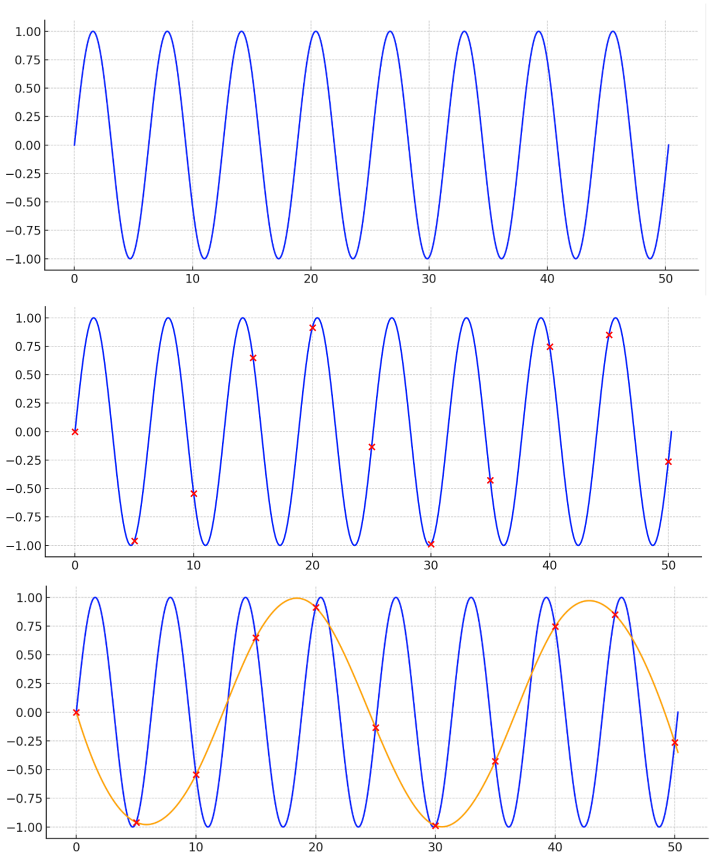
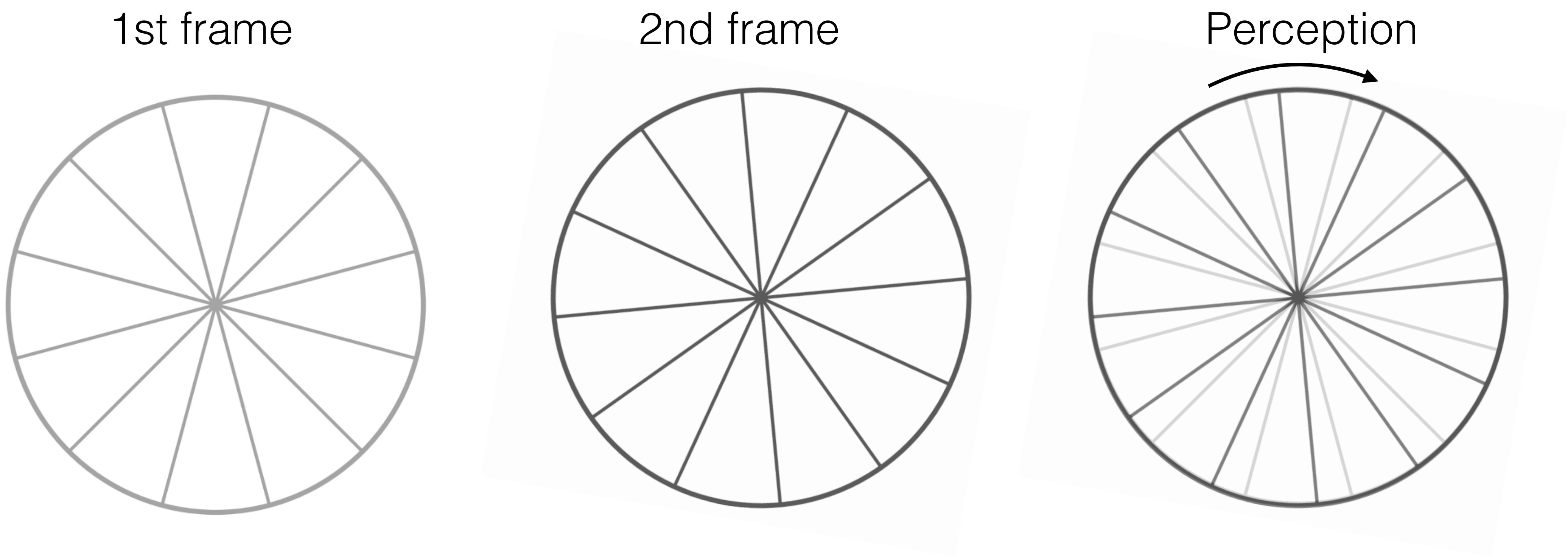
ALIASING

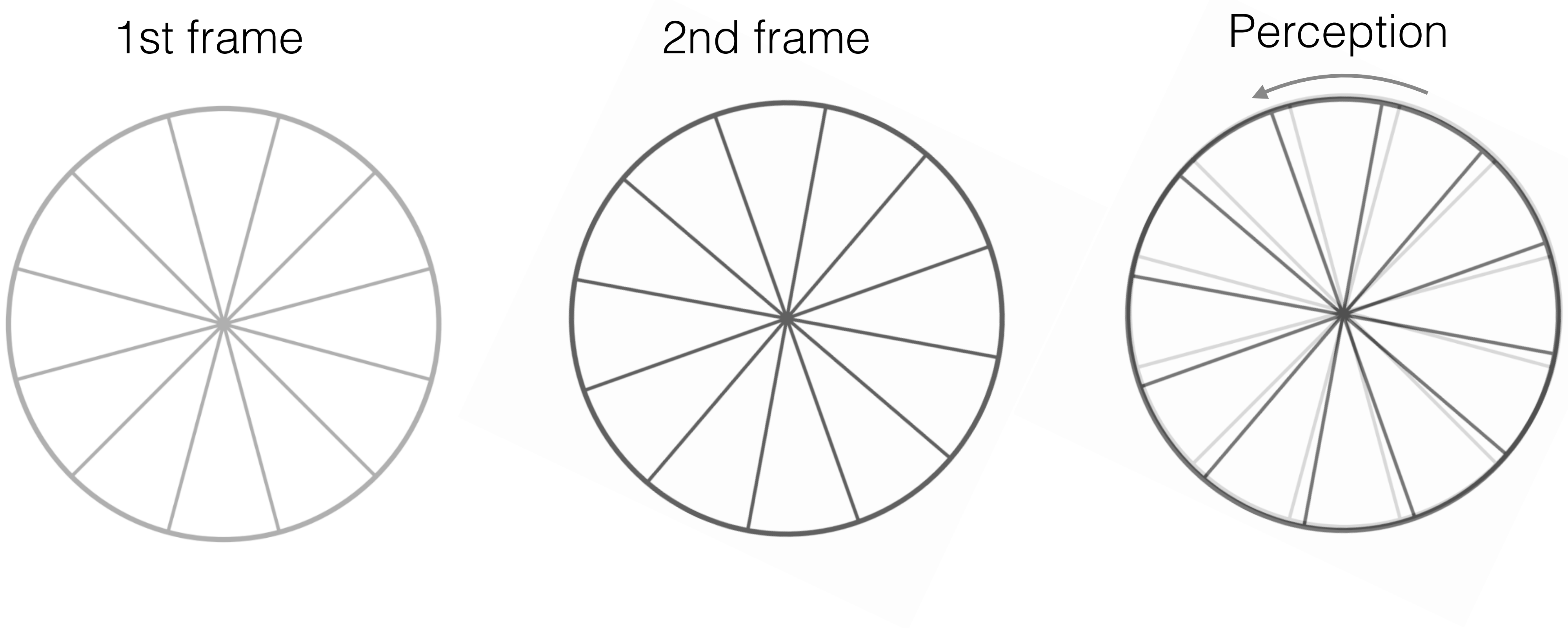
a distortion or artifact that results when the signal reconstructed from samples is different from the original continuous signal.




Wagon-Wheel Effect



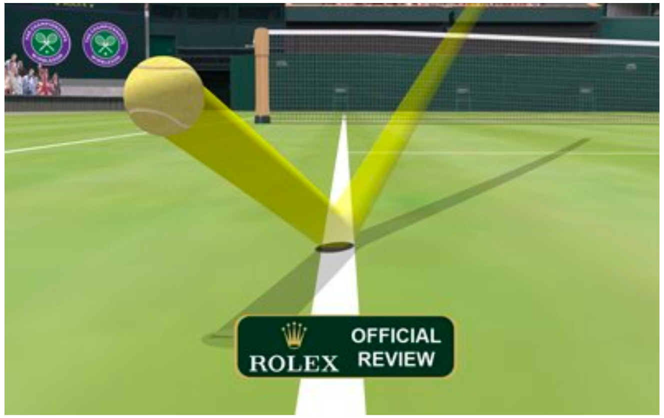
Wagon-Wheel Effect




BARNEY'S
BARNEY'S

Human Eye Sampling Rate

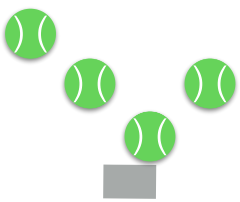
- Human eyes do not have a fixed rate
- 30~60 Hz (effective rate)

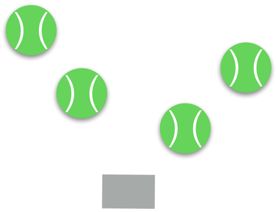


Hawk eye: 340 fps

$$\frac{1}{340} \approx 2.94 \text{ milliseconds}$$

$$44.44 \text{ m/s} \times 0.00294 \text{ s} \approx 13.1 \text{ cm per frame}$$



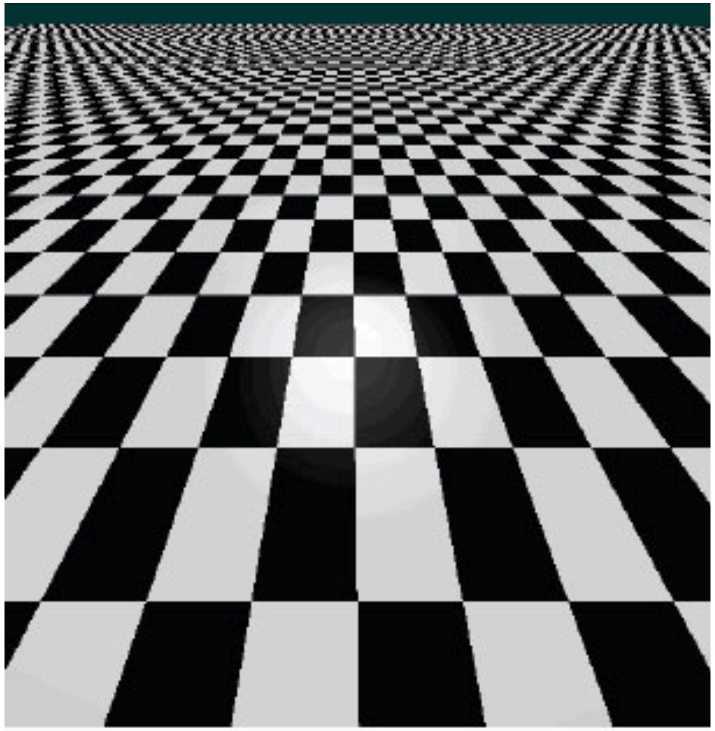


Speed = 160 km/h   Frame rate = 60 fps

$$160 \text{ km/h} = \frac{160 \times 1000}{3600} \approx 44.44 \text{ m/s} \qquad t_{\text{frame}} = \frac{1}{60} \approx 0.0167 \text{ seconds}$$

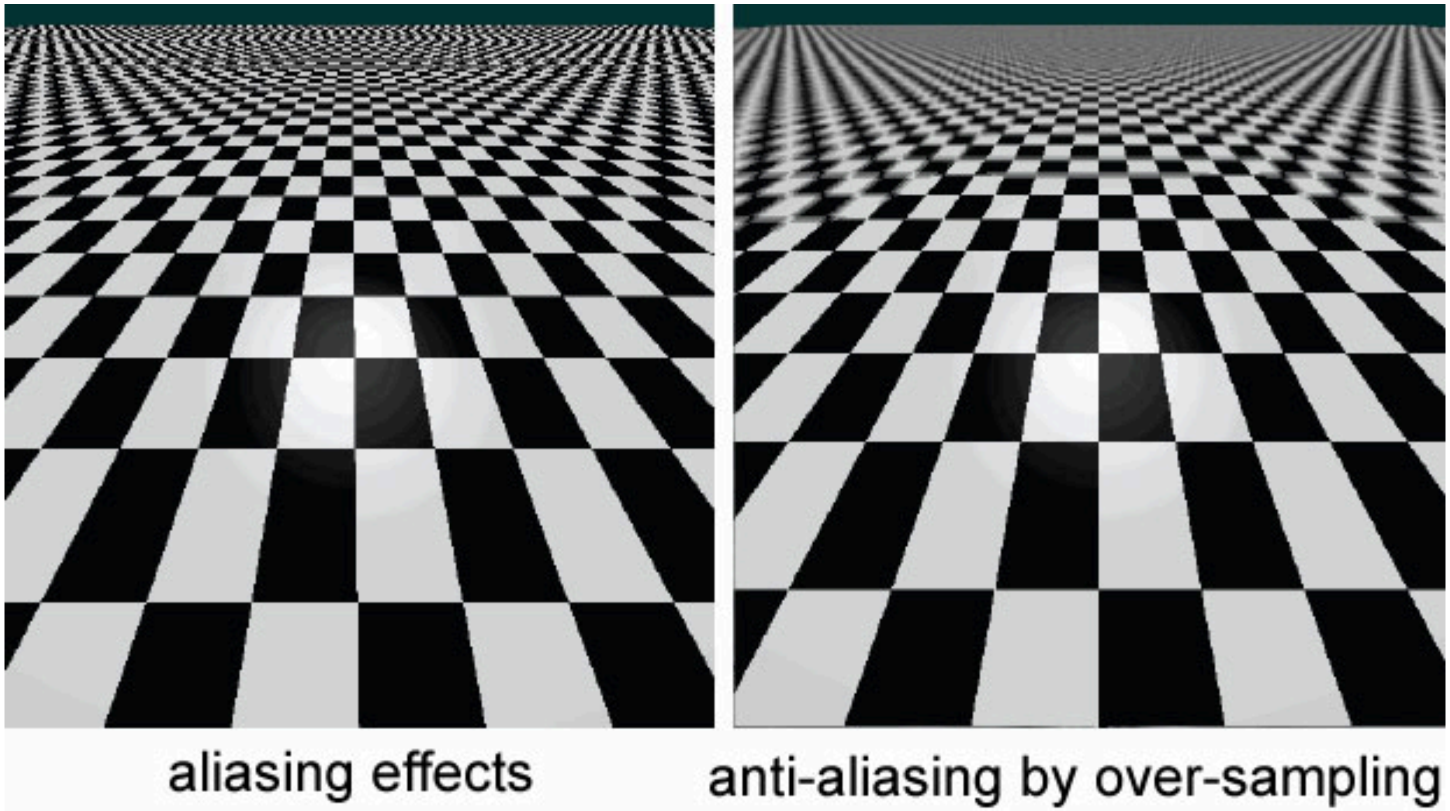
$$d = v \cdot t = 44.44 \text{ m/s} \times 0.0167 \text{ s} \approx 0.74 \text{ m}$$

Overcoming Aliasing



aliasing effects

Overcoming Aliasing



과제 11

다음 조건을 바탕으로 **Cubic Spline Interpolation**을 수행하고, 근사 정확도 및 샘플링 간격의 영향에 대해 분석하여라.

시간 [0,20]초 구간에서 다음과 같은 함수를 정의한다

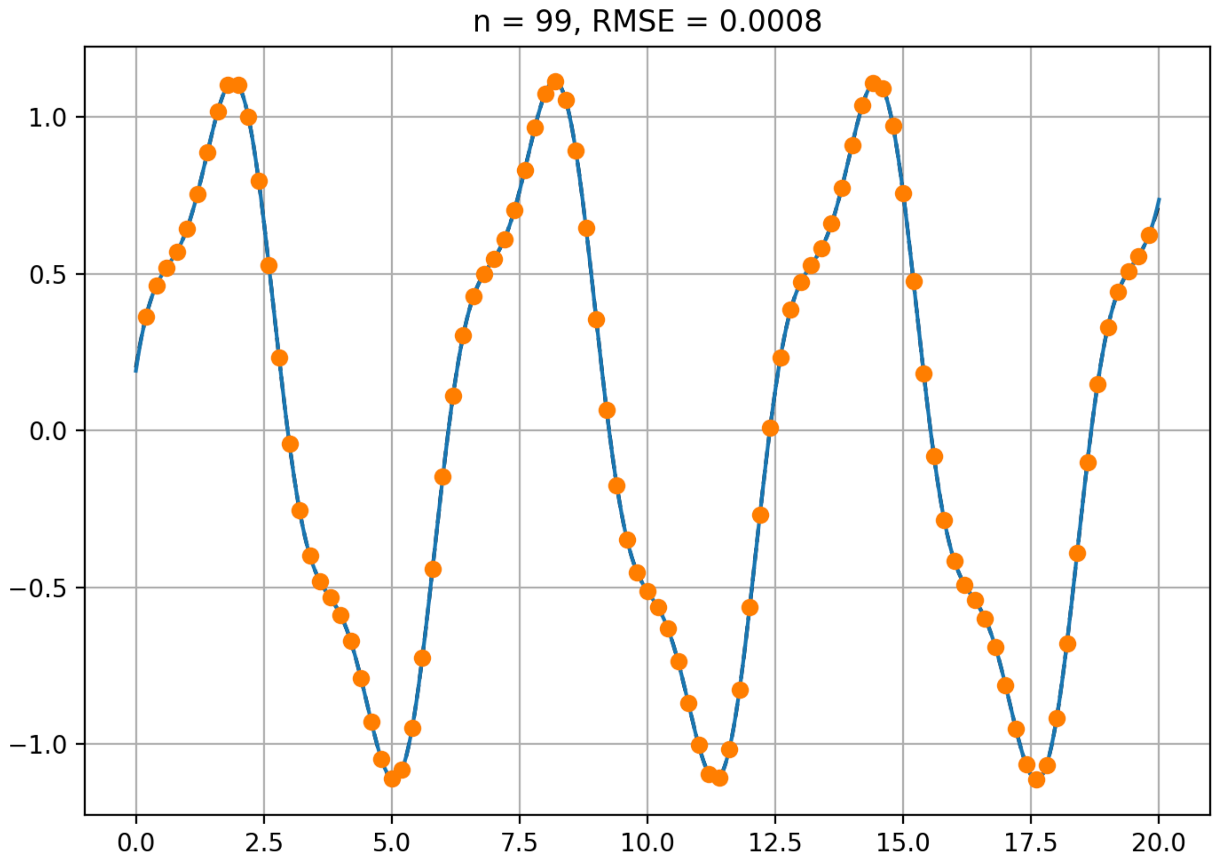
$x(t)=\sin(t)+0.2\cdot\cos(3t)$

다음 네가지 경우에 대해, 구간 [0,20]을 균등하게 분할한 지점에서 x를 샘플한다

- 99개 지점 (100등분)
- 19개 지점 (20등분)
- 9개 지점 (10등분)
- 3개 지점 (4등분)

각 경우에 대해, Cubic Spline Interpolation을 사용하여 [0, 20] 구간을 1000개 균등한 간격으로 나눈 점 t_i 에서의 함수값 x_i를 보간방법으로 추정한다.

함수의 실제값과 보간 결과의 차이를 비교한다.
Undersampling과 Oversampling 개념을 사용해 분석한다.
*scipy.interpolate.CubicSpline 같은 라이브러리의 함수 사용 허용


n = 99, RMSE = 0.0008
1.0
0.5
0.0
−0.5
−1.0
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0

Fourier Transformation

(Engineering) Definition of Fourier Transformation

Fourier Transformation

$$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x)\, e^{-i2\pi\xi x}\, dx$$

* We use a caret (^) to denote the Fourier transform.

Joseph Fourier

Inverse Fourier Transformation

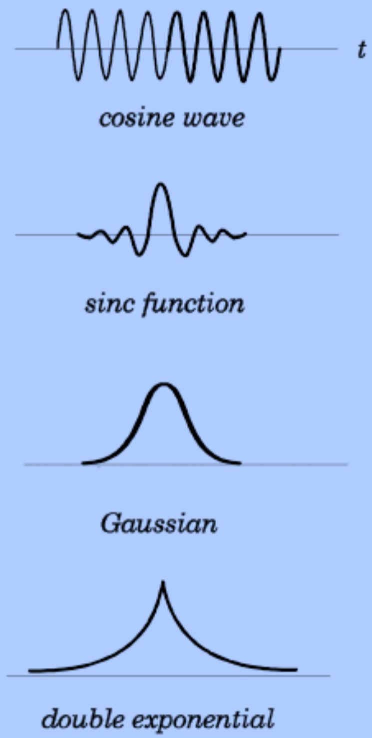
$$f(x) = \int_{-\infty}^{\infty} \hat{f}(\xi)\, e^{i2\pi\xi x}\, d\xi$$

* If we know the Fourier transform of a function f(x), we can recover the original function.

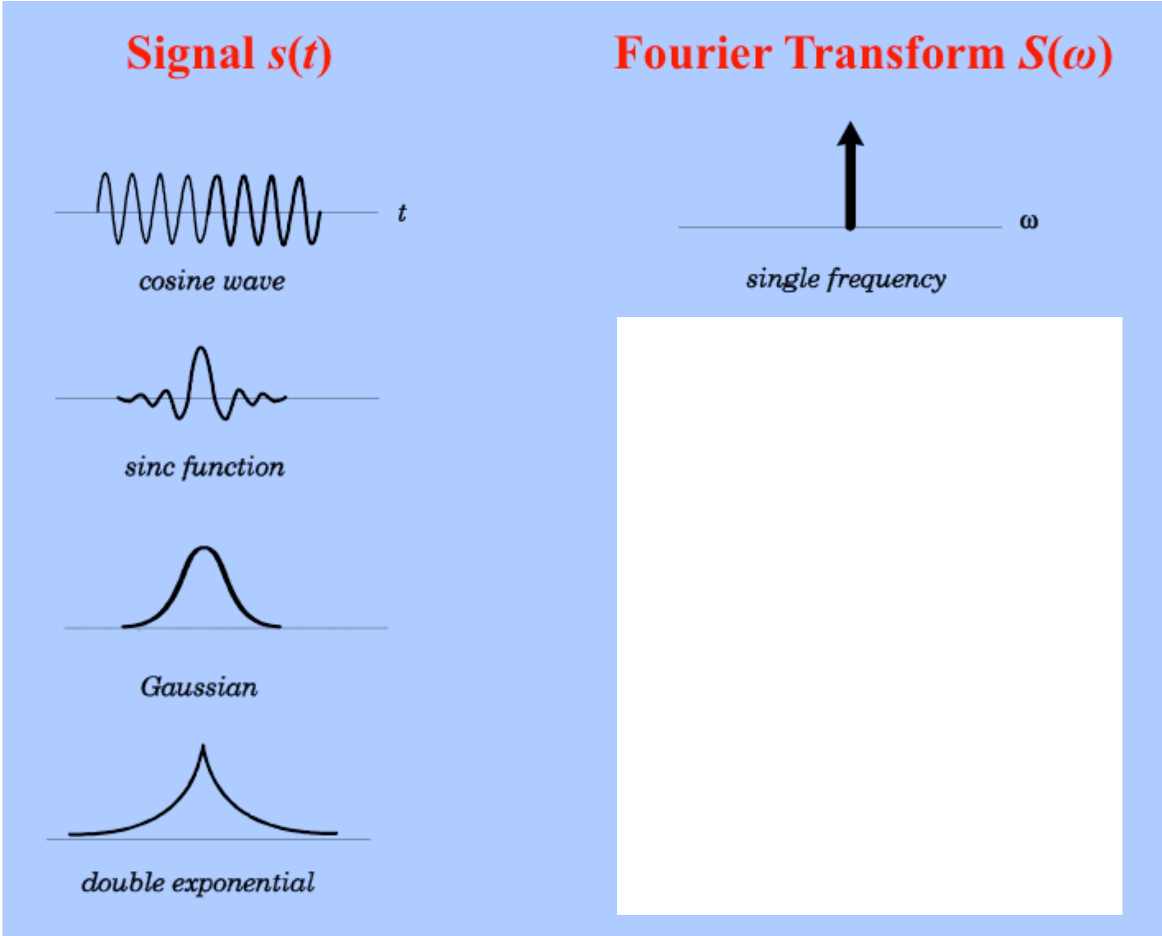
* If x is time, ξ is frequency.

$$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x)\, e^{-2\pi i x \xi}\, dx$$

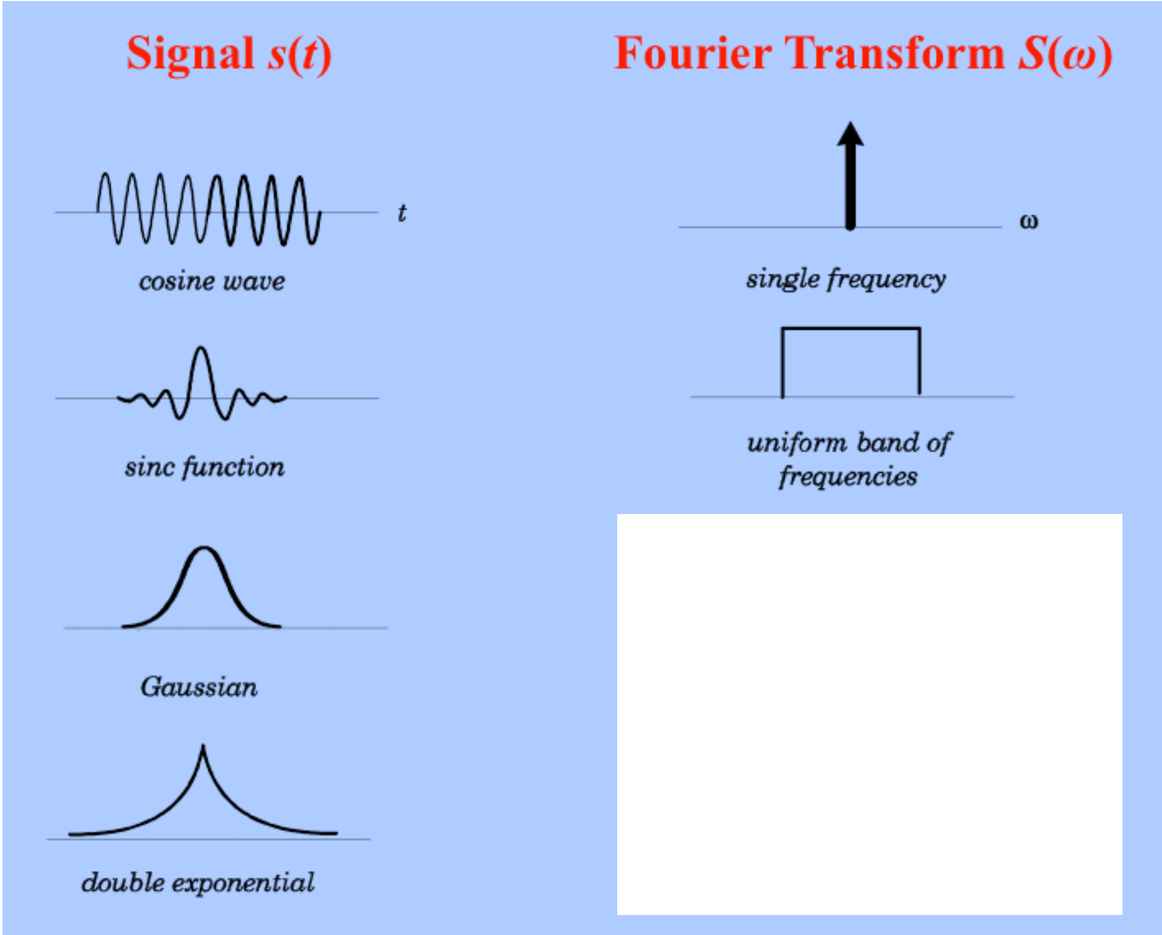
Signal $s(t)$ | **Fourier Transform $S(\omega)$**



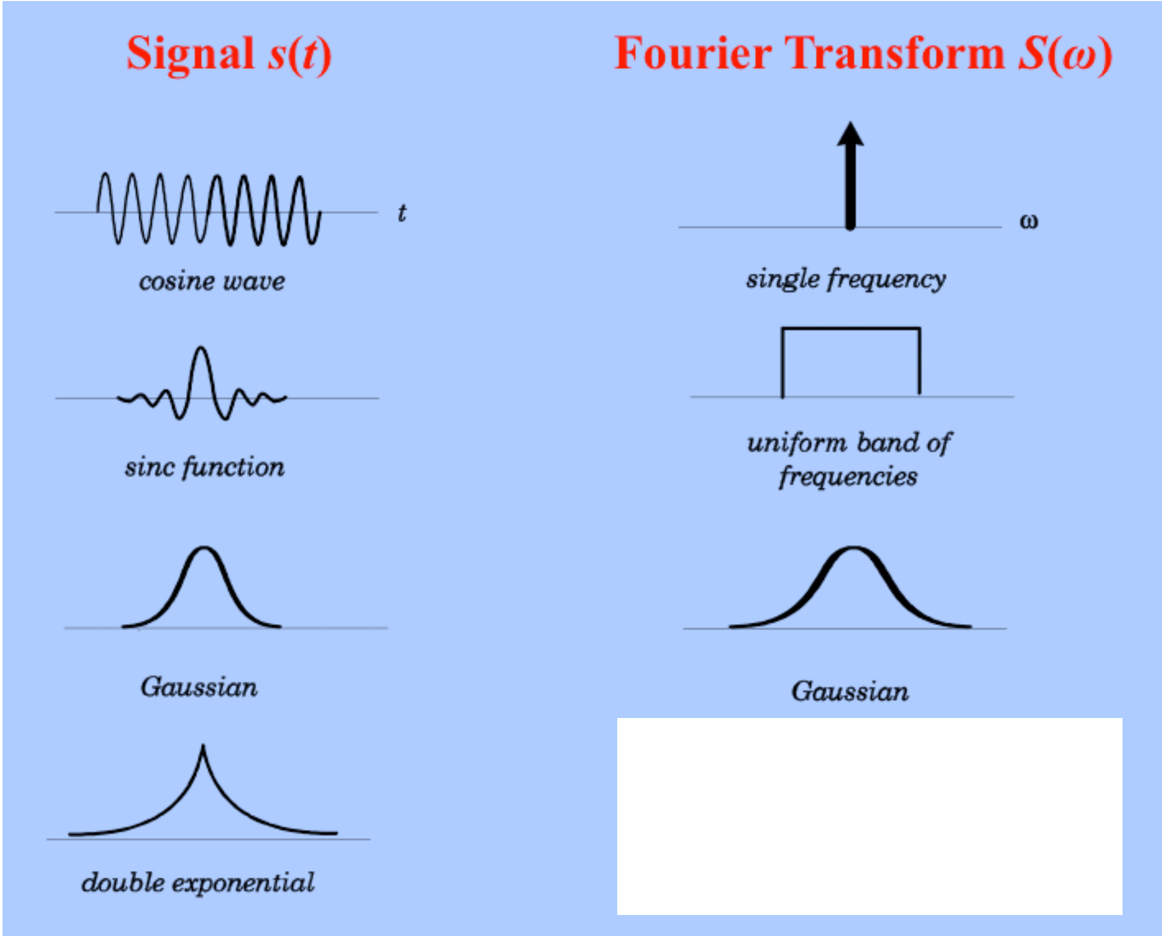
$$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x)\, e^{-2\pi i x \xi}\, dx$$



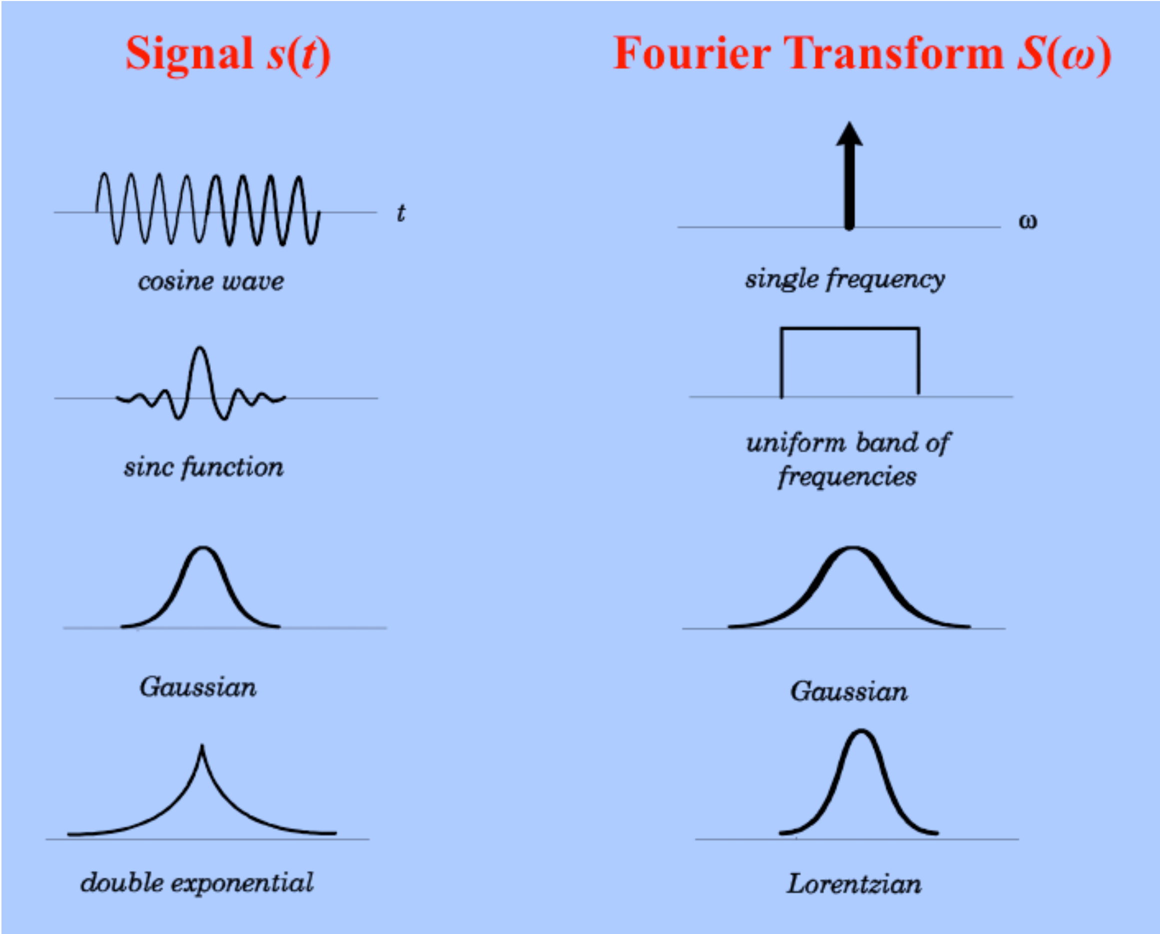
$$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x)\, e^{-2\pi i x \xi}\, dx$$



$$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x)\, e^{-2\pi i x \xi}\, dx$$



$$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x)\, e^{-2\pi i x \xi}\, dx$$



(Physicist) Definition of Fourier Transformation

Fourier Transformation with the angular frequency $k = 2\pi\xi$

$$\hat{f}(k) = \int_{-\infty}^{+\infty} f(x) e^{-ikx} dx$$

Inverse Fourier Transformation

$$\cancel{f(x) = \int_{-\infty}^{+\infty} f(k) e^{ikx} dk} \qquad f(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f(k) e^{ikx} dk$$

* The factor 1/2pi is needed to make the two transforms symmetric.

Fourier Series Expansion

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} f(k)e^{ikx}dk$$

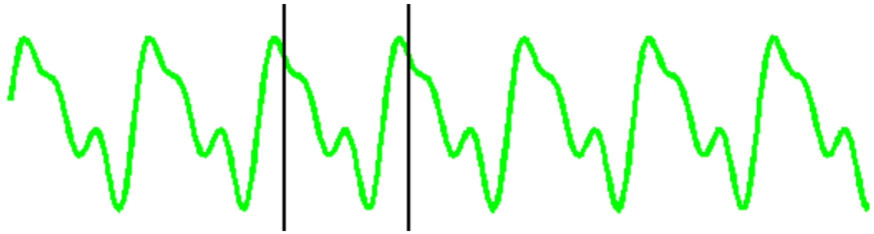
$$f(x) = \sum_{n=-\infty}^{\infty} a_n e^{inx}$$

A Fourier series is an expansion of a periodic function f(x) in terms of an infinite sum of sines and cosines.

$$f(x) = \sum_{n=-\infty}^{\infty} a_n e^{inx}$$

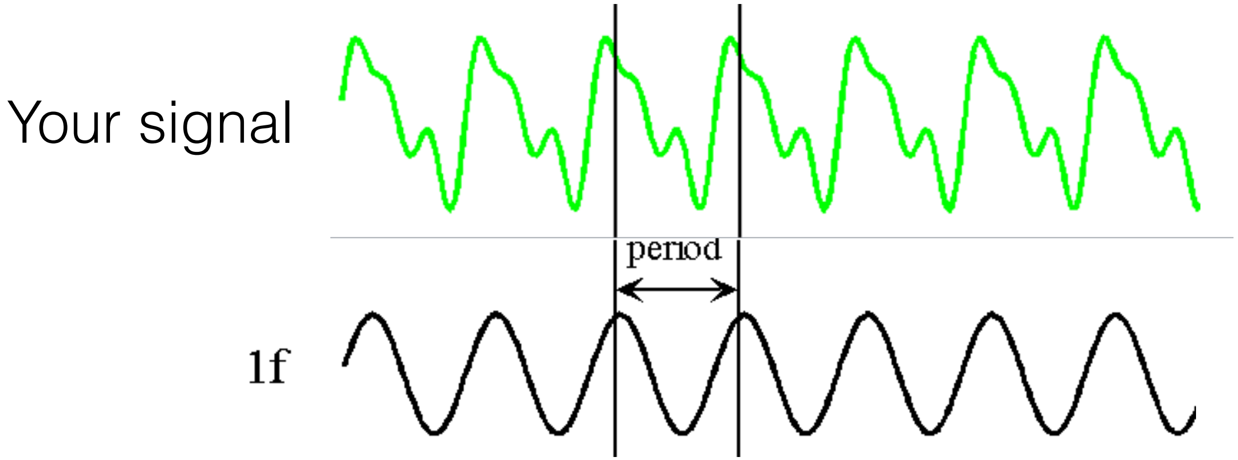
A simple case

Your signal



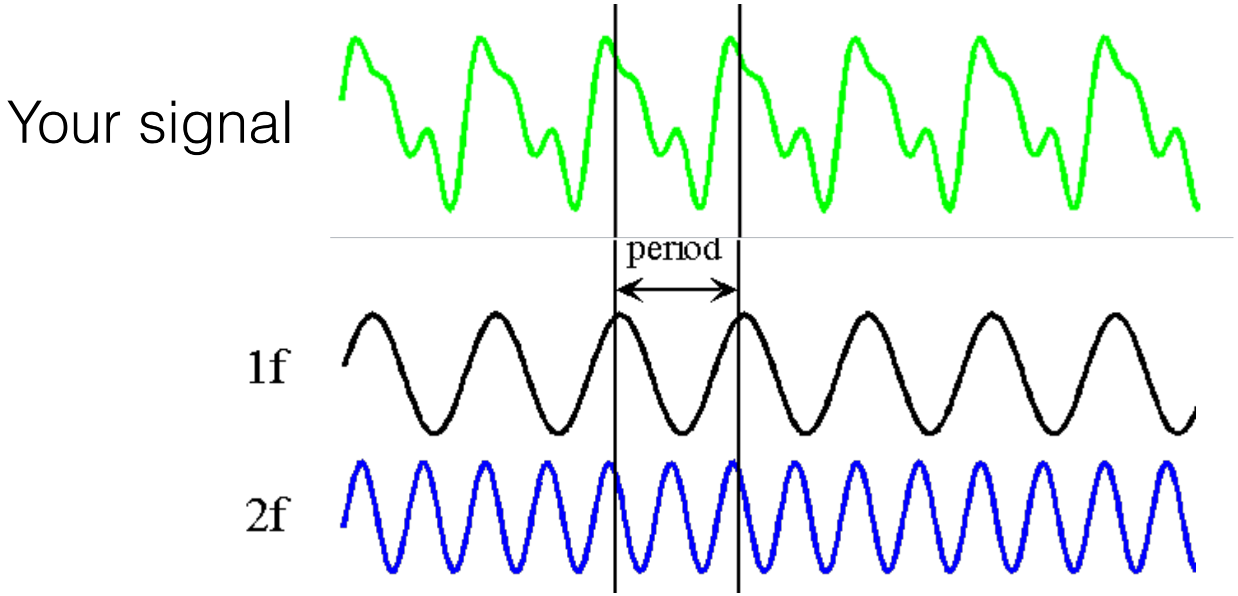
$$f(x) = \sum_{n=-\infty}^{\infty} a_n e^{inx}$$

A simple case



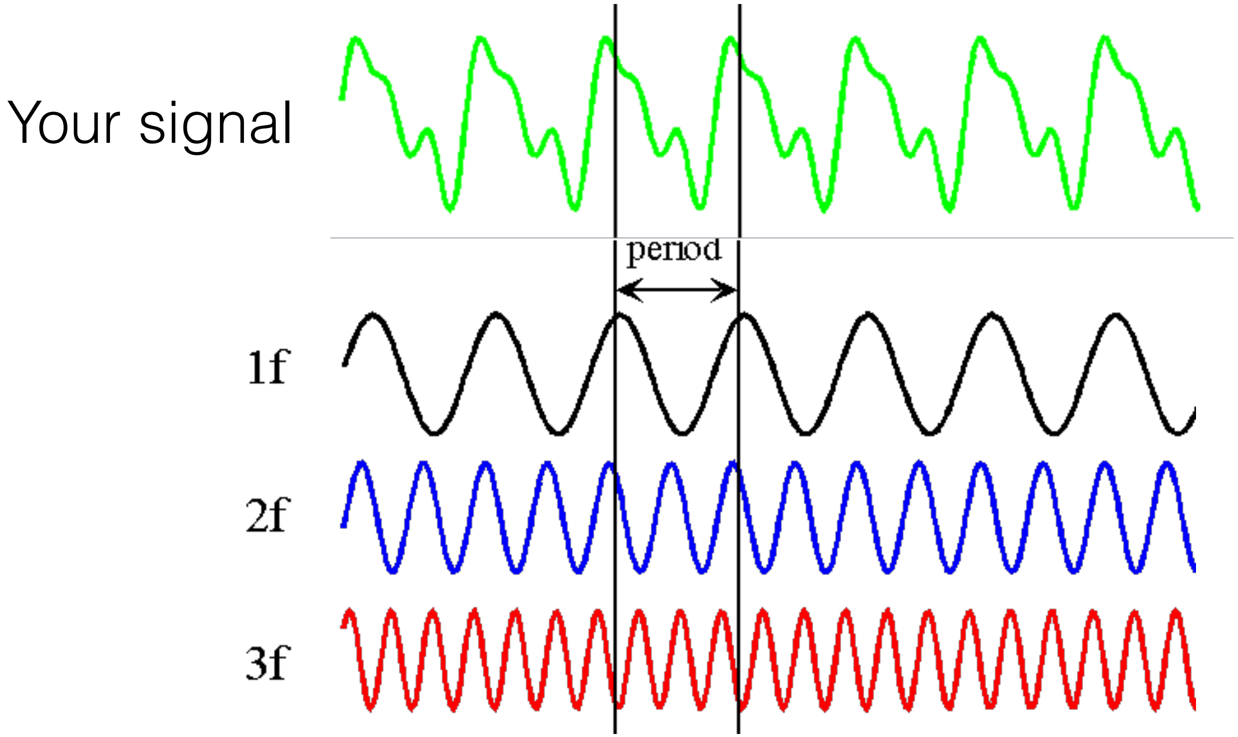
$$f(x) = \sum_{n=-\infty}^{\infty} a_n e^{inx}$$

A simple case



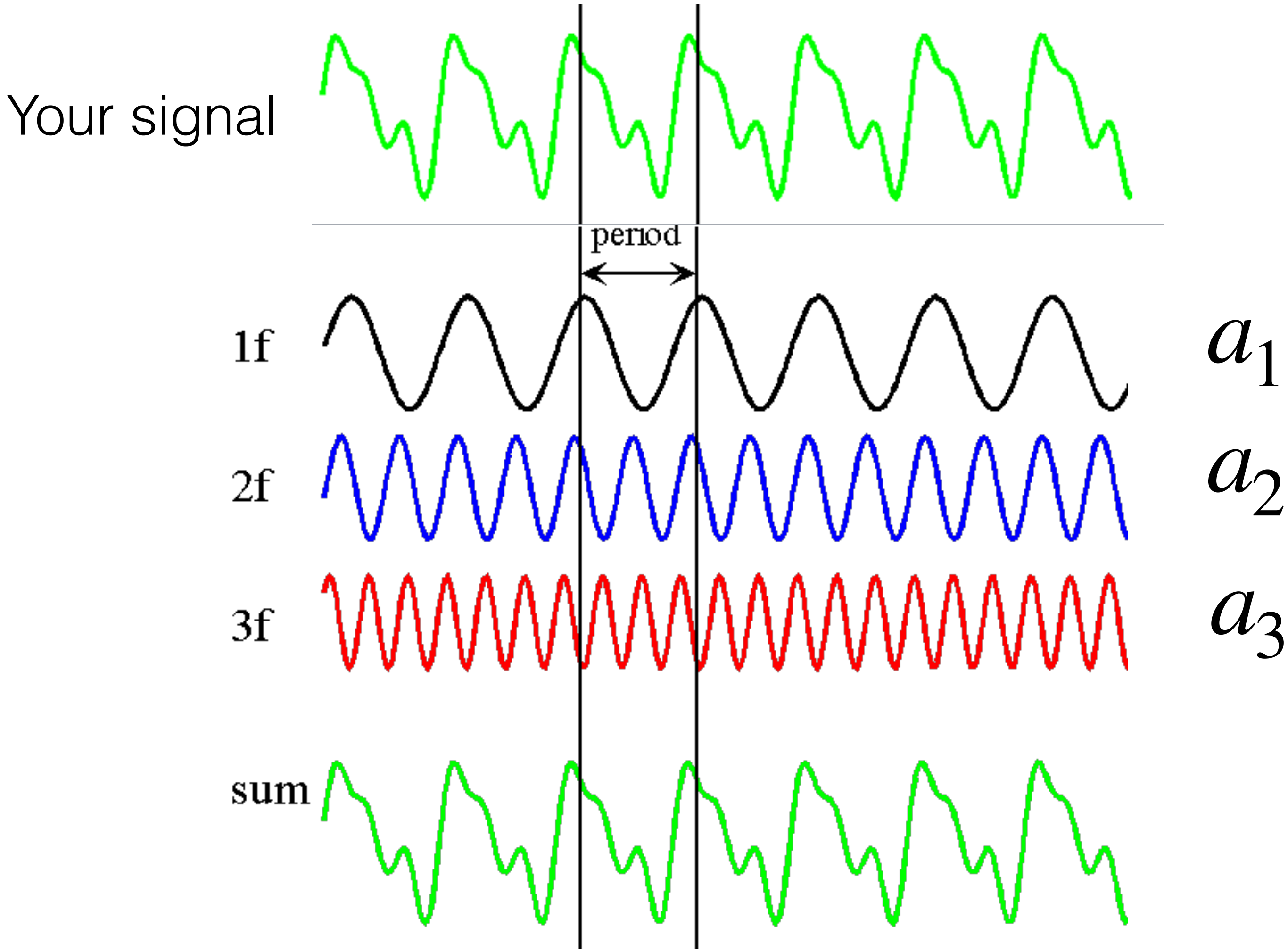
$$f(x) = \sum_{n=-\infty}^{\infty} a_n e^{inx}$$

A simple case

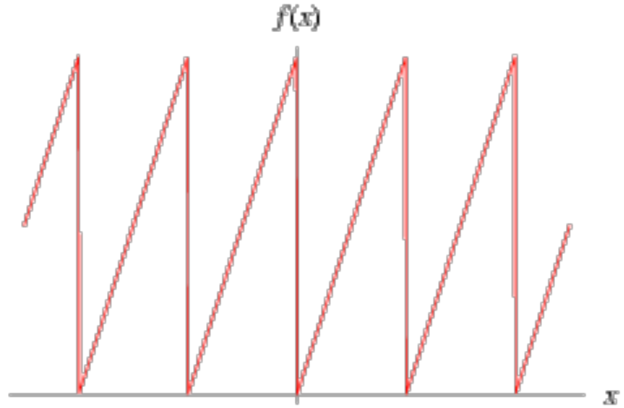


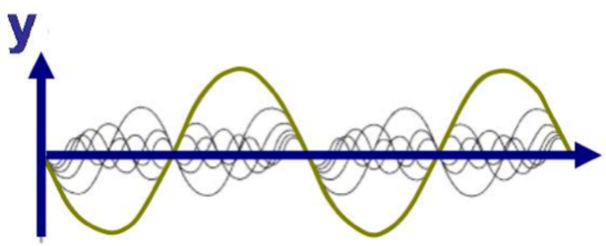
$$f(x) = \sum_{n=-\infty}^{\infty} a_n e^{inx}$$

A simple case

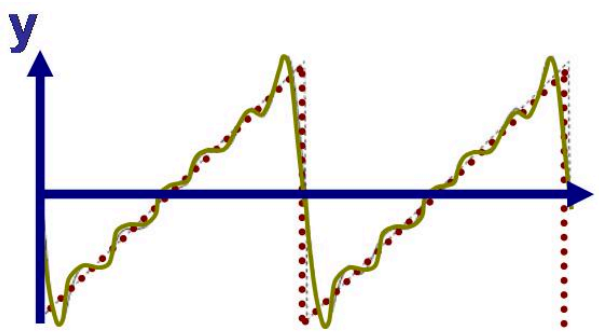


An extreme case: Sawtooth function



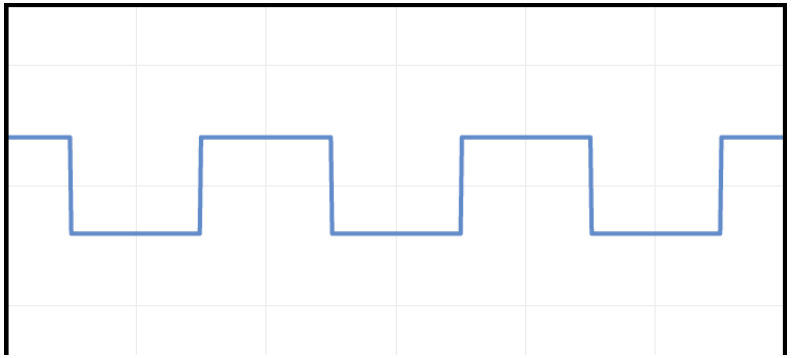


6 waves

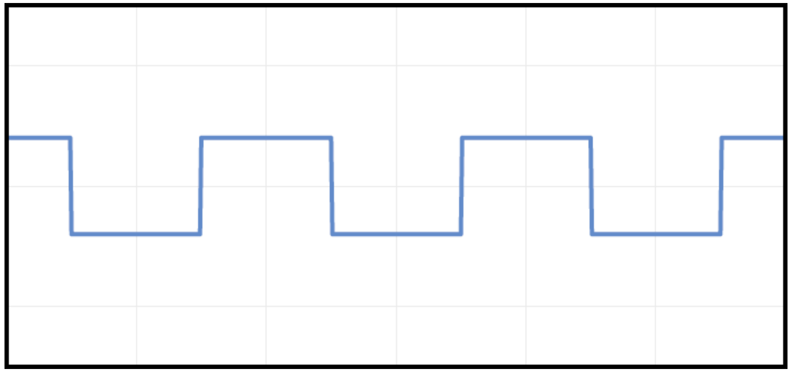


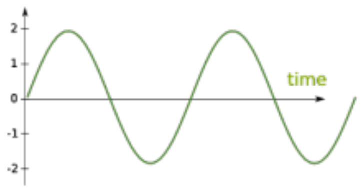
A suboptimal

Square Wave Reconstruction



Square Wave Reconstruction





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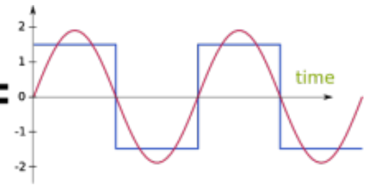
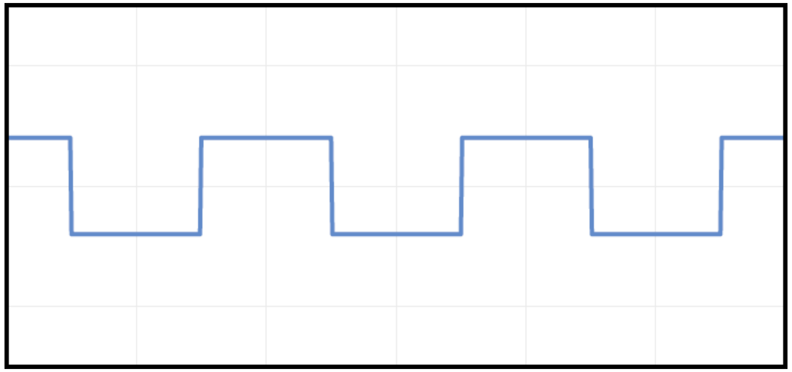
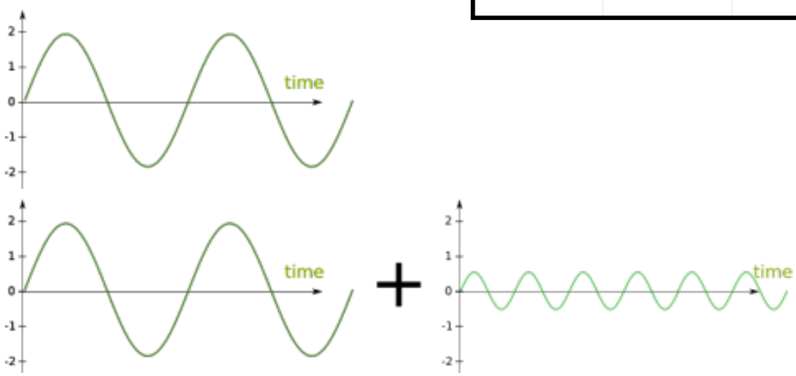
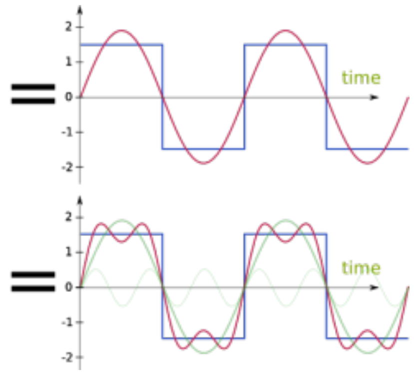


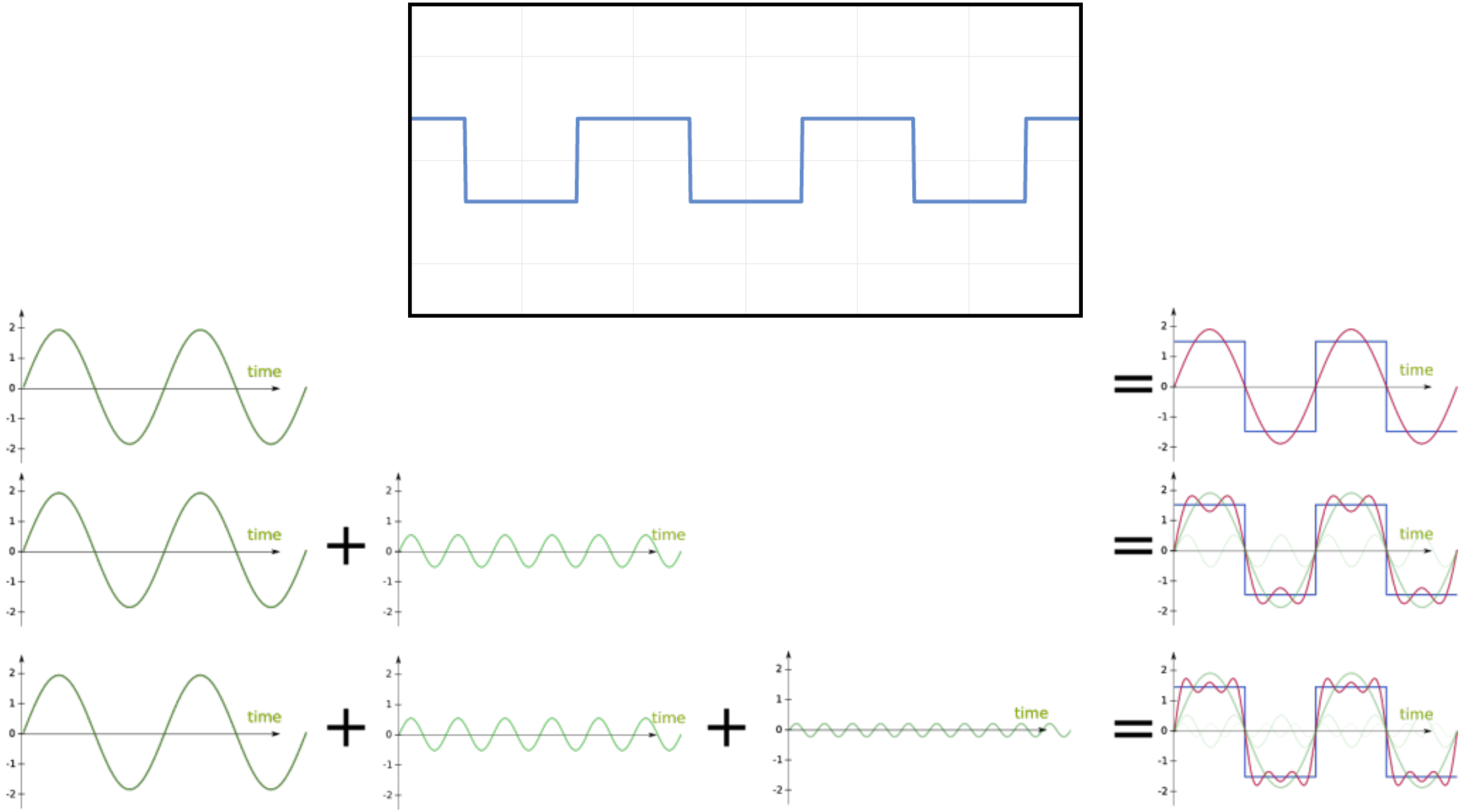
Square Wave Reconstruction



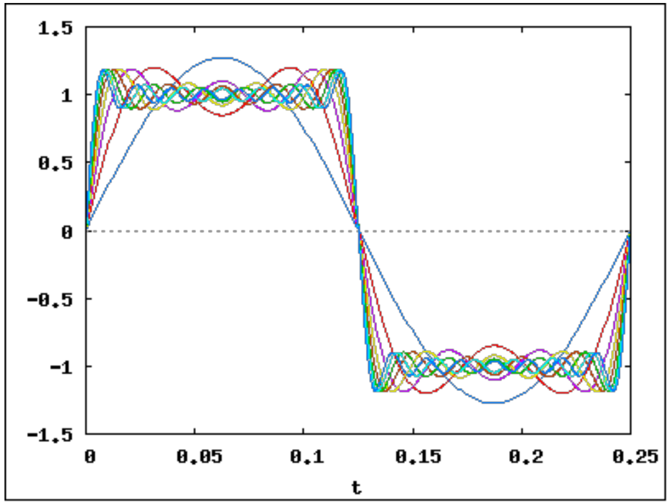


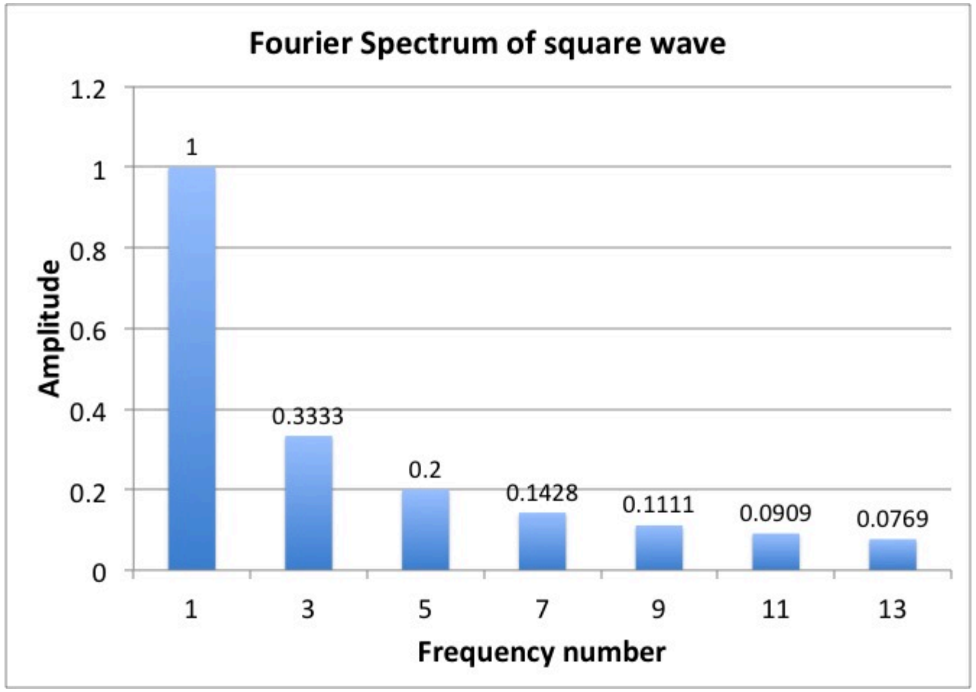


Square Wave Reconstruction



And so on...


1.5
1
0.5
0
-0.5
-1
-1.5
0
0.05
0.1
0.15
0.2
0.25
t

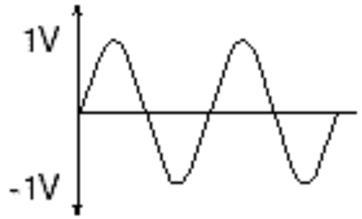

Fourier Spectrum of square wave
Amplitude
1.2
1
0.8
0.6
0.4
0.2
0
1
0.3333
0.2
0.1428
0.1111
0.0909
0.0769
1
3
5
7
9
11
13
Frequency number

Why is it useful?

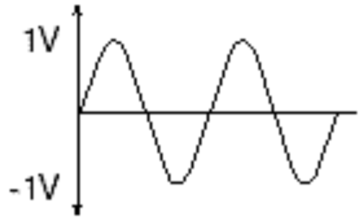
Description

A pure 5kHz
sine wave
measuring 1
volt peak

Why is it useful?

Description	Time Series
A pure 5kHz sine wave measuring 1 volt peak	

Why is it useful?

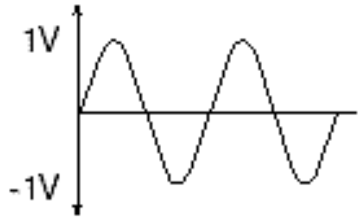
Description	Time Series	Fourier Expansion
A pure 5kHz sine wave measuring 1 volt peak		$v(t) = 1\sin(\omega 1)t$ $\omega 1 = 2\pi(5kHz)$

Why is it useful?

Description

A pure 5kHz sine wave measuring 1 volt peak

Time Series

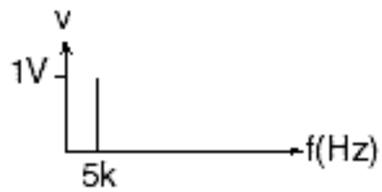


Fourier Expansion

$$v(t) = 1\sin(\omega 1)t$$

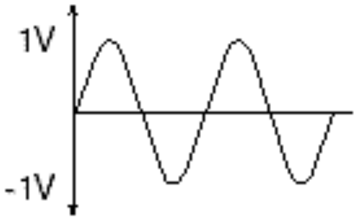
$$\omega 1 = 2\pi(5kHz)$$

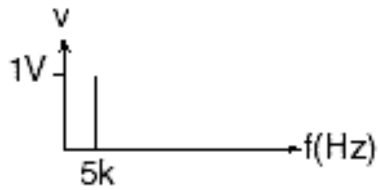
Power Spectrum



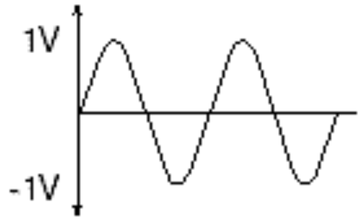
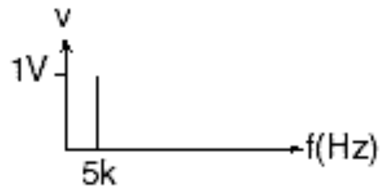
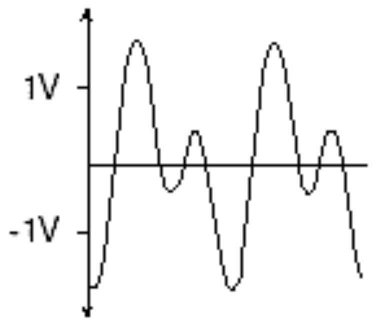
Why is it useful?

Description	Time Series	Fourier Expansion	Power Spectrum
A pure 5kHz sine wave measuring 1 volt peak	see image 1 below	$v(t) = 1\sin(\omega 1)t$ $\omega 1 = 2\pi(5kHz)$	see image 2 below
A pure 5kHz and 10kHz sine wave, each measuring 1 volt peak, added together			

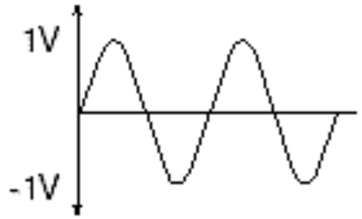
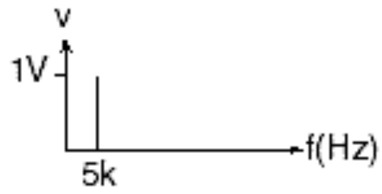
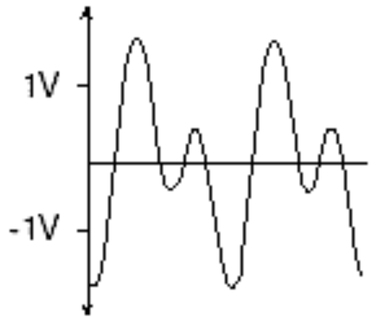




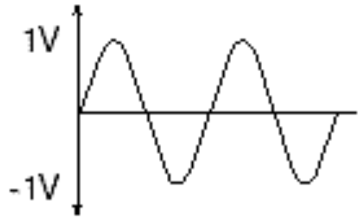
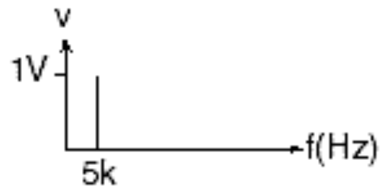
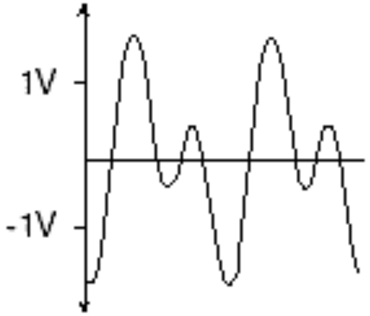
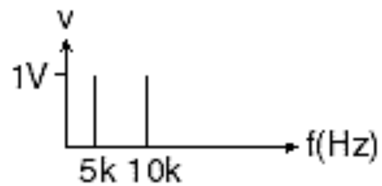
Why is it useful?

Description	Time Series	Fourier Expansion	Power Spectrum
A pure 5kHz sine wave measuring 1 volt peak		$v(t) = 1\sin(\omega 1)t$ $\omega 1 = 2\pi(5kHz)$	
A pure 5kHz and 10kHz sine wave, each measuring 1 volt peak, added together			

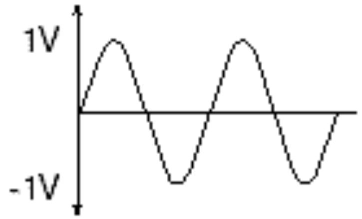
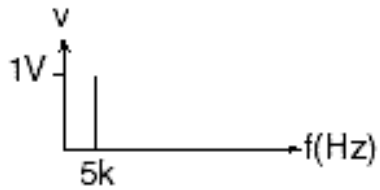
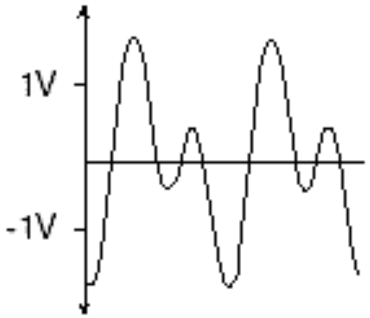
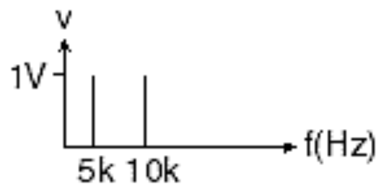
Why is it useful?

Description	Time Series	Fourier Expansion	Power Spectrum
A pure 5kHz sine wave measuring 1 volt peak		$v(t) = 1\sin(\omega 1)t$ $\omega 1 = 2\pi(5kHz)$	
A pure 5kHz and 10kHz sine wave, each measuring 1 volt peak, added together		$v(t) = 1\sin(\omega 1)t + 1\sin(\omega 2)t$ $\omega 1 = 2\pi(5kHz)$ $\omega 2 = 2\pi(10kHz)$	

Why is it useful?

Description	Time Series	Fourier Expansion	Power Spectrum
A pure 5kHz sine wave measuring 1 volt peak		$v(t) = 1\sin(\omega 1)t$ $\omega 1 = 2\pi(5kHz)$	
A pure 5kHz and 10kHz sine wave, each measuring 1 volt peak, added together		$v(t) = 1\sin(\omega 1)t + 1\sin(\omega 2)t$ $\omega 1 = 2\pi(5kHz)$ $\omega 2 = 2\pi(10kHz)$	

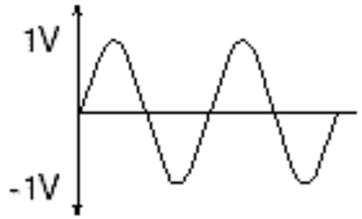
Why is it useful?

Description	Time Series	Fourier Expansion	Power Spectrum
A pure 5kHz sine wave measuring 1 volt peak		$v(t) = 1\sin(\omega 1)t$ $\omega 1 = 2\pi(5kHz)$	
A pure 5kHz and 10kHz sine wave, each measuring 1 volt peak, added together		$v(t) = 1\sin(\omega 1)t + 1\sin(\omega 2)t$ $\omega 1 = 2\pi(5kHz)$ $\omega 2 = 2\pi(10kHz)$	
A pure 5kHz, 10kHz, and 20kHz sine wave, each measuring 1 volt peak, added together			

Why is it useful?

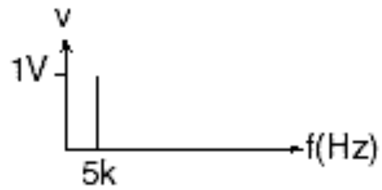
Description | Time Series | Fourier Expansion | Power Spectrum

A pure 5kHz sine wave measuring 1 volt peak

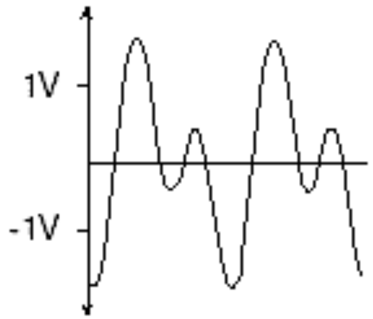


$$v(t) = 1\sin(\omega 1)t$$

$$\omega 1 = 2\pi(5kHz)$$

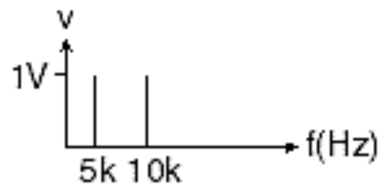


A pure 5kHz and 10kHz sine wave, each measuring 1 volt peak, added together

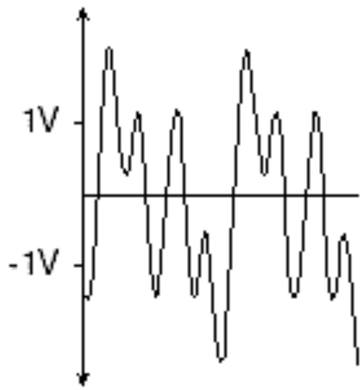


$$v(t) = 1\sin(\omega 1)t + 1\sin(\omega 2)t$$

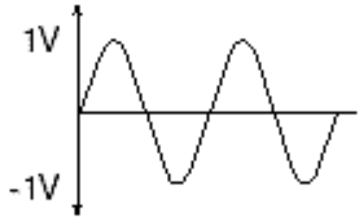
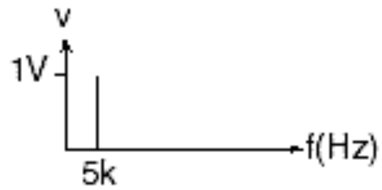
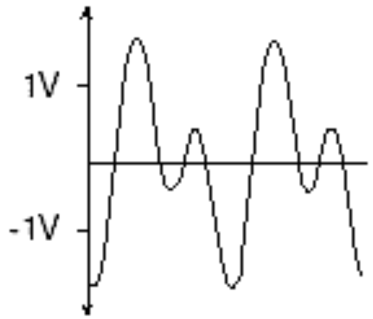
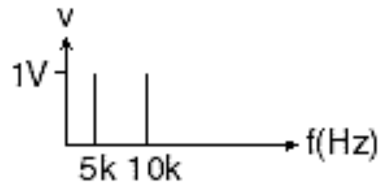
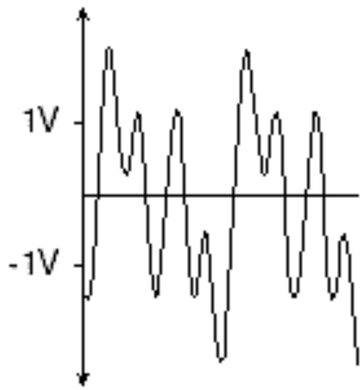
$$\omega 1 = 2\pi(5kHz)$$
$$\omega 2 = 2\pi(10kHz)$$



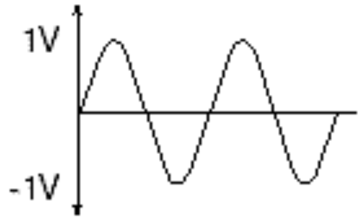
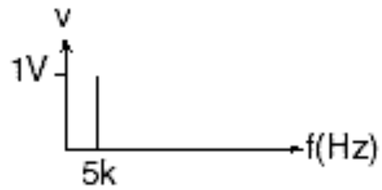
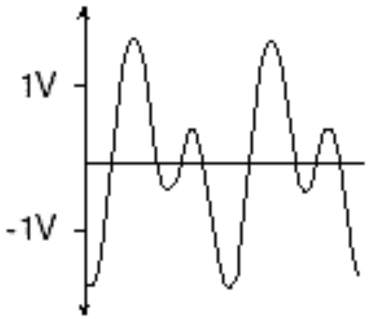
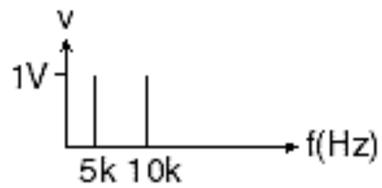
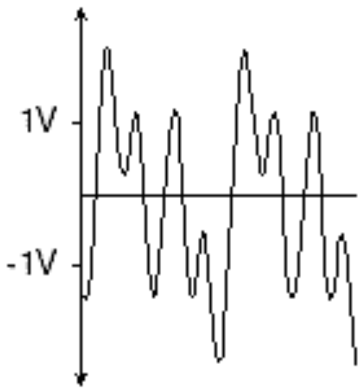
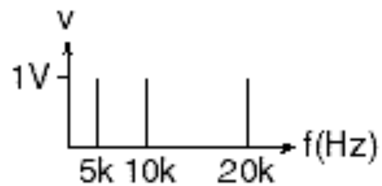
A pure 5kHz, 10kHz, and 20kHz sine wave, each measuring 1 volt peak, added together



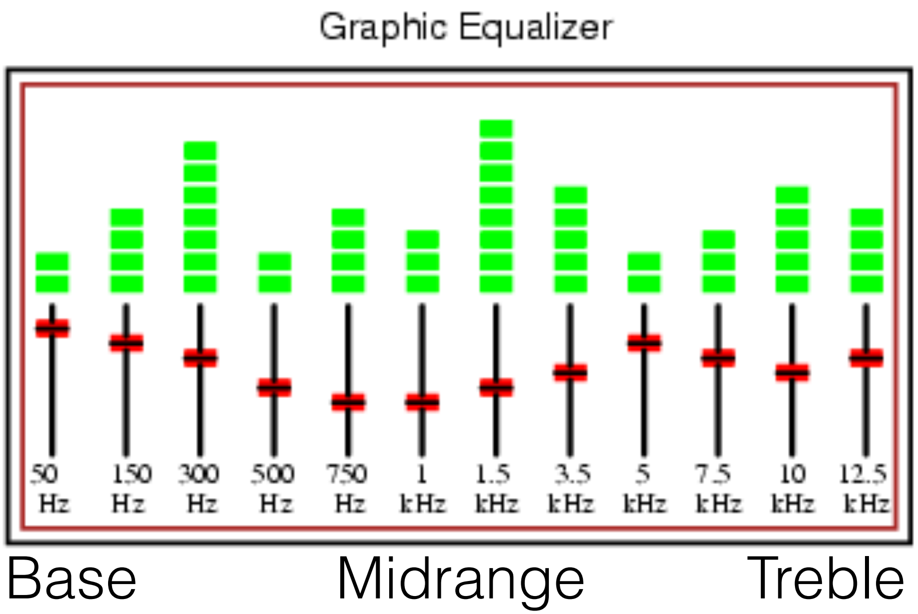
Why is it useful?

Description	Time Series	Fourier Expansion	Power Spectrum
A pure 5kHz sine wave measuring 1 volt peak		$v(t) = 1\sin(\omega 1)t$ $\omega 1 = 2\pi(5kHz)$	
A pure 5kHz and 10kHz sine wave, each measuring 1 volt peak, added together		$v(t) = 1\sin(\omega 1)t + 1\sin(\omega 2)t$ $\omega 1 = 2\pi(5kHz)$ $\omega 2 = 2\pi(10kHz)$	
A pure 5kHz, 10kHz, and 20kHz sine wave, each measuring 1 volt peak, added together		$v(t) = 1\sin(\omega 1)t + 1\sin(\omega 2)t + 1\sin(\omega 3)t$ $\omega 1 = 2\pi(5kHz)$ $\omega 2 = 2\pi(10kHz)$ $\omega 3 = 2\pi(20kHz)$	

Why is it useful?

Description	Time Series	Fourier Expansion	Power Spectrum
A pure 5kHz sine wave measuring 1 volt peak		$v(t) = 1\sin(\omega 1)t$ $\omega 1 = 2\pi(5kHz)$	
A pure 5kHz and 10kHz sine wave, each measuring 1 volt peak, added together		$v(t) = 1\sin(\omega 1)t + 1\sin(\omega 2)t$ $\omega 1 = 2\pi(5kHz)$ $\omega 2 = 2\pi(10kHz)$	
A pure 5kHz, 10kHz, and 20kHz sine wave, each measuring 1 volt peak, added together		$v(t) = 1\sin(\omega 1)t + 1\sin(\omega 2)t + 1\sin(\omega 3)t$ $\omega 1 = 2\pi(5kHz)$ $\omega 2 = 2\pi(10kHz)$ $\omega 3 = 2\pi(20kHz)$	

Audio Equalizer



50-75Hz

Boost to beef up kick drums and sub bass lines. Cut to reduce excessive low-end weight.

80-200Hz

Boost to add body to snares and guitars, punch to kick drums, roundness to bass, and general warmth. Cut to reduce low-end mud.

200-500Hz

Boost to ‘warm up’ vocals, guitars and synths, and add presence to basses. Cut to reduce muddiness.

500-800Hz

Boost (with care!) to bring out the tone of almost any instrument. Cut to reduce ‘honk’.

2-5kHz

Boost to give vocals, guitars, synths and strings clarity, definition and impact. Cut to reduce harshness.

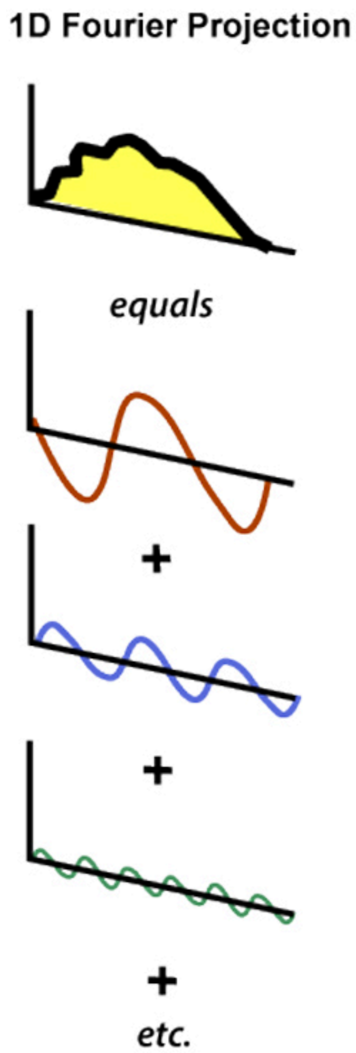
5-10kHz

Boost to add presence and sheen to drums, cymbals and guitars. Cut to reduce scratchiness and sibilance.

16kHz+

Boost for brightness and ‘air’. Cut to reduce high-end fizz.

Application to images



Application to images

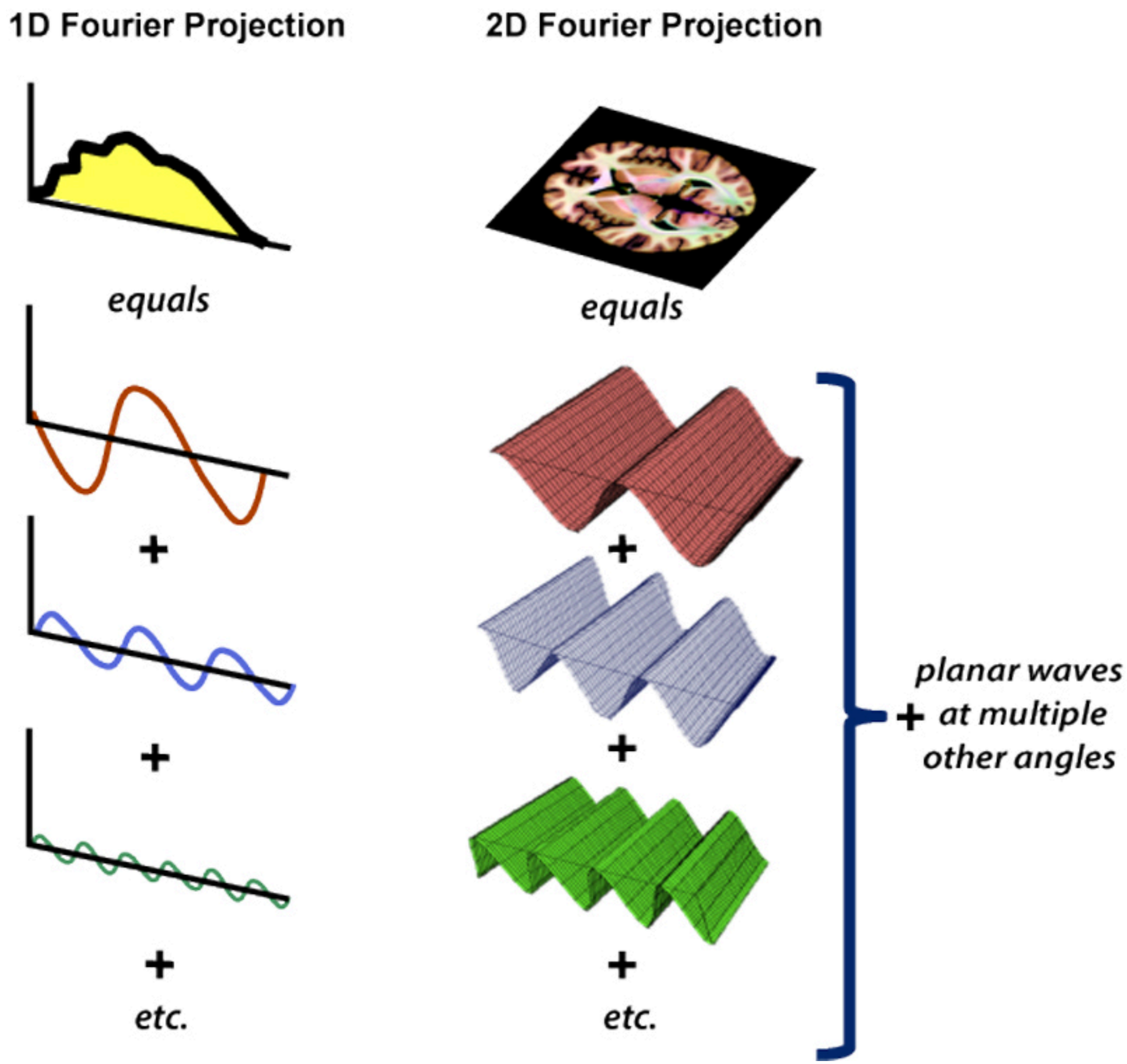
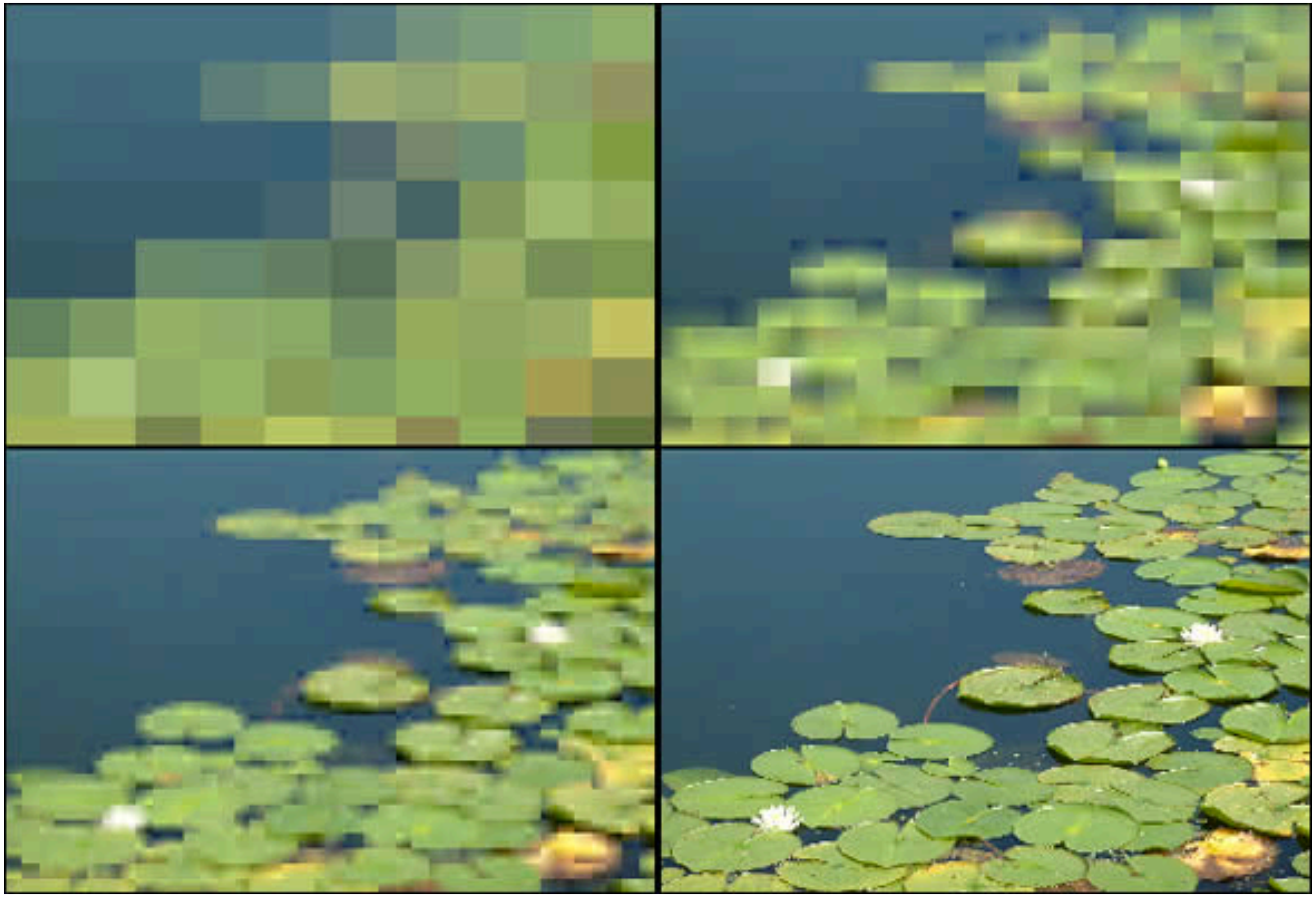


Image Equalizer?



Fourier Analysis

=Decomposition of signals with Fourier series

$$f(x) = \sum_{n=-\infty}^{\infty} a_n e^{inx}$$

Fourier analysis = Determining the coefficients

Coefficient Determination

$$f(x) = \sum_{n=-\infty}^{\infty} a_n e^{inx}$$

Let's multiply e^{-imx} and integrate.

$$\int_{-\pi}^{\pi} f(x)e^{-imx}dx = \int_{-\pi}^{\pi} \sum_{n=-\infty}^{\infty} a_n e^{inx} e^{-imx} dx$$

$$= \sum_{n=-\infty}^{\infty} a_n \int_{-\pi}^{\pi} e^{i(n-m)x} dx$$

$$\delta(x-\alpha) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ip(x-\alpha)}\, dp$$

$$= \sum_{n=-\infty}^{\infty} a_n 2\pi\delta(n-m)$$

$$= \sum_{n=-\infty}^{\infty} a_n 2\pi\delta_{n,m}$$

$$= 2\pi a_m$$

m번째 항의 coefficient를 알아냄!

$$a_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{-inx}dx$$

과제 12

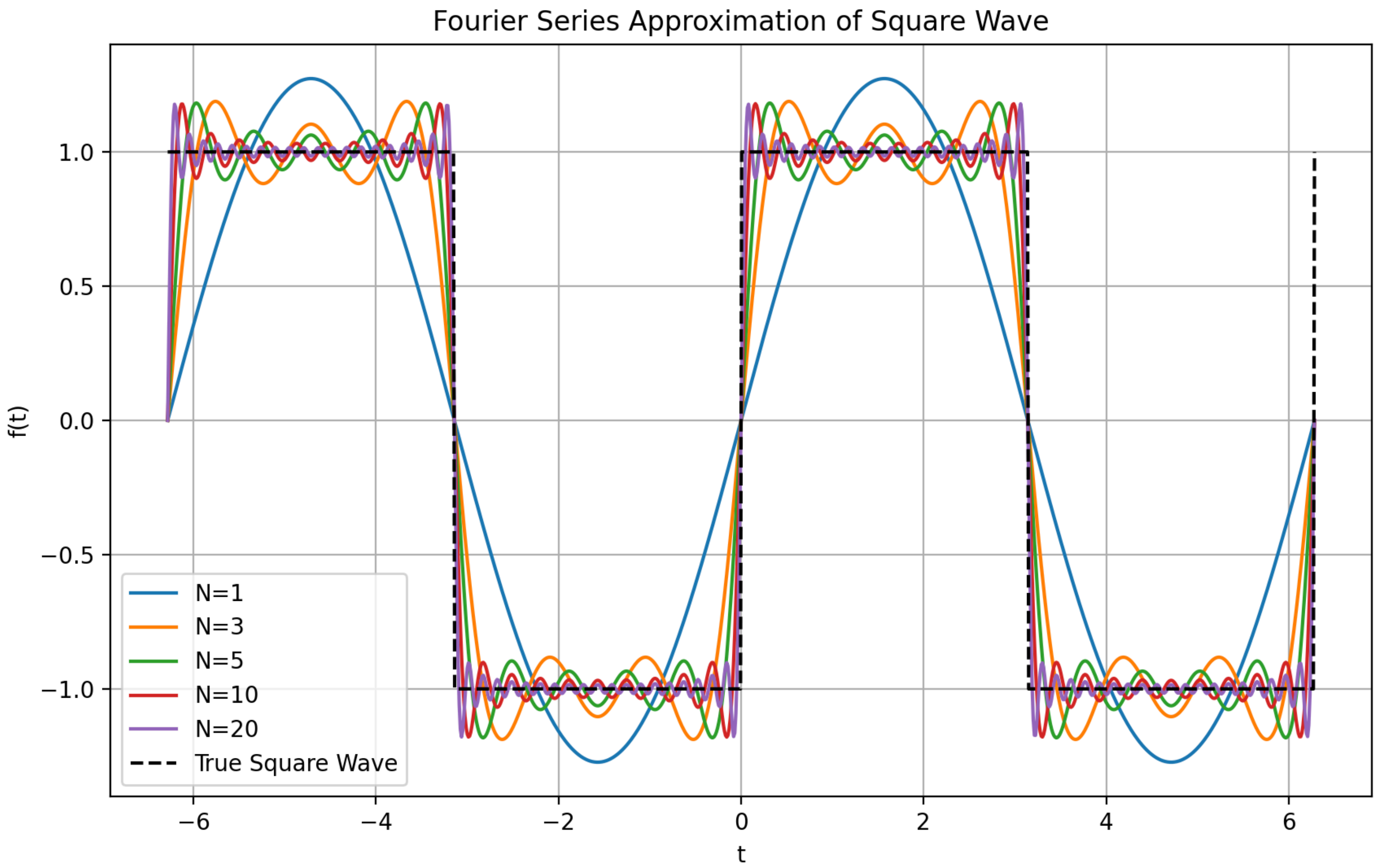
다음과 같이 정의된 주기 함수가 있다.

$$f(t) = \begin{cases} 1, & 0 < t < \pi \\ -1, & -\pi < t < 0 \end{cases}, \quad f(t + 2\pi) = f(t)$$

위 함수를 Fourier Series로 Expansion 하면 아래과 같은 결과가 나옴을 보여라.

$$f(t) = \sum_{n=1,3,5,...}^{\infty} \frac{4}{n\pi} \sin(nt)$$

term의 갯수를 점차로 증가시킬때 원래 함수에 접근해 가는 것을 보여라.


Fourier Series Approximation of Square Wave
f(t)
t
1.0
0.5
0.0
−0.5
−1.0
−6
−4
−2
0
2
4
6
N=1
N=3
N=5
N=10
N=20
True Square Wave

Shannon-Nyquist Theorem

Shannon-Nyquist Theorem

Theorem: (Shannon-Nyquist) Assume that f is band-limited by W, i.e., $\hat{f}(k) = 0$ for all $|k| \geq W$. Let $T = \pi/W$ be the Nyquist rate. Then it holds

$$\boxed{f(x) = \sum_{n=-\infty}^{\infty} f(nT)\operatorname{sinc}(\pi(x/T - n))}$$

band-limited: frequency is finite.

$$\operatorname{sinc} x = \frac{\sin x}{x}$$

f:

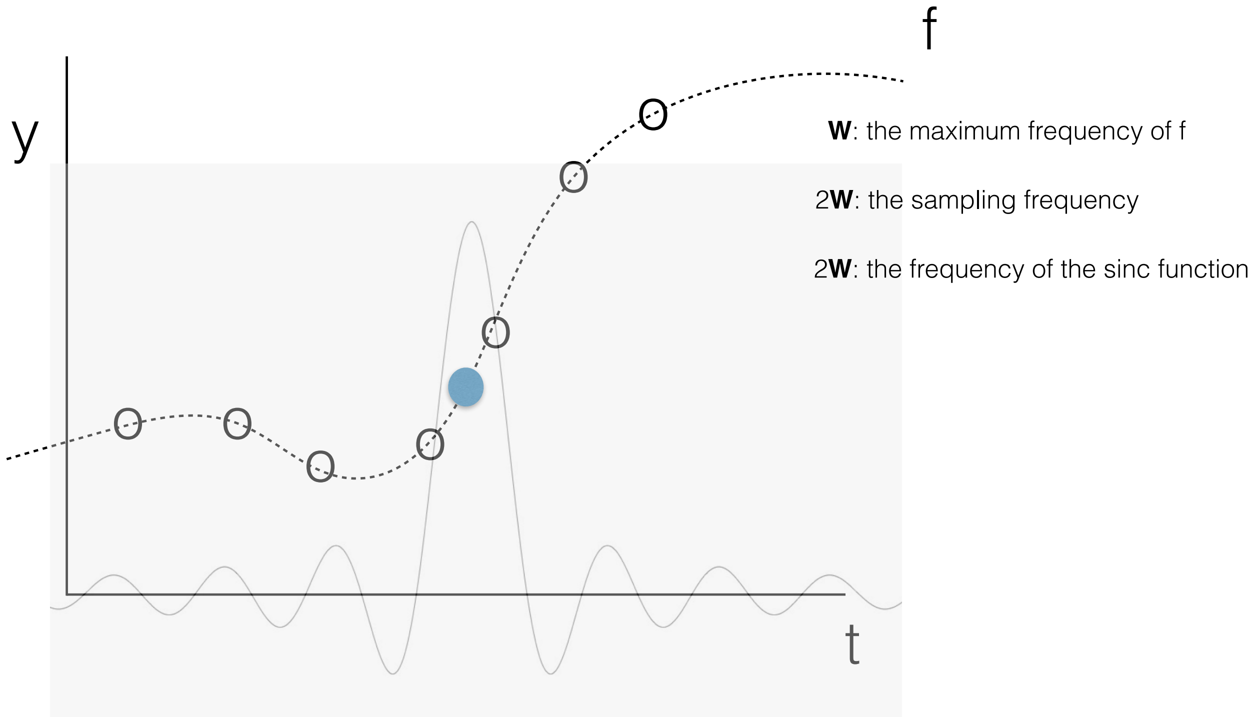
W is an angular frequency W=2πv. So the frequency of f(x) is v=W/2π.

kernel:

The sine has the same frequency v=W/2π.

We sample f(x) at …, -2T, T, 0, T, 2T, 3T, …

So the sampling frequency is v_s=1/T = W/π


f
y
t
W: the maximum frequency of f
2W: the sampling frequency
2W: the frequency of the sinc function

Proof

Assume for simplicity $W = \pi$.

Proof

Assume for simplicity $W = \pi$. That is, the maximum angular frequency is **π**.

Proof

Assume for simplicity $W = \pi$. That is, the maximum angular frequency is $\boldsymbol{\pi}$.
By the inverse Fourier transform,

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{f}(k)e^{ikx}dk = \frac{1}{2\pi}\int_{-\pi}^{\pi} \hat{f}(k)e^{ikx}dk.$$

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Define

$$\tilde{f}(k) = \begin{cases} \hat{f}(k), & \text{if } -\pi < k < \pi, \\ \text{periodic continuation}, & \text{if } |k| \geq \pi. \end{cases}$$

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$\tilde{f}$ can be represented as a Fourier series in k-space,

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where

$$\hat{\tilde{f}}_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} \tilde{f}(k)e^{-ink}dk$$

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$\tilde{f}$ can be represented as a Fourier series in k-space,

$$\tilde{f}(k) = \sum_{n=-\infty}^{\infty} \hat{\hat{f}}_n e^{ink},$$

where

$$\hat{\hat{f}}_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} \tilde{f}(k)e^{-ink}dk = \frac{1}{2\pi}\int_{-\pi}^{\pi} \hat{f}(k)e^{-ink}dk = f(-n).$$

express f(x) using inverse Fourier Transformation

$$f(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \hat{f}(k) e^{ikx} dk$$

express f(x) using inverse Fourier Transformation

$$f(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \hat{f}(k) e^{ikx} dk$$

$$\tilde{f}(k) = \sum_{n=-\infty}^{\infty} \hat{\hat{f}}_n e^{ink}$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \left(\sum_{n=-\infty}^{\infty} f(-n) e^{ink} \right) e^{ikx} dk$$

$$\begin{aligned} f(x) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} \hat{f}(k)e^{ikx}dk \\ &= \frac{1}{2\pi}\int_{-\pi}^{\pi}\left(\sum_{n=-\infty}^{\infty} f(-n)e^{ink}\right)e^{ikx}dk \\ &= \sum_{n=-\infty}^{\infty} f(-n)\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{ik(x+n)}dk \end{aligned}$$

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$$\begin{aligned}
f(x) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} \hat{f}(k) e^{ikx} dk \\
&= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left(\sum_{n=-\infty}^{\infty} f(-n) e^{ink} \right) e^{ikx} dk \\
&= \sum_{n=-\infty}^{\infty} f(-n) \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{ik(x+n)} dk \\
&= \sum_{n=-\infty}^{\infty} f(-n) \frac{\sin \pi(x+n)}{\pi(x+n)}
\end{aligned}$$

$$\tilde{f}(k) = \sum_{n=-\infty}^{\infty} \hat{\hat{f}}_n e^{ink}$$

$$\begin{aligned}
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&= \sum_{n=-\infty}^{\infty} f(-n) \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{ik(x+n)} dk \\
&= \sum_{n=-\infty}^{\infty} f(-n) \frac{\sin \pi(x+n)}{\pi(x+n)} \\
&= \sum_{n=-\infty}^{\infty} f(n) \frac{\sin \pi(x-n)}{\pi(x-n)}
\end{aligned}$$

$$\tilde{f}(k) = \sum_{n=-\infty}^{\infty} \hat{\hat{f}}_n e^{ink}$$

$$\begin{aligned}
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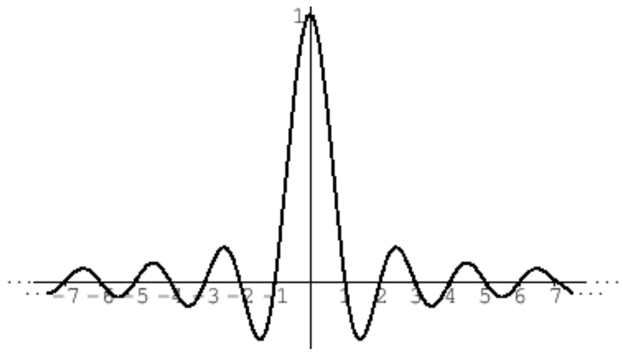
&= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left(\sum_{n=-\infty}^{\infty} f(-n) e^{ink} \right) e^{ikx} dk \\
&= \sum_{n=-\infty}^{\infty} f(-n) \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{ik(x+n)} dk \\
&= \sum_{n=-\infty}^{\infty} f(-n) \frac{\sin \pi(x+n)}{\pi(x+n)} \\
&= \sum_{n=-\infty}^{\infty} f(n) \frac{\sin \pi(x-n)}{\pi(x-n)}
\end{aligned}$$

임의의 위치 x 에서의 함수값 f(x)를 고정된 위치에서의 f(n)을 이용해 계산할수 있다.

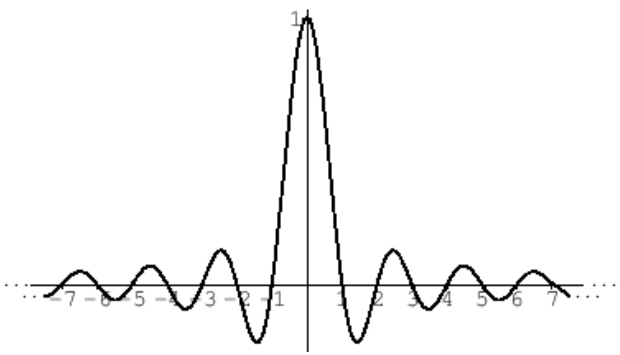
SINC Interpolation

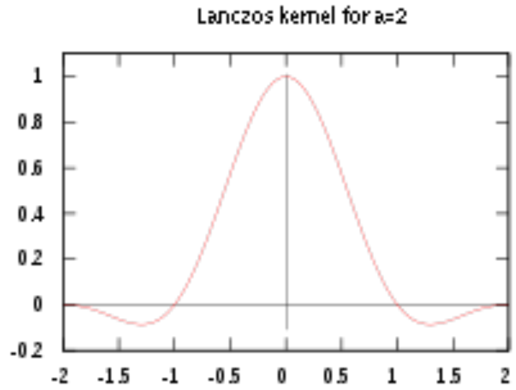
- Theoretically Ideal Interpolation Function.
- Real data are not band-limited.
- It has a power over infinite range.
- Therefore, in practice, it is impossible to apply the sinc interpolation.



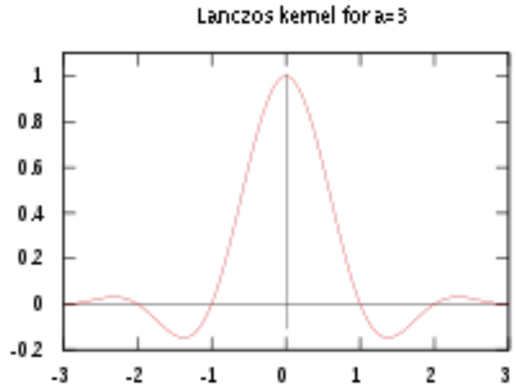
Compromised SINC Interpolation

Sinc

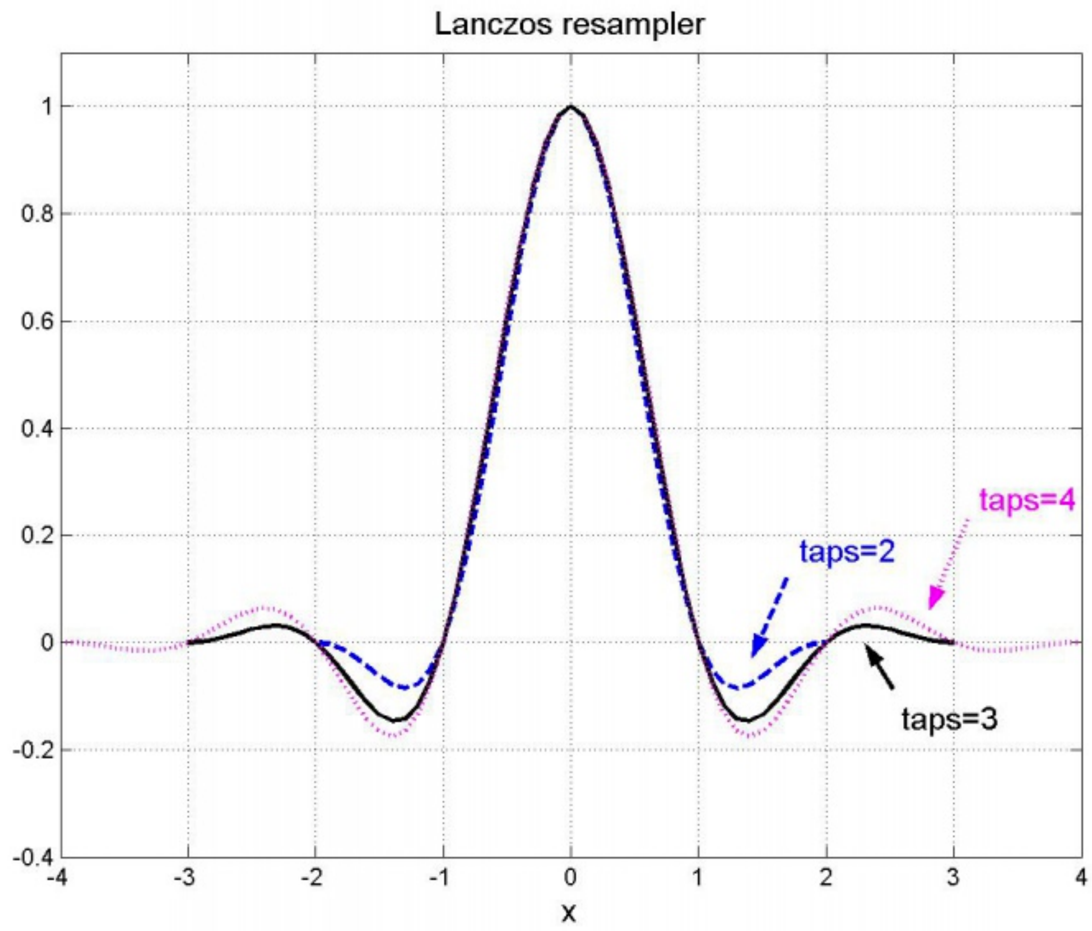




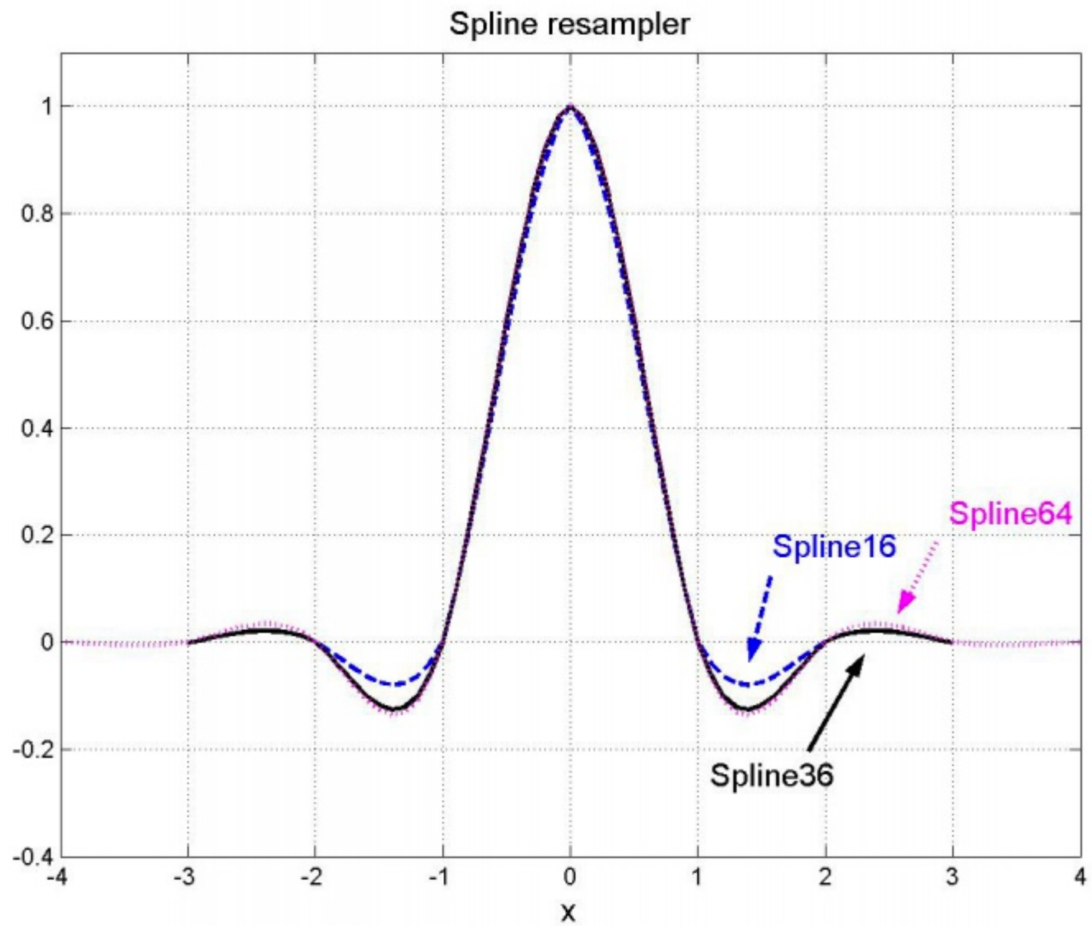
Lanczos2

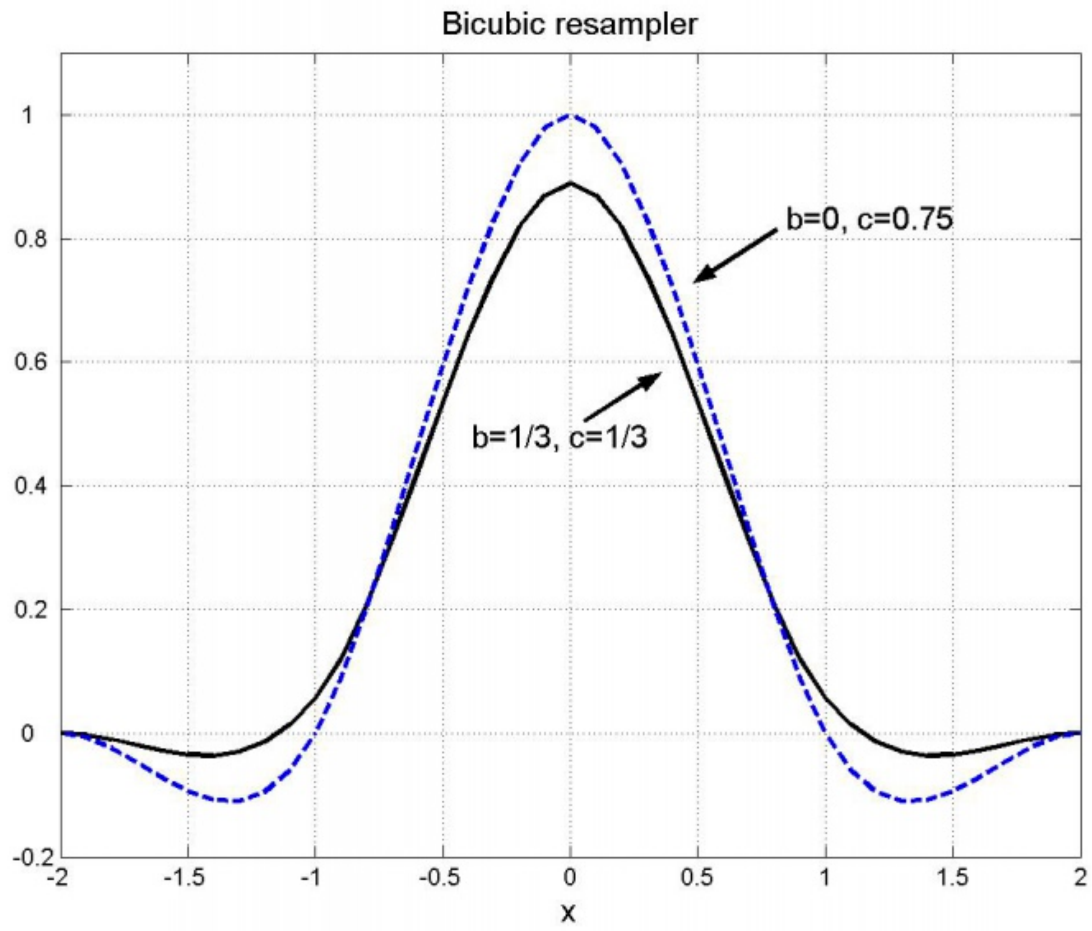


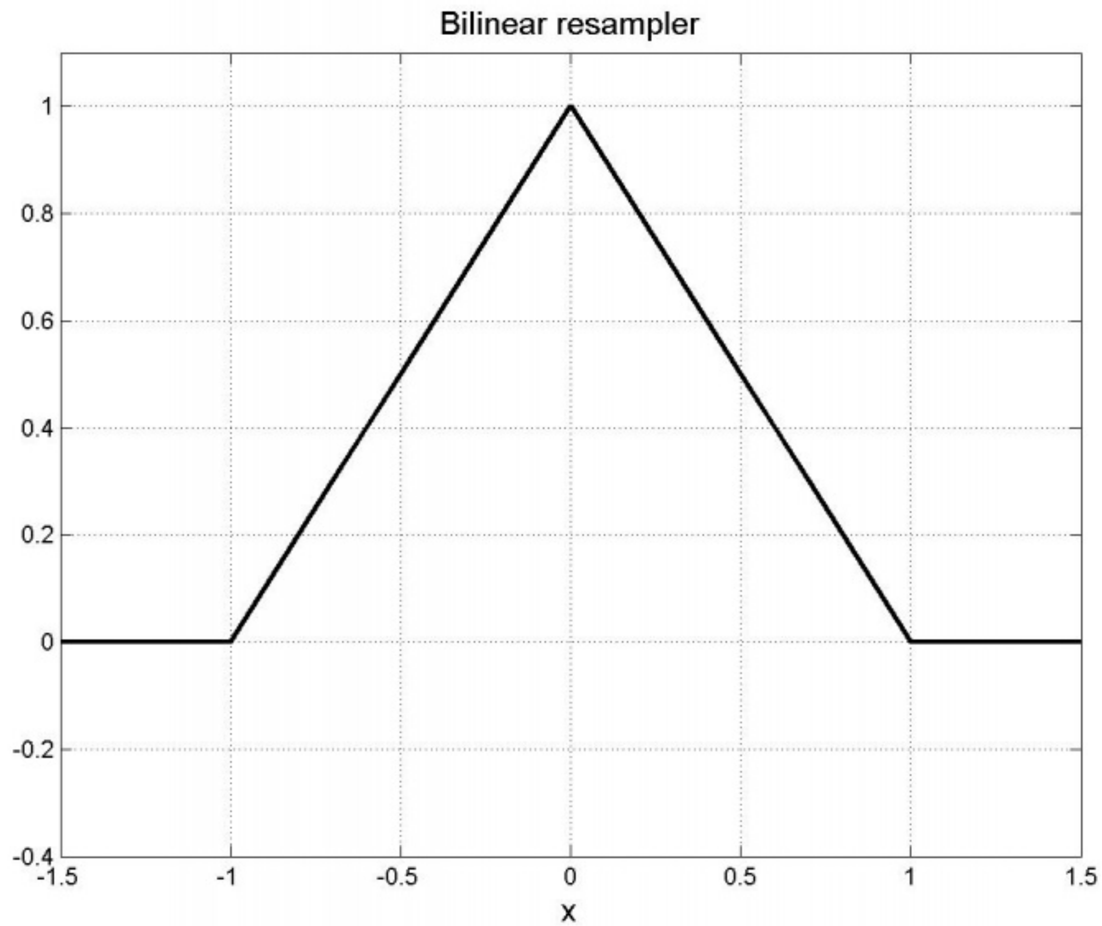
Lanczos3



Lanczos resampler
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-4
-3
-2
-1
0
1
2
3
4
x
taps=4
taps=2
taps=3


Spline resampler
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-4
-3
-2
-1
0
1
2
3
4
x
Spline16
Spline64
Spline36


Bicubic resampler
b=0, c=0.75
b=1/3, c=1/3
x
1
0.8
0.6
0.4
0.2
0
-0.2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2



Bilinear resampler
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-1.5
-1
-0.5
0
0.5
1
1.5
x