2.165 Robotics

Fall 2025

Problem Set #3

Issued : Thu 10/09/2025
Due : Thu 10/23/2025

Problem 1:

We designed a P.D. controller to achieve a point-to-point task with a planar two-link manipulator before.

For impedance control of the manipulator in this problem, we proposed a control law for the joint torque τ that can achieve in the task space a restoring force $\mathbf{F}$ corresponding to the virtual programmable cartesian spring $\mathbf{K_p}$ and damper $\mathbf{K_d}$.

With gravity compensation, the control law was formulated in terms of the end-effector cartesian displacement error $\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x_d}$ as:

$$\tau = \mathbf{g}(\mathbf{q}) - \mathbf{J}^T(\mathbf{K_p}\tilde{\mathbf{x}} + \mathbf{K_d}\dot{\tilde{\mathbf{x}}})$$

Consider now the case of impedance control of a planar four-link manipulator for achieving a point-to-point task.

In addition to the virtual cartesian spring and damper at the end-effector, let us say we also have a virtual cartesian spring and damper at joint-2 (the joint adjacent to the "shoulder" of the manipulator).

Propose a control law similar to the one shown above for the torque τ for the case of this four-link manipulator.

- The control law must be written in terms of the end-effector cartesian displacement error $\tilde{\mathbf{x}}_\mathbf{e}$ and the joint-2 cartesian displacement error $\tilde{\mathbf{x}}_\mathbf{2}$.
- Neglect the gravity compensation term $\mathbf{g}(\mathbf{q})$. (You can find this term easily, we have done a similar exercise as part of Problem 3(b) in Problem Set#2.)

- No simulations are required (i.e. just propose the control law: NO coding is required in this problem!)

[Hint: Your answer should contain two different (why?) $\mathbf{J}^T$ matrices respectively pre-multipying the restoring force vectors corresponding to the mechanical impedances at the end-effector and the joint-2. Explicity derive expressions for the Jacobian matrices in terms of joint angles and link lengths of the manipulator.]

Problem 2:

Consider the dynamic equations in the vertical plane for a two degree-of-freedom planar manipulator with two rotational joints

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \tau$$

where

$$\mathbf{q} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}, \mathbf{H} = \begin{bmatrix} H_{11} & H_{12} \\ H_{12} & H_{22} \end{bmatrix}, \mathbf{C} = \begin{bmatrix} -h\dot{\theta}_2 & -h\dot{\theta}_1 - h\dot{\theta}_2 \\ h\dot{\theta}_1 & 0 \end{bmatrix}, \mathbf{g} = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix}$$

$$\begin{aligned}
H_{11} &= m_1 l_{c1}^2 + I_1 + m_2(l_1^2 + l_{c2}^2 + 2 l_1 l_{c2} cos\theta_2) + I_2 \\
H_{22} &= m_2 l_{c2}^2 + I_2 \\
H_{12} &= m_2 l_1 l_{c2} cos\theta_2 + m_2 l_{c2}^2 + I_2 \\
h &= m_2 l_1 l_{c2} sin\theta_2 \\
G_1 &= m_1 l_{c1} g cos\theta_1 + m_2 l_{c2} g cos(\theta_1 + \theta_2) + m_2 l_1 g cos\theta_1 \\
G_2 &= m_2 l_{c2} g cos(\theta_1 + \theta_2)
\end{aligned}$$

Design and simulate an adaptive controller without any initial knowledge of the constant parameters. The desired trajectory is

$$\theta_{d1} = 2(1 - e^{-t}) \qquad \theta_{d2} = 3(1 - e^{-2t})$$

(In the simulation, you can choose the real values of the parameters and the initial conditions by yourself.)

Problem 3:

Consider the dynamic equations of a robot manipulator

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \tau$$

where the matrix $\mathbf{D}$ is constant symmetric positive definite. Consider an adaptive P.D. controller for such a robot with the control law

$$\tau = \mathbf{Y}\hat{\mathbf{a}} - \mathbf{K_d}\dot{\tilde{\mathbf{q}}} - \mathbf{K_p}\tilde{\mathbf{q}}$$

and the adaptation law

$$\dot{\hat{\mathbf{a}}} = -\mathbf{P}\mathbf{Y}^T\mathbf{s}$$

where

$$\begin{aligned}
\mathbf{Ya} &= \mathbf{H}(\mathbf{q})\ddot{\mathbf{q}}_\mathbf{r} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}_\mathbf{r} + \mathbf{D}\dot{\mathbf{q}}_\mathbf{r} + \mathbf{g}(\mathbf{q}) \\
\mathbf{s} &= \dot{\mathbf{q}} - \dot{\mathbf{q}}_\mathbf{r} = \dot{\tilde{\mathbf{q}}} + \lambda\tilde{\mathbf{q}} \\
\tilde{\mathbf{q}} &= \mathbf{q} - \mathbf{q_d}
\end{aligned}$$

The adaptation and controller gain matrices $\mathbf{P}$, $\mathbf{K_d}$ and $\mathbf{K_p}$ are all symmetric positive definite.

Show that this controller will force the tracking error $\tilde{\mathbf{q}}$ to converge to zero.

[Hint: You may want to use the Lyapunov function candidate

$$V = \frac{1}{2}\mathbf{s}^T\mathbf{H}\mathbf{s} + \frac{1}{2}\tilde{\mathbf{a}}^T\mathbf{P}^{-1}\tilde{\mathbf{a}} + \frac{1}{2}\tilde{\mathbf{q}}^T(\mathbf{K_p} + \lambda\mathbf{K_d})\tilde{\mathbf{q}}$$

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