Interpolation
in Two-dimension

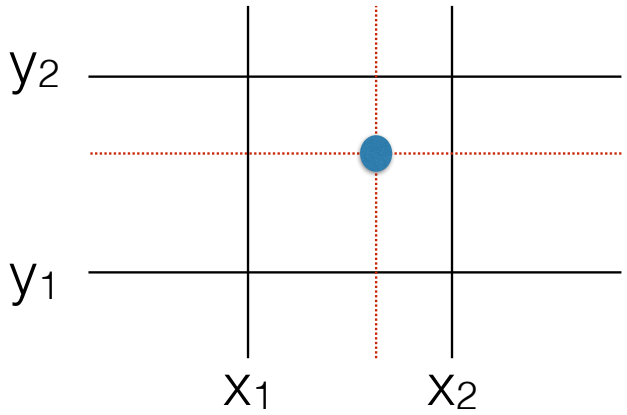
One-dimensional vs. Two-dimensional interpolations

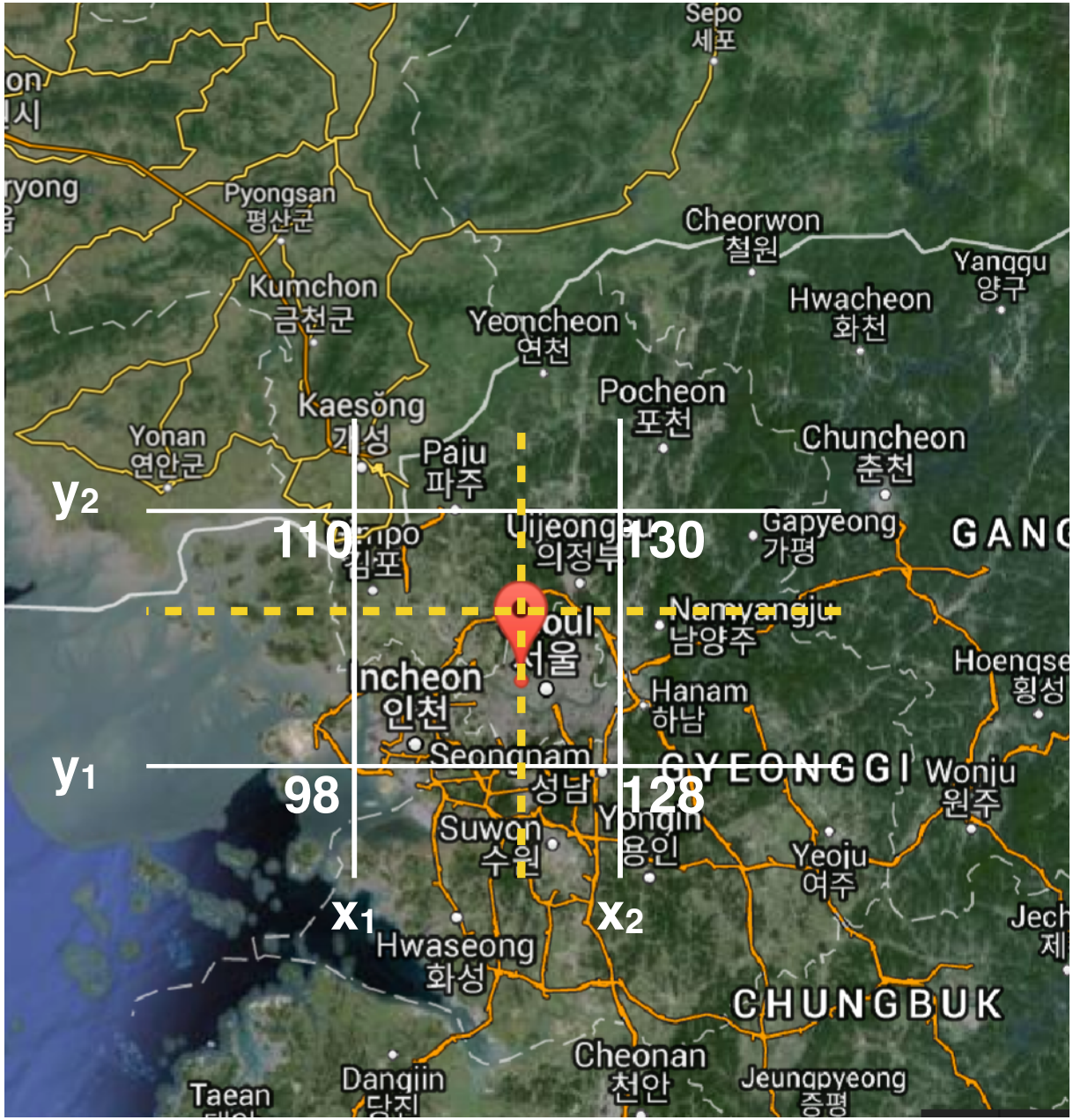
One-Dimensional Interpolation:

Given the two data points $(x_1,f(x_1))$ and $(x_2,f(x_2))$,
what is f(x)?

Two-Dimensional Interpolation:

Given the four data points $(x_1,y_1,f(x_1,y_1))$, $(x_2,y_1,f(x_2,y_1))$, $(x_1,y_2,f(x_1,y_2))$, and $(x_2,y_2,f(x_2,y_2))$ what is f(x,y)?




Sepo
세포
Pyongsan
평산군
Cheorwon
철원
Yanggu
양구
Kumchon
금천군
Yeoncheon
연천
Hwacheon
화천
Pocheon
포천
Kaesŏng
개성
Yonan
연안군
Paju
파주
Chuncheon
춘천
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Incheon
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Seongnam
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Wonju
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128
Suwon
수원
Yongin
용인
Yeoju
여주
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x2
Hwaseong
화성
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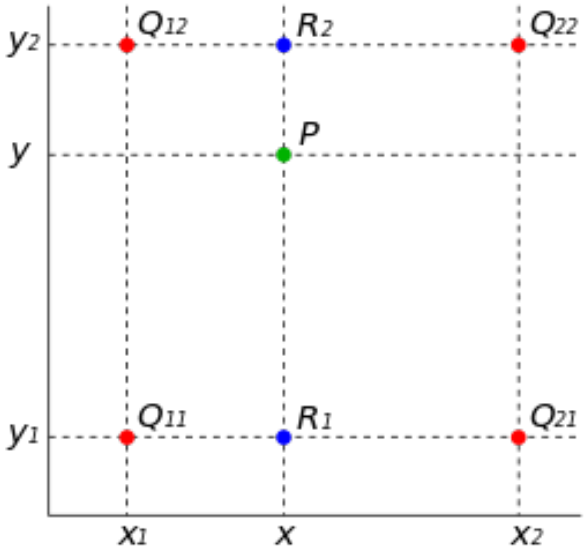
Cheonan
천안
Jeungpyeong
증평
Taean
Dangjin
당진

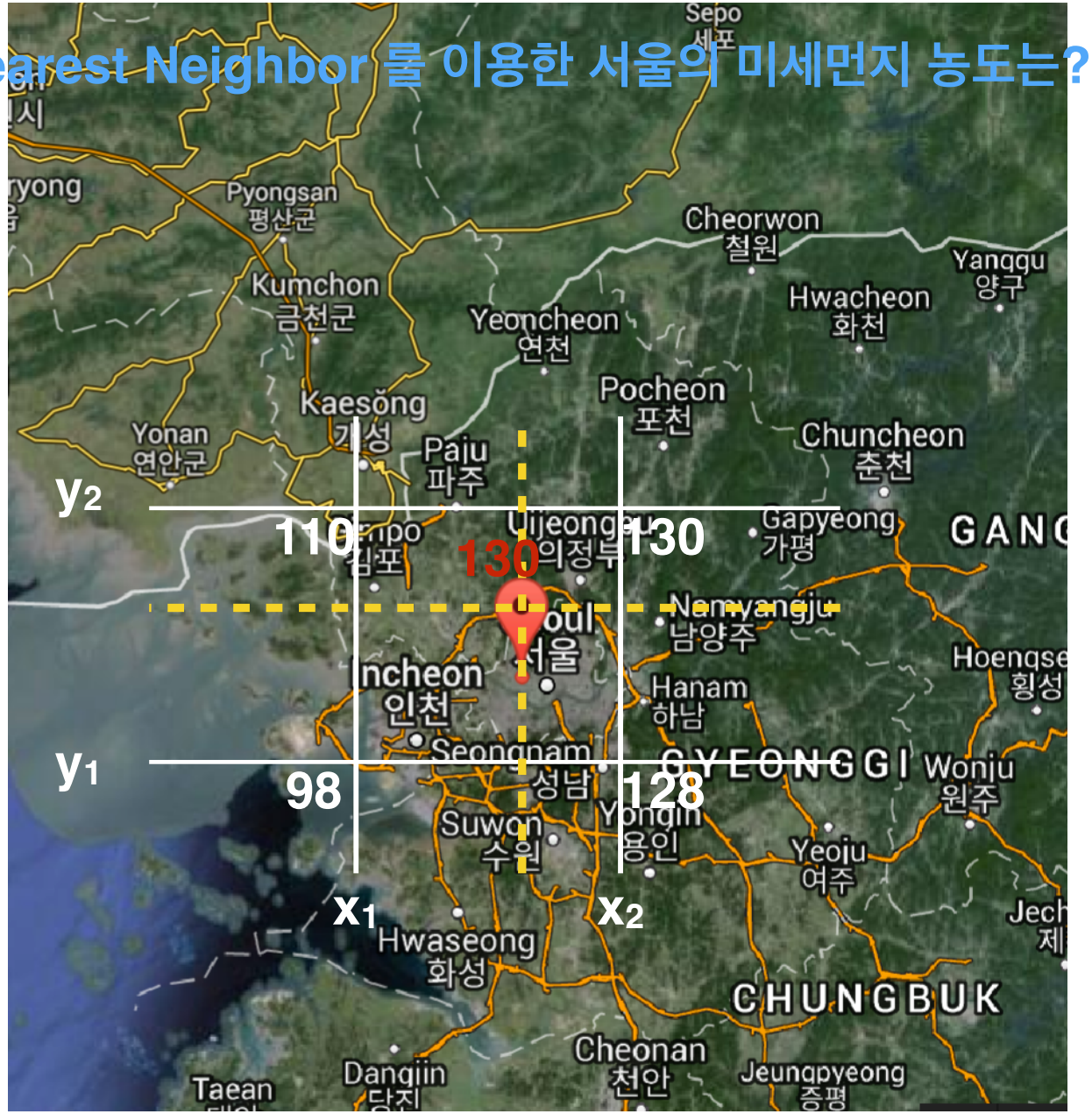
Nearest Neighbor



The nearest neighbor algorithm selects the value of the nearest point and does not consider the values of neighboring points at all.

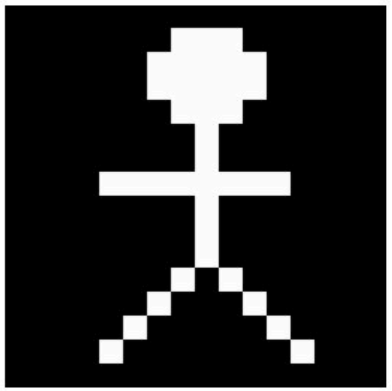
In the example on the left side, the function value at f(x,y) is equal to $f(x_1,y_2)$.

Nearest Neighbor 를 이용한 서울의 미세먼지 농도는?



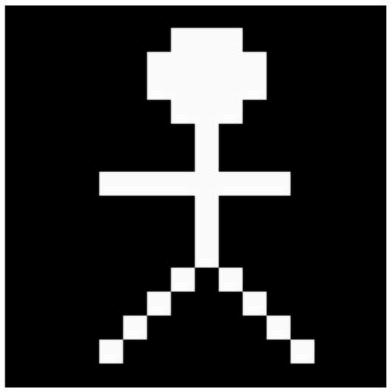
Rotation with Nearest Neighbor

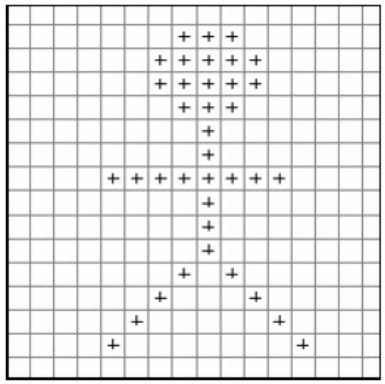
Original Image



Rotation with Nearest Neighbor

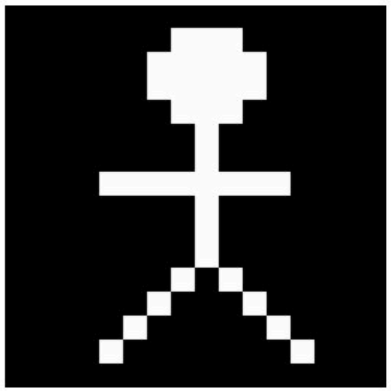
Original Image

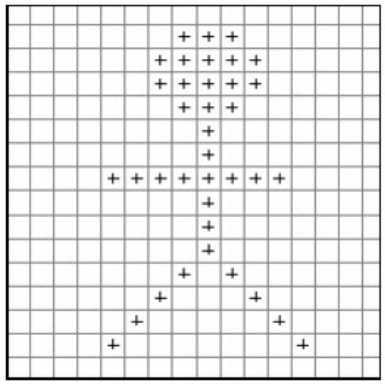


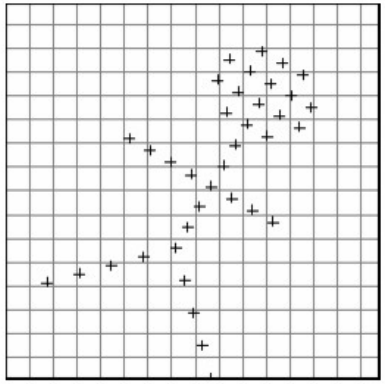


Rotation with Nearest Neighbor

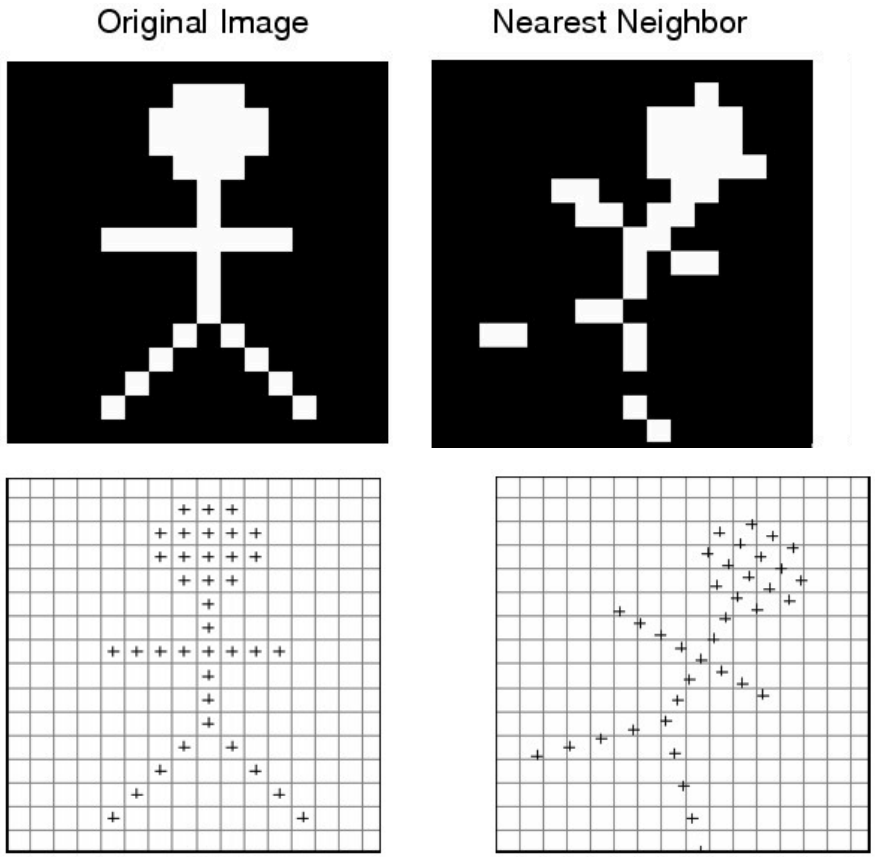
Original Image





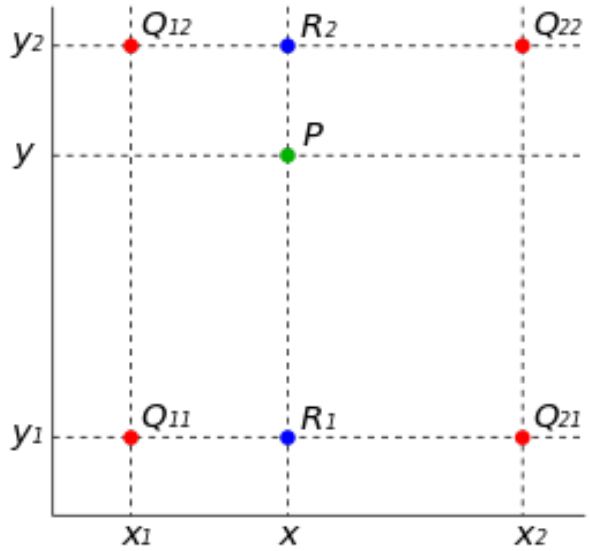


Rotation with Nearest Neighbor



artifact!

Bilinear Interpolation



First, interpolate in the x-direction.

$$f(x, y_1) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{11}) + \frac{x - x_1}{x_2 - x_1} f(Q_{21})$$
$$f(x, y_2) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{12}) + \frac{x - x_1}{x_2 - x_1} f(Q_{22})$$

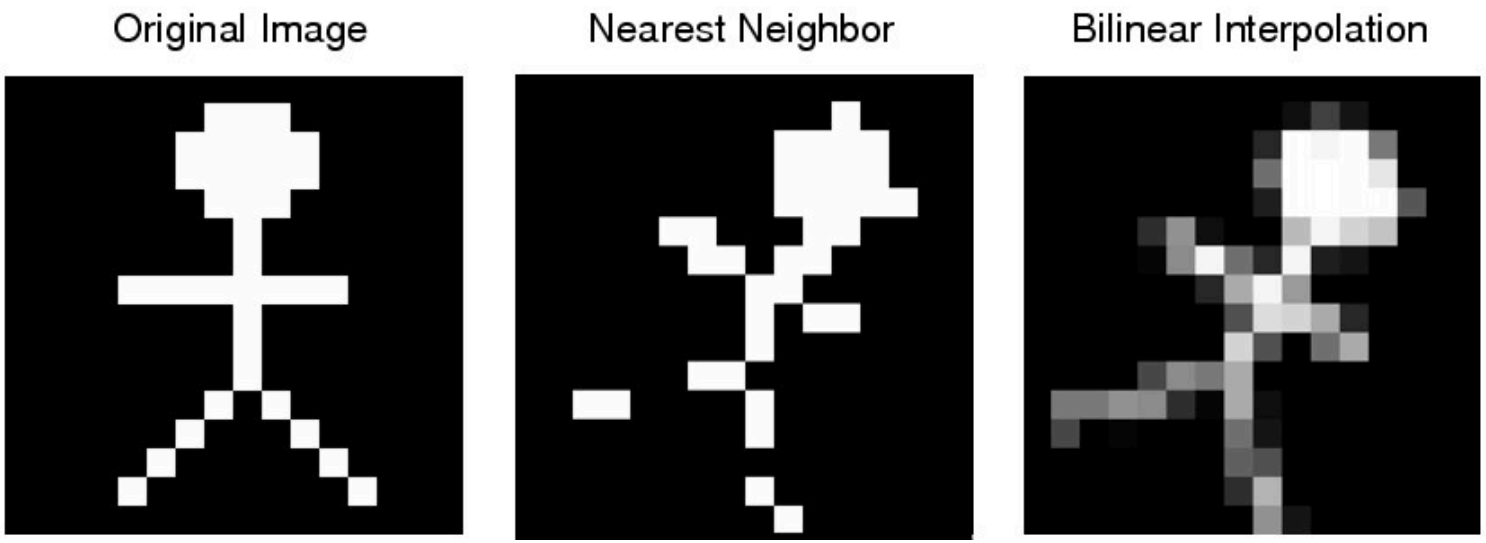
Then, interpolate in the y-direction.

$$\begin{aligned} f(x, y) &\approx \frac{y_2 - y}{y_2 - y_1} f(x, y_1) + \frac{y - y_1}{y_2 - y_1} f(x, y_2) \\ &\approx \frac{y_2 - y}{y_2 - y_1} \left(\frac{x_2 - x}{x_2 - x_1} f(Q_{11}) + \frac{x - x_1}{x_2 - x_1} f(Q_{21}) \right) + \frac{y - y_1}{y_2 - y_1} \left(\frac{x_2 - x}{x_2 - x_1} f(Q_{12}) + \frac{x - x_1}{x_2 - x_1} f(Q_{22}) \right) \\ &= \frac{1}{(x_2 - x_1)(y_2 - y_1)} \left(f(Q_{11})(x_2 - x)(y_2 - y) + f(Q_{21})(x - x_1)(y_2 - y) + f(Q_{12})(x_2 - x)(y - y_1) + f(Q_{22})(x - x_1)(y - y_1) \right) \end{aligned}$$

$$f(x, y) = Af(Q_{11}) + Bf(Q_{21}) + Cf(Q_{12}) + Df(Q_{22})$$

What is the meaning of the weight?

Rotation with Bilinear Interpolation



Interpolation for image enlargement

nearest neighbor

bilinear interpolation

bicubic interpolation

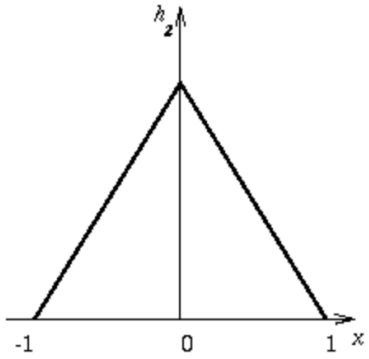
Interpolation and Kernel

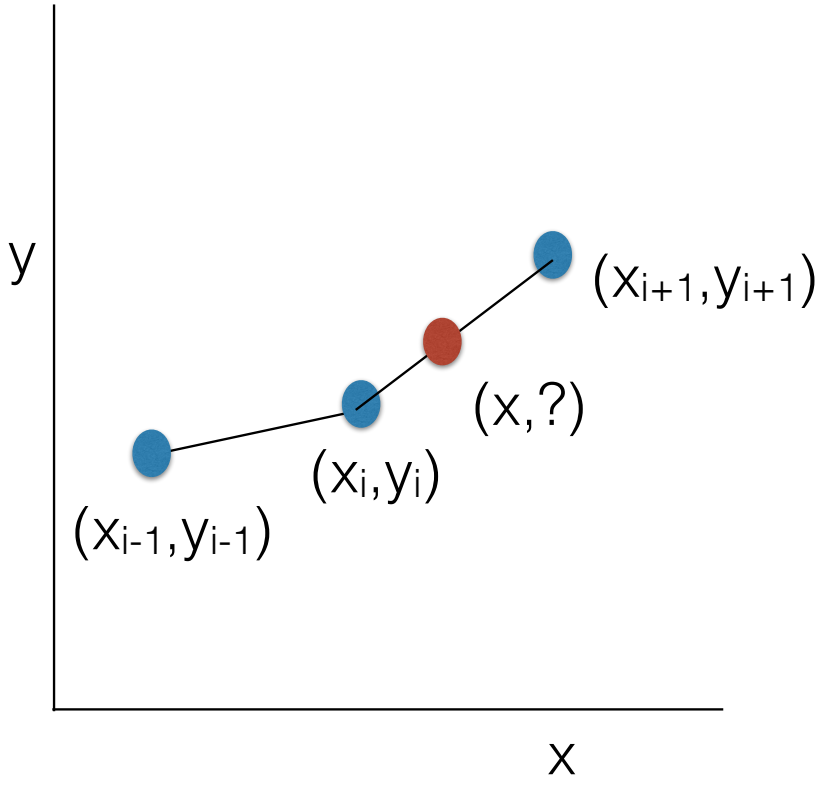
Linear

$$y = Ay_i + By_{i+1}$$

$$A = \frac{x_{j+1} - x}{x_{j+1} - x_j} \qquad B = \frac{x - x_j}{x_{j+1} - x_j}$$

Imagine the shape of A.





We call this function of the weight a **kernel**.

Interpolation and Kernel

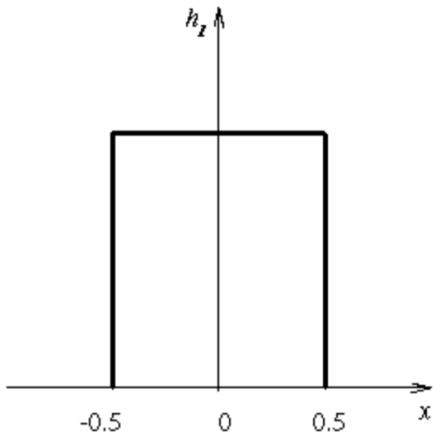
Nearest Neighbor

$$y = Ay_i + By_{i+1}$$

A=1 and B=0 if x is closer to xi.

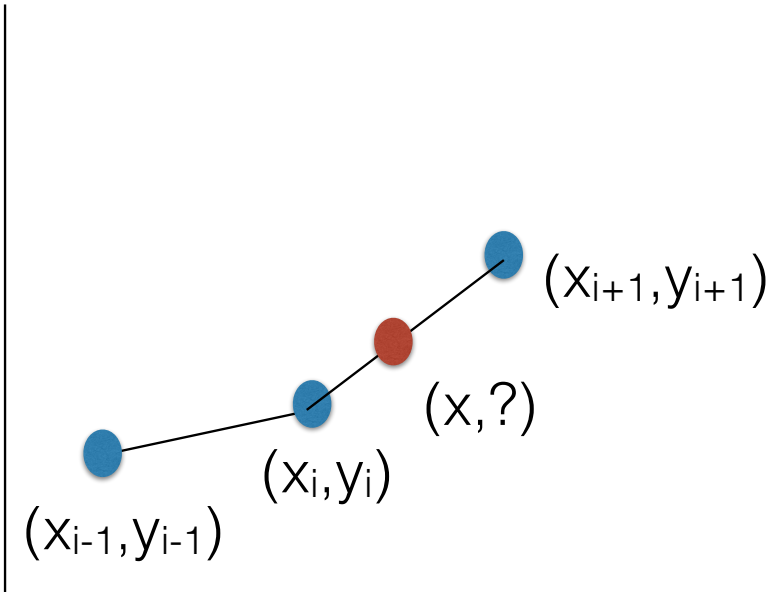
A=0 and B=1 if x is closer to xi+1.

Imagine the shape of A.



kernel

Top hat kernel



Interpolation and Kernel

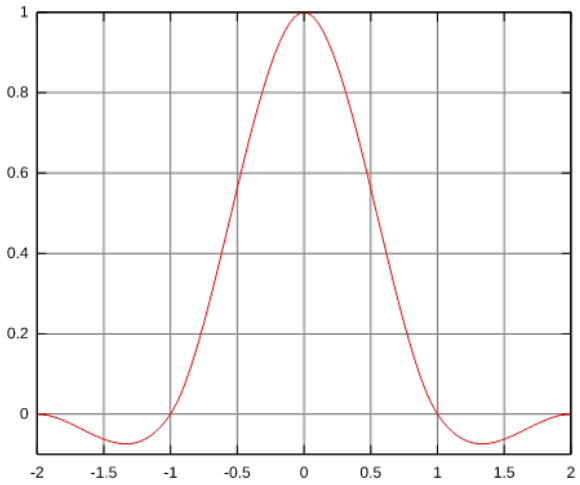
Cubic Interpolation

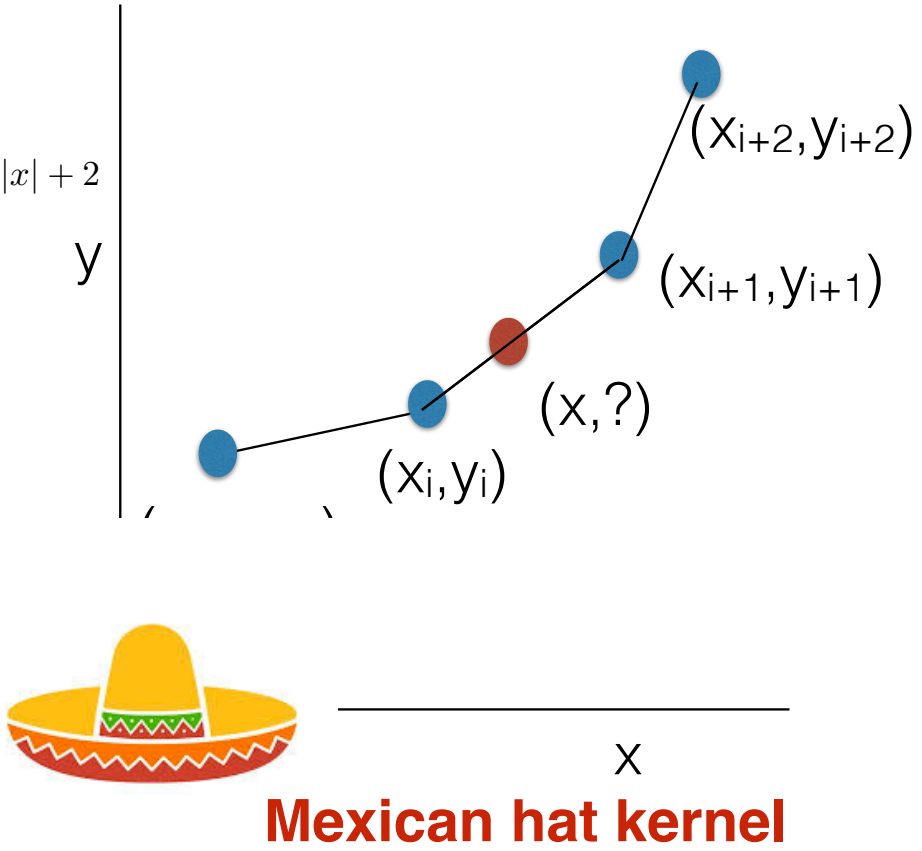
$$y = Ay_{i-1} + By_i + Cy_{i+1} + Dy_{i+2}$$

$$0 \le |x| \le 1 \quad B, C = 1.5|x|^3 - 2.5x^2 + 1$$

$$1 \le |x| \le 2 \quad C, D = -0.5|x|^3 + 2.5x^2 - 4|x| + 2$$

Imagine the kernel.

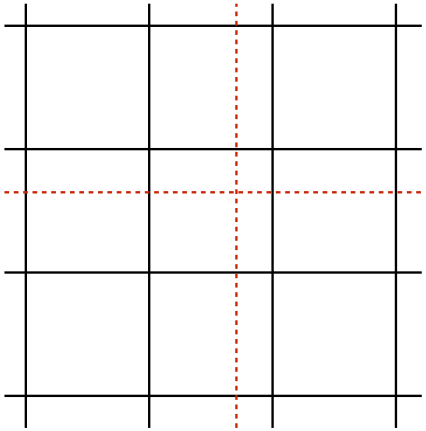




Mexican hat kernel

Bicubic Convolution Interpolation

Cubic interpolation 을 두번 실행



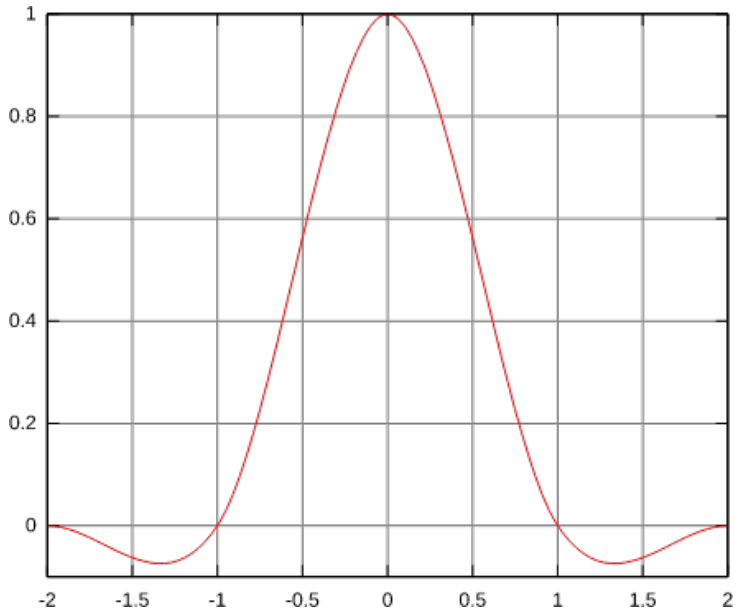
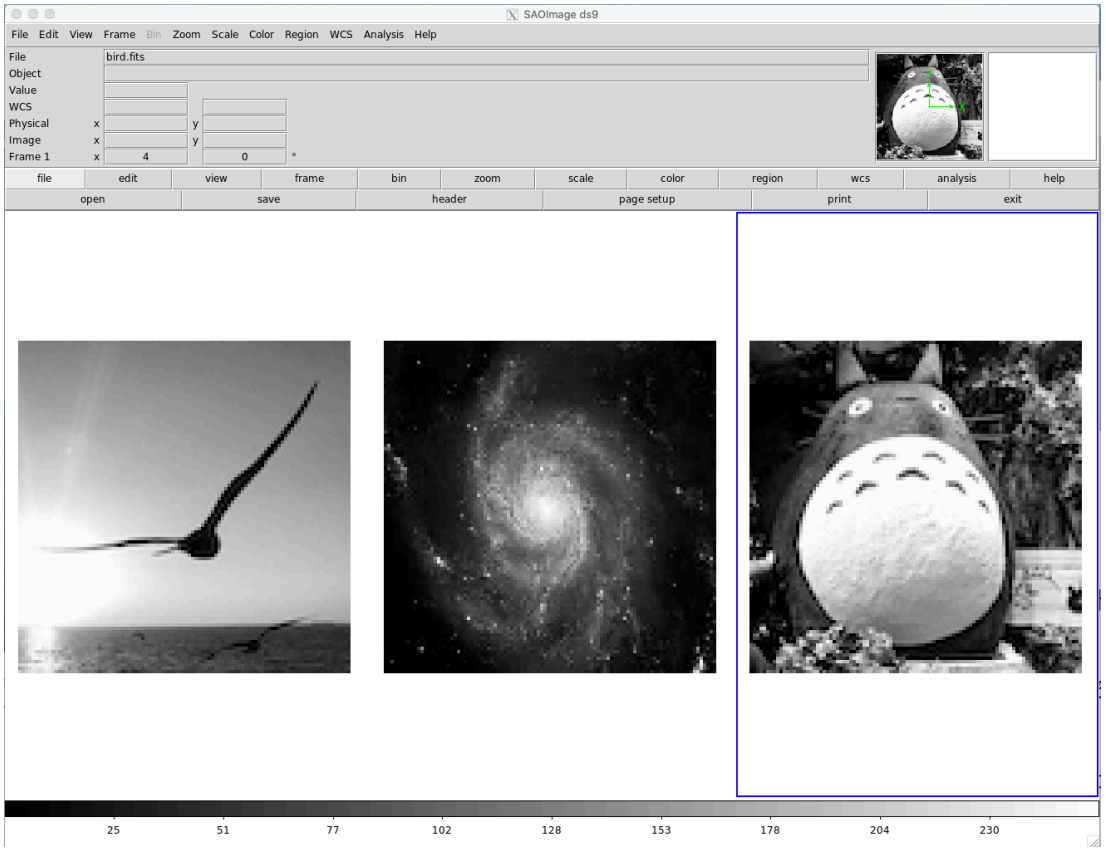


Image 경계에서는 어떻게?

과제 10

- LearnUs에서 원하는 이미지를 하나 선택하여 다운 받는다.
- 이미지의 크기를 Interpolation을 이용하여 500x500으로 증가시킨다.
- 세가지 방법 (Nearest neighbor, bilinear, bicubic)을 모두 사용하여 결과를 출력하고 차이점을 논한다 (차이점을 어떻게 정량화 할지 고민 필요).
- Interpolation Algorithm은 Library를 사용할수있다.

ds9: FITS 파일을 볼수 있는 소프트웨어

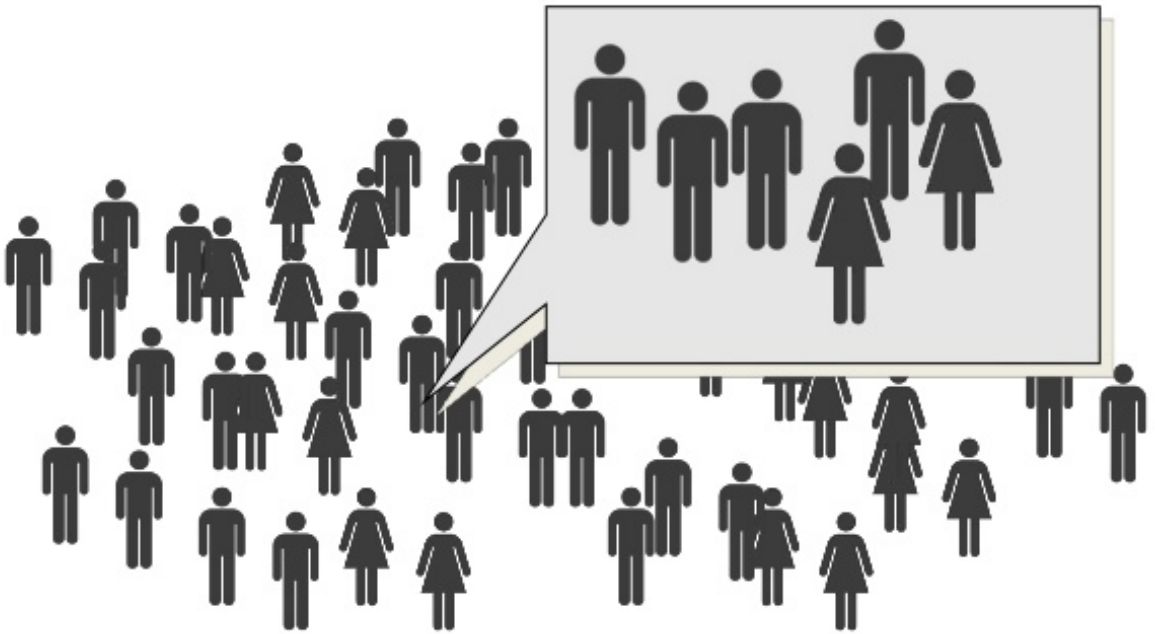


Sampling Theory

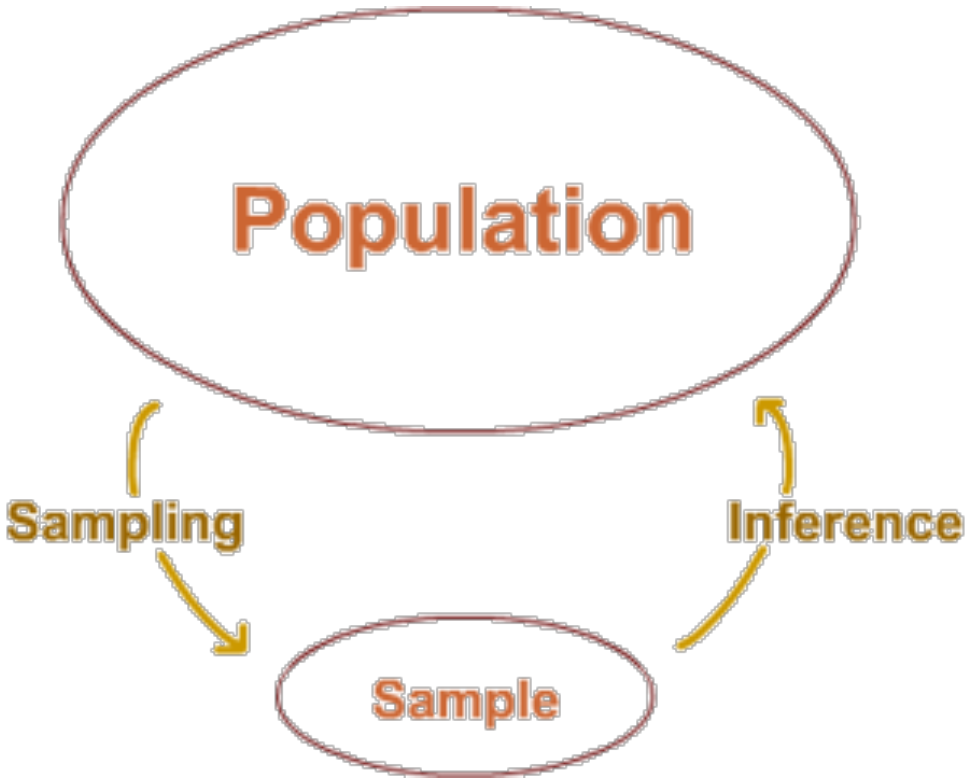
What is Sampling?

COSTCO WHOLESALE

What is sampling?

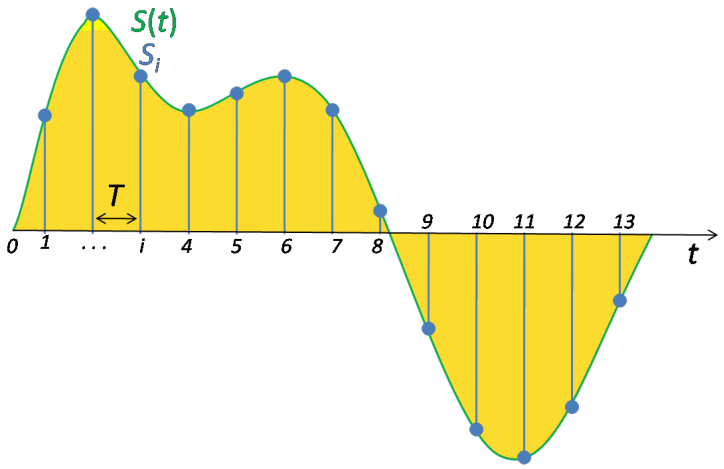


Economic Way of Understanding Population



Definition of Sampling in Signal Processing

Sampling is the process of converting a signal into a numeric sec
with finite intervals.

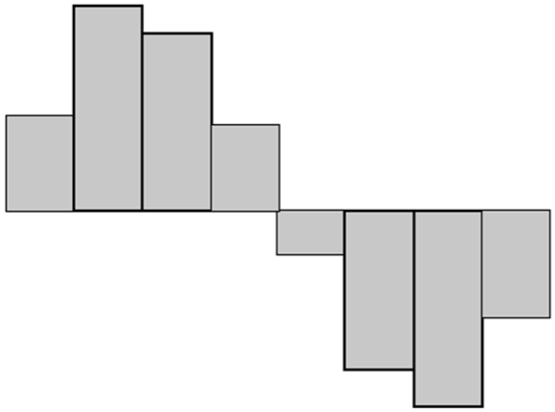


2.1, 4.4, 3.6, 2.8, … …

Sound Sampling

Analog tone

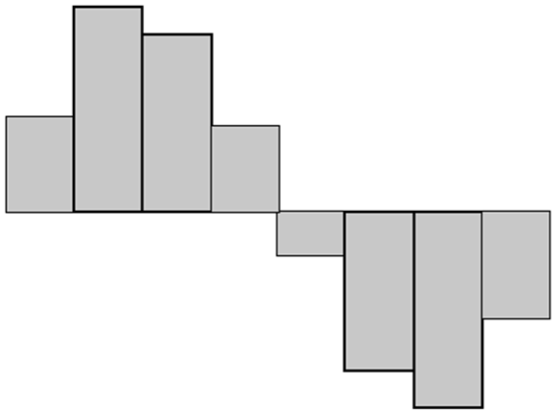
Sound Sampling



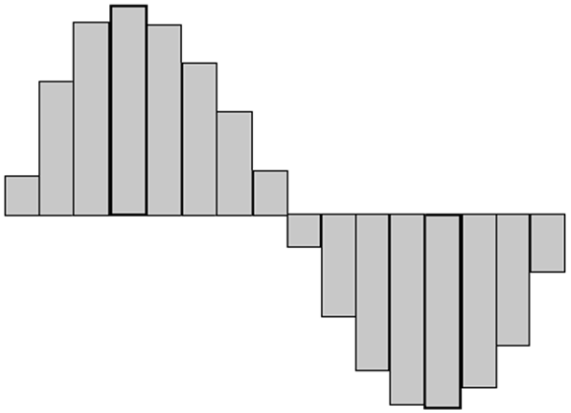
8 samples

Analog tone

Sound Sampling



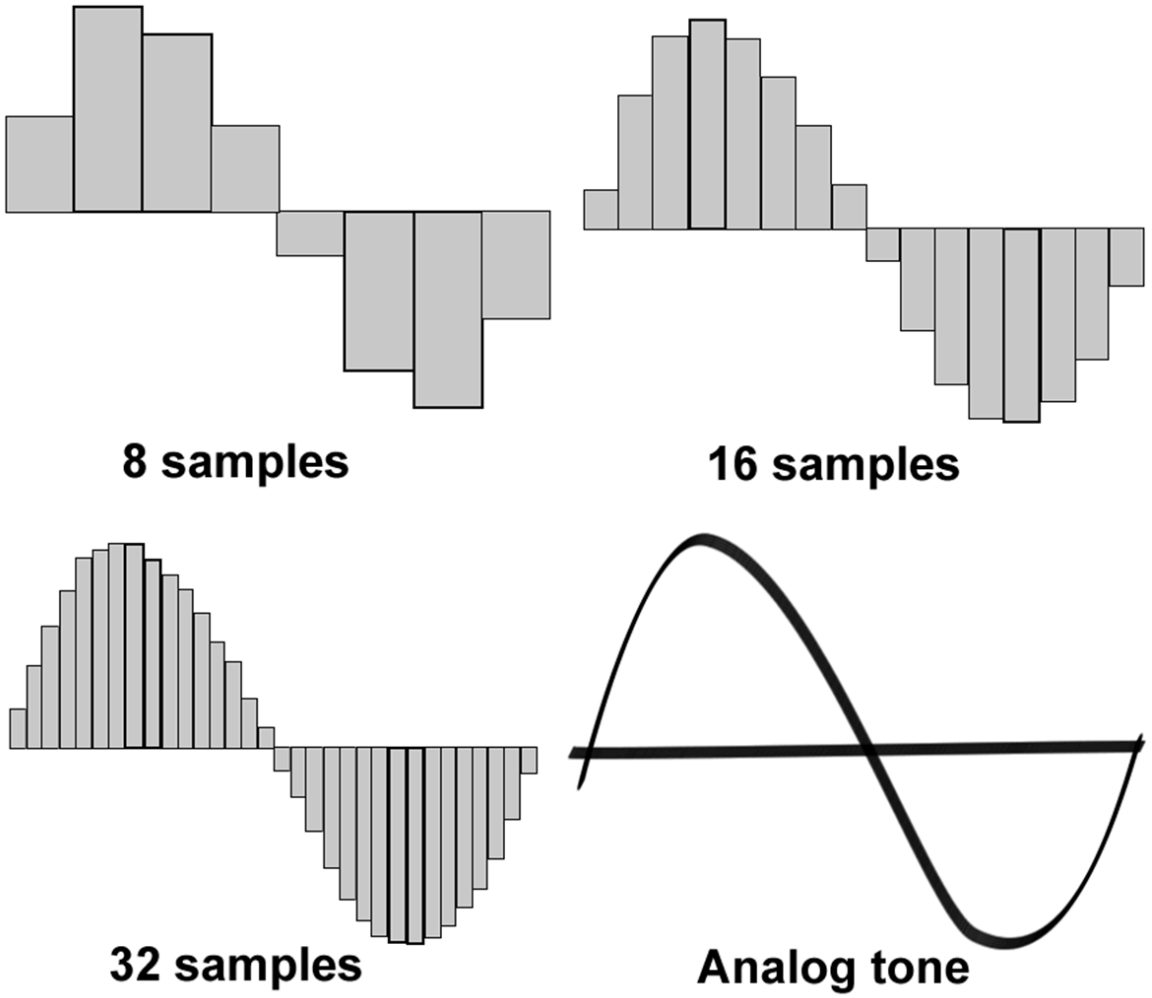
8 samples



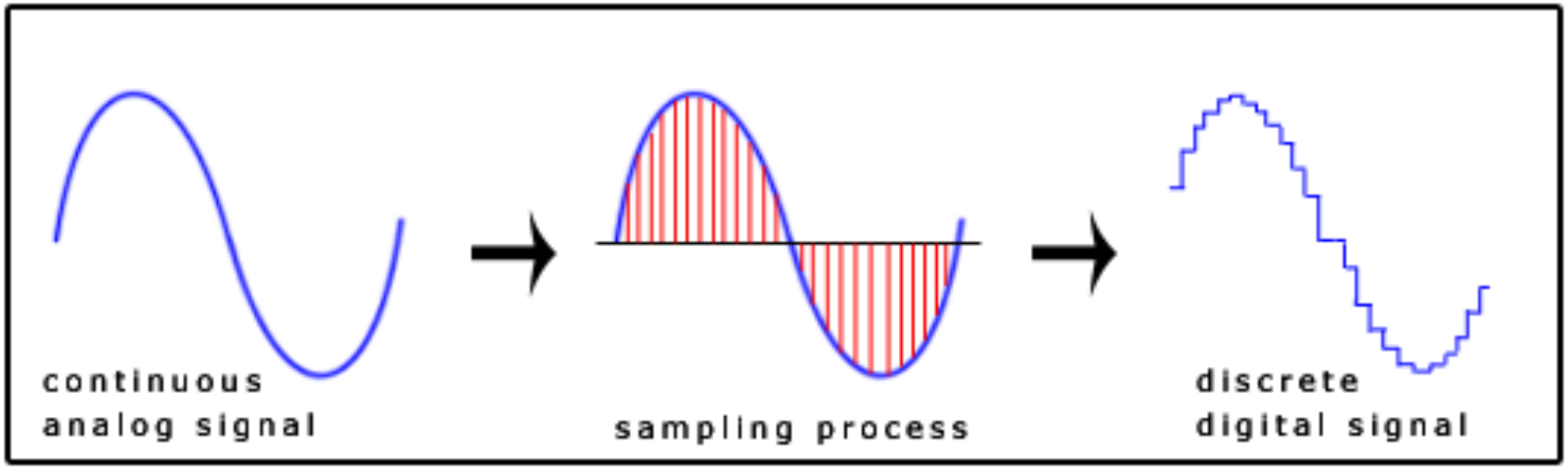
16 samples

Analog tone

Sound Sampling



Signal Processing



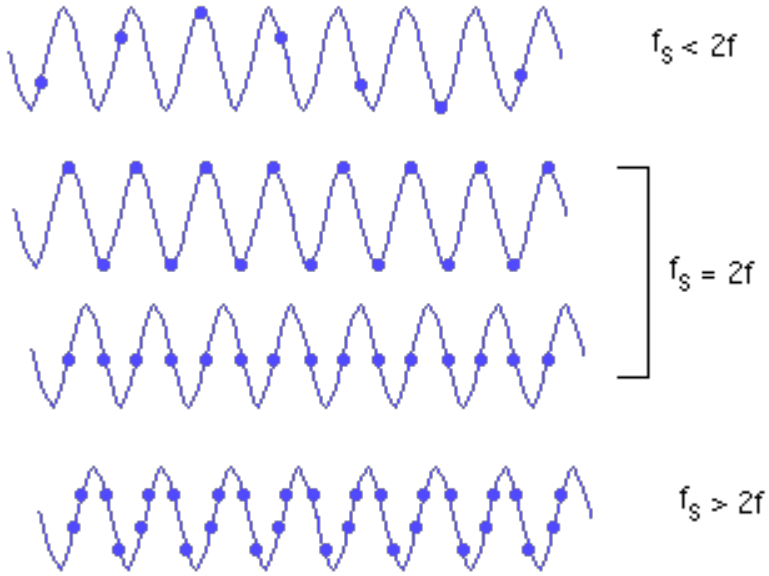
reconstructed signal

How frequently should we sample the signal?

Nyquist Sampling Theorem

If a function x(t) contains no frequencies higher than Q Hz, it is completely determined by giving its ordinates (y values) at a series of points sampled at 2Q Hz.

2 sampling points per wavelength



Nyquist Sampling of Sound

- Humans can hear sound whose frequencies range between 20 Hz and 20,000 Hz.
- The Nyquist sampling rate is twice the maximum frequency of the signal.
- Therefore, the Nyquist rate is 2 x 20,000 Hz=40,000 Hz

CD sampling rate = 44,100 Hz

Audio Sampling Rate vs. Application

Sampling Rate	Feature	Application
8,000 Hz	F or S sounds become hard to recognize.	Telephone, Wireless Microphone

Audio Sampling Rate vs. Application

Sampling Rate	Feature	Application
8,000 Hz	F or S sounds become hard to recognize.	Telephone, Wireless Microphone
16,000 Hz	Clear voice conversation	Internet Telephone

Audio Sampling Rate vs. Application

Sampling Rate	Feature	Application
8,000 Hz	F or S sounds become hard to recognize.	Telephone, Wireless Microphone
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44,100 Hz	**Theoretically**, the rate can restore the original sound quality.	Audio CD

Audio Sampling Rate vs. Application

Sampling Rate	Feature	Application
8,000 Hz	F or S sounds become hard to recognize.	Telephone, Wireless Microphone
16,000 Hz	Clear voice conversation	Internet Telephone
44,100 Hz	**Theoretically**, the rate can restore the original sound quality.	Audio CD
352,800 Hz	8 times the sampling rate of Audio CD	Digital eXtreme Definition Super Audio CD

Example of Sampling Rate Difference

FLAC: Free Lossless Audio Codec

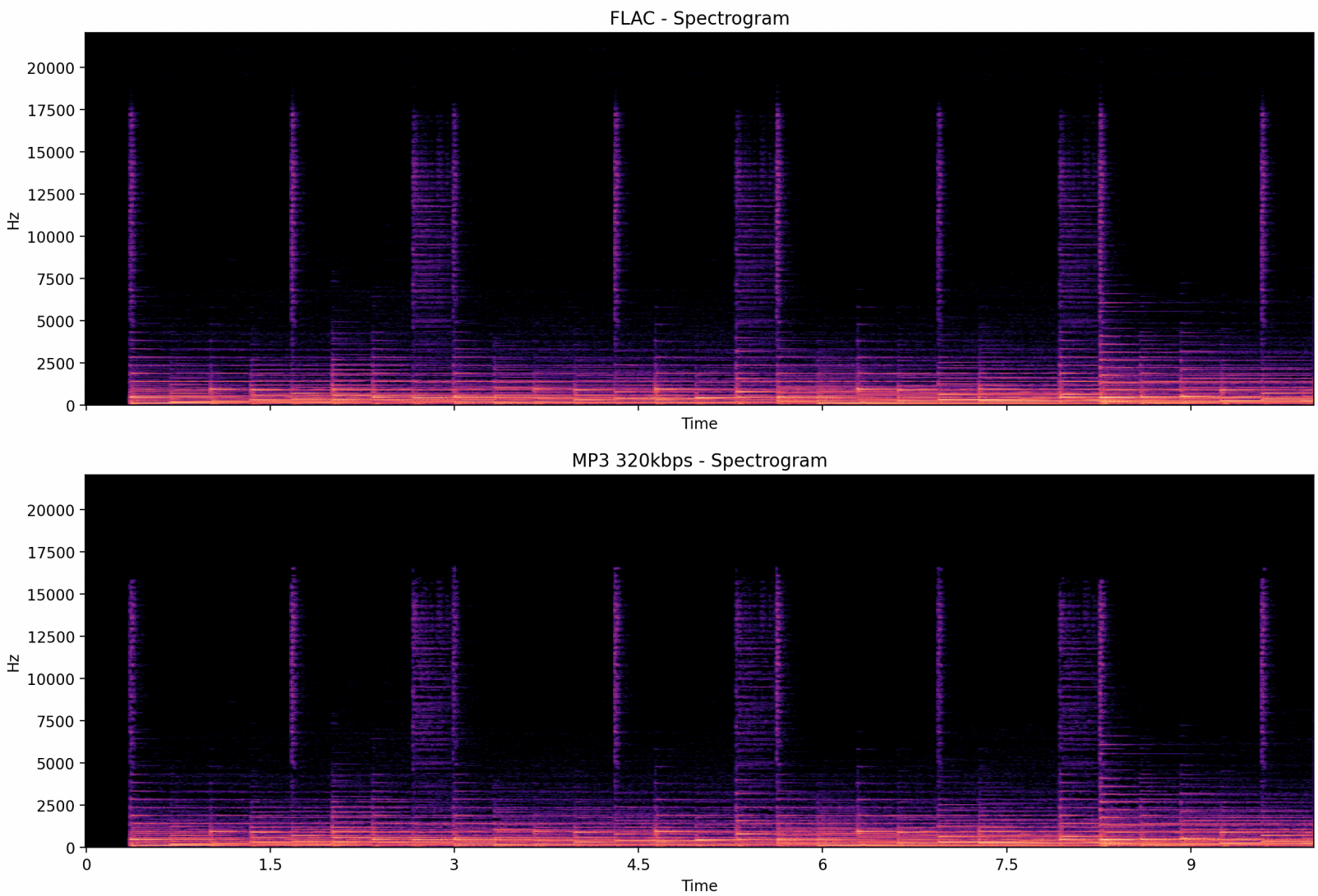
Kenny G

CD-quality: 44100 Hz Uncompressed FLAC 13MB

internet phone: 22050 Hz Uncompressed FLAC 6.5 MB

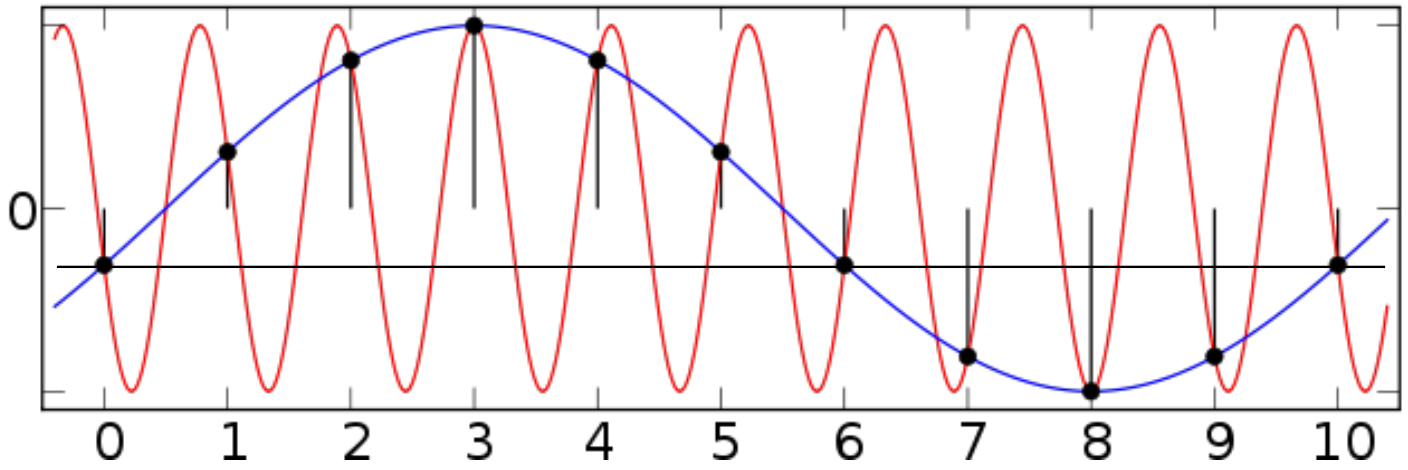
classic phone: 8000 Hz Uncompressed FLAC 2.4 MB

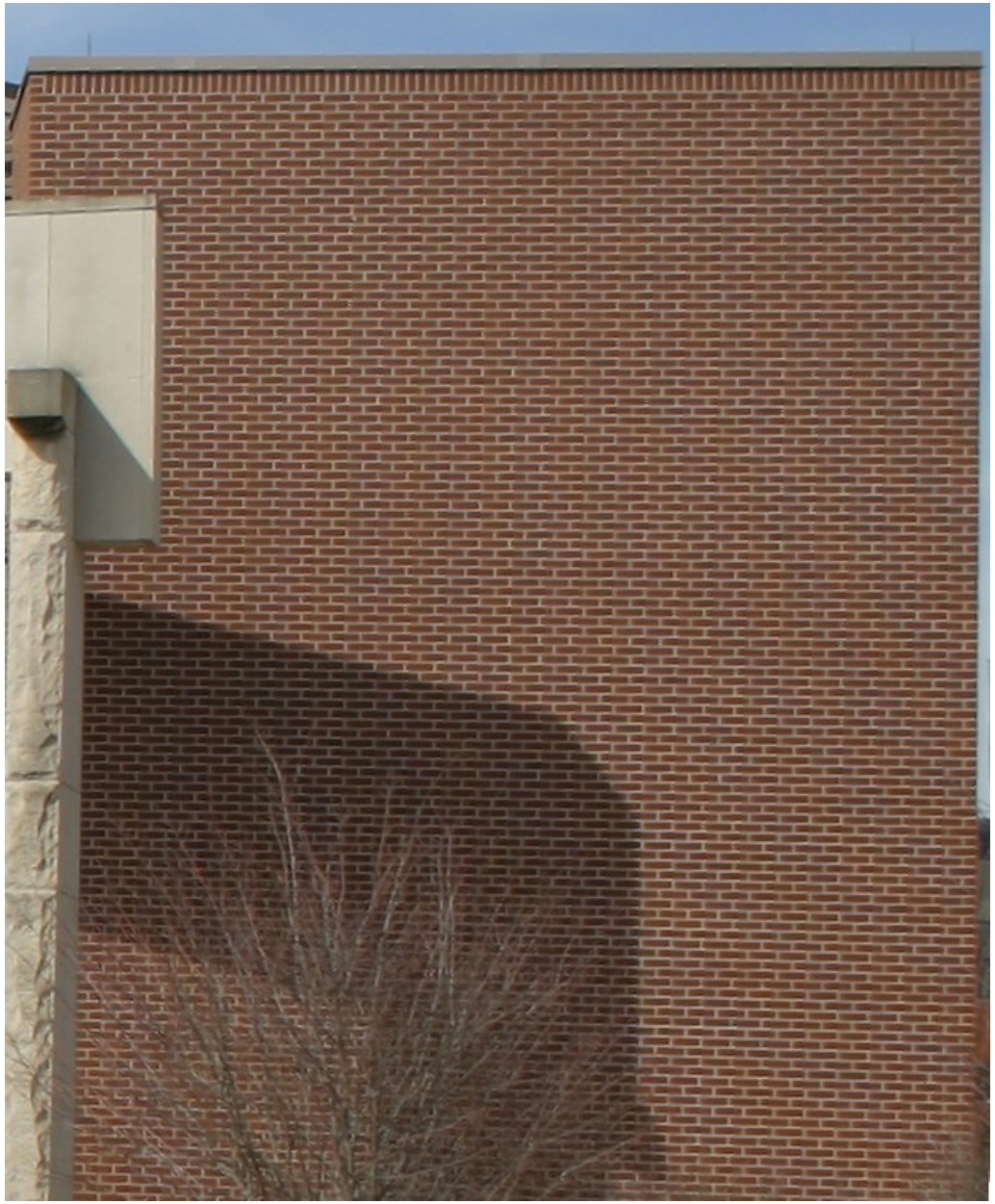
Mp3: 44100 Hz Compressed 0.6 MB


FLAC - Spectrogram
20000
17500
15000
12500
10000
7500
5000
2500
0
Hz
Time
MP3 320kbps - Spectrogram
20000
17500
15000
12500
10000
7500
5000
2500
0
Hz
0
1.5
3
4.5
6
7.5
9
Time

ALIASING

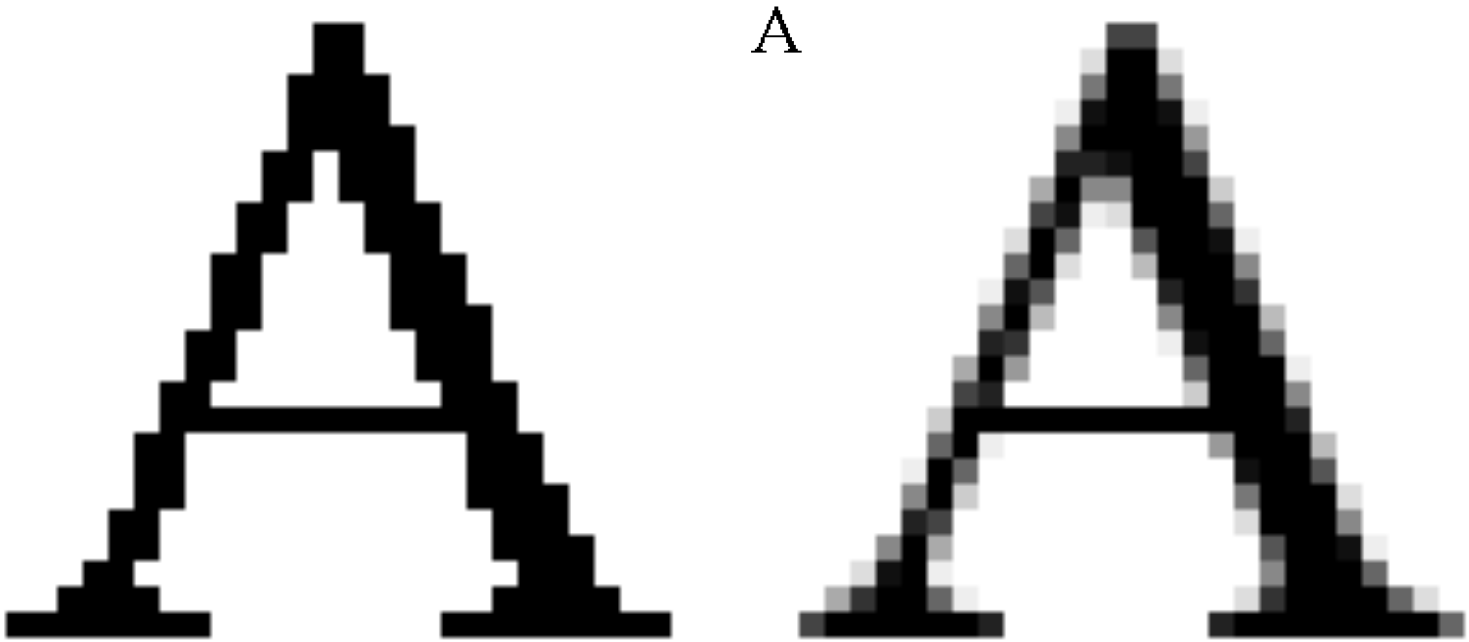
a distortion or artifact that results when the signal reconstructed from sa
different from the original continuous signal.



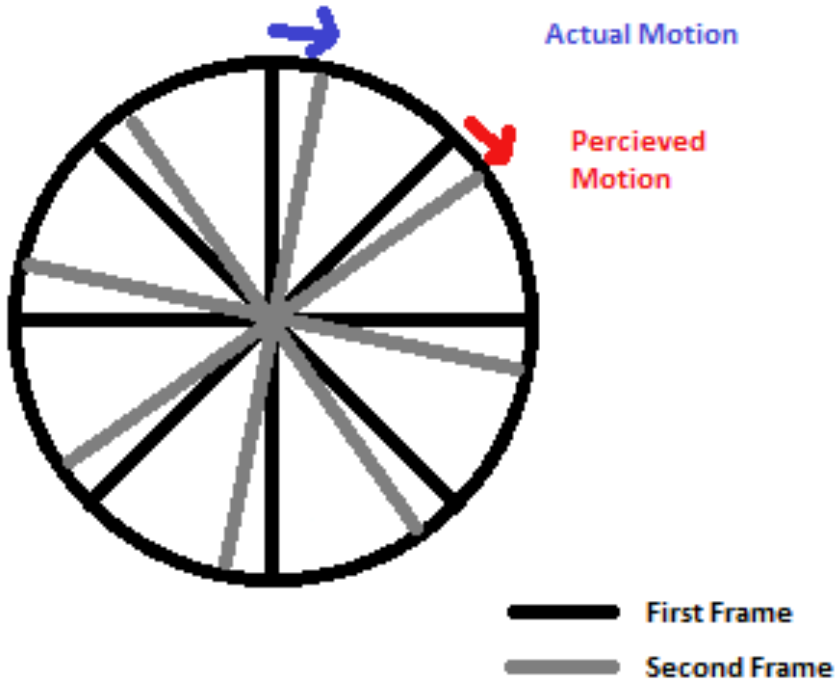


Anti-aliasing

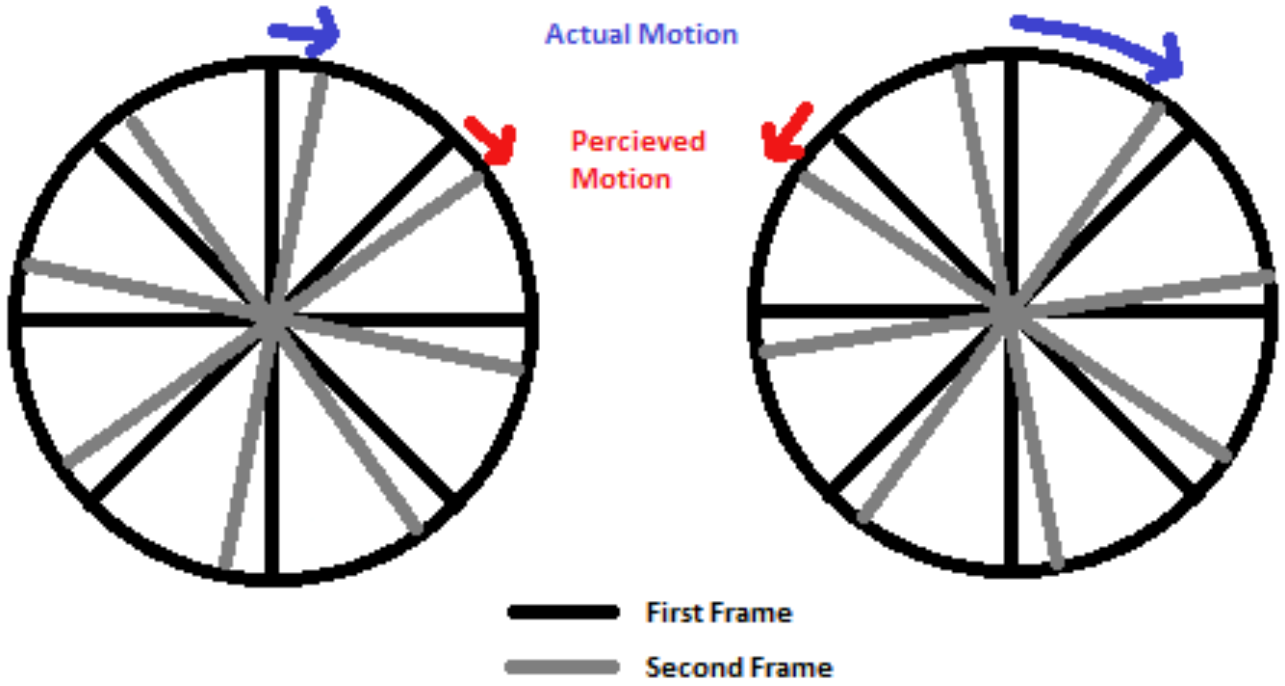
the smoothing of jagged edges in digital images by averaging the colors of the pixels at a boundary.



Wagon-Wheel Effect

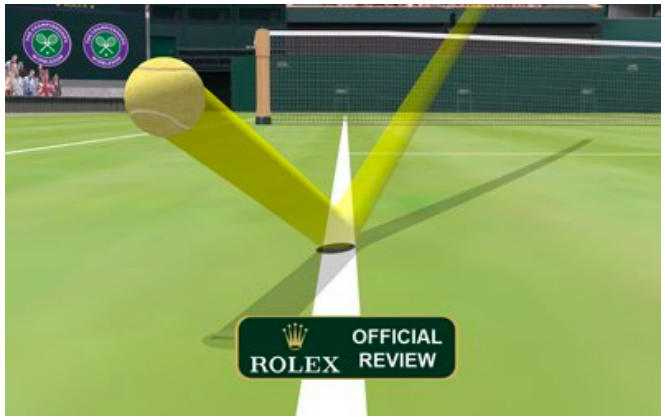


Wagon-Wheel Effect

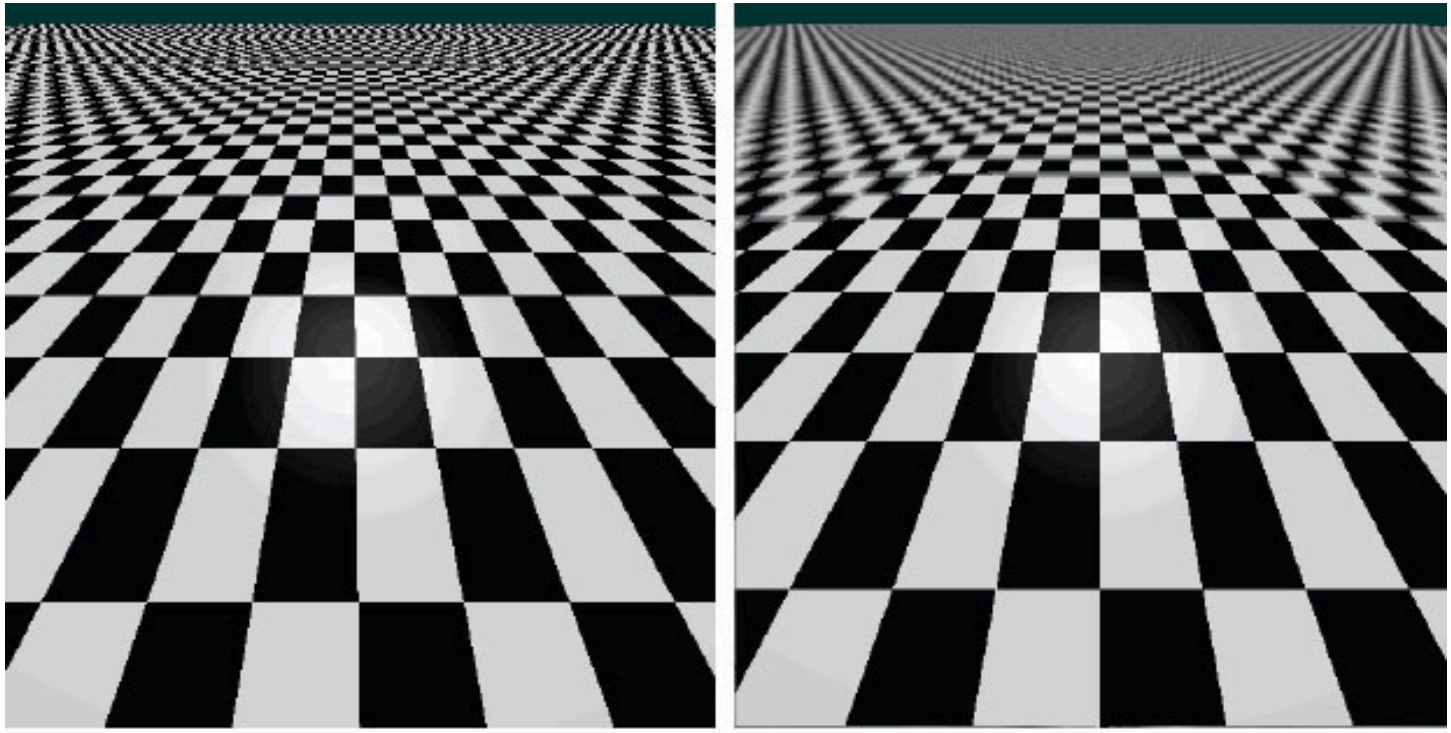


Human Eye Sampling Rate

- Human eyes do not have a fixed rate
- Can exceed hundreds of frames per second



Overcoming Aliasing



aliasing effects

anti-aliasing by over-sampling