Topic: Factoring the difference of two cubes

Question: Factor the expression.

$$x^3 - 64m^6r^9$$

Answer choices:

A $(x + 4m^2r^3)(x^2 + 4m^2r^3x)$

B $(x + 4m)(x - 4mx + r^3)$

C $(x - 4m^2r^3)(x^2 - 4m^2r^3x - 16m^4r^6)$

D $(x - 4m^2r^3)(x^2 + 4m^2r^3x + 16m^4r^6)$

Solution: D

We know we're dealing with the difference of cubes because we have two perfect cubes separated by a minus sign to indicate that the second perfect cube is to be subtracted from the first perfect cube.

When that's the case, we take the cube root of each term.

The cube root of x^3 is x.

The cube root of $64m^6r^9$ is $4m^2r^3$.

The difference of cubes $a^3 - b^3$ is always factored as

$$(a-b)(a^2+ab+b^2)$$

Since in this case $a = x$ and $b = 4m^2r^3$, we get

$$(x-4m^2r^3)\left[x^2+x(4m^2r^3)+(4m^2r^3)^2\right]$$

$$(x-4m^2r^3)(x^2+4m^2r^3x+16m^4r^6)$$

Topic: Factoring the difference of two cubes

Question: Factor the expression.

$$8y^3z^6 - 125x^3$$

Answer choices:

A $(2y^2z^2 - 5x)(4y^2z^4 + 10xyz + 25x^2)$

B $(yz^2 - 5x)(8y^2z^4 + 10xyz^2 + 25x^2)$

C $(2yz^2 - 5x)(4y^2z^4 + 10xyz^2 + 25x^2)$

D $(8yz^2 - 5x)(y^2z^4 + 10xyz^2 + 25x^2)$

Solution: C

The formula for factoring the difference of cubes is

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

In $8y^3z^6 - 125x^3$,

a is the cube root of $8y^3z^6$, which is $2yz^2$.

b is the cube root of $125x^3$, which is $5x$.

Now we'll apply the formula given above.

$$(a - b)(a^2 + ab + b^2)$$

$$(2yz^2 - 5x)(4y^2z^4 + 10xyz^2 + 25x^2)$$

We can check our work by distributing each term in the binomial factor over all the terms in the trinomial factor.

$$(2yz^2)(4y^2z^4) + (2yz^2)(10xyz^2) + (2yz^2)(25x^2)$$

$$+(-5x)(4y^2z^4) + (-5x)(10xyz^2) + (-5x)(25x^2)$$

$$8y^3z^6 + 20xy^2z^4 + 50x^2yz^2 - 20xy^2z^4 - 50x^2yz^2 - 125x^3$$

$$8y^3z^6 - 125x^3$$

Topic: Factoring the difference of two cubes

Question: Factor the expression.

$$64r^3v^3 - 216z^9$$

Answer choices:

A $(8rv - 6z^3)(8r^2v^2 + 24rvz^3 + 36z^6)$

B $(4rv - 6z^6)(16r^2v^2 + 24rvz^6 + 36z^9)$

C $(8rv - 6z^3)(8r^2v^2 + 18rvz^3 + 36z^6)$

D $(4rv - 6z^3)(16r^2v^2 + 24rvz^3 + 36z^6)$

Solution: D

The formula for factoring the difference of cubes is

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

In $64r^3v^3 - 216z^9$,

a is the cube root of $64r^3v^3$, which is $4rv$.

b is the cube root of $216z^9$, which is $6z^3$.

Now we'll apply the formula given above.

$$(a - b)(a^2 + ab + b^2)$$

$$(4rv - 6z^3)(16r^2v^2 + 24rvz^3 + 36z^6)$$

We can check our work by distributing each term in the binomial factor over all the terms in the trinomial factor.

$$(4rv)(16r^2v^2) + (4rv)(24rvz^3) + (4rv)(36z^6)$$

$$+(-6z^3)(16r^2v^2) + (-6z^3)(24rvz^3) + (-6z^3)(36z^6)$$

$$64r^3v^3 + 96r^2v^2z^3 + 144rvz^6 - 96r^2v^2z^3 - 144rvz^6 - 216z^9$$

$$64r^3v^3 - 216z^9$$