Serie 12 Aufgabe 3

Saturday, 25 December 2021 21:49

$$A = \begin{pmatrix} 2 & 0 & 1 \\ 7 & -5 & 9 \\ 6 & -6 & 9 \end{pmatrix}$$

a) $$T = \begin{pmatrix} 3 & 1 & 1 \\ -1 & 1 & 2 \\ 3 & 0 & 1 \end{pmatrix}, \quad T^{-1} = \begin{pmatrix} 1 & -1 & 1 \\ -5 & 6 & -7 \\ 3 & -3 & 4 \end{pmatrix}$$

$$D = T^{-1} A T$$

$$T^{-1}A = \begin{pmatrix} 1 & -1 & 1 \\ -5 & 6 & -7 \\ 3 & -3 & 4 \end{pmatrix} \begin{pmatrix} 2 & 0 & 1 \\ 7 & -5 & 9 \\ 6 & -6 & 9 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 1 \\ -10 & 12 & -14 \\ 9 & -9 & 12 \end{pmatrix}$$

$$T^{-1}AT = \begin{pmatrix} 1 & -1 & 1 \\ -10 & 12 & -14 \\ 9 & -9 & 12 \end{pmatrix} \cdot \begin{pmatrix} 3 & 1 & 1 \\ -1 & 1 & 2 \\ -3 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} = D$$

b) EW: $\lambda_1 = 1, \ \lambda_2 = 2, \ \lambda_3 = 3$

EV: $\begin{pmatrix} 3 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

c) $$T = \begin{pmatrix} 3 & 1 & 1 \\ -1 & 1 & 2 \\ 3 & 0 & 1 \end{pmatrix} \quad T_{neu} \begin{pmatrix} 6 & 3 & 4 \\ -2 & 3 & 8 \\ 6 & 0 & 4 \end{pmatrix}, \quad T^{-1}_{neu} = \begin{pmatrix} 6 & -6 & 6 \\ 28 & 0 & -28 \\ -9 & 9 & 12 \end{pmatrix}$$

$$T^{-1}_{neu} A = \begin{pmatrix} 6 & -6 & 6 \\ 28 & 0 & -28 \\ -9 & 9 & 12 \end{pmatrix} \begin{pmatrix} 2 & 0 & 1 \\ 7 & -5 & 9 \\ 6 & -6 & 9 \end{pmatrix} = \begin{pmatrix} 6 & -6 & 6 \\ -112 & 168 & 224 \\ 117 & -117 & 180 \end{pmatrix}$$

$$T^{-1}_{neu} A T_{neu} = \begin{pmatrix} 6 & -6 & 6 \\ -112 & 168 & 224 \\ 117 & -117 & 180 \end{pmatrix} \begin{pmatrix} 6 & 3 & 5 \\ -2 & 2 & 10 \\ 6 & 0 & 5 \end{pmatrix} = \begin{pmatrix} 84 & 0 & 0 \\ -2352 & 112 & 0 \\ 2016 & 0 & 315 \end{pmatrix}$$

$$T^{-1}_{neu} A T_{neu} = \begin{pmatrix} 6 & -6 & 6 \\ -112 & 168 & -224 \\ 117 & -117 & 180 \end{pmatrix} \begin{pmatrix} 6 & 2 & 5 \\ -2 & 2 & 10 \\ 6 & 0 & 5 \end{pmatrix} = \begin{pmatrix} 84 & 0 & 0 \\ -2352 & 112 & 0 \\ 2016 & 0 & 315 \end{pmatrix}$$

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