Lab3

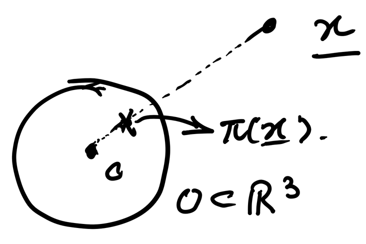
Saturday, March 12, 2022 8:15 PM

Prelab

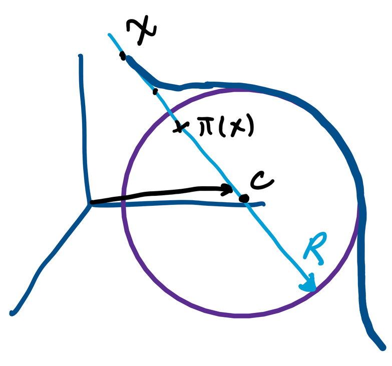
(b in lab doc.)

3) computing orthogonal projection of $\underline{x}$ onto O (obstacle), $\pi(\underline{x})$

1. Spherical O:



$O = \{ x \in \mathbb{R}^3 : \|x - c\| \leq R \}$



$c + \lambda(x - c)$ Find λ s.t. $c + \lambda(x-c) \in \partial O$, $\lambda \geq 0$

Take $x \notin O$

$\pi(x) =$

Impose that the point $c + \lambda(x - c)$ is at a distance R from c, i.e.,

$\| c + \lambda(x - c) - c \| = R$

$\lambda \|x - c\| = R \Rightarrow \lambda = \dfrac{R}{\|x - c\|}$

$$\pi(x) = \begin{cases} c + \dfrac{R}{\|x-c\|}(x-c), & \|x - c\| \geq R \\ x & \|x-c\| \leq R \end{cases}$$

$$O_i(q) - b = O_i(q) - \pi(x) = O_i(q) - c - \frac{R}{\|O_i(q) - c\|}(O_i(q) - c) \quad \text{if } \|O_i(q) - c\| \geq R$$

$$= \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} - \begin{bmatrix} c_x \\ c_y \\ c_z \end{bmatrix} - \frac{R}{\left\| \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} - \begin{bmatrix} c_x \\ c_y \\ c_z \end{bmatrix} \right\|} \begin{bmatrix} x_i - c_x \\ y_i - c_y \\ z_i - c_z \end{bmatrix}$$

$$= \begin{bmatrix} x_i - c_x \\ y_i - c_y \\ z_i - c_x \end{bmatrix} - \frac{R}{\left\| \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} - \begin{bmatrix} c_x \\ c_y \\ c_z \end{bmatrix} \right\|} \begin{bmatrix} x_i - c_x \\ y_i - c_y \\ z_i - c_z \end{bmatrix}$$

$$= \begin{bmatrix} (x_i - c_x) \\ (y_i - c_y) \\ (z_i - c_z) \end{bmatrix} \left(\frac{-R}{\sqrt{(x_i - c_x)^2 + (y_i - c_y)^2 + (z_i - c_z)^2}} + 1 \right)$$

$$= (O_i(q) - c)\left(\frac{-R}{\|O_i(q) - c\|} + 1 \right)$$

$$\boxed{\begin{aligned} \|O_i(q) - b\| &= \begin{cases} \left\| (O_i(q) - c)\left(\dfrac{-R}{\|O_i(q)-c\|} + 1 \right) \right\| & \text{if } \|O_i(q) - c\| \geq R \\ \|O_i(q) - O_i(q)\| = 0 & \text{if } \|O_i(q) - c\| \leq R \end{cases} \\ O_i(q) - b &= \begin{cases} (O_i(q) - c)\left(\dfrac{-R}{\|O_i(q)-c\|} + 1 \right) & \text{if } \|O_i(q) - c\| \geq R \\ 0 & \text{if } \|O_i(q) - c\| \leq R \end{cases} \end{aligned}}$$

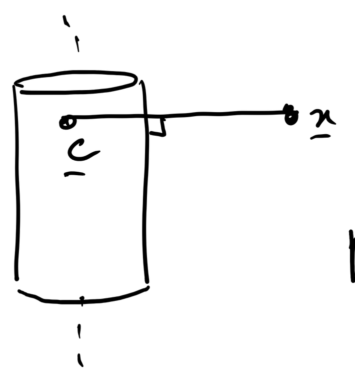
2. Cylindrical O: we first project $\underline{x}$ onto $z = x_z$ plane.

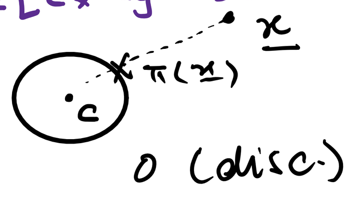
Then, we a problem treat this as in $\mathbb{R}^2$.

$c = [\underbrace{c_x \; c_y}_{\text{constant}} \; \underbrace{c_z}_{x_z}]^T$

$x \to = \begin{bmatrix} x_x \\ x_y \\ x_z \end{bmatrix}$.

Thus, c





Now, define $c = [c_x, c_y]$, $\underline{x}' = [x_x, x_y]$

$$\pi(x) = \begin{cases} \begin{bmatrix} \underline{c} + \frac{R}{\|\underline{x}' - \underline{c}\|}(\underline{x}' - \underline{c}) \\ x_z \end{bmatrix}, & \|\underline{x}' - \underline{c}\| \geq R. \\ \begin{bmatrix} x' \\ x_z \end{bmatrix} & \|x - c\| \leq R \end{cases}$$

$$O_i(q) - b = O_i(q) - \pi(x) = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} - \begin{bmatrix} \begin{bmatrix} c_x \\ c_y \end{bmatrix} - \frac{R}{\left\| \begin{bmatrix} x_i \\ y_i \end{bmatrix} - \begin{bmatrix} c_x \\ c_y \end{bmatrix} \right\|} \left(\begin{bmatrix} x_i - c_x \\ y_i - c_y \end{bmatrix} \right) \\ z_i \end{bmatrix} \quad \text{if } \|O_i(q) - c\| \geq R$$

$$= \begin{bmatrix} (x_i - c_x)\left(1 - \frac{R}{\sqrt{(x_i - c_x)^2 + (y_i - c_y)^2}}\right) \\ (y_i - c_y)\left(1 - \frac{R}{\sqrt{(x_i - c_x)^2 + (y_i - c_y)^2}}\right) \\ 0 \end{bmatrix}$$

$$O_i(q) - b = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} - \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = 0 \quad \text{if } \|O_i(q) - c\| \leq R$$

$$\boxed{\begin{aligned} O_i(q) - b &= \begin{cases} \begin{bmatrix} (x_i - c_x)\left(1 - \frac{R}{\sqrt{(x_i - c_x)^2 + (y_i - c_y)^2}}\right) \\ (y_i - c_y)\left(1 - \frac{R}{\sqrt{(x_i - c_x)^2 + (y_i - c_y)^2}}\right) \\ 0 \end{bmatrix} & \text{if } \|O_i(q) - c\| \geq R \\ 0 & \text{if } \|O_i(q) - c\| \leq R \end{cases} \\ \|O_i(q) - b\| &= \begin{cases} \left\| \begin{bmatrix} (x_i - c_x)\left(1 - \frac{R}{\sqrt{(x_i - c_x)^2 + (y_i - c_y)^2}}\right) \\ (y_i - c_y)\left(1 - \frac{R}{\sqrt{(x_i - c_x)^2 + (y_i - c_y)^2}}\right) \\ 0 \end{bmatrix} \right\| & \text{if } \|O_i(q) - c\| \geq R \\ 0 & \text{if } \|O_i(q) - c\| \leq R \end{cases} \end{aligned}}$$