Greatest common factor of trinomials

Factoring is "un-distributing," which means that we do the opposite of distributing, and take out (or "factor out") the same factor (number or other expression) from each term of the trinomial (and divide each term by that factor to get "what's left" once it's taken out). Remember that a trinomial is simply a polynomial with three terms.

The key to factoring is that every term in the trinomial needs to share the factor being taken out. Any factor that's shared by all the terms is called a **common factor**, and the factor that consists of everything which is shared by all of them is known as the **greatest common factor**.

When you're first starting to factor, it can be helpful to write out all the factors of each term. For example, you'd write $2x^3y + 4x^2y^2 + 8xy$ as

$$2 \cdot x \cdot x \cdot x \cdot y + 2 \cdot 2 \cdot x \cdot x \cdot y \cdot y + 2 \cdot 2 \cdot 2 \cdot x \cdot y$$

so that you can clearly see what factors are shared. In this case the greatest common factor is $2xy$. (Other common factors - factors that are common to all the terms of this trinomial - are 2, x, y, $2x$, and $2y$. Of course, 1 is also a common factor, but factoring out a 1 doesn't change anything, so we usually don't write 1 as a factor.) This does get tedious, but after a while you'll get used to identifying the greatest common factor without having to write it out.

Example

Factor out the greatest common factor.

$$3x + 6xy - 15ax$$

Write out all the factors of each term.

$$3 \cdot x + 2 \cdot 3 \cdot x \cdot y - 3 \cdot 5 \cdot a \cdot x$$

We can now see that each term has a 3 and an x as a factor, and there is no other factor which is shared by all the terms, so $3x$ is the greatest common factor. When we factor out the $3x$, we have to divide each term by $3x$.

$$3x(1 + 2y - 5a)$$

Notice that the first term inside the parentheses is 1, which is because

$$\frac{3x}{3x} = 1$$

Let's try another example of finding the greatest common factor of a trinomial.

Example

Factor out the greatest common factor.

$$4x^2y - 6x^4y^2 + 8x^3y^4$$

Write out all the factors of each term.

$$2 \cdot 2 \cdot x \cdot x \cdot y - 2 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y + 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y$$

The only factors that are shared by all three terms are a 2, an $x \cdot x$, and a y, so the greatest common factor is $2x^2y$. When we factor out the $2x^2y$, we have to divide each term by $2x^2y$.

$$2x^2y(2 - 3x^2y + 4xy^3)$$