※2. The average monthly sales of 2000 firms are normally distributed with mean of Rs. 38,000 and standard deviation of Rs. 10,000. Find:-

a). The numbers of firms with sales of over Rs. 50,000.

b). The percentage of firms with sales between Rs. 38,500 and 41,000.

c) The number of firms with sales between Rs. 30,000 and 50,000.

a) $\mu = 38,000$, $\sigma = 10,000$, $X = 50,000$

$$Z\text{-score} = \frac{X-\mu}{\sigma}$$

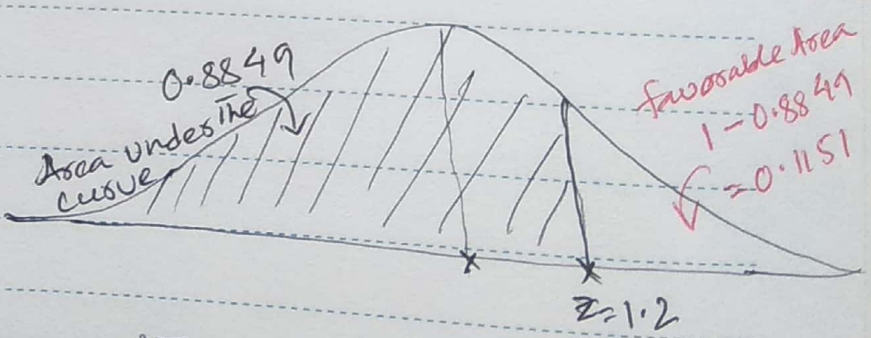
$$= \frac{50,000-38,000}{10,000} = \frac{12,000}{10,000}$$

$$= 1.2$$

Now, from Z-table,-

$$P(X<50000) = 0.8849$$

$$P(X>50000) = 1-0.8849 = 0.1151$$



The number of firms with sales over Rs. 50000 is,-

$$= 2000 \times 0.1151$$

$$= 230.2$$

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b)

$$Z\text{-Score} = \frac{x - \mu}{\sigma}$$

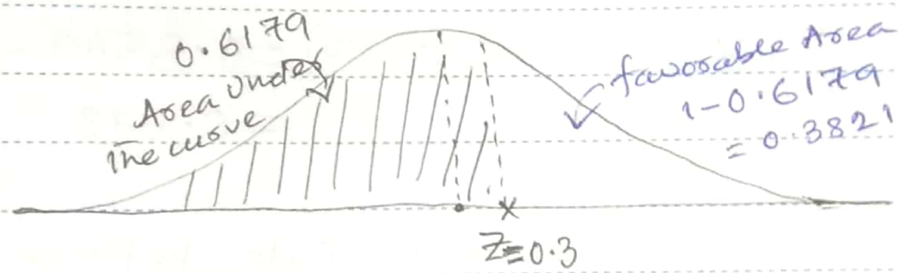
$$= \frac{41{,}000 - 38{,}000}{10{,}000} = \frac{3{,}000}{10{,}000}$$

$$= 0.3$$

from, Z-table,:-

$P(X < 40{,}000) = 0.6179$

$P(X > 40{,}000) = 1 - 0.6179 = 0.3821$



$$Z\text{-Score} = \frac{x - \mu}{\sigma} = \frac{38{,}500 - 38{,}000}{10{,}000} = \frac{500}{10{,}000}$$

$$= 0.05$$

from, Z table:-

$P(X < 38{,}500) = 0.5199$

$P(X > 38{,}500) = 1 - 0.5199 = 0.4801$

Now, $P(38{,}500 < X < 41{,}000) = 0.6179 - 0.5199$

$= 0.098$

The number of firms with sales between Rs. 38,500 and 41,000 is,— 2000×0.098

$= 196.4$

For percentage → 9.8 %

c) Z-Score = $\frac{x-\mu}{\sigma}$

$= \frac{50{,}000 - 38{,}000}{10000} = \frac{12{,}000}{10{,}000}$

$= 1.2$

$P(x < 50{,}000) = 0.8849$

Z-Score $= \frac{30{,}000 - 38{,}000}{10{,}000} = \frac{-8{,}000}{10{,}000} = -0.8$

$P(x < 30{,}000) = 0.2119$

$P(30{,}000 < x < 50{,}000) = 0.8849 - 0.2119$

$= 0.673$

The no. of firms with sales between Rs, 30,000 and 50,000 is – 2000×0.673

$= 1346$ //

*3. A test is conducted which consists of 20 MCQs with every question having 4 options. Determine the probability of a person answering exactly 5 wrong answers.

There are two ways to solve this problem.

Here, $n = 20$, $r = 5$

Success (P) = wrong = $3/4$; correct $1/4$ $(q = 1-p)$

$$^{20}C_5 \left(3/4\right)^5 \left(1/4\right)^{20-5}$$

Success = correct = $1/4$; wrong = $3/4$;

$r = 15$

$$^{20}C_{15} \left(1/4\right)^{15} \left(3/4\right)^5$$

*4. In an observational astronomy experiment, let the average rate of photons reaching the telescope is 4 photons per second (Poisson random variable with mean of 4). Find the probability that no photon reaches the telescope in a given second.

(Poisson Distribution)

$$PDF = \frac{e^{-\lambda} * \lambda^r}{r!}$$

Here, $\lambda = 4$, $r = 0$

$$P(X=0) = \frac{e^{-4} * 4^0}{0!}$$

$$= \frac{0.01831}{1} = 0.01831$$

Ans.

*5. The number of calls coming per minute into a customer support center is Poisson random variable with mean 3.

a) find the probability that no calls come in a given 1-minute period.

b) Assume that the number of calls arriving in two different minutes are independent. find the probability that atleast two calls will arrive in a given two-minute period.

a) $PDF = \dfrac{e^{-\lambda} * \lambda^{r}}{r!}$

Here, $\lambda = 3$, $r = 0$

$P(X=0) = \dfrac{e^{-3} * 3^{0}}{0!}$

$= e^{-3} = 0.0497$

b) $P(X_1 + X_2 \geq 2) = 1 - P(X_1 + X_2 = 0) - P(X_1 + X_2 = 1)$

Here, $\lambda = 3$

$P(X=0) = \dfrac{e^{-3} * 3^{0}}{0!} = e^{-3}$

$P(X=1) = \dfrac{e^{-3} * 3^{1}}{1!} = 3e^{-3}$

$P(X_1 + X_2 \geq 2) = 1 - P(X_1 + X_2 = 0) - P(X_1 + X_2 = 1)$

$= 1 - e^{-3} \cdot e^{-3} - 3e^{-3} \cdot 3e^{-3}$

*6. If a production line has a 20% defective rate. calculate the prob. of obtaining the first defected part after three good parts. What is the average numbers of inspections to obtain the first defective.

$P(X>3) = 1 - P(X=0) - P(X=1) - P(X=2) - P(X=3)$

$= 1 -$

p (Prob. of defective rate) $= 0.2$

$q = 1 - p$

$= 0.8$

r = no. of success events = 1

n = no. of trials = 3

*7. The probability that a student is accepted to a prestigious college is 0.3. If 5 students from the same school apply, what is the probability that atmost 2 are accepted?

$P(X=r) = {}^{n}C_r \, p^r q^{n-r}$

$P(X \leq r) = \sum_{i=0}^{r} {}^{n}C_i \, p^i q^{n-i}$

Here,
$n = 5$,
p (prob. of success) $= 0.3$
$q = 1 - p$

$P(X \leq 2) = {}^{5}C_0 (0.3)^0 (1-0.3)^{5-0}$
$+ {}^{5}C_1 (0.3)^1 (1-0.3)^{5-1}$
$+ {}^{5}C_2 (0.3)^2 (1-0.3)^{5-2}$

$= \frac{5!}{0!(5-0)!} (0.3)^0 (0.7)^5$
$+ \frac{5!}{1!(5-1)!} (0.3)^1 (0.7)^4$
$+ \frac{5!}{2!(5-2)!} (0.3)^2 (0.7)^3$

$= (0.3)^0 (0.7)^5 + \frac{5!}{4!} (0.3)(0.7)^4 + \frac{5!}{2!3!} (0.3)^2 (0.7)^3$

$= \cancel{(0.3)} (0.7)^5 + 5 \times (0.3)(0.7)^4 + \frac{5 \times 4}{2} (0.3)^2 (0.7)^3$

0.1681
$= \cancel{0.0504} + 0.3601 + 0.3087$

$= 0.8369$

$\approx 83.69\%$

∴ prob. that atmost 2 are accepted out of 5 students.

#8. The maximum weight that an elevator in an apartment complex can accomodate is 800 kg. The average adult weight be about 70 kg with a variance of 200. What is the probability that the lift safely reaches the ground when there are 10 different adults in the lift? What if there are 12 adults?

a) In case of 10 adults.

$$Z = \frac{X-\mu}{\sigma}$$

$X = 800$
$\mu = 70 \times 10$
$\sigma = \sqrt{200 \times 10}$

$$= \frac{800-700}{\sqrt{2000}}$$

$$= \frac{100}{44.72} = 2.236$$

from, Z-table ⇒ 0.9871

≈ 98.71 % reaches the ground safely.

b) In case of 12 adults.

$X = 800$
$\mu = 70 \times 12$
$\sigma = \sqrt{200 \times 12}$

$$Z = \frac{X-\mu}{\sigma}$$

$$= \frac{800-840}{\sqrt{200 \times 12}}$$

$$= \frac{(-40)}{48.99} = -0.816$$

*9. A student to test his luck, went to an exam unprepared. It was a MCQ type exam with two choices for each question. There were 50 questions of which atleast 20 are to be answered correctly to pass the test. What is the probability that he clears the exam? If each question had 4 choices instead of two, what is the prob. that he clears the exam?

a)

$$P(X \leq r) = \sum_{i=0}^{r} {}^{n}C_{i}\, p^{i}\, q^{n-i}$$

Here,

$n = 50$

p (prob. of success) $= \frac{1}{2}$

q (prob. of failure) $= 1-p = \frac{1}{2}$

$$P(X \leq 20) = {}^{50}C_{0}(1/2)^{0}(1/2)^{50-0} + {}^{50}C_{1}(1/2)^{1}(1/2)^{50-1} + {}^{50}C_{2}(1/2)^{2}(1/2)^{50-2} + {}^{50}C_{3}(1/2)^{3}(1/2)^{50-3} + \cdots + {}^{50}C_{19}(1/2)^{19}(1/2)^{50-19} + {}^{50}C_{20}(1/2)^{20}(1/2)^{50-20}$$

b)

$$P(X \leq 20) = {}^{50}C_{0}(1/4)^{0}(3/4)^{50-0} + {}^{50}C_{1}(1/4)^{1}(3/4)^{50-1} + {}^{50}C_{2}(1/4)^{2}(3/4)^{50-2} + \cdots + {}^{50}C_{19}(1/4)^{19}(3/4)^{50-19} + {}^{50}C_{20}(1/4)^{20}(3/4)^{50-20}$$

Here,

$n = 50$

p (prob. of success) $= 1/4$

q (prob. of failure) $= 1-p = 3/4$

*10. A company manufactures LED bulbs with a faulty rate of 30%. If I randomly select 6 bulbs, what is the probability that exactly 2 are faulty?

(Binomial Distribution)

$$P(X=2) = {}^{6}C_{2} (0.3)^{2} (1-0.3)^{6-2}$$

$$= \frac{6!}{2!(6-2)!} (0.3)^{2} (0.7)^{4}$$

$$= \frac{\cancel{6}^{3} \times 5}{\cancel{2}} \times (0.3)^{2} \times (0.7)^{4}$$

$$= 0.324$$

$$\approx 32\%$$

Prob. of faulty rate $(P) = 0.3$

$q = 1 - P$

n = no. of trials = 6

r = no. of success events = 2

*12. Executives in the New Zealand forestry Industry claim that only ~~5% of all~~ 5% of all sawmills sites contain soil residuals of dioxin (an additive previously for anti-sap-stain treatment in wood) higher than the recomended level. If Environment Canterbury randomly selects 20 old sawmill sites for inspection, assuming that the executive claim is correct:

a) Calculate the prob. that less than ~~or equal to~~ 1 site exceeds the recommended level of dioxin

b) Calculate the prob. that less than or equal to 1 site exceed the recommended level of dioxin.

c) Calculate the prob. that at most (i.e, maximum of) 2 sites exceed the recommended level of dioxin.

a) (less than 1 site)

$$P(x=0) = {}^{20}C_0 (0.05)^0 \times (0.95)^{20-0}$$

$$= \frac{20!}{0!(20-0)!} \times 1 \times (0.95)^{20}$$

$$= (0.95)^{20}$$

$$= 0.3585$$

$\approx 36\%$ exceeds the recommended level of dioxin.

Here,

n = no. of trials = 20,

p (no. of success) = 0.05

p (no. of failure) = $1-p$

= 0.95

b) Prob. of less than or equal to 1 site

$$P(X \leq 1) = \sum_{i=0}^{r} {}^{n}C_i \, p^i q^{n-i}$$

$$= {}^{20}C_0 (0.05)^0 (0.95)^{20-0} + {}^{20}C_1 (0.05)^1 (0.95)^{20-1}$$

$$= \frac{20!}{0!(20-0)!} \times (0.95)^{20} + \frac{20!}{1!(20-1)!} \times (0.05) \times (0.95)^{19}$$

$$= (0.95)^{20} + 20 \times (0.05) \times (0.95)^{19}$$

$$= 0.3585 + 0.3774$$

$$= 0.7359$$

$\approx 74\%$ exceeds recommended level of dioxin.

c) Prob. of atmost 2 sites exceeds.

$$P(X \leq 2) = \sum_{i=0}^{r} {}^{n}C_i \, p^i q^{n-i}$$

$$= {}^{20}C_0 (0.05)^0 (0.95)^{20-0} + {}^{20}C_1 (0.05)^1 (0.95)^{20-1} + {}^{20}C_2 (0.05)^2 (0.95)^{20-2}$$

$$= 0.3585 + 0.3774 + \frac{20!}{2!(20-2)!} (0.05)^2 (0.95)^{18}$$

$$= 0.3585 + 0.3774 + \frac{\overset{10}{\cancel{20}} \times 19}{\cancel{2}} (0.05)^2 (0.95)^{18}$$

$$= 0.3585 + 0.3774 + 0.1887$$

$$= 0.9246$$

$\approx 92\%$ exceeds recommended level of dioxin.

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#13. Inland Revenue audits 5% of all companies every year. The companies selected for auditing in any one year are independent of the previous year's selection.

a) What is the prob. that the company 'Ross Waste Disposal' will be selected for auditing exactly twice in the next 5 years?

b) What is the prob. that the company will be audited exactly twice in the next 2 years?

c) What is the exact prob. that this company will be audited at least once in the next 4 yrs?

a)

Here,

$n = 5$

p (prob. of success) $= 0.05$

q (prob. of failure) $= 1 - p$

$= 0.95$

$$P(X=2) = {}^{5}C_{2} (0.05)^{2} (0.95)^{5-2}$$

$$= \frac{5!}{2!(5-2)!} (0.05)^2 (0.95)^3$$

$$= \frac{5 \times 4^{2}}{2} \times (0.05)^2 (0.95)^3$$

$$= 0.0214$$

$$\approx 2\%$$

b)

$P(X=2) = {}^{2}C_{2} (0.05)^2 (0.95)^{2-2}$

$= \frac{2!}{2!(2-2)!} \times (0.05)^2 \times 1$

$= (0.05)^2 = 0.0025$

$\approx 0.25\%$

Here,
$n = 2$
$p = 0.05$
$q = 1-p = 0.95$

c)

$P(X \leq 1) = {}^{4}C_{0} (0.05)^0 (0.95)^{4-0} + {}^{4}C_{1} (0.05)^1 (0.95)^{4-1}$

(This is for at most once)

$= \frac{4!}{0!(4-0)!} \times 1 \times (0.95)^4 + \frac{4!}{1!(4-1)!} \times (0.05) \times (0.95)^3$

$= (0.95)^4 + 4 \times (0.05) \times (0.95)^3$

$= 0.8145 + 0.1715$

$= 0.986$

$\approx 97\%$

Here,
$n = 4$
$p = 0.05$
$q = 1-p = 0.95$

Atleast once

c)

$$P(X \geq 1) = 1 - P(X=0)$$

$$= 1 - {}^{4}C_{0} (0.05)^{0} (0.95)^{4-0}$$

$$= 1 - \frac{4!}{0!(4-0)!} \times (0.95)^{4}$$

$$= 1 - 0.8145$$

$$= 0.1855$$

$$\approx 18.55\ \%$$

14. The prob. that a driver must stop at any one traffic light coming to supervised learning is 0.2. There are 15 sets of traffic lights on the journey.

a) What is the prob. that a student must stop at exactly 2 of the 15 sets of traffic lights?

b) What is the probability that a student will be stopped at 1 or more of the 15 sets of traffic lights?

a)

$$P(X=2) = {}^{15}C_2 (0.2)^2 (0.8)^{15-2}$$

Here,
$n = 15$
$p = 0.2$
$q = 1-p = 0.8$

(Exactly 2)

$$= \frac{15!}{2!(15-2)!} \times (0.2)^2 \times (0.8)^{13}$$

$$= \frac{15 \times 14}{2} \times (0.2)^2 \times (0.8)^{13}$$

$$= 0.2309$$

b) $P(X \geq 1) = 1 - P(X<1)$

$= 1 - P(X=0)$

$= 1 - {}^{15}C_0 (0.2)^0 (0.8)^{15-0}$

$= 1 - \frac{15!}{0!(15-0)!} \times (0.8)^{15} = 1 - (0.8)^{15}$