P8107 MIDTERM EXAM

Fall 2021

Name:

UNI:

For this exam you allowed up to two sheets of paper (front and back). No other reference materials may be used. You have 1 hour and 30 minutes to complete this exam. Be sure to show all your work. You may attach additional pages to your exam paper if needed.

Honor code:
I have not and will not give or receive aid in this examination nor have I concealed any violation of the University Honor Code.

Sign here: ________________________________

1. (15 points) In a certain population, it is known that 1/5 of the people have hypertension and 1/10 of the people have an irregular heartbeat. Furthermore, of the people who have an irregular heartbeat, 1/3 of them also have hypertension. You choose a person at random from this population.
    a. Find the probability that the person has *neither* of these health problems.
    b. Are these events (hypertension and irregular heartbeat) independent? Justify your answer.

2. (20 points) A random variable Y has pdf $f(y) = Ky(1-2y)^2$ for $0 < y < 2$ (and 0 otherwise).
    a. Find K.
    b. Find $E[Y]$.

3. (10 points) A random variable Y has cumulative distribution function $F(y) = 1 - \frac{4}{y^2}$ for $y > 2$ (and 0 otherwise). Find the pdf for $U = Y^2$.

4. (10 points) Y_1 has a Poisson distribution with parameter λ. Y_2 is independent of Y_1 and has a binomial(n, p) distribution. Knowing that $E[Y_1] = Var(Y_1) = \lambda$, $E[Y_2] = np$, and $Var(Y_2) = np(1 - p)$ find $E[(Y_1 - Y_2)^2]$.

5. (20 points) Independent random variables Y_1, Y_2, and Y_3 each have a Bernoulli(p) distribution (i.e., $P(Y_1 = 1) = p$ and $P(Y_1 = 0) = 1 - p$).
    a. Find the distribution of $U = \min(Y_1, Y_2, Y_3)$. (Hint: you do not need to know anything about order statistics to do this. What are the possible values for U?)
    b. Find the mgf of $V = Y_1 \times Y_2$. (Hint: first find the distribution of V.)
    c. Use the mgf from (b) to find $E[V]$ and $Var(V)$.

6. (25 points) A random variable Y_1 has pdf $f_1(y_1) = \frac{1}{2}y_1$ for $0 \le y_1 \le 2$. Conditional on $Y_1 = y_1$, a random variable Y_2 has a $U(0, y_1)$ distribution.
   a. Find the joint pdf of Y_1 and Y_2.
   b. Calculate $P(1 \le Y_1 \le 2, 1 \le Y_2 \le 2)$.
   c. Find the marginal distribution of Y_2.