$\frac{Y_i - \mu}{\sigma} \sim N(0,1)$ $\left(\frac{Y_i - \mu}{\sigma}\right)^2 \sim \chi^2_1$ **You try it**

Random variables Y_1, Y_2, Y_3, Y_4 are iid $N(\mu, \sigma^2)$ random variables and $\bar{Y} = \frac{1}{4}(Y_1 + Y_2 + Y_3 + Y_4)$.

a. What is the distribution of $U = \frac{1}{\sigma^2}((Y_1 - \mu)^2 + (Y_2 - \mu)^2 + (Y_3 - \mu)^2 + (Y_4 - \mu)^2)$?

b. What is the distribution of $V = \frac{1}{\sigma^2}((Y_1 - \bar{Y})^2 + (Y_2 - \bar{Y})^2 + (Y_3 - \bar{Y})^2 + (Y_4 - \bar{Y})^2)$?

c. If Y_5 is another random variable from the same distribution, independent of the others, what is the distribution of $V + (Y_5 - \mu)^2/\sigma^2$?

(a) $U = \frac{1}{\sigma^2}(Y_1 - \mu)^2 + \frac{1}{\sigma^2}(Y_0 - \mu)^2 + \frac{1}{\sigma^2}(Y_3 - \mu)^2 + \frac{1}{\sigma^2}(Y_4 - \mu)^2$

$\qquad = \left(\frac{Y_1 - \mu}{\sigma}\right)^2 +$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \left(\frac{Y_4 - \mu}{\sigma}\right)^2$

$\qquad\qquad \chi^2_1 \qquad\qquad \chi^2_1 \qquad\qquad \chi^2_1 \qquad\qquad \chi^2_1$

$$\boxed{U \sim \chi^2_4}$$

(b) $V = \frac{(Y_1 - \bar{Y})^2}{\sigma^2} + \cdots + \frac{(Y_4 - \bar{Y})^2}{\sigma^2}$ $\boxed{V \sim \chi^2_3}$

(c) $V + \left(\frac{Y_5 - \mu}{\sigma}\right)^2$ $\boxed{\sim \chi^2_4}$

$\quad\;\; \chi^2_3 \qquad\qquad \chi^2_1$