# Graphing parabolas

In this lesson we'll learn how to identify the characteristics of a parabola and go back and forth between the equation of a parabola and its graph.

A parabola is the solution to an equation $y = f(x)$ where $f(x)$ is a quadratic polynomial. Therefore, a parabola is the solution to a non-linear equation. There are two forms of the equation of a parabola that are especially helpful when you want to know something about a parabola.

| Standard form | Vertex form |
|---|---|
| $y=ax^2+bx+c$<br><br>The equation of the axis of symmetry is $x=-b/2a$.<br><br>The vertex lies on the axis of symmetry, so its x-coordinate is $-b/(2a)$. You can find its y-coordinate by substituting $-b/(2a)$ for x in the equation of the parabola. | $y=a(x-h)^2+k$<br><br>If $\vert a\vert<1$, then the graph will be wider than the graph of $y=x^2$.<br><br>If $\vert a\vert>1$, then the graph will be narrower than the graph of $y=x^2$.<br><br>The vertex of the parabola is located at the point with coordinates $(h, k)$ and the equation of the axis of symmetry is $x=h$. |

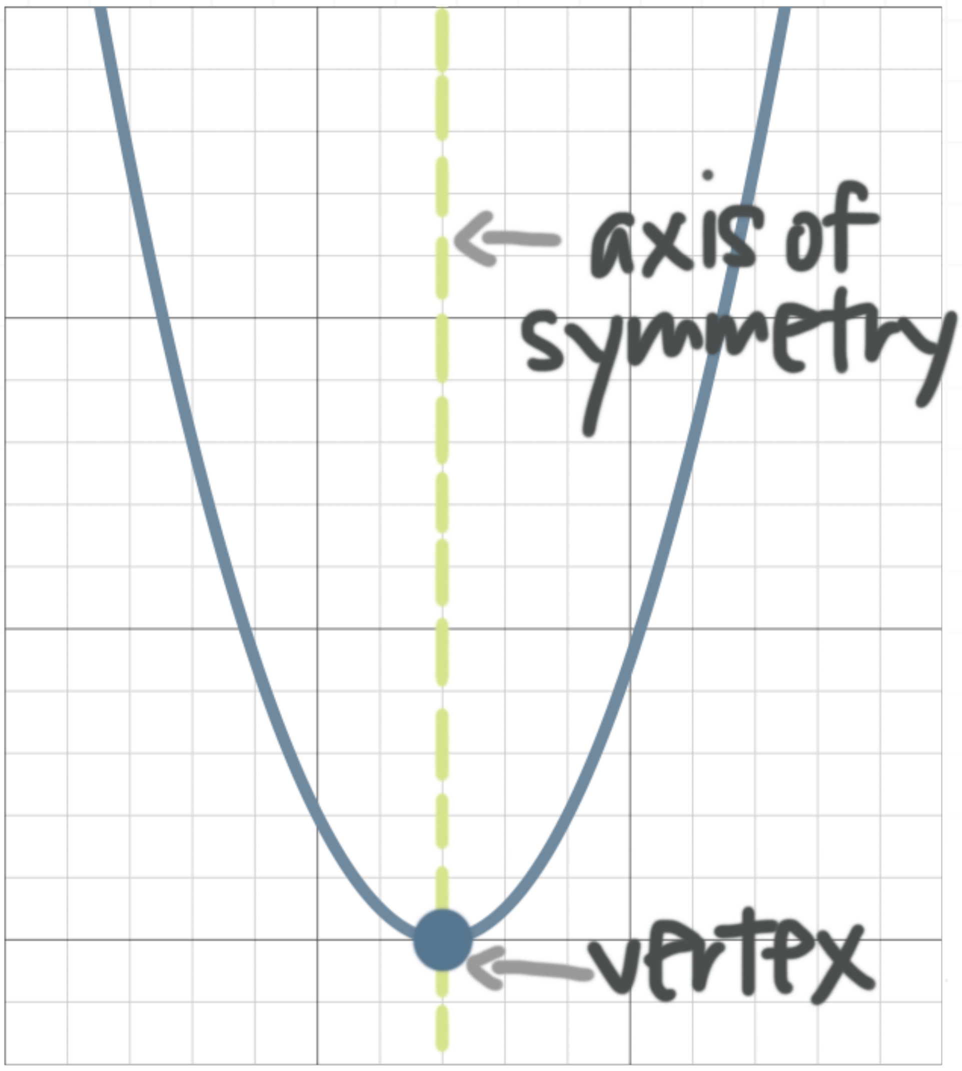
How to convert between standard form and vertex form:

To convert from standard form to vertex form, complete the square.

To convert from vertex form to standard form, expand the square, then distribute and simplify.

Let's talk about the different parts of a parabola.

In both forms (standard and vertex), if $a > 0$ the parabola opens upwards and the vertex is the point at the bottom of the parabola (the point with the minimum value of $y$).




In both forms (standard and vertex), if $a < 0$ the parabola opens downwards and the vertex is the point at the top of the parabola (the point with the maximum value of $y$).

Let's do a few problems.

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**Example**

Write the equation in vertex form.

$$y = 2x^2 + 36x + 170$$

To convert the standard equation of the parabola to vertex form from standard form, you'll need to complete the square.

Before we complete the square, we'll factor the coefficient of the $x^2$ term, which is $2$, out of the first two terms on the right-hand side of the given equation.

$$y = 2x^2 + 36x + 170$$

$$y = 2(x^2 + 18x) + 170$$

To complete the square, we need to find the number $d$ that satisfies the equation

$$x^2 + 18x + d^2 = (x + d)^2$$

That is, we need to find the number $d$ for which

$$x^2 + 18x + d^2 = x^2 + 2dx + d^2$$

This means that the coefficient of the $x$ term of the expression inside the parentheses must be equal to $2d$. That coefficient is $18$, so we'll set $2d$ to $18$ and solve for $d$.

$$2d = 18 \quad \to \quad d = 9$$

To keep our equation balanced, we need to add and subtract $d^2$ $(81)$ inside the parentheses, and then distribute, regroup, and simplify.

$$y = 2(x^2 + 18x) + 170$$

$$y = 2(x^2 + 18x + 81 - 81) + 170$$

$$y = 2(x^2 + 18x + 81) + 2(-81) + 170$$

$$y = 2(x^2 + 18x + 81) - 162 + 170$$

$$y = 2(x^2 + 18x + 81) + 8$$

Finally, we'll factor the expression that's now inside the parentheses ($x^2 + 18x + 81$). By construction ("completing the square"), that expression factors as $(x + d)^2$.

$$x^2 + 18x + 81 = (x + d)^2$$

$$x^2 + 18x + 81 = (x + 9)^2$$

The vertex form of the equation is

$$y = 2(x + 9)^2 + 8$$

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Let's try one where we need to interpret a graph.

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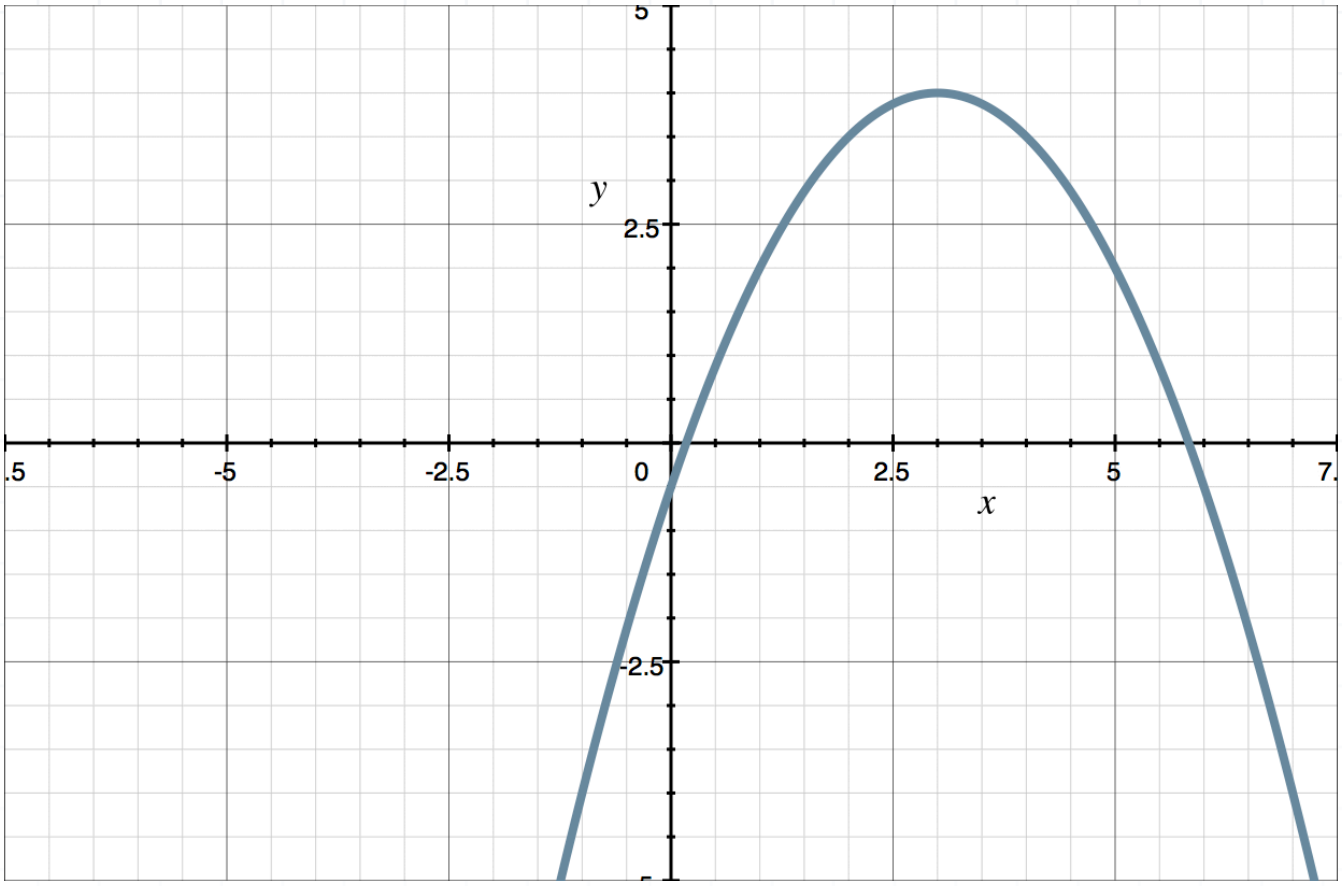
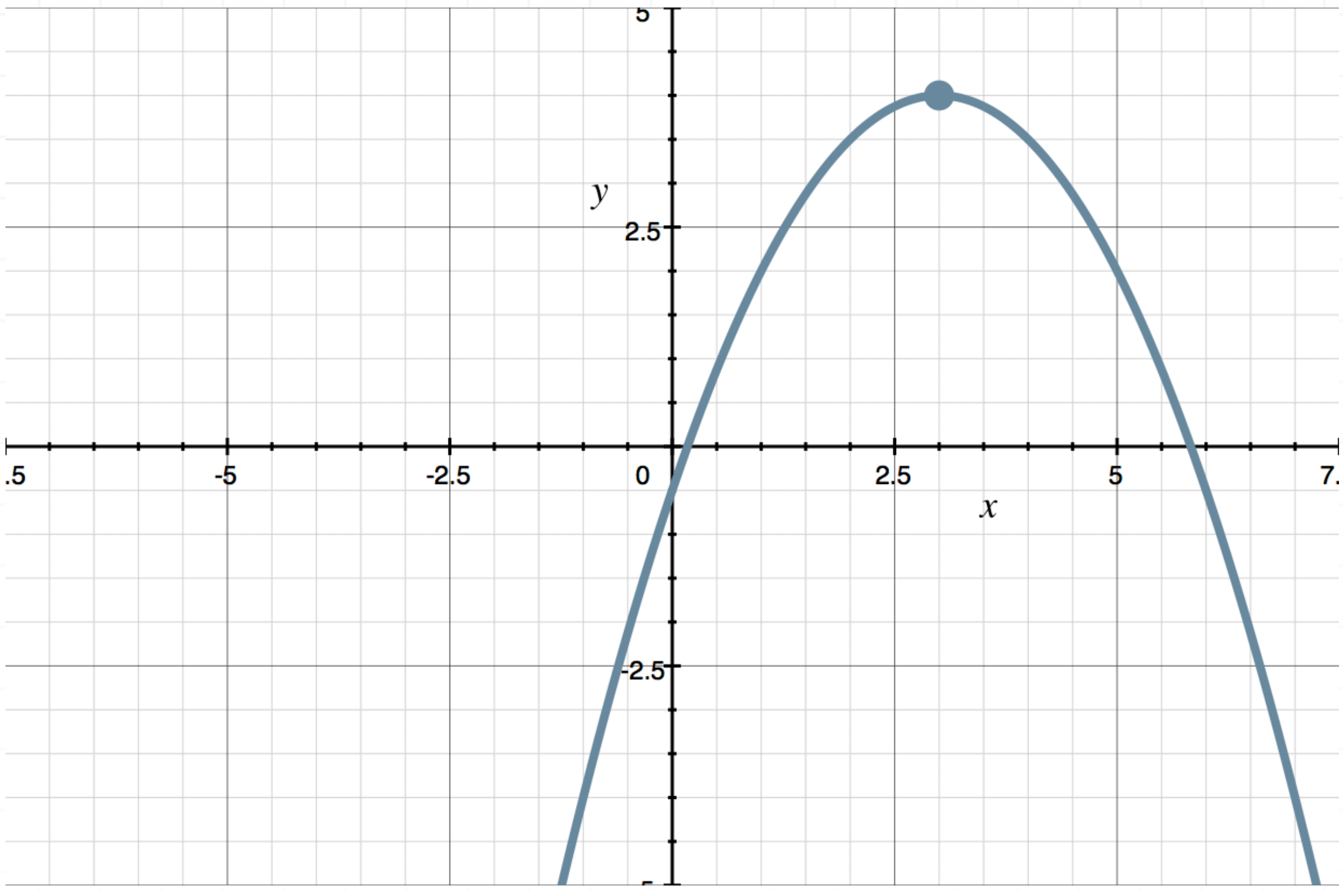
**Example**

The coefficient of the $x^2$ term in the equation of the parabola shown in the graph is $-1/2$. What is the equation of the parabola in standard form?




Remember, the vertex form of the equation of a parabola is $y = a(x - h)^2 + k$, where $(h, k)$ are the coordinates of the vertex.

We know that $a = -1/2$, and we can read the coordinates of the vertex from the graph: $(3,4)$.




So we know that $h = 3$ and $k = 4$. Let's put what we know into the vertex form of the equation of a parabola.

$$y = a(x - h)^2 + k$$

$$y = -\frac{1}{2}(x - 3)^2 + 4$$

Now we want to go from vertex form to standard form, so we'll expand the square:

$$y = -\frac{1}{2}(x - 3)(x - 3) + 4$$

$$y = -\frac{1}{2}(x^2 - 6x + 9) + 4$$

Distribute the $-1/2$ over all the terms inside the parentheses.

$$y = -\frac{1}{2}(x^2) - \frac{1}{2}(-6x) - \frac{1}{2}(9) + 4$$

$$y = -\frac{1}{2}x^2 + 3x - \frac{9}{2} + 4$$

$$y = -\frac{1}{2}x^2 + 3x - \frac{9}{2} + \frac{8}{2}$$

$$y = -\frac{1}{2}x^2 + 3x - \frac{1}{2}$$