A NOTE OF TOPOLOGICAL METHODS IN GROUP THEORY

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ABSTRACT. This is a course note of 2026 Spring MATH 569, taught by Daniel Groves at UIC, on the topic of Topological Methods in Group Theory. Reference textbooks are: Topological Methods in Group Theory, Cohomology of Groups.

CONTENTS

1. Aspherical Spaces and Sample Early Theorems

In this course we will explore the relationship between aspherical spaces and (discrete) groups. A convention: unless otherwise stated, all spaces in this course will be CW complexes.

Definition 1.1 $(K(G,1))$**.** Let G be a group. A $K(G,1)$ is a connected space such that $\pi_1(K(G,1)) \cong G$ and whose universal covering space $\widehat{K}(G,1)$ is contractible.
   It is also called *the classifying space* or *Eilenberg-Maclane space.*

Definition 1.2 (Aspherical Space)**.** If X is a connected space such that $\widetilde{X}$ is contractible, then we say X is a $K(\pi,1)$ (i.e. $K(\pi_1(X),1)$). We call such X *aspherical.*

Theorem 1.3 (Hatcher 1.B.7-9)**.** *For any group G, there exists a (CW) $K(G,1)$ which is unique up to homotopy equivalence. In fact, for any two groups G, H, there exists a bijection*

$$\hom(G,H) \leftrightarrow$$

References

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