

Understood 1

What happens when you multiply or divide something by 1, or raise something to the first power? It stays the same. This is the premise of the “understood 1.” When you have a variable such as x , you should get to a point where you almost immediately remember that “just plain x ” can be written as $1x$, because $x = 1 \cdot x = 1x$, and similarly that x can be written as $x/1$ or x^1 .

Cases of the Understood 1:

$$x = 1 \cdot x$$

$$x = 1x$$

$$x = \frac{x}{1}$$

$$x = x^1$$

If we combine the kinds of techniques that we used to get these expression, we can also say

$$x = \frac{1x^1}{1}$$

Why is this useful? There are times when x appears in a fraction, or x is added to some term in which x appears, or x is multiplied by some factor in which x appears. When something like this happens, it'll be helpful to remember the understood 1 (and maybe even write out a 1 instead of leaving it as an understood 1).



Example

Simplify the expression.

$$(x + 2x) \cdot x^3$$

Start by simplifying the expression in parentheses, using the fact that $x = 1x$.

$$(1x + 2x) \cdot x^3$$

$$(3x) \cdot x^3$$

Multiply and remember that $3x = 3x^1$.

$$3x^1 \cdot x^3$$

$$3x^4$$

This time let's go in the opposite direction, by looking at an example where one or more “unnecessary” 1's appear in an expression that we can simplify by replacing the unnecessary 1's with understood 1's.

Example

Simplify the expression.

$$\frac{1}{1(1x^1)} + 1 \left(\frac{x}{1} + 1 \right)$$



In this example an unnecessary 1 has been written out multiple times. Let's simplify by replacing each unnecessary 1 with an understood 1.

$$\frac{1}{1x^1} + 1 \left(\frac{x}{1} + 1 \right)$$

$$\frac{1}{x^1} + 1 \left(\frac{x}{1} + 1 \right)$$

$$\frac{1}{x} + 1 \left(\frac{x}{1} + 1 \right)$$

$$\frac{1}{x} + \frac{x}{1} + 1$$

$$\frac{1}{x} + x + 1$$

In our next example, we'll see how to deal with the understood 1 when we want to add two fractions where the denominator of one fraction is just plain x and the denominator of the other fraction is x raised to some power other than 1.

Example

Add the fractions.

$$\frac{2}{x} + \frac{5}{x^3}$$



In the denominator of the first fraction, use the fact that $x = x^1$.

$$\frac{2}{x^1} + \frac{5}{x^3}$$

If we multiply the top and bottom of the first fraction by x^2 , we'll find that we can use x^3 as the common denominator.

$$\frac{2}{x^1} \left(\frac{x^2}{x^2} \right) + \frac{5}{x^3}$$

$$\frac{2(x^2)}{x^1(x^2)} + \frac{5}{x^3}$$

$$\frac{2x^2}{x^3} + \frac{5}{x^3}$$

$$\frac{2x^2 + 5}{x^3}$$

