Commutative property

The commutative property is something you've been aware of for a long time. You probably just didn't know its name. It states that you can add or multiply numbers in any order. For example: $2+5=5+2$.

Commutative Property of Addition	$a+b=b+a$
Commutative Property of Multiplication	$a \cdot b=b \cdot a$

Commutative comes from the word "commute," as in "the morning commute." Since commute means to move, you can remember that when you use the commutative property the numbers will move around.

Example

Use the commutative property to write the expression a different way. Don't perform the multiplication.

$$(5 \cdot 3) \cdot 2$$

We know that when we apply the commutative property of multiplication to multiply a product of two numbers enclosed in parentheses by a number that's outside the parentheses, the numbers in parentheses can move (in this case switch places with each other) and the parentheses can stay in the same place. So we could move the numbers in parentheses to rewrite $(5 \cdot 3) \cdot 2$ as

$$(3 \cdot 5) \cdot 2$$

We didn't have to perform the multiplication to solve this problem, but we can also see that the two expressions are equal.

$$(5 \cdot 3) \cdot 2 = (15) \cdot 2 = 30$$

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Another way to do this would be to switch the parentheses (including the number enclosed in them) with the number that's outside the parentheses. So we could also rewrite $(5 \cdot 3) \cdot 2$ as

$$2 \cdot (5 \cdot 3)$$

Still another option would be to then take that version of the expression and switch the two numbers that are in parentheses:

$$2 \cdot (3 \cdot 5)$$

If we were to apply both the associative and commutative properties of multiplication, we would get even more ways to rewrite our expression. For example, we could first apply the associative property to $(5 \cdot 3) \cdot 2$ and write it as

$$5 \cdot (3 \cdot 2)$$

Then we could apply the commutative property, and get the following two expressions:

$$5 \cdot (2 \cdot 3)$$

$$(3 \cdot 2) \cdot 5$$

You should convince yourself that there are still other ways to express the original multiplication, by applying both the associative and commutative properties to the expressions we've already found.

Let's try another example, this time with the commutative property of addition.

Example

Is the equation below true or false? Explain your reasoning.

$$4 + 12 + 7 = 7 + 4 + 12$$

True, because of the commutative property of addition. We know it's the commutative property of addition since the only thing that's changed is the order of the numbers to be added.

Also we can see that both expressions simplify to 23. To simplify each expression, we'll first find the sum of the first two numbers, and then add the third number to the result.

$$4 + 12 + 7 = (4 + 12) + 7 = (16) + 7 = 23$$

$$7 + 4 + 12 = (7 + 4) + 12 = (11) + 12 = 23$$