



# Algebra 2 Workbook Solutions

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Systems of equations

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MATH

## SYSTEMS OF EQUATIONS WITH SUBSCRIPTS

- 1. Solve the system of equations using any method.

$$d_t = x_t - 3$$

$$d_t = \frac{1}{2}x_t + 2$$

*Solution:*

Substitute  $d_t = x_t - 3$  into  $d_t = (1/2)x_t + 2$  to solve for  $x_t$ .

$$d_t = \frac{1}{2}x_t + 2$$

$$x_t - 3 = \frac{1}{2}x_t + 2$$

$$\frac{1}{2}x_t - 3 = 2$$

$$\frac{1}{2}x_t = 5$$

$$x_t = 10$$

Now substitute  $x_t = 10$  into either equation. We'll use  $d_t = x_t - 3$  since it's a little simpler.

$$d_t = x_t - 3$$



$$d_t = 10 - 3$$

$$d_t = 7$$

Then the solution to the system is

$$(x_t, d_t) = (10, 7)$$

■ 2. Solve the system of equations using any method.

$$y_t = 2x_t + 5$$

$$y_t = -10x_t - 7$$

*Solution:*

Substitute  $y_t = 2x_t + 5$  into  $y_t = -10x_t - 7$  to solve for  $x_t$ .

$$y_t = -10x_t - 7$$

$$2x_t + 5 = -10x_t - 7$$

$$12x_t + 5 = -7$$

$$12x_t = -12$$

$$x_t = -1$$

Now substitute  $x_t = -1$  into either equation. We'll use  $y_t = 2x_t + 5$  since it's a little simpler.



$$y_t = 2x_t + 5$$

$$y_t = 2(-1) + 5$$

$$y_t = -2 + 5$$

$$y_t = 3$$

Then the solution to the system is

$$(x_t, y_t) = (-1, 3)$$

■ 3. Solve the system of equations using any method.

$$4A_x - B_y = 15$$

$$B_y = 12 - 5A_x$$

*Solution:*

Substitute  $B_y = 12 - 5A_x$  into  $4A_x - B_y = 15$  to solve for  $A_x$ .

$$4A_x - B_y = 15$$

$$4A_x - (12 - 5A_x) = 15$$

$$4A_x - 12 + 5A_x = 15$$

$$9A_x - 12 = 15$$



$$9A_x = 27$$

$$A_x = 3$$

Now substitute  $A_x$  into either equation. We'll use  $B_y = 12 - 5A_x$  since it's a little simpler.

$$B_y = 12 - 5A_x$$

$$B_y = 12 - 5(3)$$

$$B_y = 12 - 15$$

$$B_y = -3$$

Then the solution to the system is

$$(A_x, B_y) = (3, -3)$$

■ 4. Solve the system of equations using any method.

$$2T_x + 5T_y = 120.8$$

$$T_y = 17T_x + 17.2$$

*Solution:*

Substitute  $T_y = 17T_x + 17.2$  into  $2T_x + 5T_y = 120.8$  to solve for  $T_x$ .



$$2 T_x + 5 T_y = 120.8$$

$$2 T_x + 5(17 T_x + 17.2) = 120.8$$

$$2 T_x + 85 T_x + 86 = 120.8$$

$$87 T_x + 86 = 120.8$$

$$87 T_x = 34.8$$

$$T_x = 0.4$$

Now substitute  $T_x$  into either equation. We'll use  $T_y = 17 T_x + 17.2$  since it's a little simpler.

$$T_y = 17 T_x + 17.2$$

$$T_y = 17(0.4) + 17.2$$

$$T_y = 6.8 + 17.2$$

$$T_y = 24$$

Then the solution to the system is

$$(T_x, T_y) = (0.4, 24)$$

■ 5. Solve the system of equations using any method.

$$T_U W_U = 396$$



$$W_D = 2W_U$$

$$T_D W_D = 132$$

$$T_D = 3 - T_U$$

**Solution:**

Substitute  $W_D = 2W_U$  and  $T_D = 3 - T_U$  into  $T_D W_D = 132$ .

$$T_D W_D = 132$$

$$(3 - T_U)2W_U = 132$$

$$6W_U - 2T_U W_U = 132$$

Substitute  $T_U W_U = 396$ .

$$6W_U - 2T_U W_U = 132$$

$$6W_U - 2(396) = 132$$

$$6W_U - 792 = 132$$

Solve for  $W_U$ .

$$6W_U - 792 = 132$$

$$6W_U = 924$$

$$W_U = 154$$



Now substitute  $W_U = 154$  into the equation  $T_U W_U = 396$  to solve for  $T_U$ .

$$T_U W_U = 396$$

$$T_U(154) = 396$$

$$T_U = \frac{18}{7}$$

Substitute  $W_U = 154$  into the equation  $W_D = 2W_U$  to solve for  $W_D$ .

$$W_D = 2W_U$$

$$W_D = 2(154)$$

$$W_D = 308$$

Finally, substitute  $W_D = 308$  into  $T_D W_D = 132$  to solve for  $T_D$ .

$$T_D W_D = 132$$

$$T_D(308) = 132$$

$$T_D = \frac{3}{7}$$

Then the solution to the system is

$$(T_D, W_D) = \left(\frac{3}{7}, 308\right)$$

$$(T_U, W_U) = \left(\frac{18}{7}, 154\right)$$





- 6. Solve the system of equations using any method.

$$T_u R_u = 480$$

$$R_d = \frac{1}{2}R_u$$

$$T_d R_d = 320$$

$$T_d = 2 + T_u$$

*Solution:*

Substitute  $R_d = (1/2)R_u$  and  $T_d = 2 + T_u$  into  $T_d R_d = 320$ .

$$T_d R_d = 320$$

$$(2 + T_u) \left( \frac{1}{2} R_u \right) = 320$$

Distribute  $(1/2)R_u$ .

$$R_u + \frac{1}{2}T_u R_u = 320$$

Substitute  $T_u R_u = 480$ .

$$R_u + \frac{1}{2}T_u R_u = 320$$



$$R_u + \frac{1}{2}(480) = 320$$

$$R_u + 240 = 320$$

Solve for  $R_u$ .

$$R_u + 240 = 320$$

$$R_u = 80$$

Now substitute  $R_u = 80$  into  $T_u R_u = 480$  to solve for  $T_u$ .

$$T_u R_u = 480$$

$$T_u(80) = 480$$

$$T_u = 6$$

Substitute  $R_u = 80$  into  $R_d = (1/2)R_u$  to solve for  $R_d$ .

$$R_d = \frac{1}{2}R_u$$

$$R_d = \frac{1}{2}(80)$$

$$R_d = 40$$

Finally, substitute  $R_d = 40$  into  $T_d R_d = 320$  to solve for  $T_d$ .

$$T_d R_d = 320$$

$$T_d(40) = 320$$



$$T_d = 8$$

Then the solution to the system is

$$(R_d, T_d) = (40, 8)$$

$$(R_u, T_u) = (80, 6)$$

■ 7. Solve the system of equations using any method.

$$2X_1 + 3Y_1 = 1$$

$$X_2 = 4X_1$$

$$X_2 + Y_2 = 9$$

$$Y_2 = Y_1 + 2$$

*Solution:*

Substitute  $X_2 = 4X_1$  and  $Y_2 = Y_1 + 2$  into the equation  $X_2 + Y_2 = 9$ .

$$X_2 + Y_2 = 9$$

$$4X_1 + Y_1 + 2 = 9$$

$$4X_1 + Y_1 = 7$$

Use the method of elimination to solve for  $Y_1$ . Multiply  $2X_1 + 3Y_1 = 1$  by  $-2$  so that  $X_1$  will be eliminated.



$$-2(2X_1 + 3Y_1 = 1)$$

$$-4X_1 - 6Y_1 = -2$$

and then adding the equations together gives

$$-4X_1 - 6Y_1 + (4X_1 + Y_1) = -2 + (7)$$

$$-4X_1 - 6Y_1 + 4X_1 + Y_1 = -2 + 7$$

$$-5Y_1 = 5$$

$$Y_1 = -1$$

Substitute  $Y_1 = -1$  into  $2X_1 + 3Y_1 = 1$  to solve for  $X_1$ .

$$2X_1 + 3Y_1 = 1$$

$$2X_1 + 3(-1) = 1$$

$$2X_1 - 3 = 1$$

$$2X_1 = 4$$

$$X_1 = 2$$

Now substitute  $X_1 = 2$  into  $X_2 = 4X_1$  to solve for  $X_2$ .

$$X_2 = 4X_1$$

$$X_2 = 4(2)$$

$$X_2 = 8$$



Finally, substitute  $Y_1 = -1$  into  $Y_2 = Y_1 + 2$  to solve for  $Y_2$ .

$$Y_2 = Y_1 + 2$$

$$Y_2 = -1 + 2$$

$$Y_2 = 1$$

Then the solution to the system is

$$(X_1, Y_1) = (2, -1)$$

$$(X_2, Y_2) = (8, 1)$$

■ 8. Solve the system of equations using any method.

$$6X_1 + 12Y_1 = 78$$

$$X_2 = 2X_1$$

$$2X_2 - 3Y_2 = -24$$

$$Y_2 = 3Y_1 - 3$$

*Solution:*

Substitute  $X_2 = 2X_1$  and  $Y_2 = 3Y_1 - 3$  into  $2X_2 - 3Y_2 = -24$ .

$$2X_2 - 3Y_2 = -24$$



$$2(2X_1) - 3(3Y_1 - 3) = -24$$

$$4X_1 - 9Y_1 + 9 = -24$$

$$4X_1 - 9Y_1 = -33$$

Use the method of elimination to solve for  $Y_1$ . Multiply  $4X_1 - 9Y_1 = -33$  by  $-3$ ,

$$-3(4X_1 - 9Y_1 = -33)$$

$$-12X_1 + 27Y_1 = 99$$

and multiply  $6X_1 + 12Y_1 = 78$  by 2,

$$2(6X_1 + 12Y_1 = 78)$$

$$12X_1 + 24Y_1 = 156$$

and then add the equations together so that  $X_1$  will be eliminated.

$$-12X_1 + 27Y_1 + (12X_1 + 24Y_1) = 99 + (156)$$

$$-12X_1 + 27Y_1 + 12X_1 + 24Y_1 = 99 + 156$$

$$51Y_1 = 255$$

$$Y_1 = 5$$

Substitute  $Y_1 = 5$  into  $6X_1 + 12Y_1 = 78$  to solve for  $X_1$ .

$$6X_1 + 12Y_1 = 78$$

$$6X_1 + 12(5) = 78$$



$$6X_1 + 60 = 78$$

$$6X_1 = 18$$

$$X_1 = 3$$

Now substitute  $X_1 = 3$  into  $X_2 = 2X_1$  to solve for  $X_2$ .

$$X_2 = 2X_1$$

$$X_2 = 2(3)$$

$$X_2 = 6$$

Finally, substitute  $Y_1 = 5$  into  $Y_2 = 3Y_1 - 3$  to solve for  $Y_2$ .

$$Y_2 = 3Y_1 - 3$$

$$Y_2 = 3(5) - 3$$

$$Y_2 = 15 - 3$$

$$Y_2 = 12$$

Then the solution to the system is

$$(X_1, Y_1) = (3, 5)$$

$$(X_2, Y_2) = (6, 12)$$



## UNIFORM MOTION

■ 1. A train leaves Station 1 at a constant speed and arrives at Station 2 in 5 hours. A second train leaves Station 1 at a constant speed of 45 mph and arrives at Station 2 in 6 hours. What was the constant speed of the first train?

*Solution:*

Use the formula for distance.

$$\text{Distance} = \text{Rate} \cdot \text{Time}$$

$$D = RT$$

Use subscripts to create a separate equation for each train.

$$\text{Train 1: } D_1 = R_1 T_1$$

$$\text{Train 2: } D_2 = R_2 T_2$$

Since both trains are traveling from Station 1 to Station 2, they travel the same distance. We also know the time it took each of the trains to arrive and the rate of the second train. Organize the information we know about each train.

$$D_1 = D_2$$

$$R_1 = ?$$





$$T_1 = 5 \text{ hr}$$

and

$$D_2 = ?$$

$$R_2 = 45 \text{ mi/hr}$$

$$T_2 = 6 \text{ hr}$$

Find the distance by plugging all known values into  $D_2 = R_2T_2$ .

$$D_2 = R_2T_2$$

$$D_2 = (45 \text{ mi/hr})(6 \text{ hr})$$

$$D_2 = 270 \text{ mi}$$

Now find the rate of the first train by plugging all known values into  $D_1 = R_1T_1$  and solve for  $R_1$ .

$$D_1 = R_1T_1$$

$$270 \text{ mi} = R_1(5 \text{ hr})$$

$$R_1 = 54 \text{ mi/hr}$$

The first train travels at a constant speed of 54 mph.

■ 2. A train leaves Union Station at a constant speed and arrives at Georgetown Station in 3 hours. A second train leaves Union Station at a



constant speed of 45 mph and arrives at Georgetown Station in 4 hours.  
What was the constant speed of the first train?

*Solution:*

Use the formula for distance.

$$\text{Distance} = \text{Rate} \cdot \text{Time}$$

$$D = RT$$

Use subscripts to create a separate equation for each train.

$$\text{Train 1: } D_1 = R_1 T_1$$

$$\text{Train 2: } D_2 = R_2 T_2$$

Since both trains are traveling from Union Station to Georgetown Station, they travel the same distance. We also know the time it took each of the trains to arrive and the rate of the second train. Organize the information we know about each train.

$$D_1 = D_2$$

$$R_1 = ?$$

$$T_1 = 3 \text{ hr}$$

and

$$D_2 = ?$$



$$R_2 = 45 \text{ mi/hr}$$

$$T_2 = 4 \text{ hr}$$

Find the distance by plugging all known values into  $D_2 = R_2T_2$ .

$$D_2 = R_2T_2$$

$$D_2 = (45 \text{ mi/hr})(4 \text{ hr})$$

$$D_2 = 180 \text{ mi}$$

Now find the rate of the first train by plugging all known values into  $D_1 = R_1T_1$ .

$$D_1 = R_1T_1$$

$$180 \text{ mi} = R_1(3 \text{ hr})$$

$$R_1 = 60 \text{ mi/hr}$$

The first train travels at a constant speed of 60 mph.

■ 3. Kaitlyn is driving at a constant rate of 55 mph on the highway. 1 hour later, her friend Charlotte starts from the same point and drives at a constant rate of 65 mph. How many hours will each woman need to travel before Charlotte catches Kaitlyn. At that point, how far have each of them traveled?



*Solution:*

Use the formula for distance.

$$\text{Distance} = \text{Rate} \cdot \text{Time}$$

$$D = RT$$

Use subscripts to create a separate equation for each person.

$$\text{Kaitlyn: } D_K = R_K T_K$$

$$\text{Charlotte: } D_C = R_C T_C$$

When they're at the same spot on the highway, they'll have traveled the same distance. We also know the rate each person drove and that Charlotte left one hour later than Kaitlyn. Organize the information we know about each person.

$$D_K = D_C$$

$$R_K = 55 \text{ mi/hr}$$

$$T_K = ? \text{ hr}$$

and

$$D_C = ?$$

$$R_C = 65 \text{ mi/hr}$$

$$T_C = (T_K - 1) \text{ hr}$$



Plug all known values into both equations.

$$D_K = R_K T_K$$

$$D_C = 55 \text{ mi/hr } T_K \text{ hr}$$

$$D_C = 55T_K \text{ mi}$$

and

$$D_C = R_C T_C$$

$$D_C = 65 \text{ mi/hr } (T_K - 1) \text{ hr}$$

$$D_C = 65 (T_K - 1) \text{ mi}$$

Substitute  $D_C = 55T_K \text{ mi}$  into  $D_C = 65 (T_K - 1) \text{ mi}$ .

$$D_C = 65 (T_K - 1) \text{ mi}$$

$$55T_K \text{ mi} = 65 (T_K - 1) \text{ mi}$$

$$55T_K \text{ mi} = 65T_K \text{ mi} - 65 \text{ mi}$$

Move all  $T_K$  terms to one side of the equation by subtracting  $65T_K \text{ mi}$  from both sides.

$$55T_K \text{ mi} - 65T_K \text{ mi} = 65T_K \text{ mi} - 65T_K \text{ mi} - 65 \text{ mi}$$

$$-10T_K \text{ mi} = -65 \text{ mi}$$

$$T_K = 6.5$$

$$T_K = 6 \text{ hr } 30 \text{ min}$$



Kaitlyn drove 6 hr 30 min.

Since Charlotte left 1 hour later than Kaitlyn, we can find Charlotte's time by subtracting 1 hr.

$$T_C = (T_K - 1) \text{ hr}$$

$$T_C = 6 \text{ hr } 30 \text{ min} - 1 \text{ hr}$$

$$T_C = 5 \text{ hrs } 30 \text{ min}$$

Charlotte drove 5 hrs 30 min. Find the distance using either Kaitlyn or Charlotte's distance formula.

$$D_K = 55 \text{ mi/hr}(6.5 \text{ hr})$$

$$D_K = 357.5 \text{ mi}$$

Kaitlyn and Charlotte each traveled 357.5 miles.

■ 4. Sam walked 4 mph from the bus stop to the school. John walked 5 mph from the same bus stop to the school and it took him 1 hour to get there. How long did it take Sam to get to the school?

*Solution:*

Use the formula for distance.

$$\text{Distance} = \text{Rate} \cdot \text{Time}$$



$$D = RT$$

Use subscripts to create a separate equation for each person.

$$\text{Sam: } D_S = R_S T_S$$

$$\text{John: } D_J = R_J T_J$$

Since Sam and John both walk from the same bus stop to the school, they travel the same distance. We also know the rate each person walked and that John walked for 1 hour. Organize the information we know about each person.

$$D_S = D_J$$

$$R_S = 4 \text{ mi/hr}$$

$$T_S = ?$$

and

$$D_J = ?$$

$$R_J = 5 \text{ mi/hr}$$

$$T_J = 1 \text{ hr}$$

Find the distance by plugging all known values into  $D_J = R_J T_J$ .

$$D_J = R_J T_J$$

$$D_J = (5 \text{ mi/hr})(1 \text{ hr})$$



$$D_J = 5 \text{ mi}$$

Now find the time Sam walked by plugging all known values into  $D_S = R_S T_S$ .

$$D_S = R_S T_S$$

$$5 \text{ mi} = 4 \text{ mi/hr } T_S$$

$$T_S = 1.25 \text{ hr}$$

Sam walked 1 hr 15 min to get from the bus stop to the school.

■ 5. McKenzie and Daisy plan a trip to the mountains together. McKenzie travels at 45 mph and Daisy travels at 60 mph. Daisy's trip took 3 hours less than McKenzie's. How far did each of them travel?

*Solution:*

Use the formula for distance.

$$\text{Distance} = \text{Rate} \cdot \text{Time}$$

$$D = RT$$

Use subscripts to create a separate equation for each person.

$$\text{McKenzie: } D_M = R_M T_M$$

$$\text{Daisy: } D_D = R_D T_D$$





Since McKenzie and Daisy are going to the same place, they travel the same distance. We also know the rate at which each person drove, and we know that Daisy arrived 3 hours before McKenzie. Organize the information we know about each person.

$$D_M = D_D$$

$$R_M = 45 \text{ mi/hr}$$

$$T_M = ?$$

and

$$D_D = ?$$

$$R_D = 60 \text{ mi/hr}$$

$$T_D = (T_M - 3) \text{ hr}$$

Plug all known values into both equations.

$$D_D = R_D T_D$$

$$D_D = 60 \text{ mi/hr}(T_M \text{ hr} - 3 \text{ hr})$$

Substitute  $D_D = 45 \text{ mi/hr } T_M$  into  $D_D = 60 \text{ mi/hr}(T_M \text{ hr} - 3 \text{ hr})$ .

$$D_D = 60 \text{ mi/hr}(T_M \text{ hr} - 3 \text{ hr})$$

$$45 \text{ mi/hr } T_M = 60 \text{ mi/hr}(T_M \text{ hr} - 3 \text{ hr})$$

$$45 \text{ mi/hr } T_M = 60 \text{ mi/hr } T_M - 180 \text{ mi}$$



Move all  $T_M$  terms to one side of the equation by subtracting  $60 \text{ mi } T_M$  from both sides.

$$45 \text{ mi/hr } T_M - 60 \text{ mi/hr } T_M = 60 \text{ mi/hr } T_M - 60 \text{ mi/hr } T_M - 180 \text{ mi}$$

$$-15 \text{ mi/hr } T_M = -180 \text{ mi}$$

$$T_M = 12 \text{ hrs}$$

Find the distance using McKenzie's distance formula.

$$D_M = 45 \text{ mi/hr}(12 \text{ hrs})$$

$$D_M = 540 \text{ mi}$$

McKenzie and Daisy traveled 540 miles.

■ 6. A bus leaves Bus Stop  $A$  at a constant speed of 40 mph and arrives at Bus Stop  $B$  in 2 hours. A second bus leaves Bus Stop  $A$  at a constant speed of 25 mph. How long does it take the second bus to reach Bus Stop  $B$ ?

*Solution:*

Use the formula for distance.

$$\text{Distance} = \text{Rate} \cdot \text{Time}$$

$$D = RT$$

Use subscripts to create a separate equation for each bus.



$$\text{Bus 1: } D_1 = R_1 T_1$$

$$\text{Bus 2: } D_2 = R_2 T_2$$

Since both buses are traveling from Bus Stop  $A$  to Bus Stop  $B$ , they travel the same distance. We also know the rate each bus traveled and the time it took the first bus to make the trip. Organize the information we know about each train.

$$D_1 = ?$$

$$R_1 = 40 \text{ mi/hr}$$

$$T_1 = 2 \text{ hr}$$

and

$$D_2 = D_1$$

$$R_2 = 25 \text{ mi/hr}$$

$$T_2 = ?$$

Find the distance by plugging all known values into  $D_1 = R_1 T_1$ .

$$D_1 = R_1 T_1$$

$$D_1 = (40 \text{ mi/hr})(2 \text{ hr})$$

$$D_1 = 80 \text{ mi}$$

Now find the time the second bus travels by plugging all known values into  $D_2 = R_2 T_2$ .



$$D_2 = R_2 T_2$$

$$80 \text{ mi} = 25 \text{ mi/hr } T_2$$

$$T_2 = 3.2 \text{ hr}$$

The second bus travels for 3 hr 12 min.

■ 7. Jason and Brittany participate in a race together. Jason runs at a rate of 8 mph and it takes him 45 minutes to complete the race. It took Brittany 1 hour to complete the race. What was Brittany's average speed?

*Solution:*

Use the formula for distance.

Distance = Rate · Time

$$D = RT$$

Use subscripts to create a separate equation for each person.

Jason:  $D_J = R_J T_J$

Brittany:  $D_B = R_B T_B$

Since Jason and Brittany are running in the same race, they travel the same distance. We also know the time it took each person complete the



race, and that Jason finished it in 45 minutes, or 0.75 hours. Organize the information we know about each person.

$$D_J = ?$$

$$R_J = 8 \text{ mi/hr}$$

$$T_J = 0.75 \text{ hr}$$

and

$$D_B = D_J$$

$$R_B = 6 \text{ mi/hr}$$

$$T_B = 1 \text{ hr}$$

Find the distance by plugging all known values into  $D_J = R_J T_J$ .

$$D_J = R_J T_J$$

$$D_J = 8 \text{ mi/hr}(0.75 \text{ hr})$$

$$D_J = 6 \text{ mi}$$

Now find Brittany's rate by plugging all known values into  $D_B = R_B T_B$ .

$$D_B = R_B T_B$$

$$6 \text{ mi} = R_B(1 \text{ hr})$$

$$R_B = 6 \text{ mi/hr}$$

Brittany runs at 6 mph.



- 8. Talan and Emily participate in a race together. Talan runs 10 mph and finishes 1 hour and 18 minutes before Emily does. If Emily runs 5 mph, how long does it take each person to finish the race and how far did they run?

*Solution:*

Use the formula for distance.

$$\text{Distance} = \text{Rate} \cdot \text{Time}$$

$$D = RT$$

Use subscripts to create a separate equation for each person.

$$\text{Talan: } D_T = R_T T_T$$

$$\text{Emily: } D_E = R_E T_E$$

Since Talan and Emily are running in the same race, they travel the same distance. We also know the rate each person ran and that Talan arrived 1 hour and 18 minutes, or 1.3 hrs before Emily. Organize the information we know about each person.

$$D_T = ?$$

$$R_T = 10 \text{ mi/hr}$$

$$T_T = (T_E - 1.3) \text{ hr}$$



and

$$D_E = D_T$$

$$R_E = 5 \text{ mi/hr}$$

$$T_E = ?$$

Plug all known values into both equations.

$$D_T = R_T T_T$$

$$D_T = 10 \text{ mi/hr}(T_E \text{ hr} - 1.3 \text{ hr})$$

and

$$D_E = R_E T_E$$

$$D_T = 5 \text{ mi/hr } T_E$$

Substitute  $D_T = 5 \text{ mi/hr } T_E$  into  $D_T = 10 \text{ mi/hr}(T_E \text{ hr} - 1.3 \text{ hr})$ .

$$D_T = 10 \text{ mi/hr}(T_E \text{ hr} - 1.3 \text{ hr})$$

$$5 \text{ mi/hr } T_E = 10 \text{ mi/hr}(T_E \text{ hr} - 1.3 \text{ hr})$$

$$5 \text{ mi/hr } T_E = 10 \text{ mi/hr } T_E - 13 \text{ mi}$$

Move all  $T_E$  terms to one side of the equation by subtracting  $10 \text{ mi/hr } T_E$  from both sides.

$$5 \text{ mi/hr } T_E - 10 \text{ mi/hr } T_E = 10 \text{ mi/hr } T_E - 10 \text{ mi/hr } T_E - 13 \text{ mi}$$

$$-5 \text{ mi/hr } T_E = -13 \text{ mi}$$



$$T_E = 2.6 \text{ hr}$$

Emily ran for 2 hr 36 min.

Since Talan finished 1 hour and 18 minutes before Emily, we can find Talan's time by subtracting 1 hr 18 min.

$$T_T = T_E - 1 \text{ hr } 18 \text{ min}$$

$$T_T = 2 \text{ hrs } 36 \text{ min} - 1 \text{ hr } 18 \text{ min}$$

$$T_T = 1 \text{ hr } 18 \text{ min}$$

Talan ran for 1 hr 18 min.

Find the distance using either Talan or Emily's distance formula.

$$D_E = 5 \text{ mi/hr}(2.6 \text{ hrs})$$

$$D_E = 13 \text{ mi}$$

Talan and Emily ran 13 miles.





## SYSTEMS OF TWO EQUATIONS WITH A NON-LINEAR EQUATION

■ 1. Solve the system of equations.

$$y = x - 1$$

$$x^2 + y^2 = 1$$

*Solution:*

Use substitution to plug  $y = x - 1$  into  $x^2 + y^2 = 1$ .

$$x^2 + y^2 = 1$$

$$x^2 + (x - 1)^2 = 1$$

Use the FOIL method to expand  $(x - 1)^2$ , and then combine like terms.

$$x^2 + x^2 - x - x + 1 = 1$$

$$2x^2 - 2x + 1 = 1$$

$$2x^2 - 2x = 0$$

$$x^2 - x = 0$$

$$x(x - 1) = 0$$

$$x = 0, 1$$

Plug  $x = 0$  and  $x = 1$  into  $y = x - 1$  to find the corresponding  $y$ -values.



$$y = 0 - 1$$

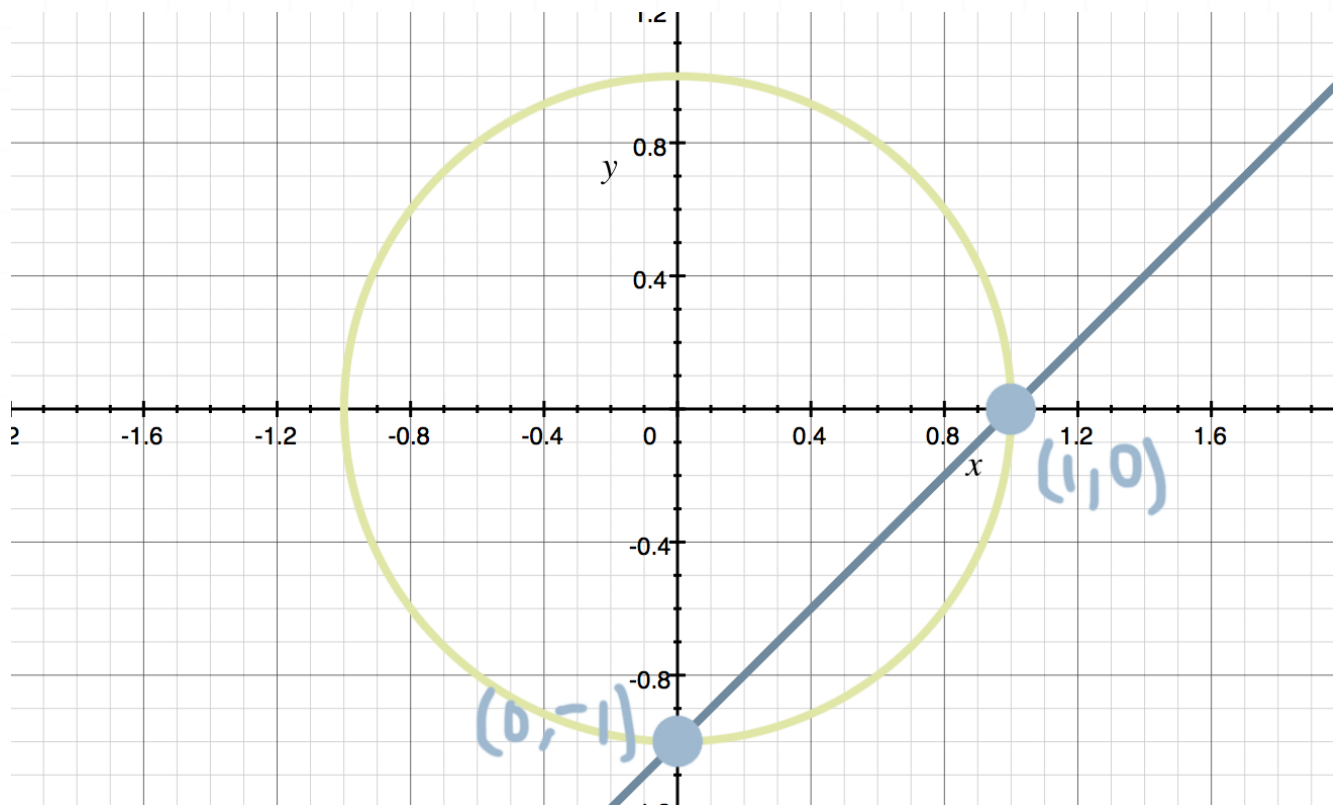
$$y = -1$$

and

$$y = 1 - 1$$

$$y = 0$$

The solutions are therefore  $(0, -1)$  and  $(1, 0)$ . If we plot both curves, we can see the points of intersection.



■ 2. Solve the system of equations.

$$y = 3x - 2$$

$$(x - 1)^2 + (y + 1)^2 = 4$$



*Solution:*

Use substitution to plug  $y = 3x - 2$  into  $(x - 1)^2 + (y + 1)^2 = 4$ .

$$(x - 1)^2 + (y + 1)^2 = 4$$

$$(x - 1)^2 + (3x - 2 + 1)^2 = 4$$

$$(x - 1)^2 + (3x - 1)^2 = 4$$

Use the FOIL method to expand and then combine like terms.

$$x^2 - x - x + 1 + 9x^2 - 3x - 3x + 1 = 4$$

$$x^2 - 2x + 1 + 9x^2 - 6x + 1 = 4$$

$$10x^2 - 8x + 2 = 4$$

$$10x^2 - 8x - 2 = 0$$

$$5x^2 - 4x - 1 = 0$$

$$(5x + 1)(x - 1) = 0$$

$$x = -1/5, 1$$

Plug  $x = -1/5$  and  $x = 1$  into  $y = 3x - 2$  to find the corresponding  $y$ -values.

$$y = 3\left(-\frac{1}{5}\right) - 2$$

$$y = -\frac{3}{5} - \frac{10}{5}$$



$$y = -\frac{13}{5}$$

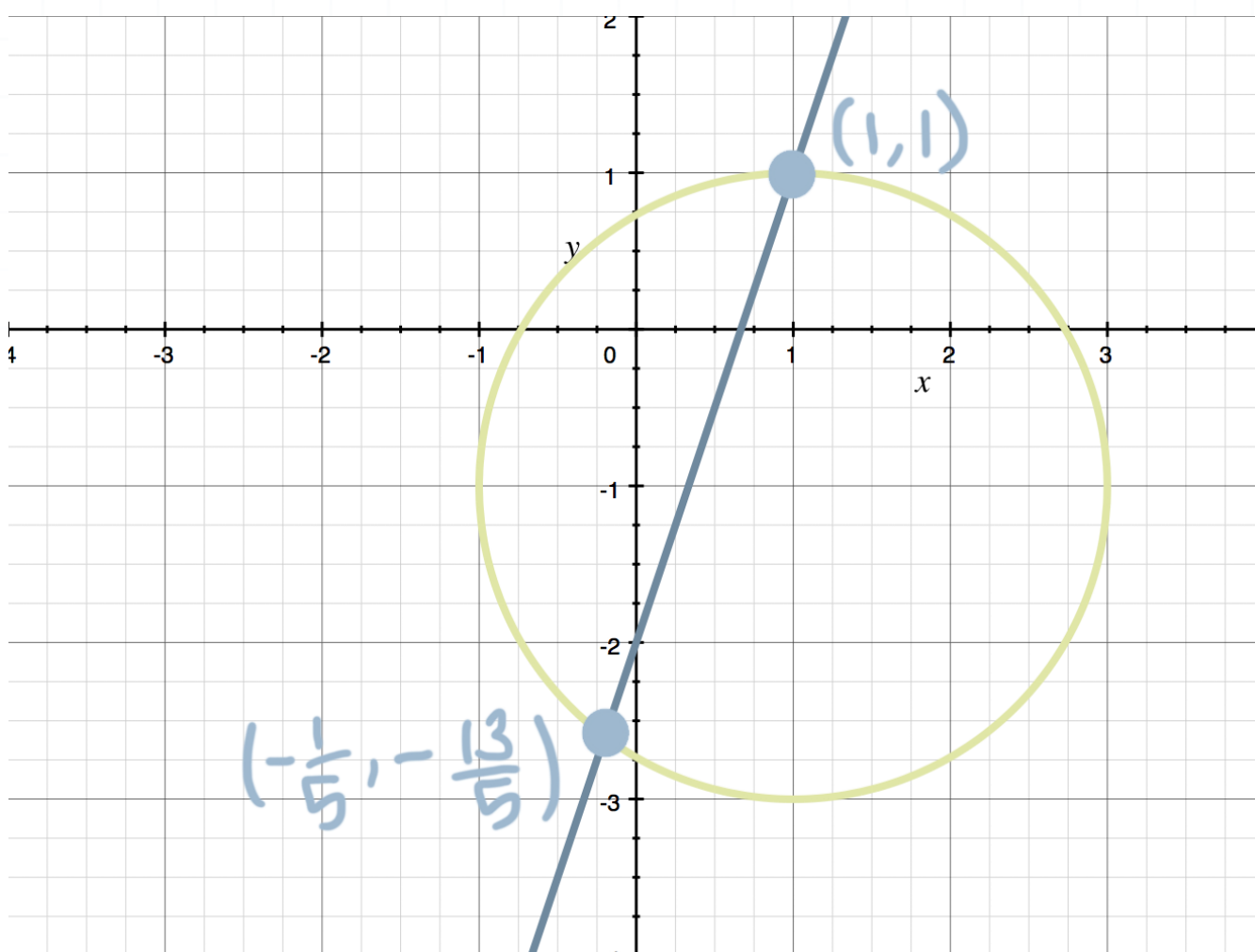
and

$$y = 3(1) - 2$$

$$y = 3 - 2$$

$$y = 1$$

The solutions are therefore  $(-1/5, -13/5)$  and  $(1,1)$ . If we plot both curves, we can see the points of intersection.



■ 3. Solve the system of equations.



$$y - 2x = 0$$

$$(x - 2)^2 + (y - 1)^2 = 9$$

*Solution:*

Solve for  $y$  by adding  $2x$  to both sides of  $y - 2x = 0$ .

$$y - 2x = 0$$

$$y = 2x$$

Use substitution to plug  $y = 2x$  into  $(x - 2)^2 + (y - 1)^2 = 9$ .

$$(x - 2)^2 + (y - 1)^2 = 9$$

$$(x - 2)^2 + (2x - 1)^2 = 9$$

Use the FOIL method to expand and then combine like terms.

$$x^2 - 2x - 2x + 4 + 4x^2 - 2x - 2x + 1 = 9$$

$$x^2 - 4x + 4 + 4x^2 - 4x + 1 = 9$$

$$5x^2 - 8x + 5 = 9$$

$$5x^2 - 8x - 4 = 0$$

Next, factor and then set each factor equal to 0 to solve for  $x$ .

$$(5x + 2)(x - 2) = 0$$

$$5x + 2 = 0$$



$$5x = -2$$

$$x = -2/5, 2$$

Plug  $x = -2/5$  and  $x = 2$  into  $y = 2x$  to find the corresponding  $y$ -values.

$$y = 2\left(-\frac{2}{5}\right)$$

$$y = -\frac{4}{5}$$

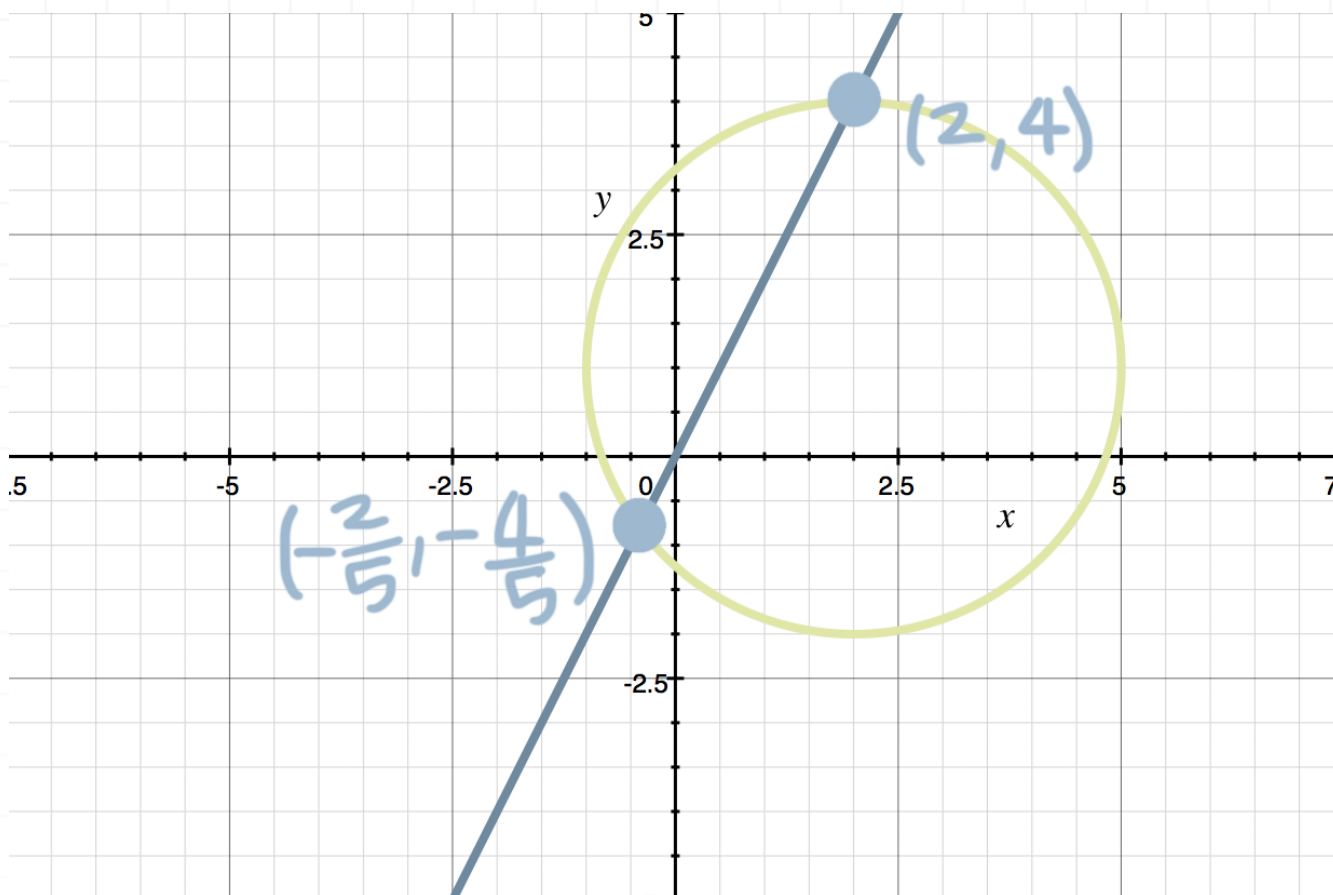
and

$$y = 2(2)$$

$$y = 4$$

The solutions are therefore  $(-2/5, -4/5)$  and  $(2,4)$ . If we plot both curves, we can see the points of intersection.





■ 4. Solve the system of equations.

$$y = 2x + 5$$

$$4x^2 + y^2 = 25$$

*Solution:*

Use substitution to plug  $y = 2x + 5$  into  $4x^2 + y^2 = 25$ .

$$4x^2 + y^2 = 25$$

$$4x^2 + (2x + 5)^2 = 25$$

Use the FOIL method to expand  $(2x + 5)^2$  and then combine like terms.



$$4x^2 + 4x^2 + 10x + 10x + 25 = 25$$

$$8x^2 + 20x + 25 = 25$$

$$8x^2 + 20x = 0$$

$$2x^2 + 5x = 0$$

$$x(2x + 5) = 0$$

$$x = 0, -5/2$$

Plug  $x = -5/2$  and  $x = 0$  into  $y = 2x + 5$  to find the corresponding  $y$ -values.

$$y = 2\left(-\frac{5}{2}\right) + 5$$

$$y = -5 + 5$$

$$y = 0$$

and

$$y = 2(0) + 5$$

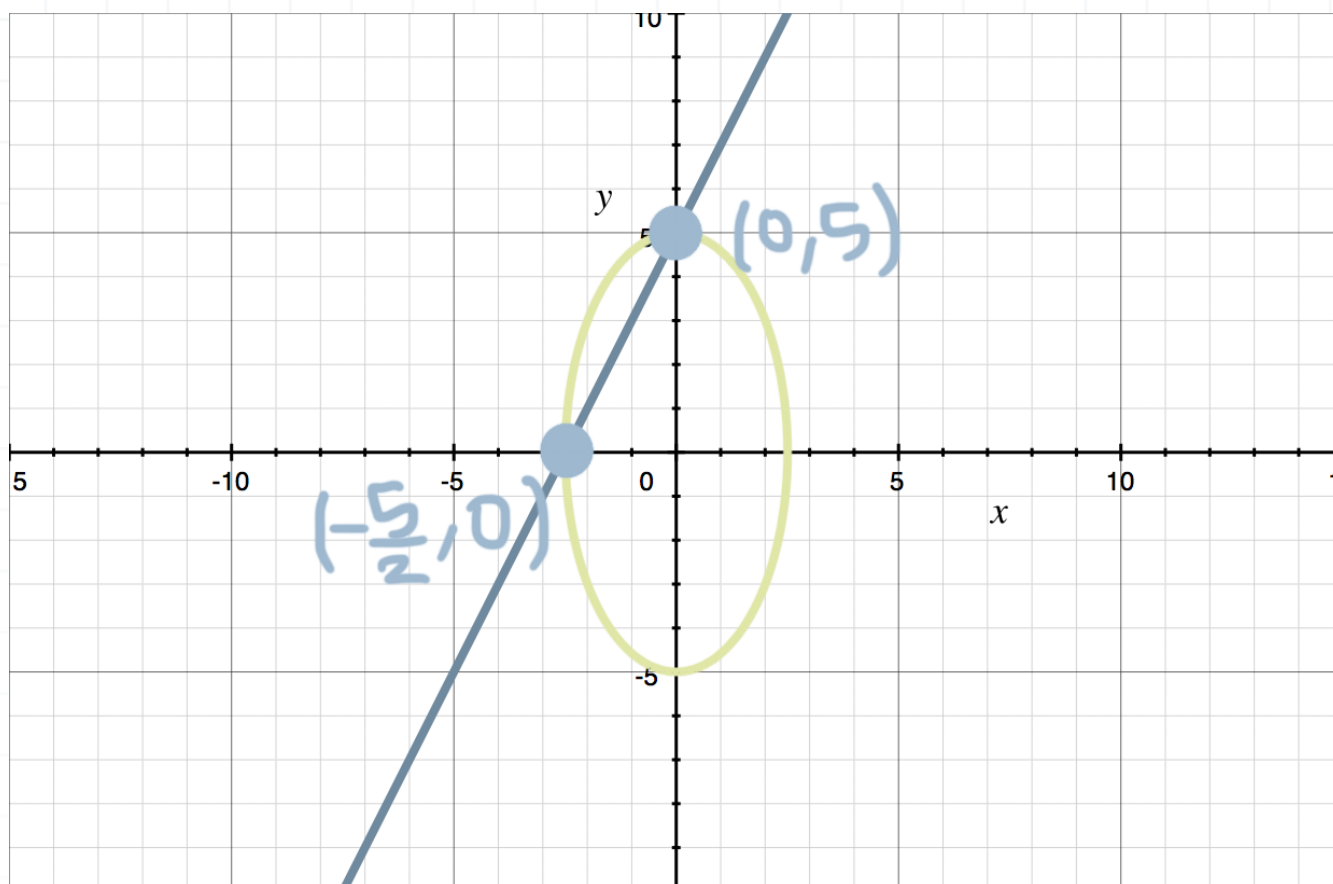
$$y = 0 + 5$$

$$y = 5$$

The solutions are therefore  $(-5/2, 0)$  and  $(0, 5)$ . If we plot both curves, we can see the points of intersection.







■ 5. Solve the system of equations.

$$y + \frac{5}{2} = -\frac{9}{2}x$$

$$9x^2 + 4y^2 = 25$$

*Solution:*

Solve for  $y$  by subtracting  $5/2$  from both sides of  $y + (5/2) = -(9/2)x$ .

$$y + \frac{5}{2} = -\frac{9}{2}x$$

$$y = -\frac{9}{2}x - \frac{5}{2}$$



Use substitution to plug  $y = -(9/2)x - (5/2)$  into  $9x^2 + 4y^2 = 25$ .

$$9x^2 + 4y^2 = 25$$

$$9x^2 + 4\left(-\frac{9}{2}x - \frac{5}{2}\right)^2 = 25$$

Use the FOIL method to expand the binomial, and then combine like terms.

$$9x^2 + 4\left(\frac{81}{4}x^2 + \frac{45}{4}x + \frac{45}{4}x + \frac{25}{4}\right) = 25$$

$$9x^2 + 4\left(\frac{81}{4}x^2 + \frac{90}{4}x + \frac{25}{4}\right) = 25$$

$$9x^2 + 81x^2 + 90x + 25 = 25$$

$$90x^2 + 90x + 25 = 25$$

$$90x^2 + 90x = 0$$

$$x^2 + x = 0$$

$$x(x + 1) = 0$$

$$x = -1, 0$$

Plug  $x = -1$  and  $x = 0$  into  $y = -(9/2)x - (5/2)$  to find the corresponding  $y$ -values.

$$y = -\frac{9}{2}(-1) - \frac{5}{2}$$



$$y = \frac{9}{2} - \frac{5}{2}$$

$$y = \frac{4}{2}$$

$$y = 2$$

and

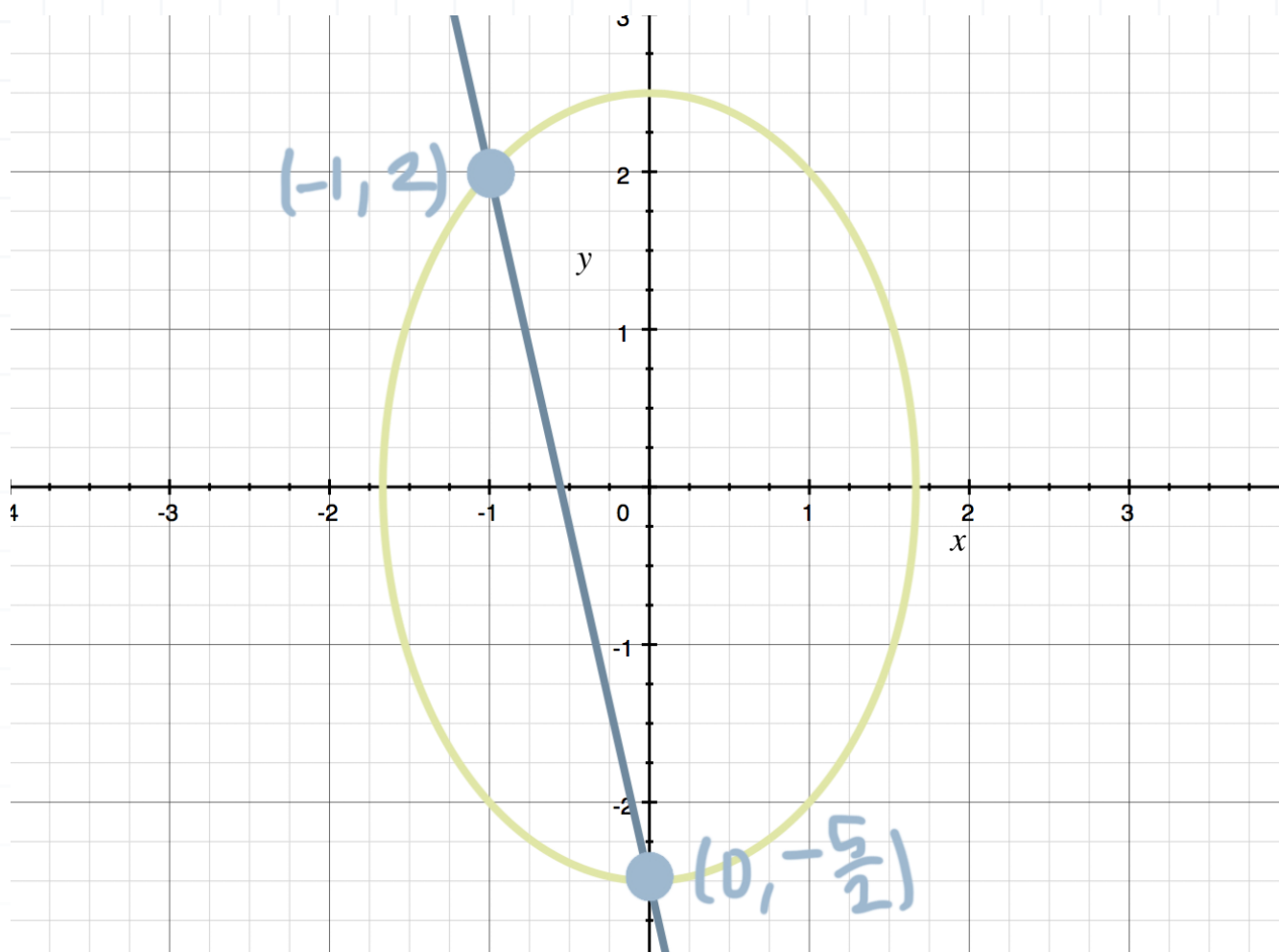
$$y = -\frac{9}{2}(0) - \frac{5}{2}$$

$$y = 0 - \frac{5}{2}$$

$$y = -\frac{5}{2}$$

The solutions are therefore  $(-1, 2)$  and  $(0, -5/2)$ . If we plot both curves, we can see the points of intersection.





■ 6. Solve the system of equations.

$$2y - x = -2$$

$$(x - 6)^2 + 4(y - 4)^2 = 16$$

*Solution:*

Solve  $2y - x = -2$  for  $x$

$$2y - x = -2$$

$$2y = x - 2$$



$$x = 2y + 2$$

Use substitution to plug  $x = 2y + 2$  into  $(x - 6)^2 + 4(y - 4)^2 = 16$ .

$$(x - 6)^2 + 4(y - 4)^2 = 16$$

$$(2y + 2 - 6)^2 + 4(y - 4)^2 = 16$$

$$(2y - 4)^2 + 4(y - 4)^2 = 16$$

Use the FOIL method to expand.

$$4y^2 - 8y - 8y + 16 + 4(y^2 - 4y - 4y + 16) = 16$$

$$4y^2 - 16y + 16 + 4(y^2 - 8y + 16) = 16$$

$$4y^2 - 16y + 16 + 4y^2 - 32y + 64 = 16$$

$$8y^2 - 48y + 80 = 16$$

$$8y^2 - 48y + 64 = 0$$

$$y^2 - 6y + 8 = 0$$

$$(y - 2)(y - 4) = 0$$

$$y = 2, 4$$

Plug  $y = 2$  and  $y = 4$  into  $x = 2y + 2$  to find the corresponding  $x$ -values.

$$x = 2(2) + 2$$

$$x = 4 + 2$$

$$x = 6$$



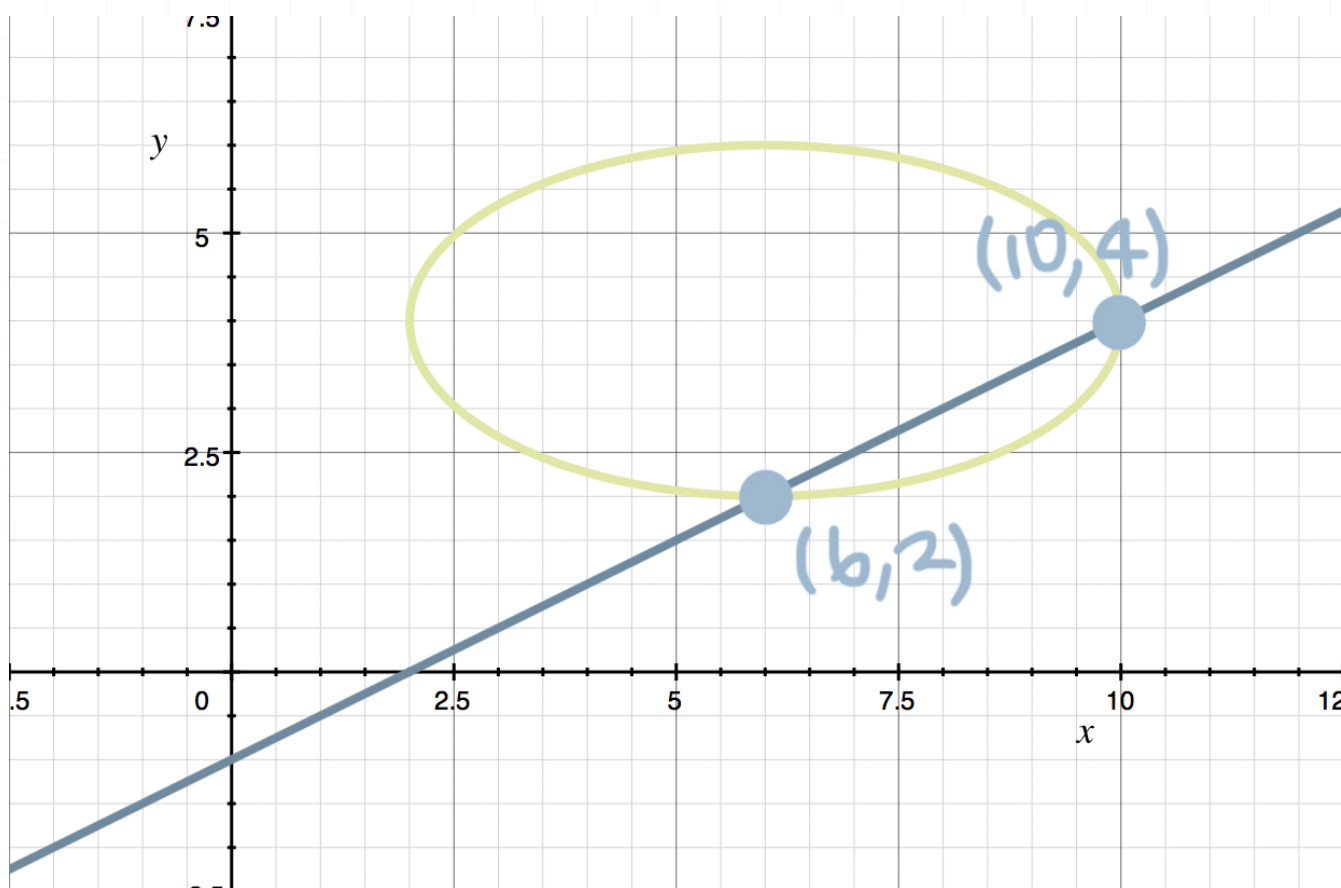
and

$$x = 2(4) + 2$$

$$x = 8 + 2$$

$$x = 10$$

The solutions are therefore (6,2) and (10,4). If we plot both curves, we can see the points of intersection.



■ 7. Solve the system of equations.

$$y = \frac{1}{2}x + 1$$

$$x^2 - y^2 = 4$$



*Solution:*

Use substitution to plug  $y = (1/2)x + 1$  into  $x^2 - y^2 = 4$ .

$$x^2 - y^2 = 4$$

$$x^2 - \left(\frac{1}{2}x + 1\right)^2 = 4$$

Use the FOIL method to expand the binomial.

$$x^2 - \left(\frac{1}{4}x^2 + \frac{1}{2}x + \frac{1}{2}x + 1\right) = 4$$

$$x^2 - \left(\frac{1}{4}x^2 + x + 1\right) = 4$$

$$x^2 - \frac{1}{4}x^2 - x - 1 = 4$$

$$\frac{3}{4}x^2 - x - 1 = 4$$

$$3x^2 - 4x - 4 = 16$$

$$3x^2 - 4x - 20 = 0$$

$$(x + 2)(3x - 10) = 0$$

$$x = -2, 10/3$$



Plug in  $x = -2$  and  $x = 10/3$  into the equation  $y = (1/2)x + 1$  to find the corresponding  $y$ -values.

$$y = \frac{1}{2}(-2) + 1$$

$$y = -1 + 1$$

$$y = 0$$

and

$$y = \frac{1}{2} \left( \frac{10}{3} \right) + 1$$

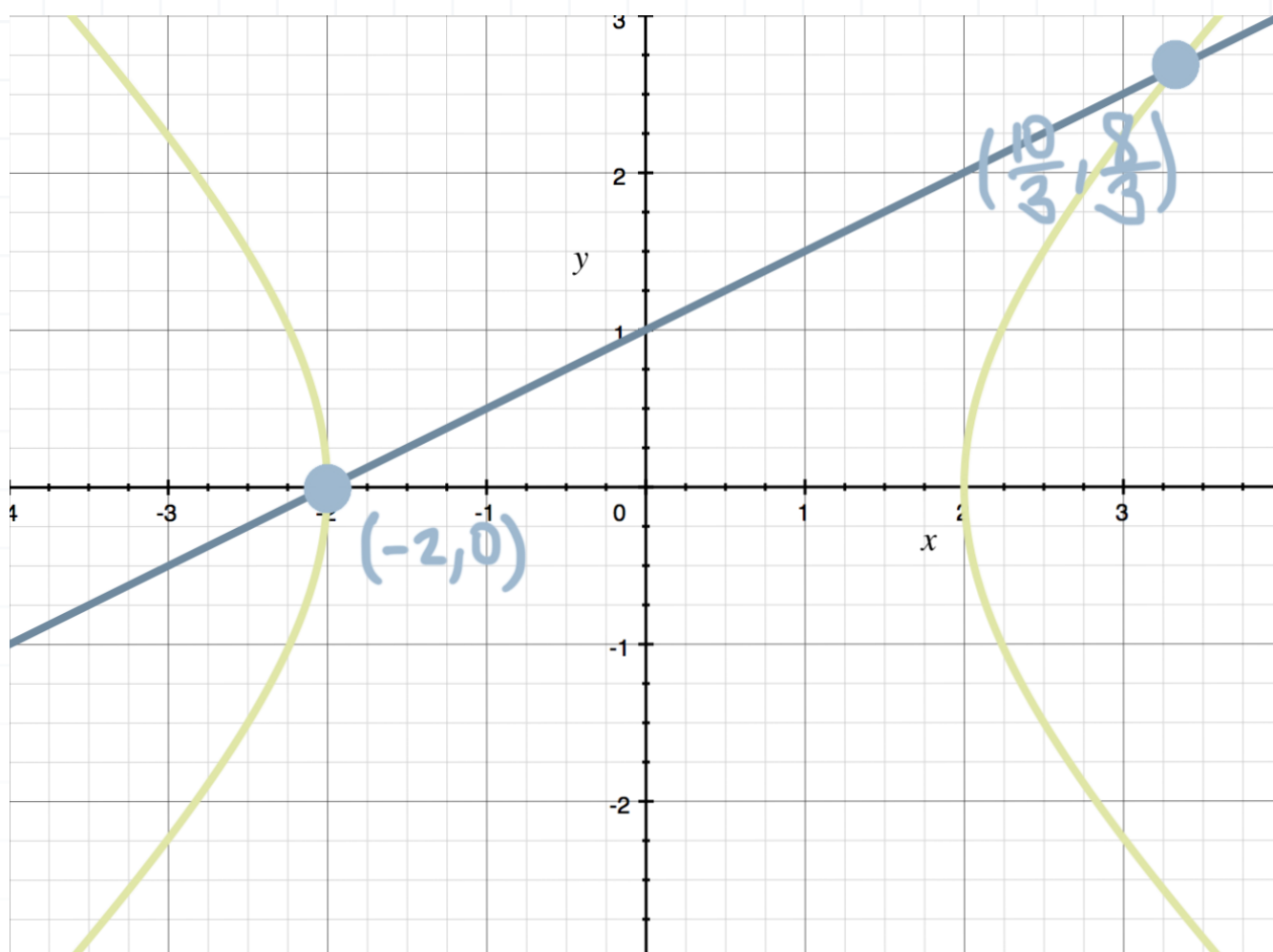
$$y = \frac{5}{3} + \frac{3}{3}$$

$$y = \frac{8}{3}$$

The solutions are therefore  $(-2,0)$  and  $(10/3,8/3)$ . If we plot both curves, we can see the points of intersection.







■ 8. Solve the system of equations.

$$y + 4x = 2$$

$$4y^2 - 16x^2 = 16$$

*Solution:*

Solve  $y + 4x = 2$  for  $y$ .

$$y = -4x + 2$$

Use substitution to plug  $y = -4x + 2$  into  $4y^2 - 16x^2 = 16$ .



$$4y^2 - 16x^2 = 16$$

$$4(-4x + 2)^2 - 16x^2 = 16$$

Use the FOIL method to expand.

$$4(16x^2 - 8x - 8x + 4) - 16x^2 = 16$$

$$4(16x^2 - 16x + 4) - 16x^2 = 16$$

$$64x^2 - 64x + 16 - 16x^2 = 16$$

$$48x^2 - 64x + 16 = 16$$

$$48x^2 - 64x = 0$$

$$3x^2 - 4x = 0$$

$$x(3x - 4) = 0$$

$$x = 0, 4/3$$

Plug  $x = 0$  and  $x = 4/3$  into  $y = -4x + 2$  to find the corresponding  $y$ -values.

$$y = -4(0) + 2$$

$$y = 0 + 2$$

$$y = 2$$

and

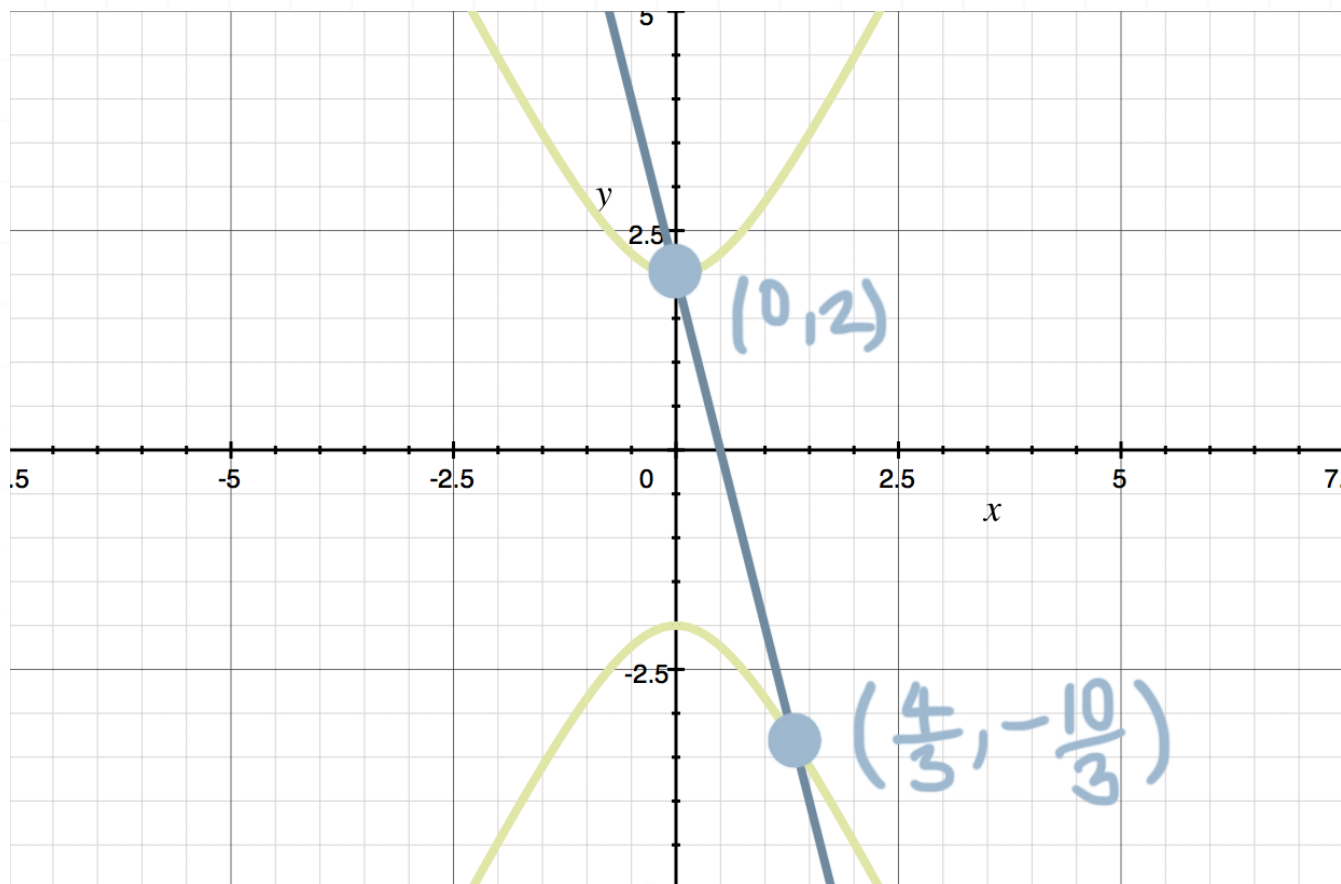
$$y = -4\left(\frac{4}{3}\right) + 2$$



$$y = -\frac{16}{3} + \frac{6}{3}$$

$$y = -\frac{10}{3}$$

The solutions are therefore  $(0, 2)$  and  $(\frac{4}{3}, -\frac{10}{3})$ . If we plot both curves, we can see the points of intersection.



■ 9. Solve the system of equations.

$$y = -\frac{1}{2}x - 3$$

$$\frac{(x-3)^2}{9} - \frac{(y+3)^2}{9} = 1$$



*Solution:*

Use substitution to plug  $y = -(1/2)x - 3$  into the second equation.

$$\frac{(x-3)^2}{9} - \frac{(y+3)^2}{9} = 1$$

$$\frac{(x-3)^2}{9} - \frac{\left(-\frac{1}{2}x - 3 + 3\right)^2}{9} = 1$$

$$\frac{(x-3)^2}{9} - \frac{\left(-\frac{1}{2}x\right)^2}{9} = 1$$

$$(x-3)^2 - \left(-\frac{1}{2}x\right)^2 = 9$$

Use the FOIL method to expand, combine like terms, and then multiply each term by 4 to simplify.

$$x^2 - 3x - 3x + 9 - \frac{1}{4}x^2 = 9$$

$$\frac{3}{4}x^2 - 6x + 9 = 9$$

$$3x^2 - 24x + 36 = 36$$

$$3x^2 - 24x = 0$$

$$x^2 - 8x = 0$$

$$x(x-8) = 0$$



$$x = 0, 8$$

Plug  $x = 0$  and  $x = 8$  into  $y = -(1/2)x - 3$  to find the corresponding  $y$ -values.

$$y = -\frac{1}{2}(0) - 3$$

$$y = 0 - 3$$

$$y = -3$$

and

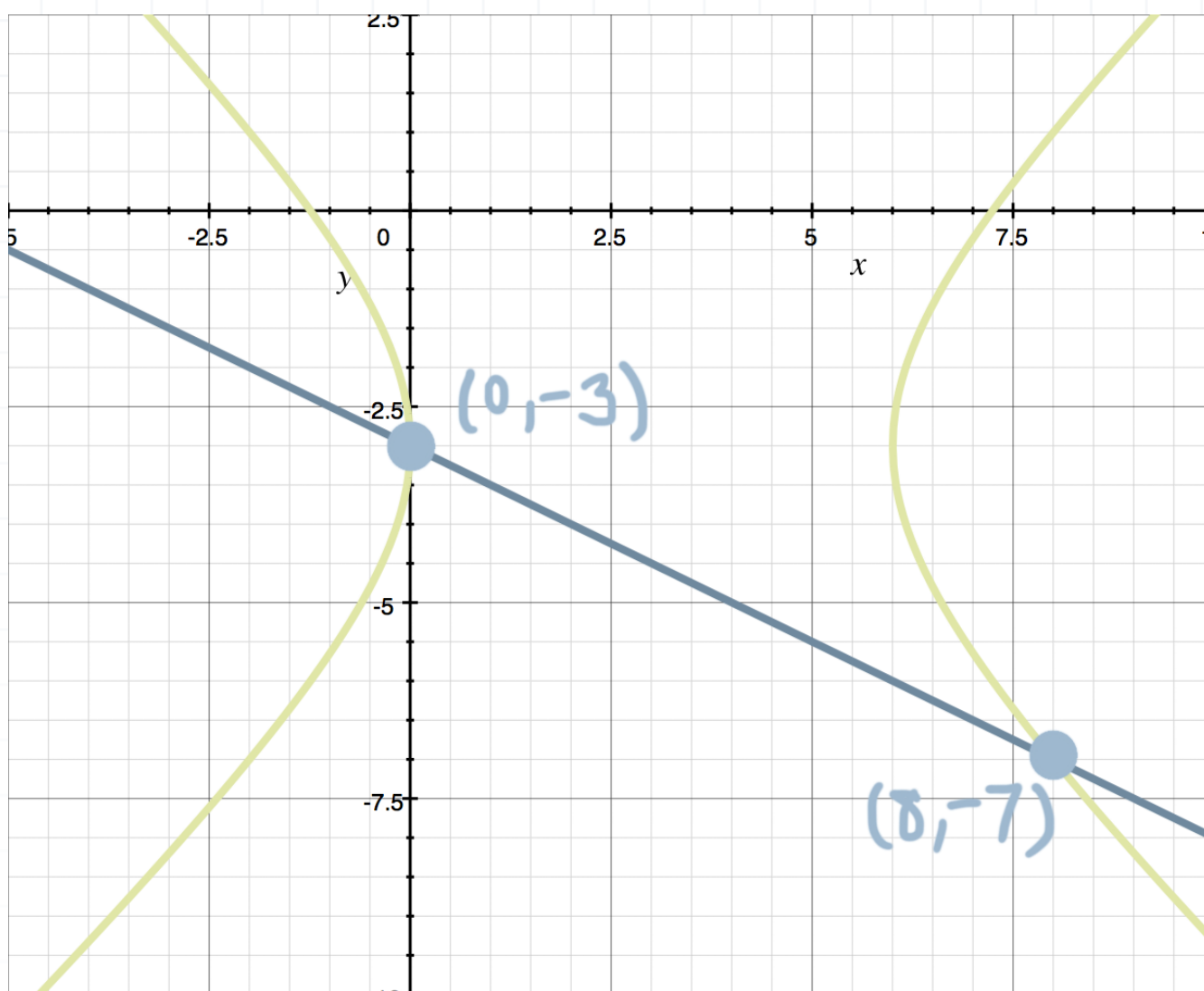
$$y = -\frac{1}{2}(8) - 3$$

$$y = -4 - 3$$

$$y = -7$$

The solutions are therefore  $(0, -3)$  and  $(8, -7)$ . If we plot both curves, we can see the points of intersection.





■ 10. Solve the system of equations.

$$2y - 3x = 14$$

$$\frac{(y + 2)^2}{9} + \frac{(x + 4)^2}{4} = 1$$

*Solution:*

Solve  $2y - 3x = 14$  for  $y$ .

$$2y - 3x = 14$$



$$2y = 3x + 14$$

$$y = \frac{3}{2}x + 7$$

Use substitution to plug  $y = (3/2)x + 7$  into the second equation in the system.

$$\frac{(y + 2)^2}{9} + \frac{(x + 4)^2}{4} = 1$$

$$\frac{\left(\frac{3}{2}x + 7 + 2\right)^2}{9} + \frac{(x + 4)^2}{4} = 1$$

$$\frac{\left(\frac{3}{2}x + 9\right)^2}{9} + \frac{(x + 4)^2}{4} = 1$$

Multiply every term by 36 to simplify.

$$\frac{\left(\frac{3}{2}x + 9\right)^2}{9}(36) + \frac{(x + 4)^2}{4}(36) = 1(36)$$

$$4\left(\frac{3}{2}x + 9\right)^2 + 9(x + 4)^2 = 36$$

Use the FOIL method to expand and then combine like terms.

$$4\left(\frac{9}{4}x^2 + \frac{27}{2}x + \frac{27}{2}x + 81\right) + 9(x^2 + 4x + 4x + 16) = 36$$

$$4\left(\frac{9}{4}x^2 + \frac{54}{2}x + 81\right) + 9(x^2 + 8x + 16) = 36$$



$$9x^2 + 108x + 324 + 9x^2 + 72x + 144 = 36$$

$$18x^2 + 180x + 468 = 36$$

$$18x^2 + 180x + 432 = 0$$

$$x^2 + 10x + 24 = 0$$

$$(x + 6)(x + 4) = 0$$

$$x = -6, -4$$

Plug  $x = -6$  and  $x = -4$  into  $y = (3/2)x + 7$  to find the corresponding  $y$ -values.

$$y = \frac{3}{2}(-6) + 7$$

$$y = -9 + 7$$

$$y = -2$$

and

$$y = \frac{3}{2}(-4) + 7$$

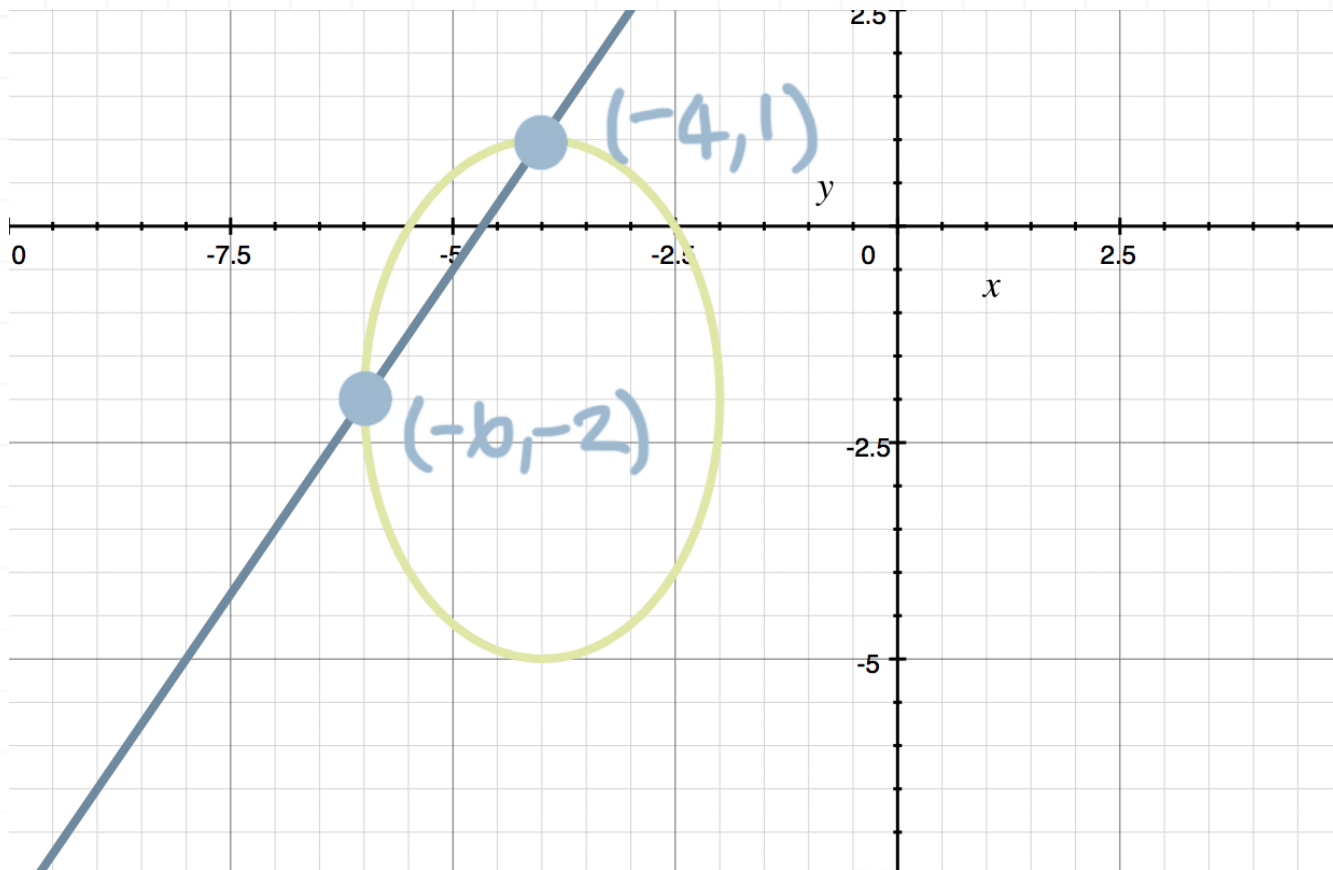
$$y = -6 + 7$$

$$y = 1$$

The solutions are therefore  $(-6, -2)$  and  $(-4, 1)$ . If we plot both curves, we can see the points of intersection.







## NUMBER WORD PROBLEMS

- 1. The sum of two consecutive integers is 37. What are the two numbers?

*Solution:*

Let  $X$  represent the first number. Because the numbers are consecutive, the next integer will be  $X + 1$ . Write the equation that represents the sum of the two numbers.

$$X + X + 1 = 37$$

Solve for  $X$ .

$$2X + 1 = 37$$

$$2X = 36$$

$$X = 18$$

If  $X = 18$ , the next consecutive integer is  $X + 1$ .

$$18 + 1 = 19$$

The integers are 18 and 19.

- 2. The sum of two consecutive odd integers is  $-8$ . What are the two numbers?



*Solution:*

Let  $X$  represent the first number. Because the numbers are consecutive odd numbers, the next integer will be  $X + 2$ . Write the equation that represents the sum of the two numbers.

$$X + X + 2 = -8$$

Solve for  $X$ .

$$2X + 2 = -8$$

$$2X = -10$$

$$X = -5$$

If  $X = -5$ , the next odd integer is  $X + 2$ .

$$-5 + 2 = -3$$

The integers are  $-5$  and  $-3$ .

■ 3. The product of two consecutive even integers is 168. What are the two numbers if they are both positive?

*Solution:*



Let  $X$  represent the first number. Because the numbers are consecutive even integers, the next integer will be  $X + 2$ . Write the equation that represents the product of the two numbers.

$$X(X + 2) = 168$$

$$X^2 + 2X = 168$$

$$X^2 + 2X - 168 = 0$$

$$(X - 12)(X + 14) = 0$$

$$X = 12, -14$$

We were asked to find two positive integers, so we'll use  $X = 12$ . If  $X = 12$ , the next even integer is  $X + 2$ .

$$12 + 2 = 14$$

The integers are 12 and 14.

■ 4. The product of two consecutive integers is 182. What are the two numbers if they are both negative?

*Solution:*

Let  $X$  represent the first number. Because the numbers are consecutive numbers, the next integer will be  $X + 1$ . Write the equation that represents the product of the two numbers.



$$X(X + 1) = 182$$

$$X^2 + X = 182$$

$$X^2 + X - 182 = 0$$

$$(X + 14)(X - 13) = 0$$

$$X = -14, 13$$

We were asked to find two negative integers, so we'll use  $X = -14$ . If  $X = -14$ , the next integer is  $X + 1$ .

$$-14 + 1 = -13$$

The integers are  $-14$  and  $-13$ .

■ 5. The sum of the digits of a two-digit counting number is 11. When the digits are reversed, the new number is 63 greater than the original number. What was the original number?

*Solution:*

Let  $T$  be the tens digit and  $U$  be the units digit of the original number. Write the equation for the sum of the digits.

$$T + U = 11$$



The value of the original number is  $10T + U$ , and reversing the digits gives us a number whose value is  $10U + T$ . The new number is 63 greater than the original number, so

$$10T + U + 63 = 10U + T$$

Move all the  $T$  and  $U$  terms to the left side of the equation and move the constant to the right side.

$$9T + U + 63 = 10U$$

$$9T - 9U + 63 = 0$$

$$9T - 9U = -63$$

$$T - U = -7$$

Add the sum of the digits equation to the equation we just found to solve this as a system of equations.

$$T - U + (T + U) = -7 + (11)$$

$$T + T - U + U = -7 + 11$$

$$2T = 4$$

$$T = 2$$

Plug  $T = 2$  into  $T + U = 11$  and solve for  $U$ .

$$2 + U = 11$$

$$U = 9$$



The value of the original number is  $10T + U$ .

$$10(2) + 9$$

$$20 + 9 = 29$$

The original number is 29.

■ 6. The sum of the digits of a two-digit counting number is 7. When the digits are reversed, the new number is 27 less than the original number. What was the original number?

*Solution:*

Let  $T$  be the tens digit and  $U$  be the units digit of the original number. Write the equation for the sum of the digits.

$$T + U = 7$$

The value of the original number is  $10T + U$ , and reversing the digits gives us a number whose value is  $10U + T$ . The new number is 27 less than the original number.

$$10T + U - 27 = 10U + T$$

Move all the  $T$  and  $U$  terms to the left-side of the equation and the constant to the right-side.

$$9T + U - 27 = 10U$$



$$9T - 9U - 27 = 0$$

$$9T - 9U = 27$$

$$T - U = 3$$

Add the sum of the digits equation to the equation we just found to solve this as a system of equations.

$$T - U + (T + U) = 3 + (7)$$

$$T + T - U + U = 3 + 7$$

$$2T = 10$$

$$T = 5$$

Plug  $T = 5$  into  $T + U = 7$  and solve for  $U$ .

$$5 + U = 7$$

$$U = 2$$

The value of the original number is  $10T + U$ .

$$10(5) + 2$$

$$50 + 2 = 52$$

The original number is 52.





■ 7. The sum of the digits of a two-digit counting number is 14. The ratio of the units digit to the tens digit is 3 to 4. What is the number?

*Solution:*

Let  $T$  be the tens digit and  $U$  be the units digit of the number. Write the equation for the sum of the digits.

$$T + U = 14$$

Write the equation for the ratio of the units digit and tens digit.

$$\frac{U}{T} = \frac{3}{4}$$

Multiply both sides by  $T$  to solve for  $U$ .

$$U = \frac{3}{4}T$$

Plug  $U = (3/4)T$  into  $T + U = 14$ .

$$T + \frac{3}{4}T = 14$$

$$\frac{7}{4}T = 14$$

$$7T = 56$$

$$T = 8$$

Plug  $T = 8$  into  $T + U = 14$  and solve for  $U$ .



$$8 + U = 14$$

$$U = 6$$

The value of the number is  $10T + U$ .

$$10(8) + 6$$

$$80 + 6 = 86$$

The number is 86.

■ 8. The sum of the digits of a two-digit counting number is 8. The ratio of the tens digit to the units digit is 3 to 1. What is the number?

*Solution:*

Let  $T$  be the tens digit and  $U$  be the units digit of the number. Write the equation for the sum of the digits.

$$T + U = 8$$

Write the equation for the ratio of the tens digit and units digit.

$$\frac{T}{U} = \frac{3}{1}$$

Multiply both sides by  $U$  to solve for  $T$ .

$$T = 3U$$



Plug  $T = 3U$  into  $T + U = 8$ .

$$3U + U = 8$$

$$4U = 8$$

$$U = 2$$

Plug  $U = 2$  into  $T + U = 14$  and solve for  $T$ .

$$T + 2 = 8$$

$$T = 6$$

The value of the number is  $10T + U$ .

$$10(6) + 2$$

$$60 + 2 = 62$$

The number is 62.

■ 9. Find three positive consecutive even integers such that the sum of the first and second is 22 less than the product of the first and third.

*Solution:*

Let  $X$  represent the first number. Because the numbers are consecutive even integers, the next integer will be  $X + 2$  and the third will be  $X + 4$ . The



sum of the first and second integers will equal the product of the first and third, minus 22.

$$X + X + 2 = X(X + 4) - 22$$

$$2X + 2 = X^2 + 4X - 22$$

Move all the terms to the left side of the equation and then factor.

$$X^2 + 2X - 24 = 0$$

$$(X + 6)(X - 4) = 0$$

$$X = -6, 4$$

We were asked to find positive integers, so we'll use  $X = 4$ . If  $X = 4$ , the next two even integers are  $X + 2$  and  $X + 4$ .

$$4 + 2 = 6$$

$$4 + 4 = 8$$

The integers are 4, 6, and 8.

■ 10. Find three negative consecutive odd integers such that the sum of the first and third is 49 less than the product of the second and third.

*Solution:*



Let  $X$  represent the first number. Because the numbers are consecutive odd integers, the next integer will be  $X + 2$  and the third will be  $X + 4$ . The sum of the first and third integers will equal the product of the second and third, minus 49.

$$X + X + 4 = (X + 2)(X + 4) - 49$$

FOIL out the binomials on the right side, then combine like terms.

$$2X + 4 = X^2 + 4X + 2X + 8 - 49$$

$$2X + 4 = X^2 + 6X - 41$$

Move all the terms to the left side of the equation and then factor.

$$X^2 + 4X - 45 = 0$$

$$(X + 9)(X - 5) = 0$$

$$X = -9, 5$$

We were asked to find negative integers, so we'll use  $X = -9$ . If  $X = -9$ , the next two odd integers are  $X + 2$  and  $X + 4$ .

$$-9 + 2 = -7$$

$$-9 + 4 = -5$$

The integers are  $-9$ ,  $-7$ , and  $-5$ .



## AGE WORD PROBLEMS

- 1. In 16 years, Thorin will be nine times older than he is now. How old is he now?

*Solution:*

Let  $T$  represent Thorin's age now. If we add 16 years to  $T$ , it will equal 9 times  $T$ .

$$T + 16 = 9T$$

Solve for  $T$ .

$$16 = 8T$$

$$T = 2$$

Thorin is 2 years old now.

- 2. In 18 years, Danica will be three times older than she is now. How old is she now?

*Solution:*



Let  $D$  represent Danica's age now. If we add 18 years to  $D$ , it will equal 3 times  $D$ .

$$D + 18 = 3D$$

Solve for  $D$ .

$$18 = 2D$$

$$D = 9$$

Danica is 9 years old now.

■ 3. In eight years, Aaron will be twice as old as he was six years ago. How old is he now?

*Solution:*

Let  $A$  represent Aaron's age now. If we add 8 years to  $A$ , it will equal 2 times  $(A - 6)$ .

$$A + 8 = 2(A - 6)$$

Solve for  $A$ .

$$A + 8 = 2A - 12$$

$$20 = A$$

Aaron is 20 years old now.



- 4. In 12 years, Jake will be four times as old as he was nine years ago. How old will he be in 12 years?

*Solution:*

Let  $J$  represent Jake's age now. If we add 12 years to  $J$ , it will equal 4 times  $(J - 9)$ .

$$J + 12 = 4(J - 9)$$

Solve for  $J$ .

$$J + 12 = 4J - 36$$

$$48 = 3J$$

$$J = 16$$

Add 12 to 16 to find out how old Jake will be in 12 years.

$$16 + 12 = 28$$

Jake will be 28 years old in 12 years.

- 5. In 16 years, Erica will be six times as old as she was 14 years ago. How old was she four years ago?





*Solution:*

Let  $E$  represent Erica's age now. If we add 16 years to  $E$ , it will equal 6 times  $(E - 14)$ .

$$E + 16 = 6(E - 14)$$

Solve for  $E$ .

$$E + 16 = 6E - 84$$

$$100 = 5E$$

$$E = 20$$

Subtract 4 from 20 to find out how old Erica was four years ago.

$$20 - 4 = 16$$

Erica was 16 years old four years ago.

■ 6. May is 6 years older than Hazel. In two years, May will be twice as old as Hazel. How old are May and Hazel now?

*Solution:*

Let  $M$  represent May's age now and let  $H$  represent Hazel's age now. Add 6 to Hazel's age to get May's age.

$$M = H + 6$$



We need to multiply Hazel's age in two years by 2 to get May's age in two years.

$$2(H + 2) = M + 2$$

Use substitution to plug  $M = H + 6$  into  $2(H + 2) = M + 2$ .

$$2(H + 2) = M + 2$$

$$2(H + 2) = H + 6 + 2$$

$$2H + 4 = H + 8$$

$$H = 4$$

Hazel is 4 years old.

Plug  $H = 4$  into  $M = H + 6$  to find May's age.

$$M = H + 6$$

$$M = 4 + 6$$

$$M = 10$$

May is 10 years old.

■ 7. Ellie is 20 years older than Clay. In four years, Ellie will be five times as old as Clay. How old are Ellie and Clay now?



***Solution:***

Let  $E$  represent Ellie's age now and let  $C$  represent Clay's age now. Add 20 to Clay's age to get Ellie's age.

$$E = C + 20$$

We need to multiply Clay's age in four years by 5 to get Ellie's age in four years.

$$5(C + 4) = E + 4$$

Use substitution to plug  $E = C + 20$  into  $5(C + 4) = E + 4$ .

$$5(C + 4) = E + 4$$

$$5(C + 4) = C + 20 + 4$$

$$5C + 20 = C + 24$$

$$4C = 4$$

$$C = 1$$

Clay is 1 year old.

Plug  $C = 1$  into  $E = C + 20$  to find Ellie's age.

$$E = C + 20$$

$$E = 1 + 20$$

$$E = 21$$



Ellie is 21 years old.

■ 8. 10 years ago, Tim was three years younger than twice Sally's age. 17 years from now Sally will be seven years older than  $\frac{5}{8}$  of Tim's age. How old are Sally and Tim now?

*Solution:*

Let  $T$  represent Tim's age now and let  $S$  represent Sally's age now. We need to multiply Sally's age 10 years ago by 2, and then subtract 3 in order to get Tim's age 10 years ago.

$$2(S - 10) - 3 = T - 10$$

$$2S - 20 - 3 = T - 10$$

$$2S - 23 = T - 10$$

$$2S - 13 = T$$

We need to subtract 7 from Sally's age in 17 years to get  $\frac{5}{8}$  of Tim's age in 17 years.

$$S + 17 - 7 = \frac{5}{8}(T + 17)$$

$$S + 10 = \frac{5}{8}(T + 17)$$

Use substitution to plug  $T = 2S - 13$  into  $S + 10 = \frac{5}{8}(T + 17)$ .



$$S + 10 = \frac{5}{8}(T + 17)$$

$$S + 10 = \frac{5}{8}(2S - 13 + 17)$$

$$S + 10 = \frac{5}{8}(2S + 4)$$

$$S + 10 = \frac{5}{4}S + \frac{5}{2}$$

$$\frac{15}{2} = \frac{1}{4}S$$

$$S = 30$$

Sally is 30 years old.

Plug  $S = 30$  into  $T = 2S - 13$  to find Tim's age.

$$T = 2S - 13$$

$$T = 2(30) - 13$$

$$T = 60 - 13$$

$$T = 47$$

Tim is 47 years old.

■ 9. Jessica is five years younger than Ryan. 12 years ago, Jessica was half the age Ryan will be in three years. How old is Jessica now?



*Solution:*

Let  $J$  represent Jessica's age now and let  $R$  represent Ryan's age now. We need to subtract 5 from Ryan's age to get Jessica's age.

$$J = R - 5$$

We need to multiply Jessica's age 12 years ago by 2 in order to get Ryan's age in three years.

$$2(J - 12) = R + 3$$

Use substitution to plug  $J = R - 5$  into  $2(J - 12) = R + 3$ .

$$2(J - 12) = R + 3$$

$$2(R - 5 - 12) = R + 3$$

$$2(R - 17) = R + 3$$

$$2R - 34 = R + 3$$

$$R = 37$$

Ryan is 37 years old.

Plug  $R = 37$  into  $J = R - 5$  to find Jessica's age.

$$J = R - 5$$

$$J = 37 - 5$$



$$J = 32$$

Jessica is 32 years old.

■ 10. Kate is five times older than Sam. Five years ago, Kate was 15 times older than Sam. How old will Sam be in three years?

*Solution:*

Let  $K$  represent Kate's age now and let  $S$  represent Sam's age now. We need to multiply Sam's age by 5 to get Kate's age.

$$K = 5S$$

We need to multiply Sam's age five years ago by 15 in order to get Kate's age five years ago.

$$15(S - 5) = K - 5$$

Use substitution to plug  $K = 5S$  into  $15(S - 5) = K - 5$ .

$$15(S - 5) = K - 5$$

$$15(S - 5) = 5S - 5$$

$$15S - 75 = 5S - 5$$

$$10S = 70$$

$$S = 7$$



Add 3 to 7 to get Sam's age in three years.

$$7 + 3 = 10$$

Sam will be 10 years old in three years.





## SYSTEMS OF THREE EQUATIONS

- 1. Find the unique solution to the system of equations.

$$2x + y - z = 3$$

$$x - y + z = 0$$

$$x - 2y - 3z = 4$$

*Solution:*

Let's number the equations to stay organized.

$$\text{[1]} \quad 2x + y - z = 3$$

$$\text{[2]} \quad x - y + z = 0$$

$$\text{[3]} \quad x - 2y - 3z = 4$$

Add equations [1] and [2] so that  $y$  and  $z$  will be eliminated.

$$(2x + y - z) + (x - y + z) = (3) + (0)$$

$$2x + y - z + x - y + z = 3$$

$$3x + y - y + z - z = 3$$

$$3x = 3$$

$$x = 1$$



Let's plug  $x = 1$  into equations [2] and [3] to put them in terms of  $y$  and  $z$ .

$$x - y + z = 0$$

$$1 - y + z = 0$$

$$\text{[4]} \quad -y + z = -1$$

and

$$x - 2y - 3z = 4$$

$$1 - 2y - 3z = 4$$

$$\text{[5]} \quad -2y - 3z = 3$$

Multiply equation [4] by 3,

$$3(-y + z = -1)$$

$$-3y + 3z = -3$$

and then add this to equation [5] so that  $z$  can be eliminated.

$$(-3y + 3z) + (-2y - 3z) = (-3) + (3)$$

$$-3y + 3z - 2y - 3z = 0$$

$$-3y - 2y = 0$$

$$-5y = 0$$

$$y = 0$$

Plug  $y = 0$  into equation [4] to solve for  $z$ .



$$-2y - 3z = 3$$

$$-2(0) - 3z = 3$$

$$-3z = 3$$

$$z = -1$$

Therefore, the solution to the system is  $(x, y, z) = (1, 0, -1)$ .

■ 2. Find the unique solution to the system of equations.

$$3x + y - z = -2$$

$$x - 2y + 3z = 23$$

$$2x + 3y + 2z = 5$$

*Solution:*

Let's number the equations to stay organized.

$$\text{[1]} \quad 3x + y - z = -2$$

$$\text{[2]} \quad x - 2y + 3z = 23$$

$$\text{[3]} \quad 2x + 3y + 2z = 5$$

Multiply equation [1] by 2,

$$2(3x + y - z = -2)$$



$$6x + 2y - 2z = -4$$

and add this to equation [3] so that  $z$  can be eliminated.

$$(6x + 2y - 2z) + (2x + 3y + 2z) = (-4) + (5)$$

$$6x + 2y - 2z + 2x + 3y + 2z = 1$$

$$6x + 2y + 2x + 3y = 1$$

$$\text{[4]} \quad 8x + 5y = 1$$

Multiply equation [1] by 3,

$$3(3x + y - z = -2)$$

$$9x + 3y - 3z = -6$$

and add this to equation [2] so that  $z$  can be eliminated.

$$(9x + 3y - 3z) + (x - 2y + 3z) = (-6) + (23)$$

$$9x + 3y - 3z + x - 2y + 3z = 17$$

$$9x + 3y + x - 2y = 17$$

$$\text{[5]} \quad 10x + y = 17$$

Multiply equation [5] by  $-5$ ,

$$-5(10x + y = 17)$$

$$-50x - 5y = -85$$

and then add this to equation [4] so that  $y$  will be eliminated.



$$(8x + 5y) + (-50x - 5y) = (1) + (-85)$$

$$8x + 5y - 50x - 5y = -84$$

$$8x - 50x = -84$$

$$-42x = -84$$

$$x = 2$$

Let's plug  $x = 1$  into equation [5] to solve for  $y$ .

$$10x + y = 17$$

$$10(2) + y = 17$$

$$20 + y = 17$$

$$y = -3$$

Plug  $x = 2$  and  $y = -3$  into any of the original equations to solve for  $z$ . We'll use equation [1].

$$3x + y - z = -2$$

$$3(2) + (-3) - z = -2$$

$$6 - 3 - z = -2$$

$$3 - z = -2$$

$$-z = -5$$

$$z = 5$$



Therefore, the solution to the system is  $(x, y, z) = (2, -3, 5)$ .

■ 3. Find the unique solution to the system of equations.

$$5x - 3y + z = -8$$

$$2x + y - 2z = -6$$

$$-3x + 2y + 4z = 19$$

*Solution:*

Let's number the equations to stay organized.

$$\text{[1]} \quad 5x - 3y + z = -8$$

$$\text{[2]} \quad 2x + y - 2z = -6$$

$$\text{[3]} \quad -3x + 2y + 4z = 19$$

Multiply equation [1] by 2,

$$2(5x - 3y + z = -8)$$

$$10x - 6y + 2z = -16$$

and then add this to equation [2] so that  $z$  will be eliminated.

$$(10x - 6y + 2z) + (2x + y - 2z) = (-16) + (-6)$$

$$10x - 6y + 2z + 2x + y - 2z = -22$$



$$10x - 6y + 2x + y = -22$$

$$\text{[4]} \quad 12x - 5y = -22$$

Multiply equation [1] by  $-4$ ,

$$-4(5x - 3y + z = -8)$$

$$-20x + 12y - 4z = 32$$

and then add this to equation [3] so that  $z$  will be eliminated.

$$(-20x + 12y - 4z) + (-3x + 2y + 4z) = (32) + (19)$$

$$-20x + 12y - 4z - 3x + 2y + 4z = 51$$

$$-20x + 12y - 3x + 2y = 51$$

$$\text{[5]} \quad -23x + 14y = 51$$

Solve for  $y$  in equation [4].

$$12x - 5y = -22$$

$$-5y = -12x - 22$$

$$\text{[6]} \quad y = \frac{12}{5}x + \frac{22}{5}$$

Plug [6] into equation [5] to solve for  $x$ .

$$-23x + 14y = 51$$

$$-23x + 14\left(\frac{12}{5}x + \frac{22}{5}\right) = 51$$



$$-23x + \frac{168}{5}x + \frac{308}{5} = 51$$

$$\frac{53}{5}x + \frac{308}{5} = 51$$

$$53x + 308 = 255$$

$$53x = -53$$

$$x = -1$$

Plug  $x = -1$  into equation [6] to solve for  $y$ .

$$y = \frac{12}{5}x + \frac{22}{5}$$

$$y = \frac{12}{5}(-1) + \frac{22}{5}$$

$$y = -\frac{12}{5} + \frac{22}{5}$$

$$y = \frac{10}{5}$$

$$y = 2$$

Plug  $x = -1$  and  $y = 2$  into any of the original equations to solve for  $z$ . We'll use equation [1].

$$5x - 3y + z = -8$$

$$5(-1) - 3(2) + z = -8$$





$$-5 - 6 + z = -8$$

$$-11 + z = -8$$

$$z = 3$$

Therefore, the solution to the system is  $(x, y, z) = (-1, 2, 3)$ .

■ 4. Find the unique solution to the system of equations.

$$-2x + 3y - 4z = 10$$

$$4x + 3y + 2z = 4$$

$$x - 6y + 4z = -19$$

*Solution:*

Let's number the equations to stay organized.

$$\text{[1]} \quad -2x + 3y - 4z = 10$$

$$\text{[2]} \quad 4x + 3y + 2z = 4$$

$$\text{[3]} \quad x - 6y + 4z = -19$$

Multiply equation [2] by 2,

$$2(4x + 3y + 2z = 4)$$

$$8x + 6y + 4z = 8$$



and then add this to equation [1] so that  $z$  will be eliminated.

$$(8x + 6y + 4z) + (-2x + 3y - 4z) = (8) + (10)$$

$$8x + 6y + 4z - 2x + 3y - 4z = 18$$

$$8x + 6y - 2x + 3y = 18$$

$$6x + 9y = 18$$

$$\text{[4]} \quad 2x + 3y = 6$$

Add equation [1] to equation [3] so that  $z$  will be eliminated.

$$(-2x + 3y - 4z) + (x - 6y + 4z) = (10) + (-19)$$

$$-2x + 3y - 4z + x - 6y + 4z = -9$$

$$-2x + 3y + x - 6y = -9$$

$$\text{[5]} \quad -x - 3y = -9$$

Solve for  $x$  in equation [5].

$$-x - 3y = -9$$

$$-x = -9 + 3y$$

$$\text{[6]} \quad x = 9 - 3y$$

Plug  $x = 9 - 3y$  into equation [4] to solve for  $y$ .

$$2x + 3y = 6$$

$$2(9 - 3y) + 3y = 6$$



$$18 - 6y + 3y = 6$$

$$18 - 3y = 6$$

$$-3y = -12$$

$$y = 4$$

Let's plug  $y = 4$  into equation [6] to solve for  $x$ .

$$x = 9 - 3y$$

$$x = 9 - 3(4)$$

$$x = 9 - 12$$

$$x = -3$$

Plug  $x = -3$  and  $y = 4$  into any of the original equations to solve for  $z$ . We'll use equation [2].

$$4x + 3y + 2z = 4$$

$$4(-3) + 3(4) + 2z = 4$$

$$-12 + 12 + 2z = 4$$

$$2z = 4$$

$$z = 2$$

Therefore, the solution to the system is  $(x, y, z) = (-3, 4, 2)$ .



- 5. Find the unique solution to the system of equations.

$$2x - y + z = 9$$

$$4x - 2y + 2z = 18$$

$$-2x + y - z = -9$$

*Solution:*

Let's number the equations to stay organized.

$$\text{[1]} \quad 2x - y + z = 9$$

$$\text{[2]} \quad 4x - 2y + 2z = 18$$

$$\text{[3]} \quad -2x + y - z = -9$$

Add equation [1] to equation [3] so that  $z$  will be eliminated.

$$(2x - y + z) + (-2x + y - z) = (9) + (-9)$$

$$2x - y + z - 2x + y - z = 0$$

$$-y + z + y - z = 0$$

$$z - z = 0$$

$$0 = 0$$

When all the variables eliminate and we get a true statement, it means all points  $(x, y, z)$  are a solution to the system. So far, this is the case with



equations [1] and [3]. If this also happens with equation [2], then the whole system is an “identity” and there are infinite solutions.

Let's check the second equation to see if this is the case. Multiply equation [3] by 2,

$$2(-2x + y - z = -9)$$

$$-4x + 2y - 2z = -18$$

and then add it to equation [2].

$$(-4x + 2y - 2z) + (4x - 2y + 2z) = (-18) + (18)$$

$$-4x + 2y - 2z + 4x - 2y + 2z = 0$$

$$2y - 2z - 2y + 2z = 0$$

$$-2z + 2z = 0$$

$$0 = 0$$

Since all the variables eliminate and we get a true statement, the system is an identity and there are infinite solutions.

■ 6. Find the unique solution to the system of equations.

$$x + 2y - z = 9$$

$$3x + y - z = 5$$

$$-x - 4y + z = 2$$



*Solution:*

Let's number the equations to stay organized.

$$\text{[1]} \quad x + 2y - z = 9$$

$$\text{[2]} \quad 3x + y - z = 5$$

$$\text{[3]} \quad -x - 4y + z = 2$$

Add equations [1] and [3] together so that  $x$  will be eliminated.

$$(x + 2y - z) + (-x - 4y + z) = (9) + (2)$$

$$x + 2y - z - x - 4y + z = 11$$

$$2y - 4y = 11$$

$$-2y = 11$$

$$\text{[4]} \quad y = -\frac{11}{2}$$

Add equation [2] to equation [3] so that  $z$  will be eliminated.

$$(3x + y - z) + (-x - 4y + z) = (5) + (2)$$

$$3x + y - z - x - 4y + z = 7$$

$$3x + y - x - 4y = 7$$

$$\text{[5]} \quad 2x - 3y = 7$$



Plug equation [4] into equation [5] to solve for  $x$ .

$$2x - 3y = 7$$

$$2x - 3\left(-\frac{11}{2}\right) = 7$$

$$2x + \frac{33}{2} = 7$$

$$4x + 33 = 14$$

$$4x = -19$$

$$x = -\frac{19}{4}$$

Plug in  $x = -19/4$  and  $y = -11/2$  into any of the original equations to solve for  $z$ . We'll use equation [3].

$$-x - 4y + z = 2$$

$$-\left(-\frac{19}{4}\right) - 4\left(-\frac{11}{2}\right) + z = 2$$

$$\frac{19}{4} + \frac{44}{2} + z = 2$$

$$19 + 88 + 4z = 8$$

$$4z = -99$$

$$z = -\frac{99}{4}$$



Therefore, the solution to the system is  $(x, y, z) = (-19/4, -11/2, -99/4)$ .

■ 7. Find the unique solution to the system of equations.

$$x - y - z = \frac{15}{2}$$

$$2x + 3y - z = 14$$

$$-2x - y + 4z = -4$$

*Solution:*

Let's number the equations to stay organized.

$$\text{[1]} \quad x - y - z = \frac{15}{2}$$

$$\text{[2]} \quad 2x + 3y - z = 14$$

$$\text{[3]} \quad -2x - y + 4z = -4$$

Add equations [2] and [3] together so that  $x$  will be eliminated.

$$(2x + 3y - z) + (-2x - y + 4z) = (14) + (-4)$$

$$2x + 3y - z - 2x - y + 4z = 10$$

$$3y - z - y + 4z = 10$$

$$\text{[4]} \quad 2y + 3z = 10$$





Multiply equation [1] by 2,

$$2 \left( x - y - z = \frac{15}{2} \right)$$

$$2x - 2y - 2z = 15$$

and then add this to equation [3] so that  $x$  will be eliminated.

$$(2x - 2y - 2z) + (-2x - y + 4z) = (15) + (-4)$$

$$2x - 2y - 2z - 2x - y + 4z = 11$$

$$-2y - 2z - y + 4z = 11$$

$$\text{[5]} \quad -3y + 2z = 11$$

Multiply equation [4] by 3,

$$3(2y + 3z = 10)$$

$$6y + 9z = 30$$

and multiply equation [5] by 2,

$$2(-3y + 2z = 11)$$

$$-6y + 4z = 22$$

then add the results together so that  $y$  will be eliminated.

$$(6y + 9z) + (-6y + 4z) = (30) + (22)$$

$$6y + 9z - 6y + 4z = 52$$



$$9z + 4z = 52$$

$$13z = 52$$

$$z = 4$$

Plug  $z = 4$  into equation [4] to solve for  $y$ .

$$2y + 3z = 10$$

$$2y + 3(4) = 10$$

$$2y + 12 = 10$$

$$2y = -2$$

$$y = -1$$

Plug  $y = -1$  and  $z = 4$  into any of the original equations to solve for  $x$ . We'll use equation [1].

$$x - y - z = \frac{15}{2}$$

$$x - (-1) - (4) = \frac{15}{2}$$

$$x + 1 - 4 = \frac{15}{2}$$

$$x - 3 = \frac{15}{2}$$

$$x = \frac{15}{2} + \frac{6}{2}$$



$$x = \frac{21}{2}$$

Therefore, the solution to the system is  $(x, y, z) = (21/2, -1, 4)$ .

■ 8. Find the unique solution to the system of equations.

$$-x + y - z = 12$$

$$x - y + z = 2$$

$$2x - 2y + 2z = 9$$

*Solution:*

Let's number the equations to stay organized.

$$\text{[1]} \quad -x + y - z = 12$$

$$\text{[2]} \quad x - y + z = 2$$

$$\text{[3]} \quad 2x - 2y + 2z = 9$$

Add equations [1] and [2] together so that  $x$  will be eliminated.

$$(-x + y - z) + (x - y + z) = (12) + (2)$$

$$-x + y - z + x - y + z = 14$$

$$y - z - y + z = 14$$



$$-z + z = 14$$

$$\text{[4]} \quad 0 = 14$$

Since all the variables were eliminated and we're left with a false statement (0 can't equal 14), it means that there's no solution to this system.

■ 9. Find the unique solution to the system of equations.

$$2x - 5y + z = 12$$

$$-3x + 10y - z = -11$$

$$x - y + 2z = 21$$

*Solution:*

Let's number the equations to stay organized.

$$\text{[1]} \quad 2x - 5y + z = 12$$

$$\text{[2]} \quad -3x + 10y - z = -11$$

$$\text{[3]} \quad x - y + 2z = 21$$

Multiply equation [2] by 2,

$$2(-3x + 10y - z = -11)$$



$$-6x + 20y - 2z = -22$$

and then add this to equation [3] so that  $z$  will be eliminated.

$$(-6x + 20y - 2z) + (x - y + 2z) = (-22) + (21)$$

$$-6x + 20y - 2z + x - y + 2z = -1$$

$$-6x + 20y + x - y = -1$$

$$\text{[4]} \quad -5x + 19y = -1$$

Add equation [1] to equation [2] so that  $z$  will be eliminated.

$$(2x - 5y + z) + (-3x + 10y - z) = (12) + (-11)$$

$$2x - 5y + z - 3x + 10y - z = 1$$

$$2x - 5y - 3x + 10y = 1$$

$$\text{[5]} \quad -x + 5y = 1$$

Multiply equation [5] by  $-5$ ,

$$-5(-x + 5y = 1)$$

$$5x - 25y = -5$$

and then add this to equation [4] so that  $x$  will be eliminated.

$$(5x - 25y) + (-5x + 19y) = (-5) + (-1)$$

$$5x - 25y - 5x + 19y = -6$$

$$-25y + 19y = -6$$



$$-6y = -6$$

$$y = 1$$

Plug  $y = 1$  into equation [5] to solve for  $x$ .

$$-x + 5y = 1$$

$$-x + 5(1) = 1$$

$$-x + 5 = 1$$

$$-x = -4$$

$$x = 4$$

Plug  $x = 4$  and  $y = 1$  into any of the original equations to solve for  $z$ . We'll use equation [1].

$$2x - 5y + z = 12$$

$$2(4) - 5(1) + z = 12$$

$$8 - 5 + z = 12$$

$$3 + z = 12$$

$$z = 9$$

Therefore, the solution to the system is  $(x, y, z) = (4, 1, 9)$ .



