Möller-Trumbore algorithm

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1 Introduction

The Möller-Trumbore (or MT for the reminder of this lesson) algorithm is a fast ray-triangle intersection algorithm that was introduced in 1997 by Tomas Möller and Ben Trumbore in a paper titled "Fast, Minimum Storage Ray/Triangle Intersection". It is still considered today a fast algorithm which is often used in benchmarks to compare performances of other methods although, a fair comparison of ray-triangle intersection algorithms is a difficult thing to do because speed can depend on many factors such as the way algorithms are implemented, the type of test scene that is used, whether values are precomputed, etc.

The Möller-Trumbore algorithm takes advantage of the parameterization of P, the intersection point in terms of barycentric coordinates (which we talked about in the previous chapter). We learned in the previous chapter to calculate P, the intersection point, using the following equation:

$$P = wA + uB + vC$$

We also learned that w=1-u-v thus we can write:

$$P = (1 - u - v)A + uB + vC$$

If we develop, we get (equation 1):

$$P = A - uA - vA + uB + vC = A + u(B - A) + v(C - A)$$

Note that (B-A) and (C-A) are the edges AB and AC of the triangle ABC. The intersection P can also be written using the ray's parametric equation (equation 2):

$$P = O + tD$$

where t is the distance from the ray's origin to the intersection P. If we replace P in equation 1 with the ray's equation we get (equation 3):

$$\begin{gathered} O + tD = A + u(B - A) + v(C - A) \\ O - A = -tD + u(B - A) + v(C - A) \end{gathered}$$

On the left side of the equal sign, we have three unknowns (t, u, v) multiplied by three known terms (B-A, C-A, D). We can rearrange these terms and present equation 3 using the following notation (equation 4):

$$\begin{bmatrix} -D & (B-A) & (C-A) \end{bmatrix} \begin{bmatrix} t \\ u \\ v \end{bmatrix} = O - A$$

The left side of the equation has been rearranged into a row-column vector multiplication. This is the simplest possible form of matrix multiplication. You just take the first element of the raw matrix (-D, B-A, C-A) and multiply it by the first element of the column vector (t, u, v). Then you add the raw matrix's second element multiplied by the column vector's second element. Then finally add the third element of the raw matrix multiplied by the third element of the column vector (which gives you equation 3 again).

In equation 3, the term on the right side of the equal sign is a vector (O-A). B-A, C-A, and D are vectors as well, and t, u, and v (unknown) are scalars. This equation is about vectors. It combines three vectors in quantities defined by t, u, and v and it gives another vector as a result of this operation (be aware that in their paper, Möller and Trumbore use the term x-, y- and z-axis instead of t, u and v when they explain the geometrical meaning of this equation).

Let's say that P is the point where the ray intersects the triangle ABC. You will probably agree that the point P Cartesian coordinates (x, y, z) change as we move or rotate the triangle and the ray together. But, on the other hand, the barycentric coordinates of P are invariant. If the triangle is rotated, scaled, stretched, or translated, the coordinates u, v defining the position of P with respect to vertices A, B, and C will not change (see figure 1).

The MT algorithm is taking advantage of this property. Instead of solving the ray-triangle intersection equation using x, y, and z coordinates for the intersection point, they express that intersection point in terms of the triangle's barycentric coordinates u and v instead. A point lying on the triangle (our intersection point if it exists) can thus be either expressed in terms of Cartesian (x, y, z) or Barycentric (u, v) coordinates.

Finally, let's recall that the u and v coordinates of the point lying on the surface of a triangle as we learned in the previous chapter, can't be greater than 1 nor lower than 0. Their sum can't be greater than 1 either (u + v ¡= 1). They express coordinates of points defined inside a unit triangle (this is the triangle defined in u, v space by the vertices (0, 0), (1, 0), (0, 1) as shown in figure 1).

Let's summarize. What do we have so far? We have re-interpreted the three-dimensional x, y, z position of point P in terms of u and v barycentric coordinates going from one parameter space to another: from xyz-space to uv-space. We have also learned that points defined in uv-space are inside a unit triangle.

Equation 3 (or 4, they are identical) has three unknowns: u, v and t. Geometrically, we have just explained the meaning of u and v. But what about t? Simple: we will consider that t is the third axis of the u and v coordinate

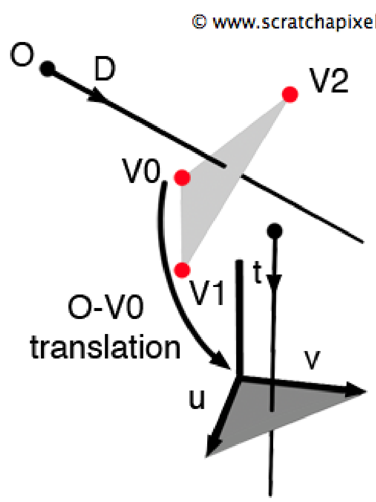

Figure 1: We can express the position of P in t, u, v space. t indicates the distance from P to the ray origin and is parallel to the y-axis. if P lies in the unit triangle it means that $0 <= u <= 1$, $0 <= v <= 1$ and $u + v <= 1$.

system we just introduced. We now have a coordinate system defined by three axes, u, v, and t. But again, geometrically let's explain what it means. We know that t expresses the distance from the ray origin to P, the intersection point. If you look at figure 1, we can see that we have created a coordinate system that allows us to fully express the position of the intersection P in terms of barycentric coordinates and distance from the ray origin to that point on the triangle.

Möller and Trumbore explain that the first part of equation 4 (the term O-A) can be looked at as a transformation moving the triangle from its original world space position to the origin (the first vertex of the triangle coincides with the origin). The other side of the equation has for effect to transform the intersection point from x, y, z space to "tuv-space" as explained above.