Solving with substitution

How do we solve a system of two equations that both contain the same two unknowns (variables)?

In the problems we'll be looking at in this section, we'll be finding the solution to a system of equations of two lines. Two lines that aren't parallel cross each other at a single point, and that point is our solution. There are several methods to accomplish this, and one of those ways is to use substitution.

Here are the steps we'll follow when we use the substitution method:

1. Get a variable by itself in one of the equations.
2. Take the expression you got for the variable in step 1, and plug it (substitute it using parentheses) into the other equation.
3. Solve the equation in step 2 for the remaining variable.
4. Use the result from step 3 and plug it into the equation from step 1.

Let's look at some examples to make things a little more clear.

Example

Find the unique solution to the system of equations.

$$y = x + 3$$

$$2x - 3y = 10$$

Since y is already solved for in the first equation, step 1 is completed, and we'll go on to step 2 by substituting $x + 3$ for y in the other equation.

$$2x - 3y = 10$$

$$2x - 3(x + 3) = 10$$

Solve for x. Start by distributing the -3.

$$2x - 3x - 9 = 10$$

Combine like terms.

$$-x - 9 = 10$$

Add 9 to both sides.

$$-x - 9 + 9 = 10 + 9$$

$$-x = 19$$

Multiply both sides by -1.

$$-x(-1) = 19(-1)$$

$$x = -19$$

To find y, we'll plug in -19 for x in the first equation.

$$y = x + 3$$

$$y = -19 + 3$$

$$y = -16$$

The unique solution is $(-19, -16)$.

Let's try another example of solving with substitution.

Example

Find the unique solution to the system of equations.

$$x - 2y = 6$$

$$4x + 5y = 32$$

First, we need to get a variable by itself. It's easiest to get x by itself in the first equation by adding $2y$ to both sides.

$$x - 2y + 2y = 6 + 2y$$

$$x = 6 + 2y$$

Now, we'll substitute $6 + 2y$ for x in the second equation and solve for y.

$$4x + 5y = 32$$

$$4(6 + 2y) + 5y = 32$$

$$24 + 8y + 5y = 32$$

$$24 + 13y = 32$$

$$24 - 24 + 13y = 32 - 24$$

$$13y = 8$$

$$\frac{13y}{13} = \frac{8}{13}$$

$$y = \frac{8}{13}$$

To find x, plug $8/13$ for y in the first equation (the one we got when we took the original first equation, $x - 2y = 6$, and solved it for x in terms of y).

$$x = 6 + 2y$$

$$x = 6 + 2\left(\frac{8}{13}\right)$$

$$x = \frac{78}{13} + \frac{16}{13}$$

$$x = \frac{94}{13}$$

The unique solution is

$$\left(\frac{94}{13}, \frac{8}{13}\right)$$