Equation of a line in slope-intercept form

You have two options for writing the equation of a line: point-slope form and slope-intercept form. Both of them require that you know at least two of the following pieces of information about the line:

1. A point
2. Another point
3. The slope, m
4. The y-intercept, b (the y-coordinate of the point at which the graph of the line crosses the y-axis)

If you know any two of these things, you can find the equation of the line.

Slope-intercept form

The equation of a line in slope-intercept form is written as

$$y = mx + b$$

where m is the slope and b is the y-intercept. Since the y-intercept, b, is the y-coordinate of the point at which the graph crosses the y-axis, and since the x-coordinate of every point on the y-axis is 0, the point where the graph crosses the y-axis is $(0,b)$.

Let's try an example where we know the slope and a point on the line.

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Example

Find the equation of the line in slope-intercept form.

$$m = -3$$

$$(0,1)$$

If you recognize that the point $(0,1)$ lies on the y-axis (since its x-coordinate is 0), then you'll realize that there's practically no work to do for this type of problem. Simply substitute -3 for m, and 1 for b, in the equation $y = mx + b$.

$$y = -3x + 1$$

If you were unsure that the given point, $(0,1)$, was the y-intercept, you could use the point-slope form of the equation of a line, $y - y_1 = m(x - x_1)$, and then solve for y.

$$y - 1 = -3(x - 0)$$

$$y - 1 = -3x$$

$$y - 1 + 1 = -3x + 1$$

$$y = -3x + 1$$

Let's try an example where we know two points on the line.

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Example

Find the slope-intercept form of the equation of the line that passes through the points $(-1, -2)$ and $(3, -4)$.

First, we need to find the slope using the formula $m = (y_2 - y_1)/(x_2 - x_1)$. It's best to label the points before you plug them in.

$$(-1, -2) = (x_1, y_1)$$

$$(3, -4) = (x_2, y_2)$$

Plug these into the formula.

$$m = \frac{-4 - (-2)}{3 - (-1)}$$

$$m = \frac{-2}{4}$$

$$m = -\frac{1}{2}$$

Next, substitute $m = -1/2$ and the coordinates of one of the points into the point-slope form of the equation of a line.

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = -\frac{1}{2}(x - (-1))$$

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$$y + 2 = -\frac{1}{2}(x + 1)$$

$$y + 2 = -\frac{1}{2}x - \frac{1}{2}$$

Finally, solve for y by subtracting 2 from both sides.

$$y + 2 - 2 = -\frac{1}{2}x - \frac{1}{2} - 2$$

$$y = -\frac{1}{2}x - \frac{5}{2}$$

Let's do another example where we know two points on the line and need to write the equation of the line in slope-intercept form. We'll use a method that's partially different than the one we used in the previous example.

Example

Find the slope-intercept form of the equation of the line that passes through the points $(1,2)$ and $(3,5)$.

We'll again start by finding the slope.

$$m = \frac{5 - 2}{3 - 1} = \frac{3}{2}$$

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This time, however, we'll plug the slope and the coordinates of one of our two given points, $(1,2)$, into the equation $y = mx + b$, and then solve for b.

$$y = mx + b$$

$$2 = \frac{3}{2}(1) + b$$

$$2 = \frac{3}{2} + b$$

$$2 - \frac{3}{2} = \frac{3}{2} - \frac{3}{2} + b$$

$$\frac{1}{2} = b$$

For the final answer, plug b and m into the equation $y = mx + b$, leaving x and y as they are.

$$y = \frac{3}{2}x + \frac{1}{2}$$

Since the slope (m) and y-intercept (b) of a given line are fixed, there can't be more than one equation of a given line in slope-intercept form:

$$y = mx + b$$

However, for every point (x_1, y_1) on a given line (with slope m), we get a different equation of that line in point-slope form:

$$y - y_1 = m(x - x_1)$$

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For example, if a line has slope 4 and we know that one point on the line is $(2,6)$, then one equation of this line in point-slope form is

$$y - 6 = 4(x - 2)$$

We can use that equation to get the coordinates of a different point on the line, by plugging in any number (other than 2) for x and then solving for y. For example, if we use $x = 5$, we get

$$y - 6 = 4(5 - 2)$$

$$y - 6 = 4(3)$$

$$y - 6 = 12$$

$$y - 6 + 6 = 12 + 6$$

$$y = 18$$

So the point $(5,18)$ is also on this line. If we let $(x_1, y_1) = (5,18)$, we get the following equation of this line in point-slope form:

$$y - 18 = 4(x - 5)$$

There are lines whose equation can't be written in point-slope form or slope-intercept form. This happens only for lines whose slope is undefined. The only thing that could make the slope of a line undefined is the existence of two (different) points on that line, (x_1, y_1) and (x_2, y_2), whose x-coordinates are equal ($x_1 = x_2$). In that case, we would find that the slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_2 - y_1}{0}$$

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which is undefined because division by 0 is undefined.

If you think about this, you'll probably realize that if there are two points on a line that have the same x-coordinate, then all the points on that line have the same x-coordinate. Therefore, the line is vertical and can be written as $x = c$ for some number c. For example, $x = 3$ is the equation of the line in which all the points have an x-coordinate of 3. Note that for any line that satisfies an equation of this type ($x = c$ for some number c), no point of the line lies on the y-axis (unless $c = 0$, in which case every point of the line lies on the y-axis).

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