P8107 MIDTERM EXAM

Fall 2024

Name:

UNI:

For this exam you allowed up to two sheets of paper (front and back). No other reference materials may be used. You have 1 hour and 30 minutes to complete this exam. Be sure to show all your work. You may attach additional pages to your exam paper if needed.

Honor code:
I have not and will not give or receive aid in this examination nor have I concealed any violation of the University Honor Code.

Sign here: _____

1. (15 points) A public health department is studying two independent health risks in a population:
   Risk A: Exposure to a certain environmental toxin
   Risk B: Having a specific genetic predisposition
   In their study, they found that the probability that a person has neither risk factor is 0.6 and the probability of having the genetic predisposition (Risk B) is 0.3. Find the probability of being exposed to the environmental toxin (Risk A).

2. (15 points) A random variable $X \sim N(\mu_1, \sigma_1^2)$ is independent of $Y \sim N(\mu_2, \sigma_2^2)$. Use moment generating functions to find the distribution of $3Y - 2X$.

3. (10 points) A researcher is monitoring fasting blood glucose levels in a population at risk for diabetes. The average fasting blood glucose level in this population is 95 mg/dL with a standard deviation of 15 mg/dL. Suppose 100 individuals are randomly selected for a new diabetes prevention program. Calculate the probability that the average fasting blood glucose level for the 100 individuals is between 93 and 99 mg/dL. (You can leave your answer in terms of the standard normal cdf $\Phi(z) = P(Z \leq z)$.)

4. (20 points) Random variables $X_1, \ldots, X_{10}$ are iid $N(\mu_1, \sigma^2)$ and, independent of the X_i's, $Y_1, \ldots, Y_{20}$ are iid $N(\mu_2, \sigma^2)$. Let s_1^2 and s_2^2 be the sample variances of the X_i's and Y_i's, respectively.
   a. Find the mean and variance of $\frac{1}{2}(s_1^2 + s_2^2)$.
   b. Find the mean and variance of $\frac{1}{28}(9s_1^2 + 19s_2^2)$.

5. (20 points) A public health researcher is studying the relationship between two health indicators in a population: blood pressure X and cholesterol level Y. These indicators are known to be correlated. The researcher wants to create a combined health risk score that is a weighted sum of these two indicators. Suppose that it is known that the mean blood pressure in mmHg is 120, and the standard deviation is 10. The mean cholesterol level in mg/dL is 180 with a standard deviation of 20, and the correlation coefficient is ¼ . The combined health risk score is defined as $Z = \frac{2}{3}X + \frac{1}{3}Y$.
    a. Calculate the expected value of the combined health risk score.
    b. Calculate the variance of the combined health risk score.
    c. What would the variance of Z be if X and Y were independent?

6. (20 points) Random variables Y_1, Y_2, Y_3 are iid with cdf $F(y) = y^2$ for $0 < y < 1$. ($F(y) = 0$ for $y \leq 0$ and $F(y) = 1$ for $y \geq 1$.). Find $E[Y_{(3)} - Y_3]$, where $Y_{(3)} = \max(Y_1, Y_2, Y_3)$.