Balancing equations

Think of an equation as a balance scale that must always be balanced. What you do to one side of an equation you must do to the other, in order for it to remain balanced.

It's important to remember that equations have an equals sign and expressions do not. We balance or solve equations, but simplify or evaluate expressions.

Here are the steps we'll use to solve equations:

1. Simplify both sides of the equation as much as possible using the order of operations (distribute, combine like terms, etc.)
2. If the variable (letter) you're trying to solve for appears on both sides of the equation, move one of them to the other side. Get all your "x's to Texas" or get all the x's to one side of the equation.
3. Move all constant values to the other side of the equation (opposite the variable) by "un-doing" each operation. Use inverse operations until the variable is alone, and remember to do the same thing to both sides of the equation so that it stays balanced.

Example

Solve for the variable by keeping the equation balanced.

$$-2(3x + 1) = 3(-5x + 11) + 1$$

Simplify both sides of the equation by distributing and then combining like terms.

$$-2(3x) - 2(1) = 3(-5x) + 3(11) + 1$$

$$-6x - 2 = -15x + 33 + 1$$

$$-6x - 2 = -15x + 34$$

Move all the x terms to one side. Use an inverse operation (in this case adding $15x$, which is the inverse of subtracting $15x$, which we know to do because of the presence of the $-15x$ term), and add $15x$ to both sides to keep the equation balanced.

$$-6x + 15x - 2 = -15x + 15x + 34$$

$$9x - 2 = 34$$

Solve by doing the opposite of -2, by adding 2 to both sides of the equation to keep it balanced.

$$9x - 2 + 2 = 34 + 2$$

$$9x = 36$$

Continue to work on getting the x alone by using an inverse operation (dividing by 9 - the inverse of multiplying by 9). Divide both sides by 9.

$$\frac{9x}{9} = \frac{36}{9}$$

$$x = 4$$

Let's try another example of balancing equations.

Example

Solve for the variable.

$$5(6a - 3) = -(1 - 9a) + 7$$

Simplify both sides of the equation by distributing and then combining like terms.

$$5(6a) + 5(-3) = -(1) - (-9a) + 7$$

$$30a - 15 = -1 + 9a + 7$$

$$30a - 15 = 9a + 6$$

Move all the a terms to one side. Use an inverse operation (subtracting $9a$), and subtract $9a$ from both sides to keep the equation balanced.

$$30a - 9a - 15 = 9a - 9a + 6$$

$$21a - 15 = 6$$

Solve by doing the opposite of -15, by adding 15 to both sides of the equation to keep it balanced.

$$21a - 15 + 15 = 6 + 15$$

$$21a = 21$$

Continue to work on getting the a alone by using an inverse operation (dividing by 21). Divide both sides by 21.

$$\frac{21a}{21} = \frac{21}{21}$$

$$a = 1$$

Let's look at another example.

Example

Solve for the variable.

$$2x - 3 = 3x + 1$$

Both sides are as simplified as they can get, so we'll start by moving all the x terms to one side. Use inverse operations, and subtract $2x$ from both sides to keep the equation balanced. It's easier to subtract $2x$ instead of $3x$, because this will give us a term with a positive coefficient of x on the right side.

$$2x - 2x - 3 = 3x - 2x + 1$$

$$-3 = x + 1$$

We need to undo the $+1$ by subtracting 1 from both sides of the equation.

$$-3-1=x+1-1$$

$$-4=x$$

Let's try another example of solving equations with variables on both sides.

Example

Solve for the variable.

$$10x-13=4x+x-6$$

Start by combining like terms to simplify the right side of the equation.

$$10x-13=5x-6$$

Move all the x terms to one side. In this case, subtract $5x$ from both sides of the equation.

$$10x-5x-13=5x-5x-6$$

$$5x-13=-6$$

We need to undo the -13 by adding 13 to both sides of the equation.

$$5x-13+13=-6+13$$

$$5x=7$$

Divide both sides by 5.

$$\frac{5x}{5} = \frac{7}{5}$$

$$x = \frac{7}{5}$$
