Apollo 11 Preliminary Report

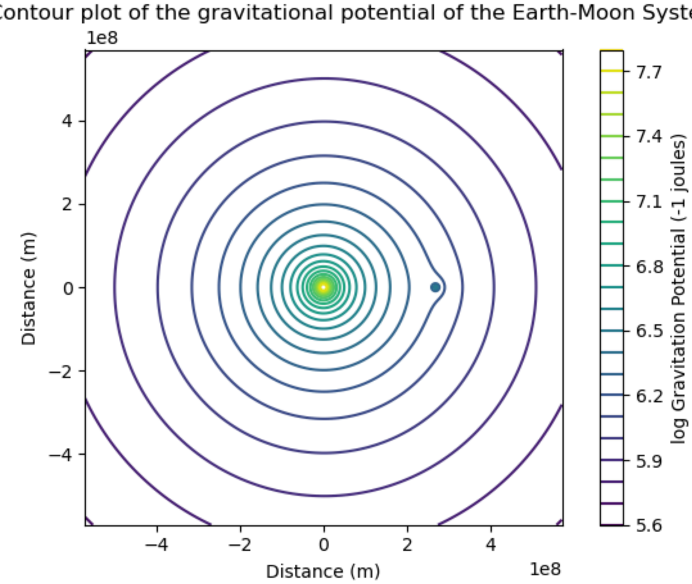
I. Introduction

The Apollo 11 mission to the Moon is a very important, but expensive endeavour that requires great funding from the United States' government. The following report highlights the current progress of research and it's key findings with regards to some of the key physics that goes into this mission. All of the mathematics and research was done using the programming language Python and some key libraries like Matplotlib and Numpy, which are libraries that allow for plotting of graphs and mathematical functions respectively. There are three key areas that this report wants to highlight: 1. The gravitational potential of the Earth-Moon system, 2. The gravitational force of the Earth-Moon system, and 3. The performance of the Saturn V rocket's first stage. These three areas are the most important and it is paramount that their understanding is very solid.

II. The Gravitational Potential of the Earth-Moon system

The potential energy of a system is simply the amount of energy it would require to do a certain task. In our case, it means the amount of energy that it would require to lift a massive object like the Saturn V rocket up to the Moon, while working against gravity. Since both the Earth is a very massive object, its gravitational potential energy outstrips that of the Moon by a very large amount, making it very difficult to use the potential of the Moon to pull the rocket towards it, unless it gets very close to it. The following figure is a contour-plot of the gravitational potential of the combined Earth-Moon system that the Saturn V will traverse through.

Fig 1. Contour plot of the gravitational potential of the Earth-Moon System

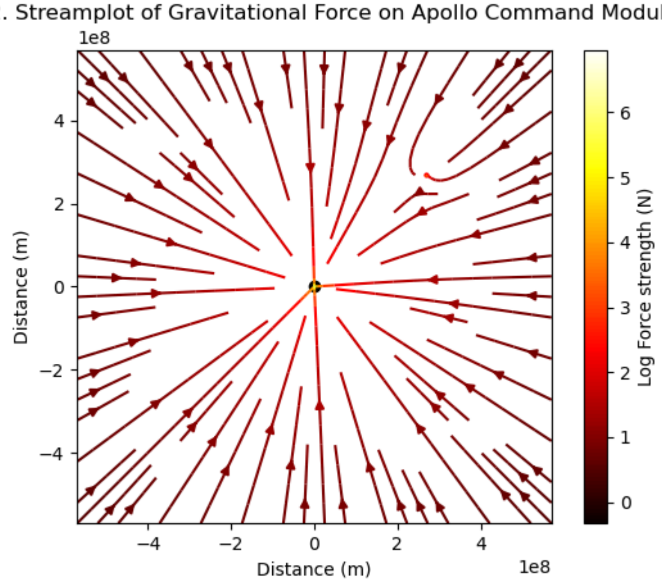


Here each line represents a constant level of potential energy, colour coded to the level of energy. Since this is a logarithmic scale the bar on the right must be read as -1 X 10^x, so that the dark purple line represents -1*10^5.6 Joules. As seen in it, the constant levels of potential form concentric circles around both the Earth and the Moon. This allows for a clever trick, wherein if we are able to expend enough energy to allow the rocket to reach a certain level, and then angle it such that it's velocity would be horizontal, it would enter a stable orbit around the Earth. At this orbit it could then use the gravitational potential energy of the Earth to "slingshot" itself towards the Moon, with just some well timed thrusts. Once closer to the Moon, the thrusters could power back up to slow the rocket down and bring it into a stable orbit around the Moon from which the Lunar Module would descend to the surface of the Moon. To understand this "slingshot" maneuver it would be very useful to understand the Gravitational Forces of the Earth-Moon system.

III. The Gravitational Force of the Earth-Moon system

Since large bodies like the Earth and the Moon impart energy onto bodies near them, they exert certain forces onto them. This force in the case of gravity is a "pulling" force that pulls things near to its center of gravity. This force can be used to move the Saturn V rocket great distances without expending much energy. As shown by the following diagram, the forces of the Earth's gravity, depicted by the arrows, are constantly pointing inwards with increasing magnitude the closer you get to the Earth, described by their colour, corresponding to the colour-bar on the right.

Fig 2. Streamplot of Gravitational Force on Apollo Command Module



This colour-bar is in a logarithmic scale as well and must be read the same way as described for Fig1. above. If a body with a high enough velocity passes by the Earth, the forces have an interesting effect, wherein the momentum of the body causes it to "miss" the Earth constantly and make it so that it is in a constant state of "falling" towards the Earth. This state is called an orbit, which is what it hoped to achieve through the first stage of the Saturn V rocket. Furthermore, this graph also shows a very interesting effect. The forces around the Moon curve inward towards it making a body near it take a curved path. This can be very useful since the gravitational forces of the Earth and the Moon together make it so that the Saturn V rocket will have some horizontal velocity, making the energy required to put the rocket into orbit just a little lower. While the graph above is made through python, and thus only serves as an approximations due to both, the limits of the computational and graphing capabilities of python, as well as certain simplifying assumptions taken which will be explained later, it nonetheless gives a good idea of how the system will behave.

IV. Projected performance of the Saturn V Stage 1

The Saturn V Stage 1 is set to be the most powerful rocket ever built by mankind and the number so far corroborates that story. Recent tests done show that the rocket will burn through its first and largest fuel tank in about 160 seconds, allowing the rocket to reach a velocity of up to about 1615 meters per second which is more than Mach 5, or 5 times the speed of sound in air. This will take the rocket to an estimated height of 70 km as per physical tests. This result is very positive since our calculations of maximum efficiency show that the rocket will burn out all of its fuel in ~157.69 seconds, and would reach a maximum height of ~74 km meaning that the rocket is very close to its peak capability when considering a situation of maximum efficiency. Certainly these figures calculated have some level of error built into them, since arithmetic with Numpy, and Scipy, work through methods that only allow a certain amount of precision. But regardless, this is a positive result.

V. Discussion and Future Work

To close out, there are a few assumptions and notes that need to be kept in mind while considering this report. Firstly the above graphs are all approximations to illustrate the more complicated situation. Python has many limitations that make it difficult to have a perfectly accurate picture of the mission, however it is an incredible tool that allows us to simplify and portray the mission, while maintaining a level of accuracy that ensures that any conclusions made are good. Secondly there were a few simplifying assumptions made that allowed us to compute numbers, without overcomplicating the calculations. The first is with regards to the burntime of the Saturn V rocket, which is the time it would take for all the fuel to burn. Our calculations give us a burntime of 157.69 seconds, as stated above. However physical tests put this to 160 seconds. This is understandable, since one of the assumptions made was that the exhaust velocity of the fuel is constantly at its peak of 2.4 x 10^3 m/s from the very beginning. In reality however, it would take the rocket some time to get up to that exhaust velocity, making the burntime slightly longer than the calculation. The second assumption is regarding the altitude of the Saturn V upon burnout. Our calculations give us a result that is 4 km greater than the ones using the data from the actual tests. This is because the assumption was made that there would be no drag force from the atmosphere. In reality it would be a significant force due to the high velocity of the rocket, meaning that a 70 km height is rather in line with the calculations.

This report is still an incomplete picture of the Apollo 11 Mission, since it does not talk about the second and third stages of the Saturn V booster. The first stage only takes the rocket into its first Earth orbit. Further work needs to be done to fully understand and illustrate the stages that will allow the rocket to perform the “slingshot” maneuver that will take the Command Module to the Moon.