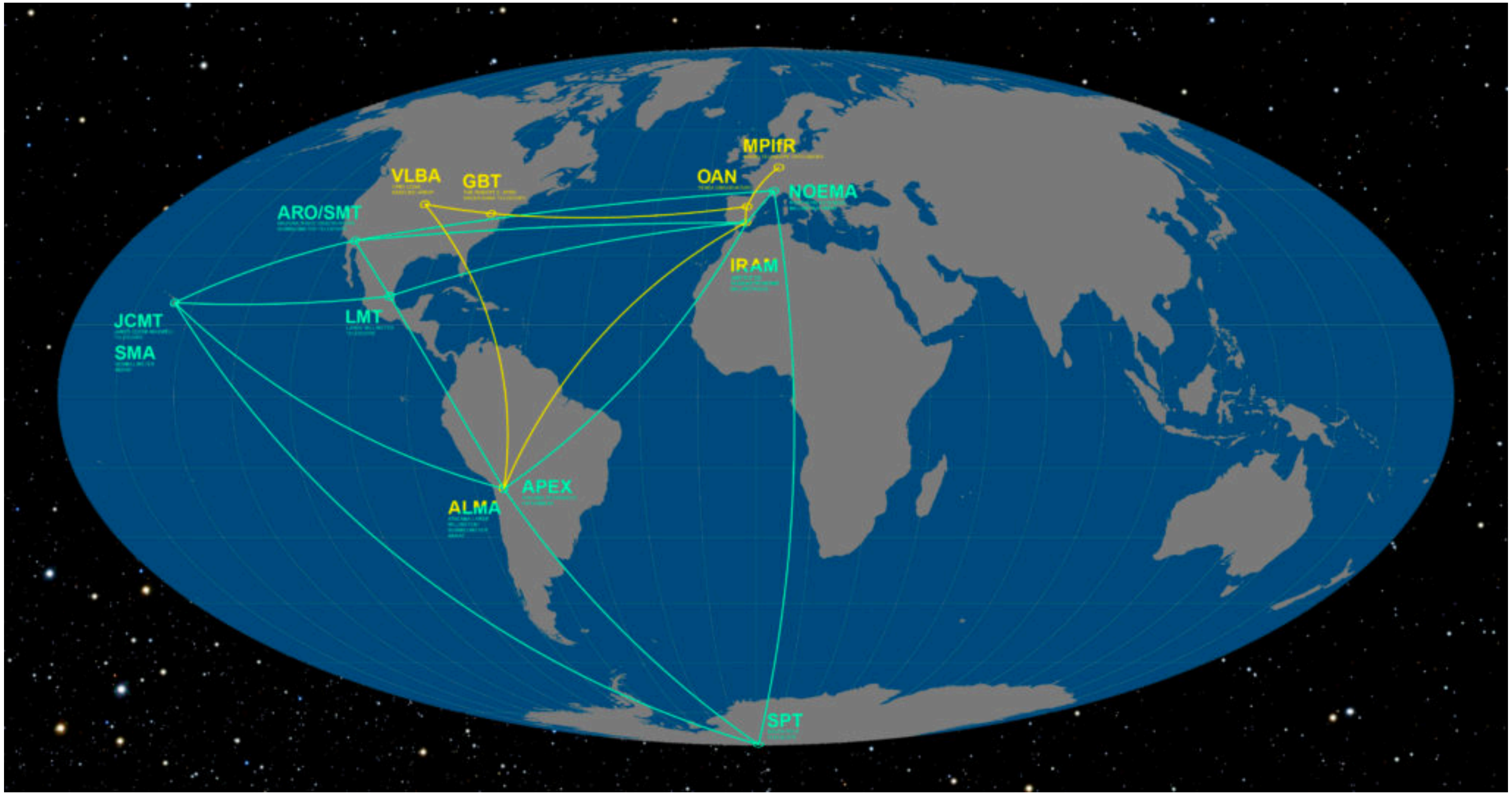
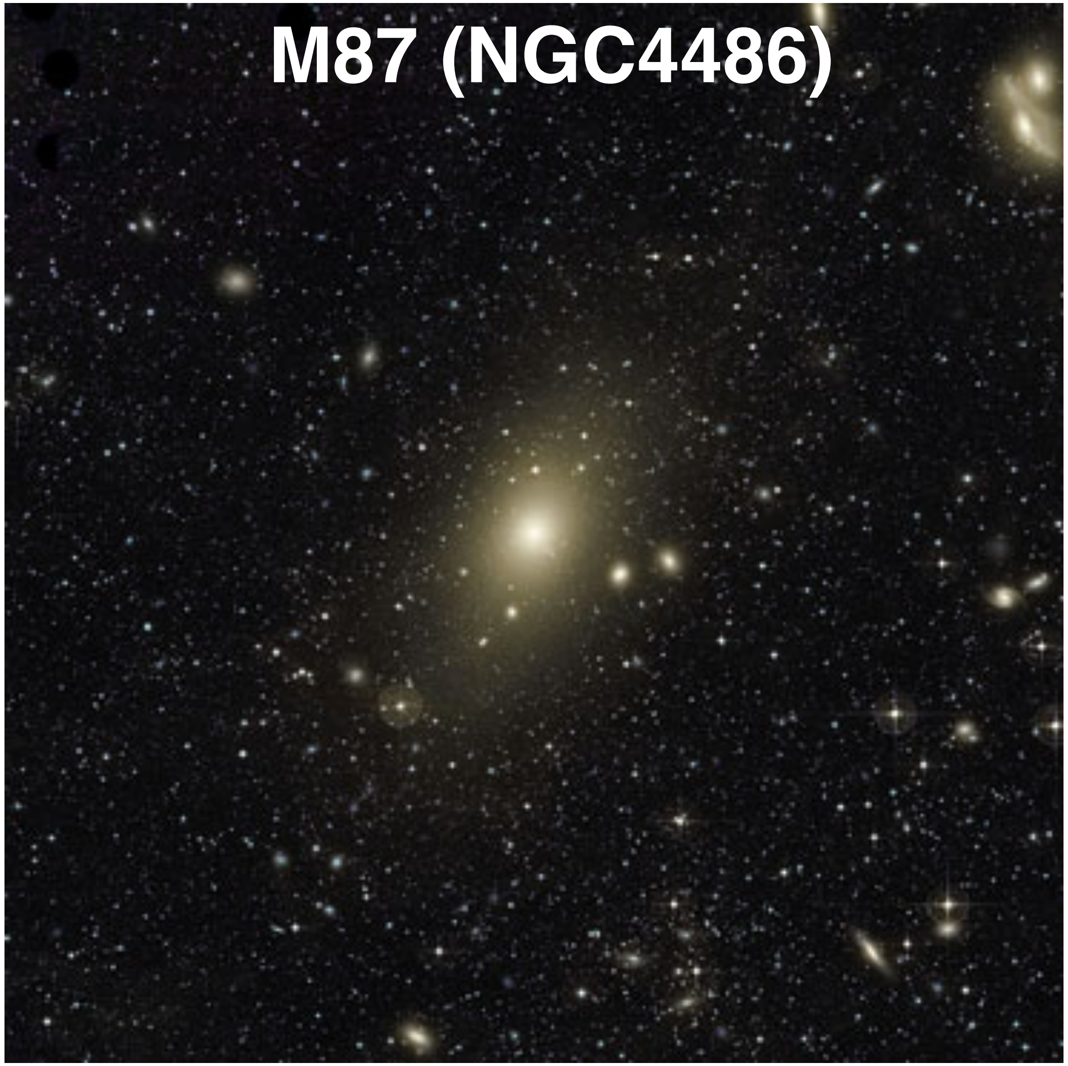
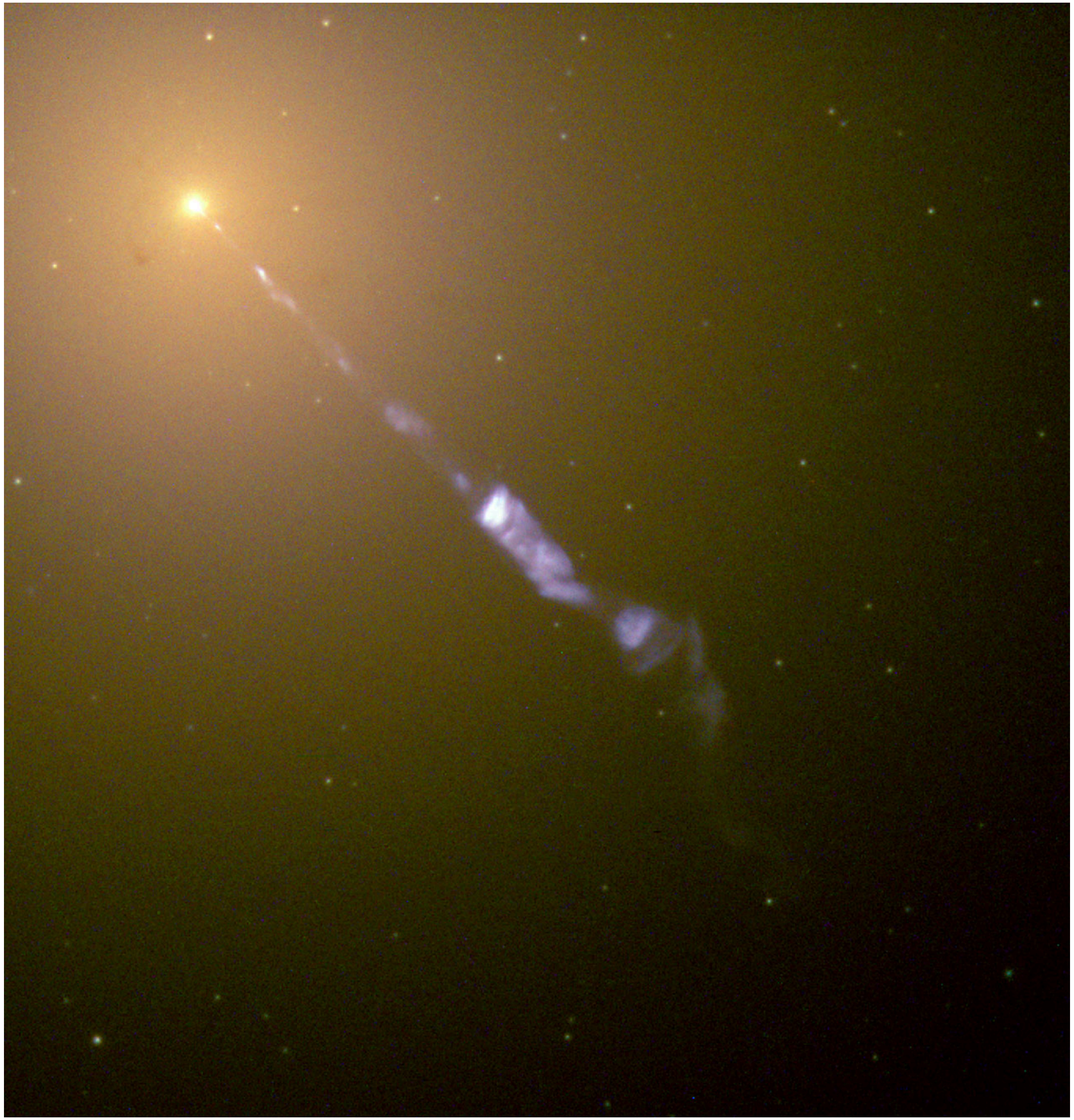
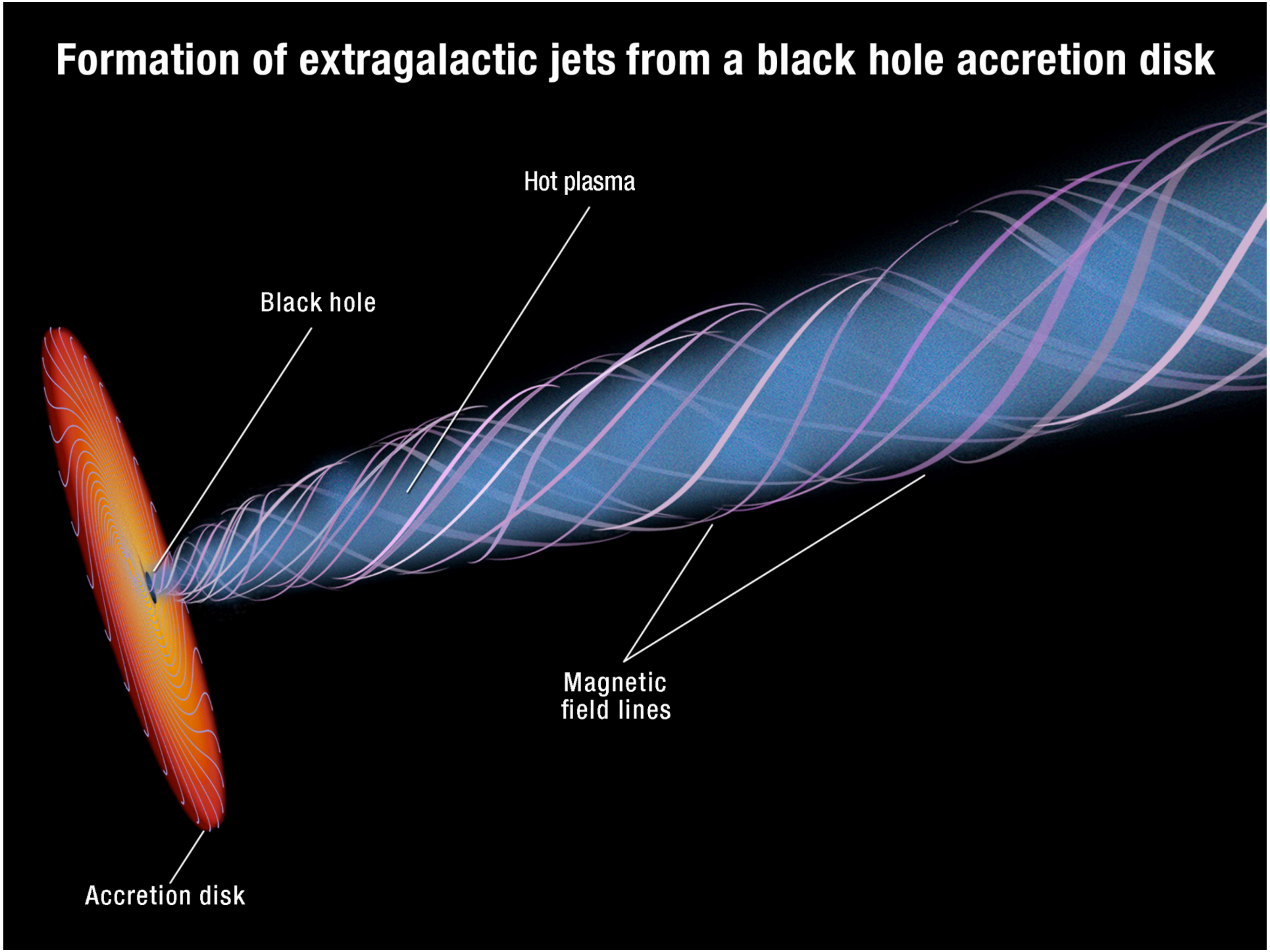
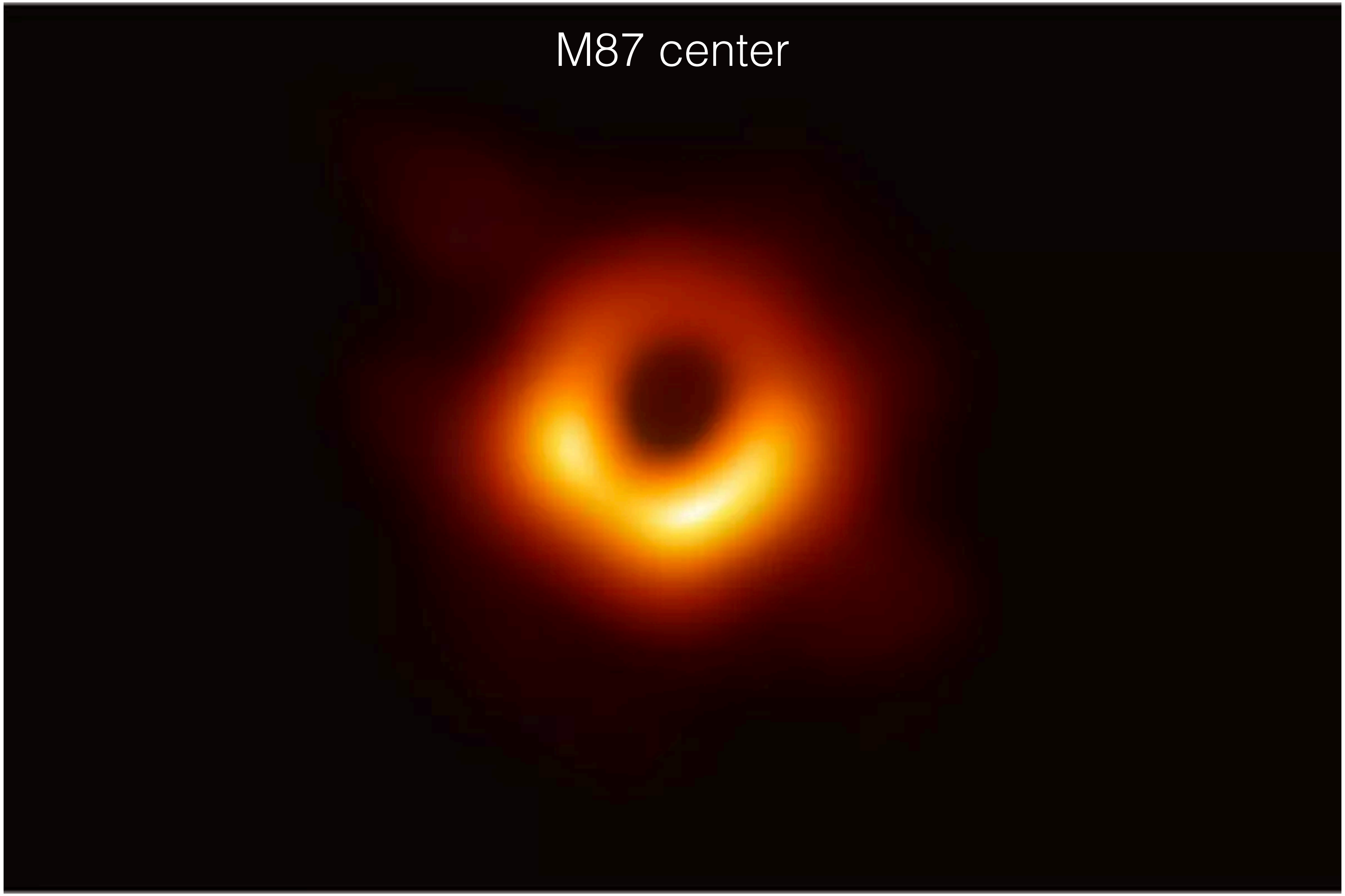
EVENT HORIZON TELESCOPE



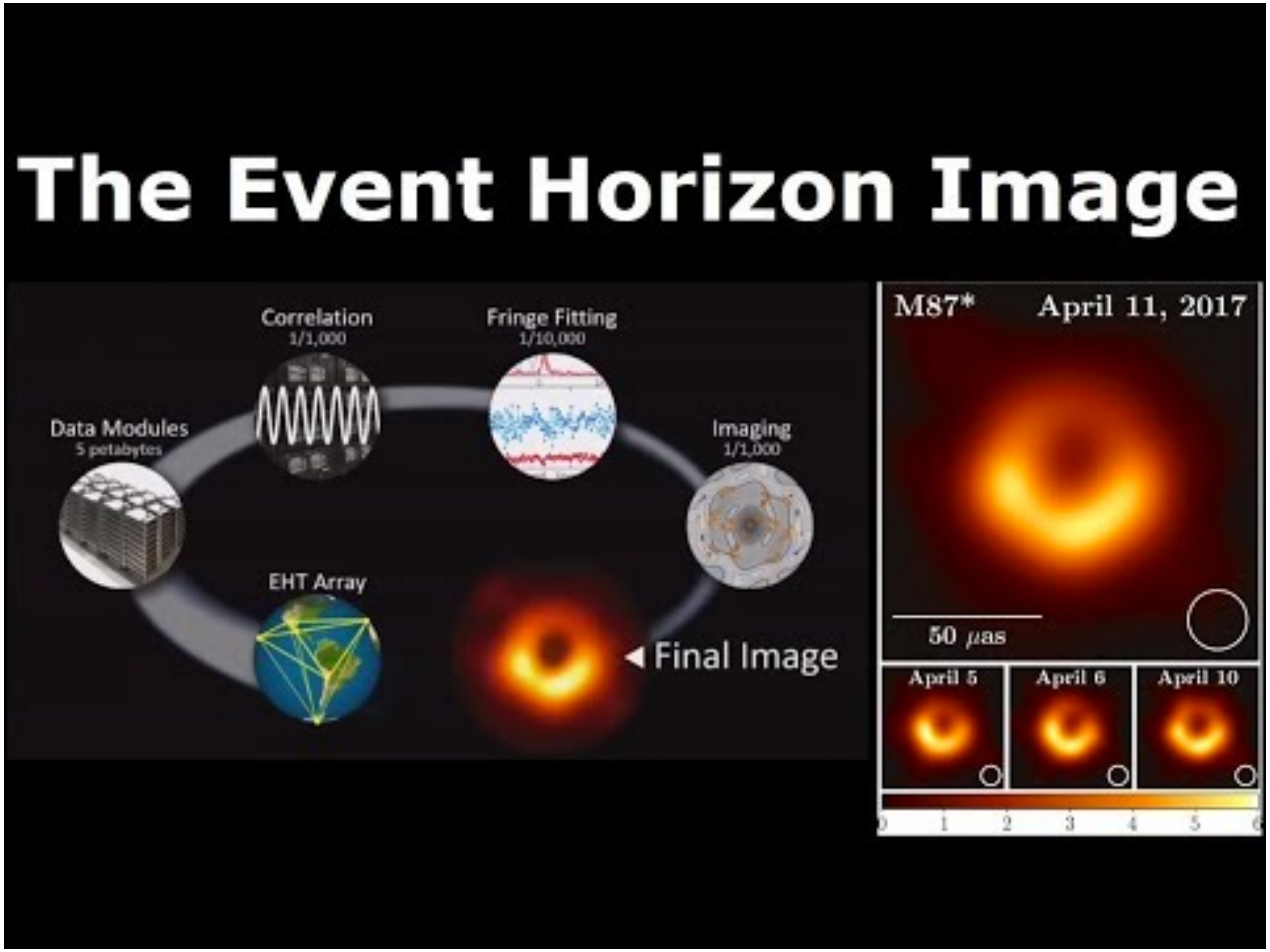

M87 (NGC4486)

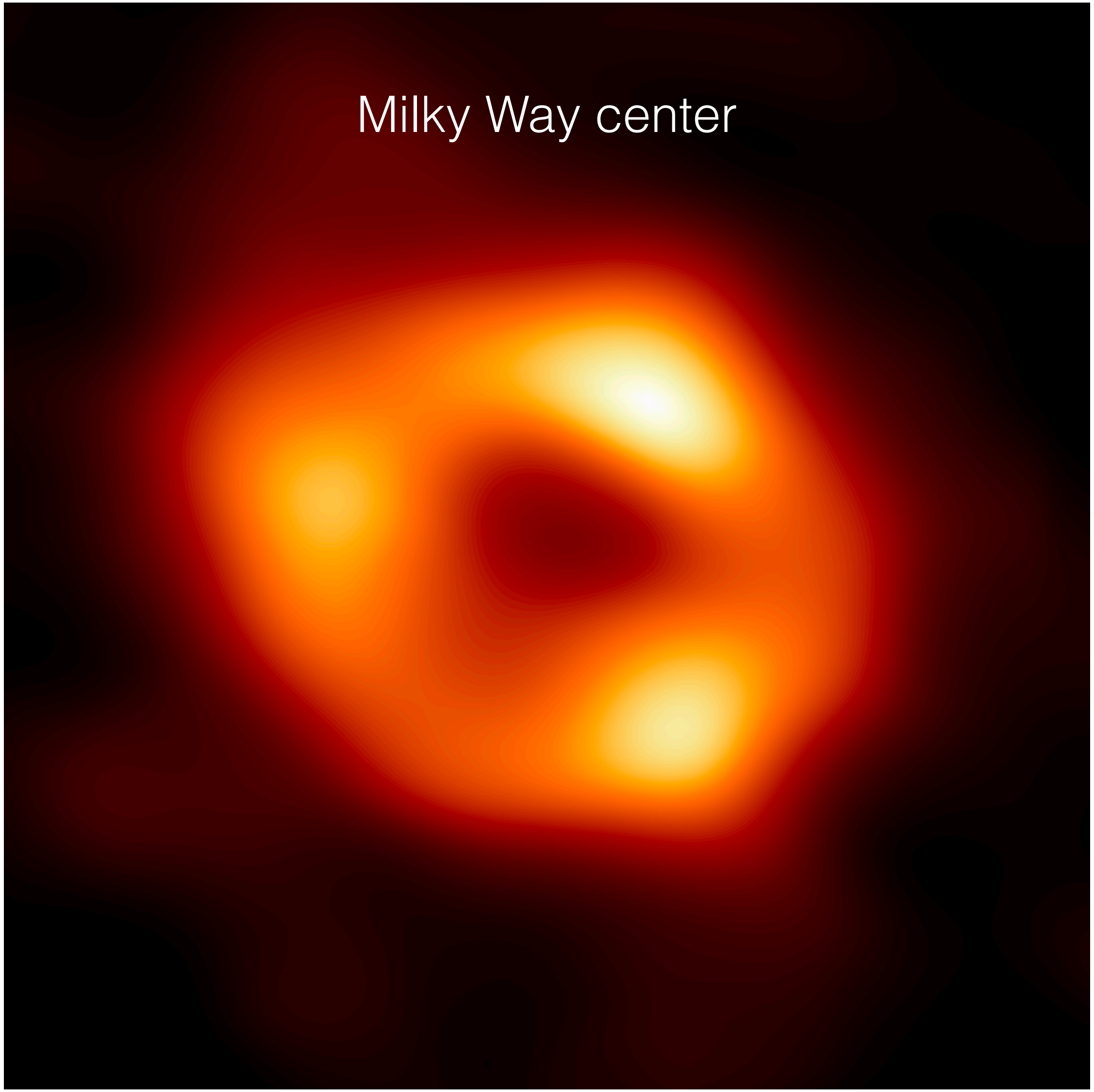



Formation of extragalactic jets from a black hole accretion disk
Hot plasma
Black hole
Magnetic
field lines
Accretion disk



M87 center


The Event Horizon Image
Correlation
1/1,000
Fringe Fitting
1/10,000
Data Modules
5 petabytes
Imaging
1/1,000
EHT Array
Final Image
M87*
April 11, 2017
50 μas
April 5
April 6
April 10
0
1
2
3
4
5
6


Milky Way center


"How to Take a Picture of the Milky Way's Black Hole"
Event Horizon Telescope

Frequency Concept in FFT

10개의 원소를 가지는 array를 아래의 방법으로 FT할 경우

$$H_n = \sum_{k=0}^{N-1} h_k e^{-2\pi ikn/N}$$

n=[0,1,2,3,4,5,6,7,8,9]

f=[0,1/10,2/10,3/10,4/10,5/10,6/10,7/10,8/10,9/10]

또는

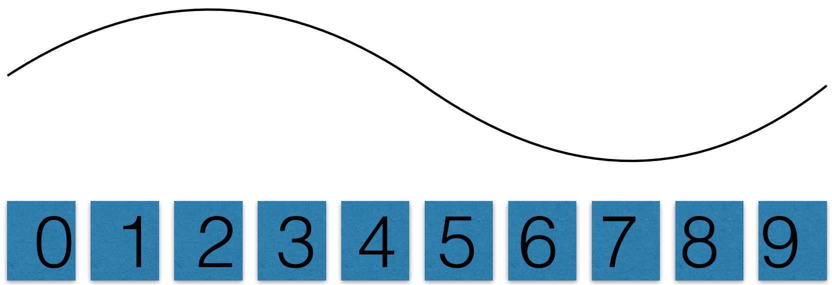
f=[0,1/10,2/10,3/10,4/10,5/10,-4/10,-3/10,-2/10,-1/10]

```
>>> np.fft.fftfreq(10)
array([ 0. ,  0.1,  0.2,  0.3,  0.4, -0.5, -0.4, -0.3, -0.2, -0.1])
```

주기=1/주파수

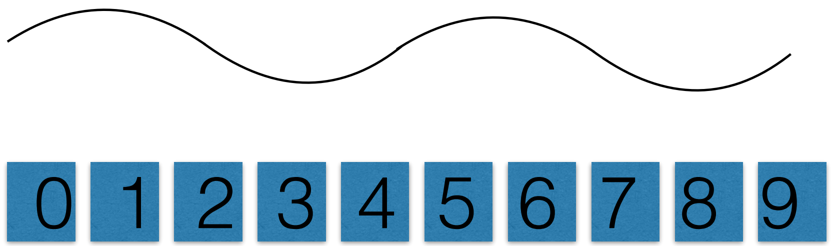
n	0	1	2	3	4	5	6	7	8	9
f	0	1/10	2/10	3/10	4/10	5/10	-4/10	-3/10	-2/10	-1/10
L	∞	10	5	3.33	2.5	2	-2.5	-3.33	-5	-10

L=10 means that the function repeats itself every 10th element
or more simply the wavelength is 10.



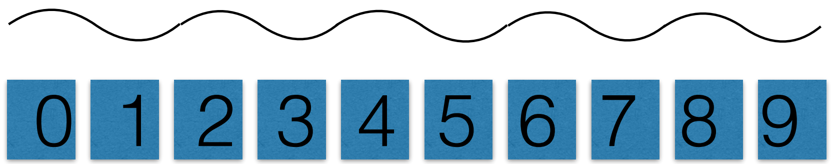
n	0	1	2	3	4	5	6	7	8	9
f	0	1/10	2/10	3/10	4/10	5/10	-4/10	-3/10	-2/10	-1/10
L	∞	10	5	3.33	2.5	2	-2.5	-3.33	-5	-10

L=5 means that the function repeats itself every 5th element.



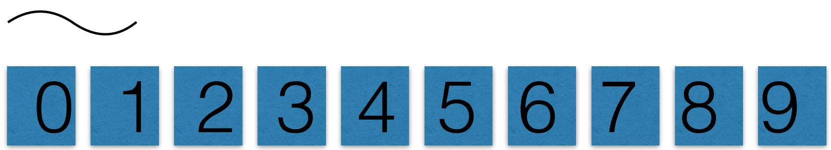
n	0	1	2	3	4	5	6	7	8	9
f	0	1/10	2/10	3/10	4/10	5/10	-4/10	-3/10	-2/10	-1/10
L	∞	10	5	3.33	2.5	2	-2.5	-3.33	-5	-10

L=2 means that the function repeats itself every 2th element.



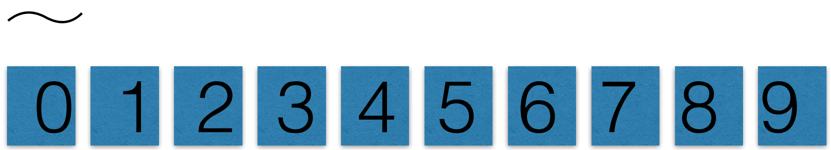
n	0	1	2	3	4	5	6	7	8	9
f	0	1/10	2/10	3/10	4/10	5/10	-4/10	-3/10	-2/10	-1/10
L	∞	10	5	3.33	2.5	2	-2.5	-3.33	-5	-10

L=-2.5 means that the function repeats itself every 1.7th element.



n	0	1	2	3	4	5	6	7	8	9
f	0	1/10	2/10	3/10	4/10	5/10	-4/10	-3/10	-2/10	-1/10
P	∞	10	5	3.33	2.5	2	-2.5	-3.33	-5	-10

P=-10 means that the function repeats itself every 1.1th element.



n	0	1	2	3	4	5	6	7	8	9
f	0	1/10	2/10	3/10	4/10	5/10	-4/10	-3/10	-2/10	-1/10
L	∞	10	5	3.33	2.5	2	-2.5	-3.33	-5	-10

L=infinite means that the function is a constant.

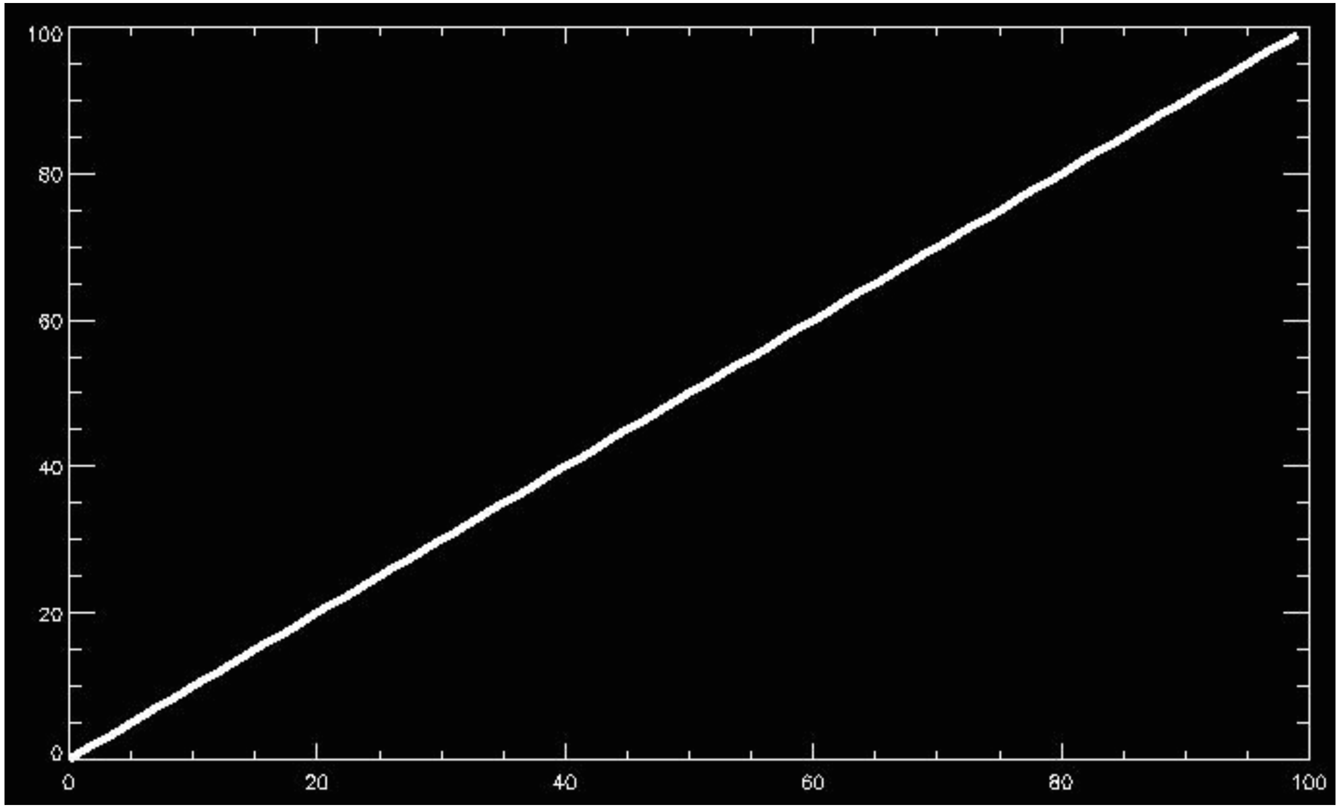
0 1 2 3 4 5 6 7 8 9

n	0	1	2	3	4	5	6	7	8	9
f	0	1/10	2/10	3/10	4/10	5/10	-4/10	-3/10	-2/10	-1/10
L	∞	10	5	3.33	2.5	2	-2.5	-3.33	-5	-10

[0,1,2,3,4,…, …, 98, 99]

f(x)

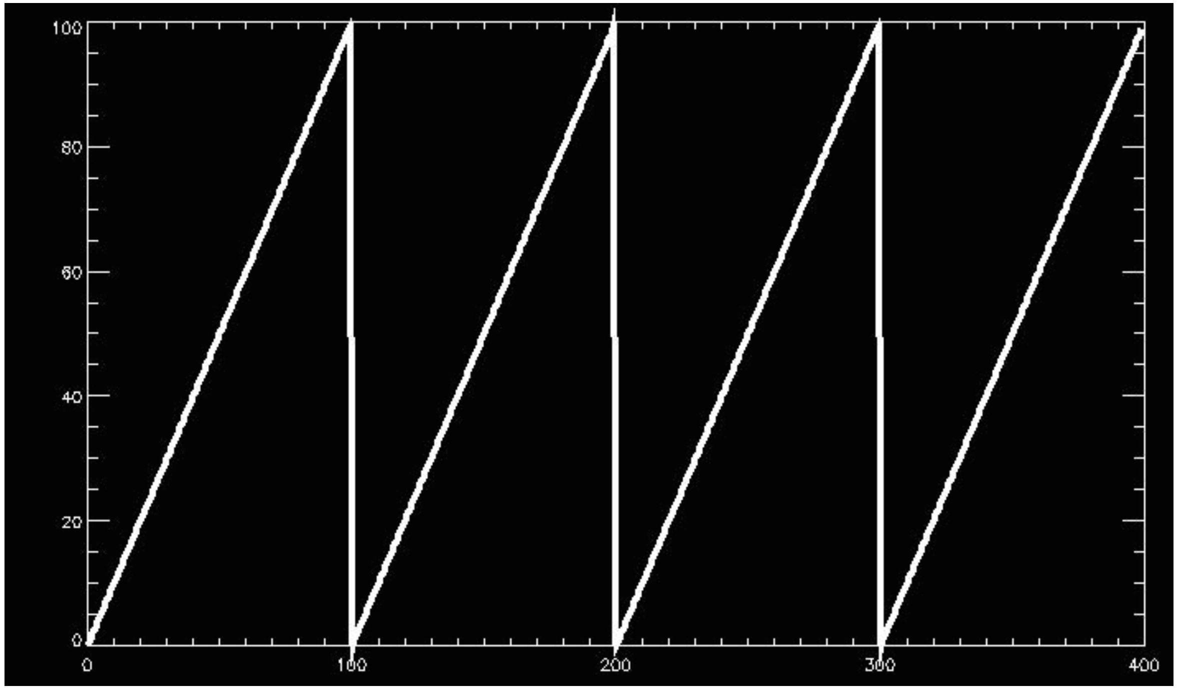
(array 값)



x

(array 번호)

FFT 수행시 우리는 이 함수가 다음과 같은 주기를 가지고 있다고 가정



$$H_n = \sum_{k=0}^{N-1} h_k e^{-2\pi i k n / N}$$

```
x=np.linspace(0,99,100)
fx=np.fft.fft(x)
print(fx.real)
```

4950.0000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
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-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000

$$H_n = \sum_{k=0}^{N-1} h_k e^{-2\pi ikn/N}$$

n=0

H_n=0+1+2+…+99=4950

n=1

$\mathrm{Re}(H_1)$=0xcos(0) + 1xcos(2πx1/100) + 2xcos(2πx2/100) + … + 99xcos(2πx99/100)

=-50

n=2

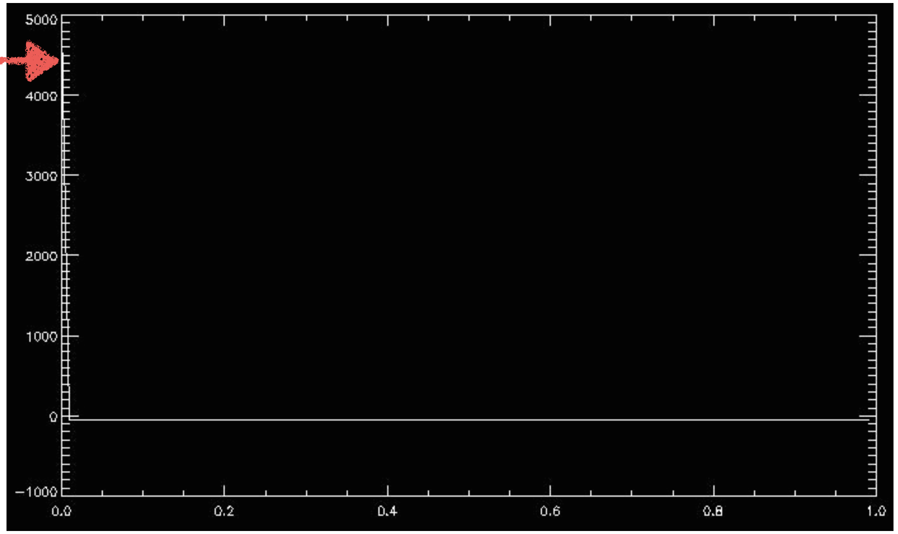
$\mathrm{Re}(H_2)$=0xcos(0) + 1xcos(2πx1x2/100) + 2xcos(2πx4x2/100) + … + 99xcos(2πx99x2/100)

=-50

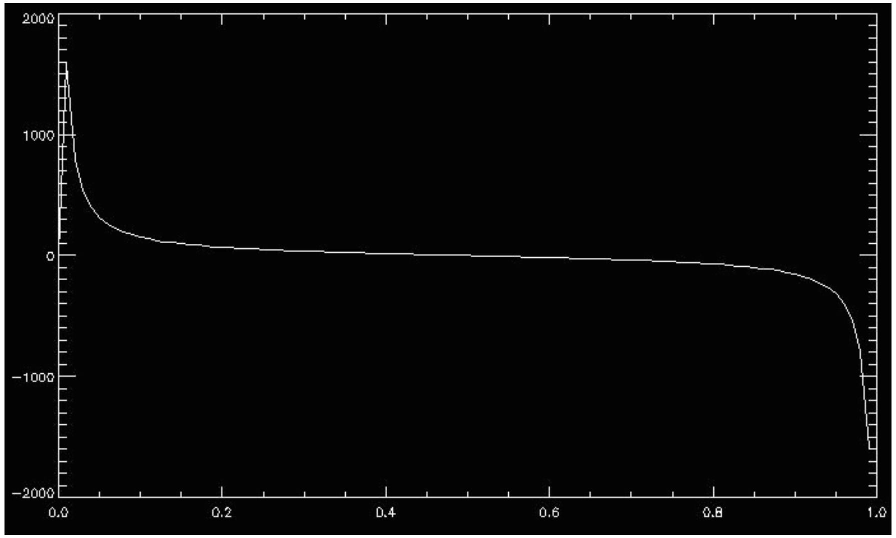
```
print(fx.imag)
     0.0000000       1591.0258       794.72724       528.94475       395.79075
     315.68758       262.10918       223.68714       194.73714       172.10113
     153.88418       138.88034       126.28558       115.54318       106.25541
     98.130525       90.949662       84.545383       78.787393       73.572766
     68.819096       64.459612       60.439618       56.713867       53.244592
     50.000000       46.953125       44.080930       41.363597       38.783976
     36.327126       33.979965       31.730965       29.569918       27.487733
     25.476272       23.528214       21.636932       19.796400       18.001108
     16.245985       14.526343       12.837818       11.176324       9.5380101
     7.9192220       6.3164689       4.7263916       3.1457334       1.5713133
  8.7214496e-12     -1.5713133      -3.1457334      -4.7263916      -6.3164689
    -7.9192220      -9.5380101      -11.176324      -12.837818      -14.526343
    -16.245985      -18.001108      -19.796400      -21.636932      -23.528214
    -25.476272      -27.487733      -29.569918      -31.730965      -33.979965
    -36.327126      -38.783976      -41.363597      -44.080930      -46.953125
    -50.000000      -53.244592      -56.713867      -60.439618      -64.459612
    -68.819096      -73.572766      -78.787393      -84.545383      -90.949662
    -98.130525      -106.25541      -115.54318      -126.28558      -138.88034
    -153.88418      -172.10113      -194.73714      -223.68714      -262.10918
    -315.68758      -395.79075      -528.94475      -794.72724      -1591.0258
```

what is this?

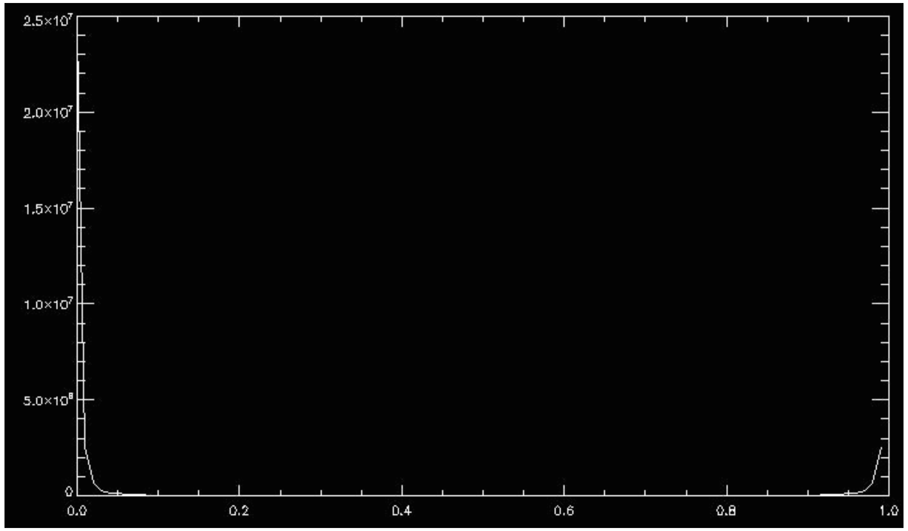
Real



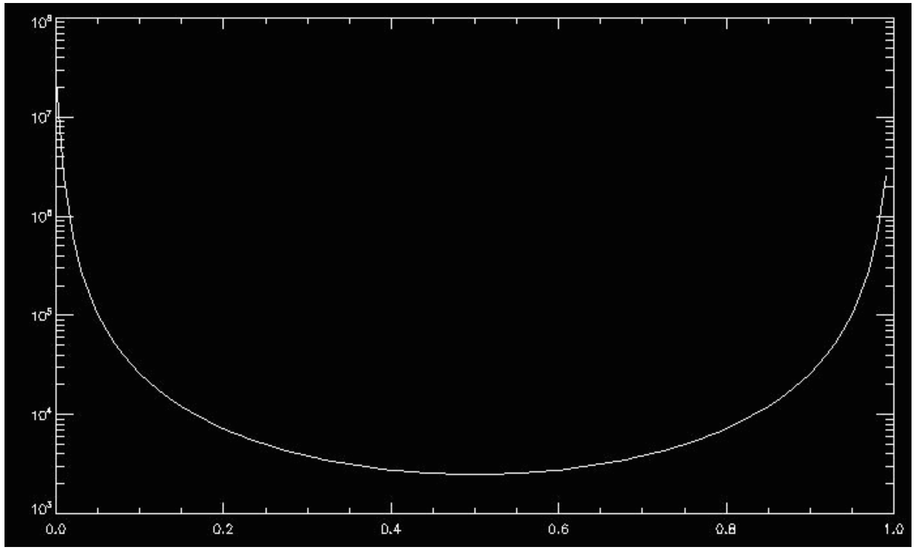
Imaginary



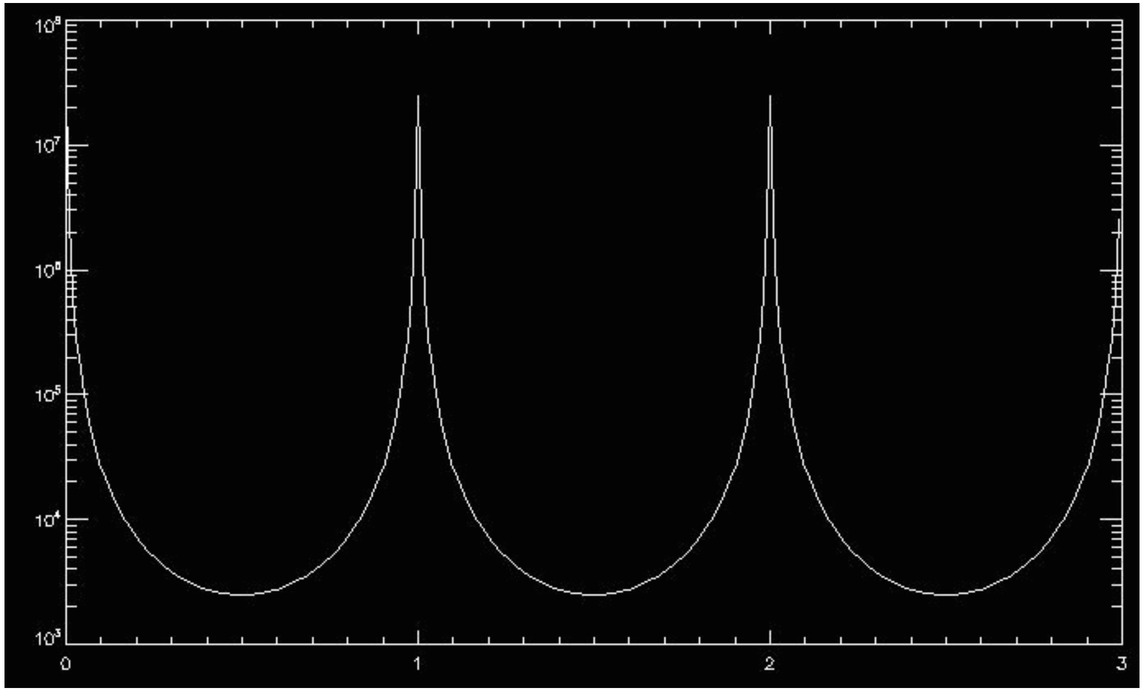
Powerspectrum
linear scale



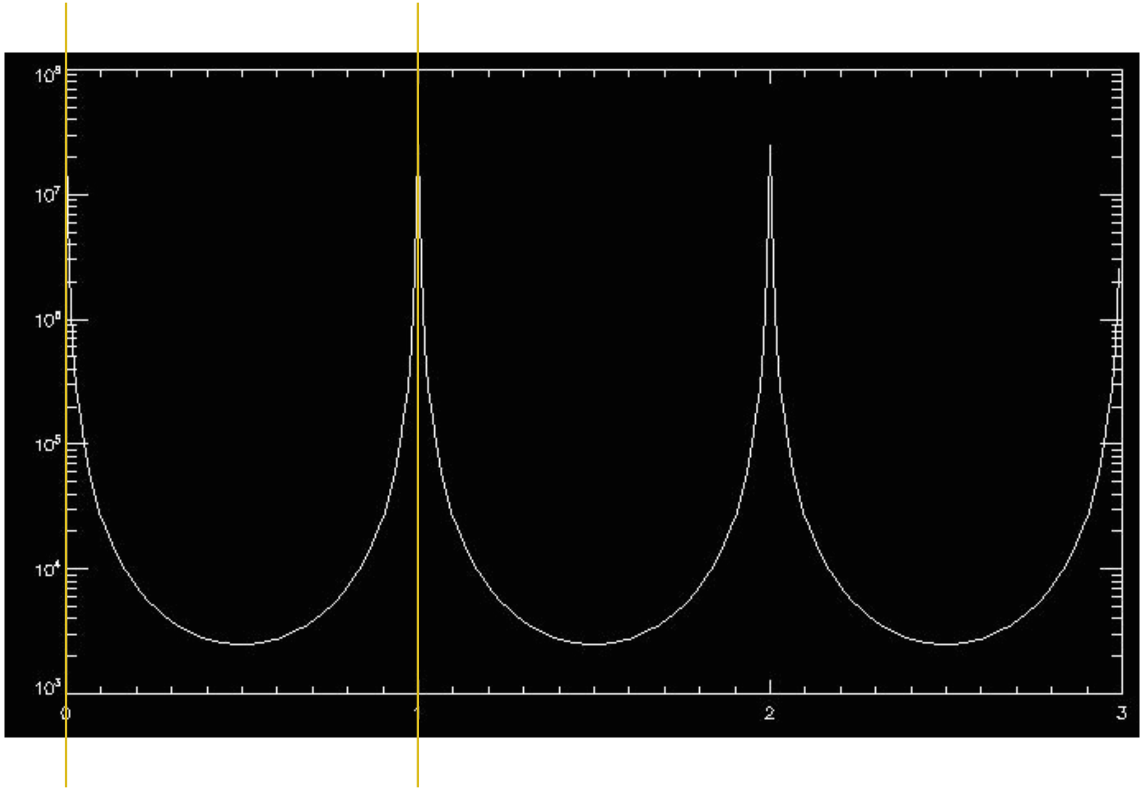
Powerspectrum
y-log scale



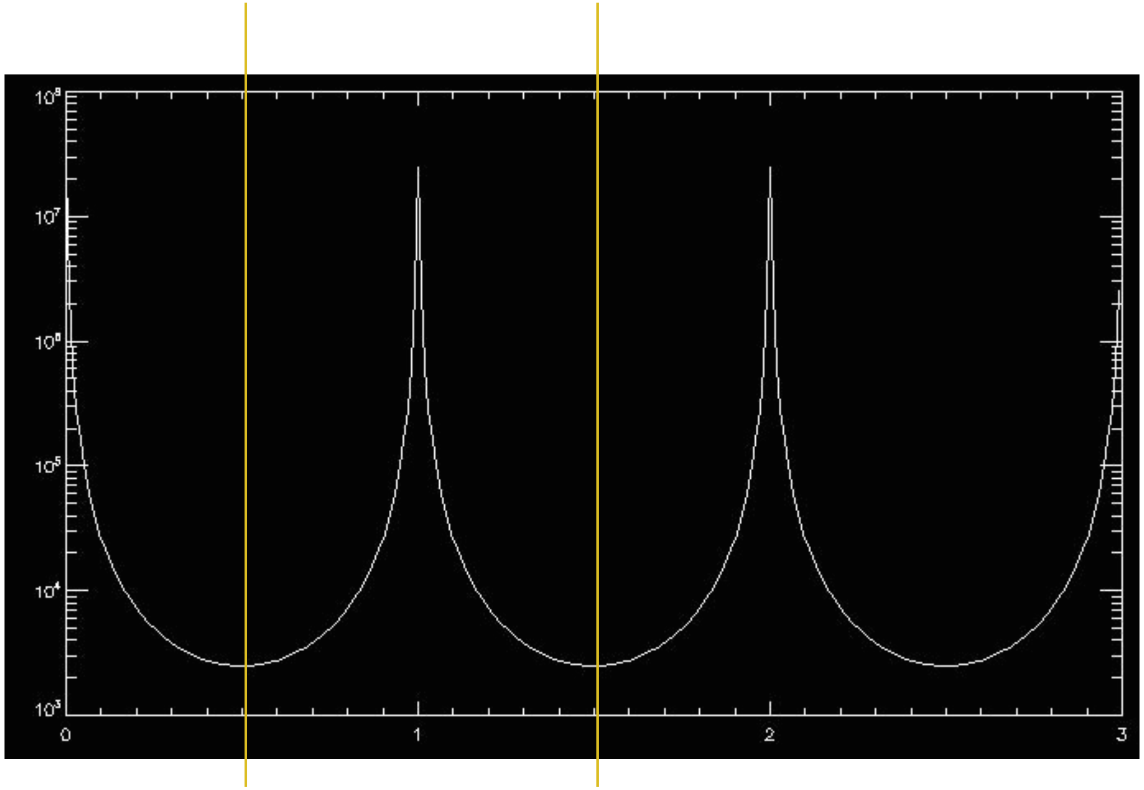
Fourier Transform 결과도 주기 함수



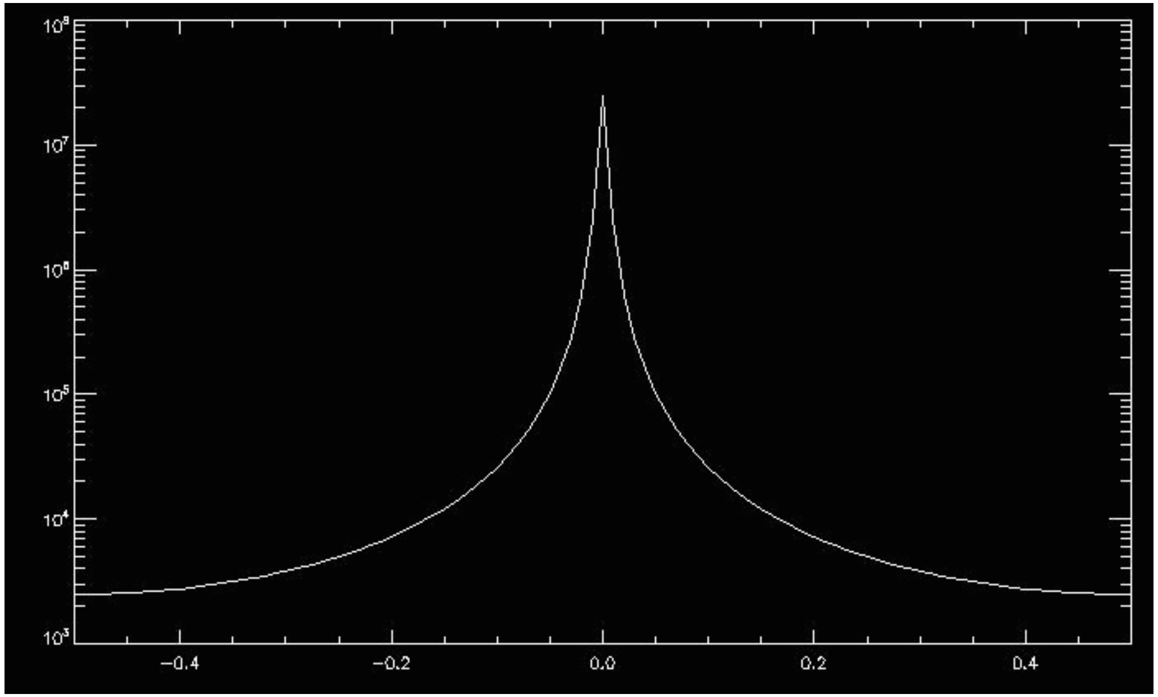
Fourier Transform 결과도 주기 함수



Fourier Transform 결과도 주기 함수



Powerspectrum centered at f=0



print,re

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-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
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-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
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-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000

print,im

0.0000000	1591.0258	794.72724	528.94475	395.79075
315.68758	262.10918	223.68714	194.73714	172.10113
153.88418	138.88034	126.28558	115.54318	106.25541
98.130525	90.949662	84.545383	78.787393	73.572766
68.819096	64.459612	60.439618	56.713867	53.244592
50.000000	46.953125	44.080930	41.363597	38.783976
36.327126	33.979965	31.730965	29.569918	27.487733
25.476272	23.528214	21.636932	19.796400	18.001108
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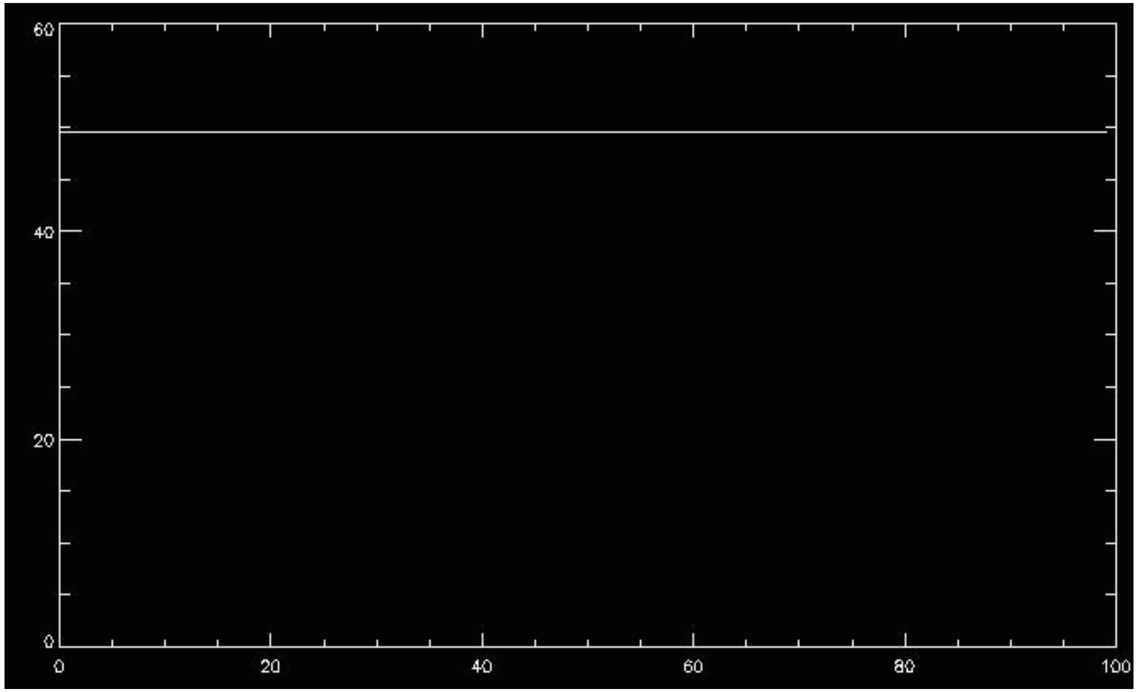
8.7214496e-12	-1.5713133	-3.1457334	-4.7263916	-6.3164689
-7.9192220	-9.5380101	-11.176324	-12.837818	-14.526343
-16.245985	-18.001108	-19.796400	-21.636932	-23.528214
-25.476272	-27.487733	-29.569918	-31.730965	-33.979965
-36.327126	-38.783976	-41.363597	-44.080930	-46.953125
-50.000000	-53.244592	-56.713867	-60.439618	-64.459612
-68.819096	-73.572766	-78.787393	-84.545383	-90.949662
-98.130525	-106.25541	-115.54318	-126.28558	-138.88034
-153.88418	-172.10113	-194.73714	-223.68714	-262.10918
-315.68758	-395.79075	-528.94475	-794.72724	-1591.0258

첫번째 frequency 만 남기고
나머지 0로 채움

그 결과를 inverse transform

Fourier 첫번째 컴포넌트의 real space 의미



```
print,re
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     -50.000000     -50.000000     -50.000000     -50.000000     -50.000000
     -50.000000     -50.000000     -50.000000     -50.000000     -50.000000
     -50.000000     -50.000000     -50.000000     -50.000000     -50.000000
     -50.000000     -50.000000     -50.000000     -50.000000     -50.000000
     -50.000000     -50.000000     -50.000000     -50.000000     -50.000000
     -50.000000     -50.000000     -50.000000     -50.000000     -50.000000
     -50.000000     -50.000000     -50.000000     -50.000000     -50.000000
     -50.000000     -50.000000     -50.000000     -50.000000     -50.000000
     -50.000000     -50.000000     -50.000000     -50.000000     -50.000000
     -50.000000     -50.000000     -50.000000     -50.000000     -50.000000
     -50.000000     -50.000000     -50.000000     -50.000000     -50.000000
     -50.000000     -50.000000     -50.000000     -50.000000     -50.000000
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     -50.000000     -50.000000     -50.000000     -50.000000     -50.000000
     -50.000000     -50.000000     -50.000000     -50.000000     -50.000000
     -50.000000     -50.000000     -50.000000     -50.000000     -50.000000
     -50.000000     -50.000000     -50.000000     -50.000000     -50.000000
```
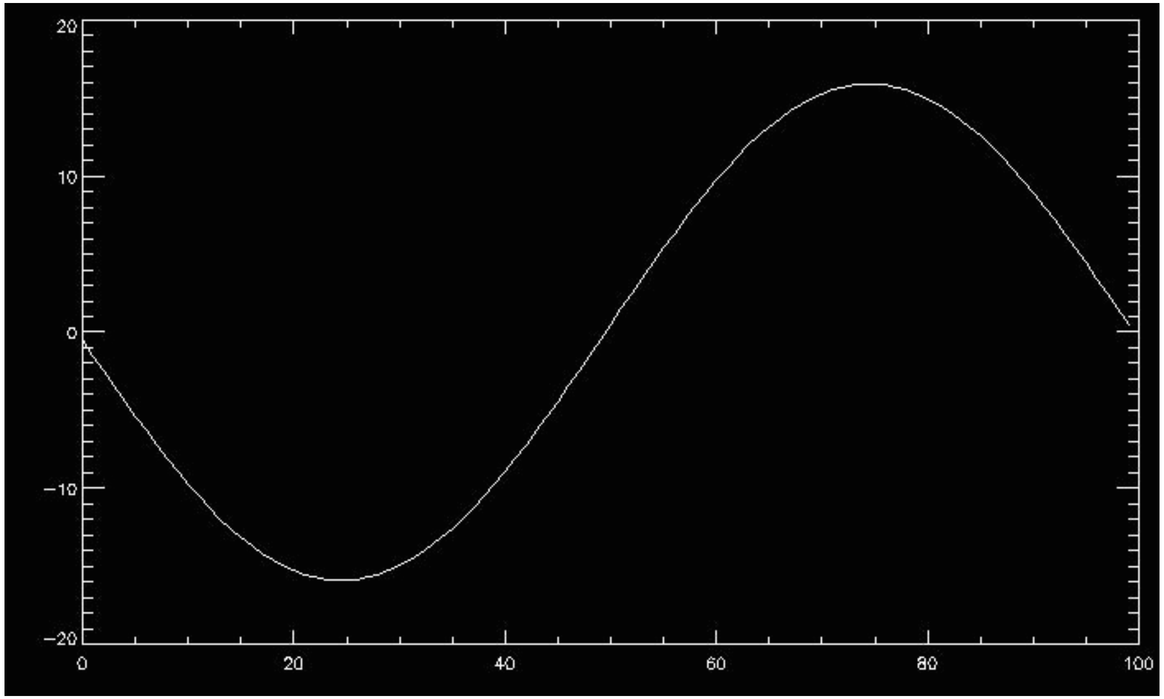

```
print,im
      0.0000000      1591.0258      794.72724      528.94475      395.79075
      315.68758      262.10918      223.68714      194.73714      172.10113
      153.88418      138.88034      126.28558      115.54318      106.25541
      98.130525      90.949662      84.545383      78.787393      73.572766
      68.819096      64.459612      60.439618      56.713867      53.244592
      50.000000      46.953125      44.080930      41.363597      38.783976
      36.327126      33.979965      31.730965      29.569918      27.487733
      25.476272      23.528214      21.636932      19.796400      18.001108
      16.245985      14.526343      12.837818      11.176324      9.5380101
      7.9192220      6.3164689      4.7263916      3.1457334      1.5713133
   8.7214496e-12     -1.5713133     -3.1457334     -4.7263916     -6.3164689
     -7.9192220     -9.5380101     -11.176324     -12.837818     -14.526343
     -16.245985     -18.001108     -19.796400     -21.636932     -23.528214
     -25.476272     -27.487733     -29.569918     -31.730965     -33.979965
     -36.327126     -38.783976     -41.363597     -44.080930     -46.953125
     -50.000000     -53.244592     -56.713867     -60.439618     -64.459612
     -68.819096     -73.572766     -78.787393     -84.545383     -90.949662
     -98.130525     -106.25541     -115.54318     -126.28558     -138.88034
     -153.88418     -172.10113     -194.73714     -223.68714     -262.10918
     -315.68758     -395.79075     -528.94475     -794.72724     -1591.0258
```

두번째 frequency 만 남기고
나머지 0로 채움

그 결과를 inverse transform

Fourier 두번째 컴포넌트의 real space 의미



print,re

4950.0000	-50.000000	-50.000000	-50.000000	-50.000000
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-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000

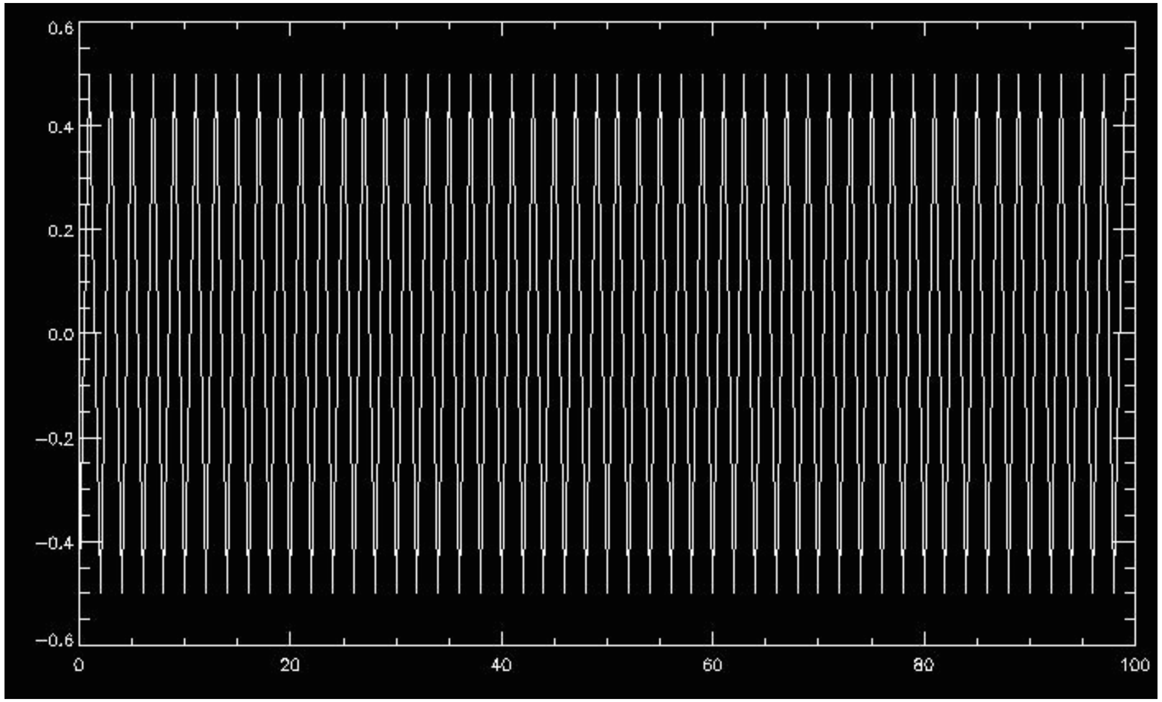
print,im

0.0000000	1591.0258	794.72724	528.94475	395.79075
315.68758	262.10918	223.68714	194.73714	172.10113
153.88418	138.88034	126.28558	115.54318	106.25541
98.130525	90.949662	84.545383	78.787393	73.572766
68.819096	64.459612	60.439618	56.713867	53.244592
50.000000	46.953125	44.080930	41.363597	38.783976
36.327126	33.979965	31.730965	29.569918	27.487733
25.476272	23.528214	21.636932	19.796400	18.001108
16.245985	14.526343	12.837818	11.176324	9.5380101
7.9192220	6.3164689	4.7263916	3.1457334	1.5713133
8.7214496e-12	-1.5713133	-3.1457334	-4.7263916	-6.3164689
-7.9192220	-9.5380101	-11.176324	-12.837818	-14.526343
-16.245985	-18.001108	-19.796400	-21.636932	-23.528214
-25.476272	-27.487733	-29.569918	-31.730965	-33.979965
-36.327126	-38.783976	-41.363597	-44.080930	-46.953125
-50.000000	-53.244592	-56.713867	-60.439618	-64.459612
-68.819096	-73.572766	-78.787393	-84.545383	-90.949662
-98.130525	-106.25541	-115.54318	-126.28558	-138.88034
-153.88418	-172.10113	-194.73714	-223.68714	-262.10918
-315.68758	-395.79075	-528.94475	-794.72724	-1591.0258

Nyquist frequency 만 남기고
나머지 0로 채움

그 결과를 inverse transform

Fourier Nyquist 컴포넌트의 real space 의미



print,re

4950.0000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000
-50.000000	-50.000000	-50.000000	-50.000000	-50.000000

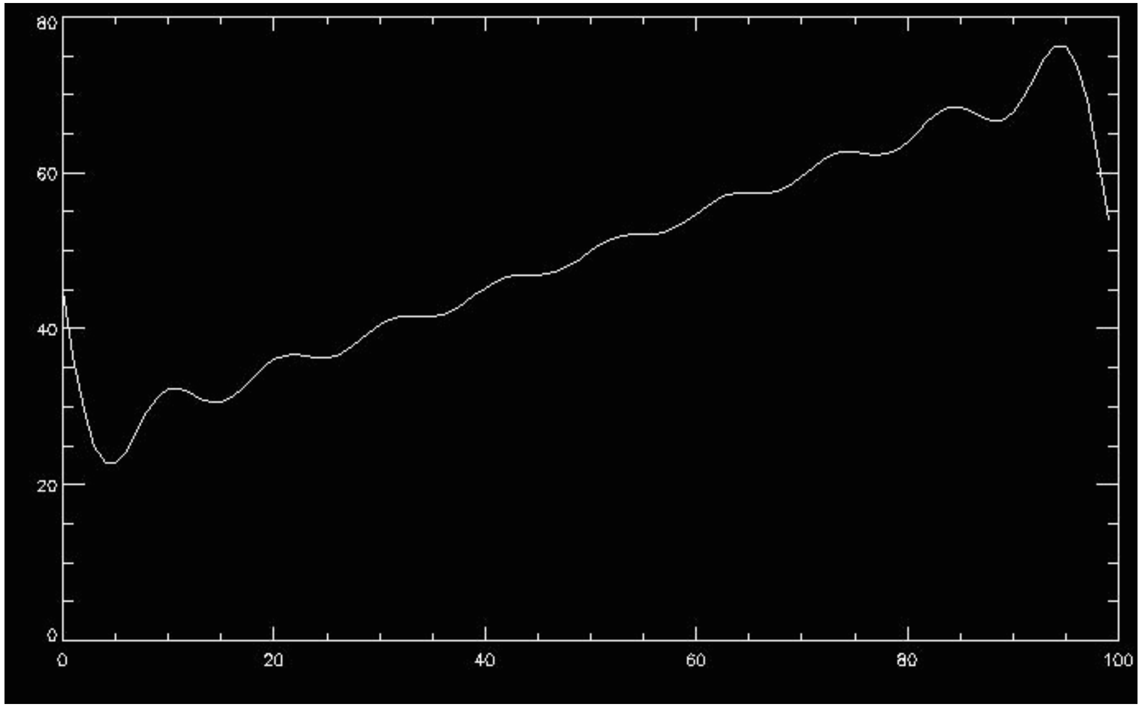
print,im

0.0000000	1591.0258	794.72724	528.94475	395.79075
315.68758	262.10918	223.68714	194.73714	172.10113
153.88418	138.88034	126.28558	115.54318	106.25541
98.130525	90.949662	84.545383	78.787393	73.572766
68.819096	64.459612	60.439618	56.713867	53.244592
50.000000	46.953125	44.080930	41.363597	38.783976
36.327126	33.979965	31.730965	29.569918	27.487733
25.476272	23.528214	21.636932	19.796400	18.001108
16.245985	14.526343	12.837818	11.176324	9.5380101
7.9192220	6.3164689	4.7263916	3.1457334	1.5713133
8.7214496e-12	-1.5713133	-3.1457334	-4.7263916	-6.3164689
-7.9192220	-9.5380101	-11.176324	-12.837818	-14.526343
-16.245985	-18.001108	-19.796400	-21.636932	-23.528214
-25.476272	-27.487733	-29.569918	-31.730965	-33.979965
-36.327126	-38.783976	-41.363597	-44.080930	-46.953125
-50.000000	-53.244592	-56.713867	-60.439618	-64.459612
-68.819096	-73.572766	-78.787393	-84.545383	-90.949662
-98.130525	-106.25541	-115.54318	-126.28558	-138.88034
-153.88418	-172.10113	-194.73714	-223.68714	-262.10918
-315.68758	-395.79075	-528.94475	-794.72724	-1591.0258

처음 10 frequency 만 남기고
나머지 0로 채움

그 결과를 inverse transform

처음 10개 Fourier 컴포넌트의 real space 의미



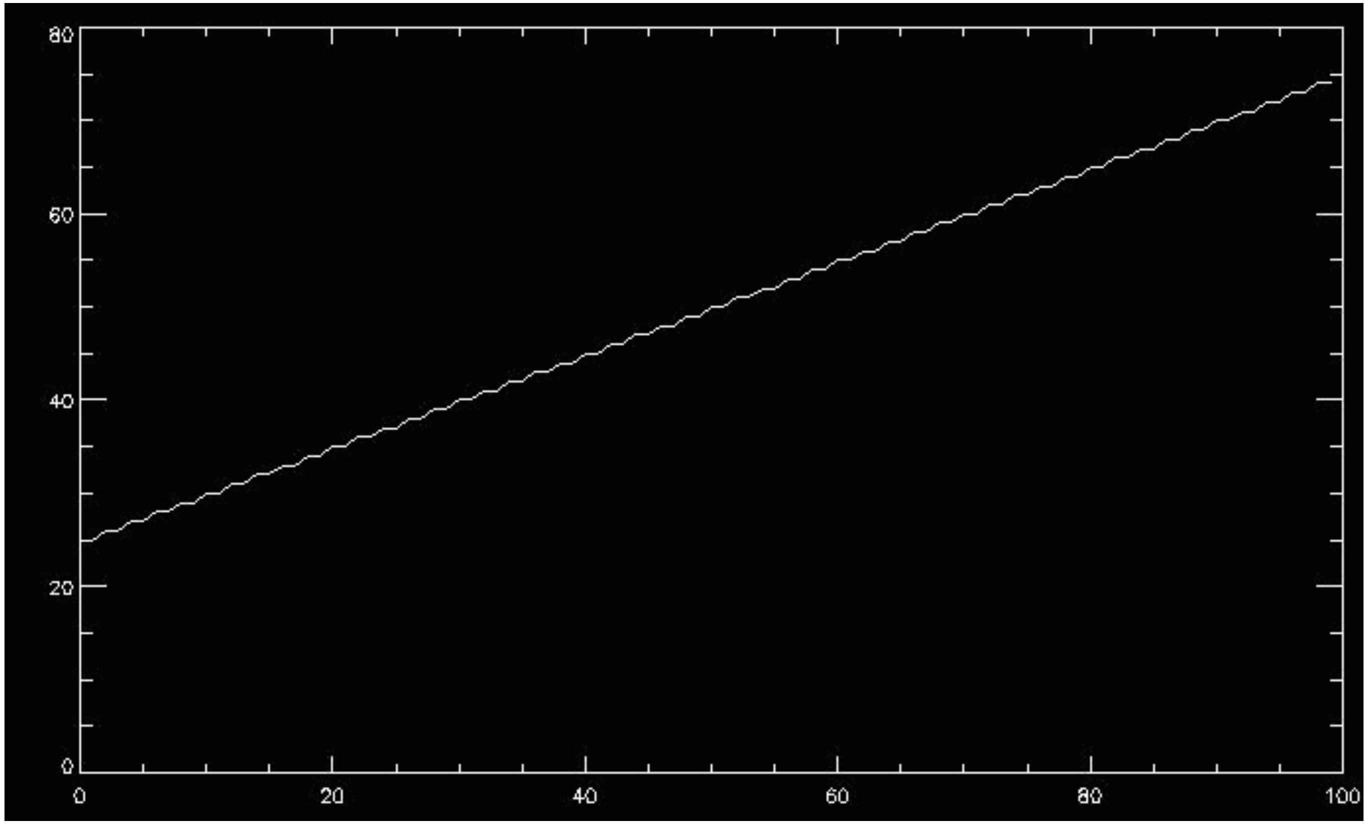
```
print,re
      4950.0000      -50.000000      -50.000000      -50.000000      -50.000000
     -50.000000      -50.000000      -50.000000      -50.000000      -50.000000
     -50.000000      -50.000000      -50.000000      -50.000000      -50.000000
     -50.000000      -50.000000      -50.000000      -50.000000      -50.000000
     -50.000000      -50.000000      -50.000000      -50.000000      -50.000000
     -50.000000      -50.000000      -50.000000      -50.000000      -50.000000
     -50.000000      -50.000000      -50.000000      -50.000000      -50.000000
     -50.000000      -50.000000      -50.000000      -50.000000      -50.000000
     -50.000000      -50.000000      -50.000000      -50.000000      -50.000000
     -50.000000      -50.000000      -50.000000      -50.000000      -50.000000
     -50.000000      -50.000000      -50.000000      -50.000000      -50.000000
     -50.000000      -50.000000      -50.000000      -50.000000      -50.000000
     -50.000000      -50.000000      -50.000000      -50.000000      -50.000000
     -50.000000      -50.000000      -50.000000      -50.000000      -50.000000
     -50.000000      -50.000000      -50.000000      -50.000000      -50.000000
     -50.000000      -50.000000      -50.000000      -50.000000      -50.000000
     -50.000000      -50.000000      -50.000000      -50.000000      -50.000000
     -50.000000      -50.000000      -50.000000      -50.000000      -50.000000
     -50.000000      -50.000000      -50.000000      -50.000000      -50.000000
     -50.000000      -50.000000      -50.000000      -50.000000      -50.000000
```

```
print,im
      0.0000000       1591.0258       794.72724       528.94475       395.79075
      315.68758       262.10918       223.68714       194.73714       172.10113
      153.88418       138.88034       126.28558       115.54318       106.25541
      98.130525       90.949662       84.545383       78.787393       73.572766
      68.819096       64.459612       60.439618       56.713867       53.244592
      50.000000       46.953125       44.080930       41.363597       38.783976
      36.327126       33.979965       31.730965       29.569918       27.487733
      25.476272       23.528214       21.636932       19.796400       18.001108
      16.245985       14.526343       12.837818       11.176324       9.5380101
      7.9192220       6.3164689       4.7263916       3.1457334       1.5713133
   8.7214496e-12      -1.5713133      -3.1457334      -4.7263916      -6.3164689
     -7.9192220      -9.5380101      -11.176324      -12.837818      -14.526343
     -16.245985      -18.001108      -19.796400      -21.636932      -23.528214
     -25.476272      -27.487733      -29.569918      -31.730965      -33.979965
     -36.327126      -38.783976      -41.363597      -44.080930      -46.953125
     -50.000000      -53.244592      -56.713867      -60.439618      -64.459612
     -68.819096      -73.572766      -78.787393      -84.545383      -90.949662
     -98.130525      -106.25541      -115.54318      -126.28558      -138.88034
     -153.88418      -172.10113      -194.73714      -223.68714      -262.10918
     -315.68758      -395.79075      -528.94475      -794.72724      -1591.0258
```

처음 50 frequency 만 남기고
나머지 0로 채움

그 결과를 inverse transform

처음 50개 Fourier 컴포넌트의 real space 의미



Speed of Fourier Transform

$$H_n = \sum_{k=0}^{N-1} h_k e^{-2\pi i k n/N}$$

How many operations do you need? N^2

FFT can reduce the number of operations to N log N.

Assume that each operation takes 10^{-3} s.

If $N=10^6$, it can make a difference between 10^{12} sec and $6\text{x}10^6$ sec!

~32 years ~1.7 hours

Danielson-Lanczos Algorithm

$$H_n = \sum_{k=0}^{N-1} h_k e^{-2\pi ikn/N}$$

$$= \sum_{k=0}^{N/2-1} h_{2k} e^{-2\pi i(2k)n/N} + \sum_{k=0}^{N/2-1} h_{2k+1} e^{-2\pi i(2k+1)n/N}$$

$$= \sum_{k=0}^{N/2-1} h_{2k} e^{-2\pi i(2k)n/N} + e^{-2\pi kn/N} \sum_{k=0}^{N/2-1} h_{2k+1} e^{-2\pi i(2k)n/N}$$

$$= \sum_{k=0}^{N/2-1} h_{2k} e^{-2\pi ikn/(N/2)} + W^n \sum_{k=0}^{N/2-1} h_{2k+1} e^{-2\pi ikn/(N/2)} \qquad (W = e^{-2\pi k/N})$$

$$= F_n^e + W^n F_n^o$$

We can repeat the above division until the number of Fourier elements becomes one.

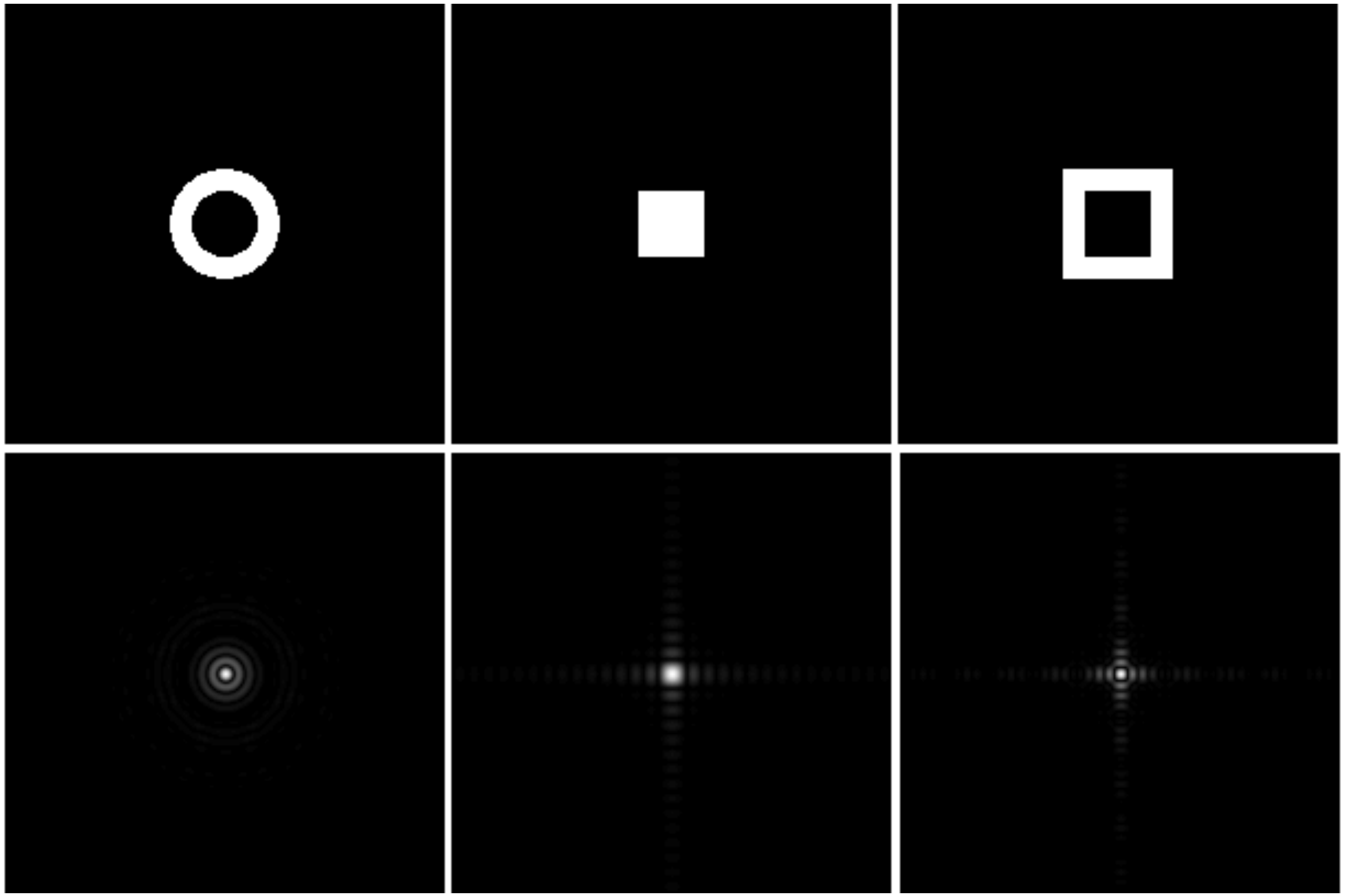
FFT in Two-Dimension

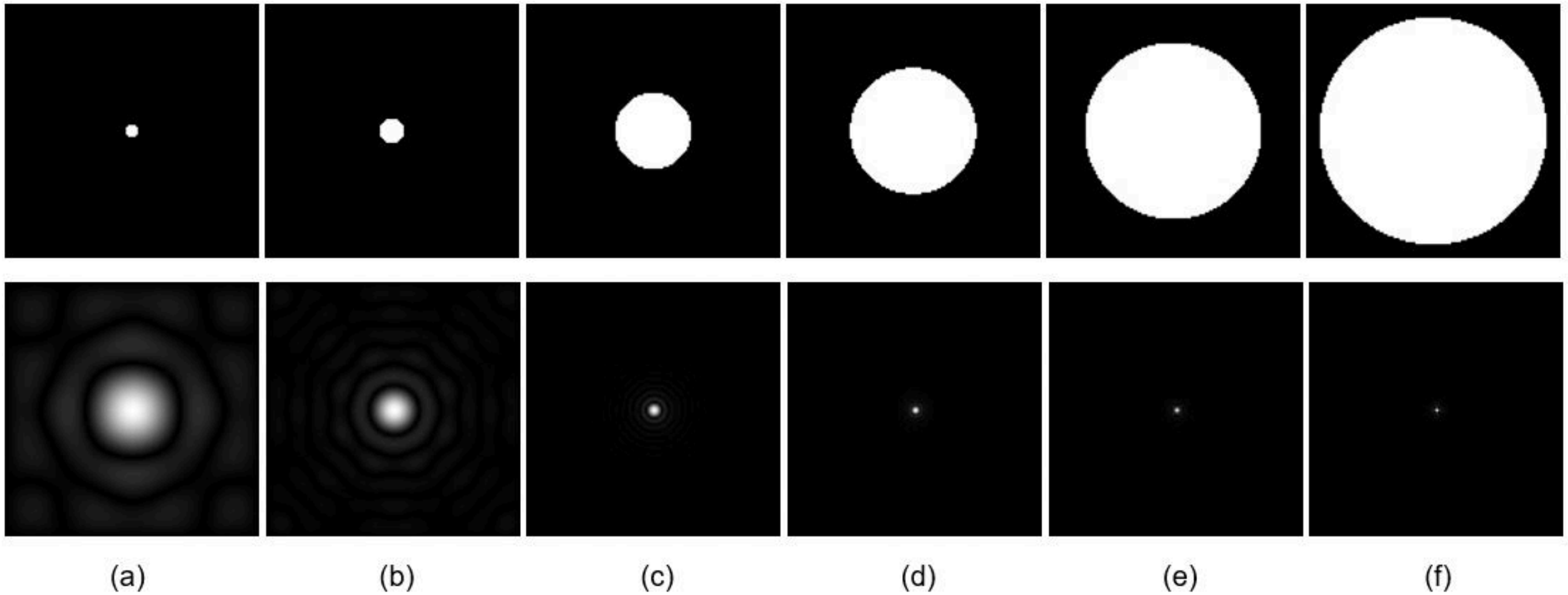
$$H(n_1, n_2) \equiv \sum_{k_2=0}^{N_2-1} \sum_{k_1=0}^{N_1-1} \exp(-2\pi i k_2 n_2/N_2) \; \exp(-2\pi i k_1 n_1/N_1) \; h(k_1, k_2)$$

$$\begin{aligned} H(n_1, n_2) &= \text{FFT-on-index-1}\left(\text{FFT-on-index-2}\left[h(k_1, k_2)\right]\right) \\ &= \text{FFT-on-index-2}\left(\text{FFT-on-index-1}\left[h(k_1, k_2)\right]\right) \end{aligned}$$

We can generalize it to an L-dimensional case.

$$\begin{aligned} H(n_1, \ldots, n_L) \equiv \sum_{k_L=0}^{N_L-1} \cdots \sum_{k_1=0}^{N_1-1} & \exp(-2\pi i k_L n_L/N_L) \times \cdots \\ & \times \exp(-2\pi i k_1 n_1/N_1) \; h(k_1, \ldots, k_L) \end{aligned}$$



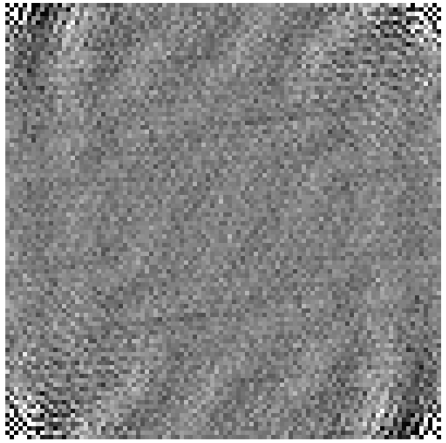


(a)
(b)
(c)
(d)
(e)
(f)

과제 15

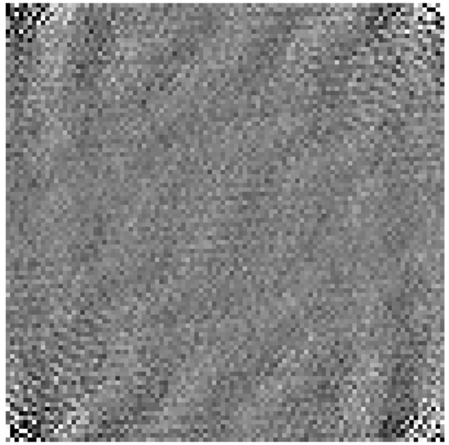
1. 과제 10에서 사용한 이미지를 사용 (100x100 해상도)
2. 2차원 Fourier Transform을 수행한다 (Library 사용할 수 있음)
3. Fourier Transform 각 원소에 해당되는 frequency 계산 (0을 중심으로 대칭이 되는 frequency 정의)
4. frequency (0<|f|<1/2) 와 power spectrum관계를 graph로 표현 (y축은 log scale을 사용하는것이 편리).
5. frequency가 1/5 보다 큰 scale의 (|f|>1/5) Fourier component를 제거 (0을 할당)
6. Inverse FT를 수행하여 결과 이미지를 출력

Real



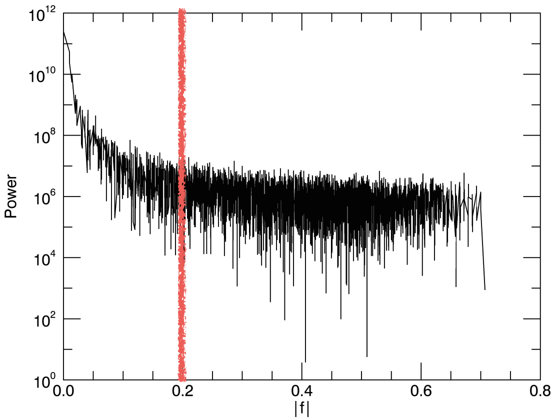
0,1/100 1/2 -1/100

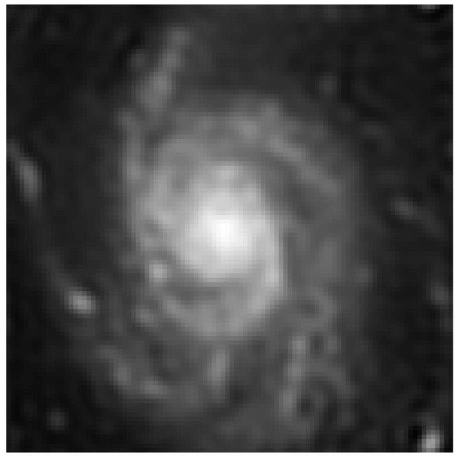
Imaginary



$|f| = (f_x^2 + f_y^2)^{1/2}$

$$P_n = |H_n|^2$$

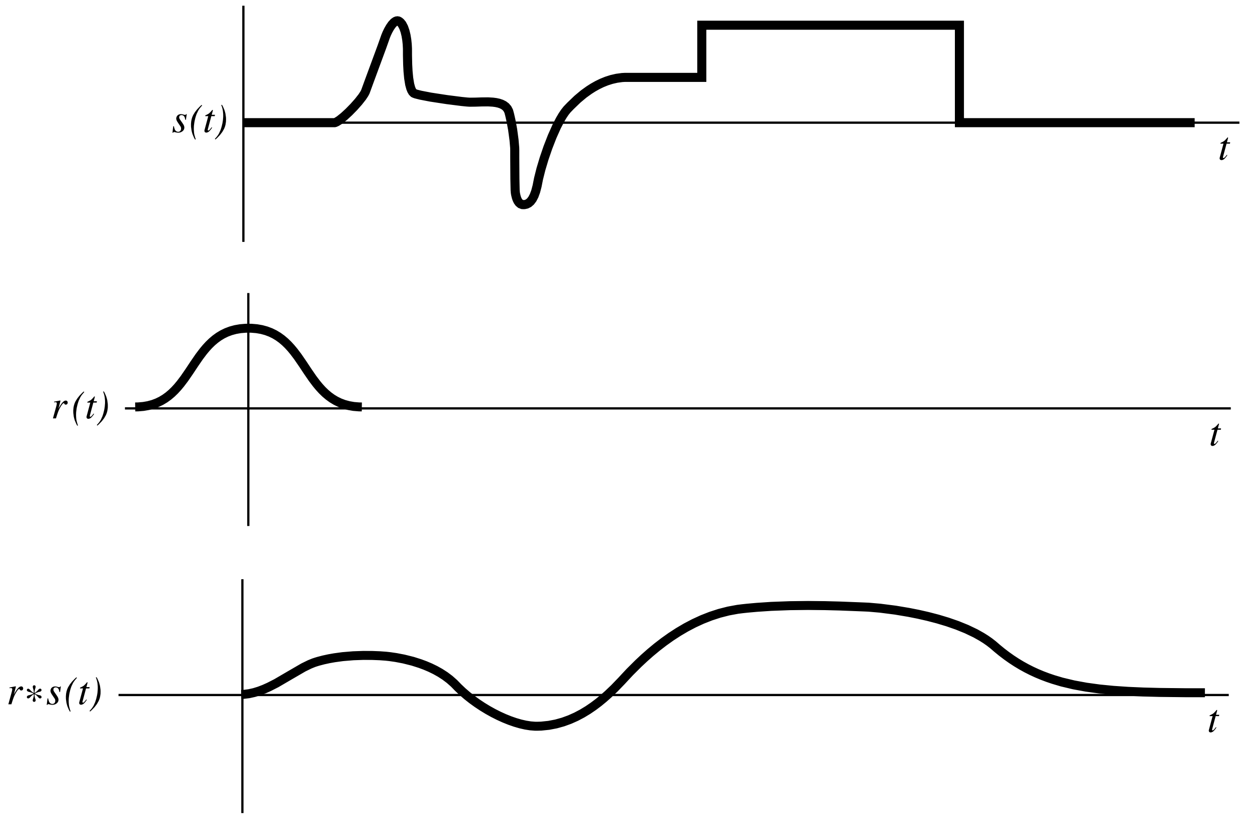




Application of FFT

Convolution

$$s * r \equiv \int_{-\infty}^{\infty} s(\tau) r(t-\tau) d\tau$$



Convolution by Hand

d=[1, 2, -1, 3, 2, 8, 5, 2, 0, 1]

k=[0.2,0.6,0.2]

d ⊛ k 은?

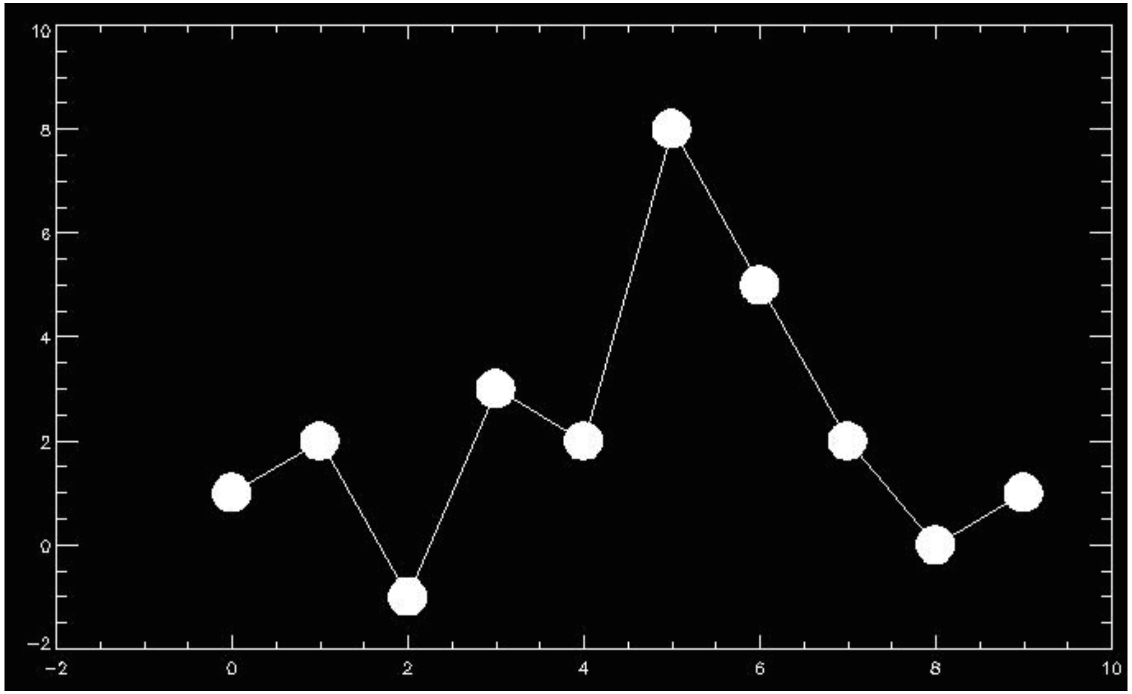
첫번째 element: 0*0.2 + 1*0.6 + 2*0.2 =1

두번째 element: 1*0.2 + 2*0.6 + (-1*0.2) =1.2

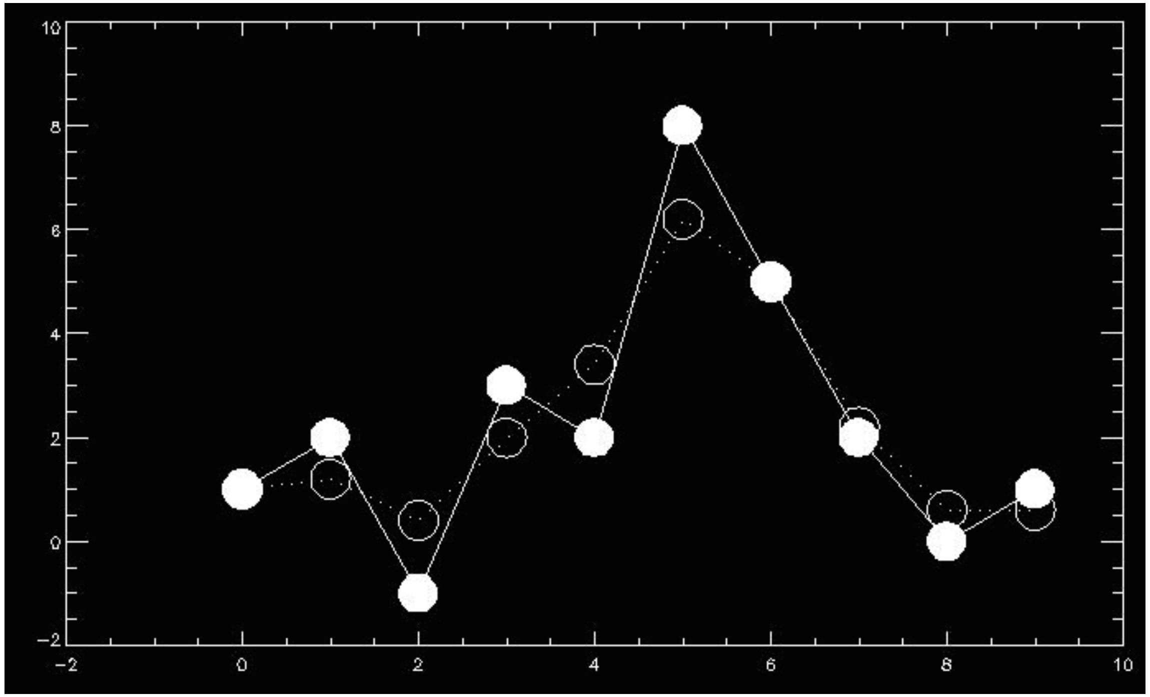
세번째 element: 2*0.2 + (-1*0.6) + 3*0.2 =0.4

마지막 element: 0*0.2 + 1*0.6 + 0*0.2 =0.6

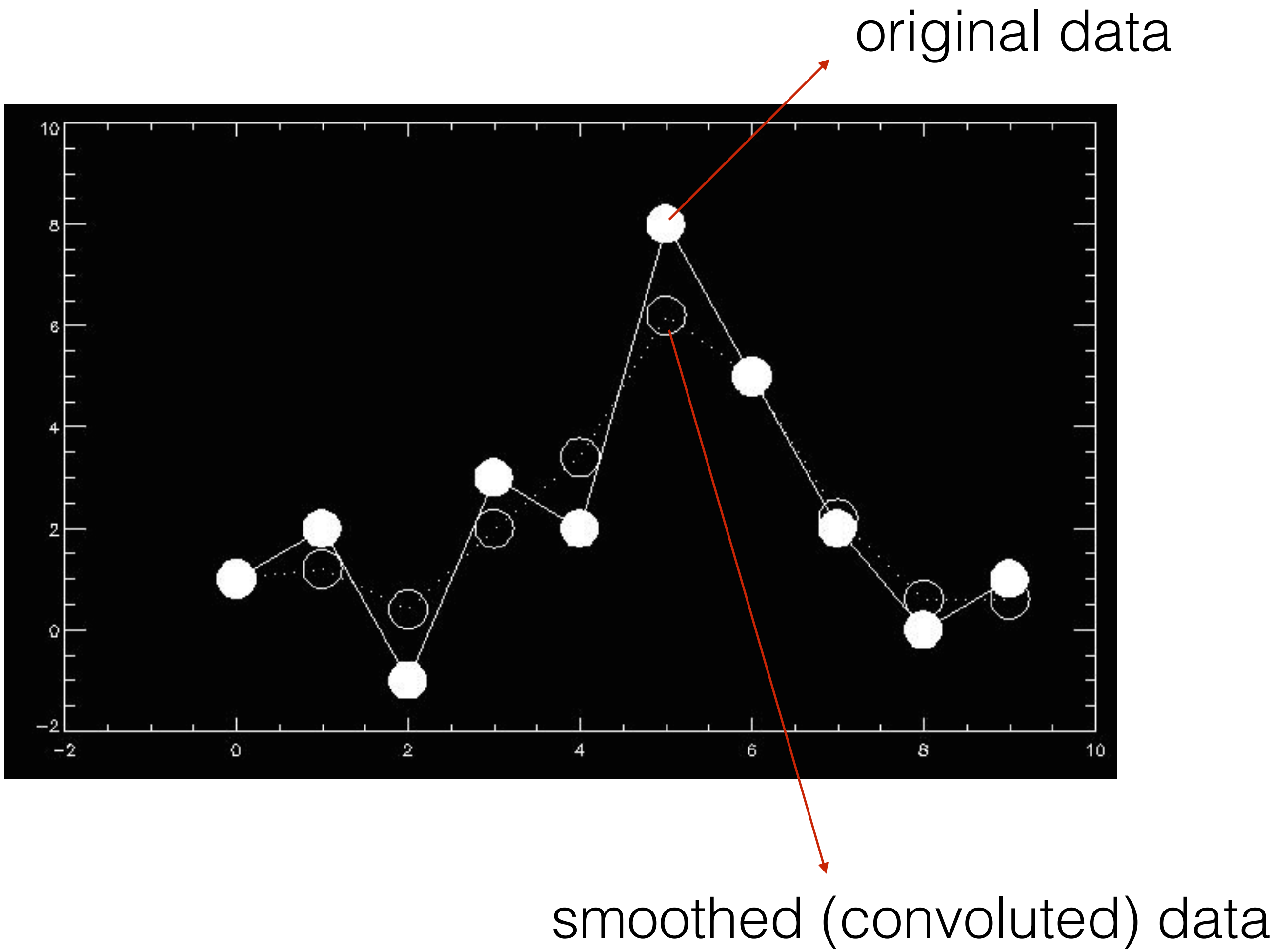
d ⊛ **k** = [1, 1.2, 0.4, 2.0, 3.4, 6.2, 5, 2.2, 0.6, 0.6]



10
8
6
4
2
0
−2
−2
0
2
4
6
8
10

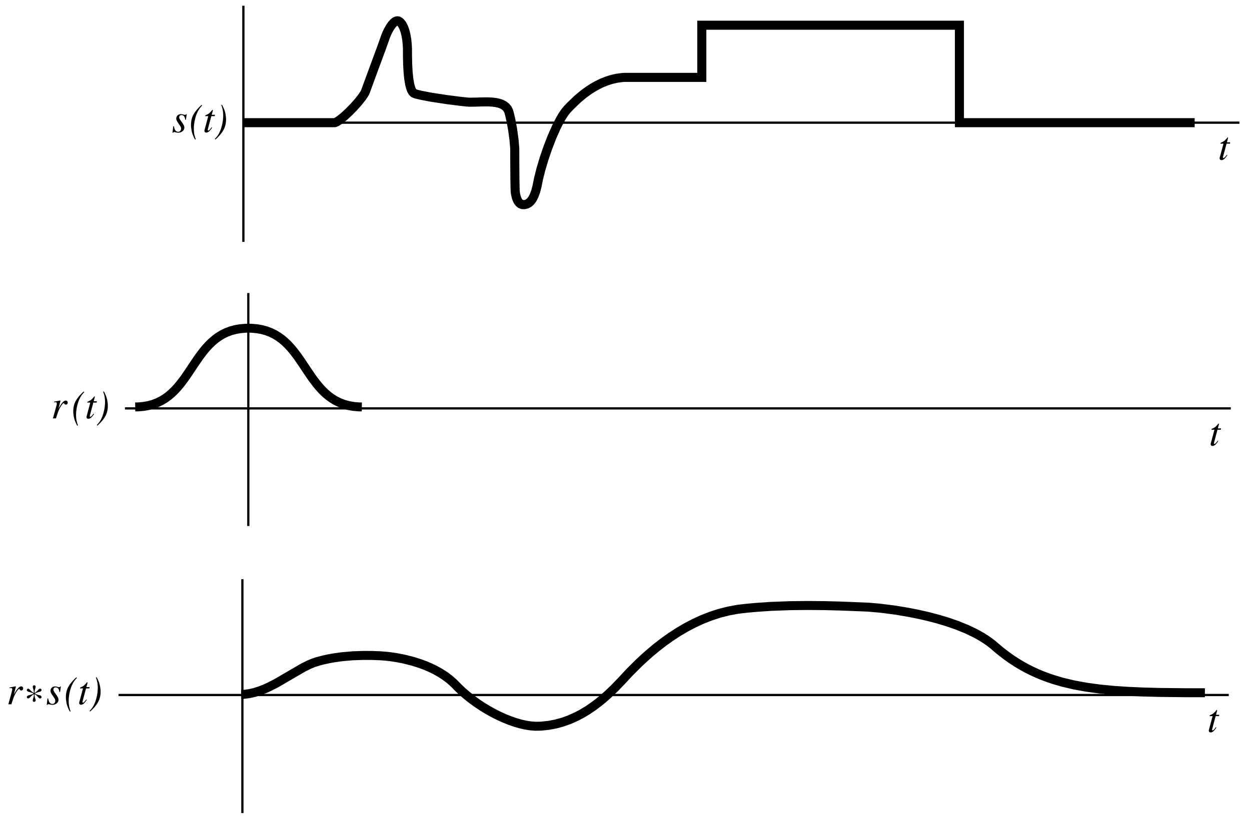


10
8
6
4
2
0
−2
−2
0
2
4
6
8
10


original data
smoothed (convoluted) data
10
8
6
4
2
0
−2
−2
0
2
4
6
8
10

Convolution

$$s * r \equiv \int_{-\infty}^{\infty} s(\tau) r(t-\tau) d\tau$$



Smoothing의 정도를 결정하는것은 ?　　　커널의 두께

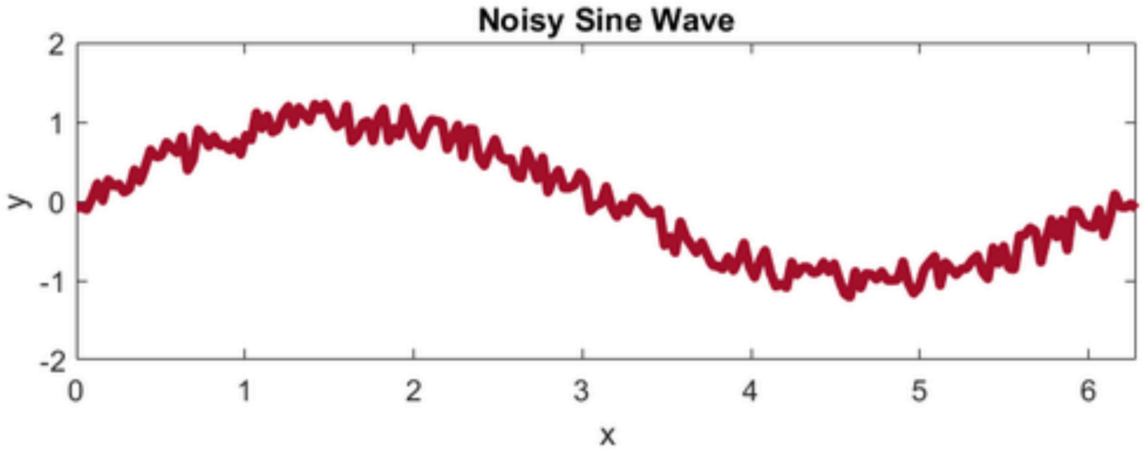
Convolution Theorem in FT

$$g * h \Longleftrightarrow G(f)H(f) \qquad \text{“Convolution Theorem”}$$

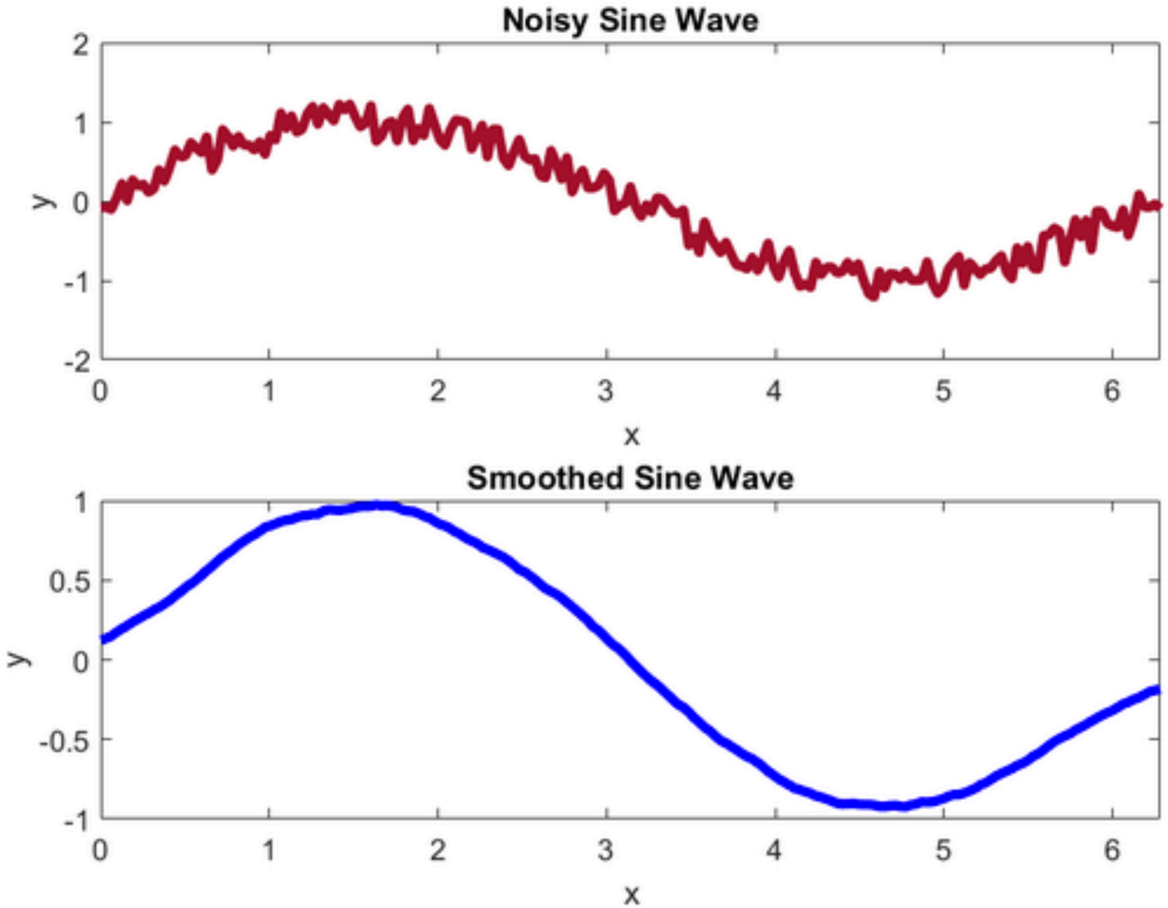
$$\sum_{k=-N/2+1}^{N/2} s_{j-k}\, r_k \quad \Longleftrightarrow \quad S_n R_n$$

Convolution theorem allows us to perform the operation very conveniently in Fourier Space.

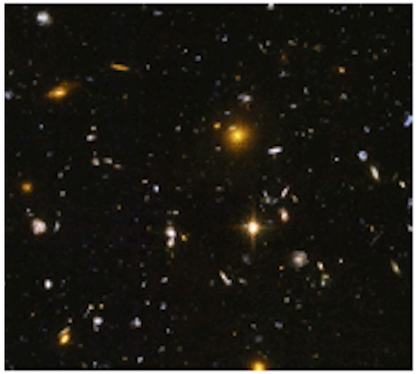
Convolution



Convolution

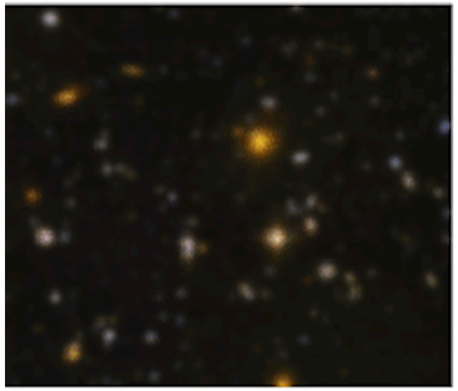


Convolution in 2D

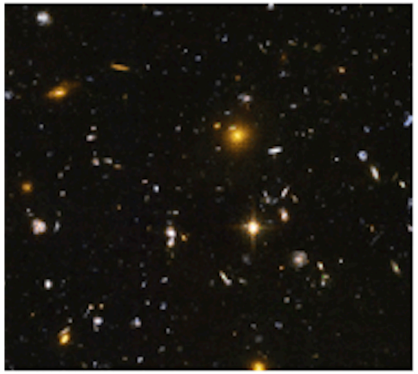


$\boxed{*}$

$=$

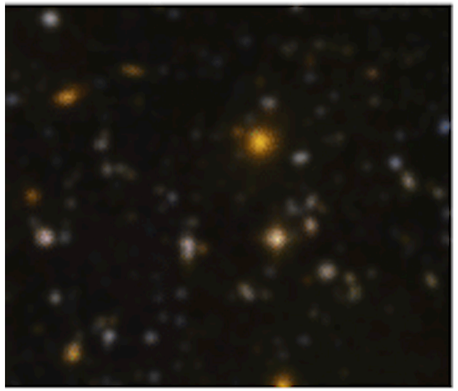


Deconvolution



$\boxed{*}$

$=$



$$AB=C$$

$$A=CB^{-1}$$

In real life,

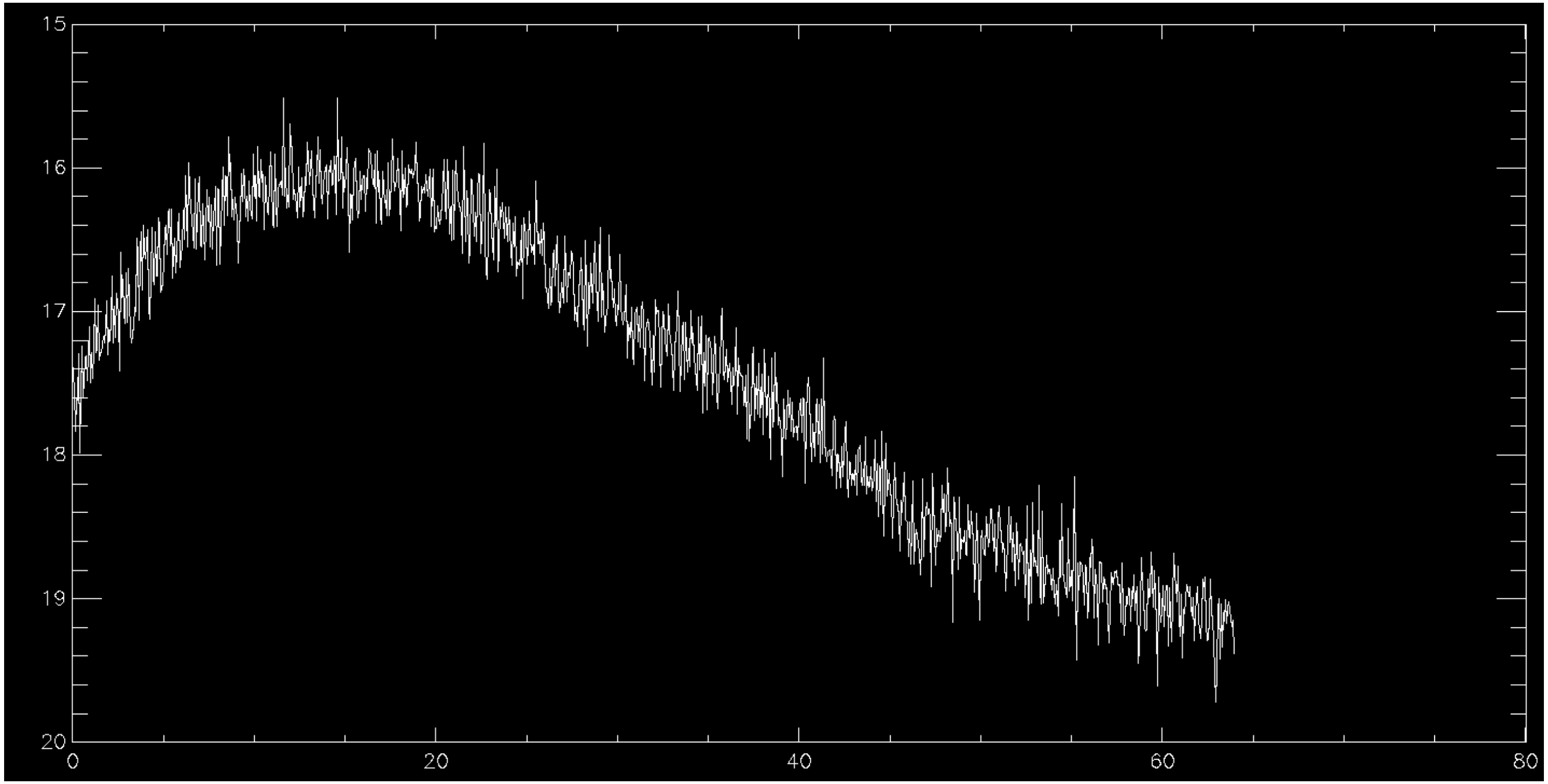
$$A(B+X)+Y=C$$

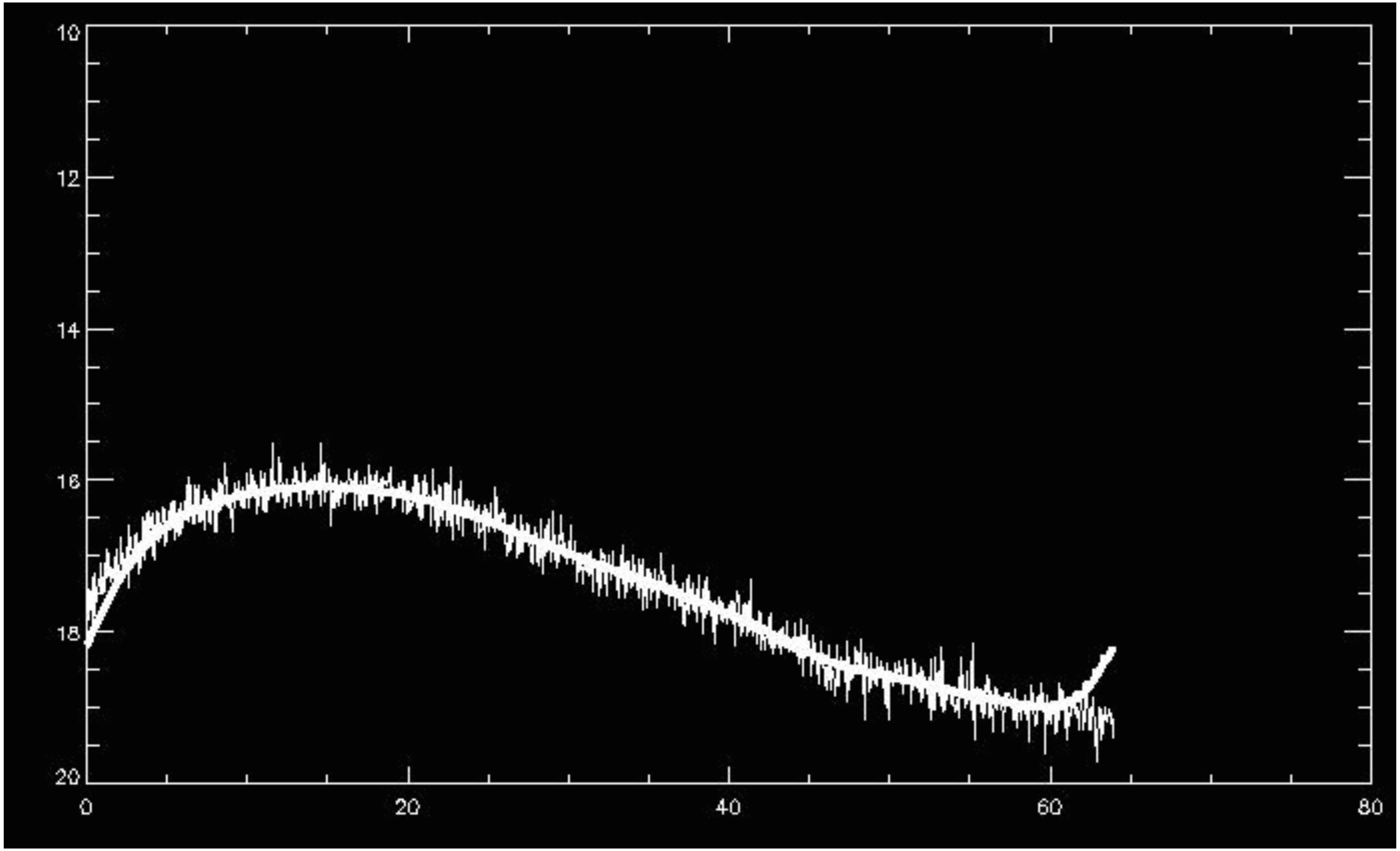
$$A=(C-Y)(B+X)^{-1}$$

X and Y are unknown.

과제 16

1. 코스 홈페이지에서 noisy supernova light curve를 다운받는다.
2. 광도곡선을 Plot 하여 (Noisy) 자료를 확인한다.
3. 4개의 Gaussian Kernel을 정의 한다 (Sigma=1, 2, 5, and 10 days).
4. 위의 4개의 Gaussian Kernel 을 사용해 Smoothing을 수행한다.
5. Real Space 에서 직접계산
6. Convolution Theorem (FFT)를 이용 (Padding 효과에 주의)
7. 5와 6의 두 결과를 비교한다.

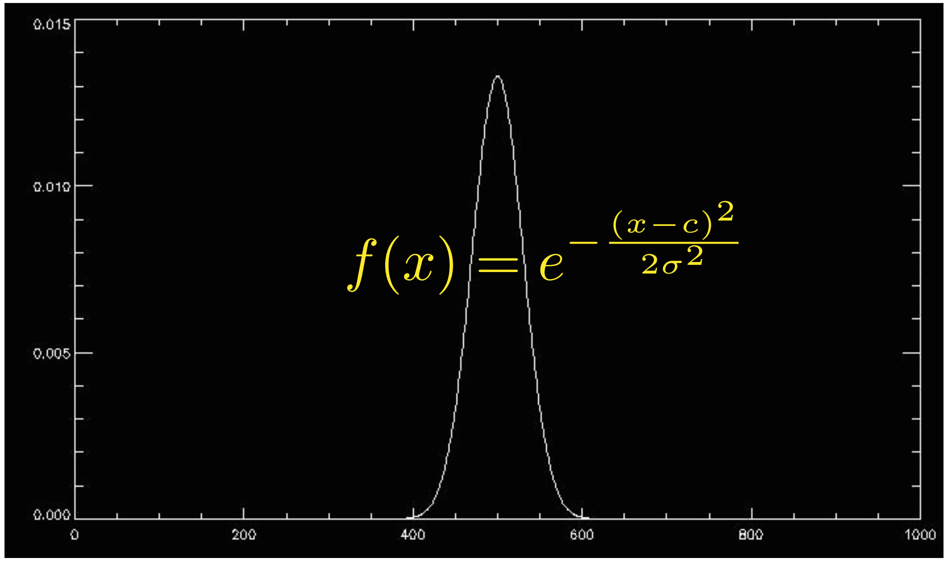

15
16
17
18
19
20
0
20
40
60
80



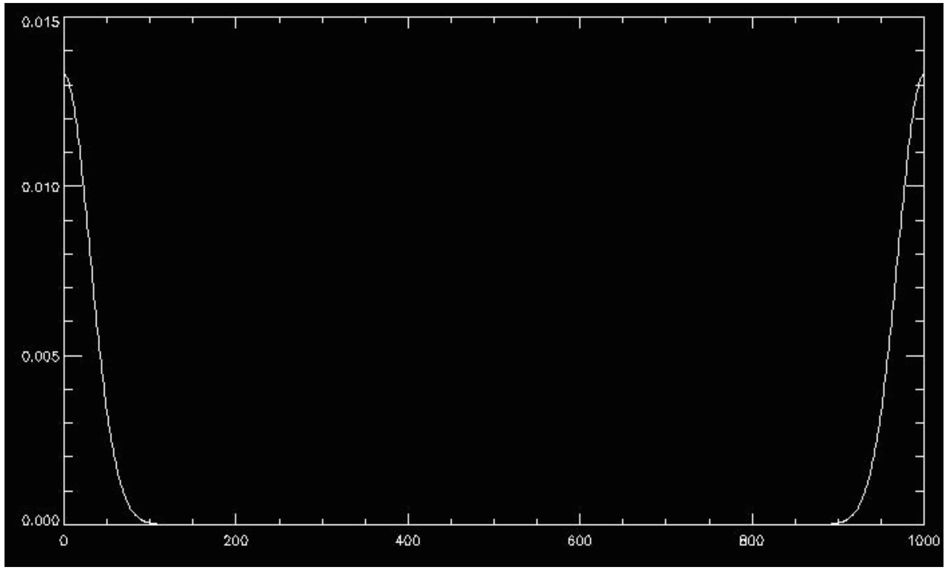
10
12
14
16
18
20
0
20
40
60
80

FFT로 Convolution Theorem 수행시 주의사항

- Gaussian Kernel Array의 원소갯수와 Supernova Light Curve Array의 원소갯수를 동일하게 만들어야 함 (1000개)
- Gaussian Kernel의 Peak의 위치는 첫번째 원소자리
- Normalization Convention에 주의 한다.

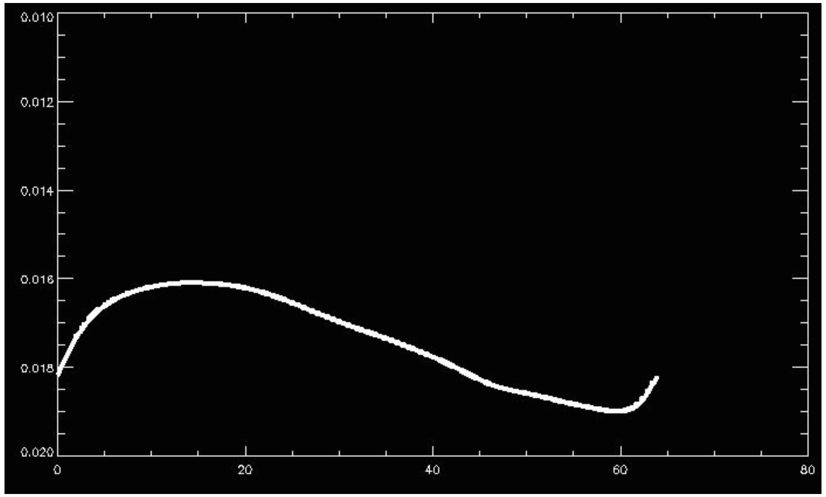

0.015
0.010
0.005
0.000
0
200
400
600
800
1000
$f(x) = e^{-\frac{(x-c)^2}{2\sigma^2}}$

X

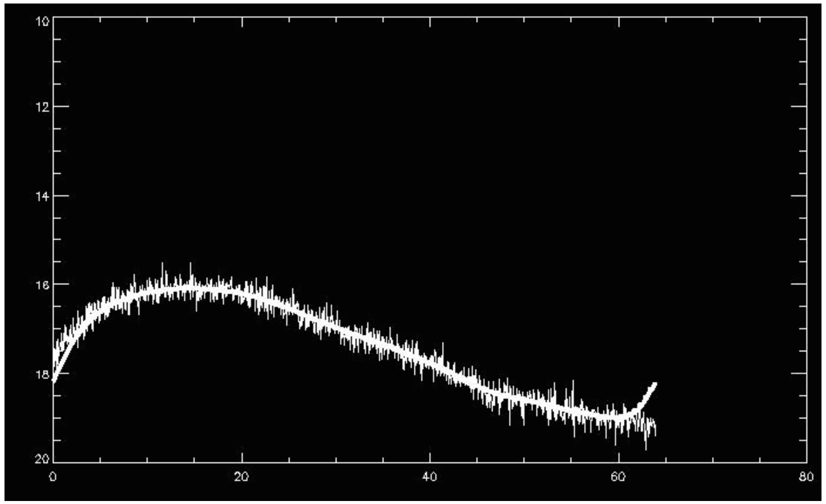

0.015
0.010
0.005
0.000
0
200
400
600
800
1000

O

convolution후
1000 배나 값이
작게 나옴



convolution후
값이 원래광도와
잘 일치



WRAP-AROUND PROBLEM