Imaginary numbers

In this lesson we'll look at the imaginary number i, what it means, and how to use it in expressions.

The imaginary number i is defined as the square root of -1, and we can use it in algebraic expressions. An imaginary number (in general) is defined as a number that can be written as a product of a real number and i. For instance, $4i$ and $-15i$ are imaginary numbers.

Things you need to remember about imaginary numbers:

1. The formulas for i and i^2 are

$$i = \sqrt{-1} \text{ and } i^2 = -1$$

You can use these formulas to express i^3 as the imaginary number $-i$, and i^4 as the real number 1:

$$i^3 = i^{2+1} = (i^2)(i^1) = (-1)(i) = -i$$

$$i^4 = i^{2+2} = (i^2)(i^2) = (-1)(-1) = 1$$

In fact, you should convince yourself that if n is any positive integer, then you can express i^n as either an imaginary number (if n is odd) or a real number (if n is even).

2. When you add or subtract expressions with i raised to the same power, treat them as like terms.

   For example, in the list

$i, 3i^2, 4, 2i, 8, 5i^2$

the like terms are

i and $2i$

$3i^2$ and $5i^2$

4 and 8

You could in turn use $i^2 = -1$ to express $3i^2$ and $5i^2$ as -3 and -5, respectively, so the like terms in the list $i, 3i^2, 4, 2i, 8, 5i^2$ would end up being

i and $2i$

-3, 4, 8, and -5

3. If you have the sum of a real number and an imaginary number, you should write the real number first and the imaginary number second.

   So $-6i + 8$ should be written as $8 - 6i$, with the real number first and the imaginary number second. A number that can be written as the sum of a real number and an imaginary number (a number that can be written in the form $a + bi$ where a and b are real numbers) is called a **complex number**.

Let's begin with a simple example.

Example

Simplify the expression.

$$-1 - 8i - 4 - i$$

Begin by grouping the like terms.

$$-1 - 4 - 8i - i$$

Remember that there's an unwritten 1 in front of the i.

$$-1 - 4 - 8i - 1i$$

Follow the usual addition and subtraction rules.

$$-5 - 9i$$

Let's look at another one.

Example

Simplify the expression.

$$\sqrt{-9} + \sqrt{9} + 5 + 3i - \sqrt{-4}$$

Remember that

$$\sqrt{-1} = i$$

Let's start with the square roots.

$$\sqrt{9 \cdot -1} + \sqrt{9} + 5 + 3i - \sqrt{4 \cdot -1}$$

$$\sqrt{9}\sqrt{-1} + \sqrt{9} + 5 + 3i - \sqrt{4}\sqrt{-1}$$

$$3i + 3 + 5 + 3i - 2i$$

Now group like terms.

$$3 + 5 + 3i + 3i - 2i$$

$$8 + 4i$$

Let's do one final example.

Example

Simplify the expression.

$$-\sqrt{-25} + 8i^3 + 2i - \sqrt{-4}\sqrt{4} + 3\sqrt{-9}$$

Let's start with the square roots.

$$-\sqrt{25 \cdot -1} + 8i^3 + 2i - \sqrt{4 \cdot -1}\sqrt{4} + 3\sqrt{9 \cdot -1}$$

$$-\sqrt{25}\sqrt{-1} + 8i^3 + 2i - \sqrt{4}\sqrt{-1}\sqrt{4} + 3\sqrt{9}\sqrt{-1}$$

$$-5i + 8i^3 + 2i - 2i \cdot 2 + 3 \cdot 3i$$

Let's simplify $8i^3$ to $8i^2i$, which is $8(-1)i$, or $-8i$.

$$-5i - 8i + 2i - 2i \cdot 2 + 3 \cdot 3i$$

Now let's do the rest of the multiplication.

$$-5i - 8i + 2i - 4i + 9i$$

Finally, let's combine like terms, by doing the addition and subtraction from left to right:

$$-13i + 2i - 4i + 9i$$

$$-11i - 4i + 9i$$

$$-15i + 9i$$

$$-6i$$