2.165 Robotics

Fall 2025

Problem Set #1

Issued : Th 09/11/2025
Due : Tu 09/23/2025

Problem 1:

Consider a two degree-freedom planar manipulator with two rotational joints with link lengths $l_1 = 4$ and $l_2 = 3$. The endpoint velocity is denoted by $V = [v_x, v_y]^T$.

(a) Given a desired endpoint velocity, find joint velocities that produce the desired endpoint velocity.

(b) Find singular configurations, and determine in which direction the endpoint can't move for each singular configuration.

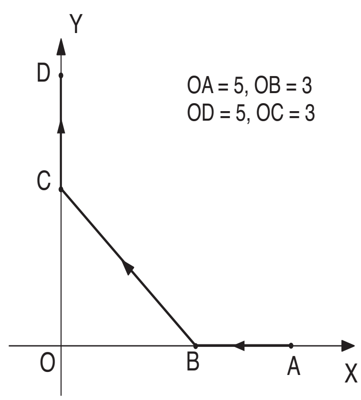

Figure 1: Trajectory for Problem 1. OA=5, OB=3, OC=3, OD=5. These dimensions have been chosen such that the trajectory lies within the workspace of the manipulator.

(c) Plot profiles of joint velocities when the endpoint is required to track a specified trajectory (shown in Figure 1) at a constant tangential speed.

Problem 2:

Consider a two degree-freedom planar manipulator with link lengths $l_1 = l_2 = 3m$. Measuring the ratio of joint torque to joint displacement, we identify the stiffness of each joint:

$$k_1 = 1 \times 10^5 Nm/rad \qquad k_2 = 2 \times 10^5 Nm/rad$$

(a) Compute the endpoint compliance matrix for the configuration of $\theta_1 = 30^0$ and $\theta_2 = 60^0$.

(b) Find the directions of maximum and minimum compliance at this configuration.

(c) Plot the maximum and minimum stiffness values as the function of θ_1 and θ_2.

Problem 3:

For the same manipulator as above, consider the problem of inverse kinematics using composite variables as we have discussed in class. Explain why defining $q_e(t)$ by the equation:

$$\dot{q}_r = \dot{q}_e - \lambda(q - q_e)$$

leads to an explicit inverse kinematics solution for q_e. Plot your result of q_e in simulation for x_d being a circle with radius 1.5 and constant tangential acceleration.