

Graphing inequalities on a number line

In previous lessons we looked at how to graph points and lines in the xy -plane along two axes: the horizontal x -axis and the vertical y -axis. In this lesson we're talking about graphing inequalities, which will be done on a number line. You can think of the number line as simply the horizontal x -axis, with the vertical y -axis stripped away. A number line looks like this:



It's simply a horizontal line, extending infinitely in both directions. You'll see negative numbers to the left of 0 and positive numbers to the right of 0. We'll be graphing inequalities on number lines like this one.

An inequality is like an equation, but instead of an equals sign, you'll have one of these inequality signs:

$>$, $<$, \neq , \leq , \geq

Before we can graph an inequality, we have to solve it. To help us in doing the graphing, we want to write the solution in a form of an inequality where only the variable is on the left side, and only a number is on the right side. That is, we want to get the solution into one of the following forms, where a is a number:

$$x > a$$

$$x < a$$

$$x \neq a$$



$$x \leq a$$

$$x \geq a$$

If you solve an inequality, and what you get is an inequality that has only a number on the left side, and only the variable on the right side, all you need to do is “turn the inequality around” (put the variable on the left side, and the number on the right side, and reverse the direction of the inequality sign). For example, if you solve an inequality and get $6 > x$, you can turn it around and write it as $x < 6$.

If the inequality is either $x \geq a$ or $x \leq a$, then (because of the “equal to” part of the inequality) the number on the right side (a) is part of the solution, so we draw a solid (filled-in) circle at a on our number line when we graph the inequality. If the inequality is either $x > a$ or $x < a$, the number on the right side (a) isn’t part of the solution, so we draw an open circle at a on our number line.

To graph an inequality $x > a$ (or the “greater than” part of an inequality $x \geq a$), we draw a ray (a line that extends infinitely in only one direction) that starts at the (solid or open) circle at a and extends infinitely to the right, since the solution of the inequality $x > a$ consists of all numbers greater than a (all the numbers to the right of a on our number line).

Similarly, to graph an inequality $x < a$ (or the “less than” part of an inequality $x \leq a$), we draw a ray that starts at the (solid or open) circle at a and extends infinitely to the left, since the solution of the inequality $x < a$ consists of all the numbers less than a (all the numbers to the left of a on our number line).



In either case, we draw an arrow to show the direction of the ray.

To summarize:

$>$ or \geq : the arrow will go to the right

$<$ or \leq : the arrow will go to the left

$>$ or $<$: the circle will be open

\geq or \leq : the circle will be solid

If the inequality is $x \neq a$ (“ x not equal to a ”), the solution consists of all the numbers other than a . If $a = 4$, for example, the solution of the inequality $x \neq 4$ consists of all the numbers other than 4. So we draw an open circle at a on our number line, and then we draw two rays: a ray that starts at the open circle at a and extends infinitely to the left, where we show the direction with an arrow that goes to the left, and a ray that starts at the open circle at a and extends infinitely to the right, where we show the direction with an arrow that goes to the right.

Let’s do a few examples so that you can see what we mean.

Example

Graph the inequality on a number line.

$$x < 5$$



First, we'll draw a number line that includes the number on the right side of the inequality (5).



Next, we'll draw an open circle at 5. The circle will be open because 5 isn't part of the solution. Finally, we'll draw a ray to the left (starting at the open circle at 5), since the solution consists of all the numbers less than 5, which means the arrow will go to the left.



Let's try another example of graphing inequalities.

Example

Graph the inequality on a number line.

$$x \geq 2$$

First, we'll draw a number line that includes the number on the right side of the inequality (2).



Next, we'll draw a solid circle at 2, because 2 is part of the solution. Finally, we'll draw a ray to the right (starting at the solid circle at 2), since the solution includes all the numbers greater than 2, which means the arrow will go to the right.

