P8107 MIDTERM EXAM

Fall 2023

Name:

UNI:

For this exam you allowed up to two sheets of paper (front and back). No other reference materials may be used. You have 1 hour and 30 minutes to complete this exam. Be sure to show all your work. You may attach additional pages to your exam paper if needed.

Honor code:
I have not and will not give or receive aid in this examination nor have I concealed any violation of the University Honor Code.

Sign here: _____

1. (15 points) A random variable Y has pdf $f(y) = \frac{1}{10}$ for $10 < y < 20$ (and zero otherwise). Find the moment generating function for Y.

2. (15 points) A random variable Y has mgf $m(t) = (1 - 3t)^{-\frac{1}{2}}$ for $t < \frac{1}{3}$.
   a. Calculate $E[Y]$.
   b. Calculate $Var(Y)$.

3. (10 points) Seven patients were recruited for a clinical trial, 4 women and 3 men. They are ordered randomly for screening. What is the probability that all four women will be screened before any of the men?

4. (15 points) In a clinical trial, 30% of participants are randomly assigned to receive a placebo, 20% are assigned to Treatment A and the remaining 50% are assigned to Treatment B. It is known that patients receiving placebo get better with probability 3/8 and patients receiving Treatment A get better with probability 5/8. If the probability that ANY patient entering the clinical trial will get better with probability 23/40, what is the probability that a patient assigned to Treatment B will get better?

5. (25 points) Random variables Y_1 and Y_2 have joint pdf $f(y_1, y_2) = \frac{1}{2} e^{\frac{1}{2}y_1 - y_2}$ for $0 < Y_1 < Y_2 < \infty$.

   a. Find the marginal distribution of Y_2.
   b. Find the distribution of Y_2 conditional on $Y_1 = 1$.
   c. Are Y_1 and Y_2 independent? Explain why or why not.

6. (20 points) Random variables Y_1 and Y_2 have joint pdf $f(y_1, y_2) = \frac{1}{12}(y_1 + 2y_2)$ for $0 \leq y_1 \leq 2$ and $0 \leq y_2 \leq 2$.
    a. Calculate $P(Y_1 > 1)$.
    b. Calculate $P(Y_2 > 1 \mid Y_1 > 1)$.