Factoring the sum of two cubes

In this lesson we'll look at how to recognize a sum of two cubes and then use a formula to factor it.

We'll know when we have a sum of cubes because we'll have two perfect cubes separated by a plus sign to indicate that the second perfect cube is to be added to the first perfect cube. When that's the case, we can take the cube root of each term and use a formula to do the factoring.

The formula for the sum of two cubes is

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Let's do an example.

Example

Factor the expression.

$$125x^3 + 512y^3z^9$$

First check to see if each term is a perfect cube.

$$\sqrt[3]{125x^3} = (125x^3)^{\frac{1}{3}} = (125)^{\frac{1}{3}}(x^3)^{\frac{1}{3}} = 5x$$

$$\sqrt[3]{512y^3z^9} = (512y^3z^9)^{\frac{1}{3}} = (512)^{\frac{1}{3}}(y^3)^{\frac{1}{3}}(z^9)^{\frac{1}{3}} = 8yz^3$$

Both terms are perfect cubes, so we can use the formula to do the factoring

$$a^3+b^3=(a+b)(a^2-ab+b^2)$$

In this case $a=5x$ and $b=8yz^3$, so we can plug these into our formula and get

$$(5x+8yz^3)\left[(5x)^2-(5x)(8yz^3)+(8yz^3)^2\right]$$

$$(5x+8yz^3)(25x^2-40xyz^3+64y^2z^6)$$

We can check our work by distributing each term in the binomial factor over all the terms in the trinomial factor.

$$5x(25x^2-40xyz^3+64y^2z^6)+8yz^3(25x^2-40xyz^3+64y^2z^6)$$

$$5x(25x^2)-5x(40xyz^3)+5x(64y^2z^6)+8yz^3(25x^2)-8yz^3(40xyz^3)+8yz^3(64y^2z^6)$$

$$125x^3-200x^2yz^3+320xy^2z^6+200x^2yz^3-320xy^2z^6+512y^3z^9$$

$$125x^3-200x^2yz^3+200x^2yz^3+320xy^2z^6-320xy^2z^6+512y^3z^9$$

$$125x^3+512y^3z^9$$

Let's do another example.

Example

Factor the expression.

$$729h^{30}j^9 + 27m^{15}n^3$$

First check to see if each term is a perfect cube.

$$\sqrt[3]{729h^{30}j^9} = (729h^{30}j^9)^{\frac{1}{3}} = (729)^{\frac{1}{3}}(h^{30})^{\frac{1}{3}}(j^9)^{\frac{1}{3}} = 9h^{10}j^3$$

$$\sqrt[3]{27m^{15}n^3} = (27m^{15}n^3)^{\frac{1}{3}} = (27)^{\frac{1}{3}}(m^{15})^{\frac{1}{3}}(n^3)^{\frac{1}{3}} = 3m^5n$$

Both terms are perfect cubes, so we can use the formula for factoring the sum of perfect cubes.

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

In this case,

$$a = 9h^{10}j^3$$

$$b = 3m^5n$$

We use the sum of cubes formula to get

$$(9h^{10}j^3 + 3m^5n)\left[(9h^{10}j^3)^2 - (9h^{10}j^3)(3m^5n) + (3m^5n)^2\right]$$

$$(9h^{10}j^3 + 3m^5n)(81h^{20}j^6 - 27h^{10}j^3m^5n + 9m^{10}n^2)$$