Quasiconvex Subgroups of Acylindrically Hyperbolic Groups

Ping Wan

Department of Mathematics, Statistics, and Computer Science
University of Illinois Chicago

May 30, 2025

Hyperbolic Groups

Hyperbolic group

A group G is hyperbolic if some Cayley graph $\Gamma(G, X)$ is connected and hyperbolic.

Quasiconvex subgroup

A subgroup H of G is quasiconvex if the inclusion map is a quasi-isometric embedding.

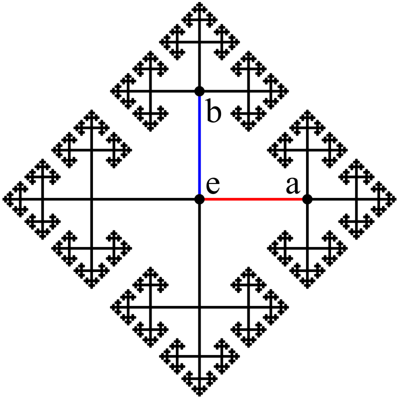

Figure: Cayley graph of $\mathbb{F}^2$

Hyperbolic Groups

Hyperbolic group

A group G is hyperbolic if some Cayley graph $\Gamma(G, X)$ is connected and hyperbolic.

Quasiconvex subgroup

A subgroup H of G is quasiconvex if the inclusion map is a quasi-isometric embedding.

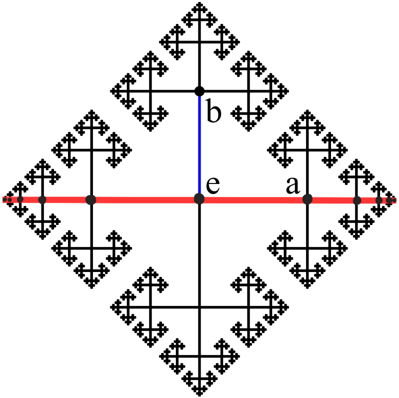

Figure: $\mathbb{Z}$ is a quasiconvex subgroup of $\mathbb{F}^2$

Acylindrically Hyperbolic Groups

$(G, \mathcal{P})$-graph [Martínez-Pedroza and Rashid, 2021]

A graph Γ is a $(G, \mathcal{P})$*–graph* if G acts on Γ and

1. Γ is connected and hyperbolic,
2. there are finitely many G-orbits of vertices,
3. G-stabilizers of vertices are finite or conjugates of $P \in \mathcal{P}$,
4. G-stabilizers of edges are finite, and
5. Γ is fine at each vertex of infinite stabilizer.

Acylindrically hyperbolic group

A group G is acylindrically hyperbolic if there exists a $(G, \mathcal{P})$-graph.

Acylindrically Hyperbolic Groups

Acylindrically hyperbolic group [2]

A group G is acylindrically hyperbolic if some coned-off Cayley graph of G along some collection of infinite subgroups $\mathcal{P}$ is connected, hyperbolic and fine at cone vertices.

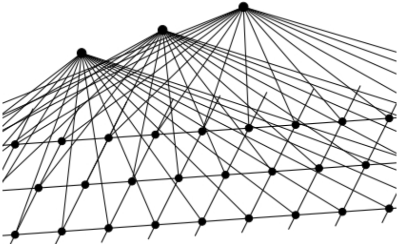

Figure: Coned-off Cayley graph of $\mathbb{Z} \oplus \mathbb{Z}$ along $\mathbb{Z}$

$(G, \mathcal{P})$-Quasiconvex Subgroups

Acylindrically hyperbolic group [1]

A group G is acylindrically hyperbolic if there exists a $(G, \mathcal{P})$-graph.

$(G, \mathcal{P})$-quasiconvex subgroup [1] [Wan]

A group H is $(G, \mathcal{P})$-quasi-convex if there exists a $(G, \mathcal{P})$–graph K, and a nonempty connected, H-invariant and quasi-isometrically embedded subgraph L of K so that L has finitely many H-orbits of vertices.

$(G, \mathcal{P})$-Quasiconvex Subgroups

Acylindrically hyperbolic group [2]

A group G is acylindrically hyperbolic if some coned-off Cayley graph of G along some collection of infinite subgroups $\mathcal{P}$ is connected, hyperbolic and fine at cone vertices.

$(G, \mathcal{P})$-quasiconvex subgroup [2] [Wan]

If a group H is $(G, \mathcal{P})$-quasi-convex, then there exists a compatible $\mathcal{D}$, and a quasi-isometric embedding $\hat{\Gamma}(H, \mathcal{D}, Y) \hookrightarrow \hat{\Gamma}(G, \mathcal{P}, X)$.

Thank You

References

Martínez-Pedroza, E. and Rashid, F. (2021).
A Note on Hyperbolically Embedded Subgroups.