Long division of polynomials

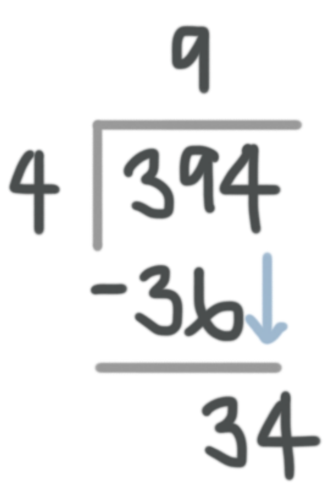
Long division of polynomials uses the same steps you learned for long division of real numbers. It might look different because of the variables, but don't worry; it's the same thing in disguise.

Let's first review long division.

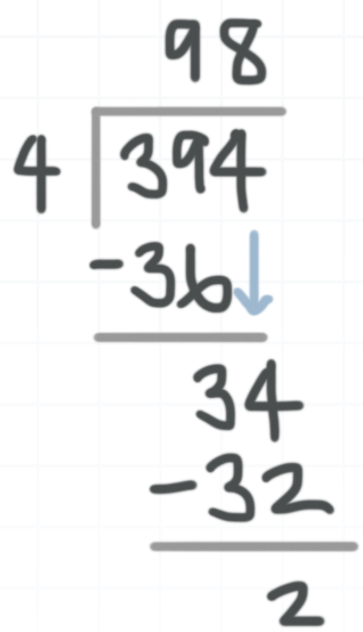
Remember this? You followed a pattern of Divide, Multiply, Subtract, Bring Down.

Ordinarily, you would start by dividing the first digit of the dividend (3) by the divisor (4). But since 4 is bigger than 3, that won't work in this case, so you need to use the first two digits of the dividend (39) instead.

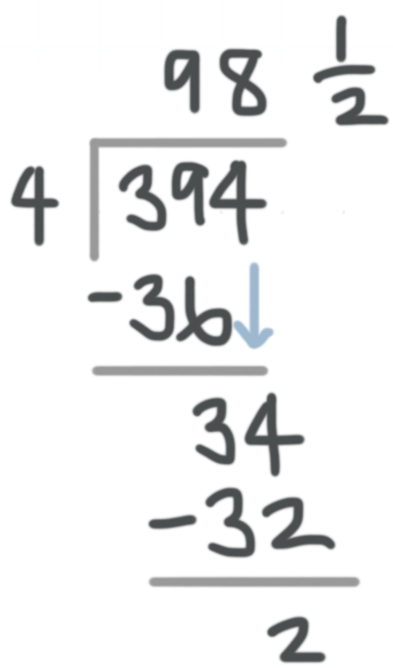
$4 \cdot 9 = 36$ but $4 \cdot 10 = 40$, so 4 times 9 is the largest product we can put into 39. Therefore, write the 9 above the ten's place and the 36 under the 39 in the division problem, then subtract and bring down the 4.



Now $4 \cdot 9 = 36$ is too big to fit in 34, but $4 \cdot 8 = 32$ does fit, so we write the 8 above the ones place and the 32 under the 34 in the division problem, then subtract.



The 2 is the remainder, so write it as a fractional part of the division $(2/4)$, which simplifies to $1/2$.



This is the same technique you use for polynomials. Let's check it out.

Example

Find the quotient.

$$\frac{m^2 - 7m - 11}{m - 6}$$

First set it up as a division problem.

$$m-6\,\overline{)\,m^2-7m-11}$$

Now divide the leading term of the dividend (m^2) by the leading term of the divisor (m), which gives m. This means we need to multiply $m - 6$ by m.

$$m(m - 6) = m^2 - 6m$$

Write the m above the $7m$ in the division problem and the $m^2 - 6m$ under the $m^2 - 7m$.

$$\begin{array}{r} m \\ m-6\,\overline{)\,m^2-7m-11} \\ \underline{m^2-6m} \end{array}$$

Remember, you're subtracting next.

$$
\begin{array}{rl}
 & \quad m \\
m-6 & \overline{\big)\, m^2-7m-11} \\
 & \underline{-(m^2-6m)}\downarrow \\
 & \qquad\quad -m-11
\end{array}
$$

Now bring down the -11, and divide the leading term of $-m-11$ (which is $-m$) by the leading term of the divisor (m), which gives -1. This means we need to multiply $m-6$ by -1.

$$-1(m-6) = -m+6$$

Write the -1 above the -11 in the division problem and the $-m+6$ under the $-m-11$.

$$
\begin{array}{rl}
 & \quad m-1 \\
m-6 & \overline{\big)\, m^2-7m-11} \\
 & \underline{-(m^2-6m)}\downarrow \\
 & \qquad\quad -m-11 \\
 & \qquad \underline{-(-m+6)}
\end{array}
$$

The next step is to subtract $-m+6$ from $-m-11$.

$$
\begin{array}{r l}
 & \quad m-1 \\
m-6 & \overline{\big)\, m^2-7m-11} \\
 & -(m^2-6m) \downarrow \\
\hline
 & \qquad\quad -m-11 \\
 & \qquad -(-m+6) \\
\hline
 & \qquad\qquad\quad -17
\end{array}
$$

Now write the remainder (-17) as a fractional part of the divisor (as $-17/(m-6)$).

$$
\begin{array}{r l}
 & \quad m-1-\frac{17}{m-6} \\
m-6 & \overline{\big)\, m^2-7m-11} \\
 & -(m^2-6m) \downarrow \\
\hline
 & \qquad\quad -m-11 \\
 & \qquad -(-m+6) \\
\hline
 & \qquad\qquad\quad -17
\end{array}
$$

Let’s do another example.

Example

Use long division to simplify the rational function.

$$f(x) = \frac{x^3 + x^2 + x + 1}{x + 1}$$

First, we should keep in mind that the divisor is $x + 1$ and the dividend is $x^3 + x^2 + x + 1$.

$$
\begin{array}{rl}
 & x^2 \qquad\quad + 1 \\ \cline{2-2}
x+1 & \big) \, x^3 + x^2 + x + 1 \\
 & -(x^3 + x^2) \\ \cline{2-2}
 & \qquad\quad 0 + x + 1 \\
 & \qquad\quad\; -(x + 1) \\ \cline{2-2}
 & \qquad\qquad\quad\; 0
\end{array}
$$

To start our long division problem, we determine what we have to multiply by x (the leading term of the divisor) to get x^3 (the leading term of the dividend). Since the answer is x^2, we put that above the x^3 in the dividend, and multiply it by the divisor, $x + 1$, which gives $x^3 + x^2$. We then subtract that $x^3 + x^2$ from the $x^3 + x^2$ in the dividend.

We bring down $x + 1$ from the dividend and do the division, multiplication, and subtraction. The division of the leading term of $x + 1$ (which is x) by the leading term of the divisor (which is x) gives 1, so we write $+1$ above the $+x$ in the dividend, and the multiplication gives $x+1$. Then we subtract and get 0, so we have a remainder of 0, which means that our original rational function reduces to

$$f(x)=x^2+1$$