

AshaSchwegler_S5_Aufg1

Sunday, 27 March 2022

21:54

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|-------|---|---|---|----|
| x_i | 4 | 6 | 8 | 10 |
| y_i | 6 | 3 | 9 | 0 |

$S_i = 0, 1, 2, 3$
Koeffizienten = a_i, b_i, c_i, d_i

Algorithmus:

1. $a_i = y_i$ 3. $c_0 = 0, c_n = 0$

2. $h_i = x_{i+1} - x_i$

4a) $i = 1$:

$$2(h_0 + h_1)c_1 + h_1 c_2 = 3 \frac{y_2 - y_1}{h_1} - 3 \frac{y_1 - y_0}{h_0}$$

4b) falls $n \geq 2 \quad \forall i = 2, \dots, n-2$

$$h_{i-1} c_{i-1} + 2(h_{i-1} + h_i) c_i + h_i c_{i+1} = 3 \frac{y_{i+1} - y_i}{h_i} - 3 \frac{y_i - y_{i-1}}{h_{i-1}}$$

4c) $i = n-1$

$$h_{n-2} c_{n-2} + 2(h_{n-2} + h_{n-1}) c_{n-1} = 3 \frac{y_n - y_{n-1}}{h_{n-1}} - 3 \frac{y_{n-1} - y_{n-2}}{h_{n-2}}$$

5. $b_i = \frac{y_{i+1} - y_i}{h_i} - \frac{h_i}{3} (c_{i+1} + 2c_i)$

6. $d_i = \frac{1}{3h_i} (c_{i+1} - c_i)$

$h_0 = x_1 - x_0 = 6 - 4 = 2, \quad h_1 = x_2 - x_1 = 8 - 6 = 2, \quad h_3 = x_4 - x_3 = 10 - 8 = 2$

$c_0 = c_3 = 0$

$$\begin{matrix} i=1 \\ i=2 \end{matrix} \begin{pmatrix} 2(h_0 + h_1) & h_1 \\ h_1 & 2(h_1 + h_2) \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = 3 \begin{pmatrix} \frac{y_2 - y_1}{h_1} - \frac{y_1 - y_0}{h_0} \\ \frac{y_3 - y_2}{h_2} - \frac{y_2 - y_1}{h_1} \end{pmatrix}$$

$$\begin{pmatrix} 8 & 2 \\ 2 & 8 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = 3 \begin{pmatrix} 4.5 \\ -7.5 \end{pmatrix} = \begin{pmatrix} 13.5 \\ -22.5 \end{pmatrix}$$

$\Rightarrow \begin{pmatrix} 8 & 2 \\ 0 & 7.5 \end{pmatrix} = \begin{pmatrix} 13.5 \\ -25.875 \end{pmatrix} \rightarrow$

$$c_1 = \frac{13.5 - 2(-3.45)}{8} = \underline{\underline{2.55}}$$

$$c_2 = \frac{-25.875}{7.5} = \underline{\underline{-3.45}}$$

$$b_0 = \frac{y_1 - y_0}{h_0} - \frac{h_0}{8} (c_1 + 2c_0) = -1.5 - \frac{4}{3} (2.55) = \underline{\underline{-3.2}}$$

$$b_1 = \frac{y_2 - y_1}{h_1} - \frac{h_1}{8} (c_2 + 2c_1) = 3 - \frac{2}{3} (-3.45 + 2(2.55)) = \underline{\underline{1.9}}$$

$$b_2 = \frac{y_3 - y_2}{h_2} - \frac{h_2}{8} (c_3 + 2c_2) = -4.5 - \frac{2}{3} (2 \cdot (-3.45)) = \underline{\underline{0.1}}$$

$$d_0 = \frac{1}{3h_0} (c_1 - c_0) = \frac{1}{6} (2.55) = \underline{\underline{0.425}}$$

$$d_1 = \frac{1}{3h_1} (c_2 - c_1) = \frac{1}{6} (-3.45 - 2.55) = \underline{\underline{-1}}$$

$$d_2 = \frac{1}{3h_2} (c_3 - c_2) = \frac{1}{6} (-3.45) = \underline{\underline{0.575}}$$

$$a_0 = 6$$

$$a_1 = 3$$

$$a_2 = 9$$

$$a_3 = 0$$