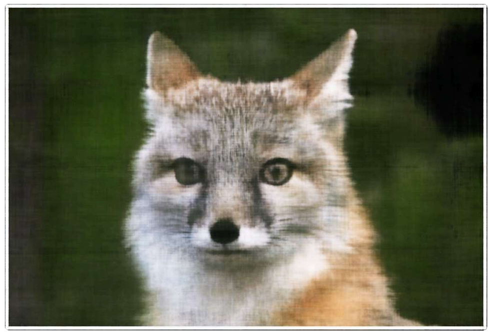
Neural Radiance Field

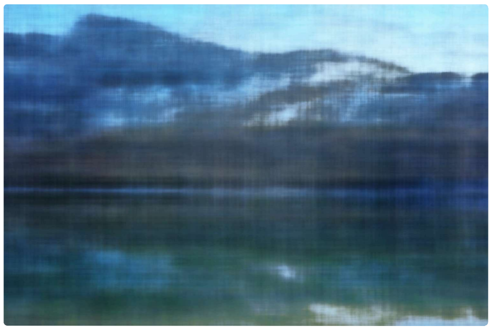
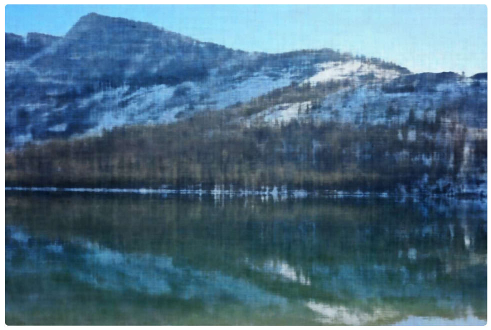
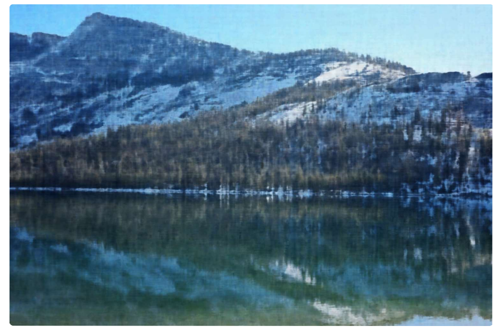
Part0: Calibrating Your Camera and Capturing a 3D Scan

Part1: Fit a Neural Field to a 2D Image

Part1.1: I use a fully connected multilayer perceptron (MLP) to regress RGB color from 2D image coordinates. The input to the network is a 2D pixel coordinate (u, v) in $[0, 1]^2$, which is first passed through a positional encoding with L = 10 frequency levels. For each dimension I apply sine and cosine at frequencies 2^k·π (k = 0 … 5), so the encoded feature has dimension 2 + 4L = 42 (original 2 coordinates plus 40 sinusoidal features). This 42-D vector is fed into a 4-layer MLP with three hidden layers of width 256: 26 → 256 → 256 → 256 → 3. Each hidden layer uses a Linear layer followed by a ReLU activation, and the final layer uses a Sigmoid activation to keep the predicted RGB values in [0, 1]. I train this network with Adam using a learning rate of 1×10^{-2}.

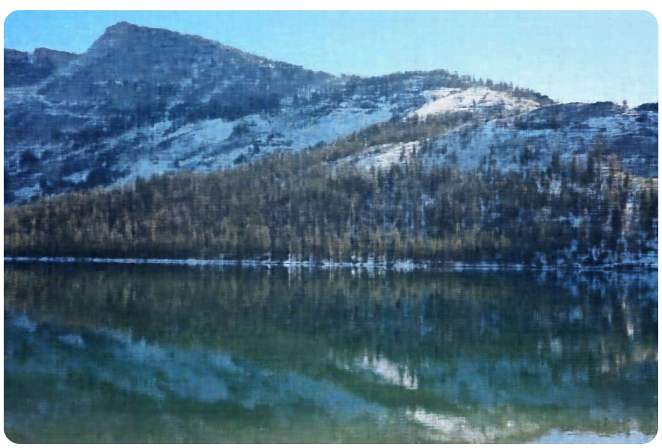
Part 1.2:

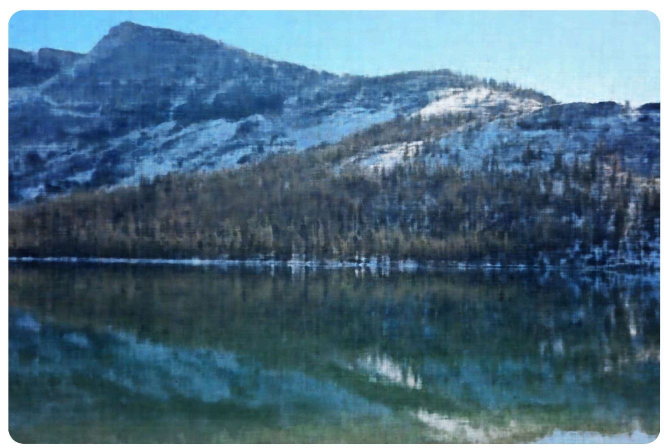





Part 1.3:

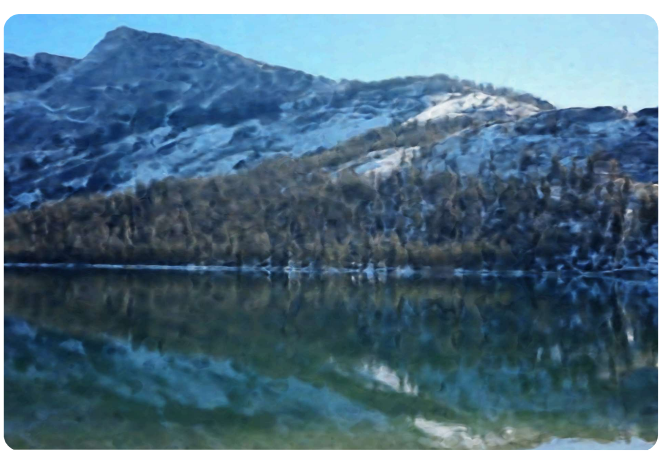
L = 10, width = 256



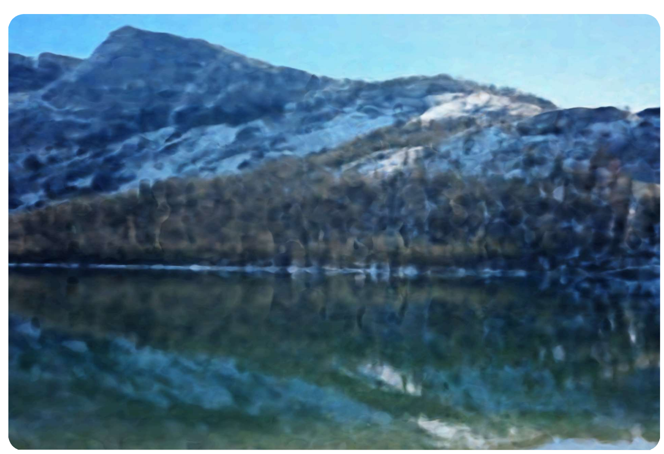
L = 10, width = 128



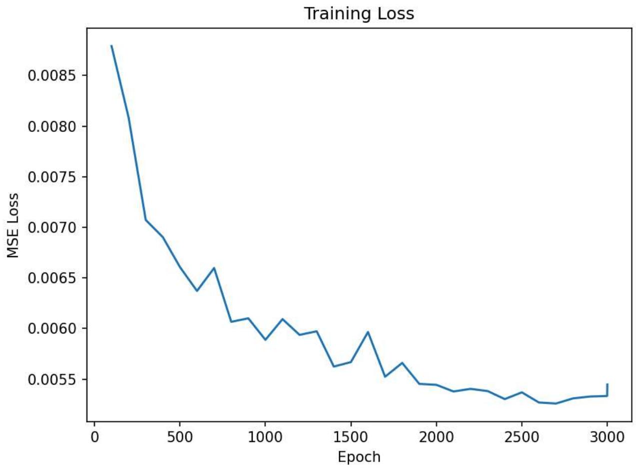
L = 6, width = 256

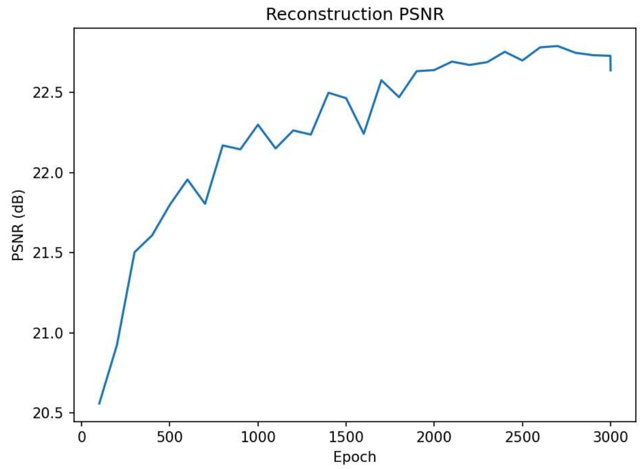


L = 6, width = 128



Part1.4: Train Loss and PSNR Loss





Part2: Fit a Neural Radiance Field from Multi-view Images

Part2.1: Create Rays from Cameras. I implemented three functions to convert between different coordinate systems and generate rays. First, `transform` takes 3D points in camera space, converts them to homogeneous coordinates, and multiplies by the camera-to-world matrix to obtain world-space points. Then, `pixel2camera` maps pixel coordinates `(u, v)` with depth `s` back to camera space using the inverse intrinsic matrix `K⁻¹`, following the pinhole camera model. Finally, `pixel2ray` combines these steps: it converts pixels to camera points, transforms them to world space, uses the camera center (translation part of `c2w`) as the ray origin, and normalizes `Xw - ro` to obtain unit ray directions `rd`. The function returns `(ro, rd)` for each input pixel.

```python
def transform(c2w: torch.Tensor, X_c: torch.Tensor):
    X_c = torch.as_tensor(X_c, device=c2w.device, dtype=c2w.dtype)
    ones = torch.ones_like(X_c[..., :1])
    xc_h = torch.cat([X_c, ones], dim=-1)

    xw_h = (c2w @ xc_h.unsqueeze(-1)).squeeze(-1)

    return xw_h[..., :3]
```

```python
def pixel2camera(K: torch.Tensor, uv: torch.Tensor, s):
    uv = uv.to(dtype=K.dtype, device=K.device)
    s  = torch.as_tensor(s, dtype=K.dtype, device=K.device)

    ones = torch.ones(*uv.shape[:-1], 1, dtype=uv.dtype, device=uv.device)
    uv_h = torch.cat([uv, ones], dim=-1)

    K_inv_T = torch.linalg.inv(K).T
    xc = (uv_h @ K_inv_T) * s[..., None]
    return xc
```

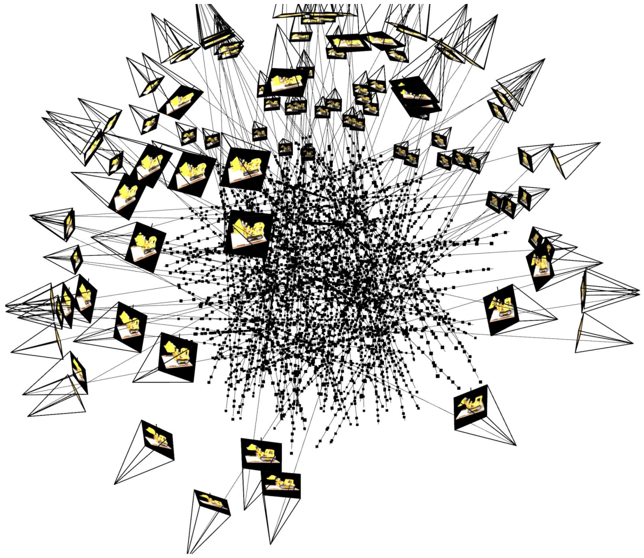
```python
def pixel2ray(K: torch.Tensor, c2w: torch.Tensor, uv: torch.Tensor, s=1.0):
    dev, dt = K.device, K.dtype
    K   = K.to(dev, dt)
    c2w = c2w.to(dev, dt)
    uv  = uv.to(dev, dt)

    Xc = pixel2camera(K, uv, s)
    Xw = transform(c2w, Xc)
    ro = c2w[..., :3, 3]
    rd = (Xw - ro) / (Xw - ro).norm(dim=-1, keepdim=True).clamp_min(1e-12)
    return ro, rd
```

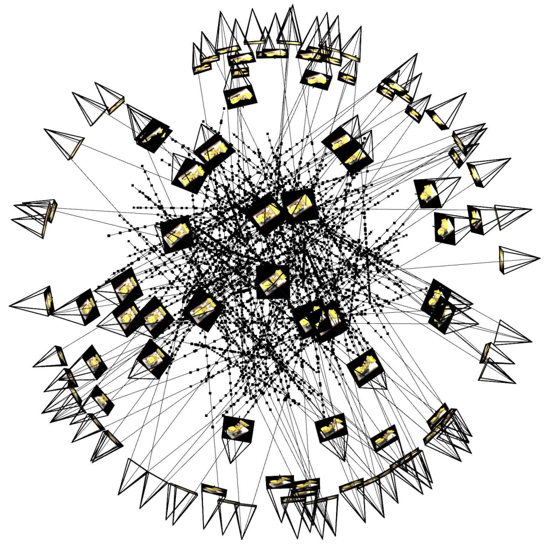
Part2.2: Sampling. I implement two routines, `sample_rays` and `sample_points`, to draw training data from the multi-view images. In `sample_rays`, I first treat all pixels from all training images as a single 1D array and randomly sample N linear indices. Each index is mapped back to an image id and (x, y) pixel coordinate, from which I read the ground-truth RGB and compute sub-pixel centers by adding 0.5 to (x, y). Together with the corresponding camera pose `c2w` and intrinsic matrix `K`, I call `pixel2ray` to obtain the ray origins and directions for those pixels, and normalize the directions to unit length. In `sample_points`, given a batch of rays, I uniformly sample `n_samples` depth values between `near` and `far` using `torch.linspace`, then add a small random jitter within each interval to perform stratified sampling. The 3D sample points are finally computed as `pts = ro + rd * t`. This gives a noisy but uniform set of samples along each ray in 3D space for NeRF training.

```
def sample_rays(self):
    total = self.total_num * self.H * self.W
    idx = torch.randint(0, total, (self.N,), device=self.device)

    img_id = idx // (self.H*self.W)
    pix_id = idx %  (self.H*self.W)
    y = pix_id // self.W
    x = pix_id %  self.W
    rgb = self.train_imgs[img_id, y, x, :]

    uv = torch.stack([x.float() + 0.5, y.float() + 0.5], dim=-1)

    c2w = self.train_c2ws[img_id]


    ro, rd = pixel2ray(self.K, c2w, uv)
    rd = rd / (rd.norm(dim=-1, keepdim=True) + 1e-9)
    ro = ro.to(self.device)
    rd = rd.to(self.device)

    return ro, rd, rgb
```

```
def sample_points(self, ro, rd, n_samples=64, near=2.0, far=6.0):
    device = ro.device
    N = ro.shape[0]
    t = torch.linspace(near, far, n_samples, device=device).repeat(N, 1)
    step = (far - near) / max(n_samples - 1, 1)
    t = t + (torch.rand_like(t) - 0.5) * step
    pts = ro[:, None, :] + rd[:, None, :] * t[:, :, None]
    return pts, t
```

Part2.3: Putting the Dataloading All Together



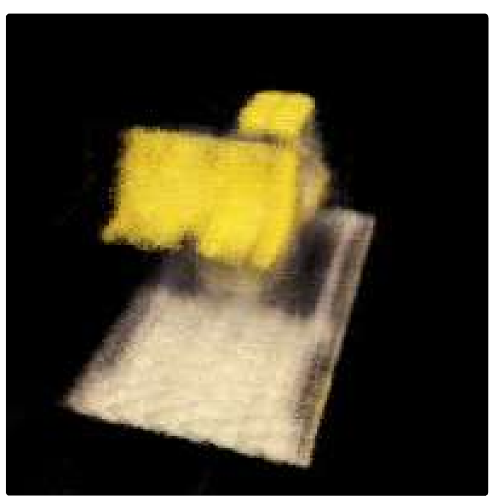


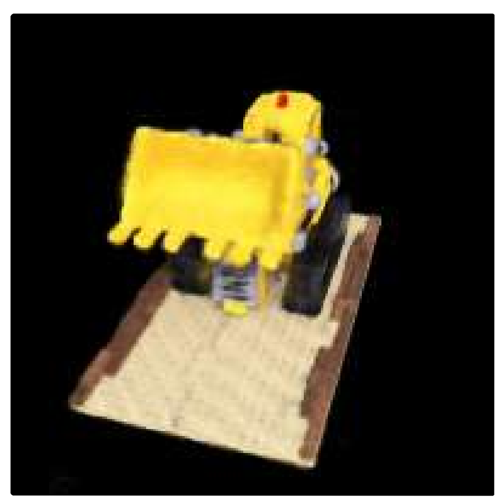
Part2.4: Lego Scene

For Part 2, I build a NeRF-style pipeline. A custom `DataLoader` reads multi-view images and camera poses, then randomly samples 4096 pixels over all training images, converts them to rays with `pixel2ray`, and takes 64 stratified samples between `near = 2.0` and `far = 6.0` along each ray. The MLP uses positional encoding (Lx = 10 for 3D points, Ld = 6 for view directions) and an 8-layer 256-width network with a density head (ReLU) and an RGB head (Sigmoid). I train with Adam (lr = 5×10^{-4}) for 8000 epochs using MSE loss, track train/val PSNR, and finally render all test views to create a spherical rotation GIF of the Lego scene.

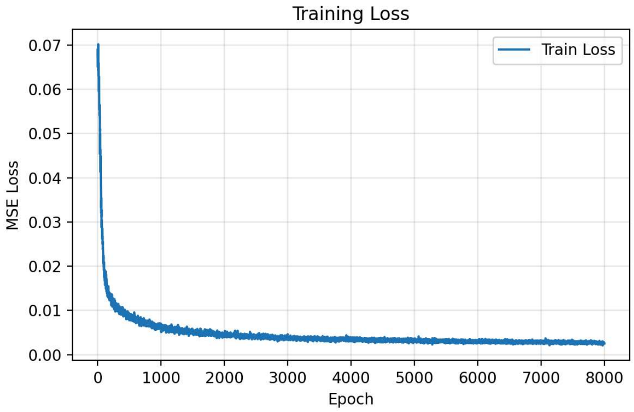
---

left is after 100-epoch-trainning, middle is 500 epochs, right is 8000 epochs





train loss



psnr loss

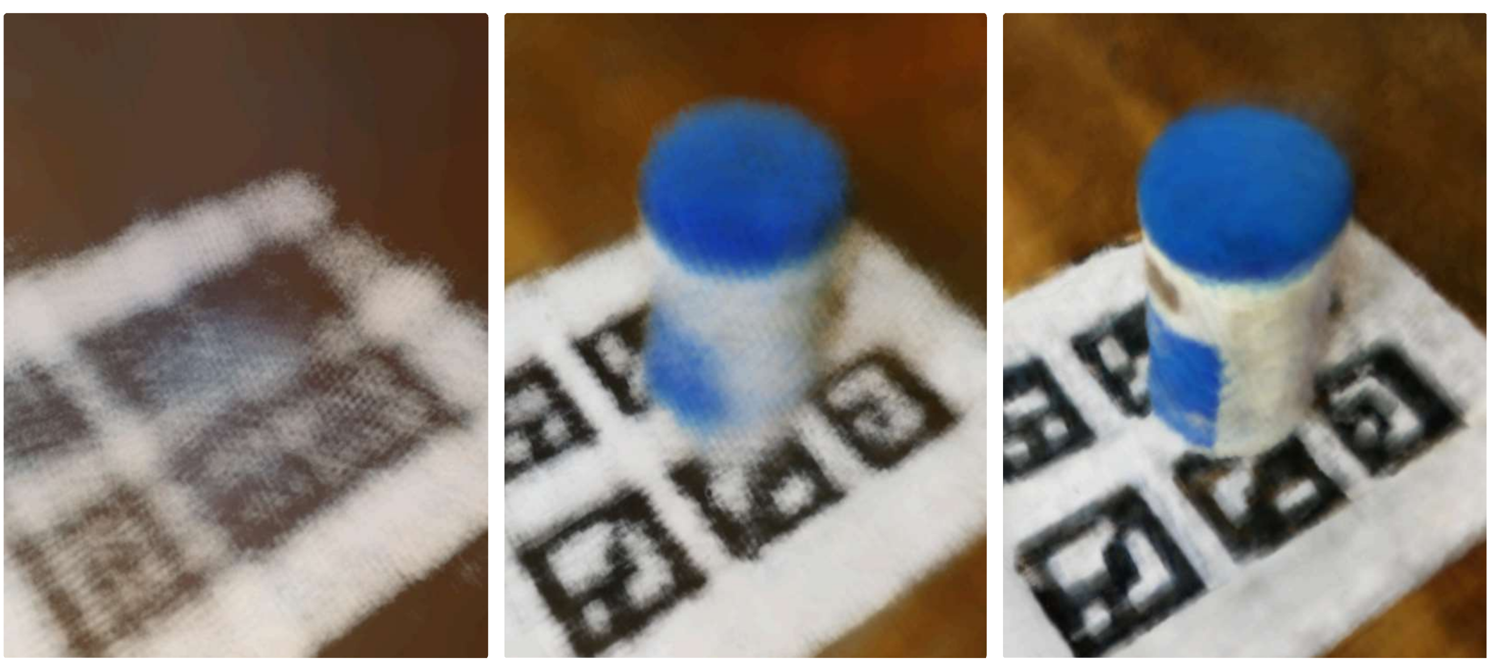
Part2.5: My Own Scene

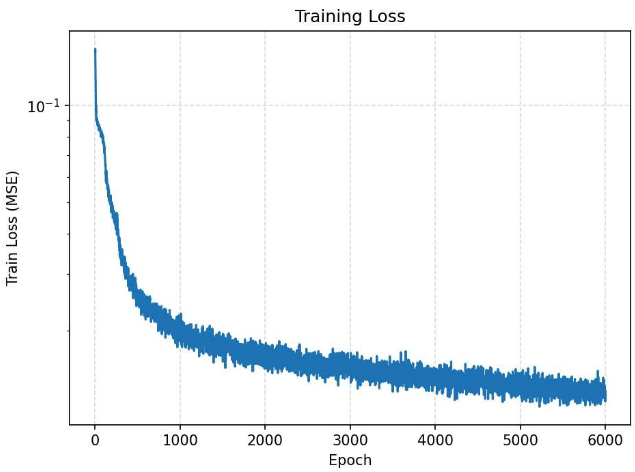
Compared to the original Lego NeRF implementation, this version is more "engineered" and feature-complete. I still use a NeRF-style MLP with positional encoding (Lx = 10 for 3D points, Ld = 4 for view directions) and hidden width 256, a density head and an RGB head. However, the density branch now uses a `softplus` activation instead of ReLU, which keeps densities strictly positive and makes training more stable. Along each ray I sample 96 points between `near = 0.1` and `far = 1.0` with stratified jitter; this higher sample count and tighter depth range improve the quality of the volume rendering. I also introduce a cosine annealing learning rate scheduler and a small weight decay (5×10^{-4} → 5×10^{-6}) for smoother convergence and better generalization.

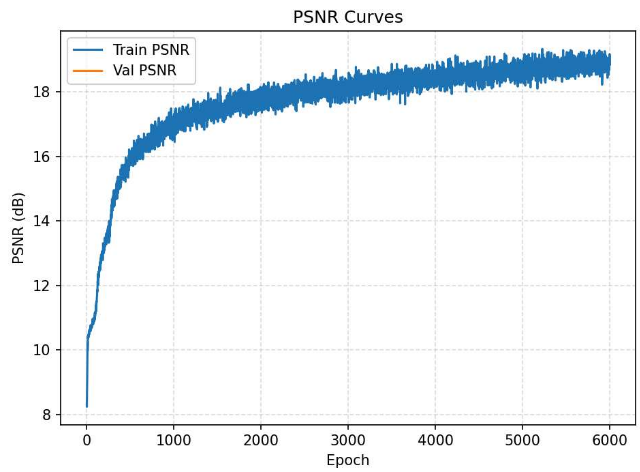
On the training side, I now keep a full `history` dictionary that records epoch, training loss, training PSNR, and validation PSNR. The helper function `save_history_and_plots` exports this data as both `.npz` and `.csv` files and automatically saves loss and PSNR curves to disk, so the whole training process can be inspected and compared across runs. Every 200 epochs I render a validation view using `rays_from_val`, compute PSNR, and save side-by-side comparisons of ground-truth and predicted images. Finally, I save a checkpoint that includes not only the model weights but also metadata such as `K`, image resolution, sampling range, and positional encoding settings. Together with the generic `render_image_from_c2w` function, this allows me to reload the model later and render arbitrary novel views by simply providing a new camera pose, without modifying the training script.

---

left is after 200-epoch-trainning, middle is 600 epochs, right is 6000 epochs




Training Loss
Train Loss (MSE)
10^{-1}
0
1000
2000
3000
4000
5000
6000
Epoch


PSNR Curves
Train PSNR
Val PSNR
PSNR (dB)
8
10
12
14
16
18
0
1000
2000
3000
4000
5000
6000
Epoch