**Topic**: Change of base

Question: Write the expression in terms of base-10 logs.

$$\log_3 12$$

# **Answer choices:**

$$A \qquad \log 12 - \log 3$$

B 
$$3^x = 12$$

$$\mathsf{D} \qquad \frac{\log 12}{\log 3}$$

#### Solution: D

To rewrite the given expression in terms of base-10 logs, use the change of base formula.

$$\log_a b = \frac{\log_c b}{\log_c a}$$

Starting with  $log_3 12$ , we get

$$\frac{\log_{10}12}{\log_{10}3}$$

The common logarithm function  $\log_{10}$  can be written as just  $\log$  without the base.

$$\frac{\log 12}{\log 3}$$



**Topic**: Change of base

**Question**: Find the exact value of the expression.

$$\frac{\log 729}{\log 9}$$

## **Answer choices:**

**A** 2

B 3

**C** 8

D 16



# Solution: B

Use the change of base formula,

$$\log_a b = \frac{\log_c b}{\log_c a}$$

to rewrite the given expression as one log.

$$\frac{\log 729}{\log 9}$$

Let  $x = \log_9 729$ , and use the general log rule to convert this to exponential form.

$$9^x = 729$$

$$9^x = 9^3$$

$$x = 3$$

**Topic**: Change of base

**Question**: Find the exact value of the expression.

$$\frac{\log 4}{\log 16} - \frac{\log 2}{\log 64}$$

## **Answer choices**:

$$A = \frac{1}{3}$$

$$\mathsf{B} \qquad \frac{1}{4}$$

$$c \frac{1}{6}$$

D 
$$\frac{1}{8}$$



#### Solution: A

Use the change of base formula,

$$\log_a b = \frac{\log_c b}{\log_c a}$$

to rewrite both fractions.

$$\frac{\log 4}{\log 16} - \frac{\log 2}{\log 64}$$

$$\log_{16} 4 - \log_{64} 2$$

Next, let  $x = \log_{16} 4$ , and use the general log rule to convert this to exponential form.

$$16^x = 4$$

$$(4^2)^x = 4^1$$

$$4^{2x} = 4^1$$

$$2x = 1$$

$$x = \frac{1}{2}$$

Now let  $x = \log_{64} 2$ , and use the general log rule to convert this to exponential form.

$$64^x = 2$$

$$(2^6)^x = 2^1$$



$$2^{6x} = 2^1$$

$$6x = 1$$

$$x = \frac{1}{6}$$

Finally, substitute the values of  $\log_{64} 2$ , and use the general log rule to convert this to exponential form.

$$\log_{16} 4 - \log_{64} 2$$

$$\frac{1}{2} - \frac{1}{6}$$

$$\frac{1}{2}\left(\frac{3}{3}\right) - \frac{1}{6}$$

$$\frac{3}{6} - \frac{1}{6}$$

$$\frac{2}{6}$$

$$\frac{1}{3}$$