Topic: Graphing log functions

Question: Will the graph of the function have a vertical asymptote or a horizontal asymptote?

$$y = \log_2(x + 2)$$

Answer choices:

A It will have a vertical asymptote at $x = -2$

B It will have a vertical asymptote at $x = 2$

C It will have a horizontal asymptote at $y = -2$

D It will have a horizontal asymptote at $y = 2$

Solution: A

Because $y = \log_2(x+2)$ is a logarithmic equation, its graph will have a vertical asymptote. To find it, we'll first use the general log rule to convert the logarithmic equation $\log_2(x+2)$ to its exponential form,

$$2^y = x + 2$$

$$x = 2^y - 2$$

We'll then plug both $y = 100$ and $y = -100$ into the equation $x = 2^y - 2$, to see what happens to the value of x as $y \to \infty$ and as $y \to -\infty$.

For $y = 100$:

$$x = 2^{100} - 2$$

$$x = \text{a very large positive number} - 2$$

$$x = \text{a very large positive number}$$

$$x = \infty$$

For $y = -100$:

$$x = 2^{-100} - 2$$

$$x = \frac{1}{2^{100}} - 2$$

$$x = \frac{1}{\text{a very large positive number}} - 2$$

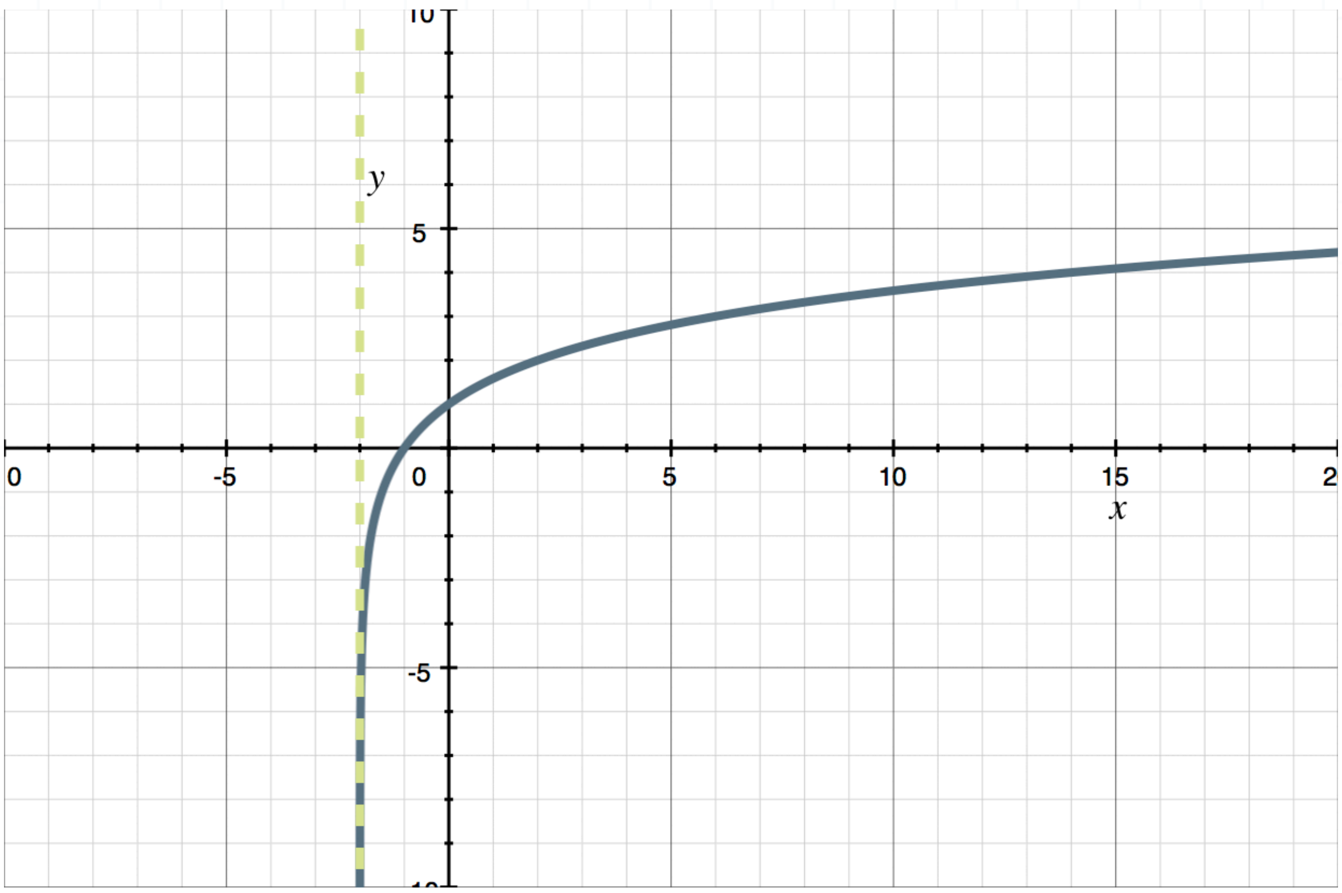
$$x = \text{a very small positive number} - 2$$

$$x = 0 - 2$$

$$x = -2$$

Plugging in $y = 100$ and $y = -100$ gives us a picture of the end behavior of the graph of the function. The results tell us that the function has a vertical asymptote at $x = -2$, and that the graph will tend toward ∞ as $x \to \infty$.

If we sketch the graph of the function, we can see this.



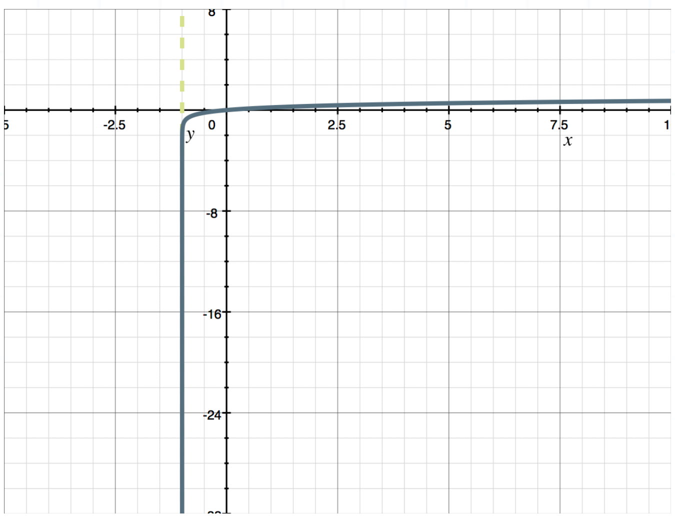
Topic: Graphing log functions

Question: Sketch the graph of the logarithmic function.

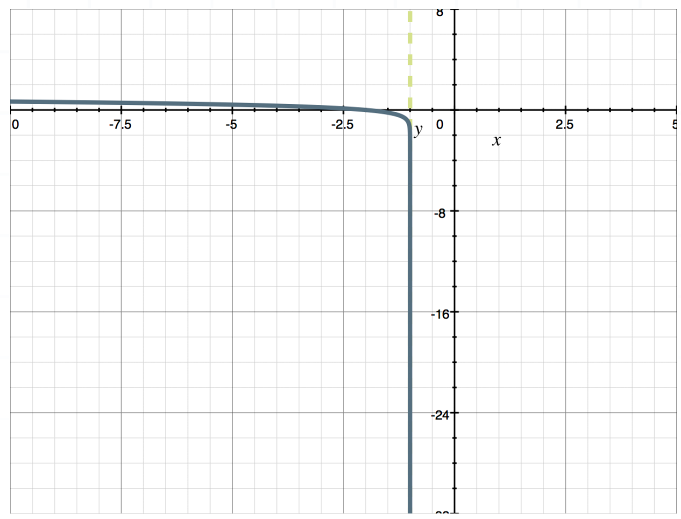
$$y = -\frac{1}{3}\log_3(x+1)$$

Answer choices:

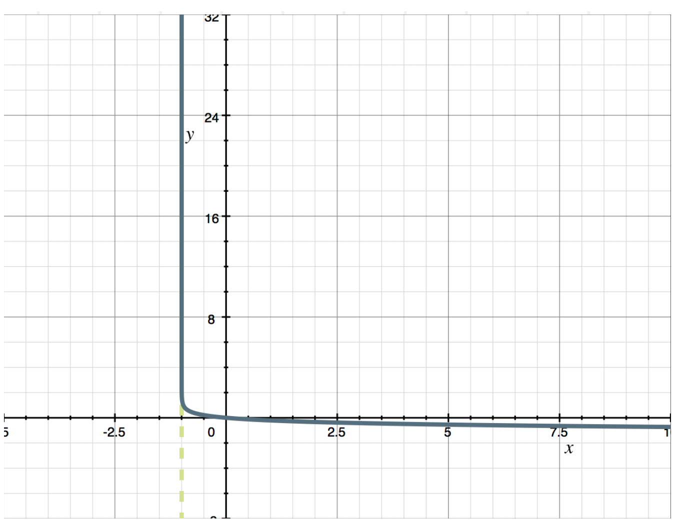
A



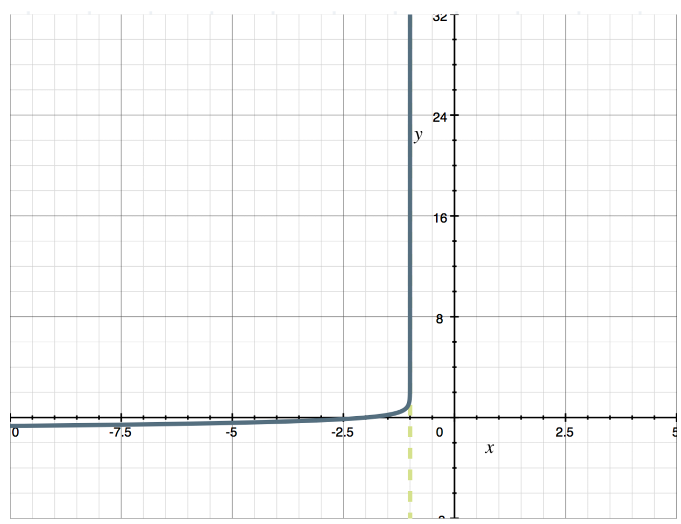
B



C



D



Solution: C

Use algebra to isolate the expression $\log_3(x+1)$.

$$y = -\frac{1}{3}\log_3(x+1)$$

$$-3y = \log_3(x+1)$$

Use the general log rule to convert this logarithmic equation to its exponential form.

$$3^{-3y} = x + 1$$

$$x = 3^{-3y} - 1$$

Plug in $y = 100$ and $y = -100$ to determine what happens to the value of x as $y \to \infty$ and as $y \to -\infty$.

For $y = 100$:

$$x = 3^{-3(100)} - 1$$

$$x = 3^{-300} - 1$$

$$x = \frac{1}{3^{300}} - 1$$

$$x = \frac{1}{\text{a very large positive number}} - 1$$

$$x = \text{a very small positive number} - 1$$

$$x = 0 - 1$$

$x = -1$

For $y = -100$:

$x = 3^{-3(-100)} - 1$

$x = 3^{300} - 1$

$x =$ a very large positive number $- 1$

$x =$ a very large positive number

$x = \infty$

Therefore, $x = -1$ will be a vertical asymptote, and as x tends toward ∞, the function will curl down toward $-\infty$.

We'll plug in a few easy-to-calculate points, like $y = -1/3, 0, 1/3$ in order to get some points of the graph of the equation $x = 3^{-3y} - 1$ that we can plot.

For $y = 0$:

$x = 3^{-3(0)} - 1$

$x = 3^0 - 1$

$x = 1 - 1$

$x = 0$

For $y = -1/3$:

$x = 3^{-3(-1/3)} - 1$

$$x = 3^1 - 1$$

$$x = 3 - 1$$

$$x = 2$$

For $y = 1/3$:

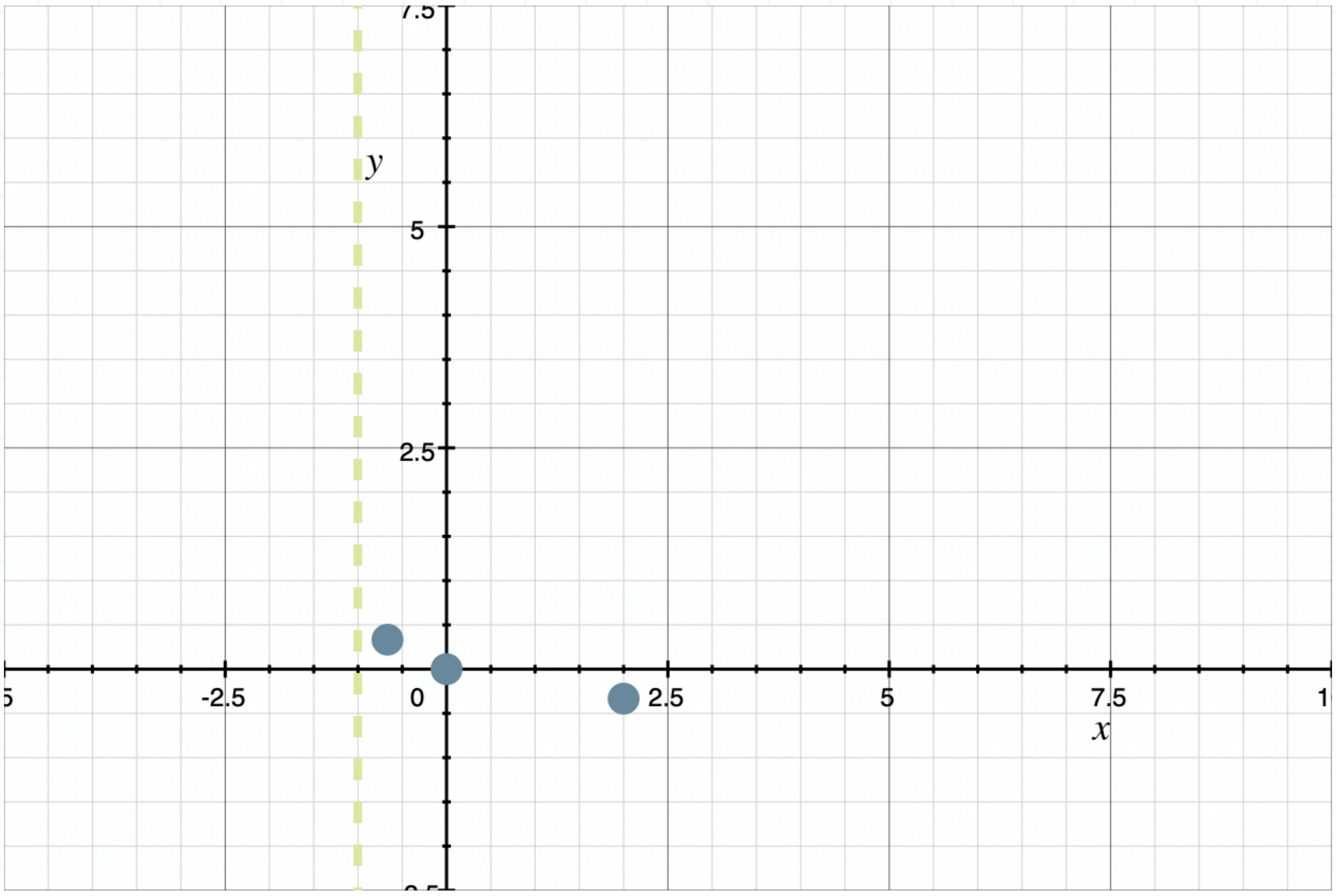
$$x = 3^{-3(1/3)} - 1$$

$$x = 3^{-1} - 1$$

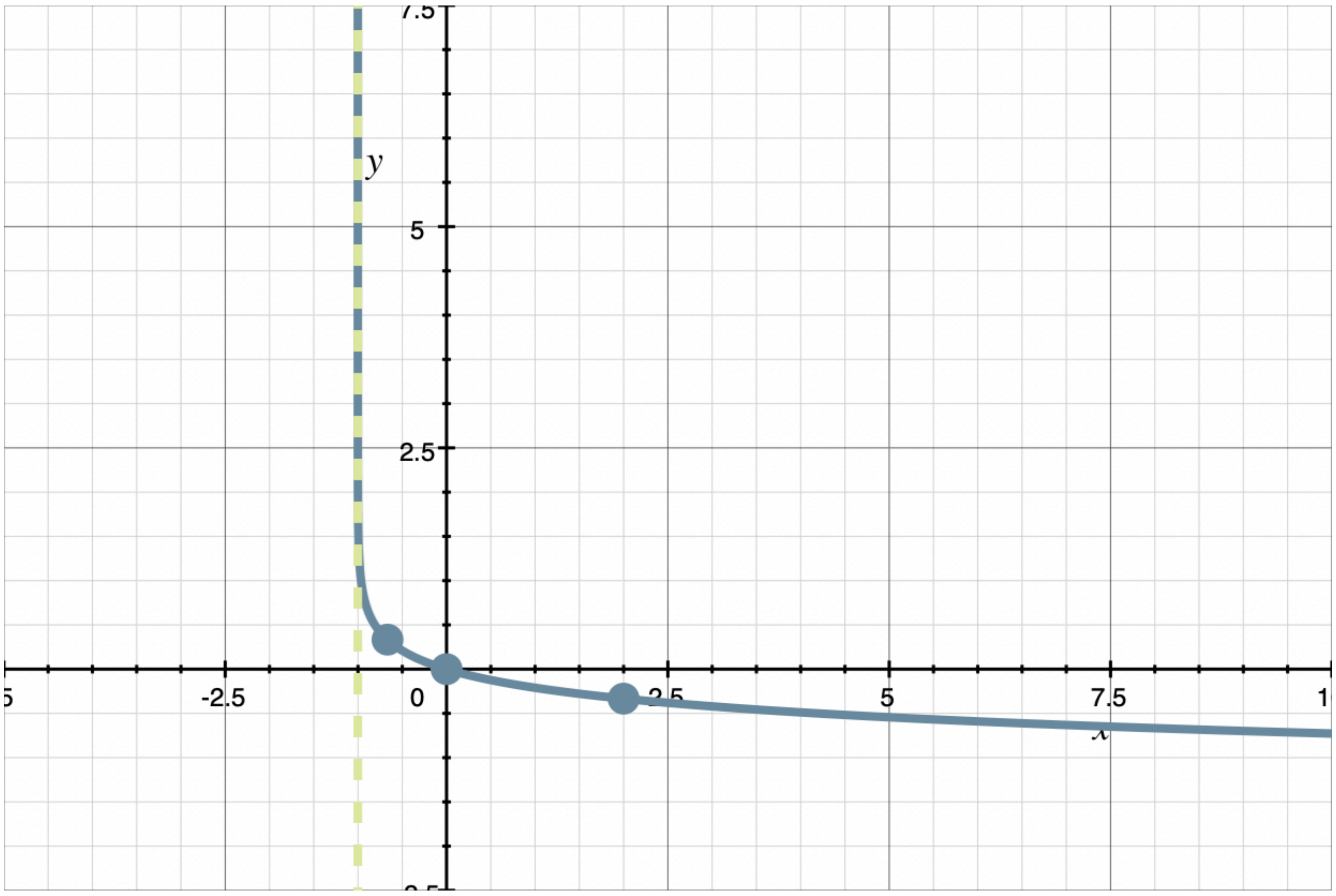
$$x = \frac{1}{3} - 1$$

$$x = -\frac{2}{3}$$

Now we have three points on the graph of the equation $x = 3^{-3y} - 1$: $(0,0)$, $(2, -1/3)$, and $(-2/3,1/3)$. If we plot these three points and draw the vertical asymptote, $x = -1$, we get



We can see, as we expected, that the exponential function will skim along the vertical asymptote $x = -1$, and then as $x \to \infty$, the function's value heads toward $-\infty$. Connecting the points on the function gives

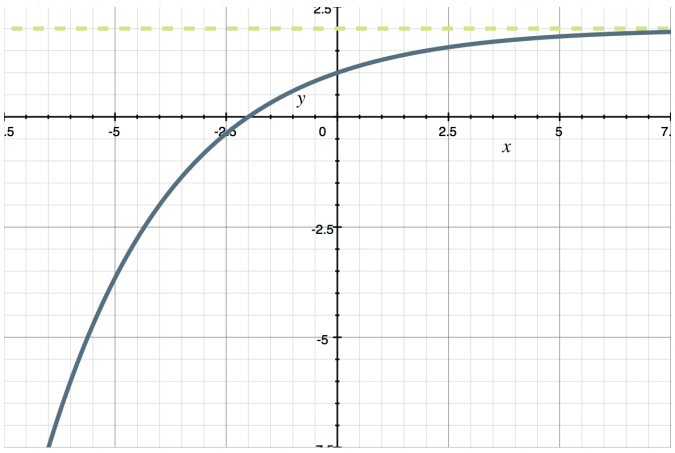


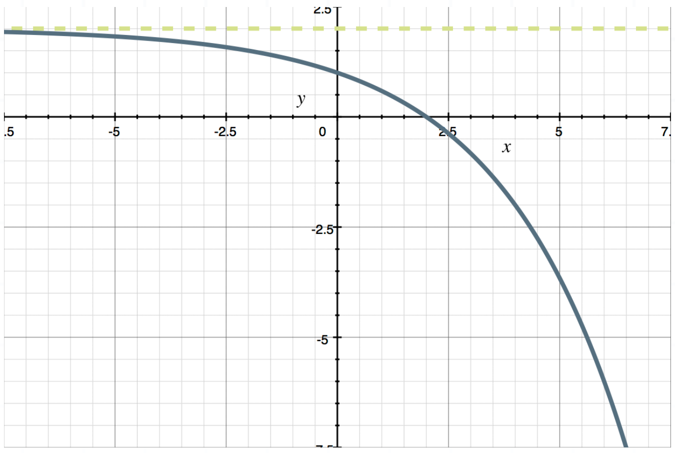
Topic: Graphing log functions

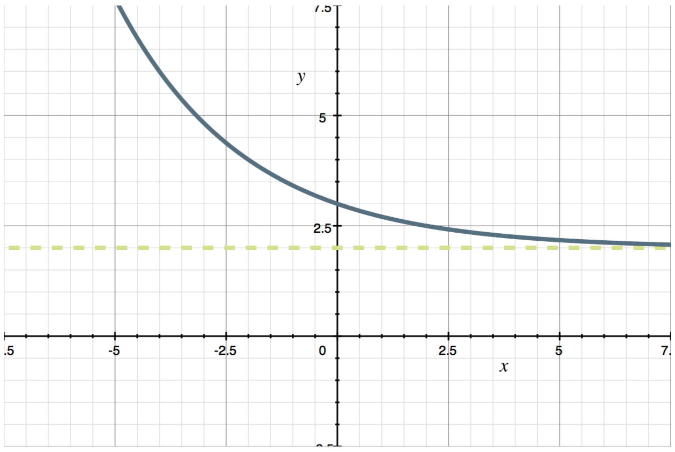
Question: Sketch the graph of the logarithmic equation.

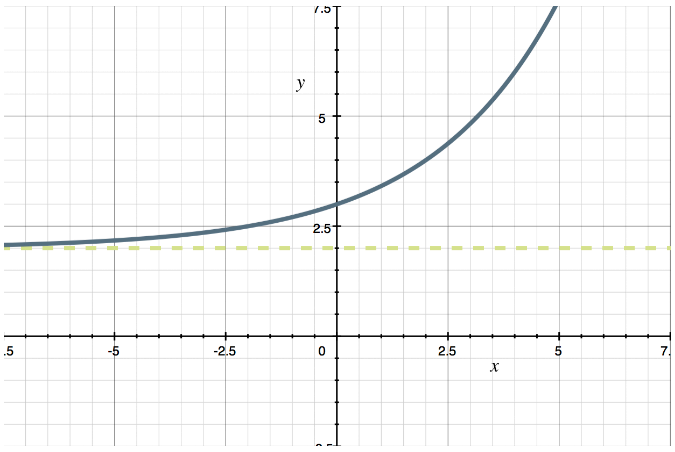
$$x = 2\log_2(y - 2)$$

Answer choices:

A


B


C


D


Solution: D

Use algebra to isolate the expression $\log_2(y-2)$.

$$x = 2\log_2(y-2)$$

$$\frac{x}{2} = \log_2(y-2)$$

Use the general log rule to convert this logarithmic equation to its exponential form.

$$2^{\frac{x}{2}} = y - 2$$

$$y = 2^{\frac{x}{2}} + 2$$

Plug in $x = 100$ and $x = -100$ to see what the function is doing as x starts getting close to $-\infty$ or $+\infty$.

For $x = 100$:

$$y = 2^{\frac{100}{2}} + 2$$

$$y = 2^{50} + 2$$

$y =$ a very large positive number $+ 2$

$y =$ a very large positive number

$$y = \infty$$

For $x = -100$:

$$y = 2^{\frac{-100}{2}} + 2$$

$$y = 2^{-50} + 2$$

$$y = \frac{1}{2^{50}} + 2$$

$$y = \frac{1}{\text{a very large positive number}} + 2$$

$$y = \text{a very small positive number} + 2$$

$$y = 0 + 2$$

$$y = 2$$

Therefore, $y = 2$ will be a horizontal asymptote, and as x tends toward ∞, the function will curl up toward ∞.

We'll plug in a few easy-to-calculate points, like $x = -1, 0, 1$ in order to get some points of the graph of the function $y = 2^{\frac{x}{2}} + 2$ that we can plot.

For $x = 0$:

$$y = 2^{\frac{0}{2}} + 2$$

$$y = 2^0 + 2$$

$$y = 1 + 2$$

$$y = 3$$

For $x = -1$:

$$y = 2^{\frac{-1}{2}} + 2$$

$$y = \frac{1}{2^{\frac{1}{2}}} + 2$$

$$y = \frac{1}{\sqrt{2}} + 2$$

$$y = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) + 2$$

$$y = \frac{\sqrt{2}}{2} + 2\left(\frac{2}{2}\right)$$

$$y = \frac{\sqrt{2} + 4}{2}$$

$$y \approx 2.7071$$

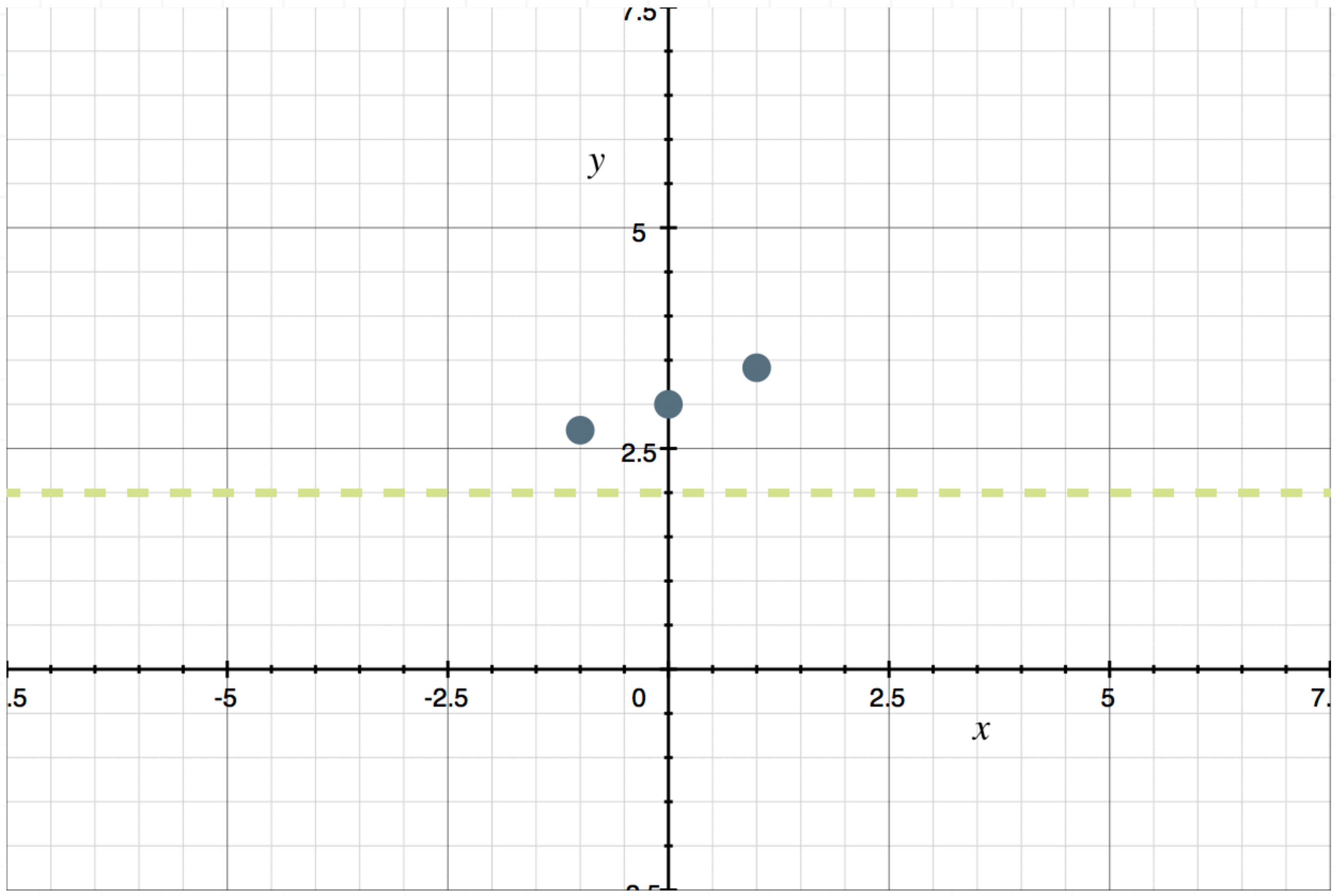
For $x = 1$:

$$y = 2^{\frac{1}{2}} + 2$$

$$y = \sqrt{2} + 2$$

$$y \approx 3.4142$$

Now we have three points on the graph of the function $y = 2^{\frac{x}{2}} + 2$: $(0,3)$, $(-1,2.7071)$, and $(1,3.4142)$. Since the graph of the function $y = 2^{\frac{x}{2}} + 2$ is identical to the graph of the equation $x = 2\log_2(y - 2)$, they are also points of the graph of the equation $x = 2\log_2(y - 2)$. If we plot these three points and draw the horizontal asymptote, $y = 2$, we get



We can see, as we expected, that the logarithmic function will skim along the horizontal asymptote $y = 2$, and then as $x \to \infty$, the function's value heads toward ∞. Connecting the points on the function gives