Topic: Number word problems

Question: The sum of the digits of a certain two-digit number is 9. When the digits are reversed, the new number is 45 greater than the original number. What is the original number?

Answer choices:

A 27

B 72

C 45

D 52

Solution: A

Let T and U be the tens digit and units digit, respectively, of the original number. Therefore, the value of the original number is $10T + U$, and the value of the new number is $10U + T$.

Since the new number is 45 greater than the original number, we have to subtract 45 from the new number to get the original number.

$$\text{second number} - 45 = \text{original number}$$

$$(10U + T) - 45 = (10T + U)$$

$$10U + T - 45 = 10T + U$$

$$9U - 9T = 45$$

$$U - T = 5$$

Also, we know that the sum of the digits is 9.

$$T + U = 9$$

Now we have the system of equations

$$T + U = 9$$

$$U - T = 5$$

We'll solve the first equation for T and then substitute the resulting expression for T in the second equation.

$$T + U = 9$$

$$T = 9 - U$$

Now we'll plug this expression for T into the second equation.

$$U - T = 5$$

$$U - (9 - U) = 5$$

$$U - 9 + U = 5$$

$$2U = 14$$

$$U = 7$$

Our next step is to plug this value of U into the equation $T = 9 - U$, and then compute the value of T.

$$T = 9 - U$$

$$T = 9 - 7$$

$$T = 2$$

The original number is 27, and the new number is 72.

Let's check these numbers against the original statement.

- The sum of the digits of the original number is 9: Yes, because $2 + 7 = 9$.
- The digits are reversed: Yes, because $27 \to 72$.
- The new number is 45 greater than the original number: Yes, because $72 = 27 + 45$.

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Question: The sum of the digits of a certain two-digit number is 8. Reversing the digits gives a number which is 18 greater than the original number. What is the original number?

Answer choices:

A 17

B 26

C 35

D 53

Solution: C

Let T and U be the tens digit and units digit, respectively, of the original number.

The value of the original number is

$$10T + U$$

Reversing the digits gives us a number whose value is

$$10U + T$$

The second number is 18 greater than the original number, so we can write

$$\text{original number} + 18 = \text{second number}$$

$$(10T + U) + 18 = (10U + T)$$

$$10T + U + 18 = 10U + T$$

$$9T - 9U + 18 = 0$$

Dividing through by 9 gives

$$T - U + 2 = 0$$

$$T = U - 2$$

We know that the sum of the digits is 8, so we'll substitute the expression we just found for T into the equation $T + U = 8$, and then solve for U.

$$T + U = 8$$

$$(U-2)+U=8$$

$$2U-2=8$$

$$2U=10$$

$$U=5$$

Our next step is to plug this value of U into the equation $T+U=8$, and then solve for T.

$$T+U=8$$

$$T+5=8$$

$$T=3$$

The original number is 35. When we reverse the digits, we get 53, which is indeed 18 greater than 35: $53=35+18$.

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Question: A certain two-digit number is 11 greater than 8 times its tens digit. The sum of the digits is 6. What is the number?

Answer choices:

A 15

B 24

C 42

D 51

Solution: D

Let T and U be the tens digit and units digit, respectively, of the number. Which means the value of the number is

$$10T + U$$

We know that that's 11 more than 8 times the tens digit, so

$$10T + U = 8T + 11$$

$$2T + U = 11$$

We also know that $T + U = 6$, so we'll solve the following system of two equations:

$$2T + U = 11$$

$$T + U = 6$$

We'll subtract the second equation from the first equation.

$$(2T + U) - (T + U) = (11) - (6)$$

$$2T + U - T - U = 11 - 6$$

$$2T - T + U - U = 11 - 6$$

$$T = 5$$

Now we'll substitute 5 for T in the equation $T + U = 6$, and then solve for U.

$$T + U = 6$$

$$5 + U = 6$$

$$U = 1$$

The number is 51. To check this, note that 8 times the tens digit of 51 is $8 \cdot 5$, or 40, and that 51 is indeed 11 greater than 40: $51 = 40 + 11$.