Algebra 1 Workbook Solutions

Operations

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MATH

IDENTIFYING MULTIPLICATION

1. Give three different examples of how you can write "a times b" mathematically.

Solution:

Possible options are: $a \times b$, $a \cdot b$, $(a)(b)$, and ab.

2. Simplify the following expression.

$$5(2 \cdot 3) \times (1)(a)$$

Solution:

$$5(6) \times a$$

$$30 \times a$$

$$30a$$

3. What number would make the expression true?

$$4 \times 3(1)(?? \cdot 1) = 24$$

Solution:

$$4 \times 3(?? \cdot 1) = 24$$

$$4 \times 3(??) = 24$$

$$12(??) = 24$$

$$?? = 2$$

$$2$$

4. What term would make the expression true?

$$??(3 \cdot x) \times (5)(2) = 60x^2$$

Solution:

$$??(3 \cdot x) \times (5)(2) = 60x^2$$

$$??(3 \cdot x) \times 10 = 60x^2$$

$$??(3 \cdot x) = 6x^2$$

$$??(3) = 6x$$

$$?? = 2x$$

$$2x$$

5. Why do we have different ways to write multiplication?

Solution:

Because $\times$ can be confused with x.

6. If $(3)(x^2) = 10 \times 2$, what does $9x^2$ equal?

Solution:

$$(3)(x^2) = 10 \times 2$$

$$3x^2 = 20$$

$$3(3x^2) = 3(20)$$

$$9x^2 = 60$$

THE ASSOCIATIVE PROPERTY

1. Give an example of an expression that demonstrates the associative property with multiplication.

Solution:

There are many correct answers. For example, the following all work:

$$2 \times (11 \times 3) = (2 \times 11) \times 3$$

$$2 \times (11 \times 3) \times 9 = (2 \times 11) \times 3 \times 9 = 2 \times 11 \times (3 \times 9)$$

However, this example does not work:

$$2 \times (11 + 3) = (2 \times 11) + 3$$

Parentheses cannot be associated when the parentheses involve both addition and multiplication.

2. What are the two main operations that the associative property works for?

Solution:

Addition and multiplication.

3. Using the associative property, rewrite and simplify $2 \times (3 \times 4)$.

Solution:

$(2 \times 3) \times 4$

6×4

24

4. According to the associative property, what number would make the most sense in the expression?

$42 + (31 + 17) = (42 + ??) + 17$

Solution:

31

5. What does the word "associate" refer to in math?

Solution:

"Associate" refers to grouping with parentheses.

6. Rearrange $(3+6)+2$ using the associative property, then simplify.

Solution:

$$3+(6+2)$$

$$3+8$$

$$11$$

7. Give an example of an expression that demonstrates the associative property with addition.

Solution:

There are many correct answers. For example, the following all work:

$$2+(11+3)=(2+11)+3$$

$$2+(11+3)+9=(2+11)+3+9=2+11+(3+9)$$

However, this example does not work:

$$2+(11\times 3)=(2+11)\times 3$$

Parentheses cannot be associated when the parentheses involve both addition and multiplication.

8. What number would make the following expression true?

$$(4 \times 2) \times 9 = ?? \times (2 \times 9)$$

Solution:

$$4$$

9. Give an example of an expression that does not demonstrate the associative property.

Solution:

There are many correct answers. These examples do not demonstrate the associative property.

$$2 \times (11 + 3) = (2 \times 11) + 3$$

$$2 \times (11 + 3) \times 9 = (2 \times 11) + 3 \times 9 = 2 \times 11 + (3 \times 9)$$

Parentheses cannot be associated when the parentheses involve both addition and multiplication.

THE COMMUTATIVE PROPERTY

1. Using the commutative property, rewrite $6 + 19$ and then simplify.

Solution:

$$19 + 6$$

$$25$$

2. Give an example of an expression that demonstrates the commutative property with multiplication.

Solution:

There are many correct answers. For example, the following all work:

$$11 \times 3 = 3 \times 11$$

$$2 \times (11 \times 3) = 2 \times (3 \times 11) = (11 \times 3) \times 2$$

However, these examples do not work:

$$3 - 4 = 4 - 3$$

$$3 \div 4 = 4 \div 3$$

The commutative property cannot be used with subtraction and division in this way.

3. According to the commutative property, what number would make the most sense in the expression?

$$11 + (23 + 6) = 11 + (6 + ??)$$

Solution:

$$23$$

4. Using the commutative property, rewrite 3×4 and then simplify.

Solution:

$$4 \times 3$$

5. Rearrange $(3 + 6) + 2$ using the commutative property and then the associative property.

Solution:

$$(6 + 3) + 2$$

$$6 + (3 + 2)$$

■ 6. Give an example of an expression that demonstrates the commutative property with addition.

Solution:

There are many correct answers. For example, the following all work:

$$11 + 3 = 3 + 11$$

$$2 + (11 + 3) = 2 + (3 + 11) = (11 + 3) + 2$$

However, these examples do not work:

$$3 - 4 = 4 - 3$$

$$3 \div 4 = 4 \div 3$$

The commutative property cannot be used with subtraction and division in this way.

■ 7. What number would make the following expression true?

$$(4 \times 2) \times 9 = (?? \times 9) \times 4$$

Solution:

2

8. What are the two main operations that the commutative property works for?

Solution:

Addition and multiplication.

THE TRANSITIVE PROPERTY

1. If $AB = CD$ and $CD = EF$ then what is the value of EF?

Solution:

$$AB$$

2. If $x = 2$ and $y = x$, then what does the transitive property tell us?

Solution:

$$y = 2$$

3. According to the transitive property, if $x = 2y$ and $2y = 5z$, then what is the value of x?

Solution:

$$5z$$

4. Give an example that demonstrates the transitive property.

Solution:

There are many correct answers. For example, the following all work:

If $x = 2$ and $y = x$ then $y = 2$.

If $z = x$ and $x = 2y$ then $z = 2y$.

However, this example does not work:

If $x = y$ and $z = 2$ then $x = 2$.

The transitive property cannot be used when the statements are not related to one another.

5. Use the transitive property to solve for z.

$$x = y$$

$$y = 3 - z$$

$$x = -2z + 7$$

Solution:

By the transitive property we have:

$$-2z + 7 = 3 - z$$

$$7 = 3 + z$$

$$4 = z$$

6. Transitive comes from the word "transit," which means, to what?

Solution:

Move from one place to another.

7. By the transitive property, what expression would make the following statement true?

If $2 + 3 = ??$ and $4 + 1 = 5$, then $2 + 3 = 5$.

Solution:

$$4 + 1$$

8. Use the transitive property to solve for x.

$$y = 2x + 3$$

$$y = z$$

$$z = 5x - 9$$

Solution:

By the transitive property we have:

$$2x + 3 = 5x - 9$$

$$3 = 3x - 9$$

$$12 = 3x$$

$$4 = x$$

9. According to the transitive property, what expression would make the most sense in the following statement?

If $x = 2y$ and $2y = ??$, then $x = 5z$.

Solution:

$$5z$$

THE DISTRIBUTIVE PROPERTY

1. Use the distributive property to solve for x.

$$5(x-2)=\frac{1}{2}(6-2x)$$

Solution:

$$5(x-2)=\frac{1}{2}(6-2x)$$

$$5x-10=3-x$$

$$6x=13$$

$$x=\frac{13}{6}$$

2. Use the distributive property to expand the expression.

$$-\frac{2}{5}(10-5x)$$

Solution:

$$-\frac{2}{5}(10)+\frac{2}{5}(5x)$$

$$-4 + 2x$$

3. Give an example that demonstrates the distributive property with subtraction.

Solution:

There are many correct answers. For example, the following all work:

$$2(x - 1) = 2x - 2$$

$$-\frac{1}{3}(9 - 2x) = -3 + \frac{2}{3}x$$

However, this example does not work:

$$2(3 - 2x) = 6 - 2x$$

The distributive property states that you must multiply the term outside of the parentheses by each term inside the parentheses.

4. What three main operations are used in the distributive property?

Solution:

Multiplication, addition, and subtraction.

5. What does distributing remove from the expression?

Solution:

Parentheses

6. Use the distributive property to solve for x.

$$2(5 - 3x) = x - 4$$

Solution:

$$2(5 - 3x) = x - 4$$

$$10 - 6x = x - 4$$

$$14 = 7x$$

$$2 = x$$

7. What value would make the following expression true?

$$2(x + 3) = ?? + 6$$

Solution:

$$2x$$

THE DISTRIBUTIVE PROPERTY WITH FRACTIONS

1. Perform the indicated operation and then simplify.

$$\frac{4y^3z^2}{3x} \times \frac{x^2y}{2z^2}$$

Solution:

$$\frac{4x^2y^4z^2}{6xz^2}$$

$$\frac{2xy^4}{3}$$

$$\frac{2}{3}xy^4$$

2. Use the distributive property to expand the expression.

$$-\frac{x^2z}{y^3}\left(\frac{y^2}{2} - \frac{xz^3}{z^2}\right)$$

Solution:

$$-\frac{x^2y^2z}{2y^3}+\frac{x^3z^4}{y^3z^2}$$

$$-\frac{x^2z}{2y}+\frac{x^3z^2}{y^3}$$

■ 3. Fill in the blank with the correct words: When we are distributing fractions, we multiply the outside numerator with the __________ of the terms inside the parentheses and the outside denominator with the __________ of the inside terms.

Solution:

numerator, denominator

■ 4. Use the distributive property to solve for x.

$$-\frac{3xy^2}{z}\left(\frac{2z}{3y^2}-z\right)=3\left(3+xy^2\right)$$

Solution:

$$-\frac{3xy^2}{z}\left(\frac{2z}{3y^2}-z\right)=3\left(3+xy^2\right)$$

$$-2x + 3xy^2 = 9 + 3xy^2$$

$$-2x = 9$$

$$x = -\frac{9}{2}$$

■ 5. Use the distributive property to show that $x = -10$.

$$\frac{2}{3}\left(\frac{x}{2} - 6\right) = 4\left(\frac{x}{3} + \frac{3}{2}\right)$$

Solution:

$$\frac{2}{3}\left(\frac{x}{2} - 6\right) = 4\left(\frac{x}{3} + \frac{3}{2}\right)$$

$$\frac{x}{3} - 4 = \frac{4x}{3} + 6$$

$$-10 = \frac{4x}{3} - \frac{x}{3}$$

$$-10 = \frac{3x}{3}$$

$$x = -10$$

6. Explain why the two sides of the equation aren't equal to one another.

$$\frac{3}{2}\left(\frac{x}{5}-\frac{y}{2}\right) \neq \frac{3x}{10}-\frac{y}{2}$$

Solution:

The outside fraction was not distributed to both terms in the parentheses. It should be:

$$\frac{3}{2}\left(\frac{x}{5}-\frac{y}{2}\right) = \frac{3x}{10}-\frac{3y}{4}$$

7. What term would make the following expression true?

$$\frac{2ab}{c^2}\left(\frac{3ac}{b}+a^2c^2\right) = \frac{6a^2}{c} + ??$$

Solution:

Simplify the left side by distributing the $2ab/c^2$ across the terms inside the parentheses.

$$\frac{2ab}{c^2}\left(\frac{3ac}{b}\right) + \frac{2ab}{c^2}(a^2c^2) = \frac{6a^2}{c} + ??$$

To multiply fractions, we multiply the numerators together, and multiply the denominators together.

$$\frac{(2ab)(3ac)}{(c^2)(b)} + \frac{(2ab)(a^2c^2)}{c^2} = \frac{6a^2}{c} + ??$$

$$\frac{6a^2bc}{bc^2} + \frac{2a^3bc^2}{c^2} = \frac{6a^2}{c} + ??$$

Cancel common factors.

$$\frac{6a^2}{c} + \frac{2a^3b}{1} = \frac{6a^2}{c} + ??$$

$$\frac{6a^2}{c} + 2a^3b = \frac{6a^2}{c} + ??$$

Now that we have matching $6a^2/c$ terms on each side, we can see that the missing term is $2a^3b$.

$$2a^3b$$

8. What term would make the following expression true?

$$\frac{??}{??}\left(\frac{2x}{z} + y^2\right) = \frac{2x^3}{3z^2} + \frac{x^2y^2}{3z}$$

Solution:

Distribute the unknown fraction across the terms in the parentheses on the left side.

$$\frac{??}{??}\left(\frac{2x}{z}\right) + \frac{??}{??}(y^2) = \frac{2x^3}{3z^2} + \frac{x^2y^2}{3z}$$

In order to turn the numerator of the first fraction from $2x$ into $2x^3$, we'd need to multiply by x^2.

$$\frac{x^2}{??}\left(\frac{2x}{z}\right) + \frac{x^2}{??}(y^2) = \frac{2x^3}{3z^2} + \frac{x^2y^2}{3z}$$

In order to turn the denominator of the first fraction from z into $3z^2$, we'd need to multiply by $3z$.

$$\frac{x^2}{3z}\left(\frac{2x}{z}\right) + \frac{x^2}{3z}(y^2) = \frac{2x^3}{3z^2} + \frac{x^2y^2}{3z}$$

Multiply out the left side to make sure that it does, in fact, match the right side.

$$\frac{2x^3}{3z^2} + \frac{x^2y^2}{3z} = \frac{2x^3}{3z^2} + \frac{x^2y^2}{3z}$$

Because the sides do match, we know the missing fraction was

$$\frac{x^2}{3z}$$

THE DISTRIBUTIVE PROPERTY AND BINOMIAL MULTIPLICATION

1. Perform the indicated operation and simplify.

$$(x-1)(x+4)$$

Solution:

$$x^2+4x-x-4$$

$$x^2+3x-4$$

2. How many terms does a binomial have?

Solution:

$$2$$

3. Use the distributive property to expand the expression.

$$4(2-x)(3+2x)$$

Solution:

$$4(6 + 4x - 3x - 2x^2)$$

$$24 + 4x - 8x^2$$

4. What term would make the following expression true?

$$(2x + 1)(5 - x) = ?? + 10x - x + 5$$

Solution:

$$-2x^2$$

5. Use the distributive property to show that $x = 3$.

$$2(x - 1)(x + 1) = 2x^2 + x - 5$$

Solution:

$$2(x - 1)(x + 1) = 2x^2 + x - 5$$

$$2(x^2 - 1) = 2x^2 + x - 5$$

$$2x^2 - 2 = 2x^2 + x - 5$$

$$-2 = x - 5$$

$$3 = x$$

6. Explain why $(x-2)(x+1) \neq x^2 - 2$.

Solution:

The terms were not fully distributed. The binomial multiplication should be

$$(x-2)(x+1) = x^2 - x - 2$$

7. Use the distributive property to expand the expression.

$$\frac{1}{2}(6x+4)(x-1)$$

Solution:

$$(3x+2)(x-1)$$

$$3x^2 - x - 2$$

8. What term would make the following expression true?

$$(3+x)(??) = 3x + 3 + x^2 + x$$

Solution:

$$x + 1$$

GROUPING SYMBOLS WITH PEMDAS AND ORDER OF OPERATIONS

■ 1. Write the expression with parentheses.

$$\frac{a}{b+c}$$

Solution:

$$a/(b+c)$$

■ 2. Simplify the expression.

$$2\left([3+1]-4\right)-[6+3]$$

Solution:

$$2(4-4)-9$$

$$2(0)-9$$

$$-9$$

■ 3. Put in grouping symbols that will make the equation true.

$$2\ x+1\ /\ 3+5\ x-2=\frac{2(x+1)}{3+5x-10}$$

Solution:

$$2(x+1)/[3+5(x-2)]$$

4. Simplify the expression.

$$|2(1-4)|-(2-5)\left[(-1)(3+2)+9\right]$$

Solution:

$$|2(-3)|-(-3)\left[-5+9\right]$$

$$|-6|-(-3)[4]$$

$$6+12$$

$$18$$

5. What number would make the expression true?

$$(3+1)\left[2(??-5)+7\right]-|(4-6)|=4\left[-2+7\right]-|-2|$$

Solution:

$$4\left[2(??-5)+7\right]-|-2|=4[5]-|-2|$$

$$4\left[2(??-5)+7\right]-2=20-2$$

$$4\left[2(??-5)+7\right]=20$$

$$2(??-5)+7=5$$

$$2(??-5)=-2$$

$$??-5=-1$$

$$??=4$$

$$4$$

6. Give three different examples of a grouping symbol.

Solution:

Possible solutions are ( ), [ ], { }, and | |.

7. Rewrite the following as a fraction.

$$[2(x+1)-3]/[5x-3(4x)]$$

Solution:

$$\frac{2(x+1)-3}{5x-3(4x)}$$

8. Simplify the following expression.

$$\sqrt{2(5-3)} - \left|3[6-7]\right|$$

Solution:

$$\sqrt{2(2)} - \left|3[-1]\right|$$

$$\sqrt{4} - |-3|$$

$$2-3$$

$$-1$$

9. Using PEMDAS, evaluate each expression separately to show that they are not equal.

$4 \times (3-1) - (4 \div 2 + 2)$ and $(4 \times 3 - 1) - 4 \div (2+2)$

Solution:

The first expression simplifies to

$$4 \times 2 - (2 + 2)$$

$$8 - 4$$

$$4$$

and the second expression simplifies to

$$(12 - 1) - 4 \div 4$$

$$11 - 1$$

$$10$$

■ 10. Use the order of operations to simplify the expression.

$$\left(10 - \left[(-1)^2 + 1 - 6 \div 6\right]\right)^{1/2} + 4 \div 2$$

Solution:

$$\left(10 - \left[2 - 1\right]\right)^{1/2} + 2$$

$$(9)^{1/2} + 2$$

$$3 + 2$$

5

11. What do the letters in PEMDAS stand for?

Solution:

Parentheses, Exponents, Multiplication/Division, Addition/Subtraction.

12. What number would make the equation true?

$$2^2 + 4 \cdot \left[(2 - ??) \div |-4|\right] = 4 + 4 \cdot \left[1 \div 4\right]$$

Solution:

$$2^2 + 4 \cdot \left[(2 - ??) \div |-4|\right] = 4 + 4 \cdot \left[1 \div 4\right]$$

$$4 + 4 \cdot \left[(2 - ??) \div |-4|\right] = 4 + 4 \cdot \left[1 \div 4\right]$$

$$4 + 4 \cdot \left[(2 - ??) \div 4\right] = 4 + 4 \cdot \left[1 \div 4\right]$$

$$4 + 4 \cdot \frac{(2 - ??)}{4} = 4 + 4 \cdot \frac{1}{4}$$

$$4 + (2 - ??) = 4 + 1$$

$$2 - ?? = 1$$

1

■ 13. Use the order of operations to simplify the expression.

$$3 - \left[(-2)^2x + (3 - 7)\right]$$

Solution:

$$3 - (4x - 4)$$

$$7 - 4x$$

■ 14. Using the order of operations, explain why $9 + 6 \div 3 \neq 5$.

Solution:

The order of operations says division comes before addition, so it would be

$$9 + 6 \div 3$$

$$9 + 2$$

$$11$$

15. Put symbols of inclusion that make the equation true.

$$4 - 5\ 3\ x + \ -1^2 \ \ = 4 - 15x - 5$$

Solution:

$$4 - 5\left(3x + (-1)^2\right) = 4 - 15x - 5$$

16. What operations must be performed before multiplication and division?

Solution:

Parentheses and exponents.

17. Using the order of operations, explain why $\sqrt{(2+7)} \neq \sqrt{2} + \sqrt{7}$.

Solution:

The order of operations says parentheses must be evaluated first, so it would be

$$\sqrt{(2+7)}$$

$$\sqrt{9}$$

$$3$$

18. Simplify this expression.

$$3\{2[4+3(7-5)-4]\}$$

Solution:

To simplify

$$3\{2[4+3(7-5)-4]\}$$

first, work inside the parentheses.

$$(7-5)=2$$

Write 2 inside the parentheses.

$$3\{2[4+3(2)-4]\}$$

Second, work inside the brackets.

Multiply $3(2)=6$ and write 6 in place of $3(2)$.

$$3\{2[4+6-4]\}$$

Add $[4+6-4]=6$ and write 6 in place of $[4+6-4]$.

$$3\{2[6]\}$$

Third, work inside the braces.

$$\{2[6]\} = 12$$

$$3\{12\}$$

Finally, multiply $3\{12\}$.

$$36$$

19. Use the order of operations to simplify the expression.

$$3 + 2(x + 1)$$

Solution:

PEMDAS tells us that we have to perform the operations in the following order:

Parentheses

Exponents

Multiplication/Division

$$3 + 2(x + 1)$$

$$3 + 2(x) + 2(1)$$

$$3 + 2x + 2$$

Addition/Subtraction

$$5 + 2x$$

20. Use order of operations to simplify the expression.

$$\frac{-2 + 3 - 10 \cdot 2 \cdot \left[(5 - 4) + 2\right]}{2}$$

Solution:

Order of operations tells us that we have to do the parentheses first, so we start with inner-most symbols of inclusion.

$$\frac{-2 + 3 - 10 \cdot 2 \cdot \left[(5 - 4) + 2\right]}{2}$$

$$\frac{-2 + 3 - 10 \cdot 2 \cdot \left[(1) + 2\right]}{2}$$

$$\frac{-2 + 3 - 10 \cdot 2 \cdot \left[3\right]}{2}$$

Since there are no exponents, we'll do multiplication next.

$$\frac{-2 + 3 - 20 \cdot \left[3\right]}{2}$$

$$\frac{-2 + 3 - 60}{2}$$

When we're dealing with fractions, we must do all of the operations in the numerator and all of the operations in the denominator, and our very last step is to divide the resulting numerator by the resulting denominator. Therefore, we'll do the addition and subtraction in the numerator next.

$$\frac{3-62}{2}$$

$$\frac{-59}{2}$$

$$-\frac{59}{2}$$

UNDERSTOOD 1

1. What is $x + 4x + x + x$?

Solution:

$$1x + 4x + 1x + 1x$$

$$7x$$

2. What happens when you multiply something by 1?

Solution:

It's value stays the same.

3. Simplify the following expression.

$$\frac{1x^1}{1(1^1)} + \frac{1}{1(1x)} - 1^1$$

Solution:

$$x + \frac{1}{x} - 1$$

4. What number would make the expression true?

$$1(2^1) - \frac{1}{1(1)^1} + \frac{??^1}{1 \times 1} = 4$$

Solution:

$$1(2) - \frac{1}{1(1)} + \frac{??}{1} = 4$$

$$2 - \frac{1}{1} + ?? = 4$$

$$2 - 1 + ?? = 4$$

$$1 + ?? = 4$$

$$?? = 3$$

$$3$$

5. Simplify the following expression.

$$x(x^2 + 3x^2) - x^3$$

Solution:

$$x(4x^2) - x^3$$

$$4x^3 - x^3$$

$$3x^3$$

6. Simplify the following expression.

$$\frac{x^1}{4x^3} + \frac{5x^4}{1x}$$

Solution:

Remove the understood 1s.

$$\frac{x}{4x^3} + \frac{5x^4}{x}$$

Cancel a common factor of x from the numerator and denominator of each fraction.

$$\frac{1}{4x^2} + \frac{5x^3}{1}$$

Remove the understood 1 in the denominator of the second fraction.

$$\frac{1}{4x^2} + 5x^3$$

7. Give an example of an expression where it would be useful to write out the understood 1.

Solution:

There are many correct answers. For example, the following all work:

$$x^2 \times x = x^2 \times x^1 = x^{2+1} = x^3$$

$$3x - x = 3x - 1x = (3-1)x = 2x$$

8. Simplify the following expression.

$$\frac{x}{1^1} \cdot \frac{x^2 + 1(1)}{5x^2}$$

Solution:

$$\frac{x}{1} \cdot \frac{x^2 + 1}{5x^2}$$

$$\frac{x(x^2 + 1)}{5x^2}$$

$$\frac{x^2 + 1}{5x}$$