Serie 9 Aufgabe 1

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$A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 10^{-4} & 0 & 10^{-4} \end{pmatrix}$ $b = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $A^{-1} \begin{pmatrix} -1 & 0 & 2 \\ 0 & 1 & 0 \\ 1 & -0 & -10^{-4} \end{pmatrix}$, $\|A\|_\infty = 3$, $\|A^{-1}\|_\infty = 3$

a) $\text{cond}(A) = \|A\|_\infty \cdot \|A^{-1}\|_\infty = 3 \cdot 3 = \underline{\underline{9}}$

b) $\tilde{b} = \begin{pmatrix} 1 \\ \varepsilon \end{pmatrix}$ $\varepsilon > 0$

$$\frac{\|\tilde{x} - x\|_\infty}{\|x\|_\infty} \leq \text{cond}(A) \cdot \frac{\|b - \tilde{b}\|_\infty}{\|b\|_\infty} \leq 9 \cdot \frac{\|\text{ell}\|_\infty}{1} = 0.01$$

$$\varepsilon = \frac{0.01}{9} = \underline{\underline{0.001111}}$$

c) $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 10^{-4} & 0 & 10^{-4} \end{pmatrix} b = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \rightarrow x = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 1 & 0 & 2 \\ 10^{-4} & 0 & 10^{-4} \end{pmatrix}$, $b = \begin{pmatrix} 1 \\ 0.001111 \end{pmatrix} \rightarrow \hat{x} = \begin{pmatrix} 21.22 \\ 1 \\ -10.11 \end{pmatrix}$

rel. Fehler: $\frac{\|\hat{x} - x\|_\infty}{\|x\|_\infty} = \frac{20.22}{1} = \underline{\underline{20.22}}$

d) $\varepsilon = 1e-7$

$A = \begin{pmatrix} -e-6 & -e-7 & -e-5 \\ -e-7 & -e-6 & -e-7 \\ -9.7182 & -e-7 & -9.7182 \end{pmatrix}$ $b = \begin{pmatrix} 1 \\ 1 \\ 0.001111 \end{pmatrix}$ $A^{-1} \begin{pmatrix} -1 & -1.9998 & 2.794 \\ -0 & 0.9999 & -0.9999 \\ 1 & 0.9998 & -1.897 \end{pmatrix}$

$\|A\|_\infty = 29.1547$ $\|A^{-1}\|_\infty = 5.7983$

$\text{cond}(A) = \|A\|_\infty \cdot \|A^{-1}\|_\infty = 29.1547 \cdot 5.7983 = \underline{169.0477}$

Kontrolle: $\text{cond}(A) \frac{\|A - \tilde{A}\|_\infty}{\|A\|_\infty} < 1$: $169.0477 \frac{e-7}{29.1547} = -24.8267 < 1$ ✓

$$\frac{\|x - \tilde{x}\|}{\|x\|} \leq \frac{\text{cond}(A)}{1 - \text{cond}(A) \cdot \frac{\|A - \tilde{A}\|}{\|A\|_\infty}} \cdot \left(\frac{\|A - \tilde{A}\|_\infty}{\|A\|_\infty} + \frac{\|b - \tilde{b}\|_\infty}{\|b\|_\infty} \right) \leq \frac{169.0477}{1 - 169.0477 \cdot (-0.1469)} \cdot (-0.1469 + 0.001111)$$

$$\leq \underline{\underline{-0.9602}}$$