii)

Let equation of the mid parallel plane is

$2x - 2y + z + \lambda = 0$ ---(iii)

Let $P(x_1, y_1, z_1)$ be any point on plane (iii),

$2x_1 - 2y_1 + z_1 + \lambda = 0$ ---(iv)

We know that, distance of a plane $ax + by + cz + d = 0$ from a point (x_1, y_1, z_1) is given by

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right| \quad ---(v)$$

Let D_1 be the distance of $P(x_1, y_1, z_1)$ from plane (i), so using equation (v),

$$D_1 = \left|\frac{2x_1 - 2y_1 + z_1 + 3}{\sqrt{(2)^2 + (-2)^2 + (1)^2}}\right|$$

$$D_1 = \left|\frac{-\lambda + 3}{\sqrt{4 + 4 + 1}}\right| \quad \text{[Using equation (iv)]}$$

$$D_1 = \left|\frac{3 - \lambda}{3}\right| \quad ---(vi)$$

Again, let D_2 be the distance of point (x_1, y_1, z_1) from plane (ii), so using equation (v),

$$D_2 = \left|\frac{2x_1 - 2y_1 + z_1 + 9}{\sqrt{(2)^2 + (-2)^2 + (1)^2}}\right|$$

$$D_2 = \left|\frac{-\lambda + 9}{\sqrt{9}}\right| \quad \text{[Using equation (iv)]}$$

$$D_2 = \left|\frac{9 - \lambda}{3}\right| \quad ---(vii)$$

Since, $P\left(x_1, y_1, z_1\right)$ is a point on mid parallel plane, so

$D_1 = D_2$

$\left|\frac{3-\lambda}{3}\right| = \left|\frac{9-\lambda}{3}\right|$ [Using (vi),(vii)]

Squaring both the sides,

$$\frac{(3-\lambda)^2}{9} = \frac{(9-\lambda)^2}{9}$$
$$9 - 6\lambda + \lambda^2 = 81 - 18\lambda + \lambda^2$$
$$-6\lambda + 18\lambda = 81 - 9$$
$$12\lambda = 72$$
$$\lambda = \frac{72}{12}$$

$$\lambda = 6$$

Put the value of λ in equation (iii),

$$2x - 2y + z + 6 = 0$$

So, equation of required plane is

$$2x - 2y + z + 6 = 0$$

The Plane Ex 29.10 Q4

Let the position vector of any point P on a plane $\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + 7$ is $\bar{a}$, so

$\bar{a}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + 7 = 0$ ---(i)

We know that distance (D) of a point $\bar{a}$ from a plane $\vec{r}.\vec{n} - d = 0$ is given by

$D = \left|\frac{\bar{a}.\bar{n} - d}{\bar{n}}\right|$ ---(ii)

Length of perpendicular from $P\left(\bar{a}\right)$ to plane $\vec{r}.\left(2\hat{i} + 4\hat{j} + 6\hat{k}\right) + 7 = 0$ is given by

$$= \left|\frac{\bar{a}.\left(2\hat{i} + 4\hat{j} + 6\hat{k}\right) + 7}{\sqrt{(2)^2 + (4)^2 + (6)^2}}\right|$$
$$= \left|\frac{2\bar{a}\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + 7}{\sqrt{4 + 16 + 36}}\right|$$

$= \left|\frac{2.(-7) + 7}{\sqrt{56}}\right|$ [Using equation (i)]

$$= \left|\frac{-14 + 7}{\sqrt{56}}\right|$$

$$= \frac{7}{\sqrt{56}}$$

Distance between two given plane

= Distance of $P\left(\bar{a}\right)$ from plane $\vec{r}.\left(2\hat{i} + 4\hat{j} + 6\hat{k}\right) + 7 = 0$

$$= \frac{7}{\sqrt{56}}$$

So, required distance $= \frac{7}{\sqrt{56}}$ units

Ex 29.11

The Plane Ex 29.11 Q1

$\vec{r} = (2\hat{i} + 3\hat{j} + 9\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$

$\vec{r}.(\hat{i} + \hat{j} + k) = 5$

Angle between line and plane is given by

$$\cos\theta = \left|\frac{2+3+4}{\sqrt{(1+1+1)(4+9+16)}}\right| = \frac{9}{\sqrt{87}}$$

The Plane Ex 29.11 Q2

We know that the angle (θ) between the line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and plane $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\sin\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \qquad ---(i)$$

Given, equation of line is

$$\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z+1}{1}$$

So, $a_1 = 1,\ b_1 = -1,\ c_1 = 1$

Given equation of plane is $2x + y - z - 4 = 0$

So, $a_2 = 2,\ b_2 = 1,\ c_2 = -1$

Put these value in equation (i),

$$\sin\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{(1)(2) + (-1)(1) + (1)(-1)}{\sqrt{(1)^2 + (-1)^2 + (1)^2}\sqrt{(2)^2 + (1)^2 + (-1)^2}}$$

$$\sin\theta = \frac{2-1-1}{\sqrt{1+1+1}\sqrt{4+1+1}}$$

$$= \frac{0}{\sqrt{3}\sqrt{6}}$$

$\sin\theta = 0$

$\theta = 0°$

angle between plane and line = 0°

The Plane Ex 29.11 Q3

We know that angle (θ) between line $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and plane $a_2x+b_2y+c_2z+d_2=0$ is given by

$$\sin\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \qquad ---(i)$$

Given that, line is passing through

$A(3,-4,-2)$ and $B(12,2,0)$, so direction ratios of line AB
$=(12-3,\ 2+4,\ 0+2)$
$=(9,\ 6,\ +2)$

So, $a_1=9,\ b_1=6,\ c_1=2$ $\qquad ---(ii)$

Given equation of plane is $3x-y+z=1$
$a_2=3,\ b_2=-1,\ c_2=1$ $\qquad ---(iii)$

Using (ii) and (iii) in equation (i),
Angle (θ) between plane and line is

$$\sin\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$
$$=\frac{(9)(3)+(6)(-1)+(2)(1)}{\sqrt{(9)^2+(6)^2+(2)^2}\sqrt{(3)^2+(-1)^2+(1)^2}}$$
$$=\frac{27-6+2}{\sqrt{81+36+4}\sqrt{9+1+1}}$$
$$=\frac{23}{\sqrt{121}\sqrt{11}}$$
$$=\frac{23}{11\sqrt{11}}$$

$$\theta=\sin^{-1}\left(\frac{23}{11\sqrt{11}}\right)$$

so, required angle between plane and line is given by

$$\theta=\sin^{-1}\left(\frac{23}{11\sqrt{11}}\right)$$

The Plane Ex 29.11 Q4

We know that, line $\vec{r}=\vec{a}+\lambda\vec{b}$ is parallel to plane $\vec{r}.\vec{n}=d$ if

$\vec{b}.\vec{n}=0$ $\qquad ---(i)$

Given, equation of line is $\vec{r}=\hat{i}+\lambda\left(2\hat{i}-m\hat{j}-3\hat{k}\right)$ and equation of plane $\vec{r}.\left(m\hat{i}+3\hat{j}+\hat{k}\right)=4$

So $\quad \vec{b}=\left(2\hat{i}-m\hat{j}-3\hat{k}\right)$
$\quad\ \ \vec{n}=\left(m\hat{i}+3\hat{j}+\hat{k}\right)$

Put $\vec{b}$ and $\vec{n}$ in equation (i),

$\left(2\hat{i}-m\hat{j}-3\hat{k}\right)\left(m\hat{i}+3\hat{j}+\hat{k}\right)=0$
$(2)(m)+(-m)(3)+(-3)(1)=0$
$2m-3m-3=0$
$-m-3=0$
$-m=3$

$m=-3$

The Plane Ex 29.11 Q5

We know that, line $\vec{r} = \vec{a} + \lambda\vec{b}$ and plane $\vec{r}.\vec{n} = d$ is parallel if

$$\vec{b}.\vec{n} = 0 \qquad ---(i)$$

Given, equation of line $\vec{r} = \left(2\hat{i} + 5\hat{j} + 7\hat{k}\right) + \lambda\left(\hat{i} + 3\hat{j} + 4\hat{k}\right)$ and equation of plane $\vec{r}\left(\hat{i} + \hat{j} - \hat{k}\right) = 7$, so

$\vec{b} = \hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{n} = \hat{i} + \hat{j} - \hat{k}$

Now,

$$\begin{aligned}\vec{b}.\vec{n} &= \left(\hat{i} + 3\hat{j} + 4\hat{k}\right)\left(\hat{i} + \hat{j} - \hat{k}\right)\\ &= (1)(1) + (3)(1) + (4)(-1)\\ &= 1 + 3 - 4\\ &= 0\end{aligned}$$

Since $\vec{b}.\vec{n} = 0$ so using (i), we get

Given line and plane are parallel

We know that, distance (D) of a plane $\vec{r}\vec{n} - d = 0$ from a point $\vec{a}$ is given by,

$$D = \left|\frac{\vec{a}.\vec{n} - d}{|\vec{n}|}\right| \qquad ---(ii)$$

We have to find distance between line and plane which is equal to the distance between point $\vec{a} = \left(2\hat{i} + 5\hat{j} + 7\hat{k}\right)$ from plane, so

$$\begin{aligned}D &= \left|\frac{\left(2\hat{i} + 5\hat{j} + 7\hat{k}\right)\left(\hat{i} + \hat{j} - \hat{k}\right) - 7}{\sqrt{(1)^2 + (1)^2 + (-1)^2}}\right|\\ &= \left|\frac{(2)(1) + (5)(1) + (7)(-1) - 7}{\sqrt{1 + 1 + 1}}\right|\\ &= \left|\frac{2 + 5 - 7 - 7}{\sqrt{3}}\right|\\ &= \left|\frac{-7}{\sqrt{3}}\right|\end{aligned}$$

$$D = \frac{7}{\sqrt{3}}$$

So, required distance between plane and line is $D = \frac{7}{\sqrt{3}}$ unit

The Plane Ex 29.11 Q6

Required line is perpendicular to plane $\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) = 3$, so line is parallel to the normal vector $\vec{n} = \hat{i} + 2\hat{j} + 3\hat{k}$ of plane.

And it is passing through point $\vec{a} = 0\hat{i} + 0\hat{j} + 0\hat{k}$.

We know that equation of a line passing through $\vec{a}$ and parallel to vector $\vec{b}$ is

$$\vec{r} = \vec{a} + \lambda\vec{b} \qquad ---(i)$$

Here, $\vec{a} = 0\hat{i} + 0\hat{j} + 0.\hat{k}$ and $\vec{b} = \vec{n} = \hat{i} + 2\hat{j} + 3\hat{k}$

So, $\vec{r} = \left(0\hat{i} + 0\hat{j} + 0.\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

Hence, equation required line is

$$\vec{r} = \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$$

The Plane Ex 29.11 Q7

We know that equation of plane passing through (x_1, y_1, z_1) is given by

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ $\qquad ---(i)$

So, equation of plane passing through $(2, 3, -4)$ is

$a(x - 2) + b(y - 3) + c(z + 4) = 0$ $\qquad ---(ii)$ $\qquad$ [Using (i)]

It is also passing through $(1, -1, 3)$, so,

$a(1 - 2) + b(-1 - 3) + c(3 + 4) = 0$
$-a - 4b + 7c = 0$
$a + 4b - 7c = 0$ $\qquad ---(iii)$

We know that line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ is parallel to plane $a_2x + b_2y + c_2z + d_2 = 0$ if
$a_1a_2 + b_1b_2 + c_1c_2 = 0$ $\qquad ---(iv)$

Here, equation (ii) is parallel to x-axis

$\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$ $\qquad ---(v)$

Using (ii) and (v) in equation (iv),

$(a)(1) + (b)(0) + c(0) = 0$
$a = 0$ $\qquad ---(vi)$

Put the value of a in equation (iii),

$a - 4b + 7c = 0$
$0 - 4b + 7c = 0$
$-4b = -7c$
$4b = 7c$

$b = \frac{7}{4}c$

Put the value of a and b in equation (ii),

$a(x - 2) + b(y - 3) + c(z + 4) = 0$
$0(x - 2) + \left(\frac{7}{4}c\right)(y - 3) + c(z + 4) = 0$
$0 + \frac{7cy}{4} - \frac{21c}{4} + \frac{cz}{1} + \frac{4c}{1} = 0$

$7cy - 21c + 4cz + 16c = 0$

Dividing by c,

$7y + 4z - 5 = 0$

Equation of required plane is

$7y + 4z - 5 = 0$

The Plane Ex 29.11 Q8

We know that equation a plane passing through the point (x_1, y_1, z_1) is given by

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0 \qquad ---(i)$$

Given the required plane is passing through $(0,0,0)$, so using (i),

$$a(x - 0) + b(y - 0) + c(z - 0) = 0$$
$$ax + by + cz = 0 \qquad ---(ii)$$

Plane (ii) is also passing through $(3,-1,2)$,

$$3a - b + 2c = 0 \qquad ---(iii)$$

We know that line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ is parallel to plane $a_2x + b_2y + c_2z + d_2 = 0$ if

$$a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(iv)$$

Given that, plane (ii) is parallel to line

$\frac{x - 4}{1} = \frac{y + 3}{-4} = \frac{z + 1}{7}$, so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(a)(1) + (b)(-4) + (c)(7) = 0$$
$$a - 4b + 7c = 0 \qquad ---(ii)$$

Solving equation (iii) and (v) by cross-multiplication, we get

$$\frac{a}{(-1)(7) - (-4)(2)} = \frac{b}{(1)(2) - (3)(7)} = \frac{c}{(3)(-4) - (1)(-1)}$$
$$\frac{a}{-7 + 8} = \frac{b}{2 - 21} = \frac{c}{-12 + 1}$$
$$\frac{a}{1} = \frac{b}{-19} = \frac{c}{-11} = \lambda \text{ (Say)}$$

$$\Rightarrow \qquad a = \lambda,\ b = -19\lambda,\ c = -11\lambda$$

Put the value of a, b, c in equation (ii),

$$ax + by + cz = 0$$
$$\lambda x - 19\lambda y - 11\lambda z = 0$$

Dividing by λ, we get

$$x - 19y - 11z = 0$$

Equation of required plane is

$$x - 19y - 11z = 0$$

The Plane Ex 29.11 Q9

We know that equation of a line passing through (x_1, y_1, z_1) is given by

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1} \quad ---(i)$$

Here, required line is passing through $(1,2,3)$, is given by, $\quad$ [Using (i)]

$$\frac{x - 1}{a_1} = \frac{y - 2}{b_1} = \frac{z - 3}{c_1} \quad ---(ii)$$

We know that, line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ is parallel to plane $a_2x + b_2y + c_2z + d_2 = 0$ if

$$a_1a_2 + b_1b_2 + c_1c_2 = 0 \quad ---(iii)$$

Given, line (ii) is parallel to plane $x - y + 2z = 5$

So,

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(a_1)(1) + (b_1)(-1) + (c_1)(2) = 0$

$$a_1 - b_1 + 2c_1 = 0 \quad ---(iv)$$

Also, given line (ii) is parallel to plane $3x + y + z = 6$

So,

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(a_1)(3) + (b_1)(1) + (c_1)(1) = 0$

$$3a_1 + b_1 + c_1 = 0 \quad ---(v)$$

Solving (iv) and (v) by cross-multiplication,

$$\frac{a_1}{(-1)(1) - (1)(2)} = \frac{b_1}{(3)(2) - (1)(1)} = \frac{c_1}{(1)(1) - (3)(-1)}$$

$$\frac{a_1}{-1 - 2} = \frac{b_1}{6 - 1} = \frac{c_1}{1 + 3}$$

$$\frac{a_1}{-3} = \frac{b_1}{5} = \frac{c_1}{4} = \lambda \text{ (Say)}$$

$\Rightarrow \quad a_1 = -3\lambda,\ b_1 = 5\lambda,\ c_1 = 4\lambda$

Put a_1, b_1, c_1 in equation (ii),

$$\frac{x - 1}{-3\lambda} = \frac{y - 2}{5\lambda} = \frac{z - 3}{4\lambda}$$

Multiplying by λ,

$$\frac{x - 1}{-3} = \frac{y - 2}{5} = \frac{z - 3}{4}$$

Equation of required line is

$$\frac{x - 1}{-3} = \frac{y - 2}{5} = \frac{z - 3}{4}$$

The vector equation of the line is

$$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(-3\hat{i} + 5\hat{j} + 4\hat{k}\right)$$

The Plane Ex 29.11 Q10

Firstly we have to find the line of section of planes $5x + 2y - 4z + 2 = 0$ and $2x + 8y + 2z - 1 = 0$
Let a_1, b_1, c_1 be the direction ratios of the line $5x + 2y - 4z + 2 = 0$ and $2x + 8y + 2z - 1 = 0$

Since, line lies in both the planes, so it is perpendicular to both planes, so

$$5a_1 + 2b_1 - 4c_1 = 0 \qquad ---(i)$$
$$2a_1 + 8b_1 + 2c_1 = 0 \qquad ---(ii)$$

Solving equation (i) and (ii), by cross-multiplication

$$\frac{a_1}{(2)(2)-(-4)(8)} = \frac{b_1}{(2)(-4)-(5)(2)} = \frac{c_1}{(5)(8)-(2)(2)}$$
$$\frac{a_1}{4+32} = \frac{b_1}{-8-10} = \frac{c_1}{40-4}$$
$$\frac{a_1}{36} = \frac{b_1}{-18} = \frac{c_1}{36}$$
$$\frac{a_1}{2} = \frac{b_1}{-1} = \frac{c_1}{2} = \lambda \text{ (Say)}$$

$\Rightarrow \quad a_1 = 2\lambda,\ b_1 = -\lambda,\ c_1 = 2\lambda$

We know that, line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ is parallel to plane $a_2x + b_2y + c_2z + d_2 = 0$ if
$$a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(iii)$$

Here line with direction ratio a_1, b_1, c_1 is parallel to plane $4x - 2y - 5z - 2 = 0$,

$$a_1a_2 + b_1b_2 + c_1c_2$$
$$= (2)(4) + (-1)(-2) + (2)(-5)$$
$$= 8 + 2 - 10$$

$$= 0$$

Therefore, line of section is parallel to the plane.

The Plane Ex 29.11 Q11

Equation of line passing through $\vec{a}$ and parallel to $\vec{b}$ is given by

$$\vec{r} = \vec{a} + \lambda\vec{b} \qquad ---(i)$$

Given that, required line is passing through $(1, -1, 2)$ is,

$$\vec{r} = \left(\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\vec{b} \qquad ---(ii)$$

Since, line (i) is perpendicular to plane $2x - y + 3z - 5 = 0$, so normal to plane is parallel to the line.

In vector form,
$\vec{b}$ is parallel to $\vec{n} = 2\hat{i} - \hat{j} + 3\hat{k}$
So, $\vec{b} = \mu\left(2\hat{i} - \hat{j} + 3\hat{k}\right)$ as μ is any scaler
Thus, equation of required line is,

$$\vec{r} = \left(\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\left(\mu\left(2\hat{i} - \hat{j} + 3\hat{k}\right)\right)$$

$$\vec{r} = \left(\hat{i} - \hat{j} + 2\hat{k}\right) + \delta\left(2\hat{i} - \hat{j} + 3\hat{k}\right)$$

The Plane Ex 29.11 Q12

We know that, equation of plane passing through (x_1, y_1, z_1) is given by

$$a(x-x_1)+b(y-y_1)+c(z-z_1)=0 \qquad ---(i)$$

Given that, required plane is passing through $(2,2,-1)$, so using (i),

$$a(x-2)+b(y-2)+c(z+1)=0 \qquad ---(ii)$$

Given, plane (ii) is passing through $(3,4,2)$,

$$a(3-2)+b(4-2)+c(2+1)=0$$
$$a+2b+3c=0 \qquad ---(iii)$$

We know that plane $a_1x+b_1y+c_1z+d_1=0$ and line $\frac{x-x_1}{a_2}=\frac{y-y_1}{b_2}=\frac{z-z_1}{c_2}$ are parallel if $a_1a_2+b_1b_2+c_1c_2=0 \qquad ---(iv)$

Given that, plane (ii) is parallel to a line whose direction ratios are $7,0,6$ so using (iv), we get

$$(a)(7)+(b)(0)+(c)(6)=0$$
$$7a+0+6c=0$$
$$7a+6c=0$$

$$a=-\frac{6c}{7}$$

Put the value of a in equation (iii),

$$a+2b+3c=0$$
$$-\frac{6c}{7}+2b+3c=0$$
$$-6c+14b+21c=0$$
$$14b+15c=0$$

$$b=-\frac{15c}{14}$$

Put the value of a and b in equation (ii),

$$a(x-2)+b(y-2)+c(z+1)=0$$
$$\left(-\frac{6c}{7}\right)(x-2)+\left(-\frac{15c}{14}\right)(y-2)+c(z+1)=0$$
$$-\frac{6cx}{7}+\frac{12c}{7}-\frac{15cy}{14}+\frac{30c}{14}+cz+c=0$$

Multiplying by $\left(\frac{14}{c}\right)$, we get

$$-12x+24-15y+30+14z+14=0$$
$$-12x+15y+14z+68=0$$

Multiplying by (-1),
Equation of required plane is,

$$12x+15y-14z-68=0$$

The Plane Ex 29.11 Q13

We know that angle (θ) between line $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and plane $a_2x+b_2y+c_2z+d_2=0$ is given by

$$\sin\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \qquad ---(i)$$

Given line is $\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-3}{2}$ and equation of plane is $3x+4y+z+5=0$, so angle between plane and line is,

$$\sin\theta=\frac{(3)(3)+(-1)(4)+(2)(1)}{\sqrt{(3)^2+(-1)^2+(2)^2}\sqrt{(3)^2+(4)^2+(1)^2}}$$

$$=\frac{9-4+2}{\sqrt{9+1+4}\sqrt{9+16+1}}$$

$$=\frac{7\times\sqrt{7}}{\sqrt{14}\sqrt{26}\times\sqrt{7}}$$

$$=\frac{7\sqrt{7}}{7\sqrt{52}}$$

$$\theta=\sin^{-1}\left(\sqrt{\frac{7}{52}}\right)$$

The Plane Ex 29.11 Q14

We know that equation of plane passing through the intersection of planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ is given by

$$(a_1x+b_1y+c_1z+d_1)+\lambda(a_2x+b_2y+c_2z+d_2)=0$$

So, equation of plane passing through the intersection of two planes $x-2y+z-1=0$ and $2x+y+z-8=0$ is given by

$$(x-2y+z-1)+\lambda(2x+y+z-8)=0$$
$$x-2y+z-1+2\lambda x+\lambda y+\lambda z-8\lambda=0$$
$$x(1+2\lambda)+y(-2+\lambda)+z(1+\lambda)-1-8\lambda=0 \qquad ---(i)$$

We know that line $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and plane $a_2x+b_2y+c_2z+d_2=0$ are parallel if $a_1a_2+b_1b_2+c_1c_2=0$ $\qquad ---(ii)$

Given that plane (i) is parallel to line with direction ratio 1,2,1, so

$$a_1a_2+b_1b_2+c_1c_2=0$$
$$(1)(1+2\lambda)+(2)(-2+\lambda)+(1)(1+\lambda)=0$$
$$1+2\lambda-4+2\lambda+1+\lambda=0$$
$$5\lambda-2=0$$

$$\lambda=\frac{2}{5}$$

Put the value of λ in equation (i),

$$x\left(1+\frac{4}{5}\right)+y\left(-2+\frac{2}{5}\right)+z\left(1+\frac{2}{5}\right)-1-\frac{16}{5}=0$$

Multiplying by 5,

$$x(5+4)+y(-10+2)+z(5+2)-5-16=0$$
$$9x-8y+7z-21=0$$

So, equation of required plane is

$$9x-8y+7z-21=0 \qquad ---(iii)$$

We know that distance (D) of a point (x_1,y_1,z_1) from plane $ax+by+cz+d=0$ is given by

$$D=\frac{ax_1+by_1+cz_1+d_1}{\sqrt{a^2+b^2+c^2}}$$

So, distance of point $(1,1,1)$ from plane (i) is given by

$$D=\left|\frac{(9)(1)+(-8)(1)+(7)(1)-21}{\sqrt{(9)^2+(-8)^2+(7)^2}}\right|$$
$$=\left|\frac{9-8+7-21}{\sqrt{81+64+49}}\right|$$
$$=\left|\frac{16-29}{\sqrt{194}}\right|$$
$$=\left|\frac{-13}{\sqrt{194}}\right|$$

$$D=\frac{13}{\sqrt{194}} \text{ units}$$

The Plane Ex 29.11 Q15

We know that line $\vec{r}=\vec{a}+\lambda\vec{b}$ is paralle to plane $\vec{r}.\vec{n}=d$ if

$$\vec{b}.\vec{n}=0$$

Given, line is $\vec{r}=\left(\hat{i}+\hat{j}\right)+\lambda\left(3\hat{i}-\hat{j}+2\hat{k}\right)$ and plane is $\vec{r}.\left(2\hat{j}+\hat{k}\right)=3$, so

$\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$, $\vec{a}=\left(\hat{i}+\hat{j}\right)$ and $\vec{n}=\left(2\hat{j}+\hat{k}\right)$

Now, $\vec{b}\vec{n}=\left(3\hat{i}-\hat{j}+2\hat{k}\right)\left(2\hat{j}+\hat{k}\right)$
$$=(3)(0)+(-1)(2)+(2)(1)$$
$$=0-2+2$$
$$=0$$

Since, $\vec{b}.\vec{n}=0$, so line is parallel to plane

Distance between point $\vec{a}$ and plane $\vec{r}.\vec{n}-d=0$ is given by

$$D=\left|\frac{\vec{a}\vec{n}-d}{\left|\vec{n}\right|}\right| \qquad ---(i)$$

$\vec{a}$ is a point on the line. So distance between line and plane is equal to the distance between $\vec{a} = \left(\hat{i} + \hat{j}\right)$ and plane $\vec{r}.\left(2\hat{j} + \hat{k}\right) = 3$, so using (i),

$$D = \left|\frac{\left(\hat{i} + \hat{j}\right)\left(2\hat{j} + \hat{k}\right) - 3}{\sqrt{(2)^2 + (1)^2}}\right|$$

$$= \left|\frac{(1)(0) + (1)(2) + (0)(1) - 3}{\sqrt{4+1}}\right|$$

$$= \left|\frac{0 + 2 + 0 - 3}{\sqrt{5}}\right|$$

$$= \left|\frac{-1}{\sqrt{5}}\right|$$

$$= \frac{1}{\sqrt{5}} \text{ unit}$$

So, required distance $= \frac{1}{\sqrt{5}}$ unit

The Plane Ex 29.11 Q16

We know that line $\vec{r} = \vec{a} + \lambda\vec{b}$ and plane $\vec{r}.\vec{n} - d = 0$ are parallel if

$$\vec{b}.\vec{n} = 0 \qquad ---(i)$$

Given, line $\vec{r} = \left(-\hat{i} + \hat{j} + \hat{k}\right) + \lambda\left(2\hat{i} + \hat{j} + 4\hat{k}\right)$ and plane is $\vec{r}.\left(\hat{i} + 2\hat{j} - \hat{k}\right) - 1 = 0$

So, $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} + 4\hat{k}$, $\vec{n} = \hat{i} + 2\hat{j} - \hat{k}$

Now, $\vec{b}.\vec{n} = \left(2\hat{i} + \hat{j} + 4\hat{k}\right)\left(\hat{i} + 2\hat{j} - \hat{k}\right)$

$= (2)(1) + (1)(2) + (4)(-1)$

$= 2 + 2 - 4$

$= 0$

Since, $\vec{b}.\vec{n} = 0$, so by equation (i), line is parallel to plane

Distance (D) between point $\vec{a}$ and plane $\vec{r}\,\vec{n} - d = 0$ is given by

$$D = \left|\frac{\vec{a}\vec{n} - d}{\left|\vec{n}\right|}\right| \qquad ---(ii)$$

Distance between given line and plane

= Distance of point $\vec{a} = \left(-\hat{i} + \hat{j} + \hat{k}\right)$ from $\vec{r}.\left(\hat{i} + 2\hat{j} - \hat{k}\right) - 1 = 0$

$$D = \left|\frac{\vec{a}\vec{n} - d}{\left|\vec{n}\right|}\right|$$

$$= \left| \frac{\left(-\hat{i} + \hat{j} + \hat{k}\right)\left(\hat{i} + 2\hat{j} - \hat{k}\right) - 1}{\sqrt{(1)^2 + (2)^2 + (-1)^2}} \right|$$

$$= \left| \frac{(-1)(1) + (1)(2) + (1)(-1) - 1}{\sqrt{1 + 4 + 1}} \right|$$

$$= \left| \frac{-1 + 2 - 1 - 1}{\sqrt{6}} \right|$$

$$= \left| \frac{-1}{\sqrt{6}} \right|$$

$$= \frac{1}{\sqrt{6}}$$

So, required distance = $\frac{1}{\sqrt{6}}$ units

The Plane Ex 29.11 Q17

We know that equation of plane passing through the line of intersection of two planes
$a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by,

$$\left(a_1x + b_1y + c_1z + d_1\right) + \lambda\left(a_2x + b_2y + c_2z + d_2\right) = 0 \qquad ---(i)$$

So, equation of plane passing through the line of intersection of planes
$3x - 4y + 5z - 10 = 0$ and $2x + 2y - 3z - 4 = 0$ is,

$$(3x - 4y + 5z - 10) + \lambda(2x + 2y - 3z - 4) = 0$$
$$(3 + 2\lambda)x + (-4 + 2\lambda)y + (5 - 3\lambda)z - 10 - 4\lambda = 0 \qquad ---(ii)$$

We know that, line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ parallel to plane
$a_2x + b_2y + c_2z + d_2 = 0$ if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ $\qquad ---(iii)$

Given that, plane (ii) is parallel to line $x = 2y = 3z$ or $\frac{x}{6} = \frac{y}{3} = \frac{z}{2}$
So,

$$(6)(3 + 2\lambda) + (3)(-4 + 2\lambda) + (2)(5 - 3\lambda) = 0$$
$$18 + 12\lambda - 12 + 6\lambda + 10 - 6\lambda = 0$$
$$12\lambda + 16 = 0$$
$$\lambda = -\frac{16}{12}$$

$$\lambda = -\frac{4}{3}$$

Put λ in equation (ii),
$$x(3 + 2\lambda) + y(-4 + 2\lambda) + z(5 - 3\lambda) - 10 - 4\lambda = 0$$
$$x\left(3 - \frac{8}{3}\right) + y\left(-4 - \frac{8}{3}\right) + z\left(5 + \frac{12}{3}\right) - 10 + \frac{16}{3} = 0$$

Multiplying by 3,

$$x(9 - 8) + y(-12 - 8) + z(15 + 12) - 30 + 16 = 0$$
$$x - 20y + 27z - 14 = 0$$

Equation of required plane is given by

$$x - 20y + 27z - 14 = 0$$

The Plane Ex 29.11 Q18

The plane passes through the point $\vec{a}(1, 2, -4)$

A vector in a direction perpendicular to

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $\vec{r} = (\hat{i} - 3\hat{j} + 5\hat{k}) + \mu(\hat{i} + \hat{j} - \hat{k})$

is $\vec{n} = (2\hat{i} + 3\hat{j} + 6\hat{k}) \times (\hat{i} + \hat{j} - \hat{k})$

$$\Rightarrow \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 1 & 1 & -1 \end{vmatrix} = -9\hat{i} + 8\hat{j} - \hat{k}$$

Equation of the plane is $(\vec{r} - \vec{a}).\vec{n} = 0$

$(\vec{r} - (\hat{i} + 2\hat{j} - 4\hat{k})).(-9\hat{i} + 8\hat{j} - \hat{k}) = 0$

$\Rightarrow \vec{r}.(-9\hat{i} + 8\hat{j} - \hat{k}) = 11$

Substituting $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, we get the Cartesian form as

$-9x + 8y - z = 11$

The distance of the point $(9, -8, -10)$ from the plane

$$= \left| \frac{-9(9) + 8(-8) - (-10) - 11}{\sqrt{9^2 + 8^2 + 1^2}} \right| = \frac{146}{\sqrt{146}} = \sqrt{146}$$

The Plane Ex 29.11 Q19

We know that equation of plane passing through (x_1, y_1, z_1) is given by

$$a(x - x_1) + b(y - y_1) + c(z - z_1) \qquad ---(i)$$

Given that, required equation of plane is passing through $(3, 4, 1)$, so

$$a(x - 3) + b(y - 4) + c(z - 1) = 0 \qquad ---(ii)$$

Plane (ii) is also passing through $(0,1,0)$, so

$a(0 - 3) + b(1 - 4) + c(0 - 1) = 0$

$-3a - 3b - c = 0$

$$3a + 3b + c = 0 \qquad ---(iii)$$

We know that, plane $a_1x + b_1y + c_1z + d_1 = 0$ and line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ are parallel if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Here, line $\frac{x + 3}{2} = \frac{y - 3}{7} = \frac{z - 2}{5}$ is parallel to plane (ii), so

$(2)(a) + (7)(b) + (c)(5) = 0$

$$2a + 7b + 5c = 0 \qquad ---(iv)$$

Solving (iii) and (iv) by cross-multiplication,

$$\frac{a}{(3)(5)-(7)(1)} = \frac{b}{(2)(1)-(3)(5)} = \frac{c}{(3)(7)-(2)(3)}$$
$$\frac{a}{15-7} = \frac{b}{2-15} = \frac{c}{21-6}$$

$$\frac{a}{8} = \frac{b}{-13} = \frac{c}{15} = \lambda \text{(Say)}$$

$\Rightarrow \qquad a = 8\lambda,\ b = -13\lambda,\ c = 15\lambda$

Put a, b, c in equation (ii),

$$a(x-3)+b(y-4)+c(z-1) = 0$$
$$8\lambda(x-3)+(-13\lambda)(y-4)+(15\lambda)(z-1) = 0$$
$$8\lambda x - 24\lambda - 13\lambda y + 52\lambda + 15\lambda z - 15\lambda = 0$$

$$8\lambda x - 13\lambda y + 15\lambda z + 13\lambda = 0$$

Dividing by λ, equation of required plane is,

$$8x - 13y + 15z + 13 = 0$$

The Plane Ex 29.11 Q20

Find the coordinates of the point where the line

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2} = r$$

$\Rightarrow x = 3r+2, y = 4r-1, z = 2r+2$

Substituting in the equation of the plane $x - y + z - 5 = 0$,

we get $(3r+2)-(4r-1)+(2r+2)-5 = 0$

$\Rightarrow r = 0$

$\Rightarrow x = 2, y = -1, z = 2$

Direction ratios of the line are $3, 4, 2$

Direction ratios of a line perpendicular to the plane are $1, -1, 1$

$$\sin\theta = \frac{3\times1+4\times-1+2\times1}{\sqrt{9+16+4}\sqrt{1+1+1}} = \frac{1}{\sqrt{87}}$$

$$\theta = \sin^{-1}\frac{1}{\sqrt{87}}$$

The Plane Ex 29.11 Q21

We know that equation of line passing through point $\vec{a}$ and parallel to vector $\vec{b}$ is given by

$$\vec{r} = \vec{a} + \lambda\vec{b} \qquad ---(i)$$

Given that, line is passing through $(1,2,3)$.

So, $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

It is given that line is perpendicular to plane $\vec{r}.\left(\hat{i}+2\hat{j}-5\hat{k}\right)+9 = 0$

So, normal to plane $(\vec{n})$ is parallel to $\vec{b}$.

So, let $\vec{b} = \mu\vec{n} = \mu\left(\hat{i}+2\hat{j}-5\hat{k}\right)$

Put $\vec{a}$ and $\vec{b}$ in (i), equation of line is,

$$\vec{r} = \left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left[\mu\left(\hat{i}+2\hat{j}-5\hat{k}\right)\right]$$
$$\vec{r} = \left(\hat{i}+2\hat{j}+3\hat{k}\right)+\delta\left(\hat{i}+2\hat{j}-5\hat{k}\right) \qquad [\text{As } \delta = \lambda\mu]$$

Equation of required line is,

$$\vec{r} = \left(\hat{i}+2\hat{j}+3\hat{k}\right)+\delta\left(\hat{i}+2\hat{j}-5\hat{k}\right)$$

The Plane Ex 29.11 Q22

Direction ratios of the line $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$

are (2,3,6)

Direction ratio of a line perpendicular to the plane

$10x+2y-11z = 3$ are $10, 2, -11$

If θ is the angle between the line and the plane, then

$$\sin\theta = \frac{2\times10+3\times2+6\times-11}{\sqrt{2^2+3^2+6^2}\sqrt{10^2+2^2+11^2}} = -\frac{40}{\sqrt{49}\sqrt{225}} = -\frac{40}{7\times15} = -\frac{8}{21}$$

$$\Rightarrow \theta = \sin^{-1}\left(-\frac{8}{21}\right)$$

The Plane Ex 29.11 Q23

We know that, equation of line passing through (x_1, y_1, z_1) is given by

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1} \qquad ---(i)$$

Given that, required line is passing through $(1, 2, 3)$, so

$$\frac{x-1}{a_1} = \frac{y-2}{b_1} = \frac{z-3}{c_1} \qquad ---(ii)$$

We know that, line $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and plane $a_2x+b_2y+c_2z+d_2=0$

are parallel if $a_1a_2+b_1b_2+c_1c_2=0$ $\qquad ---(iii)$

Given line (ii) is parallel to

$$\vec{r}.\left(\hat{i}-\hat{j}+2\hat{k}\right)=5$$

$$\Rightarrow \qquad x-y+2z-5=0, \text{ so}$$

$$a_1a_2+b_1b_2+c_1c_2=0$$

$$(a_1)(1)+(b_1)(-1)+(c_1)(2)=0$$

$$a_1-b_1+2c_1=0 \qquad ---(iii)$$

Line (ii) is also parallel to plane

$$\vec{r}.\left(3\hat{i}+\hat{j}+\hat{k}\right)=6$$

$$\Rightarrow \qquad 3x+y+z-6=0, \text{ so}$$

$a_1a_2 + b_1b_2 + c_1c_2 = 0$
$(a_1)(3) + (b_1)(1) + (c_1)(1) = 0$

$3a_1 + b_1 + c_1 = 0$ --- (iv)

Solving equation (iii) and (iv) by cross-multiplication,

$$\frac{a_1}{(-1)(1)-(2)(1)} = \frac{b_1}{(3)(2)-(1)(1)} = \frac{c_1}{(1)(1)-(3)(-1)}$$
$$\frac{a_1}{-1-2} = \frac{b_1}{6-1} = \frac{c_1}{1+3}$$
$$\frac{a_1}{-3} = \frac{b_1}{5} = \frac{c_1}{4} = \lambda \text{ (Say)}$$

$\Rightarrow \quad a_1 = -3\lambda,\ b_1 = 5\lambda,\ c_1 = 4\lambda$

Put a_1, b_1, c_1 in equation (ii), so, equation line is given by

$$\frac{x-1}{-3\lambda} = \frac{y-2}{5\lambda} = \frac{z-3}{4\lambda}$$
$$\frac{x-1}{-3} = \frac{y-2}{5} = \frac{z-3}{4}$$

So, vector equation of required line is

$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(-3\hat{i} + 5\hat{j} + 4\hat{k}\right)$$

The Plane Ex 29.11 Q24

Here, given mid line $\frac{x-2}{6} = \frac{y-1}{\lambda} = \frac{z+5}{-4}$ is perpendicular to plane $3x - y - 2z = 7$
So, normal vector of plane is parallel to line so,
Direction ratios of normal to plane are proparional to the direction ratios of line
Here,
$$\frac{6}{3} = \frac{\lambda}{-1} = \frac{-4}{-2}$$
cross multiplying the last two
$$-2\lambda = 4$$
$$\lambda = \frac{4}{-2}$$

$$\lambda = -2$$

The Plane Ex 29.11 Q25

The equation of a plane passing through (-1, 2, 0) is

$a(x+1)+b(y-2)+c(z-0)=0$.............(i)

This passes through (2, 2, -1)

$\therefore\ a(2+1)+b(2-2)+c(-1-0)=0$

$3a-c=0$............(ii)

The plane in (i) is parallel to $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{1}$.

Theefore normal to the plane is perpendicular to the line.

$\therefore\ a(1)+b(2)+c(1)=0$.........(iii)

Solving (ii) and (iii) by cross multiplication we get,

$$\frac{a}{0-(-1)(2)}=\frac{b}{(1)(-1)-(3)(1)}=\frac{c}{(3)(2)-0}$$

$$\Rightarrow \frac{a}{2}=-\frac{b}{4}=\frac{c}{6}$$

$$\Rightarrow a=-\frac{b}{2}=\frac{c}{3}=\lambda(\text{say})$$

$$\Rightarrow a=\lambda, b=-2\lambda, c=3\lambda$$

Substituting $a=\lambda, b=-2\lambda, c=3\lambda$ in (i) we get,

$\lambda(x+1)-2\lambda(y-2)+3\lambda(z-0)=0$

$x-2y+3z+5=0$

$\therefore$ The required equation of the plane is $x-2y+3z+5=0$.

Ex 29.12

The Plane Ex 29.12 Q1(i)

Direction ratios of the given line are
$(5-3, 1-4, 6-1) = (2, -3, 5)$
Hence, equation of the line is
$\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-6}{5} = r$
$\Rightarrow x = 2r+5, y = -3r+1, z = 5r+6$
For any point on the yz – plane $x = 0$
$\Rightarrow 2r+5 = 0 \Rightarrow r = -\frac{5}{2}$
$y = -3(-\frac{5}{2}) + 1 = \frac{17}{2}$
$z = 5(-\frac{5}{2}) + 6 = -\frac{13}{2}$
Hence, the coordinates of the point are $\left(0, \frac{17}{2}, -\frac{13}{2}\right)$.

The Plane Ex 29.12 Q1(ii)

Direction ratios of the given line are
$(5-3, 1-4, 6-1) = (2, -3, 5)$
Hence, equation of the line is
$\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-6}{5} = r$
$\Rightarrow x = 2r+5, y = -3r+1, z = 5r+6$
For any point on zx – plane $y = 0$
$\Rightarrow -3r+1 = 0 \Rightarrow r = \frac{1}{3}$
$x = 2\left(\frac{1}{3}\right) + 5 = \frac{17}{3}$
$z = 5\left(\frac{1}{3}\right) + 6 = \frac{23}{3}$
Hence, the coordinates of the point are $\left(\frac{17}{3}, 0, \frac{23}{3}\right)$.

The Plane Ex 29.12 Q2

Let the coordinates of the points A and B be
$(3, -4, -5)$ *and* $(2, -3, 1)$ *repectively.*
Equation of the line joining the points (x_1, y_1, z_1) *and* (x_2, y_2, z_2) *is*

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1} = r, \text{ where } r \text{ is some constant.}$$

Thus equation of AB is

$$\frac{x-3}{2-3} = \frac{y-(-4)}{(-3)-(-4)} = \frac{z-(-5)}{1-(-5)} = r$$

$$\Rightarrow \frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = r$$

Any point on the line AB is of the form
$-r+3, r-4, 6r-5$
Let P be the point of intersection of the line AB and the plane $2x+y+z=7$
Thus, we have,
$2(-r+3) + r-4+6r-5 = 7$
$\Rightarrow -2r+6+r-4+6r-5 = 7$
$\Rightarrow 5r = 10$
$\Rightarrow r = 2$
Substituting the value of r in $-r+3, r-4, 6r-5$, *the coordinates of P are:*
$(-2+3, 2-4, 6\times 2-5) = (1, -2, 7)$

The Plane Ex 29.12 Q3

The equation of the given line is

$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$ ...(1)

The equation of the given plane is

$\vec{r}.\left(\hat{i} - \hat{j} + \hat{k}\right) = 5$ ...(2)

Substituting the value of $\vec{r}$ from equation (1) in equation (2), we obtain

$$\left[2\hat{i} - \hat{j} + 2\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)\right]\cdot\left(\hat{i} - \hat{j} + \hat{k}\right) = 5$$

$$\Rightarrow \left[(3\lambda + 2)\hat{i} + (4\lambda - 1)\hat{j} + (2\lambda + 2)\hat{k}\right]\cdot\left(\hat{i} - \hat{j} + \hat{k}\right) = 5$$

$$\Rightarrow (3\lambda + 2) - (4\lambda - 1) + (2\lambda + 2) = 5$$

$$\Rightarrow \lambda = 0$$

Substituting this value in equation (1), we obtain the equation of the line as

$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k}$

This means that the position vector of the point of intersection of the line and the plane is $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k}$

This shows that the point of intersection of the given line and plane is given by the coordinates, (2, −1, 2). The point is (−1, −5, −10).

The distance d between the points, (2, −1, 2) and (−1, −5, −10), is

$$d = \sqrt{(-1-2)^2 + (-5+1)^2 + (-10-2)^2} = \sqrt{9+16+144} = \sqrt{169} = 13$$

The Plane Ex 29.12 Q4

To find the point of intersection of the line

$\vec{r} = 2\hat{i} - 4\hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r}.(\hat{i} - 2\hat{j} + \hat{k}) = 0$,

we substitute $\vec{r}$ of line in the plane.

$$[2\hat{i} - 4\hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})].(\hat{i} - 2\hat{j} + \hat{k}) = 0$$

$$\Rightarrow [(2 + 3\lambda)\hat{i} + (-4 + 4\lambda)\hat{j} + (2 + 2\lambda)\hat{k}].(\hat{i} - 2\hat{j} + \hat{k}) = 0$$

$$\Rightarrow 2 + 3\lambda + 8 - 8\lambda + 2 + 2\lambda = 0$$

$$\Rightarrow 3\lambda = 12 \Rightarrow \lambda = 4$$

$$\vec{r} = 2\hat{i} - 4\hat{j} + 2\hat{k} + 4(3\hat{i} + 4\hat{j} + 2\hat{k}) = 14\hat{i} + 12\hat{j} + 10\hat{k}$$

Hence, the distance of the point $2\hat{i} + 12\hat{j} + 5\hat{k}$ from $14\hat{i} + 12\hat{j} + 10\hat{k}$ is

$$\sqrt{(14-2)^2 + (12-12)^2 + (10-5)^2} = \sqrt{12^2 + 5^2} = \sqrt{169} = 13$$

The Plane Ex 29.12 Q5

Equation of the line through the points A(2, − 1,2)

and B(5,3,4) is $\frac{x-2}{5-2} = \frac{y+1}{3+1} = \frac{z-2}{4-2} = r$

$$\Rightarrow \frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2} = r$$

$$\Rightarrow x = 3r + 2, y = 4r - 1, z = 2r + 2$$

Substituting these in the plane equation we get

$$(3r + 2) - (4r - 1) + (2r + 2) = 5$$

$$\Rightarrow r = 0$$

$$\Rightarrow x = 2, y = -1, z = 2$$

Distance of (2, − 1,2) from (− 1, − 5, − 10) is

$$= \sqrt{(2 - (-1))^2 + (-1 - (-5))^2 + (2 - (-10))^2} = \sqrt{3^2 + 4^2 + 12^2}$$

$$= \sqrt{169} = 13$$

The Plane Ex 29.12 Q6

The equation of a line joining the points A(3, -4, -5) and B(2, -3, 1)is

$$\frac{x-3}{2-3} = \frac{y+4}{-3+4} = \frac{z+5}{1+5} = r$$

$$\Rightarrow x = 3 - r, y = -4 + r, z = -5 + 6r$$

Substituting this into the given plane equation we get,

$$2(3-r) + (-4 + r) + (-5 + 6r) = 7$$

$$\Rightarrow r = 2$$

$$\Rightarrow x = 1,\ y = -2,\ z = 7$$

Distance of (1, -2, 7) from (3, 4, 4) is

$$= \sqrt{(3-1)^2 + (4+2)^2 + (4-7)^2}$$

$$= \sqrt{4 + 36 + 9}$$

$$= \sqrt{49}$$

$$= 7$$

Ex 29.13

The Plane Ex 29.13 Q1

$\vec{r} = (2\hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

$\vec{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 4\hat{k})$

We know that the lines,

$\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ are coplanar if

$\vec{a_1} \cdot (\vec{b_1} \times \vec{b_2}) = \vec{a_2} \cdot (\vec{b_1} \times \vec{b_2})$

and the equation of the plane containing them is

$\vec{r} \cdot (\vec{b_1} \times \vec{b_2}) = \vec{a_1} \cdot (\vec{b_1} \times \vec{b_2})$

Here

$\vec{a_1} = 2\hat{j} - 3\hat{k}, \vec{b_1} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{a_2} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ and $\vec{b_2} = 2\hat{i} + 3\hat{j} + 4\hat{k}$.

Therefore, $\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix} = \hat{i}(8-9) - \hat{j}(4-6) + \hat{k}(3-4)$

$\Rightarrow \vec{b_1} \times \vec{b_2} = -\hat{i} + 2\hat{j} - \hat{k}$

$\Rightarrow \vec{a_1} \cdot (\vec{b_1} \times \vec{b_2}) = (2\hat{j} - 3\hat{k}) \cdot (-\hat{i} + 2\hat{j} - \hat{k}) = 0 + 4 + 3 = 7$

and

$\vec{a_2} \cdot (\vec{b_1} \times \vec{b_2}) = (2\hat{i} + 6\hat{j} + 3\hat{k}) \cdot (-\hat{i} + 2\hat{j} - \hat{k}) = -2 + 12 - 3 = 7$

Since $\vec{a_1} \cdot (\vec{b_1} \times \vec{b_2}) = \vec{a_2} \cdot (\vec{b_1} \times \vec{b_2})$, the lines are coplanar.

Now the equation of the plane containing the given lines is

$\vec{r} \cdot (\vec{b_1} \times \vec{b_2}) = \vec{a_1} \cdot (\vec{b_1} \times \vec{b_2})$

$\vec{r} \cdot (-\hat{i} + 2\hat{j} - \hat{k}) = 7$

$\Rightarrow \vec{r} \cdot (\hat{i} - 2\hat{j} + \hat{k}) = -7$

$\Rightarrow \vec{r} \cdot (\hat{i} - 2\hat{j} + \hat{k}) + 7 = 0$

The Plane Ex 29.13 Q2

We know that lines $\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1}$ and $\frac{x - x_2}{l_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2}$ are coplanar if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

And equation of plane containing them is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

Here, equation of lines are

$\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$

So, $x_1 = -1, y_1 = 3, z_1 = -2, l_1 = -3, m_1 = 2, n_1 = 1$
$x_2 = 0, y_2 = 7, z_2 = -7, l_2 = 1, m_2 = -3, n_2 = 2$

So,

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 0+1 & 7-3 & -7+2 \\ -3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 4 & -5 \\ -3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix}$$

$= 1(4+3) - 4(-6-1) - 5(9-2)$
$= 7 + 28 - 35$

$= 0$

So, lines are coplanar.

Equation of plane containing line is

$$\begin{vmatrix} x+1 & y-3 & z+2 \\ -3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix} = 0$$

$(x+1)(4+3) - (y-3)(-6-1) + (z+2)(9-2) = 0$
$7x + 7 + 7y - 21 + 7z + 14 = 0$
$7x + 7y + 7z = 0$

The Plane Ex 29.13 Q3

We know that the plane passing through (x_1, y_1, z_1) is given by

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ --- (i)

Required plane is passing through $(0, 7, -7)$, so

$a(x - 0) + b(y - 7) + c(z + 7) = 0$
$ax + b(y - 7) + c(z + 7) = 0$ --- (ii)

Plane (ii) also contains line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ so, it passes through point $(-1, 3, -2)$,

$a(-1) + b(3 - 7) + c(-2 + 7) = 0$
$-a - 4b + 5c = 0$ --- (iii)

Also, plane (ii) will be parallel to line
so, $a_1a_2 + b_1b_2 + c_1c_2 = 0$
$(a)(-3) + (b)(2) + (c)(1) = 0$
$-3a + 2b + c = 0$ --- (iv)

Solving (iii) and (iv) by cross-multiplication,

$$\frac{a}{(-4)(1) - (5)(2)} = \frac{b}{(-3)(5) - (-1)(1)} = \frac{c}{(-1)(2) - (-4)(-3)}$$
$$\frac{a}{-4 - 10} = \frac{b}{-15 + 1} = \frac{c}{-2 - 12}$$

$$\frac{a}{-14} = \frac{b}{-14} = \frac{c}{14} = \lambda \text{(Say)}$$

$\Rightarrow \quad a = -14\lambda,\ b = -14\lambda,\ c = -14\lambda$

Put a, b, c in equation (ii),

$$ax + b(y-7) + c(z+7) = 0$$
$$(-14\lambda)x + (-14\lambda)(y-7) + (-14\lambda)(z+7) = 0$$

Dividing by (-14λ), we get

$$x + y - 7 + z + 7 = 0$$
$$x + y + z = 0$$

So, equation of plane containing the given point and line is $x + y + z = 0$

The other line is $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$

So, $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$(1)(1) + (1)(-3) + (1)(2) = 0$$
$$1 - 3 + 2 = 0$$

$$0 = 0$$

$LHS = RHS$

So, $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ lie on plane $x + y + z = 0$

The Plane Ex 29.13 Q4

We know that equation of plane passing through (x_1, y_1, z_1) is given by

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0 \qquad ---(i)$$

Since, required plane contain lines

$$\frac{x-4}{1} = \frac{y-3}{-4} = \frac{z-2}{5} \text{ and } \frac{x-3}{1} = \frac{y+2}{-4} = \frac{z}{5}$$

So, required plane passes through $(4, 3, 2)$ and $(3, -2, 0)$, so, equation of required plane is,

$$a(x-4) + b(y-3) + c(z-2) = 0 \qquad ---(ii)$$

Plane (ii) also passes through $(3, -2, 0)$, so

$$a(3-4) + b(-2-3) + c(0-2) = 0$$
$$-a - 5b - 2c = 0$$
$$a + 5b + 2c = 0 \qquad ---(iii)$$

Now plane (ii) is also parallel to line with direction ratios $1, -4, 5$, so,

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(a)(1) + (b)(-4) + (c)(5) = 0$$
$$a - 4b + 5c = 0 \qquad ---(iv)$$

Solving equation (iii) and (iv) by cross-multiplication,

$$\frac{a}{(5)(5)-(-4)(2)}=\frac{b}{(1)(2)-(1)(5)}=\frac{c}{(1)(-4)-(1)(5)}$$
$$\frac{a}{25+8}=\frac{b}{2-5}=\frac{c}{-4-5}$$

$$\frac{a}{33}=\frac{b}{-3}=\frac{c}{-9}$$

Multiplying by 3,

$$\frac{a}{11}=\frac{b}{-1}=\frac{c}{-3}=\lambda\,(\text{Say})$$

$\Rightarrow \quad a=11\lambda,\ b=-\lambda,\ c=-3\lambda$

Put a,b,c in equation (ii),

$$a(x-4)+b(y-3)+c(z-2)=0$$
$$(11\lambda)(x-4)+(-\lambda)(y-3)+(-3\lambda)(z-2)=0$$
$$11\lambda x-44\lambda-\lambda y+3\lambda-3\lambda z+6\lambda=0$$

$$11\lambda x-\lambda y-3\lambda z-35\lambda=0$$

Dividing by λ,

$$11x-y-3z-35=0$$

So, equation of required plane is,

$$11x-y-3z-35=0$$

The Plane Ex 29.13 Q5

We have, equation of line is

$$\frac{x+4}{3}=\frac{y+6}{5}=\frac{z-1}{-2}=\lambda\,(\text{Say})$$

General point on this line is given by

$$(3\lambda-4,\ 5\lambda-6,\ -2\lambda+1) \qquad ---(i)$$

Another equation of line is

$$3x-2y+z+5=0=2x+3y+4z-4$$

Let a,b,c be the direction ratio of the line so, it will be perpendicular to normal of $3x-2y+z+5=0$ and $2x+3y+4z-4=0$, so

Using $a_1a_2+b_1b_2+c_1c_2=0$
$$(3)(a)+(-2)(b)+(1)(c)=0$$
$$3a-2b+c=0 \qquad ---(ii)$$

Again,

$$(2)(a)+(3)(b)+(4)(c)=0$$
$$2a+3b+4c=0 \qquad ---(iii)$$

Solving (ii) and (iii) by cross-multiplication,

$$\frac{a}{(-2)(4)-(3)(1)}=\frac{b}{(2)(1)-(3)(4)}=\frac{c}{(3)(3)-(-2)(2)}$$

$$\frac{a}{-8-3}=\frac{b}{2-12}=\frac{c}{9+4}$$

$$\frac{a}{-11}=\frac{b}{-10}=\frac{c}{13}$$

Direction ratios are proportional to $-11,-10,13$
Let $z=0$, so

$$3x-2y=-5 \qquad ---(A)$$
$$2x+3y=4 \qquad ---(B)$$

Solving (A) and (B),

$$\begin{array}{r} 6x-4y=-10 \\ -\;\underset{-}{6x}+\underset{-}{9y}=\underset{-}{12} \\ \hline -13y=-12 \end{array}$$

$$y=\frac{22}{13}$$

Put y in equation (A),

$$3x-2\left(\frac{22}{13}\right)=-5$$

$$3x-\frac{44}{13}=-5$$
$$3x=-5+\frac{44}{13}$$

$$3x=-\frac{21}{13}$$
$$x=-\frac{7}{13}$$

So, equation of line (2) in symmetrical form,

$$\frac{x+\frac{7}{13}}{-11}=\frac{y-\frac{22}{13}}{-10}=\frac{z-0}{13}$$

Put the general point of line (1) from equation (1)

$$\frac{3\lambda-4+\frac{7}{13}}{-11}=\frac{5\lambda-6-\frac{22}{13}}{-10}=\frac{-2\lambda+1}{13}$$

$$\frac{39\lambda-52+7}{-11\times 13}=\frac{65\lambda-78-22}{-10\times 13}=\frac{-2\lambda+1}{13}$$

$$\frac{39\lambda-45}{-11}=\frac{65\lambda-100}{-10}=\frac{-2\lambda+1}{1}$$

The equation of the plane is $45x-17y+25z+53=0$
Their point of intersection is $(2,4,-3)$

The Plane Ex 29.13 Q6

We know that plane $\vec{r}.\vec{n} = d$ contains the line $\vec{r} = \vec{a} + \lambda\vec{b}$ if

(i) $\vec{b}.\vec{n} = 0$ (ii) $\vec{a}.\vec{n} = d$ $\quad ---(i)$

Given, equation of plane $\vec{r}.\left(\hat{i} + 2\hat{j} - \hat{k}\right) = 3$ and equation of line $\vec{r} = \left(\hat{i} + \hat{j}\right) + \lambda\left(2\hat{i} + \hat{j} + 4\hat{k}\right)$

so, $\vec{n} = \hat{i} + 2\hat{j} - \hat{k}$, $\quad \vec{a} = \hat{i} + \hat{j}$

$d = 3$ $\quad \vec{b} = 2\hat{i} + \hat{j} + 4\hat{k}$

$$\vec{b}.\vec{n} = \left(2\hat{i} + \hat{j} + 4\hat{k}\right)\left(\hat{i} + 2\hat{j} - \hat{k}\right)$$
$$= (2)(1) + (1)(2) + (4)(-1)$$
$$= 2 + 2 - 4$$

$\vec{b}.\vec{n} = 0$

$$\vec{a}.\vec{n} = \left(\hat{i} + \hat{j}\right)\left(\hat{i} + 2\hat{j} - \hat{k}\right)$$
$$= (1)(1) + (1)(2) + (0)(-1)$$
$$= 1 + 2 - 0$$
$$= 3$$

$= d$

since, $\vec{b}.\vec{n} = 0$ and $\vec{a}.\vec{n} = d$, so, from (i)

Given line lie on the given plane.

The Plane Ex 29.13 Q7

Let $L_1 : \frac{x+3}{3} = \frac{y}{-2} = \frac{z-7}{6}$ and

$L_2 : \frac{x+6}{1} = \frac{y+5}{-3} = \frac{z-1}{2}$ be the equations of two lines

Let the plane be $ax + by + cz + d = 0$...(1)

Given that the required plane passes through the intersection of the lines L_1 and L_2.

Hence the normal to the plane is perpendicular to the lines L_1 and L_2.

$\therefore 3a - 2b + 6c = 0$

$a - 3b + 2c = 0$

Using cross-multiplication, we get,

$$\frac{a}{-4+18} = \frac{b}{6-6} = \frac{c}{-9+2}$$

$$\Rightarrow \frac{a}{14} = \frac{b}{0} = \frac{c}{-7}$$

$$\Rightarrow \frac{a}{2} = \frac{b}{0} = \frac{c}{-1}$$

The Plane Ex 29.13 Q8

Let the equation of the plane be $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$........(i)

Plane is passing through (3,4,2) and (7,0,6)

$\frac{3}{a}+\frac{4}{b}+\frac{2}{c}=1$

$\frac{7}{a}+\frac{0}{b}+\frac{6}{c}=1$

Required plane is perpendicular to $2x-5y-15=0$

$\frac{2}{a}+\frac{-5}{b}+\frac{0}{c}=0$

$\Rightarrow 2b=5a$

$\therefore b=2.5a$

$\frac{3}{a}+\frac{4}{2.5a}+\frac{2}{c}=1$

$\frac{7}{a}+\frac{6}{c}=1$

Solving the above 2 equations,

$a=3.4=\frac{17}{5}$, $b=\ 8.5=\frac{17}{2}$ and $c=\frac{-34}{6}=-\frac{17}{3}$

Substituting the values in (i)

$\frac{x}{\frac{17}{5}}+\frac{y}{\frac{17}{2}}+\frac{z}{-\frac{17}{3}}=1$

$\Rightarrow \frac{5x}{17}+\frac{2y}{17}-\frac{3z}{17}=1$

$\Rightarrow 5x+2y-3z=17$

$\Rightarrow (x\hat{i}+y\hat{j}+z\hat{k}).(5\hat{i}+2\hat{j}-3\hat{k})=17$

$\Rightarrow \vec{r}.(5\hat{i}+2\hat{j}-3\hat{k})=17$

Vector equation of the plane is $\vec{r}.(5\hat{i}+2\hat{j}-3\hat{k})=17$.

The line passes through $B(1,3,-2)$.

$5(1)+2(3)-3(-2)=17$

The point B lies on the plane.

$\therefore$The line $\vec{r}=\hat{i}+3\hat{j}-2\hat{k}+\lambda(\hat{i}-\hat{j}+\hat{k})$ lies on the plane $\vec{r}.(5\hat{i}+2\hat{j}-3\hat{k})=17$.

The direction ratio of the line $\frac{x-1}{-3}=\frac{y-2}{-2k}=\frac{z-3}{2}$ is

$r_1=(-3,-2k,2)$

The direction ratio of the line $\frac{x-1}{k}=\frac{y-2}{1}=\frac{z-3}{5}$ is

$r_2=(k,1,5)$

Since the lines $\frac{x-1}{-3}=\frac{y-2}{-2k}=\frac{z-3}{2}$ and $\frac{x-1}{k}=\frac{y-2}{1}=\frac{z-3}{5}$ are perpendicular so

$$r_1\cdot r_2=0$$
$$(-3,-2k,2)\cdot(k,1,5)=0$$
$$-3k-2k+10=0$$
$$-5k=-10$$
$$k=2$$

Therefore the equation of the lines are $\frac{x-1}{-3}=\frac{y-2}{-4}=\frac{z-3}{2}$ and $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{5}$

The equation of the plane containing the perpendicular lines $\frac{x-1}{-3}=\frac{y-2}{-4}=\frac{z-3}{2}$ and $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{5}$ is

$$\begin{vmatrix} x & y & z \\ -3 & -4 & 2 \\ 2 & 1 & 5 \end{vmatrix}=0$$

$$\begin{vmatrix} x & y & z \\ -3 & -4 & 2 \\ 2 & 1 & 5 \end{vmatrix}=0$$

$$(-20-2)x-y(-15-4)+z(-3+8)+d=0$$
$$-22x+19y+5z+d=0$$

The line $\frac{x-1}{-3}=\frac{y-2}{-4}=\frac{z-3}{2}$ pass through the point $(1,2,3)$ so putting $x=1, y=2, z=3$ in the equation $-22x+19y+5z+d=0$ we get

$$-22(1)+19(2)+5(3)+d=0$$
$$d=22-38-15$$
$$d=-31$$

Therefore the equation of the plane containing the lines is

$-22x+19y+5z=31$

The Plane Ex 29.13 Q10

Any point on the line

$$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}=k$$

is of the form, $(3k+2,\ 4k-1,\ 2k+2)$.

If the point $P(3k+2,\ 4k-1,\ 2k+2)$ *lies in the plane* $x-y+z-5=0$, *we have*,

$(3k+2)-(4k-1)+(2k+2)-5=0$

$\Rightarrow 3k+2-4k+1+2k+2-5=0$

$\Rightarrow k=0$

Thus, the coordinates of the point of intersection of the line and the plane are: $P(3\times 0+2,\ 4\times 0-1,\ 2\times 0+2)=P(2,-1,2)$

Let θ *be the angle between the line and the plane.*

Thus,

$$\sin\theta=\frac{al+bm+cn}{\sqrt{a^2+b^2+c^2}\sqrt{l^2+m^2+n^2}}$$, *where*, l,m *and* n *are the direction ratios of the line and* a,b *and* c *are the direction ratios of the normal to the plane.*

Here, $l=3$, $m=4$, $n=2$, $a=1$, $b=-1$, *and* $c=1$

Hence,

$$\sin\theta=\frac{1\times 3+(-1)\times 4+1\times 2}{\sqrt{1^2+(-1)^2+1^2}\sqrt{3^2+4^2+2^2}}$$

$$\Rightarrow \sin\theta=\frac{1}{\sqrt{3}\sqrt{29}}$$

$$\Rightarrow \theta=\sin^{-1}\left(\frac{1}{\sqrt{3}\sqrt{29}}\right)$$

The Plane Ex 29.13 Q11

Let A, B *and* C *be three points with position vectors*

$\hat{i}+\hat{j}-2\hat{k}$, $2\hat{i}-\hat{j}+\hat{k}$ *and* $\hat{i}+2\hat{j}+\hat{k}$.

Thus, $\overrightarrow{AB}=\vec{b}-\vec{a}=\left(2\hat{i}-\hat{j}+\hat{k}\right)-\left(\hat{i}+\hat{j}-2\hat{k}\right)=\hat{i}-2\hat{j}+3\hat{k}$

$\overrightarrow{AC}=\vec{c}-\vec{a}=\left(\hat{i}+2\hat{j}+\hat{k}\right)-\left(\hat{i}+\hat{j}-2\hat{k}\right)=\hat{j}+3\hat{k}$

Now consider $\overrightarrow{AB}\times\overrightarrow{AC}$:

$$\vec{n}=\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 3\\ 0 & 1 & 3\end{vmatrix}$$

$\vec{n}=\hat{i}(-6-3)-3\hat{j}+\hat{k}=-9\hat{i}-3\hat{j}+\hat{k}$

So, the equation of the required plane is

$(\vec{r}-\vec{a})\cdot\vec{n}=0$

$\Rightarrow(\vec{r}\cdot\vec{n})=(\vec{a}\cdot\vec{n})$

$\Rightarrow\left(\vec{r}\cdot\left(-9\hat{i}-3\hat{j}+\hat{k}\right)\right)=\left(\hat{i}+\hat{j}-2\hat{k}\right)\cdot\left(-9\hat{i}-3\hat{j}+\hat{k}\right)$

$\Rightarrow\vec{r}\cdot\left(9\hat{i}+3\hat{j}-\hat{k}\right)=14$

Also, find the coordinates of the point of intersection of this plane and the line $\vec{r}=3\hat{i}-\hat{j}-\hat{k}+\lambda\left(2\hat{i}-2\hat{j}+\hat{k}\right)$

Any point on the line $\vec{r}=3\hat{i}-\hat{j}-\hat{k}+\lambda\left(2\hat{i}-2\hat{j}+\hat{k}\right)$ *is of the form*,

$(3+2\lambda,\ -1-2\lambda,\ -1+\lambda)$

If the point $P(3+2\lambda,\ -1-2\lambda,\ -1+\lambda)$ *lies in the plane*,
$\vec{r}\cdot\left(9\hat{i}+3\hat{j}-\hat{k}\right)=14$, *we have*,
$9(3+2\lambda)-3(1+2\lambda)-(-1+\lambda)=14$
$\Rightarrow 27+18\lambda-3-6\lambda+1-\lambda=14$
$\Rightarrow 11\lambda=-11$
$\Rightarrow \lambda=-1$
Thus, the required point of intersection is
$P(3+2\lambda,\ -1-2\lambda,\ -1+\lambda)$
$\Rightarrow P(3+2(-1),\ -1-2(-1),\ -1+(-1))$
$\Rightarrow P(1,\ 1,\ -2)$

The Plane Ex 29.13 Q12

$$\frac{5-x}{-4}=\frac{y-7}{4}=\frac{z+3}{-5}$$
$$\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}\ldots\ldots\text{(i)}$$

$$\frac{x-8}{7}=\frac{2y-8}{2}=\frac{z-5}{3}$$
$$\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}\ldots\ldots\text{(ii)}$$

Here,
$a_1=4, b_1=4, c_1=-5$
$a_2=7, b_2=1, c_2=3$
$x_1=5, y_1=7, z_1=-3$
$x_2=8, y_2=4, z_2=5$
Condition for two lines to be coplanar,

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

$$\therefore \begin{vmatrix} 8-5 & 4-7 & 5+3 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix}$$

$$=\begin{vmatrix} 3 & -3 & 8 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix}$$

$=3(12+5)+3(12+35)+8(4-28)$
$=3\times17+3\times47+8\times(-24)$
$=51+141-192$
$=192-192$
$=0$

$\therefore$ The lines are coplanar to each other.

The Plane Ex 29.13 Q13

Required equation of plane is passing through the point (3, 2, 0),

$\therefore\ a(x-3)+b(y-2)+c(z-0)=0$

$\Rightarrow a(x-3)+b(y-2)+cz=0$..........(i)

Required equation of plane also contains the line $\frac{x-3}{1}=\frac{y-6}{5}=\frac{z-4}{4}$,

so it passes through the point (3, 2, 0)

$\Rightarrow a(3-3)+b(6-2)+c4=0$

$\Rightarrow 4b+4c=0$................................(ii)

Also plane will be parallel to,

$a(1)+b(5)+c(4)=0$

$a+5b+4c=0$..............(iii)

Solving (ii) and (iii) by cross multiplication,

$\frac{a}{16\text{-}20}=\frac{b}{4-0}=\frac{c}{0-4}=\lambda(\text{say})$

$-\frac{a}{4}=\frac{b}{4}=-\frac{c}{4}=\lambda(\text{say})$

$\Rightarrow a=-\lambda, b=\lambda,\ c=-\lambda$

Put $a=-\lambda, b=\lambda,\ c=-\lambda$ in equation (i) we get

$(-\lambda)(x-3)+(\lambda)(y-2)+(-\lambda)z=0$

$\Rightarrow -x+3+y-2-z=0$

$\Rightarrow x-y+z-1=0$

Ex 29.14

The Plane Ex 29.14 Q1

Consider

$$l_1 : \frac{x-2}{-1} = \frac{y-5}{2} = \frac{z-0}{3}$$

$$l_2 : \frac{x-0}{2} = \frac{y+1}{-1} = \frac{z-1}{2}$$

Clearly line l_1 passes through the point $P(2,5,0)$

The equation of a plane containing line l_2 is

$$a(x-0)+b(y+1)+c(z-1)=0 \qquad \text{...... (1)}$$

Where $2a-b+2c=0$

If it is parallel to line l_1 then

$$-a+2b+3c=0$$

Therefore

$$\frac{a}{-7} = \frac{b}{-8} = \frac{c}{3}$$

Substituting values of a, b, c in the equation (1) we obtain

$$a(x-0)+b(y+1)+c(z-1)=0$$

$$-7(x-0)-8(y+1)+3(z-1)=0$$

$$-7x-8y-8+3z-3=0$$

$$7x+8y-3z+11=0 \qquad \text{...... (2)}$$

This is the equation of the plane containing line l_2 and parallel to line l_1

Shortest distance between l_1 and l_2 = Distance between point $P(2,5,0)$ and plane (2)

$$= \left| \frac{14+40+11}{\sqrt{7^2+8^2+(-3)^2}} \right| = \frac{65}{\sqrt{122}}$$

The Plane Ex 29.14 Q2

$$l_1 : \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$$

$$l_2 : \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

Let the equation of the plane containing l_1 be $a(x+1)+b(y+1)+c(z+1)=0$

Plane is parallel to l_1: $7a-6b+c=0$......(i)

Plane is parallel to l_2: $a-2b+c=0$........(ii)

Solving (i) and (ii),

$$\frac{a}{-6+2} = \frac{b}{1-7} = \frac{c}{-14+6}$$

$$\frac{a}{-4} = \frac{b}{-6} = \frac{c}{-8}$$

$\therefore$ Equation of the plane is $-4(x+1)-6(y+1)-8(z+1)=0$

$4(x+1)+6(y+1)+8(z+1)=0$ is the equation of the plane.

The Plane Ex 29.14 Q3

The equation of a plane containing the line $3x - y - 2z + 4 = 0 = 2x + y + z + 1$ is
$x(2\lambda + 3) + y(\lambda - 1) + z(\lambda - 2) + \lambda + 4 = 0$............(i)

If it is parallel to the line then $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z+2}{1}$ then,
$2(2\lambda + 3) + 4(\lambda - 1) + (\lambda - 2) = 0$
$\Rightarrow \lambda = 0$

Putting $\lambda = 0$ in (i) we get,
$3x - y - 2z + 4 = 0$............(ii)
As this equation of the plane containing the second line and paralle to the first line.

Clearly the line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z+2}{1}$ passes through the point (1, 3, -2)
So, the shortest distance 'd' between the given lines is equal to the length of perpendicular from (1, 3, -2) on the plane (ii).

$$d = \left|\frac{3 - 3 + 4 + 4}{\sqrt{1+9+4}}\right| = \frac{8}{\sqrt{14}}$$

Ex 29.15

The Plane Ex 29.15 Q1

$3x+4y-6z+1=0$

Line passing through orgin and perpendicular to plane is given by

$\frac{x}{3}=\frac{y}{4}=\frac{z}{-6}=r(say)$

So let the image of (0,0,0) is (3r, 4r, -6r)

Midpoint of (0,0,0) and (3r, 4r, -6r) lies on plane.

$3\left(\frac{3r}{2}\right)+2(4r)-3(-6r)+1=0$

$30.5r=-1$

$r=\frac{-2}{61}$

So image is $\left(\frac{-6}{61},\frac{-8}{61},\frac{12}{61}\right)$

The Plane Ex 29.15 Q2

Here, we have to find reflection of the point $P(1,2,-1)$ in the plane $3x-5y+4z=5$

Let Q be the reflection of the point P and R be the mid-point of PQ.
Then, R lies on the plane $3x-5y+4z=5$.

Direction ratios of PQ are proportional to $3,-5,4$ and PQ is passing through $(1,2,-1)$.
So, equation of PQ is given by,

$$\frac{x-1}{3}=\frac{y-2}{-5}=\frac{z+1}{4}=\lambda \text{ (Say)}$$

Let Q be $(3\lambda+1,\ -5\lambda+2,\ 4\lambda-1)$

The coordinates of R are $\left(\frac{3\lambda+1+1}{2},\ \frac{-5\lambda+2+2}{2},\ \frac{4\lambda-1-1}{2}\right)=\left(\frac{3\lambda+2}{2},\ \frac{-5\lambda+4}{2},\ \frac{4\lambda-2}{2}\right)$

Since, R lies on the given plane $3x-5y+4z=5$

$$\therefore \quad 3\left(\frac{3\lambda+2}{2}\right)-5\left(-\frac{5\lambda+4}{2}\right)+4\left(\frac{4\lambda-2}{2}\right)=5$$

$\Rightarrow \quad 9\lambda+6+25\lambda-20+16\lambda-8=10$

$\Rightarrow \quad 50\lambda-22=10$

$\Rightarrow \quad 50\lambda=10+22$

$\Rightarrow \quad 50\lambda=32$

$\Rightarrow \quad \lambda=\frac{16}{25}$

So, $Q=(3\lambda+1,\ -5\lambda+2,\ 4\lambda-1)$

$=\left(3\left(\frac{16}{25}\right)+1,\ -5\left(\frac{16}{25}\right)+2,\ 4\left(\frac{16}{25}\right)-1\right)$

$=\left(\frac{48}{25}+1,\ -\frac{16}{5}+2,\ \frac{64}{25}-1\right)$

$=\left(\frac{73}{25},\ -\frac{6}{5},\ \frac{39}{25}\right)$

So, reflection of $P(1,2,-1)=\left(\frac{73}{25},\ -\frac{6}{5},\ \frac{39}{25}\right)$

The Plane Ex 29.15 Q3

We have to find foot of the perpendicular, say Q, drawn from $P(5,4,2)$ to the line

$$\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1} = \lambda \quad \text{(say)}$$

Let Q be $(2\lambda - 1, 3\lambda + 3, -\lambda + 1)$

Direction ratios of line PQ are $(2\lambda - 1 - 5,\ 3\lambda + 3 - 4, -\lambda + 1 - 2)$ or $(2\lambda - 6,\ 3\lambda - 1, -\lambda - 1)$

Here, line PQ is perpendicular to line given(AB).

So,

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(2\lambda - 6)(2) + (3\lambda - 1)(3) + (-\lambda - 1)(-1) = 0$$
$$4\lambda - 12 + 9\lambda - 3 + \lambda + 1 = 0$$
$$14\lambda - 14 = 0$$
$$\lambda = \frac{14}{14}$$
$$\lambda = 1$$

So, $Q = (2\lambda - 1, 3\lambda + 3, -\lambda + 1)$

$$= (2(1) - 1, 3(1) + 3, -(1) + 1)$$
$$= (2 - 1, 3 + 3, -1 + 1)$$
$$= (1, 6, 0)$$

Length of perpendicular PQ

$$= \sqrt{(5-1)^2 + (4-6)^2 + (2-0)^2} \qquad \text{[Using Distance formula]}$$
$$= \sqrt{16 + 4 + 4}$$
$$= \sqrt{24}$$
$$= 2\sqrt{6}$$

So,

Foot of perpendicular is $(1,6,0)$

Length of the perpendicular is $2\sqrt{6}$ units

The Plane Ex 29.15 Q4

Here, we have to find image of the point $P(3,1,2)$ in the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 4$ or $2x - y + z = 4$.

Let Q be the image of the point P.

So,

Direction ratios of normal to the point are $2, -1, 1$

Direction ratios of line PQ perpendicular to $2, -1, 1$ and PQ is passing through $(3,1,2)$.

So equation of PQ is

$$\frac{x-3}{2} = \frac{y-1}{-1} = \frac{z-2}{1} = \lambda \quad \text{(say)} \qquad \left[\begin{array}{l}\text{Using equation of line passing through } (x_1, y_1, z_1) \text{ is} \\ \frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}\end{array}\right]$$

General point on the line PQ is $(2\lambda + 3,\ -\lambda + 1,\ \lambda + 2)$

Let Q be $(2\lambda + 3,\ -\lambda + 1,\ \lambda + 2)$

Let R be the mid point of PQ. Then,

coordinates of R are $\left(\frac{2\lambda + 3 + 3}{2}, \frac{-\lambda + 1 + 1}{2}, \frac{\lambda + 2 + 2}{2}\right) = \left(\frac{2\lambda + 6}{2}, \frac{-\lambda + 2}{2}, \frac{\lambda + 4}{2}\right)$

Since, R lies on the plane $2x - y + z = 4$, we have,

$$2\left(\frac{2\lambda+6}{2}\right) - \left(\frac{-\lambda+2}{2}\right) + \left(\frac{\lambda+4}{2}\right) = 4$$

$\Rightarrow \quad 4\lambda + 12 + \lambda - 2 + \lambda + 4 = 8$

$\Rightarrow \quad 6\lambda = 8 - 14$

$\Rightarrow \quad \lambda = \frac{-6}{6}$

$\Rightarrow \quad \lambda = -1$

So,

Image of $P = Q\left(2(-1)+3, -(-1)+1, -1+2\right)$

Image of $P = (1, 2, 1)$

The equation of the perpendicular line through $3\hat{i}+\hat{j}+2\hat{k}$ is

$\bar{r} = 3\hat{i}+\hat{j}+2\hat{k}+\lambda\left(2\hat{i}-\hat{j}+\hat{k}\right)$

The position vector of the image point is

$3\hat{i}+\hat{j}+2\hat{k}+\lambda\left(2\hat{i}-\hat{j}+\hat{k}\right) = (3+2\lambda)\hat{i}+(1-\lambda)\hat{j}+(2+k)\hat{k}$

The position vector of the foot of the perpendicular is

$$\frac{\left[(3+2\lambda)\hat{i}+(1-\lambda)\hat{j}+(2+\lambda)\hat{k}\right]+\left[3\hat{i}+\hat{j}+2\hat{k}\right]}{2}$$

$$=(3+\lambda)\hat{i}+\left(1-\frac{\lambda}{2}\right)\hat{j}+\left(2+\frac{\lambda}{2}\right)\hat{k}$$

Putting $\lambda = -1$ the position vector of the foot of the perpendicular is

$2\hat{i}+\frac{3}{2}\hat{j}+\frac{3}{2}\hat{k}$

The Plane Ex 29.15 Q5

$2x - 2y + 4z + 5 = 0$

$(1,1,2)$

$= \left|\frac{2-2+8+5}{\sqrt{1+1+4}}\right| = \frac{13}{\sqrt{6}}$

Let the foot of perpendicular be (x,y,z). So DR's are in proportional

$\frac{x-1}{2} = \frac{y-1}{-2} = \frac{z-2}{4} = k$

$x = 2k+1$

$y = -2k+1$

$z = 4k+2$

Substitute (x,y,z)=$(2k+1,\ -2k+1, 4k+2)$ in plane equation

$2x - 2y + 4z + 5 = 0$

$4k+2+4k-2+16k+8+5 = 0$

$24k = -13$

$k = \frac{-13}{24}$

$(x,y,z) = (\frac{-1}{12}, \frac{5}{3}, \frac{-1}{6})$

The Plane Ex 29.15 Q6

Here, we have to find distance of the point $P(1,-2,3)$ from the plane

$x-y+z=5$ measured parallel to line AB, $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$

Let Q be the mid point of the line joining P to plane.

Here, PQ is parallel to line AB

$\Rightarrow$ Direction ratios of line PQ are proportional to direction ratios of line AB

$\Rightarrow$ Direction ratios of line PQ are $2,3,-6$ and PQ is passing through $P(1,-2,3)$.

So equation of PQ is given by

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

$$\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{-6}=\lambda \quad \text{(say)}$$

General point on line PQ is $(2\lambda+1,\ 3\lambda-2,-6\lambda+3)$

Suppose coordinates of Q be $(2\lambda+1,3\lambda-2,-6\lambda+3)$

General point on line PQ is $(2\lambda+1,\ 3\lambda-2,-6\lambda+3)$

Suppose coordinates of Q be $(2\lambda+1,3\lambda-2,-6\lambda+3)$

Since Q lies on the plane $x-y+z=5$

$$(2\lambda+1)-(3\lambda-2)+(-6\lambda+3)=5$$

$$2\lambda+1-3\lambda+2-6\lambda+3=5$$

$$-7\lambda=5-6$$

$$-7\lambda=-1$$

$$\lambda=\frac{1}{7}$$

Coordinate of Q = $(2\lambda+1,3\lambda-2,-6\lambda+3)=\left(\frac{9}{7},\frac{-11}{7},\frac{15}{7}\right)$

Distance between $(1,-2,3)$ and plane $= PQ$

$$=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

$$=\sqrt{\left(1-\frac{9}{7}\right)^2+\left(-2+\frac{11}{7}\right)^2+\left(3-\frac{15}{7}\right)^2}$$

$$=\sqrt{\frac{4}{49}+\frac{9}{49}+\frac{36}{49}}$$

$$=\sqrt{\frac{49}{49}}$$

$$=1$$

Required distance = 1 unit

The Plane Ex 29.15 Q7

Let Q be the foot of the perpendicular.

Here, Direction ratios of normal to plane is $3, -1, -1$

$\Rightarrow$ Line PQ is parallel to normal to plane

$\Rightarrow$ Direction ratios of PQ are proportional to $3, -1, -1$ and PQ is passing through $P(2, 3, 7)$.

So,

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

$$\frac{x - 2}{3} = \frac{y - 3}{-1} = \frac{z - 7}{-1} = \lambda \quad \text{(say)}$$

General point on line PQ

$$= (3\lambda + 2, -\lambda + 3, -\lambda + 7)$$

Coordinates of Q be $(3\lambda + 2, -\lambda + 3, -\lambda + 7)$

Point Q lies on the plane $3x - y - z = 7$.

So,

$$3(3\lambda + 2) - (-\lambda + 3) - (-\lambda + 7) = 7$$

$$9\lambda + 6 + \lambda - 3 + \lambda - 7 = 7$$

$$11\lambda = 7 + 4$$

$$11\lambda = 11$$

$$\lambda = \frac{11}{11}$$

$$\lambda = 1$$

$\therefore$ Coordinate of Q $= (3\lambda + 2, -\lambda + 3, -\lambda + 7)$

$$= (3(1) + 2, -(1) + 3, -(1) + 7)$$

$$= (5, 2, 6)$$

Length of the perpendicular PQ

$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$= \sqrt{(2 - 5)^2 + (3 - 2)^2 + (7 - 6)^2}$$

$$= \sqrt{9 + 1 + 1}$$

$$= \sqrt{11}$$

The Plane Ex 29.15 Q8

Here, we have to find image of point $P(1,3,4)$ in the plane $2x - y + z + 3 = 0$

Let Q be the image of the point.

Here, Direction ratios of normal to plane are $2,-1,1$

$\Rightarrow$ Direction ratios of PQ which is parallel to normal to the plane is proportional to $2,-1,1$ and line PQ is passing through $P(1,3,4)$.

So, equation of line PQ is given by

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

$$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = \lambda \quad (\text{say})$$

General point on line PQ

$$= (2\lambda + 1, \ -\lambda + 3, \lambda + 4)$$

Let Q be $(2\lambda + 1, -\lambda + 3, \lambda + 4)$

Q is image of P, so R is the mid point of PQ

Coordinates of R $\left(\frac{2\lambda + 1 + 1}{2}, \frac{-\lambda + 3 + 3}{2}, \frac{\lambda + 4 + 4}{2}\right)$

$$= \left(\frac{2\lambda + 2}{2}, \frac{-\lambda + 6}{2}, \frac{\lambda + 8}{2}\right)$$

$$= \left(\lambda + 1, \frac{-\lambda + 6}{2}, \frac{\lambda + 8}{2}\right)$$

Point R is on th plane $2x - y + z + 3 = 0$

$$= 2(\lambda + 1) - \left(\frac{-\lambda + 6}{2}\right) + \left(\frac{\lambda + 8}{2}\right) = 0$$

$$4\lambda + 4 + \lambda - 6 + \lambda + 8 + 6 = 0$$

$$6\lambda = -12$$

$$\lambda = -2$$

So,

$$\text{Image } Q = (2\lambda + 1, -\lambda + 3, \lambda + 4)$$

$$= (-4 + 1, 2 + 3, -2 + 4)$$

$$= (-3, 5, 2)$$

Image of P $(1,3,4)$ is $(-3,5,2)$

The Plane Ex 29.15 Q9

Here, we have to find distance of a point A with position vector $\left(-\hat{i}-5\hat{j}-10\hat{k}\right)$ from the point of intersection of line $\vec{r}=\left(2\hat{i}-\hat{j}+2\hat{k}\right)+\lambda\left(3\hat{i}+4\hat{j}+12\hat{k}\right)$ with plane $\vec{r}\cdot\left(\hat{i}-\hat{j}+\hat{k}\right)=5$.

Let the point of intersection of line and plane be $B\left(\vec{b}\right)$

The line and the plane will intersect when,

$$\left[\left(2\hat{i}-\hat{j}+2\hat{k}\right)+\lambda\left(3\hat{i}+4\hat{j}+12\hat{k}\right)\right]\left(\hat{i}-\hat{j}+\hat{k}\right)=5$$
$$\left[(2+3\lambda)\hat{i}+(-1+4\lambda)\hat{j}+(2+12\lambda)\hat{k}\right]\left(\hat{i}-\hat{j}+\hat{k}\right)=5$$
$$(2+3\lambda)(1)+(-1+4\lambda)(-1)+(2+12\lambda)(1)=5$$
$$2+3\lambda+1-4\lambda+2+12\lambda=5$$
$$11\lambda=5-5$$
$$\lambda=0$$

So, the point B is given by

$$\vec{b}=\left(2\hat{i}-\hat{j}+2\hat{k}\right)+(0)\left(3\hat{i}+4\hat{j}+12\hat{k}\right)$$
$$\vec{b}=\left(2\hat{i}-\hat{j}+2\hat{k}\right)$$

$$\overrightarrow{AB}=\vec{b}-\vec{a}$$
$$=\left(2\hat{i}-\hat{j}+2\hat{k}\right)-\left(-\hat{i}-5\hat{j}-10\hat{k}\right)\qquad=\left(2\hat{i}-\hat{j}+2\hat{k}+\hat{i}+5\hat{j}+10\hat{k}\right)=\left(3\hat{i}+4\hat{j}+12\hat{k}\right)$$
$$\left|\overrightarrow{AB}\right|=\sqrt{(3)^2+(4)^2+(12)^2}=\sqrt{9+16+144}=\sqrt{169}=13$$

Required distance = 13 units

The Plane Ex 29.15 Q10

$$x-2y+4z+5=0$$
$$(1,1,2)$$
$$=\left|\frac{1-2+4+5}{\sqrt{1+4+16}}\right|=\frac{8}{\sqrt{21}}$$

Let the foot of perpendicular be (x,y,z). So DR's are in proportional

$$\frac{x-1}{1}=\frac{y-1}{-2}=\frac{z-2}{4}=k$$
$$x=k+1$$
$$y=-2k+1$$
$$z=4k+2$$

Substitute (x,y,z)=(k+1, -2k+1, 4k+2) in plane equation

$$x-2y+4z+5=0$$
$$k+1+4k-2+16k+8+5=0$$
$$21k=-12$$
$$k=\frac{-12}{21}=\frac{-4}{7}$$
$$(x,y,z)=(\frac{3}{7},\frac{15}{7},\frac{-2}{7})$$

The Plane Ex 29.15 Q11

$2x - y + z + 1 = 0$

$(3, 2, 1)$

$= \left|\frac{6-2+1+1}{\sqrt{4+1+1}}\right| = \frac{6}{\sqrt{6}} = \sqrt{6}$

Let the foot of perpendicular be (x,y,z). So DR's are in proportional

$\frac{x-3}{2} = \frac{y-2}{-1} = \frac{z-1}{1} = k$

$x = 2k + 3$

$y = -k + 2$

$z = k - 1$

Substitute (x,y,z)=$(2k+3, -k+2, k-1)$ in plane equation

$2x - y + z + 1 = 0$

$4k + 6 + k - 2 + k - 1 + 1 = 0$

$6k = -4$

$k = \frac{-4}{6} = \frac{-2}{3}$

$(x, y, z) = (\frac{5}{3}, \frac{8}{3}, \frac{-5}{3})$

The Plane Ex 29.15 Q12

Given equation of the plane $\vec{r} \cdot (6\hat{i} - 3\hat{j} - 2\hat{k}) + 1 = 0$

Thus, the direction ratios normal to the plane are $6, -3$ *and* -2

Hence the direction cosines to the normal to the plane are

$$\frac{6}{\sqrt{6^2+(-3)^2+(-2)^2}}, \frac{-3}{\sqrt{6^2+(-3)^2+(-2)^2}}, \frac{-2}{\sqrt{6^2+(-3)^2+(-2)^2}}$$

$$= \frac{6}{7}, \frac{-3}{7}, \frac{-2}{7}$$

$$= \frac{-6}{7}, \frac{3}{7}, \frac{2}{7}$$

The direction cosines of the unit vector perpendicular to the plane are same as the direction cosines of the normal to the plane.

Thus, the direction cosines of the unit vector perpendicular to the plane are: $\frac{-6}{7}, \frac{3}{7}, \frac{2}{7}$

The Plane Ex 29.15 Q13

Consider the given equation of the plane $2x-3y+4z-6=0$
The direction ratios of the normal to the plane are $2,\ -3$ *and* 4
Thus, the directio ratios of the line perpendicular to the plane are $2,\ -3$ *and* 4.
The equation of the line passing (x_1, y_1, z_1) *having direction ratios a,b and c is*

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

Thus, the equation of the line passing through the origin with direction ratios $2,\ -3$ *and* 4 *is*

$$\frac{x-0}{2}=\frac{y-0}{-3}=\frac{z-0}{4}$$

$$\Rightarrow \frac{x}{2}=\frac{y}{-3}=\frac{z}{4}=r, \text{ where } r \text{ is some constant}$$

Any point on the line is of the form $2r,\ -3r$ *and* $4r$
If the point $P(2r, -3r, 4r)$ *lies on the plane* $2x-3y+4z-6=0$,
it should satisfies the equation, $2x-3y+4z-6=0$
Thus, we have,

$$2(2r)-3(-3r)+4(4r)-6=0$$
$$\Rightarrow 4r+9r+16r-6=0$$
$$\Rightarrow 29r=6$$
$$\Rightarrow r=\frac{6}{29}$$

Thus, the coordinates of the point of intersection of the perpendicular from the origin and the plane are:

$$P\left(2\times\frac{6}{29},\ -3\times\frac{6}{29},\ 4\times\frac{6}{29}\right)=P\left(\frac{12}{29},\ \frac{18}{29},\ \frac{24}{29}\right)$$

The Plane Ex 29.15 Q14

The length of perpendicular from the point $\left(1,\ \frac{3}{2}, 2\right)$ to the plane $2x - 2y + 4z + 5 = 0$.

$$d=\left|\frac{2-3+8+5}{\sqrt{4+4+16}}\right|=\frac{12}{2\sqrt{6}}=\sqrt{6}$$

Let the foot of perpendicular be (x, y, z). So DR's are in proportional

$$\frac{x-1}{2}=\frac{y-\frac{3}{2}}{-2}=\frac{z-2}{4}=k$$

$$x=2k+1$$
$$y=-2k+\frac{3}{2}$$
$$z=4k+2$$

So using values of x, y, z in equation of the plane we have,

$$2(2k+1)-2\left(-2k+\frac{3}{2}\right)+4(4k+2)+5=0$$
$$4k+2+4k-3+16k+8+5=0$$
$$24k=-12$$
$$k=-\frac{1}{2}$$

$$(x, y, z)=\left(0, \frac{5}{2}, 0\right)$$

Ex - 30.1

Linear Programming Ex 30.1 Q1

The given data may be put in the following tabular form: –

Gadget	Foundry	Machine-shop	Profit
A	**10**	**5**	**Rs 30**
B	**6**	**4**	**Rs 20**
Firm's capacity per week	1000	600	

Let required weekly production of gadgets A and B be x and y respectively.

Given that, profit on each gadget A is Rs 30
So, profit on x gadget of type $A = 30x$
Profit on each gadget of type B = Rs 20
So, profit on y gadget of type $B = 20y$

Let Z denote the total profit, so

$$Z = 30x + 20y$$

Given, production of one gadget A requires 10 hours per week for foundry and gadget B requires 6 hours per week for foundry.

So, x units of gadget A requires $10x$ hours per week and y units of gadget B requires by hours per week, But the maximum capacity of foundry per week is 1000 hours, so

$$10x + 6y \le 1000$$

This is first constraint.

Given, production of one unit gadget A requires 5 hours per week of machine shop and production of one unit of gadget B requires 4 hours per week of machine shop.

So, x units of gadget A requires $5x$ hours per week and y units of gadget B requires $4y$ hours per week, but the maximum capacity of machine shop is 600 hours per week
So, $5x + 4y \le 600$

This is second constraint.
Hence, mathematical formulation of LPP is:
Find x and y which
Maximize $Z = 30x + 2y$

Subject to constraints,

$$10x + 6y \le 1000$$
$$5x + 4y \le 600$$

And, $x, y \ge 0$ [Since production cannot be less than zero]

Linear Programming Ex 30.1 Q2

The given information can be written in tabular form as below:

Product	Machine hours	Labour hours	Profit
A	**1**	**1**	**Rs 60**
B	**–**	**1**	**Rs 80**
Total capacity	400 for A	500	
Minimum supply of product B is 200 units.			

Let production of product A be x units and production of product B be y units.

Given, profit on one unit of product A = Rs 60
So, profit on x unit of product A = Rs $60x$
Given, profit on one unit of product B = Rs 80
So, profit on y units of product B = Rs $80y$

Let Z denote the total profit, so

$$Z = 60x + 80y$$

Given, minimum supply of product B is 200
So, $y \geq 200$ (First constraint)

Given that, production of one unit of product A requires 1 hour of machince hours, so x units of product A requires x hours but given total machine hours available for product A is 400 hours, so

$x \leq 400$ (Second constraint)

Given, each unit of product A and B requires one hour of labour hour, so x units of product A require x hours and y units of product B require y hours of labour hours but total labour hours available are 500, so

$x + y \leq 500$ (Third constraint)

Hence, mathematical formulation of LPP is,
Find x and y which

Minimize $Z = 60x + 80y$

Subject to constraints,

$y \geq 200$
$x \leq 400$
$x + y \leq 500$
$x, y \geq 0$ [Since production of product cannot be less than zero]

Linear Programming Ex 30.1 Q3

Product	Machine (M_1)	Machine (M_2)	Profit
A	4	2	3
B	3	2	2
C	5	4	4
Capacity maximum	2000	2500	

Let required production of product A, B and C be x, y and z units respectively.

Given, profit on one unit of product A, B and C are Rs 3, Rs 2, Rs 4, so
Profit on x unit of A, y unit of B and z unit of C are given by Rs. $3x$, Rs $2y$, Rs $4z$.

Let U be the total profit, so

$$U = 3x + 2y + 4z$$

Given, one unit of product A, B and C requires 4,3 and 5 minutes on machine M_1. So, x units of product A, y units of B and z units of product C need $4x$, $3y$ and $5z$ minutes on machine M_1 is 2000 minutes, so

$$4x + 3y + 5z \le 200 \quad \text{(First constraint)}$$

Given, one unit of product A, B and C requires 2,2 and 4 minutes on machine M_2. So, x units of A, y units of B and z units of C require $2x$, $2y$ and $4z$ minutes on machine M_2 is 2500 minutes, so

$$2x + 2y + 4z \le 2500 \quad \text{(Second constraint)}$$

Also, given that firm must manufacture 100 A's, 200 B's and 50 C's but not more than 150 A's.

$$100 \le x \le 150$$
$$y \ge 200 \quad \text{(Other constraints)}$$
$$z \ge 50$$

Hence, mathematical formulation of LPP is :–
Find x, y and z which

$$\text{maximize } U = 3x + 2y + 4z$$

Subject to constraints,

$$4x + 3y + 5z \le 2000$$
$$2x + 2y + 4z \le 2500$$
$$100 \le x \le 150$$
$$y \ge 200$$
$$z \ge 50$$

And, $x, y, z \ge 0$ [Since, x, y, z are non-negative]

Linear Programming Ex 30.1 Q4

Given information can be written in tabular form as below:

Product	M_1	M_2	Profit
A	**1**	**2**	**2**
B	**1**	**1**	**3**
Capacity	6 hours 40 min = 400 min.	10 hours = 600 min.	

Let required production of product A be x units and product B be y units.

Given, profit on one unit of product A and B are Rs 2 and Rs 3 respectively, so profits on x units of product A and y units of product B will be Rs $2x$ and Rs $3y$ respectively.

Let total profit be Z, so

$$Z = 2x + 3y$$

Given, production of one unit of product A and B require 1 and 1 minute on machine M_1 respectively, so production of x units of product A and y units of product B require x minutes and y minutes on machine M_1 but total time available on machine M_1 is 600 minutes, so

$$x + y \le 400 \quad \text{(First constraint)}$$

Given, production of one unit of product A and B require 2 minutes and 1 minutes on machine M_2 respectively. So production of x units of product A and y units of product B require $2x$ minutes and y minutes respectively on machine M_2 but machine M_2 is available for 600 minutes, so

$$2x + y \le 600 \quad \text{(Second constraint)}$$

Hence, mathematical formulation of LPP is:-

Find x and y which

maximize $Z = 2x + 3y$

Subject to constraints,

$$x + y \le 400$$
$$2x + y \le 600$$

and, $x, y \ge 0$ [Since production of product can not be less than zero]

Linear Programming Ex 30.1 Q5

Plant	*A*	*B*	*C*	Cost
I	**50**	**100**	**100**	**2500**
II	**60**	**60**	**200**	**3500**
Monthly demand	2500	3000	7000	

Let plant I requires x days and plant II requires y days per month to minimize cost.

Given, plant I and II costs Rs 2500 perday and Rs 3500 perday respectively, so cost to run plant I and II is Rs $2500x$ and Rs $3500y$ per month.

Let Z be the total cost per month, so

$$Z = 2500x + 3500y$$

Given, production of tyre A from plant I and II is 50 and 60 respectively, so production of tyre A from plant I and II will be $50x$ and $60y$ respectively per month but the maximum demand of tyre A is 2500 per month so,

$$50x + 60y \geq 2500 \qquad \text{[First constraint]}$$

Given, production of tyre B from plant I and II is 100 and 60 respectively, so production of tyre B from plant I and II will be $100x$ and $60y$ per month respectively but the maximum demand of tyre B is 3000 per month, so

$$100x + 60y \geq 3000 \qquad \text{[Second constraint]}$$

Given, production of tyre C from plant I and II is 100 and 200 respectively. So production of tyre B from plant I and II will be $100x$ and $200y$ per month respectively but the maximum demand of tyre C is 7000 per day, so

$$100x + 200y \geq 7000 \qquad \text{[Third constraint]}$$

Hence, mathematical formulation of LPP is..
Find x and y which

Minimize $Z = 2500x + 3500y$

Subject to constraint,

$$50x + 60y \geq 2500$$
$$100x + 60y \geq 3000$$
$$100x + 200y \geq 7000$$

And, $x, y \geq 0$ [Since number of days can not be less than zero]

Linear Programming Ex 30.1 Q6

Product	Man hours	Maximum demand	Profit
A	**5**	**7000**	**60**
B	**3**	**10000**	**40**
Total capacity	45000		

Let required production of product A be x units and production of product B be y units.

Given, profits on one unit of product A and B are Rs 60 and Rs 40 respectively, so profits on x units of product A and y units of product B are Rs $60x$ and Rs $40y$.

Let Z be the total profit, so

$$Z = 60x + 40y$$

Given, production of one unit of product A and B require 5 hours and 3 hours respectively man hours, so x unit of product A and y units of product B require $5x$ hours and $3y$ hours of man hours respectively but total man hours available are 45000 hours, so

$$5x + 3y \le 45000 \qquad \text{(First constraint)}$$

Given, demand for product A is maximum 7000, so

$$x \le 7000 \qquad \text{(Second constraint)}$$

Hence, mathematical formulation of LPP is,

Find x and y which

$$\text{maximize } Z = 60x + 40y$$

Subject to constraints,

$$5x + 3y \le 45000$$
$$x \le 7000$$
$$y \le 10000$$

$$x, y \ge 0 \qquad \text{[Since production can not be less than zero]}$$

Linear Programming Ex 30.1 Q7

Let x and y be the packets of 25 gm of Food I and Food II purchased. Let Z be the price paid. Obviously price has to be minimized.

Take a mass balance on the nutrients from Food I and II,

Calcium $10x + 4y \geq 20$

$5x + 2y \geq 10$ ....(i)

Protein $5x + 5y \geq 20$

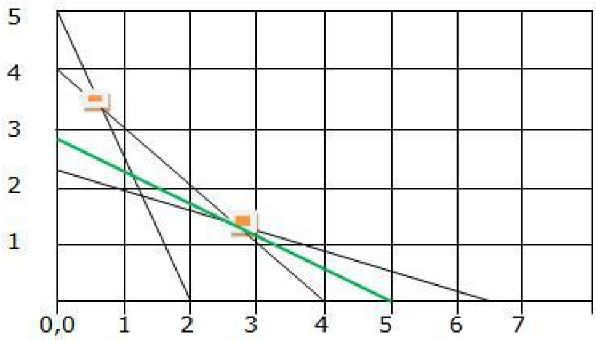
$x + y \geq 4$ ....(ii)

Calories $2x + 6y \geq 13$ ......(iii)

These become the constraints for the cost function, Z to be minimized ie., $0.6x + y = Z$, given cost of Food I is Rs 0.6/- and Rs 1/- per lb

From (i), (ii) & (iii) we get points on the X & Y-axis as [0, 5] & [2, 0] ; [0, 4] & [4, 0] ; [0, 13/6] & [6.5, 0]

Plotting these



The smallest value of Z is 2.9 at the point (2.75, 1.25). We cannot say that the minimum value of Z is 2.9 as the feasible region is unbounded.

Therefore, we have to draw the graph of the inequality $0.6x + y < 2.9$

Plotting this to see if the resulting line (in green) has any point common with the feasible region. Since there are no common points this is the minimum value of the function Z and the mix is

Food I = 2.75 lb; Food II = 1.25 lb; Price = Rs 2.9

When Z has an optimal value (maximum or minimum), where the variables *x* and *y* are subject to constraints described by linear inequalities, this optimal value must occur at a corner point (vertex) of the feasible region.

Here the feasible region is the unbounded region A-B-C-D

Computing the value of Z at the corner points of the feasible region ABHG

Point	Corner point	Value of Z = 0.6x + y
A	2, 5	6.2
B	0.67, 3.33	3.73
C	2.75, 1.25	2.9
D	6.5, 2.16	6.06

Linear Programming Ex 30.1 Q8

Given information can be tabulated as:-

Product	Grinding	Turning	Assembling	Testing	Profit
A B	1 2	3 1	6 3	5 4	2 3
Maximum capacity	30 hours	60 hours	200 hours	200 hours	

Let required production of product A and B be x and y respectively

Given, profits on one unit of product A and B are Rs 2 and Rs 3 respectively, so profits on x units of product A and y units of product B are given by $2x$ and $3y$ respectively. Let Z be total profit, so

$$Z = 2x + 3y$$

Given, production of 1 unit of product A and B require 1 hour and 2 hours of grinding respectively, so, production of x units of product A and y units of of product B require x hours and $2y$ hours of grinding respectively but maximum time available for grinding is 3 hours, so

$$x + 2y \le 30 \quad \text{(First constraint)}$$

Given, production of 1 unit of product A and B require 3 hours and 1 hours of turning respectively, so x units of product A and y units of of product B require $3x$ hours and y hours of turning respectively but total time available for turning is 60 hours, so

$$3x + y \le 60 \quad \text{(Second constraint)}$$

Given, production of 1 unit of product A and B require 6 hour and 3 hours of assembling respectively, so productinon of x units of product A and y units of product B require $6x$ hours and $3y$ hours of assembling respectively but total time available for assembling is 200 hours, so

$$6x + 3y \le 200 \quad \text{(Third constraint)}$$

Given, production of 1 unit of product A and B require 5 hours and 4 hours of testing respectively, so productinon of x units of product A and y units of product B require $5x$ hours and $4y$ hours of testing respectively but total time available for testing is 200 hours, so

$$5x + 4y \le 200 \quad \text{(Fourth constraint)}$$

Hence, mathematical formulation of LPP is,
Find x and y which

$$\text{maximize } Z = 2x + 3y$$

Subject to constraints,

$$x + 2y \le 30$$
$$3x + y \le 60$$
$$6x + 3y \le 200$$
$$5x + 4y \le 200$$

and, $x, y \ge 0$ [Since production can not be negative]

Linear Programming Ex 30.1 Q9

Given information can be tabulated as below:

Foods	Vitamin A	Vitamin B	Cost
F_1	**2**	**3**	**5**
F_2	**4**	**2**	**2.5**
Minimum daily requirement	40	50	

Let required quantity of food F_1 be x units and quantity of food F_2 be y units.

Given, costs of one unit of food F_1 and F_2 are Rs 5 and Rs 2.5 respectively, so costs of x units of food F_1 and y units of food F_2 are Rs $5x$ and Rs $2.5y$ respectively.
Let Z be the total cost, so

$$Z = 5x + 2.5y$$

Given, one unit of food F_1 and food F_2 contain 2 and 4 units of vitamin A respectively, so x unit of Food F_1 and y units of food F_2 contain $2x$ and $4y$ units of vitamin A respectively, but minimum requirement of vitamin A is 40 unit, so

$$2x + 4y \geq 40 \qquad \text{(First constraint)}$$

Given, one unit of food F_1 and food F_2 contain 3 and 2 units of vitamin B respectively, so x unit of Food F_1 and y units of food F_2 contain $3x$ and $2y$ units of vitamin B respectively, but minimum daily requirement of vitamin B is 40 unit, so

$$3x + 2y \geq 50 \qquad \text{(Second constraint)}$$

Hence, mathematical formulation of LPP is,
Find x and y which

Minimize $Z = 5x + 2.5y$

Subject to constraint,

$$2x + 4y \geq 40$$
$$3x + 2y \geq 50$$

$x, y \geq 0$ [Since requirement of food F_1 and F_2 can not be less than zero.]

Linear Programming Ex 30.1 Q10

Let the number of automobiles produced be x and let the number of trucks produced be y.

Let Z be the profit function to be maximized.
$Z = 2{,}000x + 30{,}000y$

The constraints are on the man hours worked

Shop A	$2x + 5y \le 180$	(i)	assembly
Shop B	$3x + 3y \le 135$	(ii)	finishing
	$x \ge 0$; $y \ge 0$		

Corner points can be obtained from
$2x + 5y = 180 \Rightarrow x=0;\ y=36$ and $x=90;\ y=0$
$3x + 3y \le 135 \Rightarrow x=0;\ y=45$ and $x=45;\ y=0$
Solving (i) & (ii) gives $x = 15$ & $y = 30$

Corner point	Value of Z =2,000x + 30,000y
0,0	0
0, 36	10,80,000
15, 30	9,30,000
45, 0	90,000

0 automobiles and 36 trucks will give max profit of 10,80,000/-

Linear Programming Ex 30.1 Q11

	Taylor A		Taylor B	Limit
Variable	x		y	
Shirts	6x	+	10y	≥ 60
Pants	4x	+	4y	≥ 32
Earn Rs.	150	+	200	Z

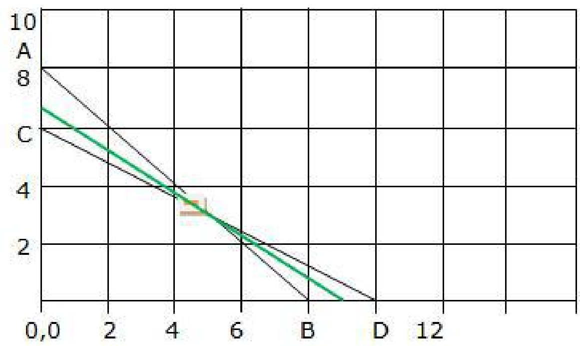
The above LPP can be presented in a table above.

To minimize labour cost means to assume minimize the earnings i.e, Min $Z = 150x + 200y$
s.t. the constraints

$x \ge 0;\ y \ge 0$	at least 1 shirt & pant is required
$6x + 10y \ge 60$	require at least 60 shirts
$4x + 4y \ge 32$	require at least 32 pants

Solving the above inequalities as equations we get,
$x = 5$ and $y = 3$

other corner points obtained are [0, 6] & [10, 0]
[0, 8] & [8,0]



The feasible region is the open unbounded region A-E-D

Point E(5, 3) may not be the minimal value. So, plot $150x + 200y < 1350$ to see if there is a common region with A-E-D

The green line has no common point , therefore

Corner point	Value of Z = 150x + 200y
0,8	0
10, 0	1500
5, 3	1350

Stitching 5 shirts and 3 pants minimizes labour cost to Rs.1350/-

Linear Programming Ex 30.1 Q12

	Model 314		Model 535	Limit
Variable	x		y	
F class	20x	+	20y	≥ 160
T class	30x	+	60y	≥ 300
Cost	1.x lakh	+	1.5y lakh	Z

The above LPP can be presented in a table above.

The flight cost is to be minimized i.e, Min $Z = x + 1.5y$
s.t. the constraints

$x \geq 2$	at least 2 planes of model 314 must be used
$y \geq 0$	at least 1 plane of model 535 must be used
$20x + 20y \geq 160$	require at least 160 F class seats
$30x + 60y \geq 300$	require at least 300 T class seats

Solving the above inequalities as equations we get,
When x=0, y=8 and when y=0, x=8
When x=0, y=5 and when y=0, x=10

We get an unbounded region 8-E-10 as a feasible solution. Plotting the corner points and evaluating we have,

Corner point	Value of $Z = x + 1.5y$
10, 0	10
0, 8	12
6, 2	9

Since we obtained an unbounded region as the feasible solution a plot of Z (x+1.5 < 9) is plotted.

Since there are no common points point E is the point that gives a minimum value.

Using 6 planes of model 314 & 2 of model 535 gives minimum cost of 9 lakh rupees.

Linear Programming Ex 30.1 Q13

Given information can be tabulated as below

Sets	Time requirement	Points
I	**3**	**5**
II	**2**	
III	**4**	**6**

Time for all three sets $= 3\frac{1}{2}$ hours

Time for Set I and Set II $= 2\frac{1}{2}$ hours

Number of questions maximum 100

Let he should x, y, z questions from set I, II and III respectively.

Given, each question from set I, II, III earn 5, 4, 6 points respectively, so x questions of set I, y questions of set II and z questions of set III earn $5x$, $4y$ and $6z$ points, let total point credit be U

So, $U = 5x + 4y + 6z$

Given, each question of set I, II and III require 3,2 and 4 minutes respectively, so x questions of set I, y questions of set II and z questions of set III require $3x$, $2y$ and $4z$ mimutes respectively but given that total time to devote in all three sets is $3\frac{1}{2}$ hours $= 210$ minutes and first two sets is $2\frac{1}{2}$ hours $= 150$ minutes

So,

$$3x + 2y + 4z \le 210 \quad \text{(First constraint)}$$

$$3x + 2y \le 150 \quad \text{(Second constraint)}$$

Given, total number of questions cannot exceed 100

So, $x + y + z \le 100$ (Third constraint)

Hence, mathematical formulation of LPP is

Find x and y which

maximize $U = 5x + 4y + 6z$

Subject to constraint,

$$3x + 2y + 4z \le 210$$

$$3x + 2y \le 150$$

$$x + y + z \le 100$$

$$x, y, z \ge 0$$

[Since number of questions to solve from each set cannot be less than zero]

Linear Programming Ex 30.1 Q14

Given information can be tabulated as below

Product	Yield	Cultivation	Price	Fertilizers
Tomatoes	2000 kg	5 days	1	100 kg
Lettuce	3000 kg	6 days	0.75	100 kg
Radishes	1000 kg	5 days	2	50 kg

Average 2000 kg/per acre
Total land $= 100$ Acre
Cost g fertilizers $=$ Rs 0.50 per kg.
A total of 400 days of cultivation labour with Rs 20 per day

Let required quantity of field for tomatoes, lettuce and radishes be x, y and z Acre respectively.

Given, costs of cultivation and harveshing of tomatoes, lettuce and radishes are 5×20 = Rs 100, 6×20 = Rs 120, 5×20 = Rs 100 respectively per acre. Cost of fertilizers for tomatoes, lettuce and radishes 100×0.50 = Rs 50, 100×0.50 = Rs 50 and 50×0.50 = Rs 25 respectively per acre.

So, total costs of production of tomatoes, lettuce and radishes are Rs $100 + 50$ = Rs $150x$, Rs $120 + 50$ = Rs $170y$ and radishes are Rs $100 + 25$ = Rs $125z$ respectively total selling price of tomatoes, lettuce and radishes, according to yield are 2000×1 = Rs $2000x$, 3000×0.75 = Rs $2250y$ and 100×2 = Rs $2000z$ respectively.

Let U be the total profit,
So,

$$U = (2000x - 150x) + (2250y - 170y) + (2000z - 125z)$$
$$U = 1850x + 2080y + 1875z$$

Given, farmer has 100 acre form
So, $x + y + z \le 100$ (First constraint)

Number of cultivation and harvesting days are 400
So, $5x + 6y + 5z \le 400$

Hence, mathematical formulation of LPP is
Find x, y, z which
maximize $U = 1850x + 2080y + 1875z$

Subject to constraint,

$$x + y + z \le 100$$
$$5x + 6y + 5z \le 400$$

$$x, y, z \ge 0$$

[Since from used for cultivation cannot be less than zero.]

Linear Programming Ex 30.1 Q15

Given information can be tabulated as below:

Product	Department 1	Department 2	Selling price	Labour cost	Raw material cost
A	3	4	25	16	4
B	2	6	30	20	4
Capacity	130	260			

Let the required product of product A and B be x and y units respectively.

Given, labour cost and raw material cost of one unit of product A is Rs 16 and Rs 4, so total cost of product A is Rs 16 + Rs 4 = Rs 20
And given selling price of 1 unit of product A is Rs 25,
So, profit on one unit of product

$A = 25 - 20 = \text{Rs } 5$

Again, given labour cost and raw material cost of one unit of product B is Rs 20 and Rs 4
So, that cost of product B is Rs 20 + Rs 4 = Rs 24
And given selling price of 1 unit of product B is Rs 30
So, profit on one unit of product $B = 30 - 24 = \text{Rs } 6$

Hence, profits on x unit of product A and y units of product B are Rs $5x$ and Rs $6y$ respectively.
Let Z be the total profit , so $Z = 5x + 6y$

Given, production of one unit of product A and B need to process for 3 and 4 hours respectively in department 1, so production of x units of product A and y units of product B need to process for $3x$ and $4y$ hours respectively in Department 1. But total capacity of Department 1 is 130 hour ,
So, $3x + 2y \le 130$ (First constraint)

Given, production of one unit of product A and B need to process for 4 and 6 hours respectively in department 2, so production of x units of product A and y units of product B need to process for $4x$ and $6y$ hours respectively in Department 2 but total capacity of Department 2 is 260 hours
So, $4x + 6y \le 260$ (Second constraint)

Hence, mathematical formulation of LPP is,
Find x and y which
Maximize $Z = 5x + 6y$

Subject to constraint,

$3x + 2y \le 130$
$4x + 6y \le 260$

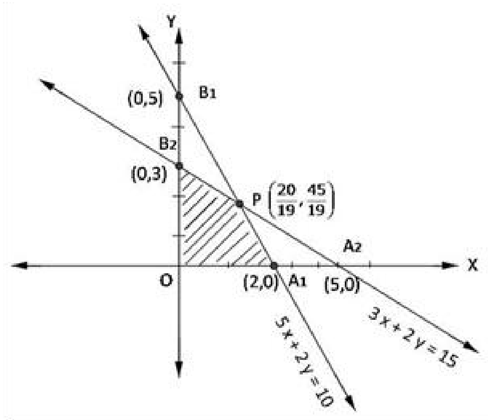
$x, y \ge 0$ [Since production cannot be less than zero]

Ex 30.2

Linear Programming Ex 30.2 Q1

Converting the given inequations into equations, we get

$3x + 5y = 15,\ 5x + 2y = 10,\ x = 0,\ y = 0$



Region represented by $5x + 2y \le 10$: The line meets coordinate axes at $A_1(2,0)$ and $B_1(0,5)$ respectively. Join these points to obtain the line $5x + 2y = 10$, clearly, $(0,0)$ satisfies the in eqation $5x + 2y \le 10$, so, the region in xy-plane that contains the origin represents the solution set if the given in equation.

Region represented by $3x + 5y \le 10$: The line meets coordinate axes at $A_2(5,0)$ and $B_2(0,3)$ respectively. Join these points to obtain the line $3x + 5y = 15$, clearly, $(0,0)$ satisfies the in eqation $3x + 5y \le 15$, so, the region in xy-plane contains the origin represents the solution set if the given in equation.

Region represented by $x \ge 0,\ y \ge 0$: It clearly represents first quadrant of xy-plane. Common region to regions represented by above in equalities.

The coordinates of the corner points of the shaded region are $O(0,0)$, $A(2,0)$, $P\left(\frac{20}{19}, \frac{45}{19}\right)$, $B_2(0,3)$.

The value of $Z = 5x + 3y$ at

$O(0,0) \quad = 5 \times +3 \times 0$

$A(2,0) \quad = 5 \times 2 + 3 \times 0 = 10$

$P\left(\frac{20}{19}, \frac{45}{19}\right) \quad = 5\left(\frac{20}{19}\right) + 3\left(\frac{45}{19}\right) = \frac{235}{19}$

$B_2(0,3) \quad = 5 \times 0 + 3 \times 3 = 9$

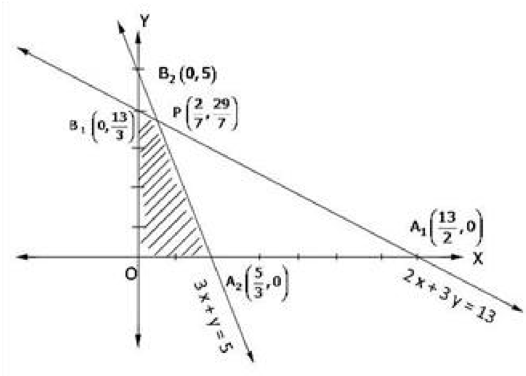
Clearly, Z is maximum at $P\left(\frac{20}{19}, \frac{45}{19}\right)$

So, $x = \frac{20}{19}$, $y = \frac{45}{19}$, maximum $Z = \frac{235}{19}$

Linear Programming Ex 30.2 Q3

Converting the given inequations into equations, we get

$2x + 3y = 13$, $3x + y = 5$, and $x = 0$, $y = 0$



Region represented by $2x + 3y \le 13$: The line meets coordinate axes at $A_1\left(\frac{13}{2},0\right)$ and $B_1\left(0,\frac{13}{3}\right)$respectively. Join these points to obtain the line $2x + 3y = 13$, clearly, $(0,0)$ satisfies the in eqation $2x + 3y \le 13$, so, the region in xy-plane that contains origin represents the solution set of $2x + 3y \le 13$.

Region represented by $3x + y \le 5$: The line meets coordinate axes at $A_2\left(\frac{5}{3},0\right)$ and $B_2(0,5)$ respectively. Join these points to obtain the line $3x + y = 5$, clearly, $(0,0)$ satisfies the in eqation $3x + y \le 5$, so, the region in xy-plane that contains origin represents the solution set of $3x + y \le 5$.

Region represented by $x, y \ge 0$: It clearly represent first quadrant of xy-plane. The common region to regions represented by above in equalities.

The coordinates of the corner points of the shaded region are $0(0,0)$, $A\left(\frac{5}{3},0\right)$, $P\left(\frac{2}{7},\frac{29}{7}\right)$, $B_2\left(0,\frac{13}{3}\right)$.

The value of $Z = 9x + 3y$ at

$0(0,0) \quad = 9(0) + 3(0) = 0$

$A_1\left(\frac{5}{3},0\right) \quad = 9\left(\frac{5}{3}\right) + 3(0) = 15$

$P\left(\frac{2}{7},\frac{29}{7}\right) \quad = 9\left(\frac{2}{7}\right) + 3\left(\frac{29}{7}\right) = 15$

$B_2\left(0,\frac{13}{3}\right) \quad = 9(0) + 3\left(\frac{13}{3}\right) = 13$

Clearly, Z is maximum at at every point on the line joining A_1 and P, so

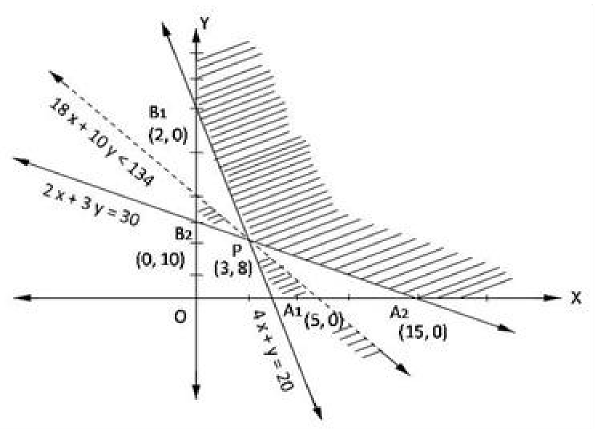
$x = \frac{5}{3}$ or $\frac{2}{7}$, $y = 0$ or $\frac{29}{7}$

and maximum $Z = 15$.

Linear Programming Ex 30.2 Q3

Converting given inequations into equations as

$4x + y = 20,\ 2x + 3y = 30,\ x = 0,\ y = 0$



Region represented by in equation $4x + y \geq 20$: The line $4x + y = 20$ meets the coordinate axes at $A_1\,(5,0)$ and $B_1\,(0,20)$. Joining A_1B_1 we get $4x + y = 20$. Clearly, $(0,0)$, also does not satisfies the in eqation, so the region does not containing the origin represents the in equality $4x + y \geq 20$ in the xy-plane.

Region represented by in equation $2x + 3y \geq 30$: The line $2x + 3y = 30$ meets the coordinate axes at $A_2\,(15,0)$ and $B_2\,(0,20)$. Obtain line $2x + 3y = 30$ by joining A_2 and B_2. Clearly, $(0,0)$, does not satisfies the in eqation $2x + 3y \geq 30$, so the region does not containing the origin represents the in equality $2x + 3y \geq 30$ in the xy-plane.

Region represented by $x, y \geq 0$: $x, y \geq 0$ represents the first quadrant of xy-plane.

The shaded region is the feasible region with corner points $A_2\,(15,0), P\,(3,8), B_1\,(0,20)$ where P is obtained by solving $2x + 3y = 30$ and $4x + y = 20$ simultaneously.

The value of $Z = 18x + 10y$ at

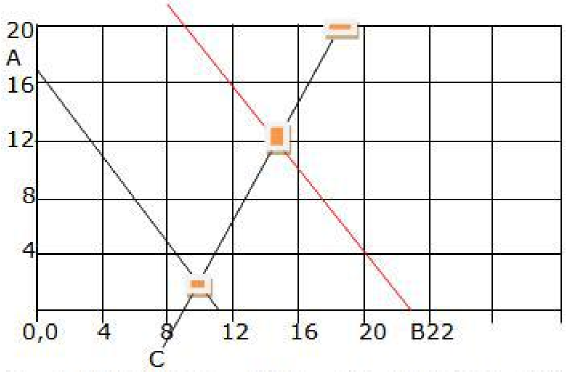
$A_2\,(15,0)$	$= 18(15) + 10(8) = 270$
$P\,(3,8)$	$= 18(3) + 10(8) = 134$
$B_1\,(0,20)$	$= 18(0) + 10(20) = 200$

Clearly, Z is manimum at $x = 3$ and $y = 8$. The minimum value of Z is 134.

We observe that open half plane represented by $18x + 10y < 134$ does not have points in common with the solution region. So Z has

Minimum value $= 134$ at $x = 3,\ y = 8$

Linear Programming Ex 30.2 Q4



$2x-y \geq 18$; when $x = 12$, $y = 6$ & when $y=0$, $x=9$
$3x+2y \leq 34$; when $x = 0$, $y = 17$ & when $y=0$, $x=34/3$

Plotting these points gives line AB and CD
The feasible area is the unbounded area D-E-12

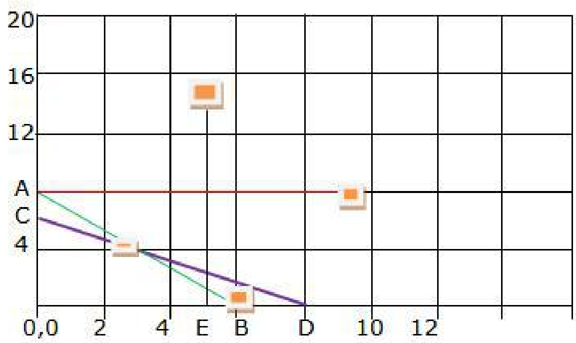
Corner point	Value of $Z = 50x + 30y$
10, 2	560
11.3, 17	1076.66

The maximize value of $Z = 50x+30y$, occurs at $x = 34/3$, $y = 17$

Since we have an unbounded region as the feasible area plot $50x + 30y > 1076.66$

Since the region D-F-B has common points with region D-E-12 the problem has no optimal maximum value.

Linear Programming Ex 30.2 Q5



$3x+4y \leq 24$; when $x = 0$, $y = 6$ & when $y=0$, $x=8$, line AB

$8x+6y \leq 48$; when $x = 0$, $y = 8$ & when $y=0$, $x=6$, line CD

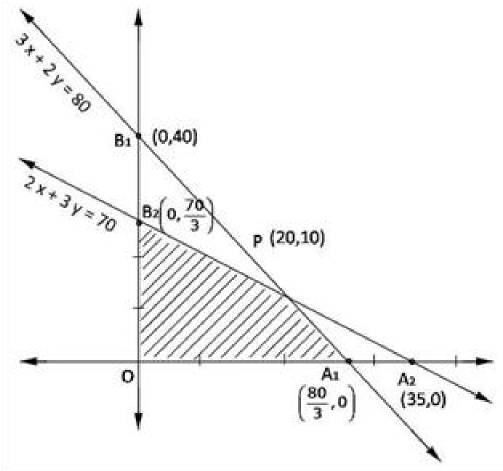
Plotting $x \leq 5$ gives line EF; Plotting $y \leq 6$ gives line AG
The feasible area is 0,0-C-H-G-E

Corner point	Value of $Z = 4x + 3y$
0, 0	0
0, 6	18
3.4, 3.4	24
5, 1	23
5,0	20

Linear Programming Ex 30.2 Q6

Converting the inequations into equations as

$3x + 2y = 80,\ 2x + 3y = 70,\ x = y = 0$



Region represented by $3x + 2y \le 80$: Line $3x + 2y = 80$ meets coordinate axes at $A_1\left(\frac{80}{3}, 0\right)$ and $B_1\ (0, 40)$, clearly, $(0,0)$ satisfies the $3x + 2y \le 80$, so, region containing the origin represents by $3x + 2y \le 80$ in xy-plane

Region represented by $2x + 3y \le 70$: Line $2x + 3y = 70$ meets the coordinate axes at $A_2\ (35,0)$ and $B_2\left(0, \frac{70}{3}\right)$, clearly, $(0,0)$ satisfies the $2x + 3y \le 70$ so, the region containing the origin represents by $2x + 3y \le 70$ in xy-plane

Region represented by $x, y \ge 0$: It represent the first quadrant in xy-plane

So, shaded area $OA_1 P B_2$ represents the feasible region.

Coordinate of $P\ (20,10)$ can be obtained by solving $3x + 2y = 80$ and $2x + 3y = 70$

Now, the value of $Z = 15x + 10y$ at

$O\ (0,0) \quad = 15(0) + 10(0) = 0$

$A_1\left(\frac{80}{3}, 0\right) \quad = 15\left(\frac{80}{3}\right) + 10(0) = 400$

$P\ (20,10) \quad = 15(20) + 10(10) = 400$

$B_2\left(0, \frac{70}{3}\right) \quad = 15(0) + 10\left(\frac{70}{3}\right) = \frac{700}{3}$

So, maximum $Z = 400$ is on each and every point on the line joining A_1P, so we can have,

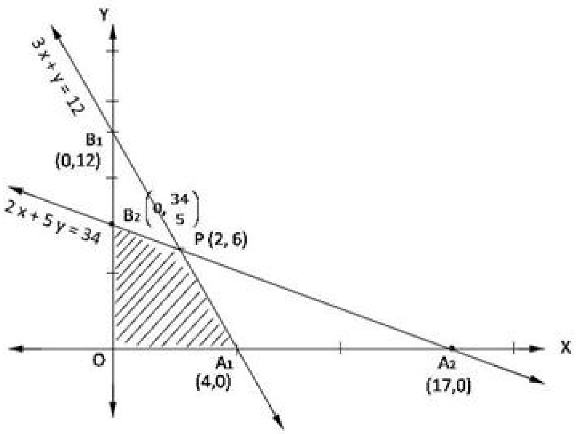
maximum $Z = 400$ at $x = \frac{80}{3}$ and $y = 0$

maximum $Z = 400$ at $x = 20$ and $y = 10$

Linear Programming Ex 30.2 Q7

Converting the given inequations into equations

$3x + y = 12,\ 2x + 5y = 34,\ x = y = 0$



Region represented by $3x + y \le 12$: Line $3x + y = 12$ meets the coordinate axes at $A_1(4,0)$ and $B_1(0,12)$, clearly, $(0,0)$ satisfies $3x + y \le 12$, so, region containing origin is represented by $3x + y \le 12$ in xy-plane

Region represented by $2x + 5y \le 34$: Line $2x + y = 34$ meets coordinate axes at $A_2(17,0)$ and $B_2\left(0,\frac{34}{5}\right)$, clearly, $(0,0)$ satisfies the $2x + 5y \le 34$ so, region containing origin represents $2x + 5y \le 34$ in xy-plane

Region represented by $x, y \ge 0$: It represent the first quadrant in xy-plane

Therefore, shaded area OA_1PB_2 is the feasible region.

The coordinate of $P(2,6)$ is obtained by solving $2x + 5y = 34$ and $3x + y = 12$

The value of $Z = 10x + 6y$ at

$O(0,0) \quad = 10(0) + 6(0) = 0$

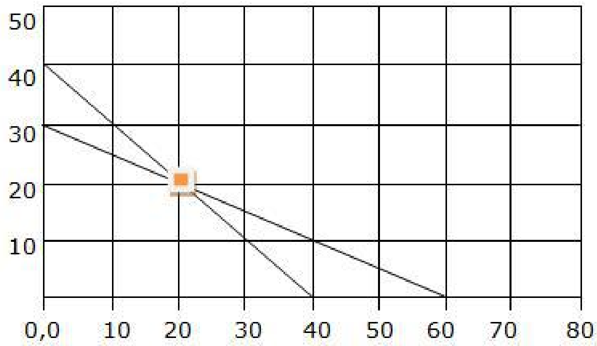
$A_1(4,0) \quad = 10(4) + 6(0) = 40$

$P(2,6) \quad = 10(2) + 6(6) = 56$

$B_2\left(0,\frac{34}{5}\right) \quad = 10(0) + 6\left(\frac{34}{5}\right) = \frac{204}{5} = 40\frac{4}{5}$

Hence, maximum $Z = 56$ at $x = 2,\ y = 6$

Linear Programming Ex 30.2 Q8



$2x+2y\leq 80$; when $x=0$, $y=40$ and when $y=0$, $x=40$
$2x+4y\leq 120$; when $x=0$, $y=30$ and when $y=0$, $x=60$

The intersection of the two plotted lines gives (20, 20)
Feasible area is 30-C-40

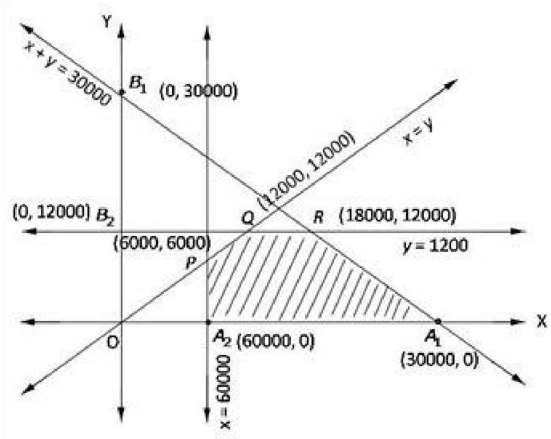
Corner point	Value of $Z = 3x + 4y$
0, 0	0
0, 30	120
20, 20	140
40, 0	120

The maxima is obtained at $x=20$, $y=20$ and is 140

Linear Programming Ex 30.2 Q9

Converting the given inequations into equations,

$x + y = 30000,\ y = 12000,\ x = 6000,\ x = y,\ x = y = 0$



Region represented by $x + y \le 30000$: Line $x + y = 30000$ meets the coordinate axes at $A_1(30000,0)$ and $B_1(0,30000)$, clearly $(0,0)$ satisfies $x + y \le 30000$, so, region containing the origin represents $x + y \le 30000$ in xy-plane

Region represented by $y \le 12000$: Line $y = 12000$ is parallel to x -axis and meets y -axis at $B_2(0,12000)$. Clearly $(0,0)$ satisfies $y \le 12000$, so, region containing origin represents $y \le 12000$ in xy -plane.

Region represented by $x \le 6000$: Line $x = 6000$ is parallel to y -axis and meets x -axis at $A_2(6000,0)$. Clearly $(0,0)$ satisfies $x \le 6000$, so, region containing origin represents $x \le 6000$ in xy -plane.

Region represented by $x \ge y$: Line $x = y$ passes through origin and point $Q(12000,12000)$. Clearly, $A_2(6000,0)$ satisfies $x \ge y$, so, region containing $A_2(6000,0)$ represents $x \ge y$ in xy -plane.

Region represented by $x, y \ge 0$: It represents the first quadrant in xy -plane.

Shaded region A_2A_1QP represents the feasible region.

Coordinates of $R(18000,12000)$ is obtained by solving $x + y = 30000$ and $y = 12000$, $Q(12000,12000)$ is obtained by solving $x = y$ and $y = 12000$, $P(6000,6000)$ is obtained by solving $x = y$ and $x = 6000$.

The value of $Z = 7x + 10y$ at

$A_2(6000,0) \quad = 7(6000) + 10(0) = 42000$

$A_1(30000,0) \quad = 7(30000) + 10(0) = 210000$

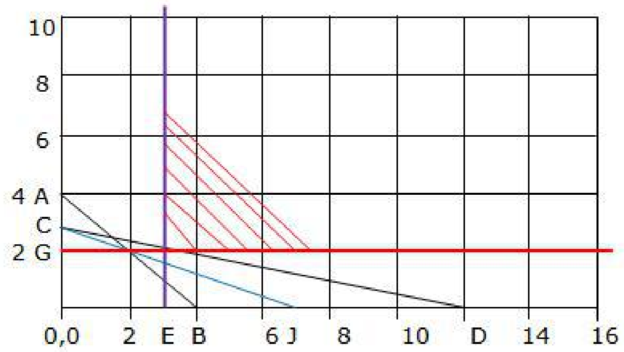
$R(18000,12000) \quad = 7(18000) + 10(12000) = 246000$

$Q(12000,12000) \quad = 7(12000) + 10(12000) = 204000$

$P(6000,6000) \quad = 7(6000) + 10(6000) = 102000$

So, maximum $Z = 246000$ at $x = 18000,\ y = 12000$

Linear Programming Ex 30.2 Q10



2x+2y≥8 ;When x=0, y=4 & when y=0, x=4 line AB
x+4y≥12; When x=0, y=3 & when y=0, x=12 line CD
x≥3, y≥2 are the lines parallel to Y-axis and X-axis resp.

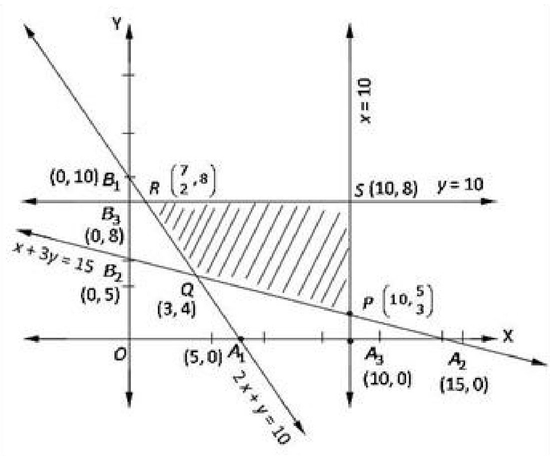
The diverging shaded area in red lines is the area of feasible solution. This area is unbounded.
Z = 2x+4y @ (3,2) = 14.

Plot 2x+4y > 14 line CJ to see if there is any common region. There is no common region so there is no optimal solution.

Linear Programming Ex 30.2 Q11

Converting the given inequations into equations,

$$2x + y = 10,\ x + 3y = 15,\ x = 10,\ y = 8,\ x = y = 0$$



Region represented by $2x + y \geq 10$: Line $2x + y = 10$ meets coordinate axes at $A_1(5,0)$ and $B_1(0,10)$. Clearly, $(0,0)$ does not satisfy $2x + y \geq 10$, so, region not containing origin represents $2x + y \geq 10$ in xy - plane.

Region represented by $x + 3y \geq 15$: Line $x + 3y = 15$ meets coordinate axes at $A_2(15,0)$ and $B_2(0,5)$. Clearly, $(0,0)$ does not satisfy $x + 3y \geq 15$, so, region not containing origin represents $x + 3y \geq 15$ in xy - plane.

Region represented by $x \leq 10$: Line $x = 10$ is parallel to y-axis and meet x-axis at $A_3(10,0)$. Clearly $(0,0)$ satisfies $x \leq 10$, so region containing origin represent $x \leq 10$ in xy-plane.

Region represented by $y \leq 8$: Line $y = 8$ is parallel to x-axis and meet y-axis at $B_3(0,8)$, clearly $(0,0)$ satisfies $y \leq 8$, so region containing origin represent $y \leq 8$ in xy-plane.

Region represented by $x, y \geq 0$: It represent the first quadrant in xy-plane.

Shaded region $QPSR$ is the feasible region. $Q(3,4)$ is obtained by solving $2x + y = 10$ and $x + 3y = 15$, $P\left(10, \frac{5}{3}\right)$ is obtained by solving $x + 3y = 15$ and $x = 10$, $R\left(\frac{7}{2}, 8\right)$ is obtained by $2x + y = 10$ and $y = 8$.

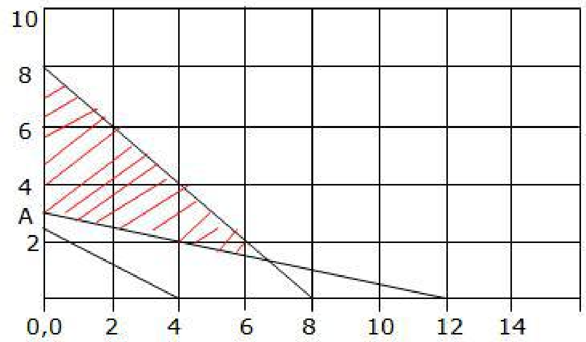
The value of $Z = 5x + 3y$ at

$$P\left(10, \frac{5}{3}\right) = 5(10) + 3\left(\frac{5}{3}\right) = 55$$
$$Q(3,4) = 5(3) + 3(4) = 27$$
$$R\left(\frac{7}{2}, 8\right) = 5\left(\frac{7}{2}\right) + 3(8) = \frac{83}{2} = 41\frac{1}{2}$$
$$S(10,8) = 5(10) + 3(8) = 74$$

So,

Minimum $Z = 27$ at $x = 3$, $y = 4$

Linear Programming Ex 30.2 Q12



$x + y \leq 8$; when x=0, y=8 & when y=0, x=8, line 8-8
$x + 4y \geq 12$; when x=0, y=3 & when y=0, x=12 line A-12
$5x+8y=20$; when x=0, y=5/2 & when y=0, x=4

The shaded area in red is the area of feasible solution.

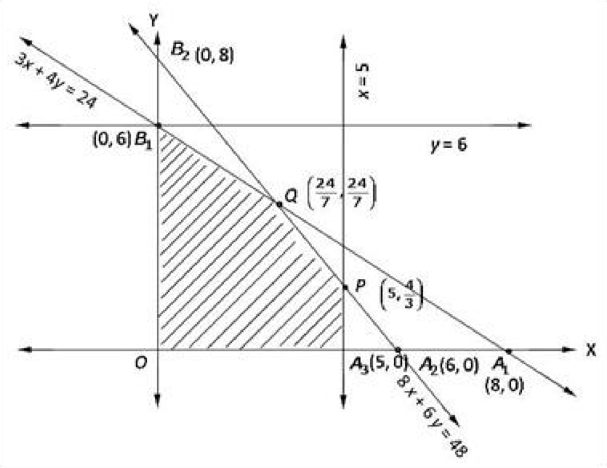
Corner point	Value of Z = 30x + 20y
0, 3	60
0, 8	160
6.66, 1.33	226.66

The maxima is obtained at x=6.66, y=1.33 and is 226.66

Linear Programming Ex 30.2 Q13

Converting the given inequations into equations,

$$3x + 4y = 24,\ 8x + 6y = 48,\ x = 5,\ y = 6,\ x = y = 0$$



Region represented by $3x + 4y \le 24$: Line $3x + 4y = 24$ meets coordinate axes at $A_1(8, 0)$ and $B_1(0, 6)$, clearly $(0,0)$ satisfies $3x + 4y \le 24$, so region containing origin represents $3x + 4y \le 24$ in xy -plane.

Region represented by $8x + 6y \le 48$: Line $8x + 6y = 48$ meets coordinate axes at $A_2(6,0)$ and $B_2(0,8)$. Clearly, $(0,0)$ satisfies $8x + 6y \le 48$, so region containing origin represents $8x + 6y \le 48$ in xy -plane.

Region represented $x \le 5$: Line $x = 5$ is parallel to y-axis and meets x-axis at $A_3(5, 0)$. Clearly $(0,0)$ satisfies $x \le 5$, so region containing origin represent $x \le 5$ in xy-plane.

Region represented by $y \le 6$: Line $y = 6$ is parallel to x-axis and meets y-axis at $B_1(0,6)$. Clearly $(0,0)$ satisfies $y \le 6$, so, region containing origin represents $y \le 6$ in xy-plane.

Region represented by $x, y \ge 0$: It represents the first quadrant in xy-plane.

So, shaded region QA_3PQB represents feasible region.

Coordinate of $P\left(5, \frac{4}{3}\right)$ is obtained by solving $8x + 6y = 48$ and $x = 5$, coordinate of $Q\left(\frac{24}{7}, \frac{24}{7}\right)$ is obtained by solving $3x + 4y = 24$ and $8x + 6y = 48$.

The value of $Z = 4x + 3y$ at

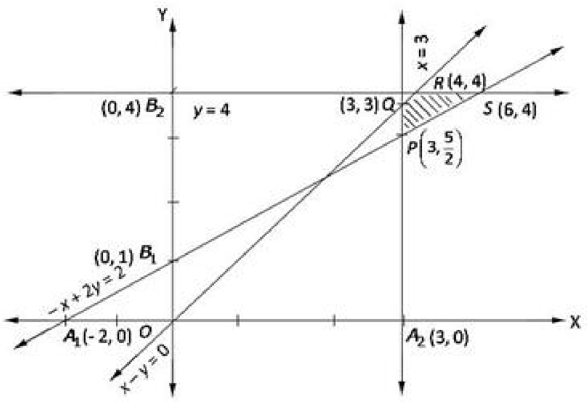
$$\begin{aligned}
&O(0, 0) && = 4(0) + 3(0) = 0\\
&A_3(5,0) && = 4(5) + 3(0) = 20\\
&P\left(5, \frac{4}{3}\right) && = 4(5) + 3\left(\frac{4}{3}\right) = 24\\
&Q\left(\frac{24}{7}, \frac{24}{7}\right) && = 4\left(\frac{24}{7}\right) + 3\left(\frac{24}{7}\right) = 24\\
&B_1(0,6) && = 4(0) + 3(6) = 18
\end{aligned}$$

So, maximum $Z = 24$ at $x = 5,\ y = \frac{4}{3}$ or $x = \frac{24}{7},\ y = \frac{24}{7}$ or at every point joining PQ.

Linear Programming Ex 30.2 Q14

Converting the given inequations into equations,

$$x - y = 0,\ -x + 2y = 2,\ x = 3,\ y = 4,\ x = y = 0$$



Region represented by $x - y \geq 0$: $x - y = 0$ is a line passing through origin and $R(4,4)$. Clearly, $(3,0)$ satisfies $x - y \geq 0$, so, region containing $(3,0)$ represents $x - y \geq 0$ in xy-plane.

Region represented by $-x + 2y \geq 2$: Line $-x + 2y = 2$ meets coordinate axes at $A_1(-2,0)$ and $B_1(0,1)$. Clearly, $(0,0)$ does not satisfy $-x + 2y \geq 2$, so, region not containing origin represents $-x + 2y \geq 2$ in xy-plane.

Region represented $x \geq 3$: Line $x = 3$ is parallel to y-axis and meets x-axis at $A_2(3,0)$. Clearly, $(0,0)$ does not satisfy $x \geq 3$, so region not containing origin represent $x \geq 3$ in xy-plane.

Region represented by $y \leq 4$: Line $y = 4$ is parallel to x-axis and meets y-axis at $B_2(0,4)$. Clearly $(0,0)$ satisfies $y \leq 4$, so region containing origin represents $y \leq 4$ in xy-plane.

Region represented by $x, y \geq 0$: It represent the first quadrant in xy-plane.

So, shaded region $PQRS$ represents feasible region.

The coordinate of $P\left(3, \frac{5}{2}\right)$ is obtained by solving $x = 3$ and $-x + 2y = 2$, $Q(3,3)$ by solving $x = 3$ and $x - y = 0$, $R(4,4)$ by solving $x = 4$ and $x - y = 0$, $S(6,4)$ by solving $y = 4$ and $-x + 2y = 2$

The value of $Z = x - 5y + 20$ at

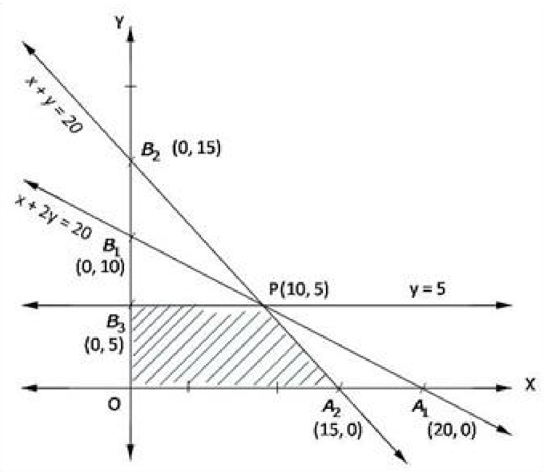
$$P\left(3, \frac{5}{2}\right) = 3 - 5\left(\frac{5}{2}\right) + 20 = \frac{21}{2} = 11\frac{1}{2}$$
$$Q(3,3) = 3 - 5(3) + 20 = 8$$
$$R(4,4) = 4 - 5(4) + 20 = 4$$
$$S(6,4) = 6 - 5(4) + 20 = 6$$

Hence,

Minimum $Z = 4$ at $x = 4$ and $y = 4$

Converting the given inequations into equations:-

$x + 2y = 20,\ x + y = 15,\ y = 5,\ x = y = 0$



Region represented by $x + 2y \le 20$: Line $x + 2y = 20$ meets coordinate axes at $A_1(20,0)$ and $B_1(0,10)$, clearly, $(0,0)$ satisfies $x + 2y \le 20$, so region containing origin represents $x + 2y \le 20$ in xy -plane.

Region represented by $x + y \le 15$: Line $x + y = 15$ meets coordinate axes at $A_2(15,0)$ and $B_2(0,15)$, clearly, $(0,0)$ satisfies $x + y \le 15$, so region containing origin represents $x + y \le 15$ in xy - plane.

Region represented by $y \le 5$: Line $y = 5$ is parallel to x-axis and meets at $B_3(0,5)$ on y-axis. Clearly $(0,0)$ satisfies $y \le 5$, so region containing origin represents $y \le 5$ in xy-plane.

Region represented by $x, y \ge 0$: It represent the first quadrant in xy-plane.

So, shaded region OA_2PB_3 represents the feasible region.

Coordinate of $P(10,5)$ is obtained by solving $x + 2y = 20$ and $y = 5$.

The value of $Z = 3x + 5y$ at

$O(0,0) = 3(0) + 5(0) = 0$

$A_2(15,0) = 3(15) + 5(0) = 45$

$P(10,5) = 3(10) + 5(5) = 55$

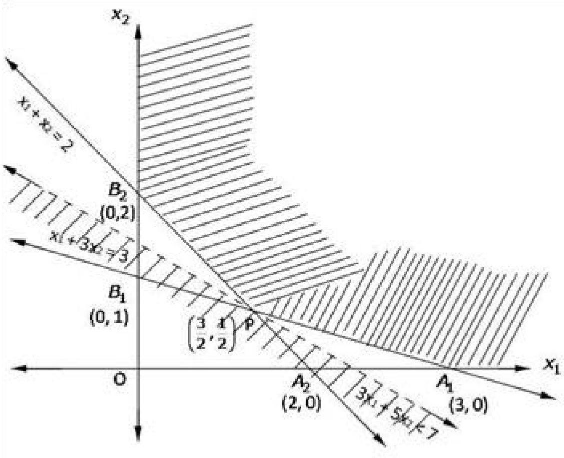
$B_3(0,5) = 3(0) + 5(5) = 25$

Hence, maximum $Z = 55$ at $x = 10$ and $y = 5$

Linear Programming Ex 30.2 Q16

Converting the given inequations into equations,

$x_1 + 3x_2 = 3,\ x_1 + x_2 = 2,\ x_1 = x_2 = 0$



Region represented by $x_1 + 3x_2 \geq 3$: Line $x_1 + 3x_2 = 3$ meets the coordinate axes at $A_1(3,0)$ and $B_1(0,1)$, clearly, $(0,0)$ does not satisfy $x_1 + 3x_2 \geq 3$, so, region not containing $(3,0)$ represents $x_1 + 3x_2 \geq 3$ in x_1x_2 -plane.

Region represented by $x_1 + x_2 \geq 2$: Line $x_1 + x_2 = 2$ meets the coordinate axes at $A_2(2,0)$ and $B_2(0,2)$, clearly, $(0,0)$ does not satisfy $x_1 + x_2 \geq 2$, so, region not containing origin represents $x_1 + x_2 \geq 2$ in x_1x_2 -plane.

Region represented $x_1, x_2 \geq 0$: It represents the first quadrant in x_1x_2-plane.

The unbounded shaded region with corner points $A_1(3,0)$, $B_2(0,2)$, and $P\left(\frac{3}{2}, \frac{1}{2}\right)$.

$P\left(\frac{3}{2}, \frac{1}{2}\right)$ is obtained by $x_1 + x_2 = 2$ and $x_1 + 3x_2 = 3$.

The value of $Z = 3x_1 + 5x_2$ at

$$A_1(3,0) \quad = 3(3) + 5(0) = 9$$
$$P\left(\frac{3}{2}, \frac{1}{2}\right) \quad = 3\left(\frac{3}{2}\right) + 5\left(\frac{1}{2}\right) = 7$$
$$B_2(0,2) \quad = 3(0) + 5(2) = 10$$

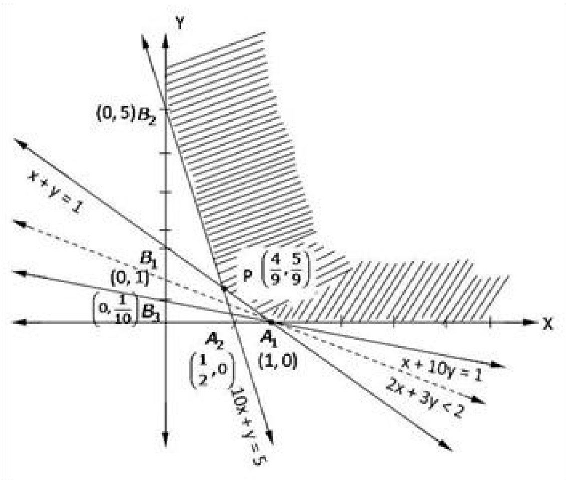
The smallest value of $Z = 7$,
region has no point in common, so smallest value is the minimum value.

Hence, minimum $Z = 7$ at $x = \frac{3}{2}$ and $y = \frac{1}{2}$

Linear Programming Ex 30.2 Q17

Converting the given inequations into equations

$x + y = 1$, $10x + y = 5$, $x + 10y = 1$, $x = y = 0$



Region represented by $x + y \geq 1$: Line $x + y = 1$ meets coordinate axes at $A_1(1, 0)$ and $B_1(0,1)$, dearly, $(0,0)$ does not satisfy $x + y \geq 1$, so region not containing origin represents $x + y \geq 1$ in xy -plane.

Region represented by $10x + y \geq 5$: Line $10x + y = 5$ meets coordinate axes at $A_2\left(\frac{1}{2}, 0\right)$ and $B_2(0,5)$. Clearly, $(0,0)$ does not satisfy $10x + y \geq 5$, so region not containing origin represents $10x + y \geq 5$ in xy -plane.

Region represented by $x + 10y \geq 1$: Line $x + 10y = 1$ meets coordinate axes $A_1(1,0)$ and $B_3\left(0, \frac{1}{10}\right)$. Clearly, $(0,0)$ does not satisfy $x + 10y \geq 1$, so, region not containing origin represents $x + 10y \geq 1$ in xy-plane.

Region represented by $x, y \geq 0$: It represents first quadrant in xy-plane.

So, unbounded shaded represents feasible region. Its corner points are $A_1(1,0)$, $P\left(\frac{4}{9}, \frac{5}{9}\right)$ and $B_2(0,5)$.

The coordinate of $P\left(\frac{4}{9}, \frac{5}{9}\right)$ is obtained by solving $10x + y = 5$ and $x + y = 1$.

The value of $Z = 2x + 3y$ at

$$A_1(1,0) \quad = 2(1) + 3(0) = 2$$

$$P\left(\frac{4}{9}, \frac{5}{9}\right) \quad = 2\left(\frac{4}{8}\right) + 3\left(\frac{5}{9}\right) = \frac{23}{9} = 2\frac{5}{9}$$

$$B_2(0,5) \quad = 2(0) + 3(5) = 15$$

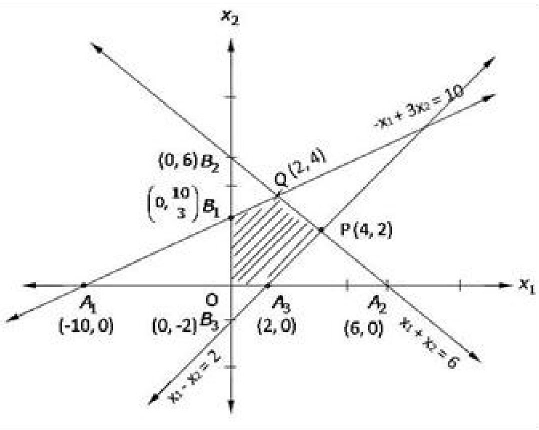
The smallest value of Z is 2. Now, open half plane $2x + 3y < 2$ has no point in common with feasible region so, smallest value of Z is the minimum value.

Hence, maximum $Z = 2$ at $x = 1$ and $y = 0$

Linear Programming Ex 30.2 Q18

Converting the given inequations into equations,

$$-x_1 + 3x_2 = 10,\ x_1 + x_2 = 6,\ x_1 = x_2 = 2,\ x_1 = x_2 = 0$$



Region represented by $-x_1 + 3x_2 \le 10$: Line $-x_1 + 3x_2 = 10$ meets coordinate axes at $A_1(-10, 0)$ and $B_1\left(0, \frac{10}{3}\right)$, clearly, $(0,0)$ satisfies $-x_1 + 3x_2 \le 10$, so region containing origin represents $-x_1 + 3x_2 \le 10$ in x_1x_2 -plane.

Region represented by $x_1 + x_2 \le 6$: Line $x_1 + x_2 = 6$ meets coordinate axes at $A_2(6, 0)$ and $B_2(0, 6)$. Clearly, $(0,0)$ satisfies $x_1 + x_2 \le 6$, so region containing origin represents $x_1 + x_2 \le 6$ in x_1x_2 -plane.

Region represented by $x_1 - x_2 \le 2$: Line $x_1 - x_2 = 2$ meets coordinate axes at $A_3(2, 0)$ and $B_3(0, -2)$. Clearly, $(0,0)$ satisfies $x_1 - x_2 \le 2$, so, region containing origin represents $x_1 - x_2 \le 2$ in x_1x_2 -plane.

Region represented $x_1, x_2 \ge 0$: It represents first quadrant in x_1x_2-plane.

So, shaded region OA_3PQB, represents feasible region.

Coordinate of $P(4,2)$ is obtained by solving $x_1 + x_2 = 6$ and $x_1 - x_2 = 2$, $Q(2, 4)$ by solving $x_1 + x_2 = 6$ and $-x_1 + 3x_2 = 10$

The value of $Z = -x_1 + 2x_2$ at

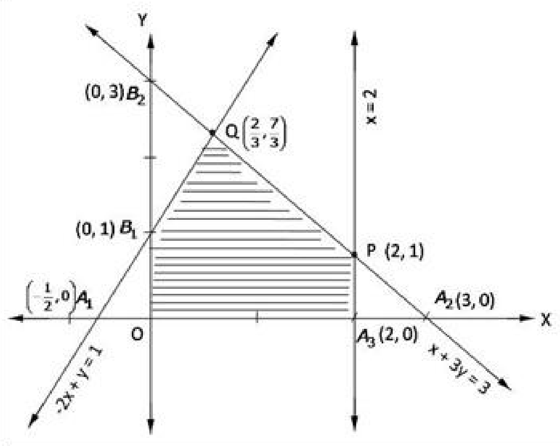
$$O(0,0) \quad = -(0) + 2(0) = 0$$
$$A_3(2,0) \quad = -(2) + 2(0) = -2$$
$$P(4,2) \quad = -(4) + 2(2) = 0$$
$$Q(2,4) \quad = -(2) + 2(4) = 6$$
$$B_1\left(0, \frac{10}{3}\right) \quad = -(0) + 2\left(\frac{10}{3}\right) = \frac{20}{3} = 6\frac{2}{3}$$

Hence, maximum $Z = \frac{20}{3}$ at $x = 0$ and $y = \frac{10}{3}$

Linear Programming Ex 30.2 Q19

Converting the given inequations into equations,

$-2x + y = 1$, $x = 2$, $x + y = 3$, $x = y = 0$



Region represented by $-2x + y \le 1$: Line $-2x + y = 1$ meets coordinate axes at $A_1\left(\frac{-1}{2},0\right)$ and $B_1(0,1)$, clearly, $(0,0)$ satisfies $-2x + y \le 1$, so region containing origin represents $-2x + y \le 1$ in xy -plane.

Region represented by $x \le 2$: Line $x = 2$ is parallel to y-axis and meets x-axis at $A_3(2,0)$. Clearly, $(0,0)$ satisfies $x \le 2$, so region containing origin represents $x \le 2$ in xy-plane.

Region represented by $x + y \le 3$: Line $x + y = 3$ meets coordinate axes at $A_2(3,0)$ and $B_2(0,3)$. Clearly, $(0,0)$ satisfies $x + y \le 3$, so region containing origin represents $x + y \le 3$ in xy -plane.

Region represented by $x, y \ge 0$: It represents first quadrant in xy-plane.

So, shaded region OA_3PQB, represents the feasible region.

Coordinates of $P(2,1)$ is obtained by solving $x + y = 3$ and $x = 2$, $Q\left(\frac{2}{3},\frac{7}{3}\right)$ by solving $-2x + y = 1$ and $x + y = 3$.

The value of $Z = x + y$ at

$O(0,0) \quad = 0 + 0 = 0$

$A_3(2,0) \quad = 2 + 0 = 2$

$P(2,1) \quad = 2 + 1 = 2$

$Q\left(\frac{2}{3},\frac{7}{3}\right) \quad = \frac{2}{3} + \frac{7}{3} = 3$

$B_1(0,1) \quad = 0 + 1 = 1$

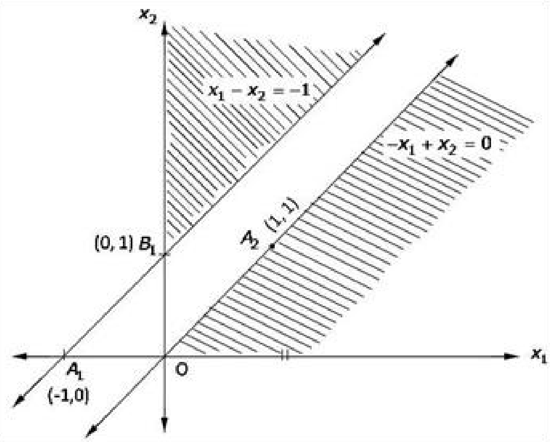
So, maximum $Z = 3$ is at every point on the line joining PQ.

Hence, maximum $Z = 3$ at $x = 2$ and $y = 1$ Or $x = \frac{2}{3}$ and $y = \frac{7}{3}$

Linear Programming Ex 30.2 Q20

Converting the given inequations into equations,

$$x_1 - x_2 = -1,\ -x_1 + x_2 = 0,\ x_1 = x_2 = 0$$



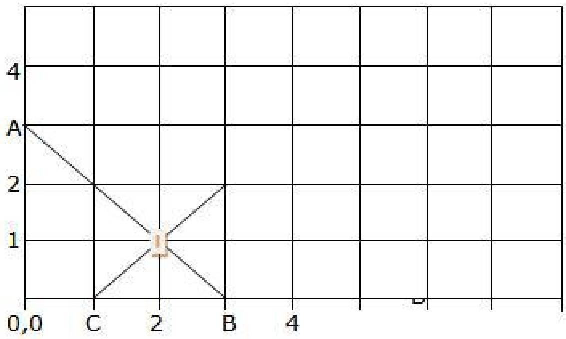
Region represented by $x_1 - x_2 \le -1$: Line $x_1 - x_2 = -1$ meets coordinate axes at $A_1(-1, 0)$ and $B_1(0, 1)$, clearly, $(0,0)$ does not satisfy $x_1 - x_2 \le -1$, so region not containing origin represents $x_1 - x_2 \le -1$ in x_1x_2-plane.

Region represented by $-x_1 + x_2 \le 0$: Line $-x_1 + x_2 = 0$ passes through origin and $A_2(1,1)$. Clearly, $(0,0)$ does not satisfy $-x_1 + x_2 \le 0$, so, region not containing $(0,1)$ represents $-x_1 + x_2 \le 0$ in x_1x_2-plane.

Since, there is not common shaded region represented by $x_1 - x_2 \le -1$ and $-x_1 + x_2 \le 0$ which can form feasible region.

Hence, maximum $Z = 3x_1 + 4x_2$ does not exists.

Linear Programming Ex 30.2 Q21



x-y≤1; when x = 0, y=1 & when y=0, x=2
x + y≥3; when x = 0, y=3 & when y=0, x=3, line AB
a unbounded region A-C-D is obtained using the constraints.

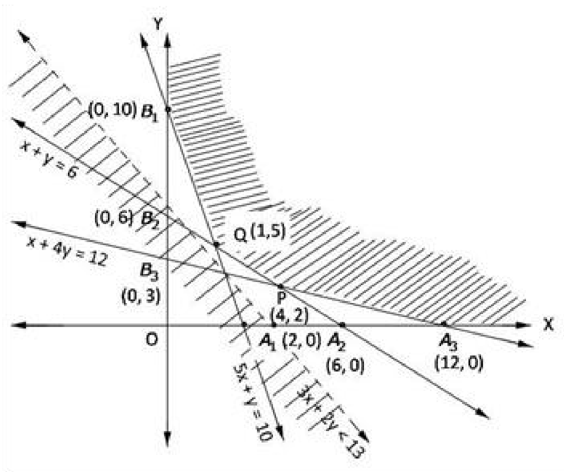
Corner point	Value of Z = 3x + 3y
0, 3	9
2, 1	9

So an optimal solution does not exist.

Linear Programming Ex 30.2 Q22

Converting the given inequations into equations

$5x + y = 10,\ x + y = 6,\ x + 4y = 12,\ x = y = 0$



Region represented by $5x + y \geq 10$: Line $5x + y = 10$ meets coordinate axes at $A_1(2,0)$ and $B_1(0,10)$. Clearly, $(0,0)$ does not satisfy $5x + y \geq 10$, so region not containing origin represents $5x + y \geq 10$ in xy -plane.

Region represented by $x + y \geq 6$: Line $x + y = 6$ meets coordinate axes at $A_2(6,0)$ and $B_2(0,6)$. Clearly, $(0,0)$ does not satisfy $x + y \geq 6$, so region not containing origin represents $x + y \geq 6$ in xy -plane.

Region represented by $x + 4y \geq 12$: Line $x + 4y = 12$ meets coordinate axes at $A_3(12,0)$ and $B_3(0,3)$. Clearly, $(0,0)$ does not satisfy $x + 4y \geq 12$, so, region not containing origin $x + 4y \geq 12$ in xy -plane.

Region represented by $x, y \geq 0$: It represents first quadrant in xy-plane.

The unbounded shaded region with corner points $A_3(12,0)$, $P(4,2)$, $Q(1,5)$, $B_1(0,10)$ represents feasible region. Point P is obtained by solving $x + 4y = 12$ and $x + y = 6$, Q by solving $x + y = 6$ and $5x + y = 10$.

The value of $Z = 3x + 2y$ at

$A_3(12,0) \quad = 3(12) + 2(0) = 36$

$P(4,2) \quad = 3(4) + 2(2) = 16$

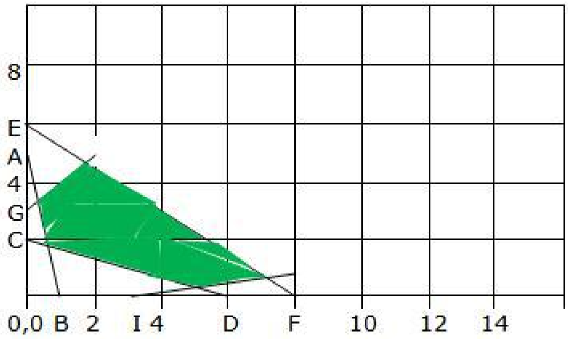
$Q(1,5) \quad = 3(1) + 2(5) = 13$

$B(0,10) \quad = 3(0) + 2(10) = 20$

Smallest value of $Z = 13$, Now open half plane $3x + 2y < 13$ has no point in common with feasible region, so, smallest value is the minimum value of Z, Hence

Minimum $Z = 13$ at $x = 1$, $y = 5$

Linear Programming Ex 30.2 Q23



x+3y≥6; or y=-0.333x+2; when x=0, y=2 & when y=0, x=6; line CD
x-3y≤3; or y=0.333x-1; when x=0, y=-1 & when y=0, x=3; line IJ
3x+4y≤24; or y=-0.75x+6; when x=0, y=6 & when y=0, x=8; line EF

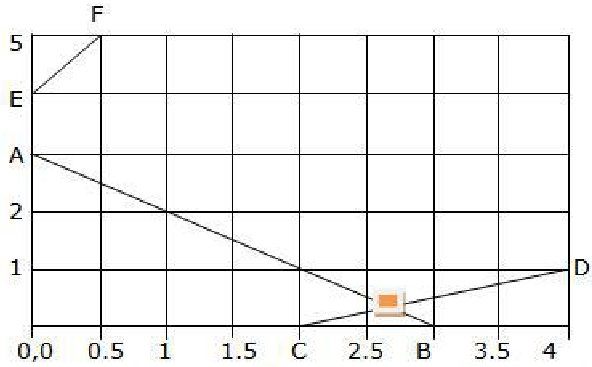
-3x+2y≤6; or y= 1.5x+3; when x=0, y=3 & when y=0, x=-2;line GH
5x+y≥5; or y=-5x+5; when x=0, y=5 & when y=0, x=1; line AB

The feasible area is shaded in green

Corner point	Value of Z = 2x + y
4.5, 0.5	9.5
0.64, 1.78	3.07
6.46, 1.15	Maximum 14.07
1.33, 5	7.6667
0.30, 3.46	4.0769

Maximum value is 14.07 at the point (6.46, 1.15)
Minimum value is 3.07 at the point (0.64, 1.78)

Linear Programming Ex 30.2 Q24



-2x+y≤4; or y=2x+4; when x=0, y=4 & when y=0, x=-2 line EF

$x+y \geq 3$; or $y=-x+3$; when $x=0$, $y=3$ & when $y=0$, $x=3$; line AB

$x-2y \leq 2$; or $y=0.5x-1$; when $x=0$, $y=-1$ & when $y=0$,

$x=2$ line CD

The feasible solution is the unbounded area with F-E-A-G-D

Corner point	Value of Z = 3x + 5y
(2.67, 0.33)	Minimum 9.66
(0, 3)	15
(0, 4)	20

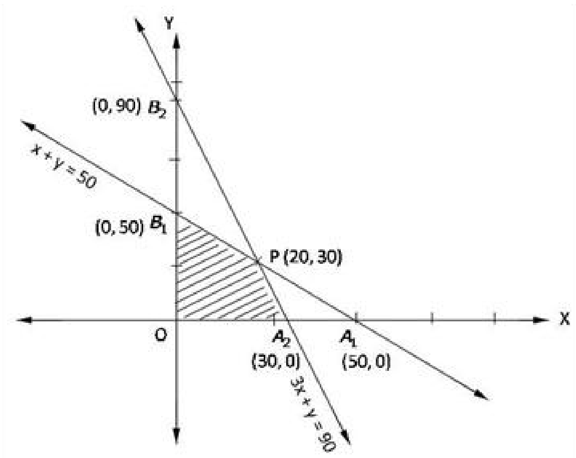
To check whether it is the minimal value plot the objective function with a value less than 9.66 or $y=-0.6x-1.932$

it can be seen that the values of x and y are always negative. So there is no optimal solution.

Linear Programming Ex 30.2 Q25

Converting the given inequations into equations,

$$x+y=50,\ 3x+y=90,\ x=y=0$$



Region represented by $x+y \leq 50$: Line $x+y=50$ meets coordinate axes at $A_1(50,0)$ and $B_1(0,50)$. Clearly, $(0,0)$ satisfies $x+y \leq 50$, so, region containing origin represents $x+y \leq 50$ in xy -plane.

Region represented by $3x+y \leq 90$: Line $3x+y=90$ meets coordinate axes at $A_2(30,0)$ and $B_2(0,90)$. Clearly, $(0,0)$ satisfies $3x+y \leq 90$, so, region containing origin represents $3x+y \leq 90$ in xy -plane.

Region represented by $x, y \geq 0$: It represents first quadrant in xy-plane.

Shaded region OA_2PB_1 represents the feasible region. $P(20,30)$ can be obtained by solving $x+y=50$ and $3x+y=90$.

The value of $Z=60x+15y$ at

$$\begin{aligned} &O(0,0) && = 60(0)+15(0) = 0 \\ &A_2(30,0) && = 60(30)+15(0) = 1800 \\ &P(20,30) && = 60(20)+15(30) = 1650 \\ &B_1(0,50) && = 60(0)+15(50) = 750 \end{aligned}$$

Hence,

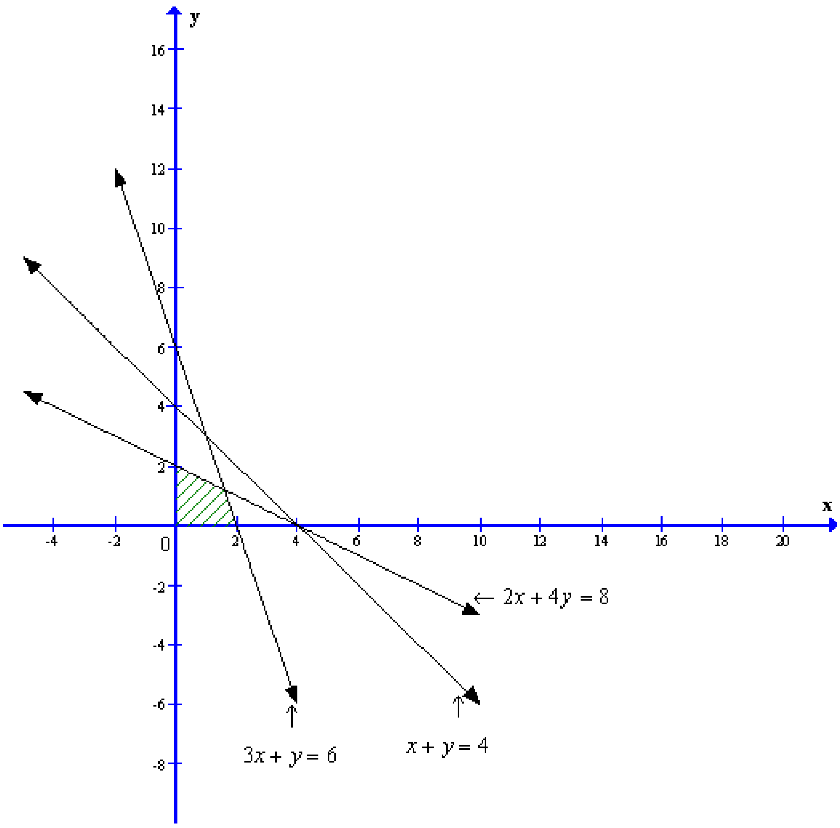
maximum Z is 1800 at $x=30$ and $y=0$.

Linear Programming Ex 30.2 Q26

Converting the inequations into equations, we obtain the lines

$2x + 4y = 8$, $3x + y = 6$, $x + y = 4$, $x = 0$, $y = 0$.

These lines are drawn on a suitable scale and the feasible region of the LPP is shaded in the graph.



From the graph we can see the corner points as (0, 2) and (2, 0).

Now solving the equations $3x + y = 6$ and $2x + 4y = 8$ we get the values of x and y as $x = \frac{8}{5}$ and $y = \frac{6}{5}$.

Substituting $x = \frac{8}{5}$ and $y = \frac{6}{5}$ in $Z = 2x + 5y$ we get,

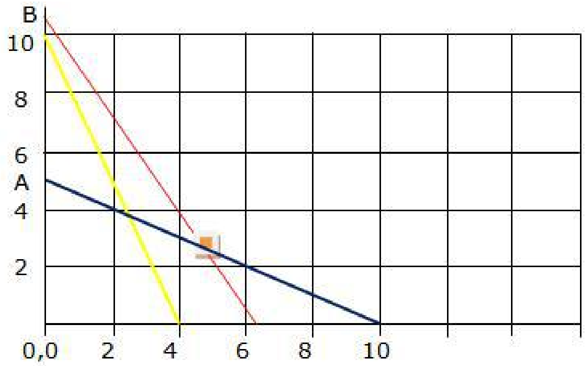
$$Z = 2\left(\frac{8}{5}\right) + 5\left(\frac{6}{5}\right)$$

$$Z = \frac{46}{5}$$

Hence maximum value of Z is $\frac{46}{5}$ at $x = \frac{8}{5}$ and $y = \frac{6}{5}$.

Ex 30.3

Linear Programming Ex 30.3 Q1



Let x and y be the No. of 25 gm packets of foods F_1 and F_2

Minimum cost of diet $Z = 0.20x + 0.15y$

The constraints are
$0.25x + 0.1y \geq 1$; when $x=0$, $y=10$ & $y=0$, $x=4$ 10-4
$0.75x + 1.5y \geq 7.5$; when $x=0$, $y=5$ & $y=0$, $x=10$ A-10
$1.6x + 0.8y \geq 10$; when $x=0$, $y=25/2$ & $y=0$, $x=25/4$

The feasible region is the open region B-E-10

The minimum cost of the diet can be checked by finding the value of Z at corner points B, E & 10

Corner point	Value of $Z = 20x + 15y$
0, 12.5	187.5
10, 0	200
5, 2.5	137.5

Since the feasible region is an open region so we plot $20x + 15y < 137.5$, to check whether the resulting open half plane has any point common with the feasible region. Since it has common points $Z = 20x + 15y$

There is no optimal minimum value subject to the given constraints.

Linear Programming Ex 30.3 Q2

Let required quantity of food A and B be x and y units respectively.

Costs of one unit of food A and B are Rs 4 and Rs 3 per unit respectively, so, costs of x unit of food A and y unit of food B are $4x$ and $3y$ respectively. Let Z be minimum total cost, so

$$Z = 4x + 3y$$

Since one unit of food A and B contain 200 and 100 units of vitamin respectively. So, x units of food A and y units of food B contain $200x$ and $100y$ units of vitamin but minimum requirement of vitamin is 4000 units, so

$$200x + 100y \geq 4000$$

$$\Rightarrow \quad 2x + y \geq 40 \quad (\text{first constraint})$$

Since one unit of food A and B contain 1 unit and 2 unit of minerals, so x units of food A and y units of food B contain x and $2y$ units of minerals respectively but minimum requirement of minerals is 50 units, so

$$x + 2y \geq 50 \quad (\text{second constraint})$$

Since one unit of food A and B contain 40 calories each, so x units of food A and y units of food B contain $40x$ and $40y$ calories respectively but minimum requirement of calories is 1400, so

$$40x + 40y \geq 1400$$

$$\Rightarrow \quad 2x + 2y \geq 70$$

$$\Rightarrow \quad x + y \geq 35 \quad (\text{third constraint})$$

So, mathematical formulation of LPP is find x and y which

$$\text{minimize} \quad Z = 4x + 3y$$

Subject to constraint,

$$2x + y \geq 40$$

$$x + 2y \geq 50$$

$$x + y \geq 35$$

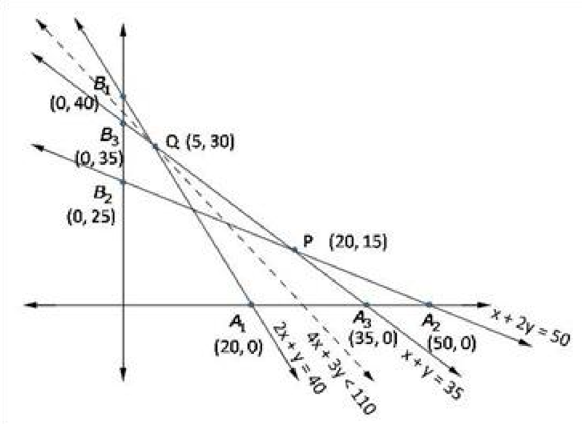
$$x, y \geq 0 \quad [\text{Since quantity of food can not be less than zero}]$$

Region $2x + y \geq 40$: Line $2x + y = 40$ meets axes at $A_1(20, 0)$, $B_1(0, 40)$ region not containing origin represents $2x + y \geq 40$ as $(0,0)$ does not satisfy $2x + y \geq 40$.

Region $x + 2y \geq 50$: Line $x + 2y = 50$ meets axes at $A_2(50,0)$, $B_2(0,25)$. Region not containing origin represents $x + 2y \geq 50$ as $(0,0)$ does not satisfy $x + 2y \geq 50$.

Region $x + y \geq 35$: Line $x + y = 35$ meets axes at $A_3(35,0)$, $B_3(0,35)$. Region not containing origin represents $x + y \geq 35$ as $(0,0)$ does not satisfy $x + y \geq 35$.

Region $x, y \geq 0$: It represent first quadrant in xy-plane.



Unbounded shaded region $A_2 P Q B_1$ represents feasible region with corner points $A_2(50,0)$, $P(20,15)$, $Q(5,30)$, $B_1(0,40)$

The value of $Z = 4x + 3y$ at

$A_2(50,0) \quad = 4(50) + 3(0) = 2000$

$P(20,15) \quad = 4(20) + 3(15) = 125$

$Q(5,30) \quad = 4(5) + 3(30) = 110$

$B_1(0,40) \quad = 4(0) + 3(40) = 110$

Smallest value of $Z = 110$

Open half plane $4x + 3y < 110$ has no point in common with feasible region, so, smallest value is the minimum value.

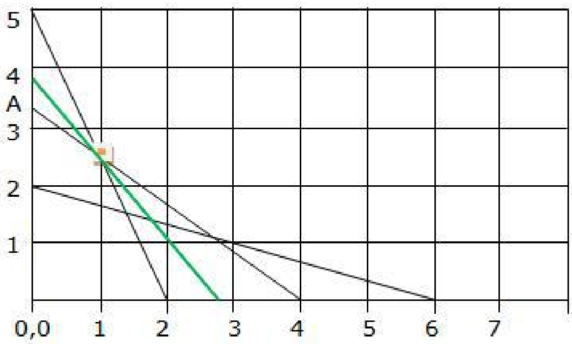
Hence,

quantity of food $A = x = 5$ unit

quantity of food $B = y = 30$ unit

minimum cost $=$ Rs 110

Linear Programming Ex 30.3 Q3



Let x & y be the units of Food I and Food II resptly.

The objective function is to minimize the function
Z = 0.6x + y such that
10x + 4y ≥ 20 requirement of calcium, line 5-2
5x + 6y ≥ 20 requirement of protein, line A-4
2x + 6y ≥ 12 requirement of calories, line 2-6

These when plotted give 5-F-E-6 an open unbounded region.

The function $20x + 15y < 57.5$ needs to be plotted to check if there are any common points. The green line shows that there are no common points. So

Corner point	Value of $Z = 0.6x + y$
0, 5	5
F(1, 2.5)	3.1
E(2.67, 1.11)	2.71
6, 0	3.6

The minimum cost occurs when Food I is 1 unit and Food II is 2.5 units. Since it is an unbounded region plotting $Z<3.1$ gives the green line which has no common points, so (1, 2.5) can be said to be a minimum point.

Linear Programming Ex 30.3 Q4

Let required quantity of food A and food B be x and y units.

Given, costs of one unit of food A and B are 10 paise per unit each, so costs of x unit of food A and y unit of food B are $10x$ and $10y$ respectively, let Z be total cost of foods, so

$$Z = 10x + 10y$$

Since one unit of food A and B contain 0.12 mg and 0.10 mg of Thiamin respectively, so, x units of food A and y units of food B contain $0.12x$ mg and $0.10y$ mg of Thiamin respectively but minimum requirement of Thiamin is 0.4 mg, so

$$0.12x + 0.10y \geq 0.5$$

$$\Rightarrow \quad 12x + 10y \geq 50$$

$$\Rightarrow \quad 6x + 5y \geq 25 \qquad \text{(first constraint)}$$

Since one unit of food A and B contain 100 and 150 Calories respectively, so x units of food A and y units of food B contain $100x$ and $150y$ units of Calories but minimum requirement of Calories is 600, so

$$100x + 150y \geq 600$$

$$\Rightarrow \quad 2x + 3y \geq 12 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

minimize $Z = 10x + 10y$

Subject to constraint,

$$6x + 5y \geq 25$$

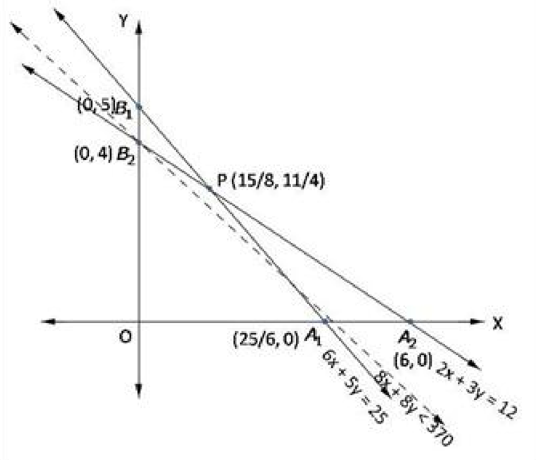
$$2x + 3y \geq 12$$

$$x, y \geq 0 \qquad [\text{Since quantity of food } A \text{ and } B \text{ can not be less than zero}]$$

Region $6x + 5y \geq 25$: $6x + 5y = 25$ meets axes at $A_1\left(\frac{25}{6}, 0\right)$, $B_1(0,5)$. Region not containing origin represents $6x + 5y \geq 25$ as $(0,0)$ does not satisfy $6x + 5y \geq 25$.

Region $2x + 3y \geq 12$: Line $2x + 3y = 12$ meets axes at $A_2(6,0)$, $B_2(0,4)$. Region not containing origin represents $2x + 3y \geq 12$ as $(0,0)$ does not satisfy $2x + 3y \geq 12$.

Region $x, y \geq 0$ represent first quadrant in xy-plane.



Unbounded shaded region $A_2 P B_1$ represents feasible region with corner points $A_2(6,0)$, $P\left(\frac{15}{8},\frac{11}{4}\right)$, $B_1(0,5)$

The value of $Z = 10x + 10y$ at

$A_2(6,0) \quad = 10(6) + 10(0) = 60$

$P\left(\frac{15}{8},\frac{11}{4}\right) \quad = 10\left(\frac{15}{8}\right) + 10\left(\frac{11}{4}\right) = \frac{370}{8} = 46\frac{1}{4}$

$B_1(0,5) \quad = 10(0) + 10(5) = 50$

Smallest value of Z is $46\frac{1}{4}$.

Now open half plane $10x + 10y < \frac{370}{8}$

$\Rightarrow$ $8x + 8y < 370$ has no point in common with feasible region, so smallest value is the minimum value.

Hence,

Required quantity of food $A = \frac{15}{8}$ units, food $B = \frac{11}{4}$ units

minimum cost = Rs 46.25

Linear Programming Ex 30.3 Q5

Let required quantity of food X and food Y be x kg and y kg.

Since costs of food X and Y are Rs 5 and Rs 8 per kg., So, costs of food X and food Y are Rs. $5x$ and Rs. $8y$ respectively. Let Z be the total cost of food, then

$$Z = 5x + 8y$$

Since one kg of food X and Y contain 1 and 2 unit of vitamin A, so, x kg of food X and y kg of food Y contain x and $2y$ units of vitamin A respectively but minimum requirement of vitamin A is 6 units, so

$$x + 2y \ge 6 \qquad \text{(first constraint)}$$

Since one kg of food X and Y contain 1 unit of vitamin B each, so x kg of food X and y kg of food Y contain x and y units of vitamin B but minimum requirement of vitamin B is 7 units, so

$$x + y \ge 7 \qquad \text{(second constraint)}$$

Since one kg of food X and food Y contain 1 unit and 3 units of vitamin C respectively, so x kg of food X and y kg of food Y contain x and $3y$ units of vitamin C respectively but minimum requirement of vitamin C is 11 units, so

$$x + 3y \ge 11 \qquad \text{(third constraint)}$$

Since 1 kg of food X and food Y contain 2 units and 1 unit of vitamin D respectively, so, x kg of food X and y kg of food Y contain $2x$ and y units of vitamin D respectively but minimum requirement of vitamin D is 9 units, so

$$2x + y \ge 9 \qquad \text{(fourth constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

minimize $Z = 5x + 8y$

Subject to constraints,

$$x + 2y \ge 6$$
$$x + y \ge 7$$
$$x + 3y \ge 11$$
$$2x + y \ge 9$$
$$x, y \ge 0 \qquad [\text{Since quantity of food } X \text{ and } Y \text{ can not be less than zero}]$$

Region $x + 2y \ge 6$: Line $x + 2y = 6$ meets axes at $A_1(6,0)$, $B_1(0,3)$. Region not containing origin represents $x + 2y \ge 6$ as $(0,0)$ does not satisfy $x + 2y \ge 6$.

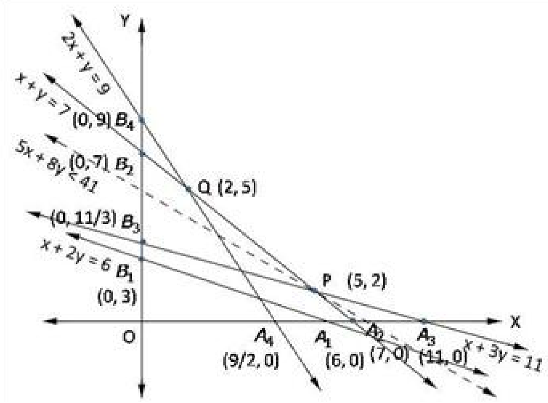
Region $x + y \ge 7$: Line $x + y = 7$ meets axes at $A_2(7,0)$, $B_2(0,7)$ respectively. Region not containing origin represents $x + y \ge 7$ as $(0,0)$ does not satisfy $x + y \ge 7$.

Region $x + 3y \ge 11$: Line $x + 3y = 11$ meets axes at $A_3(11, 0)$, $B_3\left(0, \frac{11}{3}\right)$ respectively.

Region not containing origin represents $x + 3y \ge 11$ as $(0,0)$ does not satisfy $x + 3y \ge 11$.

Region $2x + y \ge 9$: Line $2x + y = 9$ meets axes at $A_4\left(\frac{9}{2}, 0\right)$, $B_4(0, 9)$ respectively. Region not containing origin represents $2x + y \ge 9$ as $(0,0)$ does not satisfy $2x + y \ge 9$.

Region $x, y \ge 0$ it represent first quadrant.



Unbounded shaded region $A_2PQ\ B_4$ is the feasible region with corner points $A_3(11, 0)$, $P(5, 2)$, $Q(2,5)$, $B_4(0, 9)$

The value of $Z = 5x + 8y$ at

$A_3(11, 0) \quad = 5(11) + 8(0) = 55$

$P(5, 2) \quad = 5(5) + 8(2) = 41$

$Q(2,5) \quad = 5(2) + 8(5) = 50$

$B_4(0, 9) \quad = 5(0) + 8(9) = 72$

Smallest value of Z is 41.

Now open half plane $5x + 8y < 41$ has no point is common with feasible region, os,
smallest value of is the minimum value.
hence
last cost of mixture= Rs 41

Linear Programming Ex 30.3 Q6

Let quantity of food F_1 and F_2 be x and y units.
respectively.
Given, costs of one unit of food F_1 and F_2 be Rs 4 and Rs 6 per unit, So, costs of X unit of food $F_1$and Y units of food F_2 be $4x$ and $6y$ respectively,
Let Z be the total cost , so

$$Z = 4x + 8y$$

Since one unit of food F_1 and F_2 contain 3 and 6 unit of vitamin A respectively, so, x units of food F_1 and y units of food F_2 contain $3x$ and $6y$ units of vitamin A respectively, but minimum requirement
of vitamin A is 80 units, so

$$3x + 6y \geq 80 \qquad \text{(first constraint)}$$

Since one unit of food F_1 and F_2 contain 4 unit and 3 unit of mineral,
so x unit of food F_1 and y unit of food F_2 contain $4x$ and $3y$ units of mineral respectively but minimum requirement of minerals be 100 units, so

$$4x + 3y \geq 100$$

$$\Rightarrow \quad 4x + 3y \geq 100 \qquad \text{(second constraint)}$$

mathematical formulation of LPP is, Find x and y which minimum

$$Z = 4x + 6y$$

Subject to constraints,

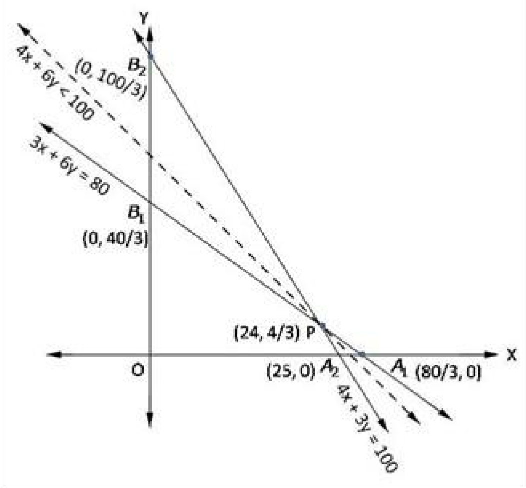
$$3x + 6y \geq 80$$
$$4x + 3y \geq 100$$
$$x, y \geq 0$$

$$\left[\text{since quantity of food can not be less than zero}\right]$$

Region $3x + 6y \geq 80$: line $3x + 6y = 80$ meets axes at $A_1\left(\frac{80}{3}, 0\right), B_1\left(0, \frac{40}{3}\right)$ respectively. Region not containing origin represents $3x + 6y \geq 80$ as $(0,0)$ does not satisfy $3x + 6y \geq 80$.

Region $4x + 3y \geq 100$ line $3x + 6y = 100$ meets axes at $A_2(25, 0), B_2\left(0, \frac{100}{3}\right)$ respectively. Region not containing origin represents $4x + 3y \geq 100$ as $(0,0)$ does not satisfy $4x + 3y \geq 100$.

Region $x, y \geq 0$ represents first quadrant



Unbounded shaded region $A_1 P\ B_2$ represents feasible region with corner points $A_1\left(\frac{80}{3}, 0\right)$, $P\left(24, \frac{4}{3}\right)$, $B_2\left(0, \frac{100}{3}\right)$.

The value of $Z = 4x + 6y$ at

$$A_1\left(\frac{80}{3}, 0\right) \quad = 4\left(\frac{80}{3}\right) + 6(0) = \frac{320}{3}$$

$$P\left(24, \frac{4}{3}\right) \quad = 4(24) + 6\left(\frac{4}{3}\right) = 104$$

$$B_2\left(0, \frac{100}{3}\right) \quad = 4(0) + 6\left(\frac{100}{3}\right) = 200$$

Smallest value of Z is 104 .Now open half plane $4x + 6y < 104$ has no point in common with feasible region so, smallest value is minimum value.

Hence,

Minimum cost of mixture = Rs 104

Linear Programming Ex 30.3 Q7

Let required quantity of bran and rice be x kg and y kg.
Given, costs of one kg of bran and rice are Rs 5 and Rs 4 per kg, So, costs of X unit of bran and Y kg of rice are $5x$ and Rs $4y$ respectively,
Let total cost of bran and rice be Z, so,

$$Z = 5x + 4y$$

Since one kg of bran and rice contain 80 and 100 mg of protien, so,
x kg of bran and y kg of rice contain $80x$ and $100y$ grms of protien respectively, but minimum requirement of protien for kelloggs is 88 gms, so

$$80x + 100y \geq 88$$

$$\Rightarrow \quad 20x + 25y \geq 22 \qquad \text{(first constraint)}$$

Since one kg of bran and rice contain 40 mg and 30 mg of iron, so,
x kg of bran and y kg of rice contain $40x$ and $30y$ mg of iron respectively, but minimum requirement of iron is 36 mg for kelloggs, so

$$40x + 30y \geq 36 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which minimize

$$Z = 5x + 4y$$

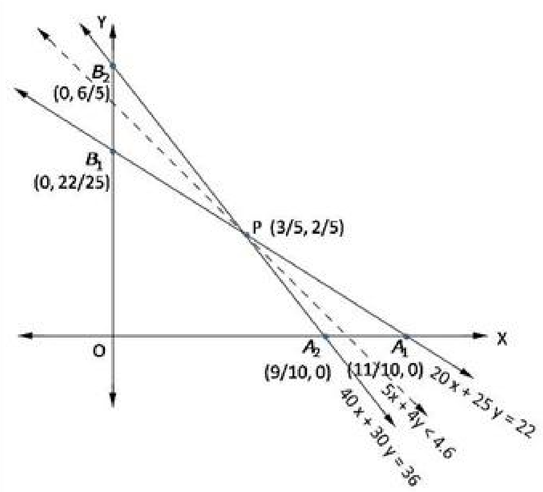
subject to constraints,

$$20x + 25y \geq 22$$
$$40x + 30y \geq 36$$
$$x, y \geq 0$$

[Since quantity of bran and rice can not be less than zero]

Region $20x + 25y \geq 22$: line $20x + 25y = 22$ meets axes at $A_1\left(\frac{11}{10}, 0\right), B_1\left(0, \frac{22}{25}\right)$ respectively. Region not containing origin represents $20x + 25y \geq 22$ as $(0,0)$ does not satisfy $20x + 25y \geq 22$.

Region $40x + 30y \geq 36$ line $40x + 30y = 36$ meets axes at $A_2\left(\frac{9}{10}, 0\right), B_2\left(0, \frac{6}{5}\right)$. Region not containing origin represents $40x + 30y \geq 36$ as $(0,0)$ does not satisfy $40x + 30y \geq 36$.



The value of $Z = 5x + 4y$ at

$$A_1\left(\frac{11}{10}, 0\right) \qquad = 5\left(\frac{11}{10}\right) + 4(0) = 5.5$$

$$P\left(\frac{3}{5}, \frac{2}{5}\right) \qquad = 5\left(\frac{3}{5}\right) + 4\left(\frac{2}{5}\right) = 4.6$$

$$B_2\left(0, \frac{6}{5}\right) \qquad = 5(0) + 4\left(\frac{6}{5}\right) = 4.8$$

Smallest value of Z is 4.6. Now open half plane $5x + 4y < 4.6$ has no point in common with feasible region so, smallest value z is the minimum value.
Hence

Minimum cost of mixture = Rs 4.6

Linear Programming Ex 30.3 Q8

Let required number of bag A and bag B be x and y respectively.

Since, costs of each bag A and bag B are Rs 8 and Rs 12 per kg., So,
cost of x number of bag A and y number of bag B are Rs $8x$ and Rs $12y$ respectively,
Let Z be total cost of bags, so,

$$Z = 8x + 12y$$

Since, each bag A and B contain 60 and 30 gms. of almonds respectively. so,
x bags of A and y bags of B contain $60x$ and $30y$ gms. of almonds respectively but,
mixtures should contain at least 240 gms almonds, so,

$$60x + 30y \geq 240$$

$$\Rightarrow \quad 2x + y \geq 8 \qquad (\text{first constraint})$$

Since, each bag A and B contain 30 and 60 gms. of cashew nuts respectively. so,
x bags of A and y bags of B contain $30x$ and $60y$ gms. of cashew nuts respectively but,
mixtures should contain at least 300 gms of cashew nuts, so,

$$30x + 60y \geq 300$$

$$\Rightarrow \quad x + 2y \geq 10 \qquad (\text{second constraint})$$

Since, each bag A and B contain 30 and 180 gms. of hazel nuts respectively. so,
x bags of A and y bags of B contain $30x$ and $180y$ gms. of hazel nuts respectively but,
mixtures should contain at least 540 gms of hazel nuts, so,

$$30x + 180y \geq 540$$

$$\Rightarrow \quad x + 6y \geq 18 \qquad (\text{third constraint})$$

Hence, mathematical formulation of LPP is, Find x and y which maximize

$$Z = 8x + 12y$$

subject to constraints,

$$2x + y \geq 8$$
$$x + 2y \geq 10$$
$$x + 6y \geq 18$$
$$x, y \geq 0$$

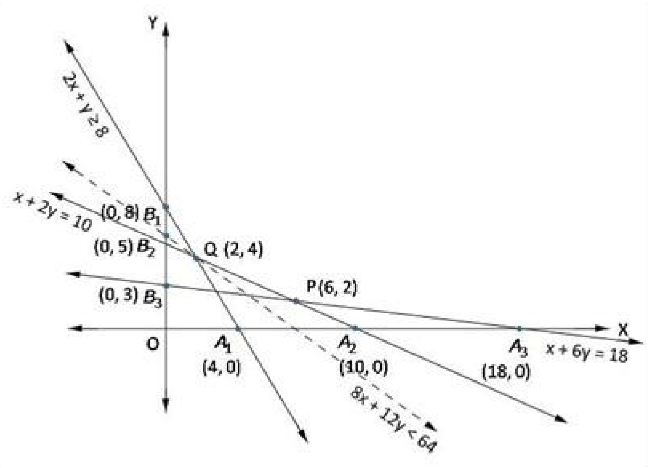
$$[\text{Since quantity of bags can not be less than zero}]$$

Region $2x + y \geq 8$: line $2x + y = 8$ meets axes at $A_1(4,0)$, $B_1(0,8)$ respectively. Region not containing origin represents $2x + y \geq 8$ as $(0,0)$ does not satisfy $2x + y \geq 8$.

Region $x + 2y \geq 10$: line $x + 2y = 10$ meets axes at $A_2(10,0)$, $B_2(0,5)$ respectively. Region not containing origin represents $2x + y \geq 10$ as $(0,0)$ does not satisfy $x + 2y \geq 10$

Region $x + 6y \geq 18$: line $x + 6y = 18$ meets axes at $A_3(18,0)$, $B_3(0,3)$ respectively. Region not containing origin represents $x + 6y \geq 8$ as $(0,0)$ does not satisfy $x + 6y \geq 8$

Region $x, y \geq 0$: it represents first quadrant.



Unbouded shaded region A_3PQB_1 is feasible region with corner point $A_3(18,0)$, $P(6,2)$ $Q(2,4)$, $B_1(0,8)$. P is obtained by solving $x + 6y = 18$ and $x + 2y = 10$, Q is obtained by solving $2x + y = 8$ and $x + 2y = 10$

The value of $z = 8x + 12y$ at

$A_3(18,0)$	$= 8(18) + 12(0) = 144$
$P(6,2)$	$= 8(6) + 12(2) = 72$
$Q(2,4)$	$= 8(2) + 12(4) = 64$
$B_1(0,8)$	$= 8(0) + 12(8) = 96$

Smallest value of Z is 64, open half plane $8x + 12y \geq 64$ has no point is common with feasible region, so, smallest value is the minimum value

Minimum cost = Rs64
quantity of mixture $A = 2$ kg.
quantity of mixture $B = 4$kg

Linear Programming Ex 30.3 Q9

Let required number of cakes of type A and B are x and y respectively.

Let Z be total number of cakes ,so,

$$Z = x + y$$

Since one unit of cake of type A and B contain 300 gm and 150 gm flour respectively, so, x unit of cake of type A and y units of cake of type B require $300x$ and $150y$ gms of flour respectivley, but maximum flour available is $7.5 \times 1000 = 7500$ gm ,so

$$300x + 150y \le 7500$$

$$\Rightarrow \quad 2x + y \le 50 \qquad \text{(first constraint)}$$

Since one unit of cake of type A and B contain 15 and 30 gm fat respectively, so, x unit of cake of type A and y units of cake of type B contain $15x$ and $30y$ gms of fat respectivley, but maximum fat available is 600 gm ,so

$$15x + 30y \le 600$$

$$\Rightarrow \quad x + 2y \le 40 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = x + y$

Subject to constriants,

$$2x + y \le 50$$

$$x + 2y \le 40$$

$$x, y \ge 0 \qquad \text{[Since number of cakes can not be less than zero]}$$

Region $2x + y \le 50$: line $2x + y = 50$ meets axes at $A_1(25,0)$, $B_1(0,50)$ respectively.

Region containing origin represents $2x + y \le 50$ as $(0,0)$ satisfies $2x + y \le 50$.

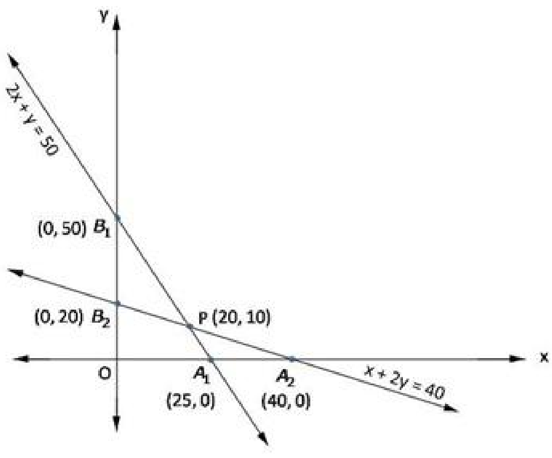
Region $x + 2y \le 40$: line $x + 2y = 40$ meets axes at $A_2(40,0)$, $B_2(0,20)$ respectively.

Region containing origin represents $x + 2y \le 40$ as $(0,0)$ satisfies $x + 2y \le 40$.

Region $x, y \ge 0$: it represent first quandrant

Shaded region OA_1PB_2 represents feasible region.

Point $P(20,10)$ is obtained by solving $x + 2y = 40$ and $2x + y = 50$



The value of $Z = x + y$ at

$O(0,0) \quad = 0 + 0 = 0$

$A_1(25,0) \quad = 25 + 0 = 25$

$P(20,10) \quad = 20 + 10 = 30$

$B_2(0,20) \quad = 0 + 20 = 20$

maximum $Z = 30$ at $x = 20$, $y = 10$

Number of books of type $A = 20$, type $B = 10$

Linear Programming Ex 30.3 Q10

Let x kg of food P and y kg of food Q are mixed together to make the mixture.

Then the mathematical model of the LPP is as follows:

Minimize $Z = 60x + 80y$

Subject to $3x + 4y \geq 8,$

$5x + 2y \geq 11$

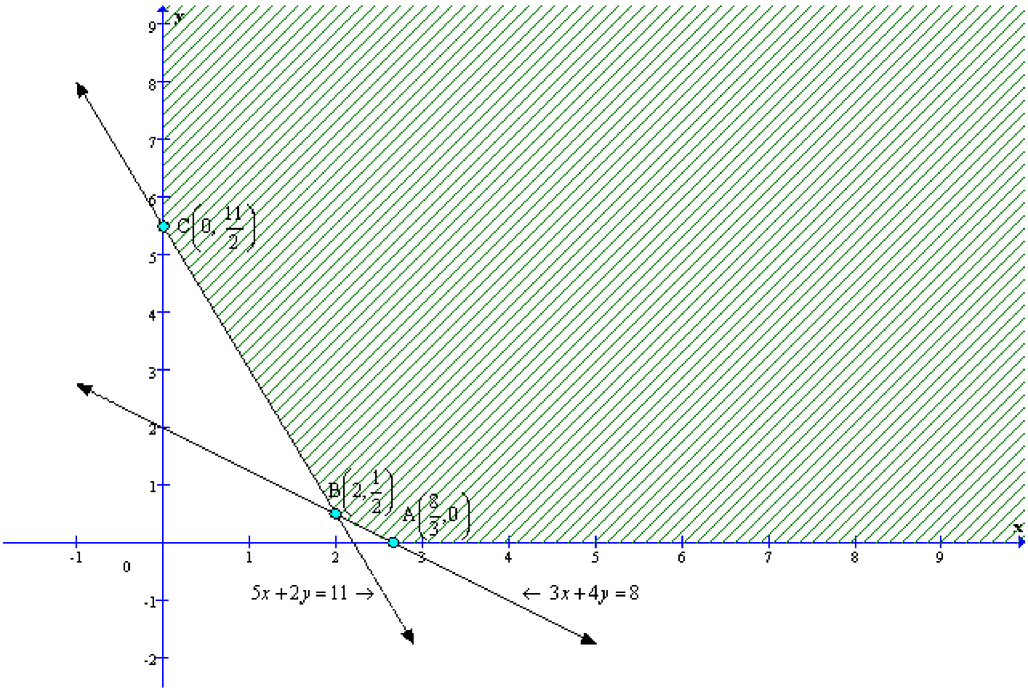
and $x \geq 0,\ y \geq 0$

To solve the LPP we draw the lines,

$3x + 4y = 8,$

$5x + 2y = 11$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are

$A\left(\frac{8}{3}, 0\right), B\left(2, \frac{1}{2}\right)$ and $C\left(0, \frac{11}{2}\right)$.

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 60x + 80y$
$A\left(\frac{8}{3}, 0\right)$	$Z = 160$
$B\left(2, \frac{1}{2}\right)$	$Z = 160$
$C\left(0, \frac{11}{2}\right)$	$Z = 440$

The minimum value of the mixture is Rs. 160 at all points on the line segment joining points $\left(\frac{8}{3}, 0\right)$ and $\left(2, \frac{1}{2}\right)$

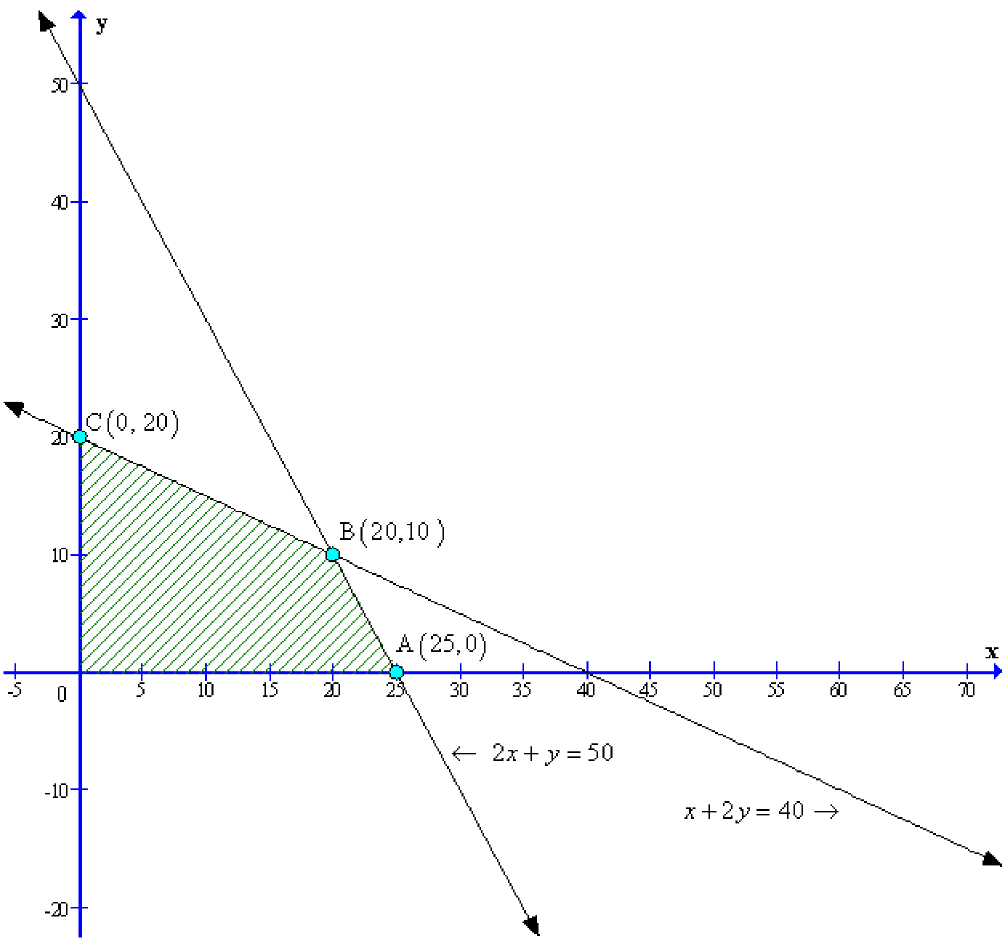
Linear Programming Ex 30.3 Q11

Let x be the number of one kind of cake and
y be the number of second kind of cakes that are made.

Then the mathematical model of the LPP is as follows:
Maximize $Z = x + y$
Subject to $200x + 100y \le 5000$,
$25x + 50y \le 1000$
and $x \ge 0$, $y \ge 0$

To solve the LPP we draw the lines,
$2x + y = 50$,
$x + 2y = 40$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(25, 0), B(20,10) and C(0,20).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = x + y$
A(25, 0)	Z = 25
B(20, 10)	Z = 30
C(0, 20)	Z = 20

The maximum of 30 cakes can be made.

Linear Programming Ex 30.3 Q12

Let x be the number of packets of food P

y be the number of packets of food Q used to minimize vitamin A.

Then the mathematical model of the LPP is as follows:

Minimize $Z = 6x + 3y$

Subject to $12x + 3y \geq 240$,

$4x + 20y \geq 460$

$6x + 4y \leq 300$

and $x \geq 0,\ y \geq 0$

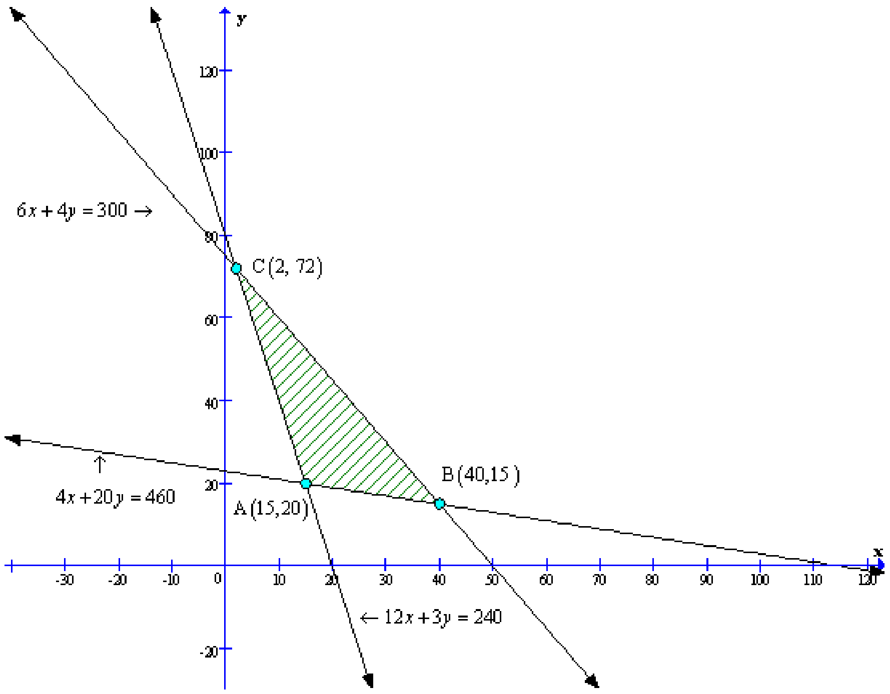
To solve the LPP we draw the lines,

$12x + 3y = 240$,

$4x + 20y = 460$,

$6x + 4y = 300$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(15, 20), B(40, 15) and C(2, 72).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 6x + 3y$
A(15, 20)	Z = 150
B(40, 15)	Z = 285
C(2, 72)	Z = 228

15 packets of food P and 20 packets of food Q should be used to minimise the amount of vitamin A.

The minimum amount of vitamin A is 150 units.

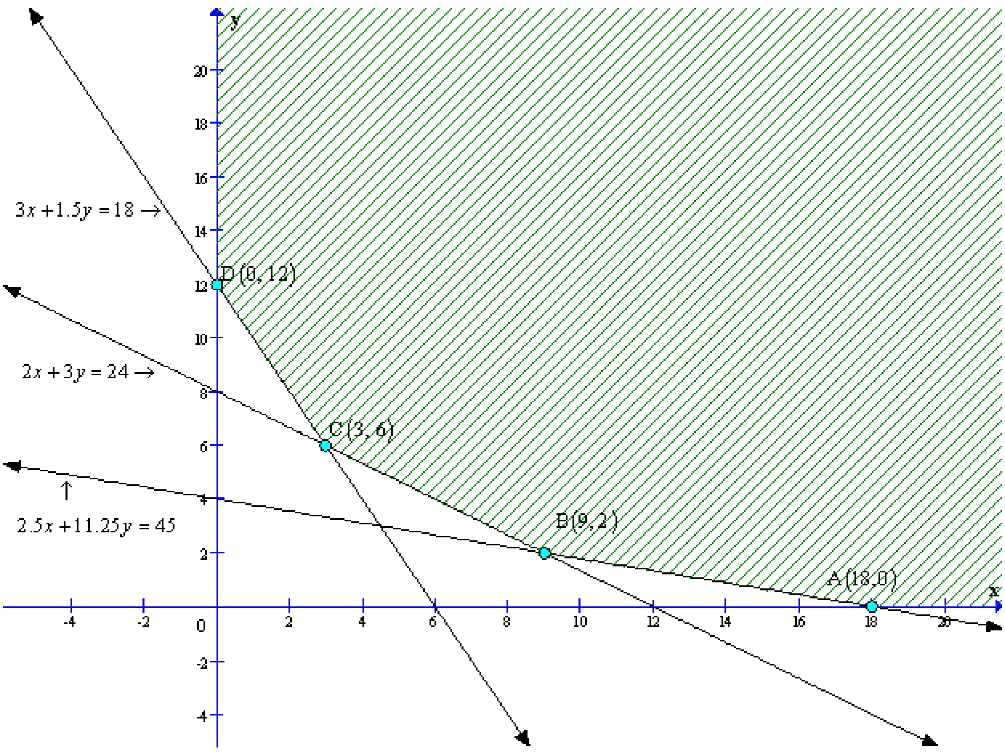
Linear Programming Ex 30.3 Q13

Let x be the number of bags of brand P
y be the number of bags of brand Q.

Then the mathematical model of the LPP is as follows:
Minimize $Z = 250x + 200y$
Subject to $3x + 1.5y \geq 18$,
$2.5x + 11.25y \geq 45$
$2x + 3y \geq 24$
and $x \geq 0$, $y \geq 0$

To solve the LPP we draw the lines,
$3x + 1.5y = 18$,
$2.5x + 11.25y = 45$
$2x + 3y = 24$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABCD are A(18, 0), B(9,2), C(3, 6) and D(0, 12).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 250x + 200y$
A(18, 0)	Z = 4500
B(9, 2)	Z = 2650
C(3, 6)	Z = 1950
D(0,12)	Z = 2400

3 bags of brand P and 6 bags of brand Q should be mixed in order to prepare the mixture having a minumum cost per bag.

Mimum cost of the mixture per bag is $= \frac{1950}{9}$ = Rs. 216.67.

Note: Answer given in the book is incorrect.

Linear Programming Ex 30.3 Q14

Let x be the amount of food X and y be the amount of food Y that is to be mixed which will produce the required diet.

Then the mathematical model of the LPP is as follows:

Minimize $Z = 16x + 20y$

Subject to $x + 2y \geq 10$,

$2x + 2y \geq 12$

$3x + y \geq 8$

and $x \geq 0,\ y \geq 0$

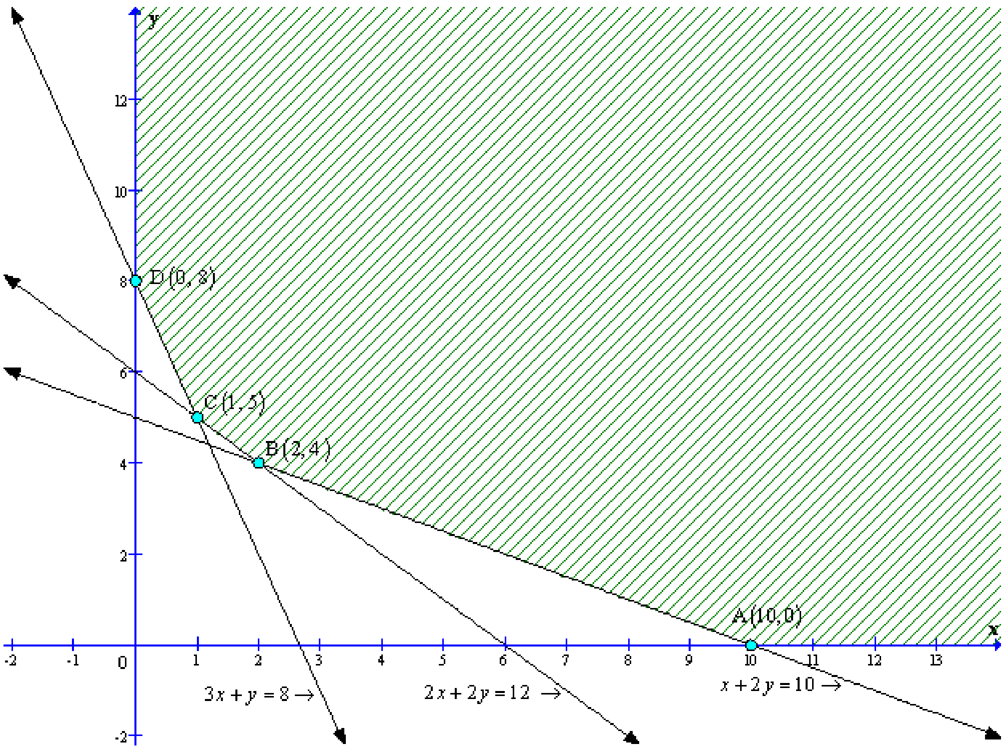
To solve the LPP we draw the lines,

$x + 2y = 10$,

$2x + 2y = 12$

$3x + y = 8$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABCD are A(10, 0), B(2, 4), C(1, 5) and D(0, 8).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 16x + 20y$
A(10, 0)	Z = 160
B(2, 4)	Z = 112
C(1, 5)	Z = 116
D(0, 8)	Z = 160

2 kg of food X and 4 kg of food y will be required to mimimize the cost of the diet.
The least cost of the mixture is Rs. 112.

Linear Programming Ex 30.3 Q15

Let x bags of fertilizer P and y bags of fertilizer Q used in the garden to minimize the usage of nitrogen.

Then the mathematical model of the LPP is as follows:

Minimize $Z = 3x + 3.5y$

Subject to $x + 2y \geq 240$,

$3x + 1.5y \geq 270$

$1.5x + 2y \leq 310$

and $x \geq 0$, $y \geq 0$

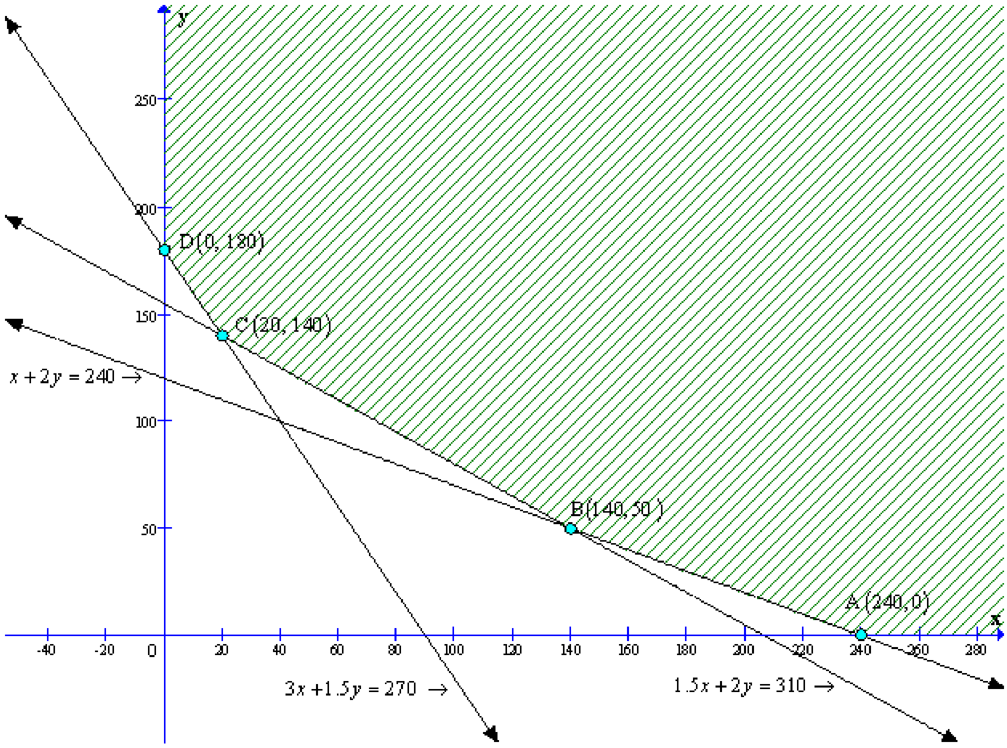
To solve the LPP we draw the lines,

$x + 2y = 240$,

$3x + 1.5y = 270$

$1.5x + 2y = 310$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(40, 100), B(140, 50) and C(20, 140).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 3x + 3.5y$
A(40, 100)	Z = 470
B(140, 50)	Z = 595
C(20, 140)	Z = 550

40 bags of brand P and 100 bags of brand Q should be used to minimize the amount of nitrogen added to the garden.

The minimum amount of nitrogen added in the garden is 470kg.

Ex 30.4

Linear Programming Ex 30.4 Q1

Let he drives x km at a speed f 25 km/hr and y km at a speed of 40 km/hr.
Let Z be total distance travelled by him, so,

$$Z = x + y$$

Since he spend Rs 2 per km on petrol when speed is 25 km/hr and Rs 5 per km on petrol when speed is 40 km/hr, so, expence on x km and y km are Rs $2x$ and Rs $5y$ respectivley, but he has only Rs 100.,so

$$2x + 5y \le 100 \qquad \text{(first constraint)}$$

$$\text{Time taken to travel } x \text{ km} = \frac{\text{Distance}}{\text{speed}} = \frac{x}{25} \text{ hr}$$

$$\text{Time taken to travel } y \text{ km} = \frac{y}{40} \text{ hr}$$

Given he has 1 hr to travel, so

$$\frac{x}{25} + \frac{y}{40} \le 1$$

$$\Rightarrow \quad 40x + 25y \le 1000$$

$$\Rightarrow \quad 8x + 5y \le 200 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = x + y$

Subject to constriants,

$$2x + 5y \le 100$$
$$8x + 5y \le 200$$
$$x, y \ge 0 \qquad [\text{Since distances can not be less than zero}]$$

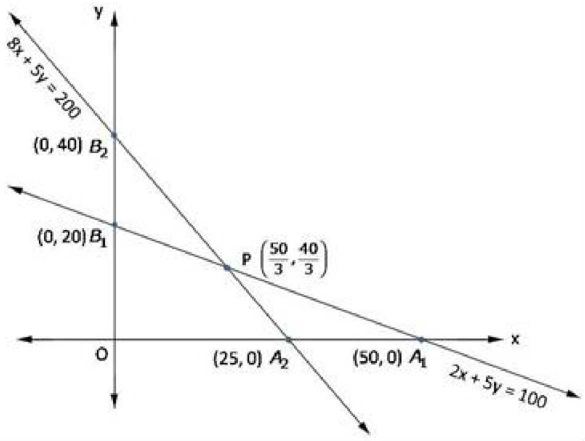
Region $2x + 5y \le 100$: line $2x + 5y = 100$ meets axes at $A_1(50,0)$, $B_1(0,20)$ respectively.
Region containing origin represents $2x + 5y \le 100$ as $(0,0)$ satisfies $2x + 5y \le 100$.

Region $8x + 5y \le 200$: line $8x + 5y = 200$ meets axes at $A_2(25,0)$, $B_2(0,40)$ respectively.
Region containing origin represents $8x + 5y \le 200$ as $(0,0)$ satisfies $8x + 5y \le 200$.

Region $x, y \ge 0$: it represent first quandrant

Shaded region OA_2PB_1 represents feasible region.

Point $P\left(\frac{50}{3}, \frac{40}{3}\right)$ is obtained by solving $8x + 5y = 200$, $2x + 5y = 100$



The value of $Z = x + y$ at

$O(0,0) = 0 + 0 = 0$

$A_2(25,0) = 25 + 0 = 25$

$P\left(\frac{50}{3}, \frac{40}{3}\right) = \frac{50}{3} + \frac{40}{3} = 30$

$B_1(0,20) = 0 + 20 = 20$

maximum $Z = 30$ at $x = \frac{50}{3}$, $y = \frac{40}{3}$

Distance travelled at speed of 25 km/hr = $\frac{50}{3}$ km

and at speed of 40 km/hr = $\frac{40}{3}$ km

maximum distance = 30 km.

Linear Programming Ex 30.4 Q2

Let required quantity of items A and B.

Given, profits on one item A and B are Rs 6 and Rs 4 respectively So, profits on X items of type A and Y items of type B are $6x$ and Rs $4y$ respectively,
Let total profit be z, so,

$$Z = 6x + 4y$$

Given, machine I works 1 hour and 2 hours on item A and B respectively, so,
x number of item A and y number of item B need x hour and $2y$ hours on machine I respectively, but machine I works at most 12 hours, so

$$x + 2y \geq 12 \qquad \text{(first constraint)}$$

Given, machine II works 2 hours and 1 hours on item A and B respectively, so,
x number of item A and y number of item B need $2x$ hours and y hour on machine II , but machine II works maximum 12 hours, so

$$2x + y \geq 12 \qquad \text{(second constraint)}$$

Given, machine III works 1 hour and $\frac{5}{4}$ hour on one item A and B respectively, so,
x number of item A and y number of item B need x hour and $\frac{5}{4}y$ hours respectively on machine III , but machine III works at least 5 hours, so

$$x + \frac{5}{4}y \geq 5$$

$$4x + 5y \geq 20 \quad \text{(third constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which miximize

$$z = 6x + 4y$$

subject to constraints,

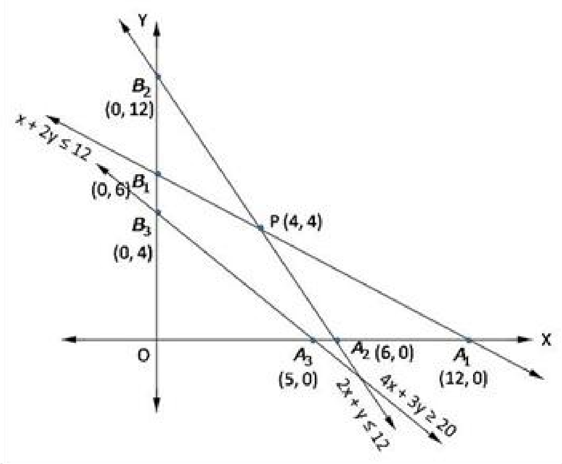
$$x + 2y \geq 12$$
$$2x + y \geq 12$$
$$4x + 5y \geq 20$$
$$x, y \geq 0$$

[Since number of item A and B not be less than zero]

Region $x + 2y \geq 12$: line $x + 2y = 12$ meets axes at $A_1(12, 0), B_1(0, 6)$ respectively. Region containing origin represents $x + 2y \geq 12$ as $(0,0)$ satisfies $2x + y \geq 12$.

Region $4x + 5y \geq 20$: line $4x + 5y = 20$ meets axes at $A_3(5,0), B_3(0, 4)$ respectively. Region not containing origin represents $4x + 5y \geq 20$ as $(0,0)$ does not satisfy $4x + 5y \geq 20$.

Region $x, y \geq 0$: it represent first quadrant.



Shaded region $A_2A_3 P B_3B_1$ represents feasible region.

The value of $Z = 6x + 4y$ at

$A_2 (6,0)$	$= 6(6) + 4(0) = 36$
$A_3 (5,0)$	$= 6(5) + 4(0) = 30$
$B_3 (0,4)$	$= 6(0) + 4(4) = 16$
$B_2 (0,6)$	$= 6(0) + 4(6) = 24$
$P (4,4)$	$= 6(4) + 4(4) = 40$

Hence, Z is maximum at $x = 4, Y = 4$

Required number of product $A = 4$, product $B = 4$

Miximum profit $=$ Rs 40

Linear Programming Ex 30.4 Q3

Suppose tailor A and B work for x and y days respectively.

Since, tailor A and B earn Rs 15 and Rs 20 respectively So, tailor A and B earn is X and Y days Rs $15x$ and $20y$ respectively, let Z denote maximum profit that gives minimum labour cost, so,

$$Z = 15x + 20y$$

Since, Tailor A and B stitch 6 and 10 shirts respectively in a day, so,
tailor A can stitch 6x and B can stitch $10y$ shirts in x and y days respectively, but it is desired to produce 60 shirts at least, so

$$6x + 10y \geq 60$$
$$3x + 5y \geq 30 \qquad \text{(first constraint)}$$

Since, Tailor A and B stitch 4 pants per day each, so,
tailor A can stitch 4x and B can stitch $4y$ pants in x and y days respectively, but it is desired to produce at least 32 pants, so

$$4x + 4y \geq 32$$
$$x + y \geq 8 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which minimize

$$Z = 15x + 20y$$

subject to constraints,

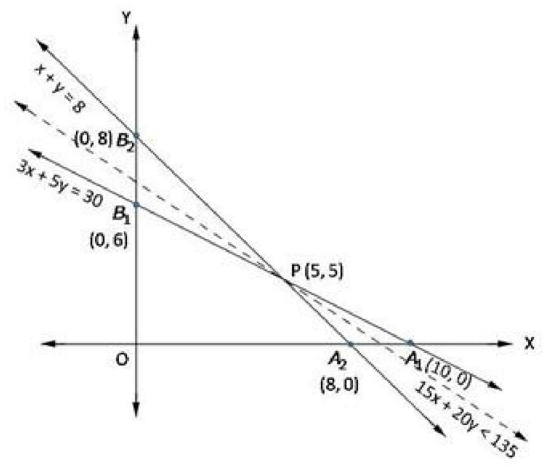
$$3x + 5y \geq 30$$
$$x + y \geq 8$$
$$x, y \geq 0$$
$$\left[\text{Since x and y not be less than zero}\right]$$

Region $3x + 5y \geq 30$: line $3x + 5y = 30$ meets axes at $A_1(10,0)$, $B_1(0,6)$ respectively. Region not containing origin represents $3x + 5y \geq 30$ as $(0,0)$ does not satisfy $3x + 5y \geq 30$.

Region $x + y \geq 8$: line $x + y = 8$ meets axes at $A_2(8,0)$, $B_2(0,8)$ respectively. Region not containing origin represents $x + y \geq 8$ as $(0,0)$ does not satisfy $x + y \geq 8$.

Region $x, y \geq 8$: it represent first quadrant.

Unbounded shaded region A_1PB_2 represents feasible region with corner points $A_1(10,0)$, $P(5,3)$, $B_2(0,8)$.



The value of $Z = 15x + 20y$ at

$A_1(10,0) \quad = 15(10)+20(0) = 150$

$P(5,3) \quad = 15(5)+20(3) = 135$

$B_2(0,8) \quad = 15(0)+20(8) = 160$

Smallest value of Z is 135 ,Now open half plane $15x + 20y < 135$ has no point in common with feasible region, so smallest value is the minimum value. So,

$Z = 135$, at $x = 5, y = 3$

Tailor A should work for 5 days and B should work for 3 days

Linear Programming Ex 30.4 Q4

Let the factory manufacture x screws of type A and y screws of type B on each day. Therefore,

$x \geq 0$ and $y \geq 0$

The given information can be compiled in a table as follows.

	Screw A	Screw B	Availability
Automatic Machine (min)	4	6	$4 \times 60 = 120$
Hand Operated Machine (min)	6	3	$4 \times 60 = 120$

The profit on a package of screws A is Rs 7 and on the package of screws B is Rs 10. Therefore, the constraints are

$4x + 6y \leq 240$

$6x + 3y \leq 240$

Total profit, $Z = 7x + 10y$

The mathematical formulation of the given problem is

Maximize $Z = 7x + 10y$ ... (1)

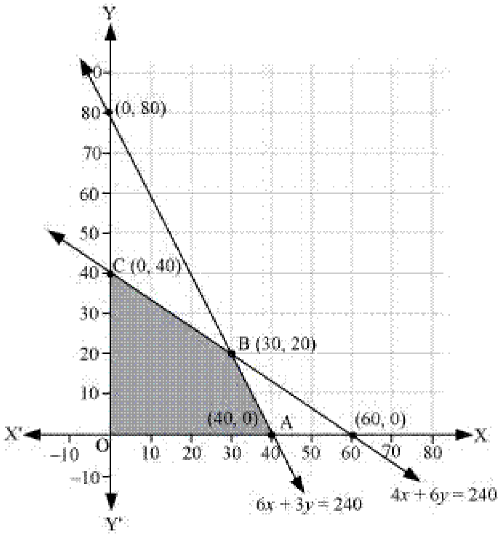
subject to the constraints,

$4x + 6y \leq 240$ ... (2)

$6x + 3y \leq 240$ ... (3)

$x, y \geq 0$ ... (4)

The feasible region determined by the system of constraints is



The corner points are A (40, 0), B (30, 20), and C (0, 40).

The values of Z at these corner points are as follows.

Corner point	**Z = 7*x* + 10*y***	
A(40, 0)	280	
B(30, 20)	410	→ Maximum
C(0, 40)	400	

The maximum value of Z is 410 at (30, 20).

Thus, the factory should produce 30 packages of screws A and 20 packages of screws B to get the maximum profit of Rs 410.

Linear Programming Ex 30.4 Q5

Let required number of belt A and B be x and y.
Given, profit on belt A and B be Rs 2 and Rs 1.50 per belt, So, profit on x belt of type A and Y belt fo type B be 2x and $1.5y$ respectively,
Let Z be total profit, so,

$$Z = 2x + 1.5y$$

Since, each belt of type A requires twice as much time as belt B. Let each belt B require 1 hour to make, so, A requires 2 hours. For x and y belts of type A and B. It required 2x and y hours to make but total time available is equal to procduction 1000 belt B that is 1000 hours, so,

$$2x + y \le 1000 \qquad \text{(first constraint)}$$

Given supply of leather only for 800 belts per day (both A and B combined), so

$$x + y \le 800 \qquad \text{(second constraint)}$$

Buckels available for A is only 400 and for B only 700, so,

$$x \le 400 \qquad \text{(third constraint)}$$

$$y \le 700 \qquad \text{(fourth constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which miximize

$$Z = 2x + 1.5y$$

subject to constraints,

$$2x + y \le 1000$$
$$x + y \le 800$$
$$x \le 400$$
$$y \le 700$$
$$x, y \ge 0$$
$$\left[\text{Since number of belt can not be less than zero}\right]$$

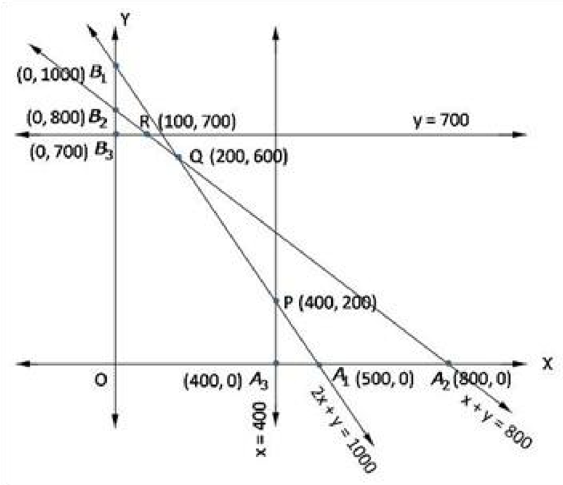
Region $2x + y \le 1000$: line $2x + y = 1000$ meets axes at $A_1(500,0), B_1(0,1000)$ respectively. Region containing origin represents $2x + y \le 1000$ as $(0,0)$ satisfies $2x + y \le 1000$.

Region $x + y \le 800$: line $x + y = 800$ meets axes at $A_2(800,0), B_2(0,800)$ respectively. Region containing origin represents $x + y \le 800$ as $(0,0)$ satisfies $x + y \le 800$.

Region Region $x \le 400$: line $x = 400$ meets axes is parallel to y axis and meet x – axis at $A_3(400,0)$. Region containing origin represents $x \le 400$ as $(0,0)$ satisfies $x \le 400$.

Region Region $y \le 400$: line $y = 700$ is parallel to x– axis and meet y – axis at $B_3(0,700)$. Region containing origin represents $y \le 700$ as $(0,0)$ satisfies $y \le 700$.

Region $x, y \ge 0$: it represent first quadrant.



Shaded region OA_3PQRB_3 is feasible region, P is points of intersections of $2x + y = 1000$ and $x = 400$, Q is the point of intersection of $x + y = 800$ and $2x + y = 1000$, R is not point of intersection of y=700, $x + y = 800$.

The value of $Z = 2x + 1.5y$ at

$O(0,0)$	$= 2(0) + 1.5(0) = 0$
$A_3(400,0)$	$= 2(400) + 1.5(0) = 800$
$P(400,200)$	$= 2(400) + 1.5(200) = 1100$
$Q(200,600)$	$= 2(200) + 1.5(600) = 1300$
$R(100,700)$	$= 2(100) + 1.5(700) = 1250$
$B_3(0,700)$	$= 2(0) + 1.5(700) = 1050$

Therefore, maximum $Z = 1300$, at $x = 200, y = 600$

Required number belt $A = 200$,belt $B = 600$
maximum profit = Rs 1300

Linear Programming Ex 30.4 Q6

Let required number of deluxe model and ordinary model be x and y respectively.

Since, profits on each model of deluxe and ordinary type model are Rs 15 and Rs 10 respectively. So, profits on x deluxe models and y ordinary models are $15x$ and $10y$

Let Z be total profit, then,

$$Z = 15x + 10y$$

Since, each deluxe and ordinary model require 2 and 1 hour of skilled men,so, x deluxe and y ordinary models required 2x and y hours of skilled men but time available by skilled men is $5 \times 8 = 40$ hours, So,

$$2x + y \le 40 \qquad \text{(first constraint)}$$

Since, each deluxe and ordinary model require 2 and 3 hours of semi-skilled men,so, x deluxe and y ordinary models require 2x and 3y hours of semi-skilled men respectively but total time available by semi-skilled men is $10 \times 8 = 80$ hours, So,

$$2x + 3y \le 80 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which maximize

$$Z = 15x + 10y$$

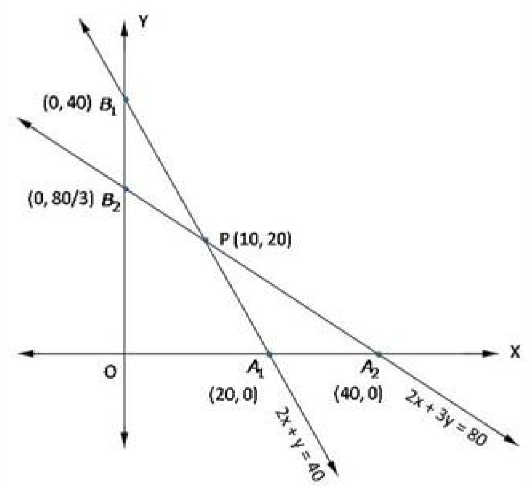
subject to constraints,

$$2x + y \le 40$$
$$2x + 3y \le 80$$
$$x, y \ge 0$$

[Since number of deluxe and ordinary models can not be less than zero]

Region $2x + y \le 40$: line $2x + y = 40$ meets axes at $A_1(20,0)$, $B_1(0,40)$ respectively. Region containing origin represents $2x + y \le 40$ as $(0,0)$ satisfies $2x + y \le 40$.

Region $2x + 3y \le 80$: line $2x + 3y = 80$ meets axes at $A_2(40,0)$, $B_2\left(0, \frac{80}{3}\right)$ respectively. Region containing origin represents $2x + 3y \le 80$.



The value of $Z = 15x + 10y$ at

$O(0,0)$	$= 15(0) + 10(0) = 0$
$A_1(20,0)$	$= 15(20) + 10(0) = 300$
$P(10,20)$	$= 15(10) + 10(20) = 350$
$B_2\left(0, \frac{80}{3}\right)$	$= 15(0) + 10\left(\frac{80}{3}\right) = \frac{800}{3}$

Therefore, maximum $Z = 350$, at $x = 10, y = 20$

Required number deluxe model = 10
number of ordinary model = 600
maximum profit =Rs 350

Linear Programming Ex 30.4 Q7

Let required number of tea-cups of type A and B are x and y respectively.

Since, profits on each tea-cups of type A and B are 75 paise and 50 paise So,
profits on x tea-cups of type A and y tea-cups of type B are $75x$ and $50y$
respectively, Let total profit on tea-cups be Z, so,

$$Z = 75x + 50y$$

Since, each tea-cup of type A and B require to work machine I for 12 and 6 minutes respectively so,
x tea cups of type B require to work on machine I for $12x$ and $6y$ minutes respectively .
Total time available on machine I is $6 \times 60 = 360$ minutes. so,

$$12x + 6y \geq 360 \qquad \text{(first constraint)}$$

Since, each tea-cup of type A and B require to work machine II for 18 and 0 minutes respectively so,
x tea cups of type A and y tea cups of B require to work on machine II for $18x$ and $0y$ minutes respectively .
but Total time available on machine II is $6 \times 60 = 360$ minutes. so,

$$18x + 0y \geq 360 \qquad \text{(second constraint)}$$
$$x \leq 20$$

Since, each tea-cup of type A and B require to work machine III for 6 and 9 minutes respectively so,
x tea cups of type A and y tea cups of B require to work on machine III for $6x$ and $9y$ minutes respectively .
Total time available on machine III is $6 \times 60 = 360$ minutes. so,

$$6x + 9y \geq 360 \qquad \text{(third constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which maximize

$$Z = 75x + 50y$$

subject to constraints,

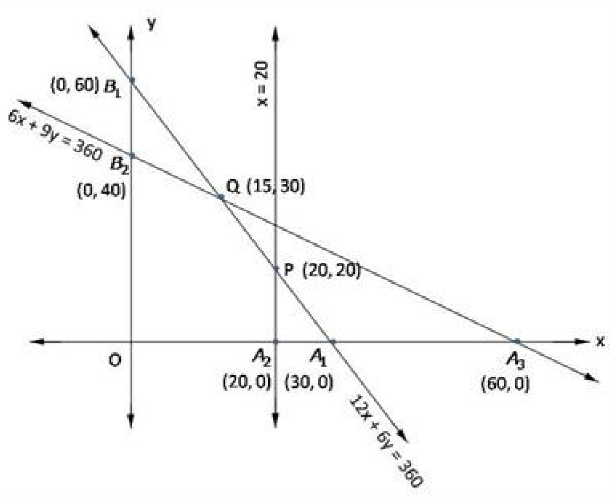
$$12x + 6y \leq 360$$
$$x \leq 20$$
$$6x + 9y \leq 360$$
$$x, y \geq 0$$
$$\left[\text{Since production of tea cups can not be less than zero}\right]$$

Region $12x + 6y \leq 360$: line $12x + 6y = 360$ meets axes at $A_1(30,0)$, $B_1(0,60)$ respectively. Region containing origin represents $12x + 6y \leq 360$ as $(0,0)$ satisfies $12x + 6y \geq 360$.

Region $x \leq 20$: line $x = 20$ is parallel to y – axes and meets x – axes at $A_2(20,0)$. Region containing origin represents $x \leq 20$ as $(0,0)$ satisfies $x \leq 20$.

Region $6x + 9y \leq 360$: line $8x + 9y = 360$ meets axes at $A_3(60,0)$, $B_2(0,40)$ respectively. Region containing origin represents $6x + 9y \leq 360$ as $(0,0)$ satisfies $6x + 9y \geq 360$.

Region $x, y \geq 0$: it represents first quadrant.



Shaded region OA_2PQB_2 is the feasible region. P is point obtained by solving $x = 20$ and $12x + 6y = 360$ and Q is point obtained by solving $12x + 6y = 360$ and $6x + 9y = 360$.

The value of $Z = 75x + 50y$ at

$O(0,0)$	$= 75(0) + 50(0) = 0$
$A_2(20,0)$	$= 75(20) + 50(0) = 1500$
$P(20,20)$	$= 75(20) + 50(20) = 2500$
$Q(15,30)$	$= 75(15) + 50(30) = 2624$
$B_2(0,40)$	$= 75(0) + 50(40) = 2000$

Hence, Z is maximum at $x = 15, Y = 30$

Therefore,

15 teacups of type A and 30 tea-cups f type B are needed to maximize profit

Linear Programming Ex 30.4 Q8

Let required number of machine A and B are x and y respectively.

Since, production of each machine A and B are 60 and 40 units daily respectively, So, productions by x number of machine A and y number of machine B are $60x$ and $40y$ respectively, Let Z denote total output daily, so,

$$Z = 60x + 40y$$

Since, each machine of type A and B require 1000 sq.m and 1200 sq.m area so, x machine of type A and y machine of type B require $100x$ and $1200y$ sq.m area but, Total area available for machine is 7600 sq.m. so,

$$1000x + 1200y \le 7600$$

$$5x + 6y \le 38 \qquad \text{(first constraint)}$$

Since, each machine of type A and B require 12 men and 8 men to work respectively so, x machine of type A and y machine of type B require $12x$ and $8y$ men to work respectively but, Total 72 men available for work so,

$$12x + 8y \le 72$$

$$3x + 2y \le 18 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which maximize

$$Z = 60x + 40y$$

subject to constraints,

$$5x + 6y \le 38$$

$$3x + 2y \le 18$$

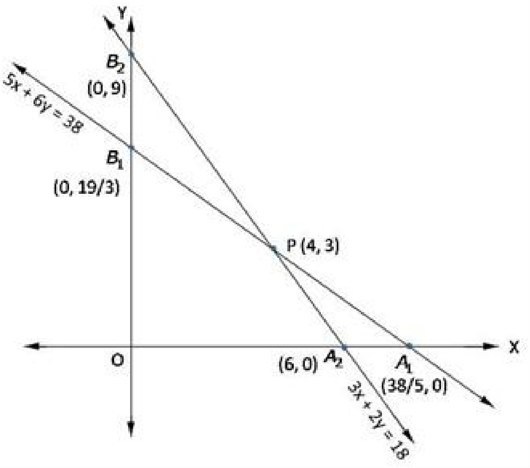
$$x, y \ge 0$$

$$\left[\text{Number of machines can not be less than zero}\right]$$

Region $5x + 6y \le 38$: line $5x + 6y = 38$ meets axes at $A_1\left(\frac{38}{5}, 0\right), B_1\left(0, \frac{19}{3}\right)$ respectively. Region containing origin represents $5x + 6y \le 38$ as origin satisfies $5x + 6y \ge 38$.

Region $3x + 2y \le 18$: line $3x + 2y = 18$ meets axes at $A_2(6,0)$, $B_2(0,9)$ respectively. Region containing origin represents $3x + 2y \le 18$ as $(0,0)$ satisfies $3x + 2y \le 18$.

Region $x, y \ge 0$: it represents first quadrant.



Shaded region OA_2PB_1 is the feasible region. $P(4,3)$ is obtained by solving $3x+2y=18$ and $5x+6y=38$

The value of $Z = 60x + 40y$ at

$O(0,0) \quad = 60(0)+40(0) = 0$

$A_2(6,0) \quad = 60(6)+40(0) = 360$

$P(4,3) \quad = 60(4)+40(3) = 360$

$B_1\left(0,\frac{19}{3}\right) \quad = 60(0)+40\left(\frac{19}{3}\right) = \frac{760}{3}$

Therefore maximum $Z = 360$ at $x = 4, Y = 3$ or $x = 6, y = 0$

Output is maximum when 4 machines of type A and 3 machine of type B or 6 machines of type A and no machine of type B.

Linear Programming Ex 30.4 Q9

Let number of goods A and B are x and y respectively.

Since, profits on each A and B are Rs 40 and Rs 50 respectively.So,
profits on x of type A and y of type B are $40x$ and $50y$ respectively,
Let Z be total profit on A and B, so,

$$Z = 40x + 50y$$

Since, each A and B require 3 gm and 1 gm of silver respectively. so,
x of type A and y type B require $3x$ and y gm silver respectively but,
Total silver available is 9 gm. so,

$$3x + y \le 9 \qquad \text{(first constraint)}$$

Since, each A and B require 1 gm and 2 gm of gold respectively. so,
x of type A and y type B require x and $2y$ gm of gold respectively but,
Total gold available is 8 gm, so,

$$x + 2y \le 8 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which maximize

$$Z = 40x + 50y$$

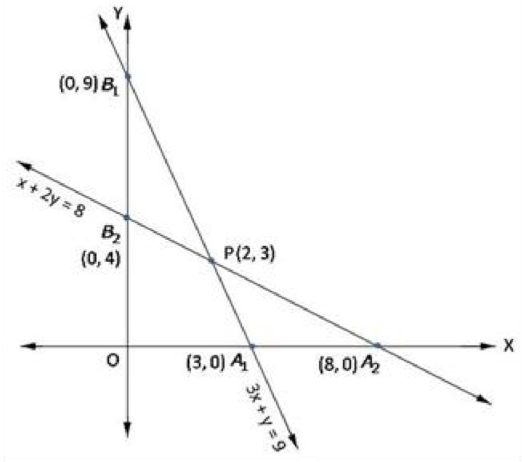
Subject to constraints,

$$3x + y \le 9$$
$$x + 2y \le 8$$
$$x, y \ge 0$$
$$\left[\text{Since production of } A \text{ and } B \text{ can not be less than zero}\right]$$

Region $3x + y \le 9$: line $3x + y = 9$ meets axes at $A_1(3,0)$, $B_1(0,9)$ respectively. Region containing origin represents $3x + y \le 9$ as $(0,0)$ satisfies $3x + y \ge 9$.

Region $x + 2y \le 8$: line $x + 2y = 8$ meets axes at $A_2(8,0)$, $B_2(0,4)$ respectively. Region containing origin represents $x + 2y \le 8$ as $(0,0)$ satisfies $x + 2y \le 8$.

Region $x, y \ge 0$: it represents first quadrant.



Shaded region OA_2PB_2 is the feasible region.Point $P(2, 3)$ is obtained by solving $3x + y = 9$ and $x + 2y = 8$

The value of $Z = 40x + 50y$ at

$O(0,0) = 40(0) + 50(0) = 0$

$A_1(3,0) = 40(3) + 50(0) = 120$

$P(2,3) = 40(2) + 50(3) = 230$

$B_2(0,4) = 40(0) + 50(4) = 200$

Therefore maximum $Z = 230$ at $x = 2, Y = 3$

Hence,

Maximum profit = Rs 230 number of goods of type $A = 2$, type $B = 3$

Linear Programming Ex 30.4 Q10

Let daily production of chairs and tables be x and y respectively.

Since, profits on each chair and table are Rs 3 and Rs 5. So,
profits on x number of chairs and y number of tables are Rs $3x$ and Rs $5y$ respectively,
Let Z be total profit on table and chair, so,

$$Z = 3x + 5y$$

Since, each chair and table require 2 hrs and 4 hrs on machine A respectively. so,
x number of chair and y number of table require $2x$ and $4y$ hrs on machine A respectively but,
maximum time available on machine A be 16 hrs, so,

$$2x + 4y \le 16$$
$$x + 2y \le 8 \qquad \text{(first constraint)}$$

Since, each chair and table require 6 hrs and 2 hrs on machine B. so,
x number of chair and y number of table require $6x$ and $2y$ hrs on machine B respectively but,
maximum time available on machine B be 30 hrs, so,

$$6x + 2y \le 30$$
$$3x + y \le 15 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which maximize

$$Z = 3x + 5y$$

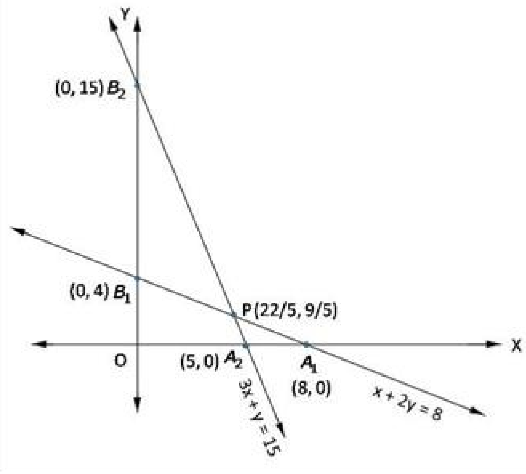
subject to constraints,

$$x + 2y \le 8$$
$$3x + y \le 15$$
$$x, y \ge 0$$
$$\left[\text{Since production of chair and table can not be less than zero}\right]$$

Region $x + 2y \le 8$: line $x + 2y = 8$ meets axes at $A_1(8,0)$, $B_1(0,4)$ respectively. Region containing origin represents $x + 2y \le 8$ as $(0,0)$ satisfies $x + 2y \le 8$.

Region $3x + y \le 15$: line $3x + y = 15$ meets axes at $A_2(5,0)$, $B_2(0,15)$ respectively. Region containing origin represents $3x + y \le 15$ as $(0,0)$ satisfies $3x + y \le 15$.

Region $x, y \ge 0$: it represents first quadrant.



Shaded region $O\ A_2 P\ B_1$ representa feasible region.Point $P\left(\frac{22}{5}, \frac{9}{5}\right)$ is obtained by solving $x + 2y = 8$ and $3x + y = 15$

The value of $Z = 3x + 5y$ at

$$
\begin{aligned}
&O\,(0,0) && = 3(0) + 5(0) = 0 \\
&A_2\,(5,0) && = 3(5) + 5(0) = 15 \\
&P\left(\frac{22}{5}, \frac{9}{5}\right) && = 3\left(\frac{22}{5}\right) + 5\left(\frac{9}{5}\right) = \frac{111}{5} = 22.2 \\
&B_1\,(0,4) && = 3(0) + 5(4) = 20
\end{aligned}
$$

Maximum $Z = 22.2$ at $x = \frac{22}{5}, y = \frac{9}{5}$

Daily production of chair $= \frac{22}{5}$, table $= \frac{9}{5}$

maximum profit = Rs 22.2

Linear Programming Ex 30.4 Q11

Let required production of chairs and tables be x and y.

Since, profits on each chair and table are Rs 45 and Rs 80, So,
profits on x number of chairs and y number of tables are Rs $45x$ and Rs $80y$,
Let Z be total profit on tables and chairs, so,

$$Z = 45x + 80y$$

Since, each chair and table require 5 sq.ft. and 20 sq.ft. of wood respectively. so,
x number of chair and y number of table require $5x$ and $20y$ sq.ft. of wood respectively but,
400 sq.ft. of wood is available, so,

$$5x + 20y \le 400$$

$$\Rightarrow \quad x + 4y \le 80 \qquad \text{(first constraint)}$$

Since, each chair and table require 10 and 25 men-hrs respectively. so,
x number of chairs and y number of tables require $10x$ and $25y$ men-hrs respectively but, only 450 men-hrs are available, so,

$$10x + 25y \le 450$$

$$\Rightarrow \quad 2x + 5y \le 90 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which maximize

$$Z = 45x + 80y$$

Subject to constraints,

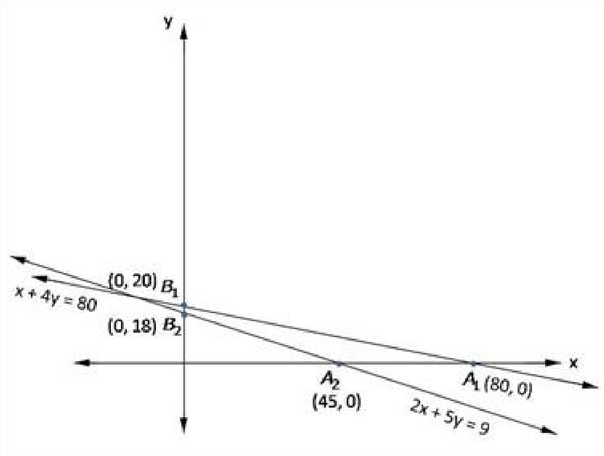
$$x + 4y \le 80$$
$$2x + 5y \le 90$$
$$x, y \ge 0$$

[Since production of tabel and chair can not be less than zero]

Region $x + 4y \le 80$: line $x + 4y = 80$ meets axes at $A_1(80,0)$, $B_1(0,20)$ respectively. Region containing origin represents $x + 4y \le 80$ as $(0,0)$ satisfies $x + 4y \le 80$.

Region $2x + 5y \le 90$: line $2x + 5y = 90$ meets axes at $A_2(45,0)$, $B_2(0,18)$ respectively. Region containing origin represents $2x + 5y \le 90$ as $(0,0)$ satisfies $2x + 5y \le 90$.

Region $x, y \geq 0$: it represents first quadrant.



Shaded region OA_2B_2 is the feasible region.

The value of $Z = 45x + 80y$ at

$O(0,0) \quad = 45(0) + 80(0) = 0$

$A_2(45,0) \quad = 45(45) + 80(0) = 2025$

$B_2(0,18) \quad = 45(0) + 80(18) = 1440$

Therefore,

Maximum $Z = 2025$ at $x = 45, y = 0$

Profit is maximum when number of chairs = 45, tables = 0

profit = Rs 2025

Linear Programming Ex 30.4 Q12

Let required production of product A and B be x and y respectively.

Since, profit on each product A and B are Rs 3 and Rs 4 respectively, So,
profit on x product A and y product B are Rs $3x$ and Rs $4y$ respectively,
Let Z be the total profit on product, so,

$$Z = 3x + 4y$$

Since, each product A and B requires 4 minutes each on machine M_1. so,
x product A and y product B require $4x$ and $4y$ minutes on machine M_1 respectively
but maximum available time on machine M_1 is 8 hrs 20 min.=500 min. so,

$$4x + 4y \leq 500$$

$$\Rightarrow \quad x + y \leq 125 \qquad \text{(first constraint)}$$

Since, each product A and B requires 8 minutes and 4 min. on machine M_2 respectively. so,
x product A and y product B require $8x$ and $4y$ min. respectively on machine M_2
but, maximum available time on machine M_2 is 10 hrs =600 min. so,

$$8x + 4y \leq 600$$

$$\Rightarrow \quad 2x + y \leq 150 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which maximize

$$Z = 3x + 4y$$

subject to constraints,

$$x + y \leq 125$$
$$2x + y \leq 150$$
$$x, y \geq 0$$

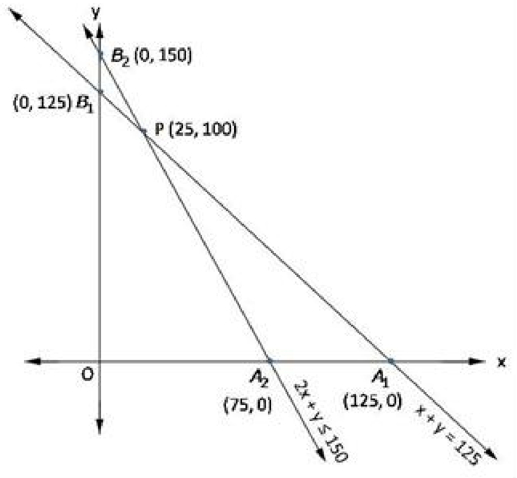
$$\left[\text{Since number of product can not be less than zero}\right]$$

Region $x + y \leq 125$: line $x + y = 125$ meets axis at $A_1(125,0)$, $B_1(0,125)$ respectively. Region $x + y \leq 125$ contains origin represents as $(0,0)$ satisfies $x + y \leq 125$.

Region $2x + y \leq 150$: line $2x + y = 150$ meets axis at $A_2(75,0)$, $B_2(0,150)$ respectively. Region containing origin represents $2x + y \leq 150$ as $(0,0)$ satisfies $2x + y \leq 150$

Region $x, y \geq 0$: it represents first quadrant.

Shaded region OA_2PB_1 is feasible region $P(25,100)$ is obtained by solving $x + y = 125$ and $2x + y = 150$



The value of $Z = 3x + 4y$ at

$O(0,0) \quad = 3(0) + 4(0) = 0$

$A_2(75,0) \quad = 3(75) + 4(0) = 225$

$P(25,100) \quad = 3(25) + 4(100) = 475$

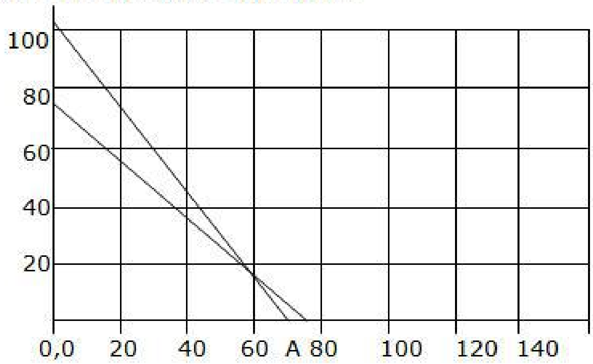
$B_1(0,125) \quad = 3(0) + 4(125) = 500$

Maximum profit = Rs 500, product $A = 0$
product $B = 125$

Linear Programming Ex 30.4 Q13

	Item A	Item B	
	x	y	
Motors	3x	2y	≤ 210
Transformer	4x	4y	≤ 300
Profit Rs.	20x	30y	Maximize

The above LPP can be presented in a table above.
Aim is to find the values of x & y that maximize the function Z = 20x + 30y, subject to the conditions
$3x + 2y \le 210$; gives x=0, y= 105 & y=0, x=70
$4x + 4y \le 300$; gives x=0, y=75 & y=0, x=75
x, y ≥ 0. Plotting the constraints,



The feasible region is 80-B-A-0,0
Tabulating the value of Z at the corner points

Corner point	Value of Z = 20x + 30y
0, 0	0
0, 75	2250
70, 0	1400
60, 15	1650

The maximum occur with the production of 0 units of Item A and 75 units of Item B, with a value of Rs. 2250/-

Linear Programming Ex 30.4 Q14

Let number of I product and II product produced are x and y respectively.

Since, profits on each unit of product I and product II are 2 and 3 monetary unit, So,
profits on x units of product I and y units of product II are $2x$ and $3y$ monetary units respectively,
Let Z be total profit, so,

$$Z = 2x + 3y$$

Since, each product I and II require 2 and 4 units of resources A, so,
x units of product I and y units of product II require $2x$ and $4y$ units of resource A respectively, but maximum available quantity of resource A is 20 units. so,

$$2x + 4y \le 20$$

$$\Rightarrow \quad x + 2y \le 10 \qquad (\text{first constraint})$$

Since, each product I and II require 2 and 4 units of resource B each, so,
x units of product I and y units of product II require $2x$ and $2y$ units of resource B respectively, but maximum available quantity of resource B is 12 units. so,

$$2x + 2y \le 12$$

$$\Rightarrow \quad x + y \le 6 \qquad (\text{second constraint})$$

Since, each units of product I require 4 units of resource C. It is not required by product II , so, x units of product I require $4x$ units of resource C, but maximum available quantity of resource C is 16 units. so,

$$4x \le 16$$

$$\Rightarrow \quad x \le 6 \qquad (\text{Third constraint})$$

Hence, mathematical formulation of LPP is, Find x and y which maximize

$$Z = 2x + 3y$$

Subject to constraints,

$$x + 2y \le 10$$
$$x + y \le 6$$
$$x \le 4$$
$$x, y \ge 0$$

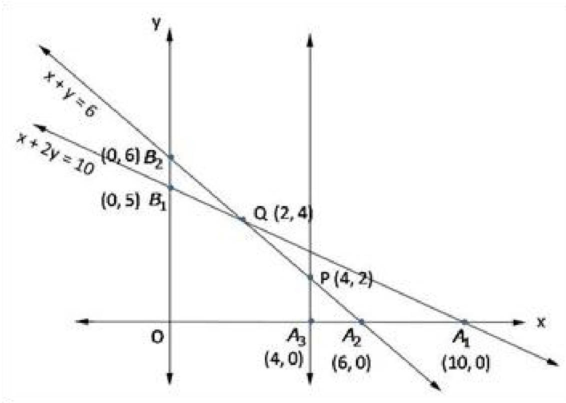
$$\left[\text{Since production fo I and II can not be less than zero}\right]$$

Region $x + 2y \le 10$: line $x + 2y = 10$ meets axes at $A_1(10,0)$, $B_1(0,5)$ respectively. Region containing origin represents $x + 2y \le 10$ as $(0,0)$ satisfies $x + 2y \le 10$.

Region $x + y \le 6$: line $x + y = 6$ meets axes at $A_2(6, 0)$, $B_2(0,6)$ respectively. Region containing origin represents $x + y \le 6$ as $(0,0)$ satisfies $x + y \le 6$.

Region $x \le 4$: line $x = 4$ is parallel to y - axis and meets y-axis at $A_3(4, 0)$. Region containing origin represents $x \le 4$ as $(0,0)$ satisfies $x \le 4$

Region $x, y \ge 0$: it represents first quadrant.



Shaded region OA_3PQB_1 represents feasible region $P(4,2)$ is obtained by solving $x = 4$ and $x + y = 6$, $Q(2,4)$ is obtained by solving $x + y = 6$ and $x + 2y = 10$.

The value of $Z = 2x + 3y$ at

$O(0, 0)$	$= 2(0) + 3(0) = 0$
$A_3(4,0)$	$= 2(4) + 3(0) = 8$
$P(4,2)$	$= 2(4) + 3(2) = 14$
$Q(12, 4)$	$= 2(12) + 3(4) = 16$
$B_1(0, 5)$	$= 2(0) + 3(5) = 15$

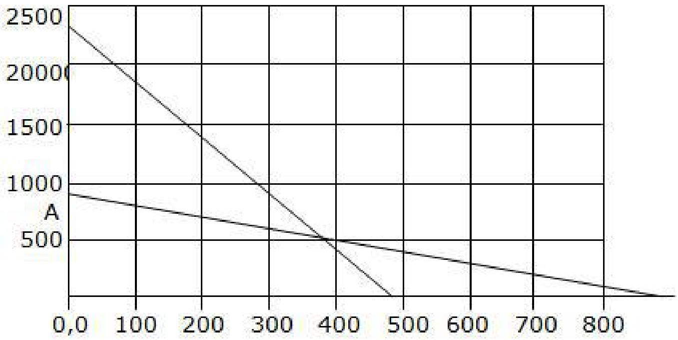
Maximum $Z = 16$ at $x = 2, y = 4$
First product = 2 units, second product = 4 unit
Maximum profit = 16 monetary units

Linear Programming Ex 30.4 Q15

	Hardcover	Paperback	
	x	y	
Printing time	5x	5y	≤ 4800
Binding time	10x	2y	≤ 4800
Selling price Rs.	72x	40y	Maximize

The above LPP can be presented in a table above.

Aim is to find the values of x & y that maximize the function $Z = 72x + 40y$, subject to the conditions
$5x + 5y \leq 4800$; gives $x=0$, $y= 960$ & $y=0$, $x=960$
$10x + 2y \leq 4800$; gives $x=0$, $y=2400$ & $y=0$, $x=480$
$x, y \geq 0$. Plotting the constraints,



The feasible region is A-B-480-0,0
Tabulating the value of Z at the corner points

Corner point	Value of Z = 72x + 40y
0, 0	0
0, 480	19200
360, 600	49920
480, 0	34560

The maximum occurs with the production of 360 units of Hardcover books and 600 units of Paperback books, with a value of Rs. 49920/-. This the selling price.

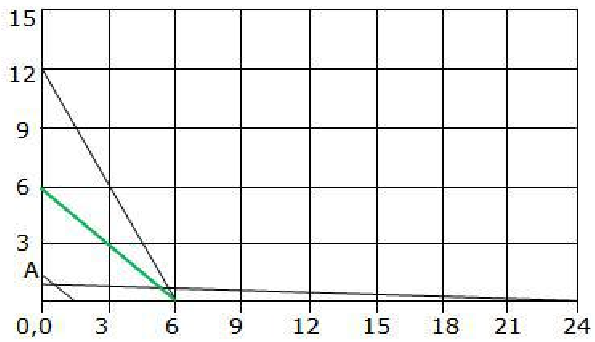
Cost price = fixed cost + variable cost
= 9600+56x360+28x600 = 46560

Profit = Selling price − cost price = 49920-46560
= Rs. 3360

Linear Programming Ex 30.4 Q16

	Pill size A	Pill size B	
	x	y	
Aspirin	2x	1.y	≥ 12
Bicarbonate	5x	8y	≥ 7.4
Codeine	1.x	66y	≥ 24
Relief	x	y	Minimize

The above LPP can be presented in a table above.
Aim is to find the values of x & y that minimize the function $Z = x + y$, subject to the conditions
$2x + y \geq 12$; gives x=0, y= 12 & y=0, x=6
$5x + 8y \geq 7.4$; gives x=0, y=7.4/8 & y=0, x=7.4/5
$x + 66y \geq 24$; gives x=0, y=4/11 & y=0, x=24
$x, y \geq 0$. Plotting the constraints,



The feasible region is 12-C-24
Tabulating the value of Z at the corner points

Corner point	Value of Z = x + y
0, 12	12
24, 0	24
5.86, 0.27	6.13

The minimum occurs with 5.86 pills of size A and 0.27 pills of size B. since the feasible region is unbounded plot x+y < 6.13. the green line shows here are no common points with the unbounded feasible region so the obtained point is the point that gives minimum pills to be consumed.

Linear Programming Ex 30.4 Q17

Let required quantity of compound A and B are x and y kg.

Since, cost of one kg of compound A and B are Rs 4 and Rs 6 per kg.So,
cost of x kg. of compound A and y kg. of compound B are Rs $4x$ and Rs $6y$
respectively, Let Z be the total cost of compounds, so,

$$Z = 4x + 6y$$

Since, compound A and B contain 1 and 2 units of ingredient C per kg. respectively, so,
x kg. of compound A and y kg. of compound B contain x and $2y$ units of ingredient C
respectively but minimum requirement of ingredient C is 80 units, so,

$$x + 2y \ge 80 \qquad \text{(first constraint)}$$

Since, compound A and B contain 3 and 1 unit of ingredient D per kg. respectively, so,
x kg. of compound A and y kg. of compound B contain $3x$ and y units of ingredient D
respectively but minimum requirement of ingredient D is 75 units, so,

$$3x + y \ge 75 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which minimize

$$Z = 4x + 6y$$

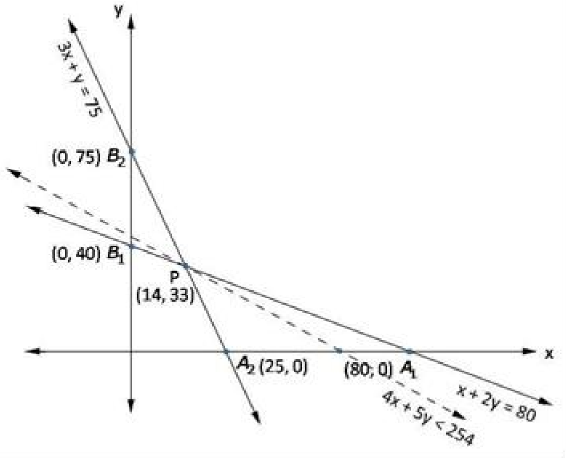
Subject to constraints,

$$x + 2y \ge 80$$
$$3x + y \ge 75$$
$$x, y \ge 0$$
$$\text{[Since production can not be less than zero]}$$

Region $x + 2y \ge 80$: line $x + 2y = 80$ meets axes at $A_1(80,0)$, $B_1(0,40)$ respectively. Region not containing origin represents $x + 2y \ge 80$ as $(0,0)$ does not satisfy $x + 2y \ge 80$.

Region $3x + y \ge 75$: line $3x + y = 75$ meets axes at $A_2(25,0)$, $B_2(0,75)$ respectively. Region not containing origin represents $3x + y \ge 75$ as $(0,0)$ does not satisfy $3x + y \ge 75$.

Region $x, y \ge 0$: it represents first quadrant.



Unbouded shaded region $A_1 P B_2$ represents feasible region. point P is obtained by solving $x + 2y = 80$ and $3x + y = 75$

The value of $Z = 4x + 6y$ at

$A_1\left(80,0\right)$	$= 4\left(80\right) + 6\left(0\right) = 320$
$P\left(14,33\right)$	$= 4\left(14\right) + 6\left(33\right) = 254$
$B_2\left(0,75\right)$	$= 4\left(0\right) + 6\left(75\right) = 450$

Smallest value of $Z =$ 254 open half plane $4x + 6y < 254$ has no point in common with feasible region. so,

Smallest value is the minimum value.

Minimum cost= Rs 254
quantity of A = 14 kg
quantity of B = 33 kg

Linear Programming Ex 30.4 Q18

Let the company manufacture x souvenirs of type A and y souvenirs of type B. Therefore,

$x \geq 0$ and $y \geq 0$

The given information can be complied in a table as follows.

	Type A	Type B	Availability
Cutting (min)	5	8	$3 \times 60 + 20 = 200$
Assembling (min)	10	8	$4 \times 60 = 240$

The profit on type A souvenirs is Rs 5 and on type B souvenirs is Rs 6. Therefore, the constraints are

$5x + 8y \leq 200$

$10x + 8y \leq 240$ i.e., $5x + 4y \leq 120$

Total profit, $Z = 5x + 6y$

The mathematical formulation of the given problem is

Maximize $Z = 5x + 6y$ ... (1)

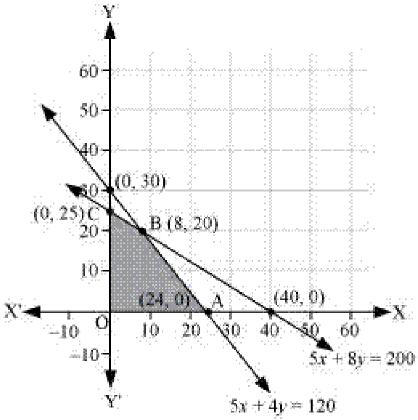
subject to the constraints,

$5x + 8y \leq 200$ ... (2)

$5x + 4y \leq 120$ ... (3)

$x, y \geq 0$ ... (4)

The feasible region determined by the system of constraints is as follows.



The corner points are A (24, 0), B (8, 20), and C (0, 25).

The values of Z at these corner points are as follows.

Corner point	**Z = 5*x* + 6*y***	
A(24, 0)	120	
B(8, 20)	160	→ Maximum
C(0, 25)	150	

The maximum value of Z is 200 at (8, 20).

Thus, 8 souvenirs of type A and 20 souvenirs of type B should be produced each day to get the maximum profit of Rs 160.

Linear Programming Ex 30.4 Q19

Let required number of product A and B be x and y respectively.

Since, profit on each product A and B are Rs 20 and Rs 30 respectively.So,
x number of product A and y number of product B gain profits of Rs $20x$ and Rs $30y$ respectively, Let Z be total profit then,

$$Z = 20x + 30y$$

Since, selling prices of each product A and B are Rs 200 and Rs 300 respectively, so, revenues earned by selling x units of product A and y units of product B are $200x$ and $300y$ respectively but weekly turnover must not be less than Rs 10000, so,

$$200x + 300y \geq 10000$$

$$2x + 3y \geq 100 \qquad \text{(first constraint)}$$

Since, each product A and B require $\frac{1}{2}$ and 1 hr.to make so, x units of product A and y units of product B are $\frac{1}{2}x$ and y hrs. to make respectively but working time available is 40 hrs maximum, so,

$$\frac{1}{2}x + y \leq 40$$

$$x + 2y \leq 80 \qquad \text{(second constraint)}$$

There is a permanent order of 14 and 16 of product A and B respectively, so,

$$x \geq 14$$

$$y \geq 16 \qquad \text{(third and fourth constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which miximize

$$Z = 20x + 30y$$

Subject to constraints,

$$2x + 3y \geq 100$$

$$x + 2y \leq 80$$

$$x \geq 40$$

$$y \geq 14$$

$$x, y \geq 0$$

$$\left[\text{Since production can not be less than zero}\right]$$

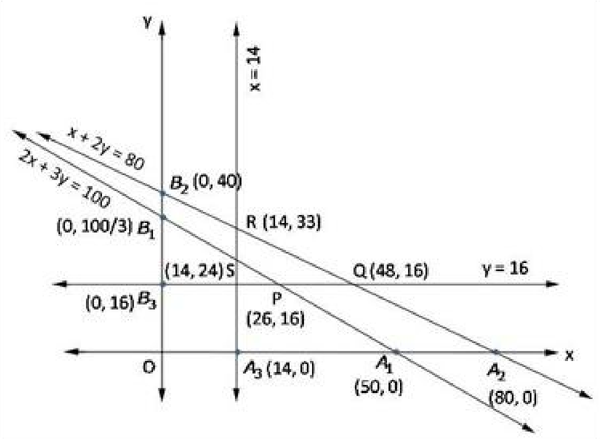
Region $2x + 3y \geq 100$: line $2x + 3y = 100$ meets axes at $A_1(50,0)$, $B_1\left(0, \frac{100}{3}\right)$ respectively. Region not containing origin represents $2x + 3y \geq 100$ as $(0,0)$ does not satisfy $2x + 3y \geq 100$.

Region $x + 2y \le 80$: line $x + 2y = 80$ meets axes at $A_2(80,0), B_2(0,40)$ respectively. Region not containing origin represents $x + 2y \le 80$ as $(0,0)$ satisfies $x + 2y \le 80$.

Region $x \ge 14$: line $x = 14$ is parallel to y-axis and meets x-axis at $A_3(14,0)$. Region not containing origin represents $x \ge 14$ as $(0,0)$ does not satisfy $x \ge 14$.

Region $y \ge 16$: line $y = 16$ is parallel to x-axis and meets y-axis at $B_3(0,16)$. Region not containing origin represents $y \ge 16$ as $(0,0)$ does not satisfy $y \ge 16$.

Region $x, y \ge 0$: it represents first quadrant.



Shaded region $PQRS$ represents feasible region. Point $P(26,16)$ is obtained by solving $y = 16$ and $2x + 3y = 100$, $Q(48,16)$ is obtained by solving $y = 16$ and $x + 2y = 80$, $R(14,33)$ is obtained by solving $x = 14$ and $x + 2y = 80$, $S(14,24)$ is obtained by solving $x = 14$ and $2x + 3y = 100$

The value of $Z = 20x + 30y$ at

$P(26,16) \quad = 20(26) + 30(16) = 1000$

$Q(48,16) \quad = 20(48) + 3(16) = 1440$

$R(14,33) \quad = 20(14) + 3(33) = 1270$

$S(14,24) \quad = 20(14) + 3(24) = 1000$

maximum $Z = 1440$ at $x = 48, y = 16$

Number product $A = 48$, product $B = 16$

maximum profit = Rs 1440

Linear Programming Ex 30.4 Q20

Let required number of trunk I and trunk II be x and y respectively.

Since, profit on each trunk I and trunk II are Rs 30 and Rs 25 respectively.So, profit on x trunk of type I and y trunk of type II are Rs $30x$ and Rs $25y$ respectively, Let total profit on trunks be Z, so,

$$Z = 30x + 25y$$

Since, each trunk I and trunk II is reequired to work 3 hrs each on machine A, so, x trunk I and y trunk II is required $3x$ and $3y$ hrs respectively to work on machine A but machine A can work for at most 18 hrs, so,

$$3x + 3y \le 18$$

$$\Rightarrow \quad x + y \le 6 \qquad \text{(first constraint)}$$

Since, each trunk I and II is reequired to work 3 hrs and 2 hrs on machine B, so, x trunk I and y trunk II is required $3x$ and $2y$ hrs to work respectively on machine B but machine B can work for at most 15 hrs, so,

$$3x + 2y \le 15 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which miximize

$$Z = 30x + 25y$$

Subject to constraints,

$$x + y \le 6$$

$$3x + 2y \le 15$$

$$x, y \ge 0$$

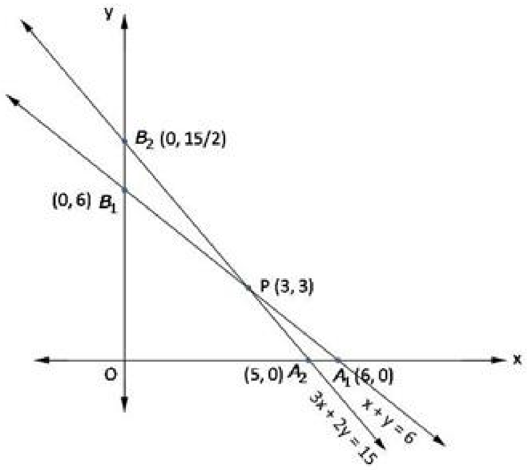
[Since production of trunk can not be less than zero]

Region $x + y \le 6$: line $x + y = 6$ meets axes at $A_1(6,0), B_1(0,6)$ respectively. Region containing origin represents $x + y \le 6$ as $(0,0)$ satisfies $x + y \le 6$.

Region $3x + 2y \le 15$: line $3x + 2y = 15$ meets axes at $A_2(5,0), B_2\left(0, \frac{15}{2}\right)$ respectively. Region containing origin represents $3x + 2y \le 15$ as $(0,0)$ satisfies $3x + 2y \le 15$.

Region $x, y \ge 0$: it represents first quadrant.

Shaded region A_2PB_1 represents feasible region. Point $P(3,3)$ is obtained by solving $x + y = 6$ and $3x + 2y = 15$,



The value of $Z = 30x + 25y$ at

$A_2(5,0) \quad = 30(5) + 25(0) = 150$

$P(3,3) \quad = 30(3) + 25(3) = 165$

$B_1(0,6) \quad = 30(0) + 25(6) = 150$

$O(0,0) \quad = 30(0) + 25(0) = 0$

maximum $Z = 165$ at $x = 3, y = 3$

Trunk of type $A = 3$, type $B = 3$

maximum profit = Rs 165

Linear Programming Ex 30.4 Q21

Let production of each bottle of A and B are x and y respectively.

Since, profits on each bottle of A and B are Rs 8 and Rs 7 per bottle respectively.So, profit on x bottles of A and y bottles of B are $8x$ and $7y$ respectively, Let Z be total profit on bottles so,

$$Z = 8x + 7y$$

Since, it takes 3 hrs and 1 hr to prepare enough material to fill 1000 bottlesof type A and B respectively, so, x bottles of A and y bottles of B are preparing is $\frac{3x}{1000}$hrs and $\frac{y}{100}$ hrs respectively but total 66 hrs are available, so,

$$\frac{3x}{1000} + \frac{y}{1000} \le 66$$

$$\Rightarrow \quad 3x + y \le 66000 \qquad \text{(first constraint)}$$

Since, row material available to make 2000 bottles of A and 4000 bottles of B but there are 45000 bottles into which either of medicines can be put so,

$$\Rightarrow \quad x \le 20000 \qquad \text{(second constraint)}$$

$$y \le 40000 \qquad \text{(third constraint)}$$

$$x + y \le 45000 \qquad \text{(fourth constraint)}$$

$$x, y \ge 0$$

[Since production of bottles can not be less than zero]

Hence, mathematical formulation of LPP is find x and y which

$$\text{maximize } Z = 8x + 7y$$

Subject to constriants,

$$3x + y \le 66000$$

$$x \le 20000$$

$$y \le 40000$$

$$x + y \le 45000$$

$$x, y \ge 0$$

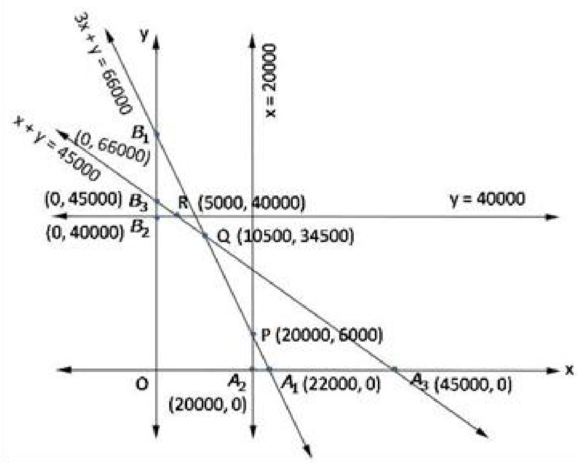
Region $3x + y \le 66000$: line $3x + y = 66000$ meets axes at $A_1(22000, 0)$, $B_1(0, 66000)$ respectively. Region containing origin represents $3x + y \le 66000$ as $(0,0)$ satisfies $3x + y \le 66000$.

Region $x \le 20000$: line $x = 20000$ is parallel to y-axis and meets x-axis at $A_2(20000, 0)$. Reg containing origin represents $x \le 20000$ as $(0,0)$ satisfies $x \le 20000$.

Region $y \le 40000$: line $y = 40000$ is parallel to x-axis and meets y-axis at $B_2(0, 40000)$. Reg containing origin represents $y \le 40000$ as $(0,0)$ satisfies $y \le 40000$.

Region $x + y \le 45000$: line $x + y = 45000$ meets axes at $A_3(45000,0)$, $B_3(0,45000)$ respectiv containing origin represents $x + y \le 45000$ as $(0,0)$ satisfies $x + y \le 45000$.

Region $x, y \ge 0$: it represents first quadrant.



Shaded region OA_2PRB_2 represents feasible region. Point $P(20000,6000)$ is obtained by solving $x = 20000$ and $3x + y = 66000$, $Q(10500, 34500)$ is obtained by solving $x + y = 45000$ and $3x + y = 66000$, $R(15000,40000)$ is obtained by solving $x + y = 45000$, $y = 40000$

The value of $Z = 8x + 7y$ at

$O(0, 0)$	$= 8(0) + 7(0) = 0$
$A_2(20000, 0)$	$= 8(20000) + 7(0) = 160000$
$P(20000, 6000)$	$= 8(20000) + 7(6000) = 202000$
$Q(10500, 34500)$	$= 8(10500) + 7(34500) = 325500$
$R(5000, 4000)$	$= 8(5000) + 7(40000) = 32000$
$B_2(0, 40000)$	$= 8(0) + 7(40000) = 250000$

maximum $Z = 325500$ at $x = 10500, y = 34500$

Number bottles A type = 10500, B type = 34500

maximum profit = Rs 325500

Linear Programming Ex 30.4 Q22

Let required number of first class and economy class tickets be x and y respectively.

Each ticket of first class and economy class make profit of Rs 400 and Rs 600 respectively.So, x ticket of first class and y tickets of economy class make profits of Rs $400x$ and Rs $600y$ respectively, Let total profit be Z, so,

$$Z = 400x + 600y$$

Given, aeroplane can carry a miximum of 200 passengers,so,

$\Rightarrow \quad x + y \le 200 \qquad$ (first constraint)

Given, airline reservesa at least 20 seats for first class,so,

$\Rightarrow \quad x \ge 20 \qquad$ (second constraint)

Given, at least 4 times as mnay passengers prefer totravel by economy class to the first class ,so,

$y \ge 4x$

$\Rightarrow \quad 4x - y \le 0 \qquad$ (third constraint)

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = 400x + 600y$

Subject to constriants,

$x + y \le 200$

$x \ge 20$

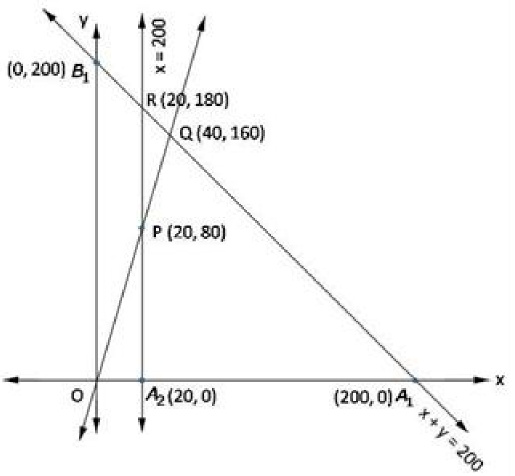
$4x - y \le 0$

$x, y \ge 0$

[Since seats of both the classes can not be less than zero]

Region $x + y \le 200$: line $x + y = 200$ meets axes at $A_1(200,0)$, $B_1(0,200)$ respectively.

Region containing origin represents $x + y \le 200$ as $(0,0)$ satisfies $x + y \le 200$.



Shaded region PQR represents feasible region. $Q(40,160)$ is obtained by solving $x + y = 200$ and $4x - y = 0$, $R(20,180)$ is obtained by solving $x = 20$ and $x + y = 200$

The value of $Z = 400x + 600y$ at

$P(20,80) \quad = 400(20) + 600(80) = 56000$

$Q(40,160) \quad = 400(40) + 600(160) = 112000$

$R(20,180) \quad = 400(20) + 600(180) = 116000$

so,

maximum Z = Rs 116000 at $x = 20, y = 180$

Number of first class ticket = 20,

Number of economy class ticket= 180

maximum profit = Rs 116000

Linear Programming Ex 30.4 Q23

	Type I	Type II	
	x	y	
Nitrogen	0.1x	0.05y	≥ 14
Bicarbonate	0.06x	0.1y	≥ 14
Cost	0.6x	0.4y	Minimize

The above LPP can be presented in a table above.
Aim is to find the values of x & y that minimize the function $Z = 0.6x + 0.4y$, subject to the conditions
$0.1x + 0.05y \geq 14$; gives $x=0$, $y= 280$ & $y=0$, $x=140$
$0.06x + 0.1y \geq 14$; gives $x=0$, $y=140$ & $y=0$, $x=233.33$
$x, y \geq 0$. Plotting the constraints,

The feasible region is the unbounded region D-C-B

Corner point	Value of $Z = 0.6x + 0.4y$
0, 280	112
233.33, 0	140
100, 80	92

The minimum occurs at $x=100$, $y=80$ with a value of 92
Since the region is unbounded plot $0.6x + 0.4y \leq 92$
Plotting the points, we get line E-100.
There are no common points so $x=100$, $y=80$ with a value of 92 is the optimal minimum.

Linear Programming Ex 30.4 Q24

Let he invests Rs x and Rs y in saving certificate (sc) and National saving bond (NSB) respectively.

Since, rate of interest on SC is 8% annual and on NSB is 10% annual,So, interest on Rs x of SC is $\frac{8x}{100}$ and Rs y of NSB is $\frac{10x}{100}$ per annum.

Let Z be total interest earned so,

$$Z = \frac{8x}{100} + \frac{10y}{100}$$

Given he wants to invest Rs 12000 is total

$$x + y \le 12000 \qquad \text{(third constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

$$\text{maximize } Z = \frac{8x}{100} + \frac{10y}{100}$$

Subject to constriants,

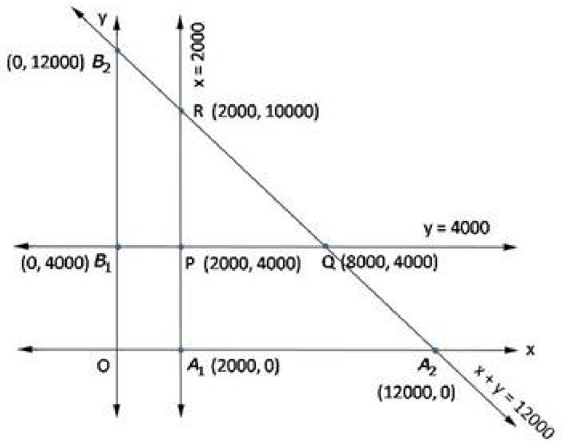
$$x \ge 2000$$
$$y \ge 4000$$
$$x + y \le 12000$$
$$x, y \ge 0 \qquad \text{[Since investment can not be less than zero]}$$

Region $x \ge 2000$: line $x = 2000$ is parallel to y-axis and meets x-axis at $A_1(2000,0)$.
Region not containing origin represents $x \ge 2000$ as $(0,0)$ does not satisfy $x \ge 2000$

Region $y \ge 4000$: line $y = 4000$ is parallel to x-axis and meets y-axis at $B_1(0, 4000)$. Region not containing origin represents $y \ge 4000$ as $(0,0)$ does not satisfy $y \ge 4000$.

Region $x + y \le 12000$: line $x + y = 12000$ meets axes at $A_2(12000,0)$, $b_2(0,1200)$ respectively.
Region containing represents $x + y \le 12000$ as $(0,0)$ satisfies $x + y \le 12000$.

Region $x, y \ge 0$: it represents first quadrant.



Shaded region PQR represents feasible region. $P(2000, 4000)$ is obtained by solving $x = 2000$ and $y = 4000$, $Q(8000, 4000)$ is obtained by solving $x + y = 12000$ and $y = 4000$ $R(2000, 10000)$ is obtained by solving $x = 2000$ and $y + x = 1200$

The value of $Z = \frac{8x}{100} + \frac{10y}{100}$ at

$P(2000, 4000) \quad = \frac{8}{100}(2000) + \frac{10}{100}(4000) = 560$

$Q(8000, 4000) \quad = \frac{8}{100}(8000) + \frac{10}{100}(4000) = 1040$

$R(2000, 10000) \quad = \frac{8}{1000}(2000) + \frac{10}{100}(10000) = 1160$

so,

maximum Z = Rs 1160 at $x = 2000, y = 10000$

He should invest Rs 2000 in Saving Certificates and 1000 in National Saving scheme, maximum Interest = Rs 1160

Linear Programming Ex 30.4 Q25

Let required number of trees of type A and B be Rs x and Rs y respectively.

Since, selling price of 1 kg of type A is Rs 2 and growth is 20 kg per tree, so, revenue from type A is Rs 40x , selling price of 1 kg of type B is Rs 1.5 and growth 40 kg per tree, so, revenue from type B is Rs $60y$. Total revenue is $(40x + 60y)$. Costs of each tree of type A and B are Rs 20 and Rs 25, so,costs of x trees of type A and y trees of type B are Rs $20x$ and $25y$ respectively. Total cost is Rs $(20x + 25y)$

Let Z be total profit so,

$$Z = (40x - 60y) - (20x + 25y)$$
$$Z = 20x + 35y$$

Since he has Rs 1400 to invest so,

cost ≤ 1400

$\Rightarrow \quad 20x + 35y \leq 1400$

$\Rightarrow \quad 4x + 5y \leq 280 \qquad$ (first constraint)

Since each tree of type A and B needs 10 sq. m and 20 sq. m of ground respectively so, x trees of type A and y trees of type B need $10x$ sq. m and $20y$ sq. m of ground respectively. but total ground available is 1000 sq. m so,

$10x + 20y \leq 1000$

$\Rightarrow \quad x + 2y \leq 100 \qquad$ (second constraint)

$x, y \geq 0$

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = 20x + 35y$

Subject to constriants,

$4x + 5y \leq 280$

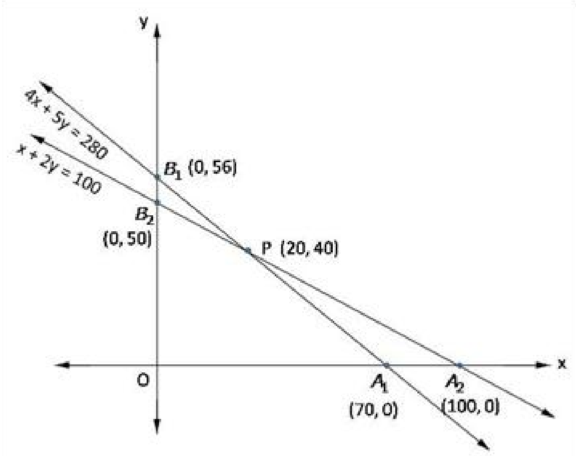
$\Rightarrow \quad x + 2y \leq 100$

$x, y \geq 0 \qquad$ [Since number of trees can not be less than zero]

Region $4x + 5y \le 280$: line $4x + 5y = 280$ meets axes at $A_1(70,0)$, $B_1(0,56)$ respectively.
Region containing origin represents $4x + 5y \le 280$ as $(0,0)$ satisfies $4x + 5y \le 280$.

Region $x + 2y \le 100$: line $x + 2y = 100$ meets axes at $A_2(100,0)$, $B_2(0,50)$ respectively.
Region containing origin represents $x + 2y \le 100$ as $(0,0)$ satisfies $x + 2y \le 100$.

Region $x, y \ge 0$: it represents first quadrant.



Shaded region OA_1PB_2 the feasible region. $P(20,40)$ is obtained by solving $x + 2y = 100$ and $4x + 5y = 280$,

The value of $Z = 20x + 35y$ at

$O(0,0)$	$= 20(0) + 35(0) = 0$
$A_1(70,0)$	$= 20(0) + 35(0) = 1400$
$P(20,40)$	$= 20(20) + 35(40) = 1800$
$B_2(0,50)$	$= 20(0) + 35(50) = 1750$

maximum $Z = 1800$ at $x = 20, y = 40$

20 trees of type A , 40 trees of type B, profit = Rs 1800

Linear Programming Ex 30.4 Q26

Let the cottage industry manufacture x pedestal lamps and y wooden shades. Therefore,

$x \ge 0$ and $y \ge 0$

The given information can be compiled in a table as follows.

	Lamps	Shades	Availability
Grinding/Cutting Machine (h)	2	1	12
Sprayer (h)	3	2	20

The profit on a lamp is Rs 5 and on the shades is Rs 3. Therefore, the constraints are

$2x + y \le 12$

$3x + 2y \le 20$

Total profit, $Z = 5x + 3y$

The mathematical formulation of the given problem is

Maximize $Z = 5x + 3y$ ... (1)

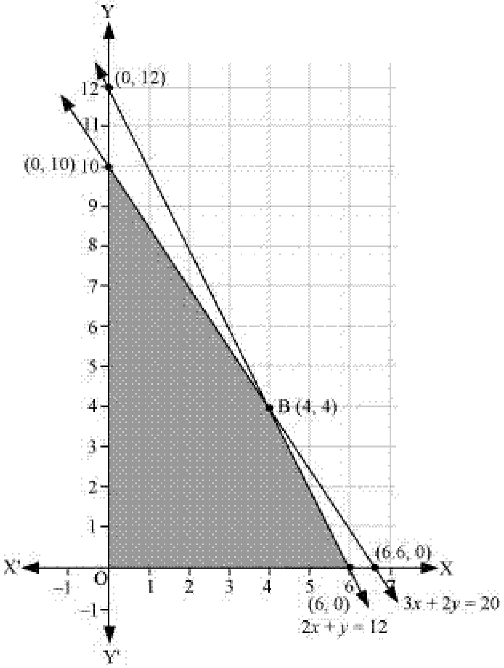
subject to the constraints,

$2x + y \le 12$ ... (2)

$3x + 2y \le 20$ ... (3)

$x, y \ge 0$ ... (4)

The feasible region determined by the system of constraints is as follows.



The corner points are A (6, 0), B (4, 4), and C (0, 10).

The values of Z at these corner points are as follows

Corner point	$Z = 5x + 3y$	
A(6, 0)	30	
B(4, 4)	32	→ Maximum
C(0, 10)	30	

The maximum value of Z is 32 at (4, 4).

Thus, the manufacturer should produce 4 pedestal lamps and 4 wooden shades to maximize his profits.

Linear Programming Ex 30.4 Q27

Let required number of goods of type x and y be x_1 and x_2 respectively.

Since, selling prices of each goods of type x and y are Rs 100 and Rs 120 respectively, so, selling price of x_1 units of goods of type x and x_2 units of goods of type y are Rs $100x$ and Rs $120y$ respective respectively

Let Z be total revenue, so

$$Z = 100x_1 + 120x_2.$$

Since each unit of goods x and y require 2 and 3 units of labour, so, x_1 unit of x and x_2 unit of y require $2x_1$ and $3x_2$ units of labour units but maximum labour units available is 30 units, so,

$$2x_1 + 3x_2 \le 30 \qquad \text{(first constraint)}$$

Since each unit of goods x and y require 3 and 1 unit of capital so, x_1 unit of x and x_2 unit of y require $3x_1$ and x_2 units of capital respectively but maximum units available for capital is 17 , so,

$$3x_1 + x_2 \le 17 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = 20x + 35y$

Subject to constriants,

$$2x_1 + 3x_2 \le 30$$

$$\Rightarrow \quad 3x_1 + x_2 \le 17$$

$$x_1, x_2 \ge 0 \qquad \text{[Since production of goods can not be less than zero]}$$

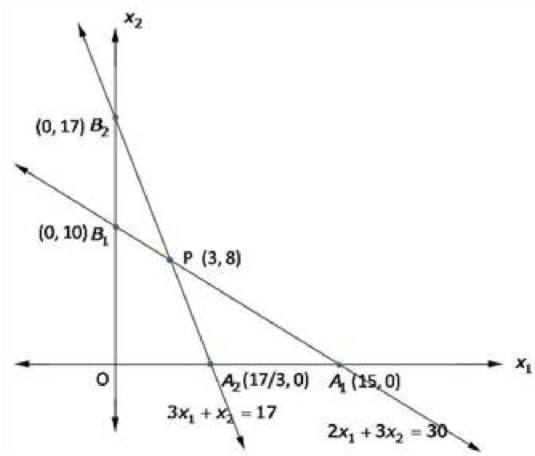
Region $2x_1 + 3x_2 \le 30$: line $2x + 3y = 30$ meets axes at $A_1(15,0)$, $B_1(0,10)$ respectively.
Region containing origin represents $2x_1 + 3x_2 \le 30$ as $(0,0)$ satisfies $2x_1 + 3x_2 = 30$.

Region $3x_1 + x_2 \le 17$: line $3x_1 + x_2 \le 17$ meets axes at $A_2\left(\frac{17}{3},0\right)$, $B_2(0,17)$ respectively.

Region containing origin represents $3x_1 + x_2 \le 17$ as $(0,0)$ satisfies $3x_1 + x_2 \le 17$.

Region $x_1, x_2 \ge 0$: it represent first quandrant shaded region OA_2PB_1 represents feasible region. Point $P(3,8)$ is obtained by solving

$2x_1 + 3x_2 = 30$ and $3x_1 + x_2 = 17$



The value of $Z = 100x_1 + 120x_2$ at

$O(0,0) = 100(0) + 120(0) = 0$

$A_2\left(\frac{17}{3},0\right) = 100\left(\frac{17}{3}\right) + 120(0) = \frac{1700}{3} = 566\frac{2}{3}$

$P(3,8) = 100(3) + 120(8) = 1260$

$B_1(0,10) = 100(0) + 120(10) = 1200$

maximum $Z = 1260$ at $x = 3, y = 8$

goods of type $x = 3$, type $y = 8$
maximum profit = Rs 12160

Linear Programming Ex 30.4 Q28

Let required number of product A and B be x and y respectively.

Since,profit on each product A and B are Rs 5 and Rs 3 respectively, so,
profits on x product A and y product B are Rs $5x$ and Rs $3y$ respectively
Let Z be total profit so

$$Z = 5x + 3y$$

Since each unit of product A and B require one min. each on machine M_1 so,
x unit of product A and y units of product B require x and y min. respectivley
on machine M_1 but M_1 can work at most $5 \times 60 = 300$ min.,so

$$x + y \leq 300 \qquad \text{(first constraint)}$$

Since each unit of product A and B require 2 and one min.respectively on machine
M_2 so, x unit of product A and y units of product B require $2x$ and y min. respectivley
on machine M_2 but M_2 can work at most $6 \times 60 = 360$ min.,so

$$2x + y \leq 360 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = 5x + 3y$

Subject to constriants,

$$x + y \leq 300$$

$$\Rightarrow \quad 2x + y \leq 360$$

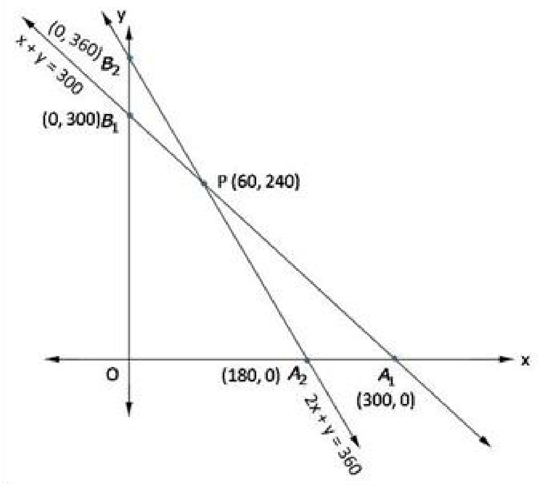
$$x, y \geq 0 \qquad \left[\text{Since production can not be less than zero}\right]$$

Region $x + y \le 300$: line $x + y = 300$ meets axes at $A_1(300,0)$, $B_1(0,300)$ respectively.
Region containing origin represents $x + y \le 300$ as $(0,0)$ satisfies $x + y = 300$.

Region $2x + y \le 360$: line $2x + y = 360$ meets axes at $A_2(180,0)$, $B_2(0,360)$ respectively.
Region containing origin represents $2x + y \le 360$ as $(0,0)$ satisfies $2x + y \le 360$.

Region $x, y \ge 0$: it represent first quandrant

Shaded region OA_2PB_1 represents feasible region.
Point $P(60,240)$ is obtained by solving
$x + y = 300$ and $2x + y = 360$



The value of $Z = 5x + 3y$ at

$O(0,0) \quad = 5(0) + 3(0) = 0$

$A_2(180,0) \quad = 5(180) + 3(0) = 900$

$P(60,240) \quad = 5(60) + 3(240) = 1020$

$B_1(0,300) \quad = 5(0) + 3(300) = 900$

maximum $Z = 1020$ at $x = 60, y = 240$

Number of product $A = 60$, product $B = 240$
maximum profit = Rs 1020

Linear Programming Ex 30.4 Q29

Let required quantity of item A and B produced be x and y respectively.

Since,profits on each item A and B are Rs 300 and Rs 160 respectively, so, profits on x unit of item A and y units of item B are Rs $300x$ and Rs $160y$ respectively
Let Z be total profit so

$$Z = 300x + 160y$$

Since one unit of item A and B require one and $\frac{1}{2}$ hr respectively, so, x units of item A and y units of item B require x and $\frac{1}{2}y$ hr. respectivley but maximum time available is 16 hours.,so

$$x + \frac{1}{2}y \le 16$$

$\Rightarrow \quad 2x + y \le 32 \qquad$ (first constraint)

Given, manufacturer can produce at most 24 items, so,

$\Rightarrow \quad x + y \le 24 \qquad$ (second constraint)

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = 300x + 160y$

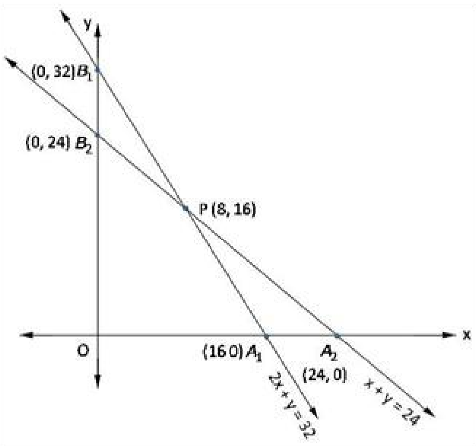
Subject to constriants,

$$2x + y \le 32$$
$$x + y \le 24$$
$$x, y \ge 0 \qquad \left[\text{Since production can not be less than zero}\right]$$

Region $2x + y \le 32$: line $2x + y = 32$ meets axes at $A_1(16,0)$, $B_1(0,32)$ respectively.
Region containing origin represents $2x + y \le 32$ as $(0,0)$ satisfies $2x + y \le 32$.

Region $x + y \le 24$: line $x + y = 24$ meets axes at $A_2(24,0)$, $B_2(0,24)$ respectively.
Region containing origin represents $x + y \le 24$ as $(0,0)$ satisfies $x + y \le 24$.

Region $x, y \ge 0$: it represent first quandrant



Shaded region OA_1PB_2 represents feasible region.
Point P is obtained by solving

$$x + y = 24 \text{ and } 2x + y = 32$$

The value of $Z = 300x + 160y$ at

$O(0,0)$	$= 300(0) + 160(0) = 0$
$A_1(16,0)$	$= 300(180) + 160(0) = 4800$
$P(8,16)$	$= 300(8) + 160(16) = 4960$
$B_2(0,24)$	$= 300(0) + 160(24) = 3640$

maximum $Z = 4960$

Number of item $A = 8$, item $B = 16$
maximum profit = Rs 4960

Linear Programming Ex 30.4 Q30

Let number of toys of type A and B produced are x and y respectively.

Since,profits on each unit of toys A and B are Rs 50 and Rs 60 respectively, so, profits on x units of toys A and y units of toy B are Rs $50x$ and Rs $60y$ respectively
Let Z be total profit so

$$Z = 50x + 60y$$

Since each unit of toy A and toy B require 5 min. and 8 min. on cutting, so, x units of toy A and y units of toy B require $5x$ and $8y$ min. respectivley but maximum time available for cutting $3 \times 60 = 180$ min.,so

$$5x + 8y \leq 180 \qquad \text{(first constraint)}$$

Since each unit of toy A and toy B require 10 min. and 8 min. for assembling, so, x units of toy A and y units of toy B require $10x$ and $8y$ min. for assembling respectivley but maximum time available for assembling is $4 \times 60 = 240$ min.,so

$$10x + 8y \leq 240$$
$$\Rightarrow \quad 5x + 4y \leq 120 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = 50x + 60y$

Subject to constriants,

$$5x + 8y \leq 180$$
$$5x + 4y \leq 120$$
$$x, y \geq 0 \qquad [\text{Since production can not be less than zero}]$$

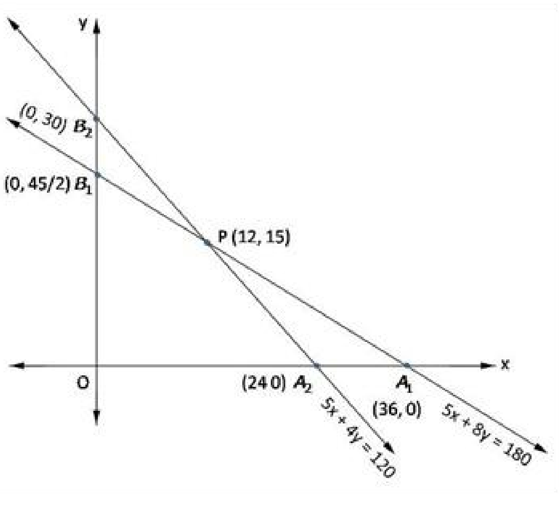
Region $5x + 8y \leq 180$: line $5x + 8y = 180$ meets axes at $A_1(36,0)$, $B_1\left(0,\frac{45}{2}\right)$ respectively.
Region containing origin represents $5x + 8y \leq 180$ as $(0,0)$ satisfies $5x + 8y \leq 180$.

Region $5x + 4y \leq 120$: line $5x + 4y = 120$ meets axes at $A_2(24,0)$, $B_2(0,30)$ respectively.
Region containing origin represents $5x + 4y \leq 120$ as $(0,0)$ satisfies $5x + 4y \leq 120$.

Region $x, y \geq 0$: it represent first quandrant

Shaded region OA_2PB_1 represents feasible region.
Point $P(12,15)$ is obtained by solving

$$5x + 8y = 180 \text{ and } 5x + 4y = 120$$



The value of $Z = 50x + 60y$ at

$O(0,0)$	$= 50(0) + 60(0) = 0$
$A_2(24,0)$	$= 50(24) + 60(0) = 1200$
$P(12,15)$	$= 50(12) + 60(15) = 1500$
$B_1\left(0,\frac{45}{2}\right)$	$= 50(0) + 60\left(\frac{45}{2}\right) = 1350$

Maximum $Z = 1500$ at $x = 12$, $y = 15$

Number of toys $A = 12$, toys $B = 15$
maximum profit = Rs 1500

Linear Programming Ex 30.4 Q31

Let required number of product A and B are x and y respectively.

Since,profits on each unit of product A and product B are Rs 6 and Rs 8 respectively, so, profits on x units of product A and y units of product B are Rs $6x$ and Rs $8y$ respectively
Let Z be total profit so

$$Z = 6x + 8y$$

Since each unit of product A and B require 4 and 2 hrs for assembling respectively, so, x units of product A and y units of product B require $4x$ and $2y$ hrs for assembling respectivley but maximum time available for assembling is 60 hrs.,so

$$4x + 2y \le 60$$
$$2x + y \le 30 \qquad \text{(first constraint)}$$

Since each unit of product A and B require 2 and 4 hrs for finishing, so, x units of product A and y units of product B require $2x$ and $4y$ hrs for finishing respectivley but maximum time available for finishing is 48 hrs.,so

$$2x + 4y \le 48$$
$$x + 2y \le 24 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = 6x + 8y$

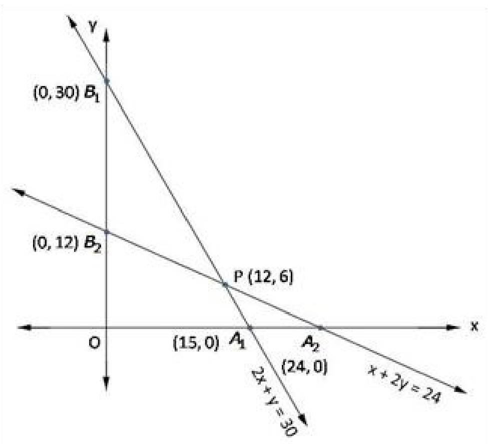
Subject to constriants,

$$2x + y \le 30$$
$$x + 2y \le 24$$
$$x, y \ge 0 \qquad [\text{Since production of both can not be less than zero}]$$

Region $2x + y \le 30$: line $2x + y = 24$ meets axes at $A_1(15,0)$, $B_1(0,30)$ respectively.
Region containing origin represents $2x + y \le 30$ as $(0,0)$ satisfies $2x + y \le 30$.

Region $x + 2y \le 24$: line $x + 2y = 24$ meets axes at $A_2(24,0)$, $B_2(0,12)$ respectively.
Region containing origin represents $x + 2y \le 24$ as $(0,0)$ satisfies $x + 2y \le 24$.

Region $x, y \ge 0$: it represent first quandrant



Shaded region OA_1PB_2 represents feasible region.
Point $P(12,6)$ is obtained by solving

$$x + 2y = 24 \text{ and } x + 2y = 30$$

The value of $Z = 6x + 8y$ at

$$O(0,0) \quad = 6(0) + 8(0) = 0$$
$$A_1(15,0) \quad = 6(15) + 8(0) = 90$$
$$P(12,6) \quad = 6(12) + 8(6) = 120$$
$$B_2(0,12) \quad = 6(0) + 8(12) = 96$$

maximum $Z = 120$ at $x = 12$, $y = 6$

Number of product A = 12, product B = 6
maximum profit = Rs 120

Linear Programming Ex 30.4 Q32

Let x & y be the No. of items of A & B respectively.

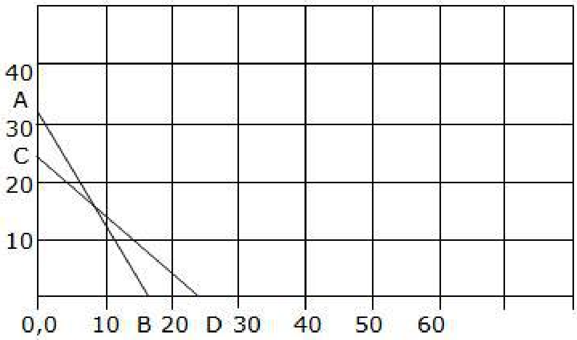
$x + y = 24$ (total No. of items constraint)

$x + 0.5y \le 16$ (time constraint)

$x, y \ge 0$

$Z = 300x + 160y$ (profit function to be maximized)

Plotting the inequalities gives,



The feasible region is 0,0-C-F-B

Corner point	Value of Z = 300x + 160y
0, 0	0
0, 24	3840
16, 0	4800
8 ,16	4960

The firm must produce 8 items of A and 16 items of B to maximize the profit at Rs. 4960/-

Linear Programming Ex 30.4 Q33

Let required number of product A and B are x and y respectively.

Since,profits on each unit of product A and product B are Rs 20 and Rs 15 respectively, so, x units of product A and y units of product B give profit of Rs $20x$ and Rs $15y$ respectively

Let Z be total profit so

$$Z = 20x + 15y$$

Since each unit of product A and B require 5 and 3 man-hrs respectively, so, x units of product A and y units of product B require $5x$ and $3y$ man-hrs respectivley but maximum time available for is 500 man-hrs., so

$5x + 3y \le 500$ (first constraint)

Since maximum number that product A and B can be sold is 70 and 125 respectively, so,

$x \le 70$ (second constraint)

$y \le 125$ (third constraint)

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = 20x + 15y$

Subject to constriants,

$5x + 3y \le 500$

$x \le 70$

$y \le 70$

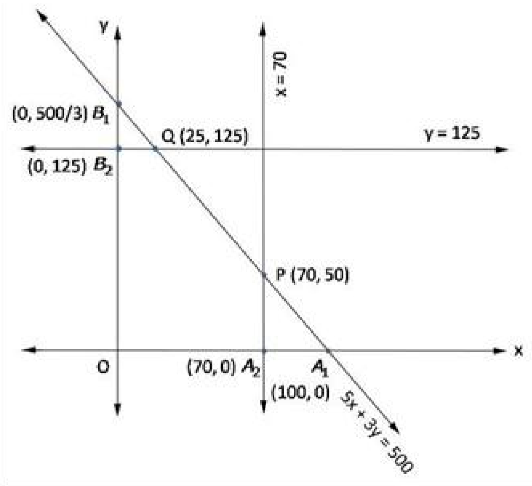
$x, y \ge 0$ [Since production of both can not be less than zero]

Region $5x + 3y \le 500$: line $5x + 3y = 500$ meets axes at $A_1(100,0)$, $B_1\left(0, \frac{500}{3}\right)$ respectively.
Region containing origin represents $5x + 3y \le 500$ as $(0,0)$ satisfies $5x + 3y \le 500$.

Region $x \le 70$: line $x = 70$ is parallel to y – axis meets x–axes at $A_2(70,0)$. Region containing origin represents $x \le 70$ as $(0,0)$ satisfies $x \le 70$.

Region $y \le 125$: line $y = 125$ is parallel to x – axis meets y–axes at $B_2(0,125)$. with y – axis. Region containing origin represents $y \le 125$ as $(0,0)$ satisfies $y \le 125$.

Region $x, y \ge 0$: it represent first quandrant.



Shaded region OA_2PQB_2 represents feasible region.
Point $P(70,50)$ is obtained by solving $x = 70$
Point $Q(25,125)$ is obtained by solving $y = 125$ and $5x + 3y = 500$.

The value of $Z = 20x + 15y$ at

$O(0,0)$	$= 20(0) + 15(0) = 0$
$A_2(70,0)$	$= 20(70) + 15(0) = 1400$
$P(70,50)$	$= 20(70) + 15(50) = 2150$
$Q(25,125)$	$= 20(25) + 15(125) = 2375$
$B_2(0,125)$	$= 20(0) + 15(125) = 1875$

maximum $Z = 2375$ at $x = 25$, $y = 125$

Number of product $A = 25$, product $B = 125$
maximum profit = Rs 2375

Linear Programming Ex 30.4 Q34

Let required quantity of large and small boxes are x and y respectively.

Since,profits on each unit of large and small boxes are Rs 3 and Rs 2 respectively, so, profit on x units of large and y units of small boxes are Rs $3x$ and Rs $2y$ respectively
Let Z be total profit so

$$Z = 3x + 2y$$

Since each large and small box require 4 sq. m. and 3 sq. m. cardboard respectively, so, x units of large and y units of small boxes require $4x$ and $3y$ sq.m. cardboard respectivley but only 60 sq. m. of cardboard is available,so

$$4x + 3y \le 60 \qquad \text{(first constraint)}$$

Since manufacturer is required to make at least three large boxes,so,

$$x \ge 3 \qquad \text{(second constraint)}$$

Since manufacturer is required to make at least twice as many small boxes as large boxes ,so,

$$y \ge 2x \qquad \text{(third constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

$$\text{maximize } Z = 20x + 15y$$

Subject to constriants,

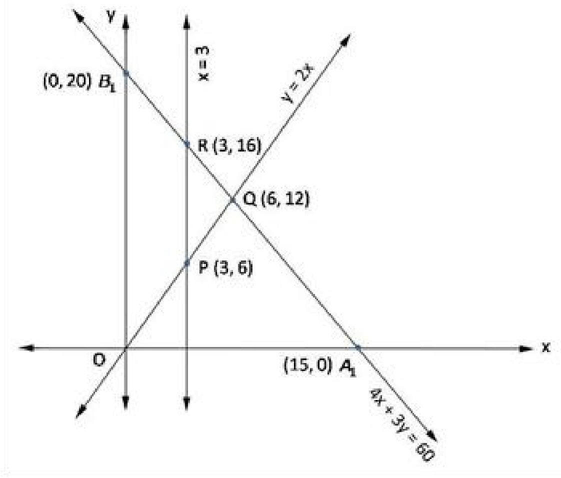
$$4x + 3y \le 60$$
$$x \ge 3$$
$$y \ge 2x$$
$$x, y \ge 0 \qquad \text{[Since production can not be less than zero]}$$

Region $4x + 3y \le 60$: line $4x + 3y = 60$ meets axes at $A_1(15,0)$, $B_1(0,20)$ respectively.
Region containing origin represents $4x + 3y \le 60$ as $(0,0)$ satisfies $4x + 3y \le 60$.

Region $x \ge 3$: line $x = 3$ is parallel to y - axis meets x-axes at $A_2(3,0)$. Region containing origin represents $x \ge 70$ as $(0,0)$ satisfies $x \ge 3$.

Region $y \ge 2x$: line $y = 2x$ is passes throgh origin and $P(3,6)$. Region containging $B_1(0,20)$ represents $y \ge 2x$ as $(0,20)$ satisfies $y \ge 2x$.

Region $x, y \ge 0$: it represent first quandrant.



Shaded region PQR represents feasible region.
Point $Q(6,12)$ is obtained by solving $y = 2x$ and $4x + 3y = 60$
Point $R(3,16)$ is obtained by solving $x = 3$ and $4x + 3y = 60$.

The value of $Z = 3x + 2y$ at

$P(3,6) \quad = 3(3) + 2(6) = 21$

$Q(6,12) \quad = 3(6) + 2(12) = 42$

$R(3,16) \quad = 3(3) + 2(16) = 41$

maximum $Z = 42$ at $x = 6,\ y = 12$

Number of large box = 6, small box =12
maximum profit = Rs 42

Linear Programming Ex 30.4 Q35

The given data can be written in the tabular form as follows:

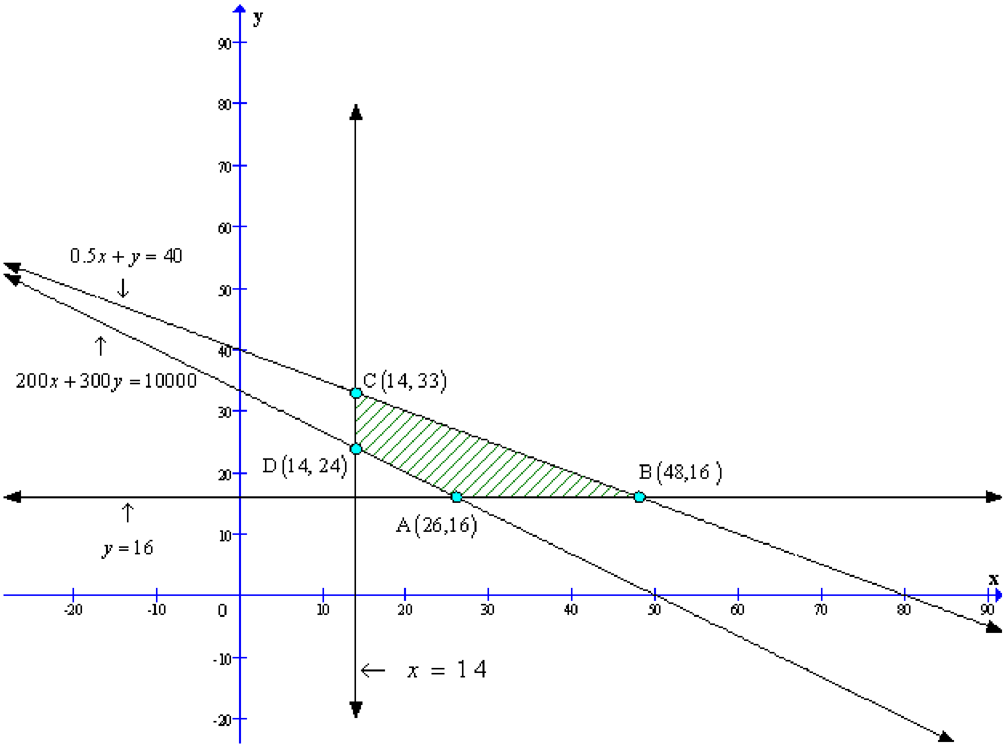
Product	A	B	Working week	Turn over
Time	0.5	1	40	
Prise	200	300		10000
Profit	20	30		
Permanent order	14	16		

Let x be the number of units of A and y be the number of units of B produced to earn the maximum profit.

Then the mathematical model of the LPP is as follows:
Maximize $Z = 20x + 30y$
Subject to $0.5x + y \le 40$,
$200x + 300y \ge 10000$

and $x \ge 14,\ y \ge 16$

To solve the LPP we draw the lines,
$0.5x + y = 40$,
$200x + 300y = 10000$
$x = 14$
$y = 16$



The coordinates of the vertices (Corner - points) of shaded feasible region ABCD are A(26, 16),B(48,16), C(14, 33) and D(14, 24).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function Z = 20x + 30y
A(26, 16)	Z = 1000
B(48, 16)	Z = 1440
C(14, 33)	Z = 1270
D(14, 24)	Z = 600

48 units of product A and 16 units of product B should be produced to earn the maximum profit of Rs. 1440.

Linear Programming Ex 30.4 Q36

Let the distance covered with the speed of 25 km/hr be x.
Let the distance covered with the speed of 40 km/hr be y.

Then the mathematical model of the LPP is as follows:
Maximize Z = x + y
Subject to $2x + 5y \le 100$,

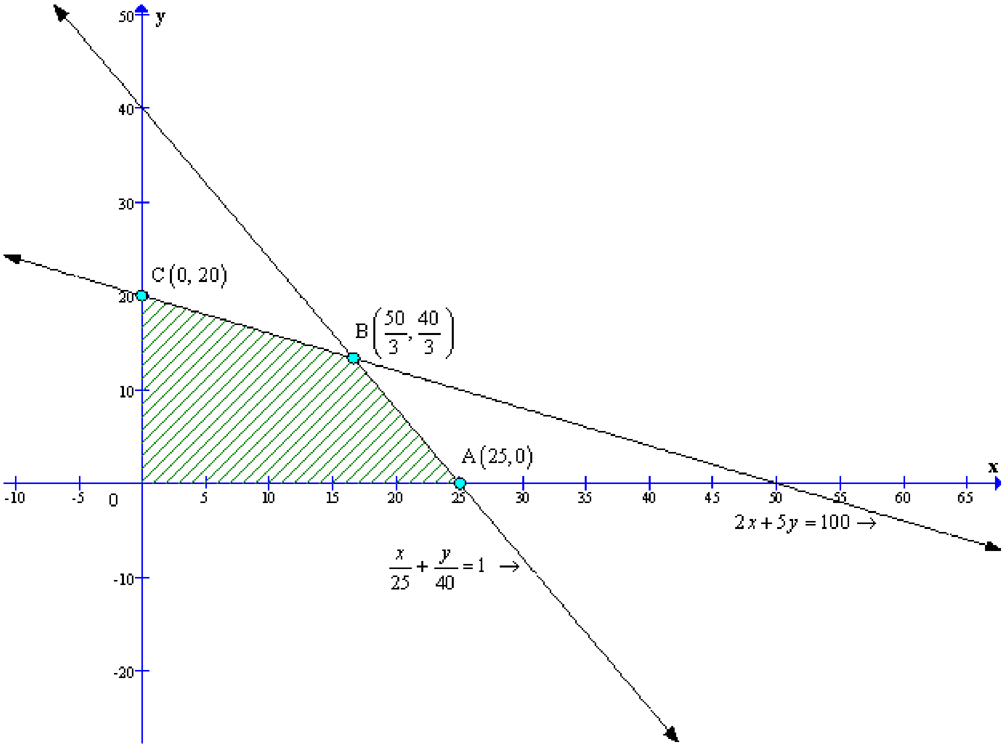
$$\frac{x}{25} + \frac{y}{40} \le 1$$

and $x \ge 0,\ y \ge 0$

To solve the LPP we draw the lines,
$2x + 5y = 100$,

$$\frac{x}{25} + \frac{y}{40} = 1$$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are $A(25, 0), B\left(\frac{50}{3}, \frac{40}{3}\right)$ and $C(0, 20)$.

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = x + y$
$A(25, 0)$	$Z = 25$
$B\left(\frac{50}{3}, \frac{40}{3}\right)$	$Z = 30$
$C(0, 20)$	$Z = 20$

The distance covered at the speed of 25km/hr is $\frac{50}{3}$km and

The distance covered at the speed of 40km/hr is $\frac{40}{3}$km.

Maximum distance travelled is 30 km.

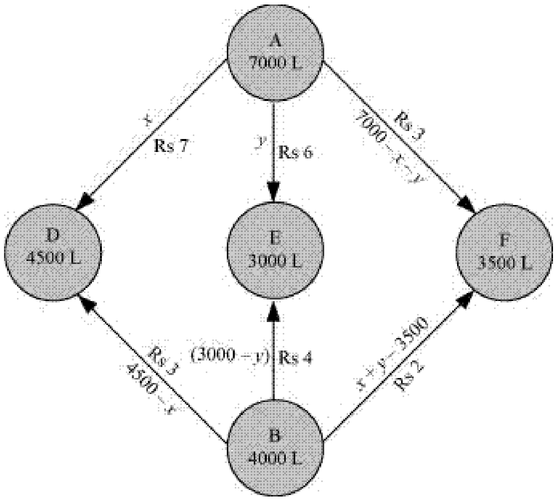
Linear Programming Ex 30.4 Q37

Let x and y litres of oil be supplied from A to the petrol pumps, D and E. Then, $(7000 - x - y)$ will be supplied from A to petrol pump F.

The requirement at petrol pump D is 4500 L. Since x L are transported from depot A, the remaining $(4500 - x)$ L will be transported from petrol pump B.

Similarly, $(3000 - y)$ L and $3500 - (7000 - x - y) = (x + y - 3500)$ L will be transported from depot B to petrol pump E and F respectively.

The given problem can be represented diagrammatically as follows.



$$x \geq 0,\ y \geq 0, \text{ and } (7000 - x - y) \geq 0$$
$$\Rightarrow x \geq 0,\ y \geq 0, \text{ and } x + y \leq 7000$$

$$4500 - x \geq 0,\ 3000 - y \geq 0, \text{ and } x + y - 3500 \geq 0$$
$$\Rightarrow x \leq 4500,\ y \leq 3000, \text{ and } x + y \geq 3500$$

Cost of transporting 10 L of petrol = Re 1

Cost of transporting 1 L of petrol = $\text{Rs } \frac{1}{10}$

Therefore, total transportation cost is given by,

$$z = \frac{7}{10} \times x + \frac{6}{10}y + \frac{3}{10}(7000 - x - y) + \frac{3}{10}(4500 - x) + \frac{4}{10}(3000 - y) + \frac{2}{10}(x + y - 3500)$$
$$= 0.3x + 0.1y + 3950$$

The problem can be formulated as follows.

Minimize $z = 0.3x + 0.1y + 3950$ ... (1)

subject to the constraints,

$x + y \leq 7000$ ...(2)

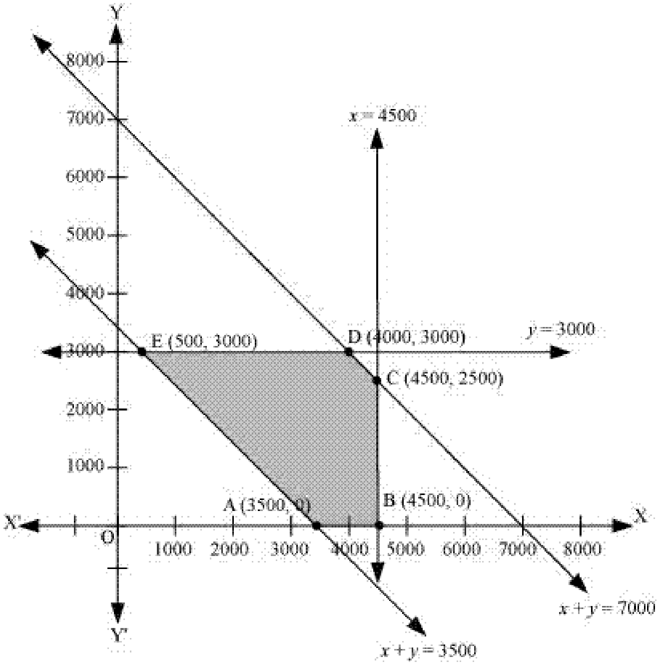
$x \leq 4500$ ...(3)

$y \leq 3000$ ...(4)

$x + y \geq 3500$ ...(5)

$x,\ y \geq 0$ ...(6)

The feasible region determined by the constraints is as follows.



The corner points of the feasible region are A (3500, 0), B (4500, 0), C (4500, 2500), D (4000, 3000), and E (500, 3000).

The values of z at these corner points are as follows.

Corner point	$z = 0.3x + 0.1y + 3950$	
A (3500, 0)	5000	
B (4500, 0)	5300	
C (4500, 2500)	5550	
D (4000, 3000)	5450	
E (500, 3000)	4400	→ Minimum

The minimum value of z is 4400 at (500, 3000).

Thus, the oil supplied from depot A is 500 L, 3000 L, and 3500 L and from depot B is 4000 L, 0 L, and 0 L to petrol pumps D, E, and F respectively.

The minimum transportation cost is Rs 4400.

Linear Programming Ex 30.4 Q38

Let required number of gold rings and chains are x and y respectively.

Since,profits on each ring and chains are Rs 300 and Rs 190 respectively, so,
profit on x units of ring and y units of chains are Rs $300x$ and Rs $190y$ respectively
Let Z be total profit so

$$Z = 300x + 190y$$

Since each unit of ring and chain require 1 hr and 30 min. to make respectively, so,
x units of rings and y units of rings require $60x$ and $30y$ min. to make respectivley,
but total time available to make is $16 \times 60 = 960$, so

$$60x + 30y \le 960$$

$$\Rightarrow \quad 2x + y \le 32 \qquad \text{(first constraint)}$$

Given, total number of rings and chains manufactured is at most 24, so,

$$x + y \le 24 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

$$\text{maximize } Z = 300x + 160y$$

Subject to constriants,

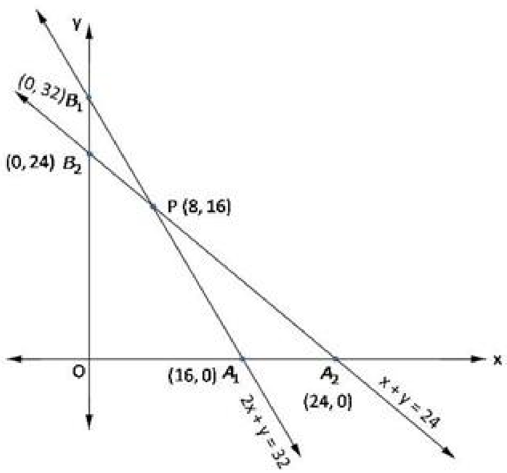
$$2x + y \le 32$$
$$x + y \le 24$$
$$x, y \ge 0 \qquad \text{[Since production can not be less than zero]}$$

Region $2x + y \le 32$: line $2x + y = 32$ meets axes at $A_1(16,0)$, $B_1(0,32)$ respectively.
Region containing origin represents $2x + y \le 32$ as $(0,0)$ satisfies $2x + y \le 32$.

Region $x + y \le 24$: line $x + y = 24$ meets axes at $A_2(24,0)$, $B_2(0,24)$ respectively.
Region containing origin represents $x + y \le 24$ as $(0,0)$ satisfies $x + y \le 24$.

Region $x, y \ge 0$: it represent first quandrant

Shaded region OA_1PB_2 represents feasible region.
Point $P(8,16)$ is obtained by solving $2x + y = 32$ and $x + y = 24$.



The value of $Z = 300x + 160y$ at

$O(0,0) \quad = 300(0) + 160(0) = 0$

$A_1(16,0) \quad = 300(16) + 160(0) = 4800$

$P(8,16) \quad = 300(8) + 160(16) = 4960$

$B_2(0,24) \quad = 300(0) + 160(24) = 3840$

maximum $Z = 4960$ at $x = 8$, $y = 16$

Number of rings = 8, chains = 16
maximum profit = Rs 4960

Linear Programming Ex 30.4 Q39

Let required number of books of type I and II be x and y respectively.

Let Z be total number of books in the shelf ,so,

$$Z = x + y$$

Since 1 book of type I and II 6 cm and 4 cm. thick respectively, so, x books of type I and y books of type II has thickness of $6x$ and $4y$ cm. respectivley, but shelf is 96 cm. long ,so

$$6x + 4y \le 96$$

$$\Rightarrow \quad 3x + 2y \le 48 \qquad \text{(first constraint)}$$

Since 1 book of type I and II weight 1 kg and $1\frac{1}{2}$ kg respectively, so, x books of type I and y books of type II weight x kg and $\frac{3}{2}y$ kg respectivley, but shelf can support at most 21 kg,so

$$x + \frac{3}{2}y \le 21$$

$$\Rightarrow \quad 2x + 3y \le 42 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = x + y$

Subject to constriants,

$$3x + 2y \le 48$$

$$2x + 3y \le 42$$

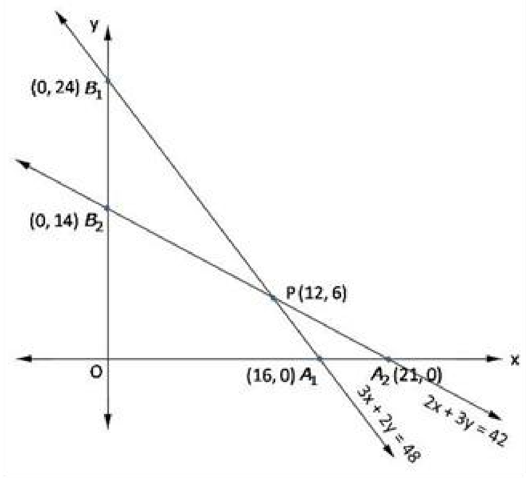
$$x, y \ge 0 \qquad \left[\text{Since number of books can not be less than zero}\right]$$

Region $3x + 2y \le 48$: line $3x + 2y = 48$ meets axes at $A_1(16,0)$, $B_1(0,24)$ respectively.
Region containing origin represents $3x + 2y \le 48$ as $(0,0)$ satisfies $3x + 2y \le 48$.

Region $2x + 3y \le 42$: line $2x + 3y = 42$ meets axes at $A_2(21,0)$, $B_2(0,14)$ respectively.
Region containing origin represents $2x + 3y \le 42$ as $(0,0)$ satisfies $2x + 3y \le 42$.

Region $x, y \ge 0$: it represent first quandrant

Shaded region OA_1PB_2 represents feasible region.
Point $P(12,6)$ is obtained by solving $2x + 3y = 42$ and $3x + 2y = 48$



The value of $Z = x + y$ at

$O(0,0) \quad = 0 + 0 = 0$

$A_1(16,0) \quad = 16 + 0 = 16$

$P(12,6) \quad = 12 + 6 = 18$

$B_2(0,14) \quad = 0 + 14 = 14$

maximum $Z = 18$ at $x = 12$, $y = 6$

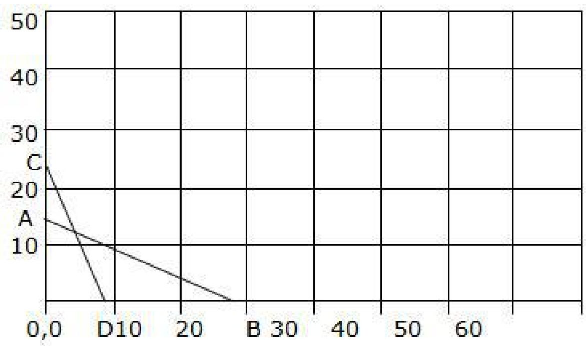
Number of books of type I = 12, type II = 6

Linear Programming Ex 30.4 Q40

Let x & y be the No. of tennis rackets and cricket bats produced.

$1.5x + 3y \le 42$ (constraint on machine time)
$3x + y \le 24$ (constraint on craftsman's time)
$Z = 20x + 10y$ (Maximize profit)
$x, y \ge 0$
plotting the inequalities we have,

when x=0, y= 14 and when y=0, x=28 and
when x=0, y= 24 and when y=0, x=8



The feasible region is given by 0,0-A-F-D
Tabulating Z and corner points we have

Corner point	Value of Z = 20x + 10y
0, 0	0
0, 14	140
4, 12	200
8, 0	160

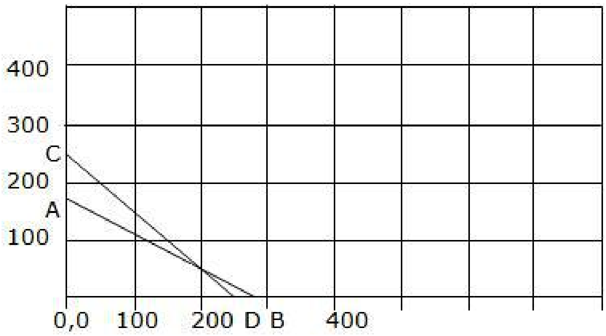
The factory must manufacture 4 tennis rackets and 12 cricket bats to earn the maximum profit of Rs. 200/-

Linear Programming Ex 30.4 Q41

Let x & y be the No. of desktop model and portable model of personal computers stocked.

$x + y \leq 250$ (constraint on total demand of computers)
$25000x + 40000y \leq 70,00,000$ (constraint on cost)
$Z = 4500x + 5000y$ (Maximize profit)
$x, y \geq 0$

plotting the inequalities we have,
when x=0, y= 250 and when y=0, x=250 and line CD
when x=0, y= 175 and when y=0, x=280



The feasible region is given by 0,0-A-E-D-0,0

Tabulating Z and corner points we have

Corner point	Value of Z = 4500x + 5000y
0, 0	0
0, 175	8,75,000
250, 0	11,25,000
200, 50	11,50,000

The merchant must stock 200 desktop models and 50 portable models to earn a maximum profit of Rs. 11,50,000/-

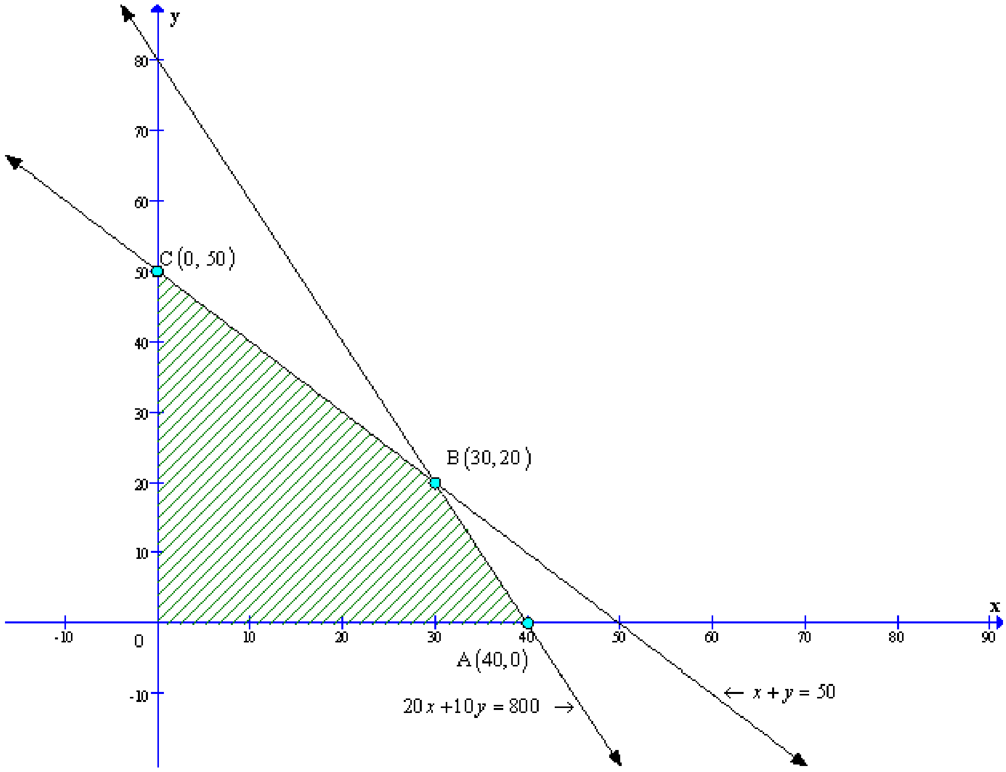
Linear Programming Ex 30.4 Q42

Let x hectores of land grows crop X.
Let y hectores of land grows crop Y.

Then the mathematical model of the LPP is as follows:
Maximize $Z = 10,500x + 9,000y$
Subject to $x + y \leq 50$,
$20x + 10y \leq 800$
and $x \geq 0$, $y \geq 0$

To solve the LPP we draw the lines,
$x + y = 50$,
$20x + 10y = 800$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(40, 0), B(30, 20) and C(0, 50).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function Z = 10,500x + 9,000y
A(40, 0)	Z = 4,20,000
B(30, 20)	Z = 4,95,000
C(0, 50)	Z = 4,50,000

30 hectors of land should be allocated to crop X and
20 hectors of land should be allocated to crop Y to maximize the profit.
The maximum profit that can be eared is Rs. 4,95,000.

Linear Programming Ex 30.4 Q43

The given data can be written in the tabular form as follows:

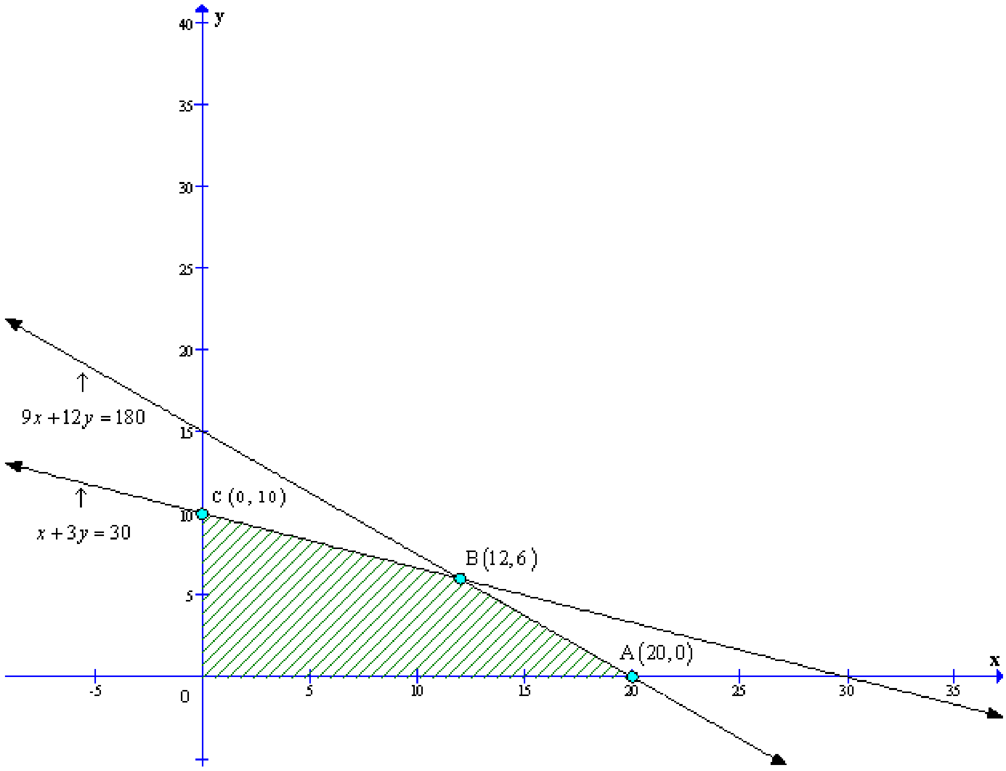
Model	A	B	Maximum hours
Fabricating	9	12	180
Finishing	1	3	30
Profit	8000	12000	

Let x be the number of pieces of A and y be the number of pieces of B manufactured to earn the maximum profit.

Then the mathematical model of the LPP is as follows:
Maximize $Z = 8000x + 12000y$
Subject to $9x + 12y \le 180$,
$x + 3y \le 30$
and $x \ge 0$, $y \ge 0$

To solve the LPP we draw the lines,
$9x + 12y = 180$,
$x + 3y = 30$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(20, 0), B(12,6) and C(0, 10).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function Z = 8,000x + 12,000y
A(20, 0)	Z = 1,60,000
B(12, 6)	Z = 1,68,000
C(0, 10)	Z = 1,20,000

12 pieces of Model A and 6 pieces of Model B should be eaned maximize the profit.
The maximum profit that can be eared is Rs. 1,68,000.

Linear Programming Ex 30.4 Q44

The given data can be written in the tabular form as follows:

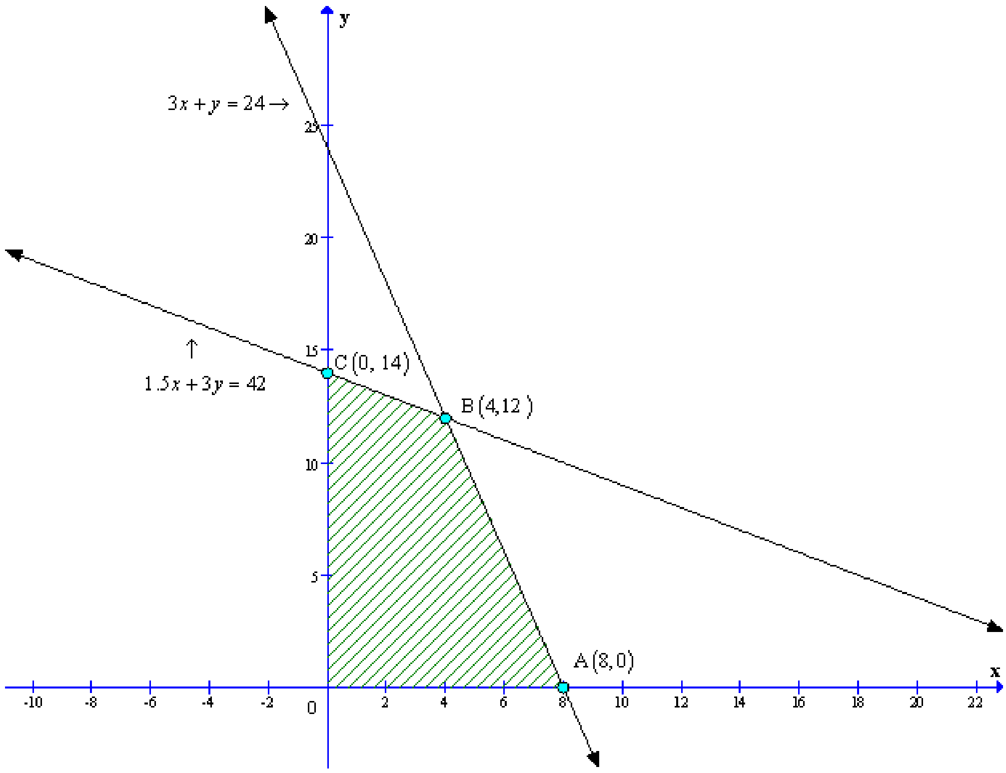
Product	Racket	Bat	Maximum hours
Machine	1.5	3	42
Craftman	3	1	24
Profit	20	10	

Let x be the number of rackets and y be the number of bats made to earn the maximum profit.

Then the mathematical model of the LPP is as follows:
Maximize Z = 20x + 10y
Subject to $1.5x + 3y \le 42$,
$3x + y \le 24$
and $x \ge 0$, $y \ge 0$

To solve the LPP we draw the lines,
$1.5x + 3y = 42$,
$3x + y = 24$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(8, 0), B(4, 12) and C(0, 14).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 20x + 10y$
A(8, 0)	$Z = 160$
B(4, 12)	$Z = 200$
C(0, 14)	$Z = 140$

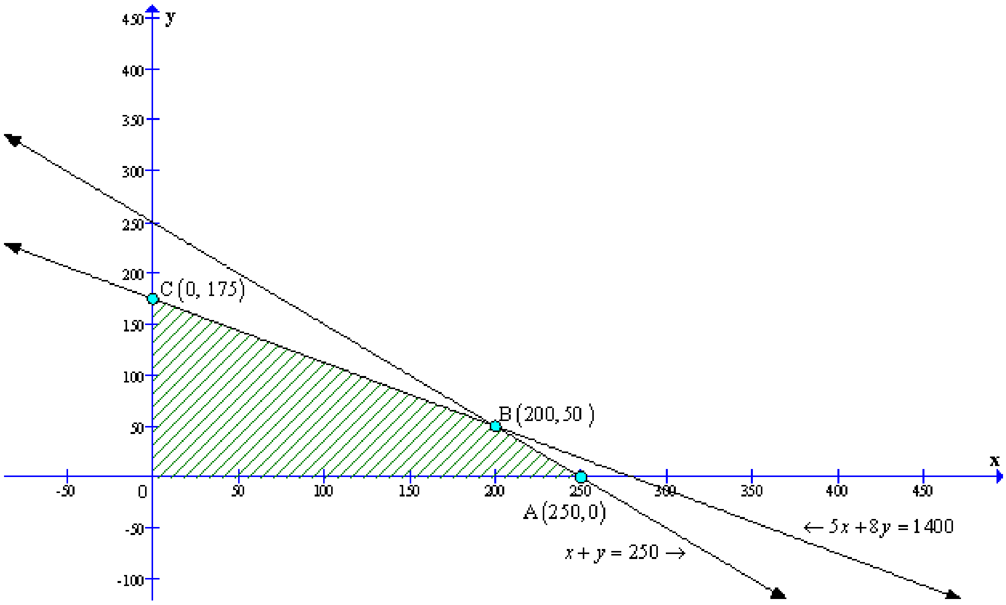
4 rackets and 12 bats must be made if the factory is to work at full capacity.
The maximum profit that can be eared is Rs. 200.

Linear Programming Ex 30.4 Q45

Let x be the number of desktop computers and y be the number of portable computers which merchant should stock to earn the maximum profit.

Then the mathematical model of the LPP is as follows:
Maximize $Z = 4500x + 5000y$
Subject to $25000x + 40000y \leq 70,00,000$
$x + y \leq 250$
and $x \geq 0$, $y \geq 0$

To solve the LPP we draw the lines,
$5x + 8y = 1,400$
$x + y = 250$
The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are
A(250, 0), B(200,50) and C(0, 175).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function Z = 4500x + 5000y
A(250, 0)	Z = 11,25,000
B(200, 50)	Z = 11,50,000
C(0, 175)	Z = 8,75,000

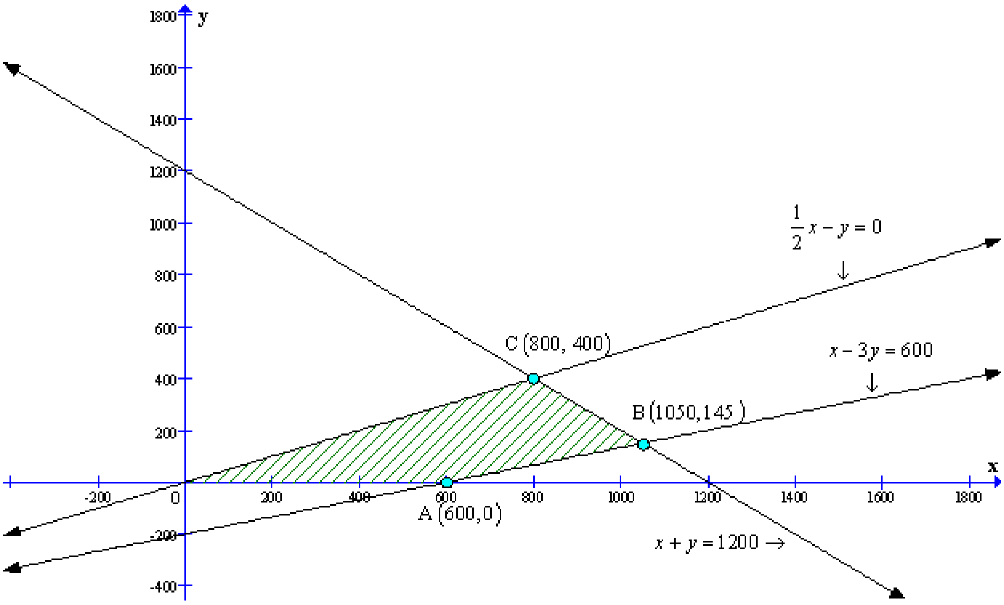
The merchant should stock 200 personal computer and 50 portable computers to earn maximum profit.
The maximum profit that can be eared is Rs. 11,50,000.

Linear Programming Ex 30.4 Q46

Let x be the number of dolls of type A and y be the number of dolls of type B should be produced to earn the maximum profit.

Then the mathematical model of the LPP is as follows:
Maximize $Z = 12x + 16y$
Subject to $x + y \leq 1200$
$\frac{1}{2}x - y \geq 0$
$x - 3y \leq 600$
and $x \geq 0, y \geq 0$

To solve the LPP we draw the lines,
$x + y = 1200$
$\frac{1}{2}x - y = 0$
$x - 3y = 600$
The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(600, 0), B(1050, 145) and C(800, 400).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function Z = 12x + 16y
A(600, 0)	Z = 7200
B(1050, 145)	Z = 14920
C(800, 400)	Z = 16000

The toy company should manufacture 800 dolls of type A and 400 dolls of type B to earn maximum profit.
The maximum profit that can be eared is Rs. 16,000.

Linear Programming Ex 30.4 Q47

Let x kg of fertiliser F_1 and y kg of fertiliser F_2 should be used to minimise the cost.

Then the mathematical model of the LPP is as follows:

Maximize Z = 6x + 5y

Subject to 10x + 5y ≥ 1400

6x + 10y ≥ 1400

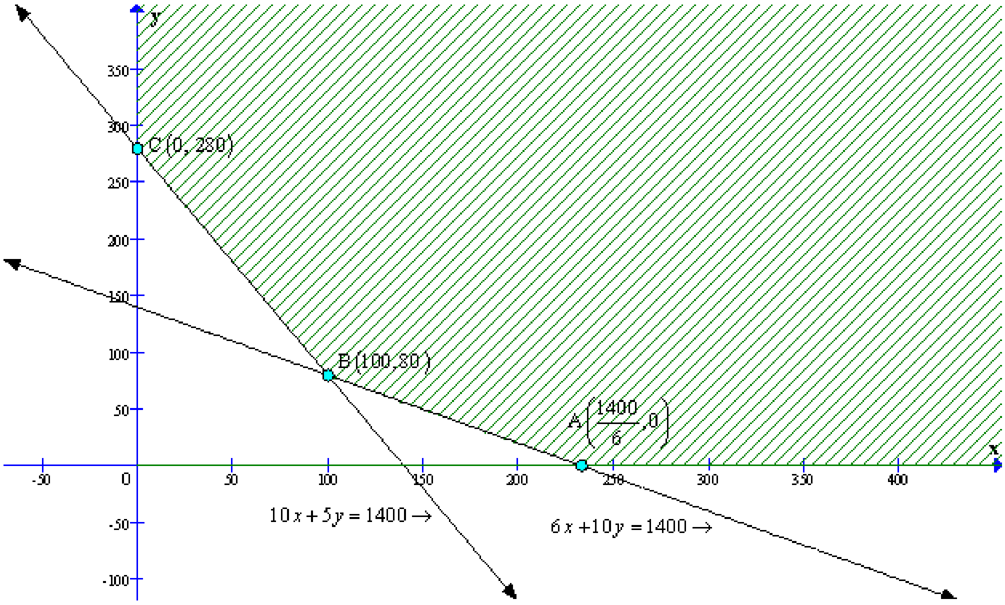
and x ≥ 0, y ≥ 0

To solve the LPP we draw the lines,

10x + 5y = 1400

6x + 10y = 1400

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are $A\left(\frac{1400}{6},\ 0\right)$, $B(100, 80)$ and $C(0,\ 280)$.

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 6x + 5y$
$A\left(\frac{1400}{6},\ 0\right)$	$Z = 1400$
$B(100,\ 80)$	$Z = 1000$
$C(0,\ 280)$	$Z = 1400$

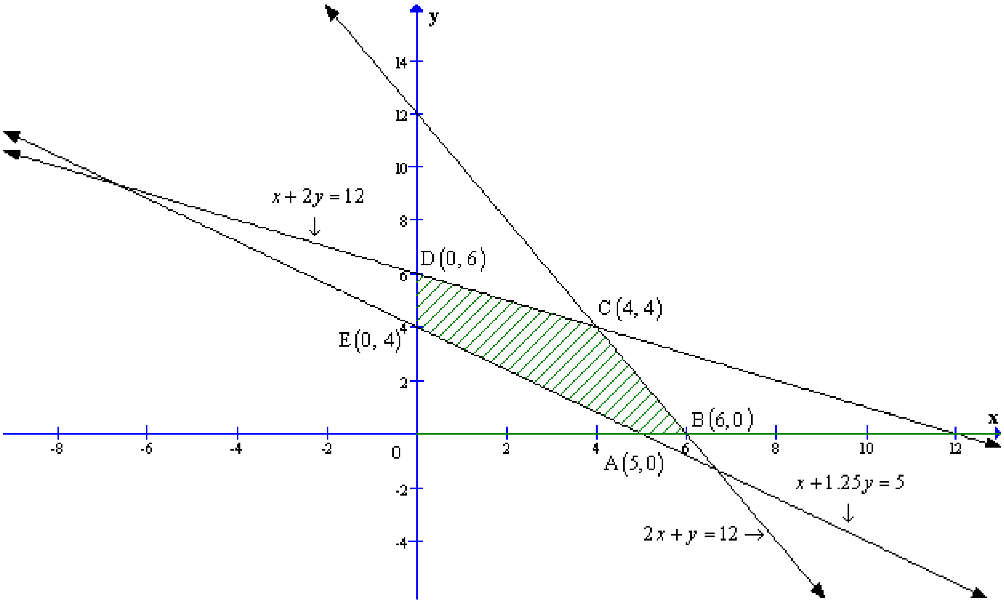
100 kg of fertiliser F_1 and 80 kg of fertiliser F_2 to earn minimise the cost.
The maximum cost Rs. 1,000.

Linear Programming Ex 30.4 Q48

Let x units of item M and y units of item N
should be produced to maximise the cost.

Then the mathematical model of the LPP is as follows:
Maximize $Z = 600x + 400y$
Subject to $x + 2y \le 12$
$2x + y \le 12$
$x + 1.25y \ge 5$
and $x \ge 0,\ y \ge 0$

To solve the LPP we draw the lines,
$x + 2y = 12$
$2x + y = 12$
$x + 1.25y = 5$
The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABCDE are A(5, 0), B(6, 0), C(4, 4), D(0, 6) and E(0, 4).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 600x + 400y$
A(5, 0)	Z = 3000
B(6, 0)	Z = 3600
C(4, 4)	Z = 4000
D(0, 6)	Z = 2400
E(0, 4)	Z = 1600

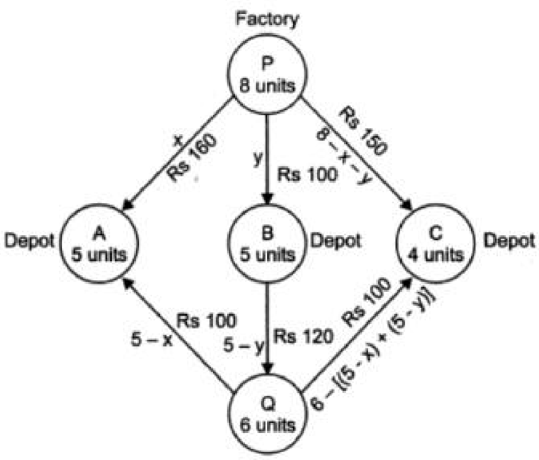
4 units of item M and 4 units of item N should be produced to maximise the profit.
The maximum profit is Rs. 4,000.

Linear Programming Ex 30.4 Q49

Let x and y units of commodity be transported from factory P to the depots at A and B respectively.

Then (8 - x - y) units will be transported to depot at C.

The flow is shown below.

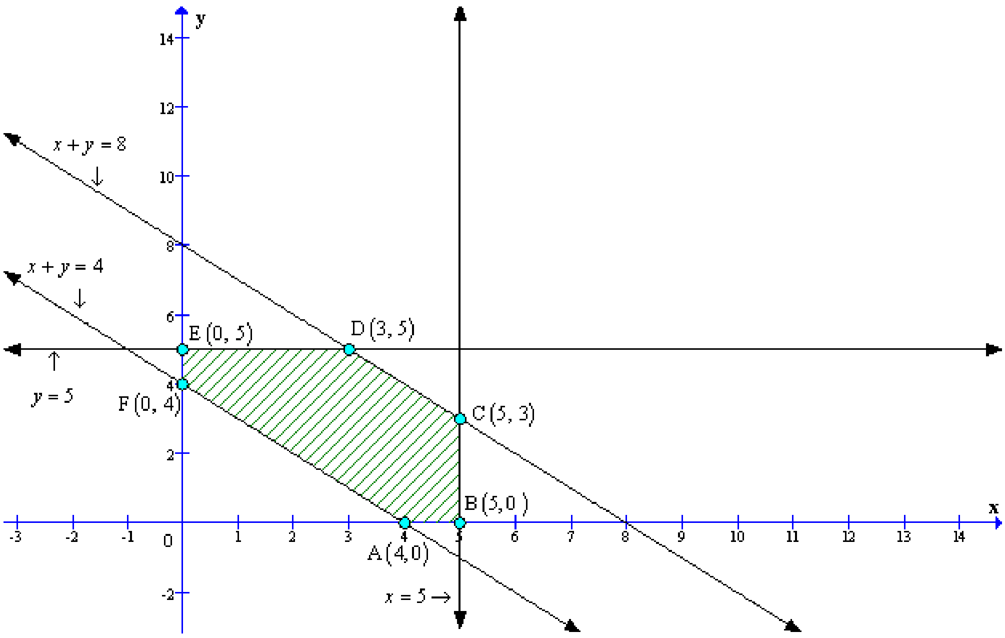


Hence we have, $x \geq 0$, $y \geq 0$ and $8 - x - y \geq 0$
i.e. $x \geq 0$, $y \geq 0$ and $x + y \geq 8$
Now, the weekly requirement of the depot at A is 5 units of the commodity.
Since x units are transported from the factory at P, remaimning $(5 - x)$ units need to be transported from the factory at Q.
$\therefore 5 - x \geq 0 \Rightarrow x \leq 5$
Simillarly, $(5 - y)$ and $6 - (5 - x + 5 - y) = x + y - 4$ units are to be transported from the factory at Q to the depots at B and C respectively.
$\therefore 5 - y \geq 0$ and $x + y - 4 \geq 0$
$\Rightarrow y \leq 5$ and $x + y \geq 4$

Total transportation cost Z is given by
$$Z = 160x + 100y + 100(5 - x) + 120(5 - y) + 100(x + y - 4) + 150(8 - x - y)$$
$$Z = 10(x - 7y + 190)$$

So the mathematical model of given LPP is as follows.
Minimize $Z = 10(x - 7y + 190)$
Subject to $x + y \leq 8$
$x \leq 5, y \leq 5$
$x + y \geq 4$
$x \geq 0, y \geq 0$

To solve the LPP we draw the lines,
$x + y = 8$
$x = 5, y = 5$
$x + y = 4$
The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABCDEF are A(4, 0), B(5, 0), C(5, 3), D(3, 5), E(0, 5) and F(0, 4).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 10(x - 7y + 190)$
A(4, 0)	Z = 1940
B(5, 0)	Z = 1950
C(5, 3)	Z = 1740
D(3,5)	Z = 1580
E(0,5)	Z = 1550
F(0, 4)	Z = 1620

Deliver 0, 5, 3 units from factory at P and 5, 0, 1 from the factory at Q to the depots at A, B and C respectively.
The minimum transportation cost is Rs. 1550.

Linear Programming Ex 30.4 Q50

Let the mixture contains x toys of type A and y toys of type B.

Type of toys	No. of toys	Machine I (in min)	Machine II (in min)	Machine III (in min)	Profit Rs.
A	x	12x	18x	6x	7.5x
B	y	6y	0	9y	5y
Total	x+y	12x + 6y	18x	6x + 9y	7.5x + 5y
Requirement		360	360	360	

Then the mathematical model of the LPP is as follows:

Maximize $Z = 7.5x + 5y$

Subject to $12x + 6y \le 360 \Rightarrow 2x + y \le 60$

$18x \le 360 \Rightarrow x \le 20$

$6x + 9y \le 360 \Rightarrow 2x + 3y \le 120$

and $x \ge 0,\ y \ge 0$

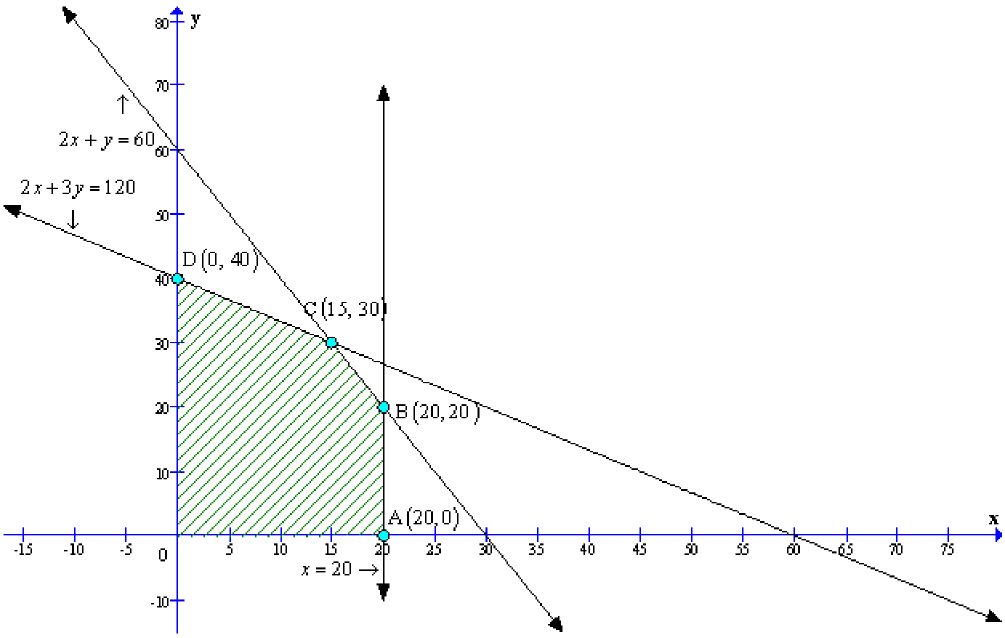
To solve the LPP we draw the lines,

$2x + y = 60$

$x = 20$

$2x + 3y = 120$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABCD are A(20, 0), B(20, 20), C(15, 30) and D(0, 40).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 7.5x + 5y$
A(20, 0)	Z = 150
B(20, 20)	Z = 250
C(15, 30)	Z = 262.5
D(0, 40)	Z = 200

Manufacturer should make 15 toys of type A and 30 toys of type B to maximize the profit.
The maximum profit that can be earned is Rs. 262.5

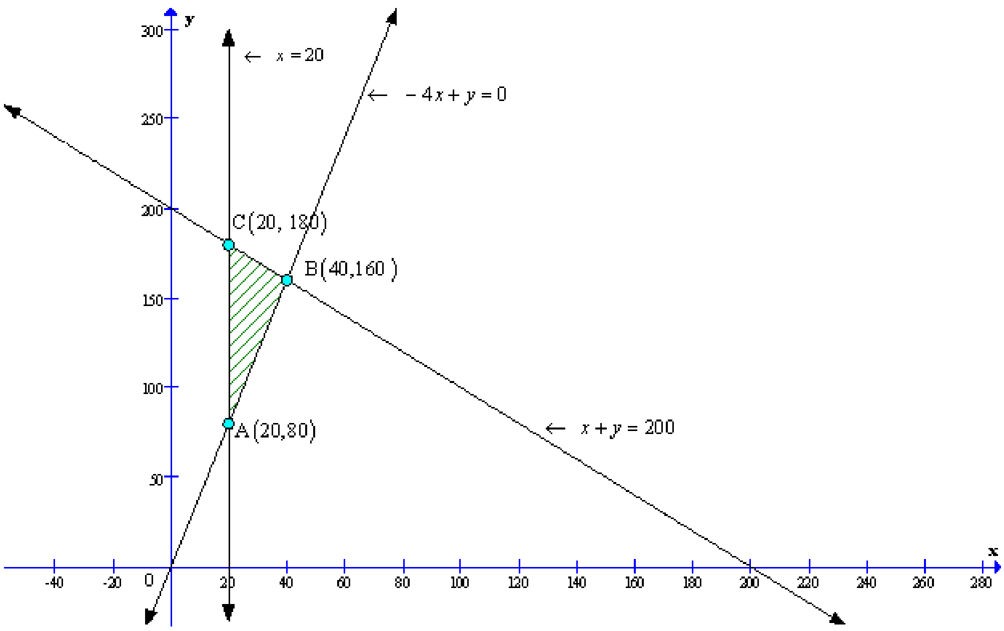
Linear Programming Ex 30.4 Q51

Let x be the number of executive class tickets and y be the number of economic class tickets.

Then the mathematical model of the LPP is as follows:
Maximize $Z = 1000x + 600y$
Subject to $x + y \le 200$
$x \ge 20$
$y \ge 4x \Rightarrow -4x + y \ge 0$
and $x \ge 0$, $y \ge 0$

To solve the LPP we draw the lines,
$x + y = 200$
$x = 20$
$-4x + y = 0$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(20, 80), B(40, 160) and C(20, 180).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 1000x + 600y$
A(20, 80)	$Z = 68{,}000$
B(40, 160)	$Z = 1{,}36{,}000$
C(20, 180)	$Z = 1{,}28{,}000$

40 tickets of executive class and 160 tickets of economic class must be sold to maximize the profit.
The maximum profit that can be earned is Rs. 1,36,000.

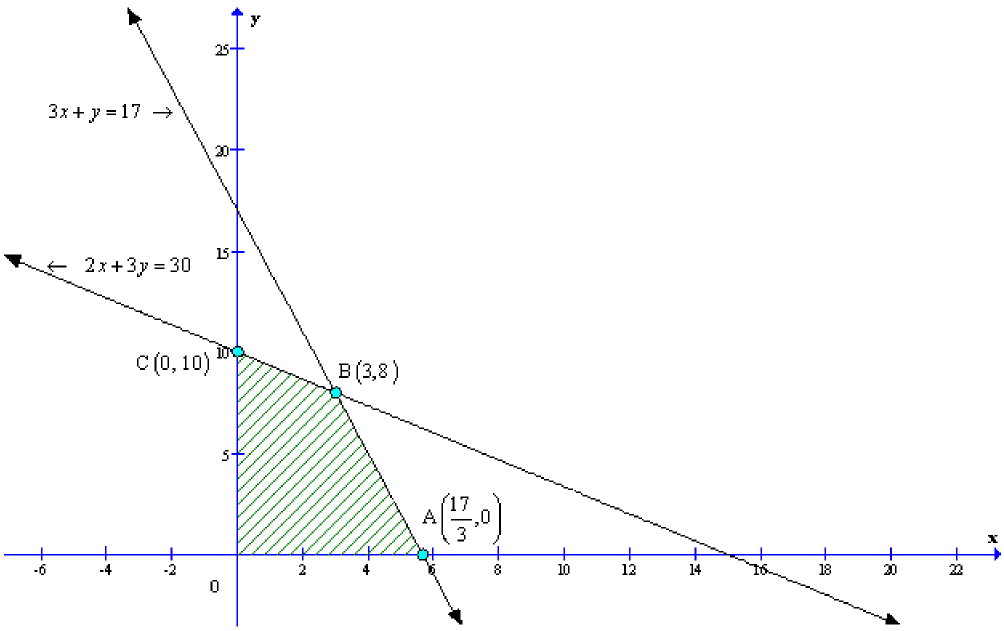
Linear Programming Ex 30.4 Q52

Then the mathematical model of the LPP is as follows:
Maximize $Z = 100x + 120y$
Subject to $2x + 3y \le 30$
$3x + y \le 17$
and $x \ge 0$, $y \ge 0$

To solve the LPP we draw the lines,
$2x + 3y = 30$
$3x + y = 17$

The feasible region of the LPP is shaded in graph.

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are $A\left(\frac{17}{3}, 0\right), B(3, 8)$ and $C(0, 10)$.

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 100x + 120y$
$A\left(\frac{17}{3}, 0\right)$	$Z = 566.67$
$B(3, 8)$	$Z = 1260$
$C(0, 10)$	$Z = 1200$

3 units of workers and 8 units of capital must be used to maximize the profit.
The maximum profit that can be earned is Rs. 1260.

Yes, because efficiency of a person does not depend on sex (male or female).

Ex - 30.5

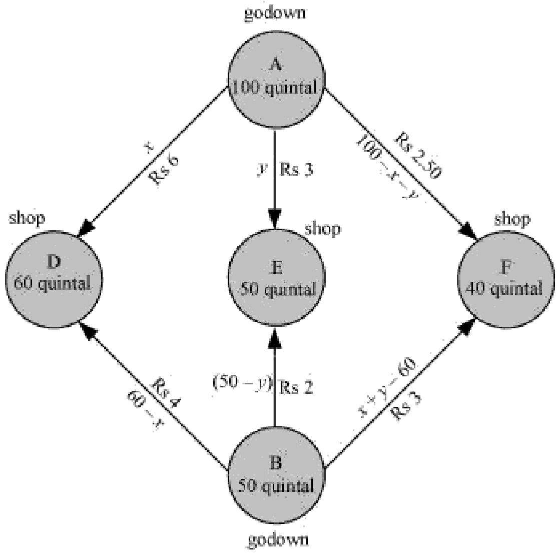
Linear Programming Ex 30.5 Q1

Let godown A supply x and y quintals of grain to the shops D and E respectively. Then, $(100 - x - y)$ will be supplied to shop F.

The requirement at shop D is 60 quintals since x quintals are transported from godown A. Therefore, the remaining $(60 - x)$ quintals will be transported from godown B.

Similarly, $(50 - y)$ quintals and $40 - (100 - x - y) = (x + y - 60)$ quintals will be transported from godown B to shop E and F respectively.

The given problem can be represented diagrammatically as follows.



$x \geq 0,\ y \geq 0,$ and $100 - x - y \geq 0$
$\Rightarrow x \geq 0,\ y \geq 0,$ and $x + y \leq 100$

$60 - x \geq 0,\ 50 - y \geq 0,$ and $x + y - 60 \geq 0$
$\Rightarrow x \leq 60,\ y \leq 50,$ and $x + y \geq 60$

Total transportation cost z is given by,

$$
\begin{aligned}
z &= 6x + 3y + 2.5(100 - x - y) + 4(60 - x) + 2(50 - y) + 3(x + y - 60) \\
&= 6x + 3y + 250 - 2.5x - 2.5y + 240 - 4x + 100 - 2y + 3x + 3y - 180 \\
&= 2.5x + 1.5y + 410
\end{aligned}
$$

The given problem can be formulated as

Minimize $z = 2.5x + 1.5y + 410$ ... (1)

subject to the constraints,

$x + y \leq 100$ ...(2)

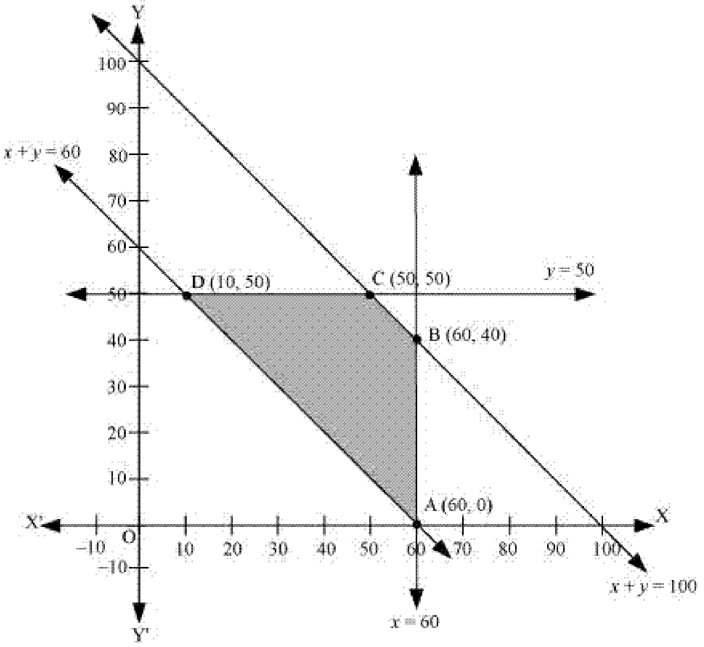
$x \leq 60$ ...(3)

$y \leq 50$ ...(4)

$x + y \geq 60$ ...(5)

$x,\ y \geq 0$ ...(6)

The feasible region determined by the system of constraints is as follows.



The corner points are A (60, 0), B (60, 40), C (50, 50), and D (10, 50).

The values of z at these corner points are as follows.

Corner point	$z = 2.5x + 1.5y + 410$	
A (60, 0)	560	
B (60, 40)	620	
C (50, 50)	610	
D (10, 50)	510	→ Minimum

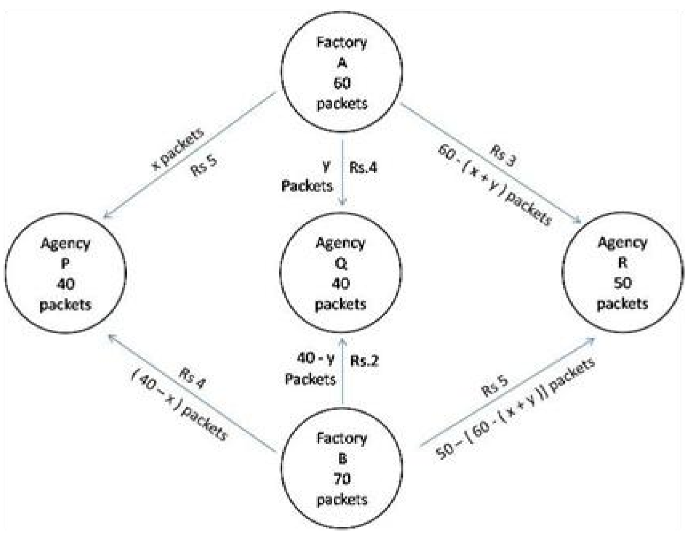
The minimum value of z is 510 at (10, 50).

Thus, the amount of grain transported from A to D, E, and F is 10 quintals, 50 quintals, and 40 quintals respectively and from B to D, E, and F is 50 quintals, 0 quintals, and 0 quintals respectively.

The minimum cost is Rs 510.

Linear Programming Ex 30.5 Q2

The given information can be exhibited diagramatically as below:



Let factory A transports x packets to agency P and y packet to agency Q. Since factory A has capacity of 60 packets so, rest $[60-(x+y)]$ packets transported to agency R.

Since requirements are always non negative so,

$\Rightarrow \quad x, y \geq 0 \qquad$ (first constraint)

and $\quad 60-(x+y) \geq 0$

$(x+y) \leq 60 \qquad$ (second constraint)

Since requirement of agency P is 40 packet but it has recieved x packet, so $(40-x)$ packets are transported from factory B, requirement of agency Q is 40 packets but it has recieved y packets, so $(40-y)$ packets are transported from factory B. Requirement of agency R is 50 packets but it has recieved $(60-x-y)$ packets from factory A, so $50-[60-x-y]=(x+y-10)$ is transported from factory B, As the requirements of agencies P, Q, R are always non negative, so,

$40-x \geq 0$

$\Rightarrow \quad x \leq 40 \qquad$ (third constraint)

$40-y \geq 0$

$\Rightarrow \quad y \leq 40 \qquad$ (fourth constraint)

$x+y-10 \geq 0$

$\Rightarrow \quad x+y \geq 10 \qquad$ (fifth constraint)

Costs of transportation of each packet from factory A to agency P, Q, R are Rs 5,4,3 respectively and costs of transportation of each packet from factory B to agency P, Q, R are Rs 4,2,5 respectively,

Let Z be total cost of transportation so,

$$Z = 5x+4y+3[60-x-y]+4(40-x)+2(40-y)+5(x+y-10)$$
$$= 5x+4y+180-3x-3y+160-4x+80-2y+5x+5y-50$$
$$= 3x+4y+370$$

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = 3x+4y+370$

subject to constraints,

$x, y \geq 0$
$x + y \leq 60$
$x \leq 40$
$y \leq 0$
$x + y \geq 10$

Region $x, y \geq 0$: It is represents first quandrant.

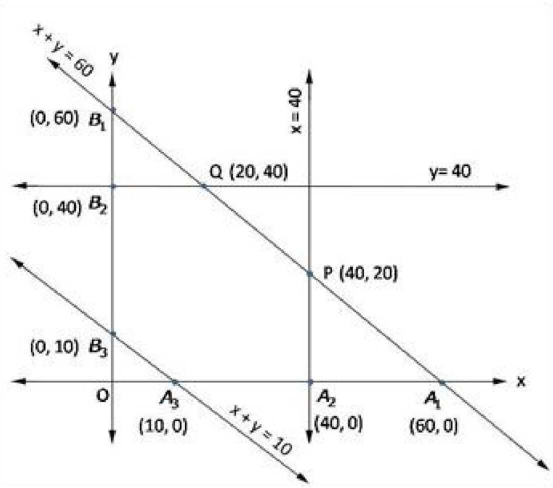
Region $x + y \leq 60$: line $x + y = 60$ meets axes at $A_1(60,0)$, $B_1(0,60)$ respectively.
Region containing origin represents $x + y \leq 60$ as $(0,0)$ satisfies $x + y \leq 60$.

Region $x \leq 40$: line $x = 40$ is parallel to y-axis and meets x-axis at $A_2(40,0)$.
Region containing origin represents $x \leq 40$ as $(0,0)$ satisfies $x \leq 40$.

Region $y \leq 40$: line $y = 40$ is parallel to x-axis and meets y-axis at $B_2(0,40)$.
Region containing origin represents $y \leq 40$ as $(0,0)$ satisfies $y \leq 40$.

Region $x + y \geq 10$: line $x + y = 10$ meets axes at $A_2(10,0)$, $B_3(0,10)$ respectively.
Region containing origin represents $x + y \geq 10$ as $(0,0)$ does not satisfy $x + y \geq 10$.

Shaded region $A_3A_2PQB_2B_3$ represents feasible region.
Point $P(40,20)$ is obtained by solving $x = 40$ and $x + y = 60$
Point $Q(20,40)$ is obtained by solving $y = 40$ and $x + y = 60$



The value of $Z = 3x + 4y + 370$ at

$A_3(10,0) \quad = 3(10) + 4(0) + 370 = 400$
$A_2(40,0) \quad = 3(40) + 4(0) + 370 = 490$
$P(40,20) \quad = 3(40) + 4(20) + 370 = 570$
$Q(20,40) \quad = 3(20) + 4(40) + 370 = 590$
$B_2(0,40) \quad = 3(0) + 4(40) + 370 = 530$
$B_3(0,10) \quad = 3(0) + 4(10) + 370 = 410$

minimum $Z = 400$ at $x = 10$, $y = 0$

From $A \to P =$ 10 packets
From $A \to Q =$ 0 packets
From $A \to R =$ 50 packets
From $B \to P =$ 30 packets
From $B \to Q =$ 40 packets
From $B \to R =$ 0 packets

minimum cost = Rs 400

Ex - 31.1

Probability Ex 31.1 Q1

The sample space $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Let,

A = Number on the card drawn is even number

$A = \{2, 4, 6, 8, 10\}$

B = Number on the card greater than 4

$B = \{4, 5, 6, 7, 8, 9, 10\}$

$A \cap B = \{4, 6, 8, 10\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$

$$= \frac{4}{7}$$

Required probability $= \frac{4}{7}$

Probability Ex 31.1 Q2

Let b and g represent the boy and the girl child respectively. If a family has two children, the sample space will be

S = {(b, b), (b, g), (g, b), (g, g)}

Let A be the event that both children are girls.

$$\therefore \mathrm{A} = \{(g, g)\}$$

(i) Let B be the event that the youngest child is a girl.

$$\therefore \mathrm{B} = [(b, g), (g, g)]$$

$$\Rightarrow \mathrm{A} \cap \mathrm{B} = \{(g, g)\}$$

$$\therefore \mathrm{P}(\mathrm{B}) = \frac{2}{4} = \frac{1}{2}$$

$$\mathrm{P}(\mathrm{A} \cap \mathrm{B}) = \frac{1}{4}$$

The conditional probability that both are girls, given that the youngest child is a girl, is given by P (A|B).

$$\mathrm{P}(\mathrm{A} \mid \mathrm{B}) = \frac{\mathrm{P}(\mathrm{A} \cap \mathrm{B})}{\mathrm{P}(\mathrm{B})} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

Therefore, the required probability is $\frac{1}{2}$.

Probability Ex 31.1 Q3

A = Two numbers on two dice are different

$$= \{(1,2),(1,3),(1,4),(1,5),(1,6)\\ (2,1),(2,3),(2,4),(2,5),(2,6)\\ (3,1),(3,2),(3,4),(3,5),(3,6)\\ (4,1),(4,2),(4,3),(4,5),(4,6)\\ (5,1),(5,2),(5,3),(5,4),(5,6)\\ (6,1),(6,2),(6,3),(6,4),(6,5)\}$$

B = Sum of numbers on the dice is 4

$B = \{(1,3),(2,2),(3,1)\}$

$A \cap B = \{(1,3),(3,1)\}$

Required probability $= P\left(\frac{B}{A}\right)$

$$= \frac{n(A \cap B)}{n(A)}$$

$$= \frac{2}{30}$$

Required probability $= \frac{1}{15}$

(ii) Let C be the event that at least one child is a girl.

$$\therefore \mathrm{C} = \{(b, g),(g, b),(g, g)\}$$
$$\Rightarrow \mathrm{A} \cap \mathrm{C} = \{g, g\}$$

$$\Rightarrow \mathrm{P(C)} = \frac{3}{4}$$
$$\mathrm{P(A \cap C)} = \frac{1}{4}$$

The conditional probability that both are girls, given that at least one child is a girl, is given by P(A|C).

$$\text{Therefore, } \mathrm{P(A \mid C)} = \frac{\mathrm{P(A \cap C)}}{\mathrm{P(C)}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

Probability Ex 31.1 Q4

A = Head on the first two toss on three tosses of coin

$A = \{HHT, HHH\}$

B = Getting ahead on third toss

$B = \{HHH, HTH, THH, TTH\}$

$A \cap B = \{HHH\}$

Required probability $= P\left(\frac{B}{A}\right)$

$$= \frac{n(A \cap B)}{n(A)}$$

Required probability $= \frac{1}{2}$

Probability Ex 31.1 Q5

A = 4 appears on third toss, if a die is thrown three times

$$\begin{aligned}
= \{&(1,1,4),(1,2,4),(1,3,4),(1,4,4),(1,5,4),(1,6,4)\\
&(2,1,4),(2,2,4),(2,3,4),(2,4,4),(2,5,4),(2,6,4)\\
&(3,1,4),(3,2,4),(3,3,4),(3,4,4),(3,5,4),(3,6,4)\\
&(4,1,4),(4,2,4),(4,3,4),(4,4,4),(4,5,4),(4,6,4)\\
&(5,1,4),(5,2,4),(5,3,4),(5,4,4),(5,5,4),(5,6,4)\\
&(6,1,4),(6,2,4),(6,3,4),(6,4,4),(6,5,4),(6,6,4)\}
\end{aligned}$$

B = 6 and 5 appears respectively on first two tosses, if die is tosses three times

$B = \{(6,5,1),(6,5,2),(6,5,3),(6,5,4),(6,5,5),(6,5,6)\}$

$A \cap B = \{(6,5,4)\}$

Required probability $= P\left(\frac{A}{B}\right)$

$$= \frac{n(A \cap B)}{n(B)}$$

$$= \frac{1}{6}$$

Required probability $= \frac{1}{6}$

Probability Ex 31.1 Q6

Given, $P(B) = 0.5$, $P(A \cap B) = 0.32$

We know that,

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$

$$= \frac{0.32}{0.5}$$

$$= \frac{32}{50}$$

$$= \frac{16}{25}$$

$$P\left(\frac{A}{B}\right) = \frac{16}{25}$$

Probability Ex 31.1 Q7

Given, $P(A) = 0.4$, $P(B) = 0.3$ and $P\left(\frac{B}{A}\right) = 0.5$

We know that,

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$0.5 = \frac{P(A \cap B)}{0.4}$$

$$P(A \cap B) = 0.5 \times 0.4$$

$$P(A \cap B) = 0.2$$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.2}{0.3}$$

$$P\left(\frac{A}{B}\right) = \frac{2}{3}$$

Probability Ex 31.1 Q8

Given, $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{5}$, $P(A \cup B) = \frac{11}{30}$

We know that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$\frac{11}{30} = \frac{1}{3} + \frac{1}{5} - P(A \cap B)$$
$$P(A \cap B) = \frac{1}{3} + \frac{1}{5} - \frac{11}{30}$$
$$= \frac{10 + 6 - 11}{30}$$
$$= \frac{5}{30}$$
$$P(A \cap B) = \frac{1}{6}$$

We know that,

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$
$$= \frac{\frac{1}{6}}{\frac{1}{5}}$$
$$P\left(\frac{A}{B}\right) = \frac{5}{6}$$

$$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$$
$$P\left(\frac{B}{A}\right) = \frac{\frac{1}{6}}{\frac{1}{3}}$$
$$= \frac{1}{6} \times \frac{3}{1}$$
$$= \frac{1}{2}$$

$$P\left(\frac{A}{B}\right) = \frac{5}{6}, \; P\left(\frac{B}{A}\right) = \frac{1}{2}$$

Probability Ex 31.1 Q9

Given, Couple has two children.

(i)

A = Both are male

$A = \{M_1M_2\}$

B = Atleast one is male

$B = \{M_1M_2, M_1F_2, F_1M_2\}$

$A \cap B = \{M_1M_2\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$
$$P\left(\frac{A}{B}\right) = \frac{1}{3}$$

$$P\left(\frac{A}{B}\right) = \frac{1}{3}$$

(ii)

A = Both are Females

$A = \{F_1F_2\}$

B = Elder child is female

$B = \{F_1M_2, F_1F_2\}$

$A \cap B = \{F_1F_2\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$
$$= \frac{1}{2}$$

$$P\left(\frac{A}{B}\right) = \frac{1}{2}$$

Ex 31.2

Probability Ex 31.2 Q1

A = first card is king
B = second card is also king

Probability of getting two kings (without replacement)

$$= P(A)P\left(\frac{B}{A}\right)$$

$$= \frac{4}{52} \times \frac{3}{51} \qquad \text{[Since, 4 kings out of 52 cards.]}$$

$$= \frac{1}{13} \times \frac{1}{17}$$

$$= \frac{1}{221}$$

Required probability $= \frac{1}{221}$

Probability Ex 31.2 Q2

A = first card Ace
B = second card Ace
C = third card Ace
D = fourth card Ace

P (All four drawn are Ace, without replacement)

$$= P(A)P\left(\frac{B}{A}\right)P\left(\frac{C}{A \cap B}\right)P\left(\frac{D}{A \cap B \cap C}\right)$$

$$= \frac{4}{52} \times \frac{3}{51} \times \frac{2}{50} \times \frac{1}{49} \qquad \text{[Since, there are four Ace in 52 cards]}$$

$$= \frac{1}{270725}$$

Required probability $= \frac{1}{270725}$

Probability Ex 31.2 Q3

Bag contains 5 red and 7 white balls

A = first ball white

B = second ball white

P (2 white balls drawn without replacement)

$$= P(A)P\left(\frac{B}{A}\right)$$

$$= \frac{7}{12} \times \frac{6}{11}$$

$$= \frac{7}{22}$$

Required probability $= \frac{7}{22}$

Probability Ex 31.2 Q4

Tickets are numbered from 1 to 25

$\Rightarrow$ Total number of tickets $= 25$

Number of tickets with even numbers on it

$= 12$ $\{2,4,6,8,10,12,14,16,18,20,22,24\}$

A = first ticket with even number

B = second ticket with even number

P (Both tickets will show even number, without replacement)

$$= P(A)P\left(\frac{B}{A}\right)$$

$$= \frac{12}{25} \times \frac{11}{24}$$

$$= \frac{11}{50}$$

Required probability $= \frac{11}{50}$

Probability Ex 31.2 Q5

We know that, Deck of 52 cards contains 13 spades.

A = first card is spade

B = second card spade

C = third card spade

P (3 cards drawn without replacement are spade)

$$= P(A)P\left(\frac{B}{A}\right)P\left(\frac{C}{A \cap B}\right)$$

$$= \frac{13}{52} \times \frac{12}{51} \times \frac{11}{50}$$

$$= \frac{11}{850}$$

Required probability $= \frac{11}{850}$

Probability Ex 31.2 Q6(i)

In a deck of 52 cards, there are 4 kings. Two cards are drawn without replacement

A = first card is king

B = second card is king

P (Both drawn cards are king)

$= P(A)P\left(\frac{B}{A}\right)$

$= \frac{4}{52} \times \frac{3}{51}$

$= \frac{1}{221}$

Required probability $= \frac{1}{221}$

Probability Ex 31.2 Q6(ii)

We know that, there are 4 kings and 4 ace in a pack of 52 cards.

Two cards are drawn without replacement

A = first card is king

B = second card an ace

P (The first card is a king and second is an ace)

$= P(A)P\left(\frac{B}{A}\right)$

$= \frac{4}{52} \times \frac{4}{51}$

$= \frac{4}{663}$

Required probability $= \frac{4}{663}$

Probability Ex 31.2 Q6(iii)

There are 13 heart and 26 red cards

Hearts are also red .

A = first card is heart

B = second card is red

P (first card is heart and second is red)

$= P(A).P\left(\frac{B}{A}\right)$

$= \frac{13}{52} \times \frac{25}{51}$

$= \frac{25}{204}$

Required probability $= \frac{25}{204}$

Probability Ex 31.2 Q7

Total number of tickets are 20 numbered from 1,2,3,...20.

Number of tickets with even numbers

$= 10 \qquad [\text{Since, even numbers are } 2,4,6,8,10,12,14,16,18,20]$

Number of tickets with odd numbers

$= 10 \qquad [\text{Since, odd numbers are } 1,3,5,7,9,11,13,15,17,19]$

Two cards are drawn without replacement.

A = tickets with even numbers

B = tickets with odd numbers

P (first ticket has even number and second has odd number)

$= P(A)P\left(\frac{B}{A}\right)$

$= \frac{10}{20} \cdot \frac{10}{19}$

$= \frac{5}{19}$

Required probability $= \frac{5}{19}$

Probability Ex 31.2 Q8

Urn contains 3 white, 4 red and 5 black balls. Total balls $= 12$

Two balls are drawn without replacement

A = first ball is black

B = second ball is black

P (Atleast one ball is black)

$= P(A \cup B)$

$= 1 - P\left(\overline{A \cup B}\right)$

$= 1 - P\left(\overline{A} \cap \overline{B}\right)$

$= 1 - P\left(\overline{A}\right)P\left(\overline{B} / \overline{A}\right)$

$= 1 - \left(\frac{7}{12} \times \frac{6}{12}\right)$

$= 1 - \frac{7}{22}$

$= \frac{15}{22}$

Required probability $= \frac{15}{22}$

Probability Ex 31.2 Q9

Bag contains 5 white, 7 red and 3 black balls.

Total number of balls = 15

Three balls are drawn without replacement

A = first ball is red

B = second ball is red

C = Third balls is red

P (Three balls are drawn, non is red)

$= P\left(\overline{A}\right) P\left(\frac{\overline{B}}{A}\right) P\left(\frac{\overline{C}}{A \cap B}\right)$

$= \frac{8}{15} \times \frac{7}{14} \times \frac{6}{13}$ [Since, number of non red balls $= 5 + 3 = 8$]

$= \frac{8}{65}$

Required probability $= \frac{8}{65}$

Probability Ex 31.2 Q10

Two cards are drawn from a pack of 52 cards without replacement.

There are 13 heart and 13 diamond in pack

A = first card is heart

B = second card is diamond

P (first card heart and second diamond)

$= P(A) P\left(\frac{B}{A}\right)$

$= \frac{13}{52} \times \frac{13}{51}$

$= \frac{13}{204}$

Required probability $= \frac{13}{204}$

Probability Ex 31.2 Q11

Let E and F denote respectively the events that first and second ball drawn are black. We have to find $P(E \cap F)$ or $P(EF)$.

Now $P(E) = P$ (black ball in first draw) $= \frac{10}{15}$

Also given that the first ball drawn is black, i.e, event E has occurred, now there are 9 black balls and five white balls left in the urn. Therefore, the probability that the second ball drawn is black, given that the ball in the first draw is black, is nothing but the conditional probability of F given that E has occurred.

i.e., $P(F \mid E) = \frac{9}{14}$

By multiplication rule of probability, we have

$P(E \cap F) = P(E)\ P(F \mid E)$

$= \frac{10}{15} \times \frac{9}{14} = \frac{3}{7}$

Multiplication rule of probability for more than two events if E.F and G are three events of sample space, we have

$P(E \cap F \cap G) = P(E)\ P(F \mid E)\ P(E)\ P(G \mid)\ (G \cap F) = P(E)\ P(F \mid E)\ P(G \mid EF)$

Similarly, the multiplication rule of probability can be extended for four or more events.

The following example illustrates the extension of multiplication rule of probability for three events.

Probability Ex 31.2 Q12

Let K denote the event that the card drawn is king
and A be the event that the card drawn is an ace.
We are to find $P(KKA)$.

Now, $P(K) = \frac{4}{52}$

Also, $P(K/K)$ is the probability of second king with the condition that one king has already been drawn.
Now, there are 3 kings in (52-1) = 51 cards.

$\therefore \quad P(K/K) = \frac{3}{51}$

Lastly, $P(A/KK)$ is the probability of third drawn card to be an ace, with the condition that two kings have already been drawn.
Now, there are four aces in left 50 cards.

$\therefore \quad P(A/KK) = \frac{4}{50}$

By multiplication law of probability, we have

$$P(KKA) = P(K)P(K/K)P(A/KK)$$
$$= \frac{4}{52} \times \frac{3}{51} \times \frac{4}{50} = \frac{2}{5525}$$

Probability Ex 31.2 Q13

There are 15 oranges out of which 12 are good and 3 are bad.

Three oranges selected without replacement are drawn and if they found good the box is approved for sale.

A = first orange good
B = second orange good
B = third orange good

P (All three oranges are good)

$$= P(A)P\left(\frac{B}{A}\right)P\left(\frac{C}{A \cap B}\right)$$
$$= \frac{12}{15} \times \frac{11}{14} \times \frac{10}{13}$$
$$= \frac{44}{91}$$

Required probability $= \frac{44}{91}$

Probability Ex 31.2 Q14

Given bag contains 4 white, 7 black and 5 red balls.

Total number of balls = 16

Three balls are drawn without replacement

A = first ball is white
B = second ball is black
C = Third balls is red

P (Three balls drawn are white, Black, red respectively)

$$= P(A)P\left(\frac{B}{A}\right)P\left(\frac{C}{A \cap B}\right)$$
$$= \frac{4}{16} \times \frac{7}{15} \times \frac{5}{14}$$
$$= \frac{1}{24}$$

Required probability $= \frac{1}{24}$

Ex 31.3

Probability Ex 31.3 Q1

Given,

$$P(A) = \frac{7}{13},\ P(B) = \frac{9}{13} \text{ and } P(A \cap B) = \frac{4}{13}$$

We know that,

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{4}{13}}{\frac{9}{13}}$$

$$= \frac{4}{9}$$

$$P\left(\frac{A}{B}\right) = \frac{4}{9}$$

Probability Ex 31.3 Q2

Given,

$$P(A) = 0.6,\ P(B) = 0.3 \text{ and } P(A \cap B) = 0.2$$

We know that,

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.2}{0.3}$$

$$P\left(\frac{A}{B}\right) = \frac{2}{3}$$

and, $$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{0.2}{0.6}$$

$$P\left(\frac{B}{A}\right) = \frac{1}{3}$$

$$P\left(\frac{A}{B}\right) = \frac{2}{3},\ P\left(\frac{B}{A}\right) = \frac{1}{3}$$

Probability Ex 31.3 Q3

Given,

$$P(A \cap B) = 0.32 \text{ and } P(B) = 0.5$$

We know that,

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.32}{0.5}$$

$$= \frac{16}{25}$$

$$= 0.64$$

$$P\left(\frac{A}{B}\right) = 0.64$$

Probability Ex 31.3 Q4

Given,

$$P(A) = 0.4,\ P(B) = 0.8,\ P\left(\frac{B}{A}\right) = 0.6$$

We know that,

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$0.6 = \frac{P(A \cap B)}{0.4}$$

$$P(A \cap B) = 0.6 \times 0.4$$

$$P(A \cap B) = 0.24$$

Now, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$$= \frac{0.24}{0.8}$$

$$P\left(\frac{A}{B}\right) = 0.3$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.4 + 0.8 - 0.24$$

$$P(A \cap B) = 0.96$$

$$P\left(\frac{A}{B}\right) = 0.3, \quad P(A \cap B) = 0.96$$

Probability Ex 31.3 Q5(i)

Given,

$$P(A) = \frac{1}{3},\ P(B) = \frac{1}{4},\ P(A \cup B) = \frac{5}{12}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{5}{12} = \frac{1}{3} + \frac{1}{4} - P(A \cap B)$$

$$P(A \cap B) = \frac{1}{3} + \frac{1}{4} - \frac{5}{12}$$

$$= \frac{4+3-5}{12}$$

$$P(A \cap B) = \frac{2}{12}$$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{2}{12}}{\frac{1}{4}}$$

$$= \frac{2}{12} \times \frac{4}{1}$$

$$P\left(\frac{A}{B}\right) = \frac{2}{3}$$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{2}{12}}{\frac{1}{3}}$$

$$= \frac{2}{12} \times \frac{3}{1}$$

$$P\left(\frac{B}{A}\right) = \frac{1}{2}$$

Hence,

$$P\left(\frac{A}{B}\right) = \frac{2}{3}$$

$$P\left(\frac{B}{A}\right) = \frac{1}{2}$$

Probability Ex 31.3 Q5(ii)

Given,

$$P(A) = \frac{6}{11},\ P(B) = \frac{5}{11} \text{ and } P(A \cup B) = \frac{7}{11}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$
$$= \frac{6}{11} + \frac{5}{11} + \frac{7}{11}$$
$$P(A \cap B) = \frac{4}{11}$$

We know that,

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$
$$= \frac{\frac{4}{11}}{\frac{5}{11}}$$
$$= \frac{4}{5}$$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$
$$= \frac{\frac{4}{11}}{\frac{6}{11}}$$
$$= \frac{2}{3}$$

Hence,

$$P\left(\frac{A}{B}\right) = \frac{4}{5},\ P\left(\frac{B}{A}\right) = \frac{2}{3}$$

Probability Ex 31.3 Q5(iii)

Given,

$$P(A) = \frac{7}{13},\ P(B) = \frac{9}{13},\ P(A \cap B) = \frac{4}{13}$$

Since, $P(A' \cap B) = P(B) - P(A \cap B)$

$$= \frac{9}{13} - \frac{4}{13}$$
$$P(A' \cap B) = \frac{5}{13}$$

$$P\left(\frac{A'}{B}\right) = \frac{P(A' \cap B)}{P(B)}$$
$$= \frac{\frac{5}{13}}{\frac{9}{13}}$$
$$= \frac{5}{9}$$

$$P\left(\frac{A'}{B}\right) = \frac{5}{9}$$

Probability Ex 31.3 Q5(iv)

$$P(A)=\frac{1}{2}, P(B)=\frac{1}{3} \text{ and } P(A\cap B)=\frac{1}{4},$$

$$P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{1}{4}}{\frac{1}{3}}=\frac{3}{4}$$

$$P(B/A)=\frac{P(B\cap A)}{P(A)}=\frac{\frac{1}{4}}{\frac{1}{2}}=\frac{1}{2}$$

$$P(\overline{A}/B)=\frac{P(B)-P(A\cap B)}{P(B)}=\frac{\frac{1}{3}-\frac{1}{4}}{\frac{1}{3}}=\frac{1}{4}$$

$$P(\overline{A}/\overline{B})=\frac{P(\overline{A}\cap\overline{B})}{P(\overline{B})}=\frac{1-P(\overline{A\cup B})}{P(A)-P(A\cap B)}=\frac{1-P(A)-P(B)+P(A\cap B)}{P(A)-P(A\cap B)}=\frac{1-\frac{1}{2}-\frac{1}{3}+\frac{1}{4}}{\frac{1}{2}-\frac{1}{4}}=\frac{5}{4}$$

Probability Ex 31.3 Q6

Given,

$$2P(A)=P(B)=\frac{5}{13}$$

$$2P(A)=\frac{5}{13}$$

$$\Rightarrow \quad P(A)=\frac{5}{26}$$

$$P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}$$

$$\frac{2}{5}=\frac{P(A\cap B)}{\frac{5}{13}}$$

$$P(A\cap B)=\frac{2}{5}\times\frac{5}{13}$$

$$P(A\cap B)=\frac{2}{13}$$

We know that,

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$=\frac{5}{26}+\frac{5}{13}-\frac{2}{13}$$

$$=\frac{5+10-4}{26}$$

$$=\frac{11}{26}$$

$$P(A\cup B)=\frac{11}{26}$$

Probability Ex 31.3 Q7

Given,

$$P(A) = \frac{6}{11},\ P(B) = \frac{5}{11},\ P(A \cup B) = \frac{7}{11}$$

(i)

Since, $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$$= \frac{6}{11} + \frac{5}{11} - \frac{7}{11}$$

$$P(A \cap B) = \frac{4}{11}$$

(ii)

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{4}{11}}{\frac{5}{11}}$$

$$P\left(\frac{A}{B}\right) = \frac{4}{5}$$

(iii)

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{4}{11}}{\frac{6}{11}}$$

$$P\left(\frac{B}{A}\right) = \frac{2}{3}$$

Probability Ex 31.3 Q8

Sample space for three coins is given by

$\{HHH, HTH, THH, TTH, HHT, HTT, THT, TTT\}$

(i)

A = Head on third toss

$A = \{HHH, HTH, THH, TTH\}$

B = Head on first two toss

$B = \{HHH, HHT\}$

$(A \cap B) = \{HHH\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$

$$P\left(\frac{A}{B}\right) = \frac{1}{2}$$

Hence, $P\left(\frac{A}{B}\right) = \frac{1}{2}$

(ii)

A = At least two heads

$A = \{HHH, HHT, HTH, THH\}$

B = At most two heads

$B = \{HHT, HTT, THT, TTT, HTH, THH, TTH\}$

$(A \cap B) = \{HHT, HTH, THH\}$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$P\left(\frac{A}{B}\right) = \frac{3}{7}$$

Hence, $P\left(\frac{A}{B}\right) = \frac{3}{7}$

(iii)

A = At most two tails

$A = \{HHH, HTH, THT, TTH, HHT, THT, HTT\}$

B = At least one tail

$B = \{HTH, THH, TTH, HHT, HTT, THT, TTT\}$

$(A \cap B) = \{HTH, THT, TTH, HHT, THT, HTT\}$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{6}{7}$$

Hence, $P\left(\frac{A}{B}\right) = \frac{6}{7}$

Probability Ex 31.3 Q9

Sample space of two coins

$\{HH, HT, TH, TT\}$

(i)

A = Tail appears on one coin

$A = \{HT, TH\}$

B = One coin shows head

$B = \{HT, TH\}$

$(A \cap B) = \{HT, TH\}$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{2}{2}$$

$$P\left(\frac{A}{B}\right) = 1$$

Hence, $P\left(\frac{A}{B}\right) = 1$

(ii)

A = No tail appears

$A = \{HH\}$

B = No head appears

$B = \{TT\}$

$(A \cap B) = \{\ \}$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0}{1}$$

$$= 0$$

$$P\left(\frac{A}{B}\right) = 0$$

Probability Ex 31.3 Q10

Die is thrown three times.

A = 4 appears on the third toss

$$A = \{(1,1,4),(1,2,4),(1,3,4),(1,4,4),(1,5,4),(1,6,4)$$
$$(2,1,4),(2,2,4),(2,3,4),(2,4,4),(2,5,4),(2,6,4)$$
$$(3,1,4),(3,2,4),(3,3,4),(3,4,4),(3,5,4),(3,6,4)$$
$$(4,1,4),(4,2,4),(4,3,4),(4,4,4),(4,5,4),(4,6,4)$$
$$(5,1,4),(5,2,4),(5,3,4),(5,4,4),(5,5,4),(5,6,4)$$
$$(6,1,4),(6,2,4),(6,3,4),(6,4,4),(6,5,4),(6,6,4)\}$$

B = 6 and 5 appear respectively on first two tosses

$$B = \{(6,5,1),(6,5,5),(6,5,3),(6,5,4),(6,5,5),(6,5,6)\}$$

$$(A \cap B) = \{(6,5,4)\}$$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$
$$= \frac{1}{6}$$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$
$$= \frac{1}{36}$$

Hence, $P\left(\frac{A}{B}\right) = \frac{1}{6}$, $P\left(\frac{B}{A}\right) = \frac{1}{36}$

Probability Ex 31.3 Q11

There are three person for photograph father (F), mother (M), son (S).

Sample space $= \{FMS, FSM, MFS, MSF, SFM, SMF\}$

A = Son on one end

$A = \{SFM, SMF, MFS, FMS\}$

B = Father in the middle

$B = \{MFS, SFM\}$

$$(A \cap B) = \{MFS, SFM\}$$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$
$$= \frac{2}{2}$$
$$P\left(\frac{A}{B}\right) = 1$$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$
$$= \frac{2}{4}$$
$$P\left(\frac{B}{A}\right) = \frac{1}{2}$$

Hence, $P\left(\frac{A}{B}\right) = 1$, $P\left(\frac{B}{A}\right) = \frac{1}{2}$

Probability Ex 31.3 Q12

The sample space of the experiment is $\{(1,1)(1,2)(1,3)\ldots(6,6)\}$ conisting of 36 outcomes.

$P(A) = P(\text{Sum} = 6) = \frac{5}{36}$

$P(B) = P(4 \text{ appears at least once}) = \frac{11}{36}$

Now, $P\left(\frac{B}{A}\right) = \frac{P(A \text{ and } B)}{P(A)}$

$= \frac{P(\text{sum is 6 and 4 has appeared at least once})}{P(A)}$

$= \frac{\frac{2}{36}}{\frac{5}{36}}$

$= \frac{2}{5}$

Probability Ex 31.3 Q13

Two dice are thrown.

A = Sum on the dice is 8

$A = \{(2,6),(3,5),(4,4),(5,3),(6,2)\}$

B = Second die always exhibits 4

$B = \{(1,4),(2,4),(3,4),(4,4),(5,4),(6,4)\}$

$(A \cap B) = \{(4,4)\}$

$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$

$= \frac{1}{6}$

Required probability $= \frac{1}{6}$

Probability Ex 31.3 Q14

Here two dice are thrown

A = Getting 7 as sum on two dice

$A = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$

B = Second die exhibits an odd number

$B = \{(1,1),(2,1),(3,1),(4,1),(5,1),(6,1)$
$(1,3),(2,3),(3,3),(4,3),(5,3),(6,3)$
$(1,5),(2,5),(3,5),(4,5),(5,5),(6,5)\}$

$(A \cap B) = \{(2,5),(4,3),(6,1)\}$

$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$

$= \frac{3}{18}$

$= \frac{1}{6}$

Hence, Required probability $= \frac{1}{6}$

Probability Ex 31.3 Q15

A pair of dice is thrown.

A = Getting 7 as sum number on 2 dice.

$A = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$

B = Second die always exhibits prime number

$$B = \{(1,2),(2,2),(3,2),(4,2),(5,2),(6,2)$$
$$(1,3),(2,3),(3,3),(4,3),(5,3),(6,3)$$
$$(1,5),(2,5),(3,5),(4,5),(5,5),(6,5)\}$$

[Since, there are 2,3,5 prime number on a die]

$(A \cap B) = \{(2,5),(4,3),(5,2)\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$
$$= \frac{3}{18}$$
$$= \frac{1}{6}$$

Hence, Required probability $= \frac{1}{6}$

Probability Ex 31.3 Q16

A die is rolled.

A = A prime number on die

$A = \{2, 3, 5\}$

B = An odd number on die

$B = (1,3,5)$

$(A \cap B) = \{3,5\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$
$$= \frac{2}{3}$$

Required probability $= \frac{2}{3}$

Probability Ex 31.3 Q17

A pair of dice is thrown

A = Getting sum 8 or more

= Getting sum 8,9,10,11 or 12 on the pair of dice

$$A = \{(2,6),(3,5),(4,4),(5,3),(6,2),(3,6)$$
$$(4,5),(5,4),(6,3),(4,6),(5,5),(6,4)$$
$$(5,6),(6,5),(6,6)$$

B = 4 on first die

$B = \{(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)\}$

$(A \cap B) = \{(4,4),(4,5),(4,6)\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$
$$= \frac{3}{6}$$
$$= \frac{1}{2}$$

Required probability $= \frac{1}{2}$

Probability Ex 31.3 Q18

Two dice are thrown

A = Sum of the numbers on dice is 8

$A = \{(2,6),(3,5),(4,4),(5,3),(6,2)\}$

B = At least one die does not show five

$$B = \{(1,1),(1,2),(1,3),(1,4),(1,6),(2,1),(2,2)$$
$$(2,3),(2,4),(2,6),(3,1),(3,2),(3,3),(3,4)$$
$$(3,6),(4,1),(4,2),(4,3),(4,4),(4,6),(6,1)$$
$$(6,2),(6,3),(6,4),(6,6)\}$$

$(A \cap B) = \{(2,6),(4,6),(6,2)\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$
$$= \frac{3}{25}$$

Required probability $= \frac{3}{25}$

Probability Ex 31.3 Q19

Two numbers are selected at random from integers 1 through 9.

A = Both numbers are odd

$$A = \{(3,1),(5,1),(7,1),(9,1),(3,5),(3,7),(9,3),(5,3),(5,7),(5,9)$$
$$(7,3),(7,5),(7,9),(9,3),(9,5),(9,7)\}$$

B = Sum of both numbers is even

= Sum of both numbers is 2, 4, 6, 8, 10, 12, 14, 16 or 18

$$= \{(1,3),(1,5),(2,4),(1,7),(2,6),(3,5),(1,9),(2,8),$$
$$(3,7),(4,6),(7,5),(8,4),(9,3),(8,6),(9,5),(9,7)\}$$

$(A \cap B) = \{(1,3),(1,5),(1,7),(3,5),(1,9),(3,7),(7,5),(9,3),(9,5),(9,7)\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$
$$= \frac{10}{16}$$

Required probability $= \frac{5}{8}$

Probability Ex 31.3 Q20

A die is thrown twice

A = The number 5 has appeared at least once

$A = \{(1,5),(2,5),(3,5),(4,5),(5,5),(6,5),(5,1),(5,2),(5,3),(5,4),(5,6)\}$

B = Sum of the numbers is 8

$= \{(2,6),(3,5),(4,4),(5,3),(6,2)\}$

$(A \cap B) = \{(3,5),(5,3)\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$
$$= \frac{2}{5}$$

Required probability $= \frac{2}{5}$

Probability Ex 31.3 Q21

Two dice are thrown

A = Sum of the numbers showing on the dice is 7

$A = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$

B = First die shows a 6

$= \{(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

$(A \cap B) = \{(6,1)\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$

$$= \frac{1}{6}$$

Required probability $= \frac{1}{6}$

Probability Ex 31.3 Q22

A pair of die is thrown

E = Sum is greater than or equal to 10

$= \{(4,6),(5,5),(6,4),(5,6),(6,5),(6,6)\}$

Case I:

F = 5 appears on first die

$= \{(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\}$

$E \cap F = \{(5,5),(5,6)\}$

$$P\left(\frac{E}{F}\right) = \frac{n(E \cap F)}{n(F)}$$

$$= \frac{2}{6}$$

$$P\left(\frac{E}{F}\right) = \frac{1}{3}$$

Case II:

F = 5 appears on at least one die

$= \{(1,5),(2,5),(3,5),(4,5),(6,5),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\}$

$E \cap F = \{(5,5),(5,6),(6,5)\}$

$$P\left(\frac{E}{F}\right) = \frac{n(E \cap F)}{n(F)}$$

$$= \frac{3}{11}$$

$$P\left(\frac{E}{F}\right) = \frac{3}{11}$$

Probability Ex 31.3 Q23

Given,

Probability to pass mathematics (M)

$$P(M) = \frac{4}{5}$$

Probability to pass in mathematics (M) and computer Science (C)

$$P(M \cap C) = \frac{1}{2}$$

To find, $P\left(\frac{C}{M}\right)$

We know that,

$$P\left(\frac{C}{M}\right) = \frac{P(M \cap C)}{P(M)}$$
$$= \frac{\frac{1}{2}}{\frac{4}{5}}$$
$$= \frac{1}{2} \times \frac{5}{4}$$
$$= \frac{5}{8}$$

Required probability $= \frac{5}{8}$

Probability Ex 31.3 Q24

Given,

Probability that a person buys a shirt $(S) = P(S) = 0.2$

Probability that he buys a trouser $(T) = P(T) = 0.3$

$$P\left(\frac{S}{T}\right) = 0.4$$

We know that,

$$P\left(\frac{S}{T}\right) = \frac{P(S \cap T)}{P(T)}$$
$$0.4 = \frac{P(S \cap T)}{0.3}$$
$$P(S \cap T) = 0.4 \times 0.3$$
$$P(S \cap T) = 0.12$$

Probability that he buys a shirt and a trouser both $= 0.12$

$$P\left(\frac{T}{S}\right) = \frac{P(S \cap T)}{P(S)}$$
$$= \frac{0.12}{0.2}$$
$$P\left(\frac{T}{S}\right) = \frac{12}{20}$$
$$= \frac{3}{5}$$
$$= 0.6$$

Probability that he buys a trouser given that he buys a shirt $= 0.6$

Probability Ex 31.3 Q25

Total students = 1000
Number of girls = 430
% of girls in class XII = 10%

Let A = Student chosen studies in class XII
B = Student chosen is a girl

Then $P(B) = \frac{430}{1000}$

$P(A \cap B) = \frac{43}{1000}$

$\therefore \quad P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{43}{430} = \frac{1}{10}$

Probability Ex 31.3 Q26

Total no. of cards = 10

Let A = drawn number is more than 3
B = drawn number is even

$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$

Now $P(A) = \frac{7}{10}$

$P(A \cap B) = \frac{4}{10}$

$\therefore \quad P\left(\frac{B}{A}\right) = \frac{4}{7}$

Probability Ex 31.3 Q27

(i) Let 'A' be the event that both the children born are girls.
Let 'B' be the event that the youngest is a girl.
We have to find conditional probability P(A/B).

$P(A/B) = \frac{P(A \cap B)}{P(B)}$

$A \subset B \Rightarrow A \cap B = A$

$\Rightarrow P(A \cap B) = P(A) = P(GG) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$P(B) = P(BG) + P(GG) = \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

Hence, $P(A/B) = \frac{1/4}{1/2} = \frac{1}{2}$

(ii) Let 'A' be the event that both the children born are girls.
Let 'B' be the event that at least one is a girl.
We have to find the conditional probability P(A/B).

$P(A/B) = \frac{P(A \cap B)}{P(B)}$

$A \subset B \Rightarrow A \cap B = A$

$\Rightarrow P(A \cap B) = P(A) = P(GG) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$P(B) = 1 - P(BB) = 1 - \frac{1}{2} \times \frac{1}{2} = 1 - \frac{1}{4} = \frac{3}{4}$

Hence, $P(A/B) = \frac{1/4}{3/4} = \frac{1}{3}$

Ex 31.4

Probability Ex 31.4 Q1(i)

A coin is tossed thrice

Sample space $= \{HHT, HTT, THT, TTT, HHH, HTH, THH, TTH\}$

A = The first throw results in head
$A = \{HHT, HTH, HHH, HTT\}$

B = The last throw in tail
$B = \{HHT, HTT, THT, TTT\}$

$A \cap B = \{HHT, HTT\}$

$P(A) = \frac{4}{8} = \frac{1}{2}$
$P(B) = \frac{4}{8} = \frac{1}{2}$

$P(A \cap B) = \frac{2}{8} = \frac{1}{4}$
$P(A).P(B) = \frac{1}{2}.\frac{1}{2}$
$P(A).P(B) = \frac{1}{4}$

$P(A).P(B) = P(A \cap B)$

So, A and B are independent events.

Probability Ex 31.4 Q1(ii)

Sample space for a coin thrown thrice is
$= \{HHT, HTT, THT, TTT, HHH, HTH, THH, TTH\}$

A = the number of head is odd
$A = \{HTT, THT, TTH, HHH\}$

B = the number of tails is odd
$B = \{THH, HTH, HHT, TTT\}$

$A \cap B = \{\ \} = \varnothing$

$P(A) = \frac{4}{8} = \frac{1}{2}$
$P(B) = \frac{4}{8} = \frac{1}{2}$

$P(A \cap B) = \frac{0}{8} = 0$

$P(A).P(B) = \frac{1}{2} \times \frac{1}{2}$
$= \frac{1}{4}$

$P(A).P(B) \neq P(A \cap B)$

So, A and B are not independent events.

Probability Ex 31.4 Q1(iii)

Sample space for throwing a coin thrice

$= \{HHT, HTT, THT, TTT, HHH, HTH, THH, TTH\}$

A = the number of heads is two

$A = \{HHT, THH, HTH\}$

B = the last throw results in head

$B = \{HHH, HTH, THH, TTH\}$

$A \cap B = \{THH, HTH\}$

$P(A) = \frac{3}{8}$

$P(B) = \frac{4}{8} = \frac{1}{2}$

$P(A \cap B) = \frac{2}{8} = \frac{1}{4}$

$$P(A).P(B) = \frac{3}{8} \times \frac{1}{2}$$
$$= \frac{3}{16}$$

$P(A).P(B) \neq P(A \cap B)$

So, A and B are not independent events.

Probability Ex 31.4 Q2

A pair of dice are thrown. It has 36 elements in its sample space.

A = Occurence of number 4 on first die

$A = \{(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)\}$

B = Occurence of 5 on second die

$B = \{(1,5), (2,5), (3,5), (4,5), (5,5), (6,5)\}$

$A \cap B = \{(4,5)\}$

$P(A) = \frac{6}{36} = \frac{1}{6}$

$P(B) = \frac{6}{36} = \frac{1}{6}$

$P(A \cap B) = \frac{1}{36}$

$$P(A).P(B) = \frac{1}{6} \times \frac{1}{6}$$
$$= \frac{1}{36}$$

$P(A).P(B) = P(A \cap B)$

So, A and B are independent events.

Probability Ex 31.4 Q3(i)

A card is drawn from 52 cards

It has 4 kings, 4 Queen, 4 Jack

A = the card drawn is a king or a queen

$$P(A) = \frac{4+4}{52}$$
$$= \frac{8}{52}$$
$$P(A) = \frac{2}{13}$$

B = the card drawn is a queen or a jack

$$P(B) = \frac{4+4}{52}$$
$$= \frac{8}{52}$$
$$= \frac{2}{13}$$

$A \cap B$ = The card drawn is a queen

$$P(A \cap B) = \frac{4}{52}$$
$$= \frac{1}{13}$$

$$P(A)P(B) = \frac{2}{13} \times \frac{2}{13}$$
$$= \frac{4}{169}$$

$$P(A)P(B) \neq P(A \cap B)$$

Hence, A and B are not independent.

Probability Ex 31.4 Q3(ii)

A card is drawn from pack of 52 cards

There are 26 black and four kings in which 2 kings are black.

A = the card drawn is black

$$P(A) = \frac{26}{52}$$
$$P(A) = \frac{1}{2}$$

B = the card drawn is a king

$$P(B) = \frac{4}{52}$$
$$= \frac{1}{13}$$

$A \cap B$ = The card drawn is a black king

$$P(A \cap B) = \frac{2}{52} = \frac{1}{26}$$

$$P(A)P(B) = \frac{1}{2} \times \frac{1}{13}$$
$$= \frac{1}{26}$$

$$P(A).P(B) = P(A \cap B)$$

So, A and B are independent events.

Probability Ex 31.4 Q3(iii)

A card is drawn from a pack of 52 cards

There are 13 spades and 4 Ace in which one card is ace of spade

A = the card drawn is spade

$$P(A) = \frac{13}{52}$$

$$P(A) = \frac{1}{4}$$

B = the card drawn is an ace

$$P(B) = \frac{4}{52}$$

$$P(B) = \frac{1}{13}$$

$A \cap B$ = The card drawn is an ace of spade

$$P(A \cap B) = \frac{1}{52}$$

$$P(A).P(B) = \frac{1}{4} \times \frac{1}{13}$$

$$= \frac{1}{52}$$

$$P(A).P(B) = P(A \cap B)$$

Hence, A and B are independent events.

Probability Ex 31.4 Q4

A coin is tossed three times,

Sample space $= \{HHH, HTH, THH, TTH, HHT, HTT, THT, TTT\}$

A = first toss is Head

$A = \{HHH, HHT, HTH, HTT\}$

$$P(A) = \frac{4}{8}$$

$$P(A) = \frac{1}{2}$$

B = second toss is Head

$= \{HHH, HHT, THH, THT\}$

$$P(B) = \frac{4}{8}$$

$$P(B) = \frac{1}{2}$$

C = exactly two Head in a row

$C = \{HHT, THH\}$

$$P(C) = \frac{2}{8}$$

$$P(C) = \frac{1}{4}$$

$A \cap B = \{HHH, HHT\}$

$$P(A \cap B) = \frac{2}{8}$$

$$= \frac{1}{4}$$

$B \cap C = \{HHT, THH\}$

$$P(B \cap C) = \frac{2}{8}$$

$$P(B \cap C) = \frac{1}{4}$$

$A \cap C = \{HHT\}$

$$P(A \cap C) = \frac{1}{8}$$

(i)

$$P(A).P(B) = \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{1}{4}$$
$$P(A).P(B) = P(A \cap B)$$

Hence, A and B are independent events.

(ii)

$$P(B)\,P(C) = \frac{1}{2} \times \frac{1}{4}$$
$$= \frac{1}{8}$$
$$P(B)\,P(C) \neq P(B \cap C)$$

So, B and C are not independent events.

(iii)

$$P(A).P(C) = \frac{1}{2} \times \frac{1}{4}$$
$$= \frac{1}{8}$$
$$P(A).P(C) = P(A \cap C)$$

Hence, A and C are independent events.

Probability Ex 31.4 Q5

Given,

$$P(A) = \frac{1}{4}, P(B) = \frac{1}{3} \text{ and } P(A \cup B) = \frac{1}{2}$$

We know that,

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$
$$= \frac{1}{4} + \frac{1}{3} + \frac{1}{2}$$
$$= \frac{3 + 4 - 6}{12}$$
$$P(A \cap B) = \frac{1}{12}$$

$$P(A)\,P(B) = \frac{1}{4} \times \frac{1}{3}$$
$$= \frac{1}{12}$$
$$P(A)\,P(B) = P(A \cap B)$$

Hence, A and B are independent events.

Probability Ex 31.4 Q6

Given that A and B are independent events and $P(A) = 0.3$, $P(B) = 0.6$

(i)

$$P(A \cap B) = P(A) P(B) \qquad [\text{Since, } A \text{ and } B \text{ are indenpendent events}]$$
$$= 0.3 \times 0.6$$

$$P(A \cap B) = 0.18$$

(ii)

$$P(A \cap \overline{B}) = P(A) - P(A \cap B)$$
$$= 0.3 - 0.18$$

$$P(A \cap \overline{B}) = 0.12$$

(iii)

$$P(\overline{A} \cap B) = P(B) - P(A \cap B)$$
$$= 0.6 - 0.18$$

$$P(\overline{A} \cap B) = 0.42$$

(iv)

$$P(\overline{A} \cap \overline{B}) = P(A) P(B)$$
$$= [1 - P(A)][1 - P(B)]$$
$$= (1 - 0.3)(1 - 0.6)$$
$$= 0.7 \times 0.4$$

$$P(\overline{A} \cap \overline{B}) = 0.28$$

(v)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.3 + 0.6 - 0.18$$

$$P(A \cup B) = 0.72$$

(vi)

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$
$$= \frac{0.18}{0.6}$$
$$P\left(\frac{A}{B}\right) = 0.3$$

(vii)

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$
$$= \frac{0.18}{0.3}$$

$$P\left(\frac{B}{A}\right) = 0.6$$

Probability Ex 31.4 Q7

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Since A, B are independent

$\therefore \quad P(A \cap B) = P(A).P(B)$

Also $P(\text{not } B) = 0.65 \Rightarrow \quad P(B) = 0.35$

Hence, we have

$$0.85 = P(A) + 0.35 - P(A)(0.35)$$

$\Rightarrow \quad 0.5 = P(A)[1 - 0.35]$

$\Rightarrow \quad \frac{0.5}{.65} = P(A)$

$\Rightarrow \quad P(A) = 0.77$

Probability Ex 31.4 Q8

We are given

$PP(\overline{A} \cap B) = \frac{2}{15}$

$P(A \cap \overline{B}) = \frac{1}{6}$

Since A, B are independent,

$\therefore \quad P(\overline{A})P(B) = \frac{2}{15} \Rightarrow \quad [1 - P(A)]P(B) = \frac{2}{15} \qquad ---(i)$

and $\quad P(A)P(\overline{B}) = \frac{1}{6} \Rightarrow \quad P(A)[1 - P(B)] = \frac{1}{6} \qquad ---(ii)$

From (i) we get

$$P(B) = \frac{2}{15} \times \frac{1}{1 - P(A)}$$

Substituting this value in equation (ii) we get,

$$P(A)\left[1 - \frac{2}{15(1 - P(A))}\right] = \frac{1}{6}$$

$\Rightarrow \quad P(A)\left[\frac{15(1 - P(A)) - 2}{15(1 - P(A))}\right] = \frac{1}{6}$

$\Rightarrow \quad 6P(A)(13 - 15P(A)) = 15(1 - P(A))$

$\Rightarrow \quad 2P(A)(13 - 15P(A)) = 5 - 5P(A)$

$\Rightarrow \quad 26P(A) - 30[P(A)]^2 + 5P(A) - 5 = 0$

$\Rightarrow \quad -30[P(A)]^2 + 31P(A) - 5 = 0$

This is a quadratic equation in $x = P(A)$ given as

$$-30x^2 + 31x - 5 = 0$$

$\Rightarrow \quad 30x^2 - 31x + 5 = 0$

$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Where $a = +30$, $b = -31$, $c = +5$

$$\Rightarrow \quad x = \frac{31 \pm \sqrt{(-31)^2 - 4(30)(5)}}{60}$$

$$= \frac{31 \pm \sqrt{961 - 600}}{60}$$

$$= \frac{31 \pm 19}{60}$$

$$= \frac{50}{60}, \frac{12}{60}$$

$$= \frac{5}{6}, \frac{1}{5}$$

$$\therefore \quad P(A) = \frac{5}{6} \text{ or } \frac{1}{5}$$

Now

$$P(A)[1 - P(B)] = \frac{1}{6}$$

Putting $P(A) = \frac{5}{6}$

$$\frac{5}{6}[1 - P(B)] = \frac{1}{6}$$

$$1 - P(B) = \frac{1}{5}$$

$$P(B) = 1 - \frac{1}{5}$$

$$P(B) = \frac{4}{5}$$

Putting $P(A) = \frac{1}{5}$

$$\frac{1}{5}[1 - P(B)] = \frac{1}{6}$$

$$1 - P(B) = \frac{5}{6}$$

$$P(B) = 1 - \frac{5}{6}$$

$$P(B) = \frac{1}{6}$$

Hence $P(B) = \frac{4}{5} \text{ or } \frac{1}{6}$

Probability Ex 31.4 Q9

Given,

$$P\left(A \cap B\right) = \frac{1}{6}$$

$$P\left(\overline{A} \cap \overline{B}\right) = \frac{1}{3}$$

We know that,

$$P\left(\overline{A} \cap \overline{B}\right) = P\left(\overline{A}\right) P\left(\overline{B}\right)$$

$$\frac{1}{3} = \left(1 - P\left(A\right)\right)\left(1 - P\left(B\right)\right)$$

$$\frac{1}{3} = 1 - P\left(B\right) - P\left(A\right) + P\left(A\right) P\left(B\right)$$

$$\frac{1}{3} = 1 - P\left(B\right) - P\left(A\right) + P\left(A \cap B\right)$$

$$\frac{1}{3} = 1 - P\left(B\right) - P\left(A\right) + \frac{1}{6}$$

$$P\left(A\right) + P\left(B\right) = \frac{1}{1} + \frac{1}{6} - \frac{1}{3}$$

$$= \frac{6 + 1 - 2}{6}$$

$$P\left(A\right) + P\left(B\right) = \frac{5}{6}$$

$$P\left(A\right) = \frac{5}{6} - P\left(B\right) \qquad ---(i)$$

Given, $P\left(A \cap B\right) = \frac{1}{6}$

$$P\left(A\right) P\left(B\right) = \frac{1}{6}$$

$$\left[\frac{5}{6} - P\left(B\right)\right] P\left(B\right) = \frac{1}{6} \qquad \left[\text{Using equation (i)}\right]$$

$\Rightarrow \quad \frac{5}{6} P\left(B\right) - \left\{P\left(B\right)\right\}^2 = \frac{1}{6}$

$\Rightarrow \quad \left\{P\left(B\right)\right\}^2 - \frac{5}{6} P\left(B\right) + \frac{1}{6} = 0$

$\Rightarrow \quad 6\left\{P\left(B\right)\right\}^2 - 5P\left(B\right) + 1 = 0$

$\Rightarrow \quad 6\left\{P\left(B\right)\right\}^2 - 3P\left(B\right) - 2P\left(B\right) + 1 = 0$

$\Rightarrow \quad 3P\left(B\right)\left[2P\left(B\right) - 1\right] - 1\left[2P\left(B\right) - 1\right] = 0$

$\Rightarrow \quad \left[2P\left(B\right) - 1\right]\left[3P\left(B\right) - 1\right] = 0$

$\Rightarrow \quad 2P\left(B\right) - 1 = 0 \text{ or } 3P\left(B\right) - 1 = 0$

Given,

$$P\left(A \cap B\right) = \frac{1}{6}$$

$$P\left(\overline{A} \cap \overline{B}\right) = \frac{1}{3}$$

We know that,

$$P\left(\overline{A} \cap \overline{B}\right) = P\left(A\right)P\left(B\right)$$

$$\frac{1}{3} = \left(1 - P\left(A\right)\right)\left(1 - P\left(B\right)\right)$$

$$\frac{1}{3} = 1 - P\left(B\right) - P\left(A\right) + P\left(A\right)P\left(B\right)$$

$$\frac{1}{3} = 1 - P\left(B\right) - P\left(A\right) + P\left(A \cap B\right)$$

$$\frac{1}{3} = 1 - P\left(B\right) - P\left(A\right) + \frac{1}{6}$$

$$P\left(A\right) + P\left(B\right) = \frac{1}{1} + \frac{1}{6} - \frac{1}{3}$$

$$= \frac{6 + 1 - 2}{6}$$

$$P\left(A\right) + P\left(B\right) = \frac{5}{6}$$

$$P\left(A\right) = \frac{5}{6} - P\left(B\right) \qquad ---(i)$$

Given, $P\left(A \cap B\right) = \frac{1}{6}$

$$P\left(A\right)P\left(B\right) = \frac{1}{6}$$

$$\left[\frac{5}{6} - P\left(B\right)\right]P\left(B\right) = \frac{1}{6} \qquad \left[\text{Using equation (i)}\right]$$

$\Rightarrow \quad \frac{5}{6}P\left(B\right) - \left\{P\left(B\right)\right\}^2 = \frac{1}{6}$

$\Rightarrow \quad \left\{P\left(B\right)\right\}^2 - \frac{5}{6}P\left(B\right) + \frac{1}{6} = 0$

$\Rightarrow \quad 6\left\{P\left(B\right)\right\}^2 - 5P\left(B\right) + 1 = 0$

$\Rightarrow \quad 6\left\{P\left(B\right)\right\}^2 - 3P\left(B\right) - 2P\left(B\right) + 1 = 0$

$\Rightarrow \quad 3P\left(B\right)\left[2P\left(B\right) - 1\right] - 1\left[2P\left(B\right) - 1\right] = 0$

$\Rightarrow \quad \left[2P\left(B\right) - 1\right]\left[3P\left(B\right) - 1\right] = 0$

$\Rightarrow \quad 2P\left(B\right) - 1 = 0 \text{ or } 3P\left(B\right) - 1 = 0$

Probability Ex 31.4 Q10

Given, A and B are independent events and $P\left(A \cup B\right) = 0.60$, $P\left(A\right) = 0.2$
A and B are independent events,

So, $\quad P\left(A \cap B\right) = P\left(A\right)P\left(B\right)$

We know that,

$$P\left(A \cup B\right) = P\left(A\right) + P\left(B\right) - P\left(A \cap B\right)$$

$$0.6 = 0.2 + P\left(B\right) - P\left(A\right)P\left(B\right)$$

$$0.6 - 0.2 = P\left(B\right) - 0.2P\left(B\right)$$

$$0.4 = 0.8P\left(B\right)$$

$$P\left(B\right) = \frac{0.4}{0.8}$$

$$P\left(B\right) = 0.5$$

Probability Ex 31.4 Q11

A die is tossed twice.

Let A = Getting a number greater than 3 on first toss
B = Getting a number greater than 3 on second toss

$P(A) = \frac{3}{6}$ [Since, number greater than 3 on die are 4, 5, 6.]

$P(A) = \frac{1}{2}$

$P(B) = \frac{3}{6}$

$P(B) = \frac{1}{2}$

P (Getting a number greater than 3 on each toss)

$= P(A \cap B)$ [Since, A and B are independent events]

$= P(A)P(B)$

$= \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4}$

Required Probability $= \frac{1}{4}$

Probability Ex 31.4 Q12

Given,

Probability that A can solve a problem $= \frac{2}{3}$

$\Rightarrow \quad P(A) = \frac{2}{3}$

$\Rightarrow \quad P(\overline{A}) = 1 - \frac{2}{3}$

$P(\overline{A}) = \frac{1}{3}$

Probability that B can solve the same problem $= \frac{3}{5}$

$\Rightarrow \quad P(B) = \frac{3}{5}$

$\Rightarrow \quad P(\overline{B}) = 1 - \frac{3}{5}$

$P(\overline{B}) = \frac{2}{5}$

P (None of them solve the problem)

$= P(\overline{A} \cap \overline{B})$

$= P(\overline{A}).P(\overline{B})$

$= \frac{1}{3} \times \frac{2}{5}$

$= \frac{2}{15}$

Required probability $= \frac{2}{15}$

Probability Ex 31.4 Q13

Given an unbiased die is tossed twice

A = Getting 4,5 or 6 on the first toss

B = 1,2,3 or 4 on second toss

$$\Rightarrow \quad P(A) = \frac{3}{6}$$

$$P(A) = \frac{1}{2}$$

and, $$P(B) = \frac{4}{6}$$

$$P(B) = \frac{2}{3}$$

P(Getting 4,5 or 6 on the first toss and 1,2,3 or 4 on second toss)

$= P(A \cap B)$

$= P(A)P(B)$

$= \frac{1}{2} \times \frac{2}{3}$

$= \frac{1}{3}$

Required probability $= \frac{1}{3}$

Probability Ex 31.4 Q14

Given bag contains 3 red and 2 black balls.

A = Getting one red ball

$$\Rightarrow \quad P(A) = \frac{3}{5}$$

B = Getting one black ball

$$\Rightarrow \quad P(B) = \frac{2}{5}$$

(i)

P(Getting two red balls)

$= P(A)P(A)$

$= \frac{3}{5} \times \frac{3}{5}$

$= \frac{9}{25}$

P(Getting two red balls) $= \frac{9}{25}$

(ii)

P(Getting two black balls)

$= P(B).P(B)$

$= \frac{2}{5} \times \frac{2}{5}$

$= \frac{4}{25}$

P(Getting two black balls) $= \frac{4}{25}$

(iii)

P(Getting first red and second black ball)

$= P(A)P(B)$

$= \frac{3}{5} \times \frac{2}{5}$

$= \frac{6}{25}$

P(Getting first red and second black ball) $= \frac{6}{25}$

Probability Ex 31.4 Q15

Three cards are drawn with replacement consider,

A = drawing a king
B = drawing a queen
C = drawing a jack

$\Rightarrow \quad P(A) = \frac{4}{52}$ [Since there are 4 kings]

$P(A) = \frac{1}{13}$

$\Rightarrow \quad P(B) = \frac{4}{52}$ [Since there are 4 queens]

$P(B) = \frac{1}{13}$

$\Rightarrow \quad P(C) = \frac{4}{52}$ [Since there are 4 jacks]

$P(C) = \frac{1}{13}$

P (Cards drawn are king, queen and jack)

$= P(A \cap B \cap C) + P(A \cap C \cap B) + P(B \cap A \cap C)$

$P(B \cap C \cap A) + P(C \cap A \cap B) + P(C \cap B \cap A)$

[Since order of drawing them may be different]

$= P(A)P(B)P(C) + P(A)P(C)P(B) + P(B)P(A)P(C)$

$+P(B)P(C)P(A) + P(C)P(A)P(B) + P(C)P(B)P(A)$

$= \frac{1}{13}.\frac{1}{13}.\frac{1}{13} + \frac{1}{13}.\frac{1}{13}.\frac{1}{13} + \frac{1}{13}.\frac{1}{13}.\frac{1}{13} + \frac{1}{13}.\frac{1}{13}.\frac{1}{13} + \frac{1}{13}.\frac{1}{13}.\frac{1}{13}$

$= \left(\frac{1}{13}.\frac{1}{13}.\frac{1}{13}\right) \times 6$

$= \frac{6}{2197}$

Required probability $= \frac{6}{2197}$

Probability Ex 31.4 Q16

Given,

Prat X has 9 out of 100 defective

$\Rightarrow$ Part X has 91 out of 100 non defective

Part Y has 5 out of 100 defective

$\Rightarrow$ Part Y has 95 out of 100 non defective

Consider,

X = A non defective part X
Y = A non defective part Y

$\Rightarrow \quad P(X) = \frac{91}{100}$ abd $P(Y) = \frac{95}{100}$

$= P$ (Assembled product will not be defective)

$= P$ (Niether X defective nor Y defective)

$= P(X \cap Y)$

$= P(X)P(Y)$

$= \frac{91}{100} \times \frac{95}{100}$

$= 0.8645$

Required probability $= 0.8645$

Probability Ex 31.4 Q17

Given,

Probability that A hits a target $= \frac{1}{3}$

$\Rightarrow \quad P(A) = \frac{1}{3}$

Probability that B hits the target $= \frac{2}{5}$

$\Rightarrow \quad P(B) = \frac{2}{5}$

P (Target will be hit)
$= 1 - P$ (target will not be hit)
$= 1 - P$ (Niether A non B hits the target)
$= 1 - P\left(\overline{A} \cap \overline{B}\right)$
$= 1 - P\left(\overline{A}\right)P\left(\overline{B}\right)$
$= 1 - \left[1 - P(A)\right]\left[1 - P\left(\overline{B}\right)\right]$
$= 1 - \left[1 - \frac{1}{3}\right]\left[1 - \frac{2}{5}\right]$
$= 1 - \frac{2}{3}.\frac{3}{5}$
$= 1 - \frac{2}{5}$
$= \frac{3}{5}$

Required probability $= \frac{3}{5}$

Probability Ex 31.4 Q18

Given,

An anti aircraft gun can take a maximum 4 shots at an enemy plane

Consider,

A = Hitting the plane at first shot
B = Hetting the plane at second shot
C = Hetting the plane at third shot
D = Hetting the plane at fourth shot

$\Rightarrow \quad P(A) = 0.4,\ P(B) = 0.3,\ P(C) = 0.2,\ P(D) = 0.1$

P (Gun hits the plane)
$= 1 - P$ (Gun does not hit the plane)
$= 1 - P$ (Non of the four shots hit the plane)
$= 1 - P\left(\overline{A} \cap \overline{B} \cap \overline{C} \cap \overline{D}\right)$
$= 1 - P\left(\overline{A}\right)P\left(\overline{B}\right)P\left(\overline{C}\right)P\left(\overline{D}\right)$
$= 1 - \left[1 - P(A)\right]\left[1 - P\left(\overline{B}\right)\right]\left[1 - P(C)\right]\left[1 - P(D)\right]$
$= 1 - \left[1 - 0.4\right]\left[1 - 0.3\right]\left[1 - 0.2\right]\left[1 - 0.1\right]$
$= 1 - (0.6)(0.7)(0.8)(0.9)$
$= 1 - 0.3024$
$= 0.6976$

Required probability $= 0.6976$

Probability Ex 31.4 Q19

Given,

The odds against a certain event $(\text{say, } A)$ are 5 to 2

$$\Rightarrow \quad P\left(\overline{A}\right) = \frac{5}{5+2}$$

$$P\left(\overline{A}\right) = \frac{5}{7}$$

The odds in favour of another event $(\text{say, } B)$ are 6 to 5

$$\Rightarrow \quad P(B) = \frac{6}{5+6}$$

$$P(B) = \frac{6}{11}$$

$$P\left(\overline{B}\right) = 1 - \frac{6}{11}$$

$$P\left(\overline{B}\right) = \frac{5}{11}$$

(a)

P (At least one of the events will occur)

$= 1 - P$ (None of events occur)

$= 1 - P\left(\overline{A} \cap \overline{B}\right)$

$= 1 - P\left(\overline{A}\right) P\left(\overline{B}\right)$ [Since events are independent]

$= 1 - \frac{5}{7} \times \frac{5}{11}$

$= 1 - \frac{25}{77}$

$= \frac{52}{77}$

Required probability $= \frac{52}{77}$

(b)

P (None of the events will occur)

$= P\left(\overline{A} \cap \overline{B}\right)$

$= P\left(\overline{A}\right) P\left(\overline{B}\right)$

$= \frac{5}{7} \times \frac{5}{11}$

$= \frac{25}{11}$

Probability Ex 31.4 Q20

Given, A die is thrown thrice.

Consider,

A = Getting an odd number in a throw of die

$P(A) = \frac{3}{6}$ [Since there are 1,3,5 odd number on die]

$P(A) = \frac{1}{2} \quad \Rightarrow P(\overline{A}) = \frac{1}{2}$

P (Getting an odd number at least once)

$= 1 - P$ (Getting no odd number)

$= 1 - P(\overline{A} \cap \overline{A} \cap \overline{A})$

$= 1 - P(\overline{A})P(\overline{A})P(\overline{A})$

$= 1 - \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

$= 1 - \frac{1}{8}$

$= \frac{7}{8}$

Required probability $= \frac{7}{8}$

Probability Ex 31.4 Q21

The box contains 10 black balls and 8red balls.

Then $P(\text{black ball}) = \frac{10}{18}$

$P(\text{red ball}) = \frac{8}{18}$

(i) $P(\text{Both balls are red}) = \frac{8}{18} \times \frac{8}{18} = \frac{16}{81}$

(ii) P (First ball is black and second is red)

$= \frac{10}{18} \times \frac{8}{18} = \frac{20}{81}$

(iii) P (one of them is black and other is red)

$= \frac{10}{18} \cdot \frac{8}{18} + \frac{8}{18} \cdot \frac{10}{18}$

$= 2\left(\frac{20}{81}\right)$

$= \frac{40}{81}$

Probability Ex 31.4 Q22

Given, Urn contains 4 red and 7 black balls.
Two balls drawn at random with replacement.

Consider,

R = Getting one red ball from urn.

$P(R) = \frac{4}{11}$

B = Getting one blue ball from urn.

$P(B) = \frac{7}{11}$

(i)

P (Getting 2 red balls)

$= P(R).P(R)$

$= \frac{4}{11} \times \frac{4}{11}$

$= \frac{16}{121}$

Required probability $= \frac{16}{121}$

(ii)

P (Getting two blue balls)

$= P(B)P(B)$

$= \frac{7}{11} \times \frac{7}{11}$

$= \frac{49}{121}$

Required probability $= \frac{49}{121}$

(iii)

P (Getting one red and one blue ball)

$= P(R)P(B) + P(B)P(R)$

$= \frac{4}{11} \times \frac{7}{11} + \frac{7}{11} \times \frac{4}{11}$

$= \frac{28}{121} + \frac{28}{121}$

$= \frac{56}{121}$

Probability Ex 31.4 Q23

Given that the events 'A coming in time' and 'B coming in time' are independent.
Let 'A' denote the event of 'A coming in time'.
Then, '$\bar{A}$' denotes the complementary event of A.
Similarly we define B and $\bar{B}$.
P(only one coming in time) $= P(A \cap \bar{B}) + P(\bar{A} \cap B)$
$= P(A) \times P(\bar{B}) + P(\bar{A}) \times P(B)$...(since A and B are independent events)
$= \frac{3}{7} \times \frac{2}{7} + \frac{4}{7} \times \frac{5}{7} = \frac{6}{49} + \frac{20}{49} = \frac{26}{49}$

The advantage of coming to school in time is that you will not miss any part of the lecture and will be able to learn more.

Probability Ex 31.4 Q24

$$S = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6), \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6), \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6), \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

$n(S) = 36$

E be the event of geting a total of 4.

$E = \{(1,3),(3,1),(2,2)\}$

$n(E) = 3$

$P(E) = \frac{n(E)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

F be the event of geting a total of 9 or more.

$F = \{(3,6),(6,3),(4,5),(5,4),(4,6),(6,4),(5,5),(5,6),(6,5),(6,6)\}$

$n(F) = 10$

$P(F) = \frac{n(F)}{n(S)} = \frac{10}{36} = \frac{5}{18}$

G be the event of geting a total divisible by 5.

$G = \{(1,4),(4,1),(2,3),(3,2),(4,6),(6,4),(5,5)\}$

$n(G) = 7$

$P(G) = \frac{n(G)}{n(S)} = \frac{7}{36}$

No pair is independent.

Probability Ex 31.4 Q25

Events are said to be independent, if the occurrence or non-occurrence
of one does not affect the probability of the occurrence or non-occurrence of the other.

(i) $p_1p_2 = P(A)P(B)$

$\Rightarrow$ Both A and B occour.

(ii) $(1-p_1)p_2 = (1-P(A))P(B) = P(\overline{A})P(B)$

$\Rightarrow$ Event A does not occour, but event B occours.

(iii) $1-(1-p_1)(1-p_2) = [1-(1-P(A))(1-P(B))] = (1-P(\overline{A})P(\overline{B}))$

$\Rightarrow$ At least one of the events A or B occours.

(iv) $p_1 + p_2 = 2p_1p_2$

$\Rightarrow P(A)+P(B) = 2P(A)P(B)$

$\Rightarrow P(A)+P(B)-2P(A)P(B) = 0$

$\Rightarrow P(A)-P(A)P(B)+P(B)-P(A)P(B) = 0$

$\Rightarrow P(A)(1-P(B))+P(B)(1-P(A)) = 0$

$\Rightarrow P(A)P(\overline{B})+P(B)P(\overline{A}) = 0$

$\Rightarrow P(A)P(\overline{B}) = P(B)P(\overline{A})$

$\Rightarrow$ Exactly one of A and B occours.

Ex 31.5

Probability Ex 31.5 Q1

There are two bags.
One bag (1) Contain 6 black and 3 white balls
other bag (2) Contain 5 black and 4 white balls

One ball is drawn from each bag

$$P(\text{One black from bag 1}) = \frac{6}{9}$$
$$P(B_1) = \frac{2}{3}$$

$$P(\text{One black from bag 2}) = \frac{5}{9}$$
$$P(B_2) = \frac{5}{9}$$

$$P(\text{One white from bag 1}) = \frac{3}{9}$$
$$P(W_1) = \frac{1}{3}$$

$$P(\text{One white from bag 2}) = \frac{4}{9}$$
$$P(W_2) = \frac{4}{9}$$

$$P(\text{Two balls of same colour})$$
$$= P\left[(W_1 \cap W_2) \cup (B_1 \cap B_2)\right]$$
$$= P(W_1 \cap W_2) + P(B_1 \cap B_2)$$
$$= P(W_1)P(W_2) + P(B_1)P(B_2)$$
$$= \frac{1}{3} \times \frac{4}{9} + \frac{2}{3} \times \frac{5}{9}$$
$$= \frac{4}{27} + \frac{10}{27}$$
$$= \frac{14}{27}$$

Required probability $= \frac{14}{27}$

Probability Ex 31.5 Q2

There are two bags.

Bag (1) contain 3 red and 5 black balls

Bag (2) contain 6 red and 4 black balls

$$P\left(\text{One red ball from bag 1}\right) = \frac{3}{8}$$
$$P\left(R_1\right) = \frac{3}{8}$$

$$P\left(\text{One black ball from bag 1}\right) = \frac{5}{8}$$
$$P\left(B_1\right) = \frac{5}{8}$$

$$P\left(\text{One red ball from bag 1}\right) = \frac{6}{10}$$
$$P\left(R_2\right) = \frac{3}{5}$$

$$P\left(\text{One black ball from bag 2}\right) = \frac{4}{10}$$
$$P\left(B_2\right) = \frac{2}{5}$$

One ball is drawn from each bag.

$$\begin{aligned}
&P\left(\text{One ball is red and the other is black}\right)\\
&= P\left[\left(R_1 \cap B_2\right) \cup \left(B_1 \cap R_2\right)\right]\\
&= P\left(R_1 \cap B_2\right) + P\left(B_1 \cap R_2\right)\\
&= P\left(R_1\right)P\left(B_2\right) + P\left(B_1\right)P\left(R_2\right)\\
&= \frac{3}{8} \times \frac{2}{5} + \frac{5}{8} \times \frac{3}{5}\\
&= \frac{6}{40} + \frac{15}{40}\\
&= \frac{21}{40}
\end{aligned}$$

$$\text{Required probability } = \frac{21}{40}$$

Probability Ex 31.5 Q3

Given, box contains 10 black and 8 red balls.
Two balls are drawn with replacement.

(i)

$$P\left(\text{Both the balls are red}\right)$$
$$= P\left(R_1 \cap R_2\right)$$
$$= P\left(R_1\right) P\left(R_2\right)$$
$$= \frac{8}{18} \times \frac{8}{18}$$
$$= \frac{16}{81}$$

Required probability $= \frac{16}{81}$

(ii)

$$P\left(\text{first ball is black and second is red}\right)$$
$$= P\left(B \cap R\right)$$
$$= P\left(B\right) P\left(R\right)$$
$$= \frac{10}{18} \times \frac{8}{18}$$
$$= \frac{20}{81}$$

Required probability $= \frac{20}{81}$

(iii)

$$P\left(\text{one of them red and other black}\right)$$
$$= P\left(\left(B \cap R\right) \cup \left(R \cap B\right)\right)$$
$$= P\left(B \cap R\right) + P\left(R \cap B\right)$$
$$= P\left(B\right) P\left(R\right) + P\left(R\right) P\left(B\right)$$
$$= \frac{10}{18} \times \frac{8}{18} + \frac{8}{18} \times \frac{10}{18}$$
$$= \frac{20 + 20}{81}$$
$$= \frac{40}{81}$$

Required probability $= \frac{40}{81}$

Probability Ex 31.5 Q4

Two cards are drawn without replacement.

There are total 4 ace.

A = Getting Ace

$$\text{P}\left(\text{Exactly one ace out of 2 cards}\right)$$
$$= P\left(\left(A \cap \overline{A}\right) \cup \left(\overline{A} \cap A\right)\right)$$
$$= P\left(A\right) P\left(\frac{\overline{A}}{A}\right) + P\left(\overline{A}\right) P\left(\frac{A}{\overline{A}}\right)$$
$$= \frac{4}{52} \cdot \frac{48}{51} + \frac{48}{52} \cdot \frac{4}{51}$$
$$= \frac{96}{663}$$
$$= \frac{32}{221}$$

Required probability $= \frac{32}{221}$

Probability Ex 31.5 Q5

Given,

A speaks truth in 75% cases.
B speaks truth in 80% cases.

$P(A) = \frac{75}{100} \quad \Rightarrow P(\overline{A}) = \frac{25}{100}$

$P(B) = \frac{80}{100} \quad \Rightarrow P(\overline{B}) = \frac{20}{100}$

$P(A \text{ and } B \text{ contradict each other})$

$= P\left[\left(A \cap \overline{B}\right) \cup \left(\overline{A} \cap B\right)\right]$

$= P\left(A \cap \overline{B}\right) + P\left(\overline{A} \cap B\right)$

$= P(A) P\left(\overline{B}\right) + P\left(\overline{A}\right) P(B)$

$= \frac{75}{100} \cdot \frac{20}{100} + \frac{25}{100} \cdot \frac{80}{100}$

$= \frac{1500}{1000} + \frac{2000}{10000}$

$= \frac{3500}{10000}$

$= 35\%$

Required probability $= 35\%$

Probability Ex 31.5 Q6

Given,

Probability of selection of Kamal $(K) = \frac{1}{3}$

$P(K) = \frac{1}{3}$

Probability of selection of Monika $(M) = \frac{1}{5}$

$P(M) = \frac{1}{5}$

(i)

$P(\text{Both of them selected})$

$= P(K \cap M)$

$= P(K) P(M)$

$= \frac{1}{3} \cdot \frac{1}{5}$

$= \frac{1}{15}$

Required probability $= \frac{1}{15}$

(ii)

$P(\text{None of them will be selected})$

$= P\left(\overline{K} \cap \overline{M}\right)$

$= P\left(\overline{K}\right) P\left(\overline{M}\right)$

$= \left[1 - P(K)\right]\left[1 - P(M)\right]$

$= \left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)$

$= \frac{2}{3} \times \frac{4}{5}$

$= \frac{8}{15}$

Required probability $= \frac{8}{15}$

(iii)

P (At least one of them selected)

$= 1 - P$ (None of them selected)

$= 1 - P\left(\overline{M} \cap \overline{K}\right)$

$= 1 - P\left(\overline{M}\right).P\left(\overline{K}\right)$

$= 1 - \left[1 - P\left(M\right)\right]\left[1 - P\left(K\right)\right]$

$= 1 - \left(1 - \frac{1}{5}\right)\left(1 - \frac{1}{3}\right)$

$= 1 - \frac{4}{5}.\frac{2}{3}$

$= 1 - \frac{8}{15}$

$= \frac{7}{15}$

Required probability $= \frac{7}{15}$

(iv)

P (Only one of them will be selected)

$= P\left[\left(K \cap \overline{M}\right) \cup \left(\overline{K} \cap M\right)\right]$

$= P\left(K \cap \overline{M}\right) + P\left(\overline{K} \cap M\right)$

$= P\left(K\right)P\left(\overline{M}\right) + P\left(\overline{K}\right)P\left(M\right)$

$= \frac{1}{3}\left[1 - P\left(M\right)\right] + \left[1 - P\left(K\right)\right]\frac{1}{5}$

$= \frac{1}{3}\left[1 - \frac{1}{5}\right] + \left[1 - \frac{1}{3}\right].\frac{1}{5}$

$= \frac{1}{3}.\frac{4}{5} + \frac{2}{3}.\frac{1}{5}$

$= \frac{4}{15} + \frac{2}{15}$

$= \frac{6}{15}$

$= \frac{2}{5}$

Required probability $= \frac{2}{5}$

Probability Ex 31.5 Q7

Bag contain 3 white, 4 red, 5 black balls.
Two balls are drawn without replacement.

P (One ball is white and other black)

$= P\left[\left(W \cap B\right) \cup \left(B \cap W\right)\right]$

$= P\left(W \cap B\right) + P\left(B \cap W\right)$

$= P\left(W\right)P\left(\frac{B}{W}\right) + P\left(B\right)P\left(\frac{W}{B}\right)$

$= \frac{3}{12} \times \frac{5}{12} + \frac{5}{12} \times \frac{3}{11}$

$= \frac{15}{132} + \frac{15}{132}$

$= \frac{30}{132}$

$= \frac{5}{22}$

Required probability $= \frac{5}{22}$

Probability Ex 31.5 Q8

A bag contains 8 red and 6 green balls.
Three balls are drawn without replacement

$$P\left(\text{at least 2 balls are green}\right)$$
$$= P\left[\left(G_1 \cap G_2 \cap R_1\right) \cup \left(G_1 \cap R_1 \cap G_2\right) \cup \left(R_1 \cap G_1 \cap G_2\right) \cup \left(G_1 \cap G_2 \cap G_3\right)\right]$$
$$= P\left(G_1 \cap G_2 \cap R_1\right) + P\left(G_1 \cap R_1 \cap G_2\right) + P\left(R_1 \cap G_1 \cap G_2\right) + P\left(G_1 \cap G_2 \cap G_3\right)$$
$$= P\left(G_1\right) P\left(\frac{G_2}{G_1}\right) P\left(\frac{R_1}{G_1 \cap G_2}\right) + P\left(G_1\right) P\left(\frac{R_1}{G_1}\right) P\left(\frac{G_2}{R_1 \cap G_1}\right) +$$
$$P\left(R_1\right) P\left(\frac{G_1}{R_1}\right) P\left(\frac{G_2}{G_1 \cap R_1}\right) + P\left(G_1\right) P\left(\frac{G_2}{G_1}\right) P\left(\frac{G_3}{G_1 \cap G_2}\right)$$
$$= \frac{6}{14} \times \frac{5}{13} \times \frac{8}{12} + \frac{6}{14} \times \frac{8}{13} \times \frac{5}{12} + \frac{8}{14} \times \frac{6}{13} \times \frac{5}{12} + \frac{6}{14} \times \frac{5}{13} \times \frac{4}{12}$$
$$= \frac{1}{14} \times \frac{1}{13} \times \frac{1}{12} \times \left(240 + 240 + 240 + 120\right)$$
$$= \frac{840}{14 \times 13 \times 12}$$
$$= \frac{5}{13}$$

Required probability $= \frac{5}{13}$

Probability Ex 31.5 Q9

Given, Probability of Arun's $\left(\text{A}\right)$ selection $= \frac{1}{4}$

$$P\left(A\right) = \frac{1}{4}$$

Probability of Tarun's$\left(\text{T}\right)$ rejection $= \frac{2}{3}$

$$P\left(\overline{T}\right) = \frac{2}{3}$$

$$P\left(\overline{A}\right) = 1 - P\left(A\right)$$
$$\Rightarrow \quad P\left(\overline{A}\right) = 1 - \frac{1}{4}$$
$$\Rightarrow \quad P\left(\overline{A}\right) = \frac{3}{4}$$

$$P\left(T\right) = 1 - P\left(\overline{T}\right)$$
$$\Rightarrow \quad P\left(T\right) = 1 - \frac{2}{3}$$
$$\Rightarrow \quad P\left(T\right) = \frac{1}{3}$$

$$P\left(\text{At least one of them will be selected}\right)$$
$$= 1 - P\left(\text{None of them selected}\right)$$
$$= 1 - P\left(\overline{A} \cap \overline{T}\right)$$
$$= 1 - P\left(\overline{A}\right) P\left(\overline{T}\right)$$
$$= 1 - \frac{2}{3} \times \frac{3}{4}$$
$$= \frac{1}{2}$$

Required probability $= \frac{1}{2}$

Probability Ex 31.5 Q10

Let E be event of occuring head in a toss of fair coin.

$$P(E) = \frac{1}{2}$$

$$P(\bar{E}) = \frac{1}{2}$$

A wins the game in first or 3rd or 5th throw, ...

Probability that A wins in first throw

$$= P(E) = \frac{1}{2}$$

Probability that A wins in 3rd throw

$$= P(\bar{E})P(\bar{E})P(E)$$

$$= \left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)$$

$$= \left(\frac{1}{2}\right)^3$$

Probability that A wins in 5th throw

$$= P(\bar{E})P(\bar{E})P(\bar{E})P(\bar{E})P(E)$$

$$= \left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)$$

$$= \left(\frac{1}{2}\right)^5$$

Hence,

Probability of winning A

$$= \frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \ldots$$

$$= \frac{1}{2}\left[1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^4 + \ldots\right]$$

$$= \frac{1}{2}\left[\frac{1}{1 - \left(\frac{1}{2}\right)^2}\right] \qquad \left[\text{Since } S_\infty = \frac{a}{1-r} \text{ for G.P.}\right]$$

$$= \frac{1}{2}\left[\frac{1}{1 - \frac{1}{4}}\right]$$

$$= \frac{1}{2} \times \frac{4}{3}$$

$$= \frac{2}{3}$$

Probability that B wins $= 1 - P(A \text{ wins})$

$$= 1 - \frac{2}{3}$$

$$= \frac{1}{3}$$

Required probability $= \frac{1}{3}$

Probability Ex 31.5 Q11

Two cards are drawn without replacement from a pack of 52 cards.
There are 26 black and 26 red cards

$$P\text{ (one red and other black card)}$$
$$= P\left[(R \cap B) \cup (B \cap R)\right]$$
$$= P(R \cap B) + P(B \cap R)$$
$$= P(R)\,P\left(\frac{B}{R}\right) + P(B)\,P\left(\frac{R}{B}\right)$$
$$= \frac{26}{52} \times \frac{26}{51} + \frac{26}{52} \times \frac{26}{51}$$
$$= \frac{13}{51} + \frac{13}{51}$$
$$= \frac{26}{51}$$

Required probability $= \frac{26}{51}$

Probability Ex 31.5 Q12

Tickets are numbered from 1 to 10.
Two tickets are drawn.

Consider, A = Multiple of 5
B = Multiple of 4

$$P(A) = \frac{2}{10} \qquad \text{[Since 5, 10 are multiple of 5]}$$
$$P(A) = \frac{1}{5}$$
$$P(B) = \frac{2}{10}$$
$$P(B) = \frac{1}{5} \qquad \text{[Since 4, 8 are multiple of 4]}$$

$$P\text{ (One number multiple of 5 and other multiple of 4)}$$
$$= P\left[(A \cap B) \cup (B \cap A)\right]$$
$$= P(A \cap B) + P(B \cap A)$$
$$= P(A)\,P\left(\frac{B}{A}\right) + P(B)\,P\left(\frac{A}{B}\right)$$
$$= \frac{1}{5} \times \frac{2}{9} + \frac{1}{5} \times \frac{2}{9}$$
$$= \frac{4}{45}$$

Required probability $= \frac{4}{45}$

Probability Ex 31.5 Q13

Given, In a family Husband (H) tells a lie in 30% cases and Wife (W) tells a lie in 35%

$P(H) = 30\%$, $P(\overline{H}) = 70\%$

$P(W) = 35\%$, $P(\overline{W}) = 65\%$

$$
\begin{aligned}
&P(\text{Both contradict each other}) \\
&= P\left[\left(H \cap \overline{W}\right) \cup \left(\overline{H} \cap W\right)\right] \\
&= P\left(H \cap \overline{W}\right) + P\left(\overline{H} \cap W\right) \\
&= P(H)P\left(\overline{W}\right) + P\left(\overline{H}\right)P(W) \\
&= \frac{30}{100} \times \frac{65}{100} + \frac{70}{100} \times \frac{35}{100} \\
&= \frac{1950 + 2450}{10000} \\
&= \frac{4400}{10000} \\
&= 0.44
\end{aligned}
$$

Required probability = 0.44

Probability Ex 31.5 Q14

Given, Probability of Husband's (H) selection = $\frac{1}{7}$

$$P(H) = \frac{1}{7}$$

Probability of Wife's (W) selection = $\frac{1}{5}$

$$P(W) = \frac{1}{5}$$

(a)

$$
\begin{aligned}
&\text{P(Both of them will be selected)} \\
&= (H \cap W) \\
&= P(H)P(W) \\
&= \frac{1}{7} \times \frac{1}{5} \\
&= \frac{1}{35}
\end{aligned}
$$

Required probability = $\frac{1}{35}$

(b)

$$
\begin{aligned}
&\text{P(Only one of them will be selected)} \\
&= P\left[\left(H \cap \overline{W}\right) \cup \left(\overline{H} \cap W\right)\right] \\
&= P\left(H \cap \overline{W}\right) + P\left(\overline{H} \cap W\right) \\
&= P(H)P\left(\overline{W}\right) + P\left(\overline{H}\right)P(W) \\
&= P(H)\left[1 - P(W)\right] + \left[1 - P(H)\right]P(W) \\
&= \frac{1}{7}\left[1 - \frac{1}{5}\right] + \left[1 - \frac{1}{7}\right]\frac{1}{5} \\
&= \frac{1}{7} \times \frac{4}{5} + \frac{6}{7} \times \frac{1}{5} \\
&= \frac{10}{35} \\
&= \frac{2}{7}
\end{aligned}
$$

Required probability = $\frac{2}{7}$

(c)

$$P\text{ (None of them selected)}$$
$$= \left(\overline{H} \cap \overline{W}\right)$$
$$= P\left(\overline{H}\right) P\left(\overline{W}\right)$$
$$= \left(1 - P\left(H\right)\right)\left(1 - P\left(W\right)\right)$$
$$= \left(1 - \frac{1}{7}\right)\left(1 - \frac{1}{5}\right)$$
$$= \frac{6}{7} \times \frac{4}{5}$$
$$= \frac{24}{35}$$

Required probability = $\frac{24}{35}$

Probability Ex 31.5 Q15

A bag contains 7 white, 5 black and 4 red balls.
Four balls are drawn without replacement

$$P\text{ (At least three balls are black)}$$
$$= P\text{ (3 black balls and one not black or 4 black balls)}$$
$$= P\text{ (3 black and one not black)} + P\text{ (4 black balls)}$$
$$= \frac{{}^{5}C_{3} \times {}^{11}C_{1}}{{}^{16}C_{4}} + \frac{{}^{5}C_{4}}{{}^{16}C_{4}}$$
$$= \frac{\frac{5!}{3!2!} \times 11 + \frac{5!}{4!1!}}{\frac{16!}{4!12!}} \qquad \left[\text{Since } {}^{n}C_{r} = \frac{n!}{r!\left(n-r\right)!}\right]$$
$$= \frac{\frac{5.4}{2} \times 11 + 5}{\frac{16.15.14.13}{4.3.2}}$$
$$= \frac{\left(110 + 5\right)}{1820}$$
$$= \frac{115}{1820}$$
$$= \frac{23}{364}$$

Required probability = $\frac{23}{364}$

Probability Ex 31.5 Q16

Given,

A speaks truth 3 out of four times
B speaks truth 4 out of five times
C speads truth 5 out of six times.

$$\Rightarrow \quad P(A) = \frac{3}{4},\ P(B) = \frac{4}{5},\ P(C) = \frac{5}{6}$$

$$P\left(\text{Reported truth fully by majority of witnesses}\right)$$
$$= P\left(\left(A \cap B \cap \overline{C}\right) \cup \left(A \cap \overline{B} \cap C\right) \cup \left(\overline{A} \cap B \cap C\right) \cup \left(A \cap B \cap C\right)\right)$$
$$= P\left(A \cap B \cap \overline{C}\right) + P\left(A \cap \overline{B} \cap C\right) + P\left(\overline{A} \cap B \cap C\right) + P\left(A \cap B \cap C\right)$$
$$= P(A)P(B)P\left(\overline{C}\right) + P(A)P\left(\overline{B}\right)P(C) + P\left(\overline{A}\right)P(B)P(C) + P(A)P(B)P(C)$$
$$= P(A)P(B)\left(1 - P(C)\right) + P(A)\left(1 - P(B)\right)P(C) + \left(1 - P(A)\right)P(B)P(C) + P(A)P(B)P(C)$$
$$= \frac{3}{4} \times \frac{4}{5}\left(1 - \frac{5}{6}\right) + \frac{3}{4}\left(1 - \frac{4}{5}\right)\frac{5}{6} + \left(1 - \frac{3}{4}\right)\frac{4}{5}.\frac{5}{6} + \frac{3}{4}.\frac{4}{5}.\frac{5}{6}$$
$$= \frac{3}{4}.\frac{4}{5}.\frac{1}{6} + \frac{3}{4}.\frac{1}{5}.\frac{5}{6} + \frac{1}{4}.\frac{4}{5}.\frac{5}{6} + \frac{3}{4}.\frac{4}{5}.\frac{5}{6}$$
$$= \frac{1}{10} + \frac{1}{8} + \frac{1}{6} + \frac{1}{2}$$
$$= \frac{12 + 15 + 20 + 60}{120}$$
$$= \frac{107}{120}$$

Required probability $= \frac{107}{120}$

Probability Ex 31.5 Q17

Bag A has 4 white balls and 2 black balls;
Bag B has 3 white balls and 5 black balls.

(i) $P\left(A_W \text{ and } B_W\right) = P\left(A_W\right)P\left(B_W\right) = \frac{4}{6}.\frac{3}{8} = \frac{1}{4}$

(ii) $P\left(A_B \text{ and } B_B\right) = P\left(A_B\right)P\left(B_B\right) = \frac{2}{6}.\frac{5}{8} = \frac{5}{24}$

(iii) $P\left(A_W \text{ and } B_B \text{ or } A_B \text{ and } B_W\right) = P\left(A_W\right)P\left(B_B\right) + P\left(A_B\right)P\left(B_W\right)$
$$= \frac{4}{6}.\frac{5}{8} + \frac{2}{6}.\frac{3}{8}$$
$$= \frac{20}{48} + \frac{6}{48}$$
$$= \frac{26}{48} = \frac{13}{24}$$

Probability Ex 31.5 Q18

Number of white balls = 4
Number of black balls = 7
Number of red balls = 5
Total balls = 16

Number of ways in which 4 balls can be drawn from 16 balls = ${}^{16}C_4$
Let A = getting at least two white ball = getting 2, 3, 4 white balls

Number of ways of choosing 2 white balls = ${}^{4}C_2 \times {}^{12}C_2$

Number of ways of choosing 3 white balls = ${}^{4}C_3 \times {}^{12}C_1$

Number of ways of choosing 4 white balls = ${}^{4}C_4 \times {}^{12}C_0$

$$\therefore \quad P(A) = \frac{{}^{4}C_2 \times {}^{12}C_2 + {}^{4}C_3 \times {}^{12}C_1 + {}^{4}C_4 \times {}^{12}C_0}{{}^{16}C_4} = \frac{67}{256}$$

Probability Ex 31.5 Q19

Three cards are drawn with replacement from a pack of cards.
There are 4 Kings, 4 Queens, 5 Jacks.

P (1 King, 1 Queen, 1 Jack)

$= P\left((K \cap Q \cap J) \cup (K \cap J \cap Q) \cup (J \cap K \cap Q) \cup (J \cap Q \cap K) \cup (Q \cap K \cap L) \cup (Q \cap J \cap K)\right)$
$= P(K \cap Q \cap J) + P(K \cap J \cap Q) + P(J \cap K \cap Q) + P(J \cap Q \cap K) + P(Q \cap K \cap J) + P(Q \cap J \cap K)$
$= P(K)P(Q)P(J) + P(K)P(J)P(Q) + P(J)P(K)P(Q) + P(J)P(Q)P(K) + P(Q)P(K)P(J)$
$+ P(Q)P(J)P(K)$
$= \frac{4}{52}.\frac{4}{52}.\frac{4}{52} + \frac{4}{52}.\frac{4}{52}.\frac{4}{52} + \frac{4}{52}.\frac{4}{52}.\frac{4}{52} + \frac{4}{52}.\frac{4}{52}.\frac{4}{52} + \frac{4}{52}.\frac{4}{52}.\frac{4}{52} + \frac{4}{52}.\frac{4}{52}.\frac{4}{52}$
$= \frac{6}{13.13.13}$
$= \frac{6}{2197}$

Required probability $= \frac{6}{2197}$

Probability Ex 31.5 Q20

Given, Bag(1) contains 4 red and 5 black balls.
Bag (2) contains 3 red and 7 black balls

One ball is drawn at random from each bag.

(i)

P (Balls are of different colours)
$= P\left((R_1 \cap B_2) \cup (B_1 \cap R_2)\right)$
$= P(R_1 \cap B_2) + P(B_1 \cap R_2)$
$= P(R_1)P(B_2) + P(B_1)P(R_2)$
$= \frac{4}{9}.\frac{7}{10} + \frac{5}{9}.\frac{9}{10}$
$= \frac{28}{90} + \frac{15}{90}$

$= \frac{43}{93}$

(ii)

P (Balls are of the same colour)
$= P\left((B_1 \cap B_2) \cup (R_1 \cap R_2)\right)$
$= P(B_1 \cap B_2) + P(R_1 \cap R_2)$
$= P(B_1)P(B_2) + P(R_1)P(R_2)$
$= \frac{5}{9}.\frac{7}{10} + \frac{4}{9}.\frac{3}{10}$
$= \frac{35}{90} + \frac{12}{90}$
$= \frac{47}{90}$

Required probability $= \frac{47}{90}$

Probability Ex 31.5 Q21

Let A be the event that "A hits the target",
B be the event that "B hits the target" and
C be the event that "C hits the target".
Then A, B and C are independent events such that
$P(A) = \frac{3}{6} = \frac{1}{2}$; $P(B) = \frac{2}{6} = \frac{1}{3}$; $P(C) = \frac{4}{4} = 1$

The target is hit by at least 2 shots in the following mutually exclusive ways :
(i) A hits, B hits and C does not hit, i.e., $A \cap B \cap C^-$
(ii) A hits, B does not hit and C hits, i.e., $A \cap B^- \cap C$
(iii) A does not hit, B hits and C hits, i.e., $A^- \cap B \cap C$
(iv) A hits, B hits and C hits, i.e., $A \cap B \cap C$
Hence, by the addition theorem for mutually exclusive events, the probability that at least 2 shots hit.
$= P(i) + P(ii) + P(iii) + P(iv)$
$= P(A \cap B \cap C^-) + P(A \cap B^- \cap C) + P(A^- \cap B \cap C) +$
$P(A \cap B \cap C)$
$= P(A)\,P(B)\,P(C^-) + P(A)\,P(B^-)\,P(C) + P(A^-)\,P(B)\,P(C) +$
$P(A)\,P(B)\,P(C)$
$= P(A)\,P(B)\,[1 - P(C)] + P(A)\,[1 - P(B)]\,P(C) +$
$[1 - P(A)]\,P(B)\,P(C) + P(A)\,P(B)\,P(C)$
$= \frac{1}{2} \times \frac{1}{3} \times (1-1) + \frac{1}{2} \times \left(1 - \frac{1}{3}\right) \times 1 + \left(1 - \frac{1}{2}\right) \times \frac{1}{3} \times 1 + \frac{1}{2} \times \frac{1}{3} \times 1$
$= \frac{1}{2} \times \frac{1}{3} \times 0 + \frac{1}{2} \times \left(\frac{2}{3}\right) \times 1 + \left(\frac{1}{2}\right) \times \frac{1}{3} \times 1 + \frac{1}{2} \times \frac{1}{3} \times 1$
$= 0 + \frac{2}{6} + \frac{1}{6} + \frac{1}{6} = \frac{4}{6}$
$= \frac{2}{3}$

Probability Ex 31.5 Q22

Given,

The probability of A passing exam $= \frac{2}{9}$

The probability of B passing exam $= \frac{5}{9}$

And they are independent.

$\Rightarrow \quad P(A) = \frac{2}{9},\ P(B) = \frac{5}{9}$

(i)

P (Only A passing the exam)
$= P\left(A \cap \bar{B}\right)$
$= P(A).P(B)$
$= P(A).\left(1 - P(B)\right)$
$= \frac{2}{9}\left(1 - \frac{5}{9}\right)$
$= \frac{2}{9}\left(\frac{4}{9}\right)$

$= \frac{8}{81}$

(ii)

P (Only one of them passing exam)
$= P\left(\left(A \cap \bar{B}\right) \cup \left(\bar{A} \cap B\right)\right)$
$= P\left(A \cap \bar{B}\right) + P\left(\bar{A} \cap B\right)$
$= P(A)\,P\left(\bar{B}\right) + P\left(\bar{A}\right)P(B)$
$= P(A)\left(1 - P(B)\right) + \left(1 - P(A)\right)P(B)$
$= \frac{2}{9}\left(1 - \frac{5}{9}\right) + \left(1 - \frac{2}{9}\right)\frac{5}{9}$
$= \frac{2}{9}.\frac{4}{9} + \frac{7}{9}.\frac{5}{9}$
$= \frac{8}{81} + \frac{35}{81}$
$= \frac{43}{81}$

Required probability $= \frac{43}{81}$

Probability Ex 31.5 Q23

Given,

Urn A contains 4 red (R_1) and 3 black (B_1) balls

Urn B contains 5 red (R_2) and 4 black (B_2) balls

Urn C contains 4 red (R_3) and 4 black (B_3) balls.

P (3 balls drawn consists of 2 red and a black ball)

$= P\left[(R_1 \cap R_2 \cap R_3) \cup (R_1 \cap B_2 \cap R_3) \cup (B_1 \cap R_2 \cap R_3)\right]$

$= P(R_1 \cap R_2 \cap R_3) + P(R_1 \cap B_2 \cap R_3) + P(B_1 \cap R_2 \cap R_3)$

$= P(R_1) + P(R_2) + P(R_3) + P(R_1) + P(B_2) + P(R_3) + P(B_1) + P(R_2) + P(R_3)$

$= \frac{4}{7}.\frac{5}{9}.\frac{4}{8} + \frac{4}{7}.\frac{4}{9}.\frac{4}{8} + \frac{3}{7}.\frac{5}{9}.\frac{4}{8}$

$= \frac{80+64+60}{504}$

$= \frac{204}{504}$

$= \frac{17}{42}$

Required probability $= \frac{17}{42}$

Probability Ex 31.5 Q24

Given,

Probability of getting A grade in mathematics (m) = 0.2

$\Rightarrow \quad P(m) = 0.2$

Probability of getting A grade in physics (p) = 0.3

$\Rightarrow \quad P(p) = 0.3$

Probability of getting A grade in chemistry (p) = 0.5

$\Rightarrow \quad P(c) = 0.5$

(i)

P (Getting A grade in all subjects)

$= P(m \cap p \cap c)$

$= P(m) + P(p) + P(c)$

$= 0.2 \times 0.3 \times 0.5$

$= 0.03$

Required probability $= 0.03$

(ii)

P (Getting A in no subject)

$= P(\overline{m} \cap \overline{p} \cap \overline{c})$

$= P(\overline{m}) + P(\overline{p}) + P(\overline{c})$

$= (1 - P(m))(1 - P(p))(1 - P(c))$

$= (1 - 0.2)(1 - 0.3)(1 - 0.5)$

$= (0.8)(0.7)(0.5)$

$= 0.28$

Required probability $= 0.28$

(iii)

P (Getting A grade in two subjects)

$= P\left((m \cap p \cap \overline{c}) \cup (m \cap \overline{p} \cap c) \cup (\overline{m} \cap p \cap c)\right)$

$= P(m)P(p)P(\overline{c}) + P(m)P(\overline{p})P(c) + P(\overline{m})P(p)P(c)$

$= P(m)P(p)(1 - P(c)) + P(m)(1 - P(p))P(c) + (1 - P(m))P(p)P(c)$

$= (0.2)(0.3)(1 - 0.5) + (0.2)(1 - 0.3)(0.5) + (1 - 0.2)(0.3)(0.5)$

Probability Ex 31.5 Q25

Sum of 9 can be obtained by

$$E = \{(3,6),(4,5),(5,4),(6,3)\}$$

Probability of throwing 9 = $\frac{4}{36}$

$$P(E) = \frac{1}{9},\ P(\overline{E}) = \frac{8}{9}$$

$\Rightarrow \quad P(A) = P(B) = \frac{1}{9}$

$\Rightarrow \quad P(\overline{A}) = P(\overline{B}) = \frac{8}{9}$

A and B take turns in throwing two dice.
Let A starts the game.

$P(A \text{ wins the game})$

$= P\left(A \cup \overline{A} \cap \overline{B} \cap A \cup \overline{A} \cap \overline{B} \cap \overline{A} \cap \overline{B} \cap A \cup \ldots\right)$

$= P(A) + P\left(\overline{A} \cap \overline{B} \cap A\right) + P\left(\overline{A} \cap \overline{B} \cap \overline{A} \cap \overline{B} \cap A\right) + \ldots$

$= P(A) + P(\overline{A})P(\overline{B})P(A) + P(\overline{A})P(\overline{B})P(\overline{A})P(\overline{B})P(A) + \ldots$

$= \frac{1}{9} + \frac{8}{9}\cdot\frac{8}{9}\cdot\frac{1}{9} + \frac{8}{9}\cdot\frac{8}{9}\cdot\frac{8}{9}\cdot\frac{8}{9}\cdot\frac{1}{9} + \ldots$

$= \frac{1}{9}\left[1 + \left(\frac{8}{9}\right)^2 + \left(\frac{8}{9}\right)^4 + \ldots\right]$

$= \frac{1}{9}\left[\frac{1}{1-\left(\frac{8}{9}\right)^2}\right]$ $\left[\begin{array}{l}\text{Since for a G.P. with first term 9 and common ratio r,}\\ S_\infty = \frac{a}{1-r}\end{array}\right]$

$= \frac{1}{9}\left[\frac{1}{1-\frac{64}{81}}\right]$

$= \frac{1}{9}\left[\frac{81}{81-64}\right]$

$= \frac{9}{17}$

$P(B \text{ wins the game}) = 1 - P(A \text{ wins the game})$

$= 1 - \frac{9}{17}$

$= \frac{8}{17}$

Chances of winning of $A : B$

$= \frac{9}{17} : \frac{8}{17}$

$= 9 : 8$

Chances of winning $A : B = 9 : 8$

Probability Ex 31.5 Q26

Let E be event of getting a head.

$$P(E)=\frac{1}{2} \qquad \Rightarrow P(\bar{E})=\frac{1}{2}$$

If A starts the game,

$\Rightarrow$ A wins the game in 1^{st}, 4^{th}, 7^{th},... toss of coin.

$P(A \text{ wins})$

$=P\left(E \cup \bar{E} \cap \bar{E} \cap \bar{E} \cap E \cup \bar{E} \cap \bar{E} \cap \bar{E} \cap \bar{E} \cap \bar{E} \cap \bar{E} \cap E \cup \ldots\right)$

$=P(\bar{E})+P\left(\bar{E} \cap \bar{E} \cap \bar{E} \cap E\right)+P\left(\bar{E} \cap \bar{E} \cap \bar{E} \cap \bar{E} \cap \bar{E} \cap \bar{E} \cap E\right)+\ldots$

$=P(E)+P(\bar{E}) P(\bar{E}) P(\bar{E}) P(E)+P(\bar{E}) P(\bar{E}) P(\bar{E}) P(\bar{E}) P(\bar{E}) P(\bar{E}) P(E)+\ldots$

$=\frac{1}{2}+\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}+\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}+\ldots$

$=\frac{1}{2}+\left(\frac{1}{2}\right)^{2}+\left(\frac{1}{2}\right)^{7}+\ldots$

$=\frac{1}{2}\left[1+\left(\frac{1}{2}\right)^{3}+\left(\frac{1}{2}\right)^{6}+\ldots\right]$

$=\frac{1}{2}\left[\frac{1}{1-\left(\frac{1}{2}\right)^{3}}\right]$ $\qquad \left[\text{Since } S_{\infty}=\frac{a}{1-r} \text{ for G.P.}\right]$

$=\frac{1}{2}\left[\frac{1}{1-\frac{1}{8}}\right]$

$=\frac{1}{2}\left[\frac{8}{7}\right]$

$=\frac{4}{7}$

B wins in 2^{nd}, 5^{th}, 8^{th},... toss of coin

$P(B \text{ wins})$

$=P\left(\bar{E} \cap E \cup \bar{E} \cap \bar{E} \cap \bar{E} \cap \bar{E} \cap E \cup \ldots\right)$

$=P(\bar{E} \cap E)+P\left(\bar{E} \cap \bar{E} \cap \bar{E} \cap \bar{E} \cap E \cup\right)+\ldots$

$=P(\bar{E}) P(E)+P(\bar{E}) P(\bar{E}) P(\bar{E}) P(\bar{E}) P(E)+\ldots$

$=\frac{1}{2} \cdot \frac{1}{2}+\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}+\ldots$

$=\left(\frac{1}{2}\right)^{2}+\left(\frac{1}{2}\right)^{5}+\ldots$

$=\left(\frac{1}{2}\right)^{2}\left[1+\left(\frac{1}{2}\right)^{3}+\ldots\right]$

$=\frac{1}{4}\left[\frac{1}{1-\left(\frac{1}{2}\right)^{3}}\right]$ $\qquad \left[\begin{array}{l}\text{Since for G.P.} \\ S_{\infty}=\frac{a}{1-r}\end{array}\right]$

$=\frac{1}{4}\left[\frac{1}{1-\frac{1}{8}}\right]$

$=\frac{1}{4}\left[\frac{8}{7}\right]$

$=\frac{2}{7}$

$P(C \text{ wins})=1-P(A \text{ wins})-P(B \text{ wins})$

$=1-\frac{4}{7}-\frac{2}{7}$

$=\frac{1}{7}$

Probabilities of winning A, B and C are $\frac{4}{7}, \frac{2}{7}$ and $\frac{1}{7}$ respectively.

Probability Ex 31.5 Q27

Let E be the even of getting a six

$$P(E)=\frac{1}{6}$$
$$P(\overline{E})=\frac{5}{6}$$

A wins if he gets a six in 1st or 4th or 7th... throw

A wins in first throw $=P(E)=\frac{1}{6}$

A wins in 4th throw if he fails in 1^{st}, B fails in 2^{nd}, C fails in 3^{rd} throw.

Probability of winning A in 4th throw

$$=P(\overline{E})P(\overline{E})P(\overline{E})P(E)$$
$$=\left(\frac{5}{6}\right)^3\cdot\frac{1}{6}$$

Similarly, Probability of winning A in 7th thrwo $=P(\overline{E})P(\overline{E})P(\overline{E})P(\overline{E})P(\overline{E})P(\overline{E})P(E)$

$$=\left(\frac{5}{6}\right)^6\cdot\frac{1}{6}$$

Hence, probability of winning of A

$$=\frac{1}{6}+\left(\frac{5}{6}\right)^3\cdot\frac{1}{6}+\left(\frac{5}{6}\right)^6\cdot\frac{1}{6}+\ldots$$
$$=\frac{1}{6}\left[1+\left(\frac{5}{6}\right)^3+\left(\frac{5}{6}\right)^6+\ldots\right]$$
$$=\frac{1}{6}\left[\frac{1}{1-\left(\frac{5}{6}\right)^3}\right] \qquad \left[\text{Using } S_\infty=\frac{a}{1-r} \text{ for G.P.}\right]$$
$$=\frac{1}{6}\left[\frac{1}{1-\frac{125}{216}}\right]$$
$$=\frac{1}{6}\times\frac{216}{91}$$
$$=\frac{36}{91}$$

B wins if he gets a six in 2nd or 5th or 8th ...throw.

B wins in 2nd throw $= P\left(\overline{E}\right)P\left(E\right)$

$$= \left(\frac{5}{6}\right)\left(\frac{1}{6}\right)$$

B wins in 5th throw if A fails in first, B fails in 2nd, C fails in 3rd, A fails in 4th.

Probability of winning B in 5th throw

$$= P\left(\overline{E}\right)P\left(\overline{E}\right)P\left(\overline{E}\right)P\left(\overline{E}\right)P\left(E\right)$$

$$= \left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right)$$

Probability of winning B in 8th thrwo

$$= \left(\frac{5}{6}\right)^7\left(\frac{1}{6}\right)$$

Hence, probability of winning B

$$= \left(\frac{5}{6}\right)\frac{1}{6} + \left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^7\left(\frac{1}{6}\right)$$

$$= \frac{5}{6}.\frac{1}{6}\left[1 + \left(\frac{5}{6}\right)^3 + \left(\frac{5}{6}\right)^6 + \ldots\right]$$

$$= \frac{5}{36}\left[\frac{1}{1-\left(\frac{5}{6}\right)^3}\right] \qquad \left[\text{Since } S_\infty = \frac{a}{1-r} \text{ for G.P.}\right]$$

$$= \frac{5}{36}\left[\frac{1}{1-\frac{125}{216}}\right]$$

$$= \frac{5}{36}\times\left[\frac{216}{91}\right]$$

$$= \frac{30}{91}$$

Probability of winning $C = 1 - P\left(A \text{ wins}\right) - P\left(B \text{ wins}\right)$

$$= 1 - \frac{36}{91} - \frac{30}{91}$$

$$= \frac{25}{91}$$

The respective probabilities of winning of A, B and C are $\frac{36}{91}$, $\frac{30}{91}$ and $\frac{25}{91}$.

Probability Ex 31.5 Q28

Let E be events of throwing 10 on a pair of dice,

$$E = \{(4,6),(5,5),(6,4)\}$$

$$P(E) = \frac{3}{37}$$

$$P(E) = \frac{1}{12}$$

$$P(\overline{E}) = \frac{11}{12}$$

A wins the game in first or 3rd or 5th throw, ...

Probability that A wins in first throw $= P(E) = \frac{1}{12}$

Probability that A wins in 3rd throw

$$= P(\overline{E})P(\overline{E})P(E)$$

$$= \left(\frac{11}{12}\right)^2\left(\frac{1}{12}\right)$$

Probability that A wins in 5th throw

$$= P(\overline{E})P(\overline{E})P(\overline{E})P(\overline{E})P(E)$$

$$= \left(\frac{11}{12}\right)^4\left(\frac{1}{12}\right)$$

Hence,

Probability of winning A

$$= \frac{1}{12} + \left(\frac{11}{12}\right)^2\left(\frac{1}{12}\right) + \left(\frac{11}{12}\right)^4\left(\frac{1}{12}\right)$$

$$= \frac{1}{12}\left[1 + \left(\frac{11}{12}\right)^2 + \left(\frac{11}{12}\right)^4 + \ldots\right]$$

$$= \frac{1}{12}\left[\frac{1}{1-\left(\frac{11}{12}\right)^2}\right] \qquad \left[\text{Since } S_\infty = \frac{a}{1-r} \text{ for G.P.}\right]$$

Probability Ex 31.5 Q29

Bag A has 3 red and 5 black balls

Bag B has 2 red and 3 black balls

One ball is drawn from bag A and two from bag B.

P(One red from bag A and 2 black from bag B Or one black from bag A and 1 red and one black from bag B)

$$= P(R_1 \cap (2B_2)) + P(B_1 \cap R_2 \cap B_2)$$

$$= P(R_1)P(2B_2) + P(B_1)P(R_2)P(B_2)$$

$$= \frac{3}{8}\cdot\frac{{}^3C_2}{{}^5C_2} + \frac{5}{8}\cdot\frac{2}{5}\cdot\frac{3}{4}$$

$$= \frac{3}{8}\cdot\frac{3}{\left(\frac{5.4}{2}\right)} + \frac{5}{8}\cdot\frac{2}{5}\cdot\frac{3}{4}$$

$$= \frac{18}{160} + \frac{30}{160}$$

$$= \frac{48}{160} = \frac{3}{10}$$

Required probability $= \frac{3}{10}$

Probability Ex 31.5 Q30

Given,

Probability of Fatima's (F) selection = $\frac{1}{7}$

$P(F) = \frac{1}{7} \qquad \Rightarrow P(\bar{F}) = \frac{6}{7}$

Probability of John's (J) selection = $\frac{1}{5}$

$P(J) = \frac{1}{5} \qquad \Rightarrow P(\bar{J}) = \frac{4}{5}$

(i)

$P(\text{Both of them selected})$

$= P(F \cap J)$

$= P(F).P(J)$

$= \frac{1}{7} \times \frac{1}{5}$

$= \frac{1}{35}$

Required probability = $\frac{1}{35}$

(ii)

$P(\text{only one of them selected})$

$= P\left((F \cap \bar{J}) \cup (\bar{F} \cap J)\right)$

$= P(F)P(\bar{J}) + P(\bar{F})P(J)$

$= \frac{1}{7} \times \frac{4}{5} + \frac{6}{7} \times \frac{1}{5}$

$= \frac{4+6}{35}$

$= \frac{10}{35}$

$= \frac{2}{7}$

Required probability = $\frac{2}{7}$

(iii)

$P(\text{None of them selected})$

$= P(\bar{F} \cap \bar{J})$

$= P(\bar{F})P(\bar{J})$

$= \frac{6}{7} \times \frac{4}{5}$

$= \frac{24}{35}$

Required probability = $\frac{24}{35}$

Probability Ex 31.5 Q31

Bag contains 3 blue, 5 red marble. One marble is drawn, its colour noted and replaced, then again a marble drawn and its colour is noted.

(i)

$P\left(\text{Blue followed by red}\right)$
$= P\left(B \cap R\right)$
$= P\left(B\right) P\left(R\right)$
$= \frac{3}{8} \times \frac{5}{8}$
$= \frac{15}{64}$

Required probability $= \frac{15}{64}$

(ii)

$P\left(\text{Blue and red in any order}\right)$
$= P\left(\left(B \cap R\right) \cup \left(R \cap B\right)\right)$
$= P\left(B \cap R\right) + P\left(R \cap B\right)$
$= P\left(B\right) P\left(R\right) + P\left(R\right) P\left(B\right)$
$= \frac{3}{8} \times \frac{5}{8} + \frac{5}{8} \times \frac{3}{8}$
$= \frac{30}{64}$
$= \frac{15}{32}$

Required probability $= \frac{15}{32}$

(iii)

$P\left(\text{of the same colour}\right)$
$= P\left(\left(R_1 \cap R_2\right) \cup \left(B_1 \cap B_2\right)\right)$
$= P\left(R_1\right) P\left(R_2\right) + P\left(B_1\right) P\left(B_2\right)$
$= \frac{5}{8} \times \frac{5}{8} + \frac{3}{8} \times \frac{3}{8}$
$= \frac{25+9}{64}$
$= \frac{34}{64}$
$= \frac{17}{32}$

Probability Ex 31.5 Q32

An urn contains 7 red and 4 blue balls.
Two balls are drawn with replacement.

(i)

$$P\left(\text{Getting 2 red balls}\right)$$
$$= P\left(R_1 \cap R_2\right)$$
$$= P\left(R_1\right).P\left(R_2\right)$$
$$= \frac{7}{11} \times \frac{7}{11}$$
$$= \frac{49}{121}$$

Required probability = $\frac{49}{121}$

(ii)

$$P\left(\text{Getting 2 blue balls}\right)$$
$$= P\left(B_1 \cap B_2\right)$$
$$= P\left(B_1\right).P\left(B_2\right)$$
$$= \frac{4}{11} \times \frac{4}{11}$$
$$= \frac{16}{121}$$

Required probability = $\frac{16}{121}$

(iii)

$$P\left(\text{Getting one red and one blue ball}\right)$$
$$= P\left(\left(R \cap B\right) \cup \left(B \cap R\right)\right)$$
$$= P\left(R\right)P\left(B\right) + P\left(B\right)P\left(R\right)$$
$$= \frac{7}{11} \times \frac{4}{11} + \frac{4}{11} \times \frac{7}{11}$$
$$= \frac{28+28}{121}$$
$$= \frac{56}{121}$$

Required probability = $\frac{56}{121}$

Probability Ex 31.5 Q33

A card is drawn, out come noted, the card is replaced, pack reshuffled, another card is drawn.

(i)

We know that, there are four suits club(C), spade(S), heart(H) diamand(D), each contains 13 cards.

$$P\,(\text{Both the cards are of same suit})$$
$$= P\left((C_1 \cap C_2) \cup (S_1 \cap S_2) \cup (H_1 \cap H_2) \cup (D_1 \cap D_2)\right)$$
$$= P\,(C_1 \cap C_2) + P\,(S_1 \cap S_2) + P\,(H_1 \cap H_2) + P\,(D_1 \cap D_2)$$
$$= P\,(C_1)\,P\,(C_2) + P\,(S_1)\,P\,(S_2) + P\,(H_1)\,P\,(H_2) + P\,(D_1)\,P\,(D_2)$$
$$= \frac{13}{52}\cdot\frac{13}{52} + \frac{13}{52}\cdot\frac{13}{52} + \frac{13}{52}\cdot\frac{13}{52} + \frac{13}{52}\cdot\frac{13}{52}$$
$$= \left(\frac{1}{4}\cdot\frac{1}{4}\right)^4$$
$$= \frac{1}{4}$$

Required probability $= \frac{1}{4}$

(ii)

We know that, there are four ace and 2 red queens.

$$P\,(\text{first card an ace and second card a red queen})$$
$$= P\,(\text{Getting an ace})\,P\,(\text{Getting a red queen})$$
$$= \frac{4}{52} \times \frac{2}{52}$$
$$= \frac{1}{338}$$

Required probability $= \frac{1}{338}$

Probability Ex 31.5 Q34

(i)

Out of 100 students two friends can enter the sections in ${}^{100}C_2$ ways.

Let A = event both enter in section A(40 students)

B = event both enter in section B(60 students)

$$P\,(A) = \frac{{}^{40}C_2}{{}^{100}C_2},\quad P\,(B) = \frac{{}^{60}C_2}{{}^{100}C_2}$$

$$P\,(A \cup B) = P\,(A) + P\,(B)$$
$$= \frac{{}^{40}C_2 + {}^{60}C_2}{{}^{100}C_2}$$
$$= \frac{\frac{40\times 39}{2} + \frac{60\times 59}{2}}{\frac{100\times 99}{2}}$$
$$= \frac{780 + 1770}{4950}$$
$$= \frac{2550}{4950}$$
$$= \frac{17}{33}$$

$$P\,(\text{Both enter same section}) = \frac{17}{33}$$

(ii)

$$P\,(\text{Both enter different section})$$
$$= 1 - P\,(\text{Both enter same section})$$
$$= 1 - \frac{17}{33}$$
$$= \frac{16}{33}$$

$$P\,(\text{Both enter different section}) = \frac{16}{32}$$

Probability Ex 31.5 Q35

Probability of getting six in any toss of a dice $= \frac{1}{6}$

Probability of not getting six in any toss of a dice $= \frac{5}{6}$

A and B toss the die alternatively.
Hence probability of A's win

$$= P(A) + P(\bar{A}\bar{B}A) + P(\bar{A}\bar{B}\bar{A}\bar{B}A) + P(\bar{A}\bar{B}\bar{A}\bar{B}\bar{A}\bar{B}A) + ...$$

$$= \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + ...$$

$$= \frac{1}{6} + \left(\frac{5}{6}\right)^2 \frac{1}{6} + \left(\frac{5}{6}\right)^4 \frac{1}{6} + \left(\frac{5}{6}\right)^6 \frac{1}{6} + ...$$

$$= \frac{1/6}{1-(5/6)^2} = \frac{1}{6} \times \frac{36}{11} = \frac{6}{11}$$

Similarly, probability of B's win

$$= P(\bar{A}B) + P(\bar{A}\bar{B}\bar{A}B) + P(\bar{A}\bar{B}\bar{A}\bar{B}\bar{A}B) + ...$$

$$= \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + ...$$

$$= \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^2 \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^4 \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^6 \frac{5}{6} \times \frac{1}{6} + ...$$

$$= \frac{\frac{5}{6} \times \frac{1}{6}}{1-\left(\frac{5}{6}\right)^2} = \frac{5}{36} \times \frac{36}{11} = \frac{5}{11}$$

Since the probabilities are not equal,
the decision of the refree was not a fair one.

Ex 31.6

Probability Ex 31.6 Q1

Given,

Bag A contains 5 white and 6 black balls

Bag B contains 4 white and 3 black balls.

There are two ways of transferring a ball from bag A to bag B

I- By transferring one white ball from bag A to bag B then drawing one black ball from bag B.

II- By transferring one black ball from bag A to bag B, then drawing one black from bag B.

Let, E_1, E_2 and A be events as below:-

E_1 = One white ball drawn from bag A

E_2 = One black ball drawn from bag B

A = One black ball drawn from bag B

$$P(E_1) = \frac{5}{11}$$

$$P(E_2) = \frac{6}{11}$$

$$P(A \mid E_1) = \frac{3}{8} \qquad [\text{Since, } E_1 \text{ has increased one white ball in bag } B]$$

$$P\left(\frac{A}{E_2}\right) = \frac{4}{8} \qquad [\text{Since, } E_2 \text{ has increased one black ball in bag } B]$$

By the law of total probability.

$$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P\left(\frac{A}{E_2}\right)$$

$$= \frac{5}{11} \times \frac{3}{8} + \frac{6}{11} \times \frac{4}{8}$$

$$= \frac{15}{88} + \frac{24}{88}$$

$$= \frac{39}{88}$$

Required probability = $\frac{39}{88}$.

Probability Ex 31.6 Q2

Purse (I) Contains 2 silver and 4 copper coins

Purse (II) Contains 4 silver and 3 copper coins

One coin is drawn from one of the two purse and it is silver

Let, E_1, E_2 and A are defined as

E_1 = Selecting purse I

E_2 = Selecting purse II

A = Drawing a silver coin

$$P(E_1) = \frac{1}{2}$$

$$P(E_2) = \frac{1}{2}$$ [Since, there are only 2 purses]

$$P(A \mid E_1) = P(\text{A} \mid \text{silver coin from purse I})$$
$$= \frac{2}{6}$$
$$= \frac{1}{3}$$

$$P\left(\frac{A}{E_2}\right) = P(\text{A} \mid \text{silver coin from purse II})$$
$$= \frac{4}{7}$$

By the law of total probability,

$$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P\left(\frac{A}{E_2}\right)$$
$$= \frac{1}{2} \times \frac{1}{3} + \frac{1}{2} \times \frac{4}{7}$$
$$= \frac{1}{6} + \frac{4}{14}$$
$$= \frac{7+12}{42}$$
$$= \frac{19}{42}$$

Required probability $= \frac{19}{42}$.

Probability Ex 31.6 Q3

Bag I contains 4 yellow and 5 red balls

Bag II contains 6 yellow and 3 red balls

Transfer can be done in two ways:-

I- A yellow ball is transferred from bag I to bag II and then one yellow ball is drawn from bag II.

II-A red ball is transferred from bag I to bag II and then one yellow ball is drawn from bag II.

Let E_1, E_2 and A be events as:

E_1 = One yellow ball drawn from bag I

E_2 = One red ball drawn from bag I

A = One yellow ball drawn from bag II.

$$P(E_1) = \frac{4}{9}$$

$$P(E_2) = \frac{5}{9}$$

$$P(A \mid E_1) = \frac{7}{10}$$ [Since E_1 has increased one yellow ball in bag II]

$$P\left(\frac{A}{E_2}\right) = \frac{6}{10}$$ [Since E_2 has increased one red ball in bag II]

By law of total probability,

$$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P\left(\frac{A}{E_2}\right)$$
$$= \frac{4}{9} \times \frac{7}{10} + \frac{5}{9} \times \frac{6}{10}$$
$$= \frac{28+30}{90}$$
$$= \frac{58}{90}$$
$$= \frac{29}{45}$$

Required probability $= \frac{29}{45}$.

Probability Ex 31.6 Q4

Bag I contains 3 white and 2 black balls

Bag II contains 2 white and 4 black balls

One bag is chosen at random, then one ball is drawn and it is white.

Let E_1, E_2 and A be events as:

E_1 = Selecting bag I

E_2 = Selecting bag II

A = Drawing one white ball

$P(E_1) = \frac{1}{2}$

$P(E_2) = \frac{1}{2}$ $\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ [Since there are only 2 bags]

$P(A \mid E_1) = P$[Drawing a white ball from bag I]

$= \frac{3}{5}$

$P\left(\frac{A}{E_2}\right) = P$[Drawing a white ball from bag II]

$= \frac{2}{6}$

By law of total probability,

$$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P\left(\frac{A}{E_2}\right)$$

$$= \frac{1}{2} \times \frac{3}{5} + \frac{1}{2} \times \frac{2}{6}$$

$$= \frac{3}{10} + \frac{2}{12}$$

$$= \frac{18+10}{60}$$

$$= \frac{28}{60}$$

$$= \frac{7}{15}$$

Required probability $= \frac{7}{15}$.

Probability Ex 31.6 Q5

Given,

Bag I contains 1 white, 2 black and 3 red balls

Bag II contains 2 white, 1 black and 1 red balls

Bag III contains 4 white, 5 black and 3 red balls.

A bag is chosen at random, then one red and one white ball is drawn.

Let E_1, E_2, E_3 and A be events as:

E_1 = Selecting bag I

E_2 = Selecting bag II

E_3 = Selecting bag III

A = Drawing one red and one white ball

$$P(E_1) = \frac{1}{3}$$

$$P(E_2) = \frac{1}{3}$$

$$P(E_3) = \frac{1}{3} \qquad \text{[Since there are only three bags]}$$

$$P(A \mid E_1) = P[\text{Drawing one red and one white ball from bag I}]$$

$$= \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2}$$

$$= \frac{1 \times 3}{\frac{6 \times 5}{2}}$$

$$= \frac{1}{5}$$

$$P\left(\frac{A}{E_2}\right) = P[\text{Drawing one red and one white ball from bag II}]$$

$$= \frac{{}^2C_1 \times {}^1C_1}{{}^4C_2}$$

$$= \frac{2 \times 1}{\frac{4 \times 3}{2}}$$

$$= \frac{1}{3}$$

$$p\left(\frac{A}{E_3}\right) = P[\text{Drawing one red and one white ball from bag III}]$$

$$= \frac{{}^4C_1 \times {}^3C_1}{{}^{12}C_2}$$

$$= \frac{4 \times 3}{\frac{12 \times 11}{2}}$$

$$= \frac{2}{11}$$

By law of total probability,

$$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)$$

$$= \frac{1}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{11}$$

$$= \frac{1}{15} + \frac{1}{9} + \frac{2}{33}$$

$$= \frac{33 + 55 + 30}{495}$$

$$= \frac{118}{495}$$

Required probability = $\frac{118}{495}$.

Probability Ex 31.6 Q6

An unbiased coin is tossed, then
I:- If head occurs, pair of dice is rolled and sum on them is either 7 or 8.
II:- If tail occurs, a card is drawn from cards numbered 2,3,...,12 and is 7 or 8.

Let E_1, E_2, A be events as

E_1 = Head occurs on the coin

E_2 = Tail occurs on the coin

A = Noted number is 7 or 8

$P\left(E_1\right) = \frac{1}{2}$

$P\left(E_2\right) = \frac{1}{2}$

$P\left(A \mid E_1\right) = P$[Pair of dice shows 7 or 8 as sum]

[Sum on dice is 7 or 8 when $(1,6), (2,5), (3,4), (4,3), (5,2), (6,1), (2,6), (3,5), (4,4), (5,3), (6,2)$]

$P\left(A \mid E_1\right) = \frac{11}{36}$

$P\left(\frac{A}{E_2}\right) = P$[7 or 8 on card drawn from 11 cards numbered $2, 3, 4, \ldots, 12$]

$= \frac{2}{11}$

By law of total probability,

$$P(A) = P\left(E_1\right) P\left(A \mid E_1\right) + P\left(E_2\right) P\left(\frac{A}{E_2}\right)$$

$$= \frac{1}{2} \times \frac{11}{36} + \frac{1}{2} \times \frac{2}{11}$$

$$= \frac{11}{72} + \frac{2}{22}$$

$$= \frac{121 + 72}{792}$$

$$= \frac{193}{792}$$

Required probability $= \frac{193}{792}$.

Probability Ex 31.6 Q7

Let E_1, E_2, A be defined as,

E_1 = Item produced by machine A

E_2 = Item produced by machine B

A = The item drawn is defective

$P\left(E_1\right) = 60\%$

$= \frac{60}{100}$

$P\left(E_2\right) = 40\%$

$= \frac{40}{100}$

$P\left(A \mid E_1\right) = P$[Defective item from machine A]

$= 2\%$

$= \frac{2}{100}$

$P\left(\frac{A}{E_2}\right) = P$[Defective item from machine B]

$= 1\%$

$= \frac{1}{100}$

By law of total probability,

$$P(A) = P\left(E_1\right) P\left(A \mid E_1\right) + P\left(E_2\right) P\left(\frac{A}{E_2}\right)$$

$$= \frac{60}{100} \times \frac{2}{100} + \frac{40}{100} \times \frac{1}{100}$$

$$= \frac{120 + 40}{10000}$$

$$= \frac{160}{10000}$$

$$= 0.016$$

Required probability = 0.016.

Probability Ex 31.6 Q8

Bag A contains 8 white and 7 black balls
Bag B contains 5 white and 4 black balls
Transfer can be done in two ways:-
I-A white ball is transferred from bag A to bag B and then one white ball is drawn from bag B.
II-A black ball is transferred from bag A to bag B, then one white ball is drawn from bag B.
Let E_1, E_2 and A be events as:-

E_1 = One white ball from bag A
E_2 = One black ball from bag A
A = One white ball from bag B

$$P(E_1) = \frac{8}{15}$$

$$P(E_2) = \frac{7}{15}$$

$$P(A \mid E_1) = \frac{6}{10} \qquad [\text{Since } E_1 \text{ has increased white balls in bag } B]$$

$$P\left(\frac{A}{E_2}\right) = \frac{5}{10} \qquad [\text{Since } E_2 \text{ has increased black ball in bag } B]$$

By law of total probability,

$$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P\left(\frac{A}{E_2}\right)$$

$$= \frac{8}{15} \times \frac{6}{10} + \frac{7}{15} \times \frac{5}{10}$$

$$= \frac{48}{150} + \frac{35}{150}$$

$$= \frac{83}{150}$$

Required probability = $\frac{83}{150}$.

Probability Ex 31.6 Q9

There are two bags.
Bag (1) contain 4 white and 5 black balls
Bag (2) contain 3 white and 4 black balls.

A ball is taken from bag (i) and without seeing its colour is put in second bag. Then a ball is drawn from bag 2 and is white in colour.

$$P(\text{White ball from bag 1}) = \frac{4}{9}$$

$$P(W_1) = \frac{4}{9}$$

$$P(\text{Black ball from bag 1}) = \frac{5}{9}$$

$$P(B_1) = \frac{5}{9}$$

P(White ball from bag 2 given B_1 transfer)

$$P\left(\frac{W_2}{B_1}\right) = \frac{3}{8}$$

P(White from bag 2 given W_1 transfer)

$$P\left(\frac{W_2}{W_1}\right) = \frac{4}{8} = \frac{1}{2}$$

P(White from bag 2)

$$= P(B_1)P\left(\frac{W_2}{B_1}\right) + P(W_1)P\left(\frac{W_2}{W_1}\right)$$

$$= \frac{5}{9} \times \frac{3}{8} + \frac{4}{9} \times \frac{1}{2}$$

$$= \frac{15}{72} + \frac{4}{18}$$

$$= \frac{31}{72}$$

Required probability $= \frac{31}{72}$

Probability Ex 31.6 Q10

There are two bags.
Bag (1) contain 4 white and 5 black balls
Bag (2) contain 6 white and 7 black balls.

A ball is taken from bag (1) and without seeing its colour is put in bag (2). Then a ball is drawn from bag (2) and is found white in colour.

$$P\left(1 \text{ white ball from bag } 1\right) = \frac{4}{9}$$

$$P\left(W_1\right) = \frac{4}{9}$$

$$P\left(1 \text{ black ball from bag } 1\right) = \frac{5}{9}$$

$$P\left(B_1\right) = \frac{5}{9}$$

$P\left(1 \text{ white ball from bag 2 given } W_1 \text{ is put in bag } 2\right)$

$$P\left(\frac{W_2}{W_1}\right) = \frac{7}{14}$$

$$P\left(\frac{W_2}{W_1}\right) = \frac{1}{2}$$

$P\left(1 \text{ white ball from bag 2 given } B_1 \text{ is put in bag } 2\right)$

$$P\left(\frac{W_2}{B_1}\right) = \frac{6}{14}$$

$P\left(1 \text{ white from bag } 2\right)$

$$= P\left(W_1\right)P\left(\frac{W_2}{W_1}\right) + P\left(B_1\right)P\left(\frac{W_2}{B_1}\right)$$

$$= \frac{4}{9} \times \frac{1}{2} + \frac{5}{9} \times \frac{6}{14}$$

$$= \frac{4}{18} + \frac{30}{126}$$

$$= \frac{58}{126}$$

$$= \frac{29}{63}$$

$$\text{Required probability } = \frac{29}{63}$$

Probability Ex 31.6 Q11

Urn '1'	Urn '2'
10W 3B	3W 5B

Let $U1_{2W}$, $U1_{1W1B}$, $U1_{2B}$ be the events of transferring 2 white balls, 1 white & 1black ball, 2 black balls from first Urn1 to second Urn2.

$P(U1_{2W}) = {}^{10}C_2/{}^{13}C_2 = 45/78$

$P(U1_{1W1B}) = {}^{10}C_1{}^3C_1/{}^{13}C_2 = 10x3/78$

$P(U1_{2B}) = {}^3C_2/{}^{13}C_2 = 3/78$

Let $U2_W$ be the event that a white ball is drawn from the Urn 2. There are three scenarios for Urn 2 based on the events $U1_{2W}$ $U1_{1W1B}$ $U1_{2B}$

	5W	4W	3W
	5B	6B	7B
Total	10	10	10

$P(U1_{2W}U2_W) = \frac{{}^5C_1}{{}^{10}C_1} = 1/2$

$P(U1_{1W1B}U2_W) = \frac{{}^4C_1}{{}^{10}C_1} = 2/5$

$P(U1_{2B}U2_W) = \frac{{}^3C_1}{{}^{10}C_1} = 3/10$

$P(U2_W) = P(U1_{2W}\,U2_W) + P(U1_{1W1B}\,U2_W) + P(U1_{2B}U2_W)$

$= P(U1_{2W})x\,P(U1_{2W}\,U2_W) + P(U1_{1W1B})x\,P(U1_{1W1B}\,U2_W) +$

$P(U1_{2B})x\,P(U1_{2B}U2_W)$

$= \frac{45}{78} \times \frac{1}{2} + \frac{30}{78} \times \frac{2}{5} + \frac{3}{78} \times \frac{3}{10} = \frac{114}{780} = \frac{59}{130}$

Probability Ex 31.6 Q12

Given,

Bag (1) contains 6 red (R_1) and 8 black (B_1) balls

Bag (2) contains 8 red (R_2) and 6 black (B_2) balls

A ball is drawn from the first bag and without noticing its colour is put in the bag (2). Then a ball is drawn from second bag and it is red.

$$P\,(\text{One red ball from bag 2})$$
$$= P\left((B_1 \cap R_2) \cup (R_1 \cap R_2)\right)$$
$$= P\,(B_1 \cap R_2) + P\,(R_1 \cap R_2)$$
$$= P\,(B_1)P\left(\frac{R_2}{B_1}\right) + P\,(R_1)P\left(\frac{R_2}{R_1}\right)$$
$$= \frac{8}{14}\cdot\frac{8}{15} + \frac{6}{14}\cdot\frac{9}{15}$$
$$= \frac{64+54}{210}$$
$$= \frac{118}{210}$$
$$= \frac{59}{105}$$

Required probability $= \frac{59}{105}$

Probability Ex 31.6 Q13

Let D be the event that the picked up tube is defective.
Let A_1, A_2 and A_3 be the events that the tube is produced on machines E_1, E_2 and E_3 respectively.

$$P(D) = P(A_1)P(D \mid A_1) + P(A_2)P(D \mid A_2) + P(A_3)P(D \mid A_3) \ldots\ldots(i)$$

$$P(A_1) = \frac{50}{100} = \frac{1}{2}, P(A_2) = \frac{25}{100} = \frac{1}{4}, P(A_3) = \frac{25}{100} = \frac{1}{4}$$

$$P(D \mid A_1) = P(D \mid A_2) = \frac{4}{100} = \frac{1}{25}$$

$$P(D \mid A_3) = \frac{5}{100} = \frac{1}{20}$$

Putting these values in (i), we get

$$P(D) = \frac{1}{2} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{20}$$

$$P(D) = \frac{17}{400}$$

Ex 31.7

Probability Ex 31.7 Q1

Urn I contains 1 white, 2 balck and 3 red balls
Urn II contains 2 white, 1 black and 1 red balls
Urn III contains 4 white, 5 black and 3 red balls.
Consider E_1, E_2, E_2 and A be events as:-

E_1 = Selecting unr I
E_2 = Selecting urn II
E_3 = Selecting urn III
A = Drawing 1 white and 1 red balls

$$P(E_1) = \frac{1}{3}$$

$$P(E_2) = \frac{1}{3}$$

$$P(E_3) = \frac{1}{3} \qquad \text{[Since there are 3 urns]}$$

$$P(A \mid E_1) = P[\text{Drawing 1 red and 1 white from urn I}]$$

$$= \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2}$$

$$= \frac{1 \times 3}{\frac{6 \times 5}{2}}$$

$$= \frac{1}{5}$$

$$P\left(\frac{A}{E_2}\right) = P[\text{Drawing 1 red and 1 white from urn II}]$$

$$= \frac{{}^2C_1 \times {}^1C_1}{{}^4C_2}$$

$$= \frac{2 \times 1}{\frac{4 \times 3}{2}}$$

$$= \frac{1}{3}$$

$$P\left(\frac{A}{E_3}\right) = P[\text{Drawing 1 red and 1 white from urn III}]$$

$$= \frac{{}^4C_1 \times {}^3C_1}{{}^{12}C_2}$$

$$= \frac{4 \times 3}{\frac{12 \times 11}{2}}$$

$$= \frac{2}{11}$$

We have to find,

$$P\left(\text{They come from urn I}\right) = P\left(\frac{E_1}{A}\right)$$

$$P\left(\text{They come from urn II}\right) = P\left(\frac{E_2}{A}\right)$$

$$P\left(\text{They come from urn III}\right) = P\left(\frac{E_3}{A}\right)$$

By baye's theorem,

$$p\left(\frac{E_1}{A}\right) = \frac{P\left(E_1\right)P\left(\frac{A}{E_1}\right)}{P\left(E_1\right)P\left(\frac{A}{E_1}\right)+P\left(E_2\right)P\left(\frac{A}{E_2}\right)+P\left(E_3\right)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{3}\times\frac{1}{5}}{\frac{1}{3}\times\frac{1}{5}+\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{11}}$$

$$= \frac{\frac{1}{5}}{\frac{1}{5}+\frac{1}{3}+\frac{2}{11}}$$

$$= \frac{\frac{1}{5}}{\frac{36+55+30}{165}}$$

$$= \frac{1}{5}\times\frac{165}{118}$$

$$= \frac{33}{118}$$

$$p\left(\frac{E_2}{A}\right) = \frac{P\left(E_2\right)P\left(\frac{A}{E_2}\right)}{P\left(E_1\right)P\left(\frac{A}{E_1}\right)+P\left(E_2\right)P\left(\frac{A}{E_2}\right)+P\left(E_3\right)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{3}\times\frac{1}{3}}{\frac{1}{3}\times\frac{1}{5}+\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{11}}$$

$$= \frac{\frac{1}{3}}{\frac{1}{5}+\frac{1}{3}+\frac{2}{11}}$$

$$= \frac{\frac{1}{3}}{\frac{33+55+30}{165}}$$

$$= \frac{1}{3}\times\frac{165}{118}$$

$$= \frac{55}{118}$$

$$p\left(\frac{E_3}{A}\right) = \frac{P\left(E_3\right)P\left(\frac{A}{E_3}\right)}{P\left(E_1\right)P\left(\frac{A}{E_1}\right)+P\left(E_2\right)P\left(\frac{A}{E_2}\right)+P\left(E_3\right)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{3}\times\frac{2}{11}}{\frac{1}{3}\times\frac{1}{5}+\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{11}}$$

$$= \frac{\frac{2}{11}}{\frac{1}{5}+\frac{1}{3}+\frac{2}{11}}$$

$$= \frac{2}{11}\times\frac{165}{118}$$

$$= \frac{30}{118}$$

Therefore, required probability $= \frac{33}{118}, \frac{55}{118}, \frac{30}{118}$.

Probability Ex 31.7 Q2

Bag A contains 2 white and 3 red balls
Bag B contains 4 white and 5 red balls.
Consider E_1, E_2 and A events as:-

E_1 = Selecting bag A
E_2 = Selecting bag B
A = Drawing one red ball

$$P(E_1) = \frac{1}{2}$$

$$P(E_2) = \frac{1}{2} \qquad \text{[Since there are 2 bags]}$$

$$P(A \mid E_1) = P[\text{Drawing one red ball from bag } A] = \frac{3}{5}$$

$$P\left(\frac{A}{E_2}\right) = P[\text{Drawing one red ball from bag } B] = \frac{5}{9}$$

To find,

$$P(\text{Drawn, one red ball is from bag } B) = P\left(\frac{E_2}{A}\right)$$

By baye's theorem,

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2}\times\frac{5}{9}}{\frac{1}{2}\times\frac{3}{5}+\frac{1}{2}\times\frac{5}{9}}$$

$$= \frac{\frac{5}{9}}{\frac{3}{5}+\frac{5}{9}}$$

$$= \frac{\frac{5}{9}}{\frac{27+25}{45}}$$

$$= \frac{5}{9}\times\frac{45}{52} = \frac{25}{52}$$

Required probability = $\frac{25}{52}$.

Probability Ex 31.7 Q3

Urn I contains 2 white and 3 black balls
Urn II contains 3 white and 2 black balls
Urn III contains 4 white and 1 black balls
Let E_1, E_2, E_3 and A be events as:-

E_1 = Selecting urn I
E_2 = Selecting urn II
E_3 = Selecting urn III
A = A white balls is drawn

$$P(E_1) = \frac{1}{2}$$

$$P(E_2) = \frac{1}{2}$$

$$P(E_3) = \frac{1}{2} \qquad \text{[Since there are 3 urns]}$$

$$P(A \mid E_1) = P[\text{Drawing 1 white ball from urn I}] = \frac{2}{5}$$

$$P\left(\frac{A}{E_2}\right) = P[\text{Drawing 1 white ball from urn II}] = \frac{3}{5}$$

$$P\left(\frac{A}{E_3}\right) = P[\text{Drawing one whtie ball from urn III}] = \frac{4}{5}$$

To find,

$$P(\text{Drawn one white ball from urn I}) = P\left(\frac{E_1}{A}\right)$$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{2} \times \frac{2}{5}}{\frac{1}{2} \times \frac{2}{5} + \frac{1}{2} \times \frac{3}{5} + \frac{1}{2} \times \frac{4}{5}}$$

$$= \frac{\frac{2}{10}}{\frac{2+3+4}{10}}$$

$$= \frac{2}{9}$$

Required probability = $\frac{2}{9}$.

Probability Ex 31.7 Q4

Urn I contains 7 white and 3 black balls
Urn II contains 4 white and 6 black balls
Urn III contains 2 white and 8 black balls
Let E_1, E_2, E_3 and A be events as:-

E_1 = Selecting urn I
E_2 = Selecting urn II
E_3 = Selecting urn III
A = Drawing 2 white balls without replacement.

Given,

$P(E_1) = 0.20$
$P(E_2) = 0.60$
$P(E_3) = 0.20$
$P(A \mid E_1) = P$[Drawing 2 white ball from urn I]

$$= \frac{{}^7C_2}{{}^{10}C_2}$$

$$= \frac{\frac{7 \times 6}{2}}{\frac{10 \times 9}{2}}$$

$$= \frac{7}{15}$$

$P\left(\frac{A}{E_2}\right) = P$[Drawing 2 white ball from urn II]

$$= \frac{{}^4C_2}{{}^{10}C_2}$$

$$= \frac{\frac{4 \times 3}{2}}{\frac{10 \times 9}{2}}$$

$$= \frac{12}{90}$$

$$= \frac{2}{15}$$

$$P\left(\frac{A}{E_3}\right) = P\left[\text{Drawing 2 whtie ball from urn III}\right]$$
$$= \frac{{}^2C_2}{{}^{10}C_2}$$
$$= \frac{1}{\frac{10 \times 9}{2}}$$
$$= \frac{1}{45}$$

To find,

$$P\left(\text{2 white balls drawn are from urn III}\right) = P\left(\frac{E_3}{A}\right)$$

By baye's theorem,

$$P\left(\frac{E_3}{A}\right) = \frac{P\left(E_3\right)P\left(\frac{A}{E_3}\right)}{P\left(E_1\right)P\left(\frac{A}{E_1}\right) + P\left(E_2\right)P\left(\frac{A}{E_2}\right) + P\left(E_3\right)P\left(\frac{A}{E_3}\right)}$$
$$= \frac{0.2 \times \frac{1}{45}}{0.2 \times \frac{7}{15} + 0.6 \times \frac{2}{15} + 0.2 \times \frac{1}{45}}$$
$$= \frac{\frac{2}{450}}{\frac{14}{150} + \frac{12}{150} + \frac{2}{450}}$$
$$= \frac{\frac{2}{450}}{\frac{42 + 36 + 2}{450}}$$
$$= \frac{2}{80}$$
$$= \frac{1}{40}$$

Required probability = $\frac{1}{40}$.

Probability Ex 31.7 Q5

Consider the following events:

E_1 = Getting 1 or 2 in a throw of die,

E_2 = Getting 3, 4, 5 or 6 in a throw of die,

A = Getting exactly one tail

Clearly,

$$P\left(E_1\right) = \frac{2}{6} = \frac{1}{3}, P\left(E_2\right) = \frac{4}{6} = \frac{2}{3}, P\left(A/E_1\right) = \frac{3}{8}, P\left(A/E_2\right) = \frac{1}{2}$$

Required probablity = $P\left(E_2/A\right)$

$$= \frac{P\left(E_2\right)P\left(A/E_2\right)}{P\left(E_1\right)P\left(A/E_1\right) + P\left(E_2\right)P\left(A/E_2\right)}$$
$$= \frac{\frac{2}{3} \times \frac{1}{2}}{\frac{1}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{1}{2}}$$
$$= \frac{8}{11}$$

Probability Ex 31.7 Q6

Consider the following events:
E_1 = First group wins, E_2 = Second group wins, A = New product is introduced.
It is given that

$$P(E_1)=0.6, P(E_2)=0.4, P(A/E_1)=0.7, P(A/E_2)=0.3$$

Required probability $=P(E_2/A)=\dfrac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}$

$$=\frac{0.4\times0.3}{0.6\times0.7+0.4\times0.3}=\frac{12}{54}=\frac{2}{9}$$

Hence required probability is $\frac{2}{9}$

Probability Ex 31.7 Q7

Given, 5 man out of 100 and 25 women out of 1000 are good orators.
Consider E_1, E_2 and A events as:-

E_1 = Selected person is male
E_2 = Selected perosn is female
E_3 = Selected person is an orator

$P(E_1)=\frac{1}{2}$

$P(E_2)=\frac{1}{2}$ [Since number of males and females are equal]

$P(A \mid E_1) = P$ (Selecting a male orator)

$$=\frac{5}{100}$$
$$=\frac{1}{20}$$

$P\left(\frac{A}{E_2}\right)=P$ (Selecting a frmale orator)

$$=\frac{25}{1000}$$
$$=\frac{1}{40}$$

To find, P (Orator selected is a male) $=P\left(\frac{E_1}{A}\right)$.

By baye's theorem,

$$P\left(\frac{E_1}{A}\right)=\frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$=\frac{\frac{1}{2}\times\frac{1}{20}}{\frac{1}{2}\times\frac{1}{20}+\frac{1}{2}\times\frac{1}{40}}$$

$$=\frac{\frac{1}{40}}{\frac{1}{40}+\frac{1}{80}}$$

$$=\frac{1}{40}\times\frac{80}{3}$$

$$=\frac{2}{3}$$

Required probability $=\frac{2}{3}$.

Probability Ex 31.7 Q8

Consider events E_1, E_2 and A events as:-

E_1 = Letters come from LONDON

E_2 = Letters come from CLIFTON

E_3 = Two consecutive letters visible on the envelope are ON

$P(E_1) = \frac{1}{2}$

$P(E_2) = \frac{1}{2}$

[Since letters came either from LONDON or CLIFTON]

$P(A \mid E_1) = P$ (Two consecutive letters ON from LONDON)

$= \frac{2}{5}$

[Since LONDON has 2-ON and 5 letters we consider one 'ON' as one letter]

$P\left(\frac{A}{E_2}\right) = P$ (Two consecutive letters ON from CLIFTON)

$= \frac{1}{6}$

[Since CLIFTON has one 'ON' nad 6 letters considering ON as one letter]

(i) To find, P (ON visible are from LONDON) $= P\left(\frac{E_1}{A}\right)$.

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2}\times\frac{2}{5}}{\frac{1}{2}\times\frac{2}{5}+\frac{1}{2}\times\frac{1}{6}}$$

$$= \frac{\frac{2}{10}}{\frac{2}{10}+\frac{1}{12}}$$

$$= \frac{2}{10}\times\frac{60}{17}$$

$$= \frac{12}{17}$$

$$P\left(\frac{E_1}{A}\right) = \frac{12}{17}$$

Required probability $= \frac{12}{17}$

(ii)
$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2}\times\frac{1}{6}}{\frac{1}{2}\times\frac{2}{5}+\frac{1}{2}\times\frac{1}{6}}$$

$$= \frac{\frac{1}{12}}{\frac{2}{10}+\frac{1}{12}}$$

$$= \frac{1}{12}\times\frac{60}{17}$$

$$= \frac{5}{17}$$

Required probability $= \frac{5}{17}$.

Probability Ex 31.7 Q9

Consider E_1, E_2 and A events as:-

E_1 = Selected student is boy

E_2 = Selected student is girl

E_3 = A student with IQ more that 150 is selected

$P(E_1) = \frac{60}{100}$

$P(E_2) = \frac{40}{100}$

$P(A \mid E_1) = P$ (Selected boy has IQ more than 150)

$= \frac{5}{100}$

$P\left(\frac{A}{E_2}\right) = P$ (Selected girl has IQ more than 150)

$= \frac{10}{100}$

To find, P (Selected student with IQ more than 150 is a boy) $= P\left(\frac{E_1}{A}\right)$.

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{60}{100} \times \frac{5}{100}}{\frac{60}{100} \times \frac{5}{100} + \frac{40}{100} \times \frac{10}{100}}$$

$$= \frac{300}{300 + 400}$$

$$= \frac{300}{700}$$

$$= \frac{3}{7}$$

Required probability $= \frac{3}{7}$.

Probability Ex 31.7 Q10

Consider E_1, E_2, E_3 and A as:-

E_1 = Bolt produced by machine X

E_2 = Bolt produced by machine Y

E_3 = Bolt produced by machine Z

A = A bolt drawn is defective.

$$P(E_1) = \frac{1000}{6000} = \frac{1}{6}$$

$$P(E_2) = \frac{2000}{6000} = \frac{1}{3}$$

$$P(E_3) = \frac{3000}{6000} = \frac{1}{2}$$

$$P(A \mid E_1) = P\left(\text{Drawing defective bolt from machine } X\right) = \frac{1}{100}$$

$$P\left(\frac{A}{E_2}\right) = P\left(\text{Drawing defective bolt from machine } Y\right) = \frac{1.5}{100} = \frac{3}{200}$$

$$P\left(\frac{A}{E_3}\right) = P\left(\text{Drawing defective bolt from machine } Z\right) = \frac{2}{100}$$

To find, $P\left(\text{Defective bolt drawn is produced by machine } X\right) = P\left(\frac{E_1}{A}\right)$.

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{6} \times \frac{1}{100}}{\frac{1}{6} \times \frac{1}{100} + \frac{1}{3} \times \frac{3}{200} + \frac{1}{2} \times \frac{2}{100}}$$

$$= \frac{\frac{1}{600}}{\frac{1}{600} + \frac{3}{600} + \frac{1}{100}}$$

$$= \frac{1}{10}$$

Required probability $= \frac{1}{10}$.

Probability Ex 31.7 Q11

Let E_1, E_2, E_3 and A be the events defined as follows

E_1 = scooters

E_2 = cars

E_3 = trucks

A = vehicle meet with an accident

Since there are 12000 vehicles, therefore

$$P(E_1)=\frac{3000}{12000}=\frac{1}{4}, P(E_2)=\frac{4000}{12000}=\frac{1}{3}, P(E_3)=\frac{5000}{12000}=\frac{5}{12}$$

It is given that $P(A/E_1)$=Probability that the accident involves a scooter $=0.02$

Similarly $P(A/E_2)=0.03$ and $P(A/E_3)=0.04$

(i)

We are required to find $P(E_1/A)$ i.e. given that the vehicle meet with an accident is a scooter

By Baye's rule

$$P(E_1/A)=\frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$$

$$=\frac{\frac{1}{4}\times 0.02}{\frac{1}{4}\times 0.02+\frac{1}{3}\times 0.03+\frac{5}{12}\times 0.04}$$

$$=\frac{3}{19}$$

(ii)

We are required to find $P(E_2/A)$ i.e. given that the vehicle meet with an accident is a car

By Baye's rule

$$P(E_2/A)=\frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$$

$$=\frac{\frac{1}{3}\times 0.03}{\frac{1}{4}\times 0.02+\frac{1}{3}\times 0.03+\frac{5}{12}\times 0.04}$$

$$=\frac{6}{19}$$

(iii)

We are required to find $P(E_3/A)$ i.e. given that the vehicle meet with an accident is a scooter

By Baye's rule

$$P(E_3/A)=\frac{P(E_3)P(A/E_3)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$$

$$=\frac{\frac{5}{12}\times 0.04}{\frac{1}{4}\times 0.02+\frac{1}{3}\times 0.03+\frac{5}{12}\times 0.04}$$

$$=\frac{10}{19}$$

Probability Ex 31.7 Q12

We need to find

$$P\left(\frac{A}{\text{Red}}\right), P\left(\frac{B}{\text{Red}}\right), P\left(\frac{C}{\text{Red}}\right)$$

Now,

$$P\left(\frac{A}{\text{Red}}\right) = \frac{P\left(\frac{\text{Red}}{A}\right) P(A)}{P\left(\frac{\text{Red}}{A}\right) P(A) + P\left(\frac{\text{Red}}{B}\right) P(B) + P\left(\frac{\text{Red}}{C}\right) P(C) + P\left(\frac{\text{Red}}{D}\right) P(D)}$$

$$= \frac{\frac{1}{10} \times \frac{1}{4}}{\frac{1}{10} \times \frac{1}{4} + \frac{6}{10} \times \frac{1}{4} + \frac{8}{10} \times \frac{1}{4} + 0}$$

$$= \frac{1}{1+6+8} = \frac{1}{15}$$

Similarly

$$P\left(\frac{B}{\text{Red}}\right) = \frac{6}{15}$$

$$P\left(\frac{C}{\text{Red}}\right) = \frac{8}{15}$$

Probability Ex 31.7 Q13

Let E_1, E_2, and E_3 be the respective events of the time consumed by machines A, B, and C for the job.

$$P(E_1) = 50\% = \frac{50}{100} = \frac{1}{2}$$

$$P(E_2) = 30\% = \frac{30}{100} = \frac{3}{10}$$

$$P(E_3) = 20\% = \frac{20}{100} = \frac{1}{5}$$

Let X be the event of producing defective items.

$$P(X|E_1) = 1\% = \frac{1}{100}$$

$$P(X|E_2) = 5\% = \frac{5}{100}$$

$$P(X|E_3) = 7\% = \frac{7}{100}$$

The probability that the defective item was produced by A is given by $P(E_1|A)$.

By using Bayes' theorem, we obtain

$$P(E_1|X)=\frac{P(E_1)\cdot P(X|E_1)}{P(E_1)\cdot P(X|E_1)+P(E_2)\cdot P(X|E_2)+P(E_3)\cdot P(X|E_3)}$$

$$=\frac{\frac{1}{2}\cdot\frac{1}{100}}{\frac{1}{2}\cdot\frac{1}{100}+\frac{3}{10}\cdot\frac{5}{100}+\frac{1}{5}\cdot\frac{7}{100}}$$

$$=\frac{\frac{1}{100}\cdot\frac{1}{2}}{\frac{1}{100}\left(\frac{1}{2}+\frac{3}{2}+\frac{7}{5}\right)}$$

$$=\frac{\frac{1}{2}}{\frac{17}{5}}$$

$$=\frac{5}{34}$$

Probability Ex 31.7 Q14

Consider the following events:

E_1 = Item is produced by machine A,

E_2 = Item is produced by machine B,

E_3 = Item is produced by machine C,

A = Item is defective

Clearly,

$$P(E_1)=\frac{50}{100}=\frac{1}{2},P(E_2)=\frac{30}{100}=\frac{3}{10},P(E_2)=\frac{20}{100}=\frac{1}{5}$$

$$P(A/E_1)=\frac{2}{100},P(A/E_2)=\frac{2}{100},P(A/E_2)=\frac{3}{100}$$

Re quired probablity $=P(E_1/A)$

$$=\frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$$

$$=\frac{\frac{1}{2}\times\frac{2}{100}}{\frac{1}{2}\times\frac{2}{100}+\frac{3}{10}\times\frac{2}{100}+\frac{1}{5}\times\frac{3}{100}}$$

$$=\frac{5}{11}$$

Probability Ex 31.7 Q15

Let E_1, E_2, E_3 be the events that we choose the first coin, second coin, and third coin respectively in a random toss.

$P(E_1)=\frac{1}{3}, P(E_2)=\frac{1}{3}, P(E_3)=\frac{1}{3}$

Let A denote the event when the toss shows heads.

It is given that

$P(A/E_1)=1, P(A/E_2)=0.75, P(A/E_3)=.60$

We have to find $P(E_1/A)$.

By Baye's theorem

$$P(E_1/A)=\frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}=$$

$$=\frac{\frac{1}{3}(1)}{\frac{1}{3}(1)+\frac{1}{3}(0.75)+\frac{1}{3}(0.60)}=\frac{1/3}{(1/3)+(1/4)+(1/5)}$$

$$=\frac{1/3}{47/60}=\frac{20}{47}$$

Probability Ex 31.7 Q16

Consider events E_1, E_2, E_3 and A as:-

E_1 = Selecting product from machine A

E_2 = Selecting product from machine B

E_3 = Selecting product from machine C

A = Selecting a standard quality product

$P(E_1) = \frac{30}{100}$

$P(E_2) = \frac{25}{100}$

$P(E_3) = \frac{45}{100}$

$P(A \mid E_1) = P$ (Selecting defective product from machine A)

$= \frac{1}{100}$

$P\left(\frac{A}{E_2}\right) = P$ (Selecting defective product from machine B)

$= \frac{1.2}{100}$

$P\left(\frac{A}{E_3}\right) = P$ (Selecting defective product from machine C)

$= \frac{2}{100}$

To find, P (Selecting defective product is produced by machine B)

By baye's theorem,

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{25}{100} \times \frac{12}{1000}}{\frac{30}{100} \times \frac{1}{100} + \frac{25}{100} \times \frac{12}{1000} + \frac{45}{100} \times \frac{2}{100}}$$

$$= \frac{300}{300 + 300 + 900}$$

$$= \frac{300}{1500}$$

$$= \frac{1}{5}$$

Required probability = $\frac{1}{5}$.

Probability Ex 31.7 Q17

Let E_1, E_2 and A be events as:-

E_1 = Selecting bicycle from first plant

E_2 = Selecting bicycle from second plant

A = Selecting a standard quality bicycle

$P(E_1) = \frac{60}{100}$

$P(E_2) = \frac{40}{100}$

$P(A \mid E_1) = P$ (Selecting standard quality bicycle from first plant)

$= \frac{80}{100}$

$P\left(\frac{A}{E_2}\right) = P$ (Selecting standard quality bicycle from second plant)

$= \frac{90}{100}$

To find, P (Selected standard quality bicycle is from second plant) $= P\left(\frac{E_2}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{40}{100} \times \frac{90}{100}}{\frac{60}{100} \times \frac{80}{100} + \frac{40}{100} \times \frac{90}{100}}$$

$$= \frac{3600}{4800 + 3600}$$

$$= \frac{3600}{8400}$$

$$= \frac{3}{7}$$

Required probability $= \frac{3}{7}$.

Probability Ex 31.7 Q18

Urn A contains 6 red and 4 white balls

Urn B contains 2 red and 6 white balls

Urn C contains 1 red and 5 white balls

Consider E_1, E_2, E_3 and A events as:-

E_1 = Selecting urn A

E_2 = Selecting urn B

E_3 = Selecting urn C

A = Selecting a red ball

$P(E_1) = \frac{1}{3}$

$P(E_2) = \frac{1}{3}$

$P(E_3) = \frac{1}{3}$ [Since there are three urns]

$P(A \mid E_1) = P$ (Selecting a red ball from urn A)

$= \frac{6}{10}$

$= \frac{3}{5}$

$P\left(\frac{A}{E_2}\right) = P$ (Selecting a red ball from urn B)

$= \frac{2}{8}$

$= \frac{1}{4}$

$P\left(\frac{A}{E_3}\right) = P$ (Selecting a red ball from urn C)

$= \frac{1}{6}$

To find, P (Selected red ball is from urn A) $= P\left(\frac{E_1}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{3} \times \frac{3}{5}}{\frac{1}{3} \times \frac{3}{5} + \frac{1}{3} \times \frac{1}{4} + \frac{1}{3} \times \frac{1}{6}}$$

Probability Ex 31.7 Q19

Let E_1, E_2, E_3 be the events that the people are smokers and non−vegetarian, smokers and vegetarian, and non−smokers and vegetarian respectively.

$P(E_1) = \frac{2}{5}, P(E_2) = \frac{1}{4}, P(E_3) = \frac{7}{20}$

Let A denote the event that the person has the special chest disease.

It is given that

$P(A/E_1) = 0.35, P(A/E_2) = 0.20, P(A/E_3) = 0.10$

We have to find $P(E_1/A)$.

By Baye's theorem

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)} =$$

$$= \frac{\frac{2}{5}(0.35)}{\frac{2}{5}(0.35) + \frac{1}{4}(0.20) + \frac{7}{20}(0.10)} = \frac{7/50}{(7/50) + (1/20) + (7/200)}$$

$$= \frac{7/50}{9/40} = \frac{28}{45}$$

Probability Ex 31.7 Q20

Let E_1, E_2, E_3 and A be events as:-

E_1 = Selecting product from machine A

E_2 = Selecting product from machine B

E_3 = Selecting product from machine C

A = Selecting a defective product

$$P(E_1) = \frac{100}{600} = \frac{1}{6}$$

$$P(E_2) = \frac{200}{600} = \frac{1}{3}$$

$$P(E_3) = \frac{300}{600} = \frac{1}{2}$$

$P(A \mid E_1) = P$ (Selecting a defective item from machine A)

$$= \frac{2}{100}$$

$P\left(\frac{A}{E_2}\right) = P$ (Selecting a defective item from machine B)

$$= \frac{3}{100}$$

$P\left(\frac{A}{E_3}\right) = P$ (Selecting a defective item machine C)

$$= \frac{5}{100}$$

To find, P (Selected defective item is produced by machine A) $= P\left(\frac{E_1}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{6} \times \frac{2}{100}}{\frac{1}{6} \times \frac{2}{100} + \frac{1}{3} \times \frac{3}{100} + \frac{1}{2} \times \frac{5}{100}}$$

$$= \frac{\frac{2}{600}}{\frac{2}{600} + \frac{3}{300} + \frac{5}{200}}$$

$$= \frac{2}{600} \times \frac{600}{23}$$

$$= \frac{2}{23}$$

Required probability $= \frac{2}{23}$.

Probability Ex 31.7 Q21

Bag I contains 1 white and 6 red balls
Bag II contains 4 white and 3 red balls
Let E_1, E_2 and A events be:-

E_1 = Selecting bag I
E_2 = Selecting bag II
A = Selecting a white ball

$$P(E_1) = \frac{1}{2}$$

$$P(E_2) = \frac{1}{2} \qquad \text{[Since there are two bags]}$$

$$P(A \mid E_1) = P\,(\text{Selecting 1 white ball from bag I})$$
$$= \frac{1}{7}$$

$$P\left(\frac{A}{E_2}\right) = P\,(\text{Selecting 1 white ball from bag II})$$
$$= \frac{4}{7}$$

To find, $P\,(\text{Drawn white ball is from bag I}) = P\left(\frac{E_1}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2}\times\frac{1}{7}}{\frac{1}{2}\times\frac{1}{7}+\frac{1}{2}\times\frac{4}{7}}$$

$$= \frac{\frac{1}{14}}{\frac{1}{14}+\frac{4}{14}}$$

$$= \frac{1}{5}$$

Required probability $= \frac{1}{5}$.

Probability Ex 31.7 Q22

Consider the following events

E_1 = The selected student is a girl
E_2 = The selected student is not a girl
A = The student is taller than 1.75 meters

We have,

$$P(E_1) = 60\% = \frac{60}{100} = 0.6$$

$$P(E_2) = 1 - P(E_1) = 1 - 0.6 = 0.4$$

$P(A/E_1)$ = Probability that the student is taller than 1.75 meters given that the student is a girl

$$P(A/E_1) = \frac{1}{100} = 0.01$$

And

$P(A/E_2)$ = Probability that the student is taller than 1.75 meters given that the student is not a girl

$$P(A/E_2) = \frac{4}{100} = 0.04$$

Now,

Required probability

$$= P(E_1/A)$$

$$= \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{0.6\times0.01}{0.6\times0.01+0.4\times0.04}$$

$$= \frac{\frac{6}{1000}}{\frac{22}{1000}}$$

$$= \frac{3}{11}$$

Probability Ex 31.7 Q23

Let E_1, E_2, E_3 and A be events as:-

E_1 = A is appointed

E_2 = B is appointed

E_3 = C is appointed

A = A change does take place

$P(E_1) = \frac{4}{7}$

$P(E_2) = \frac{1}{7}$

$P(E_3) = \frac{2}{7}$

$P(A \mid E_1) = P$ (Changes take place by A)

$= 0.3$

$P\left(\frac{A}{E_2}\right) = P$ (Changes take place by B)

$= 0.8$

$P\left(\frac{A}{E_3}\right) = P$ (Changes take place by C)

$= 0.5$

To find, P (Changes were taken place by B or C) $= P\left(\frac{E_2}{A}\right) + P\left(\frac{E_3}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_2}{A}\right) + P\left(\frac{E_3}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{7}\times\frac{8}{10} + \frac{2}{7}\times\frac{5}{10}}{\frac{4}{7}\times\frac{3}{10} + \frac{1}{7}\times\frac{8}{10} + \frac{2}{7}\times\frac{5}{10}}$$

$$= \frac{\frac{18}{70}}{\frac{30}{70}}$$

$$= \frac{18}{30}$$

$$= \frac{3}{5}$$

Required probability $= \frac{3}{5}$.

Probability Ex 31.7 Q24

Let E_1, E_2 and A be events as:-

E_1 = Vehicle is scooter

E_2 = Vehicle is motorcycle

A = An insured met with accident

$P(E_1) = \frac{2000}{5000} = \frac{2}{5}$

$P(E_2) = \frac{3000}{5000} = \frac{3}{5}$

$P(A \mid E_1) = P$ (Accident of scooter)

$= 0.01$

$P\left(\frac{A}{E_2}\right) = P$ (Accident of motorcycle)

$= 0.02$

To find, P (Accident vehicle was motorcycle) $= P\left(\frac{E_2}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{3}{5}\times\frac{2}{100}}{\frac{2}{5}\times\frac{1}{100} + \frac{3}{5}\times\frac{2}{100}}$$

$$= \frac{\frac{6}{500}}{\frac{2}{500} + \frac{6}{500}}$$

$$= \frac{6}{8}$$

$$= \frac{3}{4}$$

Required probability $= \frac{3}{4}$.

Probability Ex 31.7 Q25

Consider the following events

E_1 = The selected student is a hosteller

E_2 = The selected student is not a hosteller.

A = The student has an A grade.

We have,

$$P(E_1)=30\%=\frac{30}{100}=0.3$$

$$P(E_2)=20\%=\frac{20}{100}=0.2$$

$P(A/E_1)$ = Probability that the student has an A grade given that the student is a hosteller

$$P(A/E_1)=\frac{60}{100}=0.6$$

And

$P(A/E_2)$ = Probability that the student has an A grade given that the student is not a hosteller

$$P(A/E_2)=\frac{40}{100}=0.4$$

Now,

Required probability

$$=P(E_1/A)$$

$$=\frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}$$

$$=\frac{0.3\times0.6}{0.3\times0.6+0.2\times0.4}$$

$$=\frac{\frac{18}{100}}{\frac{26}{100}}$$

$$=\frac{9}{13}$$

Probability Ex 31.7 Q26

Let E_1, E_2, and E_3 be the respective events of choosing a two headed coin, a biased coin, and an unbiased coin.

$$\therefore P(E_1)=P(E_2)=P(E_3)=\frac{1}{3}$$

Let A be the event that the coin shows heads.

A two-headed coin will always show heads.

$$\therefore P(A|E_1)=P(\text{coin showing heads, given that it is a two-headed coin})=1$$

Probability of heads coming up, given that it is a biased coin= 75%

$$\therefore P(A|E_2)=P(\text{coin showing heads, given that it is a biased coin})=\frac{75}{100}=\frac{3}{4}$$

Since the third coin is unbiased, the probability that it shows heads is always $\frac{1}{2}$.

$$\therefore P(A|E_3)=P(\text{coin showing heads, given that it is an unbiased coin})=\frac{1}{2}$$

The probability that the coin is two-headed, given that it shows heads, is given by

P $(E_1|A)$.

By using Bayes' theorem, we obtain

$$P(E_1|A)=\frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot P(A|E_2)+P(E_3)\cdot P(A|E_3)}$$

$$=\frac{\frac{1}{3}\cdot1}{\frac{1}{3}\cdot1+\frac{1}{3}\cdot\frac{3}{4}+\frac{1}{3}\cdot\frac{1}{2}}$$

$$=\frac{\frac{1}{3}}{\frac{1}{3}\left(1+\frac{3}{4}+\frac{1}{2}\right)}$$

$$=\frac{1}{\frac{9}{4}}$$

$$=\frac{4}{9}$$

Probability Ex 31.7 Q27

Let A, E_1, and E_2 respectively denote the events that a person has a heart attack, the selected person followed the course of yoga and meditation, and the person adopted the drug prescription.

$$\therefore P(A) = 0.40$$

$$P(E_1) = P(E_2) = \frac{1}{2}$$

$$P(A|E_1) = 0.40 \times 0.70 = 0.28$$

$$P(A|E_2) = 0.40 \times 0.75 = 0.30$$

Probability that the patient suffering a heart attack followed a course of meditation and yoga is given by P (E_1|A).

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$$

$$= \frac{\frac{1}{2} \times 0.28}{\frac{1}{2} \times 0.28 + \frac{1}{2} \times 0.30}$$

$$= \frac{14}{29}$$

Probability Ex 31.7 Q28

We need to find

$$P\left(\frac{\text{Box III}}{\text{Black}}\right)$$

$$= \frac{P\left(\frac{\text{Black}}{\text{Box III}}\right) P(\text{Box III})}{P\left(\frac{\text{Black}}{\text{Box III}}\right) P(\text{Box III}) + P\left(\frac{\text{Black}}{\text{Box II}}\right) P(\text{Box II}) + P\left(\frac{\text{Black}}{\text{Box I}}\right) .P(\text{Box I}) + P\left(\frac{\text{Black}}{\text{Box Iv}}\right) .P(\text{Box Iv})}$$

$$= \frac{\frac{1}{7} \times \frac{1}{4}}{\frac{1}{7} \times \frac{1}{4} + \frac{2}{8} \times \frac{1}{4} + \frac{3}{18} \times \frac{1}{4} + \frac{4}{13} \times \frac{1}{4}}$$

$$= \frac{\frac{1}{7}}{\frac{1}{7} + \frac{1}{4} + \frac{1}{6} + \frac{4}{13}}$$

$$= \frac{1}{7} \times \frac{7 \times 4 \times 6 \times 13}{4 \times 6 \times 13 + 7 \times 6 \times 13 + 7 \times 4 \times 13 + 7 \times 4 \times 6}$$

$$= \frac{4 \times 6 \times 13}{4 \times 6 \times 13 + 7 \times 6 \times 13 + 7 \times 4 \times 13 + 7 \times 4 \times 6}$$

$= 0.165$

Probability Ex 31.7 Q29

Let A be the event that the machine produces 2 acceptable items.

Also let B_1 be the event of correct set up and B_2 represent the event of incorrect set up.

Now, $P(B_1) = 0.8$, $P(B_2) = 0.2$

$P(A/B_1) = 0.9 \times 0.9$ and $P(A/B_2) = 0.4 \times 0.4$

Therefore, $P(B_1/A) = \dfrac{P(B_1)P(A/B_1)}{P(B_1)P(A/B_1) + P(B_2)P(A/B_2)}$

$$= \frac{0.8 \times 0.9 \times 0.9}{0.8 \times 0.9 \times 0.9 + 0.2 \times 0.4 \times 0.4} = \frac{648}{680} = 0.95$$

Probability Ex 31.7 Q30

Consider events E_1, E_2 and A as

E_1 = The person selected is actually having T.B.

E_2 = The person selected is not having T.B.

E_3 = The person diagnosed to have T.B.

Given,

$$P(E_1) = \frac{1}{1000}$$

$$P(E_2) = \frac{999}{1000}$$

$P(A \mid E_1) = P$ (Person diagnosed to have T.B. and he is actually having T.B.)

$= 0.99$

$P\left(\frac{A}{E_2}\right) = P$ (Person diagnosed to have T.B. and he is not a actually having T.B.)

$= 0.001$

To find, P (Person diagnosed to have T.B. is actually having T.B.) $= P\left(\frac{E_1}{A}\right)$.

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{1000} \times 0.99}{\frac{1}{1000} \times 0.99 + \frac{999}{1000} \times 0.001}$$

$$= \frac{990}{990 + 999}$$

$$= \frac{990}{1989}$$

$$= \frac{110}{221}$$

Required probability $= \frac{110}{221}$.

Probability Ex 31.7 Q31

Consider events E_1, E_2 and A as:-

E_1 = The selected person actually has disease

E_2 = The selected person has no disease

A = Selected person has disease

$$P(E_1) = \frac{0.2}{100}$$

$$= \frac{2}{1000}$$

$$P(E_2) = \frac{998}{1000}$$

$$P(A \mid E_1) = \frac{90}{100}$$

$$P\left(\frac{A}{E_2}\right) = \frac{1}{100}$$

To find, P (Person has disease is actually diseased) $= P\left(\frac{E_1}{A}\right)$.

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{2}{1000} \times \frac{90}{100}}{\frac{2}{1000} \times \frac{90}{100} + \frac{998}{1000} \times \frac{1}{100}}$$

$$= \frac{180}{180 + 998}$$

$$= \frac{180}{1178}$$

$$= \frac{90}{589}$$

Required probability $= \frac{90}{589}$.

Probability Ex 31.7 Q32

Let E_1, E_2, E_3 and A be events as:-

E_1 = Patient has disease d_1

E_2 = Patient has disease d_2

E_3 = Patient has disease D_3

A = Selected patient has symptom S.

$$P(E_1) = \frac{1800}{5000} = \frac{18}{50}$$

$$P(E_2) = \frac{2100}{5000} = \frac{21}{50}$$

$$P(E_3) = \frac{1100}{5000} = \frac{11}{50}$$

$P(A \mid E_1) = P$ (Patient with disease d_1 and shows symptom S)

$$= \frac{1500}{1800}$$

$$= \frac{5}{6}$$

$P\left(\frac{A}{E_2}\right) = P$ (Patient with disease d_2 and symptom S)

$$= \frac{1200}{2100}$$

$$= \frac{4}{7}$$

$P\left(\frac{A}{E_3}\right) = P$ (Patient with disease d_3 and symptom S)

$$= \frac{900}{1100}$$

$$= \frac{9}{11}$$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{5}{6} \times \frac{18}{50}}{\frac{5}{6} \times \frac{18}{50} + \frac{21}{50} \times \frac{4}{7} + \frac{11}{50} \times \frac{9}{11}}$$

$$= \frac{\frac{3}{10}}{\frac{3}{10} + \frac{6}{25} + \frac{9}{50}}$$

$$= \frac{3}{10} \times \frac{50}{36}$$

$$= \frac{5}{12}$$

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{21}{50} \times \frac{4}{7}}{\frac{5}{6} \times \frac{18}{50} + \frac{21}{50} \times \frac{4}{7} + \frac{11}{50} \times \frac{9}{11}}$$

$$= \frac{\frac{6}{25}}{\frac{3}{10} + \frac{6}{25} + \frac{9}{50}}$$

$$= \frac{6}{25} \times \frac{50}{36}$$

$$= \frac{1}{3}$$

$$P\left(\frac{E_3}{A}\right) = \frac{P(E_3)P\left(\frac{A}{E_3}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{11}{50} \times \frac{9}{11}}{\frac{5}{6} \times \frac{18}{50} + \frac{21}{50} \times \frac{4}{7} + \frac{11}{50} \times \frac{9}{11}}$$

$$= \frac{\frac{9}{50}}{\frac{3}{10} + \frac{6}{25} + \frac{9}{50}}$$

$$= \frac{9}{50} \times \frac{50}{36}$$

$$= \frac{1}{4}$$

So, probabilities of d_1, d_2, d_3 diseases are $\frac{5}{12}, \frac{1}{3}, \frac{1}{4}$ respectively.

Hence, the patient is most likely to have d_1 diseased.

Probability Ex 31.7 Q33

Let E_1, E_2 and A be events as:-

E_1 = 1 occurs on die

E_2 = 1 does not occur on die

A = The man reports that it is one

$P(E_1) = \frac{1}{6}$

$P(E_2) = \frac{5}{6}$

$P\left(\frac{A}{E_1}\right) = P$ (He reports one when occurs on die)

$= P$ (He speaks truth)

$= \frac{3}{5}$

$P\left(\frac{A}{E_2}\right) = P$ (He reports one when 1 has not occured)

$= P$ (He does not speak truth)

$= 1 - \frac{3}{5}$

$= \frac{2}{5}$

To find, P (It is actually 1 when he reported that it is one on die) $= P\left(\frac{E_1}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{6} \times \frac{3}{5}}{\frac{1}{6} \times \frac{3}{5} + \frac{5}{6} \times \frac{2}{5}}$$

$$= \frac{\frac{3}{30}}{\frac{3}{30} + \frac{10}{30}}$$

$$= \frac{3}{13}$$

Required probability $= \frac{3}{13}$.

Probability Ex 31.7 Q34

Let E_1, E_2 and A events be as:-

E_1 = 5 occurs on die

E_2 = 5 does not occur on die

A = He reports that it was 5

$P(E_1) = \frac{1}{6}$

$P(E_2) = \frac{5}{6}$

$P(A \mid E_1) = P$ (He reports 5 when 5 occurs on die)

$= P$ (He speaks truth)

$= \frac{8}{10}$

$= \frac{4}{5}$

$P\left(\frac{A}{E_2}\right) = P$ (He reports 5 when 5 does not occur on die)

$= P$ (He does not speak truth)

$= \frac{1}{5}$

To find, P (It was actually 5 when he reports that it is five) $= P\left(\frac{E_1}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{6} \times \frac{4}{5}}{\frac{1}{6} \times \frac{4}{5} + \frac{5}{6} \times \frac{1}{5}}$$

$$= \frac{\frac{4}{30}}{\frac{4}{30} + \frac{5}{30}}$$

$$= \frac{4}{9}$$

Required probability $= \frac{4}{9}$.

Probability Ex 31.7 Q35

$P(\text{Knows}) = \frac{3}{4}$

$P(\text{Guesses}) = \frac{1}{4}$

$P\left(\frac{\text{Correct}}{\text{Guesses}}\right) = \frac{1}{4}$

We need to find

$$P\left(\frac{\text{Knows}}{\text{Correctly}}\right) = \frac{P\left(\frac{\text{Correctly}}{\text{knows}}\right)P(\text{Knows})}{P\left(\frac{\text{Correctly}}{\text{knows}}\right)P(\text{Knows}) + P\left(\frac{\text{Correctly}}{\text{Guesses}}\right)P(\text{Guesses})}$$

$$= \frac{1\times\frac{3}{4}}{1\times\frac{3}{4}+\frac{1}{4}\times\frac{1}{4}}$$

$$= \frac{\frac{3}{4}}{\frac{3}{4}+\frac{1}{16}}$$

$$= \frac{\frac{3}{4}}{\frac{12+1}{16}}$$

$$= \frac{12}{13}$$

Probability Ex 31.7 Q36

Let E_1 and E_2 be the respective events that a person has a disease and a person has no disease.

Since E_1 and E_2 are events complimentary to each other,

$P(E_1) + P(E_2) = 1$

$P(E_2) = 1 - P(E_1) = 1 - 0.001 = 0.999$

Let A be the event that the blood test result is positive.

$$P(E_1) = 0.1\% = \frac{0.1}{100} = 0.001$$

$$P(A|E_1) = P(\text{result is positive given the person has disease}) = 99\% = 0.99$$

$$P(A|E_2) = P(\text{result is positive given that the person has no disease}) = 0.5\% = 0.005$$

Probability that a person has a disease, given that his test result is positive, is given by $P(E_1|A)$.

By using Bayes' theorem, we obtain

$$P(E_1|A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2)}$$

$$= \frac{0.001\times 0.99}{0.001\times 0.99 + 0.999\times 0.005}$$

$$= \frac{0.00099}{0.00099 + 0.004995}$$

$$= \frac{0.00099}{0.005985}$$

$$= \frac{990}{5985}$$

$$= \frac{110}{665}$$

$$= \frac{22}{133}$$

Mean and Variance of a Random Variable Ex 32.1 Q1

EX - 32.1

(i) Here

$X:$	3	2	1	0	-1
$P(x):$	0.3	0.2	0.4	0.1	0.05

$$p(x=3)+p(x=2)+p(x=1)+p(x=0)+p(x=-1)$$
$$=0.3+0.2+0.4+0.1+0.05$$
$$=1.05 \neq 1$$

So, the given distribution of probabilities is not a probability distribution.

(ii) Here

$X:$	0	1	2
$P(x):$	0.6	0.4	0.2

$$p(x=0)+p(x=1)+p(x=2)$$
$$=0.6+0.4+0.2$$
$$=1.2 \neq 1$$

So, the given distribution of probabilities is not a probability distribution.

(iii) Here

$X:$	0	1	2	3	4
$P(x):$	0.1	0.5	0.2	0.1	0.1

$$p(x=0)+p(x=1)+p(x=2)+p(x=3)+p(x=4)$$
$$=0.1+0.5+0.2+0.1+0.1$$
$$=1$$

So, the given distribution of probabilities is a probability distribution.

(iv) Here

$X:$	0	1	2	3
$P(x):$	0.3	0.2	0.4	0.1

$$p(x=0)+p(x=1)+p(x=2)+p(x=3)$$
$$=0.3+0.2+0.4+0.1$$
$$=1$$

So, the given distribution of probabilities is a probability distribution.

Mean and Variance of a Random Variable Ex 32.1 Q2

Here

$x:$	-2	-1	0	1	2	3
$P(x):$	0.1	k	0.2	$2k$	0.3	k

We know that,

$$p(-2)+p(-1)+p(0)+p(1)+p(2)+p(3)=1$$
$$\Rightarrow \quad 0.1+k+0.2+2k+0.3+k=1$$
$$\Rightarrow \quad 4k+0.6=1$$
$$\Rightarrow \quad 4k=1-0.6$$
$$\Rightarrow \quad 4k=0.4$$
$$\Rightarrow \quad k=\frac{0.4}{4}$$
$$\Rightarrow \quad k=\frac{1}{10}$$
$$\Rightarrow \quad k=0.1$$

Mean and Variance of a Random Variable Ex 32.1 Q3

Here

$X:$	0	1	2	3	4	5	6	7	8
$P(x):$	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

Since $\sum p(x) = 1$

$$P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7) + P(8) = 1$$

$\Rightarrow \quad = a + 3a + 5a + 7a + 9a + 11a + 13a + 15a + 17a = 1$

$\Rightarrow \quad 81a = 1$

$\Rightarrow \quad a = \frac{1}{81}$

(ii) $P(x < 3) = P(0) + P(1) + P(2)$

$= a + 3a + 5a$

$= 9a$

$= 9\left(\frac{1}{81}\right)$

$\therefore P(x < 3) = \frac{1}{9}$

$P(x \geq 3) = 1 - P(x < 3) = 1 - \frac{1}{9} = \frac{8}{9}$

$P(0 < x < 5) = P(1) + P(2) + P(3) + P(4)$

$= 3a + 5a + 7a + 9a$

$= 24a$

$= 24\left(\frac{1}{81}\right)$

$\therefore P(0 < x < 5) = \frac{8}{27}$

Mean and Variance of a Random Variable Ex 32.1 Q4

Here:-

x:	0	1	2
$P(x):$	$3c^2$	$4c - 10c^2$	$5c - 1$

Where $c > 0$

(i) since $\sum P(x) = 1$

$\Rightarrow \quad P(0) + P(1) + P(2) = 1$

$\Rightarrow \quad 3c^3 + 4c - 10c^2 + 5c - 1 = 1$

$\Rightarrow \quad 3c^3 - 10c^2 + 9c - 2 = 0$

$\Rightarrow \quad 3c^3 - 3c^2 - 7c^2 + 7c + 2c - 2 = 0$

$\Rightarrow \quad 3c^2(c - 1) - 7c(c - 1) + 2(c - 1) = 0$

$\Rightarrow \quad (c - 1)(3c^2 - 7c + 2) = 0$

$\Rightarrow \quad (c - 1)(3c^2 - 6c - c + 2) = 0$

$\Rightarrow \quad (c - 1)(3c(c - 2) - 1(c - 2)) = 0$

$(c - 1)(3c - 1)(c - 2) = 0$

$c = 1, \quad c = 2, \quad c = \frac{1}{3}$

Only $c = \frac{1}{3}$ is possible. Because if c = 1, or c = 2 then P(2) will become negative.

(ii) $P(X < 2) = P(0) + P(1)$

$$= 3c^3 + 4c - 10c^2$$
$$= 3\left(\frac{1}{3}\right)^3 + 4\left(\frac{1}{3}\right) - 10\left(\frac{1}{3}\right)^2$$
$$= \frac{3}{27} + \frac{4}{3} - \frac{10}{9}$$
$$= \frac{1}{9} + \frac{4}{3} - \frac{10}{9}$$
$$= \frac{3}{9}$$

$\therefore P(X < 2) = \frac{1}{3}$

(iii) $P(1 < X \le 2) = P(2)$

$$= 5c - 1$$
$$= 5\left(\frac{1}{3}\right) - 1$$

$\therefore P(1 < X \le 2) = \frac{2}{3}$

Mean and Variance of a Random Variable Ex 32.1 Q5

Here,

$$2P(x_1) = 3P(x_2) = P(x_3) = 5P(x_4)$$

Let $P(x_3) = a$

$2P(x_1) = P(x_3) \quad \Rightarrow \quad P(x_1) = \frac{a}{2}$

$3P(x_2) = P(x_3) \quad \Rightarrow \quad P(x_2) = \frac{a}{3}$

$5P(x_4) = P(x_3) \quad \Rightarrow \quad P(x_4) = \frac{a}{5}$

Since $P(x_1) + P(x_2) + P(x_3) + P(x_4) = 1$

$\Rightarrow \quad \frac{a}{2} + \frac{a}{3} + \frac{a}{1} + \frac{a}{5} = 1$

$\Rightarrow \quad \frac{15a + 10a + 30a + 6a}{30} = 1$

$\Rightarrow \quad 61a = 30$

$\Rightarrow \quad a = \frac{30}{61}$

so,

X :	x_1	x_2	x_3	x_4
$P(X)$:	$\frac{15}{61}$	$\frac{10}{61}$	$\frac{30}{61}$	$\frac{6}{61}$

Mean and Variance of a Random Variable Ex 32.1 Q6

Here,

$$P(x=0)=P(x>0)=P(x<0)$$

Let $P(x=0)=k$

$\Rightarrow \quad P(x>0)=k=P(x<0)$

Since $\Sigma P(x)=1$

$\Rightarrow \quad P(x<0)+P(x=0)+P(x>0)=1$

$\Rightarrow \quad k+k+k=1$

$\Rightarrow \quad 3k=1$

$\Rightarrow \quad k=\frac{1}{3}$

So, $P(x<0)=$

$$P(x=-1)+P(x=-2)+P(x=-3)=\frac{1}{3}$$

$$3P(x=-1)=\frac{1}{3}, \qquad \left[\because \quad P(x=-1)=P(x=-2)=P(x=-3)\right]$$

$$P(x=-1)=\frac{1}{9}$$

$\Rightarrow \quad P(x=-1)=P(x=-2)=P(x=-3)=\frac{1}{9} ----(i)$

$\Rightarrow \quad P(x=0)=\frac{1}{3} ---------(ii)$

and

$$P(x>0)=k$$

$$P(x=1)+P(x=2)+P(x=3)=\frac{1}{3}$$

$$3P(x=1)=\frac{1}{3}, \qquad \left[\because \quad P(x=1)=P(x=2)=P(x=3)\right]$$

$\Rightarrow \quad P(x=1)=\frac{1}{9}$

$\Rightarrow \quad P(x=1)=P(x=2)=P(x=3)=\frac{1}{9} ---(iii)$

From equation (i), (ii), (iii),

x :	-3	-2	-1	0	1	2	3
$P(x)$:	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

Mean and Variance of a Random Variable Ex 32.1 Q7

Let X denote number of aces in a sample of 2 cards drawn.
There are four aces in a pack of 52 cards.
So, X can have values 0, 1, 2

Now,

$$P(x=0)=\frac{48C_2}{52C_2}=\frac{48\times 47}{2}\times\frac{2}{52\times 51}=\frac{188}{221}$$

$$P(x=1)=\frac{48C_1\times 4C_1}{52C_2}=\frac{48\times 4\times 2}{52\times 51}=\frac{32}{221}$$

$$P(x=2)=\frac{4C_2}{52C_2}=\frac{4\times 3}{2}\times\frac{2}{52\times 51}=\frac{1}{221}$$

So,

x :	0	1	2
$P(x)$:	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

Mean and Variance of a Random Variable Ex 32.1 Q8

Probability of getting a Head in one throw of a coin$=\frac{1}{2}$

$$P(H)=\frac{1}{2}$$

$$\Rightarrow \quad P(T)=1-\frac{1}{2}$$

$$\Rightarrow \quad P(T)=\frac{1}{2}$$

Let X denote the number of heads obtained is 3 throws of a coin .

Then $X=0, 1, 2, 3$

Now,

$$P(x=0)=P(T)P(T)P(T)=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$$

$$P(x=1)=P(H)P(T)P(T)+P(T)P(H)P(T)+P(T)P(T)P(H)$$

$$=\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)+\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)+\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)$$

$$P(x=1)=\frac{3}{8}$$

$$P(x=2)=P(H)P(H)P(T)+P(H)P(T)P(H)+P(T)P(H)P(H)$$

$$=\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)+\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)+\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)$$

$$P(x=2)=\frac{3}{8}$$

$$P(x=3)=P(H)P(H)P(H)$$

$$=\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)$$

$$=\frac{1}{8}$$

So,

Required probability distribution is

x :	0	1	2	3
$P(x)$:	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Mean and Variance of a Random Variable Ex 32.1 Q9

Let x denote number of aces drawn out of 4 cards drawn.

There are four ace aces in a pack of 52.

So, $X=0, 1, 2, 3, 4$

Now,

$$P(x=0)=\frac{48C_4}{52C_4}$$

$$P(x=1)=\frac{48C_3\times 4C_1}{52C_4}$$

$$P(x=2)=\frac{48C_2\times 4C_2}{52C_4}$$

$$P(x=3)=\frac{48C_1\times 4C_3}{52C_4}$$

$$P(x=4)=\frac{4C_4}{52C_4}$$

So,

Required probability distribution is

x :	0	1	2	3	4
$P(x)$:	$\frac{48C_4}{52C_4}$	$\frac{48C_3\times 4C_1}{52C_4}$	$\frac{48C_2\times 4C_2}{52C_4}$	$\frac{48C_1\times 4C_3}{52C_4}$	$\frac{4C_4}{52C_4}$

Mean and Variance of a Random Variable Ex 32.1 Q10

A bag has 4 red and 6 black balls. Three balls are drawn.

Let X denote number of red balls out of 3 drawn.

Then $X = 0,1,2,3$.

So,

$$P(\text{no red balls}) = P(X = 0) = \frac{6C_3}{10C_3} = \frac{6\times5\times4}{3\times2}\times\frac{3\times2}{10\times9\times8} = \frac{1}{6}$$

$$P(\text{one red balls}) = P(X = 1) = \frac{4C_1\times6C_2}{10C_3} = \frac{4\times6\times5}{2}\times\frac{3\times2}{10\times9\times8} = \frac{1}{2}$$

$$P(\text{two red balls}) = P(X = 1) = \frac{4C_2\times6C_1}{10C_3} = \frac{4\times3\times6}{2}\times\frac{3\times2}{10\times9\times8} = \frac{3}{10}$$

$$P(\text{all three red}) = P(X = 3) = \frac{4C_3}{10C_3} = \frac{4\times3\times2}{10\times9\times8} = \frac{1}{30}$$

The required probability distribution is

X :	0	1	2	3
$P(x)$:	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

Mean and Variance of a Random Variable Ex 32.1 Q11

Here 5 defective and 15 non-defective mangoes. Let X denote the defective mangoes drawn out of 4 mangoes drawn.

So, $X = 0,1,2,3,4.$

$$P(X=0) = \frac{15C_4}{20C_4}$$
$$= \frac{15\times14\times13\times12}{4\times3\times2\times1}\times\frac{4\times3\times2\times1}{20\times19\times18\times17}$$
$$= \frac{91}{323}$$
$$P(X=1) = \frac{5C_1\times15C_3}{20C_4}$$
$$= \frac{5\times15\times14\times13}{3\times2\times1}\times\frac{4\times3\times2\times1}{20\times19\times18\times17}$$
$$= \frac{455}{969}$$

$$P(X=2) = \frac{5C_2\times15C_2}{20C_4}$$
$$= \frac{5\times4}{2}\times\frac{15\times14}{2}\times\frac{4\times3\times2\times1}{20\times19\times18\times17}$$
$$= \frac{70}{323}$$

$$P(X=3) = \frac{5C_3\times15C_1}{20C_4}$$
$$= \frac{5\times4}{2}\times15\times\frac{4\times3\times2\times1}{20\times19\times18\times17}$$
$$= \frac{10}{323}$$

$$P(X=4) = \frac{5C_4}{20C_4}$$
$$= 5\times\frac{4\times3\times2\times1}{20\times19\times18\times17}$$
$$= \frac{1}{969}$$

So, required probability distribution is

X :	0	1	2	3	4
$P(X)$:	$\frac{91}{323}$	$\frac{455}{969}$	$\frac{70}{323}$	$\frac{10}{323}$	$\frac{1}{969}$

Mean and Variance of a Random Variable Ex 32.1 Q12

Here, X denote the number of sum of two number or two dice thrown together

So, $X = 2,3,4,5,6,7,8,9,10,11,12.$

So,

$P(x=2) = \frac{1}{36}$ [Possible pairs: $(1,1)$]

$P(x=3) = \frac{2}{36} = \frac{1}{18}$ [Possible pairs: $(1,2),(2,1)$]

$P(x=4) = \frac{3}{36} = \frac{1}{12}$ [Possible pairs: $(1,3),(2,2),(3,1)$]

$P(x=5) = \frac{4}{36} = \frac{1}{9}$ [Possible pairs: $(1,4),(2,3),(3,2),(4,1)$]

$P(x = 6) = \frac{5}{36}$ $\quad$ $[\text{Possible pairs: } (1,5),(2,4),(3,3),(4,2),(5,1)]$

$P(x = 7) = \frac{6}{36} = \frac{1}{6}$ $\quad$ $[\text{Possible pairs: } (1,6),(2,5),(3,4),(4,3),(5,2),(6,1)]$

$P(x = 8) = \frac{5}{36}$ $\quad$ $[\text{Possible pairs: } (2,6),(3,5),(4,4),(5,3),(6,2)]$

$P(x = 9) = \frac{4}{36} = \frac{1}{9}$ $\quad$ $[\text{Possible pairs: } (3,6),(4,5),(5,4),(6,3)]$

$P(x = 10) = \frac{3}{36} = \frac{1}{12}$ $\quad$ $[\text{Possible pairs: } (4,6),(5,5),(6,4)]$

$P(x = 11) = \frac{2}{36} = \frac{1}{18}$ $\quad$ $[\text{Possible pairs: } (5,6),(6,5)]$

$P(x = 12) = \frac{1}{36}$ $\quad$ $[\text{Possible pairs: } (6,6)]$

So, required probability distribution is

x :	2	3	4	5	6	7	8	9	10	11	12
$P(x)$:	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

Mean and Variance of a Random Variable Ex 32.1 Q13

There are 15 students in the class. Each student has the same chance to be chosen. Therefore, the probability of each student to be selected is $\frac{1}{15}$.

The given information can be compiled in the frequency table as follows.

X	14	15	16	17	18	19	20	21
f	2	1	2	3	1	2	3	1

$P(X = 14) = \frac{2}{15}$, $P(X = 15) = \frac{1}{15}$, $P(X = 16) = \frac{2}{15}$, $P(X = 16) = \frac{3}{15}$,

$P(X = 18) = \frac{1}{15}$, $P(X = 19) = \frac{2}{15}$, $P(X = 20) = \frac{3}{15}$, $P(X = 21) = \frac{1}{15}$

Therefore, the probability distribution of random variable X is as follows.

X	14	15	16	17	18	19	20	21
f	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

Mean and Variance of a Random Variable Ex 32.1 Q14

Here 5 defective and 20 non-defective bolts. Let X denote the number of defective bolts drawn out of 4 bolts drawn.So, X can have values 0,1,2,3,4.

$$P(X=0)=\frac{20C_4}{25C_4}$$
$$=\frac{20\times19\times18\times17}{25\times24\times23\times22}$$
$$=\frac{969}{2530}$$

$$P(X=1)=\frac{5C_1\times20C_3}{25C_4}$$
$$=\frac{5\times20\times19\times18}{3\times2\times1}\times\frac{4\times3\times2}{25\times24\times23\times22}$$
$$=\frac{114}{253}$$

$$P(X=2)=\frac{5C_2\times20C_2}{25C_4}$$
$$=\frac{5\times4}{2}\times\frac{20\times19}{2}\times\frac{4\times3\times2\times1}{25\times24\times23\times22}$$
$$=\frac{38}{253}$$

$$P(X=3)=\frac{5C_3\times20C_1}{25C_4}$$
$$=\frac{5\times4}{2}\times\frac{20\times4\times3\times2\times1}{25\times24\times23\times22}$$
$$=\frac{4}{253}$$

$$P(X=4)=\frac{5C_4}{25C_4}$$
$$=5\times\frac{4\times3\times2\times1}{25\times24\times23\times22}$$
$$=\frac{1}{2530}$$

So, required probability distribution is

x :	0	1	2	3	4
$P(x)$:	$\frac{969}{2530}$	$\frac{114}{253}$	$\frac{38}{253}$	$\frac{4}{253}$	$\frac{1}{2530}$

Mean and Variance of a Random Variable Ex 32.1 Q15

Two cards are drawn successively with replacement from a pack of 52 cards.

Let X be the number of aces obtained. Then $X = 0,1,2.$

$$P(X=0) = P(\overline{A}_1) \times P(\overline{A}_2)$$
$$= \frac{48}{52} \times \frac{48}{52}$$
$$= \frac{144}{169}$$

$$P(X=1) = P(A_1)P(\overline{A}_2) + P(\overline{A}_1)P(A_2)$$
$$= \frac{4}{52} \times \frac{48}{52} + \frac{48}{52} \times \frac{4}{52}$$
$$= \frac{24}{169}$$

$$P(X=2) = P(A_1)P(A_2)$$
$$= \frac{4}{52} \times \frac{4}{52}$$
$$= \frac{1}{169}$$

So,

Required probability distribution is

x :	0	1	2
$P(x)$:	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

Mean and Variance of a Random Variable Ex 32.1 Q16

Two cards are drawn successively with replcement from a pack of 52 cards.Let X denote the number of kings drawn out of 2 cards.
So, $X = 0,1,2.$

$$P(X = 0) = P(\overline{K}_1) \times P(\overline{K}_2)$$
$$= \frac{48}{52} \times \frac{48}{52}$$
$$= \frac{144}{169}$$

$$P(X = 1) = P(K_1)P(\overline{K}_2) + P(\overline{K}_1)P(K_2)$$
$$= \frac{4}{52} \times \frac{48}{52} + \frac{48}{52} \times \frac{4}{52}$$
$$= \frac{24}{169}$$

$$P(X = 2) = P(K_1)P(K_2)$$
$$= \frac{4}{52} \times \frac{4}{52}$$
$$= \frac{1}{169}$$

So, required probability distribution is

x :	0	1	2
$P(x)$:	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

Mean and Variance of a Random Variable Ex 32.1 Q17

Two cards are drawn without replcement from a pack of 52 cards. Let X denote the number of aces drawn from pack out of 2 cards.
So, $X = 0,1,2.$

$$P(X=0) = \frac{48C_2}{52C_2}$$
$$= \frac{48\times 47}{2}\times\frac{2\times 1}{52\times 51}$$
$$= \frac{188}{221}$$

$$P(X=1) = \frac{4C_1\times 48C_1}{52C_2}$$
$$= \frac{4\times 48\times 2}{52\times 51}$$
$$= \frac{32}{221}$$

$$P(X=2) = \frac{4C_2}{52C_2}$$
$$= \frac{4\times 3}{2}\times\frac{2}{52\times 51}$$
$$= \frac{1}{221}$$

So, required probability distribution is

x :	0	1	2
$P(x)$:	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

Mean and Variance of a Random Variable Ex 32.1 Q18

Given bag have 4 white and 6 red balls.Let X denote the number of white balls out of 3 balls drawn without replcement,
So, $X = 0,1,2,3.$

$$P(\text{No white ball}) = \frac{6C_3}{10C_3}$$
$$= \frac{6\times 5\times 4}{3\times 2}\times\frac{3\times 2}{10\times 9\times 8}$$
$$= \frac{5}{30}$$
$$P(\text{One white ball}) = \frac{4C_1\times 6C_2}{10C_3}$$
$$= \frac{4\times 6\times 5}{2}\times\frac{3\times 2}{10\times 9\times 8}$$
$$= \frac{15}{30}$$
$$P(\text{Two white balls}) = \frac{4C_2\times 6C_1}{10C_3}$$
$$= \frac{4\times 3}{2}\times\frac{6\times 3\times 2}{10\times 9\times 8}$$
$$= \frac{9}{30}$$
$$P(\text{Three white balls}) = \frac{4C_3}{10C_3}$$
$$= \frac{4\times 3\times 2\times 1}{10\times 9\times 8}$$
$$= \frac{1}{30}$$

So,

Required probability distribution is

x :	0	1	2	3
$P(x)$:	$\frac{5}{30}$	$\frac{15}{30}$	$\frac{9}{30}$	$\frac{1}{30}$

Mean and Variance of a Random Variable Ex 32.1 Q19

Since total is 9 when dice has $(3,6)(4,5)(5,4)(6,3)$

$\therefore P(\text{A total of 9 appears}) = p(A) = \frac{4}{36}$

Two dice are thrown 2 times.
Here, Y denotes the numbers of times a total of 9 appears.

So, $Y = 0,1,2$

$$P(Y=0) = P(\overline{A_1}) \times P(\overline{A_2})$$
$$= \frac{32}{36} \times \frac{32}{36}$$
$$= \frac{64}{81}$$
$$P(Y=1) = P(A_1)P(\overline{A_2}) + P(\overline{A_1})P(A_2)$$
$$= \frac{4}{36} \times \frac{32}{36} + \frac{32}{36} \times \frac{4}{36}$$
$$= \frac{16}{81}$$
$$P(Y=2) = P(A_1)P(A_2)$$
$$= \frac{4}{36} \times \frac{4}{36}$$
$$= \frac{1}{81}$$

So,

Required probability distribution is

X :	0	1	2
$P(X)$:	$\frac{64}{81}$	$\frac{16}{81}$	$\frac{1}{81}$

Mean and Variance of a Random Variable Ex 32.1 Q20

Given 25 items in the lot. 5 are defective. Good items are 20.
4 items are chosen at random.
Let X be the random variable that denotes the number of defective items in the selected lot.

P (X = 0) = P (4 non-defective and 0 defective) = $^5C_0 . ^{20}C_4 / ^{25}C_4$

= 4845/12650

P (X = 1) = P (3 non-defective and 1 defective) = $^5C_1 . ^{20}C_3 / ^{25}C_4$

=5x1140/12650

P (X = 2) = P (2 non-defective and 2 defective) = $^5C_2 . ^{20}C_2 / ^{25}C_4$

=10x190/12650

P (X = 3) = P (1 non-defective and 3 defective) = $^5C_3 . ^{20}C_1 / ^{25}C_4$

=10x20/12650

P (X = 4) = P (0 non-defective and 4 defective) = $^5C_4 . ^{20}C_0 / ^{25}C_4$

=5/12650

Mean and Variance of a Random Variable Ex 32.1 Q21

Three cards are thrown with replacement. Let X denote the numbers of hearts if three cards are drawn.
So, X has values 0,1,2,3

$$P(X=0)=P(\overline{H}_1)\times P(\overline{H}_2)\times P(\overline{H}_3)$$
$$=\frac{39}{52}\times\frac{39}{52}\times\frac{39}{52}$$
$$=\frac{27}{26}$$
$$P(X=1)=P(H_1)P(\overline{H}_2)P(\overline{H}_3)+P(\overline{H}_1)P(H_2)P(\overline{H}_3)+P(\overline{H}_1)P(\overline{H}_2)P(H_3)$$
$$=\frac{13}{52}\times\frac{39}{52}\times\frac{39}{52}+\frac{39}{52}\times\frac{13}{52}\times\frac{39}{52}+\frac{39}{52}\times\frac{39}{52}\times\frac{13}{52}$$
$$=\frac{27}{64}$$
$$P(X=2)=P(H_1)P(H_2)P(\overline{H}_3)+P(H_1)P(\overline{H}_2)P(H_3)+P(\overline{H}_1)P(H_2)P(H_3)$$
$$=\frac{13}{52}\times\frac{13}{52}\times\frac{39}{52}+\frac{13}{52}\times\frac{39}{52}\times\frac{13}{52}+\frac{39}{52}\times\frac{13}{52}\times\frac{13}{52}$$
$$=\frac{9}{64}$$
$$P(X=3)=P(H_1)P(H_2)P(H_3)$$
$$=\frac{13}{52}\times\frac{13}{52}\times\frac{13}{52}$$
$$=\frac{1}{64}$$
So,
Required probability distribution is

x :	0	1	2	3
$P(x)$:	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

Mean and Variance of a Random Variable Ex 32.1 Q22

Urn has 4 red and 3 blue balls. 3 balls are drawn with replacement.
Let X denote numbers of blue balls drawn out of 3 drawn.
So, X has values 0,1,2,3

$$P(X=0)=P(\overline{B}_1)\times P(\overline{B}_2)\times P(\overline{B}_3)$$
$$=\frac{4}{7}\times\frac{4}{7}\times\frac{4}{7}$$
$$=\frac{64}{343}$$
$$P(X=1)=P(B_1)P(\overline{B}_2)P(\overline{B}_3)+P(\overline{B}_1)P(B_2)P(\overline{B}_3)+P(\overline{B}_1)P(\overline{B}_2)P(B_3)$$
$$=\frac{3}{7}\times\frac{4}{7}\times\frac{4}{7}+\frac{4}{7}\times\frac{3}{7}\times\frac{4}{7}+\frac{4}{7}\times\frac{4}{7}\times\frac{3}{7}$$
$$=\frac{144}{343}$$
$$P(X=2)=P(B_1)P(B_2)P(\overline{B}_3)+P(B_1)P(\overline{B}_2)P(B_3)+P(\overline{B}_1)P(B_2)P(B_3)$$
$$=\frac{3}{7}\times\frac{3}{7}\times\frac{4}{7}+\frac{3}{7}\times\frac{4}{7}\times\frac{3}{7}+\frac{4}{7}\times\frac{3}{7}\times\frac{3}{7}$$
$$=\frac{108}{343}$$
$$P(X=3)=P(B_1)P(B_2)P(B_3)$$
$$=\frac{3}{7}\times\frac{3}{7}\times\frac{3}{7}$$
$$=\frac{27}{343}$$
So,
Required probability distribution is

x :	0	1	2	3
$P(x)$:	$\frac{64}{343}$	$\frac{144}{343}$	$\frac{108}{343}$	$\frac{27}{343}$

Mean and Variance of a Random Variable Ex 32.1 Q23

Two cards are drawn simultaneously .Let X denote the number of spades obtained.

So, X can have values 0,1,2.

$$P(X=0)=\frac{39C_2}{52C_2}$$
$$=\frac{39\times 38}{52\times 51}$$
$$=\frac{19}{34}$$
$$P(X=1)=\frac{39C_1\times 13C_1}{52C_2}$$
$$=\frac{13\times 39\times 2}{52\times 51}$$
$$=\frac{13}{34}$$
$$P(X=2)=\frac{13C_2}{52C_2}$$
$$=\frac{13\times 12}{52\times 51}$$
$$=\frac{2}{34}$$

So,

Required probability distribution is

x :	0	1	2
$P(x)$:	$\frac{19}{34}$	$\frac{13}{34}$	$\frac{2}{34}$

Mean and Variance of a Random Variable Ex 32.1 Q24

Let A be the event of occurence of a number less than 3.

$$P(A)=\frac{2}{6} \qquad [\because 1,2 \text{ are less than } 3.]$$
$$P(A)=\frac{1}{3}$$

Let X denote the number of success is 2 throws of die.

So, X has value 0,1,2.

$$P(X=0)=P(\overline{A}_1)\times P(\overline{A}_2)$$
$$=\frac{2}{3}\times\frac{2}{3}$$
$$=\frac{4}{9}$$
$$P(X=1)=P(A_1)P(\overline{A}_2)+P(\overline{A}_1)P(A_2)$$
$$=\frac{1}{3}\times\frac{2}{3}+\frac{2}{3}\times\frac{1}{3}$$
$$=\frac{4}{9}$$
$$P(X=2)=P(A_1)P(A_2)$$
$$=\frac{1}{3}\times\frac{1}{3}$$
$$=\frac{1}{9}$$

So,

Required probability distribution is

x :	0	1	2
$P(x)$:	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

Mean and Variance of a Random Variable Ex 32.1 Q25

Urn has 5 red and 2 black balls.2 balls are randomly selected.
Here, X denote the numbers of black balls.
So, possible values of X =0,1,2

$$P(X=0)=P(\bar{B}_1)\times P(\bar{B}_2)$$
$$=\frac{5}{7}\times\frac{5}{7}$$
$$=\frac{25}{49}$$
$$P(X=1)=P(B_1)P(\bar{B}_2)+P(\bar{B}_1)P(B_2)$$
$$=\frac{2}{7}\times\frac{5}{7}+\frac{5}{7}\times\frac{2}{7}$$
$$=\frac{20}{49}$$
$$P(X=2)=P(B_1)P(B_2)$$
$$=\frac{2}{7}\times\frac{2}{7}$$
$$=\frac{4}{49}$$

Now,
$$P(x=0)+P(x=1)+P(x=2)$$
$$=\frac{25}{49}+\frac{20}{49}+\frac{4}{49}$$
$$=\frac{49}{49}$$
$$=1$$

So, $\Sigma P(x)=1$
Therefore

X is a random variable

Mean and Variance of a Random Variable Ex 32.1 Q26

Here, coin is tossed 6 times.
So, there can have
$1H\,5T$ or $2H\,4T$ or $3H\,3T$ or
$4H\,2T$ or $5H\,1T$ or $6H$ or
$6T$
Here, X denote the difference between the number of head and number of tails.
So,

$X=6,4,2,0,-2,-4,-6$

Mean and Variance of a Random Variable Ex 32.1 Q27

It is given that out of 10 bulbs, 3 are defective.
Number of non-defective bulbs $=10-3=7$
2 bulbs are drawn from the lot with replacement.
Let X be the random variable that denotes the number of defective bulbs in the selected bulbs.

$$\therefore P(X=0)=\frac{{}^7C_2}{{}^{10}C_2}$$
$$=\frac{7}{15}$$
$$\therefore P(X=1)=\frac{{}^3C_1\times{}^7C_1}{{}^{10}C_2}$$
$$=\frac{7}{15}$$
$$\therefore P(X=2)=\frac{{}^3C_2}{{}^{10}C_2}$$
$$=\frac{1}{15}$$

Therefore, the required probability distribution is

X	0	1	2
$P(X)$	$\frac{7}{15}$	$\frac{7}{15}$	$\frac{1}{15}$

Mean and Variance of a Random Variable Ex 32.1 Q28

Clearly, X can assume values 0, 1, 2, 3, 4 such that

$$P(X = 0) = (\text{Probablity of getting no red ball}) = \frac{{}^8C_0 \times {}^4C_4}{{}^{12}C_4} = \frac{1 \times 1}{\frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}} = \frac{1}{495}$$

$$P(X = 1) = (\text{Probablity of getting one red ball}) = \frac{{}^8C_1 \times {}^4C_3}{{}^{12}C_4} = \frac{8 \times 4}{\frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}} = \frac{32}{495}$$

$$P(X = 2) = (\text{Probablity of getting two red balls}) = \frac{{}^8C_2 \times {}^4C_2}{{}^{12}C_4} = \frac{\frac{8 \times 7}{2 \times 1} \times \frac{4 \times 3}{2 \times 1}}{\frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}} = \frac{168}{495}$$

$$P(X = 3) = (\text{Probablity of getting three red balls}) = \frac{{}^8C_3 \times {}^4C_1}{{}^{12}C_4} = \frac{\frac{8 \times 7 \times 6}{3 \times 2 \times 1} \times 4}{\frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}} = \frac{224}{495}$$

$$P(X = 4) = (\text{Probablity of getting four red balls}) = \frac{{}^8C_4 \times {}^4C_0}{{}^{12}C_4} = \frac{\frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} \times 1}{\frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}} = \frac{70}{495}$$

Thus, probablity distribution of random veriable X is,

X	0	1	2	3	4
P(X)	$\frac{1}{495}$	$\frac{32}{495}$	$\frac{168}{495}$	$\frac{224}{495}$	$\frac{70}{495}$

(i) We know that,

$$P(0)+P(1)+P(2)+P(3)=1$$

$$k+\frac{k}{2}+\frac{k}{4}+\frac{k}{8}=1$$

$$15k=8$$

$$k=\frac{8}{15}$$

(ii) $P(X \le 2)$

$$=P(0)+P(1)+P(2)$$

$$=k+\frac{k}{2}+\frac{k}{4}$$

$$=\frac{8}{15}+\frac{8}{30}+\frac{8}{60}$$

$$=\frac{14}{15}$$

$$P(X>2)=P(3)=\frac{k}{8}=\frac{1}{15}$$

(iii) $P(X \le 2)+P(X>2)$

$$=P(0)+P(1)+P(2)+P(3)$$

$$=1$$

EX - 32.2

Mean and Variance of a Random Variable Ex 32.2 Q1(i)

x_i	p_i	p_ix_i	$p_ix_i^2$
2	0.2	0.4	0.8
3	0.5	1.5	4.5
4	0.3	1.2	4.8
		$\sum p_ix_i = 3.1$	$\sum p_ix_i^2 = 10.1$

$\text{Mean} = \sum p_ix_i = 3.1$

$\text{Variance} = \sum p_ix_i^2 - (\sum p_ix_i)^2 = 10.1 - (3.1)^2 = 0.49$

$\text{Standard deviation} = \sqrt{\text{Variance}} = 0.7$

Mean and Variance of a Random Variable Ex 32.2 Q1(ii)

x_i	p_i	$x_i\ p_i$	$x_i^2p_i$
1 3 4 5	0.4 0.1 0.2 0.3	0.4 0.3 0.8 1.5	0.4 0.9 3.2 7.5
		$\sum xp = 3$	$\sum x^2p = 12$

Mean = $\sum xp$
mean = 3

$$\text{Standard deviation} = \sqrt{\sum x^2p - (\text{mean})^2}$$
$$= \sqrt{12 - (3)^2}$$
$$= \sqrt{3}$$

Standard Deviation = 1.732

Mean and Variance of a Random Variable Ex 32.2 Q1(iii)

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
-5	$\frac{1}{4}$	$-\frac{5}{4}$	$\frac{25}{4}$
-4	$\frac{1}{8}$	$-\frac{1}{2}$	2
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
2	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$
		$\Sigma xp = -1$	$\Sigma x^2 p = \frac{37}{4}$

Mean = Σxp

mean = -1

Standard deviation = $\sqrt{\Sigma x^2 p - (\text{mean})^2}$

$= \sqrt{\frac{37}{4} - (-1)^2}$

$= \sqrt{\frac{33}{4}}$

$= \sqrt{8.25}$

Standard Deviation = 2.9

Mean and Variance of a Random Variable Ex 32.2 Q1(iv)

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
-1	0.3	-0.3	0.3
0	0.1	0	0
1	0.1	0.1	0.1
2	0.3	0.6	1.2
3	0.2	0.6	1.8
		$\Sigma xp = 1$	$\Sigma x^2 p = 3.4$

Mean = Σxp

mean = 1

Standard deviation = $\sqrt{\Sigma x^2 p - (\text{mean})^2}$

$= \sqrt{(3.4) - (1)^2}$

$= \sqrt{2.4}$

Standard Deviation = 1.5

Mean and Variance of a Random Variable Ex 32.2 Q1(v)

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
1	0.4	0.4	0.4
2	0.3	0.6	1.2
3	0.2	0.6	1.8
4	0.1	0.4	1.6
		$\Sigma xp = 2$	$\Sigma x^2 p = 5$

Mean = Σxp
mean = 2

Standard deviation = $\sqrt{\Sigma x^2 p - (\text{mean})^2}$
$= \sqrt{5 - (2)^2}$

Standard Deviation = 1

Mean and Variance of a Random Variable Ex 32.2 Q1(vi)

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
0	0.2	0	0
1	0.5	0.5	0.5
3	0.2	0.6	1.8
5	0.1	0.5	2.5
		$\Sigma xp = 1.6$	$\Sigma x^2 p = 4.8$

Mean = Σxp
mean = 1.6

Standard deviation = $\sqrt{\Sigma x^2 p - (\text{mean})^2}$
$= \sqrt{4.8 - (1.6)^2}$
$= \sqrt{4.8 - 2.56}$
$= \sqrt{2.24}$

Standard Deviation = 1.497

Mean and Variance of a Random Variable Ex 32.2 Q1(vii)

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
-2	0.1	-0.2	0.4
-1	0.2	-0.2	0.2
0	0.4	0	0
1	0.2	0.2	0.2
2	0.1	0.2	0.4
		$\Sigma xp = 0$	$\Sigma x^2 p = 1.2$

Mean = Σxp
mean = 0

Standard deviation = $\sqrt{\Sigma x^2p - (\text{mean})^2}$
$= \sqrt{(1.2)^2 - (0)^2}$

Standard Deviation = 1.2

Mean and Variance of a Random Variable Ex 32.2 Q1(viii)

x_i	p_i	$x_i\ p_i$	$x_i^2p_i$
-3	0.05	-0.15	0.45
-1	0.45	-0.45	0.45
0	0.20	0	0
1	0.25	0.25	0.25
3	0.05	0.15	0.45
		$\Sigma xp = -0.2$	$\Sigma x^2p = 1.6$

Mean = Σxp
mean = -0.2

Standard deviation = $\sqrt{\Sigma x^2p - (\text{mean})^2}$
$= \sqrt{1.6 - (-0.2)^2}$
$= \sqrt{1.6 - 0.04}$
$= \sqrt{1.56}$

Standard Deviation = 1.249

Mean and Variance of a Random Variable Ex 32.2 Q1(ix)

X_i	P_i	P_iX_i	$P_iX_i^2$
0	$\frac{1}{6}$	0	0
1	$\frac{5}{18}$	$\frac{5}{18}$	$\frac{5}{18}$
2	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{8}{9}$
3	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{2}$
4	$\frac{1}{9}$	$\frac{4}{9}$	$\frac{16}{9}$
5	$\frac{1}{18}$	$\frac{5}{18}$	$\frac{25}{18}$
		$\Sigma P_iX_i = \frac{35}{18}$	$\Sigma P_iX_i^2 = \frac{35}{6}$

$$\text{Mean} = \Sigma P_iX_i = \frac{35}{18}$$

$$\text{Variance} = \Sigma P_iX_i^2 - (\Sigma P_iX_i)^2 = \frac{35}{6} - \left(\frac{35}{18}\right)^2 = \frac{665}{324}$$

$$\text{Standard deviation} = \sqrt{\text{Variance}} = \frac{\sqrt{665}}{18}$$

Mean and Variance of a Random Variable Ex 32.2 Q2

(i) We know that,

$P(0.5)+P(1)+P(1.5)+P(2)=1$

$k+k^2+2k^2+k=1$

$3k^2+2k-1=0$

$3k^2+3k-k-1=0$

$(3k-1)(k+1)=0$

$k=\frac{1}{3}$ or $k=-1$

We know that $0 \le P(X) \le 1$

$\therefore k=\frac{1}{3}$

(ii)

x_i	p_i	$p_i x_i$
0.5	$\frac{1}{3}$	$\frac{1}{6}$
1	$\frac{1}{9}$	$\frac{1}{9}$
1.5	$\frac{2}{9}$	$\frac{1}{3}$
2	$\frac{1}{3}$	$\frac{2}{3}$
		$\sum p_i x_i = \frac{23}{18}$

$$\text{Mean} = \sum p_i x_i = \frac{23}{18}$$

Mean and Variance of a Random Variable Ex 32.2 Q3

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
a	p	ap	a^2p
b	q	bq	b^2q

Mean $= \sum xp$

mean $= ap + bq$

Variance $= \sum x^2p - (\text{mean})^2$

$= \left(a^2p + b^2q\right) - (ap + bq)^2$

$= a^2p + b^2q - a^2p^2 - b^2q^2 - 2abpq$

$= a^2pq + b^2pq - 2abpq \qquad [\because \quad p+q=1]$

$= pq\left(a^2 + b^2 - 2ab\right)$

Variance $= pq\,(a-b)^2$

Standard deviation $= |a-b|\sqrt{pq}$

Mean and Variance of a Random Variable Ex 32.2 Q4

We know that in a throw of coin.

$P(H)=\frac{1}{2}, \qquad P(T)=\frac{1}{2}$

Let X denote the number of heads in three tosses of coin.

So, $X = 0,1,2,3$

$$\begin{aligned}
P(x=0) &= P(T)P(T)P(T)\\
&= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\\
&= \frac{1}{8}\\
P(x=1) &= P(H)P(T)P(T)+P(T)P(H)P(T)+P(T)P(T)P(H)\\
&= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\\
&= \frac{3}{8}\\
P(x=2) &= P(H)P(H)P(T)+P(H)P(T)P(H)+P(T)P(H)P(H)\\
&= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\\
&= \frac{3}{8}\\
P(x=3) &= P(H)P(H)P(H)\\
&= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\\
&= \frac{1}{8}
\end{aligned}$$

So,

x_i	p_i	$x_i\, p_i$	$x_i^2 p_i$
0	$\frac{1}{8}$	0	0
1	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$
2	$\frac{3}{8}$	$\frac{6}{8}$	$\frac{12}{8}$
3	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{9}{8}$
		$\Sigma xp = \frac{3}{2}$	$\Sigma x^2p = 3$

Mean $= \Sigma xp$

mean $= \frac{3}{2}$

$$\begin{aligned}
\text{Variance} &= \Sigma x^2p - (\text{mean})^2\\
&= 3-\frac{9}{4}
\end{aligned}$$

Variance $= \frac{3}{4}$

Mean and Variance of a Random Variable Ex 32.2 Q5

Two cards are drawn simultneously from a pack of 52 cards.
Let X denotes the number of kings drawn.

So, $X = 0,1,2$

$$P(x=0) = \frac{48C_2}{52C_2}$$
$$= \frac{48 \times 47}{52 \times 51}$$
$$= \frac{188}{221}$$
$$P(x=1) = \frac{4C_1 \times 48C_1}{52C_2}$$
$$= \frac{4 \times 48 \times 2}{52 \times 51}$$
$$= \frac{32}{221}$$
$$P(x=2) = \frac{4C_2}{52C_2}$$
$$= \frac{4 \times 3}{52 \times 51}$$
$$= \frac{1}{221}$$

So,

x_i	p_i	$x_i\,p_i$	$x_i^2 p_i$
0	$\frac{188}{221}$	0	0
1	$\frac{32}{221}$	$\frac{32}{221}$	$\frac{32}{221}$
2	$\frac{1}{221}$	$\frac{2}{221}$	$\frac{4}{221}$
		$\Sigma xp = \frac{34}{221}$	$\Sigma x^2p = \frac{36}{221}$

Mean $= \Sigma xp$
$$\text{mean} = \frac{34}{221}$$

$$\text{Variance} = \Sigma x^2p - (\text{mean})^2$$
$$= \frac{36}{221} - \left(\frac{34}{221}\right)^2$$
$$= \frac{7956 - 1156}{48841}$$
$$= \frac{6800}{48841}$$

$$\text{Variance} = \frac{400}{2873}$$

Mean and Variance of a Random Variable Ex 32.2 Q6

We know that ,in a throw of coin.

$P(T) = \frac{1}{2}, \quad P(H) = \frac{1}{2}$

Let X denote the number of tails in three throws of coins.

So, X can take values from 0,1,2,3

$$P(x=0) = P(H)P(H)P(H)$$
$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$$
$$= \frac{1}{8}$$
$$P(x=1) = P(T)P(H)P(H) + P(H)P(T)P(H) + P(H)P(H)P(T)$$
$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$$
$$= \frac{3}{8}$$
$$P(x=2) = P(T)P(T)P(H) + P(T)P(H)P(T) + P(H)P(T)P(T)$$
$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$$
$$= \frac{3}{8}$$
$$P(x=3) = P(T)P(T)P(T)$$
$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$$
$$= \frac{1}{8}$$

So,

Mean $= \sum xp$

mean $= \frac{3}{2}$

$$\text{Variance} = \sum x^2p - (\text{mean})^2$$
$$= 3 - \left(\frac{3}{2}\right)^2$$
$$= 3 - \frac{9}{4}$$

Variance $= \frac{3}{4}$

Standard Deviation $= \sqrt{\text{Variance}}$

$$= \sqrt{\frac{3}{4}}$$

Standard Deviation = 0.87

Mean and Variance of a Random Variable Ex 32.2 Q7

Total 12 good and bad eggs. 10 are good and 2 are bad.
3 eggs are drawn from this lot
Let X be the random variable that denotes the number of bad eggs in the lot.

P (X = 0) = P (3good and 0 bad) = ${}^{3}C_0 . {}^{10}C_3 / {}^{12}C_3$

$= 1 \times 120/220 = 6/11$

P (X = 1) = P (2good and 1 bad) = ${}^{2}C_1 . {}^{10}C_2 / {}^{12}C_3$

$= 2\times 45/220 = 9/22$

P (X = 2) = P (1good and 2 bad) = ${}^{2}C_2 . {}^{10}C_1 / {}^{12}C_3$

$= 1\times10/220 = 1/22$

The probability distribution of X is

X	0	1	2
P(X)	$\frac{6}{11}$	$\frac{9}{22}$	$\frac{1}{22}$

The mean $= 0\times\frac{6}{11}+1\times\frac{9}{22}+2\times\frac{1}{22} = \frac{11}{22} = 1/2$

Mean and Variance of a Random Variable Ex 32.2 Q8

A pair of dice is thrown.And X denote minimum of the two number appeared.
So, X can have values 2,3,4,5,6.

$P(x=1)=\frac{11}{36}$ $\left[\text{Possible pairs: } (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(3,1),(4,1),(5,1),(6,1)\right]$

$P(x=2)=\frac{9}{36}$ $\left[\text{Possible pairs: } (2,2),(2,3),(2,4),(2,5),(2,6),(3,2),(4,2),(5,2),(6,2)\right]$

$P(x=3)=\frac{7}{36}$ $\left[\text{Possible pairs: } (3,3),(3,4),(3,5),(3,6),(4,3),(5,3),(6,3)\right]$

$P(x=4)=\frac{5}{36}$ $\left[\text{Possible pairs: } (4,4),(4,5),(4,6),(5,4),(6,4)\right]$

$P(x=5)=\frac{3}{36}$ $\left[\text{Possible pairs: } (5,5),(5,6),(6,5)\right]$

$P(x=6)=\frac{1}{36}$ $\left[\text{Possible pairs: } (6,6)\right]$

x_i	p_i	$x_i\,p_i$	$x_i^2 p_i$
1	$\frac{11}{36}$	$\frac{11}{36}$	$\frac{11}{36}$
2	$\frac{9}{36}$	$\frac{18}{36}$	$\frac{36}{36}$
3	$\frac{7}{36}$	$\frac{21}{36}$	$\frac{63}{36}$
4	$\frac{5}{36}$	$\frac{20}{36}$	$\frac{80}{36}$
5	$\frac{3}{36}$	$\frac{15}{36}$	$\frac{75}{36}$
6	$\frac{1}{36}$	$\frac{6}{36}$	$\frac{36}{36}$
		$\Sigma xp=\frac{91}{36}$	$\Sigma x^2p=\frac{301}{36}$

Mean $=\Sigma xp$
Mean $=\frac{91}{36}$

$$\text{Variance}=\Sigma x^2p-(\text{mean})^2$$
$$=\frac{301}{36}-\left(\frac{91}{36}\right)^2$$
$$=\frac{10836-8281}{1296}$$
$$=\frac{2555}{1296}$$

Variance = 1.97

Probability distribution is

$$\begin{array}{lcccccc} x & : & 1 & 2 & 3 & 4 & 5 & 6 \\ P(x) & : & \frac{11}{36} & \frac{9}{36} & \frac{7}{36} & \frac{5}{36} & \frac{3}{36} & \frac{1}{36} \end{array}$$

Mean and Variance of a Random Variable Ex 32.2 Q9

We know that ,In a toss of coin.

$P(T) = \frac{1}{2}, \qquad P(H) = \frac{1}{2}$

Let X denote the number of occuring head in 4 throws of coins.

So, X can take values from $X = 0,1,2,3,4$

$$P(x=0) = P(T)P(T)P(T)P(T)$$
$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$$
$$= \frac{1}{16}$$
$$P(x=1) = P(H)P(T)P(T)P(T)\times {}^4C_1$$
$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times 4$$
$$= \frac{4}{16}$$
$$P(x=2) = P(H)P(H)P(T)P(T)\times {}^4C_2$$
$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times 6$$
$$= \frac{6}{16}$$
$$P(x=3) = P(H)P(H)P(H)P(T)\times {}^4C_3$$
$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times 4$$
$$= \frac{4}{16}$$
$$P(x=4) = P(H)P(H)P(H)P(H)$$
$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$$
$$= \frac{1}{16}$$

So,

x_i	p_i	$x_i\, p_i$	$x_i^2 p_i$
0	$\frac{1}{16}$	0	0
1	$\frac{4}{16}$	$\frac{4}{16}$	$\frac{4}{16}$
2	$\frac{6}{16}$	$\frac{12}{16}$	$\frac{24}{16}$
3	$\frac{4}{16}$	$\frac{12}{16}$	$\frac{36}{16}$
4	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{16}{16}$
		$\Sigma xp = 2$	$\Sigma x^2 p = 5$

Mean $= \Sigma xp$

mean $= 2$

Variance $= \Sigma x^2 p - (\text{mean})^2$

$= 5 - (2)^2$

Variance $= 1$

Probability distribution is

x :	0	1	2	3	4
$P(x)$:	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

Mean and Variance of a Random Variable Ex 32.2 Q10

X denotes twice the number appearing on the die.
So, $X = 2,4,6,8,10,12$.

Probability distribution is

$$\begin{array}{lcccccc} X: & 2 & 4 & 6 & 8 & 10 & 12 \\ P(x): & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{array}$$

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
2	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{4}{6}$
4	$\frac{1}{6}$	$\frac{4}{6}$	$\frac{16}{6}$
6	$\frac{1}{6}$	$\frac{6}{6}$	$\frac{36}{6}$
8	$\frac{1}{6}$	$\frac{8}{6}$	$\frac{64}{6}$
10	$\frac{1}{6}$	$\frac{10}{6}$	$\frac{100}{6}$
12	$\frac{1}{6}$	$\frac{12}{6}$	$\frac{144}{6}$
		$\Sigma xp = 7$	$\Sigma x^2p = \frac{364}{6}$

Mean = Σxp
mean = 7

$$\begin{aligned} \text{Variance} &= \Sigma x^2p - (\text{mean})^2 \\ &= \left(\frac{364}{6}\right) - (7)^2 \\ &= \frac{364 - 294}{6} \\ &= \frac{70}{6} \end{aligned}$$

Variance = 11.7

Mean and Variance of a Random Variable Ex 32.2 Q11

Probability of even number = $P(E) = \frac{3}{6} = \frac{1}{2}$

$\Rightarrow \quad P(O) = \frac{1}{2}$

Here, X have values 1 or 3 according as an odd or even number.

So,

$$\begin{array}{lcc} X: & 1 & 3 \\ P(x): & \frac{1}{2} & \frac{1}{2} \end{array}$$

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
3	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{9}{2}$
		$\Sigma xp = 2$	$\Sigma x^2p = 5$

Mean = $\sum xp$
mean = 2

Variance = $\sum x^2p - (\text{mean})^2$
= 5 − 4

Variance = 1

Mean and Variance of a Random Variable Ex 32.2 Q12

Let the event of getting a head = H and getting a tail = T
Let X denote the variable longest consecutive heads occurring in 4 tosses. The possible values are

X = 0 (no head) {T, T, T, T}
X = 1 (1 heads) {H, T, T, T}
X = 2 (2 heads) {H, H, T, T}
X = 3 (3 heads) {H, H, H, T}
X = 4 (4 heads) {H, H, H, H}

n(S) = {(HHHH),(HHHT),(HHTT),(HTHH),(HTHT),(HTTH),(HTTT),
(THHH),(THTH),(THHT),(THTT),(THTT),(TTHH),(TTHT)
(TTTH),(TTTT)}

$P(X=0) = \frac{1}{16}$

$P(X=1) = \frac{7}{16}$

$P(X=2) = \frac{5}{16}$

$P(X=3) = \frac{2}{16}$

$P(X=4) = \frac{1}{16}$

Probability distribution is

X	0	1	2	3	4
$p_i = P(X)$	$\frac{1}{16}$	$\frac{7}{16}$	$\frac{5}{16}$	$\frac{2}{16}$	$\frac{1}{16}$
$p_i x_i^2$	0	$\frac{7}{16}$	$\frac{20}{16}$	$\frac{18}{16}$	1

Mean = $\sum_{i=1\ to\ n} X_i \times P(X_i)$

Mean, $\mu = 0 \times \frac{1}{16} + 1 \times \frac{7}{16} + 2 \times \frac{5}{16} + 3 \times \frac{2}{16} + 4 \times \frac{1}{16}$
$= 0 + \frac{7}{16} + \frac{10}{16} + \frac{6}{16} + \frac{4}{16}$
$= \frac{27}{16} = 1.7$
Variance Var(X) = $\sum p_i x_i^2 - (\sum p_i x_i)^2$
$= \frac{61}{16} - 1.7^2$
=3.825-2.89
= 0.935

Mean and Variance of a Random Variable Ex 32.2 Q13

Box contains five cards 1,1,2,2,3.

Here,
X denotes the sum of two number on cards drawn.
Y denotes the maximum of the two number.

So, $X = 2,3,4,5$
$Y = 1,2,3$

$$P(X=2) = P(1)P(1)$$
$$= \frac{2}{5} \times \frac{1}{4}$$
$$= 0.1$$
$$P(X=3) = P(1)P(2) + P(2)P(1)$$
$$= \frac{2}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{2}{4}$$
$$= 0.4$$
$$P(X=4) = P(2)P(2) + P(1)P(3) + P(3)P(1)$$
$$= \frac{2}{5} \times \frac{1}{4} + \frac{2}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{2}{4}$$
$$= 0.3$$
$$P(X=5) = P(2)P(3) + P(3)P(2)$$
$$= \frac{2}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{2}{4}$$
$$= 0.2$$

Probability Distribution for X

X :	2	3	4	5
$P(x)$:	0.1	0.4	0.3	0.2

x_i	p_i	$x_i\,p_i$	$x_i^2 p_i$
2	0.1	0.1	0.4
3	0.4	1.2	3.6
4	0.3	1.2	4.8
5	0.2	1.0	5.0
		$\sum xp = 3.6$	$\sum x^2 p = 13.8$

Mean = $\sum xp$

mean = 3.6

Variance = $\sum x^2 p - (\text{mean})^2$

$= 13.8 - (3.6)^2$

$= 13.8 - 12.96$

Variance = 0.84

$P(Y=1) = P(1)P(1)$

$= \frac{2}{5} \times \frac{1}{4}$

$= \frac{2}{20}$

$= 0.1$

$P(Y=2) = P(1)P(2) + P(2)P(1) + P(2)P(2)$

$= \frac{2}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{1}{4}$

$P(Y=2) = 0.5$

$P(Y=3) = P(1)P(3) + P(2)P(3) + P(3)P(1) + P(3)P(2)$

$= \frac{2}{5} \times \frac{1}{4} + \frac{2}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{2}{4} + \frac{1}{5} \times \frac{2}{4}$

$= 0.4$

Probability distribution for Y is

X :	1	2	3
$P(x)$:	0.1	0.5	0.4

Y_i	p_i	$Y_i\, p_i$	$Y_i^2 p_i$
1	0.1	0.1	0.1
2	0.5	1.0	2.0
3	0.4	1.2	3.6
		$\sum xp = 2.6$	$\sum x^2 p = 5.7$

Mean = $\sum xp$

mean = 2.3

Variance = $\sum x^2 p - (\text{mean})^2$

$= 5.7 - (2.3)^2$

Variance = 0.41

Mean and Variance of a Random Variable Ex 32.2 Q14

Probability of getting an odd number = $P(O) = \frac{1}{2}$

$\Rightarrow \quad P(E) = \frac{1}{2}$

Die is tossed twice. Let X denote the number of times an odd number occurs.

So, $X = 0,1,2.$

$$P(X=0) = P(E)P(E)$$
$$= \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{1}{4}$$
$$P(X=1) = P(O)P(E) + P(E)P(O)$$
$$= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{1}{2}$$
$$P(X=2) = P(O)P(O)$$
$$= \frac{1}{2} \times \frac{1}{2}$$
$$P(X=2) = \frac{1}{4}$$

x_i	p_i	$x_i\, p_i$	$x_i^2 p_i$
0	$\frac{1}{4}$	0	0
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
2	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{4}{4}$
		$\Sigma xp = 1$	$\Sigma x^2 p = \frac{3}{2}$

Mean = $\Sigma xp = 1$

Variance = $\Sigma x^2 p - (\text{mean})^2$
$$= \frac{3}{2} - 1$$

Variance = $\frac{1}{2}$

Mean and Variance of a Random Variable Ex 32.2 Q15

Out of 13 bulbs 5 are defective $\Rightarrow$ 8 bulbs are good.
3 bulbs are drawn without replacement ,
Let X denote number of defective bulbs,
So, X can have values 0,1,2,3

$$P(X=0) = p(\text{No defective})$$
$$= \frac{8C_3}{13C_3}$$
$$= \frac{8\times7\times6}{13\times12\times11}$$
$$= \frac{28}{143}$$
$$P(X=1) = p(\text{Only one defective})$$
$$= \frac{8C_2\times5C_1}{13C_3}$$
$$= \frac{8\times7\times5}{2}\times\frac{3\times2\times1}{13\times12\times11}$$
$$= \frac{70}{143}$$
$$P(X=2) = p(\text{Only two defective})$$
$$= \frac{8C_1\times5C_2}{13C_3}$$
$$= \frac{8\times5\times4}{2}\times\frac{3\times2\times1}{13\times12\times11}$$
$$= \frac{40}{143}$$
$$P(X=3) = p(\text{all three are defective})$$
$$= \frac{5C_3}{13C_3}$$
$$= \frac{4\times5}{2}\times\frac{3\times2\times1}{13\times12\times11}$$
$$= \frac{5}{143}$$
So, Probability distribution is

X :	0	1	2	3
$P(x)$:	$\frac{28}{143}$	$\frac{70}{143}$	$\frac{40}{143}$	$\frac{5}{143}$

Mean and Variance of a Random Variable Ex 32.2 Q16

$P(\text{win}) = \frac{1}{13} \Rightarrow \qquad P(\text{lose}) = \frac{12}{13}$

He gains Rs 90 if he wins and loses Rs 10 if his number does not appear.

Let X denote total loss or gain, so,

X :	90	-10
$P(x)$:	$\frac{1}{13}$	$\frac{12}{13}$
XP :	$\frac{90}{13}$	$\frac{-120}{13}$

$$E(X) = \Sigma XP$$
$$= \frac{90}{13} - \frac{120}{13}$$
$$E(X) = -\frac{30}{13}$$

Mean and Variance of a Random Variable Ex 32.2 Q17

Let 'X' be the random variable which can assume values from 0 to 3.

$$P(X=0)=\frac{^{26}C_3}{^{52}C_3}=\frac{2600}{22100}=\frac{2}{17}$$

$$P(X=1)=\frac{^{26}C_1\times{}^{26}C_2}{^{52}C_3}=\frac{8450}{22100}=\frac{13}{34}$$

$$P(X=2)=\frac{^{26}C_2\times{}^{26}C_1}{^{52}C_3}=\frac{8450}{22100}=\frac{13}{34}$$

$$P(X=3)=\frac{^{26}C_3}{^{52}C_3}=\frac{2600}{22100}=\frac{2}{17}$$

Probability distribution of X:

$X=x_i$	0	1	2	3
$p(X=x_i)$	$\frac{2}{17}$	$\frac{13}{34}$	$\frac{13}{34}$	$\frac{2}{17}$

$$\text{Mean}=\sum_{i=0}^{3}(x_i\times p_i)$$
$$=x_0p_0+x_1p_1+x_2p_2+x_3p_3$$
$$=0\times\frac{2}{17}+1\times\frac{13}{34}+2\times\frac{13}{34}+3\times\frac{2}{17}$$
$$=\frac{13+26+12}{34}$$
$$=\frac{51}{34}$$
$$=\frac{3}{2}$$
$$=1.5$$

Mean and Variance of a Random Variable Ex 32.2 Q18

X can assume values 0, 1, 2.
Yes X is a random variable.

$$P(X=0)=(\text{Probablity of getting no black ball})=\frac{^{2}C_0\times{}^{5}C_2}{^{7}C_2}=\frac{1\times\frac{5\times4}{2\times1}}{\frac{7\times6}{2\times1}}=\frac{20}{42}$$

$$P(X=1)=(\text{Probablity of getting one black ball})=\frac{^{2}C_1\times{}^{5}C_1}{^{7}C_2}=\frac{2\times5}{\frac{7\times6}{2\times1}}=\frac{20}{42}$$

$$P(X=2)=(\text{Probablity of getting two black balls})=\frac{^{2}C_2\times{}^{5}C_0}{^{7}C_2}=\frac{1\times1}{\frac{7\times6}{2\times1}}=\frac{2}{42}$$

Thus, probablity distribution of random veriable X is,

X	0	1	2
P(X)	$\frac{20}{42}$	$\frac{20}{42}$	$\frac{2}{42}$

X_i	p_i	p_iX_i	$p_iX_i^2$
0	$\frac{20}{42}$	0	0
1	$\frac{20}{42}$	$\frac{20}{42}$	$\frac{20}{42}$
2	$\frac{2}{42}$	$\frac{4}{42}$	$\frac{8}{42}$
		$\Sigma p_iX_i=\frac{4}{7}$	$\Sigma p_iX_i^2=\frac{2}{3}$

$$\text{Mean}=\Sigma p_iX_i=\frac{4}{7}$$

$$\text{Variance}=\Sigma p_iX_i^2-(\Sigma p_iX_i)^2=\frac{2}{3}-\left(\frac{4}{7}\right)^2=\frac{50}{147}$$

Mean and Variance of a Random Variable Ex 32.2 Q19

We can select two positive in $6 \times 5 = 30$ different ways.
X denotes tha larger number so, X can assume values 3, 4, 5, 6 and 7.
Yes X is a random variable.

$$P(X = 3) = P(\text{larger number is } 3) = \{(2,3),(3,2)\} = \frac{2}{30}$$

$$P(X = 4) = P(\text{larger number is } 4) = \{(2,4),(4,2),(3,4),(4,3)\} = \frac{4}{30}$$

$$P(X = 5) = P(\text{larger number is } 5) = \{(2,5),(5,2),(3,5),(5,3),(4,5),(5,4)\} = \frac{6}{30}$$

$$P(X = 6) = P(\text{larger number is } 6) = \{(2,6),(6,2),(3,6),(6,3),(4,6),(6,4),(5,6),(6,5)\} = \frac{8}{30}$$

$$P(X = 7) = P(\text{larger number is } 7) = \{(2,7),(7,2),(3,7),(7,3),(4,7),(7,4),(5,7),(7,5),(6,7),(7,6)\} = \frac{10}{30}$$

Thus, probablity distribution of random veriable X is,

x_i	p_i	$p_i x_i$	$p_i x_i^2$
3	$\frac{2}{30}$	$\frac{6}{30}$	$\frac{18}{30}$
4	$\frac{4}{30}$	$\frac{16}{30}$	$\frac{64}{30}$
5	$\frac{6}{30}$	$\frac{30}{30}$	$\frac{150}{30}$
6	$\frac{8}{30}$	$\frac{48}{30}$	$\frac{288}{30}$
7	$\frac{10}{30}$	$\frac{70}{30}$	$\frac{490}{30}$
		$\sum p_i x_i = \frac{17}{3}$	$\sum p_i x_i^2 = \frac{101}{3}$

$$\text{Mean} = \sum p_i x_i = \frac{17}{3}$$

$$\text{Variance} = \sum p_i x_i^2 - \left(\sum p_i x_i\right)^2 = \frac{101}{3} - \left(\frac{17}{3}\right)^2 = \frac{14}{9}$$

Binomial Distribution Ex 33.1 Q1

Ex - 33.1

Let p denote the probability of having defective item, so

$$p = 6\% = \frac{6}{100} = \frac{3}{50}$$

So, $q = 1 - p$

$$= 1 - \frac{3}{50} \qquad \left[\text{Since } p + q = 1\right]$$

$$= \frac{47}{50}$$

Let X denote the number of defective items in a sample of 8 items. Then, the pro ty of getting r defective bulks is

$$P\left(X = r\right) = {}^nC_r p^r q^{n-r}$$

$$P\left(X = r\right) = {}^8C_r \left(\frac{3}{50}\right)^r \left(\frac{47}{50}\right)^{8-r} \qquad ---(1)$$

Therefore, probability of getting not more then one defective item

$$= P\left(X = 0\right) + P\left(X = 1\right)$$

$$= {}^8C_0 \left(\frac{3}{50}\right)^0 \left(\frac{47}{50}\right)^{8-0} + {}^8C_1 \left(\frac{3}{50}\right)^1 \left(\frac{47}{50}\right)^{8-1} \qquad \left[\text{Using equation } (1)\right]$$

$$= 1.1.\left(\frac{47}{50}\right)^8 + 8.\frac{3}{50}.\left(\frac{47}{50}\right)^7$$

$$= \left(\frac{47}{50}\right)^7 \left(\frac{47}{50} + \frac{24}{50}\right)$$

$$= \left(\frac{71}{50}\right)\left(\frac{47}{50}\right)^7$$

$$= \left(1.42\right) \times \left(0.\ 94\right)^7$$

The required probability is,

$$\left(1.42\right) \times \left(0.\ 94\right)^7$$

Binomial Distribution Ex 33.1 Q2

Probability of getting head on one throw of coin $= \frac{1}{2}$

So, $p = \frac{1}{2}$

$q = 1 - \frac{1}{2}$

$q = \frac{1}{2}$ $\quad$ [Since $p + q = 1$]

The coin is tossed 5 times. Let X denote the number of getting head as 5 tosses of coins.
So probability of getting r heads in n tosses of coin is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$P(X = r) = {}^5C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{5-r} \qquad ---(1)$$

Probability of getting at least 3 heads

$$= P(X = 3) + P(X = 4) + P(X = 5)$$

$$= {}^5C_3\left(\frac{1}{2}\right)^3 . \left(\frac{1}{2}\right)^{5-3} + {}^5C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^{5-4} + {}^5C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^0 \qquad [\text{Using } (1)]$$

$$= {}^5C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 + {}^5C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right) + {}^5C_5\left(\frac{1}{2}\right)^5 .1$$

$$= \frac{5.4}{2}.\left(\frac{1}{2}\right)^5 + 5\left(\frac{1}{2}\right)^5 + 1.\left(\frac{1}{2}\right)^5$$

$$= \left(\frac{1}{2}\right)^5 [10 + 5 + 1]$$

$$= 16.\frac{1}{32}$$

$$= \frac{1}{2}$$

The required probability is $= \frac{1}{2}$

Binomial Distribution Ex 33.1 Q3

Let p be the probability getting tail on a toss of a fair coin, so

$$p = \frac{1}{2}$$

$$q = 1 - \frac{1}{2}$$

$$q = \frac{1}{2} \qquad \left[\text{Since } p + q = 1\right]$$

Let X denote the number tail obtained on the toss of coin 5 times. So probability of getting r tails in n tosses of coin is given by

$$P\left(X = r\right) = {}^{n}C_{r}p^{r}q^{n-r}$$

$$= {}^{5}C_{r}\left(\frac{1}{2}\right)^{r}\left(\frac{1}{2}\right)^{5-r} \qquad ---(1)$$

Probability of getting tail an odd number of times

$$= P\left(X = 1\right) + P\left(X = 3\right) + P\left(X = 5\right)$$

$$= {}^{5}C_{3}\left(\frac{1}{2}\right)^{1}\left(\frac{1}{2}\right)^{5-1} + {}^{5}C_{3}\left(\frac{1}{2}\right)^{3}\left(\frac{1}{2}\right)^{5-3} + {}^{5}C_{5}\left(\frac{1}{2}\right)^{5}\left(\frac{1}{2}\right)^{0} \qquad \left[\text{Using } (1)\right]$$

$$= 5.\left(\frac{1}{2}\right)^{5} + \frac{5.4}{2}.\left(\frac{1}{2}\right)^{5} + 1.\left(\frac{1}{2}\right)^{5}$$

$$= \left(\frac{1}{2}\right)^{5}\left[5 + 10 + 1\right]$$

$$= 16\left(\frac{1}{2}\right)^{5}$$

$$= 16.\frac{1}{32}$$

$$= \frac{1}{2}$$

The required probability is $= \frac{1}{2}$

Binomial Distribution Ex 33.1 Q4

Let p be the probability of getting a sum of 9 and it considered as success.
Sum of a 9 on a pair of dice

$$= \{(3,6),(4,5),(5,4),(6,3)\}$$

So, $$p = \frac{4}{36}$$

$$p = \frac{1}{9}$$

$$q = 1 - \frac{1}{9}$$

$$q = \frac{8}{9} \qquad [\text{Since } p + q = 1]$$

Let X denote the number of success in throw of a pair of dice 6 times. So probability of getting r success out of n is given by

$$P(X = r) = {}^nC_r p^r q^{n-r} \qquad ---(1)$$

Probability of getting at least 5 success

$$= P(X = 5) + P(X = 6)$$

$$= {}^6C_5\left(\frac{1}{9}\right)^5\left(\frac{8}{9}\right)^{6-5} + {}^6C_6\left(\frac{1}{9}\right)^6\left(\frac{8}{9}\right)^{6-6} \qquad [\text{Using } (1)]$$

$$= 6\left(\frac{1}{9}\right)^5\left(\frac{8}{9}\right)^1 + 1.\left(\frac{1}{9}\right)^6\left(\frac{8}{9}\right)^0$$

$$= \left(\frac{1}{9}\right)^5\left[\frac{48}{9} + \frac{1}{9}\right]$$

$$= \frac{49}{9} \times \left(\frac{1}{9}\right)^5$$

$$= \frac{49}{9^6}$$

So,

Required probability $= \dfrac{49}{9^6}$

Binomial Distribution Ex 33.1 Q5

Let p be the probability of getting head in a throw of coin. So,

$$p = \frac{1}{2}$$

$$q = 1 - \frac{1}{2}$$

$$q = \frac{1}{2} \qquad \left[\text{Since } p + q = 1\right]$$

Let X denote the number of heads on tossing the coin 6 times. Probability of getting r in tossing the coin n times is given by

$$P\left(X = r\right) = {}^nC_r p^r q^{n-r} \qquad ---(1)$$

Probability of getting at least three heads

$$= P\left(X = 3\right) + P\left(X = 4\right) + P\left(X = 5\right) + P\left(X = 6\right)$$

$$= 1 - \left[P\left(X = 0\right) + P\left(X = 1\right) + P\left(X = 2\right)\right]$$

$$= 1 - \left[{}^6C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{6-0} + {}^6C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{6-1} + {}^6C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{6-2}\right] \qquad \left[\text{Using } (1)\right]$$

$$= 1 - \left[1.\left(\frac{1}{2}\right)^6 + 6\left(\frac{1}{2}\right)^6 + \frac{6.5}{2}.\left(\frac{1}{2}\right)^6\right]$$

$$= 1 - \left[\left(\frac{1}{2}\right)^6\left(1 + 6 + 15\right)\right]$$

$$= 1 - \left[\frac{22}{64}\right]$$

$$= \frac{64 - 22}{64}$$

$$= \frac{42}{64}$$

$$= \frac{21}{32}$$

Required probability $= \dfrac{21}{32}$

Binomial Distribution Ex 33.1 Q6

Let p denote the 4 turning up in a toss of a fair die, so

$$p = \frac{1}{6}$$

$$q = 1 - \frac{1}{6}$$

$$q = \frac{5}{6} \qquad [\text{Since } p + q = 1]$$

Let X denote the variable showing the number of turning 4 up in 2 tosses of die.
Probability of getting 4, r times in n tosses of a die is given by

$$P(X = r) = {}^{n}C_r p^r q^{n-r}$$

$$= {}^{2}C_r \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{2-r} \qquad ---(1)$$

Probability of getting 4 at least once in tow tosses of a fair die

$$= P(X = 1) + P(X = 2)$$

$$= 1 - P(X = 0)$$

$$= 1 - \left[{}^{2}C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^{2-0}\right] \qquad [\text{Using } (1)]$$

$$= 1 - \left[1.1.\left(\frac{5}{6}\right)^2\right]$$

$$= 1 - \left[\frac{25}{36}\right]$$

$$= \frac{36 - 25}{36}$$

$$= \frac{11}{36}$$

So,

Required probability $= \frac{11}{36}$

Binomial Distribution Ex 33.1 Q7

Let p denote the probability of getting head in a toss of fair coin. So

$$p = \frac{1}{2}$$

$$q = 1 - \frac{1}{2}$$

$$q = \frac{1}{2} \qquad [\text{Since } p + q = 1]$$

Let X denote the variable representing number of heads on 5 tosses of a fair coin.
Probability of getting r an n tosses of a fair coin, so

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$P(X = r) = {}^5C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{5-r} \qquad ---(1)$$

Probability of getting head on an even number of tosses of coin

$$= P(X = 0) + P(X = 2) + P(X = 4)$$

$$= {}^5C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{5-0} + {}^5C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{5-2} + {}^5C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{5-4} \qquad [\text{Using } (1)]$$

$$= 1.1.\left(\frac{1}{2}\right)^5 + \frac{5.4}{2}.\left(\frac{1}{2}\right)^5 + 5.\left(\frac{1}{2}\right)^5$$

$$= \left(\frac{1}{2}\right)^5 [1 + 10 + 5]$$

$$= 16 \times \frac{1}{32}$$

$$= \frac{1}{2}$$

Required probability $= \frac{1}{2}$

Binomial Distribution Ex 33.1 Q8

Let p be the probability of hitting the target, so

$$p = \frac{1}{4}$$

$$q = 1 - p \qquad \left[\text{Since } p + q = 1\right]$$

$$= 1 - \frac{1}{4}$$

$$q = \frac{3}{4}$$

Let X denote the variable representing the number of times hittintg the target out of 7 fires. Probability of hitting the target r times out of n fires is given by,

$$P\left(X = r\right) = {}^{n}C_{r} p^{r} q^{n-r}$$

$$= {}^{7}C_{r} \left(\frac{1}{4}\right)^{r} \left(\frac{3}{4}\right)^{7-r} \qquad ---(1)$$

Probability of hitting the target at least twice

$$= P\left(X = 2\right) + P\left(X = 3\right) + P\left(X = 4\right) + P\left(X = 5\right) + P\left(X = 6\right) + P\left(X = 7\right)$$

$$= 1 - \left[P\left(X = 0\right) + P\left(X = 1\right)\right]$$

$$= 1 - \left[{}^{7}C_{0}\left(\frac{1}{4}\right)^{0}\left(\frac{3}{4}\right)^{7-0} + {}^{7}C_{1}\left(\frac{1}{4}\right)^{1}\left(\frac{3}{4}\right)^{7-1}\right] \qquad \left[\text{Using } (1)\right]$$

$$= 1 - \left[1.1.\left(\frac{3}{4}\right)^{7} + 7.\frac{1}{4}.\left(\frac{3}{4}\right)^{6}\right]$$

$$= 1 - \left(\frac{3}{4}\right)^{6}\left(\frac{3}{4} + \frac{7}{4}\right)$$

$$= 1 - \left(\frac{3}{4}\right)^{6}\left(\frac{10}{4}\right)$$

$$= 1 - \frac{7290}{16384}$$

$$= \frac{9194}{16384}$$

$$= \frac{4547}{8192}$$

Binomial Distribution Ex 33.1 Q9

Let the probability of one telephone number out of 15 is busy between 2 PM and 3 PM be 'p'. then
P = 1/15 ; probability that number is not busy, q = 1-p

Q = 14/16. Binomial distribution is $\left(\frac{14}{15}+\frac{1}{15}\right)^6$

Since 6 numbers are called we find the probability for none of the numbers are busy is P(0)
One number is busy P(1); Two numbers are busy is P(2)
Three numbers are busy is P(3); Four numbers are busy is P(4) ; Five numbers are busy is P(5); Six numbers are busy is P(6).

$P(0) = {}^6C_0\left(\frac{14}{15}\right)^6$

$P(1) = {}^6C_1\left(\frac{14}{15}\right)^5\left(\frac{1}{15}\right)^1$

$P(2) = {}^6C_2\left(\frac{14}{15}\right)^4\left(\frac{1}{15}\right)^2$

$P(3) = {}^6C_3\left(\frac{14}{15}\right)^3\left(\frac{1}{15}\right)^3$

$P(4) = {}^6C_4\left(\frac{14}{15}\right)^2\left(\frac{1}{15}\right)^4$

$P(5) = {}^6C_5\left(\frac{14}{15}\right)^1\left(\frac{1}{15}\right)^5$

$P(6) = {}^6C_6\left(\frac{14}{15}\right)^0\left(\frac{1}{15}\right)^6$

Probability that at least 3 of the numbers will be busy

P(3) + P(4) + P(5) + P(6) = 0.05

Binomial Distribution Ex 33.1 Q10

p denote the probability of success

p = Probability of getting 5 or 6 in a throw of die.

$$= \frac{2}{6}$$

$$p = \frac{1}{3}$$

$$q = 1 - \frac{1}{3} \qquad [\text{Since } p + q = 1]$$

$$q = \frac{2}{3}$$

Let X denote the number of success in six throws of a dic. Probability of getting r success in six throws of an unbiased dic is given by

$$P(X = r) = {}^{n}C_{r} p^{r} q^{n-r}$$

$$= {}^{6}C_{r} \left(\frac{1}{3}\right)^{r} \left(\frac{2}{3}\right)^{6-r} \qquad \text{---}(1)$$

$$P(X \geq 4)$$

$$= P(X = 4) + P(X = 5) + P(X = 6)$$

$$= {}^{6}C_{4}\left(\frac{1}{3}\right)^{4}\left(\frac{2}{3}\right)^{6-4} + {}^{6}C_{5}\left(\frac{1}{3}\right)^{5}\left(\frac{2}{3}\right)^{6-5} + {}^{6}C_{6}\left(\frac{1}{3}\right)^{6}\left(\frac{2}{3}\right)^{6-6}$$

$$= \frac{6.5}{2}\left(\frac{1}{3}\right)^{4}\left(\frac{2}{3}\right)^{2} + 6\left(\frac{1}{3}\right)^{5}\left(\frac{2}{3}\right) + 1.\left(\frac{1}{3}\right)^{6}.1$$

$$= 15.\frac{1}{81}.\frac{4}{9} + 6.\frac{1}{243}.\frac{2}{3} + \frac{1}{729}$$

$$= \frac{60}{729} + \frac{12}{729} + \frac{1}{729}$$

$$= \frac{73}{729}$$

Required probability $= \dfrac{73}{729}$

Binomial Distribution Ex 33.1 Q11

Let p denote the probability of getting head on a throw of fair coin, so

$$p = \frac{1}{2}$$

$$q = 1 - \frac{1}{2} \qquad \left[\text{Since } p + q = 1\right]$$

$$q = \frac{1}{2}$$

Let X denote the variable representing the number of getting heads on throw of 8 coins.
Probability of getting r heads in a throw of n coins is given by

$$P\left(X = r\right) = {}^{n}C_{r} p^{r} q^{n-r}$$

$$= {}^{8}C_{r} \left(\frac{1}{2}\right)^{r} \left(\frac{1}{2}\right)^{8-r} \qquad ---(1)$$

Probability of getting at least six heads

$$= P\left(X = 6\right) + P\left(X = 7\right) + P\left(X = 8\right)$$

$$= {}^{8}C_{6} \left(\frac{1}{2}\right)^{8} \left(\frac{1}{2}\right)^{8-6} + {}^{8}C_{7} \left(\frac{1}{2}\right)^{7} \left(\frac{1}{2}\right)^{8-7} + {}^{8}C_{8} \left(\frac{1}{2}\right)^{8} \left(\frac{1}{2}\right)^{8-8} \qquad \left[\text{Using } (1)\right]$$

$$= \frac{8.7}{2} \left(\frac{1}{2}\right)^{8} + 8 \left(\frac{1}{2}\right)^{8} + 1.\left(\frac{1}{2}\right)^{8}.1$$

$$= \left(\frac{1}{2}\right)^{8} \left[28 + 8 + 1\right]$$

$$= \frac{1}{256}\left(37\right)$$

$$= \frac{37}{256}$$

Required probability $= \dfrac{37}{256}$

Binomial Distribution Ex 33.1 Q12

Let p denote the probability of getting one spade out of a deck of 52 cards, so

$$p = \frac{13}{52}$$

$$p = \frac{1}{4}$$

$$q = 1 - \frac{1}{4} \qquad \left[\text{Since } p + q = 1\right]$$

$$q = \frac{-}{4}$$

Let X denote the radom variable of number of spades out of 5 cards. Probability of getting r spades out of n cards is given by

$$P(X = r) = {}^{n}C_{r}p^{r}q^{n-r}$$

$$= {}^{5}C_{r}\left(\frac{1}{4}\right)^{r}\left(\frac{3}{4}\right)^{5-r} \qquad ---(1)$$

(i)

Probability of getting all five spades

$$= P(X = 5)$$

$$= {}^{5}C_{5}\left(\frac{1}{4}\right)^{5}\left(\frac{3}{4}\right)^{5-5}$$

$$= \frac{1}{1024}$$

Probability of getting 5 spades $= \frac{1}{1024}$

(ii)

Probability of getting only 3 spades

$$= P(X = 3)$$

$$= {}^{5}C_{3}\left(\frac{1}{4}\right)^{3}\left(\frac{3}{4}\right)^{5-3}$$

$$= \frac{5.4}{2}\left(\frac{1}{64}\right)\left(\frac{9}{16}\right)$$

$$= \frac{45}{512}$$

Probability of getting 3 spades $= \frac{45}{512}$

(iii)

Probability that none is spade

$$= P(X = 0)$$

$$= {}^5C_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^{5-0}$$

$$= \frac{243}{1024}$$

Probability of getting non spade $= \dfrac{243}{1024}$

Binomial Distribution Ex 33.1 Q13

Let p be the probability of getting 1 white ball out of 7 red, 5 white and 8 black balls. So

$$p = \frac{5}{20}$$

$$p = \frac{1}{4}$$

$$q = 1 - \frac{1}{4} \qquad [\text{Since } p + q = 1]$$

$$q = \frac{3}{4}$$

Let X denote the random variable of number of selecting white ball with replacement out of 4 balls. Probability of getting r white balls out of n balls is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$= {}^4C_r\left(\frac{1}{4}\right)^r\left(\frac{3}{4}\right)^{4-r} \qquad ---(1)$$

(i)

Probability of getting none white ball

$$= P(X = 0)$$

$$= {}^4C_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^{4-0} \qquad [\text{Using } (1)]$$

$$= \left(\frac{3}{4}\right)^4$$

$$= \frac{81}{256}$$

Probability of getting none white ball $= \frac{81}{256}$

(ii)

Probability of getting all white balls

$$= P(X = 4)$$

$$= {}^4C_4 \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^{4-0}$$

$$= \left(\frac{1}{4}\right)^4$$

$$= \frac{1}{256}$$

Probability of getting all white balls $= \frac{1}{256}$

(iii)

Probability of getting any two are white

$$= P(X = 2)$$

$$= {}^4C_2 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^{4-2}$$

$$= \frac{4.3}{2} \cdot \frac{1}{16} \cdot \frac{9}{16}$$

$$= \frac{27}{128}$$

Probability of getting any two are white balls $= \frac{27}{128}$

Binomial Distribution Ex 33.1 Q14

Let p denote the probability of getting a ticket bearing number divisible by 10, So

$$p = \frac{10}{100} \qquad \left[\begin{array}{l}\text{Since there are } 10, 20, 30, 40, 50, 60, 70, 80, \\ 90, 100 \text{ which are divisible by } 10\end{array}\right]$$

$$p = \frac{1}{10}$$

$$q = 1 - \frac{1}{10} \qquad [\text{Since } p + q = 1]$$

$$q = \frac{9}{10}$$

Let X denote the variable representing the number of tickets bearing a number divisible by 10 out of 5 tickets. Probability of getting r tickets bearing a number divisible by 10 out of n tickets is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$= {}^5C_r \left(\frac{1}{10}\right)^r \left(\frac{9}{10}\right)^{5-r} \qquad ---(1)$$

Probability of getting all the tickets bearing a number divisible by 10

$$= {}^5C_5 \left(\frac{1}{10}\right)^5 \left(\frac{9}{10}\right)^{5-5} \qquad [\text{Using } (1)]$$

$$= 1.\left(\frac{1}{10}\right)^5 \left(\frac{9}{10}\right)^0$$

$$= \left(\frac{1}{10}\right)^5$$

Required probability $= \left(\frac{1}{10}\right)^5$

Binomial Distribution Ex 33.1 Q15

Let p denote the probability of getting a ball marked with 0. So

$$p = \frac{1}{10} \qquad \left[\text{Since balls are marked with } 0,1,2,3,4,5,6,7,8,9\right]$$

$$q = 1 - \frac{1}{10} \qquad \left[\text{Since } p + q = 1\right]$$

$$q = \frac{9}{10}$$

Let X denote the variable presenting the number of balls marked with 0 out of four balls drawn. Probability of drawing r balls out of n balls that are marked 0 is given by

$$P(X = r) = {}^{n}C_{r} p^{r} q^{n-r}$$

$$= {}^{4}C_{r} \left(\frac{1}{10}\right)^{r} \left(\frac{9}{10}\right)^{4-r} \qquad ---(1)$$

Probability of getting none balls marked with 0

$$= P(X = 0)$$

$$= {}^{4}C_{0} \left(\frac{1}{10}\right)^{0} \left(\frac{9}{10}\right)^{4-0}$$

$$= 1.1.\left(\frac{9}{10}\right)^{4}$$

$$= \left(\frac{9}{10}\right)^{4}$$

Probability of getting none balls marked with 0 $= \left(\frac{9}{10}\right)^{4}$

Binomial Distribution Ex 33.1 Q16

Let p denote the probability of getting one defective item out of hundred. So

$$p = 5\% \qquad \text{[Since 5\% are defective items]}$$

$$= \frac{5}{100}$$

$$p = \frac{1}{20}$$

$$q = 1 - \frac{1}{20} \qquad [\text{Since } p + q = 1]$$

$$q = \frac{19}{20}$$

Let X denote the random variable representing the number of defective items out of 10 items. Probability of getting r defective items out of n items selected is given by,

$$P(X = r) = {}^{n}C_r p^r q^{n-r}$$

$$= {}^{10}C_r \left(\frac{1}{20}\right)^r \left(\frac{19}{20}\right)^{10-r} \qquad ---(1)$$

Probability of getting not more than one defective items

$$= P(X = 0) + P(X = 1)$$

$$= {}^{10}C_0 \left(\frac{1}{20}\right)^0 \left(\frac{19}{20}\right)^{10-0} + {}^{10}C_1 \left(\frac{1}{20}\right)^1 \left(\frac{19}{20}\right)^{10-1}$$

$$= 1.1.\left(\frac{19}{20}\right)^{10} + 10.\frac{1}{20}\left(\frac{19}{20}\right)^9$$

$$= \left(\frac{19}{20}\right)^9 \left[\frac{19}{20} + \frac{10}{20}\right]$$

$$= \frac{29}{20}\left(\frac{19}{20}\right)^9$$

The required probability $= \frac{29}{20}\left(\frac{19}{20}\right)^9$

Binomial Distribution Ex 33.1 Q17

Let p denote the probability that one bulb produced will fuse after 150 days, so

$$p = 0.05$$

$$= \frac{5}{100} \qquad \text{[It is given]}$$

$$20$$

$$q = 1 - \frac{1}{20} \qquad [\text{Since } p + q = 1]$$

$$q = \frac{19}{20}$$

Let X denote the number of fuse bulb out of 5 bulbs. Probability that r bulbs out of n will fuse in 150 days is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$= {}^5C_r \left(\frac{1}{20}\right)^r \left(\frac{19}{20}\right)^{5-r} \qquad ---(1)$$

(i)

Probability that none is fuse $= P(X = 0)$

$$= {}^5C_0 \left(\frac{1}{20}\right)^0 \left(\frac{19}{20}\right)^{5-0}$$

$$= \left(\frac{19}{20}\right)^5$$

Probability that none will fuse $= \left(\frac{19}{20}\right)^5$

(ii)

Probability that not more than 1 will fuse

$$= P(X = 0) + P(X = 1)$$

$$= \left(\frac{19}{20}\right)^5 + {}^5C_1 \left(\frac{1}{20}\right)^1 \left(\frac{19}{20}\right)^{5-1}$$

$$= \left(\frac{19}{20}\right)^4 \left[\frac{19}{20} + \frac{5}{20}\right]$$

$$= \left(\frac{24}{20}\right)\left(\frac{19}{20}\right)^4$$

Probability not more than one will fuse $= \left(\frac{6}{5}\right)\left(\frac{19}{20}\right)^4$

(iii)

Probability that more than one will fuse

$$= P(X=2)+P(X=3)+P(X=4)+P(X=5)$$

$$= 1-\left[P(X=0)+P(X=1)\right]$$

$$= 1-\left[\frac{6}{5}\left(\frac{19}{20}\right)^{4}\right]$$

Probability that more than one will fuse $= 1-\left[\frac{6}{5}\left(\frac{19}{20}\right)^{4}\right]$

(iv)

Probability that that at least one will fuse

$$= P(X=1)+P(X=2)+P(X=3)+P(X=4)+P(X=5)$$

$$= 1-P(X=0)$$

$$= 1-\left[{}^{5}C_{0}\left(\frac{1}{20}\right)^{0}\left(\frac{19}{20}\right)^{5-0}\right]$$

$$= 1-\left[\left(\frac{19}{20}\right)^{5}\right]$$

Probability that that at least one will fuse $= 1-\left(\frac{19}{20}\right)^{5}$

Binomial Distribution Ex 33.1 Q18

A person can be either right-handed or left-handed.

It is given that 90% of the people are right-handed.

$$\therefore p = \mathrm{P}(\text{right-handed}) = \frac{9}{10}$$

$$q = \mathrm{P}(\text{left-handed}) = 1 - \frac{9}{10} = \frac{1}{10}$$

Using binomial distribution, the probability that more than 6 people are right-handed is given by,

$$\sum_{r=7}^{10} {}^{10}C_r p^r q^{n-r} = \sum_{r=7}^{10} {}^{10}C_r \left(\frac{9}{10}\right)^r \left(\frac{1}{10}\right)^{10-r}$$

Therefore, the probability that at most 6 people are right-handed

= 1 − P (more than 6 are right-handed)

$$= 1 - \sum_{r=7}^{10} {}^{10}C_r (0.9)^r (0.1)^{10-r}$$

Binomial Distribution Ex 33.1 Q19

Let p denote the probability of getting 1 red ball out of 7 green, 4 white and 5 red balls, so

$$p = \frac{5}{16}$$

$$q = 1 - \frac{5}{16} \qquad [\text{Since } p + q = 1]$$

$$q = \frac{11}{16}$$

Let X denote the number of red balls drawn out of four balls. Probability of getting r red balls out of n drawn balls is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$= {}^4C_r \left(\frac{5}{16}\right)^r \left(\frac{11}{16}\right)^{4-r} \qquad ---(1)$$

Probability of getting one red ball

$$= P(X = 1)$$

$$= {}^4C_1 \left(\frac{5}{16}\right)^1 \left(\frac{11}{16}\right)^{4-1}$$

$$= 4.\left(\frac{5}{16}\right)\left(\frac{11}{16}\right)^3$$

$$= \left(\frac{5}{4}\right)\left(\frac{11}{16}\right)^3$$

Required probability $= \left(\frac{5}{4}\right)\left(\frac{11}{16}\right)^3$

Binomial Distribution Ex 33.1 Q20

X	$P(X)$
0	$\frac{7}{9}\times\frac{6}{8}=\frac{21}{36}$
1	$\frac{7}{9}\times\frac{2}{8}\times 2=\frac{14}{36}$
2	$\frac{2}{9}\times\frac{1}{8}=\frac{1}{36}$

Binomial Distribution Ex 33.1 Q21

X	$P(X)$
0	${}^3C_0\left(\frac{3}{7}\right)^0\left(\frac{4}{7}\right)^{3-0}=\left(\frac{4}{7}\right)^3=\frac{64}{343}$
1	${}^3C_1\left(\frac{3}{7}\right)^1\left(\frac{4}{7}\right)^{3-1}=3.\left(\frac{3}{7}\right)\left(\frac{4}{7}\right)^2=\frac{144}{343}$
2	${}^3C_2\left(\frac{3}{7}\right)^2\left(\frac{4}{7}\right)^{3-2}=3.\left(\frac{3}{7}\right)^2\left(\frac{4}{7}\right)=\frac{108}{343}$
3	${}^3C_3\left(\frac{3}{7}\right)^3\left(\frac{4}{7}\right)^{3-0}=\left(\frac{3}{7}\right)^3=\frac{27}{343}$

Binomial Distribution Ex 33.1 Q22

Let p be the probability of getting doublet is a throw of a pair of dice, so

$$p=\frac{6}{36} \qquad \left[\text{Since } (1,1),(2,2),(3,3),(4,4),(5,5),(6,6) \text{ are doublets}\right]$$

$$p=\frac{1}{6}$$

$$q=1-\frac{1}{6} \qquad \left[\text{Since } p+q=1\right]$$

$$=\frac{5}{6}$$

Let X denote the number of getting doublets out of 4 times. So probability distribution is given by

Binomial Distribution Ex 33.1 Q23

X	$P(X)$
0	${}^3C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{3-0} = \left(\frac{5}{6}\right)^3 = \frac{125}{216}$
1	${}^3C_1\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^{3-1} = 3\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)^2 = \frac{25}{72}$
2	${}^3C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{3-2} = 3\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right) = \frac{5}{72}$
3	${}^3C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^{3-3} = \left(\frac{1}{6}\right)^3 = \frac{1}{216}$

Binomial Distribution Ex 33.1 Q24

We know that, probability of getting head in a toss of coin $p = \frac{1}{2}$

Probability of not getting head $q = 1 - \frac{1}{2}$

$$q = \frac{1}{2}$$

The coin is tossed 5 times. Let X denote the number of times head occur is 5 tosses.

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$= {}^5C_r\left(\frac{1}{2}\right)^r\left(\frac{1}{2}\right)^{5-r}$$

Probability distribution is given by

Binomial Distribution Ex 33.1 Q25

Let p be the probability of a getting a number greater than 4 in a toss of die, so

$$p = \frac{2}{6} \qquad \left[\text{Since, numbers greater than 4 coin a die } = 5,6\right]$$

$$p = \frac{1}{3}$$

$$q = 1 - \frac{1}{3} \qquad \left[\text{Since } p + q = 1\right]$$

$$q = \frac{2}{3}$$

Let X denote the number of success in 2 throws of a die. Probability of getting r success in n thrown of a die is given by

$$P\left(X = r\right) = {}^nC_r p^r q^{n-r}$$

$$= {}^2C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{2-r} \qquad ---(1)$$

Probability distribution of number of success is given by

X	$P(X)$
0	${}^2C_0\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^{2-0} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$
1	${}^2C_1\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^{2-1} = 2.\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) = \frac{4}{9}$
2	${}^2C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^{2-2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

Binomial Distribution Ex 33.1 Q26

Let n denote the number of throws required to get a head and X denote the amount won/lost.

He may get head on first toss or lose first and 2[nd] toss or lose first and won second toss probability distribution for X

Number of throws (n):	1	2	2
Amount won/lost (X):	1	0	-2
Probability $P(X)$:	$\frac{1}{2}$	$\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$	$\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$

So probability distribution is given by

X	$P(X)$
0	$\frac{1}{4}$
1	$\frac{1}{2}$
-2	$\frac{1}{4}$

Binomial Distribution Ex 33.1 Q27

Let p denote the probability of getting 3,4 or 5 in a throw of die. So

$$p = \text{probability of success}$$
$$= \frac{3}{6}$$

$$p = \frac{1}{2}$$
$$q = 1 - \frac{1}{2} \qquad [\text{Since } p + q = 1]$$
$$q = \frac{1}{2}$$

Let X denote the number of success in throw of 5 dice simultaneously. Probability of getting r success out of n throws of die is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$
$$= {}^5C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{5-r} \qquad ---(1)$$

Probability getting at least 3 success

$$= P(X = 3) + P(X = 4) + P(X = 5)$$
$$= {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{5-3} + {}^5C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{5-4} + {}^5C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^{5-5}$$
$$= \frac{5.4}{2}\left(\frac{1}{2}\right)^5 + 5.\left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^5$$
$$= \left(\frac{1}{2}\right)^5 [10 + 5 + 1]$$
$$= \frac{16}{32}$$
$$= \frac{1}{2}$$

Required probability $= \frac{1}{2}$

Binomial Distribution Ex 33.1 Q28

Let p denote the probability of getting defective items out of 100 items, so

$$p = 10\%$$
$$= \frac{10}{100}$$

$$p = \frac{1}{10}$$
$$q = 1 - \frac{1}{10} \qquad \left[\text{Since } p + q = 1\right]$$
$$q = \frac{9}{10}$$

Let X denote the number of defective items drawn out of 8 items. Probability of getting r defective items out of a sample of 8 items is given by

$$P\left(X = r\right) = {}^{n}C_{r} p^{r} q^{n-r}$$
$$= {}^{8}C_{r} \left(\frac{1}{10}\right)^{r} \left(\frac{9}{10}\right)^{8-r} \qquad ---(1)$$

Probability of getting 2 defective items

$$= P\left(X = 2\right)$$
$$= {}^{8}C_{2} \left(\frac{1}{10}\right)^{2} \left(\frac{9}{10}\right)^{8-2}$$
$$= \frac{8 \times 7}{2} \left(\frac{1}{10}\right)^{2} \left(\frac{9}{10}\right)^{6}$$
$$= \frac{28 \times 9^{6}}{10^{8}}$$

Required probability $= \dfrac{28 \times 9^{6}}{10^{8}}$

Binomial Distribution Ex 33.1 Q29

Let p denote the probability of drawing a heart from a deck of 52 cards, so

$$p = \frac{13}{52} \qquad \left[\because \text{ There are 13 hearts in deck}\right]$$

$$p = \frac{1}{4}$$

$$q = 1 - \frac{1}{4} \qquad \left[\text{Since } p + q = 1\right]$$

$$q = \frac{3}{4}$$

Let the card is drawn n times. So Binomial distribution is given by

$$P\left(X = r\right) = {}^{n}C_{r} p^{r} q^{n-r}$$

where X denote the number of spades drawn and $r = 0, 1, 2, 3, \ldots n$

(i)

We have to find the smallest value of n for which $P\left(X = 0\right)$ is less than $\frac{1}{4}$

$$P\left(X = 0\right) < \frac{1}{4}$$

$${}^{n}C_{0} \left(\frac{1}{1}\right)^{0} \left(\frac{3}{4}\right)^{n-0} < \frac{1}{4}$$

$$\left(\frac{3}{4}\right)^{n} < \frac{1}{4}$$

Put $n = 1, \left(\frac{3}{4}\right) \not< \frac{1}{4}$

$$n = 2, \left(\frac{3}{4}\right)^{2} \not< \frac{1}{4}$$

$$n = 3, \left(\frac{3}{4}\right)^{3} \not< \frac{1}{4}$$

So, smallest value of $n = 3$

$\therefore$ We must draw cards at least 3 times

(ii)

Given, the probability of drawing a heart $> \frac{3}{4}$

$$1 - P\left(X = 0\right) > \frac{3}{4}$$

$$1 - {}^{n}C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{n-0} > \frac{3}{4}$$

$$1 - \left(\frac{3}{4}\right)^n > \frac{3}{4}$$

$$1 - \frac{3}{4} > \left(\frac{3}{4}\right)^n$$

$$\frac{1}{4} > \left(\frac{3}{4}\right)^n$$

For $n = 1$, $\left(\frac{3}{4}\right)^1 \not< \frac{1}{4}$

$n = 2$, $\left(\frac{3}{4}\right)^2 \not< \frac{1}{4}$

$n = 3$, $\left(\frac{3}{4}\right)^3 \not< \frac{1}{4}$

$n = 4$, $\left(\frac{3}{4}\right)^4 \not< \frac{1}{4}$

$n = 5$, $\left(\frac{3}{4}\right)^5 \not< \frac{1}{4}$

So, card must be drawn 5 times.

Binomial Distribution Ex 33.1 Q30

Here $x = 8, p = \frac{1}{2}, q = \frac{1}{2}$

Let there be k desks and X be the number of students studying in office.

Then we want that

$$P(X \le k) > .90$$

$\Rightarrow \quad P(X > k) < .10$

$\Rightarrow \quad P(X = k+1, k+2, \ldots 8) < .10$

Clearly $P(X > 6) = P(X = 7 \text{ or } X = 8)$

$$= {}^8C_7\left(\frac{1}{2}\right)^8 + {}^8C_8\left(\frac{1}{2}\right)^8$$

$$= .04$$

and $\quad P(X > 5) = P(X = 6,\ X = 7 \text{ or } X = 8)$

$$= .15$$

$\therefore \quad P(X > 6) < 0.10$

$\Rightarrow$ If there are 6 desks then there is at least 90% chance for every graduate assistant to get a desk.

Binomial Distribution Ex 33.1 Q31

Binomial Distribution formula is given by

$P(x) = {}^nC_x\, p^x\, q^{n-x}$, where $x = 0, 1, 2, \ldots n$

Let x = No. of heads in a toss

We need probability of 6 or more heads

$X = 6, 7, 8$

Here $p = ½$ and $q = ½$

$P(6)$ = Prob of getting 6 heads, 2 tails $= {}^8C_6 \left(\frac{1}{2}\right)^6 \times \left(\frac{1}{2}\right)^2$

$P(7)$ = Prob of getting 7 heads, 1tails $= {}^8C_7 \left(\frac{1}{2}\right)^7 \times \left(\frac{1}{2}\right)^1$

$P(8)$ = Prob of getting 8 heads, 0 tails $= {}^8C_8 \left(\frac{1}{2}\right)^8 \times \left(\frac{1}{2}\right)^0$

The probability of getting at least 6 heads (not more than 2 tails) is then

$${}^8C_6 \left(\frac{1}{2}\right)^6 \times \left(\frac{1}{2}\right)^2 + {}^8C_1 \left(\frac{1}{2}\right)^7 \times \left(\frac{1}{2}\right)^1 + {}^8C_2 \left(\frac{1}{2}\right)^8 \times \left(\frac{1}{2}\right)^0$$

$$= \frac{1}{256} + 8\frac{1}{256} + 28\frac{1}{256} = \frac{37}{256}$$

Binomial Distribution Ex 33.1 Q32

Let p represents the probability of getting head in a toss of fair coin, so

$$p = \frac{1}{2}$$

$$q = 1 - \frac{1}{2} \qquad \left[\text{Since } p + q = 1\right]$$

$$q = \frac{1}{2}$$

Let X denote the random variable representing the number heads in 6 tosses of coin. Probability of getting r sixes in n tosses of a fair coin is given by,

$$P\left(X = r\right) = {}^{n}C_{r} p^{r} q^{n-r}$$

$$= {}^{6}C_{r}\left(\frac{1}{2}\right)^{r}\left(\frac{1}{2}\right)^{6-r} \qquad ---(1)$$

(i)

Probability of getting 3 heads

$$= P\left(X = 3\right)$$

$$= {}^{6}C_{3}\left(\frac{1}{2}\right)^{3}\left(\frac{1}{2}\right)^{6-3}$$

$$= \frac{6 \times 5 \times 4}{3 \times 2}\left(\frac{1}{2}\right)^{3}\left(\frac{1}{2}\right)^{3}$$

$$= \frac{20}{64}$$

Probability of getting 3 heads $= \frac{20}{64} = \frac{5}{16}$

(ii)

Probability of getting no heads

$$= P(X = 0)$$

$$= {}^6C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{6-0}$$

$$= \left(\frac{1}{2}\right)^6$$

$$= \frac{1}{64}$$

Probability of getting no heads $= \frac{1}{64}$

(iii)

Probability of getting at least one head

$$= P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6)$$

$$= 1 - P(X = 0)$$

$$= 1 - \frac{1}{64}$$

$$= \frac{63}{64}$$

Probability of getting at least one head $= \frac{63}{64}$

Binomial Distribution Ex 33.1 Q33

Let p be the probability that a tube function for more than 500 hours. So

$$p = 0.2$$

$$p = \frac{1}{5}$$

$$q = 1 - \frac{1}{5} \qquad \left[\text{Since } p + q = 1\right]$$

$$= \frac{4}{5}$$

Let X denote the random variable representing the number of tube that functions for more than 500 hours out of 4 tubes. Probability of functioning r tubes out n tubes selected for more than 500 hours is given by,

$$P\left(X = r\right) = {}^{n}C_{r} p^{r} q^{n-r}$$

$$= {}^{4}C_{r} \left(\frac{1}{5}\right)^{r} \left(\frac{4}{5}\right)^{4-r} \qquad ---(1)$$

Probability that exactly 3 tube will function for more than 500 hours

$$= {}^{4}C_{3} \left(\frac{1}{5}\right)^{3} \left(\frac{4}{5}\right)^{4-3}$$

$$= 4.\left(\frac{1}{5}\right)^{3} \left(\frac{4}{5}\right)$$

$$= \frac{16}{625}$$

Required probability $= \dfrac{16}{625}$

Binomial Distribution Ex 33.1 Q34

Let p be the probability that component survive the shock test. So

$$p = \frac{3}{4}$$

$$q = 1 - \frac{3}{4} \qquad \left[\text{Since } p + q = 1\right]$$

$$q = \frac{1}{4}$$

Let X denote the random variable representing the number of components that survive shock test out of 5 components. Probability of that r components that survive shock test out of n components is given by

$$P\left(X = r\right) = {}^{n}C_{r} p^{r} q^{n-r}$$

$$= {}^{5}C_{r} \left(\frac{3}{4}\right)^{r} \left(\frac{1}{4}\right)^{5-r} \qquad ---(1)$$

(i)

Probability that exactly 2 will survive the shock test

$$= P\left(X = 2\right)$$

$$= {}^{5}C_{2} \left(\frac{3}{4}\right)^{2} \left(\frac{1}{4}\right)^{5-2}$$

$$= \frac{5.4}{2}\left(\frac{9}{16}\right)\left(\frac{1}{64}\right)$$

$$= \frac{45}{512} \quad = 0.0879$$

Probability that exactly 2 survive $= 0.0879$

(ii)

Probability that at most 3 will survive

$$= P(X = 0) + P(X = 1) + P(X = 3) + P(X = 4)$$

$$= 1 - [P(X = 4) + P(X = 5)]$$

$$= 1 - \left[{}^5C_4\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right)^{5-4} + {}^5C_5\left(\frac{3}{4}\right)^5\left(\frac{1}{4}\right)^{5-5}\right]$$

$$= 1 - \left[5.\frac{81}{1024} + \frac{243}{1024}\right]$$

$$= 1 - \left[\frac{405 + 243}{1024}\right]$$

$$= \frac{1024 - 648}{1024}$$

$$= \frac{376}{1024} = 0.3672$$

Binomial Distribution Ex 33.1 Q35

Probability that bomb strikes a target p = 0.2
Probability that a bomb misses the target = 0.8
n = 6
let x = number of bombs that strike the target
P(x=2) = exactly 2 bombs strike the target

$$= {}^6C_2\left(\frac{2}{10}\right)^2 \times \left(\frac{8}{10}\right)^4 = 15\times\frac{16384}{10^6} = 0.24576$$

$P(x\geq2)$ = at least 2 bombs strike the target

$= 1- P(x<2)$

$= 1 - [P(x=0)+P(x=1)]$

$$= 1 - [\ {}^6C_0\left(\frac{2}{10}\right)^0 \times \left(\frac{8}{10}\right)^6 + {}^6C_1\left(\frac{2}{10}\right)^1 \times \left(\frac{8}{10}\right)^5]$$

$= 1 - [0.0.262144 + 0.393216] = 1 - 0.65536$
$= 0.34464$

Binomial Distribution Ex 33.1 Q36

Let p be the probability that a mouse get contract the desease. So

$$p = 40\%$$
$$= \frac{40}{100}$$
$$= \frac{2}{5}$$
$$q = 1 - \frac{2}{5} \qquad \left[\text{Since } p + q = 1\right]$$
$$q = \frac{3}{5}$$

Let X denote the variable representing number of mice contract the disease out of 5 mice.
Probability the r mice get contract the disease out of n mice inoculated is given by

$$P\left(X = r\right) = {}^nC_r p^r q^{n-r}$$
$$= {}^5C_r \left(\frac{2}{5}\right)^r \left(\frac{3}{5}\right)^{5-r} \qquad ---(1)$$

(i)

Probability that none contract the disease $= P\left(X = 0\right)$

$$= {}^5C_0 \left(\frac{2}{5}\right)^0 \left(\frac{3}{5}\right)^{5-0}$$
$$= \left(\frac{3}{5}\right)^5$$

Probability that none contract the disease $= \left(\frac{3}{5}\right)^5$

(ii)

Probability that more than 3 contract disease

$$= P\left(X = 4\right) + P\left(X = 5\right)$$
$$= {}^5C_4 \left(\frac{2}{5}\right)^4 \left(\frac{3}{5}\right)^{5-4} + {}^5C_5 \left(\frac{2}{5}\right)^5 \left(\frac{3}{5}\right)^{5-5}$$
$$= 5.\left(\frac{2}{5}\right)^4 \left(\frac{3}{5}\right) + \left(\frac{2}{5}\right)^5$$

$$= \left(\frac{}{5}\right) \left[3 + \frac{}{5}\right]$$

$$= \frac{17}{5}\left(\frac{2}{5}\right)^4$$

Binomial Distribution Ex 33.1 Q37

Let p be the probability of success is experiments, q be the probability of failure,

Given, $p = 2q$

but $\quad p + q = 1$

$$2q + q = 1$$
$$3q = 1$$
$$q = \frac{1}{3}$$
$$p = \frac{2}{3}$$

Let X denote the random variable representing the number of success out of 6 experiments.

Probability of getting r success out of n experiments is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$
$$= {}^6C_r \left(\frac{2}{3}\right)^r \left(\frac{1}{3}\right)^{6-r} \qquad ---(1)$$

Probability of getting at least 4 success

$$= P(X = 4) + P(X = 5) + P(X = 6)$$
$$= {}^6C_4 \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^{6-4} + {}^6C_5 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^{6-5} + {}^6C_6 \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^{6-6}$$
$$= \frac{6 \times 5}{2} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^2 + 6 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^1 + \left(\frac{2}{3}\right)^6$$
$$= \left(\frac{2}{3}\right)^4 \left[\frac{15}{9} + \frac{4}{3} + \frac{4}{9}\right]$$
$$= \left(\frac{2}{3}\right)^4 \left[\frac{15 + 12 + 4}{9}\right]$$
$$= \left(\frac{31}{9}\right)\left(\frac{2}{3}\right)^4$$
$$= \frac{496}{729}$$

Required probability $= \dfrac{496}{729}$

Binomial Distribution Ex 33.1 Q38

Let x = number of out of service machines
p = probability that machine will be out of service on the same day
= 2/100
q = probability that machine will be in service on the same day
= 8/100
P(x=3) = probability exactly 3 machines will be out of service on the same day

$P(x=3) = {}^{20}C_3 \times \left(\frac{2}{100}\right)^3 \left(\frac{8}{100}\right)^0$ = 1140x0.000008
= 0.00912
For low probability events Poisson' distribution is used instead of Binomial distribution. Then,
$\lambda = np$ = 20x0.02 = 0.4

$P(x=r) = \frac{e^{-\lambda} \times \lambda^3}{r!}$

$P(x=3) = \frac{e^{-0.4} \times 0.4^3}{3!}$ = 0.6703x0.064/6 = 0.0071

Binomial Distribution Ex 33.1 Q39

Let p be the probability that a student entering a university will graduate, so

$p = 0.4$

$q = 1 - 0.4$ [Since $p + q = 1$]

$= 0.6$

Let X denote the random variable representing the number of students entering a university will graduate out of 3 students of university. Probability that r students will graduate out of n entering the university is given by

$$P(X = r) = {}^nC_r p^r (q)^{n-r}$$

$$= {}^3C_r (0.4)^r (0.6)^{3-r} \quad ---(1)$$

(i)

Probability that none will graduate

$= P(X = 0)$

$= {}^3C_0 (0.4)^0 (0.6)^{3-0}$

$= 0.216$

Probability that none will graduate $= 0.216$

(ii)
Probability that one will graduate

$= P(X = 1)$

$= {}^3C_1 (0.4)^1 (0.6)^{3-1}$

$= 3 \times (0.4)(0.36)$

$= 0.432$

Probability that only one will graduate $= 0.432$

(iii)
Probability that all will graduate

$= P(X = 3)$

$= {}^3C_3 (0.4)^3 (0.6)^{3-3}$

$= (0.4)^3$

$= 0.064$

Probability that all will graduate $= 0.064$

Binomial Distribution Ex 33.1 Q40

Let X denote the number of defective eggs in the 10 eggs drawn.
Since the drawing is done with replacement, the trials are Bernoulli trials.

Clearly, X has the binomial distribution with n=10 and $p = \frac{10}{100} = \frac{1}{10}$

Therefore, $q = 1 - \frac{1}{10} = \frac{9}{10}$

Now, $P(\text{at least one defective egg}) = P(X \geq 1) = 1 - P(X = 0)$

$$= 1 - {}^{10}C_0 \left(\frac{9}{10}\right)^{10} = 1 - \frac{9^{10}}{10^{10}}$$

Binomial Distribution Ex 33.1 Q41

Let p be the probability of answering a true. So

$$p = \frac{1}{2}$$

$$q = 1 - \frac{1}{2} \qquad [\text{Since } p + q = 1]$$

$$= \frac{1}{2}$$

Thus the probability that he answers at least 12 questions correctly among 20 questions is

$$P(X \geq 12) = P(X = 12) + P(X = 13) + P(X = 14) + P(X = 15) + P(X = 16) +$$

$$P(X = 17) + P(X = 18) + P(X = 19) + P(X = 20)$$

$$= \left(\frac{1}{2}\right)^{20} \left\{ {}^{20}C_{12} + {}^{20}C_{13} + {}^{20}C_{14} + {}^{20}C_{15} + {}^{20}C_{16} + {}^{20}C_{17} + {}^{20}C_{18} + {}^{20}C_{19} + {}^{20}C_{20} \right\}$$

$$= \frac{{}^{20}C_{12} + {}^{20}C_{13} + {}^{20}C_{14} + {}^{20}C_{15} + {}^{20}C_{16} + {}^{20}C_{17} + {}^{20}C_{18} + {}^{20}C_{19} + {}^{20}C_{20}}{2^{20}}$$

Therefore, the required answer is

$$\frac{{}^{20}C_{12} + {}^{20}C_{13} + {}^{20}C_{14} + {}^{20}C_{15} + {}^{20}C_{16} + {}^{20}C_{17} + {}^{20}C_{18} + {}^{20}C_{19} + {}^{20}C_{20}}{2^{20}}$$

Binomial Distribution Ex 33.1 Q42

X is the random variable whose binomial distribution is $B\left(6, \frac{1}{2}\right)$.

Therefore, $n = 6$ and $p = \frac{1}{2}$

$$\therefore q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\text{Then, } P(X = x) = {}^{n}C_x q^{n-x} p^x$$

$$= {}^{6}C_x \left(\frac{1}{2}\right)^{6-x} \cdot \left(\frac{1}{2}\right)^x$$

$$= {}^{6}C_x \left(\frac{1}{2}\right)^6$$

It can be seen that P(X = x) will be maximum, if ${}^{6}C_x$ will be maximum.

$$\text{Then, } {}^{6}C_0 = {}^{6}C_6 = \frac{6!}{0! \cdot 6!} = 1$$

$${}^{6}C_1 = {}^{6}C_5 = \frac{6!}{1! \cdot 5!} = 6$$

$${}^{6}C_2 = {}^{6}C_4 = \frac{6!}{2! \cdot 4!} = 15$$

$${}^{6}C_3 = \frac{6!}{3! \cdot 3!} = 20$$

The value of ${}^{6}C_3$ is maximum. Therefore, for $x = 3$, P(X = x) is maximum.

Thus, X = 3 is the most likely outcome.

Binomial Distribution Ex 33.1 Q43

The repeated guessing of correct answers from multiple choice questions are Bernoulli trials. Let X represent the number of correct answers by guessing in the set of 5 multiple choice questions.

Probability of getting a correct answer is, $p = \frac{1}{3}$

$$\therefore q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$$

Clearly, X has a binomial distribution with n = 5 and $p = \frac{1}{3}$

$$\therefore \mathrm{P}(\mathrm{X} = x) = {}^{n}\mathrm{C}_{x} q^{n-x} p^{x}$$
$$= {}^{5}\mathrm{C}_{x} \left(\frac{2}{3}\right)^{5-x} \cdot \left(\frac{1}{3}\right)^{x}$$

P (guessing more than 4 correct answers) = P(X ≥ 4)

$$= \mathrm{P}(\mathrm{X} = 4) + \mathrm{P}(\mathrm{X} = 5)$$
$$= {}^{5}\mathrm{C}_{4} \left(\frac{2}{3}\right) \cdot \left(\frac{1}{3}\right)^{4} + {}^{5}\mathrm{C}_{5} \left(\frac{1}{3}\right)^{5}$$
$$= 5 \cdot \frac{2}{3} \cdot \frac{1}{81} + 1 \cdot \frac{1}{243}$$
$$= \frac{10}{243} + \frac{1}{243}$$
$$= \frac{11}{243}$$

Binomial Distribution Ex 33.1 Q44

(b) P (winning exactly once) = P(X = 1)

$$= {}^{50}C_1\left(\frac{99}{100}\right)^{49}\cdot\left(\frac{1}{100}\right)^{1}$$

$$= 50\left(\frac{1}{100}\right)\left(\frac{99}{100}\right)^{49}$$

$$= \frac{1}{2}\left(\frac{99}{100}\right)^{49}$$

(c) P (at least twice) = P(X ≥ 2)

$$= 1 - P(X < 2)$$

$$= 1 - P(X \le 1)$$

$$= 1 - [P(X = 0) + P(X = 1)]$$

$$= [1 - P(X = 0)] - P(X = 1)$$

$$= 1 - \left(\frac{99}{100}\right)^{50} - \frac{1}{2}\cdot\left(\frac{99}{100}\right)^{49}$$

$$= 1 - \left(\frac{99}{100}\right)^{49}\cdot\left[\frac{99}{100} + \frac{1}{2}\right]$$

$$= 1 - \left(\frac{99}{100}\right)^{49}\cdot\left(\frac{149}{100}\right)$$

$$= 1 - \left(\frac{149}{100}\right)\left(\frac{99}{100}\right)^{49}$$

Binomial Distribution Ex 33.1 Q45

Let the shooter fire n times.
n fires are Bernoulli trials.

In each trial, p= probability of hitting the target= $\frac{3}{4}$

And q = probability of not hitting the target= $1-\frac{3}{4}=\frac{1}{4}$

Then, $P(X=x)={}^nC_x\, q^{n-x} p^x={}^nC_x\left(\frac{1}{4}\right)^{n-x}\left(\frac{3}{4}\right)^x={}^nC_x\,\frac{3^x}{4^n}$

Now, given that

P(hitting the target atleast once) > 0.99

i.e. $P(x\ge 1)>0.99$

$\Rightarrow \quad 1-P(x=0)>0.99$

$\Rightarrow \quad 1-{}^nC_0\,\frac{1}{4^n}>0.99$

$\Rightarrow \quad {}^nC_0\,\frac{1}{4^n}<0.01$

$\Rightarrow \quad \frac{1}{4^n}<0.01$

$\Rightarrow \quad 4^n>\frac{1}{0.01}=100$

The minimum value of n to satisfy this inequality is 4
Thus, the shooter must fire 4 times.

Binomial Distribution Ex 33.1 Q46

Let the man toss the coin n times. The n tosses are n Bernoulli trials.

Probability (p) of getting a head at the toss of a coin is $\frac{1}{2}$.

$\Rightarrow p = \frac{1}{2} \Rightarrow q = \frac{1}{2}$

$$\therefore P(X = x) = {}^nC_x p^{n-x} q^x = {}^nC_x \left(\frac{1}{2}\right)^{n-x} \left(\frac{1}{2}\right)^x = {}^nC_x \left(\frac{1}{2}\right)^n$$

It is given that,

P (getting at least one head) > $\frac{90}{100}$

P ($x \geq 1$) > 0.9

1 − P ($x = 0$) > 0.9

$$1 - {}^nC_0 \cdot \frac{1}{2^n} > 0.9$$

$$ {}^nC_0 \cdot \frac{1}{2^n} < 0.1$$

$$\frac{1}{2^n} < 0.1$$

$$2^n > \frac{1}{0.1}$$

$$2^n > 10 \qquad \ldots(1)$$

The minimum value of n that satisfies the given inequality is 4.

Thus, the man should toss the coin 4 or more than 4 times.

Binomial Distribution Ex 33.1 Q47

Let the man toss the coin n times.

Probability (p) of getting a head at the toss of a coin is $\frac{1}{2}$.

So,

$$p=\frac{1}{2}$$

$$q=1-\frac{1}{2} \qquad [\text{Since } p+q=1]$$

$$=\frac{1}{2}$$

$$\therefore P(X=x)={}^{n}C_{x}p^{n-x}q^{x}$$

$$={}^{n}C_{x}\left(\frac{1}{2}\right)^{n-x}\left(\frac{1}{2}\right)^{x}$$

$$={}^{n}C_{x}\left(\frac{1}{2}\right)^{n}$$

It is given that

$$p(\text{getting atleast one head})>\frac{80}{100}$$

$$P(x\geq 1)>0.8$$

$$1-P(x=0)>0.8$$

$$1-{}^{n}C_{0}\cdot\frac{1}{2^{n}}>0.8$$

$${}^{n}C_{0}\cdot\frac{1}{2^{n}}<0.2$$

$$\frac{1}{2^{n}}<0.2$$

$$2^{n}>\frac{1}{0.2}$$

$$2^{n}>5$$

The minimum value of n that satisfies the given inequality is 3.
Thus, the man should toss the coin 3 or more than 3 times.

Binomial Distribution Ex 33.1 Q48

Let p be the probability of getting a doublet in a throw of a pair of dice, so

$$p = \frac{6}{36} \qquad \left[\text{Since } (1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\right]$$
$$= \frac{1}{6}$$
$$q = 1 - \frac{1}{6} \qquad [\text{Since } p+q=1]$$
$$= \frac{5}{6}$$

Let X denote the number of getting doublets i.e. success out of 4 times. So, probability distribution is given by

X	$P(X)$
0	${}^4C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{4-0} = \left(\frac{5}{6}\right)^4$
1	${}^4C_1\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^{4-1} = 4\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^3 = \frac{2}{3}\left(\frac{5}{6}\right)^3$
2	${}^4C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{4-2} = \frac{4\cdot 3}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2 = \frac{25}{216}$
3	${}^4C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^{4-3} = \frac{4\cdot 3}{2}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right) = \frac{5}{324}$
4	${}^4C_4\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^{4-4} = \left(\frac{1}{6}\right)^4 = \frac{1}{1296}$

Binomial Distribution Ex 33.1 Q49

Let p be the probability of defective bulbs, so

$$p = \frac{6}{30}$$
$$= \frac{1}{5}$$
$$q = 1 - \frac{1}{5} \qquad [\text{Since } p + q = 1]$$
$$= \frac{4}{5}$$

Here, 4 bulbs is drawn at random with replacement. So, probability distribution is given by

X	$P(X)$
0	${}^4C_0\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^{4-0} = \frac{256}{625}$
1	${}^4C_1\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^{4-1} = \frac{4}{5} \times \frac{4^3}{5^3} = \frac{256}{625}$
2	${}^4C_2\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^{4-2} = \frac{6}{5^2} \times \frac{4^2}{5^2} = \frac{96}{625}$
3	${}^4C_3\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^{4-3} = \frac{4}{5^3} \times \frac{4}{5} = \frac{16}{625}$
4	${}^4C_4\left(\frac{1}{5}\right)^4\left(\frac{6}{5}\right)^{4-4} = 1 \cdot \frac{1}{625} = \frac{1}{625}$

Binomial Distribution Ex 33.1 Q50

Here success is a score which is multiple of 3 i.e. 3 or 6.

$$\therefore p(3 \text{ or } 6) = \frac{2}{6} = \frac{1}{3}$$

The probability of r successes in 10 throws is given by

$$P(r) = {}^{10}C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{10-r}$$

Now P(at least 8 successes) = P(8) + P(9) + P(10)

$$= {}^{10}C_8 \left(\frac{1}{3}\right)^8 \left(\frac{2}{3}\right)^2 + {}^{10}C_9 \left(\frac{1}{3}\right)^9 \left(\frac{2}{3}\right)^1 + {}^{10}C_{10} \left(\frac{1}{3}\right)^{10} \left(\frac{2}{3}\right)^0$$

$$= \frac{1}{3^{10}}\left[45 \times 4 + 10 \times 2 + 1\right]$$

$$= \frac{201}{3^{10}}$$

Binomial Distribution Ex 33.1 Q51

Here success is an odd number i.e. 1,3 or 5.

$\therefore p(1,3 \text{ or } 5) = \frac{3}{6} = \frac{1}{2}$

The probability of r successes in 5 throws is given by

$$P(r) = {}^5C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{5-r}$$

Now P(exactly 3 times) = P(3)

$$= {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2$$

$$= \frac{10}{2^5}$$

$$= \frac{5}{16}$$

Binomial Distribution Ex 33.1 Q52

Pr obablity of a man hitting a target is 0.25.

$$\therefore p = 0.25 = \frac{1}{4}, \quad q = 1 - p = \frac{3}{4}$$

The probability of r successes in 7 shoots is given by

$$P(r) = {}^7C_r (0.25)^r (0.75)^{7-r}$$

Now P(at least twice) = 1 − P(less than 2)

$$= 1 - {}^7C_0 (0.25)^0 (0.75)^7 + {}^7C_1 (0.25)^1 (0.75)^6$$

$$= 1 - \frac{3^7}{4^7} + 7 \times \frac{3^6}{4^7}$$

$$= \frac{4547}{8192}$$

Binomial Distribution Ex 33.1 Q53

Pr obablity of a bulb to be defective is $\frac{1}{50}$.

$\therefore p = \frac{1}{50}, \; q = 1 - p = \frac{49}{50}$

The probability of r defective bulbs in 10 bulbs is given by

$P(r) = {}^{10}C_r \left(\frac{1}{50}\right)^r \left(\frac{49}{50}\right)^{10-r}$

(i) P(none of the bulb is defective) = P(0)

$= {}^{10}C_0 \left(\frac{1}{50}\right)^0 \left(\frac{49}{50}\right)^{10}$

$= \left(\frac{49}{50}\right)^{10}$

(ii) P(exactly two bulbs are defective) = P(2)

$= {}^{10}C_2 \left(\frac{1}{50}\right)^2 \left(\frac{49}{50}\right)^8$

$= 45 \times \frac{(49)^8}{(50)^{10}}$

(iii) P(more than 8 bulbs work properly)

= P(at most two bulbs are defective)

$$= {}^{10}C_0\left(\frac{1}{50}\right)^0\left(\frac{49}{50}\right)^{10} + {}^{10}C_1\left(\frac{1}{50}\right)^1\left(\frac{49}{50}\right)^9 + {}^{10}C_2\left(\frac{1}{50}\right)^2\left(\frac{49}{50}\right)^8$$

$$= \left(\frac{49}{50}\right)^{10} + 10\times\frac{(49)^9}{(50)^{10}} + 45\times\frac{(49)^8}{(50)^{10}}$$

$$= \frac{(49)^8}{(50)^{10}}\left[(49)^2 + 490 + 45\right]$$

$$= \frac{(49)^8\times 2936}{(50)^{10}}$$

Ex 33.2

Binomial Distribution Ex 33.2 Q1

Let X be a binomial variate with parameters n and p.

$$\text{Mean} = np$$
$$\text{Varience} = npq$$

$$\begin{aligned}\text{Mean} - \text{Variance} &= np - npq \\ &= np(1-q) \\ &= np.p \\ &= np^2\end{aligned}$$

$$\text{Mean} - \text{Variance} > 0$$
$$\text{Mean} > \text{Variance}$$

So, mean can never be less than varience.

Binomial Distribution Ex 33.2 Q2

Let X denote the variance with parameters n and p

$$p + q = 1$$
$$q = 1 - p$$

Goven,Mean $= np = 9$ $\quad --- (i)$

Variance $= npq = \frac{9}{4}$ $\quad --- (ii)$

$$\frac{npq}{np} = \frac{\frac{9}{4}}{9} \qquad [\text{By dividing } (i) \text{ by } (ii)]$$
$$q = \frac{1}{4}$$

So, $p = 1 - q$

$$= 1 - \frac{1}{4}$$
$$p = \frac{3}{4}$$

Put p in equation (i),

$$n\left(\frac{3}{4}\right) = 9$$
$$\Rightarrow \quad n = \frac{36}{3}$$

So, $n = 12$

The distribution is given by

$$= {}^nC_r p^r (q)^{n-r}$$

$$P(X = r) = {}^{12}C_r \left(\frac{3}{4}\right)^r \left(\frac{1}{4}\right)^{12-r}$$

for $r = 0, 1, 2, ..12$

Binomial Distribution Ex 33.2 Q3

Let n and p be parameters of binomial distribution. Here

$$\text{Mean } = np = 9 \qquad ---(i)$$

$$\text{Variance } = npq = 6 \qquad ---(ii)$$

$$\frac{npq}{np} = \frac{6}{9}$$

$$q = \frac{2}{3}$$

So, $p = 1 - \frac{2}{3}$ $\qquad$ $[\text{Since } p + q = 1]$

$$p = \frac{1}{3}$$

Using equation (i), $np = 9$

$$n\left(\frac{1}{3}\right) = 9$$

$$n = 27$$

Hence, binomial distribution is given by

$$P(X = r) = {}^{27}C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{27-r}$$

$$r = 0, 1, 2, \ldots 27$$

Binomial Distribution Ex 33.2 Q4

Given that,

$$n = 5$$

Also, $\text{Mean} + \text{Varience} = 4.8$

$$np + npq = 4.8$$

$$np(1+q) = 4.8$$

$$5p(1+q) = 4.8$$

$$5(1-q)(1+q) = 4.8 \qquad [\text{Since } p + q = 1]$$

$$5(1-q^2) = 4.8$$

$$1 - q^2 = \frac{4.8}{5}$$

$$q^2 = 1 - \frac{4.8}{5}$$

$$= \frac{0.2}{5}$$

$$q^2 = \frac{1}{25}$$

$$q = \frac{1}{5}$$

$\Rightarrow$ $p = 1 - q$

$$= 1 - \frac{1}{5}$$

$$p = \frac{4}{5}$$

So, $n = 5$, $p = \frac{4}{5}$, $q = \frac{1}{5}$

Here binomial distribution is

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$P(X = r) = 5C_r \left(\frac{4}{5}\right)^r \left(\frac{1}{5}\right)^{5-r}$$

$$r = 0, 1, 2, 3, \ldots 5$$

Binomial Distribution Ex 33.2 Q5

Given that,

$$\text{Mean } = np = 20 \qquad ---(i)$$

$$\text{Variance } = npq = 16 \qquad ---(ii)$$

Let n and p be the parameters of distribution dividing equation (ii) by (i)

$$\frac{npq}{np} = \frac{16}{20}$$

$$q = \frac{4}{5}$$

So, $p = 1 - q$ $\qquad$ [Since $p + q = 1$]

$$= 1 - \frac{4}{5}$$

$$p = \frac{1}{5}$$

Put p in equation (i),

$$np = 20$$

$$n\left(\frac{1}{5}\right) = 20$$

$$n = 20 \times 5$$

$$n = 100$$

So, binomial distribution is given by

$$P(X = r) = {}^{n}C_r p^r q^{n-r}$$

$$P(X = r) = {}^{100}C_r \left(\frac{1}{5}\right)^r \left(\frac{4}{5}\right)^{100-r}$$

$$r = 0, 1, 2, 3, \ldots 100$$

Binomial Distribution Ex 33.2 Q6

Let n and p be the parameters of distribution binomial distribution. So

$$q = 1 - p \qquad \text{as } p + q = 1$$

$$\text{Mean} + \text{Variance} = \frac{25}{3}$$

$$np + npq = \frac{25}{3}$$

$$np(1+q) = \frac{25}{3}$$

$$np = \frac{25}{3(1+q)} \qquad ---(1)$$

$$\text{Mean} \times \text{Variance} = \frac{50}{3}$$

$$np \times npq = \frac{50}{3}$$

$$n^2p^2q = \frac{50}{3}$$

$$\left[\frac{25}{3(1+q)}\right]^2 .q = \frac{50}{3} \qquad [\text{Using } (1)]$$

$$625q = \frac{50}{3}\left[9(1+q)^2\right]$$

$$625q = 150(1+q)^2$$

$$25q = 6(1+q)^2$$

$$6 + 6q^2 + 12q - 25q = 0$$

$$6q^2 - 13q + 6 = 0$$

$$6q^2 - 9q - 4q + 6 = 0$$

$$3q(2q-3) - 2(2q-3) = 0$$

$$(2q-3)(3q-2) = 0$$

$$\Rightarrow \quad 2q - 3 = 0 \quad \text{or} \quad 3q - 2 = 0$$

$$\Rightarrow \quad q = \frac{3}{2} \quad \text{or} \quad q = \frac{2}{3}$$

Since $q \le 1$, so

$$q = \frac{2}{3}$$

$$p = 1 - q$$

$$= 1 - \frac{2}{3}$$

$$p = \frac{1}{3}$$

Binomial Distribution Ex 33.2 Q7

Let n and p be the parameters of binomial distribution.

Given that,

$$\text{Mean } = np - 20 \qquad ---(i)$$

$$\text{Standard deviation } = \sqrt{npq} = 4$$

Squaring both the sides,

$$npq = 16 \qquad ---(ii)$$

Dividing equation (ii) by (i),

$$\frac{npq}{np} = \frac{16}{20}$$

$$q = \frac{4}{5}$$

So, $p = 1 - q$ $\qquad$ [Since $p + q = 1$]

$$= 1 - \frac{4}{5}$$

$$p = \frac{1}{5}$$

Put value of p in equation (i),

$$np = 20$$

$$\frac{n}{5} = 20$$

$$n = 100$$

$$p = \frac{1}{5}$$

Binomial Distribution Ex 33.2 Q8

Let p denotes the probability of selecting a defective bolt, so

$$p = 0.1$$

$$p = \frac{1}{10}$$

$$q = 1 - \frac{1}{10} \qquad \left[\text{Since } p + q = 1\right]$$

$$q = \frac{9}{10}$$

Given, $n = 400$

(i)

$$\text{Mean } = np$$

$$= 400 \times \frac{1}{10}$$

$$\text{Mean } = 40$$

(ii)

$$\text{Standard deviation } = \sqrt{npq}$$

$$= \sqrt{400 \times \frac{1}{10} \times \frac{9}{10}}$$

$$= \sqrt{36}$$

Standard deviation $= 6$

Binomial Distribution Ex 33.2 Q9

Let n and p be the parameters of binomial distribution.

Given, Mean $= np = 5$ $\quad ---(i)$

$$\text{Variance } = npq = \frac{10}{3} \quad ---(ii)$$

Dividing (ii) by (i)

$$\frac{npq}{np} = \frac{\frac{10}{3}}{5}$$

$$q = \frac{2}{3}$$

So, $\quad p = 1 - q$ $\quad$ [Since $p + q = 1$]

$$= 1 - \frac{2}{3}$$

$$p = \frac{1}{3}$$

Put the value of p in equation (i),

$$np = 5$$
$$n = 5 \times 3$$
$$n = 15$$

Hence, the binomial distribution is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$P(X = r) = {}^{15}C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{15-r}$$

$$r = 0, 1, 2, \ldots 15$$

Binomial Distribution Ex 33.2 Q10

Let p be the probability of a ship returning safely to parts, so

$$p = \frac{9}{10}$$

$$q = 1 - \frac{9}{10} \quad \text{[Since } p + q = 1\text{]}$$

$$q = \frac{1}{10}$$

Given, $n = 500$

$$\text{Mean } = np$$
$$= 500 \times \frac{9}{10}$$
$$\text{Mean } = 450$$

$$\text{Standard deviation } = \sqrt{npq}$$
$$= \sqrt{500 \times \frac{9}{10} \times \frac{1}{10}}$$
$$= \sqrt{45}$$
$$= 6.71$$

Mean $= 450$

Standard deviation $= 6.71$

Binomial Distribution Ex 33.2 Q11

Given that, parameters for binomial distribution are n and p.

Also, Mean $= np = 16$ --- (i)

Variance $= npq = 8$ --- (ii)

Dividing (ii) by (i)

$$\frac{npq}{np} = \frac{8}{16}$$

$$q = \frac{1}{2}$$

So, $p = 1 - \frac{1}{2}$ $[\text{as } p + q = 1]$

$$p = \frac{1}{2}$$

Put the value of p in equation (i),

$$np = 16$$

$$n\left(\frac{1}{2}\right) = 16$$

$$n = 32$$

Hence, binomial distribution is given by,

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$P(X = r) = {}^{32}C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{32-r} \quad \text{--- (iii)}$$

$$P(X = 0)$$

$$= {}^{32}C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{32-0} \quad [\text{Using (iii)}]$$

$$= \left(\frac{1}{2}\right)^{32}$$

$$P(X = 1)$$

$$= {}^{32}C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{32-1}$$

$$= 32 . \frac{1}{2}\left(\frac{1}{2}\right)^{31}$$

$$= \left(\frac{1}{2}\right)^{27}$$

$$P(X \geq 2)$$

$$= 1 - [P(X = 0) + P(X = 1)]$$

$$= 1 - \left[\left(\frac{1}{2}\right)^{32} + \left(\frac{1}{2}\right)^{27}\right]$$

$$= 1 - \left(\frac{1}{2}\right)^{27}\left(\frac{1}{32} + 1\right)$$

$$= 1 - \left(\frac{1}{2}\right)^{27}\left(\frac{33}{32}\right)$$

$$= 1 - \frac{33}{2^{32}}$$

Hence

$$P(X = 0) = \left(\frac{1}{2}\right)^{32}, \ P(X = 1) = \left(\frac{1}{2}\right)^{27}, \ P(X \geq 2) = 1 - \frac{33}{2^{32}}$$

Binomial Distribution Ex 33.2 Q12

Let p be the probability of success in a single throw of die

$$p = \frac{2}{6} \qquad \left[\text{Since success is occurance of 5 or 6}\right]$$

$$p = \frac{1}{3}$$

$$q = 1 - \frac{1}{3} \qquad \left[\text{Since } p + q = 1\right]$$

$$q = \frac{2}{3}$$

Given, $n = 8$

$$\begin{aligned} \text{Mean } &= np \\ &= \frac{8}{3} \\ &= 2.66 \end{aligned}$$

$$\begin{aligned} \text{Standard deviation } &= \sqrt{npq} \\ &= \sqrt{8 \times \frac{1}{3} \times \frac{2}{3}} \\ &= \frac{4}{3} \\ &= 1.33 \end{aligned}$$

Mean $= 2.66$, Standard deviation $= 1.33$

Binomial Distribution Ex 33.2 Q13

Let n and p be the parameters of binomial distribution.
Let p = probability of having a boy in the family

Given, $p = q$

Since, $p + q = 1$

$$p + p = 1$$

$$2p = 1$$

$$p = \frac{1}{2}$$

$$q = \frac{1}{2}$$

$$n = 8$$

$$\begin{aligned} \text{The expected number of boys } &= np \\ &= 8 \times \frac{1}{2} \\ &= 4 \end{aligned}$$

The expected number of boys $= 4$

Binomial Distribution Ex 33.2 Q14

Let p denot the probability of a defective item produced in the factory, so

$$p = 0.02$$
$$= \frac{2}{100}$$
$$p = \frac{1}{50}$$
$$q = 1 - \frac{1}{50} \qquad [\text{Since } p + q = 1]$$
$$= \frac{49}{50}$$

Given $n = 10{,}000$

Expected number of defective item $= np$

$$= 10000 \times \frac{1}{50}$$
$$= 200$$

Standard deviation $= \sqrt{npq}$

$$= \sqrt{10000 \times \frac{1}{50} \times \frac{49}{50}}$$
$$= 14$$

Expected No. of defective items $= 200$

Standard deviation $= 14$

Binomial Distribution Ex 33.2 Q15

Let p be the probability of success, so

$$p = \frac{2}{6} \qquad [\text{Since success in occurance of 1 or 6 on the die}]$$
$$p = \frac{1}{3}$$

Given, $n = 3$

$$q = 1 - p \qquad [\text{Since } p + q = 1]$$
$$= 1 - \frac{1}{3}$$
$$q = \frac{2}{3}$$

Mean $= np$

$$= 3\left(\frac{1}{3}\right)$$
$$= 1$$

Variance $= npq$

$$= 3 \times \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)$$
$$= \frac{2}{3}$$

Mean $= 1$

Variance $= \frac{2}{3}$

Binomial Distribution Ex 33.2 Q16

Let n and p be the parameters of binomial distribution
Given,

$$\text{Mean} = np = 3 \quad ---(\text{i})$$

$$\text{Variance} = npq = \frac{3}{2} \quad ---(\text{ii})$$

Dividing equation (ii) by (i),

$$\frac{npq}{np} = \frac{\frac{3}{2}}{3}$$

$$q = \frac{1}{2}$$

$$p = 1 - \frac{1}{2} \quad [\text{as } p + q = 1]$$

$$p = \frac{1}{2}$$

Put the value of p in equation (i)

$$np = 3$$

$$n\left(\frac{1}{2}\right) = 3$$

$$n = 6$$

Hence, binomial distribution is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$P(X = r) = {}^6C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{6-r} \quad ---(\text{iii})$$

$$P(X \le 5)$$

$$= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$$

$$= 1 - P(X = 6)$$

$$= 1 - {}^6C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^{6-6}, \quad [\text{Using (iii)}]$$

$$= 1 - \left(\frac{1}{2}\right)^6$$

$$= 1 - \frac{1}{64}$$

$$= \frac{63}{64}$$

$$P(X \le 5) = \frac{63}{64}$$

Binomial Distribution Ex 33.2 Q17

Let n and p be the parameters of binomial distribution.

Given,

$$\text{Mean } = np = 4 \qquad ---(i)$$
$$\text{Variance } = npq = 2 \qquad ---(ii)$$

Dividing equation (ii) by (i),

$$\frac{npq}{np} = \frac{2}{4}$$
$$q = \frac{1}{2}$$
$$p = 1 - \frac{1}{2} \qquad [\text{Since } p + q = 1]$$
$$p = \frac{1}{2}$$

Put the value of p in equation (i),

$$np = 4$$
$$n\left(\frac{1}{2}\right) = 4$$
$$n = 8$$

Hence, binomial distribution is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$
$$P(X = r) = {}^8C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{8-r} \qquad ---(iii)$$

$$P(X \geq 5)$$
$$= P(X = 5) + P(X = 6) + P(X = 7) + P(X = 8)$$
$$= {}^8C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^3 + {}^8C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^2 + {}^8C_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right) + {}^8C_8\left(\frac{1}{2}\right)^8$$
$$[\text{Using equation (iii)}]$$
$$= \frac{8 \times 7 \times 6}{3 \times 2}\left(\frac{1}{2}\right)^8 + \frac{8 \times 7}{2}\left(\frac{1}{2}\right)^8 + 8\left(\frac{1}{2}\right)^8 + \left(\frac{1}{2}\right)^8$$
$$= \left(\frac{1}{2}\right)^8 [56 + 28 + 8 + 1]$$
$$= \frac{93}{256}$$

$$P(X \geq 5) = \frac{93}{256}$$

Binomial Distribution Ex 33.2 Q18

$$= 1 - \left(\frac{2}{3}\right)^4$$
$$= 1 - \frac{16}{81}$$
$$= \frac{65}{81}$$

$$P(X \geq 1) = \frac{65}{81}$$

Binomial Distribution Ex 33.2 Q19

Let n and p be the parameters of binomial distribution,

Given, $n = 6$

Mean + Variance $= \frac{10}{3}$

$$np + npq = \frac{10}{3}$$

$$6p + 6pq = \frac{10}{3}$$

$$6p(1+q) = \frac{10}{3}$$

$$6(1-q)(1+q) = \frac{10}{3} \qquad [\text{Since } p+q=1]$$

$$6\left(1-q^2\right) = \frac{10}{3}$$

$$1-q^2 = \frac{10}{18}$$

$$-q^2 = \frac{5}{9} - 1$$

$$-q^2 = -\frac{4}{9}$$

$$q^2 = \frac{4}{9}$$

$$q = \frac{2}{3}$$

$$p = 1-q$$

$$= 1 - \frac{2}{3}$$

$$p = \frac{1}{3}$$

Hence, the binomial distribution is given by,

$$P(X=r) = {}^nC_r p^r q^{n-r}$$

$$P(X=r) = {}^6C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{6-r}$$

as $r = 0, 1, 2, \ldots 6$

Binomial Distribution Ex 33.2 Q20

Throwing a doublet i.e. $\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$

Total number of outcomes $= 36$

Let p be the probability of success therefore

$p = 6/36 = 1/6$

Let q be the probability of failure therefore $q = 1-p = 1-1/6 = 5/6$

Since the dice is thrown 4 times, $n = 4$

Let X be the random variable for getting doublet, therefore X can take at max 4 values.

$$P(X=0) = {}^4C_0 p^0 q^4 = \left(\frac{5}{6}\right)^4 = \frac{625}{1296}$$

$$P(X=1) = {}^4C_1 p^1 q^3 = 4 \cdot \frac{1}{6} \cdot \left(\frac{5}{6}\right)^3 = \frac{500}{1296}$$

$$P(X=2) = {}^4C_2 p^2 q^2 = \frac{4\cdot 3}{2} \cdot \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right)^2 = \frac{150}{1296}$$

$$P(X=3) = {}^4C_3 p^3 q^1 = 4 \cdot \left(\frac{1}{6}\right)^3 \cdot \frac{5}{6} = \frac{20}{1296}$$

$$P(X=4) = {}^4C_4 p^4 q^0 = 1 \cdot \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^0 = \frac{1}{1296}$$

Mean

$$\mu = \sum_{i=1}^{4} X_i P(X_i) = 0 \cdot \frac{625}{1296} + 1 \cdot \frac{500}{1296} + 2 \cdot \frac{150}{1296} + 3 \cdot \frac{20}{1296} + 4 \cdot \frac{1}{1296}$$

$$= \frac{500+300+60+4}{1296} = \frac{54}{81} = \frac{2}{3}$$

Hence the mean is $= \frac{2}{3}$

Binomial Distribution Ex 33.2 Q21

Throwing a doublet i.e. $\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$

Total number of outcomes $=36$

Let p be the probability of success therefore

$$p=6/36=1/6$$

Let q be the probability of failure therefore $q=1-p=1-1/6=5/6$

Since there is three rows of dice so $n=3$

Let X be the random variable for getting doublet, therefore X can take at max 3 values.

$$P(X=0)={}^3C_0p^0q^3=\left(\frac{5}{6}\right)^3=\frac{125}{216}$$

$$P(X=1)={}^3C_1p^1q^2=3\cdot\frac{1}{6}\cdot\left(\frac{5}{6}\right)^2=\frac{75}{216}$$

$$P(X=2)={}^3C_2p^2q^1=3\cdot\left(\frac{1}{6}\right)^2\cdot\left(\frac{5}{6}\right)=\frac{15}{216}$$

$$P(X=3)={}^3C_3p^3q^0=\left(\frac{1}{6}\right)^3=\frac{1}{216}$$

Mean

$$\mu=\sum_{i=1}^{3}X_iP(X_i)=0\cdot\frac{125}{216}+1\cdot\frac{75}{216}+2\cdot\frac{15}{216}+3\cdot\frac{1}{216}$$
$$=\frac{75+30+3}{216}=\frac{108}{216}=\frac{1}{2}$$

Hence the mean is $=\frac{1}{2}$

Binomial Distribution Ex 33.2 Q22

Out of 15 bulbs 5 are defective.

Hence, the probability that the drawn bulb is defective is

$$P(\text{Defective})=\frac{5}{15}=\frac{1}{3}$$

$$P(\text{Not defective})=\frac{10}{15}=\frac{2}{3}$$

Let X denote the number of defective bulbs out of 4.

Then, X follows binomial distribution with

$n=4$, $p=\frac{1}{3}$ and $q=\frac{2}{3}$ such that

$$P(X=r)={}^4C_r\left(\frac{1}{3}\right)^r\left(\frac{2}{3}\right)^{4-r};r=0,1,2,3,4$$

$$\text{Mean}=\sum_{r=0}^{4}rP(r)=1\times{}^4C_1\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3+2\times{}^4C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2$$
$$+3\times{}^4C_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)+4\times{}^4C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^0$$
$$=\frac{32}{81}+\frac{48}{81}+\frac{24}{81}+\frac{4}{81}=\frac{108}{81}=\frac{4}{3}$$

Binomial Distribution Ex 33.2 Q23

Let p be the probablity of getting 2 when a dice is thrown.

Then $p=\frac{1}{6}$

Clearly, X follows binomial distribution with $n=3$, $p=\frac{1}{6}$.

$\therefore$ Expectation $=E(X)=np=3\times\frac{1}{6}=\frac{1}{2}$

Binomial Distribution Ex 33.2 Q24

Let p be the probablity of getting an even number on the toss when a dice is thrown.

Let q be the probablity of not getting an even number on the toss when a dice is thrown.

Then $p=\frac{3}{6}=\frac{1}{2}$ and $q=\frac{1}{2}$

Clearly, X follows binomial distribution with $n=2$, $p=\frac{1}{2}$.

$\therefore$ Variance $=npq=2\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{2}$

Binomial Distribution Ex 33.2 Q25

Let p be the probablity of getting a spade card.
Let q be the probablity of getting a spade card.

Then $p = \frac{13}{52} = \frac{1}{4}$ and $q = \frac{3}{4}$

Clearly, X follows binomial distribution with $n = 3$, $p = \frac{1}{4}$ and $q = \frac{3}{4}$.

Probablity distribution is given by,

$$P(X = r) = {}^3C_r\left(\frac{1}{4}\right)^r\left(\frac{3}{4}\right)^{3-r};\ r = 0, 1, 2$$

$$\therefore \text{Mean} = np = 3 \times \frac{1}{4} = \frac{3}{4}$$