You try it

Each of 20 subjects walked on a treadmill while maximal oxygen intake was measured. Assume that these measurements follow a normal distribution with mean 54 and standard deviation 5.5.

$Y_i \sim N(54, (5.5)^2)$

a. calculate the probability that the mean of the 20 subjects will be between 53 and 57.
b. Find the point q such that the probability that the sample mean is less than q is 0.1.
c. Find the probability that the sample standard deviation will be at least 6.

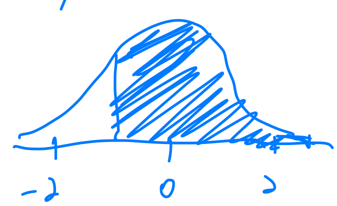
Y_i = measurement of subject i, $i = 1, \dots, 20$

$$\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i \sim N\left(54, \frac{(5.5)^2}{20}\right)$$

$$\sim N(54, 1.5125)$$

(a) $P(53 < \bar{Y} < 57) = P\left(\frac{53-54}{\sqrt{1.5125}} < \frac{\bar{Y}-54}{\sqrt{1.5125}} < \frac{57-54}{\sqrt{1.5125}}\right)$

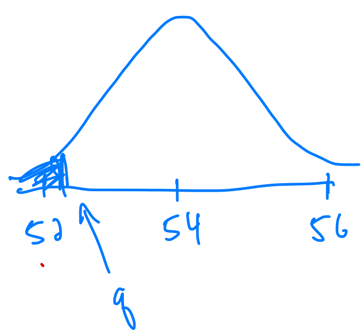
$= P(-0.8131 < Z < 2.4393)$ $\quad Z \sim N(0,1)$



In R pnorm(2.4393) - pnorm(-0.8131)

$\boxed{0.7846}$

(b)



We know (table) or qnorm(0,1)

$P(Z < -1.28) = 0.1$

$P\left(\frac{\bar{Y}-54}{\sqrt{1.5125}} < -1.28\right) = 0.1$

$P(\bar{Y} - 54 < -1.28\sqrt{1.5125}) = 0.1$

$P(\bar{Y} < \underbrace{54 - 1.28\sqrt{1.5125}}_{q}) = 0.1$

$q = \boxed{52.43}$

Solution:

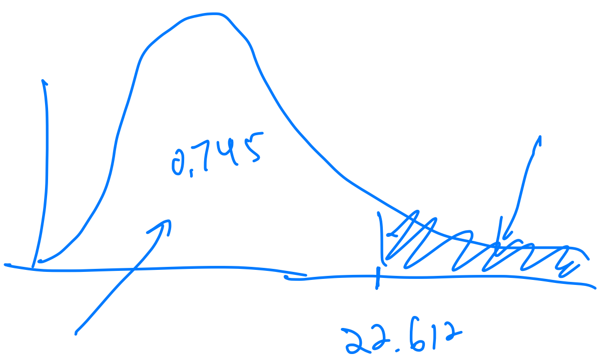
a. $P(-0.8131 < Z < 2.4393) \approx 0.7846$
b. 52.43
c. 0.255

(c) We know $\dfrac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$

$P(s > 6) = P(s^2 > 36) = P\left(\dfrac{(n-1)s^2}{\sigma^2} > \dfrac{(n-1)\,36}{\sigma^2}\right)$

χ^2_{19}

$(19)(36)/5.5^2 = 22.612$



pchisq(22.612, df = 19) $\longrightarrow$ 0.745

Area to left

$= 1 - 0.745 = \boxed{0.255}$