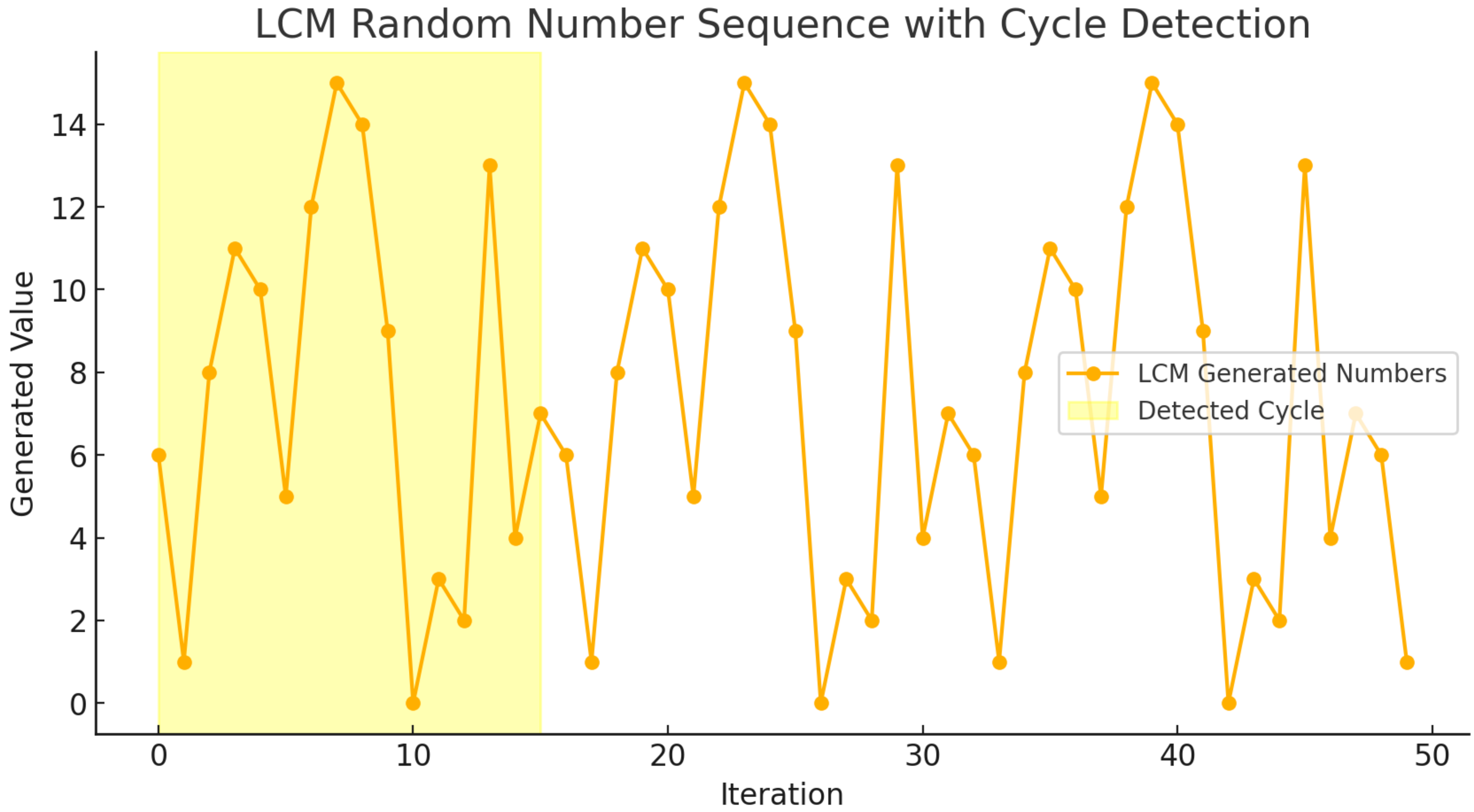
Review

1. What is a good random number generator?
2. Explain the Linear Congruence Method.
3. How does one generate random numbers following a target probability $p(y)$ from a uniform distribution $p(x)$?
4. Why is encoding through LCM susceptible to security breaches?


LCM Random Number Sequence with Cycle Detection
Generated Value
Iteration
0
2
4
6
8
10
12
14
0
10
20
30
40
50
LCM Generated Numbers
Detected Cycle

Using OS entropy

```
import secrets

def generate_random_numbers(count=5, min_val=1, max_val=100):
   return [secrets.randbelow(max_val - min_val + 1) + min_val for _ in range(count)]

random_numbers = generate_random_numbers(count=10, min_val=1, max_val=100)
print("Generated Secure Random Numbers:", random_numbers)
```

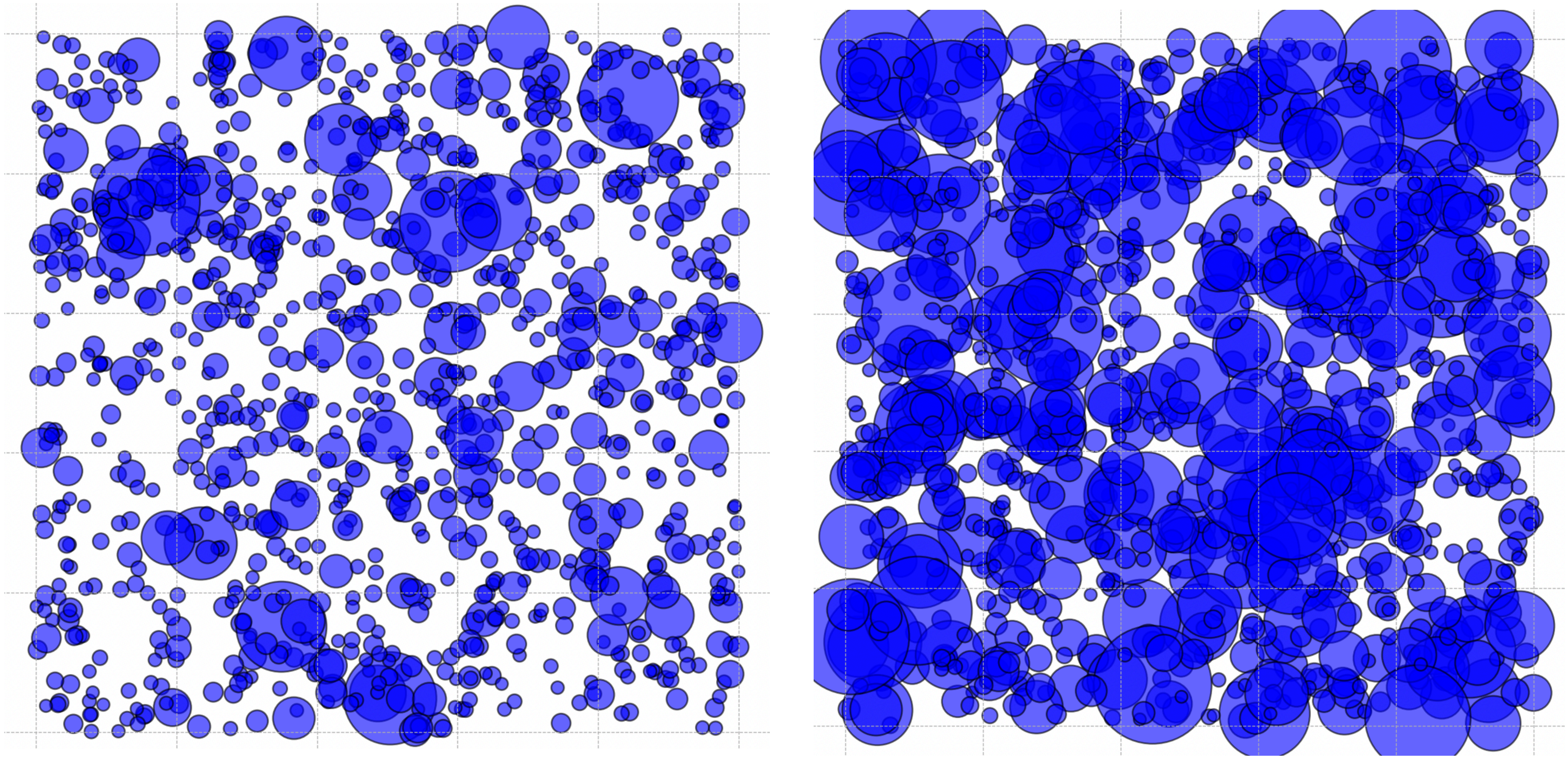
Salpeter Initial Mass Function (IMF)

$$\frac{dN}{dM} \propto M^{-\alpha}$$

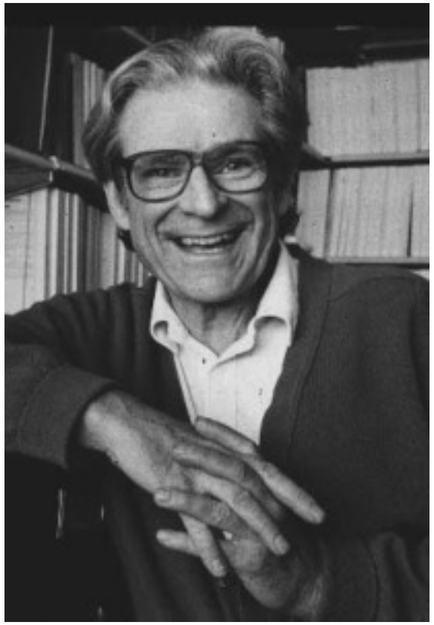
- $\frac{dN}{dM}$ is the number of stars formed per unit mass interval,
- M is the stellar mass in solar masses ($M_\odot$),
- α is the power-law exponent,
- The original **Salpeter (1955) value** for the exponent is $\alpha = 2.35$ for stars with masses greater than about $0.5 M_\odot$.

For every massive star, there are many more low-mass stars.

$\alpha = 2.35$ $\alpha = 1.5$



Derivation of IMF



Edwin E. Salpeter

Q1. How do we measure stellar masses?

$$\Phi(L) = \frac{dN}{dL} \longrightarrow \frac{dN}{dM}$$

$L \propto M^s \quad s \approx 3.5$ for main-sequence stars

Q2. Where do we measure the luminosity function?

Q3. What is the valid range of the IMF?

proton-proton repulsion: ~17 billion K

Sun's core temperature: ~15 million K

Time evolution problem for field stars

Lifetime of main sequence stars:

1 Msun	1×10^{10} yrs
3 Msun	6×10^{8} yrs
10 Msun	3×10^{7} yrs
30 Msun	2×10^{6} yrs
100 Msun	1×10^{5} yrs

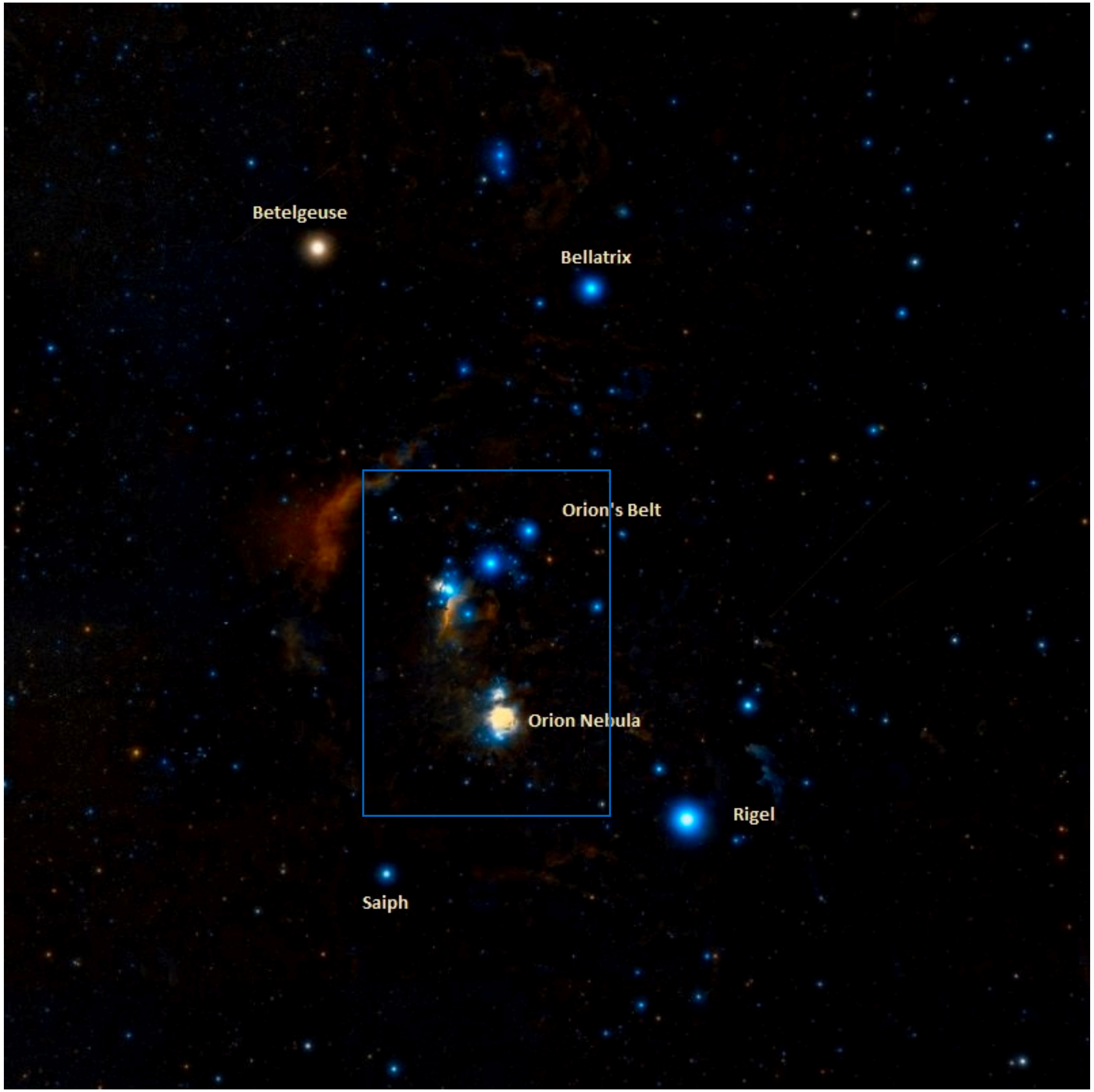
age of the universe $t \sim 10^{10}$ yrs

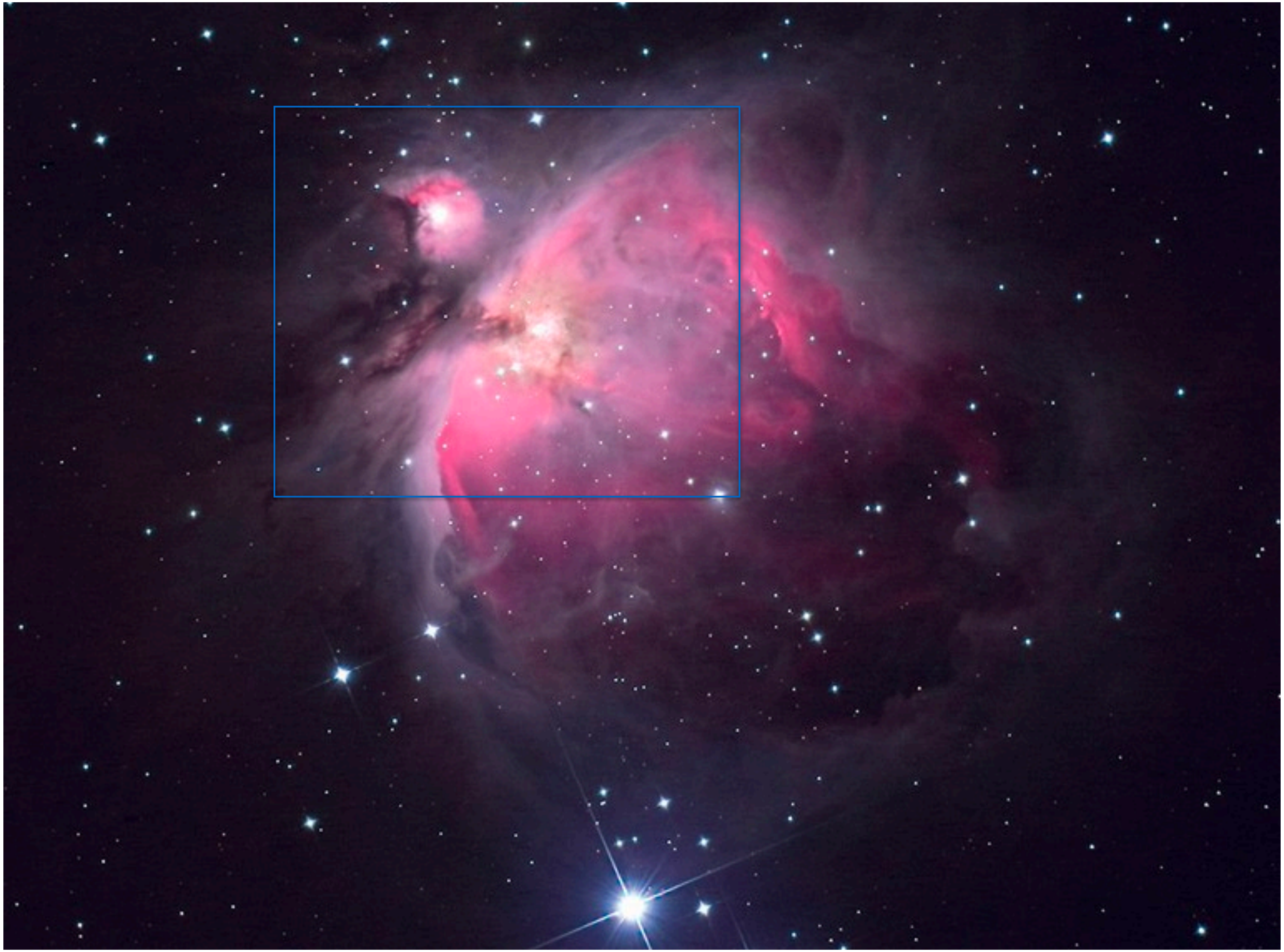
Nuclear fusion timescale:

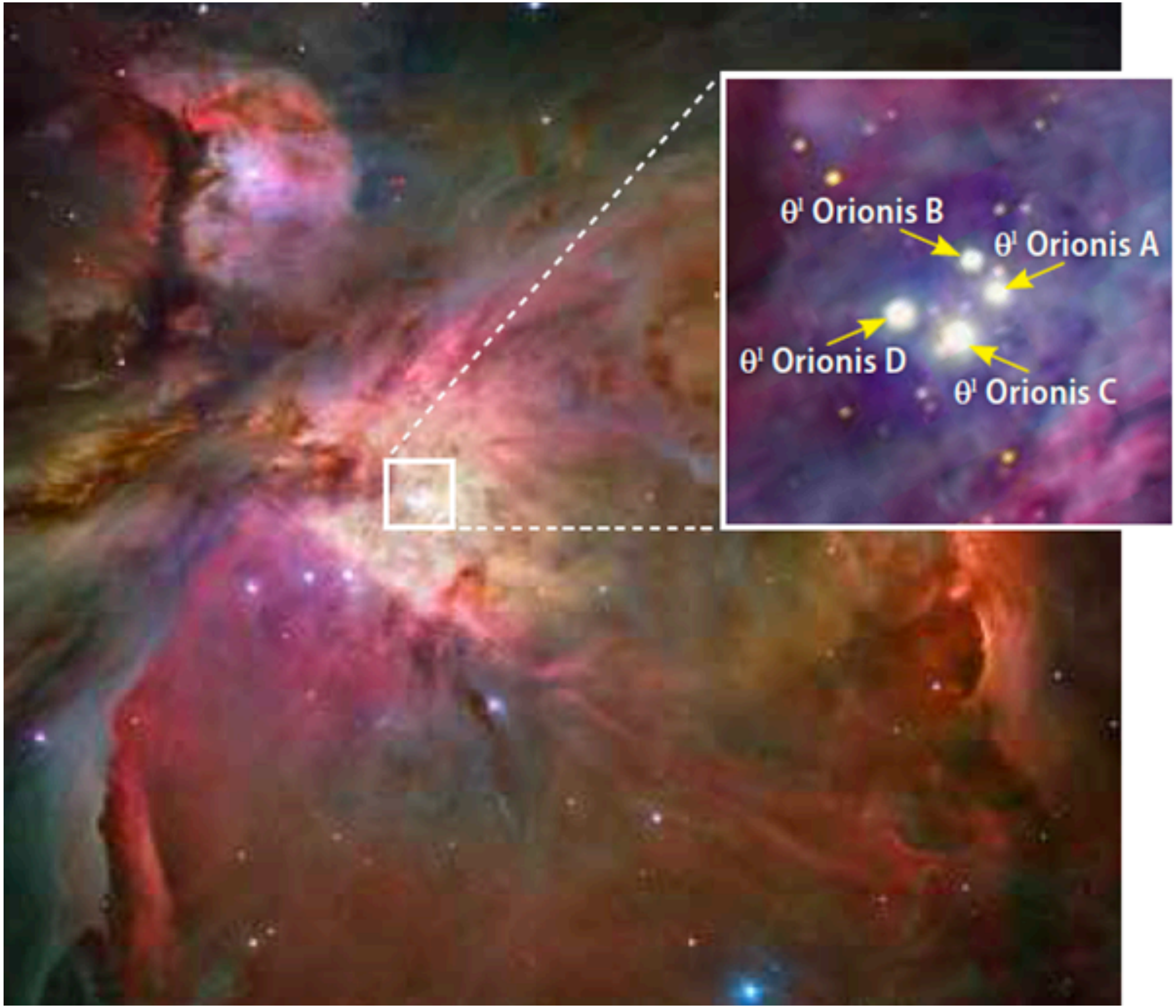
$$t_{MS} \sim 10^{10} \text{ yrs } [M/M_{sun}]\,[L_{sun}/L]$$

$$= 10^{10} \text{ yrs } (M/M_{sun})^{-2.5}$$

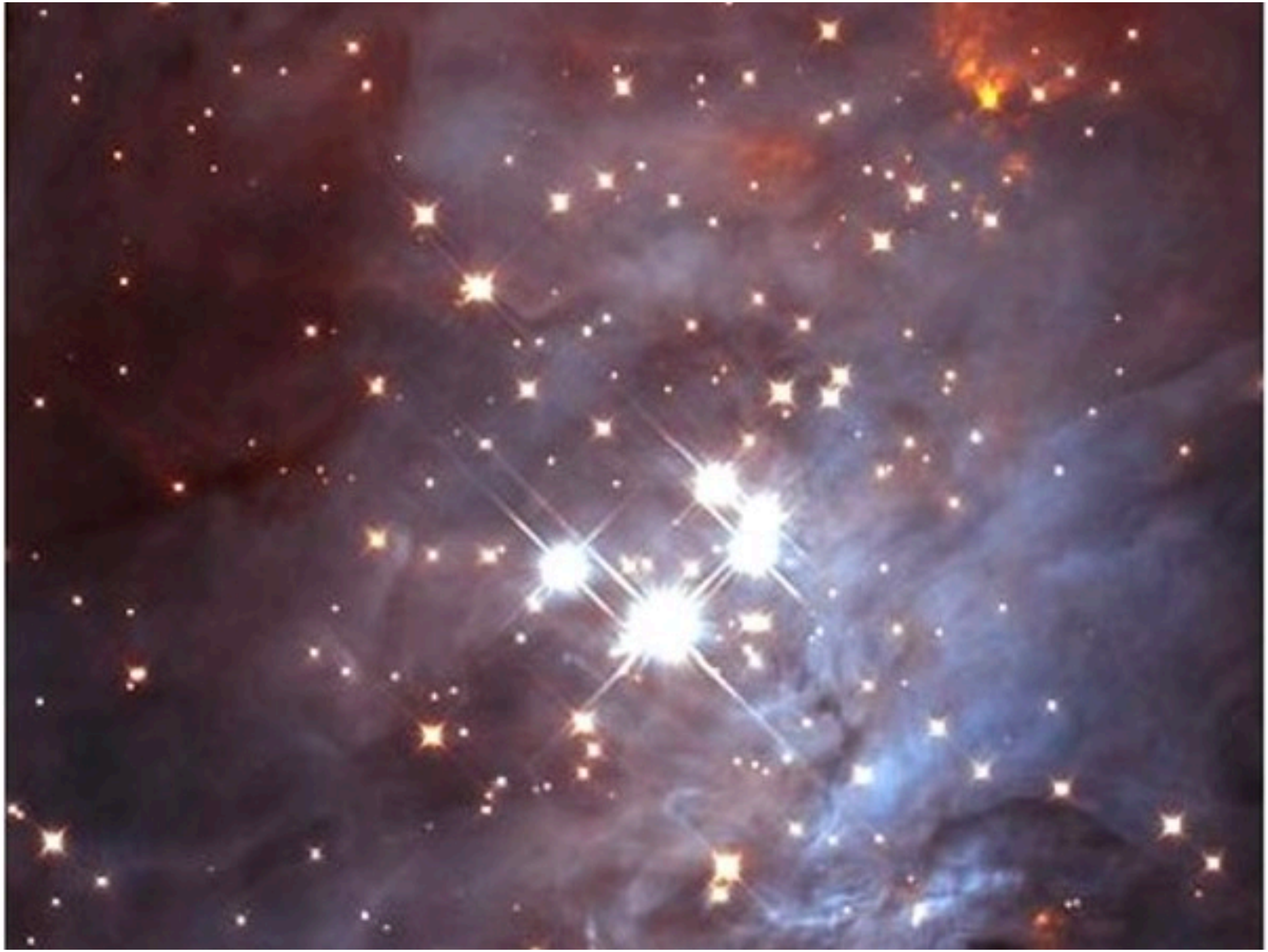
with $L \sim M^{3.5}$ for main sequence stars

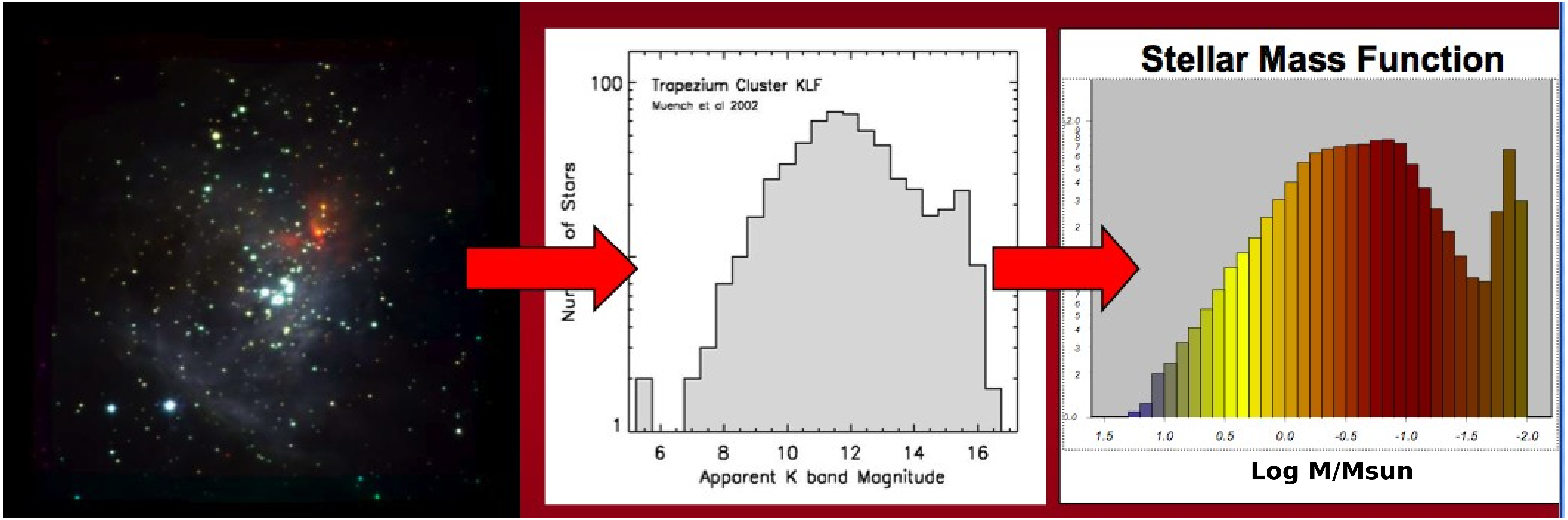

Betelgeuse
Bellatrix
Orion's Belt
Orion Nebula
Rigel
Saiph



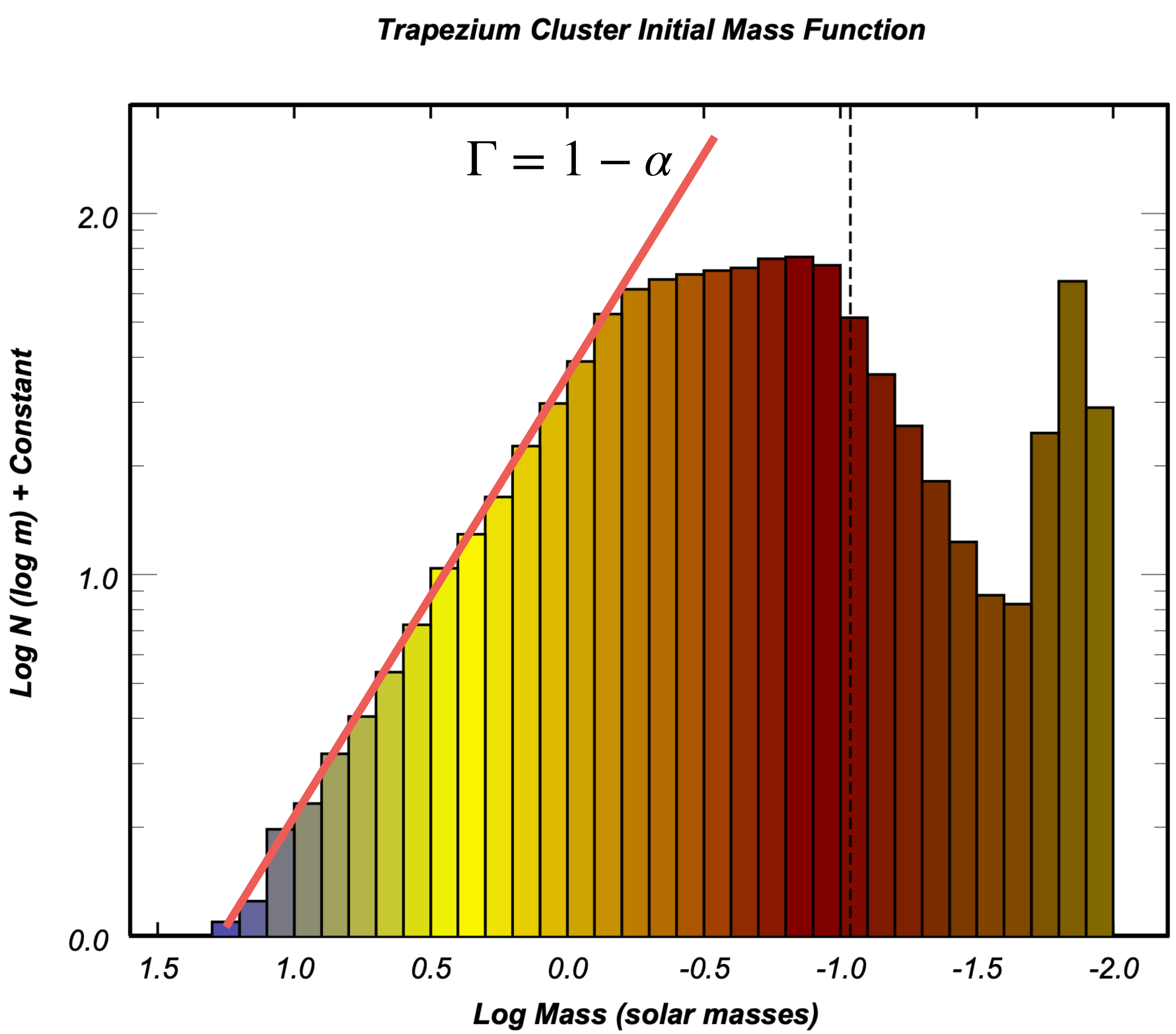


θ^1 Orionis B
θ^1 Orionis A
θ^1 Orionis D
θ^1 Orionis C





Muench et al. 2002, Lada & Lada 2003,
Slide courtesy C. Lada


Trapezium Cluster Initial Mass Function
$\Gamma = 1 - \alpha$
Log N (log m) + Constant
0.0
1.0
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
Log Mass (solar masses)

Power law= Pareto’s rule =80/20 rule

The rule suggests that roughly 80% of effects come from 20% of causes or inputs.

Examples:

- 80% of the land is owned by 20% of people.
- 80% of a company's sales typically come from 20% of its customers.
- 80% of my money is spent on 20% of items.

Identifying and focusing on the most significant factors can lead to more efficient allocation of resources and improved decision-making.

Why does power law occur?

$$P(x) \propto x^{-\alpha}$$

- Positive feedback: Rich-Get-Richer or Winner-takes-it-all
- Self-similarity or scale-invariance: no preferred scale
- Cascading effect: a small event triggers a chain reaction

과제 3

1.과제 2에서 만든 난수 발생기 사용

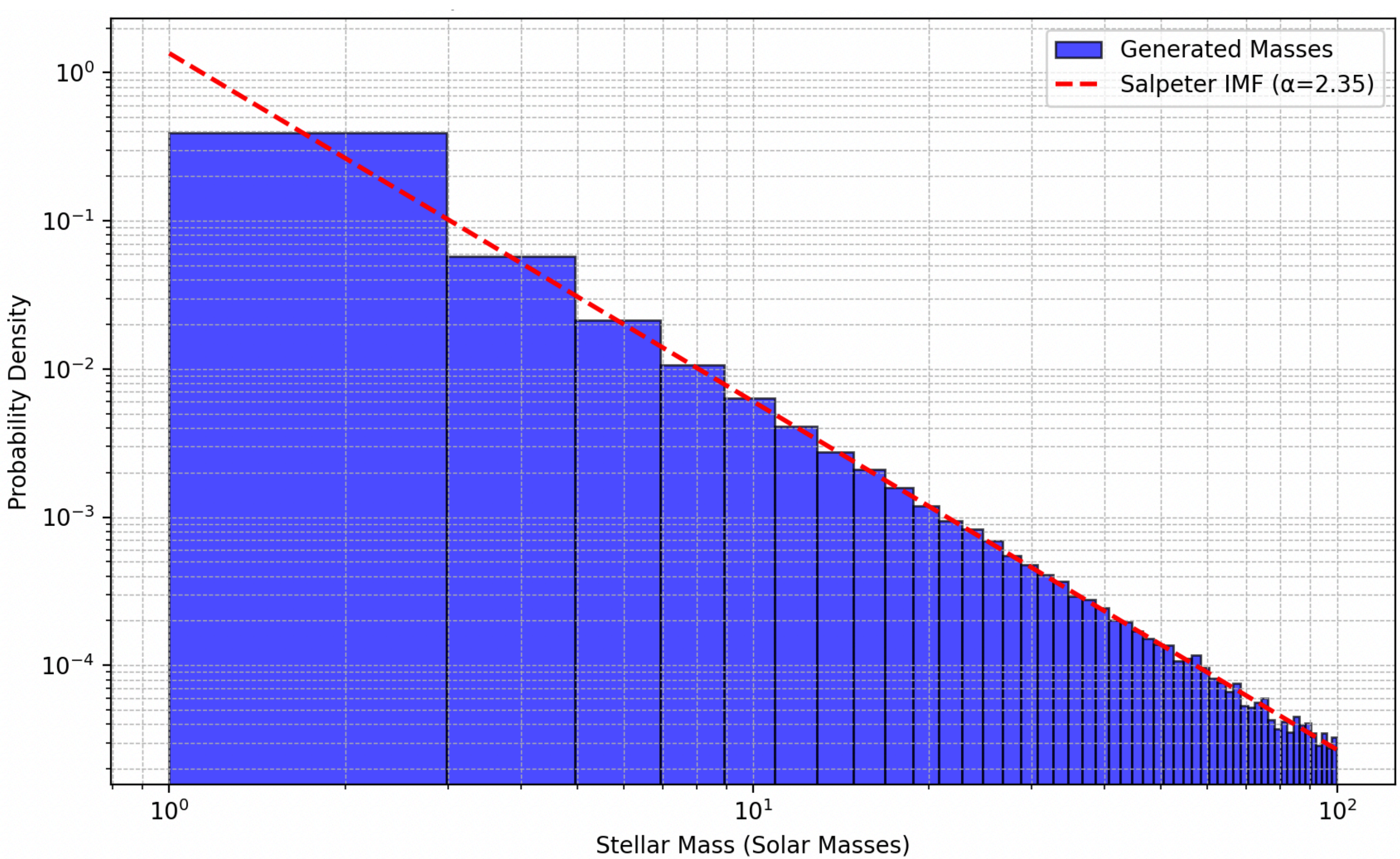
2. Salpeter IMF 분포를 가지는 별 질량 1,000,000 개 생성

3. 별의 질량은 1 solar mass 부터 100 solar mass 까지

4.역변환 방법을 직접 구현하고 수식과 코드를 설명

5. 별의 질량분포를 히스토그램으로 표현 (y-축 스케일은 로그 스케일 사용)

6. 자연 또는 일상생활에서 power-law가 관측되는 예를 찾고 원인을 설명하여라.


Generated Masses
Salpeter IMF (α=2.35)
Probability Density
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{0}
10^{1}
10^{2}
Stellar Mass (Solar Masses)

Rejection Method

Gaussian 분포를 가지는 난수발생

$$p(y)dy = \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy$$

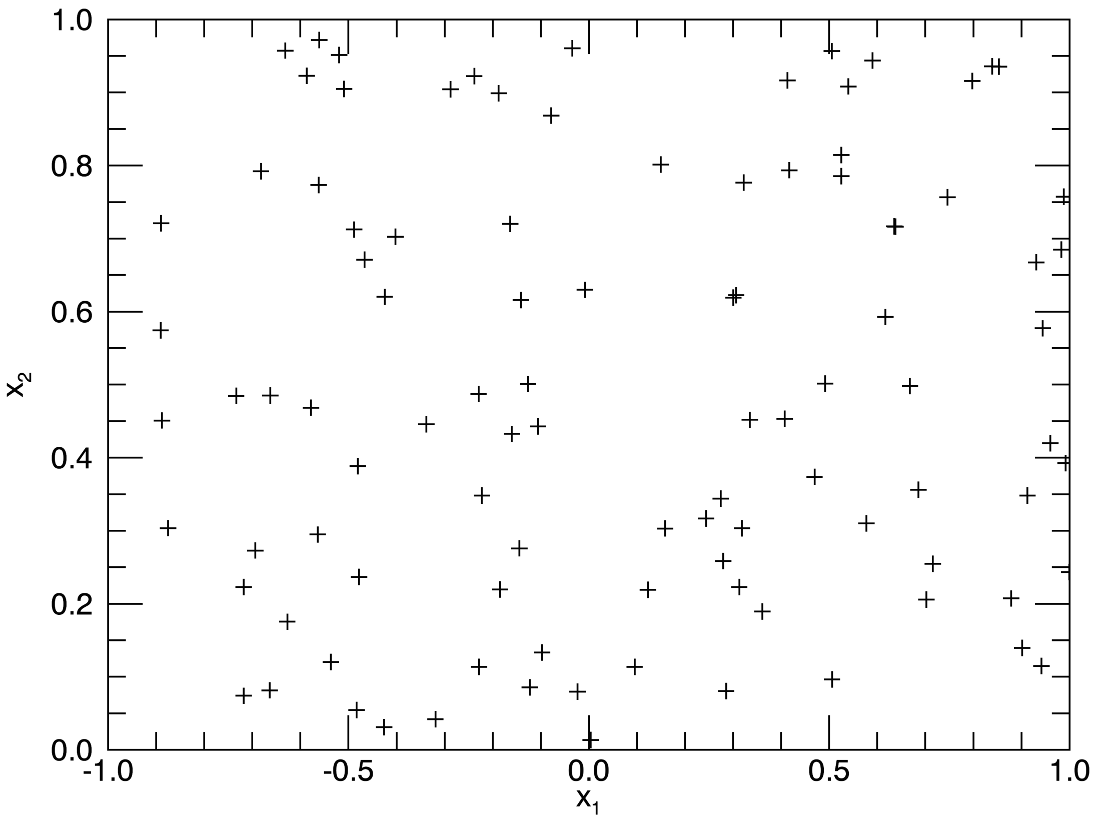
Gaussian을 적분하면?

$$\begin{aligned}\operatorname{erf}(x) &= \frac{1}{\sqrt{\pi}} \int_{-x}^{x} e^{-t^2}\, dt \\ &= \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-t^2}\, dt.\end{aligned}$$

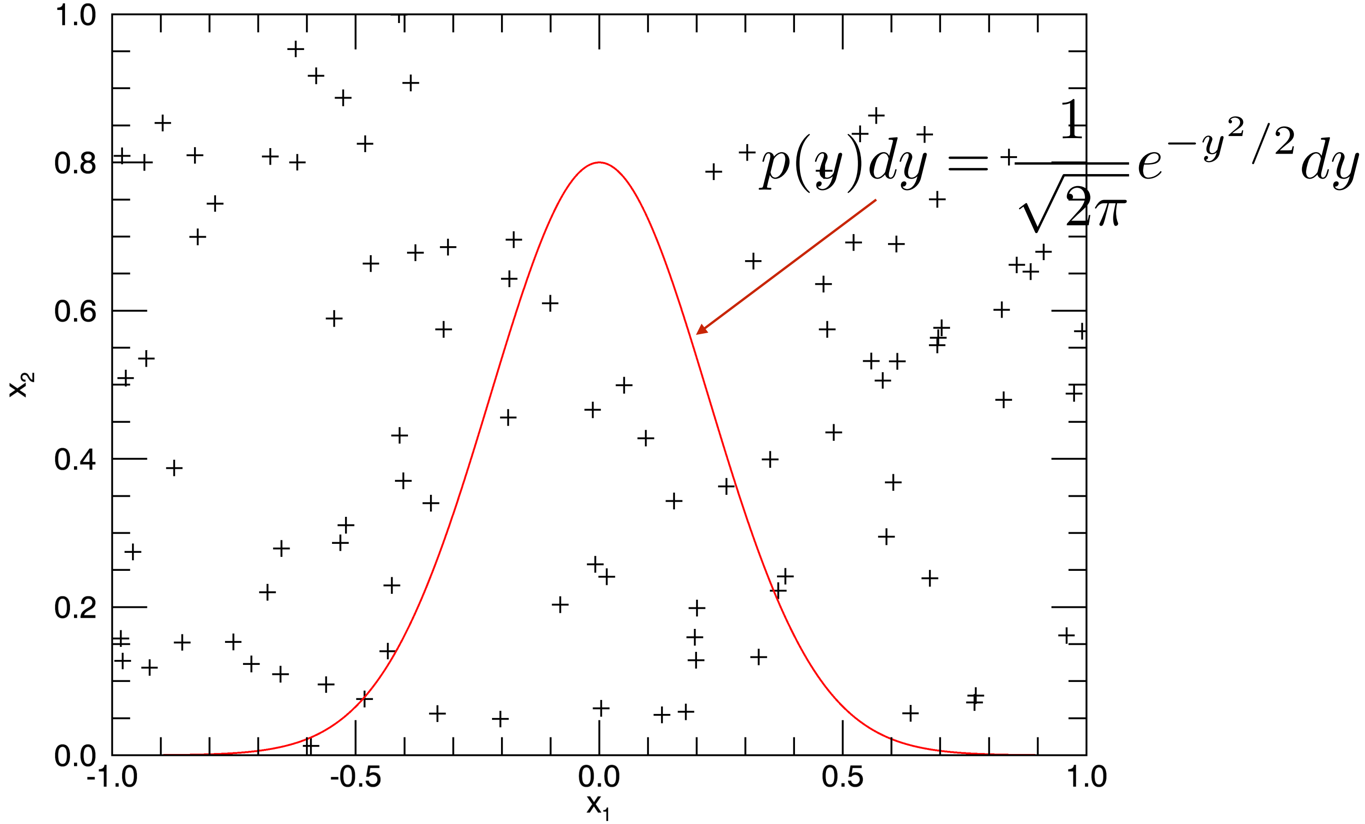
Error function!

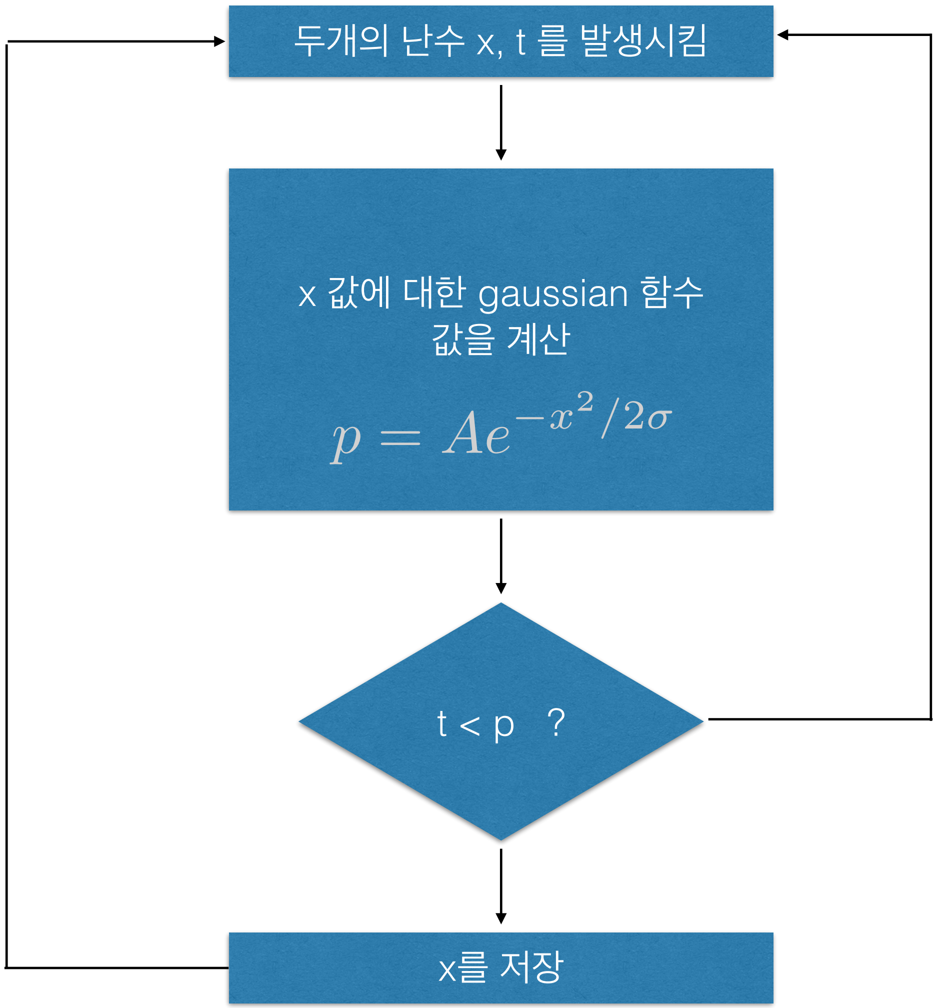
역함수를 구하기가 너무 어려움!

Rejection Method



Rejection Method



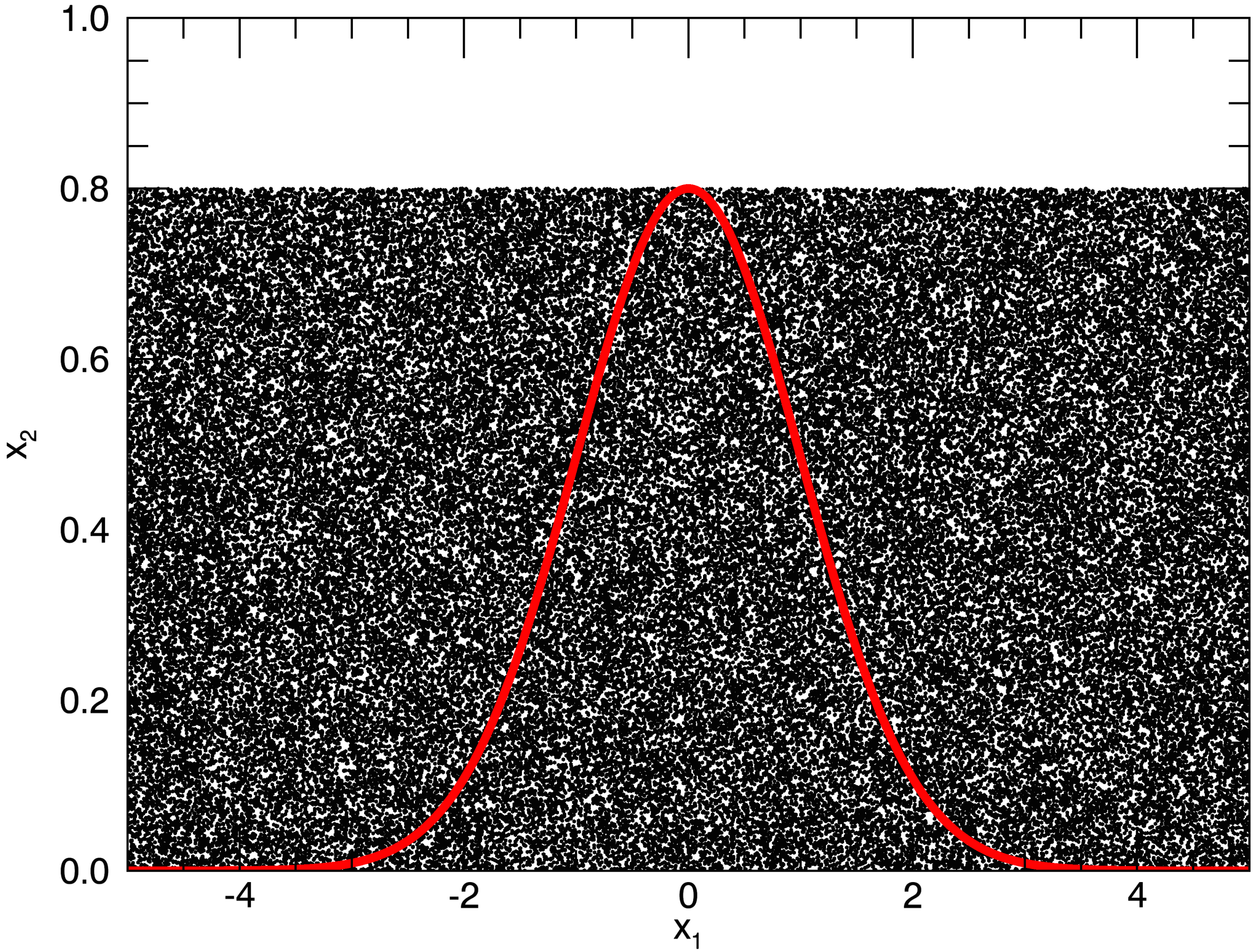

두개의 난수 x, t 를 발생시킴
x 값에 대한 gaussian 함수 값을 계산
$p = Ae^{-x^2/2\sigma}$
t < p ?
x를 저장

과제 4

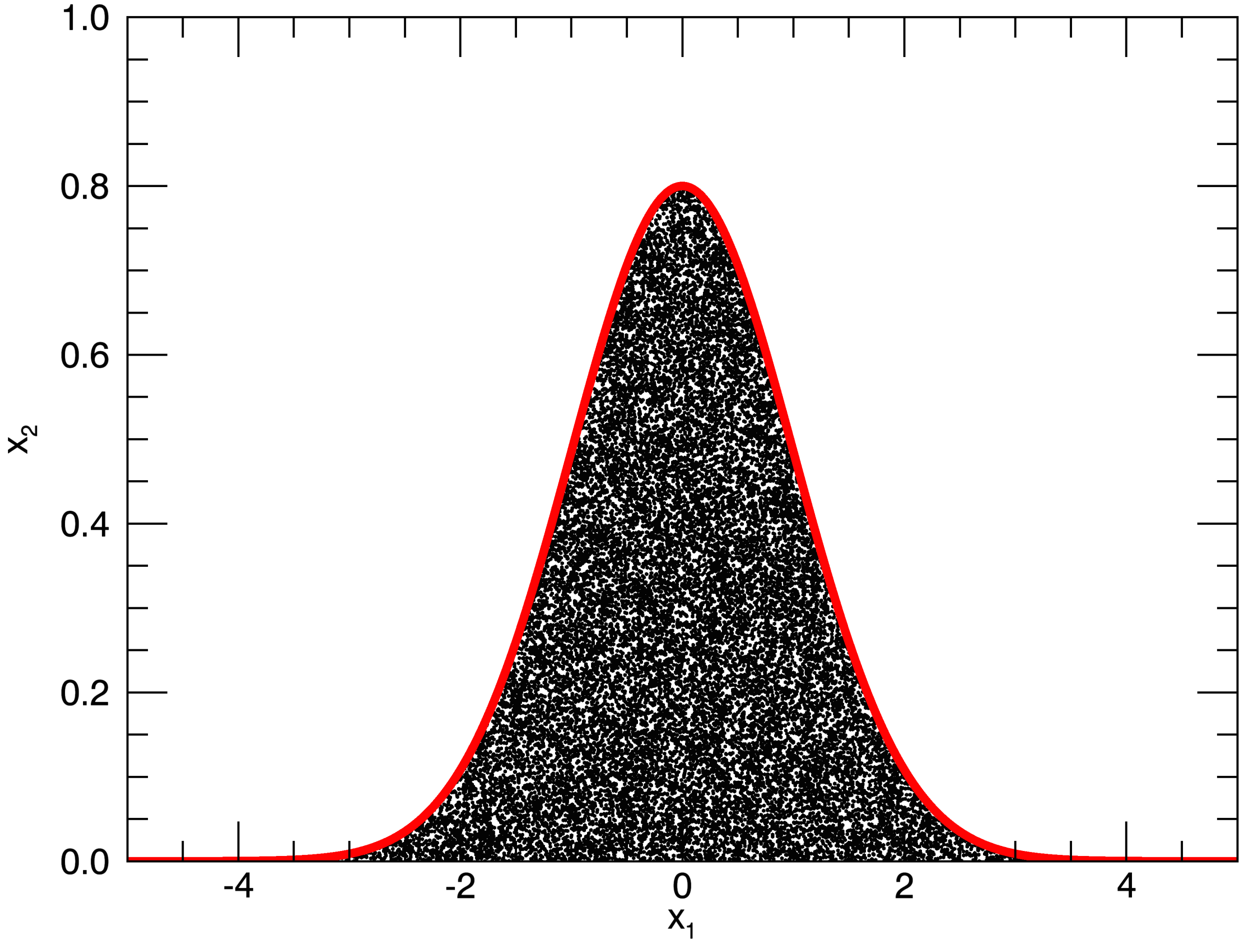
1.과제 2의 난수 발생기 사용

2. mean=0, min=-5, max=5, std=1 인 정규분포를 가지는 난수를 발생

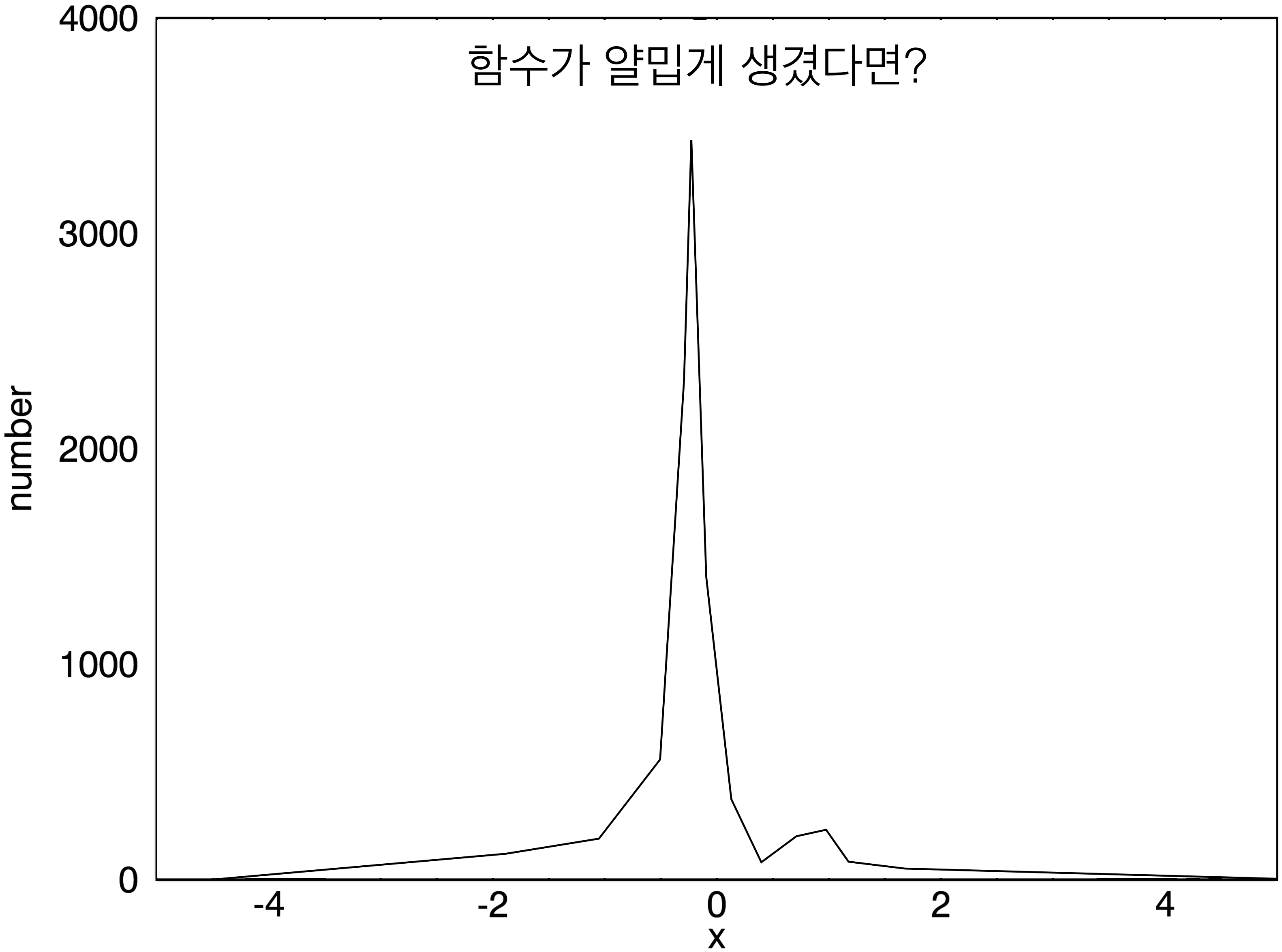
3. 결과를 히스토그램으로 표현

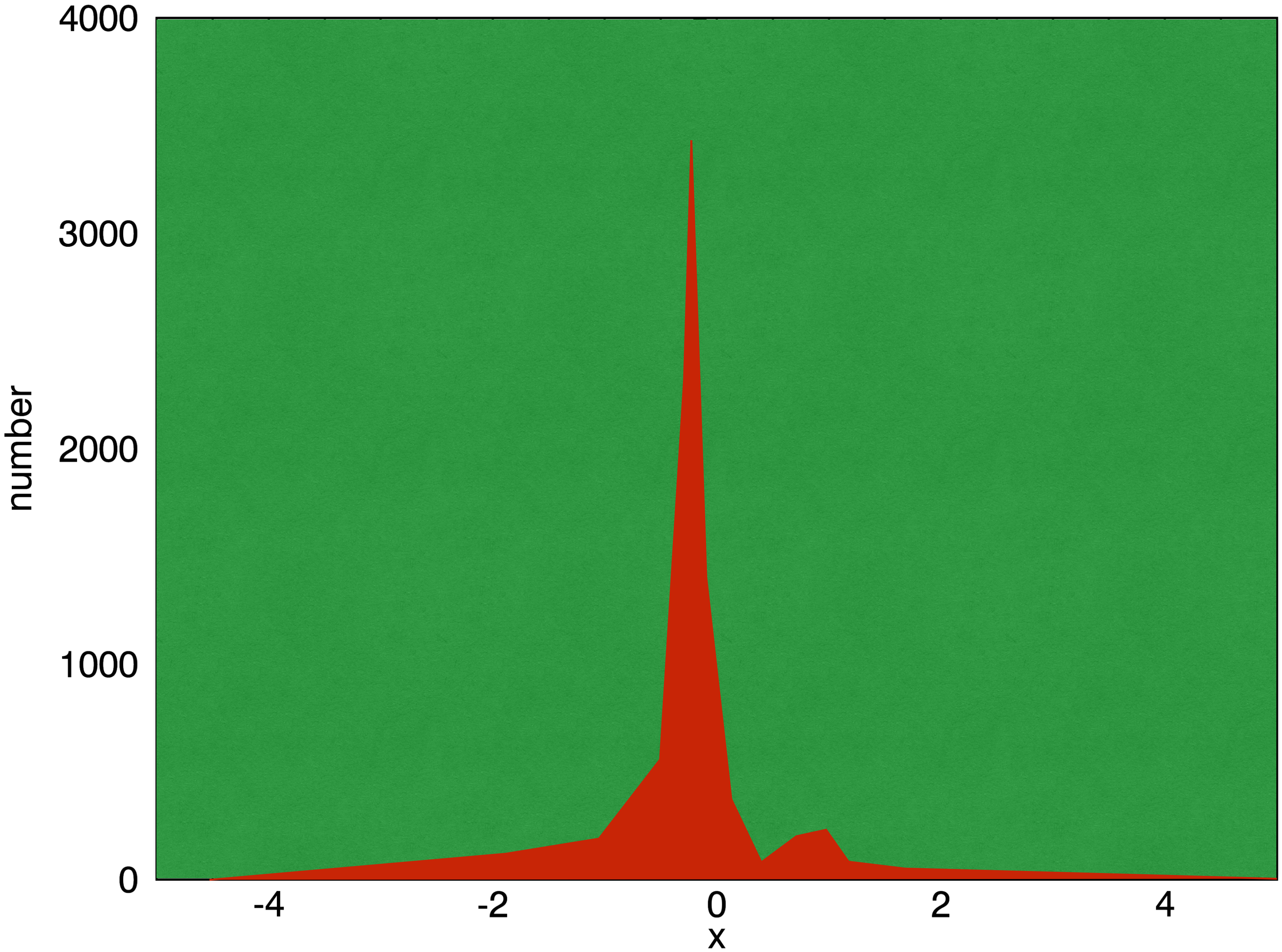


1.0
0.8
0.6
0.4
0.2
0.0
x_2
-4
-2
0
2
4
x_1

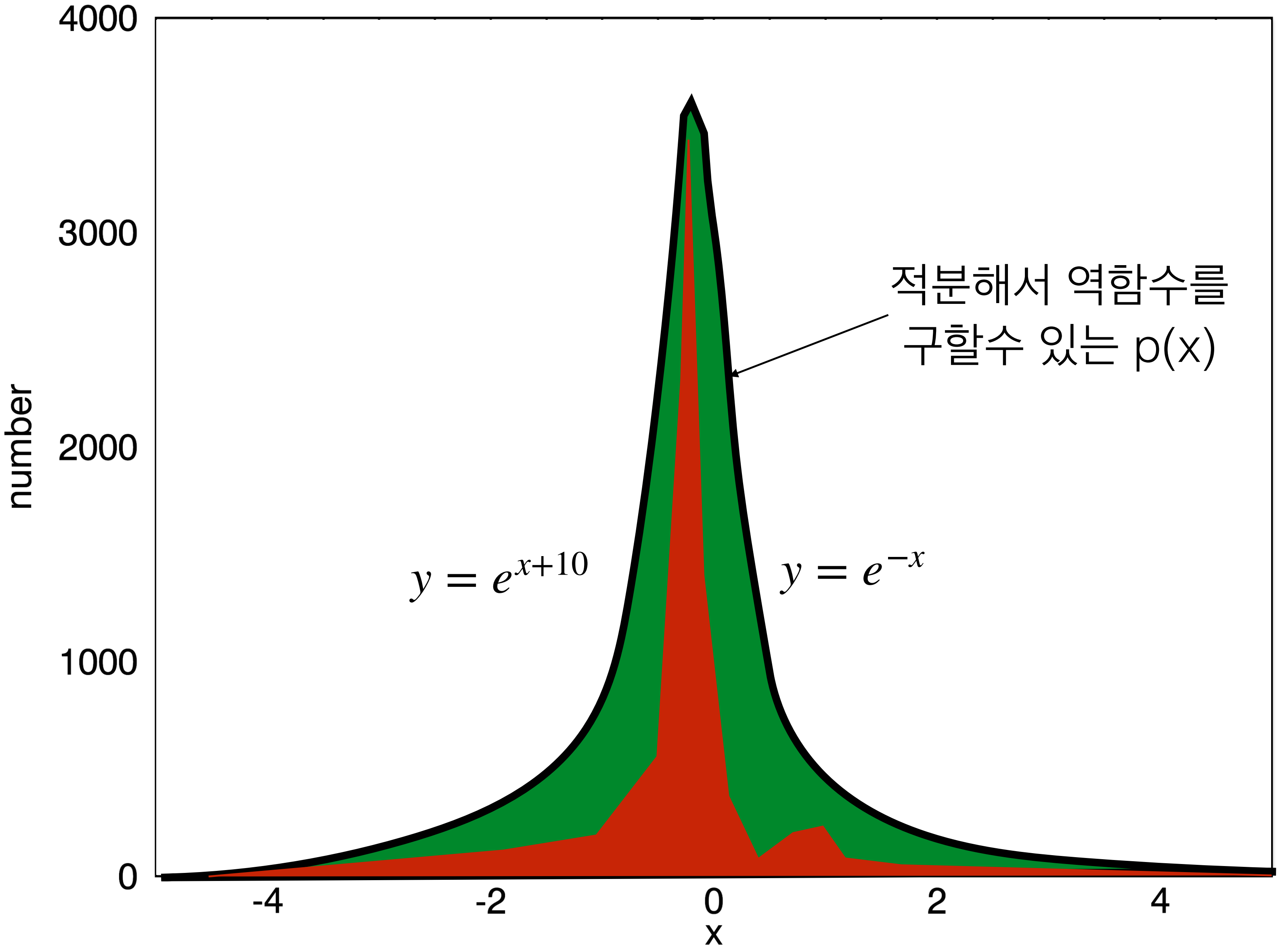


1.0
0.8
0.6
0.4
0.2
0.0
-4
-2
0
2
4
x_1
x_2


함수가 얇밉게 생겼다면?
4000
3000
2000
1000
0
-4
-2
0
2
4
x
number

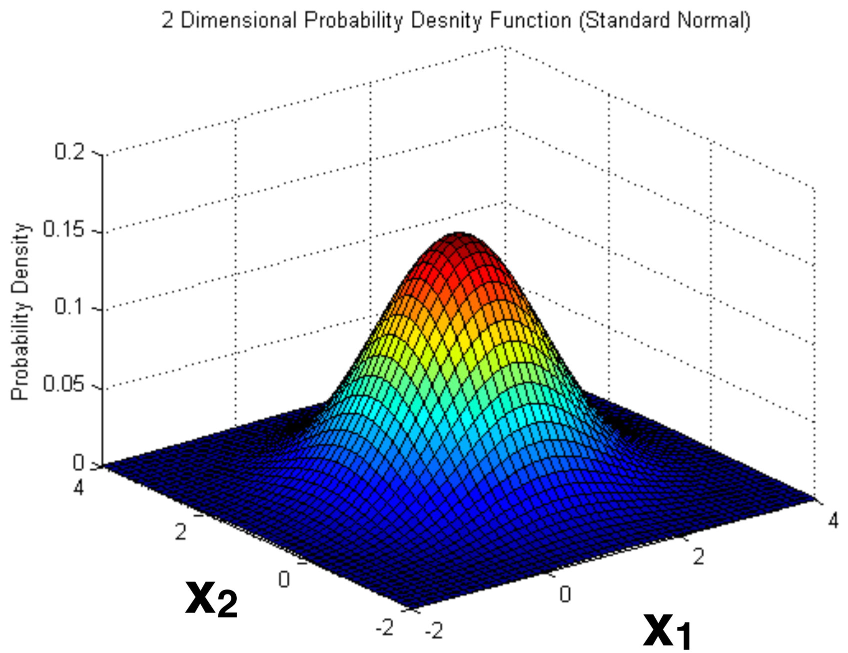


4000
3000
2000
1000
0
number
-4
-2
0
2
4
x

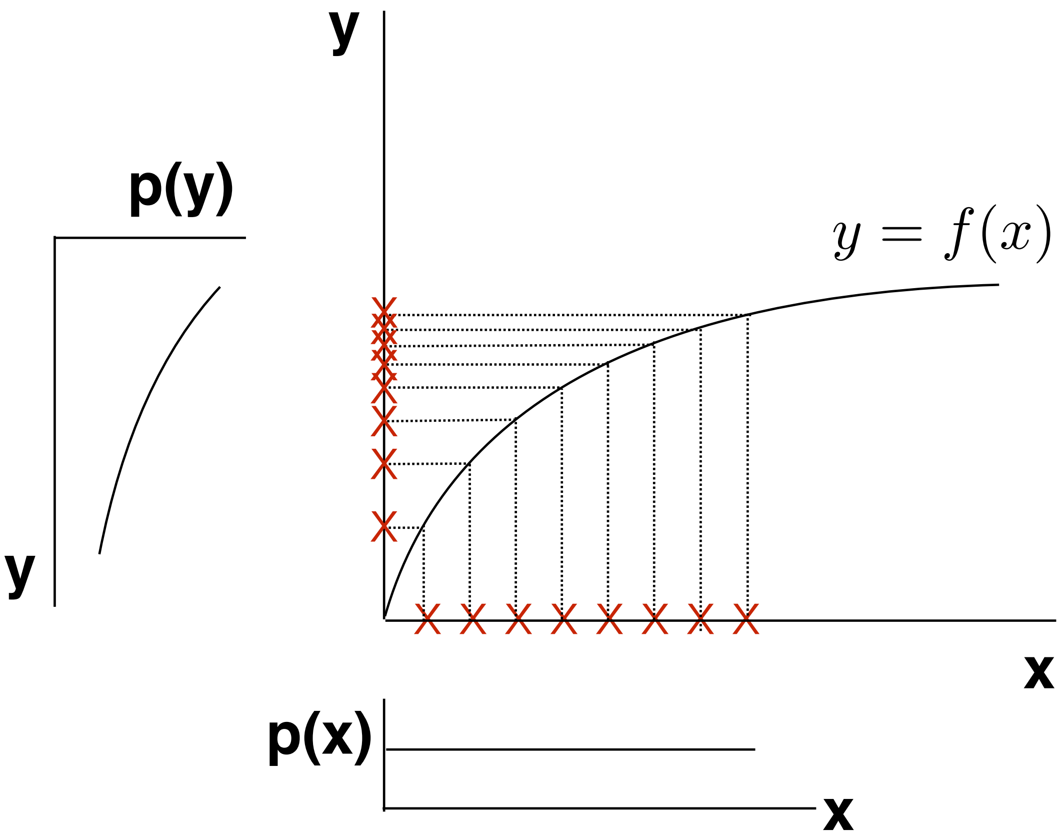


4000
3000
2000
1000
0
number
-4
-2
0
2
4
x
적분해서 역함수를
구할수 있는 p(x)
$y = e^{x+10}$
$y = e^{-x}$

2차원 확률분포


2 Dimensional Probability Desnity Function (Standard Normal)
Probability Density
0.2
0.15
0.1
0.05
0
4
2
0
-2
-2
0
2
4
x2
x1

1차원 확률변환



$$p(x)|dx| = p(y)|dy|$$

$$p(y) = \frac{dx}{dy}$$

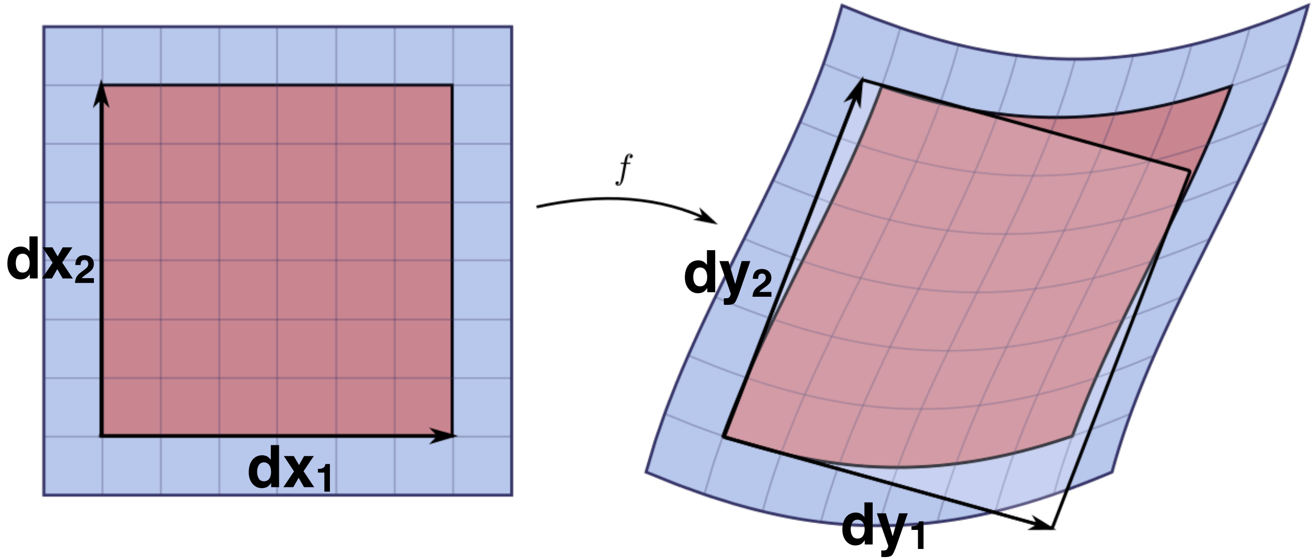
$$p(y)dy = dx$$

$$P(y) = x$$

$$y = P^{-1}(x)$$

$$f(x) = P^{-1}(x)$$

2차원 확률변환



probability conservation: $p(x_1, x_2)dx_1dx_2 = p(y_1, y_2)dy_1dy_2$

$$\frac{dx_1dx_2}{dy_1dy_2} = \left|\frac{\partial(x_1, x_2)}{\partial(y_1, y_2)}\right|$$

area ratio

$$\left|\frac{\partial(x_1, x_2)}{\partial(y_1, y_2)}\right| = \frac{\partial x_1}{\partial y_1}\frac{\partial x_2}{\partial y_2} - \frac{\partial x_1}{\partial y_2}\frac{\partial x_2}{\partial y_1}$$

$$p(y_1, y_2) = \left|\frac{\partial(x_1, x_2)}{\partial(y_1, y_2)}\right|$$

다음의 두 변환을 가정

$$y_1 = \sqrt{-2\ln x_1}\cos 2\pi x_2$$
$$y_2 = \sqrt{-2\ln x_1}\sin 2\pi x_2$$

x_1, x_2 은 0부터 1사이의 균일한 난수

$$\frac{\partial(x_1, x_2)}{\partial(y_1, y_2)} = \begin{vmatrix} \frac{\partial x_1}{\partial y_1} & \frac{\partial x_1}{\partial y_2} \\ \frac{\partial x_2}{\partial y_1} & \frac{\partial x_2}{\partial y_2} \end{vmatrix} = -\left[\frac{1}{\sqrt{2\pi}}e^{-y_1^2/2}\right]\left[\frac{1}{\sqrt{2\pi}}e^{-y_2^2/2}\right] = p(y_1, y_2) = p(y_1)p(y_2)$$

$$= p(y_1) \quad = p(y_2)$$

y_1, y_2 각각이 Gaussian 분포

낭비없는 Gaussian 난수 발생기

두개의 난수 x_1, x_2 를 발생시킴

0부터 1사이 균일한 분포

↓

$$y_1 = \sqrt{-2\ln x_1}\cos 2\pi x_2$$
$$y_2 = \sqrt{-2\ln x_1}\sin 2\pi x_2$$

↓

y_1, y_2를 사용