Topic: Factoring quadratic polynomials with coefficients

Question: Factor the quadratic.

$$5x^2 - 3x - 2$$

Answer choices:

A $(5x + 2)(x + 1)$

B $(5x - 2)(x + 1)$

C $(5x + 2)(x - 1)$

D $(5x - 2)(x - 1)$

Solution: C

The only factors of 5 are 5 and 1, so we know we'll have

$$(5x \quad)(x \quad)$$

The only factors of 2 are 2 and 1, which means we'll have one of the following:

$$(5x \quad 1)(x \quad 2) \qquad \text{or} \qquad (5x \quad 2)(x \quad 1)$$

If we do the factoring the first way, we'll need to combine $10x$ and x to get the middle term, $-3x$. There's no way we can do that, even if we make one or both of them negative. If we do the factoring the second way, we'll need to combine $5x$ and $2x$ to get $-3x$. If we make the $5x$ negative and keep the $2x$ positive, then we get $-5x + 2x = -3x$. Therefore, we have to use -1 as the constant term in the second factor (because $-5x = 5x \cdot -1$), and 2 as the constant term in the first factor (because $2x = 2 \cdot x$), so we get

$$(5x + 2)(x - 1)$$

Topic: Factoring quadratic polynomials with coefficients

Question: Factor the quadratic.

$$6x^2 - 2x - 4$$

Answer choices:

A $2(3x + 2)(x + 1)$

B $2(3x - 2)(x - 1)$

C $2(3x - 2)(x + 1)$

D $2(3x + 2)(x - 1)$

Solution: D

First, we'll factor out a 2, because 2 is the factor that's common to all three terms.

$$6x^2 - 2x - 4$$

$$2(3x^2 - x - 2)$$

The only factors of 3 are 3 and 1, so we know we'll have

$$2(3x \quad)(x \quad)$$

The only factors of 2 are 2 and 1, which means we'll have one of the following:

$$2(3x \quad 1)(x \quad 2) \qquad \text{or} \qquad 2(3x \quad 2)(x \quad 1)$$

If we do the factoring the first way, we'll need to combine $6x$ and x to get the middle term, $-x$. There's no way we can do that, even if we make one or both of them negative. If we do the factoring the second way, we'll need to combine $3x$ and $2x$ to get $-x$. If we make the $3x$ negative and keep the $2x$ positive, then we get $-3x + 2x = -x$. Therefore, we have to use -1 as the constant term in the second factor in parentheses (because $-3x = 3x \cdot -1$), and 2 as the constant term in the first factor in parentheses (because $2x = 2 \cdot x$), so we get

$$2(3x + 2)(x - 1)$$

Topic: Factoring quadratic polynomials with coefficients

Question: Factor the cubic polynomial, remembering to look first for a greatest common factor.

$$6x^3 + 11x^2 - 2x$$

Answer choices:

A $x(6x + 1)(x - 2)$

B $x(6x - 1)(x - 2)$

C $x(6x - 1)(x + 2)$

D $(6x + 1)(x + 2)$

Solution: C

Note that $6x^3 + 11x^2 - 2x$ is a trinomial (a polynomial with three nonzero terms) but not a quadratic polynomial (because it has an x^3 term). However, we can factor out an x, because x is the factor that's common to all three terms.

$$6x^3 + 11x^2 - 2x$$

$$x(6x^2 + 11x - 2)$$

The only pairs of factors of 6 are $(6,1)$ and $(3,2)$, so we'll have one of these:

$x(6x \quad)(x \quad)$ or $x(3x \quad)(2x \quad)$

The only factors of 2 are 2 and 1, which means we'll have one of the following four possibilities:

$x(6x \quad 2)(x \quad 1)$ $\qquad$ $x(3x \quad 2)(2x \quad 1)$

$x(6x \quad 1)(x \quad 2)$ $\qquad$ $x(3x \quad 1)(2x \quad 2)$

If we do the factoring as $x(6x \quad 2)(x \quad 1)$, we'll need to combine $6x$ and $2x$ to get the middle term, $11x$. There's no way we can do that, even if we make one or both of them negative.

If we do the factoring as $x(3x \quad 2)(2x \quad 1)$, we'll need to combine $3x$ and $4x$ to get $11x$. There's no way we can do that, even if we make one or both of them negative.

If we do the factoring as $x(3x \quad 1)(2x \quad 2)$, we'll need to combine $6x$ and $2x$ to get $11x$. There's no way we can do that, even if we make one or both of them negative.

Finally, if we do the factoring as $x(6x \quad 1)(x \quad 2)$, we'll need to combine $12x$ and x to get $11x$, which we can do by making $12x$ positive and x negative. Therefore, we have to use 2 as the constant term in the second factor in parentheses (because $12x = 6x \cdot 2$), and -1 as the constant term in the first factor in parentheses (because $-x = -1 \cdot x$), so we get

$$x(6x - 1)(x + 2)$$