Topic: Dividing rational functions

Question: Find the quotient of the rational functions.

$$\frac{x^2+x-20}{x^2-8x+15} \div \frac{x^2-6x+8}{x^2-7x+10}$$

Answer choices:

A $\frac{x+3}{x-5}$ with $x \neq 2,3,4$

B $\frac{x-5}{x+3}$ with $x \neq -5,2,4$

C $\frac{x-3}{x+5}$ with $x \neq -3,2,4$

D $\frac{x+5}{x-3}$ with $x \neq 2,4,5$

Solution: D

Factor the numerator and denominator of both fractions as completely as possible.

$$\frac{(x+5)(x-4)}{(x-5)(x-3)} \div \frac{(x-4)(x-2)}{(x-5)(x-2)}$$

Consider restrictions. The denominator of the dividend gives $x \neq 3,5$, the denominator of the divisor gives $x \neq 2,5$, and the numerator of the divisor gives $x \neq 2,4$. So the set of restrictions we should keep in mind until the end of the problem is $x \neq 2,3,4,5$.

Now turn the division problem into a multiplication problem and cancel common factors.

$$\frac{(x+5)(x-4)}{(x-5)(x-3)} \cdot \frac{(x-5)(x-2)}{(x-4)(x-2)}$$

$$\frac{x+5}{x-3}$$

This resulting quotient shows that $x \neq 3$, so we can eliminate that from our list of restrictions. Then the final answer is

$$\frac{x+5}{x-3} \text{ with } x \neq 2,4,5$$

Topic: Dividing rational functions

Question: Find the quotient of the rational functions.

$$\frac{x^2-6x+8}{x^2+7x+12} \div \frac{x^2-x-20}{x^2+2x-15}$$

Answer choices:

A $\frac{(x-2)(x-3)(x-4)}{(x+3)(x+4)^2}$ with $x \neq -5,3,5$

B $\frac{(x-2)(x-3)(x+5)}{(x+3)(x-5)}$ with $x \neq -5,-4,3$

C $\frac{(x-2)(x-3)(x-4)(x+5)}{(x+3)(x+4)^2(x-5)}$ with $x \neq -5,3$

D $\frac{(x-2)(x-3)(x-4)(x+5)}{2(x+3)(x+4)(x-5)}$ with $x \neq -5,-4,3$

Solution: C

Factor the numerator and denominator of both fractions as completely as possible.

$$\frac{(x-2)(x-4)}{(x+3)(x+4)} \div \frac{(x-5)(x+4)}{(x+5)(x-3)}$$

Consider restrictions. The denominator of the dividend gives $x \neq -4, -3$, the denominator of the divisor gives $x \neq -5,3$, and the numerator of the divisor gives $x \neq -4,5$. So the set of restrictions we should keep in mind until the end of the problem is $x \neq -5, -4, -3,3,5$.

Now turn the division problem into a multiplication problem and cancel common factors.

$$\frac{(x-2)(x-4)}{(x+3)(x+4)} \cdot \frac{(x+5)(x-3)}{(x-5)(x+4)}$$

$$\frac{(x-2)(x-3)(x-4)(x+5)}{(x+3)(x+4)^2(x-5)}$$

This resulting quotient shows that $x \neq -4, -3,5$, so we can eliminate that from our list of restrictions. Then the final answer is

$$\frac{(x-2)(x-3)(x-4)(x+5)}{(x+3)(x+4)^2(x-5)} \text{ with } x \neq -5,3$$

Topic: Dividing rational functions

Question: Find the quotient of the rational functions.

$$\frac{w^2 - w - 12}{w^2 + 5w + 4} \div \frac{w^2 - 9}{w^2 - 2w - 3}$$

Answer choices:

A -1

B $\frac{w+4}{w-4}$ with $w \neq -4, -3, -1, 3$

C $\frac{w-4}{w+4}$ with $w \neq -3, -1, 3$

D 1

Solution: C

Factor the numerator and denominator of both fractions as completely as possible.

$$\frac{(w-4)(w+3)}{(w+1)(w+4)} \div \frac{(w+3)(w-3)}{(w+1)(w-3)}$$

Consider restrictions. The denominator of the dividend gives $w \neq -4, -1$, the denominator of the divisor gives $w \neq -1,3$, and the numerator of the divisor gives $w \neq -3,3$. So the set of restrictions we should keep in mind until the end of the problem is $w \neq -4, -3, -1,3$.

Now turn the division problem into a multiplication problem and cancel common factors.

$$\frac{(w-4)(w+3)}{(w+1)(w+4)} \cdot \frac{(w+1)(w-3)}{(w+3)(w-3)}$$

$$\frac{w-4}{w+4}$$

This resulting quotient shows that $w \neq -4$, so we can eliminate that from our list of restrictions. Then the final answer is

$$\frac{w-4}{w+4} \text{ with } w \neq -3, -1,3$$