

Complex fractions, ratio and proportion

When you have complex fractions in a proportion that includes a variable, you can solve for the variable in one of two ways: Either use cross multiplication, or multiply by the reciprocals of the fractions in the denominators.

What is cross multiplication?

Remember that

$$\frac{a}{b} = \frac{c}{d}$$

can be rewritten as $ad = bc$.

Also remember these rules that we use for complex fractions:

A reciprocal of a fraction is just that fraction “flipped upside down.”

The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$

The reciprocal of $\frac{x}{1}$ is $\frac{1}{x}$

A fraction bar can be thought of as a division sign.

$$\frac{x}{y} = x \div y$$

To divide by a fraction, you can multiply by its reciprocal.



$$\frac{x}{\left(\frac{a}{b}\right)} = x \div \frac{a}{b} = x \cdot \frac{b}{a}$$

Any number or variable can be written as itself divided by 1.

$$x = \frac{x}{1}$$

Let's look at a few examples.

Example

Solve for the variable.

$$\frac{\left(\frac{1}{3}\right)}{x} = \frac{\left(\frac{1}{6}\right)}{\left(\frac{1}{7}\right)}$$

We'll cross multiply.

$$\frac{1}{3} \cdot \frac{1}{7} = x \cdot \frac{1}{6}$$

Now we can simplify by multiplying the fractions.

$$\frac{1 \cdot 1}{3 \cdot 7} = \frac{x}{6}$$

$$\frac{1}{21} = \frac{x}{6}$$



Multiply both sides of this equation by 6 to solve for x .

$$\frac{6}{21} = x$$

$$x = \frac{2}{7}$$

Let's look at one more example.

Example

Solve for the variable.

$$\frac{\left(\frac{x}{4}\right)}{\left(\frac{8}{3}\right)} = \frac{\left(\frac{4}{3}\right)}{\left(\frac{5}{4}\right)}$$

Instead of dividing by the fractions in the denominators, we can multiply by their reciprocals.

$$\frac{x}{4} \cdot \frac{3}{8} = \frac{4}{3} \cdot \frac{4}{5}$$

After multiplying you get

$$\frac{3x}{32} = \frac{16}{15}$$

Multiply both sides by 32.



$$3x = 32 \cdot \frac{16}{15}$$

Divide both sides by 3 to solve for x . Then multiply fractions to simplify.

$$x = \frac{32}{3} \cdot \frac{16}{15}$$

$$x = \frac{512}{45}$$

