Trichotomy

How can you describe the relationship between two numbers? There are only three ways to describe this relationship, which is why it's called the law of trichotomy ("tri" means three).

The law of trichotomy says that two numbers can have exactly one of three possible relationships:

1. The first number is smaller than the second number: $a < b$.
2. The first number is greater than the second number: $a > b$.
3. The first number is equal to the second number: $a = b$.

Because of the trichotomy law, we can make the following three statements:

1. If a is not greater than b and also not equal to b, then a must be less than b. If $a \ngeq b$, then $a < b$.
2. If a is not less than b and also not equal to b, then a must be greater than b. If $a \nleq b$, then $a > b$.
3. If a is not greater than b and also not less than b, then a must be equal to b. If $a \nless b$ and $a \ngtr b$, then $a = b$.

Example

Solve the inequality.

$$4(x-2) \ngeq 3(x+8)$$

We need to recognize that, because of the trichotomy law, if the value of one expression is not greater than or equal to the value of another expression, then it must be less than the value of that other expression. Therefore we can rewrite $4(x-2) \ngeq 3(x+8)$ with a less than sign.

$$4(x-2) < 3(x+8)$$

We basically follow the same steps to solve inequalities as we do equations, but don't accidentally change your $<$ to $=$. We'll simplify both sides of the inequality by distributing.

$$4x-8 < 3x+24$$

Now move the $3x$ so that both of the x terms are on the left side.

$$4x-3x-8 < 3x-3x+24$$

$$x-8 < 24$$

Add 8 to both sides.

$$x-8+8 < 24+8$$

$$x < 32$$

Let's try another example of trichotomy.

Example

Solve the inequality.

$$4x + 5 \nleq 2x + 7$$

This time, let's solve for x first, just so that we can see that we can do these kinds of problems in any order that we like. Start by subtracting $2x$ from both sides so that both of the x terms are on the left side.

$$4x - 2x + 5 \nleq 2x - 2x + 7$$

$$2x + 5 \nleq 7$$

Subtract 5 from both sides.

$$2x + 5 - 5 \nleq 7 - 5$$

$$2x \nleq 2$$

Divide both sides by 2.

$$\frac{2x}{2} \nleq \frac{2}{2}$$

$$x \nleq 1$$

Since x isn't less than 1 and also isn't equal to 1, it can only be greater than 1, according to the trichotomy law.

$$x > 1$$