§ 1

Certainly most of the *paradoxical* assertions which we meet with in the area of mathematics, though not all of them as *Kästner* suggests, are propositions that either contain the concept of *infinity* directly, or depend on it in some way for their attempted proof. It is even more indisputable that precisely those mathematical paradoxes which deserve our greatest attention are of this kind. This is because decisions on very important questions in many another subject, such as metaphysics and physics, depend on a satisfactory resolution of their apparent contradictions.

This is the reason why in the present work I am dealing exclusively with the consideration of the paradoxes of the infinite. But it is self-evident that it would not be possible to recognize the appearance of contradiction which is attached to these mathematical paradoxes for what it is, a mere appearance, if we did not make abundantly clear what concept we actually associate with the infinite. Therefore we do this first.

§ 2

The word already indicates that the *infinite* is contrasted [*entgegensetze*] with everything that is merely *finite*. And the fact that the *name* of the former is derived from that of the latter, shows that we think of the *concept* of the infinite as one which arises from that of the finite only through the addition of a new component (such indeed is the mere concept of *negation*). Finally, that both concepts are applied to *multitudes* [*Mengen*], or more specifically to *pluralities* [*Vielheiten*] (i.e. to multitudes of units),[d] therefore also to *quantities*, cannot be denied because it is precisely *mathematics*, i.e. the theory of quantity, where we speak most frequently of the infinite. Since here finite as well as infinite pluralities, and besides *finite* quantities not only *infinitely large* but even *infinitely small* quantities arise as objects of our consideration—and even calculation. Without assuming that both those concepts (namely of the finite and of the infinite) can always only be applied to objects to which *magnitude* and *plurality* can be referred in some respect, we may hope that a more precise investigation of the question of the circumstances in which we define a multitude as finite or as infinite, will also give us information about the *infinite in general*.

§ 3

For this purpose we must nevertheless go back to one of the simplest concepts of our understanding so as to agree first of all on the word we wish to use for its designation. It is the concept which underlies the conjunction '*and*' which I believe, if it is to stand out as clearly as required in countless cases for the purpose of mathematics as well as philosophy, can be expressed most suitably by

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[d] On the translation of *Menge* and *Vielheit* see §4 and the *Note on the Translations*.

the words: *a collection* [*Inbegriff*] *of certain things* or *a whole* [*Ganze*] *consisting of certain parts*. That is, if it is agreed that we wish to interpret these words in such a wide sense that they may be asserted in all propositions where the conjunction '*and*' is usually used, e.g. in the following: '*The sun, the earth and the moon* have a mutual effect on one another', '*the rose and the concept of a rose* are a pair of very different things', 'the names *Socrates and son of Sophroniskus* designate one and the same person'—the object which is spoken about in these propositions is *a certain collection of things*, a *whole* consisting of certain *parts*. Namely, in the first one, it is that whole which the sun, earth and moon form together of which it is stated that it is a whole whose parts have mutual effect on one another. In the second proposition it is the collection which the two objects 'the rose and the concept of a rose' jointly make up of which it is judged that they are two very different things etc. These few examples should already be enough for agreement about the concept spoken of here, at least if we add that any arbitrary object A can be combined with all the other arbitrary objects $B, C, D, \ldots$ into a collection or (to speak more correctly) already forms a collection in itself. About this collection several more or less important truths can be stated provided each of the ideas $A, B, C, D, \ldots$ does in fact represent *another* object, or provided none of the propositions 'A is the same as B', 'A is the same as E', 'B is the same as C' etc. is true. For if, for example, A is the same thing as B, then it is of course absurd to speak of a collection of the things A and B.

§ 4

There are collections, which, although containing the same parts $A, B, C, D, \ldots,$ nevertheless present themselves as *different* (we call it essentially different) according to the viewpoint (concept) under which we interpret them. For example, a complete glass and a glass broken into pieces considered as a drinking vessel. We call the basis [*Grund*] for this difference in such collections, the *mode of combination* or *arrangement* of its parts. A collection which we put under a concept so that the arrangement of its parts is unimportant (in which therefore nothing essential changes for us if we merely change this arrangement) I call a *multitude* [*Menge*]. And a multitude whose parts are all considered as *units* of a certain kind A, i.e. as objects which come under the concept A, is called a *plurality* [*Vielheit*] of A.

§ 5

It is well known that there are also collections whose parts themselves are compound, i.e. are again collections. Among these are also such as we consider from a viewpoint for which nothing essential changes in them if we conceive the parts of the parts as parts of the whole itself. I call them, with a word borrowed from mathematicians, *sums* [*Summen*]. For it is just the concept of a sum that it must be that $A + (B + C) = A + B + C$.

§ 6

If we consider an object as belonging to a kind [*Gattung*] of thing of which every two, M and N, can have no other relationship to one another than that they are either equal to one another, or that one of them presents itself as a sum which includes a part equal to the other one. That is, that either $M = N$ or $M = N + \nu$ or $N = M + \mu$, where the same must again hold of the parts ν and μ, namely that they are either equal to one another, or one is to be viewed as a part contained in the other, then we consider this object as a *quantity* [*Größe*].

§ 7

If a given collection of things $\ldots, A, B, C, D, E, F, \ldots, L, M, N, \ldots$ has the property that for every part M some one, and only one, other part N can be identified of a kind that we can *determine* by the *same rule* for all parts of the collection either N by its relationship to M, or M by its relationship to N, then I call this collection a *series* [*Reihe*] and its parts the *terms* of this series. I call that rule by which either N is determinable through its relationship to M, or M is determinable through its relationship to N, the *rule of formation* [*Bildungsgesetz*] of the series. One of these terms, whichever one wants, I call (without wishing to designate by this name the concept of an actual sequence in time or space) the *previous* or *preceding* term, the other the *following* or *succeeding* term. Every term M which has a previous term, as well as a following term, i.e. which is not only itself derivable from another but from which also again another term is derivable according to the rule of formation holding for the series, I call an *interior* term of the series, from which it is self-evident which terms, if they exist, I call *exterior*, the *first* or the *last* term.*

§ 8

Let us imagine a *series* of which the *first* term is a *unit* of the kind A, but every succeeding term is derived from its predecessor by our taking an object equal to it and combining it with a new unity of kind A into a sum. Then obviously all the terms appearing in this series—with the exception of the first which is a *mere unit* of the kind A—are *pluralities of the kind A* and in fact these are such as I call *finite* or *countable pluralities*, indeed I call them straightforwardly (and even including the first term) *numbers* [*Zahlen*], and more definitely, *whole* numbers.

* More precise discussions about this, as also about some of the concepts put forward in the previous paragraphs, are found in the *Wissenschaftslehre*.[e]

[e] Explanations of the terms for collections are given in *WL* §§ 82–86.

§ 9

According to the different nature of the concept designated here by *A* there may sometimes be a greater and sometimes a smaller multitude of objects which it comprehends, i.e. the units of the kind *A*. And therefore there is sometimes a greater and sometimes a smaller multitude of terms in the series being discussed. In particular there can even be so many of them that this series, to the extent that it is to exhaust *all* these units (taken in themselves), may have absolutely *no last term*. We shall prove this in more detail in what follows. Therefore assuming this for the time being I shall call a plurality which is greater than every finite one, i.e. a plurality which has the property that every finite multitude represents only a part of it, an *infinite plurality*.

§ 10

I hope it will be granted that the definition put forward here of both the concepts of a *finite* and of an *infinite* plurality truly determine the difference between them as intended by those who have used these expressions in a strict sense. It will also be granted that there is no hidden circularity in these definitions. Therefore it only a question of whether through a mere definition of what is called an infinite *plurality* we are in a position to determine what is [the nature of] the *infinite* in general. This would be the case if it should prove that, strictly speaking, there is nothing other than pluralities to which the concept of infinity may be applied in its true meaning, i.e. if it should prove that infinity is really only a property of a plurality or that everything which we have defined as *infinite* is only called so because, and in so far as, we discover a property in it which can be regarded as an infinite plurality. Now it seems to me that is really the case. The mathematician obviously never uses this word in any other sense. For generally it is nearly always quantities with whose determination he is occupied and for which he makes use of the assumption of one of those of the same kind for the *unit*, and then of the concept of a number. If he finds a quantity greater than every number of the unit taken, then he calls it *infinitely large*; if he finds one so small that every multiple of it is smaller than the unit, then he calls it *infinitely small*. Outside these two classes of infinities and the kinds further derived from them of infinitely greater and infinitely smaller *quantities of higher order*, which all proceed from the same concepts, there is no other infinity for him.

§ 11

Now some philosophers, particularly of more recent times, like *Hegel* and his followers, are not satisfied with this infinity so well known to mathematicians. They call it contemptuously 'the bad infinity' and claim to know a much higher one, the true, the *qualitative infinity* which they find especially in *God* and generally only in the *absolute*. If they, like *Hegel*, *Erdmann* and others, imagine the mathematical infinity only as a quantity which is *variable* and has no limit to its growth (which is, of course, as we shall soon see, what some mathematicians have put forward as the definition of their concept), then I would agree with them in their criticism of this concept of a quantity itself never *reaching* but only *growing* into infinity. A *truly infinite* quantity, e.g. the length of the whole straight line unbounded in both directions (i.e. the magnitude of that spatial thing which contains all points which are determined by their merely conceptual relationship to two given points), needs precisely not to be variable, as it is in fact not in the example mentioned. A quantity which can always be taken greater than it has already been taken, and may become greater than every given (finite) quantity can nevertheless always remain a merely finite quantity, as holds in particular of every number quantity [*Zahlgröße*] 1, 2, 3, 4, ... What I do not concede is merely that the philosopher may know an object on which he is justified in conferring the predicate of being infinite without first having identified in some respect an infinite magnitude or plurality in this object. If I can prove that even in God as that being which we consider as the most perfect unity, viewpoints can be identified from which we see in him an infinite plurality, and that it is only from these viewpoints that we attribute infinity to him, then it will hardly be necessary to demonstrate further that similar considerations underlie all other cases where the concept of infinity is well justified. Now I say we call God infinite because we concede to him powers of more than one kind that have an infinite magnitude. Thus we must attribute to him a power of knowledge that is true omniscience, that therefore comprehends an infinite multitude of truths because all truths in general etc. And what would be the concept that anyone would want to press upon us in place of the concept of true infinity put forward here? It should be the universe [*das All*], which comprehends every possible thing, the absolute universe, apart from which there is nothing. According to this statement there would be an infinity which included, according to our definition, infinitely many things. It would be a collection of not only all actual things, but also all those things which have no reality, the propositions and truths in themselves. And thus even with all the other errors in mind which are mixed up in this theory of the universe there should be no basis for abandoning our concept of the infinite so as to adopt that other one.

§ 12

I cannot also help rejecting as incorrect many other definitions of infinity, which have been proposed even by mathematicians in the opinion that they present only the components of this one and the same concept.

1. In fact, as I have just mentioned earlier, some mathematicians, among them even *Cauchy* (in his *Cours d'Analyse* and many other writings), and the author of the article '*Unendlich*' in *Klügel*'s *Wörterbuch*, have believed infinity to be defined if they describe it as a variable quantity whose value increases *without bound* and which can be proved to become greater than *every given quantity however large*. The *limit* of this unbounded increase is the *infinitely large quantity*. Thus the tangent of a right angle, thought of as a continuous quantity, is unbounded, without end,

and in the *proper sense infinite*. The mistakenness of this definition is clear from the fact that what mathematicians call a *variable quantity* is not really a quantity but is the mere concept, the mere *idea* of a quantity, and in fact such an idea that is concerned not with a single quantity but an infinite multitude of quantities differing from one another in value, i.e. quantities distinguishable by their *magnitude* [*Großheit*]. What is called infinite are indeed not those *different* values which, in the example mentioned here, are represented by the expression tang. ϕ for different values of ϕ, but only that single value which is imagined (although wrongly in this case) that that expression takes for the value $\phi = \frac{\pi}{2}$. Also it is certainly a contradiction to speak of the limit of an unbounded increase, and equally for the definition of the infinitely small, of the limit of an unbounded decrease. And if the infinitely large is defined by the former, then by analogy the infinitely small should be defined by the latter, i.e. the mere *zero* (a nothing). But this is certainly incorrect and neither *Cauchy* nor *Grunert* allow themselves to say it.

2. If the definition just considered was too wide, in contrast that adopted by Spinoza and many other philosophers as well as mathematicians, that *only that is infinite which is capable of no further increase*, or to which nothing more can be attached (added), is much too narrow. The mathematician is allowed to add to every quantity, even infinitely large ones, other quantities, and not only finite ones but even other quantities which are already infinite. Indeed he may even multiply the infinitely large infinitely many times etc. And if some dispute whether this procedure is even a legitimate one, which mathematicians, providing they do not reject everything infinite, will not have to admit that the length of a straight line which is bounded on only one side while on the other side it continues indefinitely, is infinitely large and nevertheless can be increased by additions on the first side?

3. No more satisfactory is the definition of those who adhere precisely to the components of the word and say infinite is *what has no end*. If they think thereby only of an end in time, a cessation, then they could only call things which are in time, finite or infinite. However we also ask about things which are not in time, e.g. lines or quantities in general, whether they are finite or infinite. But if they take the word in a wider sense, roughly equivalent to *limits* in general, then I point out *firstly* that there many objects for which one cannot reasonably show that a limit exists for them without attributing to the word a highly unreliable, confused meaning, and which nevertheless nobody counts as infinite. Thus every simple part of time or space (a point in time or space) has no limit, but is instead usually considered itself only as a limit (of a time interval or line), indeed most of them are directly defined so that this belongs to their nature. But it occurs to nobody (unless they are Hegel) to wish to see an infinity in a mere point. Just as little does the mathematician regard the circumference of a circle and many other lines and surfaces which turn back on themselves as a limit and consider them only as finite things. (It would have to be that he may come to speak of the infinite multitude of the points contained in them, and in that respect he must also recognize in every bounded line something infinite.) *Secondly*, I remark that there are many objects which are undeniably bounded, but are regarded as quantities belonging to the

infinite. It is so not only with the straight line already mentioned earlier, which only extends into infinity on one side, but also with the surface area which a pair of infinite parallel lines encloses between them, or the two indefinitely extended arms of an angle drawn in the plane, and several others. Thus also in rational psychology we shall call an intellect infinitely large if, even without being omniscient, it is just capable of surveying some infinite multitude of truths, e.g. just the complete infinite series of decimal places which the single quantity $\sqrt{2}$ contains.

4. Most commonly what is called infinitely large is what is greater than every quantity that *could be given* [*angebliche Größe*]. Here we need most of all a more exact determination of what is in mind with the words '*could be given*'. Should it only mean that something is *possible*, i.e. *can* have reality, or only that it is *nothing contradictory*? In the first case, the concept of *finite thing* is limited solely to that kind of thing which has *real existence* [*Wirklichkeit*], either they are real at all times, or have been or will be real at certain times, or at least *could* become real at some time. In fact it is in this sense which *Fries* (*Naturphilosophie*, §47) seems to have taken the infinite when he calls it the *incompletable*. But usage applies the concept of finite and also that of infinite both to objects which have real existence, like God, and also to others which cannot be spoken of as having any existence at all such as the pure propositions and truths in themselves, together with their components the ideas in themselves, since we assume finite as well as infinite multitudes of them. But if by 'what could be given' is understood everything which is just *not contradictory*, then one already puts into the definition of the concept that there may be no infinity, for a quantity which is to be greater than every [quantity] which is not contradictory, would also have to be greater than itself, which is, of course, absurd. However there is still a third meaning in which the words '*could be given*' could be taken, if one understood by them only such a thing as *can only be given to us*, i.e. can become an object of *our experience*. But I ask everyone whether—if a beneficial use is to be made of it in science—he does not in any case take the words '*finite*' and '*infinite*' in a sense, and he must necessarily adopt only such a sense, that they refer to a certain *internal* property of the object which we call thus, but in no way do they refer to a mere relationship of it to our *perception*, even to our *sense awareness* (whether we may be able, or not, to have *experiences* of it). Thus the question of whether something is finite or infinite can certainly not depend upon whether the object in question possesses a quantity which we are able to perceive (for example, to look at, or not).

§ 13

If we have now come to agreement on which concept we shall associate with the word '*infinite*' and if we have also made clear the components from which we compose this concept, then the next question is whether it also has *objectivity*,[f] i.e. whether there are also things to which it can be applied, multitudes, which we

[f] See the footnote on p. 596.

may call infinite in the sense defined? And I venture to *affirm* this categorically. In the *realm of those things which make no claim to reality but only to possibility*, there are indisputably multitudes which are infinite. *The multitude of propositions and truths in themselves* is, as may very easily be seen, infinite. For if we consider some truth, perhaps the proposition that there are actually truths, or otherwise any arbitrary truth, which I shall designate by *A*, then we find that the proposition that the words '*A is true*' express is different from *A* itself, for the latter obviously has a completely different subject from the former. Namely, its subject is the whole proposition *A* itself. However, by the rule by which we derived from the proposition *A*, this different one, which I shall call *B*, we can again derive from *B* a third proposition *C*, and continue in this way without end. The collection of all these propositions in which each successive one stands in the relationship just given to the one immediately before it, in that it makes it its subject and states of it that it is a true proposition, this collection—I say—comprises a multitude of parts (propositions) which are greater than every finite multitude. For without my reminder the reader may notice the similarity between the series of these propositions formed by the rule just given, and the *series of numbers* considered in §8. This is a similarity consisting in this, that to every term of the latter there is a term of the former corresponding to it, that therefore for every number, however large, there is also a number of distinct propositions equal to it, and that we can always form new propositions, or to say it better, that there are such propositions in themselves regardless of whether we form them or not. Whence it follows that the collection of all these propositions has a plurality which is greater than every number, i.e. is infinite.

§ 14

Nevertheless, simple and clear as the proof just given is, there are a considerable number of scholarly and intelligent men who declare the proposition which I believe I have proved here, to be not only paradoxical but downright false. They deny that there *is any infinity*. According to their claim, not only among things which have reality but also among the others, there is no single thing, not even a collection of several things, for which an infinite multitude of parts could in any respect be assumed. We shall consider later the arguments which they raise against infinity in the realm of reality because we shall also bring forward later the reasons for the existence of such an infinity. Therefore let us examine here the arguments through which it is to be proved that there may never be something infinite, not even among the things which make no claim to being real.

1. They say, 'There can never be an infinite multitude, just for this reason because an infinite multitude *can never be united into a whole, can never be gathered together in thought*.' I must immediately call this assertion an error which is produced by the false view that in order to think of a whole consisting of certain objects *a*, *b*, *c*, *d*, ... one would first have to have formed *ideas* which represent each one of

these objects individually (individual ideas of them). It is definitely not so; I can imagine the multitude, or the collection if preferred, the *whole* [*Ganze*] of the inhabitants of Prague or of Beijing without imagining each of these inhabitants individually, i.e. through an idea corresponding exclusively to each one. I am actually doing this just now, since I am speaking of this very multitude, and, for example, make the judgement, that their number in Prague lies between the two numbers 100 000 and 120 000. That is, as soon as we possess an idea A which represents each of the objects $a, b, c, d, \ldots$, but nothing else, it is extremely easy to reach an idea which represents the *collection* which all these objects make up together. In fact nothing extra is needed other than the concept which the word '*collection*' denotes, connected with the idea A in such a way as indicated by the words: *the collection of all A*. By this single remark, whose correctness I believe must be clear to everyone, all difficulty which may be found with the concept of a multitude if it consists of infinitely many parts, is removed. As soon as a category [*Gattungsbegriff*] for each of these parts exists, but which covers nothing else, as is the case with the concept: '*The multitude of all propositions or truths in themselves*', where the required category is already: 'a proposition or truth in itself'. However, I cannot leave uncriticized a *second* error which is revealed in that objection.

It is the opinion, 'that a multitude would not exist unless first somebody were to exist who *conceives* it'. Whoever asserts this, in order to be as consistent as one can actually be with an error, should not only assert that there may be no *infinite* multitude of propositions and truths in themselves, but he should assert that actually there may *not be any* propositions and truths in themselves *at all*. For if we have brought about a clear awareness in ourselves of the concept of propositions and truths in themselves and do not in fact doubt the objectivity of them, then we could hardly make assertions like the one just mentioned, and could certainly not persist with them. In order to show this in a way clear to everybody, I permit myself to put the question whether there do not exist at the poles of the earth fluid as well as solid bodies, air, water, rocks and such like, whether these bodies do not act upon another according to certain laws, e.g. that the speeds which they impart to one another on impact are inversely proportional to their masses and such like, and whether all these things occur even if no person, or any other thinking being, is there to observe it? If one agrees with this (and who would not have to agree?) then there are also propositions and truths in themselves which express all these proceedings without anyone knowing and thinking them. And in these propositions there is frequent reference to wholes and multitudes, for every body is a whole and produces many of its effects only through the multitude of parts of which it consists. Therefore there are multitudes and wholes without the presence of a being which conceives them. And if this were not so, if these multitudes were not there themselves, how could the judgements which we make about them be true? Or rather, what would be the meaning of these judgements if they should only become true if somebody is there who perceives these proceedings? If I say, 'This boulder broke off from that cliff in front of my eyes, cut through the air, and crashed down below,' this would have to have roughly the following meaning: While I thought of certain simple entities together up there, a combination of them arose which I call a boulder, this combination withdrew from certain others, which, while I think of them together, united into a whole which I call a cliff, etc.

2. However, one might say, 'for all this it remains true that it is only *our act* [*Werk*], and in fact a largely very arbitrary act, whether we want to think of certain simple objects together in a collection or not, and only if we do this first do relationships arise between them. The central particle in this button on my coat and the central particle in the top of that tower there have nothing to do with one another and have no connection with one another at all, only through my present thinking of them together does any kind of connection between them originate.' Even this I must contradict. The two particles were, even before the thinking being put together their ideas, in mutual effect on one another, e.g. through the force of attraction and such like, and if, on the other hand, that thinking being does not, by virtue of his thoughts, also adopt actions which produce a change in the relationships between the two particles, then it is absolutely untrue that it is only through that thinking of them together that relationships arise among them, which apart from this would not be there. If I should judge truly that the former [particle] is lower, and the latter is higher, and that therefore the latter may be pulled up by the former by some small amount in height etc, then all this would have to be the case even if I had not thought about it, etc.

3. Other people say, 'It is not the case that for a collection to exist it is necessary for it to have *actually been thought* by a thinking being, but rather it is necessary that it *could* be thought. Now because no being is possible that can imagine each one of an infinite multitude of things individually and then connect these ideas, then also no collection which comprises an infinite multitude of things as parts in itself is possible.'

We have already seen in no.1 how much in error is the assumption that is repeated here, that for the thinking of a collection the thinking of all its parts individually, i.e. the thinking of each individual part by means of a single idea representing it, is required. Also we do not need to refer at the outset to the omniscient being as such a being for which the conception of an infinite multitude of things, each one individually, causes no trouble. However, we may not even grant the first assumption, namely that the existence of a collection of things rests on the condition that such a collection *can be thought of*. For the '*capacity of a thing to be thought of*' can never include the basis of its possibility, instead it is exactly conversely that the possibility of a thing is firstly the basis on which a reasonable being, providing it is not mistaken, and the thing is *possible*, *can think* it, or as we say (but improperly) finds it *thinkable*. One will be even more convinced of the complete correctness of this remark and the fact that the admittedly very widespread view which I am attacking here is completely untenable, if one tries to clarify the components of which the highly important concept of possibility consists. That one calls that which is *possible*, what *can* be, is obviously not an analysis of this concept, for the concept of possibility is altogether involved in the word '*can*'. But it would be still more incorrect to wish to set up the definition that that is possible *which can be thought*. We can even think, in the true sense of the word where it concerns mere *representation*, of the impossible, and we actually think it whenever we judge about it and e.g. explain something as impossible, as when we say that there is, and can be, no quantity which represented by 0 or $\sqrt{-1}$. But even if one understands by thinking here not a mere representing but an actual *asserting* [*Fürwahrhalten*], it is false that everything is possible which we can assert as true. By mistake we sometimes even hold the impossible, e.g. that we had found the square of the circle, as true. Therefore it would have to be said (as I already adopted in modified form above) that is possible about which a thinking being, if it judged the truth appropriately, expresses the judgement that it can be, i.e. that it is possible. A definition which contains an obvious circularity! We are therefore required to drop completely the reference to a thinking being for the definition of the possible and look for another characteristic. One sometimes hears people say that 'possible' is '*what does not contradict itself*'. Of course, everything which already contains a contradiction within itself, e.g. that a sphere is not a sphere, is impossible. But not everything impossible is of such a kind that the contradiction which is already in the components from which we have composed the idea of it, is found. It is impossible that a solid which is enclosed by seven plane polygonal surfaces, may be enclosed by equal polygonal surfaces. But the contradictory nature does not lie open to view in the words which are connected here. We must therefore extend our definition further. But if we want to say that the impossible is what stands in contradiction to some truth, then we would by this be defining everything which is not, as impossible, because the proposition that it is, would contradict the truth, that it is not. We would therefore admit no difference between the possible and the actual, and even the necessary, which nevertheless we all do distinguish. Accordingly we see the domain of truths which the impossible contradicts must be limited only to a certain class, and now we can hardly fail to notice which class of truths this is. They are the pure conceptual truths [*Begriffswahrheiten*]. Whatever some pure conceptual truth contradicts is called the *impossible*. Therefore the possible is what stands in contradiction with no pure conceptual truth. Whoever has once realized that this is the correct concept of possibility, to them it can hardly occur to make the assertion that something is only possible if it is thought, i.e. is viewed as possible by a thinking being which does not err in its judgement. For this is to say: 'A proposition contradicts no pure conceptual truth if it contradicts no pure conceptual truth that there is a thinking being which judges of this proposition the truth that it contradicts no pure conceptual truth.' Who does not see how irrelevant here is this addition of a thinking being? But if it is decided that the *thinking* does not make the possibility, where is there some reason for concluding from the supposed circumstance that an infinite multitude of things cannot be *thought together* that such multitudes cannot exist?