Serie 12 Aufgabe 1

Monday, 20 December 2021 15:23

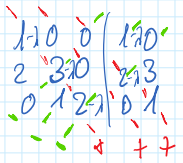
$p(\lambda) = \det(A - \lambda I_n) = 0$

Lw: $\lambda^2 - \text{tra}(A) + \det(A)$

$\text{tra}(A) = a_{11} + a_{22} + \dots + a_{nn} = \lambda_1 + \lambda_2 + \dots + \lambda_n$

$\det(A) = \lambda_1 \cdot \lambda_2 \cdot \dots \cdot \lambda_n$

a) $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{pmatrix}$ $A - \lambda I_3 = \begin{pmatrix} 1-\lambda & 0 & 0 \\ 2 & 3-\lambda & 0 \\ 0 & 1 & 2-\lambda \end{pmatrix}$



$\text{tra}\, A = 1-\lambda + 3-\lambda + 2-\lambda = 6 - 3\lambda$

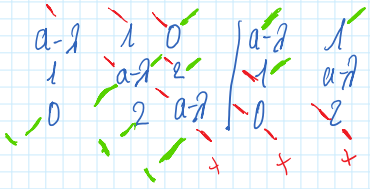
$\det(A) = (1-\lambda)(3-\lambda)(2-\lambda)$

$p(\lambda) = (1-\lambda)(3-\lambda)(2-\lambda) = 0$ $\quad \lambda_1 = 1, \ \lambda_2 = 2, \ \lambda_3 = 3$

$\sigma(A) = \{1, 2, 3\}$

Da Matrix A eine Obere Dreiecksmatrix ist, sind die EW gleich der Elemente der Diagonale.

b) $A = \begin{pmatrix} a & 1 & 0 \\ 1 & a & 2 \\ 0 & 2 & a \end{pmatrix}$; $A - \lambda I_3 = \begin{pmatrix} a-\lambda & 1 & 0 \\ 1 & a-\lambda & 2 \\ 0 & 2 & a-\lambda \end{pmatrix}$



$\text{tra}\, A = (a-\lambda) + (a-\lambda) + (a-\lambda) = 3a - 3\lambda$

$\det(A) = (a-\lambda)(1-\lambda)(a^2-4-2\lambda) = (a-\lambda)^3 - (2 \cdot 2 \cdot a - \lambda) - (1 \cdot 1 \cdot a - \lambda) = (a-\lambda)^3 - 4(a-\lambda) - 1(a-\lambda)$

$= (a-\lambda)^3 - 5(a-\lambda)$

$p(\lambda) = (a-\lambda)^3 - 5(a-\lambda) = 0$

$\lambda_1 = a$

$(a-\lambda)^3 - 5(a-\lambda) : (\lambda - a) = -(a-\lambda)^2 + 5$

$(a-\lambda)^3$

$-5a + 5\lambda$

$-5a + 5\lambda$

0

$-(a-\lambda)^2 + 5 = 0$

$-(a^2 - 2\lambda a + \lambda^2) + 5 = 0$

$-a^2 + 2\lambda a - \lambda^2 + 5 = 0$

$-\lambda^2 + 2\lambda a + 5 - a^2 = 0$

$(a-\lambda)(a-\lambda) = a^2 - \lambda a - \lambda a + \lambda^2$

$\lambda_2 = a - \sqrt{5}$, $\lambda_3 = a + \sqrt{5}$