Topic: Domain and range

Question: What are the domain and range of the function?

(3,4), (4,1), (5,2), (7,1)

Answer choices:

A   The domain is 3, 4, 5, 7.   The range is 1, 2, 4.

B   The domain is 3, 7.   The range is 1, 4.

C   The domain is 3, 4, 5, 7.   The range is 1, 2.

D   None of these

Solution: A

Remember that the coordinates of points in the Cartesian coordinate system are given in the form (x, y).

Since the domain of a function is all of the x-values, we can see that the domain of this function is

$$3, \ 4, \ 5, \ 7$$

The range of a function is all of the y-values, so we can see that the range of this function is

$$4, \ 1, \ 2, \ 1$$

We don't need to include the same value more than once, so we'll list 1 only once, and rearrange the numbers so that they are in ascending order. The range is

$$1, \ 2, \ 4$$

Topic: Domain and range

Question: What are the domain and range of the function?

$$y = \frac{2}{x}$$

Answer choices:

A   Domain: all real numbers except 2 Range: all real numbers except 2

B   Domain: all real numbers except 0 Range: all real numbers except 0

C   Domain: all real numbers except 0 Range: all real numbers except 2

D   Domain: all real numbers except 2 Range: all real numbers except 0

Solution: B

The domain of a function is all of the x-values for which the function is defined. The range of a function is all of the y-values that correspond to the x-values in the domain. To solve for the domain of a function, we look for any places where the function is not defined. For example, this can happen if there's a variable in the denominator of the function or if a radical has a negative number under it.

The function

$$y = \frac{2}{x}$$

is undefined for $x = 0$, because division by 0 is undefined. However, this function is defined for all other values of x, so its domain consists of all real numbers except 0.

To solve for the range of the function, we need to look for the y-values that correspond to numbers in the domain and for those that don't.

For every nonzero real number y, there's some nonzero real number x such that

$$y = \frac{2}{x}$$

To see this, multiply both sides of this equation by x/y.

$$y\left(\frac{x}{y}\right) = \left(\frac{2}{x}\right)\left(\frac{x}{y}\right)$$

$$x = \frac{2}{y}$$

So for any nonzero real number y, we divide 2 by y to get a nonzero real number x for which $y = 2/x$.

However, there's no nonzero real number x such that

$$0 = \frac{2}{x}$$

To see this, multiply both sides of this equation by x.

$$0(x) = \left(\frac{2}{x}\right)(x)$$

$$0 = 2$$

This gives us the false equation $0 = 2$.

Combining these results, we find that the range of this function is all real numbers except 0.

Topic: Domain and range

Question: What is the domain of the function?

$$f(x) = \sqrt{4x^3}$$

Answer choices:

A   The domain is all values of x that make $4x^3$ positive

B   The domain is all values of x that make $4x^3$ negative

C   The domain is all values of x that make $4x^3$ either 0 or positive

D   The domain is all values of x that make $4x^3$ either 0 or negative

Solution: C

When we're dealing in real numbers, we can only take the square root of 0, or of positive values.

In other words, we won't be able to find the square root of $4x^3$ unless the value of $4x^3$ is positive, or equal to 0.

Therefore, any values of x that make $4x^3$ equivalent to 0, or equivalent to any positive value, will be included in the domain of the function.