# Serie 10 Aufgabe1

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$A = \begin{pmatrix} 8 & 5 & 2 \\ 5 & 9 & 1 \\ 4 & 2 & 7 \end{pmatrix}, \quad b = \begin{pmatrix} 19 \\ 5 \\ 34 \end{pmatrix}$

A ist Diagonaldominant:

$8 > (5+2) = 7$

$9 > (1+5) = 6$

$7 > (4+2) = 6$

a) $D = \begin{pmatrix} 8 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 7 \end{pmatrix}, \quad L = \begin{pmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 4 & 2 & 0 \end{pmatrix}, \quad R = \begin{pmatrix} 0 & 5 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

$B = -D^{-1}(L+R)$

$\begin{pmatrix} -1/8 & 0 & 0 \\ 0 & -1/9 & 0 \\ 0 & 0 & -1/7 \end{pmatrix} \begin{pmatrix} 0 & 5 & 2 \\ 5 & 0 & 1 \\ 4 & 2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -5/8 & -2/8 \\ -5/9 & 0 & -1/9 \\ -4/7 & -2/7 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -0.625 & -0.25 \\ -0.556 & 0 & -0.11 \\ -0.571 & -0.286 & 0 \end{pmatrix}$

$+ \begin{pmatrix} 19/8 \\ 5/9 \\ 34/7 \end{pmatrix}$ $\quad \|B\|_\infty = \underline{0.875} < 1$ also konvergiert sie ✓.

$x^{(0)} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$x^{(1)} = \begin{pmatrix} 2.375 \\ 0.556 \\ 4.857 \end{pmatrix}$

$x_1^{(k+1)} = -0.625x_2 - 0.25x_3 + \underline{2.375}.$

$x_2^{(k+1)} = -0.556x_1 - 0.11x_3 + 0.556$

$x_3^{(k+1)} = -0.571x_1 - 0.286x_2 + 4,857$

$x^2(1) = \underbrace{(-0.625 \cdot 0.556)}_{-0.3475} + \underbrace{(-0.25 \cdot 4.857)}_{-1.2143} + 2.375 = 0.8132$

$x^2(2) = (-0.556 \cdot 2.375) + (-0.11 \cdot 4.857) + 0.556 = -1.29877$

$x^2(3) = (-0.571 \cdot 2.375) + (-0.286 \cdot 0.556) + 4.857 = 3.6454$

$x_1^{(3)} = (-0.625 \cdot (-1.2988)) + (-0.25 \cdot 3.6454) + 2.375 = 2.2754$

$x_2^{(3)} = (-0.556 \cdot 0.8132) + (-0.11 \cdot 3.6454) + 0.556 = -0.2971$

$x_3^{(3)} = (-0.571 \cdot 0.8132) + (-0.286 \cdot (-1.2988)) + 4.857 = 4.756$

Ja konvergiert gegen $\begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix}$

b) $x^0 = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$

$A = \begin{pmatrix} 8 & 5 & 2 \\ 5 & 9 & 1 \\ 4 & 2 & 7 \end{pmatrix}, \quad b = \begin{pmatrix} 19 \\ 5 \\ 34 \end{pmatrix}$

$D = \begin{pmatrix} 8 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 7 \end{pmatrix}, \quad L = \begin{pmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 4 & 2 & 0 \end{pmatrix}, \quad R = \begin{pmatrix} 0 & 5 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \quad D^{-1} = \begin{pmatrix} 1/8 & 0 & 0 \\ 0 & 1/9 & 0 \\ 0 & 0 & 1/7 \end{pmatrix}$

$$D = \begin{pmatrix} 8 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 7 \end{pmatrix}, \quad L = \begin{pmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 4 & 2 & 0 \end{pmatrix}, \quad R = \begin{pmatrix} 0 & 5 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \quad D^{-1} = \begin{pmatrix} 1/8 & 0 & 0 \\ 0 & 1/9 & 0 \\ 0 & 0 & 1/7 \end{pmatrix}$$

$B = -D^{-1}(L+R)$:

$$\begin{pmatrix} -1/8 & 0 & 0 \\ 0 & -1/9 & 0 \\ 0 & 0 & -1/7 \end{pmatrix} \begin{pmatrix} 0 & 5 & 2 \\ 5 & 0 & 1 \\ 4 & 2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -5/8 & -2/8 \\ -5/9 & 0 & -1/9 \\ -4/7 & -2/7 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -0.625 & -0.25 \\ -0.556 & 0 & -0.11 \\ -0.571 & -0.286 & 0 \end{pmatrix}$$

$C = D^{-1} \cdot b$: $\begin{pmatrix} 19/8 \\ 5/9 \\ 34/7 \end{pmatrix} = \begin{pmatrix} 2.375 \\ 0.556 \\ 4.857 \end{pmatrix}$

$$x_1^{(k+1)} = 0.625 x_2 - 0.25 x_3 + 2.375$$
$$x_2^{(k+1)} = -0.556 x_1 - 0.11 x_3 + 0.556$$
$$x_3^{(k+1)} = -0.571 x_1 - 0.286 x_2 + 4.857$$

$$x^{(3)} = \begin{pmatrix} 1.6882 \\ -1.1585 \\ 3.7807 \end{pmatrix}$$

c) ges: a-posteriori Absch. abs. Fehler von $x^{(3)}$

$$\|B\|_\infty = 0.875$$

$$\|x^3 - x^2\|_\infty = \begin{pmatrix} |1.6882 - 2.0981| = 0.4098 \\ |-1.1585 - -0.6521| = 0.5064 \\ |3.7807 - 4.3776| = 0.5969 \end{pmatrix} = 0.5969$$

$$\|x^3 - \bar{x}\|_\infty \leq \frac{\|B\|_\infty}{1 - \|B\|_\infty} \cdot \|x^3 - x^2\|_\infty = \frac{0.875}{0.125} \cdot 0.5969 = \underline{\underline{4.1783}}$$

d) ges: A-priori Absch. $\bar{x} \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$, Toleranz: $10^{-4}$

$$\|x^1 - x^0\|_\infty = \begin{pmatrix} |2.25 - 2| = 0.25 \\ |-0.333 - 1| = 0.6667 \\ |4.5714 - 4| = 0.5714 \end{pmatrix} = \underline{0.6667}$$

$$\frac{0.875^k}{0.125} \cdot 0.6667 \leq 10^{-4}$$

$$0.875^k \leq \frac{10^{-4} \cdot 0.125}{0.6667} = 19 \cdot 10^{-6}$$

$k \geq \log(19 \cdot 10^{-6})$ ... $= 81.4$ ... $\rightarrow$ Mind. 82 Schritte dann Toleranz erreicht

$$0.6667$$

$$k \geq \frac{\log(19 \cdot 10^{-6})}{\log(0.875)} = \underline{81.4121} \rightarrow \text{Mind. 82 Schritte dann Toleranz erreicht}$$

e) $\|x^3 - x^2\|_\infty = 0.5969$

$$\frac{0.875^k}{0.125} \cdot 0.5969 \leq 10^{-4}$$

$$0.875^k \leq \frac{10^{-4} \cdot 0.125}{0.5969} = 21 \cdot 10^{-6}$$

$$k \geq \frac{\log(21 \cdot 10^{-6})}{\log(0.875)} = \underline{80.6626} \rightarrow \underline{\underline{\text{Mind. 81 Schritte}}}$$