Zero theorem

In this lesson we'll learn how to use the zero theorem to calculate the zeroes of a polynomial function $f(x)$, that is, the values of x at which the polynomial function is equal to 0. The zeroes of $f(x)$ are also called the roots of the polynomial equation $f(x) = 0$.

We can use the zero theorem to find the zeroes of a polynomial function once it's been factored. When a polynomial is factored, the zero theorem tells us that, in order for the polynomial function to be equal to 0, at least one of the factors must be 0.

For example, suppose you have the following equation, where (as you can probably tell) the expression on the left-hand side is the factored form of a polynomial:

$$(2x + 5)(x - 3) = 0$$

Then according to the zero theorem, we can set the factors separately to 0 to find the solutions to the equation.

To find the solutions (or roots) of this equation we'll set the factors ($2x + 5$ and $x - 3$) separately to 0 and solve for x.

$$2x + 5 = 0 \quad \rightarrow \quad 2x = -5 \quad \rightarrow \quad x = -\frac{5}{2}$$

$$x - 3 = 0 \quad \rightarrow \quad x = 3$$

The solutions (roots) of the equation are $x = -5/2$ and $x = 3$.

Let's look at how a question like this might appear on an exam.

Example

Find the roots of the equation.

$$x^2 - 13x + 36 = 0$$

The roots of this equation are the values of x at which the polynomial on the left-hand side is equal to 0.

We'll factor the left-hand side of the equation

$$x^2 - 13x + 36 = 0$$

$$(x - 4)(x - 9) = 0$$

The zero theorem tells us that, in order for the left-hand side to be equal to 0, one or both of the factors must be 0. Therefore, we'll set the factors ($x - 4$ and $x - 9$) separately to 0 and solve for x.

$$x - 4 = 0 \quad \rightarrow \quad x = 4$$

$$x - 9 = 0 \quad \rightarrow \quad x = 9$$

The roots are $x = 4$ and $x = 9$.

Let's do another example.

Example

Find the zeros of the function.

$$f(x) = 5x^2 - 8x + 3$$

Finding the zeros of a function means finding the values of x at which the function is equal to 0.

Let's set the function equal to 0 and factor.

$$5x^2 - 8x + 3 = 0$$

The only factors of 5 are 5 and 1, so we know we'll have the following:

$$(5x \quad)(x \quad) = 0$$

And the only factors of 3 are 3 and 1, so we know we'll have one of the following:

$$(5x \quad 3)(x \quad 1) = 0$$

or

$$(5x \quad 1)(x \quad 3) = 0$$

If we use the first way, the terms we get (apart from the signs) are

$$5x^2 \quad 5x \quad 3x \quad 3$$

If we use the second way, the terms we get (apart from signs) are

$$5x^2 \quad 15x \quad x \quad 3$$

The only way we can get the middle term of our polynomial $(-8x)$ by combining the x terms (including the proper signs) is by adding $-5x$ and $-3x$, so our polynomial factors as

$$(5x - 3)(x - 1)$$

This means that we need to solve the equation

$$(5x - 3)(x - 1) = 0$$

The zero theorem tells us that, in order for the left-hand side to be equal to 0, one or both of the factors must be 0. Therefore, we'll set the factors ($5x - 3$ and $x - 1$) separately to 0 and solve for x.

$$5x - 3 = 0 \quad \rightarrow \quad 5x = 3 \quad \rightarrow \quad x = \frac{3}{5}$$

$$x - 1 = 0 \quad \rightarrow \quad x = 1$$

The zeros are $x = 3/5$ and $x = 1$.