Ch-04 R Codes

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Textbook: Montgomery, D. C. (2012). *Design and analysis of experiments*, 8th Edition. John Wiley & Sons.

Online handouts: https://github.com/PingYangChen/ANOVA_Course_R_Code

Chapter 4

Randomized Complete Block Designs

Read the csv file `4_VascularGraft.csv` in R. Make sure that in the `data.frame` the variables `ExtPressure` and `ResinBatchNo` are the type of factor. If not sure, apply `as.factor()` on those variables after reading the dataset.

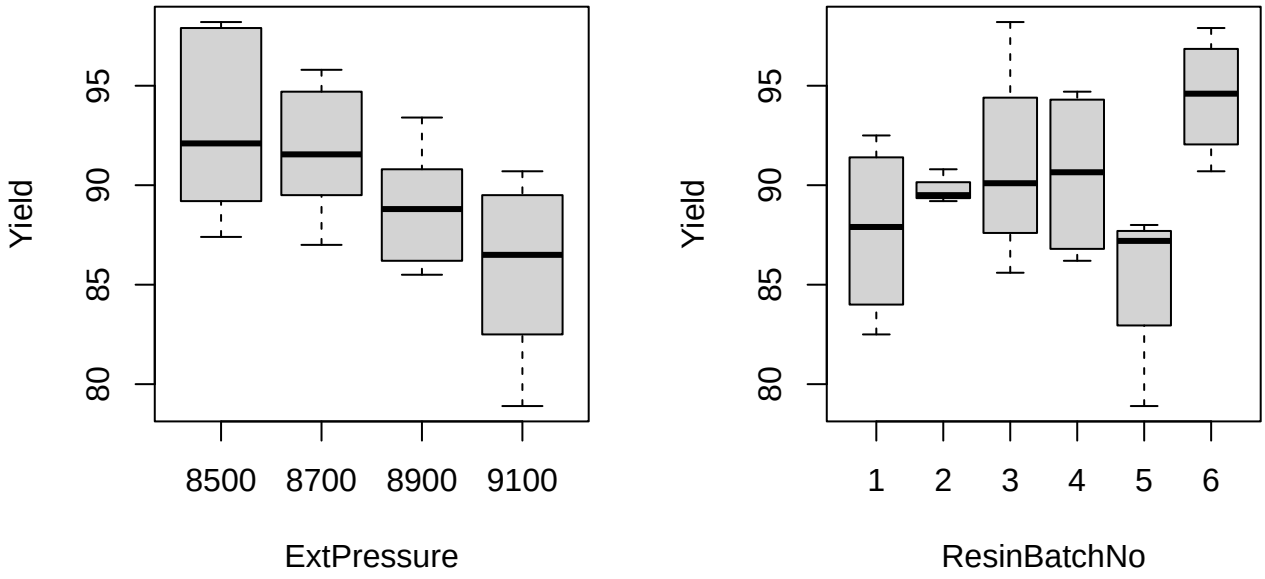
```
df1 <- read.csv(file.path("data", "4_VascularGraft.csv"))
df1$ExtPressure <- as.factor(df1$ExtPressure)
df1$ResinBatchNo <- as.factor(df1$ResinBatchNo)
```

```
tapply(df1$Yield, df1$ExtPressure, summary)
```

```
## $`8500`
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   87.40   89.47   92.10   92.82   96.90   98.20
##
## $`8700`
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   87.00   89.78   91.55   91.68   94.15   95.80
##
## $`8900`
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   85.50   86.65   88.80   88.92   90.50   93.40
##
## $`9100`
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   78.90   83.28   86.50   85.77   88.97   90.70
```

Use boxplots to observe the differences of `Yield` among four treatments of `ExtPressure`, and, the differences of `Yield` among six resin batches `ResinBatchNo`. The average `Yield` tends to be lower for higher `ExtPressure` and varies in different `ResinBatchNo`. Thus, to investigate the `ExtPressure` effect on the `Yield`, we must eliminate the effect caused by `ResinBatchNo`. That is, we treat `ResinBatchNo` as the block effect.

```r
# Draw the grouped boxplot
par(mfrow = c(1, 2))
boxplot(Yield ~ ExtPressure, data = df1)
boxplot(Yield ~ ResinBatchNo, data = df1)
```



The function `aov()` fits the ANOVA model, and the ANOVA table is obtained by calling `summary()`. For Randomized Complete Block Design (RCBD), the R model formula includes two items on its right-hand-side: `ExtPressure + ResinBatchNo`. It's important to note that the order of placing the factor and block effects is somewhat arbitrary for RCBD. However, this order matters if the design is incomplete (see BIBD), as some sums of squares are adjusted accordingly.

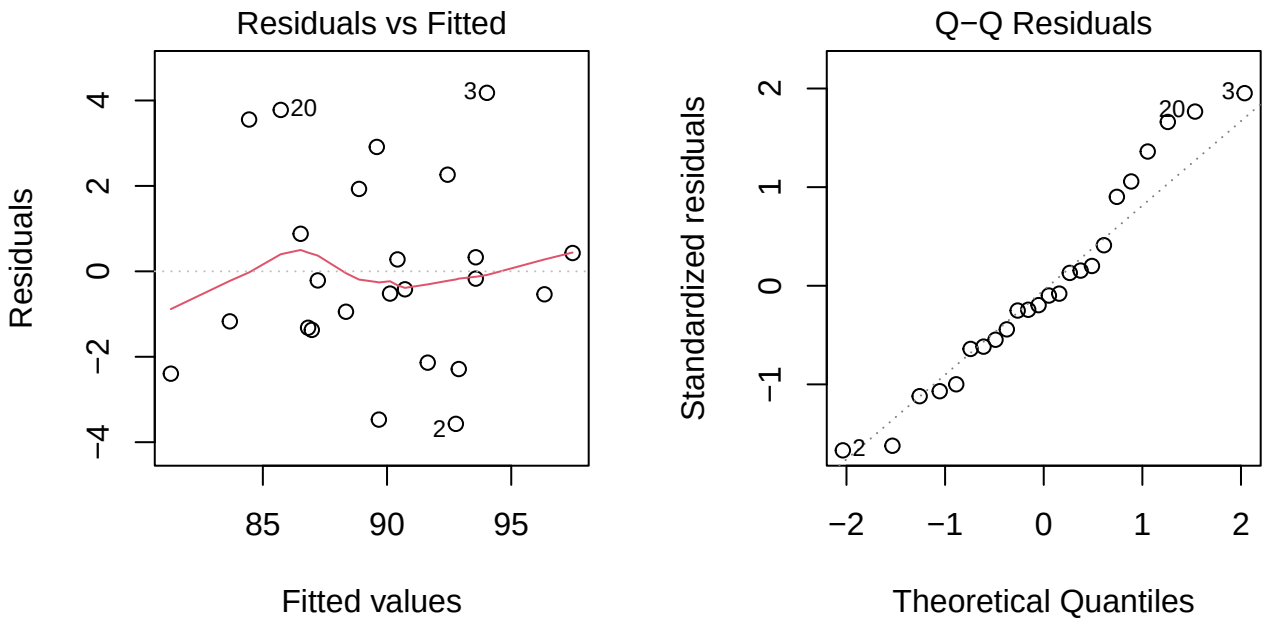
```r
fit1 <- aov(Yield ~ ExtPressure + ResinBatchNo, data = df1)
summary(fit1)
```

```
##              Df Sum Sq Mean Sq F value  Pr(>F)
## ExtPressure   3  178.2   59.39   8.107 0.00192 **
## ResinBatchNo  5  192.2   38.45   5.249 0.00553 **
## Residuals    15  109.9    7.33
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Let the significance level $\alpha = 0.05$. The ANOVA shows the P-value for the effect of extrusion pressure on yield is 0.0019, and therefore, we conclude that extrusion pressure affects the mean yield. Also, the mean yields from resin batches (blocks) differ significantly, because the corresponding P-value is 0.0055.

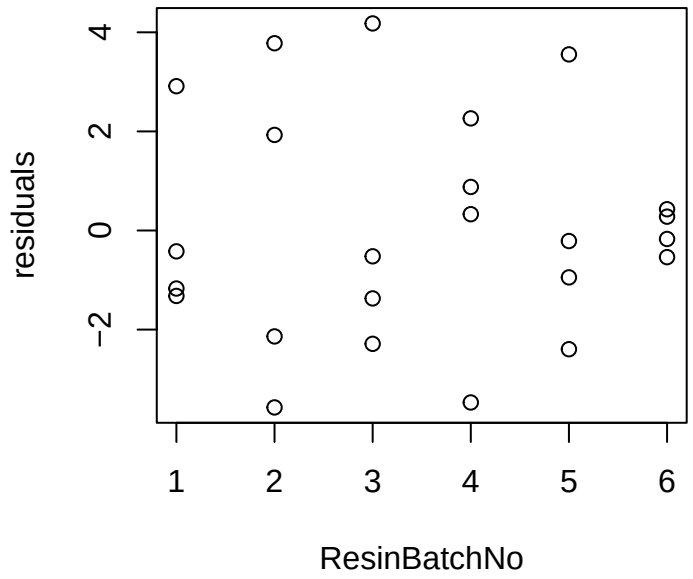
The procedure of diagnosing the residual is similar to that for the one-way ANOVA model. Please refer to the handout of R codes in Chapter 3 for more details of interpreting the residual plots.

```r
par(mfrow = c(1, 2))
plot(fit1, which = 1:2)
```



One additional plot is to draw the scatter plot of the residual against the levels of the block. A lack of any visually obvious pattern in the dots on the plot is desired.

```
plot(
  as.numeric(df1$ResinBatchNo), fit1$residuals,
  xlab = "ResinBatchNo", ylab = "residuals"
)
```



Multiple comparison is performed for the treatment effect. The following codes demonstrate the use of Tukey's test and Fisher's LSD method. For Tukey's test, add the input argument `which = "ExtPressure"` to only show the test results of comparing differences among the `ExtPressure` levels. For Fisher's LSD method, specify `trt = "ExtPressure"` as the input argument to the `LSD.test()` function to show the comparison results among the `ExtPressure` levels. For information of interpreting the results, please refer to the handout of R codes in Chapter 3.

```r
TukeyHSD(fit1, which = "ExtPressure")
```

```
##   Tukey multiple comparisons of means
##     95% family-wise confidence level
##
## Fit: aov(formula = Yield ~ ExtPressure + ResinBatchNo, data = df1)
##
## $ExtPressure
##                 diff        lwr       upr     p adj
## 8700-8500 -1.133333  -5.637161  3.370495 0.8854831
## 8900-8500 -3.900000  -8.403828  0.603828 0.1013084
## 9100-8500 -7.050000 -11.553828 -2.546172 0.0020883
## 8900-8700 -2.766667  -7.270495  1.737161 0.3245644
## 9100-8700 -5.916667 -10.420495 -1.412839 0.0086667
## 9100-8900 -3.150000  -7.653828  1.353828 0.2257674
```

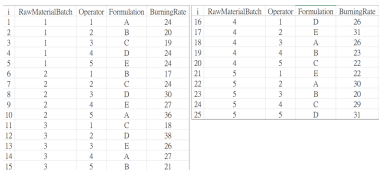
```r
if (!("agricolae" %in% rownames(installed.packages()))) {
  install.packages("agricolae")
}
library(agricolae)
out <- LSD.test(fit1, trt = "ExtPressure", p.adj = "bonferroni")
out$group
```

```
##         Yield groups
## 8500 92.81667      a
## 8700 91.68333      a
## 8900 88.91667     ab
## 9100 85.76667      b
```

The multiple comparison results show that the effect on `Yield` when `ExtPressure = 9100` is significantly lower than effects on `Yield` when `ExtPressure = 8500` and `ExtPressure = 8700`, and seems no different from the effect on `Yield` when `ExtPressure = 8900`.

Latin Square Designs

Create an csv file to store the data and then read it in R.



	RawMaterialBatch	Operator	Formulation	BurningRate
1	1	1	A	24
2	1	2	B	20
3	1	3	C	19
4	1	4	D	24
5	1	5	E	24
6	2	1	B	17
7	2	2	C	24
8	2	3	D	30
9	2	4	E	27
10	2	5	A	36
11	3	1	C	18
12	3	2	D	38
13	3	3	E	26
14	3	4	A	27
15	3	5	B	21

	RawMaterialBatch	Operator	Formulation	BurningRate
16	4	1	D	26
17	4	2	E	31
18	4	3	A	26
19	4	4	B	23
20	4	5	C	22
21	5	1	E	22
22	5	2	A	30
23	5	3	B	20
24	5	4	C	29
25	5	5	D	31

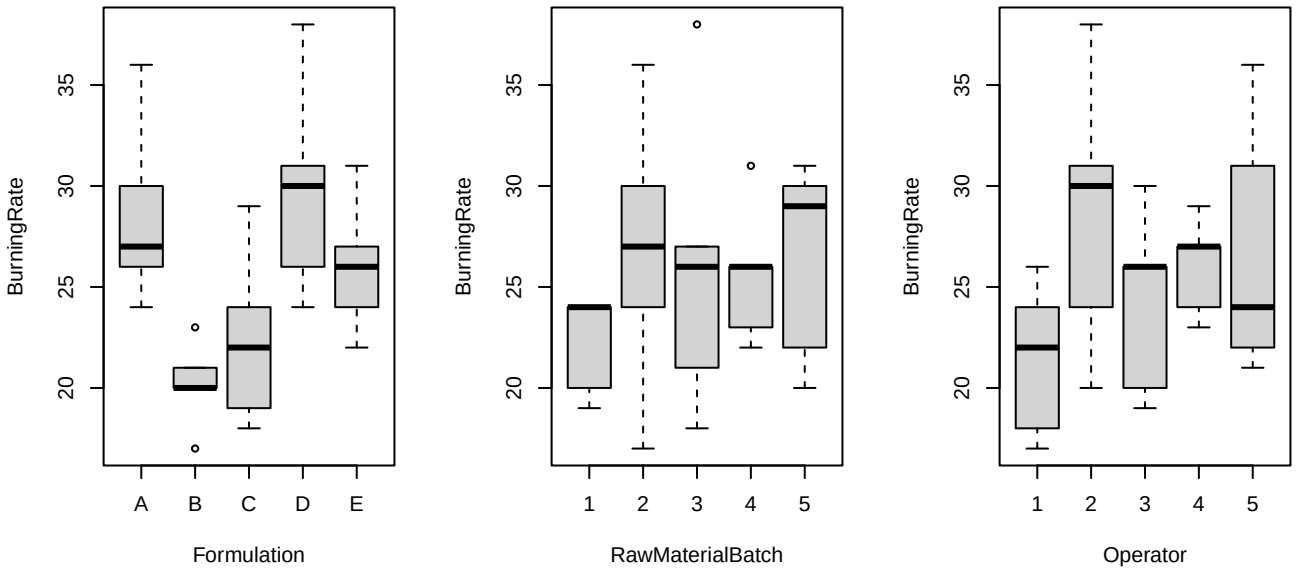
Figure 1: Rocket Propellant Latin Square Experiment Data

Read the csv file `4_RocketPropellant.csv` in R. Make sure that in the `data.frame` the variables `RawMaterialBatch`, `Operator` and `Formulation` are the type of factor. If not sure, apply `as.factor()` on those variables after reading the dataset.

```
df2 <- read.csv(file.path("data", "4_RocketPropellant.csv"))
df2$RawMaterialBatch <- as.factor(df2$RawMaterialBatch)
df2$Operator <- as.factor(df2$Operator)
df2$Formulation <- as.factor(df2$Formulation)
```

The boxplots show the distributions of `BurningRate` under subgroups of (left) five treatments of `Formulation`; (middle) five levels of the first block `RawMaterialBatch`; (right) five levels of the second block `Operator`. The left plot suggests that there may be differences in the `BurningRate` among the different `Formulations` used. The middle and right plots, show the `BurningRate` behaves differently within each level of blocks. Thus, the AONVA model must take the block effects into account.

```
# Draw the grouped boxplot
par(mfrow = c(1, 3))
boxplot(BurningRate ~ Formulation, data = df2)
boxplot(BurningRate ~ RawMaterialBatch, data = df2)
boxplot(BurningRate ~ Operator, data = df2)
```



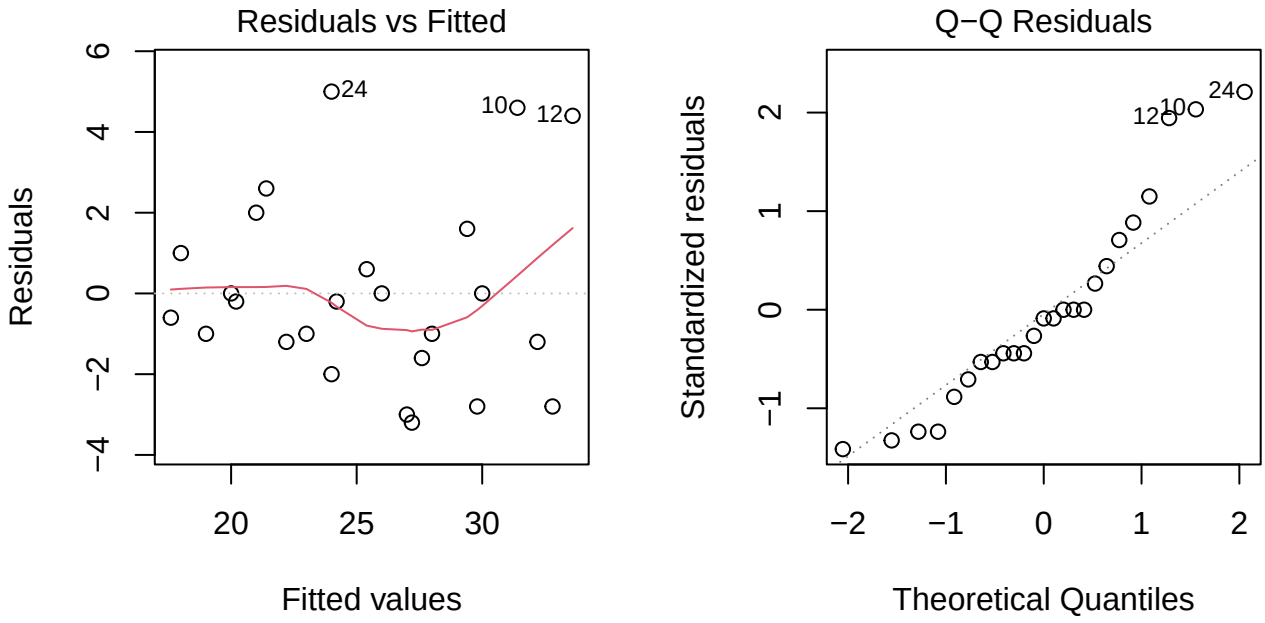
Again, functions `aov()` fits the ANOVA model followed by calling `summary()` to obtain the ANOVA table. For Latin Square Designs, the R model formula includes three items on its right-hand-side: `Formulation + RawMaterialBatch + Operator`. Similar to RCBD, the order of placing the factor and block effects is arbitrary here.

```
fit2 <- aov(BurningRate ~ Formulation + RawMaterialBatch + Operator, data = df2)
summary(fit2)
```

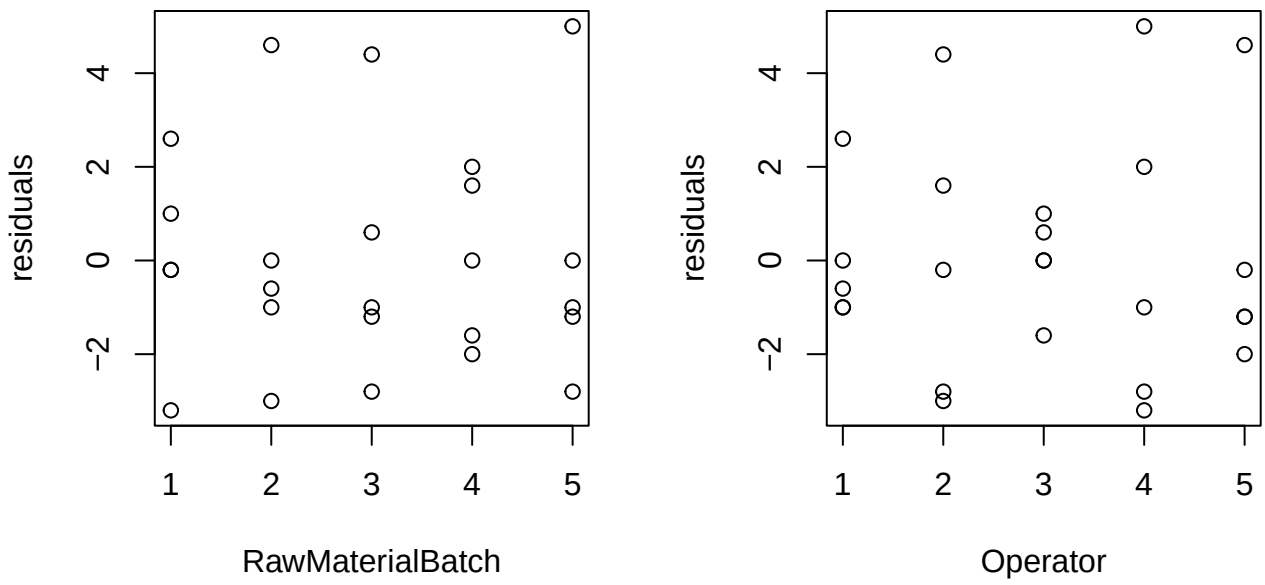
```
##                   Df Sum Sq Mean Sq F value  Pr(>F)
## Formulation        4    330   82.50   7.734 0.00254 **
## RawMaterialBatch   4     68   17.00   1.594 0.23906
## Operator           4    150   37.50   3.516 0.04037 *
## Residuals         12    128   10.67
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The procedures of diagnosing the residual and multiple comparison are similar to those for the RCBD. Please refer to the previous section for more details.

```
par(mfrow = c(1, 2))
plot(fit2, which = 1:2)
```




```
par(mfrow = c(1, 2))
plot(
  as.numeric(df2$RawMaterialBatch), fit2$residuals,
  xlab = "RawMaterialBatch", ylab = "residuals"
)
plot(
  as.numeric(df2$Operator), fit2$residuals,
  xlab = "Operator", ylab = "residuals"
)
```



Multiple comparison is performed solely for the treatment effect, `Formulation`. The following codes demonstrate the use of Tukey's test and Fisher's LSD method. For Tukey's test, add the input argument `which = "Formulation"` to only show the test results of comparing differences among the `Formulation` levels. For Fisher's LSD method, specify `trt = "Formulation"` as the input argument to the `LSD.test()` function to show the comparison results among the `Formulation` levels. For information of interpreting the results, please refer to the handout of R codes in Chapter 3.

```
TukeyHSD(fit2, which = "Formulation")
```

```
##   Tukey multiple comparisons of means
##     95% family-wise confidence level
##
## Fit: aov(formula = BurningRate ~ Formulation + RawMaterialBatch + Operator, data = df2)
##
## $Formulation
##     diff         lwr        upr     p adj
## B-A -8.4 -14.9839317 -1.8160683 0.0110827
## C-A -6.2 -12.7839317  0.3839317 0.0684350
## D-A  1.2  -5.3839317  7.7839317 0.9754380
## E-A -2.6  -9.1839317  3.9839317 0.7194121
## C-B  2.2  -4.3839317  8.7839317 0.8204614
## D-B  9.6   3.0160683 16.1839317 0.0041583
## E-B  5.8  -0.7839317 12.3839317 0.0944061
## D-C  7.4   0.8160683 13.9839317 0.0254304
## E-C  3.6  -2.9839317 10.1839317 0.4461852
## E-D -3.8 -10.3839317  2.7839317 0.3966727
```

```
if (!("agricolae" %in% rownames(installed.packages()))) {
  install.packages("agricolae")
}
library(agricolae)
out <- LSD.test(fit2, "Formulation", p.adj = "bonferroni")
out$group
```

```
##   BurningRate groups
## D        29.8      a
## A        28.6     ab
## E        26.0    abc
## C        22.4     bc
## B        20.2      c
```

The multiple comparison results indicate that using `Formulation = D` and `Formulation = A` produces the highest mean `BurningRates`, both of which are significantly higher than the `BurningRate` obtained with `Formulation= B`. However, there is no statistical evidence to conclude that the mean `BurningRate` under `Formulation = D` is higher than that under `Formulation = A`.

Balanced Incomplete Block Design

Read the csv file `4_Catalysts.csv` in R. Make sure that in the `data.frame` the variables `RawMaterialBatch` and `Catalysts` are the type of factor. If not sure, apply `as.factor()` on those variables after reading the dataset.

```
df3 <- read.csv(file.path("data", "4_Catalysts.csv"))
df3$RawMaterialBatch <- as.factor(df3$RawMaterialBatch)
df3$Catalysts <- as.factor(df3$Catalysts)
```

This catalysts experiment data is obtained by the balanced incomplete block design (BIBD) with $a = 4$ `Catalysts` treatment levels and $b = 4$ `RawMaterialBatch` block levels. Each treatment levels are performed in $r = 3$ blocks, and in each block, $k = 3$ treatment levels are performed. The effect model is

$$y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij}, \ i = 1, ..., 4, \ j = 1, ..., 4$$

where τ_i denotes the treatment effect of types of `Catalysts` and β_j denotes the block effect of the `RawMaterialBatch`.

If the design is BIBD, the adjusted treatment sum of squares is

$$SS_{Treatment(adj)} = \frac{k \sum_{i=1}^{b} Q_i^2}{\lambda a} = \frac{3 \sum_{i=1}^{4} Q_i^2}{4\lambda}$$

where

$$\lambda = \frac{r(k-1)}{a-1} = \frac{3 \cdot 2}{3} = 2$$

and

$$Q_i = y_{i\cdot} - \frac{1}{k} \sum_{j=1}^{b} n_{ij} y_{\cdot j} = y_{i\cdot} - \frac{1}{3} \sum_{j=1}^{4} n_i j y_{\cdot j}$$

for $i = 1, ..., 4$, and, $n_{ij} = 1$ if the ith treatment level appears in block j and $n_{ij} = 0$ otherwise. Let's compute $SS_{Treatment(adj)}$ by ourselves in R first.

```
appears <- tapply(df3$RawMaterialBatch, df3$Catalyst, table)
y_i_dot <- tapply(df3$ReactionTime, df3$Catalysts, sum)
y_dot_j <- tapply(df3$ReactionTime, df3$RawMaterialBatch, sum)
q <- numeric(4)
for (i in 1:4) {
  n_i_j <- appears[[i]]
  q[i] <- y_i_dot[i] - (1/3)*sum(n_i_j*y_dot_j)
}
SS_Treatment_adj <- 3*sum(q^2)/(2*4)
SS_Treatment_adj
```

```
## [1] 22.75
```

To obtain the ANOVA table with the adjusted treatment sum of squares for BIBD in R, **keep the block effect in the first place of the R model formula** for the `aov` function.

```
fit3_treatment <- aov(ReactionTime ~ RawMaterialBatch + Catalysts, data = df3)
summary(fit3_treatment)
```

```
##                   Df Sum Sq Mean Sq F value  Pr(>F)
## RawMaterialBatch  3  55.00  18.333   28.20 0.00147 **
## Catalysts         3  22.75   7.583   11.67 0.01074 *
## Residuals         5   3.25   0.650
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Likewise, if one is interested to test the significance of the block effect, then the block sum of squares should be adjusted. In R, the adjusted block sum of squares is obtained by placing the ''treatment effect'' in the first place of the R model formula for the `aov` function.

```
fit3_block <- aov(ReactionTime ~ Catalysts + RawMaterialBatch, data = df3)
summary(fit3_block)
```

```
##                   Df Sum Sq Mean Sq F value   Pr(>F)
## Catalysts         3  11.67   3.889   5.983 0.041463 *
## RawMaterialBatch  3  66.08  22.028  33.889 0.000953 ***
## Residuals         5   3.25   0.650
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

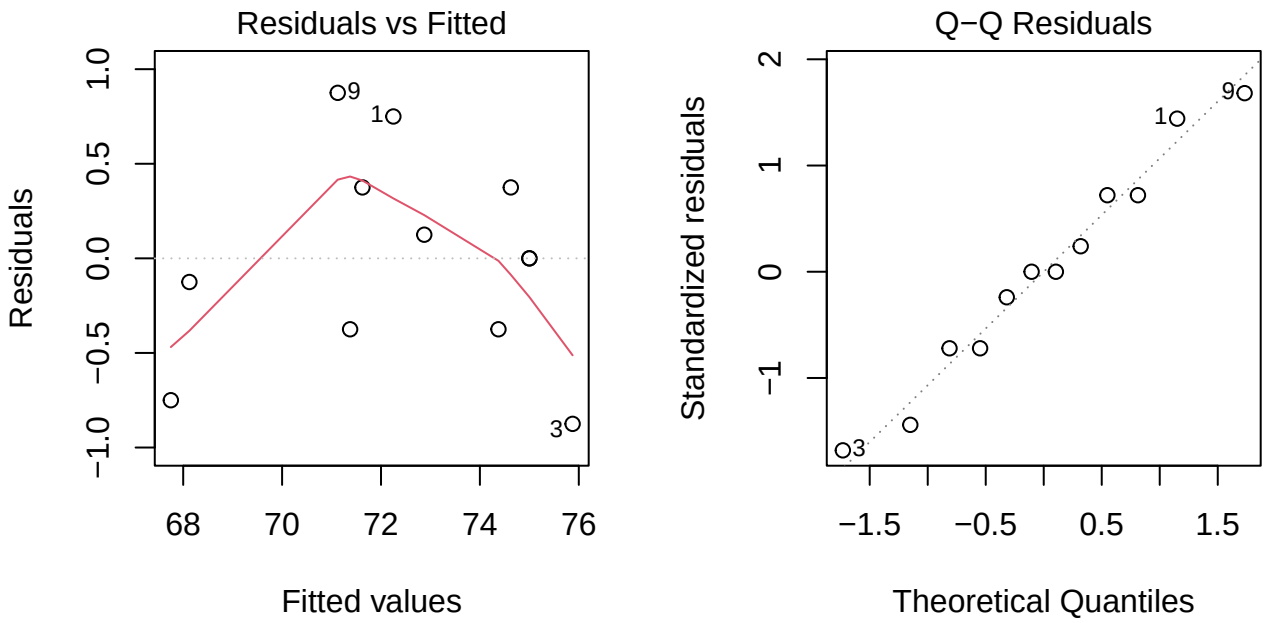
The R package **ibd** provides the function `aov.ibd` to compute the adjusted treatment and block sum of squares at the same time.

```
if (!("ibd" %in% rownames(installed.packages()))) {
  install.packages("ibd")
}
library(ibd)
fit3 <- aov.ibd(ReactionTime ~ Catalysts + RawMaterialBatch, data = df3)
fit3
```

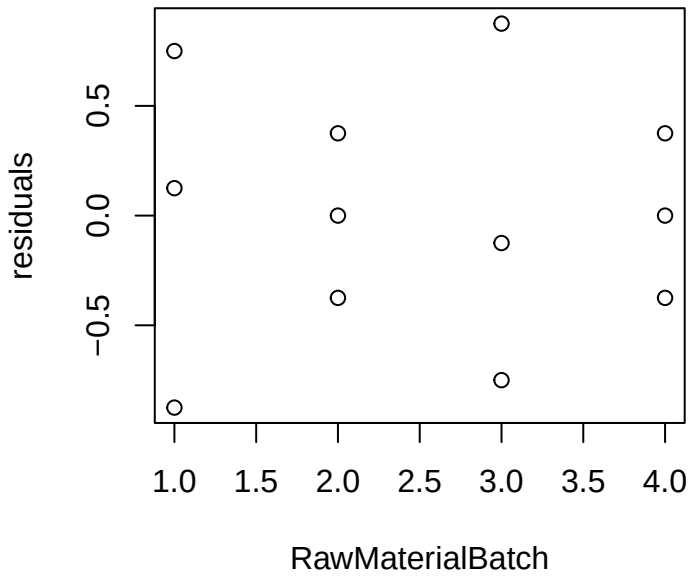
```
## $ANOVA.table
## Anova Table (Type III tests)
##
## Response: ReactionTime
##                   Sum Sq Df   F value    Pr(>F)
## (Intercept)      8948.7  1 13767.198 8.528e-10 ***
## Catalysts          22.7  3    11.667 0.0107387 *
## RawMaterialBatch   66.1  3    33.889 0.0009528 ***
## Residuals           3.3  5
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The procedures of diagnosing the residual and multiple comparison are similar to those for the RCBD. Please refer to the previous section for more details.

```
# Residual Checking:
#   can use any aov model fit3_treatment or fit3_block
par(mfrow = c(1, 2))
plot(fit3_treatment, which = 1:2)
```




```
plot(
  as.numeric(df3$RawMaterialBatch), fit3_treatment$residuals,
  xlab = "RawMaterialBatch", ylab = "residuals"
)
```



Multiple comparison is performed solely for the treatment effect, `Catalysts`, here. The following codes demonstrate the use of Tukey's test and Fisher's LSD method. For Tukey's test, add the input argument `which = "Catalysts"` to only show the test results of comparing differences among the `Catalysts` levels. For Fisher's LSD method, specify `trt = "Catalysts"` as the input argument to the `LSD.test()` function to show the comparison results among the `Catalysts` levels. For information of interpreting the results, please refer to the handout of R codes in Chapter 3.

```r
TukeyHSD(fit3_treatment, which = "Catalysts")
```

```
##   Tukey multiple comparisons of means
##     95% family-wise confidence level
##
## Fit: aov(formula = ReactionTime ~ RawMaterialBatch + Catalysts, data = df3)
##
## $Catalysts
##          diff        lwr      upr     p adj
## 2-1 0.2222222 -2.2067758 2.651220 0.9852477
## 3-1 0.5555556 -1.8734425 2.984554 0.8323947
## 4-1 3.2222222  0.7932242 5.651220 0.0165827
## 3-2 0.3333333 -2.0956647 2.762331 0.9540767
## 4-2 3.0000000  0.5710020 5.428998 0.0222012
## 4-3 2.6666667  0.2376686 5.095665 0.0352698
```

```r
if (!("agricolae" %in% rownames(installed.packages()))) {
  install.packages("agricolae")
}
library(agricolae)
out <- LSD.test(fit3_treatment, "Catalysts", p.adj = "bonferroni")
out$group
```

```
##   ReactionTime groups
## 4     74.00000      a
## 1     72.66667      a
## 3     72.00000      a
## 2     71.33333      a
```

Although the ANOVA table indicates a statistically significant difference among the treatment means, the post-hoc multiple comparison test shows that all catalysts (1–4) share the same grouping (denoted by letter "a"). This means there is no statistical evidence to conclude that the mean `ReactionTime` differs between any pair of `Catalysts`.