

Topic: Systems of two equations with a non-linear equation

Question: Use any method to find the solution(s) to the system of equations.

$$x^2 + y^2 = 25$$

$$3x - y = 5$$

Answer choices:

A (0,5) and $(-3, -4)$

B $(0, -5)$ and $(-3, -4)$

C (0,5) and (3,4)

D $(0, -5)$ and (3,4)



Solution: D

We'll solve the second equation for y , and then substitute the resulting expression for y into the first equation.

$$3x - y = 5$$

$$-y = 5 - 3x$$

$$y = 3x - 5$$

Plug this expression for y into the first equation, and then solve for x .

$$x^2 + y^2 = 25$$

$$x^2 + (3x - 5)^2 = 25$$

$$x^2 + (3x - 5)(3x - 5) = 25$$

$$x^2 + 9x^2 - 15x - 15x + 25 = 25$$

$$10x^2 - 30x + 25 = 25$$

$$10x^2 - 30x = 0$$

$$x^2 - 3x = 0$$

$$x(x - 3) = 0$$

$$x = 0, 3$$

Now we'll plug these values of x into the expression we found for y , to get the corresponding values of y .



For $x = 0$:

$$y = 3x - 5$$

$$y = 3(0) - 5$$

$$y = 0 - 5$$

$$y = -5$$

For $x = 3$:

$$y = 3x - 5$$

$$y = 3(3) - 5$$

$$y = 9 - 5$$

$$y = 4$$

The points $(0, -5)$ and $(3,4)$ satisfy the linear equation in this system. We have to make sure they also satisfy the non-linear equation.

For $(0, -5)$:

$$x^2 + y^2 = 25$$

$$(0)^2 + (-5)^2 = 25$$

$$0 + 25 = 25$$

$$25 = 25$$

For $(3,4)$:



$$x^2 + y^2 = 25$$

$$(3)^2 + (4)^2 = 25$$

$$9 + 16 = 25$$

$$25 = 25$$

Both $(0, -5)$ and $(3,4)$ are solutions to the system of equations.



Topic: Systems of two equations with a non-linear equation

Question: Solve the system for x and y .

$$(x - 2)^2 + y^2 = 4$$

$$\frac{x}{2} - y = -1$$

Answer choices:

A $(2,2)$ and $\left(\frac{6}{5}, \frac{2}{5}\right)$

B $\left(\frac{6}{5}, 2\right)$ and $\left(\frac{2}{5}, 2\right)$

C $\left(2, \frac{2}{5}\right)$ and $\left(2, \frac{6}{5}\right)$

D $(2,2)$ and $\left(\frac{2}{5}, \frac{6}{5}\right)$



Solution: D

Solve the second equation for y .

$$\frac{x}{2} - y = -1$$

$$-y = -\frac{x}{2} - 1$$

$$y = \frac{x}{2} + 1$$

Substitute this expression for y into the first equation.

$$(x - 2)^2 + y^2 = 4$$

$$(x - 2)^2 + \left(\frac{x}{2} + 1\right)^2 = 4$$

Expand the squares.

$$(x - 2)(x - 2) + \left(\frac{x}{2} + 1\right)\left(\frac{x}{2} + 1\right) = 4$$

$$x^2 - 4x + 4 + \frac{x^2}{4} + x + 1 = 4$$

$$x^2 + \frac{x^2}{4} - 4x + x + 4 + 1 = 4$$

$$x^2 + \frac{x^2}{4} - 3x + 1 = 0$$

Clear the fraction by multiplying both sides by 4.



$$4x^2 + x^2 - 12x + 4 = 0$$

$$5x^2 - 12x + 4 = 0$$

Factor, and then solve for x .

$$(5x - 2)(x - 2) = 0$$

$$5x - 2 = 0 \text{ gives } x = 2/5$$

$$x - 2 = 0 \text{ gives } x = 2$$

Plug these values of x into the expression we found for y , to get the corresponding values of y .

For $x = 2/5$:

$$y = \frac{x}{2} + 1$$

$$y = \frac{1}{2} \left(\frac{2}{5} \right) + 1$$

$$y = \frac{1}{5} + 1$$

$$y = \frac{6}{5}$$

For $x = 2$:

$$y = \frac{x}{2} + 1$$

$$y = \frac{1}{2}(2) + 1$$



$$y = \frac{2}{2} + 1$$

$$y = 1 + 1$$

$$y = 2$$

This tells us that the points (2,2) and (2/5,6/5) satisfy the linear equation in this system. If you plug the coordinates of these points into the non-linear equation, you'll find that they satisfy that one as well.



Topic: Systems of two equations with a non-linear equation

Question: Solve the system for x and y .

$$2x^2 - 12x - y + 19 = 0$$

$$2x + y = 11$$

Answer choices:

- A (1,3) and (4,9)
- B (4,3) and (1,9)
- C (9,3) and (1,4)
- D (3,1) and (4,9)



Solution: B

Solve the second equation for y .

$$2x + y = 11$$

$$y = -2x + 11$$

Substitute this expression for y into the first equation, and then simplify.

$$2x^2 - 12x - y + 19 = 0$$

$$2x^2 - 12x - (-2x + 11) + 19 = 0$$

$$2x^2 - 12x + 2x - 11 + 19 = 0$$

$$2x^2 - 10x + 8 = 0$$

Divide both sides by 2.

$$x^2 - 5x + 4 = 0$$

Factor, and then solve for x .

$$(x - 4)(x - 1) = 0$$

$$x - 4 = 0 \text{ gives } x = 4$$

$$x - 1 = 0 \text{ gives } x = 1$$

Plug these values of x into the expression we found for y , to get the corresponding values of y .

For $x = 4$:



$$y = -2x + 11$$

$$y = -2(4) + 11$$

$$y = -8 + 11$$

$$y = 3$$

For $x = 1$:

$$y = -2x + 11$$

$$y = -2(1) + 11$$

$$y = -2 + 11$$

$$y = 9$$

So the points (4,3) and (1,9) satisfy the linear equation in this system.

If you plug the coordinates of these points into the non-linear equation, you will find that they satisfy that one as well.

