Input: a polynomial that will be represented as a sequence of numbers, each time an exponent followed by a coefficient as a list. The exponents will be in decreasing order. So for example,

3 5 1 10 0 5

represents the polynomial $5x^3+10x+5$. While the exponents are always integers, the coefficients may be rational numbers. Now, write programs to do the following:

(1) Prompt the user for a polynomial input. Once that is entered, prompt the user for the input of some number a. Now compute the quotient and remainder obtained when the input polynomial $P(x)$ is divided by $x-a$. For example, suppose the user enters the polynomial $3x^4 + 7x^2 - x + 3$ and enters $a = 1$, the result should be,

Quotient : $3x^3 + 3x^2 + 10x + 9$, Remainder : 12.

Make sure this runs in $O(n)$ time where n is the degree of the polynomial.

(2) Write code to compute $(x - a_1)(x - a_2)...(x - a_n)$ for n given numbers, in $O(n^2)$ time.

(3) Now write code to do interpolation in $O(n^2)$ time. The problem input be a set of pairs of (x,y) values like

3 4 7 2 4 10

where the list is the x value followed by the y value. Here, the output will be a polynomial of degree 2. It is computed as follows. Suppose the inout is $(x_1,a_1),(x_2,a_2),...,(x_n,a_n)$. Then the interpolated polynomial is given by $\sum_{i=1}^{n} a_i P_i(x)$, where the polynomial $P_i(x)$ is defined as,

$$\frac{\prod_{j\neq i}(x - x_j)}{\prod_{j\neq i}(x_i - x_j)}.$$

Notice that $P_i(x_i) = 1$ and $P_i(x_j) = 0$ for j 6= i.