



# Algebra 2 Workbook Solutions

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Rational functions

## LONG DIVISION OF POLYNOMIALS

- 1. Find the quotient.

$$\frac{x^2 + 2x - 1}{x + 3}$$

*Solution:*

Use long division to find the quotient.

$$\begin{array}{r} x - 1 + \frac{2}{x+3} \\ x+3 \overline{) x^2 + 2x - 1} \\ \underline{-(x^2 + 3x)} \phantom{-1} \\ -x - 1 \\ \underline{-(-x - 3)} \\ 2 \end{array}$$

- 2. Find the quotient.

$$\frac{2x^3 - x^2 - 4x + 5}{x - 2}$$



*Solution:*

Use long division to find the quotient.

$$\begin{array}{r}
 2x^2 + 3x + 2 + \frac{9}{x-2} \\
 x-2 \overline{) 2x^3 - x^2 - 4x + 5} \\
 \underline{-(2x^3 - 4x^2)} \phantom{+ 5} \\
 3x^2 - 4x \phantom{+ 5} \\
 \underline{-(3x^2 - 6x)} \phantom{+ 5} \\
 2x + 5 \phantom{+ 5} \\
 \underline{-(2x - 4)} \\
 9
 \end{array}$$

■ 3. Find the quotient.

$$\begin{array}{r}
 2x^4 + 4x^3 - x^2 + 5x - 150 \\
 \hline
 x + 4
 \end{array}$$

*Solution:*

Use long division to find the quotient.



$$\begin{array}{r}
 2x^3 - 4x^2 + 15x - 55 + \frac{70}{x+4} \\
 x+4 \overline{) 2x^4 + 4x^3 - x^2 + 5x - 150} \\
 \underline{-(2x^4 + 8x^3)} \\
 -4x^3 - x^2 \\
 \underline{-(-4x^3 - 16x^2)} \\
 15x^2 + 5x \\
 \underline{-(15x^2 + 60x)} \\
 -55x - 150 \\
 \underline{-(-55x - 220)} \\
 70
 \end{array}$$

■ 4. Find the quotient.

$$\frac{3x^3 - x^2 - 7x + 5}{x - 1}$$

*Solution:*

Use long division to find the quotient.



$$\begin{array}{r}
 3x^2 + 2x - 5 \\
 x-1 \overline{) 3x^3 - x^2 - 7x + 5} \\
 \underline{-(3x^3 - 3x^2)} \\
 2x^2 - 7x \\
 \underline{-(2x^2 - 2x)} \\
 -5x + 5 \\
 \underline{-(-5x + 5)} \\
 0
 \end{array}$$

■ 5. Find the quotient.

$$\frac{-x^2 + 3x + 15}{x + 5}$$

*Solution:*

Use long division to find the quotient.



$$\begin{array}{r}
 -x + 8 - \frac{25}{x+5} \\
 x+5 \overline{) -x^2 + 3x + 15} \\
 \underline{-(-x^2 - 5x)} \phantom{+ 15} \\
 8x + 15 \\
 \underline{-(8x + 40)} \\
 -25
 \end{array}$$

■ 6. Find the quotient.

$$\frac{x^4 + x - 3}{x - 2}$$

*Solution:*

Use long division to find the quotient. Remember to represent the missing  $x^3$  and  $x^2$  terms.



$$\begin{array}{r}
 x^3 + 2x^2 + 4x + 9 + \frac{15}{x-2} \\
 x-2 \overline{) x^4 + 0x^3 + 0x^2 + x - 3} \\
 \underline{-(x^4 - 2x^3)} \phantom{+ 0x^2 + x - 3} \\
 2x^3 + 0x^2 \phantom{+ x - 3} \\
 \underline{-(2x^3 - 4x^2)} \phantom{+ x - 3} \\
 4x^2 + x \phantom{- 3} \\
 \underline{-(4x^2 - 8x)} \phantom{- 3} \\
 9x - 3 \\
 \underline{-(9x - 18)} \\
 15
 \end{array}$$

■ 7. Find the quotient.

$$\frac{x^3 + 6}{x + 6}$$

*Solution:*

Use long division to find the quotient. Remember to represent the missing  $x^2$  and  $x$  terms.



$$\begin{array}{r}
 x^2 - bx + 3b - \frac{210}{x+b} \\
 x+b \overline{) x^3 + 0x^2 + 0x + b} \\
 \underline{-(x^3 + bx^2)} \\
 -bx^2 + 0x \\
 \underline{-(-bx^2 - 3bx)} \\
 3bx + b \\
 \underline{-(3bx + 21b)} \\
 -210
 \end{array}$$

■ 8. Find the quotient.

$$\frac{x^2 + x}{x - 3}$$

*Solution:*

Use long division to find the quotient. Remember to represent the missing constant term.





$$\begin{array}{r}
 x + 4 + \frac{12}{x-3} \\
 x-3 \overline{) x^2 + x + 0} \\
 \underline{-(x^2 - 3x)} \phantom{+ 0} \\
 4x + 0 \\
 \underline{-(4x - 12)} \\
 12
 \end{array}$$

■ 9. Find the quotient.

$$\frac{x^4 - 2x^2}{x - 4}$$

*Solution:*

Use long division to find the quotient. Remember to represent the missing terms.



$$\begin{array}{r}
 x^3 + 4x^2 + 14x + 56 + \frac{224}{x-4} \\
 x-4 \overline{) x^4 + 0x^3 - 2x^2 + 0x + 0} \\
 \underline{-(x^4 - 4x^3)} \\
 4x^3 - 2x^2 \\
 \underline{-(4x^3 - 16x^2)} \\
 14x^2 + 0x \\
 \underline{-(14x^2 - 56x)} \\
 56x + 0 \\
 \underline{-(-56x - 224)} \\
 224
 \end{array}$$

■ 10. Find the quotient.

$$\frac{-2x^3 + 8x}{x + 2}$$

*Solution:*

Use long division to find the quotient. Remember to represent the missing terms.



$$\begin{array}{r} -2x^2 + 4x \\ x+2 \overline{) -2x^3 + 0x^2 + 8x + 0} \\ \underline{-(-2x^3 - 4x^2)} \phantom{+ 0} \\ 4x^2 + 8x \\ \underline{-(4x^2 + 8x)} \\ 0 \end{array}$$



## SIMPLIFYING RATIONAL FUNCTIONS

- 1. Reduce the fraction to its lowest terms.

$$\frac{4x^3 + 12x^2 + 8x}{12x^2}$$

*Solution:*

Look for a common factor in each term. In this case the common factor is  $4x$ .

$$\frac{4x^3 + 12x^2 + 8x}{12x^2}$$

$$\frac{4x(x^2 + 3x + 2)}{4x(3x)}$$

$$\frac{x^2 + 3x + 2}{3x}$$

- 2. Reduce the fraction to its lowest terms.

$$\frac{10x^2 - 5x + 20}{15x^2}$$



*Solution:*

Look for a common factor in each term. In this case the common factor is 5.

$$\frac{10x^2 - 5x + 20}{15x^2}$$

$$\frac{5(2x^2 - x + 4)}{5(3x^2)}$$

$$\frac{2x^2 - x + 4}{3x^2}$$

■ 3. Reduce the fraction to its lowest terms.

$$\frac{3x^4 + 6x^3 - 18x^2}{12x^3}$$

*Solution:*

Look for a common factor in each term. In this case the common factor is  $3x^2$ .

$$\frac{3x^4 + 6x^3 - 18x^2}{12x^3}$$

$$\frac{3x^2(x^2 + 2x - 6)}{3x^2(4x)}$$



$$\frac{x^2 + 2x - 6}{4x}$$

- 4. Reduce the fraction to its lowest terms.

$$\frac{18y^2 + 6y}{8y}$$

*Solution:*

Look for a common factor in each term. In this case the common factor is  $2y$ .

$$\frac{18y^2 + 6y}{8y}$$

$$\frac{2y(9y + 3)}{2y(4)}$$

$$\frac{9y + 3}{4}$$

- 5. Simplify each expression in the difference.

$$\frac{3ab + 2a^2b^2}{5ab} - \frac{12a^3b^3 + 3a^2b^2}{6a^2b^2}$$



*Solution:*

Look for a common factor in each term. In this case the common factor of the first fraction is  $ab$  and the common factor of the second fraction is  $3a^2b^2$ .

$$\frac{3ab + 2a^2b^2}{5ab} - \frac{12a^3b^3 + 3a^2b^2}{6a^2b^2}$$

$$\frac{ab(3 + 2ab)}{ab(5)} - \frac{3a^2b^2(4ab + 1)}{3a^2b^2(2)}$$

$$\frac{3 + 2ab}{5} - \frac{4ab + 1}{2}$$

■ 6. Simplify each expression in the sum.

$$\frac{2ab^2 + 3a^2b^3}{a^3b^3} + \frac{2ab^3 + b^4}{ab^3}$$

*Solution:*

Look for a common factor in each term. In this case the common factor of the first fraction is  $ab^2$  and the common factor of the second fraction is  $b^3$ .

$$\frac{2ab^2 + 3a^2b^3}{a^3b^3} + \frac{2ab^3 + b^4}{ab^3}$$

$$\frac{ab^2(2 + 3ab)}{ab^2(a^2b)} + \frac{b^3(2a + b)}{b^3(a)}$$



$$\frac{2 + 3ab}{a^2b} + \frac{2a + b}{a}$$

■ 7. Simplify each expression in the sum.

$$\frac{8mn + 20m}{4mn} + \frac{m^3n - 3m^2n}{6m^2n}$$

*Solution:*

Look for a common factor in each term. In this case the common factor of the first fraction is  $4m$  and the common factor of the second fraction is  $m^2n$ .

$$\frac{8mn + 20m}{4mn} + \frac{m^3n - 3m^2n}{6m^2n}$$

$$\frac{4m(2n + 5)}{4m(n)} + \frac{m^2n(m - 3)}{m^2n(6)}$$

$$\frac{2n + 5}{n} + \frac{m - 3}{6}$$

■ 8. Simplify each expression in the difference.

$$\frac{21x^2y^2}{14x^3y} - \frac{24xy + 12y}{96y}$$





*Solution:*

Look for a common factor in each term. In this case the common factor of the first fraction is  $7x^2y$  and the common factor of the second fraction is  $12y$ .

$$\frac{21x^2y^2}{14x^3y} - \frac{24xy + 12y}{96y}$$

$$\frac{7x^2y(3y)}{7x^2y(2x)} - \frac{12y(2x + 1)}{12y(8)}$$

$$\frac{3y}{2x} - \frac{2x + 1}{8}$$



ADDING AND SUBTRACTING RATIONAL FUNCTIONS

■ 1. Simplify the expression.

$$\frac{2}{3ab} + \frac{b}{4a} + \frac{ab}{6}$$

*Solution:*

In order to combine the three fractions in the expression we need to find a common denominator. Make a chart of with each denominator to find the least common multiple.

	Coefficients	a-terms	b-terms
3ab	3	a	b
4a	2*2	a	
6	2*3		

To find the least common multiple, find the largest multiple from each column. For the coefficients multiply each unique factor together to get  $2 \cdot 2 \cdot 3 = 12$ . The largest common multiple in the  $a$  column is  $a$  and the largest common multiple in the  $b$  column is  $b$ . The least common multiple is therefore  $12ab$ .

Now we need to multiply each fraction by a well-chosen 1 that will make the denominator  $12ab$ .



$$\frac{2}{3ab} \cdot \frac{4}{4} + \frac{b}{4a} \cdot \frac{3b}{3b} + \frac{ab}{6} \cdot \frac{2ab}{2ab}$$

$$\frac{8}{12ab} + \frac{3b^2}{12ab} + \frac{2a^2b^2}{12ab}$$

$$\frac{8 + 3b^2 + 2a^2b^2}{12ab}$$

■ 2. Simplify the expression.

$$\frac{1}{2xy} + \frac{2}{3x^2} + \frac{3}{4xy^2}$$

*Solution:*

In order to combine the three fractions in the expression we need to find a common denominator. Make a chart of with each denominator to find the least common multiple.

	Coefficients	x-terms	y-terms
2xy	2	x	y
3x <sup>2</sup>	3	x <sup>2</sup>	
4xy <sup>2</sup>	2*2	x	y <sup>2</sup>

To find the least common multiple, find the largest multiple from each column. For the coefficients multiply each unique factor together to get  $2 \cdot 2 \cdot 3 = 12$ . The largest common multiple in the  $x$  column is  $x^2$  and the

largest common multiple in the  $y$  column is  $y^2$ . The least common multiple is therefore  $12x^2y^2$ .

Now we need to multiply each fraction by a well-chosen 1 that will make the denominator  $12x^2y^2$ .

$$\frac{1}{2xy} \cdot \frac{6xy}{6xy} + \frac{2}{3x^2} \cdot \frac{4y^2}{4y^2} + \frac{3}{4xy^2} \cdot \frac{3x}{3x}$$

$$\frac{6xy}{12x^2y^2} + \frac{8y^2}{12x^2y^2} + \frac{9x}{12x^2y^2}$$

$$\frac{6xy + 8y^2 + 9x}{12x^2y^2}$$

■ 3. Simplify the expression.

$$\frac{a}{3xy} + \frac{b}{15y^2} + \frac{c}{5x^3y^2}$$

*Solution:*

In order to combine the three fractions in the expression we need to find a common denominator. Make a chart of with each denominator to find the least common multiple.



	Coefficients	x-terms	y-terms
<b>3xy</b>	3	x	y
<b>15x<sup>2</sup></b>	3*5		y <sup>2</sup>
<b>5x<sup>3</sup>y<sup>2</sup></b>	5	x <sup>3</sup>	y <sup>2</sup>

To find the least common multiple, find the largest multiple from each column. For the coefficients multiply each unique factor together to get  $3 \cdot 5 = 15$ . The largest common multiple in the  $x$  column is  $x^3$  and the largest common multiple in the  $y$  column is  $y^2$ . The least common multiple is therefore  $15x^3y^2$ .

Now we need to multiply each fraction by a well-chosen 1 that will make the denominator  $15x^3y^2$ .

$$\frac{a}{3xy} \cdot \frac{5x^2y}{5x^2y} + \frac{b}{15y^2} \cdot \frac{x^3}{x^3} + \frac{c}{5x^3y^2} \cdot \frac{3}{3}$$

$$\frac{5ax^2y}{15x^3y^2} + \frac{bx^3}{15x^3y^2} + \frac{3c}{15x^3y^2}$$

$$\frac{5ax^2y + bx^3 + 3c}{15x^3y^2}$$

#### ■ 4. Simplify the expression.

$$\frac{x}{2x^2y} + \frac{y}{3z} + \frac{z}{5yz^2}$$



*Solution:*

In order to combine the three fractions in the expression we need to find a common denominator. Make a chart of with each denominator to find the least common multiple.

	Coefficient s	x-terms	y-terms	z-terms
$2x^2y$	2	$x^2$	y	
$3z$	3			z
$5yz^2$	5		y	$z^2$

To find the least common multiple, find the largest multiple from each column. For the coefficients multiply each unique factor together to get  $2 \cdot 3 \cdot 5 = 30$ . The largest common multiple in the  $x$  column is  $x^2$  and the largest common multiple in the  $y$  column is  $y$ . The largest common multiple in the  $z$  column is  $z^2$ . The least common multiple is therefore  $30x^2yz^2$ .

Now we need to multiply each fraction by a well-chosen 1 that will make the denominator  $30x^2yz^2$ .

$$\frac{x}{2x^2y} \cdot \frac{15z^2}{15z^2} + \frac{y}{3z} \cdot \frac{10x^2yz}{10x^2yz} + \frac{z}{5yz^2} \cdot \frac{6x^2}{6x^2}$$

$$\frac{15xz^2}{30x^2yz^2} + \frac{10x^2y^2z}{30x^2yz^2} + \frac{6x^2z}{30x^2yz^2}$$

$$\frac{15xz^2 + 10x^2y^2z + 6x^2z}{30x^2yz^2}$$



■ 5. Simplify the expression.

$$\frac{3ab}{4c} + \frac{2bc}{6a^3} + \frac{5}{8ab^2c^3}$$

*Solution:*

In order to combine the three fractions in the expression we need to find a common denominator. Make a chart of with each denominator to find the least common multiple.

	Coefficient s	a-terms	b-terms	c-terms
<b>4c</b>	2*2			c
<b>6a<sup>3</sup></b>	2*3	a <sup>3</sup>		
<b>8ab<sup>2</sup>c<sup>3</sup></b>	2*2*2	a	b <sup>2</sup>	c <sup>3</sup>

To find the least common multiple, find the largest multiple from each column. For the coefficients multiply each unique factor together to get  $2 \cdot 2 \cdot 2 \cdot 3 = 24$ . The largest common multiple in the  $a$  column is  $a^3$  and the largest common multiple in the  $b$  column is  $b^2$ . The largest common multiple in the  $c$  column is  $c^3$ . The least common multiple is therefore  $24a^3b^2c^3$ .

Now we need to multiply each fraction by a well-chosen 1 that will make the denominator  $24a^3b^2c^3$ .

$$\frac{3ab}{4c} \cdot \frac{6a^3b^2c^2}{6a^3b^2c^2} + \frac{2bc}{6a^3} \cdot \frac{4b^2c^3}{4b^2c^3} + \frac{5}{8ab^2c^3} \cdot \frac{3a^2}{3a^2}$$



$$\frac{18a^4b^3c^2}{24a^3b^2c^2} + \frac{8b^3c^4}{24a^3b^2c^3} + \frac{15a^2}{24a^3b^2c^3}$$

$$\frac{18a^4b^3c^2 + 8b^3c^4 + 15a^2}{24a^3b^2c^3}$$





## MULTIPLYING RATIONAL FUNCTIONS

- 1. Simplify the expression.

$$\frac{25x^2 - 4}{x^2 - 36} \cdot \frac{x + 6}{5x - 2}$$

*Solution:*

Factor and cancel whatever you can, then simplify.

$$\frac{(5x - 2)(5x + 2)}{(x - 6)(x + 6)} \cdot \frac{x + 6}{5x - 2}$$

$$\frac{5x + 2}{x - 6}$$

We cancelled factors of  $5x - 2$  and  $x + 6$  which means

$$5x - 2 \neq 0, \text{ or } x \neq 2/5$$

$$x + 6 \neq 0, \text{ or } x \neq -6$$

So the simplified expression is

$$\frac{5x + 2}{x - 6} \text{ with } x \neq -6, \frac{2}{5}$$

- 2. Simplify the expression.



$$\frac{4x^2 - 49}{9x^2 - 16} \cdot \frac{3x + 4}{2x + 7}$$

*Solution:*

Factor and cancel whatever you can, then simplify.

$$\frac{(2x - 7)(2x + 7)}{(3x - 4)(3x + 4)} \cdot \frac{3x + 4}{2x + 7}$$

$$\frac{2x - 7}{3x - 4}$$

We cancelled factors of  $2x + 7$  and  $3x + 4$  which means

$$2x + 7 \neq 0, \text{ or } x \neq -7/2$$

$$3x + 4 \neq 0, \text{ or } x \neq -4/3$$

So the simplified expression is

$$\frac{2x - 7}{3x - 4} \text{ with } x \neq -\frac{7}{2}, -\frac{4}{3}$$

■ 3. Simplify the expression.

$$\frac{x^2 - 25}{81x^2 - 64} \cdot \frac{9x - 8}{x - 5}$$



*Solution:*

Factor and cancel whatever you can, then simplify.

$$\frac{(x-5)(x+5)}{(9x-8)(9x+8)} \cdot \frac{9x-8}{x-5}$$

$$\frac{x+5}{9x+8}$$

We cancelled factors of  $x-5$  and  $9x-8$  which means

$$x-5 \neq 0, \text{ or } x \neq 5$$

$$9x-8 \neq 0, \text{ or } x \neq 8/9$$

So the simplified expression is

$$\frac{x+5}{9x+8} \text{ with } x \neq \frac{8}{9}, 5$$

■ 4. Simplify the expression.

$$\frac{x^2 + 8x + 16}{9x^2 + 36x + 36} \cdot \frac{3x + 6}{x + 4}$$

*Solution:*

Factor and cancel whatever you can, then simplify.



$$\frac{(x+4)(x+4)}{(3x+6)(3x+6)} \cdot \frac{3x+6}{x+4}$$

$$\frac{x+4}{3x+6}$$

We cancelled factors of  $x+4$  and  $3x+6$  which means

$$x+4 \neq 0, \text{ or } x \neq -4$$

$$3x+6 \neq 0, \text{ or } x \neq -2$$

So the simplified expression is

$$\frac{x+4}{3x+6} \text{ with } x \neq -4, -2$$

However, the remaining factor of  $3x+6$  in the denominator still shows us that  $x \neq -2$ , so we don't need to exclude that value in our answer.

$$\frac{x+4}{3x+6} \text{ with } x \neq -4$$

■ 5. Simplify the expression.

$$\frac{16x^2 + 16x + 4}{x^2 + 18x + 81} \cdot \frac{x^2 - 81}{16x^2 - 4}$$

*Solution:*

Factor and cancel whatever you can, then simplify.



$$\frac{(4x+2)(4x+2)}{(x+9)(x+9)} \cdot \frac{(x-9)(x+9)}{(4x-2)(4x+2)}$$

$$\frac{(4x+2)(x-9)}{(x+9)(4x-2)}$$

We cancelled factors of  $4x+2$  and  $x+9$  which means

$$4x+2 \neq 0, \text{ or } x \neq -1/2$$

$$x+9 \neq 0, \text{ or } x \neq -9$$

So the simplified expression is

$$\frac{(4x+2)(x-9)}{(x+9)(4x-2)} \text{ with } x \neq -9, -\frac{1}{2}$$

However, the remaining factor of  $x+9$  in the denominator still shows us that  $x \neq -9$ , so we don't need to exclude that value in our answer.

$$\frac{(4x+2)(x-9)}{(x+9)(4x-2)} \text{ with } x \neq -\frac{1}{2}$$

■ 6. Simplify the expression.

$$\frac{x^2+5x-14}{x^2+2x-3} \cdot \frac{x^2+4x-5}{x^2+9x+14}$$

*Solution:*



Factor and cancel whatever you can, then simplify.

$$\frac{(x-2)(x+7)}{(x+3)(x-1)} \cdot \frac{(x-1)(x+5)}{(x+7)(x+2)}$$

$$\frac{(x-2)(x+5)}{(x+3)(x+2)}$$

We cancelled factors of  $x+7$  and  $x-1$  which means

$$x+7 \neq 0, \text{ or } x \neq -7$$

$$x-1 \neq 0, \text{ or } x \neq 1$$

So the simplified expression is

$$\frac{(x-2)(x+5)}{(x+3)(x+2)} \text{ with } x \neq -7, 1$$

■ 7. Simplify the expression.

$$\frac{x^2 + 3x + 2}{x^2 - 7x + 12} \cdot \frac{x^2 + 2x - 15}{x^2 - 4x - 12}$$

*Solution:*

Factor and cancel whatever you can, then simplify.

$$\frac{(x+1)(x+2)}{(x-3)(x-4)} \cdot \frac{(x-3)(x+5)}{(x+2)(x-6)}$$



$$\frac{(x+1)(x+5)}{(x-4)(x-6)}$$

We cancelled factors of  $x+2$  and  $x-3$  which means

$$x+2 \neq 0, \text{ or } x \neq -2$$

$$x-3 \neq 0, \text{ or } x \neq 3$$

So the simplified expression is

$$\frac{(x+1)(x+5)}{(x-4)(x-6)} \text{ with } x \neq -2, 3$$

■ 8. Simplify the expression.

$$\frac{x^2 + x - 56}{x^2 - x - 90} \cdot \frac{x^2 + 8x - 9}{x^2 - 5x - 14}$$

*Solution:*

Factor and cancel whatever you can, then simplify.

$$\frac{(x-7)(x+8)}{(x+9)(x-10)} \cdot \frac{(x+9)(x-1)}{(x-7)(x+2)}$$

$$\frac{(x+8)(x-1)}{(x-10)(x+2)}$$

We cancelled factors of  $x-7$  and  $x+9$  which means



$$x - 7 \neq 0, \text{ or } x \neq 7$$

$$x + 9 \neq 0, \text{ or } x \neq -9$$

So the simplified expression is

$$\frac{(x+8)(x-1)}{(x-10)(x+2)} \text{ with } x \neq -9, 7$$

■ 9. Simplify the expression.

$$\frac{2x^2 - 13x - 24}{3x^2 - x - 4} \cdot \frac{3x^2 - 7x + 4}{x^2 - 6x - 16}$$

*Solution:*

Factor and cancel whatever you can, then simplify.

$$\frac{(2x+3)(x-8)}{(3x-4)(x+1)} \cdot \frac{(3x-4)(x-1)}{(x+2)(x-8)}$$

$$\frac{(2x+3)(x-1)}{(x+1)(x+2)}$$

We cancelled  $x - 8$  and  $3x - 4$  which means

$$x - 8 \neq 0, \text{ or } x \neq 8$$

$$3x - 4 \neq 0, \text{ or } x \neq 4/3$$

So the simplified expression is





$$\frac{(2x + 3)(x - 1)}{(x + 1)(x + 2)} \text{ with } x \neq \frac{4}{3}, 8$$



## DIVIDING RATIONAL FUNCTIONS

- 1. Simplify the expression.

$$\frac{2x + 16}{9x^2 + 27x} \div \frac{3x + 24}{x + 3}$$

*Solution:*

Factor the numerator and denominator of both fractions as completely as possible.

$$\frac{2(x + 8)}{9x(x + 3)} \div \frac{3(x + 8)}{x + 3}$$

Consider restrictions. The denominator of the dividend gives  $x \neq -3, 0$ , the denominator of the divisor gives  $x \neq -3$ , and the numerator of the divisor gives  $x \neq -8$ . So the set of restrictions we should keep in mind until the end of the problem is  $x \neq -8, -3, 0$ .

Now turn the division problem into a multiplication problem and cancel common factors.

$$\frac{2(x + 8)}{9x(x + 3)} \cdot \frac{x + 3}{3(x + 8)}$$

$$\frac{2}{9x} \cdot \frac{1}{3}$$



$$\frac{2}{27x}$$

This resulting quotient shows that  $x \neq 0$ , so we can eliminate that from our list of restrictions. Then the final answer is

$$\frac{2}{27x} \text{ with } x \neq -8, -3$$

■ 2. Simplify the expression.

$$\frac{6x + 15}{12x^2 + 24x} \div \frac{2x + 5}{5x + 10}$$

*Solution:*

Factor the numerator and denominator of both fractions as completely as possible.

$$\frac{3(2x + 5)}{12x(x + 2)} \div \frac{2x + 5}{5(x + 2)}$$

Consider restrictions. The denominator of the dividend gives  $x \neq -2, 0$ , the denominator of the divisor gives  $x \neq -2$ , and the numerator of the divisor gives  $x \neq -5/2$ . So the set of restrictions we should keep in mind until the end of the problem is  $x \neq -5/2, -2, 0$ .

Now turn the division problem into a multiplication problem and cancel common factors.



$$\frac{3(2x+5)}{12x(x+2)} \cdot \frac{5(x+2)}{2x+5}$$

$$\frac{3}{12x} \cdot \frac{5}{1}$$

$$\frac{15}{12x}$$

$$\frac{5}{4x}$$

This resulting quotient shows that  $x \neq 0$ , so we can eliminate that from our list of restrictions. Then the final answer is

$$\frac{5}{4x} \text{ with } x \neq -5/2, -2$$

### ■ 3. Simplify the expression.

$$\frac{3x^3 - 3x^2 - 6x}{2x^2 - 14x + 24} \div \frac{3x^2 + 21x}{x^2 - 8x + 15}$$

*Solution:*

Factor the numerator and denominator of both fractions as completely as possible.

$$\frac{3x(x^2 - x - 2)}{2(x^2 - 7x + 12)} \div \frac{3x(x+7)}{(x-3)(x-5)}$$



$$\frac{3x(x+1)(x-2)}{2(x-3)(x-4)} \div \frac{3x(x+7)}{(x-3)(x-5)}$$

Consider restrictions. The denominator of the dividend gives  $x \neq 3, 4$ , the denominator of the divisor gives  $x \neq 3, 5$ , and the numerator of the divisor gives  $x \neq -7, 0$ . So the set of restrictions we should keep in mind until the end of the problem is  $x \neq -7, 0, 3, 4, 5$ .

Now turn the division problem into a multiplication problem and cancel common factors.

$$\frac{3x(x+1)(x-2)}{2(x-3)(x-4)} \cdot \frac{(x-3)(x-5)}{3x(x+7)}$$

$$\frac{(x+1)(x-2)(x-5)}{2(x-4)(x+7)}$$

This resulting quotient shows that  $x \neq -7, 4$ , so we can eliminate those from our list of restrictions. Then the final answer is

$$\frac{(x+1)(x-2)(x-5)}{2(x-4)(x+7)} \text{ with } x \neq 0, 3, 5$$

#### ■ 4. Simplify the expression.

$$\frac{2x^2 - 13x - 7}{12x + 6} \div \frac{3x - 2}{3x^2 - 17x + 10}$$

*Solution:*



Factor the numerator and denominator of both fractions as completely as possible.

$$\frac{(2x+1)(x-7)}{6(2x+1)} \div \frac{3x-2}{(3x-2)(x-5)}$$

Consider restrictions. The denominator of the dividend gives  $x \neq -1/2$ , the denominator of the divisor gives  $x \neq 2/3, 5$ , and the numerator of the divisor gives  $x \neq 2/3$ . So the set of restrictions we should keep in mind until the end of the problem is  $x \neq -1/2, 2/3, 5$ .

Now turn the division problem into a multiplication problem and cancel common factors.

$$\frac{(2x+1)(x-7)}{6(2x+1)} \cdot \frac{(3x-2)(x-5)}{3x-2}$$

$$\frac{(x-7)(x-5)}{6}$$

This resulting quotient doesn't show any restrictions, so we need to keep our entire list of them. Then the final answer is

$$\frac{(x-7)(x-5)}{6} \text{ with } x \neq -1/2, 2/3, 5$$

## ■ 5. Simplify the expression.

$$\frac{4x^2 + 13x + 10}{3x + 6} \div \frac{3x - 1}{3x^2 - 13x + 4}$$



*Solution:*

Factor the numerator and denominator of both fractions as completely as possible.

$$\frac{(4x + 5)(x + 2)}{3(x + 2)} \div \frac{3x - 1}{(3x - 1)(x - 4)}$$

Consider restrictions. The denominator of the dividend gives  $x \neq -2$ , the denominator of the divisor gives  $x \neq 1/3, 4$ , and the numerator of the divisor gives  $x \neq 1/3$ . So the set of restrictions we should keep in mind until the end of the problem is  $x \neq -2, 1/3, 4$ .

Now turn the division problem into a multiplication problem and cancel common factors.

$$\frac{(4x + 5)(x + 2)}{3(x + 2)} \cdot \frac{(3x - 1)(x - 4)}{3x - 1}$$

$$\frac{(4x + 5)(x - 4)}{3}$$

This resulting quotient doesn't show any restrictions, so we need to keep our entire list of them. Then the final answer is

$$\frac{(4x + 5)(x - 4)}{3} \text{ with } x \neq -2, 1/3, 4$$

■ 6. Simplify the expression.



$$\frac{x^2 - 9}{x^2 + x - 2} \div \frac{x^2 + 4x + 3}{x^2 - 16}$$

*Solution:*

Factor the numerator and denominator of both fractions as completely as possible.

$$\frac{(x - 3)(x + 3)}{(x + 2)(x - 1)} \div \frac{(x + 3)(x + 1)}{(x - 4)(x + 4)}$$

Consider restrictions. The denominator of the dividend gives  $x \neq -2, 1$ , the denominator of the divisor gives  $x \neq -4, 4$ , and the numerator of the divisor gives  $x \neq -3, -1$ . So the set of restrictions we should keep in mind until the end of the problem is  $x \neq -4, -3, -2, -1, 1, 4$ .

Now turn the division problem into a multiplication problem and cancel common factors.

$$\frac{(x - 3)(x + 3)}{(x + 2)(x - 1)} \cdot \frac{(x - 4)(x + 4)}{(x + 3)(x + 1)}$$

$$\frac{(x - 3)(x - 4)(x + 4)}{(x + 2)(x - 1)(x + 1)}$$

This resulting quotient shows that  $x \neq -2, -1, 1$ , so we can eliminate those from our list of restrictions. Then the final answer is

$$\frac{(x - 3)(x - 4)(x + 4)}{(x + 2)(x - 1)(x + 1)} \text{ with } x \neq -4, -3, 4$$





■ 7. Simplify the expression.

$$\frac{4x^2 - 9}{x^2 + 12x + 36} \div \frac{4x^2 - 12x + 9}{x^2 + 7x + 6}$$

*Solution:*

Factor the numerator and denominator of both fractions as completely as possible.

$$\frac{(2x - 3)(2x + 3)}{(x + 6)(x + 6)} \div \frac{(2x - 3)(2x - 3)}{(x + 6)(x + 1)}$$

Consider restrictions. The denominator of the dividend gives  $x \neq -6$ , the denominator of the divisor gives  $x \neq -6, -1$ , and the numerator of the divisor gives  $x \neq 3/2$ . So the set of restrictions we should keep in mind until the end of the problem is  $x \neq -6, -1, 3/2$ .

Now turn the division problem into a multiplication problem and cancel common factors.

$$\frac{(2x - 3)(2x + 3)}{(x + 6)(x + 6)} \cdot \frac{(x + 6)(x + 1)}{(2x - 3)(2x - 3)}$$

$$\frac{(2x + 3)(x + 1)}{(x + 6)(2x - 3)}$$

This resulting quotient shows that  $x \neq -6, 3/2$ , so we can eliminate those from our list of restrictions. Then the final answer is



$$\frac{(2x+3)(x+1)}{(x+6)(2x-3)} \text{ with } x \neq -1$$

■ 8. Simplify the expression.

$$\frac{15x^2 + 75x + 90}{5x^2 + 50x + 125} \div \frac{x^2 - 3x + 2}{x^2 - 25}$$

*Solution:*

Factor the numerator and denominator of both fractions as completely as possible.

$$\frac{15(x^2 + 5x + 6)}{5(x^2 + 10x + 25)} \div \frac{(x-1)(x-2)}{(x-5)(x+5)}$$

$$\frac{15(x+3)(x+2)}{5(x+5)(x+5)} \div \frac{(x-1)(x-2)}{(x-5)(x+5)}$$

Consider restrictions. The denominator of the dividend gives  $x \neq -5$ , the denominator of the divisor gives  $x \neq -5, 5$ , and the numerator of the divisor gives  $x \neq 1, 2$ . So the set of restrictions we should keep in mind until the end of the problem is  $x \neq -5, 1, 2, 5$ .

Now turn the division problem into a multiplication problem and cancel common factors.

$$\frac{15(x+3)(x+2)}{5(x+5)(x+5)} \cdot \frac{(x-5)(x+5)}{(x-1)(x-2)}$$



$$\frac{3(x+3)(x+2)(x-5)}{(x+5)(x-1)(x-2)}$$

This resulting quotient shows that  $x \neq -5, 1, 2$ , so we can eliminate those from our list of restrictions. Then the final answer is

$$\frac{3(x+3)(x+2)(x-5)}{(x+5)(x-1)(x-2)} \text{ with } x \neq 5$$



