COMP9020 Task2 - Question 4

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1 Question 1

Task Details:

Prove, using the laws of set operations, for all sets $A : A \cap \emptyset = \emptyset$.

Solution:

$A \cap \emptyset = A \cap (A \cap A^c)$ $\quad (Complementation)$
$= (A \cap A) \cap A^c$ $\quad (Assoxciatitity)$
$= A \cap A^c$ $\quad (Idempotence)$
$= \emptyset$ $\quad (Complementation)$

2 Question 2

Task Details:

Prove, using the laws of set operations, for all sets $A : A \cap (\mathcal{U} \oplus A) = \emptyset$.

Solution:

$A \cap (\mathcal{U} \oplus A)$
$= A \cap ((\mathcal{U} \backslash A) \cup (A \backslash \mathcal{U}))$
$= A \cap ((\mathcal{U} \cap A^c) \cup (A \cap \mathcal{U}^c))$
$= A \cap ((A^c \cap \mathcal{U}) \cup (A \cap \mathcal{U}^c))$ $\quad (Commutativity)$
$= (A \cap (A^c \cap \mathcal{U})) \cup (A \cap (A \cap \mathcal{U}^c))$ $\quad (Distribution)$
$= ((A \cap A^c) \cap \mathcal{U}) \cup ((A \cap A) \cap \mathcal{U}^c)$ $\quad (Associativity)$
$= (\emptyset \cap \mathcal{U}) \cup (A \cap \emptyset)$ $\quad (Complementation)$
$= (\mathcal{U} \cap \emptyset) \cup (A \cap \emptyset)$ $\quad (Commutativity)$
$= \emptyset \cup \emptyset$ $\quad (Annihilation)$
$= \emptyset$

3 Question 3

Task Details:

Prove, using the laws of set operations, for all sets A and B: $A \cup (B \cap A) = A$.

Solution:

$A \cup (B \cap A)$
$= (A \cap \mathcal{U}) \cup (B \cap A)$ $\quad (Identity)$
$= (A \cap \mathcal{U}) \cup (A \cap B)$ $\quad (Commutativiy)$
$= A \cap (\mathcal{U} \cup B)$ $\quad (Distribution)$
$= A \cap \mathcal{U}$
$= A$

4 Question 4

Task Details:

Prove, using the laws of set operations, for all sets A and B: $(A \cup B)^c = A^c \cap B^c$.

Solution:

$(A \cup B)^c$
$= U \backslash (A \cup B)$
$= (U \backslash A) \cap (U \backslash B)$
$= A^c \cap B^c$
$(A \cup B)^c$
$= U \backslash (A \cup B)$
$= U \cup (A \cup B)^c$