P8110 Applied Regression II - Homework 6

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2025-11-12

Contents

Introduction

This homework analyzes a motor vehicle safety study where 300 drivers were asked to rate the importance of air conditioning and power steering in cars. We will compare ordinal logistic regression and multinomial logistic regression models.

Data Preparation

```r
# Load the data
cars <- read.csv("cars.csv", header = FALSE)
colnames(cars) <- c("sex", "age", "response", "count")

# Convert to factors with appropriate labels
cars$sex <- factor(cars$sex, levels = c(1, 2),
                   labels = c("Women", "Men"))
cars$age <- factor(cars$age, levels = c(1, 2, 3),
                   labels = c("18-23", "24-40", ">40"))
cars$response <- factor(cars$response, levels = c(1, 2, 3),
                        labels = c("No/Little", "Important", "Very Important"),
                        ordered = TRUE)

# Display the data
kable(cars, caption = "Motor Vehicle Safety Study Data")
```

Table 1: Motor Vehicle Safety Study Data

sex	age	response	count
Women	18-23	No/Little	26
Women	18-23	Important	12
Women	18-23	Very Important	7
Women	24-40	No/Little	9
Women	24-40	Important	21
Women	24-40	Very Important	15
Women	>40	No/Little	5
Women	>40	Important	14
Women	>40	Very Important	41
Men	18-23	No/Little	40
Men	18-23	Important	17
Men	18-23	Very Important	8
Men	24-40	No/Little	17
Men	24-40	Important	15
Men	24-40	Very Important	12
Men	>40	No/Little	8
Men	>40	Important	15
Men	>40	Very Important	18

```r
# Expand the data based on count
cars_expanded <- cars[rep(row.names(cars), cars$count), 1:3]
rownames(cars_expanded) <- NULL

# Summary statistics
cat("\nTotal Sample Size:", nrow(cars_expanded), "\n")
```

```
##
## Total Sample Size: 300
```

```r
table(cars_expanded$sex, cars_expanded$response)
```

```
##
##          No/Little Important Very Important
##   Women         40        47             63
##   Men           65        47             38
```

Question 1: Ordinal Logistic Regression

1.1 Model Specification [2 points]

The **ordinal logistic regression model** (proportional odds model) is:

$$\text{logit}[P(Y \leq j)] = \log\left(\frac{P(Y \leq j)}{P(Y > j)}\right) = \alpha_j - \beta_1 \text{sex} - \beta_2 \text{age2} - \beta_3 \text{age3}$$

where:

- $j = 1, 2$ (response categories: 1 = "No/Little", 2 = "Important")
- Y is the ordinal response variable
- α_j are the intercepts for each cumulative logit
- β_1 is the coefficient for sex (Men vs Women, with Women as reference)
- β_2 is the coefficient for age category 24-40 vs 18-23
- β_3 is the coefficient for age category >40 vs 18-23

The model assumes **proportional odds**, meaning the effect of predictors is the same across all cumulative logits.

```r
# Fit the ordinal logistic regression model
ordinal_model <- polr(response ~ sex + age,
                      data = cars_expanded,
                      weights = NULL,
                      Hess = TRUE)

summary(ordinal_model)
```

```
## Call:
## polr(formula = response ~ sex + age, data = cars_expanded, weights = NULL,
##     Hess = TRUE)
##
## Coefficients:
##            Value Std. Error t value
## sexMen   -0.5762     0.2262  -2.548
## age24-40  1.1471     0.2776   4.132
## age>40    2.2325     0.2915   7.659
##
## Intercepts:
##                           Value   Std. Error t value
```

```
## No/Little|Important       0.0435  0.2323     0.1874
## Important|Very Important  1.6550  0.2556     6.4744
##
## Residual Deviance: 581.2956
## AIC: 591.2956
```

1.2 Test for Proportional Odds Assumption [2 points]

We test the proportional odds assumption by comparing the ordinal model with a multinomial model.

Hypotheses:

- H_0: The proportional odds assumption holds (ordinal model is appropriate)
- H_a: The proportional odds assumption does not hold (multinomial model is needed)

```r
# Fit multinomial logistic regression for comparison
multinomial_model <- multinom(response ~ sex + age,
                              data = cars_expanded,
                              trace = FALSE)

# Likelihood ratio test
# Ordinal model has fewer parameters due to proportional odds constraint
logLik_ordinal <- logLik(ordinal_model)
logLik_multinomial <- logLik(multinomial_model)

# Test statistic
G <- -2 * (as.numeric(logLik_ordinal) - as.numeric(logLik_multinomial))

# Degrees of freedom
# Multinomial has 2*(k-1) coefficients for each predictor
# Ordinal has (k-1) coefficients for each predictor
# df = difference in number of parameters
df_ordinal <- length(coef(ordinal_model)) + length(ordinal_model$zeta)
df_multinomial <- length(coef(multinomial_model))
df <- df_multinomial - df_ordinal

# P-value
p_value <- 1 - pchisq(G, df)

cat("\n=== Test for Proportional Odds Assumption ===\n")
```

```
##
## === Test for Proportional Odds Assumption ===
```

```r
cat("H0: Proportional odds assumption holds\n")
```

```
## H0: Proportional odds assumption holds
```

```r
cat("Ha: Proportional odds assumption does not hold\n\n")
```

```
## Ha: Proportional odds assumption does not hold
```

```r
cat("Test Statistic (G):", round(G, 4), "\n")
```

```
## Test Statistic (G): 0.5934
```

```r
cat("Degrees of Freedom:", df, "\n")
```

```
## Degrees of Freedom: 3
```

```r
cat("P-value:", round(p_value, 4), "\n")
```

```
## P-value: 0.898
```

```r
cat("\nConclusion:",
    ifelse(p_value > 0.05,
           "We fail to reject H0 at  =0.05. The proportional odds assumption is reasonable.",
           "We reject H0 at  =0.05. The proportional odds assumption is violated."))
```

```
##
## Conclusion: We fail to reject H0 at  =0.05. The proportional odds assumption is reasonable.
```

1.3 Odds Ratio for Sex Effect [4 points]

We estimate the odds ratio of a **lower rating** (rating less important) between men and women.

```r
# Get coefficients and confidence intervals
coef_summary <- summary(ordinal_model)
coef_sex <- coef(ordinal_model)["sexMen"]

# Calculate 95% CI
se_sex <- coef_summary$coefficients["sexMen", "Std. Error"]
ci_lower <- coef_sex - 1.96 * se_sex
ci_upper <- coef_sex + 1.96 * se_sex

# Odds ratio and CI for LOWER rating
# Note: polr models logit[P(Y j)] = theta - beta*X
# So exp(-beta) gives OR for lower rating (P(Y j))
or_sex <- exp(-coef_sex)
or_ci_lower <- exp(-ci_upper)  # Note: CI bounds are reversed when taking negative
or_ci_upper <- exp(-ci_lower)

cat("\n=== Odds Ratio for Sex (Men vs Women) ===\n")
```

```
##
## === Odds Ratio for Sex (Men vs Women) ===
```

```r
cat("Coefficient ( 1):", round(coef_sex, 4), "\n")
```

```
## Coefficient ( 1): -0.5762
```

```r
cat("Odds Ratio (OR):", round(or_sex, 4), "\n")
```

```
## Odds Ratio (OR): 1.7793
```

```r
cat("95% CI:", "(", round(or_ci_lower, 4), ",", round(or_ci_upper, 4), ")\n")
```

```
## 95% CI: ( 1.1421 , 2.772 )
```

```r
cat("\n")
```

```r
# Interpretation
if (or_sex > 1) {
  cat("Interpretation:\n")
  cat("Men have", round(or_sex, 2), "times the odds of giving a LOWER rating\n")
  cat("(less important) compared to women, adjusting for age.\n\n")
  cat("This means women care MORE about air conditioning and power steering\n")
  cat("features compared to men, as men are more likely to rate these features\n")
  cat("as less important.\n\n")
  if (or_ci_lower > 1) {
    cat("The 95% CI does not include 1, indicating this difference is\n")
    cat("statistically significant at  =0.05 level.\n")
  }
} else {
  cat("Interpretation:\n")
  cat("Men have", round(or_sex, 2), "times the odds of giving a LOWER rating\n")
  cat("compared to women, adjusting for age.\n\n")
  cat("This means women care LESS about air conditioning and power steering\n")
  cat("features compared to men.\n")
}
```

```
## Interpretation:
## Men have 1.78 times the odds of giving a LOWER rating
## (less important) compared to women, adjusting for age.
##
## This means women care MORE about air conditioning and power steering
## features compared to men, as men are more likely to rate these features
## as less important.
##
## The 95% CI does not include 1, indicating this difference is
## statistically significant at  =0.05 level.
```

1.4 Probability of "Very Important" for Women Aged 18-23 [3 points]

We estimate $P(Y = 3)$ for women aged 18-23 using: $P(Y = 3) = 1 - P(Y \leq 2)$.

```r
# Create data for prediction: Women aged 18-23
newdata <- data.frame(sex = factor("Women", levels = c("Women", "Men")),
                      age = factor("18-23", levels = c("18-23", "24-40", ">40")))

# Predict cumulative probabilities
```

```r
cumulative_probs <- predict(ordinal_model, newdata = newdata, type = "probs")

# P(Y = 3) = 1 - P(Y <= 2)
prob_very_important <- cumulative_probs[3]

# Alternative calculation using formula
# Extract intercepts and coefficients
alpha1 <- ordinal_model$zeta[1]  # Intercept for Y <= 1
alpha2 <- ordinal_model$zeta[2]  # Intercept for Y <= 2

# For Women (sex=1, reference) and age 18-23 (reference), all predictors = 0
# P(Y <= 2) = exp(alpha2) / (1 + exp(alpha2))
p_Y_le_2 <- exp(alpha2) / (1 + exp(alpha2))
p_Y_eq_3 <- 1 - p_Y_le_2

cat("\n=== Probability Calculation for Women Aged 18-23 ===\n")
```

```
##
## === Probability Calculation for Women Aged 18-23 ===
```

```r
cat("Cumulative probabilities:\n")
```

```
## Cumulative probabilities:
```

```r
cat("P(Y = No/Little):", round(cumulative_probs[1], 4), "\n")
```

```
## P(Y = No/Little): 0.5109
```

```r
cat("P(Y = Important):", round(cumulative_probs[2], 4), "\n")
```

```
## P(Y = Important): 0.3287
```

```r
cat("P(Y = Very Important):", round(cumulative_probs[3], 4), "\n\n")
```

```
## P(Y = Very Important): 0.1604
```

```r
cat("Using formula: P(Y = 3) = 1 - P(Y   2)\n")
```

```
## Using formula: P(Y = 3) = 1 - P(Y   2)
```

```r
cat("Intercept  2:", round(alpha2, 4), "\n")
```

```
## Intercept  2: 1.655
```

```r
cat("P(Y   2) = exp( 2)/(1 + exp( 2)) =", round(p_Y_le_2, 4), "\n")
```

```
## P(Y   2) = exp( 2)/(1 + exp( 2)) = 0.8396
```

```
cat("P(Y = 3) = 1 - P(Y   2) =", round(p_Y_eq_3, 4), "\n")
```

```
## P(Y = 3) = 1 - P(Y   2) = 0.1604
```

Question 2: Multinomial Logistic Regression

2.1 Model Specification [2 points]

The **multinomial logistic regression model** with "No/Little importance" as the reference category:

$$\log\left(\frac{P(Y=j)}{P(Y=1)}\right) = \beta_{j0} + \beta_{j1}\text{sex} + \beta_{j2}\text{age2} + \beta_{j3}\text{age3}$$

where $j = 2, 3$ (Important, Very Important).

This gives us two equations:

For "Important" vs "No/Little":

$$\log\left(\frac{P(Y=2)}{P(Y=1)}\right) = \beta_{20} + \beta_{21}\text{sex} + \beta_{22}\text{age2} + \beta_{23}\text{age3}$$

For "Very Important" vs "No/Little":

$$\log\left(\frac{P(Y=3)}{P(Y=1)}\right) = \beta_{30} + \beta_{31}\text{sex} + \beta_{32}\text{age2} + \beta_{33}\text{age3}$$

```
# Relevel to make "No/Little" the reference category (it already is)
cars_expanded$response_unordered <- factor(cars_expanded$response,
                                           ordered = FALSE)
cars_expanded$response_unordered <- relevel(cars_expanded$response_unordered,
                                            ref = "No/Little")

# Fit multinomial logistic regression
multinom_model <- multinom(response_unordered ~ sex + age,
                           data = cars_expanded,
                           trace = FALSE)

summary(multinom_model)
```

```
## Call:
## multinom(formula = response_unordered ~ sex + age, data = cars_expanded,
##     trace = FALSE)
##
## Coefficients:
##                (Intercept)     sexMen  age24-40   age>40
## Important       -0.5907992 -0.3881301 1.128268 1.587709
## Very Important  -1.0390726 -0.8130202 1.478104 2.916757
##
```

```
## Std. Errors:
##                (Intercept)    sexMen  age24-40    age>40
## Important        0.2839756 0.3005115 0.3416449 0.4028997
## Very Important   0.3305014 0.3210382 0.4009256 0.4229276
##
## Residual Deviance: 580.7022
## AIC: 596.7022
```

```
# Display coefficients more clearly
cat("\n=== Model Coefficients ===\n")
```

```
##
## === Model Coefficients ===
```

```
print(round(coef(multinom_model), 4))
```

```
##                (Intercept)  sexMen age24-40 age>40
## Important          -0.5908 -0.3881   1.1283 1.5877
## Very Important     -1.0391 -0.8130   1.4781 2.9168
```

2.2 Odds Ratio for "Very Important" vs "No/Little" [2 points]

```
# Get coefficients for "Very Important" (row 2 in the coefficient matrix)
coef_matrix <- coef(multinom_model)
se_matrix <- summary(multinom_model)$standard.errors

# Coefficient for sex in "Very Important" equation
coef_sex_VeryImp <- coef_matrix[2, "sexMen"]
se_sex_VeryImp <- se_matrix[2, "sexMen"]

# Calculate OR and 95% CI
or_sex_VeryImp <- exp(coef_sex_VeryImp)
ci_lower_VeryImp <- exp(coef_sex_VeryImp - 1.96 * se_sex_VeryImp)
ci_upper_VeryImp <- exp(coef_sex_VeryImp + 1.96 * se_sex_VeryImp)

cat("\n=== Odds Ratio: Very Important vs No/Little (Men vs Women) ===\n")
```

```
##
## === Odds Ratio: Very Important vs No/Little (Men vs Women) ===
```

```
cat("Coefficient ( 31):", round(coef_sex_VeryImp, 4), "\n")
```

```
## Coefficient ( 31): -0.813
```

```
cat("Odds Ratio (OR):", round(or_sex_VeryImp, 4), "\n")
```

```
## Odds Ratio (OR): 0.4435
```

```r
cat("95% CI:", "(", round(ci_lower_VeryImp, 4), ",", round(ci_upper_VeryImp, 4), ")\n\n")
```

```
## 95% CI: ( 0.2364 , 0.8321 )
```

```r
cat("Interpretation:\n")
```

```
## Interpretation:
```

```r
if (or_sex_VeryImp < 1) {
  cat("Men have", round(or_sex_VeryImp, 3), "times the odds of rating 'Very Important'\n")
  cat("versus 'No/Little importance' compared to women, adjusting for age.\n")
  cat("This indicates women are more likely to rate these features as 'Very Important'.\n")
} else {
  cat("Men have", round(or_sex_VeryImp, 3), "times the odds of rating 'Very Important'\n")
  cat("versus 'No/Little importance' compared to women, adjusting for age.\n")
}
```

```
## Men have 0.444 times the odds of rating 'Very Important'
## versus 'No/Little importance' compared to women, adjusting for age.
## This indicates women are more likely to rate these features as 'Very Important'.
```

2.3 Probability of "Very Important" for Women Aged 18-23 [3 points]

```r
# Create prediction data
newdata_multinom <- data.frame(
  sex = factor("Women", levels = c("Women", "Men")),
  age = factor("18-23", levels = c("18-23", "24-40", ">40"))
)

# Predict probabilities
probs_multinom <- predict(multinom_model, newdata = newdata_multinom,
                          type = "probs")

cat("\n=== Probability from Multinomial Model for Women Aged 18-23 ===\n")
```

```
##
## === Probability from Multinomial Model for Women Aged 18-23 ===
```

```r
cat("P(Y = No/Little):", round(probs_multinom[1], 4), "\n")
```

```
## P(Y = No/Little): 0.5242
```

```r
cat("P(Y = Important):", round(probs_multinom[2], 4), "\n")
```

```
## P(Y = Important): 0.2903
```

```r
cat("P(Y = Very Important):", round(probs_multinom[3], 4), "\n\n")
```

```
## P(Y = Very Important): 0.1855
```

```r
# Manual calculation using coefficients
# For Women (reference) and age 18-23 (reference), all predictors = 0
intercept_Important <- coef_matrix[1, "(Intercept)"]
intercept_VeryImp <- coef_matrix[2, "(Intercept)"]

# exp(linear predictor)
exp_Important <- exp(intercept_Important)
exp_VeryImp <- exp(intercept_VeryImp)

# Denominator
denom <- 1 + exp_Important + exp_VeryImp

# Probabilities
p_NoLittle <- 1 / denom
p_Important <- exp_Important / denom
p_VeryImp <- exp_VeryImp / denom

cat("Manual calculation verification:\n")
```

```
## Manual calculation verification:
```

```r
cat("P(Y = No/Little) =", round(p_NoLittle, 4), "\n")
```

```
## P(Y = No/Little) = 0.5242
```

```r
cat("P(Y = Important) =", round(p_Important, 4), "\n")
```

```
## P(Y = Important) = 0.2903
```

```r
cat("P(Y = Very Important) =", round(p_VeryImp, 4), "\n")
```

```
## P(Y = Very Important) = 0.1855
```

Question 3: Model Selection [2 points]

```r
# Compare the two models
cat("\n=== Model Comparison ===\n\n")
```

```
##
## === Model Comparison ===
```

```r
# 1. Proportional odds assumption test result
cat("1. Proportional Odds Assumption Test:\n")
```

```
## 1. Proportional Odds Assumption Test:
```

```r
cat("   P-value:", round(p_value, 4), "\n")
```

```
##    P-value: 0.898
```

```r
if (p_value > 0.05) {
  cat("   Result: Assumption holds (p > 0.05)\n\n")
} else {
  cat("   Result: Assumption violated (p < 0.05)\n\n")
}
```

```
##    Result: Assumption holds (p > 0.05)
```

```r
# 2. Model fit comparison
cat("2. Model Fit Statistics:\n")
```

```
## 2. Model Fit Statistics:
```

```r
cat("   Ordinal Model:\n")
```

```
##    Ordinal Model:
```

```r
cat("     AIC:", round(AIC(ordinal_model), 2), "\n")
```

```
##      AIC: 591.3
```

```r
cat("     Log-likelihood:", round(as.numeric(logLik(ordinal_model)), 2), "\n\n")
```

```
##      Log-likelihood: -290.65
```

```r
cat("   Multinomial Model:\n")
```

```
##    Multinomial Model:
```

```r
cat("     AIC:", round(AIC(multinom_model), 2), "\n")
```

```
##      AIC: 596.7
```

```r
cat("     Log-likelihood:", round(as.numeric(logLik(multinomial_model)), 2), "\n\n")
```

```
##      Log-likelihood: -290.35
```

```r
# 3. Interpretation
cat("3. Probability Comparison for Women Aged 18-23:\n")
```

```
## 3. Probability Comparison for Women Aged 18-23:
```

```r
cat("   Ordinal Model P(Y=3):", round(prob_very_important, 4), "\n")
```

```
##    Ordinal Model P(Y=3): 0.1604
```

```r
cat("   Multinomial Model P(Y=3):", round(probs_multinom[3], 4), "\n")
```

```
##    Multinomial Model P(Y=3): 0.1855
```

```r
cat("   Difference:", round(abs(prob_very_important - probs_multinom[3]), 4), "\n\n")
```

```
##    Difference: 0.025
```

Conclusion and Recommendation

Model Choice:

I would choose the **Ordinal Logistic Regression Model**.

Reasons:

1. **Proportional Odds Assumption:** The test for proportional odds shows p-value = 0.898 , which is greater than 0.05. This indicates that the proportional odds assumption is reasonable for this data.
2. **Model Parsimony:** The ordinal model is more parsimonious, using fewer parameters (5 vs 8 parameters). It estimates one set of coefficients for the predictors, rather than separate coefficients for each response level.
3. **Better AIC:** The ordinal model has a lower AIC (591.3 vs 596.7), suggesting better model fit while accounting for model complexity.
4. **Meaningful Interpretation:** Since the response variable (importance rating) has a natural ordering, the ordinal model respects this structure and provides a more interpretable cumulative odds ratio across all levels.
5. **Consistent Results:** Both models yield similar probability estimates (e.g., P(Y=3) for women aged 18-23: 0.16 vs 0.185), supporting the use of the simpler ordinal model.

Appendix: Additional Diagnostics

```r
# Observed vs predicted frequencies
cat("\n=== Cross-tabulation of Observed Data ===\n")
```

```
##
## === Cross-tabulation of Observed Data ===
```

```
observed <- xtabs(count ~ sex + age + response, data = cars)
print(ftable(observed))
```

```
##              response No/Little Important Very Important
## sex   age
## Women 18-23                 26        12              7
##       24-40                  9        21             15
##       >40                    5        14             41
## Men   18-23                 40        17              8
##       24-40                 17        15             12
##       >40                    8        15             18
```

```
cat("\n=== Summary by Sex ===\n")
```

```
##
## === Summary by Sex ===
```

```
summary_sex <- cars_expanded %>%
  group_by(sex, response) %>%
  summarise(n = n(), .groups = "drop") %>%
  group_by(sex) %>%
  mutate(prop = n / sum(n))
print(summary_sex)
```

```
## # A tibble: 6 x 4
## # Groups:   sex [2]
##   sex   response           n  prop
##   <fct> <ord>          <int> <dbl>
## 1 Women No/Little         40 0.267
## 2 Women Important         47 0.313
## 3 Women Very Important    63 0.42
## 4 Men   No/Little         65 0.433
## 5 Men   Important         47 0.313
## 6 Men   Very Important    38 0.253
```

```
cat("\n=== Summary by Age ===\n")
```

```
##
## === Summary by Age ===
```

```
summary_age <- cars_expanded %>%
  group_by(age, response) %>%
  summarise(n = n(), .groups = "drop") %>%
  group_by(age) %>%
  mutate(prop = n / sum(n))
print(summary_age)
```

```
## # A tibble: 9 x 4
## # Groups:   age [3]
##   age   response           n  prop
```

```
##   <fct> <ord>          <int> <dbl>
## 1 18-23 No/Little         66 0.6
## 2 18-23 Important         29 0.264
## 3 18-23 Very Important    15 0.136
## 4 24-40 No/Little         26 0.292
## 5 24-40 Important         36 0.404
## 6 24-40 Very Important    27 0.303
## 7 >40   No/Little         13 0.129
## 8 >40   Important         29 0.287
## 9 >40   Very Important    59 0.584
```

Session Information

```r
sessionInfo()
```

```
## R version 4.3.3 (2024-02-29)
## Platform: x86_64-pc-linux-gnu (64-bit)
## Running under: Ubuntu 24.04.1 LTS
##
## Matrix products: default
## BLAS:   /usr/lib/x86_64-linux-gnu/blas/libblas.so.3.12.0
## LAPACK: /usr/lib/x86_64-linux-gnu/lapack/liblapack.so.3.12.0
##
## locale:
##  [1] LC_CTYPE=C.UTF-8       LC_NUMERIC=C           LC_TIME=C.UTF-8
##  [4] LC_COLLATE=C.UTF-8     LC_MONETARY=C.UTF-8    LC_MESSAGES=C.UTF-8
##  [7] LC_PAPER=C.UTF-8       LC_NAME=C              LC_ADDRESS=C
## [10] LC_TELEPHONE=C         LC_MEASUREMENT=C.UTF-8 LC_IDENTIFICATION=C
##
## time zone: America/New_York
## tzcode source: system (glibc)
##
## attached base packages:
## [1] stats     graphics  grDevices utils     datasets  methods   base
##
## other attached packages:
## [1] knitr_1.50      dplyr_1.1.4     broom_1.0.10    nnet_7.3-19
## [5] MASS_7.3-60.0.1
##
## loaded via a namespace (and not attached):
##  [1] vctrs_0.6.5       cli_3.6.5         rlang_1.1.6       xfun_0.53
##  [5] purrr_1.1.0       generics_0.1.4    glue_1.8.0        backports_1.5.0
##  [9] htmltools_0.5.8.1 rmarkdown_2.30    evaluate_1.0.5    tibble_3.3.0
## [13] fastmap_1.2.0     yaml_2.3.10       lifecycle_1.0.4   compiler_4.3.3
## [17] pkgconfig_2.0.3   tidyr_1.3.1       digest_0.6.37     R6_2.6.1
## [21] utf8_1.2.6        tidyselect_1.2.1  pillar_1.11.1     magrittr_2.0.4
## [25] tools_4.3.3
```