主观贝叶斯方法
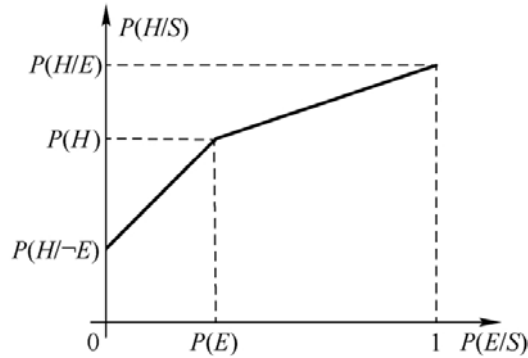
一、问题描述

在证据不确定的情况下，根据充分性度量 LS、必要性度量 LN、E 的先验概率 P(E)和 H 的先验概率 P(H)作为前提条件，作出 P(H|S)和 P(E|S)的关系图（示意图如下）。

二、算法描述

由公式：

$$P(H|S) = P(H|E) \times P(E|S) + P(H|\neg E) \times P(\neg E|S)$$

可得：

1) 当 $P(E|S) = 1$ 时

$$P(H|S) = P(H|E) = \frac{LS \times P(H)}{(LS - 1) \times P(H) + 1}$$

这就是证据肯定存在的情况。

2) 当 $P(E|S) = 0$ 时

$$P(H|S) = P(H|\neg E) = \frac{LN \times P(H)}{(LN - 1) \times P(H) + 1}$$

这就是证据肯定不存在的情况。

3) 当 $P(E|S) = P(E)$ 时

$$P(H|S) = P(H|E) \times P(E) + P(H|\neg E) \times P(\neg E) = P(H)$$

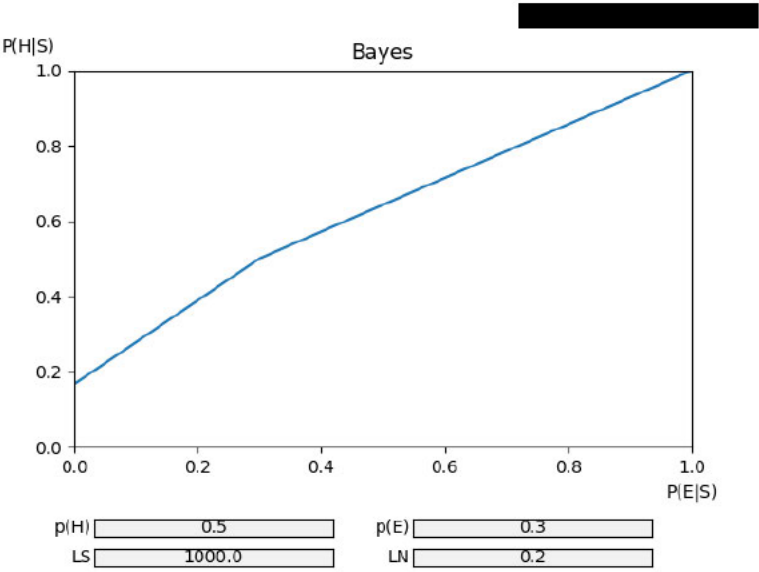即观察与证据无关，观察与结论无关。也就是说，该观察不影响结论，所以在该观察下，结论的概率没有变，还是原来的先验概率。
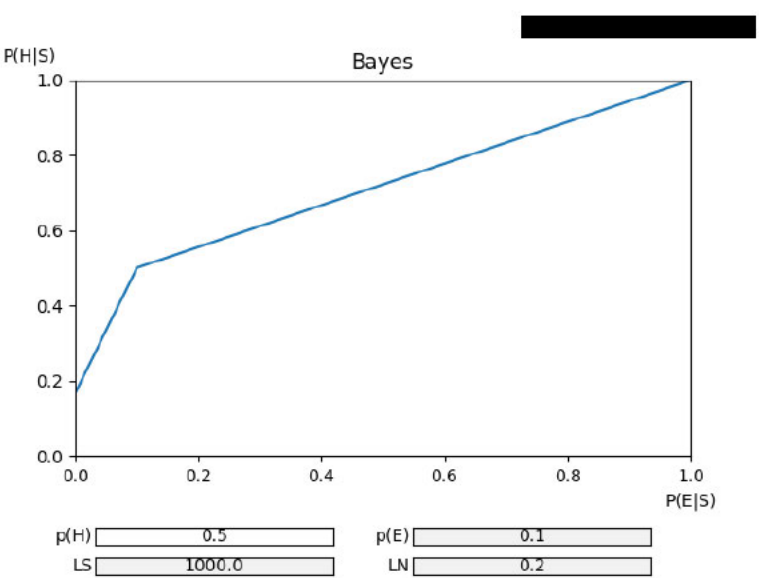
4) 其它情况

通过分段线性插值可得 EH 公式如下：

$$P(H|S) = \begin{cases} P(H|\neg E) + \dfrac{P(H) - P(H|\neg E)}{P(E)} \times P(E|S), 0 \le P(E|S) \le P(E) \\ P(H) + \dfrac{P(H|E) - P(H)}{1 - P(E)} \times [P(E|S) - P(E)], P(E) \le P(E|S) \le 1 \end{cases}$$

三、实验结果

1)

2)

四、源代码

```python
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.widgets import TextBox


pH = 0.5
pE = 0.3
LN = 0.2
LS = 1000.0


def EH(pE_S):
    pH_E = (LS * pH) / ((LS - 1) * pH + 1)
    pH_negE = (LN * pH) / ((LN - 1) * pH + 1)
    if 0 <= pE_S <= pE:
        pH_S = pH_negE + (pH - pH_negE) / pE * pE_S
    elif pE < pE_S <= 1:
        pH_S = pH + (pH_E - pH) / (1 - pE) * (pE_S - pE)
    return pH_S


func = np.vectorize(EH)
x = np.linspace(0, 1, 1000)
y = func(x)
fig, ax = plt.subplots()
ax.set_title('Bayes')
l, = plt.plot(x, y)

plt.xlim(0, 1)
plt.ylim(0, 1)
plt.xlabel('P(E|S)', x=1)
plt.ylabel('P(H|S)', y=1.04, rotation=0)
plt.subplots_adjust(bottom=.25)


def draw():
    y = func(x)
    l.set_ydata(y)
    plt.draw()


def submit1(expression):
    global pH
    pH = float(expression)
    draw()


pH_box = fig.add_axes([.15, .1, .3, .03])
pH_text_box = TextBox(pH_box, "p(H)", textalignment="center")
pH_text_box.set_val("0.5")
pH_text_box.on_submit(submit1)


def submit2(expression):
    global pE
    pE = float(expression)
    draw()
```

```python
pE_box = fig.add_axes([.55, .1, .3, .03])
pE_text_box = TextBox(pE_box, "p(E)", textalignment="center")
pE_text_box.set_val("0.3")
pE_text_box.on_submit(submit2)


def submit3(expression):
    global LS
    LS = float(expression)
    draw()


LS_box = fig.add_axes([.15, .05, .3, .03])
LS_text_box = TextBox(LS_box, "LS", textalignment="center")
LS_text_box.set_val("1000.0")
LS_text_box.on_submit(submit3)


def submit4(expression):
    global LN
    LN = float(expression)
    draw()


LN_box = fig.add_axes([.55, .05, .3, .03])
LN_text_box = TextBox(LN_box, "LN", textalignment="center")
LN_text_box.set_val("0.2")
LN_text_box.on_submit(submit4)

plt.rc("font", family='YouYu
plt.text(0.45, 30.25, '██████ ████████████', fontsize=10,
color='black')
plt.show()
```