Sum of functions

Did you know that, just as you can add numbers, you can add function? Yes, you can do this as long as the functions you're adding have the same domain.

There are two ways to add functions for which you have mathematical expressions (expressions that include a variable such as x). One way to add them is to input the value of x into the expression for each function and compute the outputs for the two functions separately, and then add the outputs. The other way is to first add the expressions for the functions, and then input the value of x and compute the output. You can write the sum of functions f and g as

$$(f+g)(x) = f(x) + g(x)$$

Let's work through an example of function addition.

Example

Find $(f+g)(3)$ if $f(x) = x^2 - x + 4$ and $g(x) = x - 2$.

We need to find $(f+g)(3)$, which we could rewrite as $f(3) + g(3)$. So we can input the value of x (3) into the expression for each function and then add the outputs.

First, let's find $f(3)$.

$$f(x) = x^2 - x + 4$$

$$f(3) = (3)^2 - 3 + 4$$

Simplify using the order of operations.

$$f(3) = 9 - 3 + 4$$

$$f(3) = 6 + 4$$

$$f(3) = 10$$

Now let's find $g(3)$.

$$g(x) = x - 2$$

$$g(3) = 3 - 2$$

$$g(3) = 1$$

Now we can add the outputs to find the sum.

$$(f + g)(3) = f(3) + g(3)$$

$$(f + g)(3) = 10 + 1$$

$$(f + g)(3) = 11$$

We could also have added the expressions for the functions, and then plugged in 3 for x to get the answer.

$$(f + g)(x) = (x^2 - x + 4) + (x - 2)$$

$$(f + g)(x) = x^2 - x + 4 + x - 2$$

$$(f + g)(x) = x^2 + 2$$

Plug in 3 for x.

$$(f+g)(3) = 3^2 + 2$$

$$(f+g)(3) = 9 + 2$$

$$(f+g)(3) = 11$$

Let's try another example of a sum of functions.

Example

Find $(g+h)(-2)$ if $g(x) = x^2 + 5x$ and $h(x) = 3 - x$.

We need to find $(g+h)(-2)$, which we could rewrite as $g(-2) + h(-2)$. So we can input the value of x (-2) into the expression for each function and then add the outputs.

First, let's find $g(-2)$.

$$g(x) = x^2 + 5x$$

$$g(-2) = (-2)^2 + 5(-2)$$

Simplify using the order of operations.

$$g(-2) = 4 - 10$$

$$g(-2) = -6$$

Now let's find $h(-2)$.

$$h(x) = 3 - x$$

$$h(-2) = 3 - (-2)$$

$$h(-2) = 5$$

Now we'll find the sum of the functions at $x = -2$.

$$(g + h)(-2) = g(-2) + h(-2)$$

$$(g + h)(-2) = -6 + 5$$

$$(g + h)(-2) = -1$$

We could also have added the expressions for the functions, and then plugged in -2 for x to get the answer.

$$(g + h)(x) = (x^2 + 5x) + (3 - x)$$

$$(g + h)(x) = x^2 + 5x + 3 - x$$

$$(g + h)(x) = x^2 + 4x + 3$$

Plug in -2 for x.

$$(g + h)(-2) = (-2)^2 + 4(-2) + 3$$

$$(g + h)(-2) = 4 - 8 + 3$$

$$(g + h)(-2) = -4 + 3$$

$$(g + h)(-2) = -1$$

If the only information you have are the pairs of inputs and outputs for each function (if you have no mathematical expressions for them), there's only one way to add them, and that's to add the outputs. Suppose, for example, that you have functions f and g:

$$f: (1,-9),\ (-2,8),\ (-5,16),\ (3,2)$$

$$g: (1,12),\ (-2,10),\ (-5,9),\ (3,-4)$$

Then the only way to find $(f+g)(-5)$ is to add the values of $f(-5)$ and $g(-5)$:

$$(f+g)(-5) = f(-5) + g(-5)$$

$$(f+g)(-5) = 16 + 9$$

$$(f+g)(-5) = 25$$