Multiplying multivariable polynomials

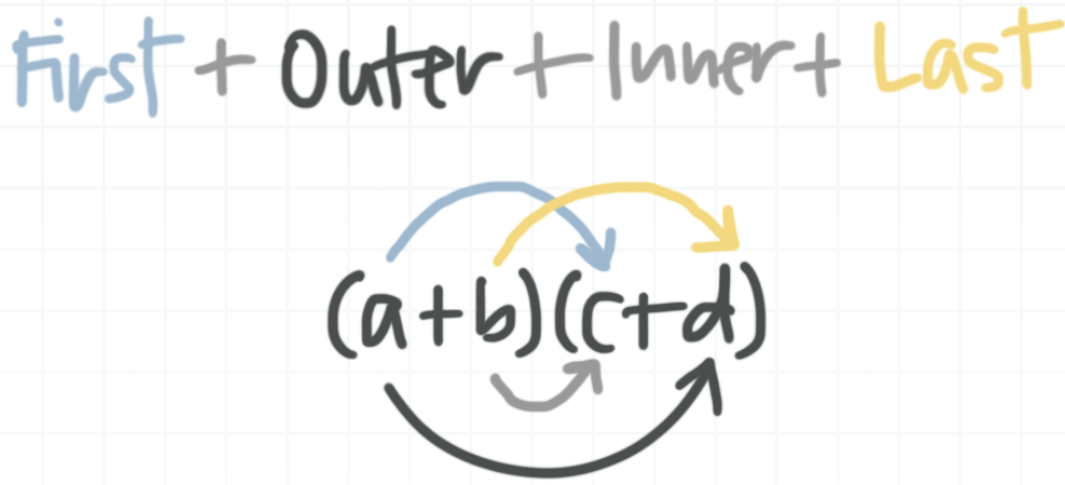
Multiplying multivariable polynomials (polynomials with two or more different variables) is very similar to multiplying single-variable polynomials (those that have just one variable). You'll just need to be careful about combining like terms. In the case of a multivariable polynomial, two terms aren't "like terms" unless each variable has the same exponent in both of them.

For example, $3x^2y^3$ and $-5x^2y^3$ are like terms: In both terms, the variable x has an exponent of 2 and the variable y has an exponent of 3. However, $-9x^2y$ and $8xy$ are **not** like terms: In $-9x^2y$ the variable x has an exponent of 2, but in $8xy$ the variable x has an exponent of 1.

Here are some reminders about multiplying polynomials.

FOIL

FOIL is a way to help you remember to multiply each term in the first set of parentheses by each term in the second set of parentheses. FOIL stands for **Firsts**, **Outsides**, **Insides**, **Lasts**, which is the order of the four terms in the result of the multiplication; it also indicates which terms in the given binomials are multiplied to produce each term in the result.



Chart

The example below shows binomial multiplication (two terms by two terms), but a chart can be used when multiplying by more than two terms as well.

	c	d
a	ac	ad
b	bc	bd

$$ac + ad + bc + bd$$

Let's look at a couple of examples where we multiply multivariable polynomials.

Example

Simplify the expression.

$$(x - 2y)(2x^3 - 3xy - y^2)$$

A chart will be useful to make sure we distribute every term in the first set of parentheses across all the terms in the second set of parentheses.

	$2x^3$	$-3xy$	$-y^2$
x	$2x^4$	$-3x^2y$	$-xy^2$
-2y	$-4x^3y$	$6xy^2$	$2y^3$

$$2x^4 - 3x^2y - xy^2 - 4x^3y + 6xy^2 + 2y^3$$

Next, we'll rearrange the terms in descending order of powers of x (a common practice when dealing with multivariable polynomials).

$$2x^4 - 4x^3y - 3x^2y - xy^2 + 6xy^2 + 2y^3$$

Now we'll group like terms together, and then combine like terms.

$$2x^4 - 4x^3y - 3x^2y + (-xy^2 + 6xy^2) + 2y^3$$

$$2x^4 - 4x^3y - 3x^2y + 5xy^2 + 2y^3$$

Let's try another example of multiplying multivariable polynomials.

Example

Simplify the expression.

$$(2x + 3y)(x - y) + (x + y)(4x - 2y)$$

Multiply the first pair of binomials by using either a chart or FOIL.

$$2x^2 - 2xy + 3xy - 3y^2 + (x + y)(4x - 2y)$$

Combine like terms.

$$2x^2 + xy - 3y^2 + (x + y)(4x - 2y)$$

Multiply the other pair of binomials by using either a chart or FOIL.

$$2x^2 + xy - 3y^2 + 4x^2 - 2xy + 4xy - 2y^2$$

Group like terms together, and then combine like terms.

$$(2x^2 + 4x^2) + (xy - 2xy + 4xy) + (-3y^2 - 2y^2)$$

$$6x^2 + 3xy - 5y^2$$