$\boxed{6.50}$

$U = 15$

$T = 20$

η

$$\lambda = \frac{h}{p} \qquad T = \frac{m \cdot v^2}{2} = \frac{p^2}{2m} \qquad p = \sqrt{2T \cdot m}$$

до перехода через барьер $U_1 \quad T_1$

после перехода через барьер $U_2 \quad T_2$

$$U_1 + T_1 = U_2 + T_2 \qquad U_1 = 0 \quad U_2 = U$$

$$T_1 = T \qquad T_2 = U_1 + T_1 - U_2 = T - U$$

$$\lambda_1 = \frac{h}{p_1} = \frac{h}{\sqrt{2T_1 \cdot m}} \qquad \lambda_2 = \frac{h}{p_2} = \frac{h}{\sqrt{2T_2 \cdot m}}$$

$$\eta = \frac{\lambda_2}{\lambda_1} = \frac{\dfrac{h}{\sqrt{2T_2 \cdot m}}}{\dfrac{h}{\sqrt{2T_1 \cdot m}}} = \frac{\dfrac{h}{\sqrt{2(T-U) \cdot m}}}{\dfrac{h}{\sqrt{2T \cdot m}}} = \sqrt{\frac{T-U}{T}}$$

$\boxed{6.49}$

$T = 100 \cdot 1.6 \cdot 10^{-19}$

$m_e = 0.911 \cdot 10^{-30}$

$m_p = 1.672 \cdot 10^{-27}$

m_U

$$\lambda = \frac{h}{p} \qquad T = \frac{m \cdot v^2}{2} = \frac{p^2}{2m} \qquad p = \sqrt{2T \cdot m} \qquad \lambda = \frac{h}{\sqrt{2T \cdot m}}$$

$$\lambda_e = \frac{h}{\sqrt{2T \cdot m_e}} \qquad \lambda_e = \frac{6.626 \cdot 10^{-34}}{\sqrt{2\left(100 \cdot 1.6 \cdot 10^{-19}\right) \cdot \left(0.911 \cdot 10^{-30}\right)}} \rightarrow \lambda_e = 1.2272053895159848636 \cdot 10^{-1}$$

$$\lambda_p = \frac{h}{\sqrt{2T \cdot m_p}} \qquad \lambda_p = \frac{6.626 \cdot 10^{-34}}{\sqrt{2\left(100 \cdot 1.6 \cdot 10^{-19}\right) \cdot \left(1.672 \cdot 10^{-27}\right)}} \rightarrow \lambda_p = 2.8645625234853816677 \cdot 10^{-1}$$

$\boxed{6.66}$

$$\lambda = \frac{h}{p} \qquad p = m \cdot v = \frac{h}{\lambda} \qquad m \cdot v \cdot 2 \cdot \pi \cdot R = \frac{h}{\lambda} \cdot (2 \cdot \pi \cdot R) \quad (2)$$

$m \cdot v \cdot 2 \cdot \pi \cdot R = n \cdot h$ (постулат Бора) (1)

$(1) + (2)$ $\quad \dfrac{h}{\lambda} \cdot (2 \cdot \pi \cdot R) = n \cdot h \qquad 2 \cdot \pi \cdot R = n \cdot \lambda \qquad R = \dfrac{\lambda}{2\pi} \cdot n$ (резрешены такие орбиты, на которых укладывается целое число волн)

$\boxed{6.75}$

$U(x) = \kappa \cdot \dfrac{x^2}{2}$ обозначим $\xi = \dfrac{h}{2\pi}$

E_{min}

$$\Delta p \approx \frac{\xi}{2} \cdot \frac{1}{\Delta x} \qquad \begin{matrix} \Delta p = p - p_0 & p_0 = 0 & \Delta p = p \\ \Delta x = x - x_0 & x_0 = 0 & \Delta x = x \end{matrix} \qquad p \approx \left(\frac{\xi}{2} \cdot \frac{1}{x}\right)$$

$$E(x) = T + U = \frac{p^2}{2m} + U(x) = \left(\frac{\xi}{2x}\right)^2 \frac{1}{2m} + \kappa \cdot \frac{x^2}{2}$$

$$\frac{d}{dx}E(x) = 0 \qquad \frac{d}{dx}\left[\left(\xi \cdot \frac{1}{2x}\right)^2 \cdot \frac{1}{2 \cdot m} + \kappa \cdot \frac{x^2}{2}\right] = 0 \rightarrow \frac{-1}{4} \cdot \frac{\xi^2}{x^3 \cdot m} + \kappa \cdot x = 0 \qquad x_{min} = \frac{1}{2} \cdot \frac{\sqrt{2}}{\kappa \cdot m} \cdot \left(\xi^2 \cdot \kappa^3 \cdot m^3\right)^{\frac{1}{4}}$$

$$E(x_{min}) = \left[\xi\cdot\frac{1}{2\cdot\left[\frac{1}{2}\cdot\frac{\sqrt{2}}{\kappa\cdot m}\cdot\left(\xi^2\cdot\kappa^3\cdot m^3\right)^{\frac{1}{4}}\right]}\right]^2\cdot\frac{1}{2\cdot m}+\frac{\kappa}{2}\cdot\left[\frac{1}{2}\cdot\frac{\sqrt{2}}{\kappa\cdot m}\cdot\left(\xi^2\cdot\kappa^3\cdot m^3\right)^{\frac{1}{4}}\right]^2 = \frac{\frac{1}{4}\cdot\xi^2\cdot\kappa^2\cdot m}{\left(\xi^2\cdot\kappa^3\cdot m^3\right)^{\frac{1}{2}}}+\frac{1}{4}\cdot\kappa\cdot m^2\cdot\frac{\left(\xi^2\cdot\kappa^3\cdot m^3\right)^1}{2} = \frac{1}{2}\cdot\xi\cdot\sqrt{\frac{\kappa}{m}}$$

$$E_{min} = \frac{1}{2}\cdot\xi\cdot\sqrt{\frac{\kappa}{m}}$$

$\boxed{6.80}$

n_1

n_2

$\Delta E = E_2 - E_1$

L

$U = 0$ стационарное ур-ие Шредингера: $\frac{d^2}{dx^2}\psi + \frac{2m}{\xi^2}\cdot E\cdot\psi = 0$ $\quad\sqrt{\frac{2m}{\xi^2}\cdot E} = k\quad$ $\psi'' + k^2\psi = 0$

решение вида: $\psi(x) = A\cdot\cos(k\cdot x) + B\cdot\sin(k\cdot x)$

$\psi(0) = 0\quad A\cdot\cos(0) + B\cdot\sin(0) = 0\quad A = 0$

$\psi(L) = 0\quad A\cdot\cos(L\cdot x) + B\cdot\sin(L\cdot x) = 0\quad \sin(k\cdot L) = 0\quad k\cdot L = \pi\cdot n\quad k = \pi\cdot\frac{n}{L}$

$$\sqrt{\frac{2m}{\xi^2}\cdot E} = \pi\cdot\frac{n}{L}\qquad E = \frac{1}{2}\cdot\pi^2\cdot n^2\cdot\frac{\xi^2}{m\cdot L^2}\qquad \Delta E = E_2 - E_1 = \frac{1}{2}\cdot\pi^2\cdot\left(n_2{}^2 - n_1{}^2\right)\cdot\frac{\xi^2}{m\cdot L^2}\qquad L = \pi\,\xi\cdot\sqrt{\frac{n_2{}^2 - n_1{}^2}{2\cdot\Delta E\cdot m}}$$

$\boxed{6.81}$

$U = 0$ стационарное ур-ие Шредингера: $\frac{d^2}{dx^2}\psi + \frac{2m}{\xi^2}\cdot E\cdot\psi = 0$ $\quad\sqrt{\frac{2m}{\xi^2}\cdot E} = k\quad$ $\psi'' + k^2\psi = 0$

$n = 1$

$L \qquad \frac{L}{3} < x < \frac{2L}{3}$

решение вида: $\psi(x) = A\cdot\cos(k\cdot x) + B\cdot\sin(k\cdot x)$

$\psi(0) = 0\quad A\cdot\cos(0) + B\cdot\sin(0) = 0\quad A = 0$

$\psi(L) = 0\quad A\cdot\cos(L\cdot x) + B\cdot\sin(L\cdot x) = 0\quad \sin(k\cdot L) = 0\quad k\cdot L = \pi\cdot n\quad k = \pi\cdot\frac{n}{L}$

$$\psi(x) = B\cdot\sin(k\cdot x) = B\cdot\sin\left(\pi\cdot\frac{n}{L}\cdot x\right)$$

P

коэффициент B найдем из условия нормировки $\int_0^L \psi(x)^2\,dx = 1$

$$\int_0^L B^2\cdot\sin\left(\pi\cdot\frac{n}{L}\cdot x\right)^2 dx = 1 \rightarrow \frac{1}{2}\cdot L\cdot B^2\cdot\frac{\left(-\cos(\pi\cdot n)\cdot\sin(\pi\cdot n) + \pi\cdot n\right)}{\pi\cdot n} = 1\qquad B = \sqrt{\frac{2}{L}}\qquad \psi(x) = \sqrt{\frac{2}{L}}\cdot\sin\left(\pi\cdot\frac{n}{L}\cdot x\right)$$

вероятность P находим аналогично (из смысла пси-функции)

$$P(n) = \int_{\frac{L}{3}}^{2\cdot\frac{L}{3}} \frac{2}{L}\cdot\sin\left(\pi\cdot\frac{n}{L}\cdot x\right)^2 dx \rightarrow P(n) = \frac{1}{3}\cdot\frac{\left(-3\cdot\cos\left(\frac{2}{3}\cdot\pi\cdot n\right)\cdot\sin\left(\frac{2}{3}\cdot\pi\cdot n\right) + 2\cdot\pi\cdot n\right)}{\pi\cdot n} - \frac{1}{3}\cdot\frac{\left(-3\cdot\cos\left(\frac{1}{3}\cdot\pi\cdot n\right)\cdot\sin\left(\frac{1}{3}\cdot\pi\cdot n\right) + \pi\cdot n\right)}{\pi\cdot n}$$

$$n = 1\qquad P(1) = \frac{1}{12}\cdot\frac{\left(3\cdot\sqrt{3} + 8\cdot\pi\right)}{\pi} - \frac{1}{12}\cdot\frac{\left(-3\cdot\sqrt{3} + 4\cdot\pi\right)}{\pi}\qquad P(1) = \frac{1}{3} + \frac{1}{2}\cdot\frac{\sqrt{3}}{\pi}$$

$\boxed{6.84}$ из уравнения Шредингера:

L

m

$\frac{dN}{dW}$

$$W = \frac{1}{2}\cdot\pi^2\cdot N^2\cdot\frac{\xi^2}{m\cdot L^2}\qquad N = \frac{1}{\pi}\cdot\sqrt{2}\cdot(W\cdot m)^{\frac{1}{2}}\cdot\frac{L}{\xi}\qquad \frac{dN}{dW} = \frac{d}{dW}\left[\frac{1}{\pi}\cdot\sqrt{2}\cdot(W\cdot m)^{\frac{1}{2}}\cdot\frac{L}{\xi}\right] \rightarrow \frac{dN}{dW} = \frac{1}{2\cdot\pi}\cdot\frac{\sqrt{2}}{(W\cdot m)^{\frac{1}{2}}}\cdot\frac{L}{\xi}\cdot m$$

$$\frac{dN}{dW} = \frac{1}{\pi}\cdot\frac{1}{\left(\sqrt{2W\cdot m}\right)}\cdot\frac{L}{\xi}\cdot m$$

$\boxed{6.95}$

$\psi(x) = A\cdot\exp\left(-\alpha\cdot x^2\right)$

$U(x) = k\cdot\dfrac{x^2}{2}$

E α

стационарное ур-ие Шредингера: $\dfrac{d^2}{dx^2}\psi + \dfrac{2m}{\xi^2}\cdot(E - U)\cdot\psi = 0$

$$\frac{d^2}{dx^2}\left(A\cdot\exp\left(-\alpha\cdot x^2\right)\right) + \frac{2m}{\xi^2}\cdot\left(E - k\cdot\frac{x^2}{2}\right)\cdot\left(A\cdot\exp\left(-\alpha\cdot x^2\right)\right) = 0$$

$$-2\cdot A\cdot\alpha\cdot\exp\left(-\alpha\cdot x^2\right) + 4\cdot A\cdot\alpha^2\cdot x^2\cdot\exp\left(-\alpha\cdot x^2\right) + 2\cdot\frac{m}{\xi^2}\cdot\left(E - k\cdot\frac{x^2}{2}\right)\cdot A\cdot\exp\left(-\alpha\cdot x^2\right) = 0$$

$$-\alpha + 2\cdot\alpha^2\cdot x^2 + \frac{m}{\xi^2}\cdot\left(E - k\cdot\frac{x^2}{2}\right) = 0 \qquad -\alpha + 2\cdot\alpha^2\cdot x^2 + \frac{m}{\xi^2}\cdot E - \frac{m}{\xi^2}\cdot k\cdot\frac{x^2}{2} = 0$$

$$\left(2\cdot\alpha^2 - \frac{1}{2}\cdot\frac{m}{\xi^2}\cdot k\right)\cdot x^2 - \alpha + \frac{m}{\xi^2}\cdot E = 0 \qquad -\alpha + \frac{m}{\xi^2}\cdot E = 0 \qquad E = \frac{\alpha}{m}\cdot\xi^2$$

$$2\cdot\alpha^2 - \frac{1}{2}\cdot\frac{m}{\xi^2}\cdot k = 0 \qquad \alpha = \frac{1}{2}\cdot\frac{\sqrt{m\cdot k}}{\xi}$$

$$E = \frac{\frac{1}{2}\cdot\frac{\sqrt{m\cdot k}}{\xi}}{m}\cdot\xi^2 = \frac{1}{2}\cdot\sqrt{\frac{k}{m}}\cdot\xi$$