Composite functions, domain

In this lesson we'll look at how to find the domain of a composite function.

The domain of a function is the set of x-values where the function is defined. To determine the domain of a composite function, you need to consider the domains of the original functions.

Remember that the composite function of $f(x)$ and $g(x)$ is written as $f \circ g$ or $f(g(x))$, and is found by plugging $g(x)$ into $f(x)$.

The domain of a composite must exclude all values of x that aren't in the domain of the "inside" function (g), and all values of x such that $g(x)$ isn't in the domain of the "outside" function (f). In other words, given the composite $f(g(x))$, the domain will exclude all values of x where $g(x)$ is undefined, and all values of x where $g(x)$ is defined but $f(g(x))$ is undefined.

Let's look at a few examples.

Example

What is the domain of $f \circ g$?

$$f(x) = x^2 - 3$$

$$g(x) = \sqrt{x+9}$$

First, find the domain of $g(x)$. The expression $\sqrt{x+9}$ is undefined where $x+9$ is negative. For example, if $x = -10$, then $x+9$ is -1. In general, if x is any number less than -9, then $x+9$ is negative. However, -9 itself is okay, because $\sqrt{-9+9}=0$. Therefore, the domain of $g(x)$ is all real numbers x such that $x \geq -9$.

The algebraic expression for the composite function is

$$f(g(x)) = \left(\sqrt{x+9}\right)^2 - 3$$

$$f(g(x)) = (x+9) - 3$$

$$f(g(x)) = x + 6$$

For this simple binomial $(x+6)$, no real numbers are excluded, so its domain is all real numbers. But because the domain of $g(x)$ excludes all $x < -9$, those values of x also have to be excluded from the domain of the composite function $f(g(x))$.

That means the domain of $f(g(x))$ is $x \geq -9$.

Let's try another example.

Example

What is the domain of $f \circ g$?

$$f(x) = \frac{2}{2x+4}$$

$$g(x) = \frac{3}{x-5}$$

First, find the domain of $g(x)$. The expression $3/(x-5)$ is undefined if the denominator is 0. That means $x = 5$ isn't in the domain of $g(x)$. Therefore, the domain of $g(x)$ is all real numbers x such that $x \neq 5$.

The algebraic expression for the composite function is

$$f(g(x)) = \frac{2}{2\left(\frac{3}{x-5}\right) + 4}$$

$$f(g(x)) = \frac{2}{\left(\frac{6}{x-5}\right) + 4\left(\frac{x-5}{x-5}\right)}$$

$$f(g(x)) = \frac{2}{\left(\frac{6+4x-20}{x-5}\right)}$$

$$f(g(x)) = \frac{2}{\left(\frac{4x-14}{x-5}\right)}$$

$$f(g(x)) = 2\left(\frac{x-5}{4x-14}\right)$$

$$f(g(x)) = \frac{2(x-5)}{2(2x-7)}$$

$$f(g(x)) = \frac{x-5}{2x-7}$$

For this rational function ($(x-5)/(2x-7)$), any numbers that make the denominator 0 are excluded from the domain.

$$2x - 7 = 0 \quad \to \quad 2x = 7 \quad \to \quad x = \frac{7}{2}$$

Putting both exclusions together, the domain of the composite is all real numbers except $7/2$ and 5, so

$$f(g(x)) = \frac{x-5}{2x-7}, \ x \neq \frac{7}{2}, \ 5$$