Multiplying and dividing like terms

Remember that **like terms** are terms with the same base and same exponent. For instance, x^2 and x^2 are like terms, because they have the same base, x, and the same exponent, 2. The terms $3x^2$ and $-4x^2$ are also like terms; they just have different coefficients, 3 and -4.

On the other hand, $3x^2$ and $-4x^5$ are *not* like terms, because, while they have the same base, the exponents 2 and 5 are different.

Addition vs. multiplication

When adding and subtracting, only like terms can be combined. So the sum of $3x^2$ and $-4x^2$ is

$$3x^2 + (-4x^2)$$

$$3x^2 - 4x^2$$

$$(3 - 4)x^2$$

$$(-1)x^2$$

$$-x^2$$

But the sum of the unlike terms $3x^2$ and $-4x^5$ is simply $3x^2 - 4x^5$, and the sum can't be simplified any further, since x^2 and x^5 are unlike terms.

So when it comes to addition and subtraction, only like terms can be combined. But when it comes to multiplication and division, even unlike terms can be multiplied and divided.

If we're multiplying terms with the same base, we apply product rule, keeping the base the same, and adding the exponents. The coefficients also get multiplied. For example,

$$4x^2 \cdot 3x^5$$

$$(4 \cdot 3)x^{2+5}$$

$$12x^7$$

If we're dividing terms with the same base, we apply quotient rule, keeping the base the same, and subtracting the exponents. The coefficients also get divided. For example,

$$\frac{4x^3}{2x}$$

$$\frac{4}{2}x^{3-1}$$

$$2x^2$$

When the bases of two terms are unlike, like x^2 and y^3, the terms don't "combine" in the same way. So $x^2 \cdot y^3$ would simply be x^2y^3. And $(x^2)/(y^3)$ doesn't simplify at all, and the quotient is still just $(x^2)/(y^3)$.

Let's work through some more examples to see this in action.

Example

Simplify the expression.

$$\frac{x(x + 2x - a + b)}{x}$$

Notice that the x outside of the parentheses in the numerator is being multiplied by all the other terms in the numerator. Also notice that there's an x in the denominator, which means we need to divide by x. Since division undoes multiplication, we can cancel the x outside of the parentheses in the numerator against the x in the denominator, so now we have just

$$x + 2x - a + b$$

Combine like terms (the x terms in this case).

$$(1 + 2)x - a + b$$

$$3x - a + b$$

Let's try another example of multiplying and dividing like terms.

Example

Simplify the expression.

$$\frac{(2x-4)(x^2-x+3)}{2}$$

Start by expanding the numerator.

$$\frac{2x^3-2x^2+6x-4x^2+4x-12}{2}$$

Combine like terms in the numerator. Remember to group terms with the same exponents.

$$\frac{2x^3+(-2x^2-4x^2)+(6x+4x)-12}{2}$$

$$\frac{2x^3-6x^2+10x-12}{2}$$

Since the coefficient of every term in the numerator is even, we can factor out a 2 in the numerator, and then cancel that 2 against the 2 in the denominator.

$$\frac{2(x^3-3x^2+5x-6)}{2}$$

$$x^3-3x^2+5x-6$$

There's another way we could have done this problem. We could have started out by simplifying the original expression.

$$\frac{(2x-4)(x^2-x+3)}{2}$$

Notice that in the first factor in the numerator, $(2x-4)$, the coefficients of both terms are even. So we can factor out a 2 there, and immediately cancel that 2 against the 2 in the denominator.

$$\frac{2(x-2)(x^2-x+3)}{2}$$

$$(x-2)(x^2-x+3)$$

Now we'll expand this remaining expression.

$$x^3-x^2+3x-2x^2+2x-6$$

Grouping like terms and then combining them:

$$x^3+(-x^2-2x^2)+(3x+2x)-6$$

$$x^3-3x^2+5x-6$$

The advantage of this approach is that we get to multiply smaller numbers in the numerator (as a result of first factoring out the 2 in the numerator), so it may be less likely that we'll make a mistake in multiplication, or get an incorrect sign for one or more of the terms. Sometimes it pays to be on the lookout for opportunities to factor an expression before proceeding to do any other algebraic operations with it.