Factoring the difference of two cubes

In this lesson we'll look at how to recognize a difference of two cubes and then use a formula to factor it.

We know we're dealing with the difference of cubes, because we have two perfect cubes separated by a minus sign to indicate that the second perfect cube is to be subtracted from the first perfect cube. When that's the case, we can take the cube root of each term.

The formula for factoring a difference of cubes is

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Example

Factor the expression.

$$c^3 - 8d^{12}$$

To check whether the terms to the left and right of the minus sign are perfect cubes, we'll take the cube root of each of them (we'll raise each of them to the $1/3$ power).

$$\sqrt[3]{c^3} = (c^3)^{\frac{1}{3}} = c$$

$$\sqrt[3]{8d^{12}} = (8d^{12})^{\frac{1}{3}} = (8)^{\frac{1}{3}}(d^{12})^{\frac{1}{3}} = 2d^4$$

We can see that both terms are perfect cubes. The difference of cubes formula says $a^3 - b^3$ is always factored as

$$(a-b)(a^2+ab+b^2)$$

a is the cube root of the first term (in this case $a = c$), and b is the cube root of the second term (in this case $b = 2d^4$), so we get

$$(c-2d^4)\left[(c)^2+(c)(2d^4)+(2d^4)^2\right]$$

$$(c-2d^4)(c^2+2cd^4+4d^8)$$

Let's do one more.

Example

Factor the expression.

$$27x^3y^9 - 216z^{15}$$

To check whether the terms to the left and right of the minus sign are perfect cubes, we'll take the cube root of each of them (we'll raise each of them to the $1/3$ power).

$$\sqrt[3]{27x^3y^9} = (27x^3y^9)^{\frac{1}{3}} = (27)^{\frac{1}{3}}(x^3)^{\frac{1}{3}}(y^9)^{\frac{1}{3}} = 3xy^3$$

$$\sqrt[3]{216z^{15}} = (216z^{15})^{\frac{1}{3}} = (216)^{\frac{1}{3}}(z^{15})^{\frac{1}{3}} = 6z^5$$

We can see that both terms are perfect cubes. The difference of cubes formula says $a^3 - b^3$ is always factored as

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

a is the cube root of the first term, and b is the cube root of the second term, so

$$a = 3xy^3$$

$$b = 6z^5$$

Now we'll apply the formula.

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$(3xy^3 - 6z^5)\left[(3xy^3)^2 + (3xy^3)(6z^5) + (6z^5)^2\right]$$

$$(3xy^3 - 6z^5)(9x^2y^6 + 18xy^3z^5 + 36z^{10})$$

We can check our work by distributing each term in the binomial factor over all the terms in the trinomial factor.

$$(3xy^3)(9x^2y^6) + (3xy^3)(18xy^3z^5) + (3xy^3)(36z^{10})$$

$$+(-6z^5)(9x^2y^6) + (-6z^5)(18xy^3z^5) + (-6z^5)(36z^{10})$$

$$27x^3y^9 + 54x^2y^6z^5 + 108xy^3z^{10} - 54x^2y^6z^5 - 108xy^3z^{10} - 216z^{15}$$

$$27x^3y^9 + 54x^2y^6z^5 - 54x^2y^6z^5 + 108xy^3z^{10} - 108xy^3z^{10} - 216z^{15}$$

$$27x^3y^9 - 216z^{15}$$