

Greatest common factor of polynomials

Finding the greatest common factor of polynomials uses the same techniques as finding the greatest common factor of trinomials. The difference is that trinomials have three terms, and polynomials can have any (finite) number of terms.

Factoring is “un-distributing,” which means that we do the opposite of distributing and take out (or “factor out”) the same factor from each term of the polynomial (and divide each term by that factor to get “what’s left” once it’s taken out).

The key is that all the terms of the polynomial need to share the factor being taken out. Any factor that’s shared by all the terms is called a **common factor**, and the factor that consists of everything which is shared by all of them is known as the **greatest common factor**.

When you’re first starting to factor, it can be helpful to write out all the factors of each term. For example, you’d write $2x^3y + 4x^2y^2 + 8xy$ as

$$2 \cdot x \cdot x \cdot x \cdot y + 2 \cdot 2 \cdot x \cdot x \cdot y \cdot y + 2 \cdot 2 \cdot 2 \cdot x \cdot y$$

so that you can clearly see what factors are shared. In this case, the greatest common factor is $2xy$. (Other common factors - factors that are common to all the terms of this polynomial - are 2, x , y , $2x$, and $2y$.) This does get tedious, but after a while you’ll get used to identifying the greatest common factor without having to write it out.

Example



Factor the polynomial in the numerator and simplify the resulting expression.

$$\frac{3x^2 - 15xy}{3}$$

Write out all the factors of each term in the numerator.

$$3 \cdot x \cdot x - 3 \cdot 5 \cdot x \cdot y$$

We can see that each term in the numerator has a factor of 3 and a factor of x , and that there's no factor which is shared by both terms, so the greatest common factor is $3x$. Factoring the numerator, we find that the given expression can be written as

$$\frac{3x(x - 5y)}{3}$$

Now we can simplify this by canceling the 3 in the numerator against the 3 in the denominator.

$$x(x - 5y)$$

Let's try another example of finding the greatest common factor of a polynomial.

Example

Find the greatest common factor of the polynomial.



$$4s^4t^3 + 16s^2t^2 - 24st^4$$

Write out all the factors of each term.

$$2 \cdot 2 \cdot s \cdot s \cdot s \cdot s \cdot t \cdot t \cdot t + 2 \cdot 2 \cdot 2 \cdot 2 \cdot s \cdot s \cdot t \cdot t - 2 \cdot 2 \cdot 2 \cdot 3 \cdot s \cdot t \cdot t \cdot t$$

The only factors that are shared by all three terms are a $2 \cdot 2$, an s , and a $t \cdot t$. Therefore, the greatest common factor is $4st^2$.

This problem asked only for the greatest common factor, but factoring out $4st^2$ will give

$$4st^2(s^3t + 4s - 6t^2)$$

