Topic: Rationalizing complex denominators

Question: Use the conjugate method to simplify the expression.

$$\frac{\sqrt{-3}\sqrt{-3}-2i^3}{i+3}$$

Answer choices:

A $\frac{7}{10}+\frac{9i}{10}$

B $-\frac{7}{10}+\frac{9i}{10}$

C $\frac{7}{10}-\frac{9i}{10}$

D $-\frac{7}{10}-\frac{9i}{10}$

Solution: B

Remember that

$$i = \sqrt{-1}$$

and

$$i^2 = -1$$

First, we'll rewrite the numerator using $\sqrt{-3} = \sqrt{3(-1)} = \sqrt{3}\sqrt{-1} = \sqrt{3}(i)$ and $i^2 = -1$.

$$\frac{\sqrt{-3}\sqrt{-3} - 2i^3}{i+3}$$

$$\frac{\sqrt{3}(i) \cdot \sqrt{3}i - 2i^3}{i+3}$$

$$\frac{(\sqrt{3} \cdot \sqrt{3})(i^2) - 2i^3}{i+3}$$

$$\frac{3(-1) - 2i^3}{i+3}$$

$$\frac{-3 - 2i^3}{i+3}$$

We can use the conjugate method to get the imaginary number i out of the denominator.

$$\frac{-3 - 2i^3}{i+3} \cdot \frac{3-i}{3-i}$$

$$\frac{(-3-2i^3)(3-i)}{(i+3)(3-i)}$$

Now that we have a binomial multiplication problem, we need to make sure that (in the numerator and denominator separately) we multiply the first terms, outer terms, inner terms, and last terms.

$$\frac{-9+3i-6i^3+2i^4}{3i-i^2+9-3i}$$

$$\frac{2i^4-6i^3+3i-9}{-i^2+9}$$

Replacing i^2 with -1, we get

$$\frac{2(-1)(-1)-6(-1)i+3i-9}{-(-1)+9}$$

$$\frac{2+6i+3i-9}{10}$$

$$\frac{9i-7}{10}$$

Split the fraction.

$$\frac{9i}{10}-\frac{7}{10}$$

$$-\frac{7}{10}+\frac{9i}{10}$$

Topic: Rationalizing complex denominators

Question: Use the conjugate method to simplify the expression.

$$\frac{3i - i^2}{2i^2 - i^3}$$

Answer choices:

A $\frac{1}{5} - \frac{7}{5}i$

B $-\frac{2}{3} - \frac{10}{3}i$

C $\frac{2}{3} - \frac{10}{3}i$

D $-10 + 3i$

Solution: A

Remember that

$$i = \sqrt{-1}$$

and

$$i^2 = -1$$

First, we'll rewrite the expression using $i^3 = i^2 i$ and $i^2 = -1$.

$$\frac{3i - i^2}{2i^2 - i^3}$$

$$\frac{3i - i^2}{2i^2 - i^2 i}$$

$$\frac{3i - (-1)}{2(-1) - (-1)i}$$

$$\frac{3i + 1}{-2 + i}$$

Now we can use the conjugate method to get the imaginary number i out of the denominator.

$$\frac{3i + 1}{-2 + i} \cdot \frac{-2 - i}{-2 - i}$$

$$\frac{(3i + 1)(-2 - i)}{(-2 + i)(-2 - i)}$$

Now that we have a binomial multiplication problem, we need to make sure that (in the numerator and denominator separately) we multiply the first terms, outer terms, inner terms, and last terms.

$$\frac{-6i-3i^2-2-i}{4+2i-2i-i^2}$$

$$\frac{-3i^2-7i-2}{4-i^2}$$

Replacing i^2 with -1, we get

$$\frac{-3(-1)-7i-2}{4-(-1)}$$

$$\frac{3-7i-2}{4+1}$$

$$\frac{1-7i}{5}$$

$$\frac{1}{5}-\frac{7i}{5}$$

$$\frac{1}{5}-\frac{7}{5}i$$

Topic: Rationalizing complex denominators

Question: Use the conjugate method to simplify the expression.

$$\frac{4+4i}{5-3i}$$

Answer choices:

A $\quad \dfrac{4+16i}{17}$

B $\quad \dfrac{1+4i}{2}$

C $\quad \dfrac{16+16i}{17}$

D $\quad \dfrac{1+4i}{4}$

Solution: A

The complex conjugate of the denominator of

$$\frac{4+4i}{5-3i}$$

is $5+3i$, so we multiply the numerator and denominator by that and simplify.

$$\frac{4+4i}{5-3i}\cdot\frac{5+3i}{5+3i}$$

$$\frac{20+12i+20i+12i^2}{25+15i-15i-9i^2}$$

$$\frac{20+32i-12}{25+9}$$

$$\frac{8+32i}{34}$$

$$\frac{4+16i}{17}$$