

Rational equations

At times we'd like to take an equation that has at least one fraction with a variable in its denominator and write the equation in a different way. We'll call an equation like this an **abstract fractional equation**. This lesson will look at how to do that.

There are a few things we want to remember about rational functions in general.

1. Multiplying a fraction by its reciprocal will always give you a value of 1.

For example x/y has a reciprocal of y/x because

$$\frac{x}{y} \cdot \frac{y}{x} = 1$$

Remember that you can't divide by 0, so you should explicitly state that neither x nor y can be 0, like this:

$$\frac{x}{y} \cdot \frac{y}{x} = 1, x, y \neq 0$$

2. To clear a fraction from an equation, multiply both sides of the equation by the fraction's denominator.

For example, to clear the b from the fraction in

$$ax + \frac{m}{b} = c$$

multiply both sides of the equation by b .



$$ax + \frac{m}{b} = c$$

$$b \left(ax + \frac{m}{b} \right) = b(c)$$

$$b(ax) + b \left(\frac{m}{b} \right) = b(c)$$

$$abx + m = bc$$

Remember that you can't divide by 0, so this new equation is true only if $b \neq 0$.

Let's solve some abstract fractional equations for specific variables.

Example

Solve the equation for n , if $n \neq 0$.

$$\frac{m}{n} + x + ab = c$$

In order to get rid of the fraction, we have to multiply both sides of the equation by the denominator of m/n .

$$\frac{m}{n} + x + ab = c$$

$$n \left(\frac{m}{n} + x + ab \right) = n(c)$$



$$n \cdot \frac{m}{n} + n(x) + n(ab) = n(c)$$

$$m + nx + nab = nc$$

To solve for n , we'll need to collect all terms containing n on one side of the equation, and then factor out the n .

Let's move m to the right side and nc to the left side.

$$nx + nab - nc = -m$$

Now factor out n .

$$n(x + ab - c) = -m$$

Divide both sides by $(x + ab - c)$.

$$n = \frac{-m}{x + ab - c}, n \neq 0$$

We could also write this with the negative sign in front.

$$n = -\frac{m}{x + ab - c}, n \neq 0$$

Let's try another one.

Example

Solve for x if $x \neq 0$ and $y \neq 0$.



$$\frac{1}{x} - \frac{m}{y} = p$$

In order to get rid of the fractions, we have to multiply both sides of the equation by the denominators of both fractions, x and y .

$$\frac{1}{x} - \frac{m}{y} = p$$

$$xy \left(\frac{1}{x} - \frac{m}{y} \right) = xy(p)$$

$$xy \left(\frac{1}{x} \right) - xy \left(\frac{m}{y} \right) = xy(p)$$

$$y - mx = xyp$$

To solve for x we'll need to collect all terms containing x on one side of the equation, and then factor out the x .

Let's move mx to the right side.

$$y = mx + xyp$$

It's nice to end up with the variable that we're solving for (in this case x) on the left side, so we'll switch the two sides of this equation.

$$mx + xyp = y$$

Now factor out the x .

$$x(m + yp) = y$$



Divide both sides by $m + yp$.

$$\frac{x(m + yp)}{m + yp} = \frac{y}{m + yp}$$

$$x = \frac{y}{m + yp}, x, y \neq 0$$

