Algebra 1 Workbook Solutions

Polynomials

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ADDING AND SUBTRACTING POLYNOMIALS

1. What stays the same when adding and subtracting like terms?

Solution:

The exponent stays the same when adding and subtracting like terms.

2. Simplify the expression.

$$(2x^3 - 5x^2 + x - 3) - (x^2 - 2x + 7)$$

Solution:

Simplify the expression by combining like terms.

$$(2x^3 - 5x^2 + x - 3) - (x^2 - 2x + 7)$$

$$2x^3 - 5x^2 + x - 3 - x^2 + 2x - 7$$

$$2x^3 - 6x^2 + 3x - 10$$

3. What went wrong in the following set of steps?

$$6x^3 + 7 + x^2$$

$$7x^3 + 7$$

Solution:

The terms $6x^3$ and x^2 were added together but they are not like terms. The exponents are not the same, so they cannot be added together.

4. What is the coefficient in the following expression?

$$5x^8$$

Solution:

The coefficient is 5.

5. Simplify the expression.

$$(10a^2b + 3ab^2 - ab) + (2ab^2 - a^2b + ab)$$

Solution:

Simplifying the expression by combining like terms.

$$(10a^2b + 3ab^2 - ab) + (2ab^2 - a^2b + ab)$$

$$10a^2b + 3ab^2 - ab + 2ab^2 - a^2b + ab$$

$$9a^2b + 5ab^2$$

6. What is the exponent in the following expression?

$$3z^8$$

Solution:

The exponent is 8.

7. Simplify the expression.

$$(x^4 - 5y^3 + z - xy) - (2y^4 + 6xy - z + x^4)$$

Solution:

Simplifying the expression by combining like terms.

$$(x^4 - 5y^3 + z - xy) - (2y^4 + 6xy - z + x^4)$$

$$x^4 - 5y^3 + z - xy - 2y^4 - 6xy + z - x^4$$

$$-5y^3 + 2z - 7xy - 2y^4$$

8. What is the variable in the following expression?

$$-y^4$$

Solution:

The variable is y.

9. What went wrong in the following set of steps?

$$9 - x^3 + 3 + 4x^3$$

$$12 + 3x^6$$

Solution:

The terms $-x^3$ and $4x^3$ were added together. That's correct since they are like terms, but when the terms were added, the exponents were added as well. The term should be $3x^3$, not $3x^6$.

MULTIPLYING POLYNOMIALS

1. Expand the expression.

$$(2x - y)^2$$

Solution:

The expression is expanded and simplified as

$$(2x - y)(2x - y)$$

$$4x^2 - 2xy - 2xy + y^2$$

$$4x^2 - 4xy + y^2$$

2. What does FOIL stand for?

Solution:

FOIL stands for **F**irst, **O**uter, **I**nner, **L**ast.

3. What went wrong in the following set of steps?

$$(a - 2)^2$$

$$a^2 - 4$$

Solution:

The expression was not interpreted correctly. The exponent was just distributed to both terms directly, but the expression should have been expanded as

$$(a-2)^2$$

$$(a-2)(a-2)$$

From there, you would then need to FOIL in order to expand and simplify.

4. Expand the expression.

$$(3x+2y)(3x-2y)$$

Solution:

The expression is expanded and simplified as

$$(3x+2y)(3x-2y)$$

$$9x^2 - 6xy + 6xy - 4y^2$$

$$9x^2 - 4y^2$$

5. Fill in the blank.

$$(3-a)(5+a) = 15 + \_\_\_ - a^2$$

Solution:

If we FOIL the product on the left, we get

$$(3-a)(5+a)$$

$$15 + 3a - 5a - a^2$$

$$15 - 2a - a^2$$

The value that goes in the blank is therefore $-2a$.

6. Expand the expression.

$$(x^2-3)(2-x)$$

Solution:

The expression is expanded and simplified as

$$(x^2-3)(2-x)$$

$$2x^2 - x^3 - 6 + 3x$$

$$-x^3 + 2x^2 + 3x - 6$$

7. What went wrong in the following set of steps?

$$(x - y)(x + y)$$

$$x^2 - 2xy - y^2$$

Solution:

When using FOIL to expand and simplify the expression, the like terms were not combined correctly. It should be

$$(x - y)(x + y)$$

$$x^2 + xy - xy - y^2$$

$$x^2 - y^2$$

DIVIDING POLYNOMIALS

1. In words, what is the first question you should ask when solving the problem using long division?

$$(2x^2 + 4x - 4) \div (x - 1)$$

Solution:

"What do I multiply by x in order to get $2x^2$?"

2. Simplify the expression using polynomial long division.

$$(3x^3 - x^2 + 5) \div (x + 2)$$

Solution:

$$
\begin{array}{r|l}
 & 3x^2-7x+14 \\
\hline
x+2 & 3x^3-x^2 \quad +5 \\
 & -3x^3-6x^2 \\
\hline
 & -7x^2 \\
 & 7x^2+14x \\
\hline
 & 14x+5 \\
 & -14x-28 \\
\hline
 & -23
\end{array}
$$

Therefore, the solution is

$$3x^2 - 7x + 14 - \frac{23}{x+2}$$

3. What went wrong in setting up the long division problem?

$$(5x^4 - 3x^2 + x - 2) \div (x^2 + 1)$$

$$5x^4-3x^2+x-2 \;\big|\; x^2+1$$

Solution:

The dividend and divisor were placed incorrectly. It should be

$$x^2+1 \overline{)\,5x^4-3x^2+x-2}$$

4. Given the following long division, write the answer as

$$\text{quotient} + \frac{\text{remainder}}{\text{divisor}}$$

$$\begin{array}{r} 3x-1 \\ x^2-3 \overline{)\,3x^3-x^2+x-5} \\ \underline{-3x^3 \quad +9x} \\ -x^2+10x-5 \\ \underline{x^2 \quad -3} \\ 10x-8 \end{array}$$

Solution:

The answer is written as

$$3x-1+\frac{10x-8}{x^2-3}$$

5. Use long division to simplify the expression.

$$(2x^5 - 3x^3 + x^2 + 4x - 1) \div (x^2 + 2)$$

Solution:

The long division would be

$$\begin{array}{r} 2x^3 \quad -7x+1 \\ x^2+2 \overline{)\,2x^5-3x^3+x^2+4x-1} \\ \underline{-2x^5-4x^3} \\ -7x^3+x^2+4x \\ \underline{7x^3 \quad +14x} \\ x^2+18x-1 \\ \underline{-x^2 \quad -2} \\ 18x-3 \end{array}$$

Therefore, the solution is

$$2x^3 - 7x + 1 + \frac{18x - 3}{x^2 + 2}$$

6. How would you rewrite the expression before starting the long division process?

$$(6x^3 - x + 7) \div (x + 1)$$

Solution:

You would rewrite the expression and set up the long division as

$$x+1 \overline{)\, 6x^3+0x^2-x+7}$$

7. Set up but do not solve the following division problem.

$$\frac{x^5 - x^3 + 4x^2 - x + 6}{2x^3 - 5}$$

Solution:

The above problem is set up as

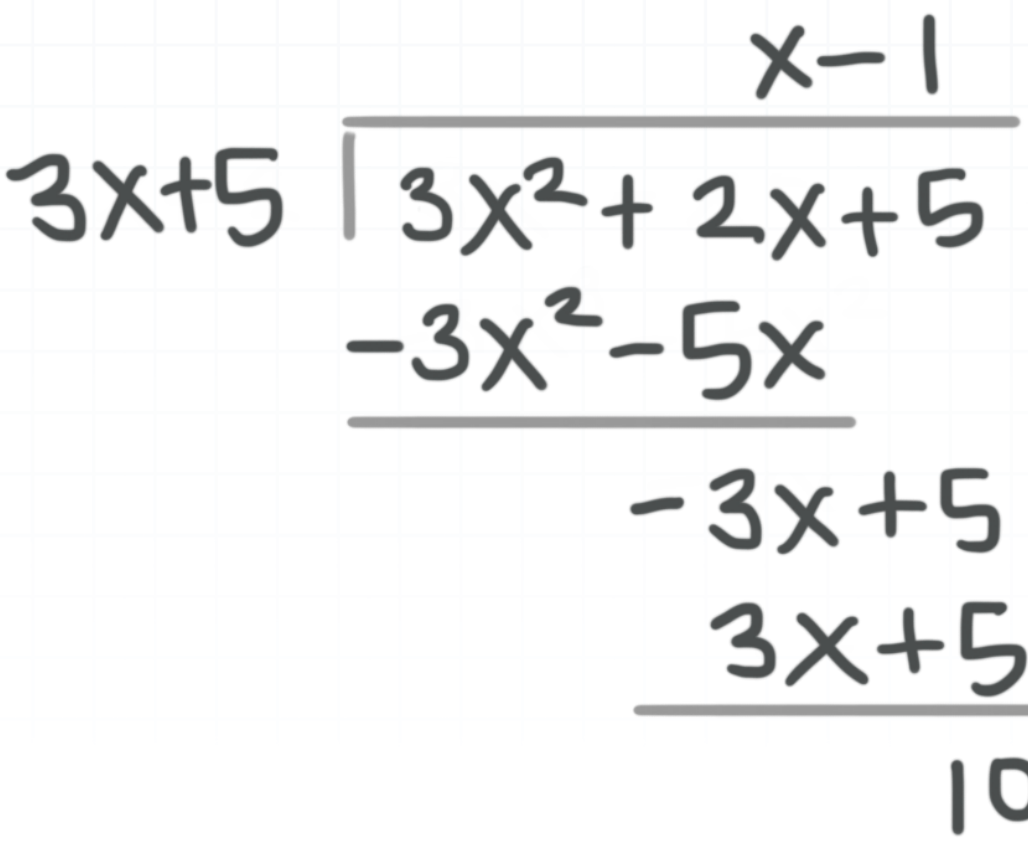
$$2x^3-5 \overline{)\, x^5+0x^4-x^3+4x^2-x+6}$$

8. Simplify the expression using polynomial long division.

$$(3x^2 + 2x + 5) \div (3x + 5)$$

Solution:

The long division is set up as



Therefore, the solution is

$$x - 1 + \frac{10}{3x + 5}$$

MULTIPLYING MULTIVARIABLE POLYNOMIALS

1. Why can we not add the following two terms?

$$2x^3y + x^3y^2$$

Solution:

We can't add these terms because they aren't like terms. The exponents on both x and y do not match since there is a y in the first term and a y^2 in the second term.

2. Simplify the expression.

$$(a - 3y)(2a + y)$$

Solution:

The expression is simplified as

$$2a^2 + ay - 6ay - 3y^2$$

$$2a^2 - 5ay - 3y^2$$

3. What went wrong in the following set of steps?

$$(x+3b)(-2x-b)$$

$$-2x^2-bx-6bx+3b^3$$

Solution:

The negative sign was not used when multiplying the last terms $+3b$ and $-b$, in order to give the last term of the product, $-3b^2$.

4. Simplify the expression.

$$(x-2y)(x+y)+(3x-y)(4x+4y)$$

Solution:

The expression is simplified as

$$(x-2y)(x+y)+(3x-y)(4x+4y)$$

$$(x^2+xy-2xy-2y^2)+(12x^2+12xy-4xy-4y^2)$$

$$13x^2+7xy-6y^2$$

5. Fill in the blanks with the correct terms.

$$(5a - b)(7b - 3a)$$

$$35ab - 15a^2 + \_\_\_ + 3ab$$

$$\_\_\_ - 15a^2 + \_\_\_$$

Solution:

Expanding and simplifying the expression gives

$$(5a - b)(7b - 3a)$$

$$35ab - 15a^2 - 7b^2 + 3ab$$

$$38ab - 15a^2 - 7b^2$$

The first blank should be filled with $-7b^2$, the second blank with $38ab$, and the last blank with $-7b^2$.

6. What does FOIL stand for when used in multiplying multivariable polynomials?

Solution:

FOIL stands for **F**irst, **O**uter, **I**nner, **L**ast.

7. Fill in the following chart for the multiplication of the following two expressions.

$$(2x - 3y)(x^2 + y)$$

	2x	-3y
x^2		
y		

Solution:

The chart is filled in as

	2x	-3y
x^2	$2x^3$	$-3x^2y$
y	$2xy$	$-3y^2$

8. What went wrong in the following set of steps?

$$(a^2 + 6b)(-a - b^2)$$

$$-a^3 - a^2b^2 - 6ab - b^3$$

$$-a^3 - 7ab - b^3$$

Solution:

In the first step, the terms $6b$ and $-b^2$ were multiplied incorrectly. Their product was shown as $-b^3$, but it should have been $-6b^3$. In the second step, the terms $-a^2b^2$ and $-6ab$ were added, but they shouldn't have been added because they're not like terms.

9. Fill in the blanks of the multiplication chart with the correct terms when given the following problem.

$$(4a + 3b)(-a + 2b^2)$$

		3b
-a		-3ab

Solution:

The chart is filled in as

	4a	**3b**
-a	$-4a^2$	$-3ab$
$2b^2$	$8ab^2$	$6b^3$

10. Simplify the following expression.

$$(5ax - 3by)(a + y) - (a - y)(2ax + 4by)$$

Solution:

The expression is simplified as

$$(5ax - 3by)(a + y) - (a - y)(2ax + 4by)$$

$$(5a^2x + 5axy - 3aby - 3by^2) - (2a^2x + 4aby - 2axy - 4by^2)$$

$$5a^2x + 5axy - 3aby - 3by^2 - 2a^2x - 4aby + 2axy + 4by^2$$

$$3a^2x + 7axy - 7aby + by^2$$

11. What went wrong in this set of steps?

$$(-2x)(3y - x^2)$$

$$-6xy - 2x^3$$

Solution:

The negative sign wasn't distributed to the second term. The product should be

$$-6xy + 2x^3$$

DIVIDING MULTIVARIABLE POLYNOMIALS

1. Set up but do not solve the long division problem.

$$\frac{y^3 - 3yx^2 + x^3}{y - x}$$

Solution:

The division is set up as

$$y - x \overline{\big| \; y^3 - 3yx^2 + x^3}$$

2. Find the quotient.

$$\frac{3x^2 + 6xy - 2y^2}{x - 2y}$$

Solution:

The quotient is given by

$$
\begin{array}{r|l}
 & 3x + 12y + \frac{22y^2}{x-2y} \\
\cline{2-2}
x-2y & 3x^2+6xy-2y^2 \\
 & -(3x^2-6xy) \\
\cline{2-2}
 & \quad 12xy-2y^2 \\
 & -(12xy-24y^2) \\
\cline{2-2}
 & \quad 22y^2
\end{array}
$$

3. Given the following long division, identify the quotient, remainder, and divisor.

$$
\begin{array}{r|l}
 & x^2-xy+y^2 \\
\cline{2-2}
x+y & x^3+0x^2y+0xy^2+y^3 \\
 & -(x^3+x^2y) \\
\cline{2-2}
 & \quad -x^2y+0xy^2 \\
 & -(-x^2y-xy^2) \\
\cline{2-2}
 & \quad xy^2+y^3 \\
 & -(xy^2+y^3) \\
\cline{2-2}
 & \quad 0
\end{array}
$$

Solution:

The quotient is $x^2 - xy + y^2$, the remainder is 0, and the divisor is $x + y$.

4. How would you rewrite the expression before starting the long division process?

$$\frac{2y^3 - xy^2 + x^3}{x - y}$$

Solution:

The quotient would be rewritten as

$$x - y \overline{)\, x^3 + 0x^2y - xy^2 + 2y^3}$$

5. Find the quotient.

$$\frac{6x^2 - xy + 2y^2}{2x - y}$$

Solution:

The quotient is given by

$$
\begin{array}{r}
3x + y + \frac{3y^2}{2x-y} \\
2x-y \;\overline{)\; 6x^2 - xy + 2y^2} \\
\underline{-(6x^2 - 3xy)} \\
2xy + 2y^2 \\
\underline{-(2xy - y^2)} \\
3y^2
\end{array}
$$

6. In words, what is the first question you should ask when solving this long division problem?

$$2x+3y \;\overline{)\; 6x^4 - x^2y + xy^2 + 4y^4}$$

Solution:

"What do I need to multiply $2x$ by to get $6x^4$?"

7. What went wrong in setting up the long division?

$$\frac{7x^3 + x^2y - 2xy^2 + y^3}{x - 2y}$$

$$7x^3+x^2y-2xy^2+y^3 \overline{\big|\, x-2y}$$

Solution:

The dividend was written as the divisor. Instead, it should be

$$x-2y \overline{\big|\, 7x^3+x^2y-2xy^2+y^3}$$

8. Fill in the blanks with the correct terms.

$$(2x-y)(\quad\_\_\_\_\quad) = 6x^2-3xy$$

Solution:

The blank should be filled in with $3x$.

9. Find the quotient.

$$(y^2+xy-3x^2) \div (y+x)$$

Solution:

The quotient is

$$
\begin{array}{r}
-3x+4y-\dfrac{3y^2}{x+y} \\
x+y \;\overline{)\; -3x^2+xy+y^2} \\
\underline{-(-3x^2-3xy)} \\
4xy+y^2 \\
\underline{-(4xy+4y^2)} \\
-3y^2
\end{array}
$$


FOR THE LOVE OF MATH