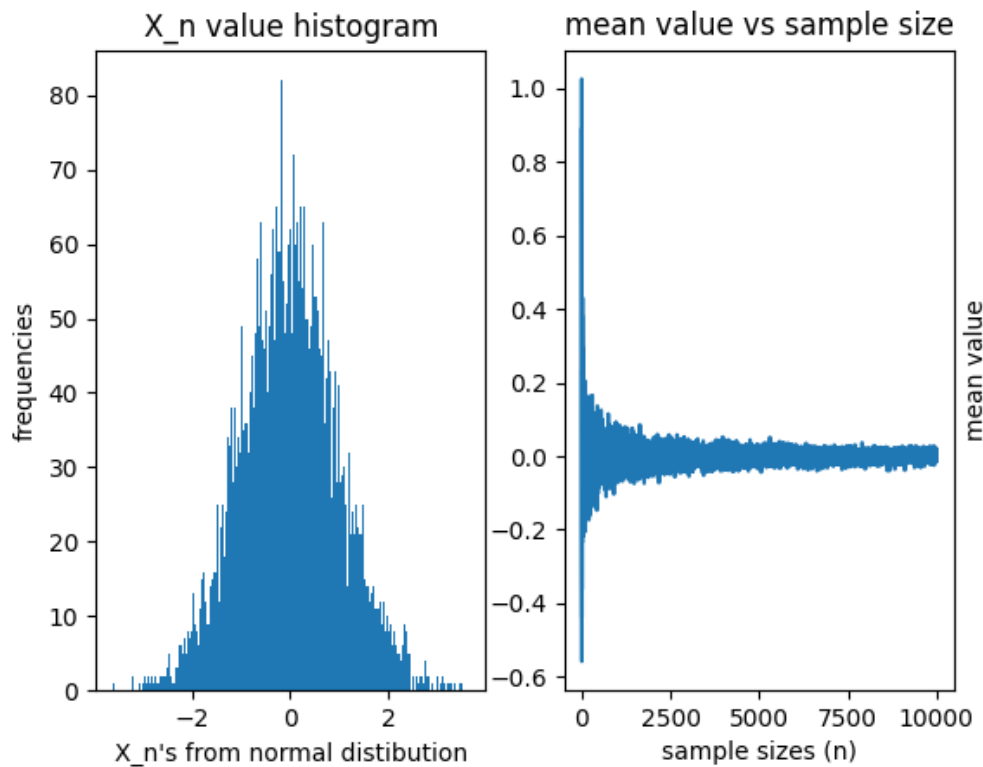


### **Simulation of Random Variables**

This report consists of three parts, each addressing the tasks specified in the handout. All simulation and plotting are done via Python.

#### **Part 1**

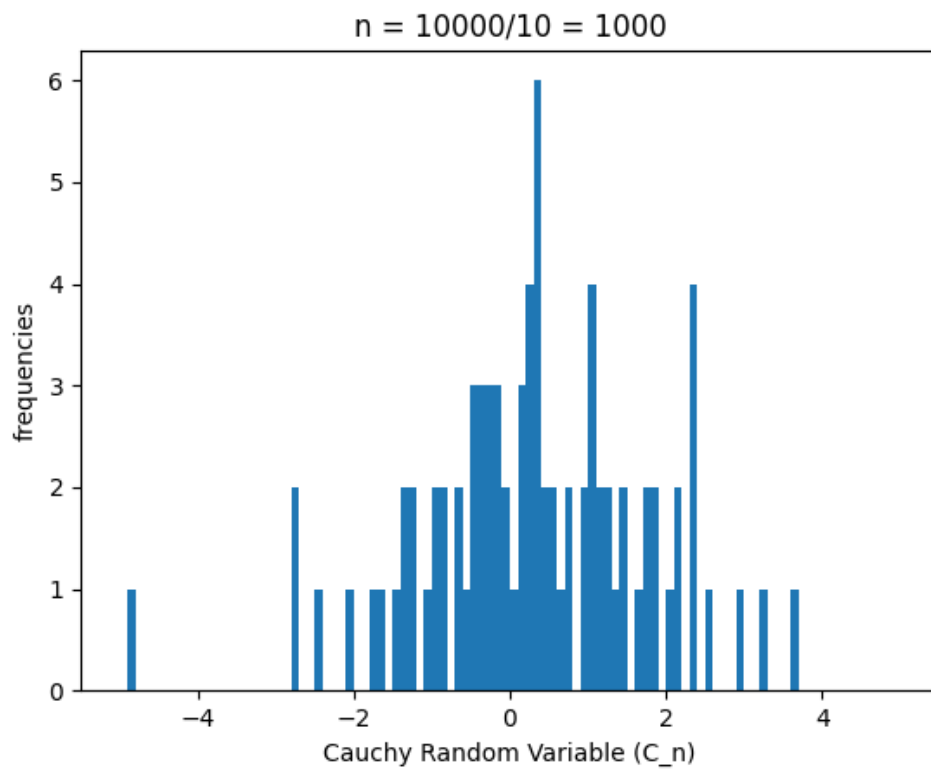
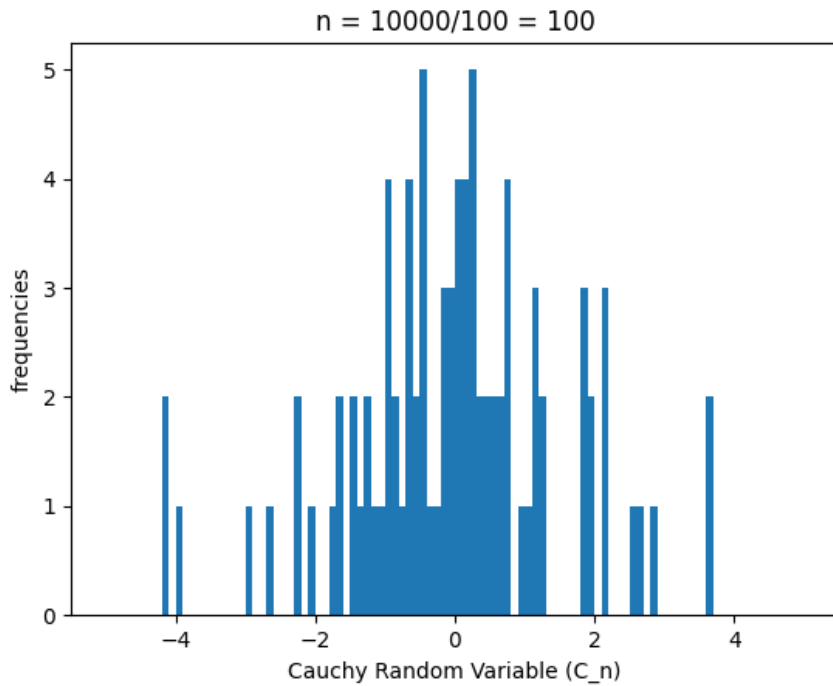
The program generates 10,000 samples of pseudo random numbers from a zero-mean, unit-variance Gaussian random variable, plots the histogram of these samples and a line graph comparing how the sample means changes as sample size increases from “re-sampling” from the generated 10,000 samples. Graphs are shown below.

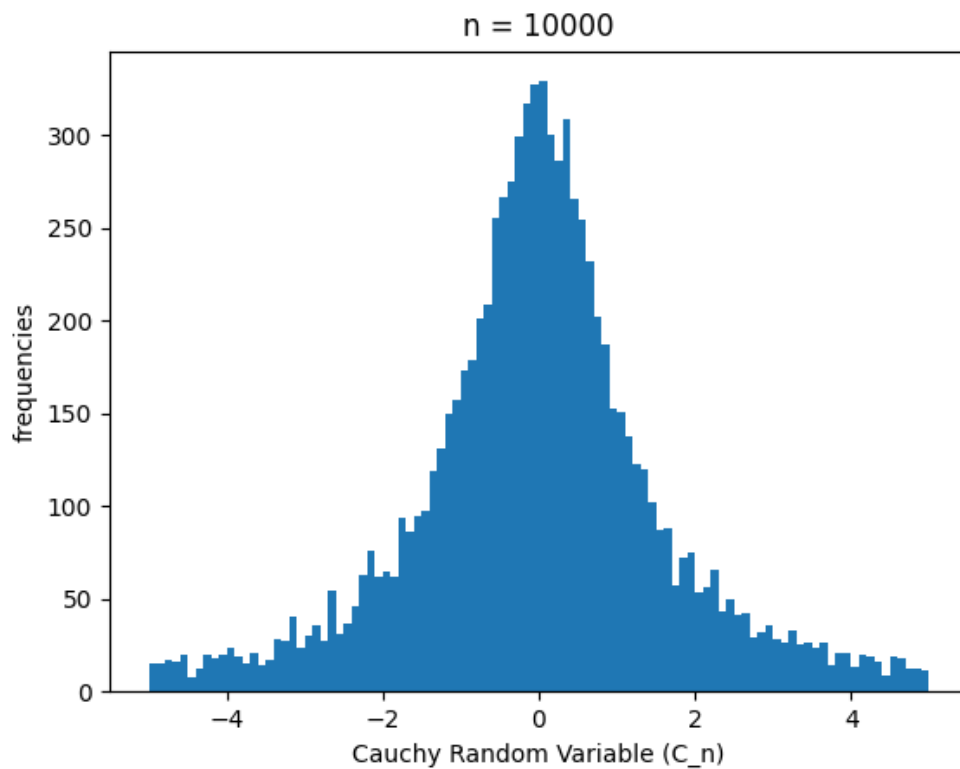


As shown in the histogram on the left,  $X_n$  samples tends to cluster towards 0 and the standard deviation tends to be 1, which is expected as they are sampled from a Gaussian random variable of mean 0 and variance of 1. The line graph on the right shows that the variation of sample means from these 10,000 samples tend to “converge” to 0 with decreasing variation, this demonstrates central limit theorem: as the sample size of each sample mean increases, the mean of these sample means converges toward the population mean.

## Part2

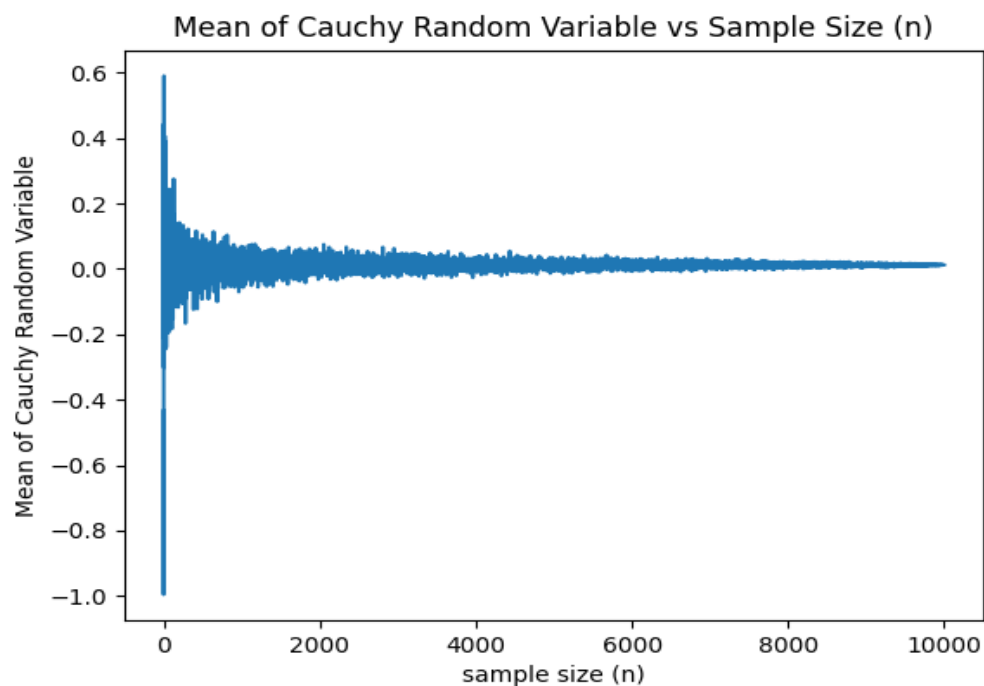
The program simulates Cauchy random variable by first generate 10, 000 samples from a uniform random distribution on interval  $[-1,1]$ , then apply equation  $\tan(\pi U_n)$  to each sample  $U_n$ . Histograms of sample sizes 100, 1,000, and 10,000 are plotted and shown below.





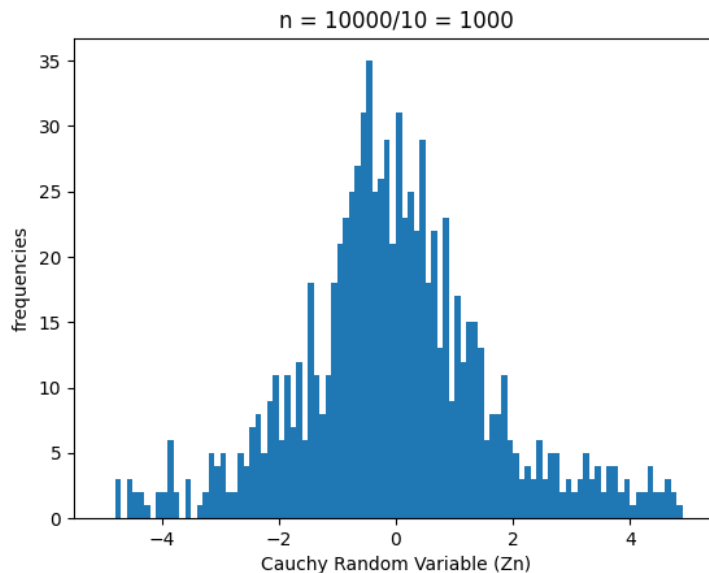
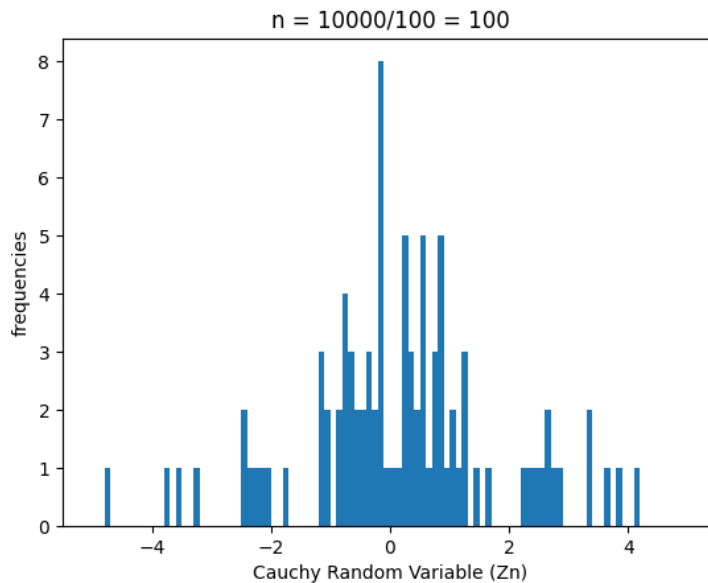
As sample size increases, the histogram tends to a narrow bell curve that could be modeled by  $f_Y(y) = \frac{1}{\pi} * \frac{1}{1+y^2}$ , the pdf of Cauchy random variable.

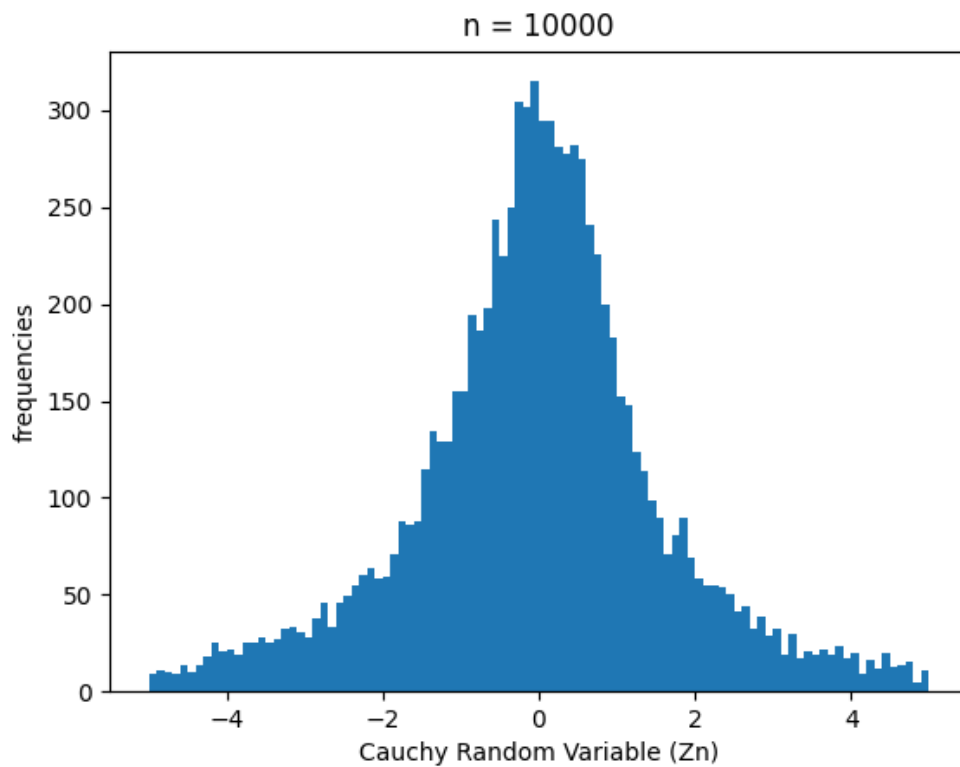
A plot displaying how the means of samples of sizes varying from 1 to 10,000 is also plotted and shown below.



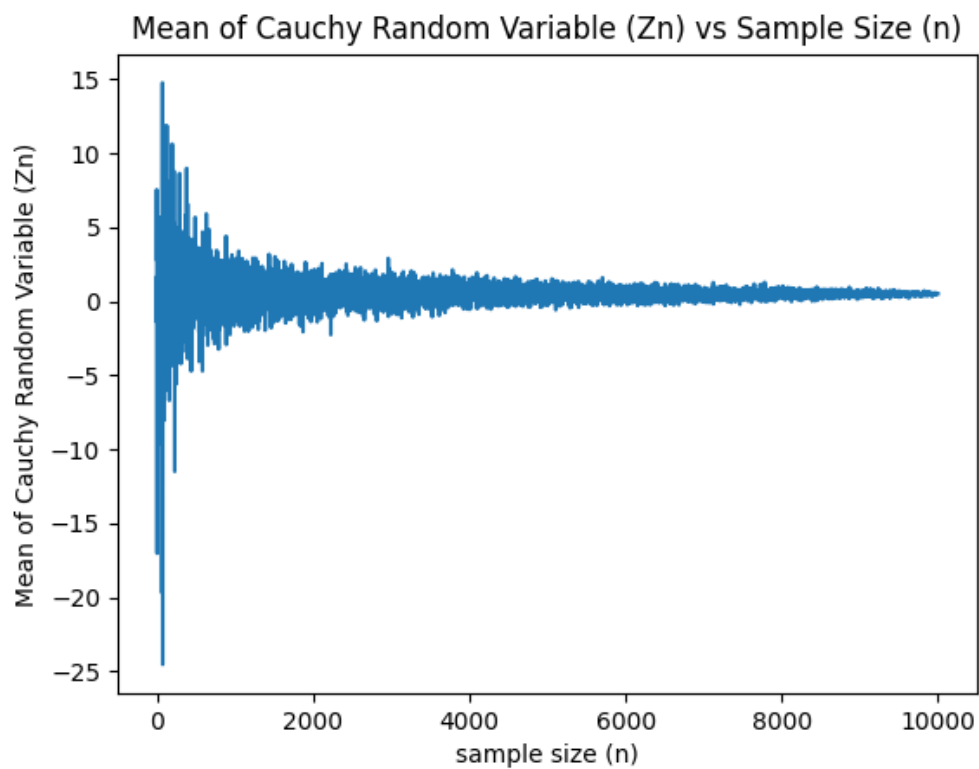
### Part3

The program now simulates the Cauchy random variable with a different approach. It first generates 2 zero-mean, unit-variance Gaussian random variable, then the Cauchy random variable is generated by taking the quotient of these random variables sample by sample. The histogram of the Cauchy random variable is again plotted with varying sample sizes of 100, 1,000, and 10,000 are again plotted and shown below.





The plot below shows how the sample means of Cauchy random variable could change as the size of sample means change, the sizes ranges from 1 to 10,000.



Code available at the following github repo:

<https://github.com/Ping-Hung/Purdue-ECE-302-final-project.git>