Powers of fractions

This lesson will cover how to find the power of a fraction as well as introduce how to work with fractional exponents.

Powers of fractions

Say we have something like

$$\left(\frac{a}{b}\right)^c$$

where a, b, and c are integers. This is like saying that we're doing a multiplication in which a/b appears as a factor c times (and there are no other factors). This turns the power problem into a fraction multiplication problem, where you multiply the numerators and the denominators separately. In this example, a is the numerator in each factor, and b is the denominator in each factor.

Example

Simplify the expression.

$$\left(\frac{3}{4}\right)^2$$

This is an example of a power of a fraction. The way the problem is written, it's like saying that we're multiplying $3/4$ by itself, since the base is $3/4$ and the exponent is 2. So the problem becomes

$$\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)$$

Now we've got a fraction multiplication problem. When we multiply fractions, we multiply the numerators and the denominators separately.

$$\frac{3 \cdot 3}{4 \cdot 4} = \frac{9}{16}$$

Let's look at an example with variables.

Example

Simplify the expression.

$$\left(\frac{x}{y^3}\right)^4$$

This is an example of a power of a fraction. The way the problem is written, it's like saying that we're doing a multiplication in which x/y^3 appears as a factor four times (and there are no other factors), since the base is x/y^3 and the exponent is 4. So the problem becomes

$$\left(\frac{x}{y^3}\right)\left(\frac{x}{y^3}\right)\left(\frac{x}{y^3}\right)\left(\frac{x}{y^3}\right)$$

Now we've got a fraction multiplication problem. Remember, when we multiply fractions, we multiply the numerators and the denominators separately.

$$\frac{x \cdot x \cdot x \cdot x}{y^3 \cdot y^3 \cdot y^3 \cdot y^3}$$

Now we have a like base of x in the numerator and a like base of y in the denominator.

In the numerator we can write x^4 because x appears as a factor four times.

Remember when you have a like base you can add the exponents. We'll need to do this for the denominator. Let's look at the calculation for the denominator:

$$y^3 \cdot y^3 \cdot y^3 \cdot y^3 = y^{3+3+3+3} = y^{12}$$

So the simplified expression is

$$\frac{x^4}{y^{12}}$$

Another way you can think of simplifying an expression of the form $(a/b)^c$ is that you rewrite it as a fraction in which the numerator and the denominator are separately raised to the power c.

$$\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$$

Fractional exponents with like bases

If we start with something like $x^a \cdot x^{c/d}$ (where a, c, and d are integers and x is a variable or a real number), we have like bases because the base of both factors is x. When that's the case, we add the exponents.

$$x^a \cdot x^{\frac{c}{d}} = x^{a+\frac{c}{d}}$$

Now the problem is just about fraction addition.

Let's look at an example.

Example

Simplify the expression.

$$a^3 \cdot a^{\frac{1}{4}}$$

We have like bases because the base of both factors is a. When that's the case, we add the exponents.

$$a^{3+\frac{1}{4}}$$

Now the problem is just about fraction addition in the exponent. To add the fractions, we have to find a common denominator.

$$a^{3(\frac{4}{4})+\frac{1}{4}}$$

$$a^{\frac{12}{4}+\frac{1}{4}}$$

$$a^{\frac{13}{4}}$$
