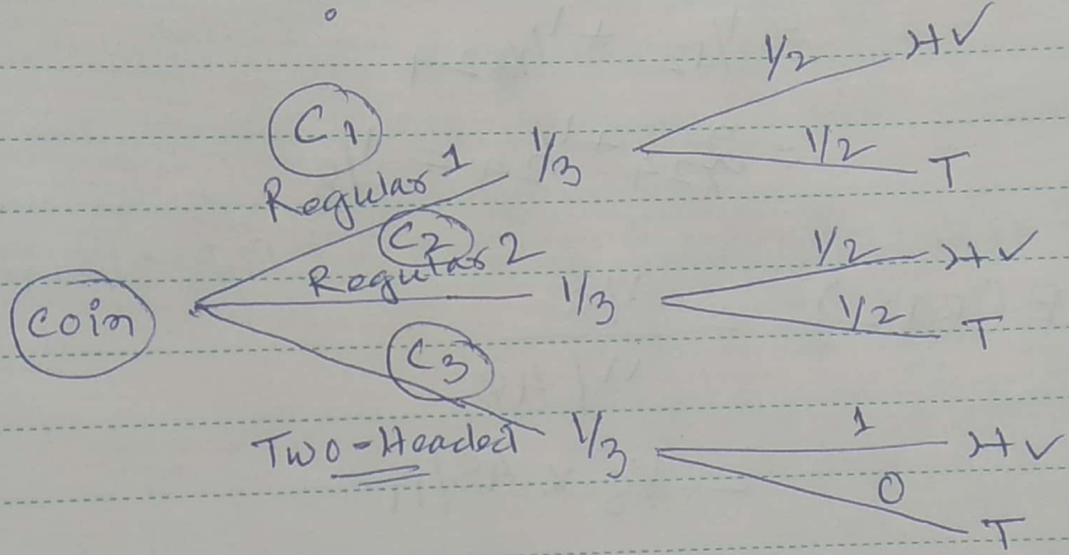
*6. A box contains three coins: two regular coins and fake two-headed coin ($P(\text{Heads}) = 1$), You pick a coin at random and toss it.

a) What is the prob. that it lands heads up?

b) You pick a coin at random and toss it and get heads, what is the prob. that it is the two headed coin?



a) $P(H) = P(H|C_1)P(C_1) + P(H|C_2)P(C_2) + P(H|C_3)P(C_3)$

$= \frac{1}{2} \times \frac{1}{3} + \frac{1}{2} \times \frac{1}{3} + 1 \times \frac{1}{3}$

$= \frac{1}{6} + \frac{1}{6} + \frac{1}{3} \cdot \frac{2}{2} = \frac{2}{3}$

b) P(Two-Headed | Head)

$$= \frac{P(\text{Head} \mid \text{Two Headed}) \times P(\text{Two-Headed})}{P(\text{Head})}$$

$$= \frac{1 \times 1/3}{2/3}$$

$= 1/2$ 4

Scanned with CamScanner

#7. Suppose that, of all the customers at a coffee shop,

a) 70% purchase a cup of coffee

b) 40% purchase a piece of cake.

c) 20% purchase both a cup of coffee and a piece of cake. Given that a randomly chosen customer has purchased a piece of cake, what is the prob. that he/she also purchased a cup of coffee.

$$P(\text{Coffee}) = 0.7$$

$$P(\text{cake}) = 0.4$$

$$P(\text{Coffee} \cap \text{cake}) = 0.2$$

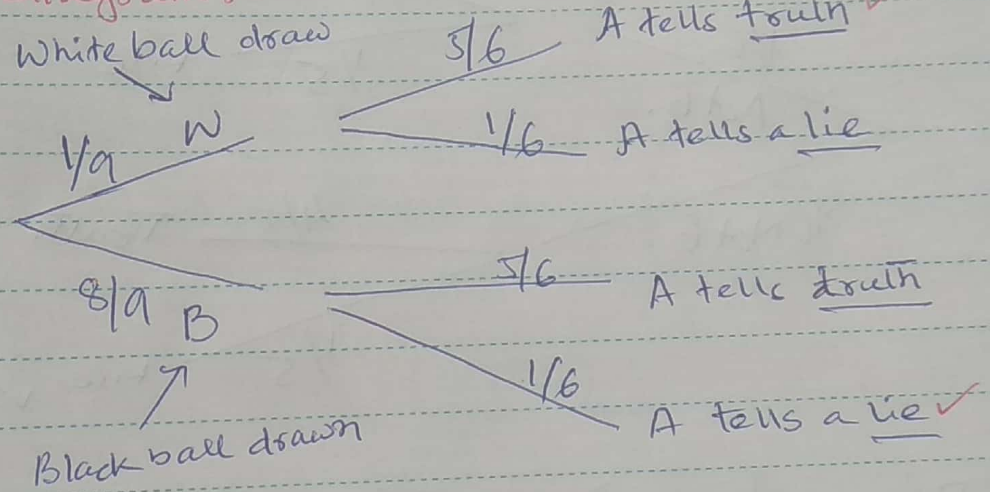
$$P(\text{Coffee}/\text{cake}) = \frac{P(\text{coffee} \cap \text{Cake})}{P(\text{cake})}$$

$$= \frac{0.2}{0.4}$$

$$= \frac{1}{2}$$

*8. A is known to tell the truth in 5 cases out of 6 and he states that a white ball was drawn from a bag containing 8 blacks and 1 white ball. find the prob. that the white ball was drawn.

Tree diagram:-



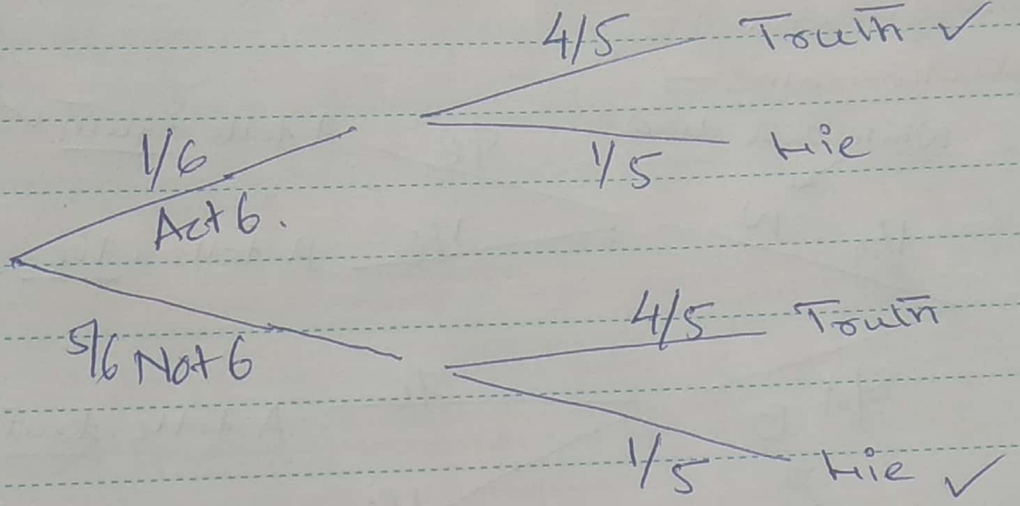
$$P(W/\text{Truth}) = \frac{P(\text{Truth}/W) \times P(W)}{P(\text{Truth})}$$

$$= \frac{P(\text{Truth}/W) \times P(W)}{P(\text{Truth}/W) \times P(W) + P(\text{Lie}/B) \times P(B)}$$

$$= \frac{5/6 \times 1/9}{(5/6 \times 1/9) \times (1/6 \times 8/9)}$$

$$= 5/13.$$

*9. A speaks the truth 4 out of 5 times. A die is tossed. A report that it is a 6. What are the chances that there actually was a 6?



$$P(\text{Act}6 \mid \text{Truth}) = \frac{P(\text{Truth} \mid \text{Act}6) \times P(\text{Act}6)}{P(\text{Truth})}$$

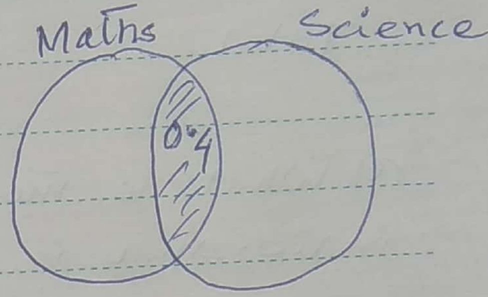
$$= \frac{P(\text{Truth} \mid \text{Act}6) \times P(\text{Act}6)}{P(\text{Truth} \mid \text{Act}6) \times P(\text{Act}6) + P(\text{Lie} \mid \text{Not}6) \times P(\text{Not}6)}$$

$$= \frac{4/5 \times 1/6}{(4/5 \times 1/6) + (1/5 \times 5/6)}$$

$$= \frac{4}{9}$$

10:-

* In a class, 40% of the student's study math and science. 60% of the students study math. What is the prob. of a student studying science given he/she is already studying math?



$$P(M) = 0.6$$
$$P(M \cap S) = 0.4$$

$$P(S/M) = \frac{P(M \cap S)}{P(M)}$$

$$= \frac{0.4}{0.6}$$

$$= \frac{0.2}{0.3}$$

$$= 0.67$$

※ 11. Below is a table of graduates and post graduates,—

	Graduate	Post Graduate	Total
Male	19	41	60
female	12	28	40
Total	31	69	100

a) What is the prob. that a randomly selected individual is a male and a graduate? What kind of prob. is this? (Marginal / Joint / Conditional)

b) What is the prob. that a randomly selected individual is a male?

c) What is the prob. of a randomly selected individual being a graduate? What kind of prob. is this?

d) What is the prob. that a randomly selected person is a female given that the selected person is a post graduate? What kind of prob. is this?

Scanned with CamScanner

a) $P(\text{Male} \cap \text{Graduate}) = P(\text{Male}/\text{Graduate}) \times P(\text{Graduate})$

$= 19/31 \times 31/100$

$= 19/100$

$= 0.19$

This is a Joint Probability.

b) $P(\text{Male}) = \dfrac{\text{~~\#~~ of fav. outcomes}}{\text{All possible outcomes.}}$

$= 60/100 = 0.6$

c) $P(\text{Graduate}) = 31/100$

$= 0.31$

This is a marginal probability.

d) $P(\text{female}/PG) = \dfrac{P(\text{female} \cap PG)}{P(PG)}$

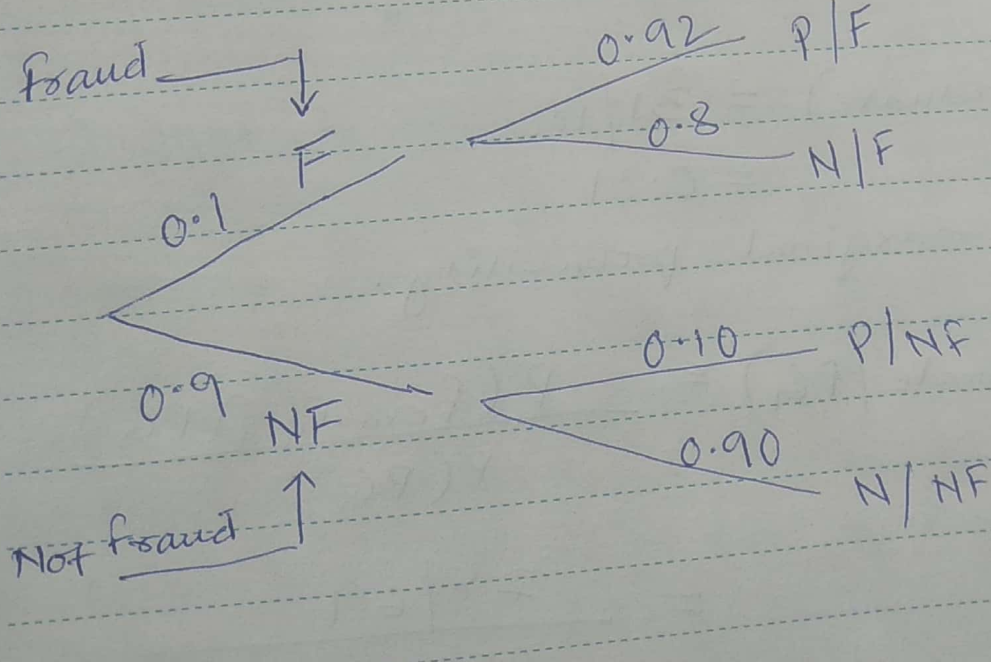
$= \dfrac{28/69}{69/100}$

$= 7/25 = 0.28$

This is a conditional Probability.

Bayes Theorem :-

*12. You need to figure out whether a company is fraud based on the legal charges they filed. We have the knowledge that, the chances a company submitting fraudulent fillings is 0.1. There exists an algorithm that can predict fraud. This algorithm returns a correct positive result in 92% of the cases in which the fraud is present and correct negative results in 90% of the cases where the fraud is not present. Suppose we observe a company for whom the algorithm test returns a fraud result. Calculate the posterior prob. that this company truly did fraud in their fillings.



P(Fraud | Positive)

$$P(F/P) = \frac{P(P|F) \times P(F)}{P(P)}$$

$$= \frac{P(P|F) \times P(F)}{P(P|F) \times P(F) + P(P|NF) \times P(NF)}$$

$$= \frac{0.92 \times 0.1}{(0.92 \times 0.1) + (0.10 \times 0.9)}$$

$$= \frac{0.092}{0.092 + 0.09}$$

$$= 0.505 = 51\%$$