NekROM Report

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1 NekROM - Model Order Reduction Framework for Nek5000

This package include tools to help apply model-order reduction (MOR) to data produced by Nek5000 to generate reduced-order models (ROM). The generated ROMs can be run either in the Fortran driver embedded inside the Nek5000 userchk subroutine or can be run separately by provided driver scripts in Matlab, Python, and Julia. Users can also provide their own drivers which read the ROM operators and QOI factors from the ops/ and qoi/ directories.

1.1 Setup & Procedure

Set shell variables (in .bashrc for BASH users):

```
$ export MOR_DIR='/path/to/NekROM'
$ export PATH='$MOR_DIR/bin:$PATH'
```

Required files in NekROM case directory:

- Nek5000 case files e.g., .rea, .map, SIZE
- $caserom.usr, .usr file specific for NekROM cases (see '$MOR_DIR/examples')
- LMOR, specifies compile-time parameters
- $case.mor, specifies run-time parameters
- file.list, contains list of paths to the snapshots (relative path)

In addition to the .rea support for setting internal parameters, .mor files are supported for par-like dictionary. The possible key/values are described in templates/mpar.template.

Optional file:

- avg.list, contains list of paths to the average files

After ensuring the required files are in the case directory, run "makerom $caserom" to make a Nek5000 executable for ROM.

1.2 Parameters

Compile-time parameters (for setting memory allocation size) can be found in LMOR.

- ls: maximum number of snapshots
- lb: maximum number of total modes

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run-time parameters can be found in $case.mor.

- [general], header for general parameters
  - mode: off = offline, on = online, all = offline + online
  - field: v = velocity, t = temperature, vt = velocity + temperature
  - nb: number of POD modes (must be less than lb, default == lb)
- [pod], header for pod parameters
  - type: l2 = L^2 POD modes, h10, H^1_0 POD modes
  - mode0: avg = average 0th mode, state = user-defined in ub,vb,wb,tb
  - augment: 0 = no ABM, 1 = 0th interactions, 2 = diagonals, 3 = 1 + 2 [1]
- [copt], header for constrained optimization parameters [2][3]
  - mode: off, on, hybrid
- [filter], header for Leray filtering parameters [2][3]
  - location: none, convecting field, entire field
  - type: transfer function, differentiation
  - modes: > 0 number of modes to filter for tfunc < 0 percentage of nb
  - radius: radius of filter for the differentiation filter
- [qoi], header for qoi parameters
  - freq: frequency of QOI dump, if < 1 freq=iostep
  - drag: drag based on OBJ data

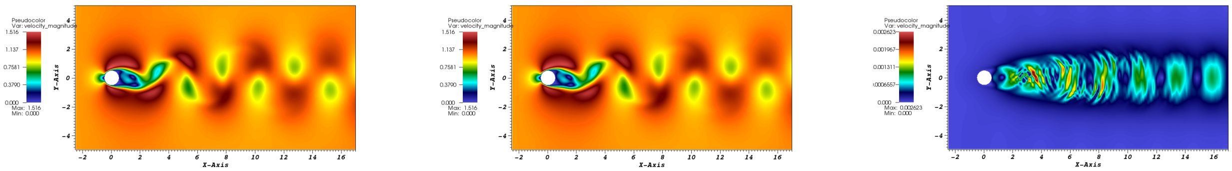
2 Example Cases

This section provides a collection of NekROM examples illustrating basic approaches and results. The examples here can be found in '$MOR DIR/examples' directory.

2.0.1 Flow pass a cylinder

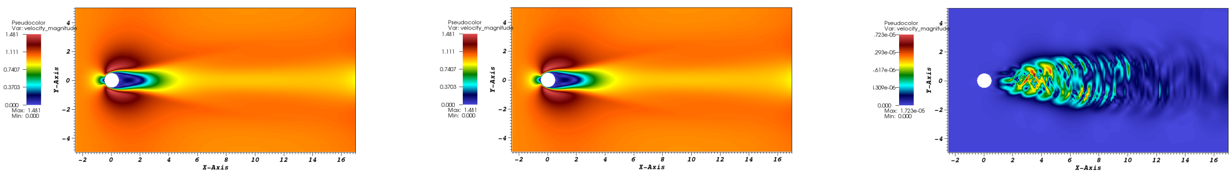
As a first example, we consider the problem of 2D flow past a cylinder. This is a canonical test case for ROMs because of its robust and low-dimensional attractor, which is manifest as a von Karman vortex street for $\mathrm{Re} = \mathrm{UD}/\nu > 34.37$ [4]. The Reynolds number in our test case Re = 100 and domain is $\Omega = [-2.5 : 17]D \times [-5 : 5]D$, with the unit-diameter cylinder centered at $[0, 0]$.

The reproduction results with ROM using $N = 20$ at Re = 100 is shown in figures 1–2



(a) FOM (b) ROM with $N = 20$ (c) Point-wise difference

Figure 1: Reconstruction results with ROM with $N = 20$ at Re $= 100$. a–b: Magnitude of instantaneous velocity of FOM and ROM at $t = 1000$. c: Point-wise difference between FOM and ROM with maximum difference $\approx 3e - 3$.



(a) FOM (b) ROM with $N = 20$ (c) Point-wise difference, $u = 1$

Figure 2: Reconstruction results with ROM with $N = 20$ at Re $= 100$. a–b: Magnutide of averaged velocity of FOM and ROM . c: Point-wise difference between FOM and ROM with maximum difference $\approx 1.5e - 5$.

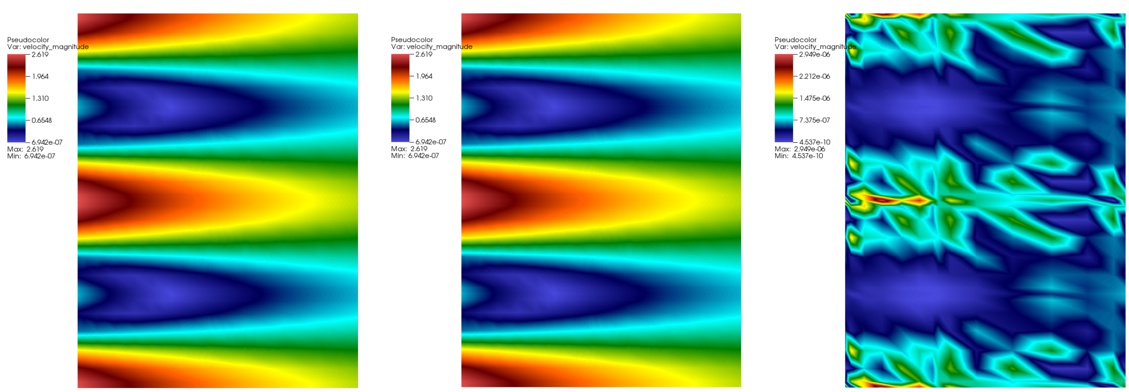
2.1 Kovasznay Solution

Kovasznay gives an analytical solution to the steady-state Navier-Stokes equations that is similar to the two-dimensional flow-field behind a periodic array of cylinders,

$$u_x = 1 - e^{\lambda x}\cos(2\pi y), \quad u_y = \frac{\lambda}{2\pi} e^{\lambda x}\sin(2\pi y), \tag{1}$$

where $\lambda \coloneqq \mathrm{Re}/2 - \sqrt{\mathrm{Re}^2/4 + 4\pi^2}$.

The reproduction results with ROM at Re $= 40$ is shown in figure 3.



(a) FOM (b) ROM (c) Point-wise difference

Figure 3: Reconstruction results with ROM at Re $= 40$.

	time	istep	current growth rate	total energy growth	$\frac{E(t)}{E(0)}$	$e^{(2\omega_i t)}$	$e^{(2\omega_i t)} - \frac{E(t)}{E(0)}$	error in the growth rate
FOM	200	10000	2.23368536E-03	2.23371912E-03	2.44363736	2.44486586	-1.22850700E-03	5.77317855E-04
ROM	200	10000	2.24312097E-03	2.24312098E-03	2.45284455	2.44486586	7.97868317E-03	3.64447902E-03
ROM without forcing	200	10000	2.32059287E-03	2.34185537E-03	2.55165495	2.44486586	1.06789083E-01	3.83078984E-02

2.2 Convection in 2D Annulus

We consider a natural convection between the two concentric cylinders as considered by Grigull & Hauf and presented by Van Dyke. The inner cylinder (diameter D) is slightly heated with with respect to the outer one (diameter $3D$). The Boussinesq approximation is used to formulate the equations of motion, valid in situations where density differences are small enough to be neglected everywhere except in the gravitational forcing. Normalizing the Navier-Stokes and energy equations with D for the length scale and D/U for the time scale ($U \approx \sqrt{\alpha g D(T_1 - T_0)}$) is the characteristic velocity in the given problem), and introducing nondimensional temperature $\theta = (T - T_0)/(T_1 - T_0)$, where T_0 and T_1 are the respective temperatures of the outer and inner cylinders, the governing equations take the nondimensional form

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{\sqrt{Gr}} \nabla^2 \mathbf{u} + \theta, \quad \nabla \mathbf{u} = 0, \tag{2}$$

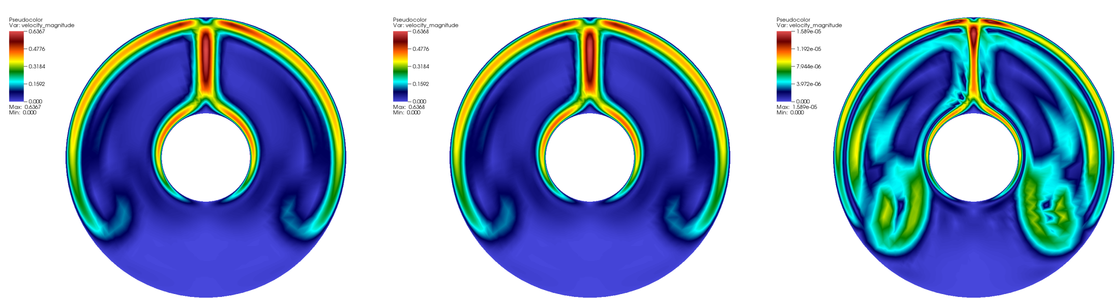
$$\frac{\partial \theta}{\partial t} + \mathbf{u} \cdot \nabla \theta = \frac{1}{\sqrt{Gr} Pr} \nabla^2 \theta. \tag{3}$$

Here $\mathbf{u}$ is the velocity vector, p is the pressure, θ is the temperature. The Grashof and Prandtl numbers are defined as

$$\mathrm{Gr} = \frac{\alpha g (T_1 - T_0) D^3}{\nu^2}, \quad \mathrm{Pr} = \frac{\nu}{\alpha}, \tag{4}$$

where ν is the kinematic viscosity, α is the volumetric thermal expansion coefficient, and g is the acceleration due to gravity.

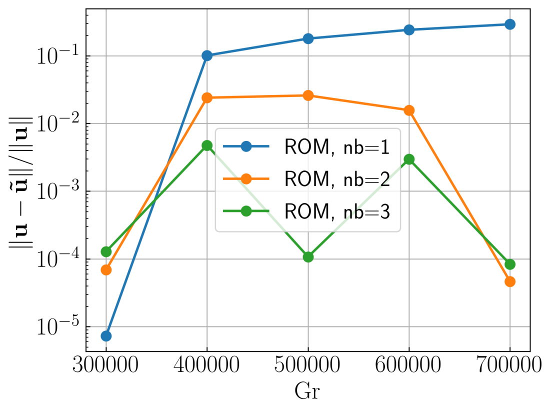
The reproduction results with ROM at Gr = 700000 is shown in figure 4.



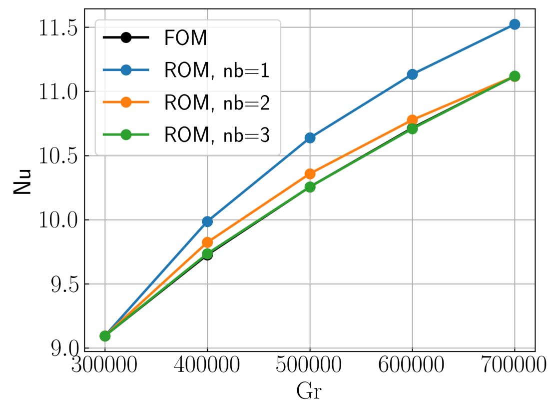
(a) FOM (b) ROM (c) Point-wise difference

Figure 4: Reconstruction results with ROM at Gr = 700000.

The pMOR results with one to three anchor points are shown in figure 5.



(a)



(b)

Figure 5: a: Relative H^1 error in the predicted solution with $N = 1,\ 2,\ 3$. b: Relative error in the predicted Nu with $N = 1,\ 2,\ 3$; the ROM prediction values with $N = 3$ are overlapped with the FOM values.

References

[1] Kento Kaneko and Paul Fischer. Augmented reduced order models for turbulence. *Frontiers in Physics*, page 808, 2022.

[2] Kento Kaneko, Ping-Hsuan Tsai, and Paul Fischer. Towards model order reduction for fluid-thermal analysis. *Nuclear Engineering and Design*, 370:110866, 2020.

[3] Ping-Hsuan Tsai and Paul Fischer. Parametric model-order-reduction development for unsteady convection. *Frontiers in Physics*, page 711, 2022.

[4] Ke Ding and Arne Pearlstein. Flow past a linearly-sprung, freely-rotatable circular cylinder with an eccentric center of mass. In *APS Division of Fluid Dynamics Meeting Abstracts*, pages P08–001, 2021.

[5] P Fischer, J Kruse, J Mullen, H Tufo, J Lottes, and S Kerkemeier. Nek5000: Open source spectral element cfd solver. *Argonne National Laboratory, Mathematics and Computer Science Division, Argonne, IL, see https://nek5000.mcs.anl.gov*, 2008.