Assignment On Central Limit Theorem.

1. Let X be a random variable with $\mu = 10$ and $\sigma = 4$. A sample of size 100 is taken from this population. Find the probability that the sample mean of these 100 observations is less than 9.

Given,

$\mu_{pop} = 10$

Hence, $\mu_{SD} \simeq 10$

$$\sigma_{SD} = \frac{\sigma_{pop}}{\sqrt{N}}$$

$$= \frac{4}{\sqrt{100}}$$

$$P(\bar{x} < 9) = P\left(Z < \frac{9-10}{\frac{4}{\sqrt{100}}}\right)$$

$$= P\left(Z < -\frac{(\sqrt{100})}{4}\right)$$

$$= P(Z < -2.5)$$

$$= 0.0062 \quad \{\Rightarrow \text{from Z table}\}$$

$$\approx 0.6\%$$

*2. The Data Science classes are being conducted at 8th floor of a building and students must use lift to reach there (No bonus for stairs climbing!). An elevator can transport a maximum of 550 Kgs. Based on overall students data, it has shown that the weight of students follows a distribution with mean 50 Kgs and standard deviation 15. Suppose a team of 10 students waiting for lift to reach the 8th floor via the elevator. Based on this information, what is the prob. that all 10 students can be safely reach the 8th floor?

Here,

$n = 10$

$\mu_{SD} = 50$

$\mu_{pop} = 50 \times 10$

$\sigma_{SD} = 15$

$\sigma_{pop} = 15 \times \sqrt{10}$

$$Z = \frac{X - \mu_{pop}}{\sigma_{pop}}$$

$$= \frac{550 - (50 \times 10)}{15 \times \sqrt{10}} = \frac{50}{47.43} = 1.05$$

from Z-table = 0.8531

$\approx 85\%$

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3. from past experience, it is known that the number of tickets purchased by a passenger travelling from Hyderabad to Delhi in AP Express from IRCTC website on TATKAL Quota follows a distribution that has mean $\mu = 2.4$ and sample standard deviation $\sigma = 2.0$. Suppose that before the start of booking the tickets there are 100 eager passengers logged into the website to purchase tickets. If only 250 tickets remain, what is the prob. that 100 passengers will be able to purchase all the 250 tickets for their journey.

Here,

$\mu_{SD} = 2.4$

$\sigma_{SD} = 2.0$

$n = 100$

$$Z = \frac{X - \mu}{\sigma}$$

$$= \frac{250 - 100 \times 2.4}{2.0\sqrt{100}}$$

$$= \frac{10}{20} = 0.5$$

from z-table = 0.6915

$\approx 69\%$

*4.1 An officer in the army needs 35 men for a mission. He wants these soldiers to be smart ~~enogth~~ enough to understand the details of the mission so the average IQ score of the 35 men must be greater than 98 pt. It is known that average IQ of a soldier is 96 with a standard deviation of 16 point. If the officer is given a random sample of 35 soldiers for the mission, what is the prob. that he will get what he wants.

Here,

$n = 35$

$\mu_{pop} = 96, \ \mu_{SD} \cong 96$

$\sigma_{pop} = 16$

$$\sigma_{SD} = \frac{16}{\sqrt{35}}$$

$$Z = \frac{\bar{x} - \mu_{SD}}{\sigma_{SD}}$$

$$= \frac{98 - 96}{\frac{16}{\sqrt{35}}}$$

$$= \frac{\cancel{2} \times \sqrt{35}}{\cancel{16}\,8} = 0.739$$

from Z table $\rightarrow 0.7673$

$\approx 77\%$

5. Engineers must consider the breadths of male heads when designing motor cycle helmets. Men have head breadths that are normally distributed with a mean of 6.0 inch and standard deviation of 1.0 inch.

a) If One male is randomly selected, find the prob that his head breadth is less than 6.2 inch.

b) find the prob. that 100 randomly selected men have a mean breadth that is less than 6.2 inch.

a) $P(\bar{x} < 6.2)$

$$\Downarrow z = \frac{\bar{x} - \mu_{SD}}{\frac{\sigma_{pop}}{\sqrt{n}}}$$

$n = 1$

$\mu_{pop} = 6.0$, $\mu_{SD} \cong 6.0$

$\sigma_{pop} = 1.0$

$\sigma_{SD} = \frac{1.0}{\sqrt{1}}$

$$= \frac{6.2 - 6.0}{1}$$

$$= 0.2$$

from z-table $\Rightarrow 0.5793 \approx 58\%$

b) $$Z = \frac{6.2 - 6.0}{1.0/\sqrt{100}}$$

Here,

$n = 100$

$\sigma_{SD} = 1.0/\sqrt{100}$

$$= 2$$

from, z-table $\Rightarrow 0.9772$

$\approx 98\%$

*6. A production Manager for Safeguard Helmet company plans an initial run of 100 helmets. Seeing the result from part (b), the manager reasons that all helmets should be made for men with head breadths less than 6.2 inch, because they would fit all but a few men. What is wrong with that reasoning?

P(an individual head breadth greater than 6.2)

$= 1 - 0.5793$ (from part (a)) $(P(\bar{X} < 6.2))$

$= 0.4207$

$\approx 42\%$

With this it shows almost 42% men would have head breadth greater than 6.2 inch, and they would not find helmet that fits.

*7. The lengths of pregnancies are normally distributed with a mean of 268 days and a standard deviation of 15 days. If 25 women are randomly selected, find the prob. that their lengths of pregnancy have a mean that is less than 260 days.

$n = 25$

$\mu_{SD} \cong 268$

$\sigma_{SD} = 15/\sqrt{25}$

$\bar{X} = 260$

$$P(X < 260) = P\left(z < \frac{260-268}{15/\sqrt{25}}\right)$$

$$= P\left(z < \frac{-8 \times 5}{15}\right)$$

$$= P(z < -2.66)$$

from Z-table $\cong 0.0039$

$\approx 0.4\%$

*8. If 25 women are put on a special diet just before they become pregnant and they end up having a mean length of pregnancy of less than 260 days, does it appear that the diet has an effect on the length of pregnancy?

~~Yes~~, As we have seen in earlier example, $P(X<260)$ for 25 women is only 0.4%. So, it is very unlikely that diet would have an effect on the length of pregnancy.

*9. The weight of adult males are normally distributed with a mean of 172 pounds and a standard deviation of 29 pounds. Based on this info, solve the following problems:-

a) What is the prob. that one randomly selected adult male will weigh more than 190 pounds)

b) What is the prob. that 25 randomly selected adult male will have a mean weight of more than 190 pounds?

a)

$$P(X>190)=\cancel{1}-P\left(Z\cancel{<}\frac{190-172}{29}\right)$$

$n=1$

$\mu=172$

$\sigma=29/\sqrt{1}$

$$=1-P(Z<\cancel{0.413})\quad 0.6207$$

$$=1-0.7326\ (\Rightarrow z\text{-score})$$

~~from z-table ⇒ 0.659 ≈ 66%~~
~~(P-value)~~

$$=0.2674\approx 27\%$$

b)

$n=25$

$$P(X>190)=\cancel{1}-P\left(Z\cancel{<}\frac{190-172}{29/\sqrt{25}}\right)$$

$\sigma=29/\sqrt{25}$

$$=1-P(Z<\cancel{2.07})\quad 3.103\ \ \cancel{3.013}$$

$$=1-0.9990\quad (\text{P-value})$$

~~P-value from z-score ⇒ 0.9807 ≈ 98%~~

$$=0.001\approx 0.1\%$$

*9. (c) An elevator at a men fitness centre has a sign that the maximum allowable weight is 4750 pounds. If 25 randomly selected men cram into the elevator, what is the prob. it will be over the maximum allowable weigh?

$P(T > 4750) = ?$

$$Z = \frac{X - \mu}{\sigma}$$

$$= \frac{4750 - (25 \times 172)}{(29 \times \sqrt{25})}$$

$$= \frac{450 \cancel{\times 5}}{29 \times 5}$$

$$= \frac{90}{29} = 3.103$$

P-value $\Rightarrow 0.9990$

$P(T > 4750) = 1 - 0.9990$

$= 0.001$

$\approx 0.1\%$

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* 10. The amount of impurity in a batch of a chemical product is a random variable with a mean value 4.0g and a standard deviation of 1.5g. If 50 batched are independently prepared, what is the (approx) prob. that the average amount of impurity in these 50 batches is between 3.5g and 3.8g?

$$P(3.5 \le X \le 3.8)$$

$$\approx P\left(\frac{3.5-4.0}{1.5/\sqrt{50}} \le Z \le \frac{3.8-4.0}{1.5/\sqrt{50}}\right)$$

$$= P\left(\frac{-0.5}{0.2121} \le z \le \frac{-0.2}{0.2121}\right)$$

$$= P(-2.36 \le z \le -0.94)$$

$\mu = 4.0$

$\sigma = 1.5$

$n = 50$

Pvalue from z-table

$$= P(0.0091 \le z \le 0.1736)$$

$$\approx 0.1645$$

$$\approx 17\%$$

*11. Suppose the age a student graduates from Salem State is normally distributed. If the mean age is 23.1 years and the standard deviation is 3.1 years, what is the prob. that 6 randomly selected students had a mean age at graduation that was greater than 27?

$$P(\bar{X} > 27) = 1 - P\left(Z < \frac{27 - 23.1}{3.1/\sqrt{6}}\right)$$

$$= 1 - P(Z < 3.08)$$

$$= 1 - 0.9990$$

$$= 0.0010$$

$$\approx \cancel{0.01}\ 0.1\%$$

12. While checking receipts at Reds, it was determined that the average amount spent on food per table was \$21.50 with a standard deviation of \$2.22. If we can assume that the amount of money spent was normally distributed, what is the prob. that the average of 8 checks is between \$20 and \$23?

$n = 8, \mu = 21.50, \sigma = 2.22$

$$P(\$20 \le x \le \$23) = P\left(\frac{20 - 21.50}{2.22/\sqrt{8}} \le z \le \frac{23 - 21.50}{2.22/\sqrt{8}}\right)$$

$$= P\left(\frac{-1.50}{0.7848} \le z \le \frac{1.5}{0.7848}\right)$$

$$= P(-1.91 \le z \le 1.91)$$

P-value z-table

$0.02807 \le z \le 0.9719$

≈ 0.9438

$\approx 94.38\%$

*13. Suppose the grades in a finite mathematics class are normally distributed with a mean of 75 and a standard deviation of 5.

a) What is the prob. that a randomly selected student had a grade of atleast 83?

b) What is the prob. that the average grade of 5 randomly selected students was atleast 83?

$\mu = 75$, $\sigma = 5$, $n = 1$

$\sigma_{SD} = 5/\sqrt{1}$

a) $P(\bar{x} \geq 83) = 1 - P\left(z \geq \frac{83-75}{5/\sqrt{1}}\right)$

$= 1 - P\left(z \geq \frac{8}{5}\right)$

$= 1 - P(z \geq 1.6)$

P-value from z-table

$= 1 - 0.9452$

$= 0.0548 \simeq$ ~~60%~~ 5%

b) $P(\bar{x} \geq 83) = 1 - P\left(z \geq \frac{83-75}{5/\sqrt{5}}\right)$

$= 1 - P(z \geq 3.57)$

$= 1 - 0.9998$

$= 0.0002$

$\approx 0.02\%$

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*14. The average age of major league baseball players is 28.3 years and has a standard deviation of 2.3 years. If we can assume that ages are normally distributed, what is the prob. that the average age of 10 randomly selected Red Sox players are less than 27 years?

$n = 10, \quad \mu = 28.3, \quad \sigma = 2.3$

$\sigma_{SD} = 2.3/\sqrt{10}$

$$P(\bar{X} < 27) = P\left(Z < \frac{27 - 28.3}{2.3/\sqrt{10}}\right)$$

$$= P\left(Z < \frac{-1.3}{0.7273}\right)$$

$$= P(Z < -1.787)$$

P-value from Z-table

$= \cancel{0.9633} = 0.0367$

$\approx \cancel{96\%} \approx 3.67\%$