# MULTIVARIATE DISTRIBUTIONS

Expectations

# Expectations for multivariate distributions

- For a univariate random variable $Y$ with pdf $f(y)$, the expected value of a function of $Y$ is just

$$E[g(Y)] = \int_{-\infty}^{\infty} g(y)f(y)dy$$

- In the multivariate case, it is similar – for $Y_1$ and $Y_2$ with joint pdf $f(y_1, y_2)$, the expected value of a function of $Y_1$ and $Y_2$ is

$$E[g(Y_1, Y_2)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(y_1, y_2)f(y_1, y_2)dy_1dy_2$$

# Rules of expected values of functions of random vectors

- Theorem 6: If $c$ is a constant, then $E[c] = c$.
- Theorem 7: If $g(Y_1, Y_2)$ is a function of two random variables and $c$ is a constant, then $E[cg(Y_1, Y_2)] = cE[g(Y_1, Y_2)]$.
- Theorem 8: If $g_1(Y_1, Y_2)$, $g_2(Y_1, Y_2)$, …, $g_k(Y_1, Y_2)$ are functions of two random variables, then

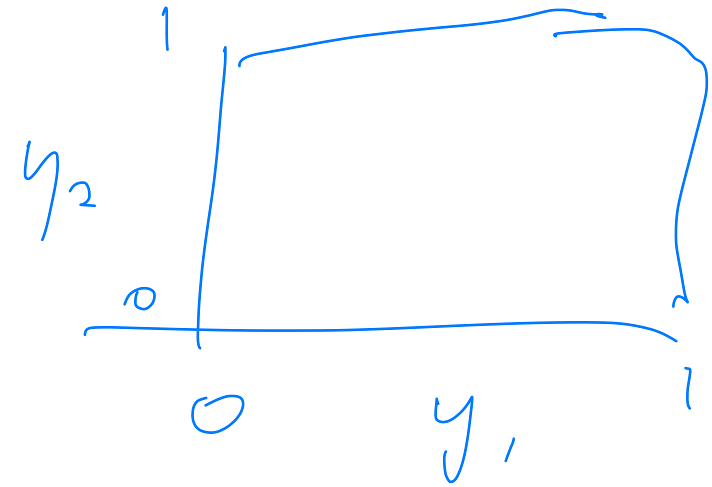
$$E[g_1(Y_1, Y_2) + g_2(Y_1, Y_2) + \cdots + g_k(Y_1, Y_2)] = E[g_1(Y_1, Y_2)] + E[g_2(Y_1, Y_2)] + \cdots + E[g_k(Y_1, Y_2)].$$

Example 9: If $Y_1$ and $Y_2$ have joint pdf $f(y_1, y_2) = 2(1 - y_1)$, $0 \le y_1 \le 1$, $0 \le y_2 \le 1$, find

a. $E[Y_1]$

b. $E[Y_1 Y_2]$

c. $Var(Y_1)$




(a) $E[Y_1] = \int_0^1 \int_0^1 y_1 \, 2(1 - y_1) \, dy_2 \, dy_1$

$= \boxed{\frac{1}{3}}$

(b) $E[Y_1 Y_2] = \int_0^1 \int_0^1 y_1 y_2 \, 2(1-y_1) \, dy_2 \, dy_1$

$= \boxed{\frac{1}{6}}$

(c) $Var(Y_1) = \underbrace{E[Y_1^2]} - (\underbrace{E[Y_1]}_{1/3})^2$

$E[Y_1^2] = \int_0^1 \int_0^1 y_1^2 \, 2(1-y_1) \, dy_2 \, dy_1$

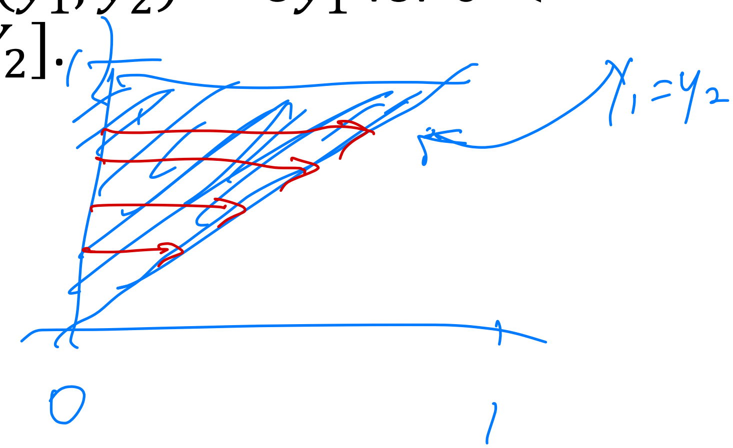
$= \boxed{\frac{1}{6}}$

$Var(Y_1) = \frac{1}{6} - \frac{1}{3^2} = \frac{1}{6} - \frac{1}{9} = \frac{3-2}{18}$

$= \boxed{\frac{1}{18}}$

Example 7 revisited: Random variables $Y_1$ and $Y_2$ are both proportions and we know that $Y_1 < Y_2$. If their joint pdf is $f(y_1, y_2) = 6y_1$ for $0 < y_1 < y_2 < 1$ (and zero otherwise), find $E[Y_1 - Y_2]$.

$$E[Y_1 - Y_2] = \int_0^1 \int_0^{y_2} (y_1 - y_2)\, 6y_1 \, dy_1 \, dy_2$$




$$= -\frac{1}{4}$$

# Rules of expected values of functions of random vectors

- Theorem 9: If $Y_1$ and $Y_2$ are independent, and if $g(Y_1)$ is a function only of $Y_1$ and $h(Y_2)$ is a function only of $Y_2$, then

$$E[g(Y_1)h(Y_2)] = E[g(Y_1)]E[h(Y_2)]$$

Corollary: If $Y_1$ and $Y_2$ are independent, then $E[Y_1Y_2] = E[Y_1]E[Y_2]$.