Radical equations

In this lesson we'll look at how to solve for the variable in a radical equation by isolating the radical, squaring both sides, and then using inverse operations.

The thing to remember about solving a radical equation is that if you can get the radical by itself, then you just need to square both sides and solve for the variable. However, because of the squaring, you could introduce "solutions" that aren't actually solutions of the original equation. (They're called **extraneous solutions**.)

So after you think you've found the solutions, you need to check them by plugging them into the original equation. If the equation you get for a potential solution is true, then that solution is indeed a solution; otherwise, it isn't a solution.

Let's look at a few examples.

Example

Solve for the variable.

$$\sqrt{x} - 3 = 2$$

We have to keep the equation balanced, so when we add 3 to the left side, we'll also add it to the right side.

$$\sqrt{x} - 3 = 2$$

$$\sqrt{x} - 3 + 3 = 2 + 3$$

$$\sqrt{x} = 5$$

Squaring both sides, we get

$$\left(\sqrt{x}\right)^2 = 5^2$$

$$x = 25$$

To determine whether this is actually a solution, we'll substitute 25 for x in the original equation:

$$\sqrt{x} - 3 = 2$$

$$\sqrt{25} - 3 = 2$$

$$5 - 3 = 2$$

$$2 = 2$$

This equation is true, so $x = 25$ is indeed a solution.

Let's do another one.

Example

Solve for the variable.

$$\sqrt{x-2}+5=9$$

We have to keep the equation balanced, so when we subtract 5 from the left side, we'll also subtract it from the right side.

$$\sqrt{x-2}+5=9$$

$$\sqrt{x-2}+5-5=9-5$$

$$\sqrt{x-2}=4$$

Squaring both sides, we get

$$\left(\sqrt{x-2}\right)^2=4^2$$

$$x-2=16$$

Now add 2 to both sides.

$$x-2+2=16+2$$

$$x=18$$

To determine whether this is actually a solution, we'll substitute 18 for x in the original equation.

$$\sqrt{x-2}+5=9$$

$$\sqrt{18-2}+5=9$$

$$\sqrt{16}+5=9$$

$$4 + 5 = 9$$

$$9 = 9$$

This equation is true, so $x = 18$ is indeed a solution.

Let's look at an example where we'll have an x^2 term once we do the squaring.

Example

Solve for the variable.

$$2x + \sqrt{x + 1} = 8$$

We'll first get the radical by itself.

$$2x + \sqrt{x + 1} = 8$$

$$2x - 2x + \sqrt{x + 1} = 8 - 2x$$

$$\sqrt{x + 1} = 8 - 2x$$

Squaring both sides, we get

$$\left(\sqrt{x + 1}\right)^2 = (8 - 2x)^2$$

$$x + 1 = 64 - 32x + 4x^2$$

Now let's get all of the terms to one side of the equation. Once we do that, we'll have a quadratic polynomial on one side of the equation. As usual, we'd like the coefficient of the x^2 term in that quadratic polynomial to be positive, so we want to keep the $4x^2$ on the right side of the equation. Therefore, we'll move the $x + 1$ to the right side.

$$x + 1 - (x + 1) = 64 - 32x + 4x^2 - (x + 1)$$

$$0 = 64 - 32x + 4x^2 - x - 1$$

Now we'll combine like terms on the right side.

$$0 = 4x^2 + (-32x - x) + (64 - 1)$$

$$0 = 4x^2 - 33x + 63$$

Factor the quadratic polynomial $4x^2 - 33x + 63$, and then solve the resulting equation for x.

$$0 = (4x - 21)(x - 3)$$

Now we'll set each factor equal to 0 and solve for x.

$$4x - 21 = 0$$

$$4x = 21$$

$$x = \frac{21}{4}$$

and

$$x - 3 = 0$$

$$x = 3$$

Now we'll check to determine whether $x = 21/4$ and $x = 3$ are actually solutions, by plugging each of them into the original equation.

For $x = 21/4$:

$$2x + \sqrt{x+1} = 8$$

$$2\left(\frac{21}{4}\right) + \sqrt{\frac{21}{4} + 1} = 8$$

$$\frac{21}{2} + \sqrt{\frac{21}{4} + \frac{4}{4}} = 8$$

$$\frac{21}{2} + \sqrt{\frac{25}{4}} = 8$$

$$\frac{21}{2} + \frac{5}{2} = 8$$

$$\frac{26}{2} = 8$$

$$13 = 8$$

This equation is false, so $x = 21/4$ is not a solution.

For $x = 3$:

$$2x + \sqrt{x+1} = 8$$

$$2(3) + \sqrt{3+1} = 8$$

$$6 + \sqrt{4} = 8$$

$$6 + 2 = 8$$

$$8 = 8$$

This equation is true, so $x = 3$ is indeed a solution.
