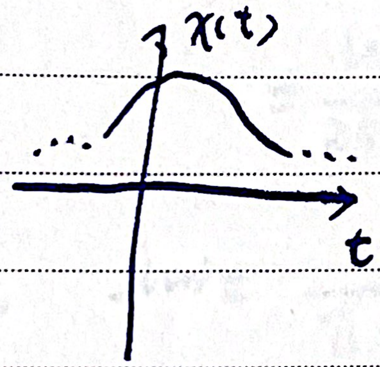
前置知识:

1. Sampling:

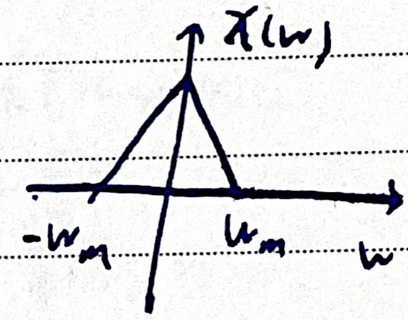
CTS ⇄ DTS

continuous time signals　　discrete time signals

2. band-limited:

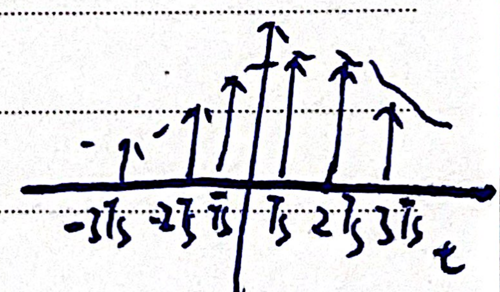


fourier $\longrightarrow$

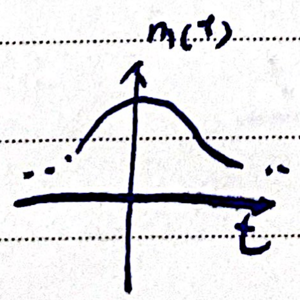


fourier 变换后，信号在有限频率内

$s(t) = m(t) \cdot c(t)$

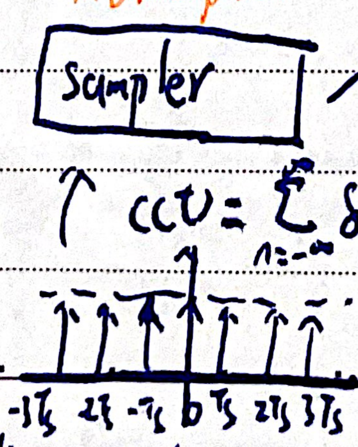


3. Sample basic concept:



$\longrightarrow$

multitiplier



$c(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$

T_s = sampling period, $w_s = \frac{2\pi}{T_s}$ = sampling freq.

4. sampling theory:

S M T W T F S

NO.

DATE / /

$$s(t) \rightleftarrows S(\omega)$$

$$s(t) = m(t)\cdot c(t)$$

$\downarrow$ FT

convolution

$$S(\omega) = \frac{1}{2\pi}\left[M(\omega) * C(\omega)\right]$$

Also, $C(\omega) = \omega_s \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s)$

So:

$$S(\omega) = \frac{1}{2\pi}\left[M(\omega) * \omega_s \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s)\right]$$

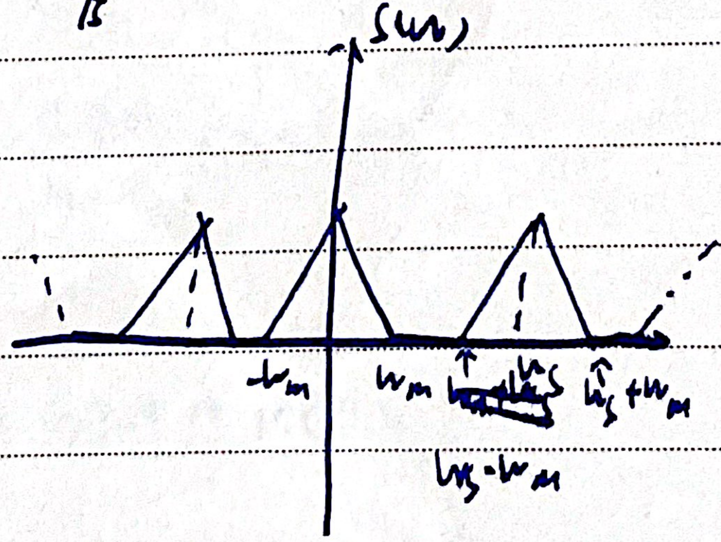
$$= \frac{\omega_s}{2\pi}\left[M(\omega) * \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s)\right]$$

$$\Rightarrow \quad S(\omega) = \frac{1}{T_s}\left[\sum_{n=-\infty}^{\infty} M(\omega) * \delta(\omega - n\omega_s)\right]$$

$$= \frac{1}{T_s}\left[\sum_{n=-\infty}^{\infty} M(\omega - n\omega_s)\right]$$

$$= \cancel{\frac{1}{T_s}}$$

$S(\omega)$:



采样定理

So we need: $\omega_s - \omega_m > \omega_m \longrightarrow \omega_s > 2\omega_m$

S M T W T F S

NO.

DATE / /

5. Poisson Process

泊松过程是一系列离散事件的模型，其中平均时间事件之间是已知的，但事件之~~间~~的确切时间是随机的，且事件到来与之前事件无关。

Abstract:

1. 事件相互独立
2. 平均速率恒定
3. 两个事件不能同时发生

→ 许多建模为 Poisson Process 的真实不符合标准

经典的例子可以是 $\boxed{\text{流星}}$

泊松过程 → 泊松分布

$\boxed{\text{泊松过程下，一段时间内多个事件的概率}}$

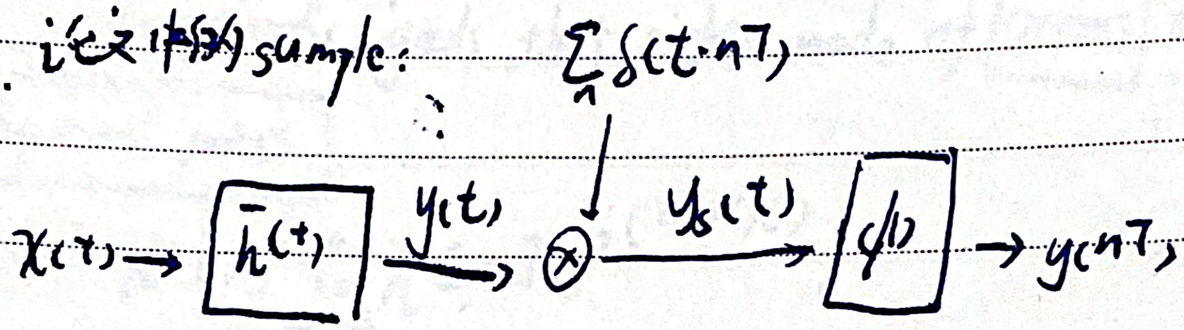
$$P(k \text{ events in time period}) = e^{-\frac{\text{events}}{\text{time}} * \text{time period}} * \frac{\left(\frac{\text{events}}{\text{time}} * \text{time period}\right)^k}{k!}$$

简化：$\frac{\text{events}}{\text{time}} * \text{time period} = \lambda$ 速率参数，rate parameter

$$P(k \text{ events in interval}) = e^{-\lambda}\frac{\lambda^k}{k!}$$

论文剖析:

1. 论文中的sample:



跟前面没区别,只是多了个 smoothing kernal $\bar{h}(t)$

$$y_s(t)=\sum_{n\in Z} y(nT)\,\delta(t-nT)$$

2. finite rate of innovation.

单位时间的自由度数称为信号的创新率,用ρ表示.

在bandlimited的信号下, $\rho=1/T=\omega_m/\pi$

3. 有新息率的信号:

$$x(t)=\sum_{n\in Z} c_n\,\varphi\left(\frac{t-nT}{T}\right)$$

$$x(t)=\sum_{n\in Z} c_n\,\varphi\left(\frac{t-t_n}{T}\right)$$, allow arbitrary shifts

~~when $c_n=1$, $\varphi(t)=\delta(t)$, $T=1/u$, and t_n-t_{n+1} are i.i.d. with~~

$$x(t)=\sum_{n\in Z}\sum_{r=0}^{R} c_{nr}\,\varphi_r\left(\frac{t-t_n}{T}\right)$$

$$\rho=\lim_{\tau\to\infty}\frac{1}{\tau}C_x\left(-\frac{\tau}{2},\frac{\tau}{2}\right)$$

计算自由度的函数

4. 例子

τ-periodic signals that are either made of $\boxed{\text{Diracs}}$ or $\boxed{\text{polynomial pieces}}$.

$$x(t)=\sum_{m\in\mathbb{Z}} x[m]\,e^{j(2\pi m t/\tau)},$$

~~A. Diracs~~

A. 狄拉克流
B. 非均匀样条曲线 } → 有精确推导见论文
C. 分段多项式

有点复杂，我从另篇文献上调查了一下：

<u>FRI信号指能够用有限参数确定的信号</u>，比如上面A,B,C

采样过程和前面一样，重点就是多一个 smoothing kernel.

然后，重建 FRI信号没有统一的方法，要根据信号和采样核函数重建.