Factoring quadratic polynomials

Factoring of quadratic polynomials (second-degree polynomials) is done by "un-FOILing," which means we start with the result of a FOIL problem and work backwards to find the two binomial factors.

To factor a quadratic polynomial in which the x^2 term has a coefficient of 1 and the constant term is nonzero (in other words, a quadratic polynomial of the form $x^2 + ax + b$ where $b \neq 0$), you'll be considering pairs of factors of the last term (the constant term) and finding the pair of factors whose sum is the coefficient of the middle term (the x-term).

Example

Factor the quadratic polynomial.

$$x^2 - x - 20$$

Start by listing the pairs of factors of the constant term, -20, and their sums. We're looking for the pair of factors whose sum is -1 (the coefficient of the x-term).

Factors of -20	Sum
-1 and 20	19
1 and -20	-19
-2 and 10	8
2 and -10	-8
-4 and 5	1
4 and -5	-1

Since 4 and -5 have a sum of -1, they're the factors we need. The answer is

$$(x+4)(x-5)$$

To check our answer, we can FOIL $(x+4)(x-5)$.

$$x^2-5x+4x-20$$

$$x^2-x-20$$

Let's try another example of factoring a quadratic polynomial.

Example

Factor the quadratic polynomial.

$$x^2-8x+15$$

Start by listing the pairs of factors of 15 and their sums. We're looking for the pair of factors whose sum is -8 (the coefficient of the x-term).

Factors of 15	Sum
1 and 15	16
-1 and -15	-16
3 and 5	8
-3 and -5	-8

The factors -3 and -5 have a sum of -8, so they're the correct factors.

$$(x-3)(x-5)$$

To check our answer, we can FOIL $(x-3)(x-5)$.

$$x^2-5x-3x+15$$

$$x^2-8x+15$$

If the coefficient of the x^2 term in a quadratic polynomial is either -1 or the greatest common factor of the polynomial, we can first factor that out and then use the procedure described above to factor what's left over.

Example

Factor the quadratic polynomial.

$$4x^2-20x+24$$

The greatest common factor of this polynomial is 4, so we first factor out a 4.

$$4(x^2 - 5x + 6)$$

Since $(-3)(-2) = 6$ and $(-3) + (-2) = -5$, we see that $x^2 - 5x + 6$ can be factored as follows:

$$(x - 3)(x - 2)$$

So the given quadratic polynomial can be factored as

$$4(x - 3)(x - 2)$$

In later lessons, you'll learn how to factor more complicated quadratic polynomials - those in which all of the following conditions are satisfied:

- The coefficient of the x^2 term is neither 1 nor -1.
- The coefficient of the x^2 term isn't the greatest common factor of the polynomial.
- The constant term is nonzero.