Combinations of functions

In this lesson you'll learn how to combine functions and use combination notation.

Let's say you have two functions. We'll call them $f(x)$ and $g(x)$, since those seem to be the most commonly used names. Then you can perform addition, subtraction, multiplication, and division of the two functions.

When you perform these operations, the resulting function is called a combination. Let's look at each operation and how it's defined and written when it's used for functions.

Sum	$(f+g)(x)=f(x)+g(x)$
Difference	$(f-g)(x)=f(x)-g(x)$
Product	$(f\cdot g)(x)=f(x)\cdot g(x)$
Quotient	$(f\div g)(x)=\dfrac{f(x)}{g(x)}$

Just as with these operations on numbers, the order of the functions in an addition or multiplication doesn't matter, but the order of the functions in a subtraction or division does matter. In a subtraction or division of two functions, if the f comes first, then you need to start with $f(x)$; if the g comes first, then you need to start with $g(x)$.

Also, we will look at composite functions in the next lesson, and the notation for a composite function can look very similar to the notation for a product. Look closely here to be sure you see the difference.

Product	$(f \cdot g)(x) = f(x) \cdot g(x)$
Composite	$(f \circ g)(x) = f(g(x))$

See the difference? There's a big difference in meaning between an open circle (for a composite function) and a closed circle (for a product of functions).

Now let's do an example of each of the four operations.

Example

Find $(f - g)(x)$.

$$f(x) = 3x^2 + 2x - 4$$

$$g(x) = x^2 - 3x + 2$$

The combination $(f - g)(x)$ means the same thing as $f(x) - g(x)$, so we can find the difference.

$$(f - g)(x) = (3x^2 + 2x - 4) - (x^2 - 3x + 2)$$

$$(f - g)(x) = 3x^2 + 2x - 4 - x^2 + 3x - 2$$

$$(f - g)(x) = 3x^2 - x^2 + 2x + 3x - 4 - 2$$

$$(f - g)(x) = 2x^2 + 5x - 6$$

How about addition?

Example

Find $(f+g)(x)$.

$$f(x) = 3x - 4$$

$$g(x) = 3x^2 + 5x + 3$$

Remember that $(f+g)(x)$ is the same thing as $f(x) + g(x)$. Therefore,

$$(f+g)(x) = (3x - 4) + (3x^2 + 5x + 3)$$

$$(f+g)(x) = 3x - 4 + 3x^2 + 5x + 3$$

$$(f+g)(x) = 3x^2 + 3x + 5x - 4 + 3$$

$$(f+g)(x) = 3x^2 + 8x - 1$$

It's possible to use names other than f and g for the functions in a combination. Let's use some different function names for the product.

Example

Find $(h \cdot m)(x)$.

$$h(x) = 4x - 3$$

$$m(x) = -3x^2 - 1$$

The combination $(h \cdot m)(x)$ is the same as $h(x) \cdot m(x)$. Therefore,

$$(h \cdot m)(x) = (4x - 3)(-3x^2 - 1)$$

We can find this product using the FOIL method.

$$(h \cdot m)(x) = -12x^3 - 4x + 9x^2 + 3$$

$$(h \cdot m)(x) = -12x^3 + 9x^2 - 4x + 3$$

Now let's try a problem where we find the quotient.

Example

Find $(b \div w)(x)$.

$$w(x) = 2x$$

$$b(x) = 3$$

Remember that $(b \div w)(x)$ means the same thing as $b(x)/w(x)$. Therefore,

$$(b \div w)(x) = \frac{3}{2x}$$