

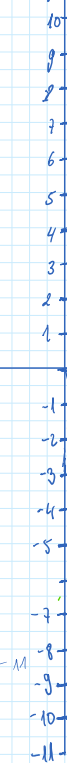
# Serie 11 Aufgabe 1

Monday, 6 December 2021 15:54

a)  $z = 3 - 11i$

ges: Zeiger zeichnen, exp. form, trig. form &  $\bar{z}$

Im(z)



$$r = \sqrt{3^2 + 11^2} = \sqrt{130} = 11.4018$$

$$\varphi = 2\pi - \tan^{-1}\left(\frac{11}{3}\right) = 360 - \tan^{-1}\left(\frac{11}{3}\right) = 360 - 74.75 = 285.25^\circ$$

$$\text{Exp. Form} = z = r e^{i\varphi} = 11.4018 \cdot e^{i \cdot 285.25^\circ}$$

$$\text{Trig. Form} = z = r (\cos(\varphi) + i \sin(\varphi)) = 11.4018 (\cos(285.25^\circ) - i \sin(285.25^\circ))$$

$$\bar{z} = 3 + 11i$$

b)  $z = 4 (\cos(-40^\circ) + i \sin(-40^\circ)) + 2 e^{i 30^\circ} - 3 + 1.5i$

Normalform =  $z = x + iy$

$$z = 4 (\cos(-40^\circ) + i \sin(-40^\circ)) = 4 (0.76604 + i (-0.642788)) = 3.06418 - 2.57115i$$

$$z = 2 e^{i 30^\circ} = 2 (\cos(30^\circ) + i \sin(30^\circ)) = 2 (0.866025 + i (0.5)) = 1.73205 + i$$

$$z = 3 + 1.5i$$

$$z = (3.06418 - 2.57115i) + (1.73205 + i) - (3 + 1.5i)$$

$$\bar{z} = (3.06418 + i 2.57115) + (1.73205 - i) - (3 - 1.5i)$$

c)  $z_1 = \frac{2+i}{1-2i}$ ,  $z_2 = 2 e^{-i \frac{\pi}{3}}$ ,  $z_3 = 4 (\cos(30^\circ) + i \sin(30^\circ))$

$$z_1 = \frac{(2+i)(1+2i)}{(1-2i)(1+2i)} = \frac{2+4i+i+2i^2}{1+2i-2i-4i^2} = \frac{2+5i-2}{1+4} = \frac{5i}{5} = i$$

$$z_2 = 2 (\cos(60^\circ) + i \sin(60^\circ)) = 2 (0.5 + i \cdot 0.866025) = 1 + 1.73205i$$

$$z_3 = 4 (0.866025 + i \cdot 0.5) = 3.4641 + 2i$$

$$\bar{z}_1 = -i$$

$$0.5 z_2 = \frac{z_2}{2} = \frac{1 + 1.73205i}{2} = 0.5 + 0.866025i$$

$$\frac{\bar{z}_1 \cdot z_3}{0.5 z_2} = \frac{-i (3.4641 + 2i)}{0.5 + 0.866025i} = \frac{-3.4641i - 2i^2}{0.5 + 0.866025i} = \frac{2 - 3.4641i}{0.5 + 0.866025i} = \frac{(2 - 3.4641i)(0.5 - 0.866025i)}{(0.5 + 0.866025i)(0.5 - 0.866025i)}$$

$$= \frac{1 - 1.73205i - 1.73205i + 3i^2}{0.25 - 0.433013 + 0.433013 - 0.75i^2} = \frac{-2 - 3.4641i}{1} = -2 - 3.4641i$$

$$0.5 z_2 = \frac{2 e^{-i \frac{\pi}{3}}}{2} = e^{-i \frac{\pi}{3}}$$

$$\bar{z}_1 \cdot z_3 = -4 (e^{i 30^\circ}) \cdot e^{-i \frac{\pi}{3}} = -4 e^{i (-30^\circ)} = -3.46 + ei$$

$$\begin{aligned}
 d) \quad (1 - \sqrt{2}i)^3 &= z^3, \quad r = \sqrt{1^2 + (-\sqrt{2})^2} = \sqrt{1+2} = \sqrt{3} = 1.73 \\
 \varphi &= 2\pi - \tan^{-1}\left(\frac{\sqrt{2}}{1}\right) = 360 - \tan^{-1}\left(\frac{\sqrt{2}}{1}\right) = 360 - 54.74 = 305.26^\circ \\
 z^n &= r^n \cdot e^{in\varphi} = r^n (\cos(n\varphi) + i \cdot \sin(n\varphi)) = z^3 = (1.73)^3 (\cos(3 \cdot 305.26^\circ) + i \cdot \sin(3 \cdot 305.26^\circ)) \\
 &= 5 (\cos(915.78) + i \cdot \sin(915.78)) = 5(-0.96 + i \cdot -0.27) = \underline{\underline{-4.8 - 1.35i}}
 \end{aligned}$$