Topic: Composite functions

Question: Find the composite function.

$$g(f(x))$$

$$f(x) = \frac{1}{x^2}$$

$$g(x) = \sqrt{x - 3}$$

Answer choices:

A $g(f(x)) = \dfrac{1}{x - 3}$

B $g(f(x)) = \sqrt{\dfrac{1}{x^2} - 3}$

C $g(f(x)) = \sqrt{\dfrac{1}{(x - 3)^2}}$

D $g(f(x)) = \dfrac{1}{\sqrt{x - 3}}$

Solution: B

To find the composite function $g(f(x))$, we plug $f(x)$ into $g(x)$, which means that we take the algebraic expression for $f(x)$ and substitute it for x in the algebraic expression for $g(x)$.

$$g(f(x)) = \sqrt{\frac{1}{x^2} - 3}$$

Topic: Composite functions

Question: Find $g(h(x))$.

$$g(x) = x^2 - x - 4$$

$$h(x) = x\sqrt{2} + 1$$

Answer choices:

A $\sqrt{2}(x^2 - x - 4) + 1$

B $2x^2 + x\sqrt{2} - 4$

C $\sqrt{2}x^2 - 2x + 3$

D $2x^2 + 3x\sqrt{2} + 5$

Solution: B

To find $g(h(x))$, we have to plug $h(x)$ into $g(x)$. Given

$$g(x) = x^2 - x - 4$$

$$h(x) = x\sqrt{2} + 1$$

we get

$$g(h(x)) = \left(x\sqrt{2} + 1\right)^2 - \left(x\sqrt{2} + 1\right) - 4$$

$$g(h(x)) = 2x^2 + 2x\sqrt{2} + 1 - x\sqrt{2} - 1 - 4$$

$$g(h(x)) = 2x^2 + x\sqrt{2} - 4$$

Topic: Composite functions

Question: Find $f(g(x)) - g(f(x))$.

$$f(x) = x^2 - 2x$$

$$g(x) = 3x + 1$$

Answer choices:

A $6x^2 - 6x + 2$

B $6x^2 + 6x + 2$

C $6x^2 - 6x - 2$

D $6x^2 + 6x - 2$

Solution: D

To find $f(g(x))$, we have to plug $g(x)$ into $f(x)$, and to find $g(f(x))$, we have to plug $f(x)$ into $g(x)$. Given

$$f(x) = x^2 - 2x$$

$$g(x) = 3x + 1$$

we get

$$f(g(x)) = (3x + 1)^2 - 2(3x + 1)$$

$$f(g(x)) = 9x^2 + 6x + 1 - 6x - 2$$

$$f(g(x)) = 9x^2 - 1$$

and

$$g(f(x)) = 3(x^2 - 2x) + 1$$

$$g(f(x)) = 3x^2 - 6x + 1$$

Therefore, the function $f(g(x)) - g(f(x))$, which is the difference of the composite functions $f(g(x))$ and $g(f(x))$, is

$$f(g(x)) - g(f(x)) = (9x^2 - 1) - (3x^2 - 6x + 1)$$

$$f(g(x)) - g(f(x)) = 9x^2 - 1 - 3x^2 + 6x - 1$$

$$f(g(x)) - g(f(x)) = 6x^2 + 6x - 2$$