Grouping symbols with PEMDAS and order of operations

In a math expression, grouping symbols are used to group factors or terms together. When you evaluate a math expression, you have to perform the operations that are enclosed in grouping symbols before you perform other kinds of operations in that expression. You're most familiar with parentheses as one kind of grouping symbol, but there are others.

Symbols of Inclusion:

Parentheses	( )
Brackets (square brackets)	[ ]
Braces (curly braces)	{ }
Absolute Value	| |

In some expressions, a division symbol (either the ÷ symbol or the "fraction line" that separates the numerator of a fraction from the denominator) is used as a grouping symbol. In the fraction

$$\frac{a}{b+c}$$

for example, the division symbol tells us that we have to first perform the addition in the denominator $(b+c)$, and then divide the numerator a by the result.

Similarly, the absolute-value signs in $a|b-c|$ tell us that we have to first perform the subtraction $b-c$, and then take the absolute value of the result, before multiplying by a.

PEMDAS and order of operations

When we're given an expression and we want to evaluate it, we have to perform the indicated operations in the correct order; that order has come to be known as the **order of operations**, or **PEMDAS**.

The first letter P tells us that the first thing we have to do is perform operations that are enclosed in grouping symbols; the reason why P is used for this is that parentheses are the most commonly used grouping symbol. The order of operations for PEMDAS is

Parentheses	(all grouping symbols)
Exponents	(powers and roots)
Multiplication/**D**ivision	(going through the expression from left to right and top to bottom, performing each multiplication/division as you come to it)
Addition/**S**ubtraction	(going through the expression from left to right and top to bottom, performing each addition/subtraction as you come to it)

All grouping symbols other than the division symbol actually consist of a pair of symbols:

- an opening (left) parenthesis and a closing (right) parenthesis

- an opening (left) bracket and a closing (right) bracket
- an opening (left) brace and a closing (right) brace
- an opening absolute-value sign and a closing absolute-value sign

But keep in mind that when we refer in general to a “pair” of grouping symbols, in some cases we could be referring to just a division symbol.

Sometimes one pair of grouping symbols is inside another pair. When that happens, you have to start by performing the operation that’s enclosed in the “innermost” pair of grouping symbols and work your way “outwards.”

Example

Simplify the expression.

$$[15 - (2 + 4)] \cdot 5 - 3$$

Start by performing the operation that’s enclosed in the innermost pair of grouping symbols (the addition in the parentheses).

$$[15 - (6)] \cdot 5 - 3$$

$$[15 - 6] \cdot 5 - 3$$

Now perform the operation that’s enclosed in the remaining pair of grouping symbols (the subtraction in the brackets).

$$[9] \cdot 5 - 3$$

$$9 \cdot 5 - 3$$

There are no exponents, so move on to multiplication/division.

$$45 - 3$$

Finally, do addition/subtraction.

$$42$$

Let's try another example using grouping symbols.

Example

Simplify the expression.

$$3[(4 - 1) + 7] - (8 + 2)$$

Start by performing the operation that's enclosed in the innermost pair of grouping symbols (the subtraction $4 - 1$ in the parentheses).

$$3[(3) + 7] - (8 + 2)$$

$$3[3 + 7] - (8 + 2)$$

Neither of the two remaining pairs of grouping symbols is inside the other pair, so perform the operations enclosed in those two pairs of grouping symbols (the addition $3 + 7$ in the square brackets, and the addition $8 + 2$ in the parentheses) separately.

$$3[10] - (10)$$

$$3[10] - 10$$

There are no exponents, so move on to multiplication/division.

$$30 - 10$$

Finally, do addition/subtraction.

$$20$$

We also want to make sure we know how to deal with grouping symbols in the denominator of a fraction.

Example

Simplify the expression.

$$\frac{3}{(4-1)+7} + \frac{(8+2)-4}{(12+2)-4}$$

When you have grouping symbols within a fraction, you want to first simplify any grouping in the numerator,

$$\frac{3}{(4-1)+7} + \frac{10-4}{(12+2)-4}$$

then simplify grouping in the denominator.

$$\frac{3}{3+7}+\frac{10-4}{14-4}$$

Then perform any other operations that remain.

$$\frac{3}{3+7}+\frac{6}{14-4}$$

$$\frac{3}{10}+\frac{6}{10}$$

$$\frac{9}{10}$$

Let's look at an example that includes exponents.

Example

Use the order of operations to simplify the expression.

$$3^3+9\div(5-2)\cdot(4)^2$$

Follow the order of operations (PEMDAS) to simplify.

Parentheses

$$3^3+9\div(5-2)\cdot(4)^2$$

$$3^3+9\div(3)\cdot(4)^2$$

Exponents

$$27 + 9 \div (3) \cdot (16)$$

Multiplication/Division

$$27 + 3 \cdot 16$$

$$27 + 48$$

Addition/Subtraction

$$75$$

Let's try another example using the order of operations.

Example

Use the order of operations to simplify the expression.

$$6 + 2(3x + 1)$$

Follow the order of operations (PEMDAS) to simplify.

Parentheses

Exponents

Multiplication/Division

$$6 + 2(3x + 1)$$

$$6 + 2(3x) + 2(1)$$

$$6 + 6x + 2$$

Addition/Subtraction

$$6x + 8$$
