Topic: Laws of logarithms

Question: Write the expression as a rational number if possible, or if not, as a single logarithm.

$$\log_3 54 - \log_3 2$$

Answer choices:

A $-\log_3 9$

B 9

C $\log_3 27$

D 3

Solution: D

First, use the rule

$$\log_a x - \log_a y = \log_a \frac{x}{y}$$

to rewrite the given expression.

$$\log_3 54 - \log_3 2$$

$$\log_3 \frac{54}{2}$$

$$\log_3 27$$

To simplify further, use this rule:

If $\log_a y = x$, then $a^x = y$.

If we let $x = \log_3 27$, then $3^x = 27$. Therefore, $x = 3$, because $27 = 3 \cdot 3 \cdot 3 = 3^3$.

Topic: Laws of logarithms

Question: Which expression is equal to 1?

Answer choices:

A $\log_5 20 - \log_5 10$

B $\log_3 18 - \log_3 6$

C $\log_2 8 - \log_2 7$

D $\log_8 128 - \log_8 2$

Solution: B

Use these two rules to evaluate each expression.

$$\log_a x - \log_a y = \log_a \frac{x}{y}$$

If $\log_a y = x$, then $a^x = y$.

Applying the first rule to the answer choices gives

A $\quad \log_5 20 - \log_5 10 = \log_5 \frac{20}{10} = \log_5 2$

B $\quad \log_3 18 - \log_3 6 = \log_3 \frac{18}{6} = \log_3 3 = 1$

C $\quad \log_2 8 - \log_2 7 = \log_2 \frac{8}{7}$

D $\quad \log_8 128 - \log_8 2 = \log_8 \frac{128}{2} = \log_8 64 = 2$

Now we'll show that the values of the expressions we found for answer choices A and C are not equal to 1.

For answer choice A, let $x = \log_5 2$. By the second rule given above, $5^x = 2$. We know that $5^1 = 5$, so $5^x \neq 5^1$. Since $5^x \neq 5^1$, this tells us that $x \neq 1$. So $\log_5 2 \neq 1$.

For answer choice C, we can use similar reasoning. Let $x = \log_2(8/7)$. By the second rule given above, $2^x = 8/7$. We know that $2^1 = 2$, so $2^x \neq 2^1$. Therefore, $x \neq 1$, and $\log_2(8/7) \neq 1$.

Topic: Laws of logarithms

Question: Write the expression as a rational number if possible, or if not, as a single logarithm.

$$\log_2 \frac{1}{4} + \log_2 16$$

Answer choices:

A 2

B 4

C $\log_2 8$

D $\log_2 64$

Solution: A

Use these two rules to evaluate the expression.

$$\log_a x + \log_a y = \log_a xy$$

If $\log_a y = x$, then $a^x = y$.

Applying the first rule to the given expression gives

$$\log_2 \frac{1}{4} + \log_2 16$$

$$\log_2 \left(\frac{1}{4} \cdot 16 \right)$$

$$\log_2 4$$

It's probably obvious from this that $\log_2 4 = 2$, but if not, use the second rule above. If we let $x = \log_2 4$, then

$$2^x = 4$$

$$x = 2$$