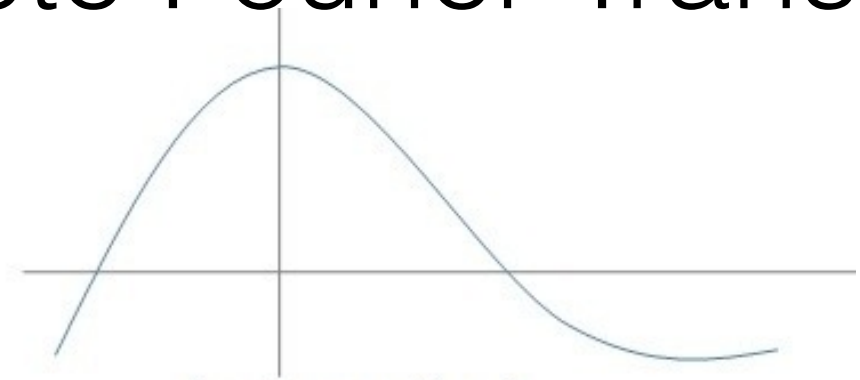
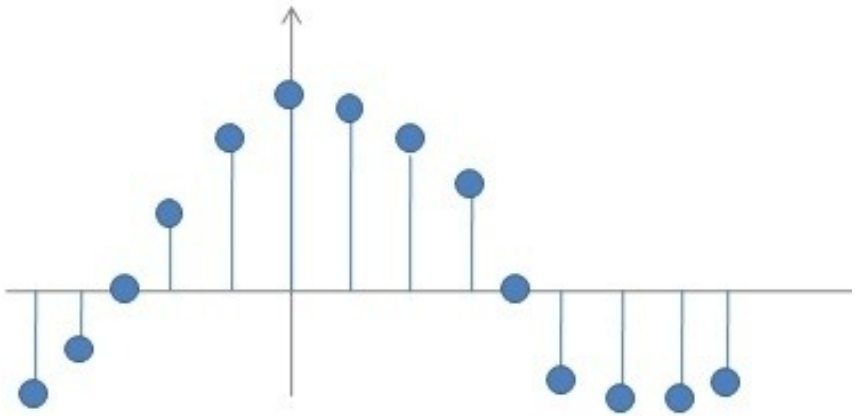
Discrete Fourier Transform



Continuous Signal



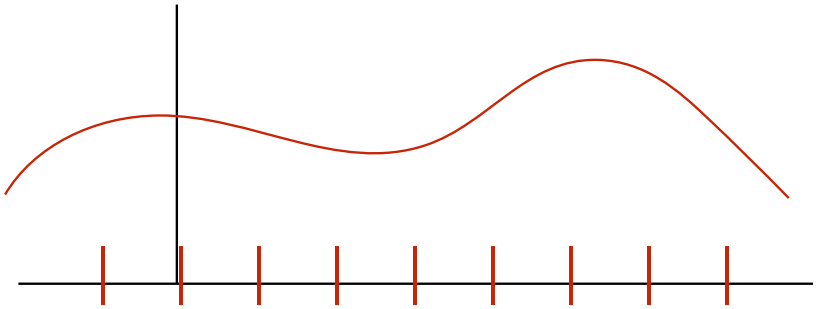
Discrete Signal

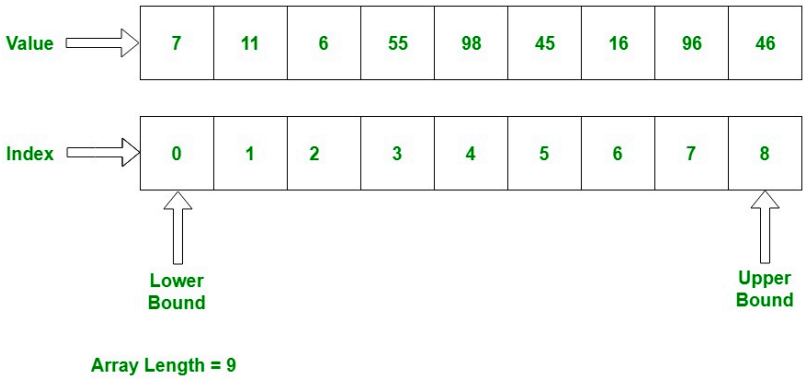
Discrete Fourier Transform

$$H(f) = \int_{-\infty}^{\infty} h(t) e^{-2\pi f t} dt$$

The problem is that $h(t)$ is discrete.
That is, we know $h(t)$ only at $t_1, t_2, t_3, \ldots, t_{n-1}, t_n$.

1. The range is finite.
2. The function values are known at discrete, uniform intervals.





Discrete Fourier Transform

We need to define the Discrete Fourier Transform.

$$H(f) = \int_{-\infty}^{\infty} h(t) e^{-2\pi i f t} dt \longrightarrow H(f) = \sum_{k=0}^{N-1} h_k e^{-2\pi i f t_k} \Delta$$

where N is the number of data points.

Let $\Delta = 1$ and $t_0, t_1, t_2, \ldots = 0,1,2,...$

$$H(f) = \sum_{k=0}^{N-1} h_k e^{-2\pi i f k}$$

We make f and H have N elements.

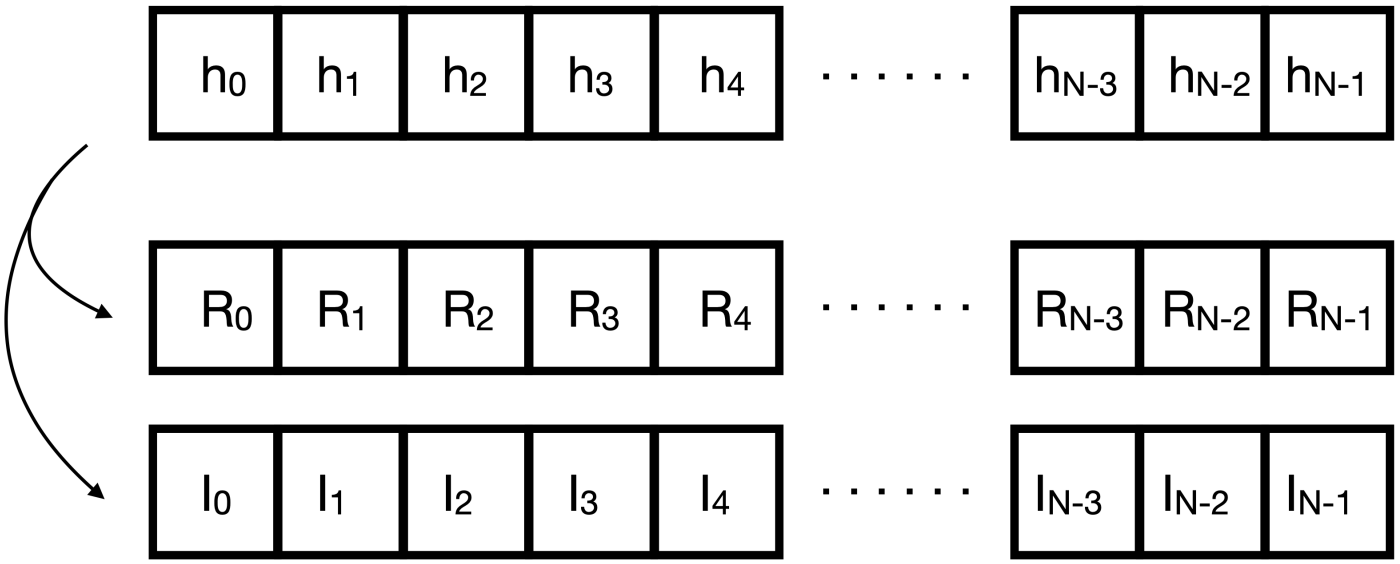
Let $f_n = \frac{n}{N}$, where $n = 0,1,2,...,N-1$

$$H_n = \sum_{k=0}^{N-1} h_k e^{-2\pi i k \frac{n}{N}}$$

Discrete Fourier Transform

$$H_n \equiv \sum_{k=0}^{N-1} h_k e^{-2\pi i k n/N}$$

$$\equiv \sum_{k=0}^{N-1} h_k [\cos(2\pi k n/N) - i \sin(2\pi k n/N)]$$



Negative Frequency

$$H_n = \sum_{k=0}^{N-1} h_k e^{-2\pi i k n/N}$$

Transformation of *N* h_k's to *N* H_n (complex)

$$f = [0, 1/N, 2/N, ..., (N-2)/N, (N-1)/N]$$

The above is periodic in *n* with period *N.*

For even N, this is equal to

$$f = [0, 1/N, 2/N, ...1/2, ..., -(N/2-1)/N, ..., -1/N]$$

Negative Frequency Example

10개의 원소를 가지는 array를 아래의 방법으로 FT할 경우

$$H_n = \sum_{k=0}^{N-1} h_k e^{-2\pi i k n / N}$$

f=[0,1/10,2/10,3/10,4/10,5/10,6/10,7/10,8/10,9/10]

또는

f=[0,1/10,2/10,3/10,4/10,5/10,-4/10,-3/10,-2/10,-1/10]

FT and Power Spectrum

Power Spectrum= power/frequency

$$H_n = \sum_{k=0}^{N-1} h_k e^{-2\pi i k n / N}$$

$$P_n = |H_n|^2$$

Discrete Inverse Fourier Transform

$$h_k = \frac{1}{N} \sum_{n=0}^{N-1} H_n e^{2\pi i k n / N}$$

We can almost use the same Fourier Transformation code!

과제 13

1. 0부터 99까지의 정수를 가지는 array를 정의 한다. x=[0,1,2,3,...,98,99]

2. x의 Fourier Transform(FT)을 수행한다 (Library 사용할 수 없음)

3. Fourier Transform후 각 원소에 해당되는 frequency 계산 (0을 중심으로 대칭이 되는 frequency 정의, -1/2<f<1/2)

4. FT 결과를 plot으로 표현 (e.g., power spectrum, image, etc.)

5. frequency=0 에서의 FT값은 어떤 의미를 가지는가?

6. 2의 FT 결과를 Inverse FT시켜 원래 Data가 나오는지 확인한다.

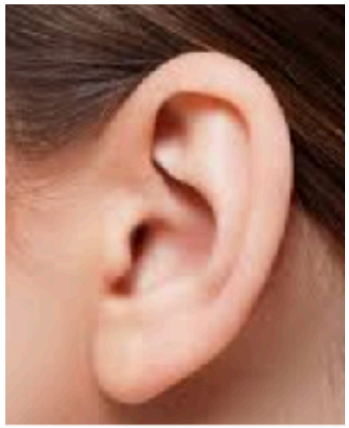
7. 2의 FT 결과에서 처음 10개의 frequency만 남기고 나머지를 frequency에 해당되는 FT값을 0으로 만든다음 Inverse FT 시켜 그 결과를 그래프로 표현한다. 어째서 이런 결과가 나오는지 설명한다.

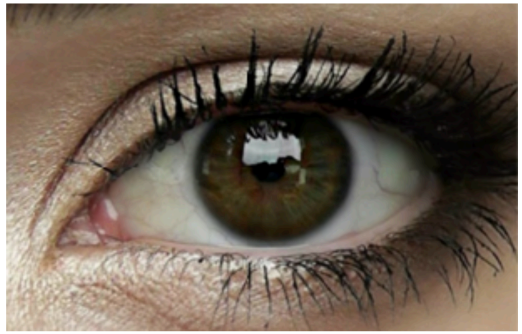
8. (삭제되지 않은 온전한) FT 결과를 이용하여 power spectrum을 계산하여 그래프로 출력한다.

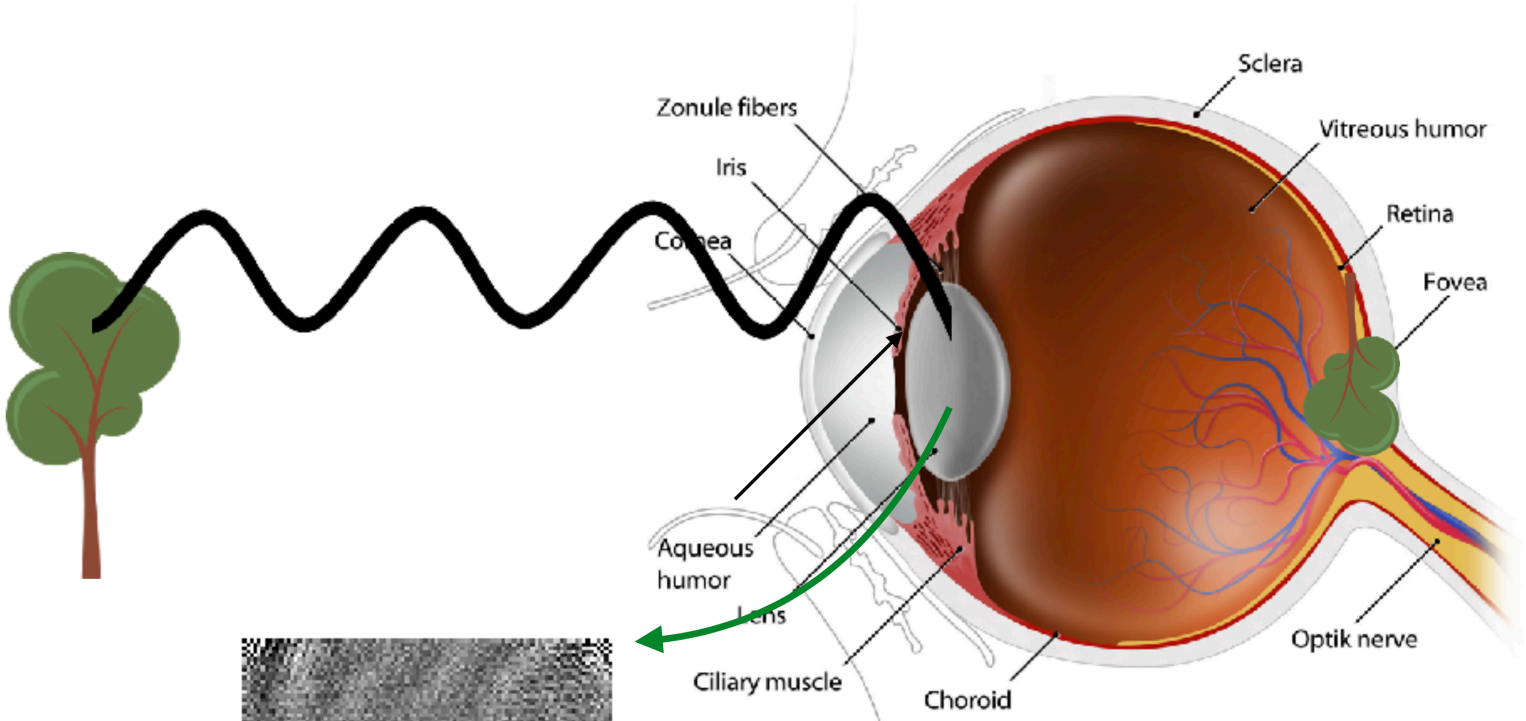
과제 13

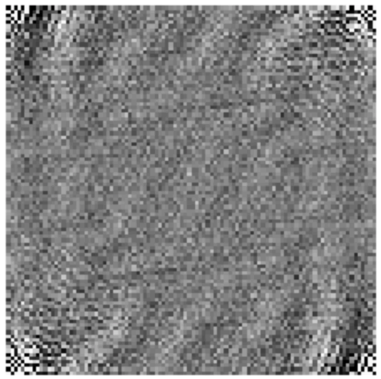
1. 0부터 99까지의 정수를 가지는 (길이가 100인) array를 정의 한다. x=[0,1,2,3,...,98,99]
2. x의 Fourier Transform(FT) **X**를 구한다 (numpy.fft, scipy.fftpack등의 Library 사용할 수 없음)
3. **X** 각 성분에 대응되는 주파수를 계산한다. 주파수는 -1/2<f<1/2 범위를 가진다.
4. FT 결과를 plot으로 표현한다(real과 imaginary를 모두 보여줌)
5. 파워스펙트럼을 그래프로 그리고 설명한다
6. 주파수=0 에서의 FT값은 어떤 의미를 가지는지 설명한다
7. 2의 FT 결과를 Inverse FT시켜 원래 Data가 나오는지 확인한다(Library 사용불가)
8. 2의 FT 결과에서 낮은 주파수 영역만 남기고 나머지 주파수에 해당되는 푸리에 계수를 0으로 만든 후 inverse FT를 수행한다
9. 원래의 x와 비교하고 어째서 이런 결과가 나오는지 설명한다.

Natural Fourier Transformer



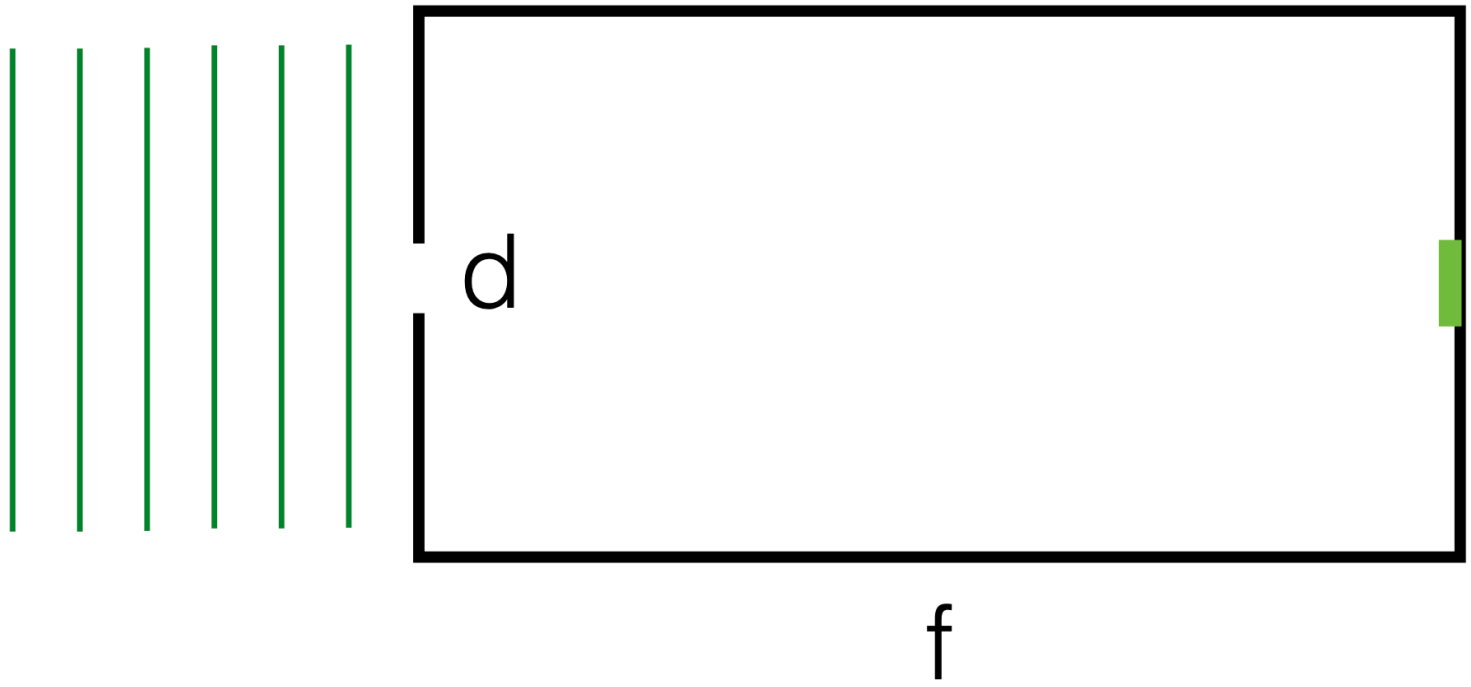






every telescope = FT machine

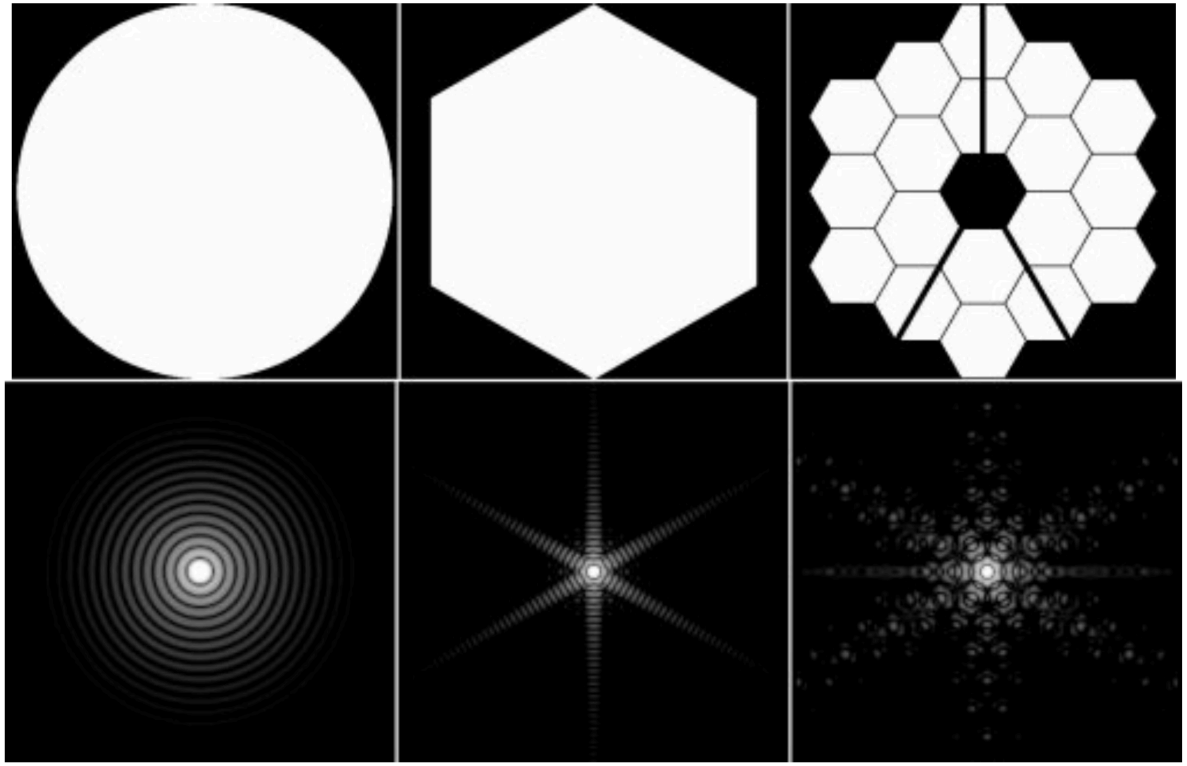
better resolution = big eyes

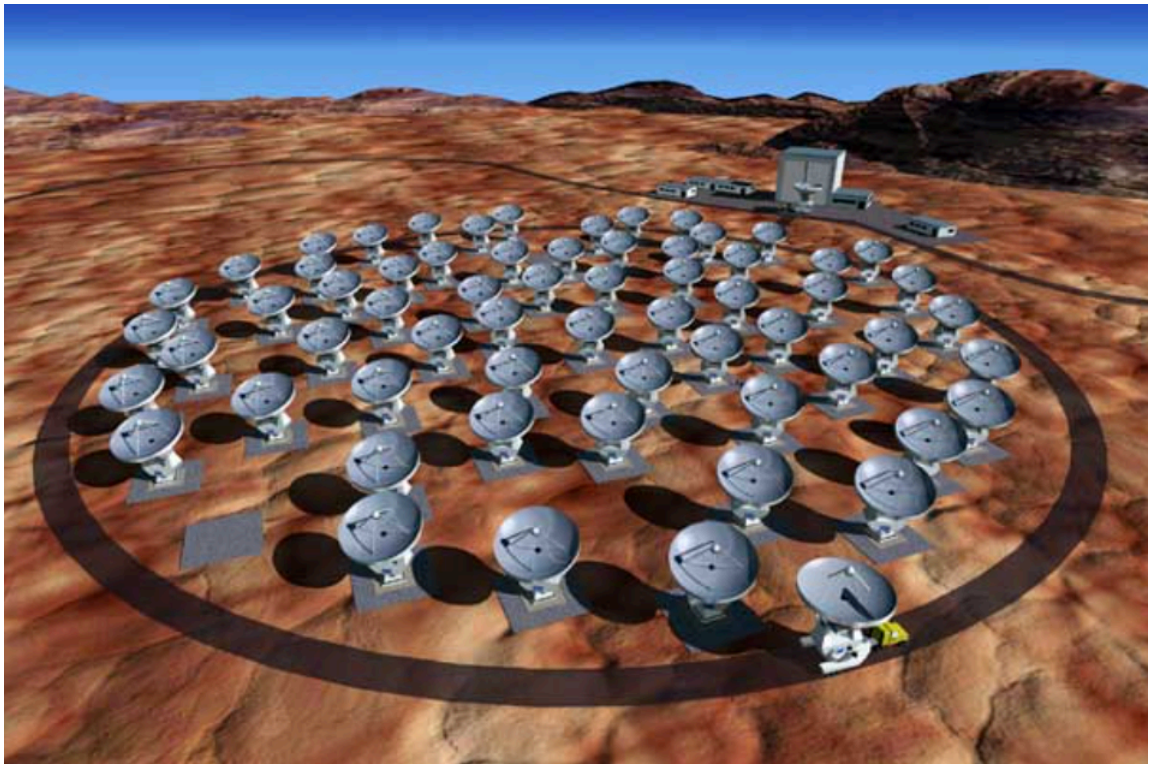


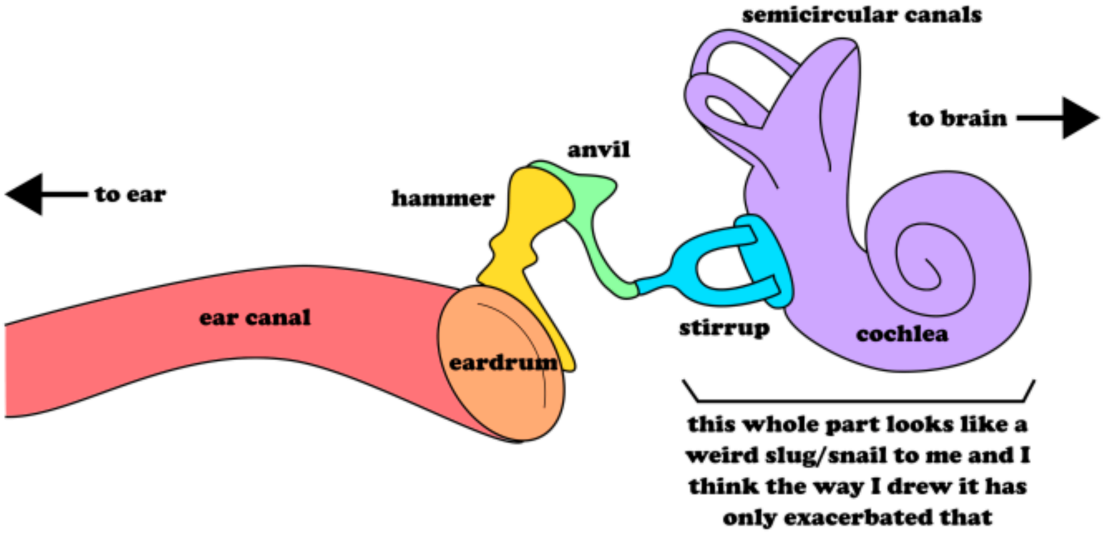
$$U(x,y,f) \propto \int \quad \exp\left[-i\pi \frac{(x-x')^2+(y-y')^2}{\lambda d}\right] dx'dy'$$

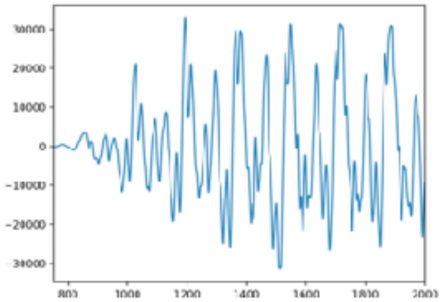
wave propagation from pupil to detector

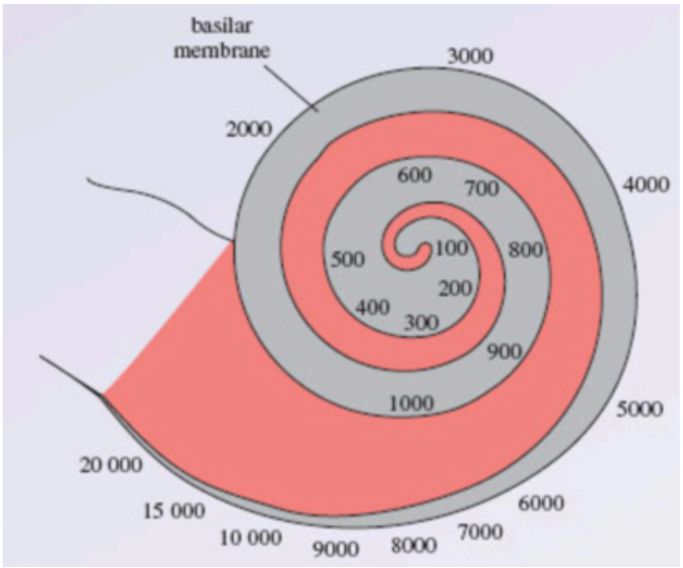
Telescope = Fourier Transformer

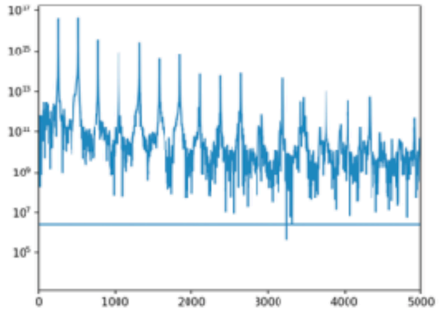




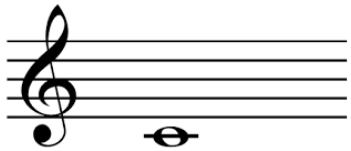

semicircular canals
to brain
anvil
to ear
hammer
ear canal
eardrum
stirrup
cochlea
this whole part looks like a weird slug/snail to me and I think the way I drew it has only exacerbated that



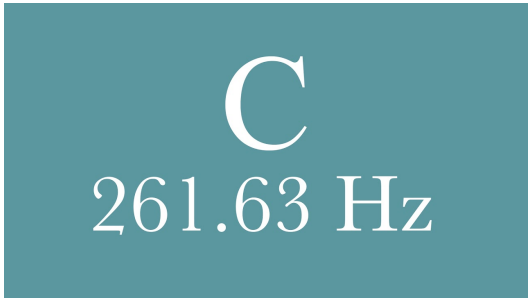

basilar membrane
3000
2000
4000
600
700
500
100
800
200
400
300
900
1000
5000
20 000
15 000
10 000
9000
8000
7000
6000

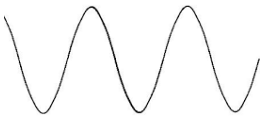


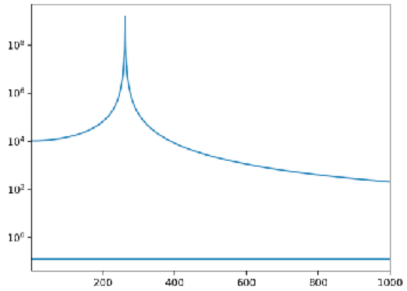
Middle C

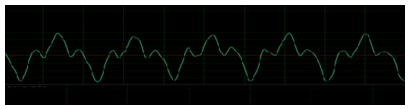


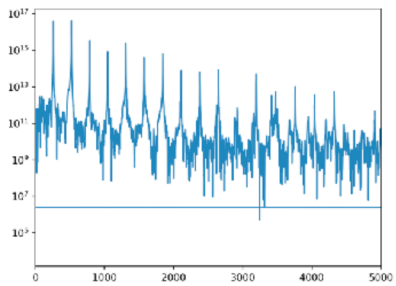
261.63 Hz







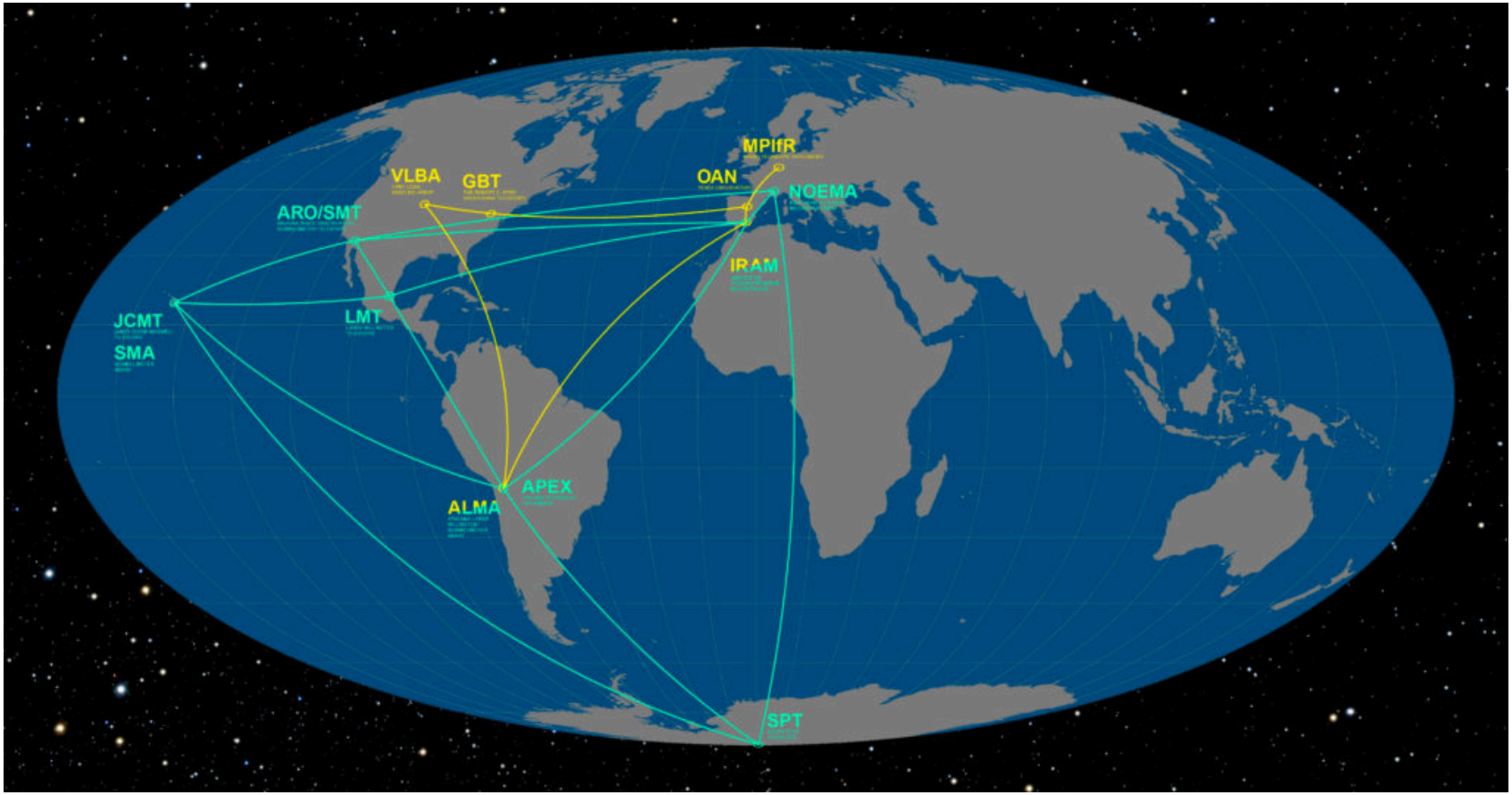


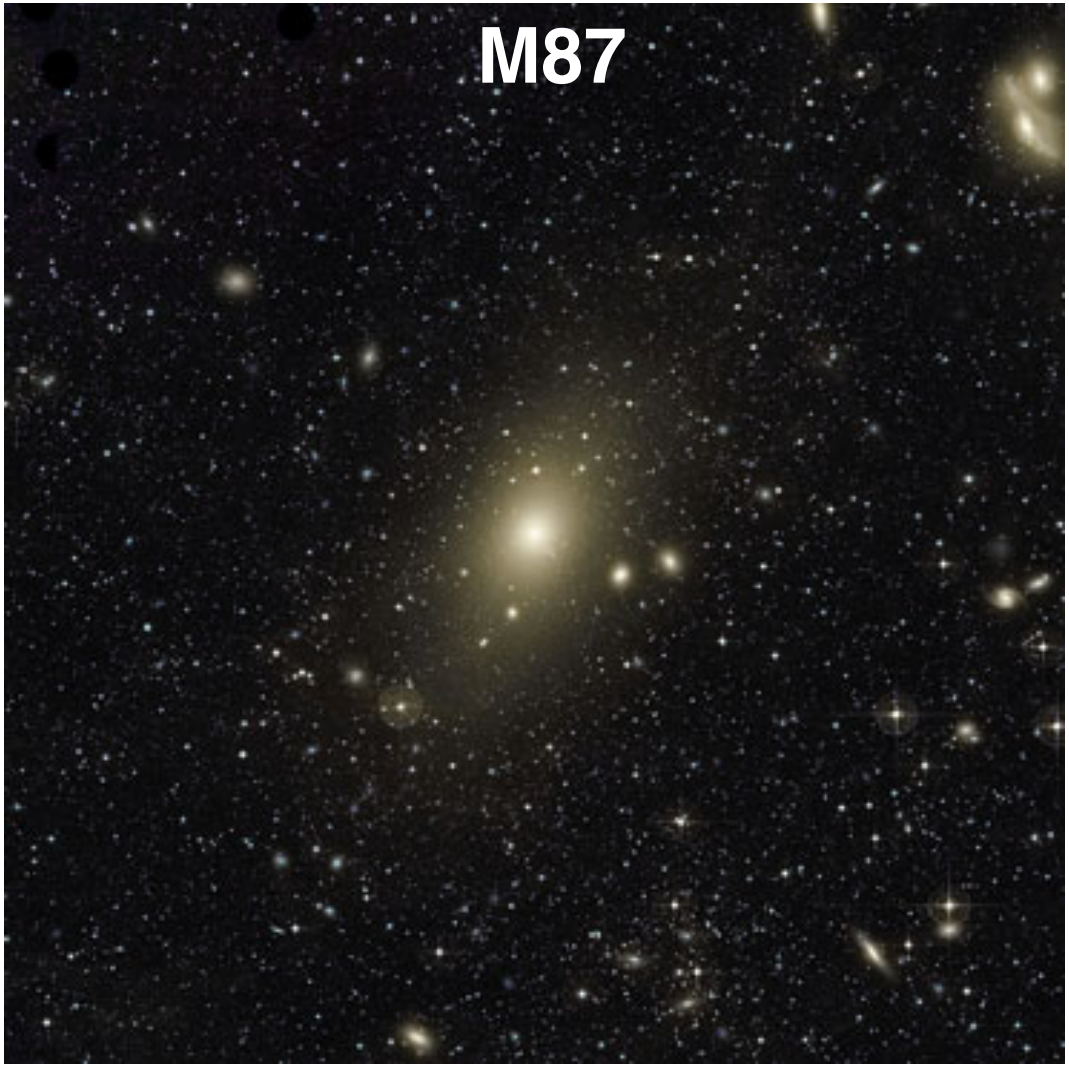


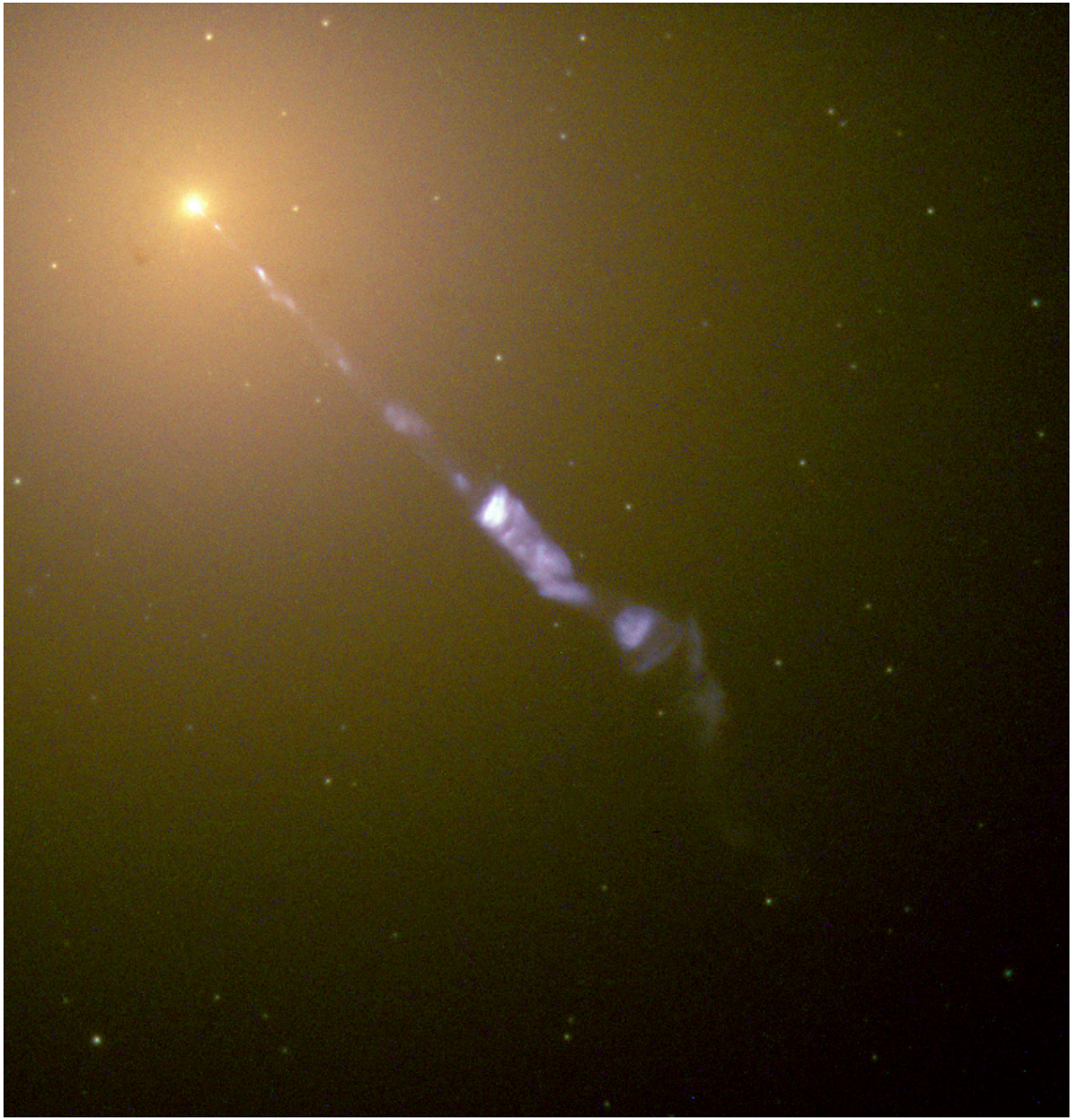
과제 14

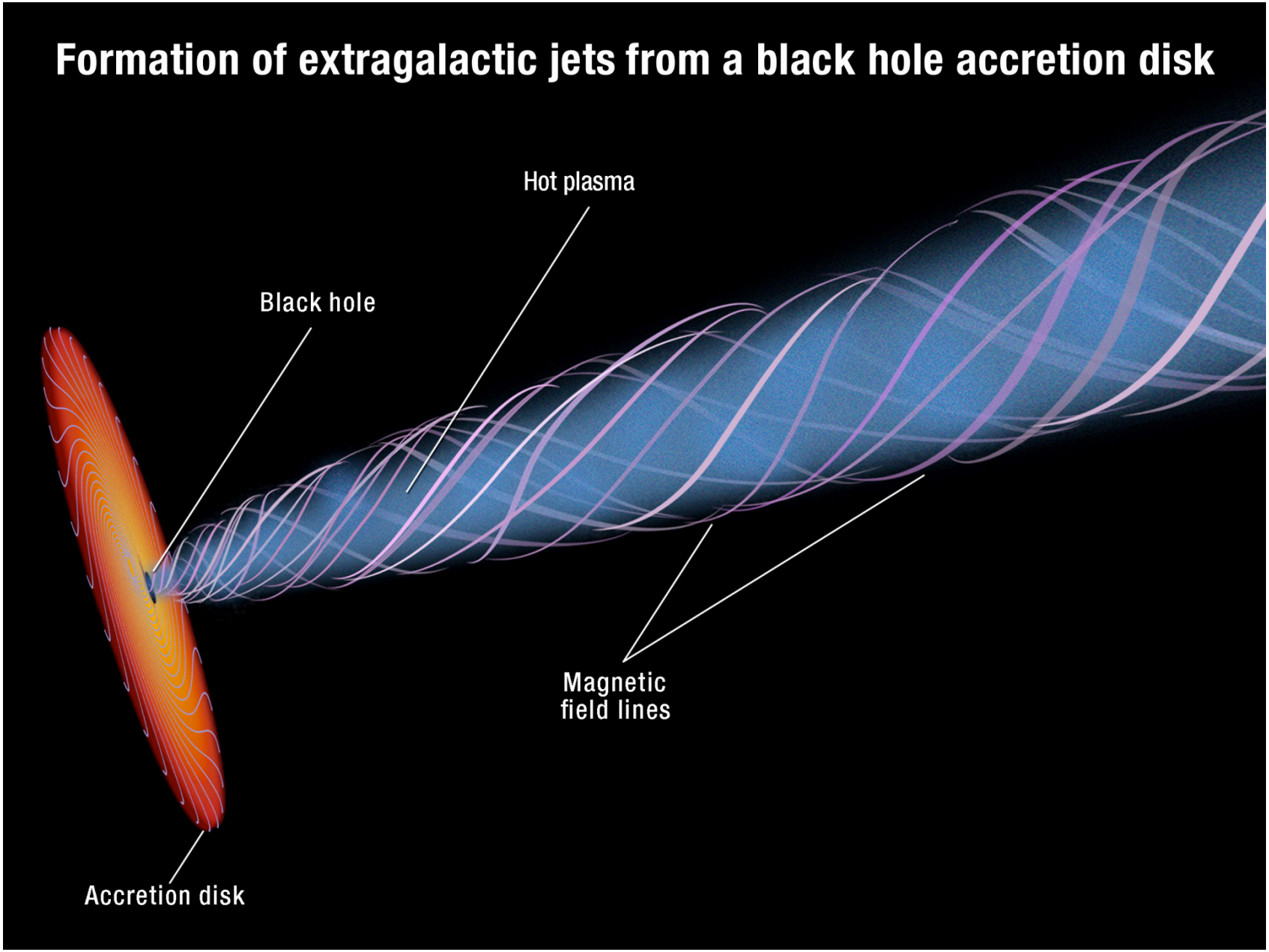
1. LearnUs에서 피아노의 가온다 wav file을 다운 받는다.
2. Real space에서 (가로축이 시간) 소리의 모양을 그래프로 나타낸다.
3. Fourier space에서 (가로축이 주파수) 소리의 모양을 그래프로 나타낸다.
4. 3번 Fourier space의 분석을 통해 어떠한 사실을 알 수 있는지 논한다.
5. 자신의 목소리로 "도"음을 1초 동안 소리내어 wav file로 저장한다.
6. 2-4번의 작업을 수행한다.
7. piano의 음과 비교해 보았을때 자신의 목소리는 어떠한 특징을 가지는가?

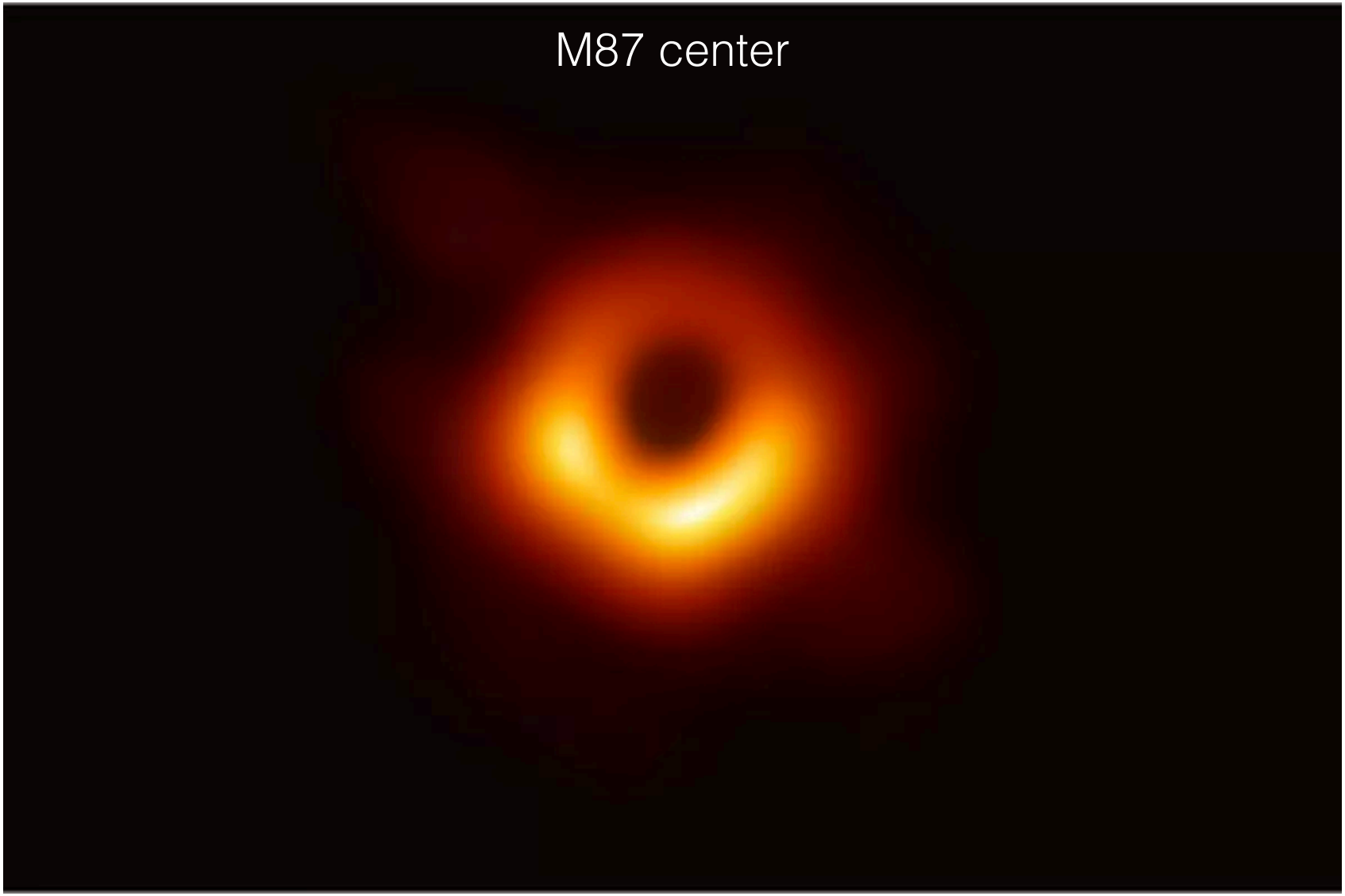
EVENT HORIZON TELESCOPE



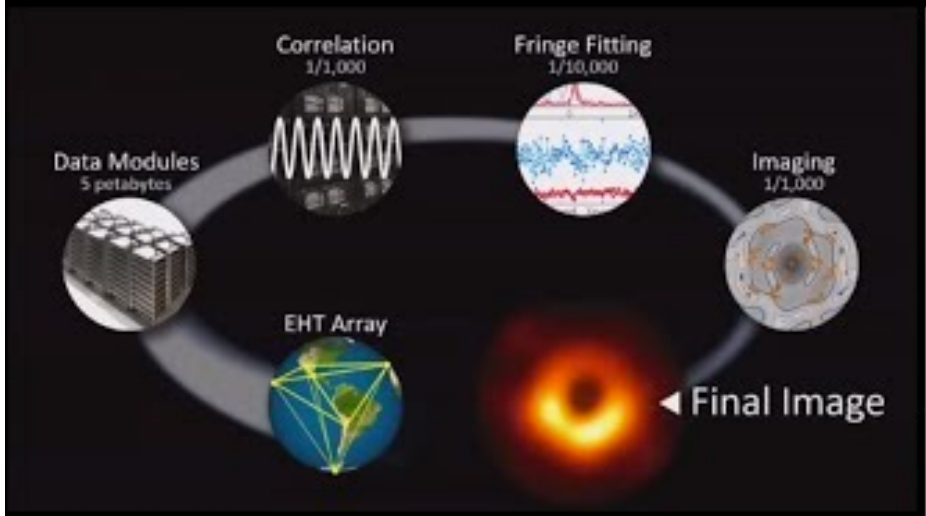

M87

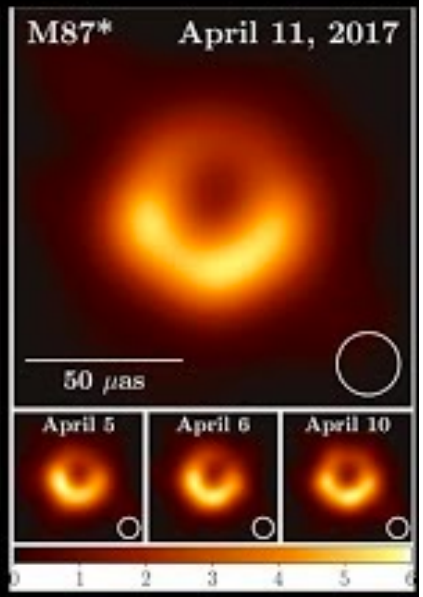


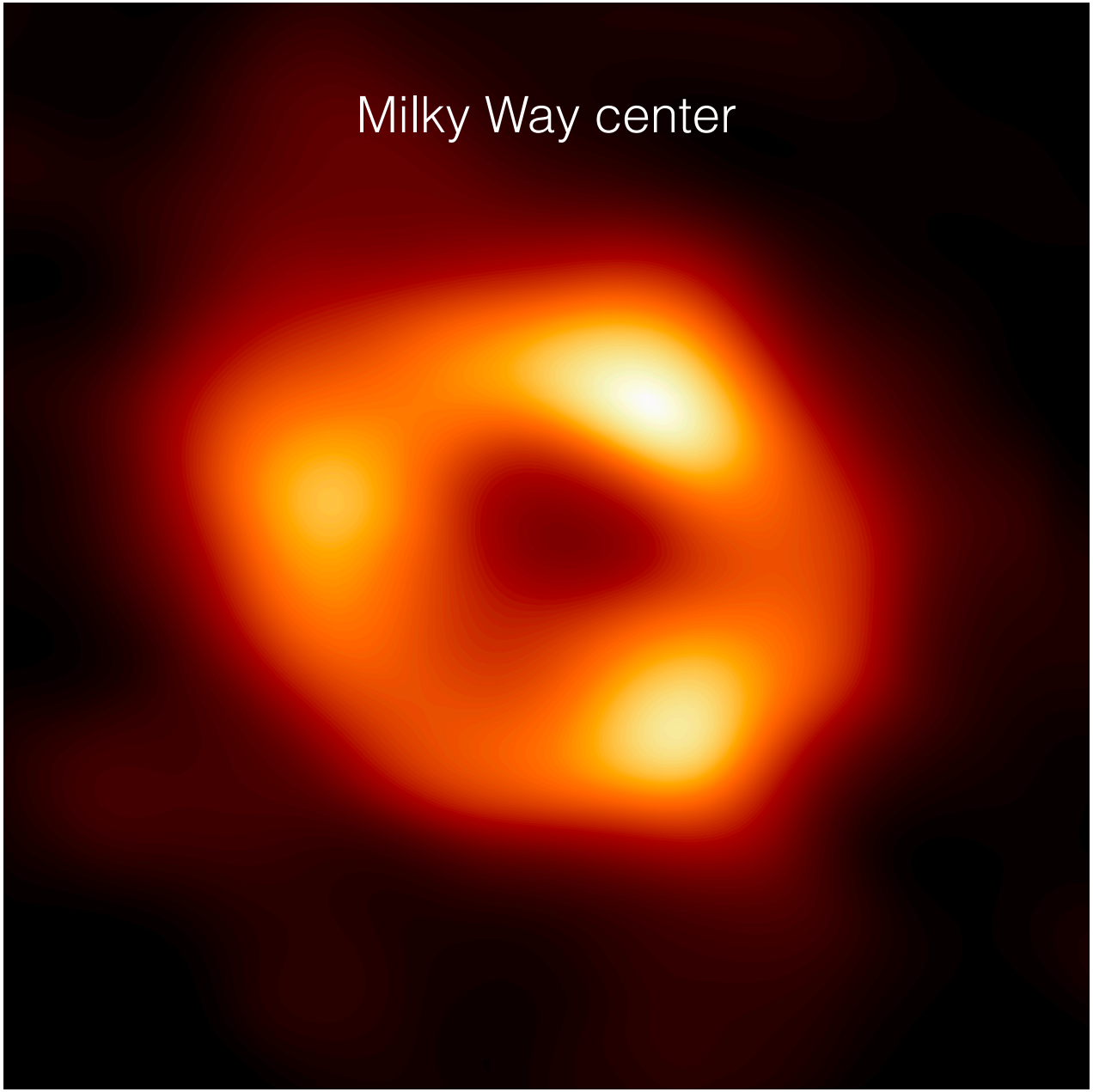

Formation of extragalactic jets from a black hole accretion disk
Hot plasma
Black hole
Magnetic
field lines
Accretion disk


M87 center

The Event Horizon Image






Milky Way center


“How to Take a Picture of the Milky Way’s Black Hole”
Event Horizon Telescope