You try it

Random variables $X_1, X_2, X_3, X_4, Y_1, Y_2, Y_3, Y_4, Y_5, Z_1, Z_2$ are all iid $N(0,4)$ random variables. $= \sigma^2$
Let $\bar{X}$ be the average of the X's and $\bar{Y}$ be the average of the Y's, with s_X and s_Y representing the sample standard deviation of the X's and the Y's, respectively.

a. Find $P(\bar{X} > 2\bar{Y} + 1)$.
b. What is the distribution of $(Y_1^2 + Y_2^2 + Y_3^2 + Y_4^2 + Y_5^2)/4$?
c. Find the expected value of $\bar{X}^2/s_Y^2$. (Hint: the expected value of a random variable that has an F_{ν_1,ν_2} distribution is $\nu_2/(\nu_2 - 2)$ (provided that $\nu_2 > 2$).
d. Find the expected value of $\bar{Y}^2/(4\bar{X}^2 + Z_1^2 + Z_2^2)$. (Same hint as for part (c))

(a) $P(\bar{X} > 2\bar{Y} + 1) = P(\bar{X} - 2\bar{Y} > 1)$

$Var(aY) = a^2 Var(Y)$

$Var(X - Y)$

$\bar{X} \sim N(0, \frac{4}{4}) \Rightarrow \bar{X} \sim N(0,1)$

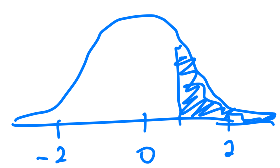
$\bar{Y} \sim N(0, \frac{4}{5}) \Rightarrow 2\bar{Y} \sim N(0, \frac{16}{5})$

$\bar{X} - 2\bar{Y} \sim N(0, 1 + \frac{16}{5})$

$\bar{X} - 2\bar{Y} \sim N(0, \frac{21}{5})$

$P(\bar{X} - 2\bar{Y} > 1) = P\left(\frac{\bar{X} - 2\bar{Y} - 0}{\sqrt{21/5}} > \frac{1-0}{\sqrt{21/5}}\right)$

$= P(Z > 0.4880)$



1- pnorm(0.4880) = $\boxed{0.3128}$

(b) $Y_1 \sim N(0,4)$ $\quad Y_2 \sim N(0,4)$ $\quad \ldots$ $\quad Y_5 \sim N(0,4)$

$\frac{Y_1}{2} \sim N(0,1)$ $\quad \ldots \quad$ $\frac{Y_5}{2} \sim N(0,1)$

$\frac{Y_1^2}{4} \sim \chi^2_1$ $\quad \ldots \quad$ $\frac{Y_5^2}{4} \sim \chi^2_1$

$\frac{Y_1^2}{4} + \ldots + \frac{Y_5^2}{4} \sim \chi^2_5$

$\boxed{\frac{Y_1^2 + \ldots + Y_5^2}{4} \sim \chi^2_5}$

$\boxed{\frac{\bar{X}^2}{s_Y^2}}$

(c) $\bar{X} \sim N(0,1)$ $\quad \bar{X}^2 \sim \chi^2_1$

$\boxed{\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}}$ $\quad \frac{\cancel{4} s_Y^2}{\cancel{4}} \sim \chi^2_4$

$\frac{\bar{X}^2/1}{s_Y^2/4} \sim F_{1,4}$

$E\left[\frac{4\bar{X}^2}{s_Y^2}\right] = \frac{4}{4-2} = 2$

$4E\left[\frac{\bar{X}^2}{s_Y^2}\right] = 2$

$E\left[\frac{\bar{X}^2}{s_Y^2}\right] = \frac{2}{4} = \boxed{\frac{1}{2}}$

Solution:

a. 0.2676
b. χ^2_5
c. $1/2$
d. [redacted] 1/5

(d) $\dfrac{\bar{Y}^2}{4\bar{X}^2 + Z_1^2 + Z_2^2}$

$\bar{Y} \sim N(0, \frac{4}{5})$

$\dfrac{\bar{Y}}{\sqrt{4/5}} \sim N(0,1) \Rightarrow \dfrac{5\bar{Y}^2}{4} \sim \chi^2_1$

$\bar{X} \sim N(0,1)$ $\quad \bar{X}^2 \sim \chi^2_1$

$\cancel{2\bar{X} \sim N(0,4)}$

$Z_1 \sim N(0,4)$

$\dfrac{Z_1}{2} \sim N(0,1)$ $\quad \dfrac{Z_1^2}{4} \sim \chi^2_1$ $\quad \dfrac{Z_2^2}{4} \sim \chi^2_1$

denom $= 4\left(\bar{X}^2 + \dfrac{Z_1^2}{4} + \dfrac{Z_2^2}{4}\right)$

$$\bar{X}^2 + \frac{Z_1^2}{4} + \frac{Z_2^2}{4} \sim \chi^2_3$$

$$\frac{\left(5\bar{Y}^2/4\right)/1}{\left(\bar{X}^2 + \frac{Z_1^2}{4} + \frac{Z_2^2}{4}\right)/3} \sim F_{1,3}$$

$$E\left[\frac{5\bar{Y}^2 \cdot 3}{4\left(\bar{X}^2 + \frac{Z_1^2}{4} + \frac{Z_2^2}{4}\right)}\right] = \frac{3}{3-2} = 3$$

$$15\,E\left[\frac{\bar{Y}^2}{4\bar{X}^2 + Z_1^2 + Z_2^2}\right] = 3$$

$$E\left[\frac{\bar{Y}^2}{4\bar{X}^2 + Z_1^2 + Z_2^2}\right] = \frac{3}{15} = \boxed{\frac{1}{5}}$$