Вариант – 26

Задание 3.

Для графа произвести кратчайшую раскраску вершин графа.

Решение:

Методом граничного перебора строятся все внутренне устойчивые множества вершин графа:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x1 | x2 | x3 | x4 | x5 | x6 | x7 | x8 | x9 |  |
| 1 |  | 1 |  |  | 1 |  |  |  | A |
| 1 |  |  |  | 1 |  |  |  |  | B |
| 1 |  |  |  |  |  |  |  | 1 | C |
|  | 1 |  | 1 |  | 1 |  |  |  | D |
|  | 1 |  |  |  |  |  | 1 |  | E |
|  | 1 |  |  |  |  |  |  | 1 | F |
|  |  | 1 |  |  |  | 1 |  |  | G |
|  |  | 1 |  |  |  |  | 1 |  | H |
|  |  |  | 1 | 1 |  | 1 |  |  | I |
|  |  |  | 1 |  | 1 |  |  |  | J |
|  |  |  |  | 1 |  | 1 |  |  | K |
|  |  |  |  | 1 |  |  | 1 |  | L |
|  |  |  |  |  |  | 1 |  | 1 | M |

Составляем таблицу покрытий:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **х1** | **х2** | **х3** | **х4** | **х5** | **х6** | **x7** | **x8** | **x9** |
| A | 1 |  | 1 |  |  | 1 |  |  |  |
| B | 1 |  |  |  | 1 |  |  |  |  |
| C | 1 |  |  |  |  |  |  |  | 1 |
| D |  | 1 |  | 1 |  | 1 |  |  |  |
| E |  | 1 |  |  |  |  |  | 1 |  |
| F |  | 1 |  |  |  |  |  |  | 1 |
| G |  |  | 1 |  |  |  | 1 |  |  |
| H |  |  | 1 |  |  |  |  | 1 |  |
| I |  |  |  | 1 | 1 |  | 1 |  |  |
| J |  |  |  | 1 |  | 1 |  |  |  |
| K |  |  |  |  | 1 |  | 1 |  |  |
| L |  |  |  |  | 1 |  |  | 1 |  |
| M |  |  |  |  |  |  | 1 |  | 1 |

Разложение по столбцу x1:

{A}

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **х2** | **х4** | **х5** | **x7** | **x8** | **x9** |
| B |  |  | 1 |  |  |  |
| C |  |  |  |  |  | 1 |
| D | 1 | 1 |  |  |  |  |
| E | 1 |  |  |  | 1 |  |
| F | 1 |  |  |  |  | 1 |
| G |  |  |  | 1 |  |  |
| H |  |  |  |  | 1 |  |
| I |  | 1 | 1 | 1 |  |  |
| J |  | 1 |  |  |  |  |
| K |  |  | 1 | 1 |  |  |
| L |  |  | 1 |  | 1 |  |
| M |  |  |  | 1 |  | 1 |

Разложение по столбцу x4:

{A,D}

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **х5** | **x7** | **x8** | **x9** |
| E |  |  | 1 |  |
| F |  |  |  | 1 |
| I | 1 | 1 |  |  |
| L | 1 |  | 1 |  |
| M |  | 1 |  | 1 |

Покрытие: {A,D,L,M}

{A,I}

|  |  |  |  |
| --- | --- | --- | --- |
|  | **х2** | **x8** | **x9** |
| D | 1 |  |  |
| E | 1 | 1 |  |
| F | 1 |  | 1 |
| L |  | 1 |  |
| M |  |  | 1 |

Покрытие: {A,I,E,F}

{B}

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **х2** | **х3** | **х4** | **х6** | **x7** | **x8** | **x9** |
| A |  | 1 |  | 1 |  |  |  |
| C |  |  |  |  |  |  | 1 |
| D | 1 |  | 1 | 1 |  |  |  |
| E | 1 |  |  |  |  | 1 |  |
| F | 1 |  |  |  |  |  | 1 |
| G |  | 1 |  |  | 1 |  |  |
| H |  | 1 |  |  |  | 1 |  |
| I |  |  | 1 |  | 1 |  |  |
| J |  |  | 1 | 1 |  |  |  |
| K |  |  |  |  | 1 |  |  |
| L |  |  |  |  |  | 1 |  |
| M |  |  |  |  | 1 |  | 1 |

Разложение по столбцу x4:

{B,D}

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **х3** | **x7** | **x8** | **x9** |
| A | 1 |  |  |  |
| E |  |  | 1 |  |
| F |  |  |  | 1 |
| G | 1 | 1 |  |  |
| H | 1 |  | 1 |  |
| I |  | 1 |  |  |
| M |  | 1 |  | 1 |

Покрытие: {B,D,H,M}

{B,I}

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **х2** | **х3** | **х6** | **x8** | **x9** |
| A |  | 1 | 1 |  |  |
| D | 1 |  | 1 |  |  |
| E | 1 |  |  | 1 |  |
| F | 1 |  |  |  | 1 |
| G |  | 1 |  |  |  |
| H |  | 1 |  | 1 |  |
| M |  |  |  |  | 1 |

Покрытие: {B,I,F,A,H}

{C}

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **х2** | **х3** | **х4** | **х5** | **х6** | **x7** | **x8** |
| A |  | 1 |  |  | 1 |  |  |
| B |  |  |  | 1 |  |  |  |
| D | 1 |  | 1 |  | 1 |  |  |
| E | 1 |  |  |  |  |  | 1 |
| F | 1 |  |  |  |  |  |  |
| G |  | 1 |  |  |  | 1 |  |
| H |  | 1 |  |  |  |  | 1 |
| I |  |  | 1 | 1 |  | 1 |  |
| J |  |  | 1 |  | 1 |  |  |
| K |  |  |  | 1 |  | 1 |  |
| L |  |  |  | 1 |  |  | 1 |
| M |  |  |  |  |  | 1 |  |

Разложение по столбцу x2:

{C,D}

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **х3** | **х5** | **x7** | **x8** |
| A | 1 |  |  |  |
| E |  |  |  | 1 |
| G | 1 |  | 1 |  |
| H | 1 |  |  | 1 |
| I |  | 1 | 1 |  |
| J |  |  |  |  |
| K |  | 1 | 1 |  |
| L |  | 1 |  | 1 |

Покрытие: {C,D,H,I}

{C,E}

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **х3** | **х4** | **х5** | **х6** | **x7** |
| A | 1 |  |  | 1 |  |
| D |  | 1 |  | 1 |  |
| G | 1 |  |  |  | 1 |
| H | 1 |  |  |  |  |
| I |  | 1 | 1 |  | 1 |
| J |  | 1 |  | 1 |  |
| K |  |  | 1 |  | 1 |
| L |  |  | 1 |  |  |

Покрытие: {C,E,J,K,A}

Кратчайшее покрытие {C,D,H,I}:

C = {x1,x9} – краска 1

D = {x2,x4,x6} – краска 2

H = {x3,x8} – краска 3

I = {x5,x7} – краска 4