

Probability Assignment

Conditional, Joint and Marginal Probability

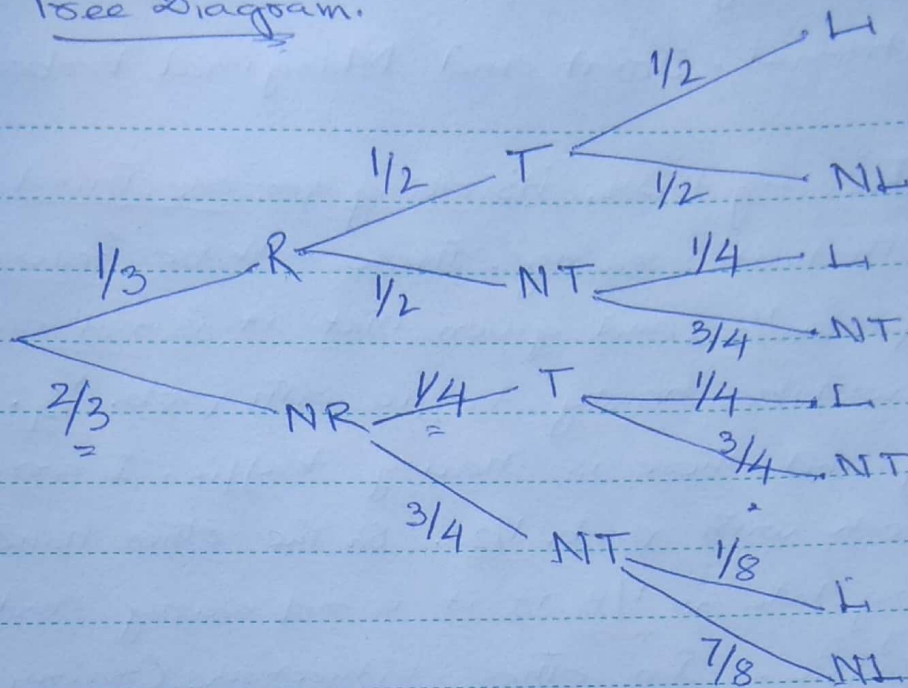
#5. In my town, it's raining for one third of the days. Given that it is raining, there will be heavy traffic with prob. $\frac{1}{2}$, and given that it is not raining, there will be heavy traffic with prob. $\frac{1}{4}$. If it's raining and there is heavy traffic, I arrive late for work with prob. $\frac{1}{2}$. On the other hand, the prob. of being late is $\frac{1}{8}$ if it is not raining and there is heavy traffic. In other situations (raining and no traffic, not raining and traffic) the prob. of being late is 0.25. You pick a random day.

a) What is the prob. that it's not raining and there is heavy traffic and I am not late?

b) What is the prob. that I am late?

c) Given that I arrived late at work, what is the prob. that it rained that day.

Tree Diagram.



$$\begin{aligned}
 a) P(NR \cap T \cap NL) &= P(NR) \times P(T|NR) \times P(NL|NR \cap T) \\
 &= \frac{2}{3} \times \frac{1}{4} \times \frac{3}{4} \\
 &= \frac{6}{48} = \frac{1}{8}
 \end{aligned}$$

$$\begin{aligned}
 b) P(L) &= P(R \cap T \cap L) + P(R \cap NT \cap L) + P(NR \cap T \cap L) + P(NR \cap NT \cap L) \\
 &= \left(\frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{2}\right) + \left(\frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{4}\right) + \left(\frac{2}{3} \cdot \frac{1}{4} \cdot \frac{1}{4}\right) + \left(\frac{2}{3} \cdot \frac{3}{4} \cdot \frac{1}{8}\right)
 \end{aligned}$$

$$= \frac{1}{12} + \frac{1}{24} + \frac{1}{24} + \frac{1}{16}$$

$$= \frac{1 \times 4}{12 \times 4} + \frac{1 \times 2}{24 \times 2} + \frac{1 \times 2}{24 \times 2} + \frac{1 \times 3}{16 \times 3}$$

$$= \frac{11}{48}$$

$$c) P(R|L) = \frac{P(R \cap L)}{P(L)}$$

$$P(R \cap L) = P(RTL) + P(RNTL)$$

$$= \left(\frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{2}\right) + \left(\frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{4}\right)$$

$$= \frac{1}{12} + \frac{1}{24}$$

$$= \frac{2}{24} + \frac{1}{24} = \frac{1}{8}$$

$$P(R|L) = \frac{\frac{1}{8}}{\frac{11}{48}}$$

$$= \frac{1}{8} \times \frac{48}{11}$$

$$= \frac{6}{11}$$