Simple equations with subscripts

Sometimes equations may have variables with subscripts (tiny numbers just after - and at a lower level than - the variable). This is especially the case in science subjects such as chemistry or physics.

Why use subscripts instead of different variables? Well, the letters used for variables in science often represent something specific. For example, P stands for pressure, and in some equations you'll have more than one pressure represented. In that case we use subscripts: P_1, P_2, P_3, and so on.

How do we handle subscripts in equations? Just like we do any other variable. But be careful to copy the subscripts down carefully as you work with the equation, to avoid mistaking the subscript for a non-subscripted number.

Example

In chemistry we learn that under certain conditions the pressure and volume of a gas are related according to the equation $P_1V_1 - P_2V_2 = 0$, where P_1 and V_1 are the original pressure and volume, and P_2 and V_2 are the new pressure and volume. If the original pressure is 1.4 and the original volume is 210, and if the new pressure is 28, what is the new volume?

Start by plugging in what you do know (use parentheses when plugging numbers in). We know that $P_1 = 1.4$, $V_1 = 210$, and $P_2 = 28$, so we get

$$P_1V_1 - P_2V_2 = 0$$

$$(1.4)(210) - (28)V_2 = 0$$

Simplify the left side using the order of operations.

$$294 - 28V_2 = 0$$

Solve by working backwards from the order of operations. In this equation, $28V_2$ is being subtracted, so add $28V_2$ to both sides.

$$294 - 28V_2 + 28V_2 = 0 + 28V_2$$

$$294 = 28V_2$$

Divide both sides by 28, because division undoes multiplication.

$$\frac{294}{28} = \frac{28V_2}{28}$$

$$10.5 = V_2$$

Let's try another example of solving simple equations with subscripts.

Example

Suppose a car travels at a constant speed of 50 mph for 125 miles, then speeds up and travels at a new constant speed for another 153 miles. If the total time for the trip is 4.75 hours, how fast does the car go during the second part of the trip?

We'll use an equation that relates distance, rate, and time for an object in motion. The equation is

$$\text{distance} = \text{rate} \times \text{time}$$

and tells us that multiplying how fast something is moving and the amount of time it's been moving is equal to the distance that it's moved. We can manipulate this equation to solve for any of the three values in the equation. For example, we can divide both sides by rate to get an equation for time.

$$\frac{\text{distance}}{\text{rate}} = \text{time}$$

The total time for the trip (which we'll call t) is the sum of the times for the two parts. Let d_1 and r_1 be the distance and rate on the first part of the trip, and let d_2 and r_2 be the distance and rate on the second part. If we divide d_1 by r_1, we get the time for the first part; similarly if we divide d_2 by r_2, we get the time for the second part. Therefore,

$$\frac{d_1}{r_1} + \frac{d_2}{r_2} = t$$

Start by plugging in what you do know, which is $d_1 = 125$, $r_1 = 50$, $d_2 = 153$, and $t = 4.75$.

$$\frac{d_1}{r_1} + \frac{d_2}{r_2} = t$$

$$\frac{125}{50} + \frac{153}{r_2} = 4.75$$

Simplify the left side using the order of operations.

$$2.5 + \frac{153}{r_2} = 4.75$$

Solve by working backwards from the order of operations. In this equation, 2.5 is being added, so subtract 2.5 from both sides.

$$2.5 - 2.5 + \frac{153}{r_2} = 4.75 - 2.5$$

$$\frac{153}{r_2} = 2.25$$

Since the variable r_2 is in the denominator, multiply both sides by r_2 to move it to the numerator.

$$\frac{153}{r_2} \cdot r_2 = 2.25 \cdot r_2$$

$$153 = 2.25r_2$$

Divide both sides by 2.25.

$$\frac{153}{2.25} = \frac{2.25r_2}{2.25}$$

$$68 = r_2$$

The car travels at 68 mph for the second part of the trip.