PROJECT 2

DENOISED A SIGNAL WITH SPECTRAL SUBSTRACTION

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INTRODUCTION

This paper will explain how to implement algorithm able to denoised a signal based on the spectral substraction. This is based one the work of S.F. Böll. The aim of the algorithm is to evaluate the power spectral density of the noise and then extract it from the noisy signal in order to recover a noisyless signal.

I DATA HYPOTHESIS

First of all we had to make some assumption about the signal which is going to be unnoised. Let $y(t) = x(t) + z(t)$ our signal with $x(t)$ the recovered signal and $z(t)$ the noise.

locally wide-sense stationary random processes

$x(t)$ the recovered signal and $z(t)$ the noise must be locally wide-sense stationary random processes ie $\forall t \in \mathbb{R}+ \ \exists d \in \mathbb{R}$ with $\forall k \in [t-d, t+d]$, we have $x(k)$ with ts mean function constant and its autocorrelation function a function only of τ

Also, $x(t)$ and $z(t)$ must be orthogonal ie <x,z> = 0

Noise available

We must be able to know the spectral density function of the noise. This algorithm need either to know it before either to be able to compute it thanks to noise only part in the signal. We will choose the second option. The first second of the signal will be noise only.

frames

The signal we need to unnoised is divised into many frame of approximatly 15ms. and we apply the algorithm on each frame separately (because we supposed that the hypothesis are locally true non always true) and we recover the signal by putting together at the end the recovered frames.

II First Step : Estimation of the power spectral density of the noise and the signal

The aim of this part is to evaluate the power spectral density of the noise. For this we will used the fact that the first second of our signal is noise only so we can compute for each frame that is noise the power spectral density and then compute the mean value. ie Let S = y(1), ..., y(N) the signal split into frames. $k \in [0, N]$ with $\forall i \in [0, k]\, y(i) = z(t)$ (noise only) We need to compute the power spectral density. First we compute the discrete fourier analysis for each frame wich is noise only. ie $Y(w_i) = \sum_{t=0}^{N-1}(x(t).e^{-jw_it}$ with $w_i = \frac{2\pi i}{N}$

Then we compute the power spectral density with

$\forall i \in [0, \frac{N}{2}], S_Z(w_i) = \frac{1}{N}|Z(w_i|^2$

Once this done, we compute for each frame the spectral power density if the signal (even the frame with the noise). We can compute this with the exact same formula as before.

III Second Step : Estimation of the power spectral density of the noiseless signal

Now we have an estimated power spectral density of the noise $S_Z(w_i)$ and the power spectral density of every frame of the signal $S_Y(w_i)$. We can compute for each frame separately the estimation of the power spectral density of the noiseless signal $S_X(w_i)$ with :
$S_X(w_i) = S_Y(w_i)$ - $S_Z(w_i)$ if nonnegative , 0 otherwise

IV Third Step, the filter

With all the previous step do before, we can compute the filter that will be able to denoised the signal. Let $A \in \mathbb{R}^N$ be the filter.

$\forall i \in [0, \frac{N}{2}]$, $A(w_i) = \sqrt{\frac{S_X(w_i)}{S_Y(w_i)}}$
And $A(w_{N-i-1}) = A(w_i)$

V STEP IV : EVALUTATION OF THE RECOVER SIGNAL

We can finally compute the denoised signal thanks to the filter. We have $X(w_i) = A(w_i)Y(w_i) for i \in [0, N-1]$
And the last step, leave the frequencie domain by compute the inverse frequency transformation and recover our signal $x(t)$. ie $x(t) = \frac{1}{N}\sum_{t=0}^{N-1}(x(t).e^{jw_i t}$