Solution Manual for Switching and Statistical Multiplexing in Telecommunications
Text Book: Telecommunication Switching Systems and Networks – Thiagarajan Viswanathan

1. Introduction

Q1. A fully connected network supports full duplex communications using unidirectional links. Show that the total number of links in such a network with n nodes is given by 2 x $^{n}C_2$

Ans: Number of links in a point to point link with n nodes is given by:

$$\frac{n(n-1)}{2}$$

For a full duplex unidirectional links, total number of links required is:

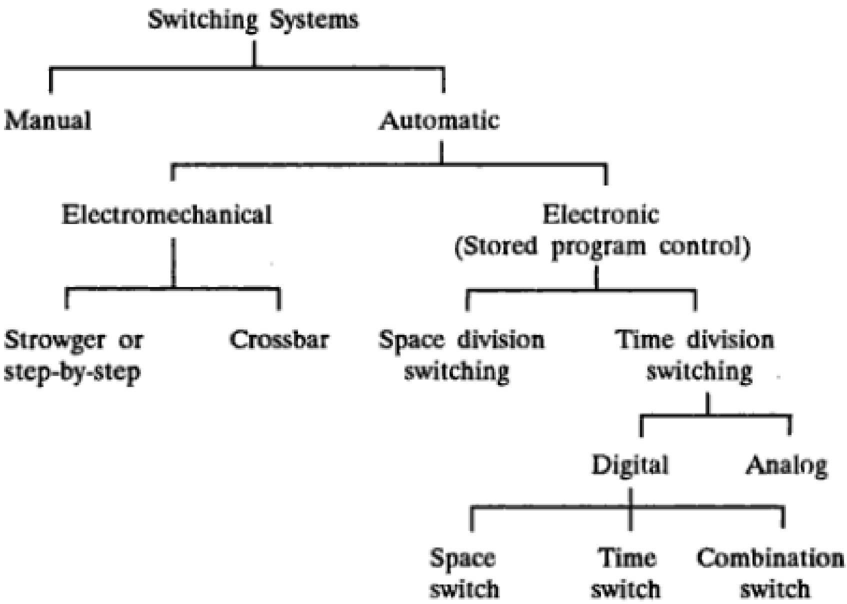
$$2 \times \frac{n(n-1)}{2}$$

$$^{n}C_2 = \frac{n!}{2!\,(n-2)!} = \frac{n(n-1)(n-2)!}{2!\,(n-2)!} = \frac{n(n-1)}{2}$$

Therefore, total number of links = 2 x $^{n}C_2$

Q2. How are switching systems classified? In what way is stored program control superior to hard-wired control?

Ans: Switching systems are classified as follows:



1. New facilities can be added to SPC program by changing control parameters but it is virtually impossible to modify the hard-wired subsystems to provide new functionalities.
2. Hard-wired subsystems have limited capacity, but SPC has higher capacity.
3. In SPC control functions are performed by the computer or processor rather than hardware.

Q3. Estimate the bandwidth requirements of a single satellite that is to support 20 million telephone conversations simultaneously.

Ans: Each telephone conversation takes 64 kbps bandwidth.

For 20 million telephone conversations the bandwidth required is given by:

Bandwidth = 20 X 10^6 X 64 kbps = 20 X 10^6 X 64 X 10^3

= 1280 X 10^9 = 1.280 Gbps

Q4. An electrical communication system uses a channel that has 20 dB loss. Estimatethe received power if transmitted power is 1 watt.

Ans: Channel loss = 20 dB

Transmitted power = 1 watt

Loss = $10 \log_{10}(P_{in}/P_{out})$

$20 \text{ dB} = 10 \log_{10}(1/P_{out})$

$\log_{10}(1/P_{out}) = (20/10) = 2$

$(1/P_{out}) = 10^2$

$P_{out}=1/100 = 0.01 = 10 \text{ mW}$

Q5. If the signal input to an amplifier is 0 dBm, what is the power output in mW if the gain of the amplifier is 20 dB?

Ans: We know that dBm = $10 \log_{10}(P_{in})$ mW

$0 \text{ dBm} = 10 \log_{10}(P_{in})$

$P_{in} = 1 \text{ mW}$

Gain = P_{out}/P_{in}

$100 = P_{out} / 1 \text{ mW}$

$P_{out} = 100 \text{ X } 10^{-3} = 100 \text{ mW}$

Q6. The channel interfaces in a point-to-point communication system attenuate the signal by 3 dB each. The channel has a loss of 30 dB. If the received signal is to be amplified such that the overall loss is limited to 20 dB, estimate the amplifier gain.

Ans: Overall loss at the amplifier = sum of all losses = gain of the amplifier gain = -20 dB

-20 dB = -(3 dB + 3 dB + 30 dB) + gain

Gain = 16 dB

Q7. If the noise power in a channel is 0.1 dBm and the signal power is 10 mW, what is the SNR?

Ans: $0.1 \text{ dBm} = 10 \log_{10}(P)$

$\log_{10}(P_n) = 0.1 / 10 = 0.01$

$P_n = 10^{0.01} = 1.02329 \text{ mW}$

Signal power $P_s = 10$ mW

SNR = P_s / P_n

SNR = 10 mw / 1.02392 mW = 9.77 Watt

Q8. What is the significance of SNR being -3 dB?

Ans: SNR will be -3 dB when the ratio of $P_{in}/P_{out} = 0.5$

Or the output power is half of the input power

i.e. $P_{in} = 2 P_{out}$

Q9. For a carbon granule microphone, determine a suitable value for m, if the contribution from each of the higher order terms is to be less than 0.01 I_0.

Ans: For each of the higher order terms is to be less than 0.01

$I_0 m^2 \sin^2 wt < 0.01 I_0$

i.e. $m < \sqrt{0.01} / \sin wt$

Since the maximum value for sin*wt* = 1, the value for m must be less than 0.1

Q10. What is the importance of a steady current flowing through a carbon microphone? Is the harmonic distortion affected by a change in the energizing current?

Ans: The carbon granule microphone acts as a modulator of direct current I_0 which is analogues to the carrier wave in AM systems. The quantity m is equivalent to the modulation index in AM. The higher order terms, represents harmonic distortions and hence it is essential that the value of m be kept sufficiently low.

$i = I_0 (1 + \sin wt)$

The alternating current output is zero if the quiescent current I_0 is zero.

Hence the flow of steady current through the microphone is essential and the current is known as energizing current.

Q11. Why it is necessary to keep the magnetic diaphragm in an earphone displaced from its unstressed position? How is this achieved?

Ans: The voice frequency current in the microphone causes variations in the force exerted by the electromagnet, thus vibrating the diaphragm and producing sound waves. Faithful reproduction of the signals by the receiver requires that the magnetic diaphragm be displaced in one direction from its unstressed position. The quiescent current provides this bias. In some circuits, a permanent magnet is used to provide the necessary displacement instead of the quiescent current.

Q12. What happens if the ratio $Ø/Ø_0$ is not very small in the case of an earphone?

Ans: The instantaneous force $F = K (Ø_0^2 + Ø^2 \sin wt + 2 Ø_0 Ø \sin wt)$

If $Ø/Ø_0 << 1$ then

$F = K Ø_0^2 (1 + K_1 I_0 \sin wt)$

The ratio $Ø/Ø_0$ is not very small then the value of F increases hence the force on the diaphragm increases.

Q13. What is the significance of side tone in a telephone conversation?

Ans: The audio signal, heard at the generating end, is called sidetone. A certain amount of sidetone is useful, or even essential. Human speech and hearing system is a feedback system in which the volume of speech is automatically adjusted, based on the sidetone heard by the ear. If no side tone is present, the person tends to shout, and if too much of side tone is present, there is a tendency to reduce the speech to a very low level.

Q14. In the circuit of Fig. 1.8, it is desired that 10% of the microphone signal is heard as sidetone. If the number of turns in the coil P is 200, determine the number of turns in the coil Q and the secondary winding in the earphone circuit. Assume that Z_b is exactly matched to the line impedance on the exchange side.

Ans: 10% of the microphone signal is heard as the sidetone.

Number of turns in the coil P is $N_P = 200$, N_Q =?

Number of turns in the secondary winding, N_X =?

10% of $N_P = 20$

$N_Q = N_P + N_P \times 0.01 = 200 + 200 \times 0.01 = 200 + 20 = 220$ turns

$N_X = N_P + N_Q = 200 + 220 = 420$ turns.

Q15. In a 100 line folded network, how many switching elements are required for nonblocking operation?

Ans: We know that the number of switching elements required for a nonblocking operation is n/2.

n = 100, therefore we need n/2 = 100/2 = 50 switching elements.

Q16. A 1000 line exchange is partly folded and partly nonfolded. 40% of the subscribers are active during peak hour. If the ratio of local to external traffic is 4:1, estimate the number of trunk lines required.

Ans: 4 : 1 = 800 : 200

Switching pattern = 200

400 (folded) + 200 (Unfolded) = 600 patterns

Therefore 800 lines of local traffic and 200 lines of external traffic. For 800 lines of folded traffic, the number of switching pattern = 400 / 2 = 200

Out of 600 patterns, 40% are active.

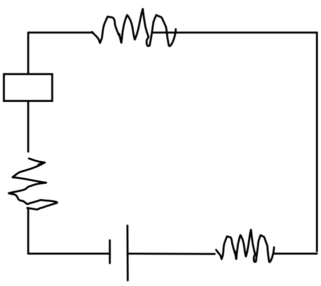
No. of busy lines = 0.4 X 600 = 240

4 : 1 = 192 : 48

192 lines are used for local traffic. 48 lines are used for trunk lines.

Q17. A central battery exchange is powered with a 48 V battery. The carbon microphone requires a minimum of 24 mA as energizing current. The battery has a 400 Ω resistance in series for short circuit protection. The dc resistance of the microphone is 50 Ω. If the cable used for subscriber lines offers a resistance of 50 Ω/km, determine the maximum distance at which a subscriber station can be located.

Ans: Given V = 48 V, I = 24 mA, Rb = 400 Ω, Rm = 50 Ω, Rc = 50 Ω/km. All resistances are in series.



V = IR ➔ R = V / I

$R = 48 / (24 \times 10^{-3}) = 2000\ \Omega$

Total resistance, R = Rb + Rm + Rc * X

2000 = 400 + 50 + 50 * X ➔ X = 31 km

31 km is the distance from the exchange to the subscriber and back to the exchange.

Therefore the maximum distance between the subscriber and exchange = 31/2 = 15.5 km.

Q18. A manual switchboard system needs to support 900 subscribers, numbered 100 – 999. Average peak hour traffic is 250 calls. 130 of them are within the range 400 – 699, 20 of them are between this range and other range of numbers and the remaining are uniformly distributed in the other number ranges. The average lean traffic is 60 calls, of which no call is originated/destined from/to the number range 400 – 699 but uniformly distributed otherwise. An operator is capable of handling 30 simultaneous calls. Suggest a suitable manual switchboard system design that minimizes the total number of terminations at the switchboards and employ the minimum number of operators. Estimate the number of terminations in your design.

Ans: Peak hour = 250 calls. Lean hour = 60 calls

Number of calls in the range 100 – 399 = 50 calls

Number of calls in the range 400 – 699 = 130 + 20 = 150 calls

Number of calls in the range 700 – 999 = 50 calls

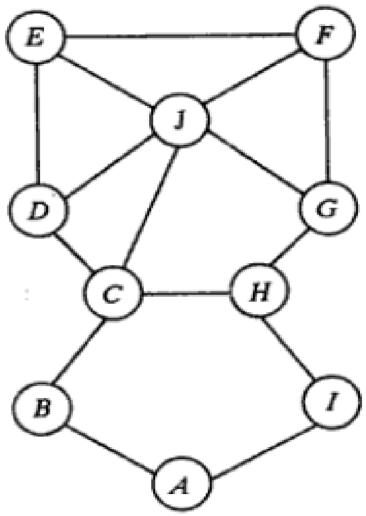
Each operator can handle 30 calls simultaneously.

50	operators	=	1500 calls (Peak Hour)
4	operators	=	120 calls (Peak Hour)
2	operators	=	60 calls (Lean Hour)

2. Crossbar Switching

1. Numbering plan in a telephone network must be independent of call routing. Why? Explain.

Ans: Consider a multi exchange network as shown below:



If a subscriber in exchange A wants to call a subscriber in exchange F, the call has to be routed via at least 3 exchanges and there are two routes possible.

A-B-C-J-F and A-I-H-G-F

And the outlets are assigned as sown below:

From exchange	Outlet	To exchange
A	01	*B*
A	02	*I*
B	04	*C*
C	03	*J*
I	05	*H*
H	01	*G*
G	02	*F*
J	01	*F*

Let 1457 be the subscriber to be called in exchange F and can be reached by calling the following numbers of sequence:

For route *A-B-C-J-F*	01-04-03-01-1457
For route *A-I-H-G-F*	02-05-01-02-1457

Difficulties with this type of network are as follows;

1. Identification number of a subscriber is route dependent.
2. The user must have the knowledge of the topology of the network and outlet assignment in each exchange.
3. Depending upon from which exchange the call originates, the number and its size vary for the same called subscriber.

These difficulties can be overcome by the following:

1. Let exchange the routing.
2. Uniform numbering scheme as far as user is concerned.
3. A number with two parts: An exchange identifier and subscriber line identifier.

Thus the numbering plan in a telephone network must be independent of call routing.

2. What are the differences between common control and direct control?

Common Control	Direct Control
Control subsystems are outside of switching system	Control subsystems are integral part of switching system
Ex. Strowger exchange	Ex. Stored program control
Hard wired control subsystems	Computer or processor control susbsystems
Limited capability	High capability
Impossible to add new features	Easy to add new features by changing control program

3. List 6 events that may occur in a telephone system and the corresponding actions to be taken by the common control system.

Ans: The six events are:

1. Call request/release event at line unit
2. Event sensing
3. Register finding
4. Choosing appropriate register
5. Initial translator

6. Final translator

The occurrence of events is signaled by operating relay which initiate control action. The control subsystem may operate relays in the junctors, receivers/senders and the line units, and thus command these units perform certain functions.

1. Line units handle the call request and call release signals. These line units themselves may initiate control actions on the occurrence of certain line events.
2. When a subscriber goes off hook, the event is sensed; the calling location is determined and marked for dial tone.
3. The register finder is activated to seize a free register. Identity of the calling line is used to determine line category and the class of service to which the subscriber belongs.
4. A register appropriate to the line category is chosen, which then sends out the dial tone to the subscriber, in readiness to receive the dialing information. As soon as the initial digits (usually 2-5) are received in the register, they are passed on to the initial translator for processing.
5. The initial translator determines the route for the call through the network and decides whether a call should be put through or not. It also determines the charging method and the rates applicable to the subscriber. Such decisions are based on the class of service information of the subscriber which specifies details such as the following:
   Call barring, Call priority, Call charging, Origin based routing, No dialing calls
6. If a call is destined to number in an exchange other than the present one processing the digits, the initial translator generates the required routing digits and passes them on to the register sender. Register sender uses appropriate signaling technique, depending on the requirements of the destination exchange. If a call is destined to a subscriber within the same exchange, the digits are processed by the final translator.

4. Calculate the time taken to dial a 12-digit number in a DTMF telephone when

a) The exchange is capable of receiving a DTMF signals; and

b) The exchange can receive only pulse dialing

Compare the result with a rotary telephone dialing.

a) Median tone deviation = 160 ms and inter-digit gap = 350 ms
   Total duration = 510 ms for 1 digit
   For 12 digits, duration = 510 X 12 ms = 6120 ms = 6.120 seconds
b) In pulse the minimum gap requirement is 400-500 ms
   Time taken to receive pulse is,
   If we consider the maximum (worse) case = 500 X 12 ms = 6000 ms = 6 seconds
   Minimum case = 400 X 12 ms = 4800 ms = 4.8 seconds

5. 'Contact bounce' can be a problem in DTMF telephone, i.e. a single press of a push button may be interpreted as more than one press. How does the DTMF dial design take this into account?

Ans: The probability of talk-off can be reduced by increasing the duration of the test applied to a signal by the receiver before accepting the signal as valid. But, it is clearly unacceptable to expect the user to extend the push button operation for this purpose, beyond an interval that is natural to his dialing habit. Fortunately, this requirement doesn't arise as even the fast dialer pauses for about 200 ms between digits, and efficient circuits can be designed to accurately determine the signaling frequencies by testing for a much smaller duration. A minimum of 40 ms has been chosen for both signal and inter-signal intervals, allowing for a dialing rate of over 10 signals per second.

6. Show that the harmonic frequencies of any two adjacent base frequencies in DTMF telephone cannot match within the 15 harmonics.

Ans: Having decided on the frequency band and the spacing, the specific values of the frequencies can be so chosen as to avoid simple harmonic relationships like 1:2 and 2:3 between two adjacent frequencies in the same band and between pairs of frequencies in the two different bands, respectively. The adjacent frequencies in the same band have fixed ratio of 21:19, i.e. only the 21^{st} and 19^{th} harmonic components have the same frequency values. Across the bands, the frequencies that lie along the diagonals have the ratio of 59:34. Thus the chosen frequency values are such that they almost eliminate talk-off possibility due to harmonics.

7. If the transmitted power of the low band frequency signal from a DTMF telephone is 1 mW, what should be the power in mW of the high band frequencies?

Ans: We know that high band frequency is 3 dB more the lower band

3 dB = 10 $\log_{10}$ SNR ➔ $\log_{10}$ SNR = 0.3

SNR = $10^{0.3}$ = 1.995

Power of the high band frequency = 2 mW.

8. A telephone exchange supporting 5000 subscribers uses a DTMF dialing and a common control subsystem with 100 digit receivers. Each digit receiver is assigned for duration of 5 seconds per subscriber call processing. If 20 % of the subscribers attempt to call simultaneously, what is the worst case wait time for a subscriber before he receives the dial tone?

Ans: 20 % of 5000 = 1000 subscribers

Number of digits received by each receiver = 1000/100 = 10 digits

Assuming he is the caller, actually one would on call so, 9 X 5 = 45 seconds.

9. A diagonal cross point matrix exchange supports 500 users. On an average 1000 calls are put through everyday. If the cross point contacts have a mean life of 10000 breaks and makes, estimate as to how often a cross point may be replaced in this exchange.

Ans: In a diagonal cross point matrix, the number of cross points = $\frac{n(n-1)}{2}$ = (500 X 499) / 2 = 124750 cross points

No. of days to replace a cross point = 10000/1000 = 10 days.

10. Estimate the number of cross points required to design an exchange that supports 500 users on a non-blocking basis and 50 transit, outgoing or incoming calls simultaneously.

Ans: N(N+L) = 500(500+50) = 275000

N(2K+L) = 500(2 X 50 + 50) ➔ 2K + L = 650 x 500 = 325000

13. A blocking crossbar switch is to be designed to support 1000 subscribers. If the estimated peak traffic is 10 Erlangs with average holding times of 3 minutes/call, estimate the number of cross points required.

Ans: 1000 subscribers, 10 Erlangs with average holding times of minutes/call

Traffic Intensity = Total Traffic / Total Time

10 = Total Traffic / Total Time

10 = Busy Period / Total Duration

Busy Period = 180x = 10 X 60 / 3

60 + 10 - 3 = 67

10 X 3 = 30 minutes in 1 hour

No. of calls put through = 30

3. Electronic Space Division Switching

1. Define each of the following terms: Program, Procedure, processor, process, user, task, job and subroutine.

Ans: Program: A program is a set of instructions executed in a predefined order and execution is dependent on the conditions in the program.

Procedure: It is a part of the code written in a larger program which performs a specific task and is relatively independent of the remaining code.

Processor: The part of a computer system that interprets the instruction.

Process: A process is an active entity. It can be thought of as a program under execution.

User: A user is a person or a system that uses the system to perform some operation to get the results.

Task: A task is an execution pattern through address space or a set of processes that are loaded in memory.

Job: Job is a single instance of a program.

Subroutine: A section of code that implements a task, while it may be used at more than one point in a program.

2. Blocked processes list is not maintained in priority order. Why?

Ans: Because sometimes processes are waiting for some resources or some I/O services, in such a situation if we give priority to processes that are waiting for some resources that are available or may be some processes are using all the resources, then low level priority processes are blocked, this may lead to deadlock or starvation.

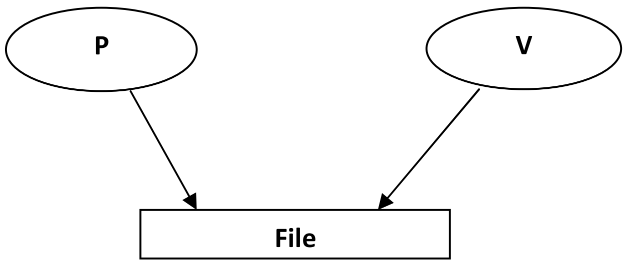
3. In a switching system running thousands of processes, it cannot be easily determined that a process is in infinite loop. What safeguards can be built into the operating system to prevent processes running indefinitely?

Ans: To prevent processes running indefinitely or getting into deadlock following safeguards can be built into operating system.

a. Mutual Exclusion
   - Must hold for non-sharable resources
   - When one process is using the resources the other processes should wait
b. Hold and Wait
   - If a process is requesting a resource then make sure that it doesn't hold any other resources
c. No Preemption (rights to purchase)
   - If a process is requesting for a resource not available immediately then it should release all the resources it usually holds
   - Process will restart after it gets all of the resources
d. Circuit Wait
   - Improve vulnerability on all the resources
   - Process must request all the resources in increasing order of enumeration

4. Show that if P and V operations are not executed atomically (indivisibly), the mutual exclusion may be violated.

Ans: Consider that P and V are write operations. Both P and V are making use of a same file. First P locks the file and writes to it and modifies the file, then V wants to read the original file, but it has been modified by the P modifier and this modification of file is not known to V. It reads the modified file and its results or outcomes are wrong. After that V modifies the file and then P reads the file and its results are also wrong. This situation is called violation of mutual exclusion.



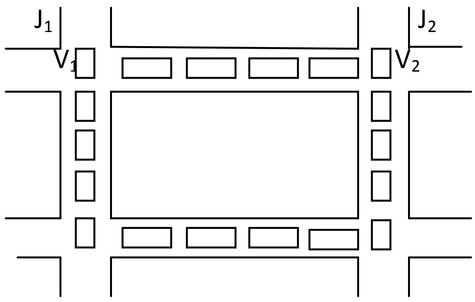
5. Show that a general semaphore can be implemented using binary semaphores.

Ans: Binary Semaphore

Declare: S_1, S_2, int C
Initialize: $S_1 = 1$, $S_2 = 0$, C = Initial value of semaphore
wait(S_1)
C--;
if(C < 0) {
    signal(S_1);
    wait(S_2);
}
signal(S_1);
wait(S_1);
C++;
if(C <= 0)
    signal(S_1);
else
    signal(S_2);

6. Deadlock may occur in a road traffic junction. Illustrate with the help of a diagram.

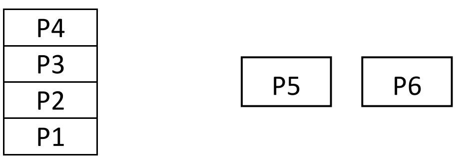
Ans: Let J_1, J_2, J_3, J_4 are traffic junction and V_1, V_2, V_3, V_4 are the vehicles.



At the junction J1, vehicle V1 is blocking the vehicles that are going towards top direction at the same time at junction J2, vehicle V2 is blocking vehicles that are going towards right or down. The same way other two junctions are also blocked be vehicles V3 and V4 at junction J3 and J4. No in this situation none of the vehicles have a chance to move either ways, thus deadlock occurs.

8. Semaphore waiting lists are usually implemented in FIFO order. What problems do you foresee if they are implemented in LIFO order?

Ans: As the name indicates the processes that are last, will be removed from the list first. Processes that are entered first will be removed last.



In the above figure P1, P2, P3 and P4 are waiting to enter into the semaphore. As P1 comes first, it is inserted into the stack. Similarly P2, P3, P4 are pushed into the stack as they come. The stack becomes full. At some time P4 gets channel to enter the semaphore and it removed from stack. Next P5 and P6 enter into the stack. P2, P1 remain in the stack. Next P6 enters the semaphore. If the same continues for further more processes, then P1 and P2 will remain in the stack forever. This situation is called starvation.

9. A local switching system has a capacity of SC. When there are no calls the SPC processor is idle and is activated when a call arises. If a call arrives when the processor is busy, it is put on a wait provided there is a spare capacity available in the system. Otherwise, the call is considered lost. Write a program to coordinate the SPC processor function and the call arrivals.

Ans: Program to simulate the SPC processor function:

```
SEMAPHORE SPC_BUSY = SC
SPC_USAGE() {
        while(true) {
                if(Active Call == true)  {
                        wait(SPC_BUSY);
                        Process Call();
                        signal(SPC_BUSY);
                } else {  // Idle
                        signal(SPC_BUSY);
                        // do nothing
                }
                wait(Semaphore S);
        }
        if(SC == 0) {
                ref = semaphore();
                if(ref == 0) {
                        // could not use the case
                        // drop it
                        Return;
                }
        }
}
```

11. A three stage switching structure supports 128 inlets and 128 outlets. It is proposed to use 16 first stage and third stage matrices.

A) What is the number of switching elements in the network if is nonblocking.

B) At peak periods, the occupancy rate of an inlet is 10%. If the number of switching elements required for nonblocking operation is reduced by a factor of 3, what is the blocking probability of the network?

Ans: Number of inlets n = 128, number of blocks r = 16,

Number of inlets in each block, p = N/r = 128/16 = 8

Number of switching elements S = 2p-1 = 2*8 – 1 = 15

We know that $S = spr + sr^2 + spr = 15*8*16 = 15*16^2 + 15*8*16 = 7680$

Occupancy $\alpha = 10/100 = 0.1$

Blocking probability $P_B = [1 - (1 - \alpha/k)^2]^{s'}$

S' = 7680/3 = 2560   and k = 3

$P_B = 0.002$

12. Determine the switch advantage ratio of a three stage network with N inlets and N outlets for the cases when (a) N = 128 (b) N = 32768

Ans:

(a) N = 128

For single stage network

$S_1 = N^2 = 28^2 = 16384$

For three stage networks

$S_3 = 4N * \text{sqrt}(2N) = 4*128*\text{sqrt}(2*128) = 8192$

Switch advantage ratio = 16384/8192 = 2

(b) N = 32768

For single stage network

$S_1 = N^2 = 32768^2 = 1.07 * 10^9$

For three stage networks

$S_3 = 4N * \text{sqrt}(2N) = 4*32768*\text{sqrt}(2*32768) = 33.55 * 10^6$

Switch advantage ratio = $1.07 * 10^9/33.55 * 10^6 = 31.88 = 32$

13. A three stage network is designed with the following parameters: M = N = 512, p = q = 16 and α = 0.7. Calculate the blocking probability of the network if (a) s = 16 (b) s = 24 (c) s = 31 using the Lee equation. Determine the inaccuracy of the result in case of (c).

Ans:

α = probability that a given inlet is active

k = No. of links between first-second stage pairs

M = N = 512 p = q = 16 α = 0.7

(a) s = 16

s = k * p

k = s/p = 16/16 = 1

$P_B = [1 - (1 - \alpha/k)^2]^s = 0.22$

(b) s = 24

k = s/p = 24/16 = 1.5

$P_B = [1 - (1 - \alpha/k)^2]^s = 0.0032$

(c) s = 31

k = s/p = 31/16 = 1.93

$P_B = [1 - (1 - \alpha/k)^2]^s = 8.6 * 10^{-8}$

14. The Jacobaeus equation for the blocking probability in a 3 stage network is given by

$$P_B = \frac{(p\,!)^2}{s\,!\,(2p-s)\,!}\alpha^s(2-\alpha)^{2p-s}$$

Where symbols have the same meaning as in Eqs. 4.18 and 4.19. For the three stage parameters and the cases given in Exercise 13, calculate the blocking probabilities. Compare the results with those obtained in Exercise 13.

Ans: M = N = 512 p = q = 16 α = 0.7

(a) s = 16

$$P_B = \frac{(16!)^2}{16! * (32-16)!} * 0.7^{16} * (2-0.7)^{2*16-16}$$

$P_B = 0.22$

(b) s = 24

$$P_B = \frac{(16!)^2}{24! * (32-24)!} * 0.7^{24} * (2-0.7)^{2*16-24}$$

$P_B = 2.73 * 10^{-3}$

(c) s = 31

$P_B = 1.09 * 10^{-12}$

15. Determine the design parameters of a three-stage switch with inlet utilization of 0.1 to achieve a PB = 0.002 for (a) N = 128 (b) N = 2048 and (c) N = 8192.

Ans:

(a) N = 128

$P_B = [1 - (1 - \alpha/k)^2]^s$

$0.002 = [1 - (1 - 0.1/k)^2]^{10}$

k = 0.322

16. Using the Lee graph, show that the blocking probability of a Five-stage network is given by

$$P_B = [1 - (1 - \alpha_1)^2 [1 - \{1 - (1 - \alpha_2^2)s_1\}]^s$$

Where $\alpha_1 = \alpha(p/s)$, $\alpha 2 = \alpha_1(r/r_1s_1)$, α is the probability that an inlet is active and r, r_1 and s_1 have same significance as in Fig. 4.27

Ans: For the 5 stage we have considered we have 1st stage, 1 intermediate stage, 3 intermediate stages and one output stage.

Let β = probability that a link is busy

β' = probability that a link is free

$\beta = 1 - \beta'$

If there are 's' parallel links in 2nd stage, the blocking probability that all the links are busy is

$P_B = \beta^S$ $\quad Q_B = 1 - P_B = 1 - \beta^S$

When a series of s links are needed to complete a connection, the blocking probability is determined one minus the probability that they are available

$P_B = 1 - (\beta')^S = 1 - (1 - \beta)^S$

There are 2 links in a series for every path and there are s parallel paths. Therefore

$$P_B = [1 - (\beta')^2]^s = [1 - (1 - \beta)^2]^s$$

Let $\beta = \alpha_1 = p\alpha/s$, where α is the probability that an input line is active. Therefore

$P_B = [1 - (1 - \alpha_1)^2]^s$

The probability indicates only blocking at the 1st and last stage. The intermediate stage consists of a 3 stage network, since the 1st intermediate and last stage form a 3 stage network. The same result can be used for the intermediate 3 stage network.

For this intermediate stage p i.e. input to each stage is equal to r and s is same as r_1s_1.

Therefore probability that a link is busy = $\alpha_2 = \alpha_1 r/r_1s_1$

Probability that a link is available = $(1 - \alpha_2)^2$ with 2 links in series. Also the 3rd stage links to be available links are available.

Therefore probability that a link is available in 3rd stage network is

$P_B = (1 - \alpha_2)^2 s_1$

Probability that a line is not available in 3rd stage is

$P_B = 1 - (1 - \alpha_2)^2 s_1$

Probability that a line is not available in 2nd stage is

$P_B = [1 - \{1 - (1 - \alpha_2)^2 s_1\}]$

Since a series of s links are needed to complete a connection at the 2nd stage, the blocking probability is

$P_B = [1 - \{1 - (1 - \alpha_2)^2 s_1\}]^s$

So the probability of the 5 stage network is given by multiplying the equations. The multiplication yields the following result

$$P_B = [1 - (1 - \alpha_1)^2 [1 - \{1 - (1 - \alpha_2^2)s_1\}]^s$$