

Serie 12 Aufgabe 1

Monday, 20 December 2021 15:23

$$p(\lambda) = \det(A - \lambda I_n) = 0$$

$$\text{zw: } \lambda^2 - \text{tr}(A) + \det(A)$$

$$\text{tr}(A) = a_{11} + a_{22} + \dots + a_{nn} = \lambda_1 + \lambda_2 + \dots + \lambda_n$$

$$\det(A) = \lambda_1 \cdot \lambda_2 \cdot \dots \cdot \lambda_n$$

$$a) A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{pmatrix} \quad A - \lambda I_3 = \begin{pmatrix} 1-\lambda & 0 & 0 \\ 2 & 3-\lambda & 0 \\ 0 & 1 & 2-\lambda \end{pmatrix}$$

$$\begin{vmatrix} 1-\lambda & 0 & 0 \\ 2 & 3-\lambda & 0 \\ 0 & 1 & 2-\lambda \end{vmatrix} = (1-\lambda) \begin{vmatrix} 3-\lambda & 0 \\ 1 & 2-\lambda \end{vmatrix} = (1-\lambda)(3-\lambda)(2-\lambda)$$

$$\text{tr } A = 1 + 3 + 2 = 6$$

$$\det(A) = (1-2)(3-2)(2-2) = 0$$

$$p(\lambda) = (1-\lambda)(3-\lambda)(2-\lambda) = 0$$

$$\sigma(A) = \{1, 2, 3\}$$

$$\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 3$$

Da Matrix A eine Oberdreiecksmatrix ist, sind die EW gleich den Elementen der Diagonale.

$$b) A = \begin{pmatrix} a & 1 & 0 \\ 1 & a & 2 \\ 0 & 2 & a \end{pmatrix}; \quad A - \lambda I_3 = \begin{pmatrix} a-\lambda & 1 & 0 \\ 1 & a-\lambda & 2 \\ 0 & 2 & a-\lambda \end{pmatrix}$$

$$\begin{vmatrix} a-\lambda & 1 & 0 \\ 1 & a-\lambda & 2 \\ 0 & 2 & a-\lambda \end{vmatrix} = (a-\lambda) \begin{vmatrix} a-\lambda & 2 \\ 2 & a-\lambda \end{vmatrix} - 1 \cdot (a-\lambda) = (a-\lambda)^3 - 4(a-\lambda) - 1(a-\lambda)$$

$$\text{tr } A = (a-\lambda) + (a-\lambda) + (a-\lambda) = 3a - 3\lambda$$

$$\det(A) = (a-\lambda)(1-\lambda)(a-2) = (a-\lambda)^3 - (2 \cdot 2 \cdot a - 2) - (1 \cdot 1 \cdot a - 2) = (a-\lambda)^3 - 4(a-\lambda) - 1(a-\lambda)$$

$$= (a-\lambda)^3 - 5(a-\lambda)$$

$$p(\lambda) = (a-\lambda)^3 - 5(a-\lambda) = 0$$

$$\lambda_1 = a$$

$$(a-\lambda)^3 - 5(a-\lambda) : (a-\lambda) = -(a-\lambda)^2 + 5$$

$$\begin{array}{r} -5a+5\lambda \\ -5a+5\lambda \\ \hline 0 \end{array}$$

$$-5a+5\lambda$$

$$-5a+5\lambda$$

$$0$$

$$-(a-\lambda)^2 + 5 = 0$$

$$-(a^2 - 2a\lambda + \lambda^2) + 5 = 0$$

$$-a^2 + 2a\lambda - \lambda^2 + 5 = 0$$

$$-\lambda^2 + 2a\lambda + 5 - a^2 = 0$$

$$\lambda_2 = a - \sqrt{5}, \quad \lambda_3 = a + \sqrt{5}$$

$$(a-2)(a-2) = a^2 - 2a - 2a + 4 = a^2 - 4a + 4$$