Distributive property and binomial multiplication

The distributive property can be used even when there are two sets of parentheses with two terms each. It's called binomial multiplication (remember that a bicycle has two wheels and a binomial has two terms).

Binomial Multiplication:

$$(a+b)(c+d) = ac + ad + bc + bd$$

$$(a+b)(c-d) = ac - ad + bc - bd$$

$$(a-b)(c+d) = ac + ad - bc - bd$$

$$(a-b)(c-d) = ac - ad - bc + bd$$

Notice that in the binomial multiplication $(a+b)(c+d)$, first a is multiplied by both terms in the second set of parentheses and then b is multiplied by both terms in the second set of parentheses.

We can also make a chart for the binomial multiplication $(a+b)(c+d)$ in which the terms a and b from the first set of parentheses go along the left side, and the terms c and d from the second set of parentheses go across the top. Then we multiply each term along the left side by both terms across the top, and write the individual results in the chart. The four results all get added together to make the expanded expression.

	c	**d**
a	ac	ad
b	bc	bd

When we add all the results in the chart together, we get

$$ac + ad + bc + bd$$

When we have negative signs in one or both of the binomials, we keep each negative sign with the corresponding term. Then using the fact that multiplication of two terms with negative signs (like $-b$ and $-d$) gives us a positive result, our chart for the binomial multiplication $(a - b)(c - d)$ looks like

	c	-d
a	ac	-ad
-b	-bc	bd

When we add all the results in the chart together, we get

$$ac - ad - bc + bd$$

These charts are another way to keep track of the different multiplications that happen during binomial multiplication.

A third way to keep track of them is a method called “FOIL,” which stands for First, Outer, Inner, Last. Those words tell us which terms in each set of parentheses we should multiply together to get the corresponding term in the expanded expression.

F + O + I + L

$$(a + b)(c + d) = ac \ + \ ad \ + \ bc \ + \ bd$$

Here’s how you break that down with FOIL:

- The **Firsts** are the first terms in each set of parentheses: a in $a+b$ and c in $c+d$.

- The **Outers** are the terms on the "outside" in the expression: a in $a+b$ and d in $c+d$.

- The **Inners** are the terms on the "inside" in the expression: b in $a+b$ and c in $c+d$.

- The **Lasts** are the last terms in each set of parentheses: b in $a+b$ and d in $c+d$.

Let's work through an example.

Example

Use the distributive property to expand the expression.

$$5(x-2)(x+3)$$

Start by distributing the 5 across $x-2$.

$$[5(x)-5(2)](x+3)$$

$$(5x-10)(x+3)$$

Now distribute each term in the first set of parentheses across both of the terms in the second set of parentheses. You may use a chart to help organize your work.

	x	3
5x	5x^2	15x
-10	-10x	-30

When we add all the results in the chart together, we get

$$5x^2 + 15x - 10x - 30$$

Combine like terms: $15x - 10x = 5x$.

$$5x^2 + 5x - 30$$

Alternatively, you could use FOIL to organize your work.

- First: $5x$ in $5x - 10$, and x in $x + 3$
- Outer: $5x$ in $5x - 10$, and 3 in $x + 3$
- Inner: -10 in $5x - 10$, and x in $x + 3$
- Last: -10 in $5x - 10$, and 3 in $x + 3$

The multiplication would be

F	+	O	+	I	+	L
$(5x)(x)$	$+$	$(5x)(3)$	$+$	$(-10)(x)$	$+$	$(-10)(3)$

This gives

$$5x^2 + 15x - 10x - 30$$

$$5x^2 + 5x - 30$$

Let's try another example of binomial multiplication.

Example

Use the distributive property to expand the expression.

$$3x(x+4)(x+1)(x-2)$$

Start by distributing the $3x$ across $x+4$.

$$(3x^2+12x)(x+1)(x-2)$$

Now distribute $3x^2+12x$ across $x+1$. You may use a chart to help organize your work.

	x	**1**
3x²	$3x^3$	$3x^2$
12x	$12x^2$	$12x$

When we add all the results in the chart together, we get

$$3x^3+3x^2+12x^2+12x$$

Combine like terms: $3x^2+12x^2=15x^2$.

$$3x^3+15x^2+12x$$

Finally, distribute $3x^3+15x^2+12x$ across $x-2$. Here, we're actually multiplying a trinomial (an expression with three terms) by a binomial (an expression with two terms). The trinomial is $3x^3 + 15x^2 + 12x$, and the binomial is $x - 2$. We use the same general approach that we used for multiplication of two binomials: Each term of the trinomial must be multiplied by both terms of the binomial. You can use a chart to help organize your work.

	x	**-2**
3x³	$3x^4$	$-6x^3$
15x²	$15x^3$	$-30x^2$
12x	$12x^2$	$-24x$

When we add all the results in the chart together, we get

$$3x^4 - 6x^3 + 15x^3 - 30x^2 + 12x^2 - 24x$$

Combine like terms: $-6x^3 + 15x^3 = 9x^3$ and $-30x^2 + 12x^2 = -18x^2$.

$$3x^4 + 9x^3 - 18x^2 - 24x$$