CONTINUOUS RANDOM VARIABLES

Expected values

discrete r.v.:

$$E[Y] = \sum_y y P(Y=y) = \sum_y y p(y)$$

Expected values for continuous random variables

- The expected value of a continuous random variable Y having pdf f is

$$E[Y] = \int_{-\infty}^{\infty} yf(y)dy$$

provided that the integral exists.

- If Y is a continuous random variable with pdf f, then the expected value of $g(Y)$ is

$$E[g(Y)] = \int_{-\infty}^{\infty} g(y)f(y)dy$$

provided that the integral exists.

Expected values of functions of a continuous random variable

- Theorem 2: If c is a constant and $g, g_1, g_2, \ldots, g_k$ are functions of a continuous random variable Y then
  1. $E[c] = c$
  2. $E[cg(Y)] = cE[g(Y)]$ $= cE[g(Y)]$
  3. $E[g_1(Y) + g_2(Y) + \cdots + g_k(Y)] = E[g_1(Y)] + E[g_2(Y)]$ +...+$E[g_k(Y)]$

- Q: If Y is a continuous random variable, what is $Var(Y)$?

$$Var(Y) = E[(Y - E[Y])^2]$$
$$= E[Y^2] - (E[Y])^2$$
$$= E[Y^2] - \mu^2$$

$$\mu = E[Y]$$

Expected values

- Example 4: If Y is a random variable with pdf $f(y) = 3y^2$ for $0 < y < 1$ ($f(y)$ is zero outside of this interval), find $E[Y]$ and $Var(Y)$.

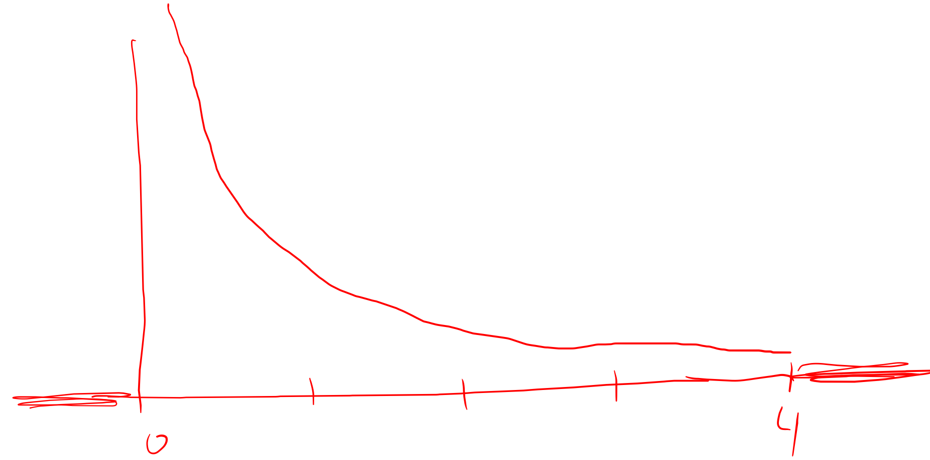
$$E[Y] = \int_{-\infty}^{\infty} y f(y)\,dy = \int_0^1 y \cdot 3y^2\,dy = \int_0^1 3y^3\,dy$$

$$= 3\left(\frac{1}{4}y^4\Big|_0^1\right) = 3\left(\frac{1}{4} - 0\right) = \boxed{\frac{3}{4}}$$

$$E[Y^2] = \int_{-\infty}^{\infty} y^2\, 3y^2\,dy = \int_0^1 3y^4\,dy = 3\left(\frac{1}{5}y^5\Big|_0^1\right) = 3\frac{1}{5} - 0 = \frac{3}{5}$$

$$Var(Y) = E[Y^2] - (E[Y])^2 = \frac{3}{5} - \left(\frac{3}{4}\right)^2 = \ldots = \boxed{0.0375}$$

Expected values



- Example 5: If Y is a random variable with pdf $f(y) = \frac{1}{4}y^{-\frac{1}{2}}$ for $0 < y < 4$ ($f(y)$ is zero outside of this interval), find $E[Y]$ and $Var(Y)$.

$$E[Y] = \int_{-\infty}^{\infty} y \frac{1}{4} y^{-1/2}\, dy = \frac{1}{4}\int_0^4 y^{\frac{1}{2}}\, dy = \frac{1}{4}\left(\frac{2}{3} y^{3/2}\Big|_0^4\right)$$

$$= \frac{1}{4}\frac{2}{3}4^{3/2} = \frac{4}{3}$$

$$E[Y^2] = \int_{-\infty}^{\infty} y^2 \frac{1}{4} y^{-\frac{1}{2}}\, dy = \frac{1}{4}\int_0^4 y^{3/2}\, dy$$

$$= \frac{1}{4}\left(\frac{2}{5} y^{5/2}\Big|_0^4\right) = \frac{1}{\cancel{4}}\frac{2}{5}\cancel{4}\,4\,2 = \frac{16}{5}$$

$$Var(Y) = \frac{16}{5} - \left(\frac{4}{3}\right)^2 = \ldots = 1{,}422$$