Solving three ways

There are three ways to solve systems of linear equations: substitution, elimination, and graphing. Let's review the steps for each method.

Substitution

1. Get a variable by itself in one of the equations.

2. Take the expression you got for the variable in step 1, and plug it (substitute it using parentheses) into the other equation.

3. Solve the equation in step 2 for the remaining variable.

4. Use the result from step 3 and plug it into the equation from step 1.

Elimination

1. If necessary, rearrange both equations so that the x-terms are first, followed by the y-terms, the equals sign, and the constant term (in that order). If an equation appears to have not constant term, that means that the constant term is 0.

2. Multiply one (or both) equations by a constant that will allow either the x-terms or the y-terms to cancel when the equations are added or subtracted (when their left sides and their right sides are added separately, or when their left sides and their right sides are subtracted separately).

3. Add or subtract the equations.

4. Solve for the remaining variable.

5. Plug the result of step 4 into one of the original equations and solve for the other variable.

Graphing

1. Solve for y in each equation.

2. Graph both equations on the same Cartesian coordinate system.

3. Find the point of intersection of the lines (the point where the lines cross).

Now let's look at a few examples in which we need to decide which of these three methods to use.

Example

Which method would you use to solve the following problem? Explain why you picked the method that you did.

$$x = y + 2$$

$$3y - 2x = 15$$

The easiest way to solve this system would be to use substitution since x is already isolated in the first equation. Whenever one equation is already solved for a variable, substitution will be the quickest and easiest method.

Even though you're not asked to solve, these are the steps to solve the system:

Substitute $y+2$ for x in the second equation.

$$3y-2(y+2)=15$$

Distribute the -2 and then combine like terms.

$$3y-2y-4=15$$

$$y-4=15$$

Add 4 to both sides.

$$y-4+4=15+4$$

$$y=19$$

Plug 19 for y into the first equation.

$$x=y+2$$

$$x=19+2$$

$$x=21$$

The unique solution is $(21,19)$.

Let's try another example of solving three ways.

Example

To solve the system by elimination, what would be a useful first step?

$$x + 3y = 12$$

$$2x - y = 5$$

When we use elimination to solve a system, it means that we're going to get rid of (eliminate) one of the variables. So we need to be able to add the equations, or subtract one from the other, and in doing so cancel either the x-terms or the y-terms.

Any of the following options would be a useful first step:

Multiply the first equation by -2 or 2. This would give us $2x$ or $-2x$ in both equations, which will cause the x-terms to cancel when we add or subtract.

Multiply the second equation by 3 or -3. This would give us $3y$ or $-3y$ in both equations, which will cause the y-terms to cancel when we add or subtract.

Divide the second equation by 2. This would give us x or $-x$ in both equations, which will cause the x-terms to cancel when we add or subtract.

Divide the first equation by 3. This would give us y or $-y$ in both equations, which will cause the y-terms to cancel when we add or subtract.

Let's re-do the last example, but instead of the elimination method, use a graph to find the solution.

Example

Graph both equations to find the solution to the system.

$$x + 3y = 12$$

$$2x - y = 5$$

In order to graph these equations, let's put both of them into slope-intercept form. We get

$$x + 3y = 12$$

$$3y = -x + 12$$

$$y = -\frac{1}{3}x + 4$$

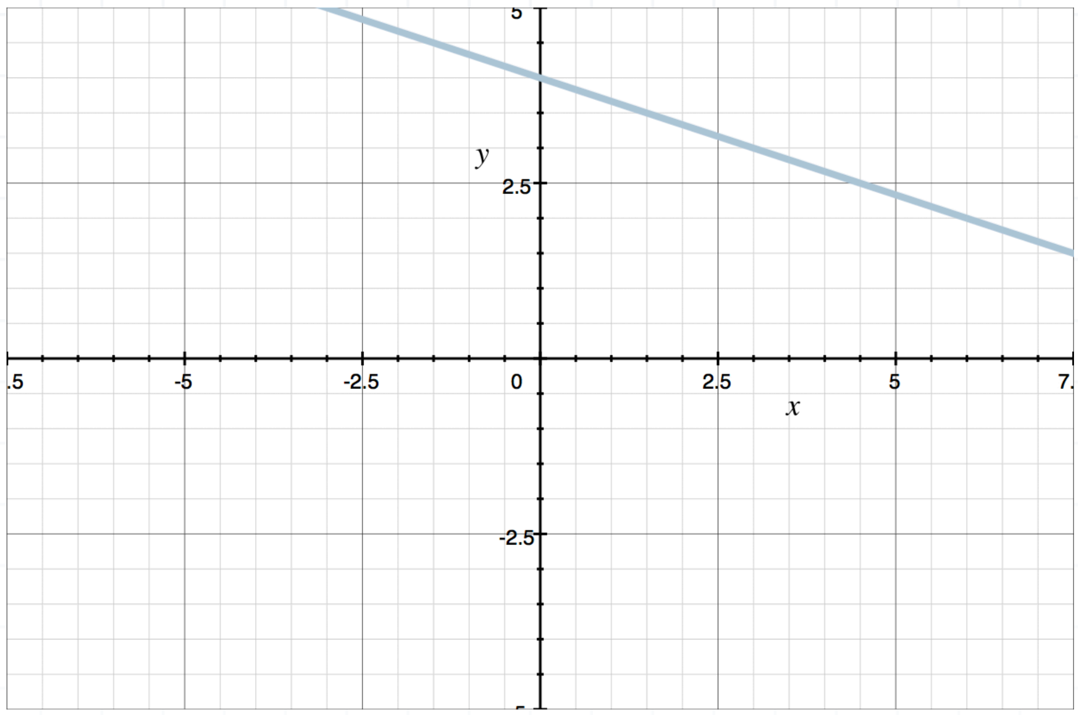
and

$$2x - y = 5$$

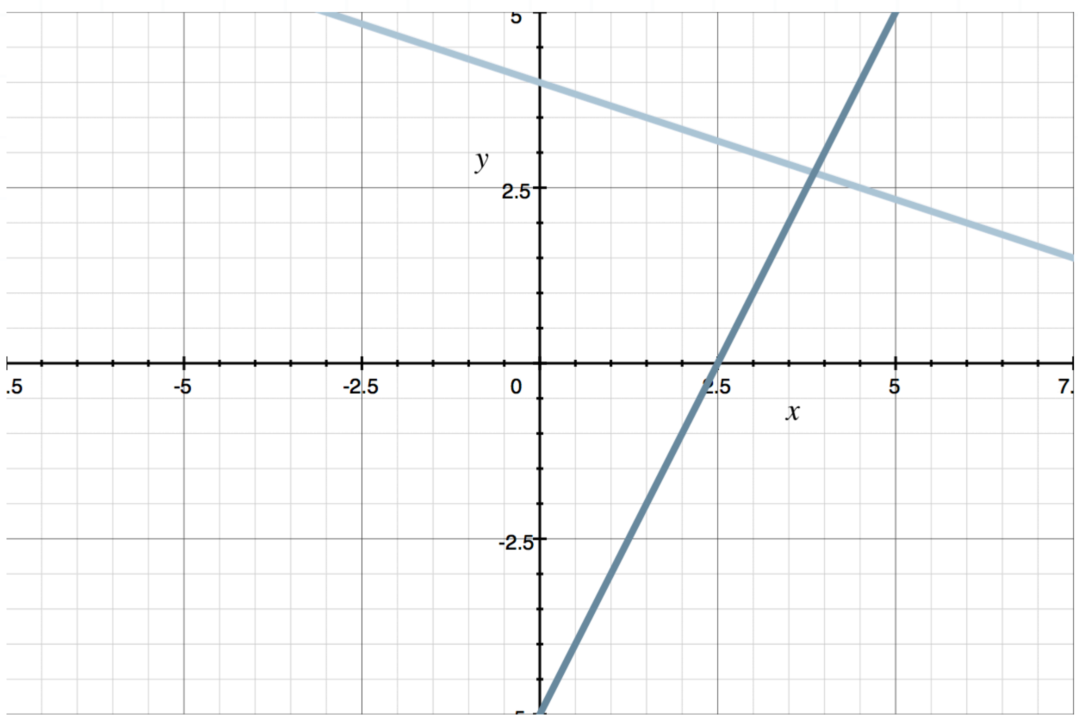
$$-y = -2x + 5$$

$$y = 2x - 5$$

The line $y = -(1/3)x + 4$ intersects the y-axis at 4, and then has a slope of $-1/3$, so its graph is



The line $y = 2x - 5$ intersects the y-axis at -5, and then has a slope of 2, so if you add its graph to the graph of $y = -(1/3)x + 4$, you get



Looking at the intersection point, it appears as though the solution is approximately $(3.75,2.75)$. In actuality, the solution is $(27/7,19/7) \approx (3.86,2.71)$, so our visual estimate of $(3.75,2.75)$ wasn't that far off.