Serie 12 Aufgabe 2

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$A = \begin{pmatrix} 2 & 5 \\ -1 & -2 \end{pmatrix}$

$p(\lambda) = (2-\lambda)(-2-\lambda)+5 = \lambda^2+1=0$

$\lambda_1 = i \quad , \quad \lambda_2 = -i$

$E: (A - i\, I_2)\,\vec{x} = 0$

$\left(\begin{array}{cc|c} 2-i & 5 & 0 \\ -1 & -2-i & 0 \end{array}\right) \xrightarrow{z_2 - \frac{-1}{2-i}\cdot z_1} \left(\begin{array}{cc|c} 2-i & 5 & 0 \\ 0 & 0 & 0 \end{array}\right) \Rightarrow x \begin{pmatrix} -2-i \\ 1 \end{pmatrix} \quad Rg(A-I_2)=1$

$\left(\begin{array}{cc|c} 2+i & 5 & 0 \\ -1 & -2+i & 0 \end{array}\right) \xrightarrow{z_2 - \frac{-1}{2+i}\cdot z_1} \left(\begin{array}{cc|c} 2+i & 5 & 0 \\ 0 & 0 & 0 \end{array}\right) = x \begin{pmatrix} -2+i \\ 1 \end{pmatrix} \quad Rg(A-I_2)=1$

$Dim = n - Rg(A-I_2) = 2-1 = 1$

$\|x\| = \sqrt{(-2-i)(-2+i)+1^2} = \sqrt{6} \qquad \frac{1}{\|x\|} = \frac{1}{\sqrt{6}}\begin{pmatrix} -2-i \\ 1 \end{pmatrix}, \quad \frac{1}{\sqrt{6}}\begin{pmatrix} -2+i \\ 1 \end{pmatrix}$

Eigenspaces: $\begin{pmatrix} (-2-i) \\ 1 \end{pmatrix}$ für $\lambda = i$, $\quad \lambda = -i \begin{pmatrix} -2+i \\ 1 \end{pmatrix}$

b) $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0-\lambda & 1 & 1 \\ 1 & 0-\lambda & 1 \\ 1 & 1 & 0-\lambda \end{pmatrix} = A - \lambda I_3$

$\left(\begin{array}{ccc|cc} 0-\lambda & 1 & 1 & 0-\lambda & 1 \\ 1 & 0-\lambda & 1 & 1 & 0-\lambda \\ 1 & 1 & 0-\lambda & 1 & 1 \end{array}\right)$

$tra(A) = -\lambda - \lambda - \lambda = -3\lambda$

$\det(A) = (-\lambda)^3 + 1 + 1 - (-\lambda) - (-\lambda) - (-\lambda) = (-\lambda)^3 + 2 + \lambda + \lambda + \lambda = (-\lambda)^3 + 2 + 3\lambda =$

$p(\lambda) = -\lambda^3 + 3\lambda + 2 = 0$

$= -(\lambda-2)(\lambda+1)^2 = 0$

$\lambda_1 = 2$

$-\lambda^3 + 3\lambda + 2 : (\lambda - 2) = -\lambda^2 - 2\lambda - 1$

$-\lambda^3 + 2\lambda^2$

$0 + 2\lambda^2 + 3\lambda$

$-2\lambda^2 - 4\lambda$

$-\lambda + 2$

$-\lambda^2 - 2\lambda - 1 = 0$

$\lambda_1 = \frac{-2 + \sqrt{(-2)^2 - 4(-1\cdot -1)}}{2}, \quad \lambda_2 = \frac{-2 - \sqrt{(-2)^2 - 4(-1\cdot -1)}}{2} =$

$$\lambda_2 = \frac{-2 + \sqrt{4-4}}{2} \, , \quad \lambda_3 = \frac{-2 - \sqrt{0}}{2} = \underline{\underline{-1}}$$

$$\underline{\underline{\lambda_1 = 2}} \, , \quad \underline{\underline{\lambda_2 = -1}}$$

$$\left(\begin{array}{ccc|c} -2 & 1 & 1 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 1 & -2 & 0 \end{array}\right) \begin{array}{l} z_2 - \frac{1}{-2} \cdot z_1 \\ z_3 - \frac{1}{-2} \cdot z_1 \end{array} \left(\begin{array}{ccc|c} -2 & 1 & 1 & 0 \\ 0 & -3/2 & 3/2 & 0 \\ 0 & 3/2 & -3/2 & 0 \end{array}\right) \rightsquigarrow \left(\begin{array}{ccc|c} -2 & 1 & 1 & 0 \\ 0 & -3/2 & 3/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\rightsquigarrow \left(\begin{array}{ccc|c} -2 & 1 & 1 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \qquad \begin{array}{l} x_3 = 1 \\ x_2 = 1 \\ x_1 = 1 \end{array} \to \vec{x} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right) \rightsquigarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Substitution: $x_2 = s$, $x_3 = t$

$$\begin{array}{l} x_1 = -s-t \\ x_2 = s \\ x_3 = t \end{array}$$

$$\vec{x} \begin{pmatrix} -s-t \\ s \\ t \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} s + \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} t$$