Serie 10 Aufgabe 2

Monday, 6 December 2021 16:58

a) A ist Diagonaldominant also ja konvergiert

b) $B = -(D+L)^{-1}R = -\left(\begin{pmatrix}8&0&0\\0&9&0\\0&0&7\end{pmatrix} + \begin{pmatrix}0&0&0\\5&0&0\\4&2&0\end{pmatrix}\right)^{-1} \begin{pmatrix}0&5&2\\0&0&1\\0&0&0\end{pmatrix} = -\begin{pmatrix}8&0&0\\5&9&0\\4&2&7\end{pmatrix}^{-1}\begin{pmatrix}0&5&2\\0&0&1\\0&0&0\end{pmatrix} = \begin{pmatrix}0.125&0&0\\-0.0694&0.1111&0\\-0.0516&-0.0317&0.1429\end{pmatrix}\begin{pmatrix}0&5&2\\0&0&1\\0&0&0\end{pmatrix} =$

$$= \begin{pmatrix}0 & 1.0854 & 0.3238\\ 0 & -0.5516 & -0.0498\\ 0 & -0.4626 & -0.1708\end{pmatrix}$$

$$C = (D+L)^{-1} b = \begin{pmatrix}2\\-1\\4\end{pmatrix},$$

$$x_1^{(k+1)} = 1.0854\, x_2^{(k)} - 0.3238 x_3^{(k)} + 2$$
$$x_2^{(k+1)} = -0.0498 x_3^{(k)} - 1$$
$$x_3^{(k+1)} = -0.4626^{(k+1)} + 4$$

Startvektor $x^0 = \begin{pmatrix}1\\1\\3\end{pmatrix}$

$$x^3 = \begin{pmatrix}2.0051\\-1.0021\\3.9977\end{pmatrix}$$

c) ges. a-posteriori Absch. abs. Fehler von x^3

$$\|B\|_\infty = 1.4092$$

$$\|x^3 - x^2\|_\infty = \begin{pmatrix}|2.0051 - 2.0147| = 0.0096\\ |-1.0021 - 1.0054| = 0.0033\\ |3.9977 - 3.9931| = 0.0046\end{pmatrix} = \underline{0.0096}$$

$$\|x^3 - \bar{x}\| \le \frac{\|B\|_\infty}{1-\|B\|_\infty} \cdot \|x^3 - x^2\|_\infty = \frac{1.4092}{0.4092} \cdot 0.0096 \Rightarrow \underline{\underline{0.03306}}$$

d) ges: A-priori Absch. $\bar{x}$ $\begin{pmatrix}2\\-1\\4\end{pmatrix}$, Toleranz: 10^{-4}

$$\|x^1 - x^0\|_\infty = \begin{pmatrix}|2.0511 - 2.25| = 0.1989\\ |1.0134 - 1.0278| = 0.0144\\ |3.9746 - 3.8651| = 0.1095\end{pmatrix} = \underline{0.1989}$$

$$\frac{1.4092^k}{-0.4092} \cdot 0.1989 \le 10^{-4}$$

$$1.4092^k \le \frac{10^{-4} \cdot (-0.4092)}{0.1989} = 20.6 \cdot 10^{-5}$$

$$k \ge \frac{\log(20.6 \cdot 10^{-5})}{\log(1.4092)} = -24.7437 \rightarrow \underline{\underline{\text{Mind. 25 Schritte}}}$$

e) $\|x^3 - x^2\|_\infty = 0.0096$

$$\frac{1.4092^k}{0.4092} \cdot 0.0096 \leq 10^{-4}$$

$$1.4092^k \leq \frac{10^{-4} \cdot 0.4092}{0.0096} = 0.004263$$

$$k \geq \frac{\log(0.004263)}{\log(1.4092)} = \underline{-15.91} \rightarrow \underline{\underline{\text{Mind. 16 Schritte}}}$$