Fractional exponents

In this lesson we'll work with both positive and negative fractional exponents.

When you have a fractional exponent, the numerator (of the exponent) is the power and the denominator is the root. In the expression $x^{\frac{a}{b}}$, where a and b are positive integers, a is the power and b is the root.

$$x^{\frac{a}{b}} = \sqrt[b]{x^a}$$

For instance,

$$x^{\frac{1}{3}} = \sqrt[3]{x}$$

$$x^{\frac{2}{5}} = \sqrt[5]{x^2}$$

$$x^{\frac{4}{3}} = \sqrt[3]{x^4}$$

Let's look at a quick example.

Example

Simplify the expression.

$$4^{\frac{3}{2}}$$

In the fractional exponent $3/2$, 3 is the power and 2 is the root, which means we can rewrite the expression as

$$\sqrt{4^3}$$

$$\sqrt{4 \cdot 4 \cdot 4}$$

$$\sqrt{64}$$

$$8$$

In other words, to make a problem easier to solve, you can break up the exponent by rewriting it as a product of two positive real numbers. For a fractional exponent a/b, two “natural” ways to do this are as follows:

$$\frac{a}{b} = a \cdot \left(\frac{1}{b}\right) \text{ and } \frac{a}{b} = \left(\frac{1}{b}\right) \cdot a$$

There’s a power rule for exponents which tells us that if c and d are positive real numbers, then

$$(x^c)^d = x^{c \cdot d}$$

In the expression $x^{a/b}$, therefore, you can let $c = a$ and $d = 1/b$, so you can rewrite $x^{a/b}$ as

$$\left[(x)^a\right]^{\frac{1}{b}}$$

But if you let $c = 1/b$ and $d = a$, you can rewrite $x^{a/b}$ as

$$\left[(x)^{\frac{1}{b}}\right]^a$$

Let’s do a few more examples.

Example

Simplify the expression.

$$\left(\frac{1}{9}\right)^{\frac{3}{2}}$$

9 is a perfect square, so we can simplify the problem by finding the square root first. We can rewrite the expression by breaking up the exponent.

$$\left[\left(\frac{1}{9}\right)^{\frac{1}{2}}\right]^3$$

Raising a number to the power $1/2$ is the same as taking the square root of that number, so we get

$$\left[\sqrt{\frac{1}{9}}\right]^3$$

$$\left(\frac{\sqrt{1}}{\sqrt{9}}\right)^3$$

$$\left(\frac{1}{3}\right)^3$$

This is the same as

$$\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)$$

$$\frac{1}{27}$$

Let's look at another example.

Example

Write the expression with no fractional exponents.

$$\left(\frac{1}{6}\right)^{\frac{3}{2}}$$

We can rewrite the expression by breaking up the exponent.

$$\left[\left(\frac{1}{6}\right)^{3}\right]^{\frac{1}{2}}$$

$$\left(\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}\right)^{\frac{1}{2}}$$

Raising a number to the power $1/2$ is the same as taking the square root of that number, so we get

$$\sqrt{\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}}$$

$$\sqrt{\frac{1}{216}}$$

$$\frac{\sqrt{1}}{\sqrt{216}}$$

$$\frac{1}{\sqrt{36\cdot 6}}$$

$$\frac{1}{\sqrt{36}\sqrt{6}}$$

$$\frac{1}{6\sqrt{6}}$$

What happens if you have a negative fractional exponent?

You should deal with the negative sign first, then use the rules for fractional exponents.

Example

Write the expression with no fractional exponents.

$$4^{-\frac{2}{5}}$$

First, we'll deal with the negative exponent. Remember that when a is a positive real number, both of these equations are true:

$$x^{-a} = \frac{1}{x^a}$$

$$\frac{1}{x^{-a}} = x^a$$

Therefore, we can rewrite $4^{-2/5}$ as

$$\frac{1}{4^{\frac{2}{5}}}$$

In the fractional exponent $2/5$, 2 is the power and 5 is the root, which means we can rewrite the expression as

$$\frac{1}{\sqrt[5]{4^2}}$$

$$\frac{1}{\sqrt[5]{16}}$$