# Solving with elimination

Another method to solve a system of linear equations (equations of lines) is elimination. Remember that two lines that aren't parallel cross each other at a single point, and that point is our solution.

Here are the steps we'll follow when we use the elimination method to solve a system of two linear equations:

1. If necessary, rearrange both equations so that the $x$-terms are first, followed by the $y$-terms, the equals sign, and the constant term (in that order) If an equation appears to have no constant term, that means that the constant term is $0$.

2. Multiply one (or both) equations by a constant that will allow either the $x$-terms or the $y$-terms to cancel when the equations are added or subtracted (when their left sides and their right sides are added separately, or when their left sides and their right sides are subtracted separately).

3. Add or subtract the equations.

4. Solve for the remaining variable.

5. Plug the result of step 4 into one of the original equations and solve for the other variable.

Let's look at some examples to make things more clear.

**Example**

Find the unique solution to the system of equations.

$$3x + 4y = 12$$

$$-3x + 2y = 18$$

Steps 1 and 2 are done, since the individual parts of each equation are in the correct places, and the $x$-terms ($3x$ in the first equation and $-3x$ in the second equation) will cancel when we add the equations. So we'll skip to step $3$ and add the equations.

$$3x + 4y = 12 \qquad + \qquad -3x + 2y = 18$$

$$3x + 4y + (-3x + 2y) = 12 + (18)$$

$$3x + 4y - 3x + 2y = 12 + 18$$

$$3x - 3x + 4y + 2y = 30$$

$$0 + 6y = 30$$

$$6y = 30$$

Next, we'll solve for $y$ by dividing both sides by $6$.

$$\frac{6y}{6} = \frac{30}{6}$$

$$y = 5$$

Now, we'll plug in $5$ for $y$ in the original first equation and solve for $x$.

$$3x + 4y = 12$$

$$3x + 4(5) = 12$$

$$3x + 20 = 12$$

Subtract $20$ from both sides.

$$3x + 20 - 20 = 12 - 20$$

$$3x = -8$$

Divide both sides by $3$.

$$\frac{3x}{3} = \frac{-8}{3}$$

$$x = -\frac{8}{3}$$

The unique solution is

$$\left(-\frac{8}{3}, 5\right)$$

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Let's try another example of solving with elimination.

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**Example**

Find the unique solution to the system of equations.

$$y = 3x - 4$$

$$-x + 2y = 12$$

First, we'll rearrange the first equation so that its individual parts are in the correct places for elimination. Subtract $3x$ from both sides.

$$y = 3x - 4$$

$$-3x + y = 3x - 3x - 4$$

$$-3x + y = -4$$

Next, multiply the result above by $2$ so that the $y$-terms will cancel when we subtract the equations.

$$2(-3x + y) = 2(-4)$$

$$-6x + 2y = -8$$

Now we'll subtract the equations.

$$-6x + 2y = -8 \qquad - \qquad -x + 2y = 12$$

$$-6x + 2y - (-x + 2y) = -8 - (12)$$

$$-6x + 2y + x - 2y = -8 - 12$$

$$-6x + x + 2y - 2y = -20$$

$$-5x + 0 = -20$$

$$-5x = -20$$

Divide both sides by $-5$.

$$\frac{-5x}{-5} = \frac{-20}{-5}$$

$$x = 4$$

To solve for $y$, we'll plug in $4$ for $x$ in the original first equation.

$$y = 3x - 4$$

$$y = 3(4) - 4$$

$$y = 12 - 4$$

$$y = 8$$

The unique solution is $(4,8)$.