

Inequalities and negative numbers

We solve inequalities the same way we solve equations, except that when we multiply or divide both sides of an inequality by a negative number, we have to “flip” (reverse) the direction of the inequality.

So if we started with a less than sign ($<$) and we multiply or divide through the inequality by negative number, we need to change the less than sign $<$ to a greater than sign $>$. Likewise, if we start with $>$ and multiply or divide through by a negative number, we need to flip the inequality to $<$.

If we have a greater than or equal to sign (\geq), or a less than or equal to sign (\leq), the “equals” part of the inequality sign is unaffected when we multiply or divide by a negative number, but the less than or greater than part still flips. In other words, multiplying or dividing by a negative number changes \geq to \leq , and changes \leq to \geq .

Let's do an example.

Example

Solve the inequality.

$$-x + 3 > 12$$

Subtract 3 from both sides.

$$-x + 3 - 3 > 12 - 3$$



$$-x > 9$$

Now we have to multiply both sides by -1 , so we have to change the direction of the inequality sign at the same time that we do the multiplication.

$$(-1)(-x) < 9(-1)$$

$$x < -9$$

Let's try another example of solving inequalities with negatives.

Example

Solve the inequality.

$$-2x + 4 \geq -6$$

Subtract 4 from both sides.

$$-2x + 4 - 4 \geq -6 - 4$$

$$-2x \geq -10$$

Now we have to divide both sides by -2 , so we have to change the direction of the inequality sign at the same time that we do the division.

$$\frac{-2x}{-2} \leq \frac{-10}{-2}$$



$x \leq 5$

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