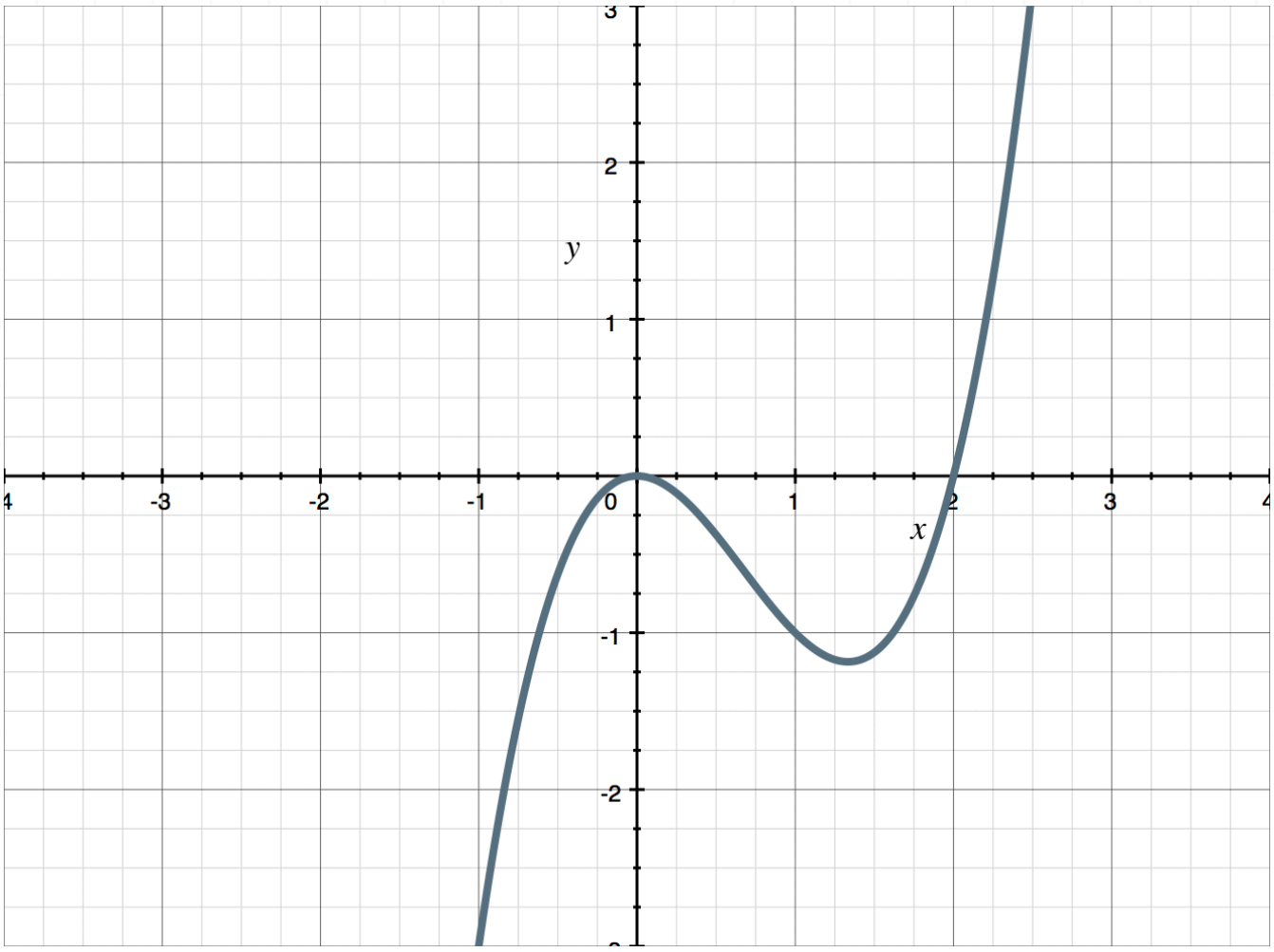
Topic: One-to-one functions and the horizontal line test

Question: This is a graph of $y = x^3 - 2x^2$. Which of these describes the relation it represents?



Answer choices:

A Not a function

B A function, but not one-to-one

C A one-to-one function

D An exponential function

Solution: B

A look at the graph tells us that there's no vertical line that intersects the graph at more than one point, so the relation is a function.

On the other hand, it's easy to find a horizontal line that intersects the graph at more than one point (the x-axis is one example), which indicates that the function is not one-to-one.

The given graph doesn't represent a quadratic function, because the graph of a quadratic function is a parabola (not the type of curve given here), and the function represented by the graph ($x^3 - 2x^2$) is a cubic polynomial (not a quadratic polynomial). Therefore the only answer choice that works is B.

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Question: Which function is not one-to-one?

Hint: Picture the graphs of these functions if you can, or use the method $f(a) = f(b)$ implies $a = b$ to determine which function isn't one-to-one.

Answer choices:

A $j(x) = \sqrt{x + 2}$

B $g(x) = 7x - 2$

C $h(x) = (x - 2)(x + 3)$

D $f(x) = \dfrac{x + 7}{x + 5}$

Solution: C

Answer choice A: $j(x) = \sqrt{x+2}$

Use the method $j(a) = j(b)$ implies $a = b$.

$$\sqrt{a+2} = \sqrt{b+2}$$

$$a + 2 = b + 2$$

$$a = b$$

That worked, so $j(x)$ is a one-to-one function.

Answer choice B: $g(x) = 7x - 2$

This is a linear function whose graph is a line that has a slope of 7, so so any horizontal or vertical line will intersect it at only one point. Therefore, $g(x)$ is a one-to-one function.

Answer choice C: $h(x) = (x-2)(x+3)$

This can be written as $h(x) = x^2 + x - 6$, a quadratic function. The graph will be a parabola that opens upward. There are horizontal lines that will intersect the parabola at two points, so $h(x)$ is not one-to-one.

Answer choice D: $f(x) = (x+7)/(x+5)$

Use the method $f(a) = f(b)$ implies $a = b$.

$$\frac{a+7}{a+5} = \frac{b+7}{b+5}$$

$$(a+7)(b+5) = (b+7)(a+5)$$

$$ab + 5a + 7b + 35 = ab + 5b + 7a + 35$$

$$5a + 7b = 5b + 7a$$

$$5a - 7a = 5b - 7b$$

$$-2a = -2b$$

$$a = b$$

That worked, so $f(x)$ is a one-to-one function.

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Question: Which function below is one-to-one?

Answer choices:

A $f(x) = 5x(x - 3)$

B $g(x) = \dfrac{2 - x^3}{4}$

C $h(x) = x^4$

D $j(x) = x^2 - 4$

Solution: B

Choices A and D are both quadratic functions, and their graphs are parabolas that open upward. For each of those functions, there are horizontal lines that will intersect its graph at two points, so neither function is one-to-one.

Answer choice B: $g(x) = \left(2 - x^3\right)/4$

Use the method $f(a) = f(b)$ implies $a = b$.

$$\frac{2 - a^3}{4} = \frac{2 - b^3}{4}$$

$$2 - a^3 = 2 - b^3$$

$$-a^3 = -b^3$$

$$a^3 = b^3$$

If the cubes of two real numbers are equal, then the numbers themselves are equal, so

$$a = b$$

That worked, so $g(x)$ is one-to-one.

Answer choice C: $h(x) = x^4$

Use the method $h(a) = h(b)$ implies $a = b$.

$$a^4 = b^4$$

$$(a^2)^2 = (b^2)^2$$

Since a and b are both real numbers, we know that a^2 and b^2 are both nonnegative real numbers. Also, if the squares of two nonnegative real numbers are equal, then the numbers themselves are equal, so

$$a^2 = b^2$$

From this it follows that if $a = 0$ then $b = 0$, and vice versa, so $a = b$. If a and b are nonzero real numbers, then

$$a^2 = b^2 \text{ if and only if } |a| = |b|$$

This solution allows a to be negative while b is positive, or vice versa, so it doesn't follow that $a = b$. Therefore, the function $h(x)$ isn't one-to-one.