Graphing exponential functions

We want to be able to graph exponential functions. The kinds of functions we're talking about are those in which the variable is in the exponent, like these:

$$3^x$$

$$-3^{x-1}-2$$

$$2\cdot\left(\frac{1}{3}\right)^{-x+2}-4$$

$$6\cdot 2^{-x}+1$$

In general, I like to use the following procedure for graphing these kinds of functions:

1. Plug in $x=100$ and $x=-100$, and use the values $f(100)$ and $f(-100)$ to determine the "end behavior" of the function, that is, what happens to the value of the function as $x\to\infty$ and as $x\to-\infty$.

2. One of these will result in an infinite value, the other will give a real-number value. The real-number value is the horizontal asymptote of the exponential function.

3. Plug in a few easy-to-calculate values of x, like $x=-1, 0, 1$, in order to get a couple of points that we can plot.

4. Connect the points with an exponential curve, and draw the horizontal asymptote.

Let's do an example where we walk through each of these steps.

Example

Graph the exponential function.

$$f(x) = -3^{x-1} - 2$$

We'll start by plugging in $x = 100$ and $x = -100$.

For $x = 100$:

$$f(100) = -3^{100-1} - 2$$

$$f(100) = -3^{99} - 2$$

$$f(100) = -(\text{very large positive number}) - 2$$

$$f(100) = \text{very large negative number} - 2$$

$$f(100) = \text{very large negative number}$$

$$f(100) = -\infty$$

For $x = -100$:

$$f(-100) = -3^{-100-1} - 2$$

$$f(-100) = -3^{-101} - 2$$

$$f(-100) = -\frac{1}{3^{101}} - 2$$

$$f(-100) = -\frac{1}{\text{very large positive number}} - 2$$

$$f(-100) = -(0) - 2$$

$$f(-100) = 0 - 2$$

$$f(-100) = -2$$

We'll plug in a few values of x for which the value of $f(x)$ will be easy to calculate.

For $x = 0$:

$$f(0) = -3^{0-1} - 2$$

$$f(0) = -3^{-1} - 2$$

$$f(0) = -\frac{1}{3^1} - 2$$

$$f(0) = -\frac{1}{3} - \frac{6}{3}$$

$$f(0) = -\frac{7}{3}$$

For $x = -1$:

$$f(-1) = -3^{-1-1} - 2$$

$$f(-1) = -3^{-2} - 2$$

$$f(-1) = -\frac{1}{3^2} - 2$$

$$f(-1) = -\frac{1}{9} - \frac{18}{9}$$

$$f(-1) = -\frac{19}{9}$$

For $x = 1$:

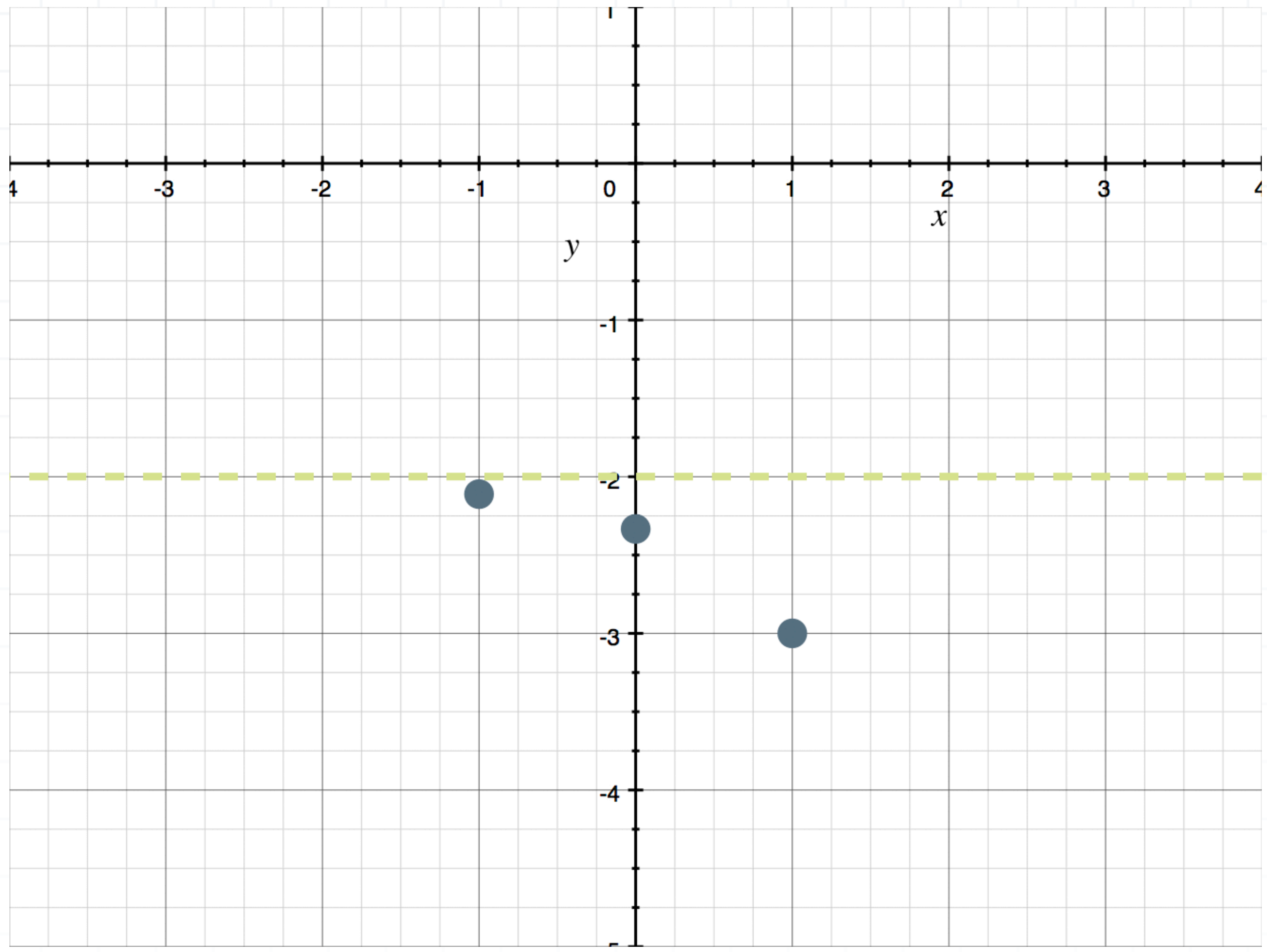
$$f(1) = -3^{1-1} - 2$$

$$f(1) = -3^0 - 2$$

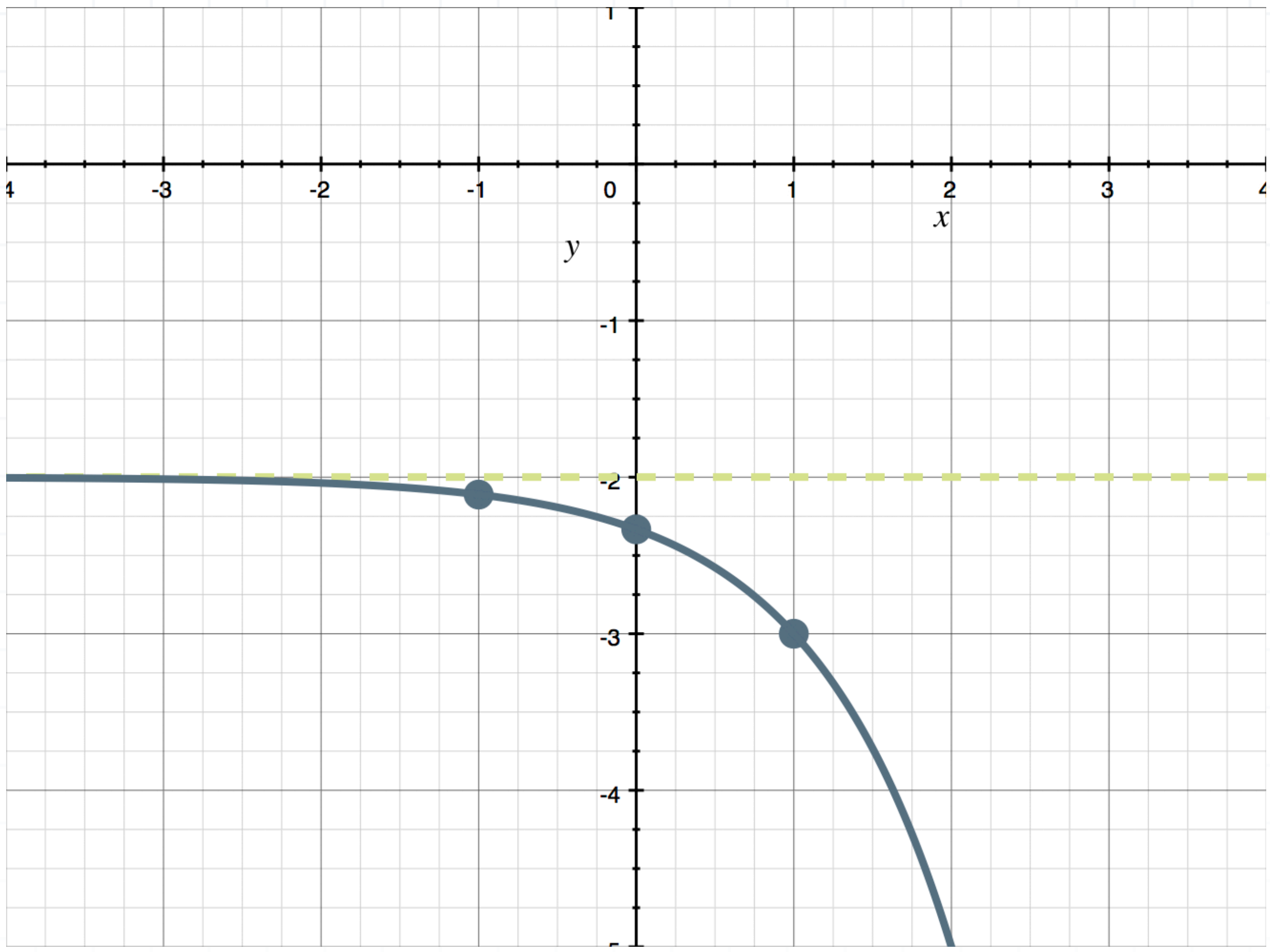
$$f(1) = -1 - 2$$

$$f(1) = -3$$

Now we have three points of the graph of f: $(0, -7/3)$, $(-1, -19/9)$, and $(1, -3)$. If we plot these three points and draw the horizontal asymptote $y = -2$, we get



Based on the asymptote and the points we've found, we can already see what the graph is going to do. We'll simply connect the points to sketch the graph.



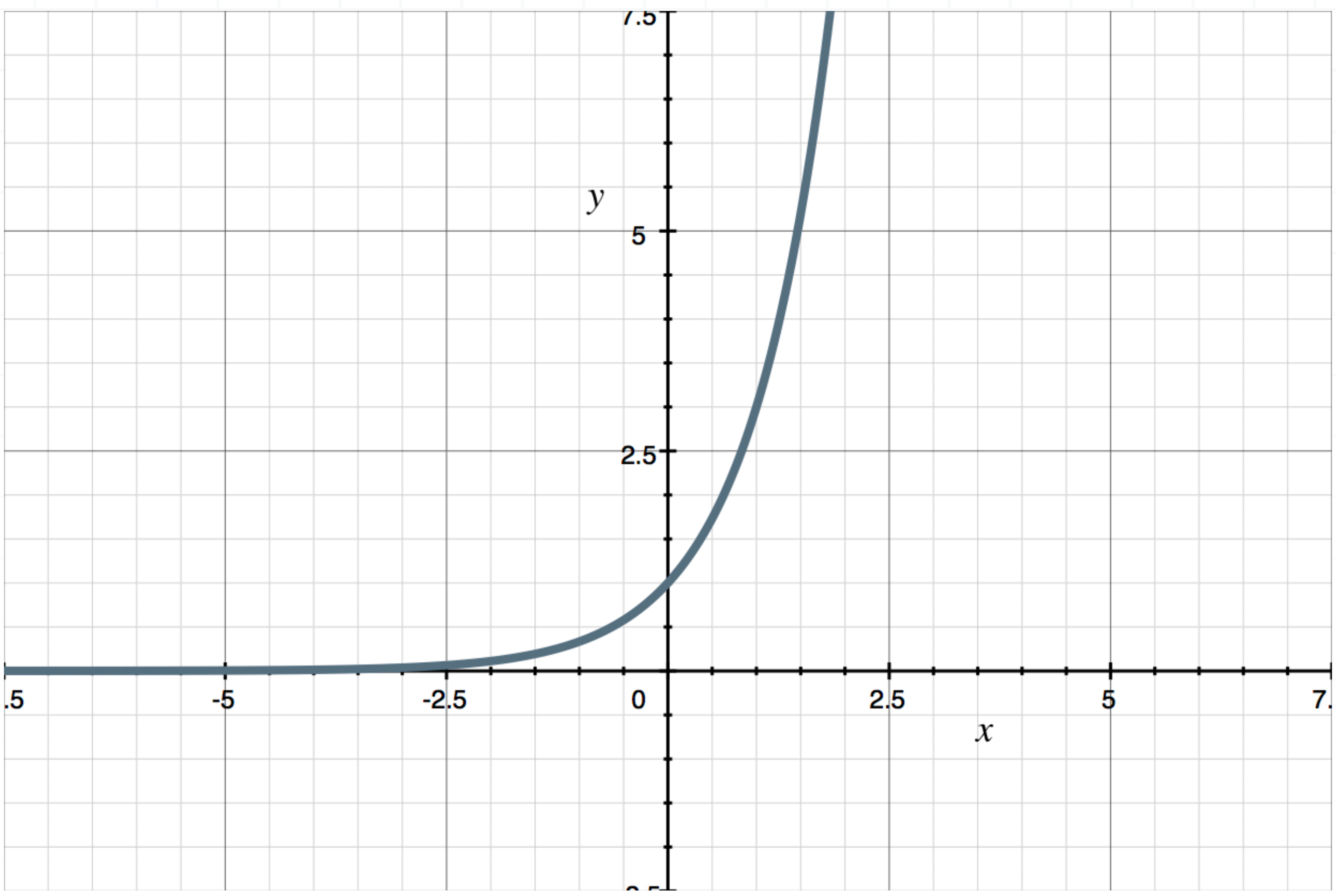
Sometimes we're given the graph of one exponential function, and asked to sketch the graph of a similar function which is just a few "transformations" away from the given function. Each such transformation consists of one of the following operations:

- addition of a constant to the exponent
- addition of a constant to the exponential function
- multiplication of the exponent by a nonzero constant
- multiplication of the exponential function by a nonzero constant

Let's do an example.

Example

The graph of the function 3^x is shown. Use this graph to sketch the graph of the function $6 \cdot 3^{-x} + 1$.



The function $6 \cdot 3^{-x} + 1$ is the result of applying several transformations to the function 3^x in turn. We'll treat each transformation in a separate step, and we'll give different names to the functions we obtain in different steps.

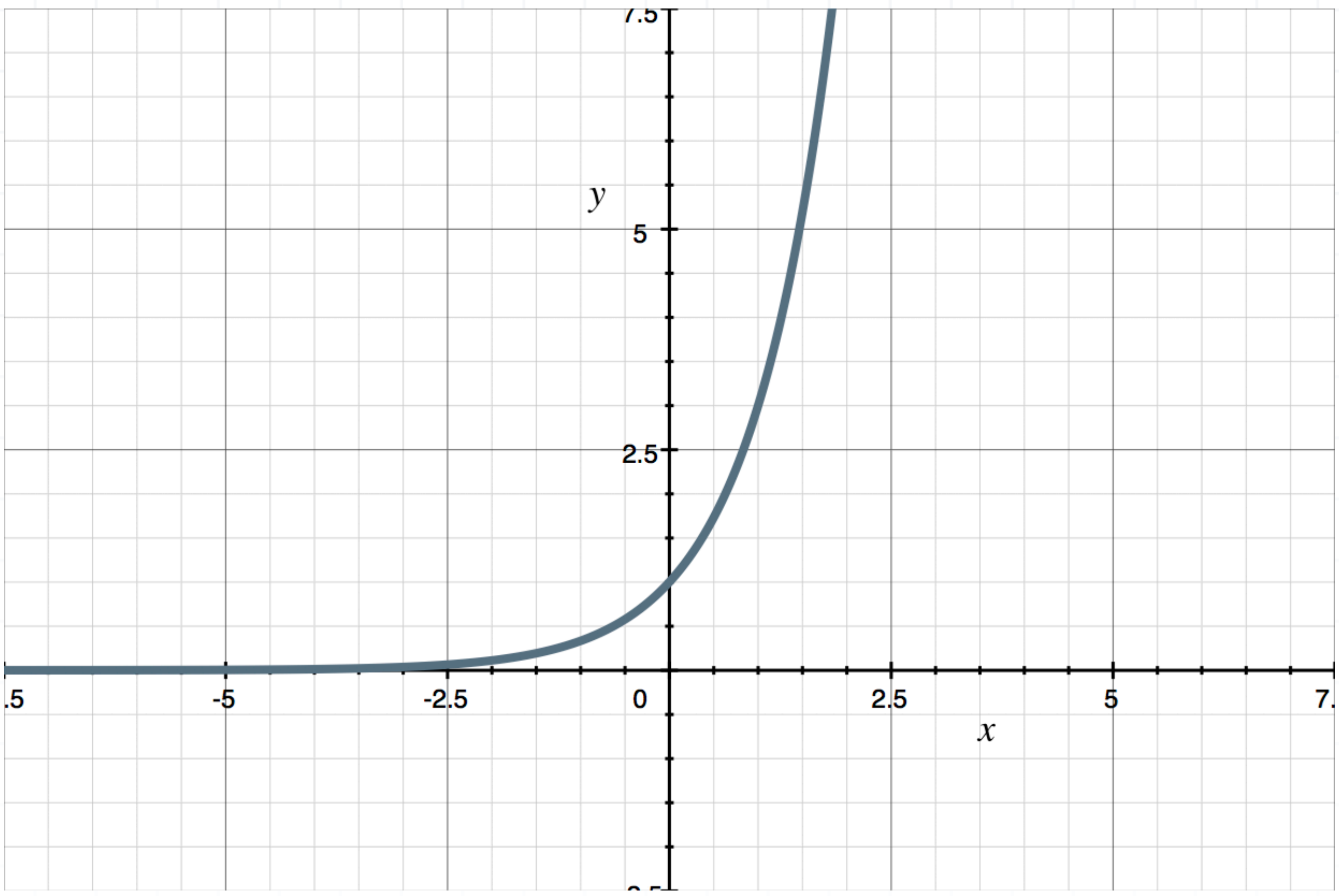
[1] $f(x) = 3^x$

[2] $g(x) = 3^{-x}$

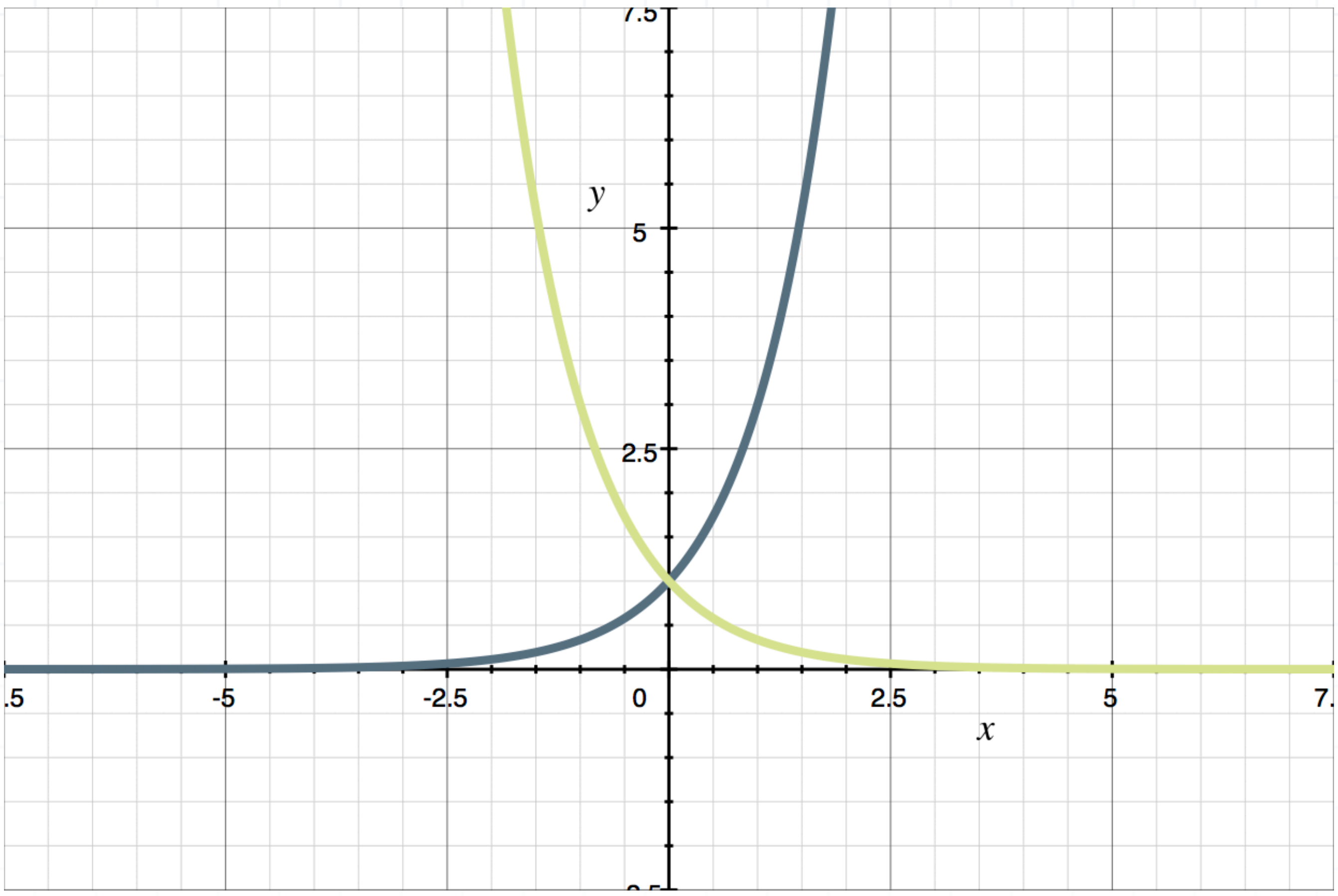
[3] $h(x) = 6 \cdot 3^{-x}$

[4] $k(x) = 6 \cdot 3^{-x} + 1$

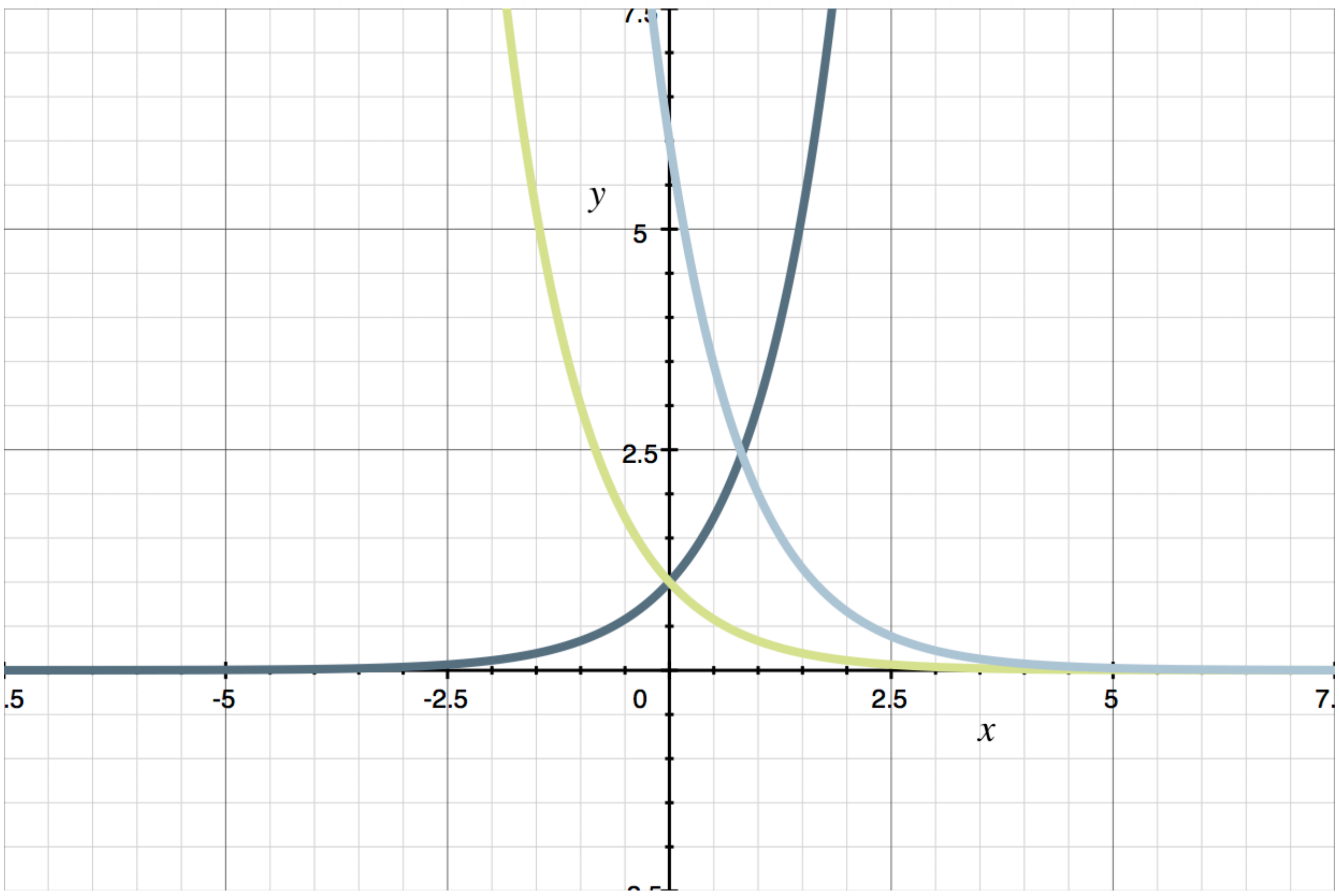
We were given the graph of $f(x)$.



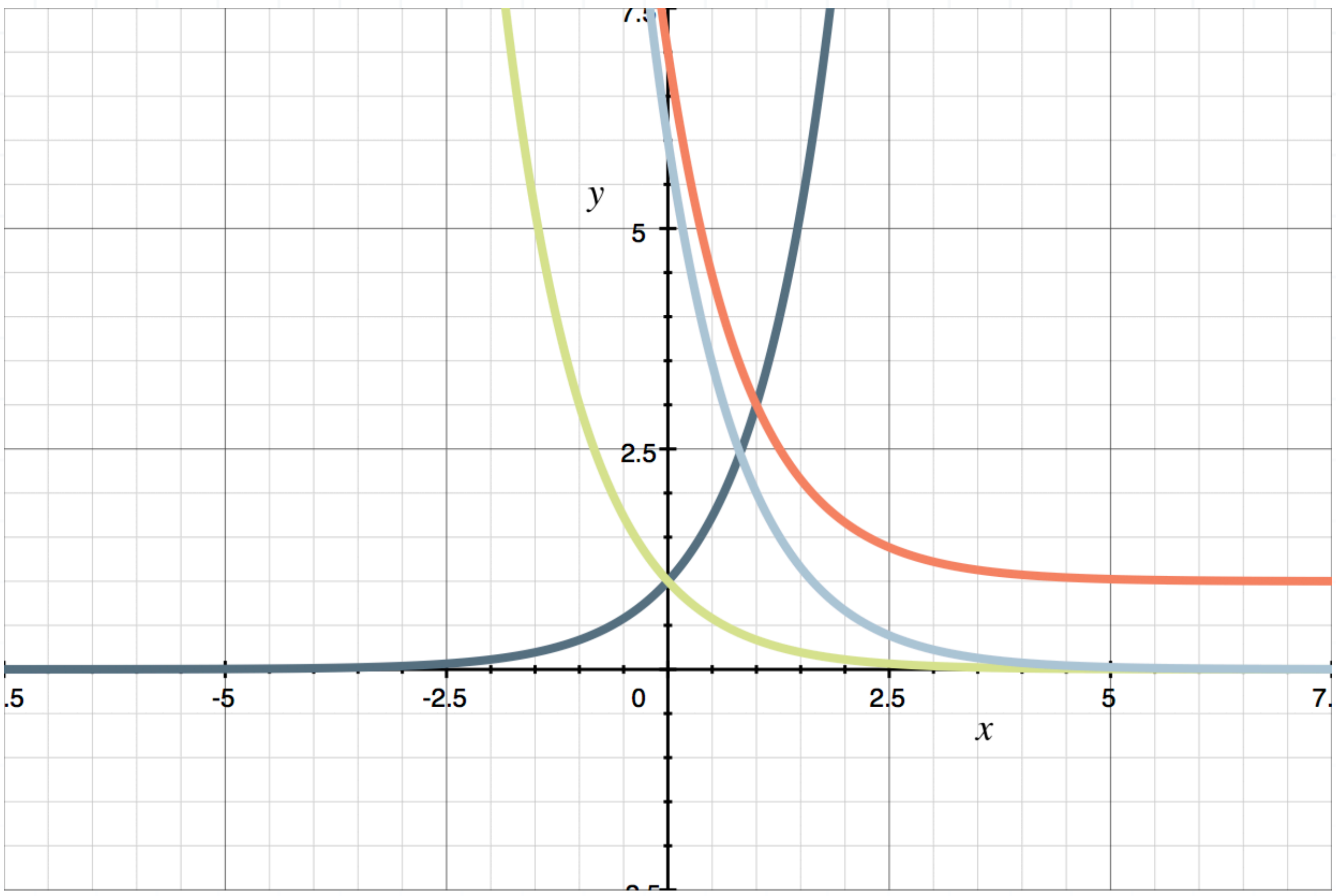
We can think of $g(x) = 3^{-x}$ as substituting $-x$ for x in the expression for $f(x)$. This means that to get the graph of $g(x)$, we're reflecting the graph of $f(x)$ with respect to the y-axis. If you're not sure about this, try plugging a few values of x into the function $g(x) = e^{-x}$. If we graph $g(x)$ on the same set of axes as $f(x)$, we get



To get $h(x) = 6 \cdot 3^{-x}$, we multiply the value of $g(x)$ by 6. Since the graph of $g(x)$ crosses the y-axis at 1, the graph of $h(x)$ crosses the y-axis at $1 \cdot 6 = 6$.



To get $k(x)$, we add 1 to the value of $h(x)$, so we take the graph of $h(x)$ and shift it upward by 1 unit. Since the graph of $h(x)$ crosses the y-axis at 6, the graph of $k(x)$ crosses the y-axis at $6+1=7$.



To summarize, we started with 3^x and its graph. To get the graph of $6 \cdot 3^{-x}+1$, we applied one transformation at a time.

[1] $f(x) = 3^x$

[2] $g(x) = f(-x) = 3^{-x}$

[3] $h(x) = 6 \cdot g(x) = 6 \cdot 3^{-x}$

[4] $k(x) = h(x) + 1 = 6 \cdot 3^{-x} + 1$