



COLUMBIA

MAILMAN SCHOOL
OF PUBLIC HEALTH

Introduction to Mathematical Statistics

3 credits

P8107

COURSE SCHEDULE

Mondays 10:00 – 11:20am, Hammer 322

Wednesdays 10:00 – 11:20am, Hammer LL110

INSTRUCTOR

R. Todd Ogden, PhD

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Office hours: by appointment

TEACHING ASSISTANT(S)

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COURSE DESCRIPTION

The first portion of this course provides an introductory-level mathematical treatment of the fundamental principles of probability theory, providing the foundations for statistical inference. Students will learn how to apply these principles to solve a range of applications. The second portion of this course provides a mathematical treatment of (a) point estimation, including evaluation of estimators and methods of estimation; (b) interval estimation; and (c) hypothesis testing, including power calculations and likelihood ratio testing.

CEPH DEPARTMENTAL DEGREE PROGRAM COMPETENCIES

This course is designed to help students attain mastery of the following degree competencies. Student achievement of these competencies will be measured through performance on the corresponding assessments.

CEPH Competency	Primary Assessment(s)	Secondary Assessments
Describe the basic concepts of probability, random variation and commonly used statistical probability distributions	Midterm and final exam	In-class group assignments, homework assignments, IRAs
Describe preferred methodological alternatives to commonly used statistical methods when assumptions are not met	Midterm and final exam	In-class group assignments, homework assignments, IRAs
Interpret results of statistical analyses found in public health studies	Midterm and final exam	In-class group assignments, homework assignments, IRAs

COURSE LEARNING OBJECTIVES

By the time you complete this course, you should be able to

- Understand and apply basic principles and laws of probability
- Calculate probabilities, means, variances using distributions of random variables.
- Know and apply common discrete and continuous distributions
- Make calculations based on joint, marginal, and conditional distributions of random variables
- Understand and apply the central limit theorem
- Compare properties of estimators and tests
- Derive point estimators, interval estimators, and hypothesis tests based on fundamental statistical principles

COURSE REQUIREMENTS

Required Text:

Mathematical Statistics with Applications (2008) by Wackerly D, Mendenhall W, and Scheaffer RL, 7th edition (ISBN: 9780495110811)

COURSE STRUCTURE

This course will be presented in a “flipped” format. All lectures are pre-recorded, and the videos will be made available in advance of class. Students are responsible for watching the videos before class, and class time will be devoted to assessments, discussion, problem sessions, and group work.

ASSESSMENT AND GRADING POLICY

Student grades will be based on:

Individual Readiness Assessments (IRA).....	10%
In-class group assignments	15%
Homework assignments	15%
Participation	10%
Midterm exam	25%
Final exam	25%

Individual Readiness Assessments (IRA)

The first 10 minutes of each class will be used for an assessment that is designed to determine each student's competency in the fundamental principles covered in the recorded lecture assigned for that session. At the end of the semester, the lowest three IRA grades will be dropped before computing the average.

IRAs must be handed in **no later than 10:10** each class period to receive any credit. Thus, a student arriving 5 minutes late to class will only have 5 minutes to complete the IRA.

During an IRA, students may access the textbook, their notes, course slides – any static source. They may not receive help from any other person (or artificial intelligence) during an IRA.

In-class Group Assessments

Groups of size 3 will be assigned randomly in advance of each class. A problem set will be released near the beginning of each class period, and groups will work together on the problem during the rest of the class period. Each group should submit one set of solutions, either hard copy or uploaded to Courseworks, by the end of class. Late submissions will not receive any credit. At the end of the semester, the lowest two group grades will be dropped before computing the average.

Homework Assessments

Homework will be assigned weekly. Students are encouraged to consult with classmates about homework solutions, but each student must independently write up and turn in his/her own solutions for each problem. Late homework will be accepted up to two days late with a 25% penalty per day late. After two days, homework will not be accepted. At the end of the semester, the lowest homework grade will be dropped before computing the homework average.

Participation

The participation score will be based on the average of two sources: Peer assessment and teaching team assessment (see rubric below). Scores will be converted to percentages (0% for an average of 1.0; 100% for an average of 5.0). The final participation score will be the average of these assessments multiplied by a measure of consistency/validity (scaled to be between 0 and 1; see rubric below).

After each class period, students should score their teammates (see rubric). They can refer to these when completing their peer assessments

Exams

The midterm and final will be written exams administered in class. Students will be allowed two formula sheets (front and back) for the midterm and an additional two for the final exam.

Grading

- A+ Reserved for highly exceptional achievement.
- A Excellent. Outstanding achievement.
- A- Excellent work, close to outstanding.
- B+ Very good. Solid achievement expected of most graduate students.
- B Good. Acceptable achievement.
- B- Acceptable achievement, but below what is generally expected of graduate students.
- C+ Low achievement, above minimally acceptable level.
- C Low achievement, but only minimally acceptable.
- C- Very low performance.
- F Failure. Course usually may not be repeated unless it is a required course.

Please refer to the [School Handbook](#) for further details on grading and good academic standing.

Courseworks

Individual- and course-level activity data are collected and maintained in CourseWorks, Panopto and other educational technology tools, and may be analyzed or monitored by the course faculty, teaching team, and/or the Office of Education to improve course experience and student support. Details about the information collected can be found here:

<https://community.canvaslms.com/t5/Instructor-Guide/How-do-I-view-analytics-for-a-student-in-a-course/ta-p/969>.

MAILMAN SCHOOL POLICIES AND EXPECTATIONS

Students and faculty have a shared commitment to the School's mission, values and oath.
mailman.columbia.edu/about/mission-history

Academic Integrity

Students are required to adhere to the Mailman School [Conduct and Community Standards](#), which includes the Code of Academic Integrity. Columbia Mailman and Columbia University take academic integrity very seriously. This instructor and course are no different. Should any student be suspected of an academic integrity violation, there will be a report submitted to the Center for Student Success & intervention/Student Conduct. After these offices conduct their process, if a student is found responsible for violating an academic integrity policy (see [Standards & Discipline/Academic Violations](#) and [Student Honor Code & Professional Guidelines](#)), they will receive 0 points on the assignment and there will be no opportunity to make up the points.

Please review the university, school, and course policies, as you are responsible for behaving according to the outlined expectations.

Personal Support

Students sometimes experience life challenges that require additional support and connection to resources. If you are experiencing difficult circumstances, please reach out for help and support. Student Support Services in the Office of Student Affairs is poised to connect with students, provide resource referrals, and provide ongoing, non-clinical support. They are a good place to start if you do not know where to turn.

If you would like to make a referral to student support, whether for yourself or someone else, the best way to do so is to fill out the form linked below, and student support will reach out directly. You may also email Meurcie Zignoli at mz3047@cumc.columbia.edu to connect to Student Support Services. Please note that this form is separate from other reporting structures in place for code of conduct violations or Title VI/ Title IX concerns. This form is also not an emergency response mechanism. In case of an emergency, please contact Public Safety at 212-305-7972: [Mailman Student Support Team Referral Form \(maxient.com\)](#)

Disability Access

In order to receive disability-related academic accommodations, students must first be registered with the Office of Disability Services (ODS). Students who have or think they may have a disability are invited to contact ODS for a confidential discussion at 212.854.2388 (V) 212.854.2378 (TTY), or by email at disability@columbia.edu. If you have already registered with ODS, please speak to your instructor to ensure that they have been notified of your recommended accommodations by Meredith Ryer (mr4075@cumc.columbia.edu), Assistant Director of Student Support and Mailman's liaison to the Office of Disability Services.

Bias Incidents

Our community at Columbia University's Mailman School of Public Health is committed to creating an inclusive working, learning, and living environment where all are respected. The occurrence of bias related incidents, involving conduct, speech, or expressions reflecting prejudice are an opportunity for learning and growing as a community.

As part of our efforts to create as inclusive a community as possible, when bias incidents occur at Columbia, we provide an opportunity for those involved to engage in education, advocacy and conversation. In this way, we work to address the incident and minimize the potential for future occurrences. Our community's tools to address bias include a reporting process and the Bias Incident Resource Team, plus resources within schools and various offices. You can access information about the Bias Reporting Process and FAQs [here](#).

Why Reporting Matters and How to File a Report

Our priority is ensuring that Columbia University is a safe community and workplace where we can learn, live, work and express ourselves. As members of the community, we have a shared

responsibility to uphold these standards and report behavior that violates these standards. The reporting options provide Columbia University community members an opportunity to share important information directly with appropriate offices. If you or a member of the community needs support please take the time to complete a report so we may provide support, care, and accountability: <https://universitylife.columbia.edu/report>

COURSE SCHEDULE

The schedule laid out here is approximate. Adaptations to this schedule will be made as necessary.

Week 1 – Introduction to Probability

Learning Objectives:

- Understand and apply basic principles of probability
- Calculate probabilities of events based on counting points
- Understand and apply conditional probability and independence of events

Reading:

Wackerly et al Sections 2.2-2.7 (36 pp.)

Week 2 – Some probability laws

Learning Objectives:

- Understand and apply multiplicative and additive laws of probability
- Understand and apply the law of total probability
- Understand and apply Bayes' rule
- Understand random variables

Reading:

Wackerly et al Sections 2.8-2.12 (20 pp.)

Week 3 – Discrete random variables

Learning Objectives:

- Understand and apply basic calculations using discrete random variables
- Calculate expected values of discrete random variables
- Make probability calculations based on random variables having binomial, geometric, hypergeometric, or Poisson distributions

Reading:

Wackerly et al Sections 3.1-3.5, 3.7, 3.8 (41 pp.)

Week 4 – Moments, moment generating functions, introduction to continuous random variables

Learning Objectives:

- Derive and apply moment generating functions for a given distribution
- Understand and apply basic calculations using continuous random variables
- Calculate expected values of continuous random variables
- Make probability calculations based on random variables having uniform or normal distributions

Reading:

Wackerly et al Sections 3.9, 4.1-4.5 (26 pp.)

Week 5 – Some continuous random variables and Chebyshev's Theorem

Learning Objectives:

- Make probability calculations based on random variables having gamma or beta distributions
- Derive and apply moment generating functions for continuous distributions
- Understand and apply Chebyshev's Theorem

Reading:

Wackerly et al Sections 4.6-4.10 (25 pp.)

Week 6 – Joint, marginal, and conditional probability distributions

Learning Objectives:

- Calculate probabilities of events using joint distributions
- Derive marginal and conditional distributions

Reading:

Wackerly et al Sections 5.1-5.4 (32 pp.)

Week 7 –Means, variances, covariances for bivariate distributions

Learning Objectives:

- Calculate expected value of functions of random variables
- Calculate expectations of sums and products of random variables
- Calculate means, variances, and covariances using joint distributions
- Calculate conditional expectations

Reading:

Wackerly et al Sections 5.5-5.9, 5.11 (29 pp.)

Week 8 – MIDTERM EXAM

In-class written exam

Week 9 – Functions of random variables

Learning Objectives:

- Calculate distributions of functions of random variables

Reading:

Wackerly et al Sections 6.1-6.5, 6.7 (37 pp.)

Week 10 – Sampling distributions and the central limit theorem

Learning Objectives:

- Identify sampling distributions
- Understand and apply the central limit theorem
- Calculate approximate probabilities using the normal approximation to the binomial distribution

Reading:

Wackerly et al Sections 7.1-7.3, 7.5 (38 pp.)

Week 11 – Estimation

Learning Objectives:

- Compare estimators based on bias, variance, and mean squared error
- Calculate and interpret confidence intervals
- Make sample size calculations

Reading:

Wackerly et al Sections 8.1-8.9 (47 pp.)

Week 12 – Properties of point estimation, methods of estimation

Learning Objectives:

- Understand and apply principles of relative efficiency, consistency, and sufficiency
- Derive estimators based on the method of moments and the maximum likelihood principle

Reading:

Wackerly et al Sections 9.1-9.8 (41 pp.)

Week 13 – Hypothesis testing

Learning Objectives:

- Understand and apply fundamental principles of statistical tests of hypotheses
- Calculate power of hypothesis tests and make sample size calculations
- Understand and apply the connection between hypothesis testing and interval estimation
- Calculate and properly interpret p-values

Reading:

Wackerly et al Sections 10.1-10.7 (32 pp.)

Week 14 – Testing for means and variances, likelihood ratio tests

Learning Objectives:

- Understand and apply the Neyman-Pearson lemma
- Carry out hypothesis testing for means and variances
- Construct likelihood ratio tests

Reading:

Wackerly et al Sections 10.8-10.11 (29 pp.)

PEER ASSESSMENT RUBRIC

Criteria	Poor (1)	Below Average (2)	Average (3)	Above Average (4)	Excellent (5)
Knowledge of Material	Had no apparent knowledge of the material. Was not able to contribute meaningfully to completion of the assignment.	Minimally understood the topic, missing several facts/concepts. Minimally contributed to the completion of the assignment.	Understood some of the fundamentals of the topic. Was able to comment on facts relevant to the completion of the assignment.	Understood the fundamentals of the topic. Was able to substantially contribute to the completion of the assignment.	Completely understood the fundamentals of the topic. Was able to contribute numerous relevant facts that shaped the direction of the completion of the assignment.
Leadership/Cooperation	Uncooperative and would not participate. Overall was a hinderance to the team.	Generally cooperative but helped little in moving the others and worked team forward.	Consistently listened to others and worked cooperatively to complete the assignment.	Showed initiative in helping the team. Was a key part of completing the assignment	Positive force in completing the assignment. Sought input and actively listened to others. Helped motivate team members.
Overall Assessment	If given a choice, I would refuse to work with this person on a team again.	If given a choice, I would be reluctant to work with this person on a team again.	If given a choice, I would be indifferent about working with this person on a team again.	If given a choice, I would be pleased to work with this person on a team again.	If given a choice, I would actively seek to work with this person on a team again.

VALIDITY RUBRIC

Criteria	Poor (0)	Sufficient (1/2)	Exemplary (1)
Quality of Assessment	Response to peer assessments do not show validity; have gross inconsistencies relative to other assessments	Response to peer assessments demonstrate limited validity; some inconsistencies without clear justification.	Response to peer assessments demonstrate complete validity; any inconsistencies are fully justified.