Topic: Center and radius of a circle

Question: Find the center and radius of the circle.

$$4x^2 + 4y^2 + 4x - 12y + 1 = 0$$

Answer choices:

A Center at $\left(\frac{1}{2}, \frac{3}{2}\right)$ Radius of $\frac{3}{2}$

B Center at $\left(-\frac{1}{2}, \frac{3}{2}\right)$ Radius of $\frac{3}{2}$

C Center at $\left(\frac{1}{2}, \frac{3}{2}\right)$ Radius of $\frac{1}{2}$

D Center at $\left(-\frac{1}{2}, \frac{3}{2}\right)$ Radius of $\frac{1}{2}$

Solution: B

In order to find the center and radius, we need to convert the equation of the circle to standard form, $(x-h)^2+(y-k)^2=r^2$, where h and k are the coordinates of the center and r is the radius.

We'll begin by grouping the x terms separately from the y terms, and moving the constant term to the right side of the equation.

$$4x^2+4y^2+4x-12y+1=0$$

$$4x^2+4y^2+4x-12y=-1$$

In standard from, the coefficients of the x^2 term and the y^2 term must be equal to 1. Since the coefficient of each of those terms is now 4, we'll factor out a 4 on the left side of the equation and then divide both sides by 4.

$$4(x^2+y^2+x-3y)=-1$$

$$x^2+y^2+x-3y=-\frac{1}{4}$$

In order to get this equation into standard form, we need to complete the square on both x and y.

$$(x^2+x)+(y^2-3y)=-\frac{1}{4}$$

To complete the square on x, we need to find the number a that satisfies the equation

$$x^2+x+a^2=(x+a)^2$$

That is, we need to find the number a for which

$$x^2 + x + a^2 = x^2 + 2ax + a^2$$

This means that the coefficient of the x term of the expression inside the first set of parentheses must be equal to $2a$. That coefficient is 1, so we'll set $2a$ to 1 and solve for a.

$$2a = 1 \quad \to \quad a = \frac{1}{2}$$

To keep our equation balanced, we need to add and subtract a^2 $(1/4)$ inside that set of parentheses and then regroup.

$$(x^2 + x) + (y^2 - 3y) = -\frac{1}{4}$$

$$\left(x^2 + x + \frac{1}{4} - \frac{1}{4}\right) + (y^2 - 3y) = -\frac{1}{4}$$

$$\left(x^2 + x + \frac{1}{4}\right) - \frac{1}{4} + (y^2 - 3y) = -\frac{1}{4}$$

To complete the square on y, we need to find the number b that satisfies the equation

$$y^2 - 3y + b^2 = (y + b)^2$$

That is, we need to find the number b for which

$$y^2 - 3y + b^2 = y^2 + 2by + b^2$$

This means that the coefficient of the y term of the expression inside the second set of parentheses must be equal to $2b$. That coefficient is -3, so we'll set $2b$ to -3 and solve for b.

$$2b = -3 \quad \rightarrow \quad b = -\frac{3}{2}$$

To keep our equation balanced, we need to add and subtract b^2 $(9/4)$ inside that set of parentheses and then regroup.

$$\left(x^2 + x + \frac{1}{4}\right) - \frac{1}{4} + (y^2 - 3y) = -\frac{1}{4}$$

$$\left(x^2 + x + \frac{1}{4}\right) - \frac{1}{4} + \left(y^2 - 3y + \frac{9}{4} - \frac{9}{4}\right) = -\frac{1}{4}$$

$$\left(x^2 + x + \frac{1}{4}\right) - \frac{1}{4} + \left(y^2 - 3y + \frac{9}{4}\right) - \frac{9}{4} = -\frac{1}{4}$$

Moving the $-1/4$ and $-9/4$ to the right side, we have

$$\left(x^2 + x + \frac{1}{4}\right) + \left(y^2 - 3y + \frac{9}{4}\right) = -\frac{1}{4} + \frac{1}{4} + \frac{9}{4}$$

Factoring the expressions in parentheses and simplifying the right side, we get

$$\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 = \frac{9}{4}$$

If you think of $x + (1/2)$ and $9/4$ as $x - (-1/2)$ and $(3/2)^2$, respectively, you'll see that the center of the circle is at

$$(h,k)=\left(-\frac{1}{2},\frac{3}{2}\right)$$

and its radius is

$$r=\frac{3}{2}$$

Topic: Center and radius of a circle

Question: Find the center and radius of the given circle.

$$x^2 + y^2 - 6y = 5$$

Answer choices:

A Center is $(0,3)$. Radius is $\sqrt{14}$.

B Center is $(0, -3)$. Radius is $\sqrt{14}$.

C Center is $(0,3)$. Radius is 14.

D Center is $(0, -3)$. Radius is 14.

Solution: A

x^2 is already a perfect square, so we'll complete the square on y.

$$x^2 + (y^2 - 6y) = 5$$

To do that, we need to find the number a that satisfies the equation

$$y^2 - 6y + a^2 = (y + a)^2$$

That is, we need to find the number a for which

$$y^2 - 6y + a^2 = y^2 + 2ay + a^2$$

This means that the coefficient of the y term of the expression inside the parentheses must be equal to $2a$. That coefficient is -6, so we'll set $2a$ equal to -6 and solve for a.

$$2a = -6 \quad \to \quad a = -3$$

To keep our equation balanced, we need to add and subtract a^2 (9) inside the parentheses and then regroup.

$$x^2 + (y^2 - 6y) = 5$$

$$x^2 + (y^2 - 6y + 9 - 9) = 5$$

$$x^2 + (y^2 - 6y + 9) - 9 = 5$$

Moving the -9 to the right side, we have

$$x^2 + (y^2 - 6y + 9) = 5 + 9$$

Factoring the expression in parentheses and simplifying the right side, we get

$$x^2 + (y - 3)^2 = 14$$

If you think of x and 14 as $x - 0$ and $(\sqrt{14})^2$, respectively, you'll see that the center of the circle is at $(h, k) = (0,3)$ and the radius is $\sqrt{14}$.

Topic: Center and radius of a circle

Question: Find the center and radius of the given circle.

$$x^2 + y^2 + 10x - 4y + 13 = 0$$

Answer choices:

A Center is $(-5,2)$. Radius is 16.

B Center is $(5, -2)$. Radius is 4.

C Center is $(-5,2)$. Radius is 4.

D Center is $(5, -2)$. Radius is 16.

Solution: C

Starting with

$$x^2 + y^2 + 10x - 4y + 13 = 0$$

we'll group the terms in x separately from the terms in y, and subtract 13 from both sides.

$$x^2 + 10x + y^2 - 4y = -13$$

We need to complete the square on both x and y.

$$(x^2 + 10x) + (y^2 - 4y) = -13$$

To complete the square on x, we need to find the number a that satisfies the equation

$$x^2 + 10x + a^2 = (x + 1)^2$$

That is, we need to find the number a for which

$$x^2 + 10x + a^2 = s^2 + 2ax + a^2$$

This means that the coefficient of the x term of the expression inside the first set of parentheses must be equal to $2a$. That coefficient is 10, so we'll set $2a$ equal to 10 and solve for a.

$$2a = 10 \qquad \rightarrow \qquad z = 5$$

To keep our equation balanced, we need to add and subtract a^2 (25) inside that set of parentheses and then regroup.

$$(x^2 + 10x) + (y^2 - 4y) = -13$$

$$(x^2 + 10x + 25 - 25) + (y^2 - 4y) = -13$$

$$(x^2 + 10x + 25) - 25 + (y^2 - 4y) = -13$$

To complete the square on y, we need to find the number b that satisfies the equation

$$y^2 - 4y + b^2 = (y + b)^2$$

That is, we need to find the number b for which

$$y^2 - 4y + b^2 = y^2 + 2by + b^2$$

This means that the coefficient of the y term of the expression inside the second set of parentheses must be equal to $2b$. That coefficient is -4, so we'll set $2b$ equal to -4 and solve for b.

$$2b = -4 \quad \to \quad b = -2$$

To keep our equation balanced, we need to add and subtract b^2 (4) inside that set of parentheses and then regroup.

$$(x^2 + 10x + 25) - 25 + (y^2 - 4y) = -13$$

$$(x^2 + 10x + 25) - 25 + (y^2 - 4y + 4 - 4) = -13$$

$$(x^2 + 10x + 25) - 25 + (y^2 - 4y + 4) - 4 = -13$$

Moving the -25 and -4 to the right side, we have

$$(x^2 + 10x + 25) + (y^2 - 4y + 4) = -13 + 25 + 4$$

Factoring the expressions in parentheses and simplifying the right side, we get

$$(x+5)^2 + (y-2)^2 = 16$$

If you think of $x+5$ and 16 as $x-(-5)$ and 4^2, respectively, you'll see that the center of the circle is at $(h,k) = (-5,2)$ and the radius is 4.