

Concepts and Mental Model

DP is about solving problem with overlapping subproblems.

The SRTBOT Framework (For Logically Rigorous Algorithm Design)

1. Subproblem definition

- Describe the meaning of subproblem in **words**, in terms of input parameters
- Subproblems usually lie in **subsets of input**
 - e.g. prefix, suffix, running (continuous) substring of a sequence
- Subproblems usually record **partial state** (of the entire solution):
 - could add subproblems by incrementing some auxiliary variables.

2. Relate subproblem solutions recursively

- look for recursive relation between subproblems
- DP recurrence must satisfy:

1. define **only one** state at a time
2. decisions must be made by comparing **complete subproblem solutions** instead of local costs.

3. Topological order on subproblems

- For arguing relation is **acyclic** and **subproblems form a DAG**

4. Base cases of relation

- Solution to ultra simple cases (for all reachable subproblems) when recursion (or dependency on subproblems) is not needed

5. Original problem solution via subproblem(s)

- Show how the original problem could be solved by reusing solutions to subproblem(s)

6. Time and space analysis

- Source: [MIT 6.006 Introduction to Algorithms: DP Lecture Notes](#)

Greg Hogg's Approach (More Interview-Style, Time Limited Approach)

- This is essentially compressed SRTBOT

recursive backtrack -> top-down memoization -> bottom-up tabulation -> bottom-up without table (or only partial table)

- source: <https://www.youtube.com/shorts/uUjFL0C-vY0?feature=share>

How to Verify Recursion

Pause and ask these guiding questions (assume $f(x)$ is the recursive function):

1. What exactly does $f(x)$ mean?
2. Do my base cases match reality?
3. What is the last decision?
4. Are the subproblems disjoint and complete?

5. Does this match small hand-calculated cases?

Cache/Memoization Notes:

1. Cache/Memo only remembers **past computation**, which are computed by recursion.
2. Memoization stores answers to states, not transitions between states.