# The kinematic calibration of industrial manipulators using Force/Torque sensor

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Abstract—This document is a model and instructions for LaTeX. This and the IEEEtran.cls file define the components of your paper [title, text, heads, etc.]. \*CRITICAL: Do Not Use Symbols, Special Characters, Footnotes, or Math in Paper Title or Abstract.

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### I. INTRODUCTION

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### II. MODEL AND PROBLEM FORMULATION

The only four unique DH parameters  $\theta_i$ ,  $d_i$ ,  $a_i$ ,  $\alpha_i \in \mathbb{R}$  for each i joint are commonly used to describe the serial chain manipulator kinematic [?]. The transform matrix  $T_{i-1}^i$  contains this parameters and describes transformation from the frame i-1 to i. The forward kinematic solution gives us the end-effector (frame n) transformation relative the base of manipulator frame 0, using multiplication of transform matrices  $i=\overline{1,n}$  we get this solution

$$T_{i-1}^{i} = \begin{bmatrix} R_{i-1}^{i} & o_{i_{1}}^{i} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{\theta_{i}} & -s_{\theta_{i}}c_{\alpha_{i}} & s_{\theta_{i}}s_{\alpha_{i}} & a_{i}c_{\theta_{i}} \\ s_{\theta_{i}} & c_{\theta_{i}}c_{\alpha_{i}} & -c_{\theta_{i}}s_{\alpha_{i}} & a_{i}s_{\theta_{i}} \\ 0 & s_{\alpha_{i}} & c_{\alpha_{i}} & d_{i} \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$T_0^n = \prod_{i=1}^n T_{i-1}^i,$$
 where  $c_{(\cdot)} = \cos(\cdot)$  and  $s_{(\cdot)} = \sin(\cdot)$ , see Fig.  $\ref{eq:sin}$ . 
$$z_{i-1}$$
 DH\_Frames.pdf 
$$\theta_i \qquad z_i$$
 
$$d_i \qquad a_i \qquad \alpha_i$$
 
$$z_{i-1} \qquad x_i$$
 
$$y_{i-1}$$
 
$$x_{i-1}$$

Fig. 1. DH convention visualization

Let us consider the revolute joint serial manipulator with Force/Torque sensor (Fig. ??). Relation between force and joint torques

$$\tau = J^T \mathcal{F}, \qquad \qquad \mathcal{F} = \begin{bmatrix} F_e \\ \tau_e \end{bmatrix}, \qquad \qquad \boxed{(2)}$$

where  $\tau_e, F_e \in \mathbb{R}^3$  is the torque and force applied to endeffector that combined in the generalized force vector  $\mathcal{F}$ ,  $\tau \in \mathbb{R}^n$  in the joint torque vector and  $J \in \mathbb{R}^{6 \times 6}$  is manipulator Jacobian.

Manipulator\_with\_FTsensor.pdf 
$$\begin{array}{c} \text{Force/Torque} \\ \text{sensor} \end{array}$$
 
$$\begin{array}{c} \text{object} \\ \\ \tau_e \\ \\ F_e \end{array}$$

Fig. 2. End-effector scheme

In this work we solve the **kinematic model DH parameters estimation problem**. Consider the serial manipulator with revolute joint, the forward kinematic solution (??) and the force - torque relation (??). Define the unknown parameters vector  $p_i$  for each joint  $i = \overline{1,n}$  that combined in the vector  $p_i$ 

$$p_i = \begin{bmatrix} d_i \\ a_i \end{bmatrix}, \qquad P = \begin{bmatrix} p_0 \\ p_1 \\ \vdots \\ p_n \end{bmatrix}, \qquad i = \overline{1, n}.$$
 (3)

Design the estimator

$$\hat{P} = f(\mathcal{F}, \tau, \theta, \alpha) \tag{4}$$

such that

(1)

$$\lim_{t \to \infty} P - \hat{P} = 0 \tag{5}$$

As usual to design the estimator we need the following assumptions

Assumption 1: The only measurable signals are  $\mathcal{F}$ ,  $\tau$ ,  $\theta$ , the  $\alpha$  parameter is known

\*\*Problem of the section above

Say that we estimate only 
$$d_i$$
 and  $a_i$ 

$$\text{Define the vectors } \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}, \ \alpha = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix}$$

## III. KINEMATIC PARAMETERS ESTIMATION

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## IV. SIMULATION AND EXPERIMENTAL RESULTS

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# V. CONCLUSION AND FUTURE WORK

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