Topic: Radical equations

Question: Solve for the variable.

$$\sqrt{x} + 5 = 9$$

Answer choices:

A $\sqrt{x}$

B   16

C $\pm\sqrt{4}$

D   4

Solution: B

We have to keep the equation balanced, so when we subtract 5 from the left side, we'll also subtract it from the right side.

$$\sqrt{x} + 5 = 9$$

$$\sqrt{x} + 5 - 5 = 9 - 5$$

$$\sqrt{x} = 4$$

Squaring both sides, we get

$$\left(\sqrt{x}\right)^2 = 4^2$$

$$x = 16$$

To determine whether this is actually a solution, we'll substitute 16 for x in the original equation:

$$\sqrt{x} + 5 = 9$$

$$\sqrt{16} + 5 = 9$$

$$4 + 5 = 9$$

$$9 = 9$$

This equation is true, so $x = 16$ is indeed a solution.

Topic: Radical equations

Question: Solve for the variable.

$$\sqrt{x^2 + 3x - 10} + 2 = x$$

Answer choices:

A   2

B   4

C   6

D   8

Solution: A

We'll first get the radical by itself.

$$\sqrt{x^2 + 3x - 10} + 2 = x$$

$$\sqrt{x^2 + 3x - 10} + 2 - 2 = x - 2$$

$$\sqrt{x^2 + 3x - 10} = x - 2$$

Squaring both sides, we get

$$\left(\sqrt{x^2 + 3x - 10}\right)^2 = (x - 2)^2$$

$$x^2 + 3x - 10 = x^2 - 4x + 4$$

Next, we'll subtract x^2 from both sides.

$$x^2 - x^2 + 3x - 10 = x^2 - x^2 - 4x + 4$$

$$3x - 10 = -4x + 4$$

To solve for x, we'll need to collect all terms containing x on one side of the equation. to do this, we'll move the $-4x$ to the left side, and the -10 to the right side.

$$3x + 4x - 10 = -4x + 4x + 4$$

$$7x - 10 = 4$$

$$7x - 10 + 10 = 4 + 10$$

$$7x = 14$$

$$\frac{7x}{7} = \frac{14}{7}$$

$$x = 2$$

To determine whether this is actually a solution, we'll substitute 2 for x in the original equation:

$$\sqrt{x^2 + 3x - 10} + 2 = x$$

$$\sqrt{2^2 + 3(2) - 10} + 2 = 2$$

$$\sqrt{4 + 6 - 10} + 2 = 2$$

$$\sqrt{0} + 2 = 2$$

$$0 + 2 = 2$$

$$2 = 2$$

This equation is true, so $x = 2$ is indeed a solution.

Topic: Radical equations

Question: Solve the equation.

$$\sqrt{3 - 2x} - x = 0$$

Answer choices:

A 1 and -3

B -3

C 1

D -1 and 3

Solution: C

Isolate the radical by adding x to both sides.

$$\sqrt{3 - 2x} - x = 0$$

$$\sqrt{3 - 2x} = x$$

Square both sides.

$$3 - 2x = x^2$$

Collect all terms on one side. Once we do that, we'll have a quadratic polynomial on one side of the equation. We'd like the coefficient of the x^2 term in that quadratic polynomial to be positive, so we want to keep the x^2 on the right side of the equation. Therefore, we'll move the $3 - 2x$ to the right side.

$$3 - 2x - (3 - 2x) = x^2 - (3 - 2x)$$

$$0 = x^2 - 3 + 2x$$

Now factor the quadratic polynomial $x^2 - 3 + 2x$, and then solve the resulting equation for x.

$$0 = (x - 1)(x + 3)$$

Set each factor equal to 0 and solve for x.

$$x - 1 = 0 \text{ gives } x = 1$$

$$x + 3 = 0 \text{ gives } x = -3$$

Check each result by plugging into the original equation.

For $x = 1$:

$$\sqrt{3 - 2x} - x = 0$$

$$\sqrt{3 - 2(1)} - 1 = 0$$

$$\sqrt{1} - 1 = 0$$

$$1 - 1 = 0$$

$$0 = 0$$

This equation is true, so $x = 1$ is a solution.

For $x = -3$:

$$\sqrt{3 - 2x} - x = 0$$

$$\sqrt{3 - 2(-3)} - (-3) = 0$$

$$\sqrt{9} + 3 = 0$$

$$3 + 3 = 0$$

$$6 = 0$$

This equation is false, so $x = -3$ is not a solution.