Q. $T(n) = 2T(n/2) + n$

$T(1) = 1$

Soln:-

$T(n) = 2T(n/2) + n$ —— (1)

$T(n/2) = 2T\left(\frac{n/2}{2}\right) + n/2$ {Putting n/2 in Place of n}.

$T(n/2) = 2T\left(\frac{n}{2^2}\right) + \frac{n}{2}$.

Now,

$T(n) = 2T(n/2) + n$. {Putting value of T(n/2) in (1)}

$= 2\left[2 \cdot T\left(\frac{n}{2^2}\right) + \frac{n}{2}\right] + n$

$= 2^2 T\left(\frac{n}{2^2}\right) + 2\frac{n}{2} + n$

$= 2^2 T\left(\frac{n}{2^2}\right) + 2n$.

Now,

$T\left(\frac{n}{2^2}\right) = 2T\left(\frac{n}{2^3}\right) + \frac{n}{2^2}$

Again Putting this value Calculate T(n).

$T(n) = 2^2\left[2T\left(\frac{n}{2^2}\right) + \frac{n}{2^2}\right] + 2n$

$= 2^3 T\left(\frac{n}{2^3}\right) + n + 2n = 2^3 T\left(\frac{n}{2^3}\right) + 3n$

Doing this for 'i' times

$T(n) = 2^i T(n/2^i) + in$ [Assuming $\frac{n}{2^i} = 1$] ; $\Rightarrow n = 2^i$

$n = 2^i$

Now, taking log on both sides.

$\log_2 n = \log_2 2^i$ { By using Property of log }.

$\boxed{i = \log_2 n}$

$T(n) = 2^i \, T(1) + i n.$

$= n \, T(1) + (\log_2 n)(n)$

$= n \cdot 1 + n \log n.$

$T(n) = O(n \log n)$

Q. $T(n) = 3T(n/4) + n$, $T(1) = 1$.

Solun:- $T(n) = 3T(n/4) + n$ ——— (1)

Now,

$$T(n/4) = \frac{n}{4} + 3T\left(\frac{n/4}{4}\right)$$

$$T(n/4) = n/4 + 3T(n/16)$$

$$T(n) = n + 3\left[n/4 + 3T(n/16)\right]$$ {Substituting $T(n/4)$ in (1)}.

$$= n + \frac{3n}{4} + 9T(n/16)$$

$$T(n/16) = n/16 + 3T\left(\frac{n/16}{4}\right).$$

$$= n/16 + 3T(n/64)$$

Again,

$$T(n) = n + 3n/4 + 9\left[n/16 + 3T(n/64)\right].$$

$$= n + \frac{3n}{4} + \frac{9n}{16} + 27T\left(\frac{n}{64}\right)$$

$$T(n) = n + 3^1 \cdot \frac{n}{4^1} + 3^2 \cdot \frac{n}{4^2} + 3^3 \cdot \frac{n}{4^3} + \dots + 3^k \cdot T\left(\frac{n}{4^k}\right).$$

The Series will terminate if,

$\frac{n}{4^k} = 1$ or, $n = 4^k$.

Now, taking log on both Sides.

$$\log n = \log 4^k$$

$$k = \log_4 n \qquad \cancel{\log n}$$

Now,

$$T(n) = n + \frac{3n}{4} + \frac{3^2 n}{4^2} + \frac{3^3 n}{4^3} + \cdots + 3^k T\left(\frac{n}{4^k}\right)$$

$$\leq n + \frac{3n}{4} + \frac{3^2 n}{4^2} + \frac{3^3 n}{4^3} + \cdots + 3^{\log_4 n} T(1)$$

$T(n) \leq 5n$ (Solving above Series).

$$\boxed{O(n) = n}$$