AshaSchwegler_S9_Aufg2

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$$\int_0^{\pi} \cos(x^2)\,dx \quad , \quad h_j = \frac{b-a}{2^j} \;, (j = 0, 1, 2) \;, \; i = a + i \cdot h_j$$

$$T_{jk} = \frac{4^k \cdot T_{j+1,k-1} - T_{j,k-1}}{4^k - 1} \quad (j = 0, .., m-k) \; (k = 1, 2, ... m)$$

$$T_{00} = j = 0 \;, \; h_0 = \frac{\pi - 0}{2^0} = \pi \;, \; n_0 = 2^0 = 1$$

$$= h_0 \cdot \left(\frac{f(a)+f(b)}{2} + \sum_{i=1}^{0} f(x_i)\right)$$

$$= h_0 \cdot \left(\frac{f(a)+f(b)}{2}\right) = \pi \cdot \left(\frac{f(0) + f(\pi)}{2}\right)$$

$$= \pi \left(\frac{1 + -0.903}{2}\right) = \pi \left(\frac{0.097}{2}\right) = \underline{0.0485\,\pi}$$

$$T_{10} = j = 1 \;, \; h_0 = \frac{\pi}{2} \;, \; n_0 = 2^1 = 2$$

$$= h_1 \cdot \left(\left(\frac{f(a)+f(b)}{2}\right) + \sum_{i=1}^{1} f(x_i)\right)$$

$$= h_1 \cdot \left(\frac{f(a)+f(b)}{2} + f(x_1)\right)$$

$$= \frac{\pi}{2}\left(0.0485 + f\left(0 + 1 \cdot \frac{\pi}{2}\right)\right) = \frac{\pi}{2}\left(0.0485 - 0.78\right) = \frac{-0.66\,\pi}{2}$$

$$= \underline{-0.33\,\pi}$$

$$T_{20} = j = 2 \;, \; h_0 = \frac{\pi}{4} \;, \; n_0 = 4$$

$$= h_2 \cdot \left(\frac{f(a)+f(b)}{2} + \sum_{i=1}^{3} f(x_i)\right)$$

$$= h_2 \cdot \left(0.0485 + f(x_1) + f(x_2) + f(x_3)\right)$$

o

$$x_1 = 0 + 1 \cdot \frac{\pi}{4} \;, \; x_2 = 0 + 2 \cdot \frac{\pi}{4} \;, \; x_3 = 0 + 3 \cdot \frac{\pi}{4}$$

$$\frac{\pi}{4}\left(0.0485 + 0.82 - 0.7912 + 0.744\right) = 0.8313\,\pi/4 = \underline{0.208\,\pi}$$

T_{j0}	T_{j1}	T_{j2}
T_{00}	$T_{01} = \frac{4^1 \cdot 0.0485\pi - 0.033\pi}{3} = -1.585$	$T_{02} = \frac{16 \cdot 1.25 + 1.585}{63} = \underline{1.439}$ ✓

$$\Rightarrow T_{02} = \frac{40 \cdot 1.25 + 1.088}{63} = \underline{\underline{1.439}}$$ ✓

T10

$$\Rightarrow T_{11} = \frac{4 \cdot 0.209\pi + 0.33\pi}{3} = 1.25$$

T20