Domain and range

We often define a function by an equation in which the variables x and y represent real numbers, and where "y" (just the variable y) is all by itself on one side of the equation, and an expression that contains no variable other than x is on the other side. An example of an equation like that is $y = x^2$.

Think of the domain of a function as all the real numbers you can plug in for x without causing the function to be undefined. Things to look out for are values of x that would cause a fraction's denominator to equal 0 and values that would force a negative number under a square root sign.

The range of a function is then the real numbers that would result for y from plugging in the real numbers in the domain for x. In other words, the domain is all x-values or inputs of a function, and the range is all y-values or outputs of a function.

There are a few functions we'll use a lot that have domain restrictions:

$y = \frac{1}{x}$	x cannot equal 0
$y = \sqrt{x}$	x must be nonnegative (either positive or 0)

Example

Describe the domain of the function.

$$y = x + \frac{2}{x}$$

In this function, x cannot be equal to 0, because that value causes the denominator of the fraction to equal 0. Because setting x equal to 0 is the only way to make this function undefined, its domain is all real numbers except 0.

We can also define a function by a set of coordinates (x, y) of points in the Cartesian coordinate system. In this case, the domain of the function consists of the x-coordinates of all the points, and the range consists of the y-coordinates of all the points.

Let's look at an example where the function is defined by a set of coordinates of points in the Cartesian coordinate system.

Example

What are the domain and range of the function?

$$(-2,4),\ (1,3),\ (2,5),\ (4,3)$$

The domain consists of all the x-coordinates. Remember that inside each set of parentheses, the x-coordinate is the first number and the y-coordinate is the second number.

Domain: $-2,\ 1,\ 2,\ 4$

The range consists of all the y-coordinates.

Range: $4, 3, 5, 3$

We don't need to list numbers more than once, and we'd prefer to arrange the numbers in ascending order, so we can give the range as follows:

Range: $3, 4, 5$

Let's try another example of domain and range.

Example

What are the domain and range of the function?

$$y = \frac{6}{x}$$

In this example we have x in the denominator, which means we're dividing by x. We need to remember that we can't divide by 0, but x could be any number except 0. So the domain is all real numbers except 0.

If y is any real number other than 0, there is some nonzero real number x such that

$$y = \frac{6}{x}$$

To see this, multiply both sides of this equation by x/y.

$$y\left(\frac{x}{y}\right) = \left(\frac{6}{x}\right)\left(\frac{x}{y}\right)$$

$$x = \frac{6}{y}$$

So for any nonzero real number y, we divide 6 by y to get a nonzero real number x for which $y = 6/x$.

However, there is no nonzero real number x such that

$$0 = \frac{6}{x}$$

To see this, multiply both sides of this equation by x.

$$0(x) = \left(\frac{6}{x}\right)(x)$$

$$0 = 6$$

This gives us the false equation $0 = 6$.

Combining these results, we find that the range of this function is all real numbers except 0.