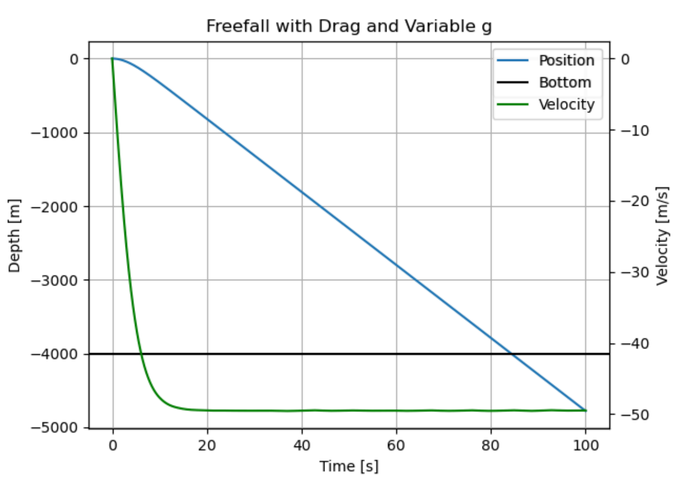
Report on the Depth Measurement using test mass drop

I. Introduction

This report is a summary of an investigation made into the feasibility of measuring the depth of a vertical mineshaft by dropping a test mass and measuring the time it takes to reach the bottom. This method, while initially appealing due to its simplicity, presents certain challenges and limitations that will make such a test rather difficult. Various factors come into play that, when accounted for, complicate the picture drastically. When looking at this experiment, we realise that two major factors affect the viability of this experiment. Firstly, and most importantly, would be the effect of drag on the fall. Making certain assumptions about the air in the mine shaft, as well as the shape of the mass dropped, we see that drag creates a very significant impact on the time for the mass to fall. Secondly, due to the Earth's rotation, a coriolis sideways force is exerted on any object travelling downwards, potentially causing the mass to hit the side of the mineshaft. Together these two factors add a large amount of uncertainty in calculating the distance of the fall. This report aims to model these effects using python, to solve and then graphically display the different models and the effects of different factors on the freefall.

II. Calculating fall time.

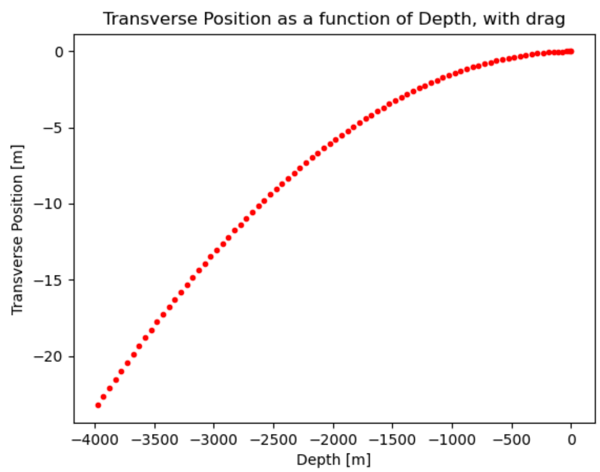
Freefall, under a constant gravitational field and no drag, is governed by a simple differential equation: $\frac{d^2y}{dy^2} = g$ where $\frac{d^2y}{dy^2}$ is the second derivative of the position of the test mass, which is just its acceleration. Solving this differential equation and inputting g = 9.81 and depth y = 4000m gives us a direct output of 28.6 seconds, which is confirmed by our python model, giving us the same time of 28.6 seconds. We understand that as one goes deeper into the Earth, the gravitational acceleration decreases slightly due to the mass of the Earth "under" the test mass decreasing. Accounting for this the free fall time increases by 0.002 seconds. This gets complicated, however, when including the atmospheric drag. This would add a drag term to our above equation, making it $\frac{d^2y}{dy^2} = g \ - \alpha v^{\gamma}$ where α and γ are determined by atmospheric conditions and the shape of the object. Upon adding this factor, its importance becomes apparent. Here $\gamma = \ 2$ and



$\alpha = 0.004$ are taken, which are reasonable assumptions to make. The above figure illustrates the time taken for the mass to fall to the bottom, with 0 being the surface of the Earth and -4000m being the bottom of the mine, with the black line representing the bottom of the mine. Further we can see that the velocity drops drastically at first and then abruptly becomes constant due to the drag resistance. When factoring in both drag and variable g, we get the time for the test mass to reach the bottom to be 84.3 seconds. When using the free fall method, the drag force on the mass would thus have to be taken into account.

III. Coriolis force and feasibility of method

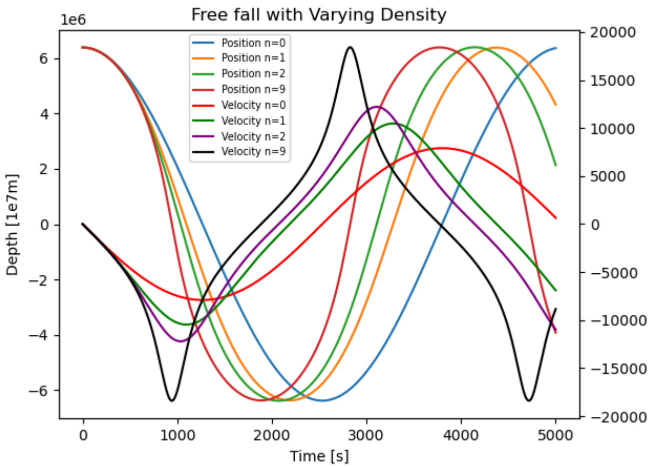
As highlighted earlier, a coriolis force due to the rotation of the Earth would cause the mass to have some sideways velocity and potentially cause it to hit the side of the mineshaft. This can be modelled using the $\frac{d^2x}{dx^2} = 2\Omega v_y$ where v_y is the downward velocity of the mass towards the centre. Solving this equation gives us the following graph, which displays the transverse position as a function of the depth, since as the depth and time increases, so does the transverse position. Here the mass begins at the top right at the top with (0,0) meaning that the mass is at the surface of the Earth and at the East-most point in the shaft, since the coriolis force acts towards the west in the northern hemisphere. This graph and our calculations tell us that the mass will hit the side of the mineshaft before it reaches the bottom. More precisely, the mass will hit the side of the shaft after 40.7 seconds at a depth of 1827.5m below the surface. This highly impacts the feasibility of this method, since the mass hitting the side of the shaft will considerably slow down its velocity in an unpredictable manner. From this result, it is unadvisable to use this method to measure the depth of the mineshaft.



IV. Crossing time

As an exercise, this report also considered the time it may take for a test mass to fall all the way to the other side of the Earth and back, as well as the same for the moon. This highlighted the variation of gravitational acceleration, not due to the proportional mass of Earth under the test mass decreasing, but rather due to the non-homogenous nature of the Earth, and its variable density. A simple model is that of increasing density as you get deeper, due to the pressure increasing. This model is governed by the equation $M = 4\pi\rho_n \int_0^R (1 - \frac{r}{R})^n r^2 dr$ where M is the mass of the Earth,

ρ_n is a normalising constant, n dictates the increase in density with depth, R is the radius of the Earth, and r is the radius measured from the centre, as opposed to the depth measured from the surface, as we were doing earlier. For a range of n values, [0, 1, 2, 9] this produces a drastic difference in the crossing times. For n = 0, which is the case where the Earth is assumed to be homogenous, we get the crossing time to be



2533.1 seconds, or about 42 minutes. In comparison, when assuming a very drastic density curve, with n = 9 we get the crossing time to be 1886.2 seconds, nearly half the time as n = 0. This shows the drastic impact of the density of the Earth on the free fall time. Assuming n = 2, which is close to the real density curve of the Earth, we get a crossing time of 2189.0 seconds, or about 36 minutes.

Seeing as density is such an important factor, we also calculated the crossing time for the Moon, which has a density about 1/6th of that of the Earth. Looking at the equations for gravity, this tells us that the crossing time for any body is inversely proportional to the root of its density. Doing some calculations gives us the crossing time for the moon which is about 1464.9 seconds. The ratio of the crossing times of the Earth and the Moon is 1.16, and the ratio of the square root of the densities, times the ratio of the Radii, is 1.5, which shows the relationship of crossing time with the density.

V. Discussion and Future Work

In summary, the free fall time, without drag would be about 28.6 seconds, however upon factoring in drag, it rises to 84.3 seconds. For the case of an infinitely long mineshaft, the crossing time is around 2533.1 seconds for a homogenous Earth, 1886.2 for n =9, and 2189 seconds for n = 2 which seems to be the most reasonable estimate. While this report takes into account many of the factors that affect this method, there is still more that can be done to increase the accuracy of the above calculations. Some of the assumptions made greatly increase the uncertainty of the calculations. Firstly, when determining the effect of drag on the object, it was assumed that alpha = 0.004. While this assumption is a reasonable estimate, it can be greatly improved upon by determining the real drag coefficient of the mass, using its cross sectional area and its shape. As seen in the beginning, drag has a very large effect on the free fall time, and a better estimate of the drag factors will greatly improve the accuracy of the free fall time. Secondly, a better estimate of the n value of the Earth, will give a much better result of the Mass curve. Thirdly, it was assumed that the Earth is a sphere in getting a variable g. However, in reality, the Earth is an oblate sphere, meaning it is wider at the radius than the poles. This not only affects the variable gravity, but also affects the density approximations. These and more areas can be improved upon in order to understand the free fall of a test mass better.