Negative exponents and power rule

This lesson will cover how to use the power rule for negative exponents to find the value of a positive number raised to a negative power.

Negative exponents in the numerator and denominator

If you have two positive real numbers a and b, then

$$a^{-b} = \frac{1}{a^b}$$

In order to change the exponent in a^{-b} from $-b$ to b, you move the a^{-b} from the numerator to the denominator to get $1/a^b$.

Alternatively, if you have two positive real numbers a and b, then

$$\frac{1}{a^{-b}} = a^b$$

In order to change the exponent in a^{-b} from $-b$ to b, you move the a^{-b} from the denominator to the numerator to get $1 \cdot a^b$, or just a^b.

In other words, if you have a negative exponent in the numerator, you can make it positive by moving that term to the denominator. Or if you have a negative exponent in the denominator, you can make it positive by moving that term to the numerator.

Reciprocals

By the way, a^b and a^{-b} are reciprocals. As you may recall, two numbers whose product is equal to 1 are reciprocals (of each other). Sometimes you'll hear or read about negative exponents and their relationship to reciprocals, and that relationship follows from the power rule for negative exponents. If we multiply a^b by a^{-b}, we can see that the product is 1, which means a^b and a^{-b} are reciprocals of each other.

$$(a^b) \cdot (a^{-b})$$

$$a^{b+(-b)}$$

$$a^{b-b}$$

$$a^0$$

$$1$$

So these two pairs of reciprocals of one another:

$$a^b \text{ and } \frac{1}{a^b}$$

$$a^{-b} \text{ and } \frac{1}{a^{-b}}$$

Let's look at a few examples.

Example

Rewrite the expression with no negative exponents.

$$2^{-1}$$

In order to get rid of the negative exponent, we change the exponent in 2^{-1} from -1 to 1 and move the resulting expression from the numerator to the denominator, so we get

$$\frac{1}{2^1}$$

Since $2^1 = 2$, we can write this as

$$\frac{1}{2}$$

Let's look at an example with a variable.

Example

Write the expression with no negative exponents.

$$x^{-5}$$

In order to get rid of the negative exponent, we change the exponent in x^{-5} from -5 to 5 and move the resulting expression from the numerator to the denominator. We get

$$\frac{1}{x^5}$$

Let's look at another example.

Example

Write this expression with no negative exponents.

$$\frac{1}{b^{-7}}$$

In order to get rid of the negative exponent, we change the exponent in b^{-7} from -7 to 7 and move the resulting expression from the denominator to the numerator. We get $1 \cdot b^7$, which is equal to b^7.

Let's look at a final example, this time with a number other than 1 in the numerator.

Example

Write the expression with no negative exponents.

$$\frac{3}{x^{-5}}$$

In order to get rid of the negative exponent, we change the exponent in x^{-5} from -5 to 5 and move the resulting expression from the denominator to the numerator. We get

$$3x^5$$
