Algebra 1 Workbook Solutions

Functions

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DOMAIN AND RANGE

1. Find the domain of $f(x)$.

$$f(x) = \frac{3}{x(x+1)} + x^2$$

Solution:

In this function, the denominator cannot be equal to 0. The values of x that make the denominator 0 are $x = 0$ and $x = -1$. So the domain of the function is all $x \neq 0, -1$, which we can write in interval notation as

$$(-\infty, -1) \cup (-1,0) \cup (0,\infty)$$

2. Find the domain and range of the given set.

$$(-1, -3), \quad (0,5), \quad (-3,6), \quad (0, -3)$$

Solution:

The domain is all the x-values and the range is all the y-values. Therefore the domain and range are

Domain: $-1,\ 0,\ -3$

Range: $-3, 5, 6$

3. Find the domain and range of $g(x)$.

$$g(x) = \frac{\sqrt{x-2}}{3}$$

Solution:

In this function, the radicand (the expression under the square root) must be 0 or positive. So $x - 2 \geq 0$, which tells us that $x \geq 2$. Therefore the domain of the function in interval notation is $[2,\infty)$. Since the square root function cannot be negative, the range in interval notation is $[0,\infty)$.

4. Find the domain and range of the function.

$$f(x) = \frac{2}{x} + 1$$

Solution:

In this function, the denominator cannot be 0, which means $x \neq 0$. Therefore the domain of the function in interval notation is

$$(-\infty,0) \cup (0,\infty)$$

Since the term $2/x$ is never 0, $f(x)$ can never be 1. Therefore the range of the function in interval notation is

$$(-\infty,1) \cup (1,\infty)$$

5. Give an example of a function that has a domain of $[1,\infty)$.

Solution:

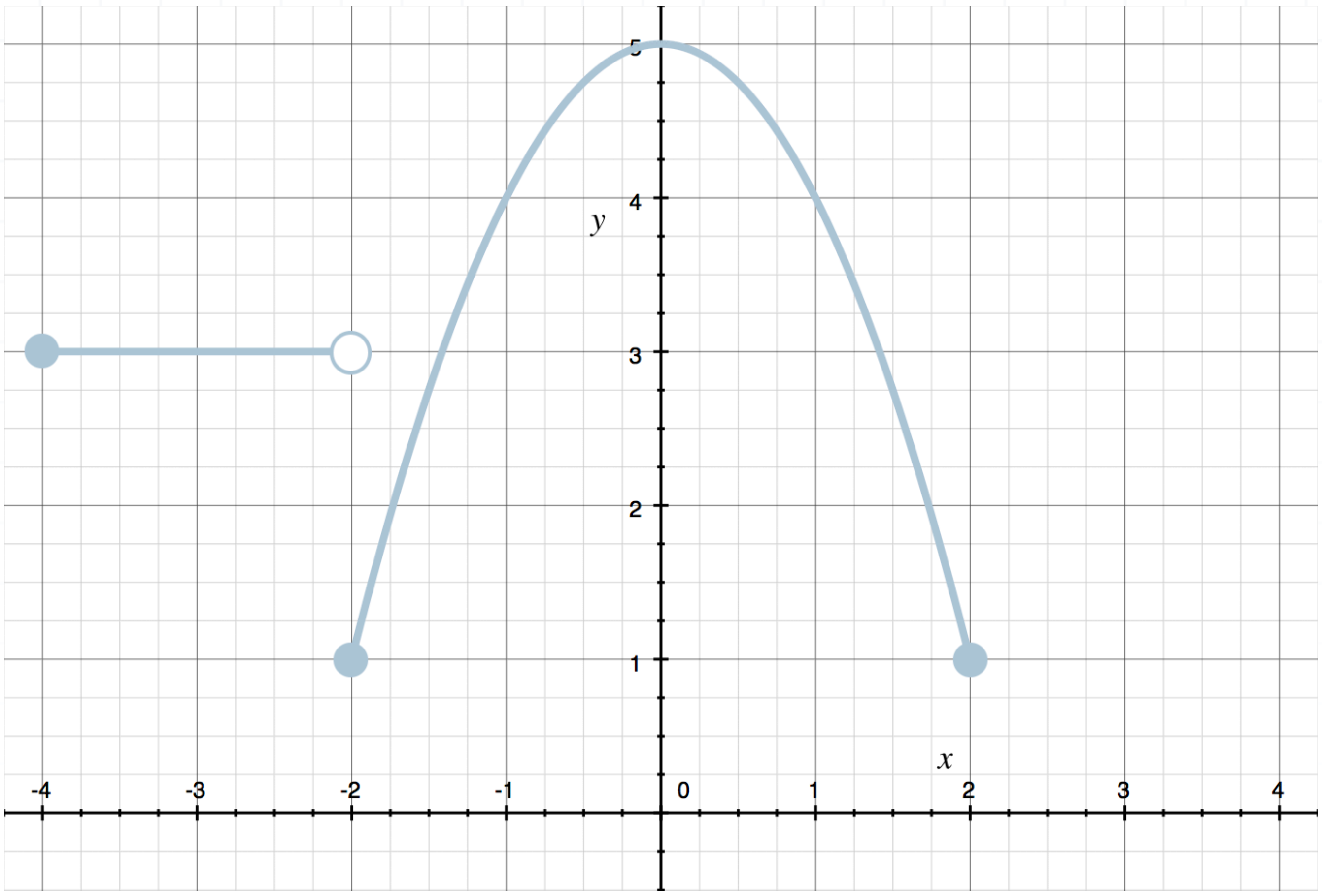
There are many correct answers. The simplest one is

$$f(x) = \sqrt{x-1}$$

Note that since the 1 is included in the domain, the function $f(x) = \ln(x-1)$ would not work.

DOMAIN AND RANGE FROM A GRAPH

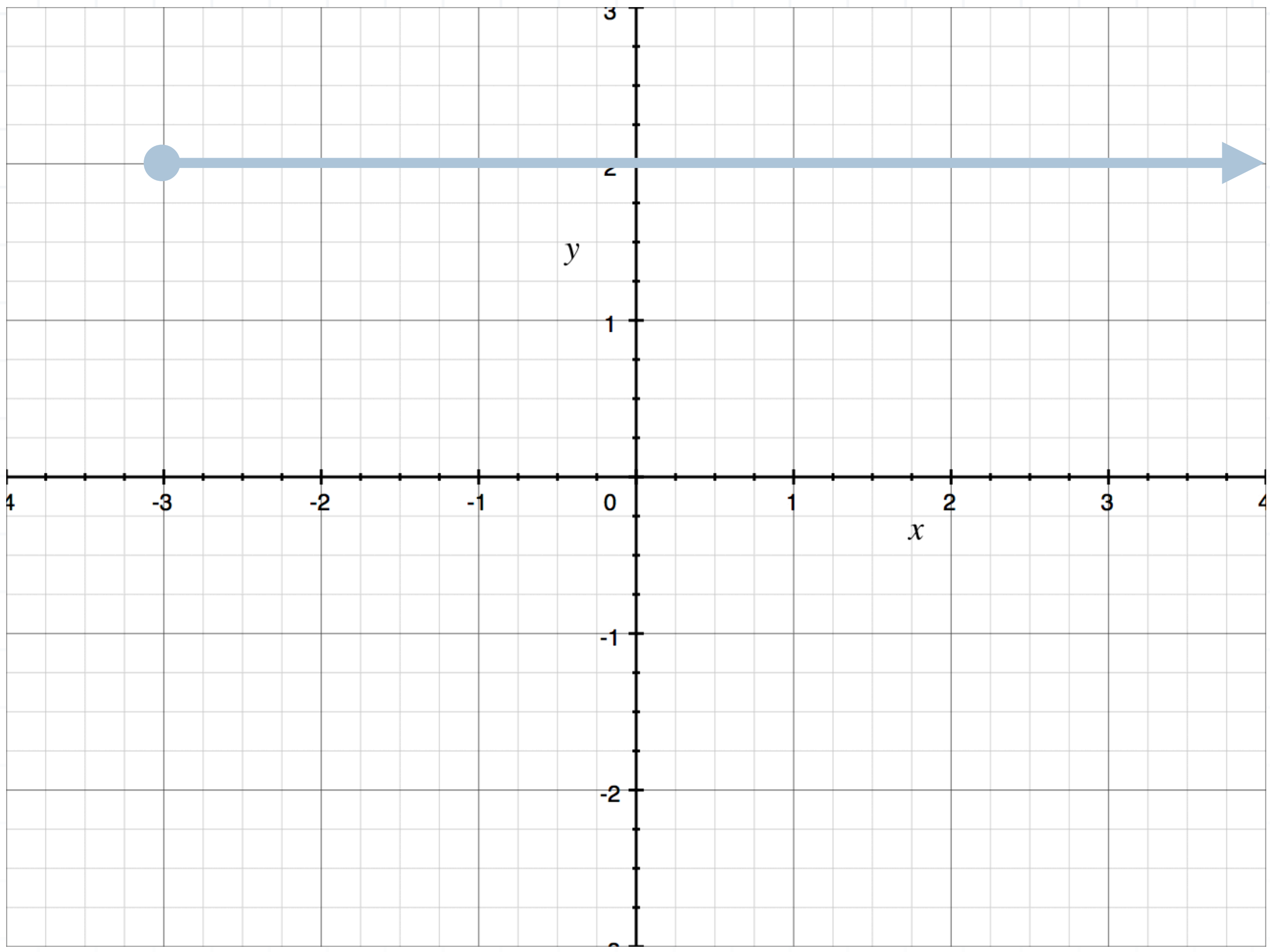
1. What is the domain and range of the function? Assume the graph does not extend beyond the graph shown.



Solution:

Solution: The domain of the function given in the graph is determined by the x-values, which are defined on the interval $[-4,2]$. The range is determined by the y-values, which are defined on the interval $[1,5]$.

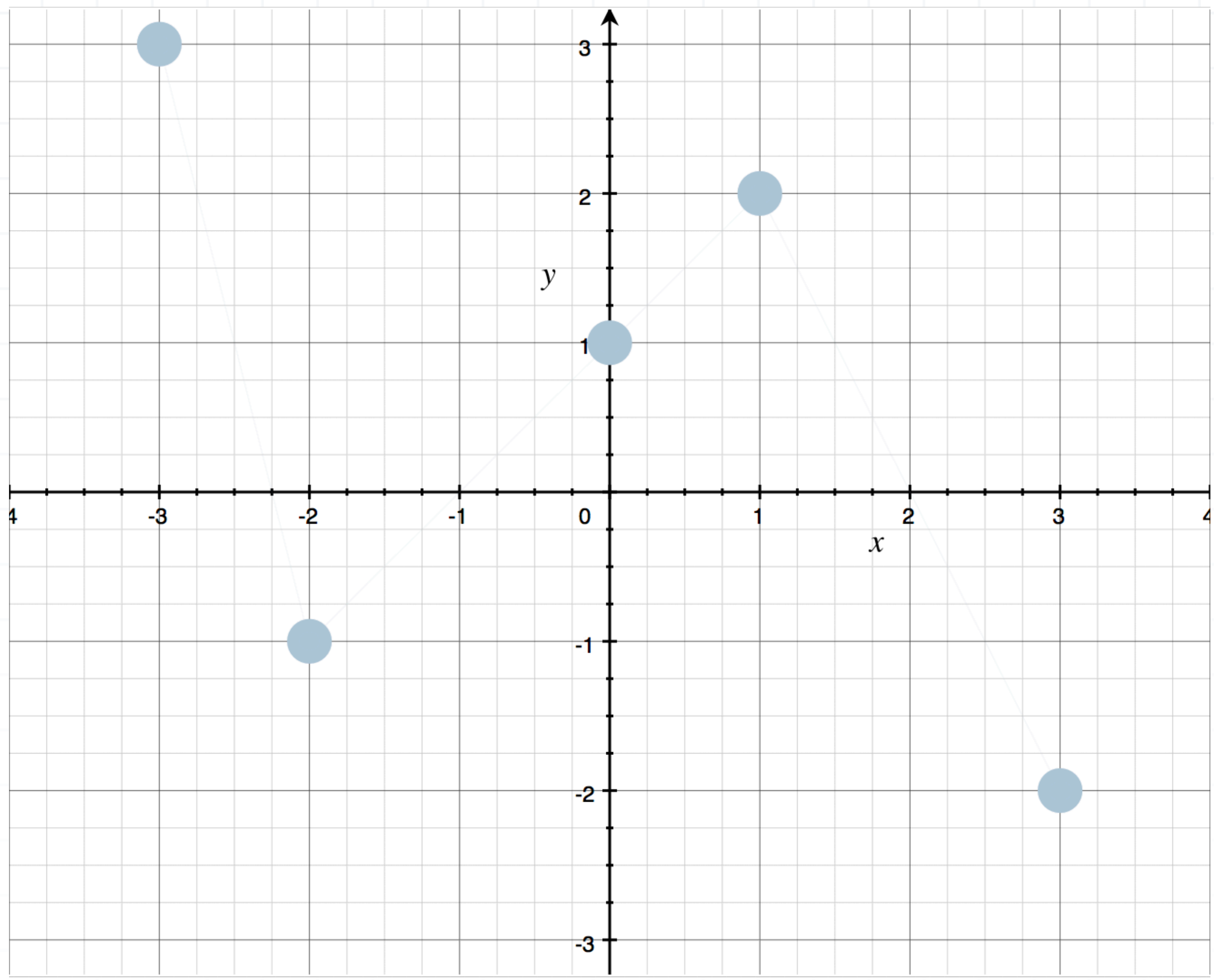
2. What is the domain and range of the function?



Solution:

The domain of the function given in the graph is determined by the x-values, which are defined by the ray on the interval $[-3,\infty)$. The range is determined by the y-values, which is only $y=2$.

3. Determine the domain and range of the function.



Solution:

The domain of the function given in the graph is determined by the x-values, which are $\{-3,-2,0,1,3\}$. The range is determined by the y-values, which are $\{-2,-1,1,2,3\}$.

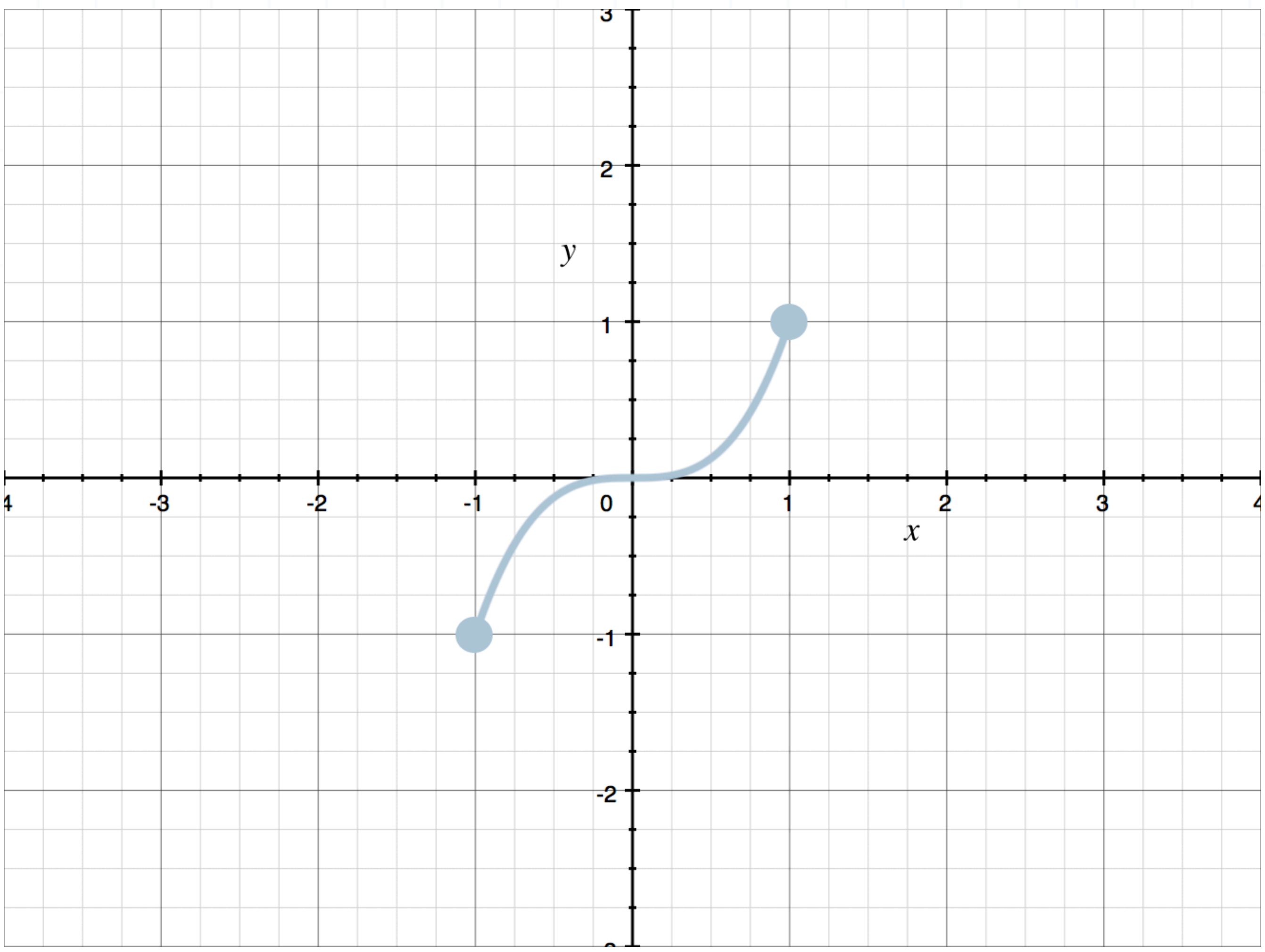
4. Fill in the blanks in the following description of the domain of a graph.

"The domain is all the values of the graph from __________ to __________."

Solution:

left, right

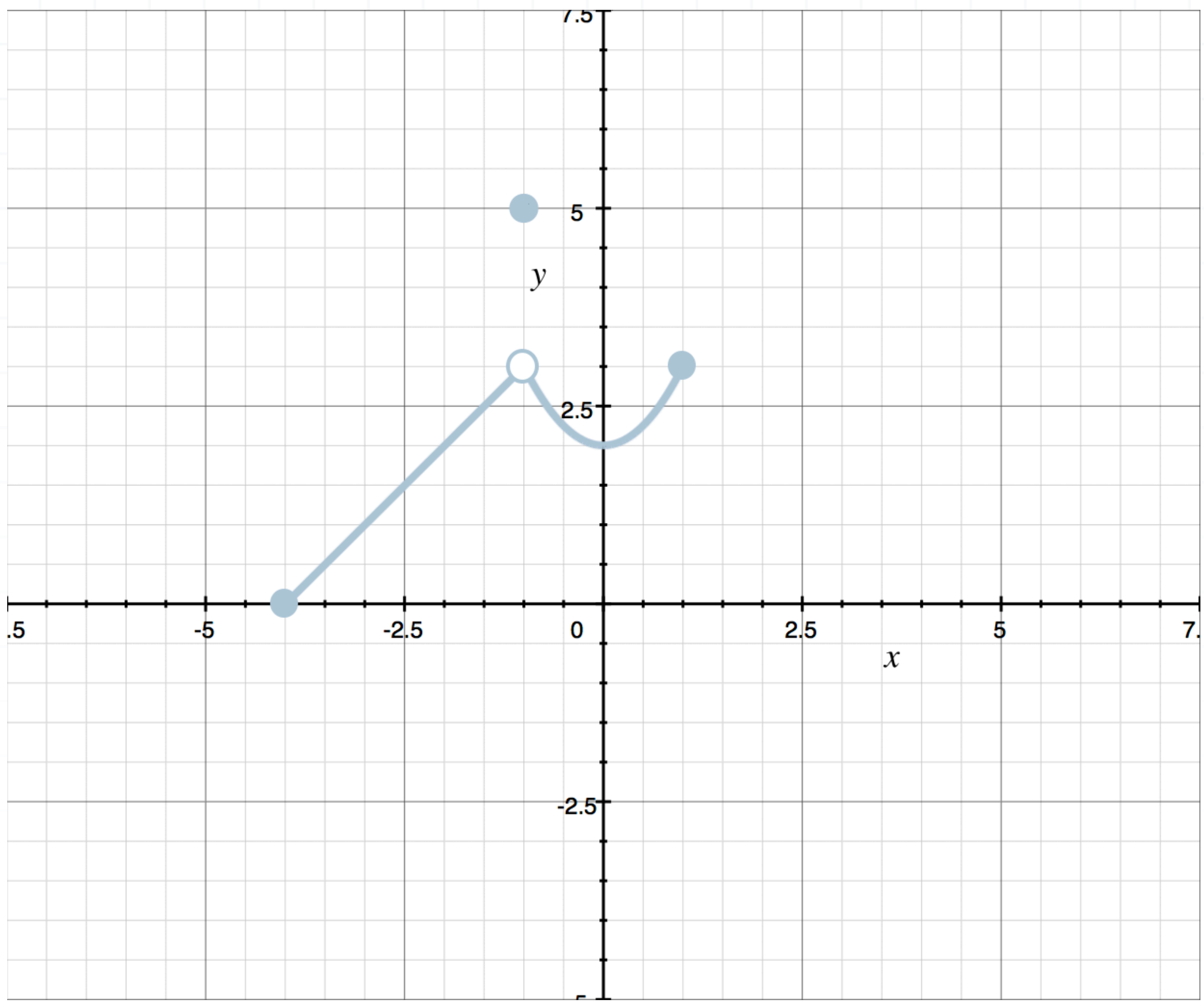
5. What is the domain and range of the function? Assume the graph does not extend beyond the graph shown.



Solution:

The domain of the function given in the graph is determined by the x -values, which are defined by the interval $[-1,1]$. The range is determined by the y-values, which are defined by the interval $[-1,1]$.

6. What is the domain and range of the function? Assume the graph does not extend beyond the graph shown.



Solution:

The domain of the function given in the graph is determined by the x-values, which are defined on the interval $[-4,1]$. The range is determined by the y-values, which are defined on the interval $[0,3]$ and at $y=5$.

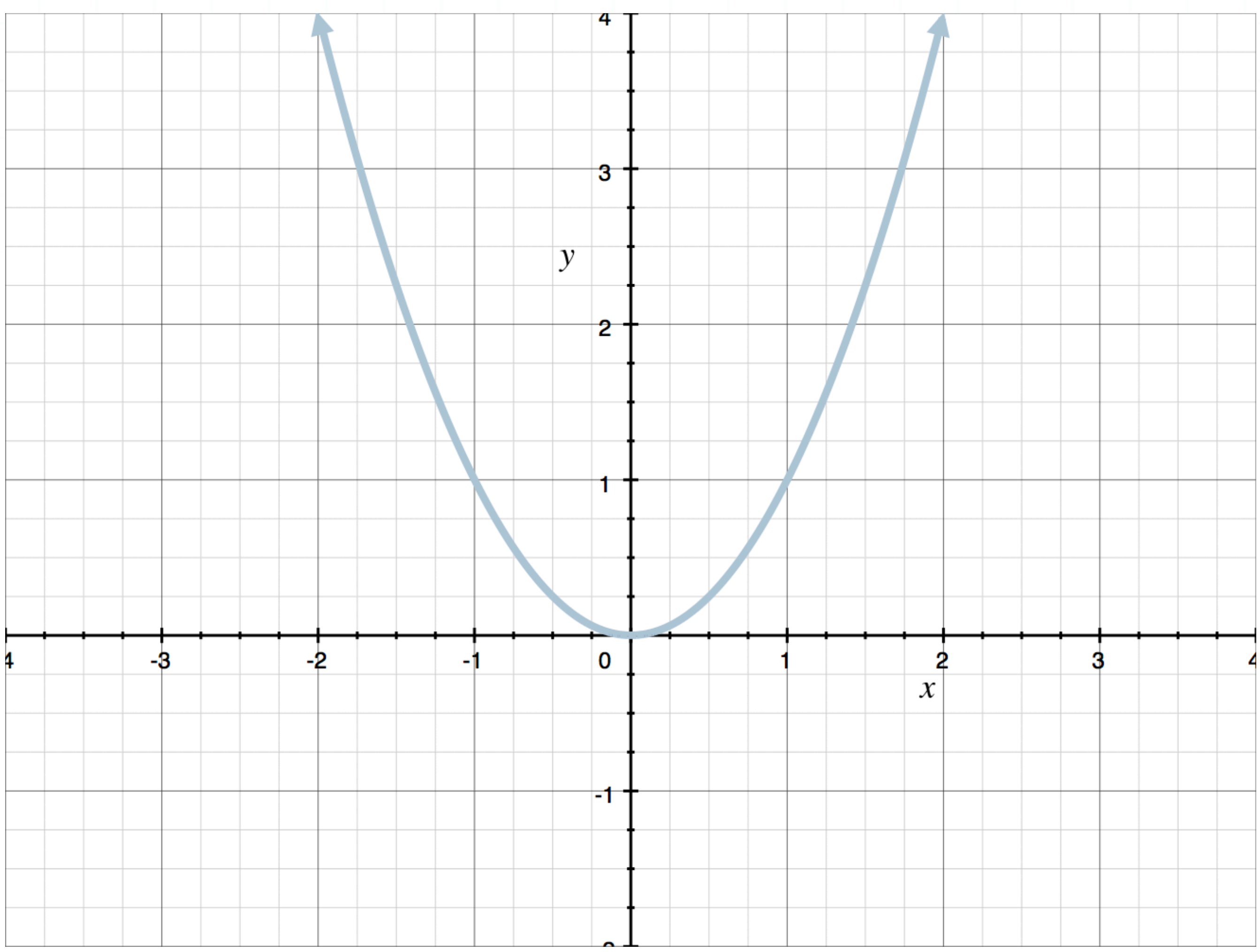
7. Fill in the blanks in the following description of the range of a graph.

"The range is all the values of the graph from __________ to __________."

Solution:

down, up

8. What is the domain and range of the function?



Solution:

The domain of the function given in the graph is determined by the x-values, which are defined on the interval $(-\infty, \infty)$. The range is determined by the y-values, which are defined on the interval $[0,\infty)$.

FUNCTIONAL NOTATION

1. If $f(x)=11-5x$, find $f(-2)$.

Solution:

To find $f(-2)$, we plug $x=-2$ into $f(x)$, which gives

$$f(-2)=11-5(-2)$$

$$f(-2)=11+10$$

$$f(-2)=21$$

2. Find and simplify $f(x+1)$ if $f(x)=4x-5$.

Solution:

To find $f(x+1)$, we plug $x+1$ into $f(x)$ in place of x.

$$f(x+1)=4(x+1)-5$$

$$f(x+1)=4x+4-5$$

$$f(x+1)=4x-1$$

3. Correct what went wrong in the following set of steps.

At $x = -2$ and $f(x) = x^2 + 1$, then

$$f(-2) = -2^2 + 1$$

$$f(-2) = -4 + 1$$

$$f(-2) = -3$$

Solution:

To find $f(-2)$, we plug $x = -2$ into $f(x)$, which is

$$f(-2) = (-2)^2 + 1$$

$$f(-2) = (-2)(-2) + 1$$

$$f(-2) = 4 + 1$$

$$f(-2) = 5$$

4. If $g(t) = t^2 - t + 3$, find $g(-1)$.

Solution:

To find $g(-1)$ we plug $t = -1$ into $g(t)$, which gives

$$g(-1) = (-1)^2 - (-1) + 3$$

$$g(-1) = 1 + 1 + 3$$

$$g(-1) = 5$$

5. Find and simplify $h(s^2)$ if $h(s) = -s^2 + 3s - 1$.

Solution:

To find $h(s^2)$, we plug s^2 into $h(s)$ in place of s.

$$h(s^2) = -(s^2)^2 + 3(s^2) - 1$$

$$h(s^2) = -s^4 + 3s^2 - 1$$

6. If $g(x) = x^3 - x + 1$, figure out what you need to plug into the function in order to get the following expression.

$$g(??) = (2x + 1)^3 - (2x + 1) + 1$$

Solution:

Notice that everywhere there's an x in $g(x)$, there's a $2x + 1$ in the given function. Therefore, the value that got plugged in was $2x + 1$.

7. If $f(x) = x^2 + x - 1$, find $f(x+h)$ and expand as much as possible.

Solution:

To find $f(x+h)$, we plug $x+h$ into $f(x)$ in place of x.

$$f(x+h) = (x+h)^2 + (x+h) - 1$$

$$f(x+h) = x^2 + 2xh + h^2 + x + h - 1$$

8. Correct what went wrong in the following set of steps.

If $f(x) = x^3 + 3x^2 - 5x + 2$, then $f(1)$ is

$$f(1) = (1)^3 + 3(1)^2 - 5(1) + 2$$

$$f(1) = 1 + 9 - 5 + 2$$

$$f(1) = 7$$

Solution:

When evaluating $f(1)$, notice that instead of evaluating $3(1)^2$ as $3(1)^2 = 3(1) = 3$, it was evaluated as $3(1)^2 = 9$. However, the 3 is not being squared, so that's incorrect.

TESTING FOR FUNCTIONS

1. Determine if the following represents a function. Explain your answer.

$$(2,-1),\ (-1,0),\ (0,-1),\ (3,2)$$

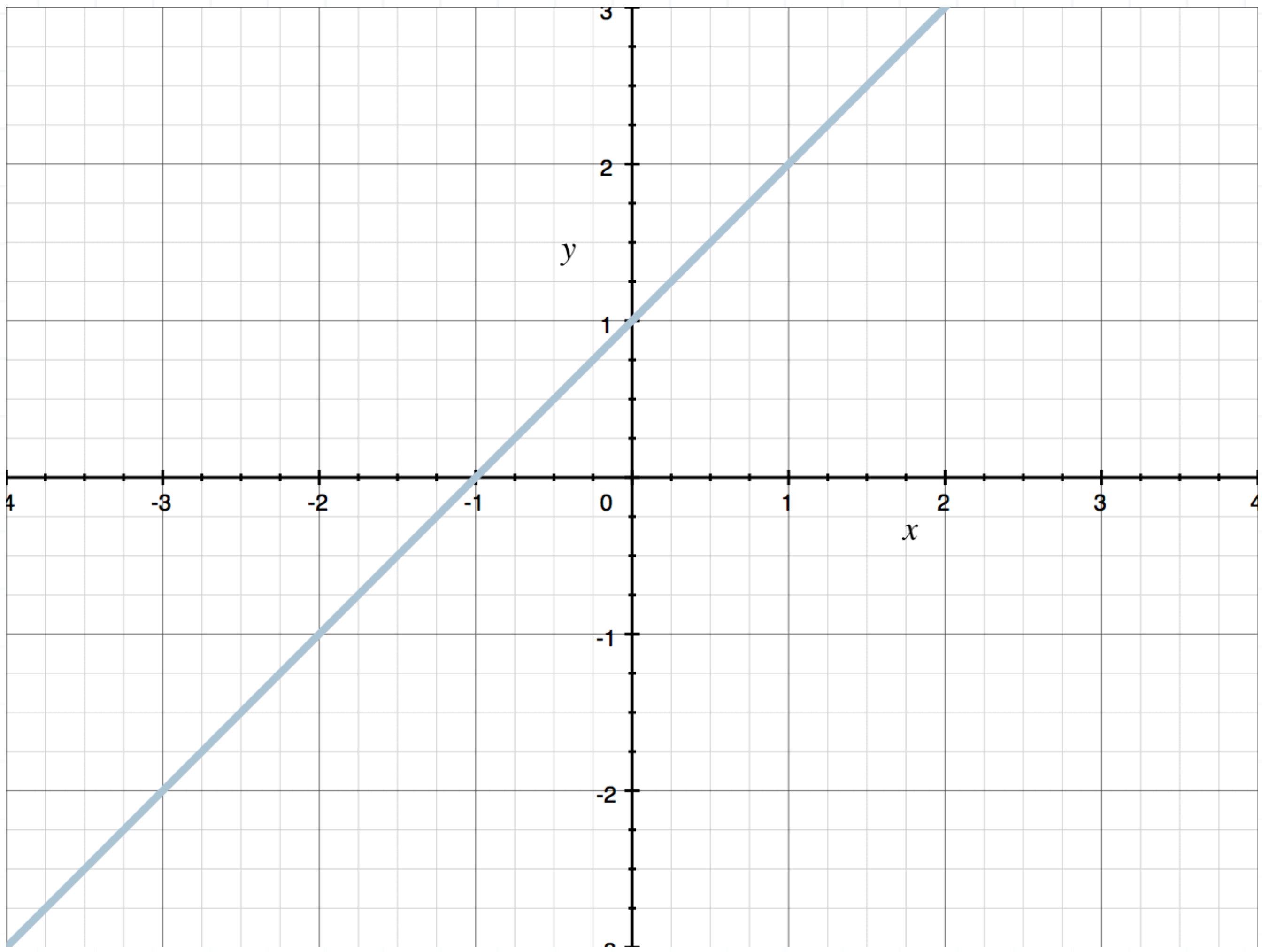
Solution:

For every x-value, there is only one y-value, so the set of points represents a function.

2. Draw a graph that represents a function. Explain why it's a function.

Solution:

There are many correct answers. Below is an example of a function because, for every input x, there is only one output y.



3. Fill in the blanks in the following definition of a function.

For every __________, there is only one unique __________.

Solution:

x (or input), y (or output)

4. Give two different y-values that have the same output value for x.

$$y^2 = x$$

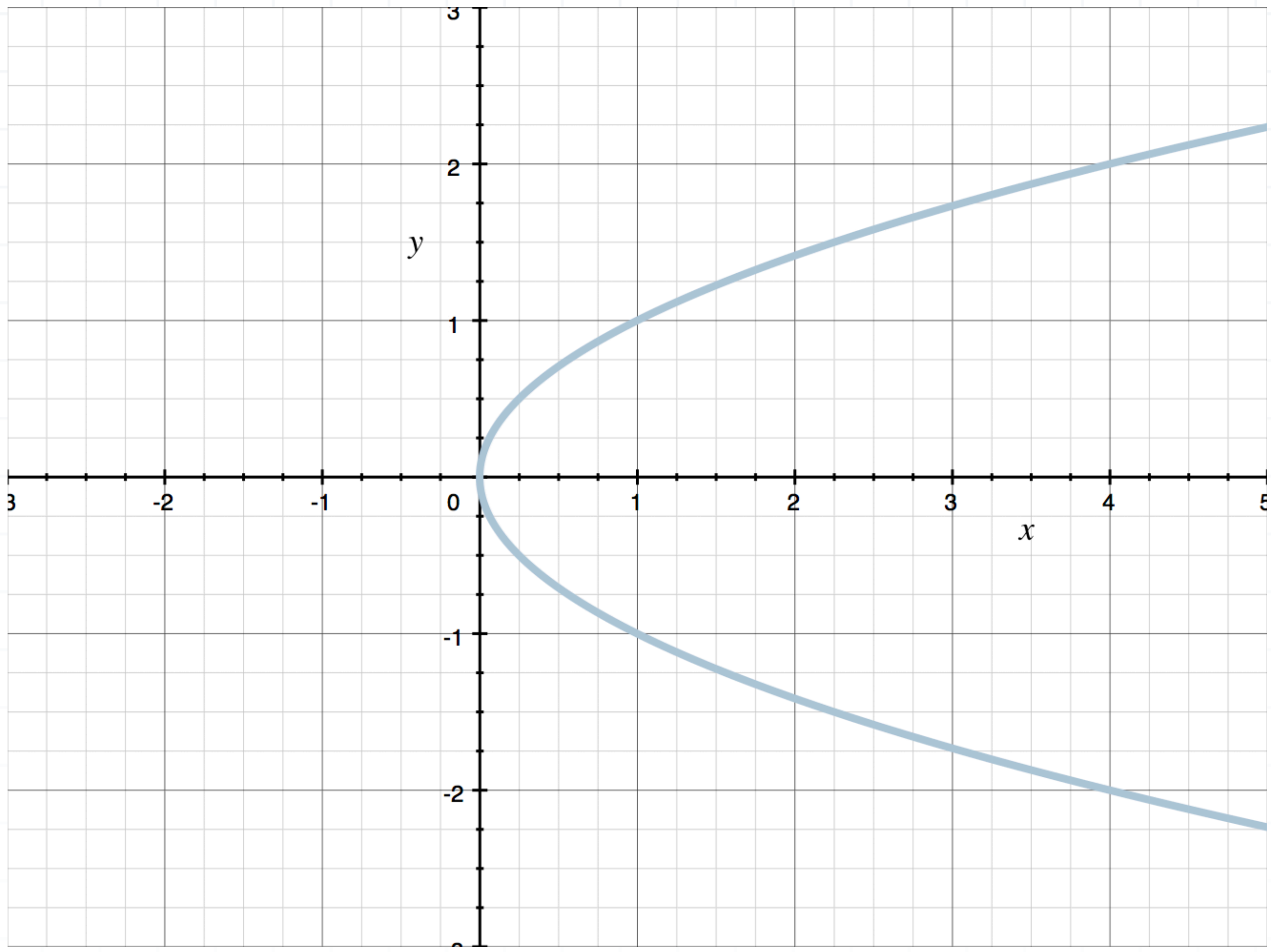
Solution:

There are many correct answers. One example is that, for both $y = 2$ and $y = -2$, the value of x is $x = 4$.

5. Draw a graph that does not represent a function. Explain why it's not a function.

Solution:

There are many correct answers. Below is an example of a graph that's not a function, because for every input $x > 0$, there are two output values for y.



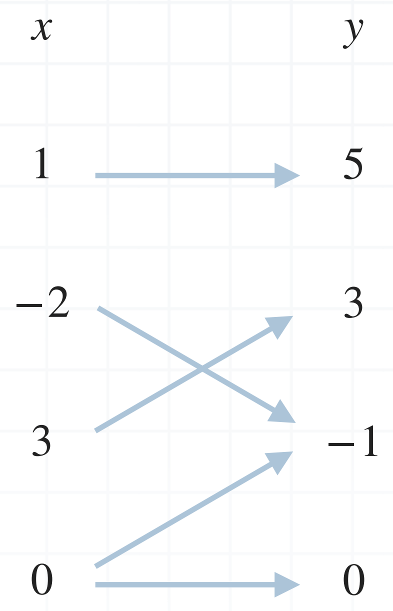
6. Determine whether or not the following set of points represents a function. Explain your answer.

$$(1,2),\ (-1,5),\ (1,-3),\ (0,1)$$

Solution:

Notice that for $x=1$, there are two y-values: $y=2$ and $y=-3$. Since for one input there are two different outputs, this does not represent a function.

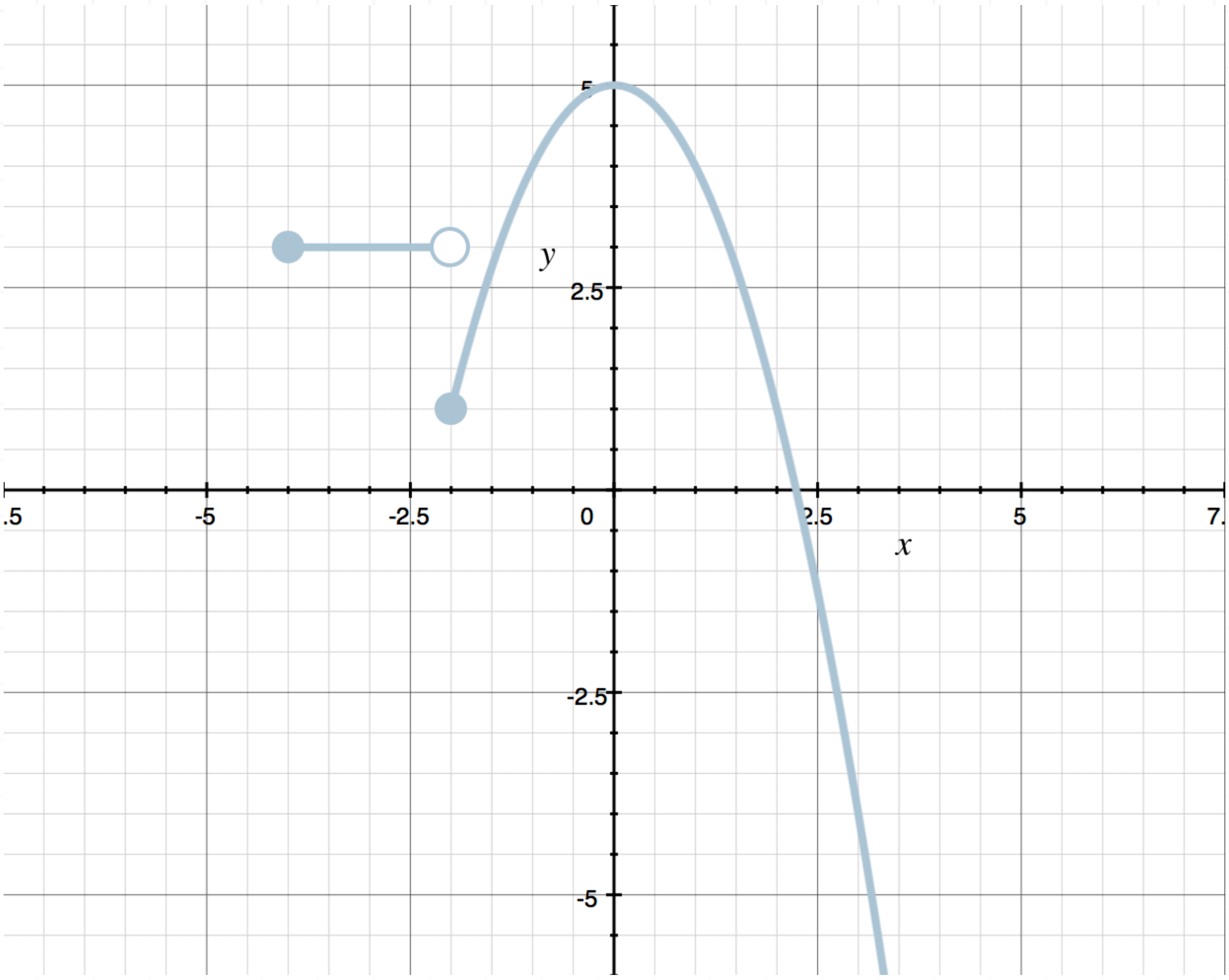
7. Determine if the following represents a function. Explain your answer.



Solution:

Notice that for $x = 0$, there are two y-values: $y = 0$ and $y = -1$. Since for one input there are two different outputs, this does not represent a function.

8. Determine if the following represents a function. Explain your answer.



Solution:

For every x-value, there's a unique y-value. So this graph represents a function.

VERTICAL LINE TEST

1. Determine algebraically whether or not the equation represents a function.

$$(x-1)^2+y=3$$

Solution:

Solve the equation for y.

$$(x-1)^2+y=3$$

$$y=3-(x-1)^2$$

Simplify the right side.

$$y=3-(x-1)(x-1)$$

$$y=3-(x^2-x-x+1)$$

$$y=3-(x^2-2x+1)$$

$$y=3-x^2+2x-1$$

$$y=2-x^2+2x$$

$$y=-x^2+2x+2$$

Each value we plug in for x will give a unique value for y, so the equation represents a function.

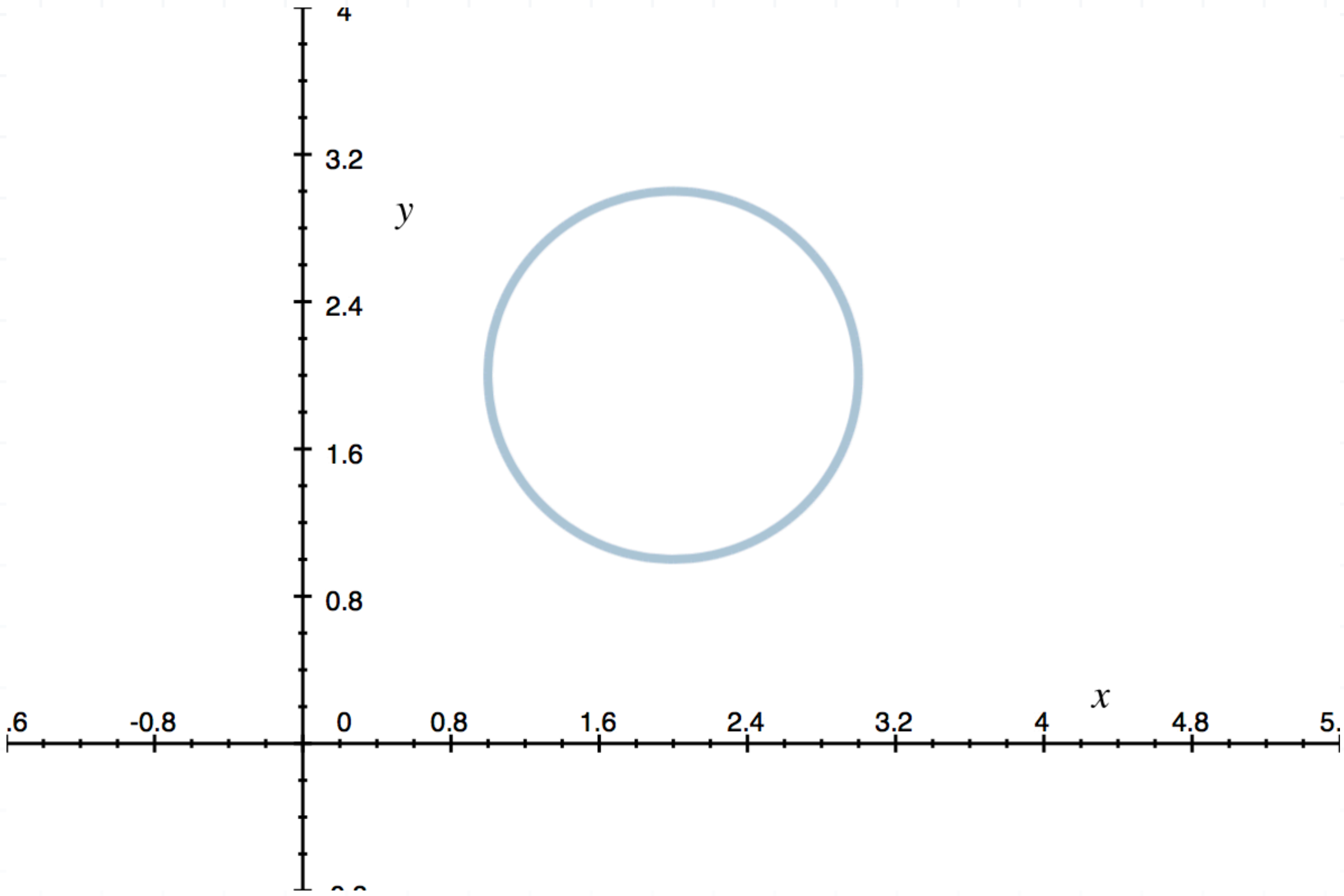
2. Fill in the blanks in the following statement using “equations,” and “functions.”

All __________ are __________.

Solution:

functions, equations

3. Use the Vertical Line Test to determine whether or not the graph is the graph of a function.



Solution:

The graph does not pass the Vertical Line Test, because any vertical line between the left edge of the circle and the right edge of the circle intersects the graph more than once. Therefore, the graph doesn't represent a function.

■ 4. Determine algebraically whether or not the equation represents a function.

$$y^2 = x + 1$$

Solution:

Solve the equation for y.

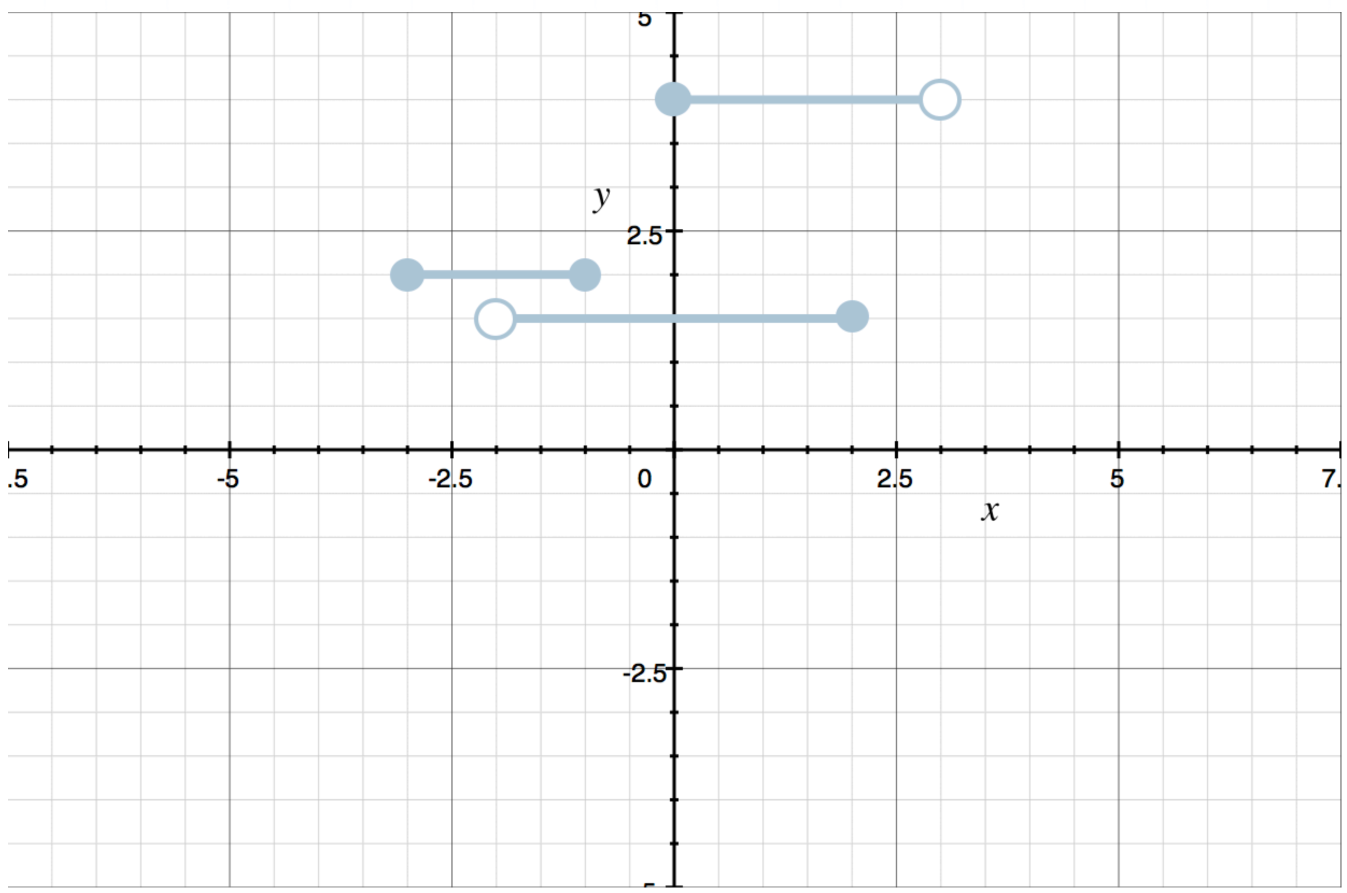
$$y^2 = x + 1$$

$$\sqrt{y^2} = \sqrt{x + 1}$$

$$y = \pm\sqrt{x + 1}$$

Given this equation for y, there are values of x that will give multiple values for y. For instance, at $x = 0$, y takes on values of $y = -1$ and $y = 1$. So for one input x, there are two outputs for y, so the equation does not represent a function.

5. Use the Vertical Line Test to determine whether or not the graph represents a function.



Solution:

There are different vertical lines that intersect the graph more than once. An example would be $x = 0$, which intersects the graph at $y = 3/2$ and $y = 4$. So by the Vertical Line Test, the graph is not a graph of a function.

■ 6. Explain why the Vertical Line Test determines whether or not a graph represents a function.

Solution:

There are many correct answers. But they should all more or less say something like:

> "The Vertical Line Test can show whether or not a graph represents a function, because if any perfectly vertical line crosses the graph more than once, it proves that there are two output values of y for the one input value of x."

■ 7. Fill in the blanks in the following statement using: equations, functions.

Not all __________ are __________.

Solution:

equations, functions

8. Determine algebraically whether or not the equation represents a function.

$$x^3 + y = 5$$

Solution:

Solve the equation for y.

$$x^3 + y = 5$$

$$y = 5 - x^3$$

Each value we plug in for x will give a unique value for y, so the equation represents a function.

SUM OF FUNCTIONS

1. Find $(f+h)(-1)$ if $f(x) = x^2 + 1$ and $h(x) = 2x - 2$.

Solution:

Find $(f+h)(x)$.

$$(f+h)(x) = (x^2 + 1) + (2x - 2)$$

$$(f+h)(x) = x^2 + 2x - 1$$

To find $(f+h)(-1)$, we plug $x = -1$ into $(f+h)(x)$.

$$(f+h)(-1) = (-1)^2 + 2(-1) - 1$$

$$(f+h)(-1) = 1 - 2 - 1$$

$$(f+h)(-1) = -2$$

2. Find and simplify $(h+g)(x)$ if $g(x) = x^2 + 3x - 1$ and $h(x) = -2x^2 + 4x - 5$.

Solution:

To find $(h+g)(x)$, we'll start by finding $h(x) + g(x)$.

$$(h+g)(x) = h(x) + g(x) = (-2x^2 + 4x - 5) + (x^2 + 3x - 1)$$

which simplifies as

$$(h+g)(x) = -2x^2 + 4x - 5 + x^2 + 3x - 1$$

$$(h+g)(x) = -x^2 + 7x - 6$$

3. If $f(-2) = 6$, $g(-2) = -3$, and $h(-2) = 4$, find $(f+g+h)(-2)$.

Solution:

By the definition of the sum of functions, we get

$$(f+g+h)(-2) = f(-2) + g(-2) + h(-2)$$

$$(f+g+h)(-2) = 6 + (-3) + 4$$

$$(f+g+h)(-2) = 7$$

4. Describe two ways you can add two functions together.

Solution:

You can either input the value for x into each function and then add the outputs together, or you can add the functions together and then input the value for x and simplify.

5. Find $(h+g)(t)$ if $h(t) = 4t^2 - 3$ and $g(t) = -3t^2 + 4$.

Solution:

By the definition of the sum of functions, we get

$$(h+g)(t) = (4t^2 - 3) + (-3t^2 + 4)$$

$$(h+g)(t) = t^2 + 1$$

6. Given the expression below, determine $f(x)$ and $g(x)$.

$$(f+g)(x) = (-x^2 + 3x + 2) + (x - 7)$$

Solution:

By the definition of the sum of functions, we can see that

$f(x) = -x^2 + 3x + 2$ and $g(x) = x - 7$

It could also be correct to say that

$g(x) = -x^2 + 3x + 2$ and $f(x) = x - 7$

7. Let $a(x) = x^3 - x^2 + x - 1$ and $b(x) = -x^3 + x^2 + x - 1$. Determine the value of $(a + b)(-1)$.

Solution:

First, we find the values for $a(-1)$ and $b(-1)$.

$$a(-1) = (-1)^3 - (-1)^2 + (-1) - 1$$

$$a(-1) = -1 - 1 - 1 - 1$$

$$a(-1) = -4$$

and

$$b(-1) = -(-1)^3 + (-1)^2 + (-1) - 1$$

$$b(-1) = 1 + 1 - 1 - 1$$

$$b(-1) = 0$$

Therefore,

$$(a + b)(-1) = a(-1) + b(-1)$$

$$(a + b)(-1) = -4 + 0$$

$$(a + b)(-1) = -4$$

8. What went wrong in the following set of steps?

$$(x^2 + x - 9) + (x - 1)$$

$$(3x - 9) + (x - 1)$$

$$3x - 9 + x - 1$$

$$4x - 10$$

Solution:

When adding the two expressions, the first function $x^2 + x - 9$ was simplified to $3x - 9$ by adding the $x^2 + x$ as $2x + x$, which is incorrect.

■ 9. If $g(1) = 5$ and $h(1) = -3$, find $(g + h)(1)$.

Solution:

By the definition of the sum of functions, we get

$$(g + h)(1) = g(1) + h(1)$$

$$(g + h)(1) = 5 + (-3)$$

$$(g + h)(1) = 2$$

■ 10. If $f(0) = 3$ and $(f + g)(0) = 8$, find $g(0)$.

Solution:

By the definition of the sum of functions, we get

$$(f+g)(0) = f(0) + g(0)$$

$$(f+g)(0) = 3 + g(0)$$

Since $(f+g)(0) = 8$, we get

$$8 = 3 + g(0)$$

$$g(0) = 5$$

PRODUCT OF FUNCTIONS

1. Find and simplify $(ab)(x)$ if $a(x) = x + 3$ and $b(x) = 5x - 4$.

Solution:

By the definition of the product of two functions, we have

$$(ab)(x) = a(x)b(x)$$

$$(ab)(x) = (x + 3)(5x - 4)$$

$$(ab)(x) = 5x^2 + 15x - 4x - 12$$

$$(ab)(x) = 5x^2 + 11x - 12$$

2. Find $(fg)(-1)$ if $f(x) = x^2 + 3$ and $g(x) = x - 5$.

Solution:

We'll find $f(-1)$ and $g(-1)$.

$$f(-1) = (-1)^2 + 3 = 1 + 3 = 4$$

$$g(-1) = (-1) - 5 = -6$$

Then the product of these functions is

$$(fg)(-1) = f(-1)g(-1)$$

$$(fg)(-1) = (4)(-6)$$

$$(fg)(-1) = -24$$

■ 3. If $g(0) = -2$ and $(gh)(0) = -14$, find $h(0)$.

Solution:

From the product of two functions, we have

$$-14 = (gh)(0)$$

$$-14 = g(0)h(0)$$

$$-14 = (-2)h(0)$$

$$h(0) = 7$$

■ 4. What went wrong in the following set of steps?

$$(x+1)(x+2)$$

$$x \cdot x + 2 \cdot x + 2$$

Solution:

When FOILing the parentheses, the $1 \cdot x$ (inner term) was forgotten.

5. Given the expanded expression below, determine $f(x)$ and $g(x)$.

$$(gf)(x) = x^2(x-7) - x(x-7) + 5(x-7)$$

Solution:

Factor the $(x-7)$ out of the expression.

$$(gf)(x) = (x-7)(x^2 - x + 5)$$

Then the two functions are $f(x) = (x-7)$ and $g(x) = x^2 - x + 5$. We could also define the functions as $g(x) = (x-7)$ and $f(x) = x^2 - x + 5$.

6. Find $(fh)(5)$ if $f(x) = -x^2 + 2x$ and $h(x) = 2x + 7$.

Solution:

By the definition of the product of functions, we have

$$(fh)(x) = f(x)h(x)$$

$$(fh)(x) = (-x^2 + 2x)(2x + 7)$$

$$(fh)(x) = -2x^3 + 4x^2 - 7x^2 + 14x$$

$$(fh)(x) = -2x^3 - 3x^2 + 14x$$

Evaluating the product at $x = 5$ gives

$$(fh)(5) = -2(5)^3 - 3(5)^2 + 14(5)$$

$$(fh)(5) = -250 - 75 + 70$$

$$(fh)(5) = -255$$

■ 7. Describe two different ways that you can multiply two functions together and evaluate the product at a particular point.

Solution:

You can either 1) input the value for x into each function and then multiply the outputs together, or you can 2) multiply the functions together and then input the value for x and simplify.

■ 8. Find and simplify $(gh)(x)$ if $g(x) = x^2 + 1$ and $h(x) = 2x^2 + 3$.

Solution:

By the product of two functions, we have

$$(gh)(x) = g(x)h(x)$$

$$(gh)(x) = (x^2 + 1)(2x^2 + 3)$$

$$(gh)(x) = 2x^4 + 3x^2 + 2x^2 + 3$$

$$(gh)(x) = 2x^4 + 5x^2 + 3$$

EVEN, ODD, OR NEITHER

1. Is the function even, odd, or neither?

$$f(x) = -x^5 + 2x^2 - 1$$

Solution:

Substitute $-x$ for x.

$$f(-x) = -(-x)^5 + 2(-x)^2 - 1$$

$$f(-x) = x^5 + 2x^2 - 1$$

Because $f(-x) \neq f(x)$, the function is not even. To see if it's odd, we check

$$-f(x) = -(-x^5 + 2x^2 - 1) = x^5 - 2x^2 + 1$$

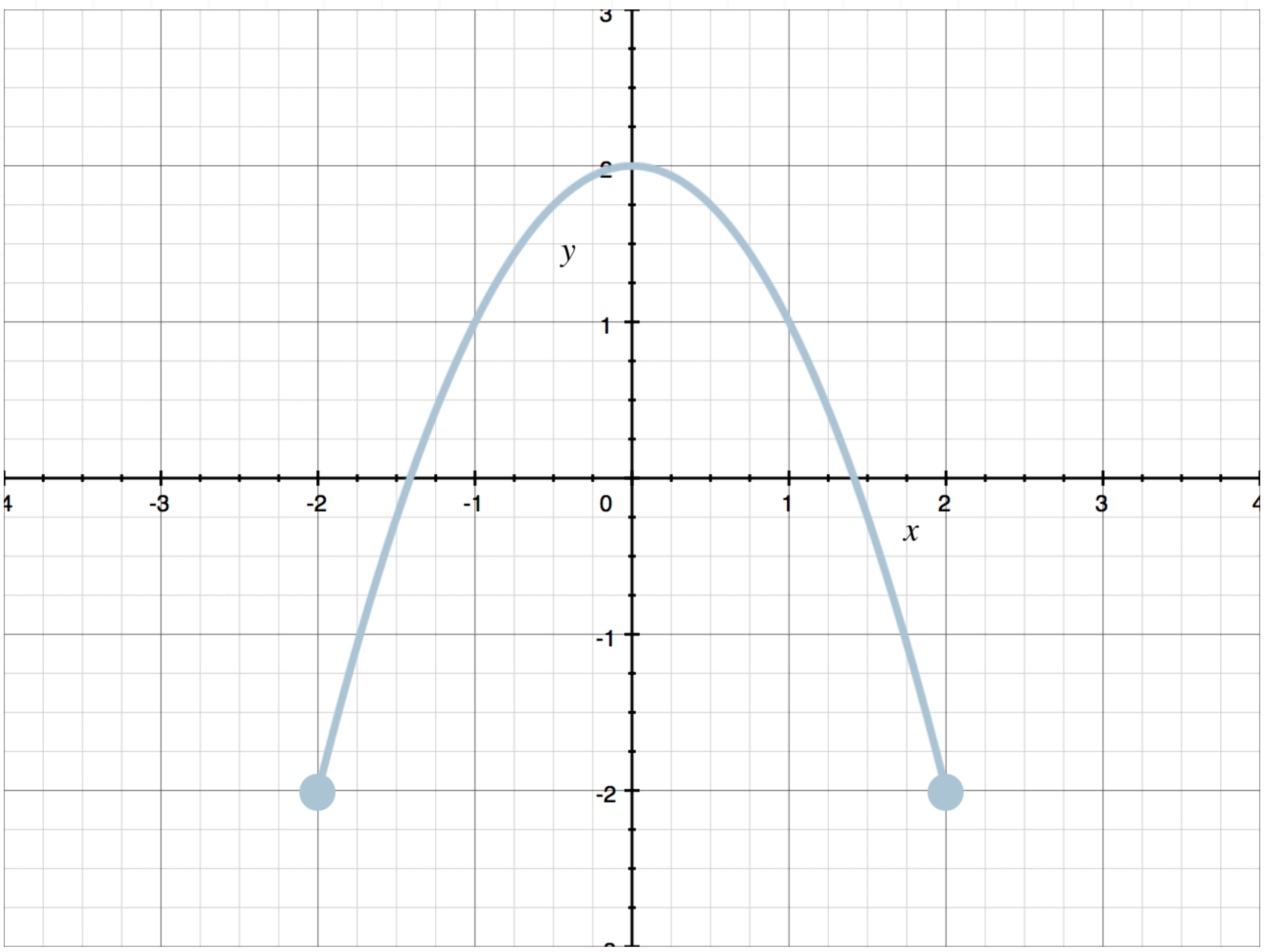
Because $f(-x) \neq -f(x)$, the function is not odd. Therefore, the function is neither even nor odd.

2. Describe the symmetry of an even function, and give an example of an even function.

Solution:

An even function is symmetric about the y-axis. There are many examples of even functions, one being $f(x) = x^2$.

3. Determine if the graph is the graph of a function that is even, odd, or neither.



Solution:

Notice that the graph is symmetric about the y-axis and therefore the graph is the graph of an even function.

4. Is the function even, odd, or neither?

$$g(x) = -3x^2 + 5x^6$$

Solution:

Substitute $-x$ for x.

$$g(-x) = -3(-x)^2 + 5(-x)^6$$

$$g(-x) = -3x^2 + 5x^6$$

Because $f(-x) = f(x)$, the function is even.

5. Show that the function is neither even nor odd.

$$f(x) = x^2 - 5x + 7$$

Solution:

Substitute $-x$ for x.

$$f(-x) = (-x)^2 - 5(-x) + 7$$

$$f(-x) = x^2 + 5x + 7$$

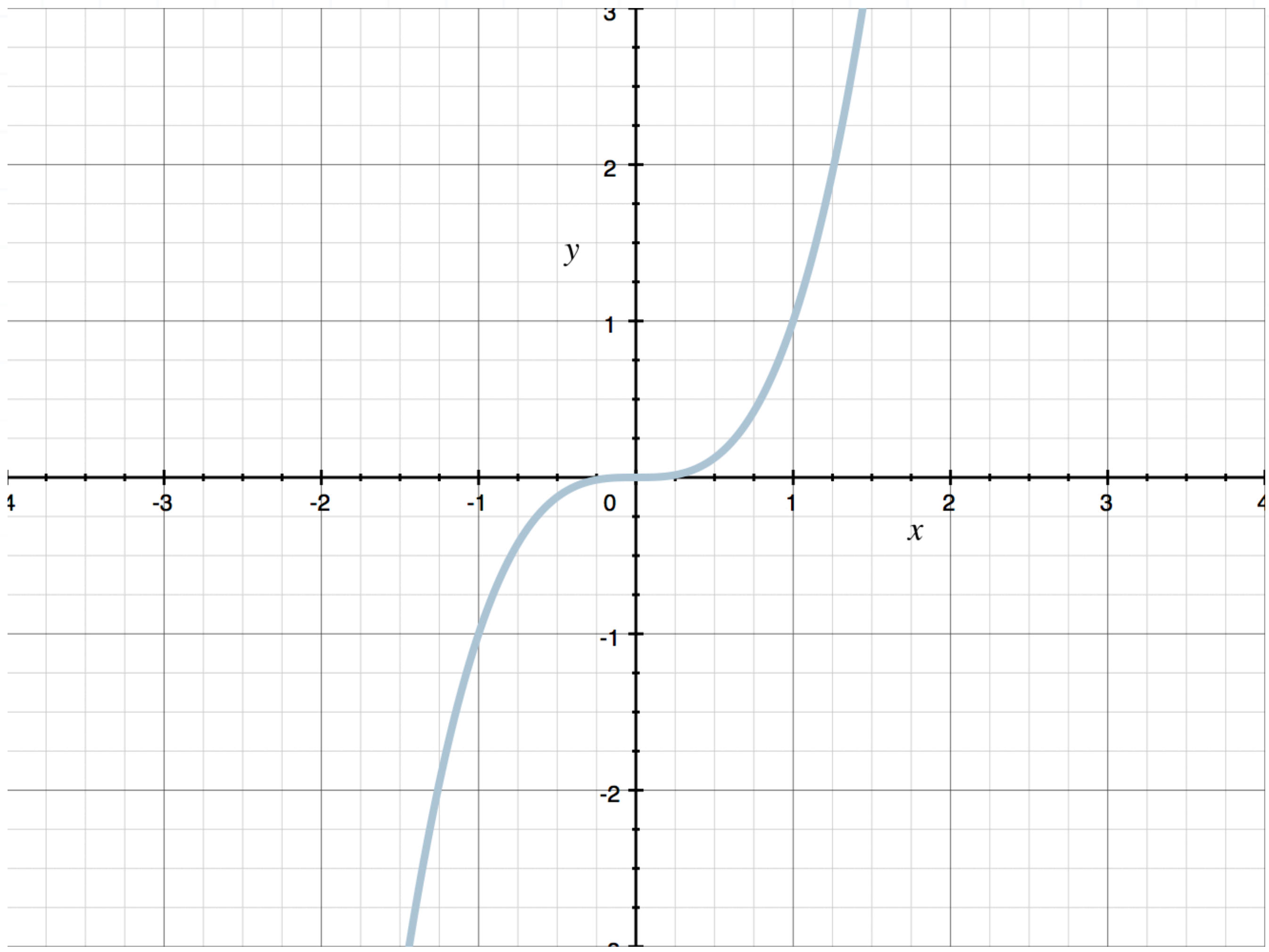
Because $f(-x) \neq f(x)$, the function is not even. To see if it's odd, we check

$$-f(x) = -(x^2 - 5x + 7)$$

$$-f(x) = -x^2 + 5x - 7$$

Because $f(-x) \neq -f(x)$, the function is not odd. Therefore, the function is neither even nor odd.

6. Determine if the graph is the graph of a function that is even, odd, or neither.



Solution:

Notice that the graph is symmetric about the origin, and therefore the graph is the graph of an odd function.

7. Is the function even, odd, or neither?

$$h(x) = x^3 - 3x$$

Solution:

Substitute $-x$ for x.

$$h(-x) = (-x)^3 - 3(-x)$$

$$h(-x) = -x^3 + 3x$$

Because $f(-x) \neq f(x)$, the function is not even. To see if it's odd, we check

$$-f(x) = -(x^3 - 3x)$$

$$-f(x) = -x^3 + 3x$$

Because $f(-x) = -f(x)$, the function is odd.

8. Describe the symmetry of an odd function, and give an example of an odd function.

Solution:

An odd function is symmetric about the origin. There are many correct examples of odd functions, one being $f(x) = x^3$.

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