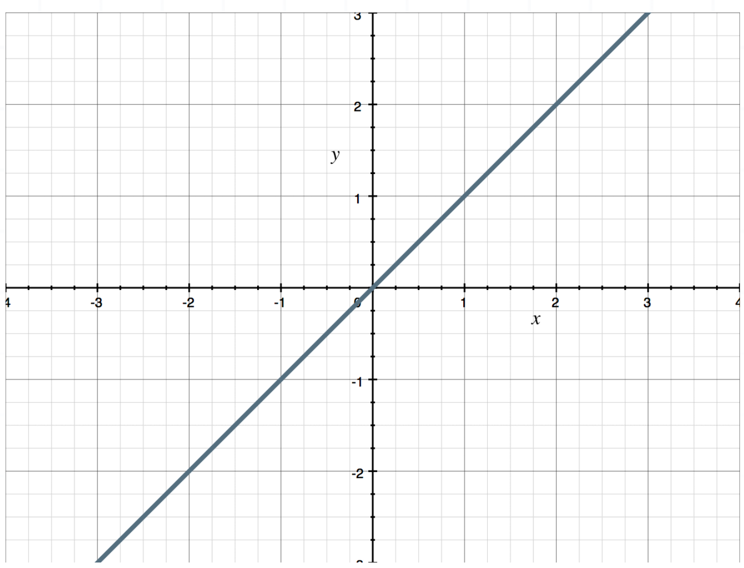
Topic: Graphing parabolas

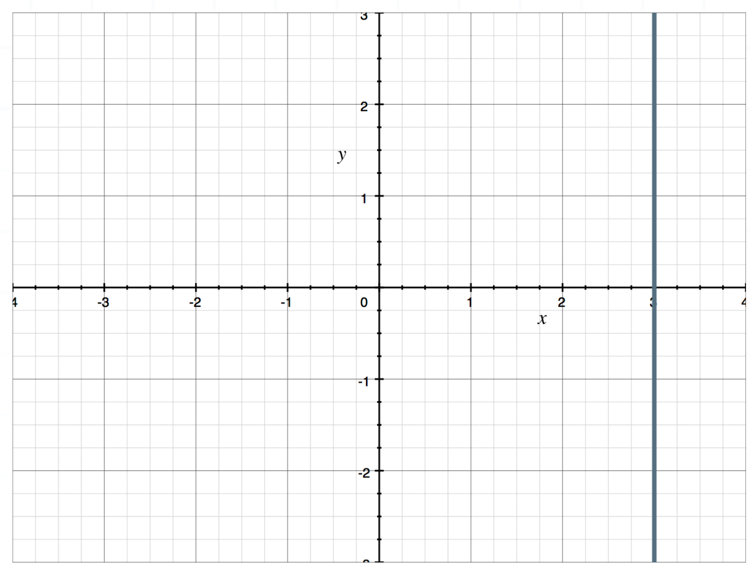
Question: Which graph represents a non-linear function?

Answer choices:

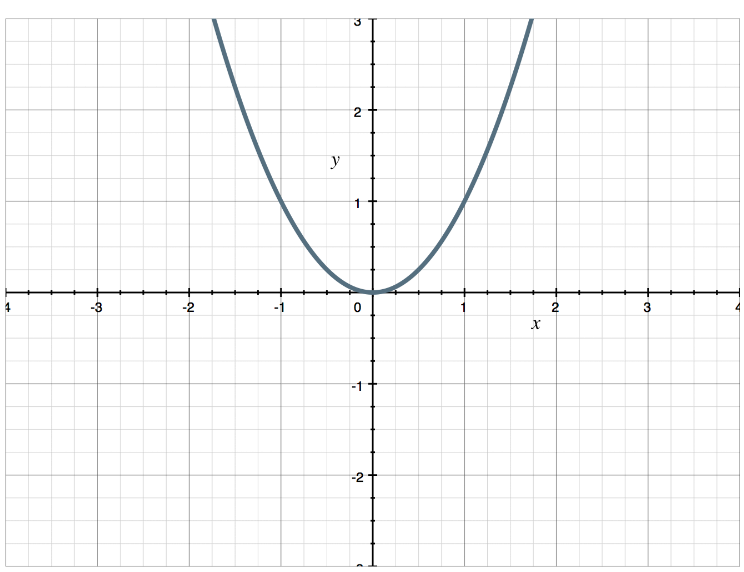
A $y = x$



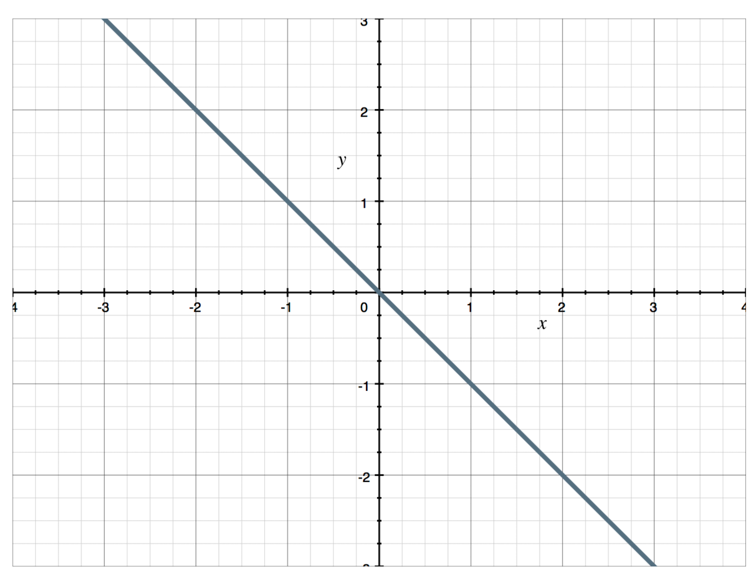
B $x = 3$



C $y = x^2$



D $y = -x$



Solution: C

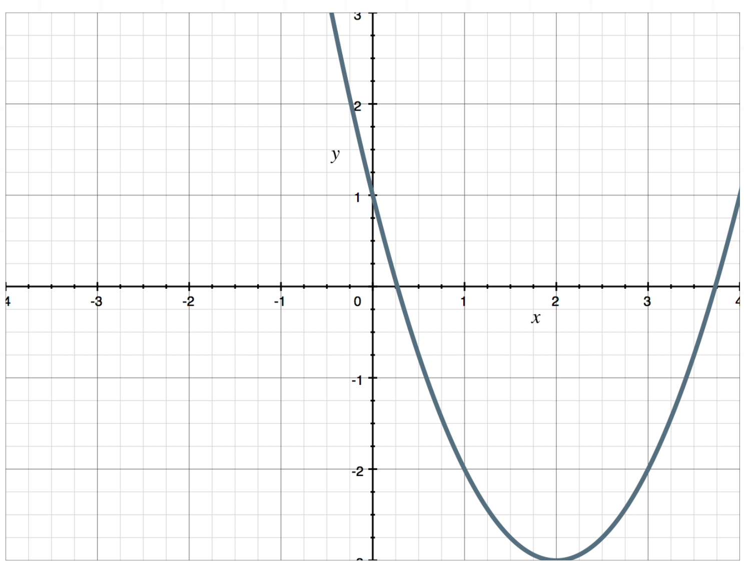
The graph in answer choice C is the only graph that isn't a line, which means it's the only one that represents a non-linear function.

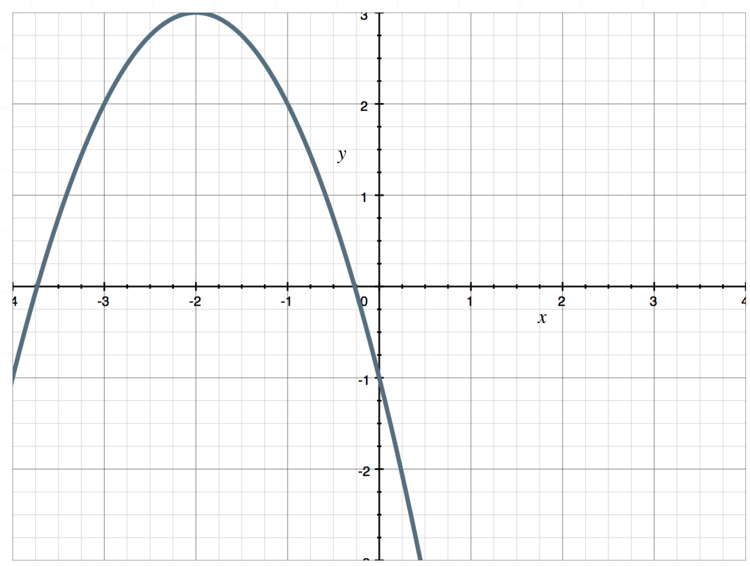
Topic: Graphing parabolas

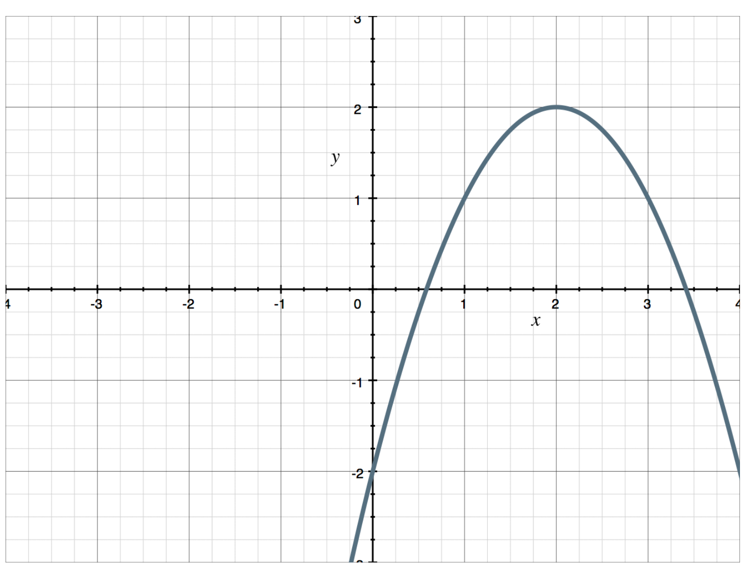
Question: Which graph represents the function?

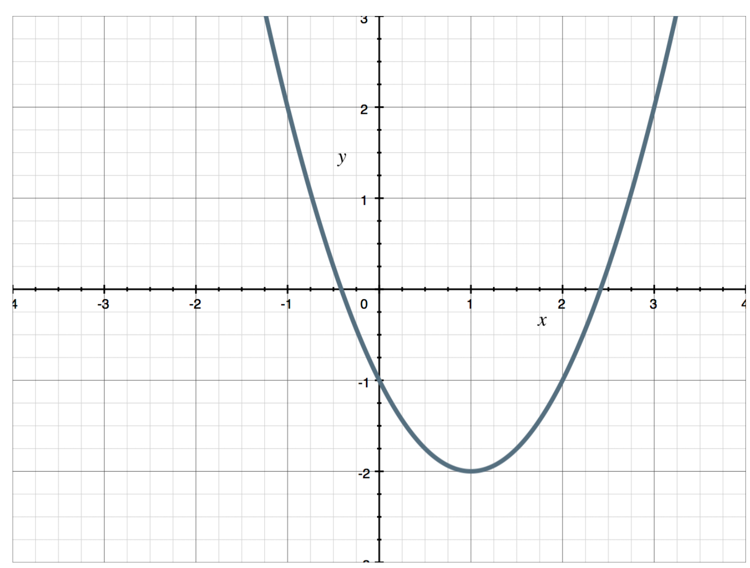
$$y = -x^2 - 4x - 1$$

Answer choices:

A


B


C


D


Solution: B

We'll first convert the equation of the parabola to vertex form,

$$y = a(x - h)^2 + k$$

where (h, k) are the coordinates of the vertex of the parabola.

We'll do this by completing the square.

Before we complete the square, we'll factor a -1 out of the expression on the right-hand side of the given equation, because it's easier to deal with a polynomial in which the leading term has a positive coefficient.

$$y = -x^2 - 4x - 1$$

$$y = -(x^2 + 4x + 1)$$

To complete the square, we need to find the number d that satisfies the equation

$$x^2 + 4x + d^2 = (x + d)^2$$

That is, we need to find the number d for which

$$x^2 + 4x + d^2 = x^2 + 2dx + d^2$$

This means that the coefficient of the x term of the expression inside the parentheses must be equal to $2d$. That coefficient is 4, so we'll set $2d$ equal to 4 and solve for d.

$$2d = 4 \quad \to \quad d = 2$$

To keep our equation balanced, we need to add and subtract d^2 (4) inside the parentheses, and then regroup and simplify.

$$y = -(x^2 + 4x + 1)$$

$$y = -(x^2 + 4x + 4 - 4 + 1)$$

$$y = -\left[(x^2 + 4x + 4) - 4 + 1\right]$$

$$y = -\left[(x^2 + 4x + 4) - 3\right]$$

$$y = -(x^2 + 4x + 4) + 3$$

Finally, we'll factor the expression that's now inside the parentheses $(x^2 + 4x + 4)$. By construction ("completing the square"), that expression factors as $(x + d)^2$.

$$x^2 + 4x + 4 = (x + d)^2$$

$$x^2 + 4x + 4 = (x + 2)^2$$

Therefore, the vertex form of the equation of the parabola is

$$y = -(x + 2)^2 + 3$$

Now that we've got the equation of the parabola in vertex form, we can identify its characteristics.

1. The negative sign in front of the parentheses indicates that the parabola opens downwards.

2. The coordinates of the vertex (in this case the point at the top of the parabola) are $(h, k) = (-2,3)$.

3. The y-coordinate of the y-intercept, which is the point of the parabola whose x-coordinate is 0, is found by substituting 0 for x in the equation of the parabola.

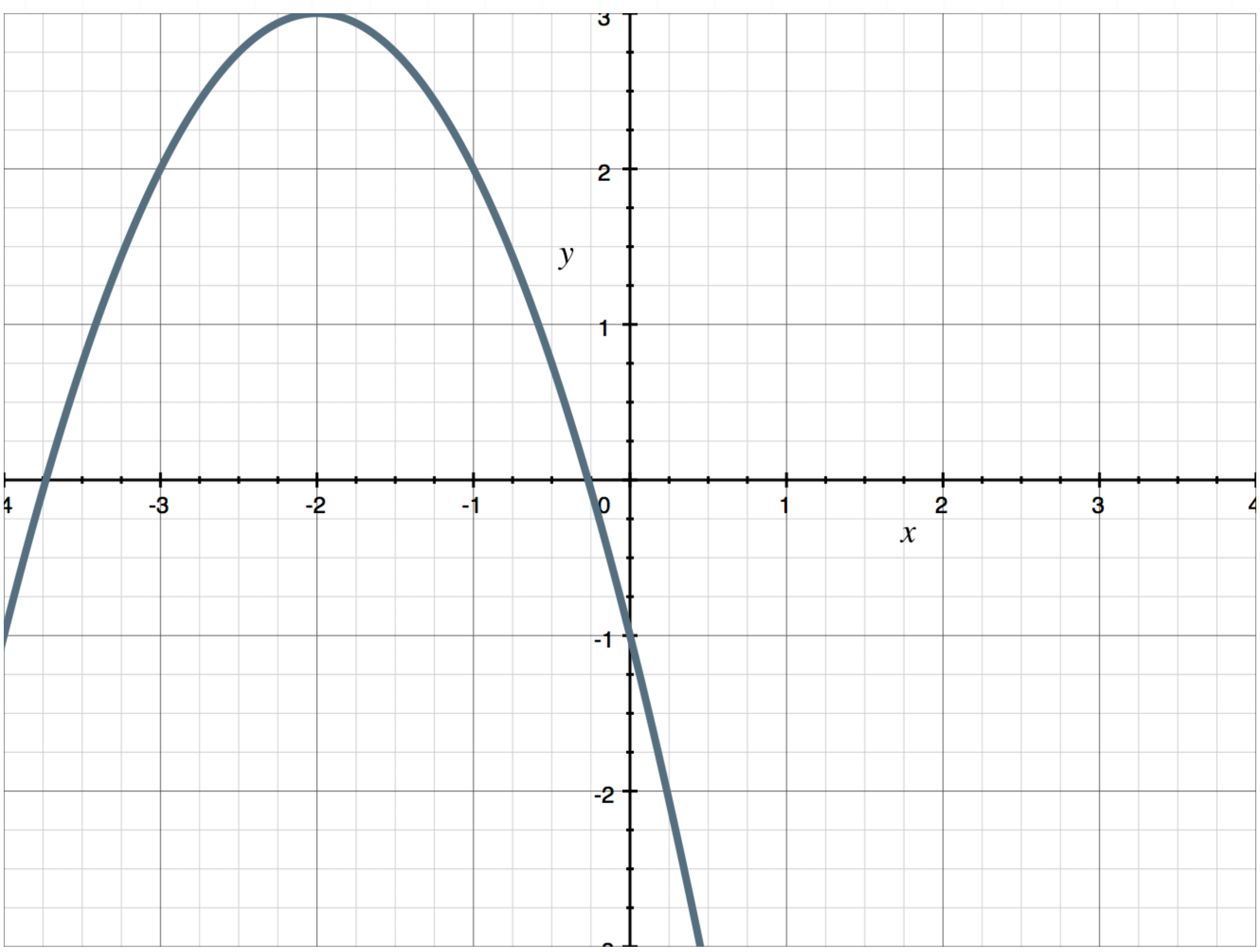
$$y = -(x+2)^2 + 3$$

$$y = -(0+2)^2 + 3$$

$$y = -2^2 + 3$$

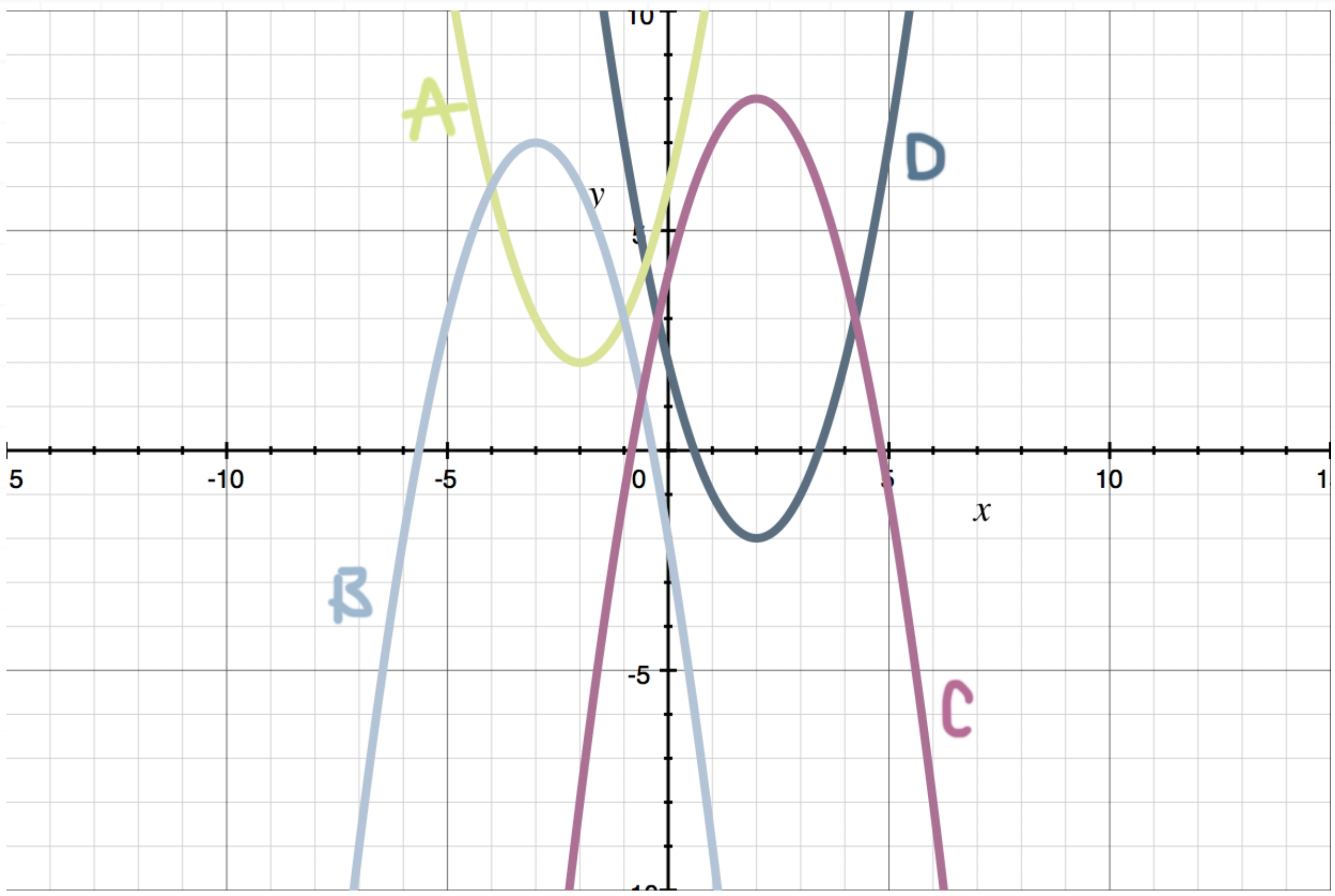
$$y = -4 + 3$$

$$y = -1$$



Topic: Graphing parabolas

Question: Which parabola is the graph of $y = x^2 - 4x + 2$?



Answer choices:

A A

B B

C C

D D

Solution: D

The easiest way to do this is to find the y-intercept. Remember, the y-intercept is the point of the parabola whose x-coordinate is 0. We can substitute 0 for x in the given equation to find the y-coordinate of the y-intercept.

$$y = x^2 - 4x + 2$$

$$y = 0^2 - 4(0) + 2$$

$$y = 2$$

The y-intercept is $(0,2)$.

Although the graphs are a little crowded there, you can see that the graph in answer choice D is the only one that passes through $(0,2)$.

Another method of doing the problem is to find the vertex by matching the given equation to the standard form of the equation of a parabola, $y = ax^2 + bx + c$, and then finding the coordinates of the vertex. We know that the x-coordinate of the vertex is $-b/(2a)$. For this parabola, $a = 1$ and $b = -4$, so

$$-\frac{b}{2a} = -\frac{(-4)}{2(1)} = 2$$

The y-coordinate of the vertex can be found by substituting 2 for x in the equation of the parabola.

$$y = x^2 - 4x + 2$$

$$y = (2^2) - 4(2) + 2$$

$$y = 4 - 8 + 2$$

$$y = -2$$

Therefore, the coordinates of the vertex are $(2, -2)$. The graph in answer choice D is the only one whose vertex is at the point $(2, -2)$.