Topic: Vertical Line Test

Question: If a perfectly straight vertical line crosses a graph at more than one point, the graph fails the Vertical Line Test.

Answer choices:

A   True

B   False

Solution: A

A graph passes the Vertical Line Test if it's impossible to draw a perfectly straight vertical line that crosses the graph more than once.

If you can draw a perfectly straight vertical line that crosses the graph more than once, then the graph fails the Vertical Line Test, and the graph does not represent a function.

Topic: Vertical Line Test

Question: Which of the following will never pass the Vertical Line Test and therefore can never represent a function?

Answer choices:

A   A horizontal line

B   A graph that consists of six points all of which have different x-coordinates

C   A "slanted" line (neither vertical nor horizontal)

D   A circle

Solution: D

A graph fails the Vertical Line Test when you can draw a vertical line that crosses the graph more than once. Since you'll always be able to draw a vertical line that crosses the graph of a circle more than once, a circle will always fail the Vertical Line Test, and therefore can never represent a function.

Topic: Vertical Line Test

Question: Does the graph of the equation pass the Vertical Line Test?

$$2x^2 + 2y^2 = 18$$

Answer choices:

A   Yes, because $2x^2 + 2y^2 = 18$ is a function.

B   No, because $2x^2 + 2y^2 = 18$ is a function.

C   No, because $2x^2 + 2y^2 = 18$ is not a function.

D   Yes, because $2x^2 + 2y^2 = 18$ is not a function.

Solution: C

In order for a graph to pass the Vertical Line Test, it must be the graph of a function, because only functions pass the Vertical Line Test. The test is simply that a vertical line drawn at any point in the graph must only pass through it once.

The equation $2x^2 + 2y^2 = 18$ is a circle. The graph of a circle allows for a vertical line to pass through it twice at many points, which means it automatically fails the Vertical Line Test. This means that a circle is not a function.

We could also solve the equation algebraically to prove that it doesn't represent a function.

$$2x^2 + 2y^2 = 18$$

$$\frac{2x^2}{2} + \frac{2y^2}{2} = \frac{18}{2}$$

$$x^2 + y^2 = 9$$

$$x^2 - x^2 + y^2 = 9 - x^2$$

$$y^2 = 9 - x^2$$

$$\sqrt{y^2} = \sqrt{9 - x^2}$$

$$y = \pm\sqrt{9 - x^2}$$

Now that we have the equation in this form, we can find values of x that return multiple y-values. For instance, at $x = 1$,

$$y = \pm \sqrt{9 - 1^2}$$

$$y = \pm \sqrt{8}$$

$$y = \pm 2\sqrt{2}$$

Because the equation takes on the values $y = -2\sqrt{2}$ and $y = 2\sqrt{2}$ at the single value $x = 1$, we know the equation doesn't represent a function.