You try it

Random variables Y_1, Y_2, Y_3, Y_4, Y_5 are iid with pdf $f(y) = y/2$ for $0 < y < 2$ (0 otherwise).

a. Find the pdf of $\boxed{Y_{(4)}}$, the 4[th] smallest (2[nd] largest) order statistic.

b. Use the result in (a) to find $P(Y_{(4)} < 1)$.

pdf of $Y_{(k)}$ is $\dfrac{n!}{(k-1)!(n-k)!}\left[F(y)\right]^{k-1}\left(1-F(y)\right)^{n-k} f(y)$

$k=4, \quad n=5 \quad f(y) = \dfrac{y}{2} \quad 0<y<2$

$F(y) = \int_0^y f(t)\,dt = \int_0^y \dfrac{t}{2}\,dt = \dfrac{t^2}{4}\Big|_0^y = \dfrac{y^2}{4} \quad 0<y<2$

(a) pdf of $Y_{(4)}$ is $\dfrac{5!}{3!\,1!}\left(\dfrac{y^2}{4}\right)^3\left(1-\dfrac{y^2}{4}\right)\dfrac{y}{2}$

$= \dfrac{5\cdot\cancel{4}\cdot\cancel{3!}}{\cancel{3!}}\;\dfrac{y^7}{4^3}\;\dfrac{(4-y^2)}{\cancel{4}\,2} = \dfrac{5y^7(4-y^2)}{128} = \boxed{\dfrac{20y^7-5y^9}{128} \quad 0<y<2}$

(b) $P(Y_{(4)} < 1) = \int_0^1 \dfrac{1}{128}(20y^7 - 5y^9)\,dy$

$= \dfrac{1}{128}\left(\dfrac{20}{8}y^8 - \dfrac{5}{10}y^{10}\right)\Big|_0^1$

$= \dfrac{1}{128}\left(\dfrac{5}{2} - \dfrac{1}{2}\right) = \boxed{\dfrac{1}{64}}$