Fractional equations

In this lesson we'll look at how to solve equations with numerical fractions as coefficients and terms.

Remember:

1. Multiplying a fraction by its reciprocal will always give you a value of 1.

For example, 4/5 has a reciprocal of 5/4 because

$$\frac{4}{5} \cdot \frac{5}{4} = 1$$

2. To clear a fraction from an equation, multiply both sides of the equation by the fraction's denominator.

For example, to clear the 2 from the fraction in 5x + 1/2 = 12, multiply both sides of the equation by 2.

$$2\left(5x + \frac{1}{2}\right) = 2(12)$$

$$2(5x) + 2\left(\frac{1}{2}\right) = 2(12)$$

$$10x + 1 = 24$$

Let's do a few examples where we solve an equation with a fraction. First let's look at an equation that has a fractional coefficient.

Example

Solve for the variable.

$$\frac{4}{5}n = 20$$

To get rid of a fractional coefficient, we'll multiply both sides of the equation by the fraction's reciprocal, because that'll change the coefficient to 1.

$$\frac{4}{5}n = 20$$

$$\frac{5}{4} \cdot \frac{4}{5}n = \frac{5}{4} \cdot 20$$

$$\frac{20}{20}n = \frac{100}{4}$$

$$1n = 25$$

$$n = 25$$

If you have a fractional coefficient and another term on the same side of the equation, you can isolate the term with the variable and then multiply both sides by the reciprocal of the fractional coefficient.

Example



Solve for the variable.

$$\frac{4}{7}x + 14 = 22$$

First isolate the fractional term.

$$\frac{4}{7}x + 14 - 14 = 22 - 14$$

$$\frac{4}{7}x = 8$$

Now get rid of the fractional coefficient by multiplying both sides of the equation by the reciprocal of 4/7.

$$\frac{7}{4} \cdot \frac{4}{7}x = \frac{7}{4} \cdot 8$$

$$\frac{28}{28}x = \frac{56}{4}$$

$$1x = 14$$

$$x = 14$$

We could also solve this problem by first clearing the fraction. In order to get rid of the fraction, we have to multiply both sides of the equation by the fraction's denominator.

$$7\left(\frac{4}{7}x + 14\right) = 7(22)$$



$$7 \cdot \frac{4}{7}x + 7 \cdot 14 = 7 \cdot 22$$

$$4x + 98 = 154$$

Now we can solve for the variable using inverse operations.

$$4x + 98 - 98 = 154 - 98$$

$$4x = 56$$

$$\frac{4x}{4} = \frac{56}{4}$$

$$x = 14$$

