Algebra 2 Workbook Solutions

Factoring

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FACTORING QUADRATIC POLYNOMIALS WITH COEFFICIENTS

■ 1. Factor the quadratic.

$$6x^2 + 11x - 10$$

Solution:

Remember that a quadratic is written in the form $ax^2 + bx + c$. In this case, $a = 6$, $b = 11$, and $c = -10$. Multiply a and c together, then find factors of ac that combine to equal b.

$$ac = 6(-10) = -60$$

Since b is positive, the larger factor of -60 will be positive.

Factors of -60	Sum
-1 and 60	59
-2 and 30	28
-3 and 20	17
-4 and 15	11

Once we've found the correct factors (-4 and 15) we don't need to continue the table. Divide each factor by a, which is 6 in this problem, and reduce if possible.

$$\frac{-4}{6} = \frac{-2}{3}$$

The denominator becomes the coefficient to x in this factor and the numerator is the constant, so this factor is $(3x-2)$.

$$\frac{15}{6}=\frac{5}{2}$$

The denominator becomes the coefficient to x in this factor and the numerator is the constant, so this factor is $(2x+5)$. Write the factors next to each other for the solution.

$$(3x-2)(2x+5)$$

■ 2. Factor the quadratic.

$$3x^2-8x-35$$

Solution:

Remember that a quadratic is written in the form ax^2+bx+c. In this case, $a=3$, $b=-8$, and $c=-35$. Multiply a and c together, then find factors of ac that combine to equal b.

$$ac=3(-35)=-105$$

Since b is negative, the larger factor of -105 will be negative.

Factors of -105	Sum
1 and -105	-104
3 and -35	-32
5 and -21	-16
7 and -15	-8

Once we've found the correct factors (7 and -15) we don't need to continue the table. Divide each factor by a, which is 3 in this problem, and reduce if possible.

$$\frac{7}{3}$$

The denominator becomes the coefficient to x in this factor and the numerator is the constant, so this factor is $(3x+7)$.

$$\frac{-15}{3} = -5$$

The denominator becomes the coefficient to x in this factor and the numerator is the constant, so this factor is $(x+5)$. Write the factors next to each other for the solution.

$$(3x+7)(x-5)$$

■ 3. Factor the quadratic.

$$20x^2 - 23x + 6$$

Solution:

Remember that a quadratic is written in the form $ax^2 + bx + c$. In this case, $a = 20$, $b = -23$, and $c = 6$. Multiply a and c together, then find factors of ac that combine to equal b.

$$ac = 20(6) = 120$$

Since b is negative and c is positive, both factors of 120 will be negative.

Factors of -120	Sum
-1 and -120	-121
-2 and -60	-62
-3 and -40	-43
-4 and -30	-34
-5 and -24	-29
-6 and -20	-26
-8 and -15	-23

Once we've found the correct factors (-8 and -15) we don't need to continue the table. Divide each factor by a, which is 20 in this problem, and reduce if possible.

$$\frac{-8}{20} = \frac{-2}{5}$$

The denominator becomes the coefficient to x in this factor and the numerator is the constant, so this factor is $(5x - 2)$.

$$\frac{-15}{20} = \frac{-3}{4}$$

The denominator becomes the coefficient to x in this factor and the numerator is the constant, so this factor is $(4x-3)$. Write the factors next to each other for the solution.

$$(5x-2)(4x-3)$$

4. Factor the quadratic.

$$4x^2+26x+36$$

Solution:

Divide through by 2.

$$2(2x^2+13x+18)$$

Remember that a quadratic is written in the form ax^2+bx+c. In this case, $a=2$, $b=13$, and $c=18$. Multiply a and c together, then find factors of ac that combine to equal b.

$$ac=2(18)=36$$

Since both b and c are positive, both factors of 36 will be positive.

Factors of 36	Sum
1 and 36	37
2 and 18	20
3 and 12	15
4 and 9	13

Once we've found the correct factors (4 and 9) we don't need to continue the table. Divide each factor by a, which is 2 in this problem, and reduce if possible.

$$\frac{4}{2}=2$$

The denominator becomes the coefficient to x in this factor and the numerator is the constant, so this factor is $(x+2)$.

$$\frac{9}{2}$$

The denominator becomes the coefficient to x in this factor and the numerator is the constant, so this factor is $(2x+9)$. Write the factors next to each other for the solution, remembering to include the 2 that we pulled out at the beginning.

$$2(x+2)(2x+9)$$

■ 5. Factor the quadratic.

$$14x^2+15x+4$$

Solution:

Remember that a quadratic is written in the form $ax^2 + bx + c$. In this case, $a = 14$, $b = 15$, and $c = 4$. Multiply a and c together, then find factors of ac that combine to equal b.

$$ac = 14(4) = 56$$

Since both b and c are positive, both factors of 56 will be positive.

Factors of 56	Sum
1 and 56	57
2 and 28	30
4 and 14	18
7 and 8	15

Once we've found the correct factors (7 and 8) we don't need to continue the table. Divide each factor by a, which is 14 in this problem, and reduce if possible.

$$\frac{7}{14} = \frac{1}{2}$$

The denominator becomes the coefficient to x in this factor and the numerator is the constant, so this factor is $(2x + 1)$.

$$\frac{8}{14} = \frac{4}{7}$$

The denominator becomes the coefficient to x in this factor and the numerator is the constant, so this factor is $(7x + 4)$. Write the factors next to each other for the solution.

$$(2x + 1)(7x + 4)$$

■ 6. Factor the quadratic.

$$15x^2 + 26x + 8$$

Solution:

Remember that a quadratic is written in the form $ax^2 + bx + c$. In this case, $a = 15$, $b = 26$, and $c = 8$. Multiply a and c together, then find factors of ac that combine to equal b.

$$ac = 15(8) = 120$$

Since both b and c are positive, both factors of 120 will be positive.

Factors of 120	Sum
1 and 120	121
2 and 60	62
3 and 40	43
4 and 30	34
5 and 24	29
6 and 20	26

Once we've found the correct factors (6 and 20) we don't need to continue the table. Divide each factor by a, which is 15 in this problem, and reduce if possible.

$$\frac{6}{15}=\frac{2}{5}$$

The denominator becomes the coefficient to x in this factor and the numerator is the constant, so this factor is $(5x+2)$.

$$\frac{20}{15}=\frac{4}{3}$$

The denominator becomes the coefficient to x in this factor and the numerator is the constant, so this factor is $(3x+4)$. Write the factors next to each other for the solution.

$$(5x+2)(3x+4)$$

■ 7. Factor the quadratic.

$$12x^2+4x-1$$

Solution:

Remember that a quadratic is written in the form ax^2+bx+c. In this case, $a=12$, $b=4$, and $c=-1$. Multiply a and c together, then find factors of ac that combine to equal b.

$$ac = 12(-1) = -12$$

Since b is positive, the larger factor of -12 will be positive.

Factors of -12	Sum
-1 and 12	11
-2 and 6	4

Once we've found the correct factors (-2 and 6) we don't need to continue the table. Divide each factor by a, which is 12 in this problem, and reduce if possible.

$$\frac{-2}{12} = \frac{-1}{6}$$

The denominator becomes the coefficient to x in this factor and the numerator is the constant, so this factor is $(6x - 1)$.

$$\frac{6}{12} = \frac{1}{2}$$

The denominator becomes the coefficient to x in this factor and the numerator is the constant, so this factor is $(2x + 1)$. Write the factors next to each other for the solution.

$$(6x - 1)(2x + 1)$$

■ 8. Factor the quadratic.

$$8x^2 - 10x - 63$$

Solution:

Remember that a quadratic is written in the form $ax^2 + bx + c$. In this case, $a = 8$, $b = -10$, and $c = -63$. Multiply a and c together, then find factors of ac that combine to equal b.

$$ac = 8(-63) = -504$$

Since b is negative, the larger factor of -504 will be negative.

Factors of -504	Sum
1 and -504	-503
2 and -252	-250
3 and -168	-165
4 and -126	-122
6 and -84	-78
7 and -72	-65
8 and -63	-55
9 and -56	-47
12 and -42	-30
14 and -36	-22
18 and -28	-10

Once we've found the correct factors (18 and -28) we don't need to continue the table. Divide each factor by a, which is 8 in this problem, and reduce if possible.

$$\frac{18}{8} = \frac{9}{4}$$

The denominator becomes the coefficient to x in this factor and the numerator is the constant, so this factor is $(4x+9)$.

$$\frac{-28}{8}=\frac{-7}{2}$$

The denominator becomes the coefficient to x in this factor and the numerator is the constant, so this factor is $(2x-7)$. Write the factors next to each other for the solution.

$$(4x+9)(2x-7)$$

FACTORING BY GROUPING

■ 1. Factor the expression by grouping.

$$2x + 3y - 4ax - 6ay$$

Solution:

Find terms that have factors in common, and group those terms.

$$(2x - 4ax) + (3y - 6ay)$$

Factor the common factor out of the first group, and then factor the common factor out of the second group.

$$2x(1 - 2a) + (3y - 6ay)$$

$$2x(1 - 2a) + 3y(1 - 2a)$$

Now because both terms happen to have $(1 - 2a)$ in common, we're able to factor $(1 - 2a)$ out of each term, leaving only $2x$ from the first term, and $3y$ from the second term.

$$(1 - 2a)(2x + 3y)$$

■ 2. Factor the quadratic by grouping.

$$4x^2 + 2xy + 10x + 5y$$

Solution:

Find terms that have factors in common, and group those terms.

$$(4x^2 + 2xy) + (10x + 5y)$$

Factor the common factor out of the first group, and then factor the common factor out of the second group.

$$2x(2x + y) + (10x + 5y)$$

$$2x(2x + y) + 5(2x + y)$$

Now because both terms happen to have $(2x + y)$ in common, we're able to factor $(2x + y)$ out of each term, leaving only $2x$ from the first term, and 5 from the second term.

$$(2x + y)(2x + 5)$$

■ 3. Factor the expression by grouping.

$$7y^2 - 6y^3 - 6y + 7$$

Solution:

Find terms that have factors in common, and group those terms.

$$(7y^2 + 7) + (-6y^3 - 6y)$$

Factor the common factor out of the first group, and then factor the common factor out of the second group.

$$7(y^2 + 1) + (-6y^3 - 6y)$$

$$7(y^2 + 1) - 6y(y^2 + 1)$$

Now because both terms happen to have $(y^2 + 1)$ in common, we're able to factor $(y^2 + 1)$ out of each term, leaving only 7 from the first term, and $-6y$ from the second term.

$$(y^2 + 1)(7 - 6y)$$

■ 4. Factor the expression by grouping.

$$8ab + 2b - 4a - 1$$

Solution:

Find terms that have factors in common, and group those terms.

$$(8ab + 2b) + (-4a - 1)$$

Factor the common factor out of the first group, and then factor the common factor out of the second group.

$$2b(4a + 1) + (-4a - 1)$$

$$2b(4a + 1) - 1(4a + 1)$$

Now because both terms happen to have $(4a+1)$ in common, we're able to factor $(4a+1)$ out of each term, leaving only $2b$ from the first term, and -1 from the second term.

$$(4a+1)(2b-1)$$

■ 5. Factor the expression by grouping.

$$9z+9qr+5ayz+5ayqr$$

Solution:

Find terms that have factors in common, and group those terms.

$$(9z+9qr)+(5ayz+5ayqr)$$

Factor the common factor out of the first group, and then factor the common factor out of the second group.

$$9(z+qr)+(5ayz+5ayqr)$$

$$9(z+qr)+5ay(z+qr)$$

Now because both terms happen to have $(z+qr)$ in common, we're able to factor $(z+qr)$ out of each term, leaving only 9 from the first term, and $5ay$ from the second term.

$$(z+qr)(9+5ay)$$

6. Factor the quadratic by grouping.

$$3k^2 + 7k - 6$$

Solution:

First find the factors of $a \cdot c$ that combine to equal b. Start with the fact that $a = 3$, $b = 7$, and $c = -6$.

$$a \cdot c = 3(-6) = -18$$

The factors of -18 that combine to equal 7 are 9 and -2. Rewrite the quadratic by replacing $7k$ with $9k - 2k$.

$$3k^2 + 9k - 2k - 6$$

Find terms that have factors in common, and group those terms.

$$(3k^2 + 9k) + (-2k - 6)$$

Factor the common factor out of the first group, and then factor the common factor out of the second group.

$$3k(k + 3) + (-2k - 6)$$

$$3k(k + 3) - 2(k + 3)$$

Now because both terms happen to have $(k+3)$ in common, we're able to factor $(k+3)$ out of each term, leaving only $3k$ from the first term, and -2 from the second term.

$$(k+3)(3k-2)$$

■ 7. Factor the quadratic by grouping.

$$6x^2+13x-5$$

Solution:

First find the factors of $a \cdot c$ that combine to equal b. Start with the fact that $a=6$, $b=13$, and $c=-5$.

$$a \cdot c = 6(-5) = -30$$

The factors of -30 that combine to equal 13 are 15 and -2. Rewrite the quadratic by replacing $13x$ with $15x-2x$.

$$6x^2+15x-2x-5$$

Find terms that have factors in common, and group those terms.

$$(6x^2-2x)+(15x-5)$$

Factor the common factor out of the first group, and then factor the common factor out of the second group.

$$2x(3x-1)+(15x-5)$$

$$2x(3x-1)+5(3x-1)$$

Now because both terms happen to have $(3x-1)$ in common, we're able to factor $(3x-1)$ out of each term, leaving only $2x$ from the first term, and 5 from the second term.

$$(3x-1)(2x+5)$$

FACTORING THE DIFFERENCE OF TWO CUBES

■ 1. Factor the polynomial.

$$x^3 - y^3$$

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{x^3} = x$$

$$\sqrt[3]{y^3} = y$$

Since they're both perfect cubes, we can use the difference of cubes formula

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

with $a = x$ and $b = y$. Therefore, we get

$$(x - y)(x^2 + xy + y^2)$$

■ 2. Factor the polynomial.

$$x^3 - 27y^9$$

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{x^3} = x$$

$$\sqrt[3]{27y^9} = 3y^3$$

Since they're both perfect cubes, we can use the difference of cubes formula

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

with $a = x$ and $b = 3y^3$. Therefore, we get

$$(x - 3y^3)(x^2 + x(3y^3) + (3y^3)^2)$$

$$(x - 3y^3)(x^2 + 3xy^3 + 9y^6)$$

■ 3. Factor the polynomial.

$$8x^3y^6 - 64z^{21}$$

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{8x^3y^6} = 2xy^2$$

$$\sqrt[3]{64z^{21}} = 4z^7$$

Since they're both perfect cubes, we can use the difference of cubes formula

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

with $a = 2xy^2$ and $b = 4z^7$. Therefore, we get

$$(2xy^2 - 4z^7)((2xy^2)^2 + (2xy^2)(4z^7) + (4z^7)^2)$$

$$(2xy^2 - 4z^7)(4x^2y^4 + 8xy^2z^7 + 16z^{14})$$

■ 4. Factor the polynomial.

$$a^3b^{12} - 125c^6$$

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{a^3b^{12}} = ab^4$$

$$\sqrt[3]{125c^6} = 5c^2$$

Since they're both perfect cubes, we can use the difference of cubes formula

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

with $a = ab^4$ and $b = 5c^2$. Therefore, we get

$$(ab^4 - 5c^2)((ab^4)^2 + (ab^4)(5c^2) + (5c^2)^2)$$

$$(ab^4 - 5c^2)(a^2b^8 + 5ab^4c^2 + 25c^4)$$

■ 5. Factor the polynomial.

$$x^{15} - 8m^3r^9$$

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{x^{15}} = x^5$$

$$\sqrt[3]{8m^3r^9} = 2mr^3$$

Since they're both perfect cubes, we can use the difference of cubes formula

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

with $a = x^5$ and $b = 2mr^3$. Therefore, we get

$$(x^5 - 2mr^3)((x^5)^2 + (x^5)(2mr^3) + (2mr^3)^2)$$

$$(x^5 - 2mr^3)(x^{10} + 2x^5mr^3 + 4m^2r^6)$$

■ 6. Factor the polynomial.

$$27y^6z^3 - 216x^9$$

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{27y^6z^3} = 3y^2z$$

$$\sqrt[3]{216x^9} = 6x^3$$

Since they're both perfect cubes, we can use the difference of cubes formula

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

with $a = 3y^2z$ and $b = 6x^3$. Therefore, we get

$$(3y^2z - 6x^3)((3y^2z)^2 + (3y^2z)(6x^3) + (6x^3)^2)$$

$$(3y^2z - 6x^3)(9y^4z^2 + 18x^3y^2z + 36x^6)$$

■ 7. Factor the polynomial.

$$64a^3b^3 - 27c^9$$

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{64a^3b^3} = 4ab$$

$$\sqrt[3]{27c^9} = 3c^3$$

Since they're both perfect cubes, we can use the difference of cubes formula

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

with $a = 4ab$ and $b = 3c^3$. Therefore, we get

$$(4ab - 3c^3)((4ab)^2 + (4ab)(3c^3) + (3c^3)^2)$$

$$(4ab - 3c^3)(16a^2b^2 + 12abc^3 + 9c^6)$$

■ 8. Factor the polynomial.

$$8x^{15} - 27y^9$$

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{8x^{15}} = 2x^5$$

$$\sqrt[3]{27y^9} = 3y^3$$

Since they're both perfect cubes, we can use the difference of cubes formula

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

with $a = 2x^5$ and $b = 3y^3$. Therefore, we get

$$(2x^5 - 3y^3)((2x^5)^2 + (2x^5)(3y^3) + (3y^3)^2)$$

$$(2x^5 - 3y^3)(4x^{10} + 6x^5y^3 + 9y^6)$$

■ 9. Factor the polynomial.

$$216a^3b^6 - 125c^{24}d^3$$

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{216a^3b^6} = 6ab^2$$

$$\sqrt[3]{125c^{24}d^3} = 5c^8d$$

Since they're both perfect cubes, we can use the difference of cubes formula

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

with $a = 6ab^2$ and $b = 5c^8d$. Therefore, we get

$$(6ab^2 - 5c^8d)((6ab^2)^2 + (6ab^2)(5c^8d) + (5c^8d)^2)$$

$$(6ab^2 - 5c^8d)(36a^2b^4 + 30ab^2c^8d + 25c^{16}d^2)$$

FACTORING THE SUM OF TWO CUBES

■ 1. Factor the polynomial.

$$x^3 + y^3$$

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{x^3} = x$$

$$\sqrt[3]{y^3} = y$$

Since they're both perfect cubes, we can use the sum of cubes formula

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

with $a = x$ and $b = y$. Therefore, we get

$$(x + y)(x^2 - xy + y^2)$$

■ 2. Factor the polynomial.

$$8x^3 + 64y^6$$

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{8x^3} = 2x$$

$$\sqrt[3]{64y^6} = 4y^2$$

Since they're both perfect cubes, we can use the sum of cubes formula

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

with $a = 2x$ and $b = 4y^2$. Therefore, we get

$$(2x + 4y^2)((2x)^2 - (2x)(4y^2) + (4y^2)^2)$$

$$(2x + 4y^2)(4x^2 - 8xy^2 + 16y^4)$$

■ 3. Factor the polynomial.

$$27z^{18} + x^6y^{12}$$

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{27z^{18}} = 3z^6$$

$$\sqrt[3]{x^6y^{12}} = x^2y^4$$

Since they're both perfect cubes, we can use the sum of cubes formula

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

with $a = 3z^6$ and $b = x^2y^4$. Therefore, we get

$$(3z^6 + x^2y^4)((3z^6)^2 - (3z^6)(x^2y^4) + (x^2y^4)^2)$$

$$(3z^6 + x^2y^4)(9z^{12} - 3x^2y^4z^6 + x^4y^8)$$

■ 4. Factor the polynomial.

$$216a^{21} + 64b^{15}c^9$$

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{216a^{21}} = 6a^7$$

$$\sqrt[3]{64b^{15}c^9} = 4b^5c^3$$

Since they're both perfect cubes, we can use the sum of cubes formula

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

with $a = 6a^7$ and $b = 4b^5c^3$. Therefore, we get

$$(6a^7 + 4b^5c^3)((6a^7)^2 - (6a^7)(4b^5c^3) + (4b^5c^3)^2)$$

$$(6a^7 + 4b^5c^3)(36a^{14} - 24a^7b^5c^3 + 16b^{10}c^6)$$

■ 5. Factor the polynomial.

$$512z^{24} + 125m^6r^3$$

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{512z^{24}} = 8z^8$$

$$\sqrt[3]{125m^6r^3} = 5m^2r$$

Since they're both perfect cubes, we can use the sum of cubes formula

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

with $a = 8z^8$ and $b = 5m^2r$. Therefore, we get

$$(8z^8 + 5m^2r)((8z^8)^2 - (8z^8)(5m^2r) + (5m^2r)^2)$$

$$(8z^8 + 5m^2r)(64z^{16} - 40m^2rz^8 + 25m^4r^2)$$

■ 6. Factor the polynomial.

$$64j^3k^6 + 8r^{12}t^6$$

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{64j^3k^6} = 4jk^2$$

$$\sqrt[3]{8r^12t^6} = 2r^4t^2$$

Since they're both perfect cubes, we can use the sum of cubes formula

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

with $a = 4jk^2$ and $b = 2r^4t^2$. Therefore, we get

$$(4jk^2 + 2r^4t^2)((4jk^2)^2 - (4jk^2)(2r^4t^2) + (2r^4t^2)^2)$$

$$(4jk^2 + 2r^4t^2)(16j^2k^4 - 8jk^2r^4t^2 + 4r^8t^4)$$

■ 7. Factor the polynomial.

$$27a^6b^3 + 64c^3$$

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{27a^6b^3} = 3a^2b$$

$$\sqrt[3]{64c^3} = 4c$$

Since they're both perfect cubes, we can use the sum of cubes formula

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

with $a = 3a^2b$ and $b = 4c$. Therefore, we get

$$(3a^2b + 4c)((3a^2b)^2 - (3a^2b)(4c) + (4c)^2)$$

$$(3a^2b + 4c)(9a^4b^2 - 12a^2bc + 16c^2)$$

■ 8. Factor the polynomial.

$$729x^{18} + 216y^6$$

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{729x^{18}} = 9x^6$$

$$\sqrt[3]{216y^6} = 6y^2$$

Since they're both perfect cubes, we can use the sum of cubes formula

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

with $a = 9x^6$ and $b = 6y^2$. Therefore, we get

$$(9x^6 + 6y^2)((9x^6)^2 - (9x^6)(6y^2) + (6y^2)^2)$$

$$(9x^6 + 6y^2)(81x^{12} - 54x^6y^2 + 36y^4)$$

■ 9. Factor the polynomial.

$$125a^3b^6 + 27c^{24}d^3$$

Solution:

Check to see if each term is a cube.

$$\sqrt[3]{125a^3b^6} = 5ab^2$$

$$\sqrt[3]{27c^{24}d^3} = 3c^8d$$

Since they're both perfect cubes, we can use the sum of cubes formula

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

with $a = 5ab^2$ and $b = 3c^8d$. Therefore, we get

$$(5ab^2 + 3c^8d)((5ab^2)^2 - (5ab^2)(3c^8d) + (3c^8d)^2)$$

$$(5ab^2 + 3c^8d)(25a^2b^4 - 15ab^2c^8d + 9c^{16}d^2)$$

ZERO THEOREM

■ 1. Find the zeros of the function.

$$y = x^2 - 5x + 6$$

Solution:

The zeros are the x-values when $y = 0$. Set the equation equal to 0 and then factor the left side.

$$x^2 - 5x + 6 = 0$$

$$(x - 2)(x - 3) = 0$$

The zero theorem tells us that one or both factors must equal 0 in order for the equation to equal 0. Set each factor equal to 0 and solve for x.

$$x - 2 = 0$$

$$x = 2$$

and

$$x - 3 = 0$$

$$x = 3$$

The roots are $x = 2$ and $x = 3$.

2. Find the zeros of the function.

$$y = x^2 - 4x - 5$$

Solution:

The zeros are the x-values when $y = 0$. Set the equation equal to 0 and then factor the left side.

$$x^2 - 4x - 5 = 0$$

$$(x - 5)(x + 1) = 0$$

The zero theorem tells us that one or both factors must equal 0 in order for the equation to equal 0. Set each factor equal to 0 and solve for x.

$$x - 5 = 0$$

$$x = 5$$

and

$$x + 1 = 0$$

$$x = -1$$

The roots are $x = 5$ and $x = -1$.

■ 3. Solve for the variable.

$$f(x) = x^2 + 10x + 24$$

Solution:

The zeros are the x-values when $f(x) = 0$. Set the equation equal to 0 and then factor the left side.

$$x^2 + 10x + 24 = 0$$

$$(x + 6)(x + 4) = 0$$

The zero theorem tells us that one or both factors must equal 0 in order for the equation to equal 0. Set each factor equal to 0 and solve for x.

$$x + 6 = 0$$

$$x = -6$$

and

$$x + 4 = 0$$

$$x = -4$$

The solutions are $x = -6$ and $x = -4$.

■ 4. Solve for the variable.

$$f(x) = 3x^2 + 7x - 6$$

Solution:

The zeros are the x-values when $f(x) = 0$. Set the equation equal to 0 and then factor the left side.

$$3x^2 + 7x - 6 = 0$$

$$(3x - 2)(x + 3) = 0$$

The zero theorem tells us that one or both factors must equal 0 in order for the equation to equal 0. Set each factor equal to 0 and solve for x.

$$3x - 2 = 0$$

$$x = \frac{2}{3}$$

and

$$x + 3 = 0$$

$$x = -3$$

The solutions are $x = -3$ and $x = 2/3$.