Graphing parallel and perpendicular lines

In this lesson we'll learn about the characteristics of parallel and perpendicular lines and how to identify them on a graph or in an equation.

The equation of a line in slope-intercept form is $y = mx + b$, where m is the slope of the line and b is the y-coordinate of the y-intercept. Remember, the y-intercept is the point at which the line crosses the y-axis. The x-coordinate of every point on the y-axis is 0, so the x-coordinate of the y-intercept is always 0. Therefore, we sometimes call b the y-intercept, even though (technically speaking) b is just the y-coordinate of the y-intercept.

Parallel lines

For two lines to be parallel, their slopes must be equal but their y-intercepts must be different (otherwise, they are the same line).

Algebraically, in slope-intercept form the equations of a pair of parallel lines would look like

$$\begin{cases} y = mx + b_1 \\ y = mx + b_2 \end{cases}$$

where $b_1 \neq b_2$.

Here are two examples of equations of a pair of parallel lines in slope-intercept form:

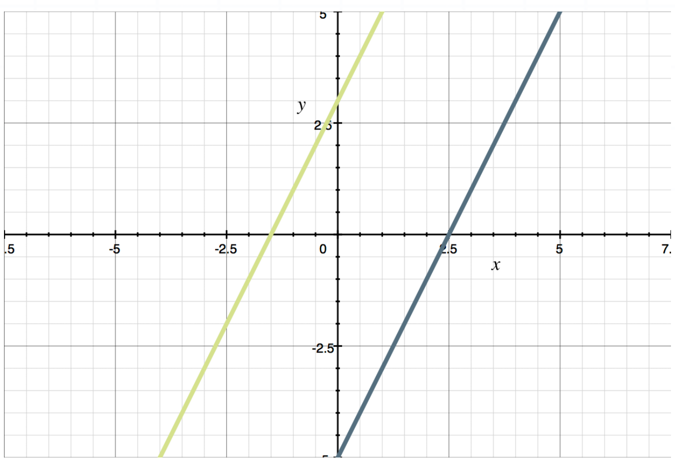
$$\begin{cases} y = 2x - 5 \\ y = 2x + 3 \end{cases}$$

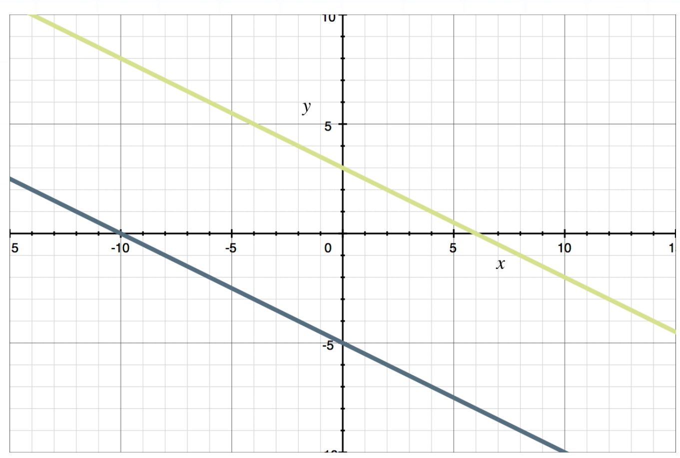
$$\begin{cases} y = -\frac{1}{2}x - 5 \\ y = -\frac{1}{2}x + 3 \end{cases}$$

Parallel lines go on forever in the same direction and never cross each other. Here are the graphs of those two examples of equations of a pair of parallel lines.

$$\begin{cases} y = 2x - 5 & \text{blue} \\ y = 2x + 3 & \text{green} \end{cases}$$

$$\begin{cases} y = -\frac{1}{2}x - 5 & \text{blue} \\ y = -\frac{1}{2}x + 3 & \text{green} \end{cases}$$





Perpendicular lines

Perpendicular lines have slopes that are negative reciprocals of each other. Remember, two numbers c, d are reciprocals of each other if $d = 1/c$. Therefore, if the slope of one of the lines in a pair of perpendicular lines is m, the slope of the other line is $-1/m$. Since the slopes are different, the y-intercepts could be the same.

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Algebraically, in slope-intercept form the equations of a pair of perpendicular lines would look like

$$\begin{cases} y = mx + b_1 \\ y = -\frac{1}{m}x + b_2 \end{cases}$$

where b_1 and b_2 could be the same.

Here are two examples of equations of a pair of perpendicular lines in slope-intercept form:

$$\begin{cases} y = 2x - 3 \\ y = -\frac{1}{2}x - 3 \end{cases}$$

$$\begin{cases} y = -5x + 2 \\ y = \frac{1}{5}x - 4 \end{cases}$$

Perpendicular lines cross each other at a single point and form four right angles (four 90-degree angles) at their point of intersection. Be aware that checking this on your calculator might not always help, because the graphing window settings can disguise right angles.

Here are the graphs of those two examples of equations of a pair of perpendicular lines.

$$\begin{cases} y = 2x - 3 & \text{blue} \\ y = -\frac{1}{2}x - 3 & \text{green} \end{cases} \qquad \begin{cases} y = -5x + 2 & \text{blue} \\ y = \frac{1}{5}x - 4 & \text{green} \end{cases}$$

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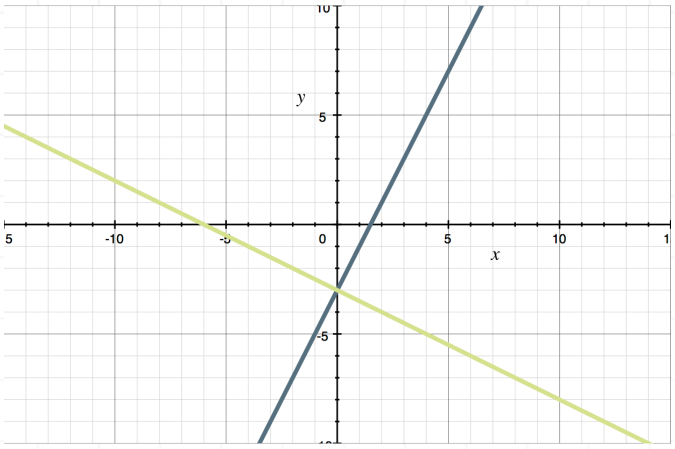

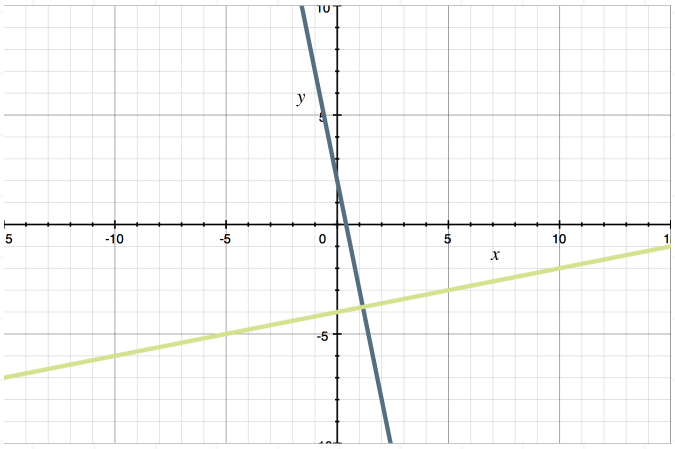

Let’s go ahead and look at a few of the types of problems involving parallel or perpendicular lines that you’ll need to know how to solve.

Example

Write the equation of the line which is parallel to the line $5x + 2y = 10$ a y-intercept of 4.

For two lines to be parallel, their slopes must be equal.

Remember that the equation of a line in slope-intercept form is given by

$$y = mx + b$$

where m is the slope and b is the y-intercept. To get the slope of the original line (which will also be the slope of the new line), we’ll first convert its equation to slope-intercept form.

$$5x + 2y = 10$$

$$5x - 5x + 2y = -5x + 10$$

$$2y = -5x + 10$$

$$\frac{1}{2} \cdot 2y = \frac{1}{2} \cdot -5x + \frac{1}{2} \cdot 10$$

$$y = -\frac{5}{2}x + 5$$

Therefore, the slope is $-5/2$.

We want to write the equation of the line that has a slope of $-5/2$ and a y-intercept of 4. So $m = -5/2$ and $b = 4$. Therefore, the equation is

$$y = -\frac{5}{2}x + 4$$

Let's look at another example.

Example

Graph the line which is parallel to the line shown in the graph and has a y-intercept of -2.

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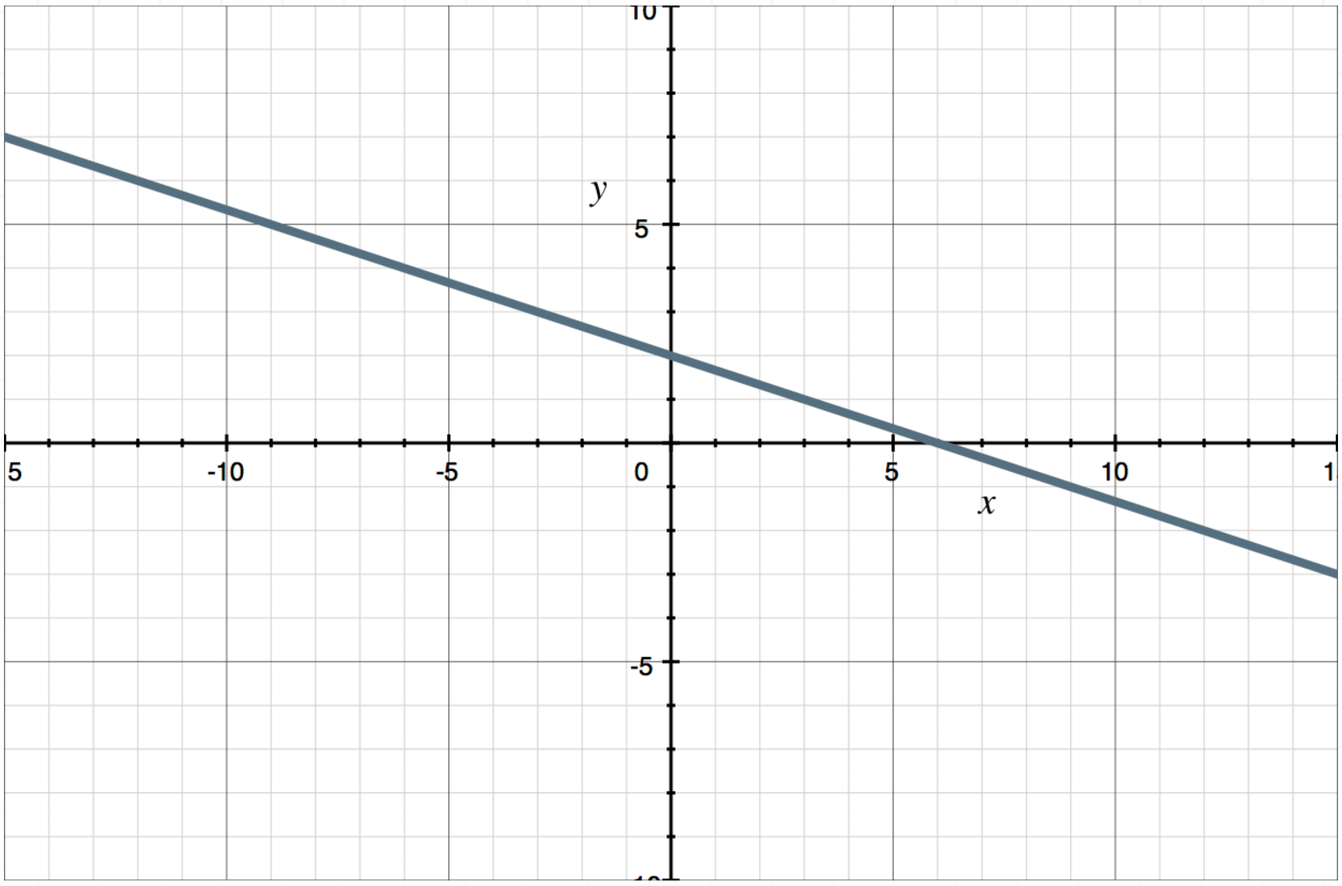

Parallel lines have the same slope. We want to draw the line that has the same slope as this one but goes through the point $(0,-2)$.

Start by graphing the point $(0,-2)$. To get the slope of the original line, we'll start from its y-intercept (the point $(0,2)$) and use the fact that the slope of a line is given by the ratio of the "rise" to the "run." Note that the point $(3,1)$ is on the original line, and that we get there from the point $(0,2)$ by going 1 unit down (from $y=2$ to $y=1$, which is a rise of -1) and 3 units to the right (from $x=0$ to $x=3$, which is a run of 3 units). Therefore, the slope of the given line (and also of the new line) is $-1/3$.

Then we'll start from the y-intercept of the new line (the point $(0,-2)$) and find the point that we get to by using that same rise and run (1 unit down and 3 units to the right). The coordinates of that point are $(0+3,-2-1)=(3,-3)$.

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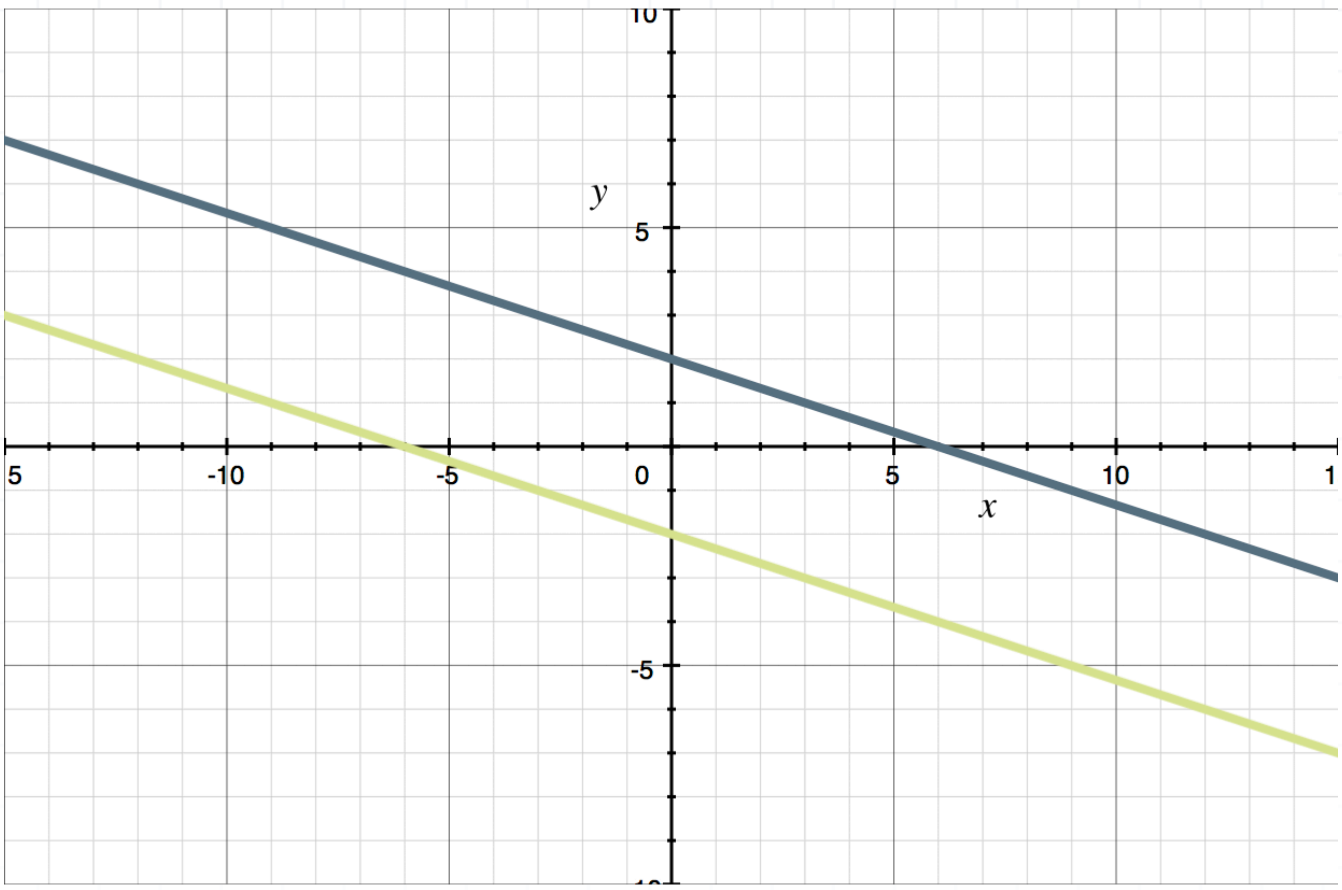
Connect the points $(0, -2)$ and $(3, -3)$, and draw the new line.



The equation of the new line is

$$y = -\frac{1}{3}x - 2$$

Let's look at an example of perpendicular lines.

Example

Write the equation of the line that passes through the point $(-2,5)$ and is perpendicular to the line

$$y = -\frac{4}{7}x - 2$$

Remember, perpendicular lines have slopes that are negative reciprocals of each other. In other words, the slope of the new line needs to be the negative reciprocal of $-4/7$, which means the slope of the new line is

$$\frac{7}{4}$$

The equation of a line in slope-intercept form is $y = mx + b$. For our new line, we know the slope, $7/4$, and one point on it, $(-2,5)$.

We can plug the slope and the coordinates of that point into the equation $y = mx + b$ and solve for b.

$$y = mx + b$$

$$5 = \frac{7}{4}(-2) + b$$

$$5 = -\frac{7}{2} + b$$

$$5 + \frac{7}{2} = -\frac{7}{2} + \frac{7}{2} + b$$

$$\frac{10}{2} + \frac{7}{2} = -\frac{7}{2} + \frac{7}{2} + b$$

$$\frac{17}{2} = b$$

The equation of the new line is

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$$y = \frac{7}{4}x + \frac{17}{2}$$

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