Serie 11 Aufgabe 2

Sunday, 12 December 2021 16:23

$z^4 + 4z^2 + 16 = 0$

Substitution: $u = z^2$

$\rightarrow u^2 + 4u + 16 = 0$

$$u_1 = \frac{b + \sqrt{b^2 - 4ac}}{2a} \qquad u_2 = \frac{b - \sqrt{b^2 - 4ac}}{2a}$$

$$u_1 = \frac{4 + \sqrt{16 - 4(16)}}{2} = \frac{4 + \sqrt{-48}}{2} = \frac{4 + 6.93i}{2} = \underline{2 + 3.465i}$$

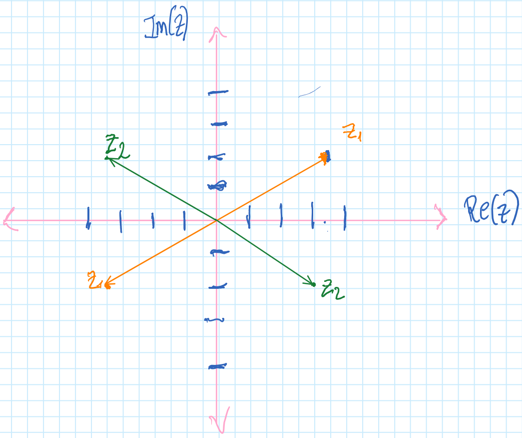
$$u_2 = \frac{4 - \sqrt{16 - 4(16)}}{2} = \frac{4 - \sqrt{-48}}{2} = \frac{4 - 6.93i}{2} = \underline{2 - 3.465i}$$

$$z_{1/2} = \sqrt{u_1}, \sqrt{u_2} = \sqrt{2 + 3.465i}, \sqrt{2 - 3.465i}$$

$$z_k = \sqrt[n]{r}\, e^{i\left(\frac{\varphi_0}{n} + \frac{2k\pi}{n}\right)} = \sqrt[n]{r}\left(\cos\left(\frac{\varphi_0}{n} + \frac{2k\pi}{n}\right) + i \sin\left(\frac{\varphi_0}{n} + \frac{2k\pi}{n}\right)\right)$$

$$\varphi_1 = \tan^{-1}\left(\frac{3.465}{2}\right) = \underline{60^\circ} \qquad \varphi_2 = 360 - \tan^{-1}\left(\frac{3.465}{2}\right) = \underline{300^\circ}$$

$$r_{1/2} = \sqrt{2^2 + (3.465)^2} = \sqrt{4 + 12} = \sqrt{16} = 4$$



z_1:

$K = 0$: $4\left(\cos(30^\circ) + i \sin(30^\circ)\right) = \underline{3.46 + 2i}$

$K = 1$: $4\left(\cos(30^\circ + \pi) + i \cdot \sin(30^\circ + \pi)\right) \approx \underline{\underline{-3.46 - 2i}}$

z_2:

$K = 0$: $4\left(\cos(150^\circ) + i \cdot \sin(150^\circ)\right) \approx \underline{-3.46 + 2i}$

$K = 1$: $4\left(\cos(150^\circ + \pi) + i \cdot \sin(150^\circ + \pi)\right) = \underline{\underline{3.46 - 2i}}$