INTRODUCTION TO PROBABILITY:

Bayes' Rule

Bayes' rule

- If $\{B_1, B_2, \ldots, B_k\}$ is a partition of S then for any event A,

$$P(B_j|A) = \frac{P(A|B_j)P(B_j)}{\sum_{i=1}^{k} P(A|B_i)P(B_i)}$$

- A more common version (two sets in the partition, i.e., $\{B, \bar{B}\}$):

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \boxed{\frac{P(A|B)P(B)}{P(A)}}$$

Example 3

- A hospital receives supplies from three vendors, with vendor A1 providing 60%, vendor A2 providing 30%, and A3 providing 10%. Of these, 2% of those from A1 are defective, 1% of those from A2 are defective, and 4% of those from A3 are defective.
  - What is the probability of a randomly selected item being defective?
  - If a randomly selected item is defective, what is the probability that it came from vendor A3?

B = event of item being defective

Want: $P(B)$

partition

A_1 = event supplied by A_1

A_2 = " " A_2

A_3 " " A_3

Know: $P(A_1) = 0.6$

$P(A_2) = 0.3$

$P(A_3) = 0.1$

$P(B|A_1) = 0.02$

$P(B|A_2) = 0.01$

$P(B|A_3) = 0.04$

$P(B) = ?$

(partition)

LOTP $\quad P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + P(B|A_3)P(A_3)$

$= (0.02)(0.6) + (0.01)(0.3) + (0.04)(0.1)$

$= \boxed{0.019}$

1.9% overall defective

$$P(A_3|B) = \frac{P(B|A_3)P(A_3)}{P(B)} \qquad \frac{(0.04)(0.1)}{0.019} = 0.21$$

21% chance came from vendor A_3.

Example 2

- A diagnostic test gives a positive result 90% of the time for patients who have the condition and gives a negative result 90% of the time for patients who do not. The condition is present in 1% of the population. If a person chosen at random from the population tests positive, what is the probability s/he has the condition?

	cond	not cond	TOTAL
pos	9,000	99,000	108,000
neg	1,000	891,000	892,000
TOTAL	10,000	990,000	1,000,000

$$\frac{9{,}000}{108{,}000} = 0.0833$$

8.33%