Topic: Simplifying rational functions

Question: Simplify the rational function to lowest terms.

$$\frac{5ab + 25a^2b^2}{5ab}$$

Answer choices:

A $2 + 10a^2b^2$

B $1 + 5ab$

C $5ab$

D $1 + 5a^2b^2$

Solution: B

First, we'll look at the coefficients. We can factor a 5 out of all the terms of the numerator and all the terms of the denominator, and then cancel it.

$$\frac{5ab + 25a^2b^2}{5ab}$$

$$\frac{5\left(ab + 5a^2b^2\right)}{5(ab)}$$

$$\frac{ab + 5a^2b^2}{ab}$$

Next, we'll factor an a out of all the terms and cancel it.

$$\frac{a\left(b + 5ab^2\right)}{a(b)}$$

$$\frac{b + 5ab^2}{b}$$

Finally, we'll factor a b out of all the terms and cancel it.

$$\frac{b\left(1 + 5ab\right)}{b(1)}$$

$$\frac{1 + 5ab}{1}$$

$$1 + 5ab$$

For the sake of illustration, we factored out the 5, the a, and the b separately, but you could just as easily factor out the entire $5ab$ and cancel it all at once.

Topic: Simplifying rational functions

Question: Simplify the rational functions in the difference.

$$\frac{3mx + 9m^2x^2}{12mx} - \frac{64m^3x^3 - 8m^2x^2}{4mx}$$

Answer choices:

A $\frac{x + 3mx^2}{4x} - \frac{16m^2x^3 - 2mx^2}{x}$

B $\frac{mx + 3m^2x^2}{4mx} - \frac{16m^3x^3 - 2m^2x^2}{mx}$

C $\frac{1 + 3mx}{4} - 2mx(8mx - 1)$

D $\frac{3mx(1 + 3mx)}{3mx(4)} - \frac{8m^2x^2(8mx - 1)}{4mx}$

Solution: C

For each rational function, we want to factor everything that we can out of all the terms of its numerator and then factor everything that we can out of all the terms of its denominator. That means we need to take out every constant, every factor of m, and every factor of x that's common.

$$\frac{3mx + 9m^2x^2}{12mx} - \frac{64m^3x^3 - 8m^2x^2}{4mx}$$

$$\frac{3mx(1 + 3mx)}{3mx(4)} - \frac{8m^2x^2(8mx - 1)}{4mx}$$

Then for each rational function, we want to cancel what's common to its numerator and denominator. Since we have a factor of $8m^2x^2$ in the numerator of the second function and a factor of only $4mx$ in its denominator, we'll first factor a $4mx$ out of the $8m^2x^2$ in the numerator.

$$\frac{3mx(1 + 3mx)}{3mx(4)} - \frac{4mx(2mx)(8mx - 1)}{4mx}$$

$$\frac{1 + 3mx}{4} - \frac{2mx(8mx - 1)}{1}$$

$$\frac{1 + 3mx}{4} - 2mx(8mx - 1)$$

Topic: Simplifying rational functions

Question: Simplify the rational function.

$$\frac{3x^4 - 9x^3}{6x^2}$$

Answer choices:

A $\frac{x^3 - 3x^2}{2x}$

B $\frac{x^2 - 3x}{2}$

C $\frac{3x^2 - 9x}{6}$

D $\frac{x^2 - 3x}{3}$

Solution: B

To simplify the rational function, we'll factor out the greatest common factor, which is $3x^2$. Note that the greatest common factor of the terms in the numerator is $3x^3$, but that x^3 isn't a factor of the denominator.

$$\frac{3x^4 - 9x^3}{6x^2}$$

$$\frac{3x^2(x^2 - 3x)}{3x^2(2)}$$

$$\frac{x^2 - 3x}{2}$$