2.165 Robotics

Fall 2025

Problem Set #2

Issued : Thu 09/25/2025
Due : Tue 10/07/2025

Problem 1:

In class, we have often come across derivatives of the form shown below:

$$\frac{\partial}{\partial \mathbf{x}}(\mathbf{x}^T\mathbf{G}\mathbf{x} + \mathbf{v}^T\mathbf{x})$$

For a constant symmetric matrix $\mathbf{G}$ and a constant vector $\mathbf{v}$, simplify explicitly the above expression containing a variable vector $\mathbf{x}$. How would your simplified answer change if $\mathbf{G}$ is not symmetric?

[Hint: Start with a vector $\mathbf{x}$ with 2, or maybe 3, components, (and appropriate $\mathbf{G}$ and $\mathbf{v}$) to do the explicit matrix algebra.]

Problem 2:

Consider the dynamic equations that we have examined for a two degree-of-freedom planar manipulator in the vertical plane with two rotational joints:

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \tau$$

where

$$\mathbf{q} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}, \mathbf{H} = \begin{bmatrix} H_{11} & H_{12} \\ H_{12} & H_{22} \end{bmatrix}, \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} = \begin{bmatrix} -2h\dot{q}_1\dot{q}_2 - h\dot{q}_2^2 \\ h\dot{q}_1^2 \end{bmatrix},$$

$$\begin{aligned} H_{11} &= m_1 l_{c1}^2 + I_1 + m_2(l_1^2 + l_{c2}^2 + 2l_1 l_{c2} cos q_2) + I_2 \\ H_{22} &= m_2 l_{c2}^2 + I_2 \\ H_{12} &= m_2 l_1 l_{c2} cos q_2 + m_2 l_{c2}^2 + I_2 \\ h &= m_2 l_1 l_{c2} sin q_2 \end{aligned}$$

(a) The Coriolis and centripetal torque vector $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}$ is a uniquely defined physical quantity. Show that, however, given $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}$ alone, a unique solution cannot be obtained for the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$. Identify two possible solutions of $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$.

(b) You have been given the Coriolis and centripetal torque vector $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}$. In addition, now use the condition that $\dot{\mathbf{H}} - 2\mathbf{C}$ is a skew symmetric matrix to solve for the unique value of $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$.

(c) Verify that $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ that you found above satisfies the following component-wise definition for the case of a n-link manipulator:

$$C_{ij} = \frac{1}{2}\dot{H}_{ij} + \frac{1}{2}\sum_{k=1}^{n}\left(\frac{\partial H_{ik}}{\partial q_j} - \frac{\partial H_{jk}}{\partial q_i}\right)\dot{q}_k$$

Problem 3:

(a) Consider a 2-link manipulator, in the horizontal plane. Assume that the manipulator is subject to a unit force at its endpoint, pointing towards its "shoulder". Compute and plot the joint torques required so that the manipulator does not move, as a function of configuration.

(b) Same question as (a), but in the vertical plane. (You may need some extra numerical assumptions, please make them simple and explicit.)

Problem 4:

Consider a 2-link manipulator in the vertical plane. Choosing arbitrary initial conditions, simulate the dynamics of the manipulator. Here, we assume no joint torque, no friction, only gravity, so that you can imagine this manipulator as a 2-link free pendulum.