Powers of negative bases

There are two cases to think of when you're simplifying powers of negative bases.

Case 1: Actually not a negative base at all

If you have something that looks like this:

$$-b^a$$

where a and b are both positive real numbers, it's really the same as $-1 \cdot b^a$. Notice how the -1 can be pulled out in front, leaving just b^a. Because b is positive, this base actually isn't negative at all. This is just a case of raising a positive number to a power, and then changing the sign of the result by multiplying by -1. So when you see something like -4^2, it means the same thing as

$$-1(4^2)$$

$$-1(4 \cdot 4)$$

$$-1(16)$$

$$-16$$

This is because PEMDAS and the order of operations tells us that you need to take care of the exponent first, and then multiply by the negative sign.

Case 2: A negative sign included in the parenthesis

If you have something that looks like $(-b)^a$, where a and b are both positive real numbers, then raise the $-b$ inside the parentheses to the power of a. In other words, this is the multiplication in which the negative integer $-b$ appears as a factor a times (and there are no other factors).

This is the case most people think of when they perform operations with exponents. It means, for example, that $(-4)^2$ equals $(-4)(-4)$ or 16.

Be careful not to confuse an expression like $(-b)^a$ with an expression like $-b^a$ (which was our Case 1 example). The $-b^a$ case means that we first raise the positive integer b to the power of a, and then change the sign of the result (by placing a negative sign in front of it).

Example

Simplify the expression.

$$-2^3$$

By PEMDAS and the order of operations, we have to take care of the exponent first, and then apply the negative sign. Remember that applying a negative sign is the same as multiplying by -1.

$$-2^3$$

$$-(2 \cdot 2 \cdot 2)$$

$$-(8)$$

$$-8$$

Let's take a look at an example with a negative number inside parentheses.

Example

Simplify the expression.

$$(-1)^4$$

Remember that -1^4 is different than $(-1)^4$. When we have $(-1)^4$, the negative sign is included in the parentheses. This means we need to raise the -1 inside the parentheses to the power of 4, so it's the same thing as having four factors of -1.

$$(-1)^4$$

$$(-1)(-1)(-1)(-1)$$

Doing the multiplication form left to right (according to the order of operations), we get the following steps:

$$(1)(-1)(-1)$$

$$(-1)(-1)$$

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