

# Morning



手机关机或静音放包/兜里

# *Formal Language And Automata*

- ♥ Languages
- ♥ Automata
- ♥ Computation



# Language

What is a language ?

This is a sentence.

This is also a sentence.

So we have

{ sentence 1, sentence 2, sentence 3, ... }

the set of sentences  $\Leftrightarrow$  Language

# Sentence/String

Sentence, String, Word

$(1 + 2) * (13 - 7)$

To stay at home and save lives.

不聚集，戴口罩，勤洗手。

0, 1, 00, 01, 10, 000, 001, 1010, 00111100

**String** = sequence of symbols

chosen from the alphabet  $\Sigma$

# Alphabet

**Alphabet** = set of symbols

$\{a, b, c, \dots, x, y, z\}$

$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, +, -, *, /, (, )\}$

ASCII, 中文国标,  $\Sigma = \{0, 1\}$

symbols  $\Rightarrow$  strings  $\Rightarrow$  language

# Rules/Grammar

**Rules** : by which strings are composed/generated  
rules for English :

$\langle \text{sentence} \rangle \rightarrow \langle \text{noun-phrase} \rangle \langle \text{predicate} \rangle$

$\langle \text{noun-phrase} \rangle \rightarrow \langle \text{article} \rangle \langle \text{noun} \rangle$

$\langle \text{predicate} \rangle \rightarrow \langle \text{verb} \rangle$

$\langle \text{article} \rangle \rightarrow a|an|the$

$\langle \text{noun} \rangle \rightarrow \text{wolf}|\text{sheep}$

$\langle \text{verb} \rangle \rightarrow \text{love}|\text{eat}$

I have a dream.

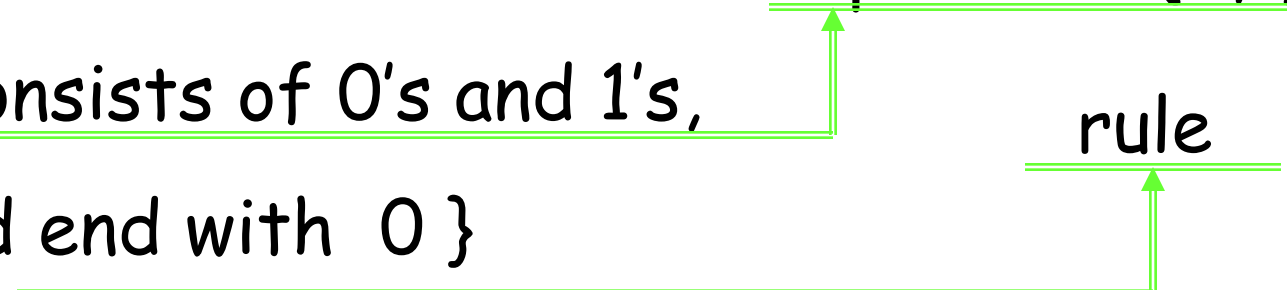
# Example 1

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$L = \{w \mid w \text{ consists of 0's and 1's, and end with 0}\}$

alphabet : {0,1}

rule

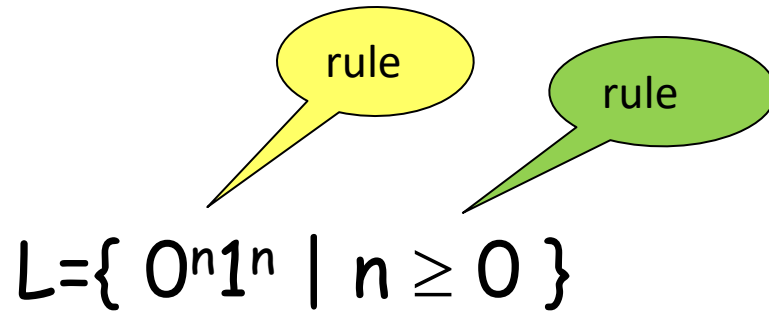


$L = \{ 0, 00, 10, 000, 010, 100, 110, 0000, \dots \}$

$1111100 \in L, 1 \notin L, 0001 \notin L, 20 \notin L$

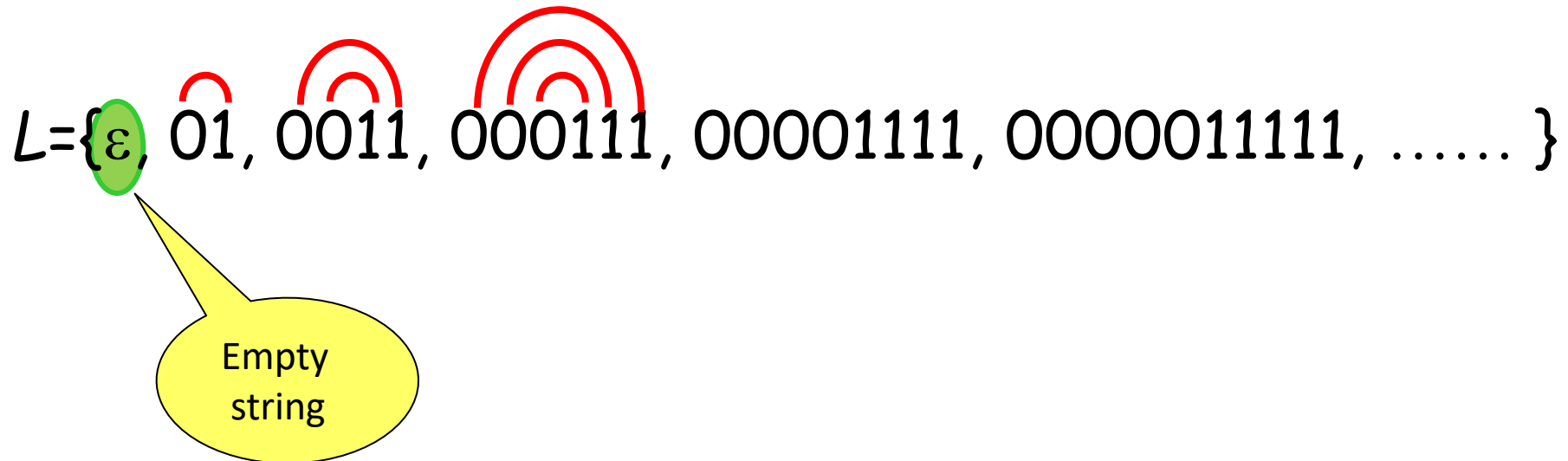
## Example 2

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$L = \{ 0^n 1^n \mid n \geq 0 \}$

alphabet = {0,1}



$L = \{ \varepsilon, 01, 0011, 000111, 00001111, 0000011111, \dots \}$

Empty string



## Example 3

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$L = \{ w \mid w \text{ is a sentence in English} \}$

Everyone loves his/her motherland. ✓

Dog bites man. ✓

Man bites dog. ?

- ♣ Formal language focus on form of string  
not semantics/meaning

# Operations of String

$$w = a_1a_2\ldots a_m$$

abc

$$v = b_1b_2\ldots b_n$$

123456

## ◆ Concatenation

$$wv = a_1a_2\ldots a_mb_1b_2\ldots b_n$$

abc123456

$$\begin{array}{c} \text{✂} \\ vw = b_1b_2\ldots b_na_1a_2\ldots a_m \end{array}$$

123456abc

## ◆ Reverse

$$w^R = a_na_{n-1}\ldots a_1$$

cba

# Operations on Languages

## ◆ Usual set operations

$$L_1 \cup L_2 = \{ w \mid w \in L_1 \text{ or } w \in L_2 \}$$

$$L_1 \cap L_2 = \{ w \mid w \in L_1 \text{ and } w \in L_2 \}$$

$$L_1 - L_2 = \{ w \mid w \in L_1 \text{ and } w \notin L_2 \}$$

## ◆ Reverse

$$L^R = \{ w^R \mid w \in L \}$$

## ◆ Concatenation

$$L_1 L_2 = \{ wv \mid w \in L_1 \text{ and } v \in L_2 \}$$

## Example 4

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$$L = \{ ab, abc, abcd \} \Rightarrow L^R = \{ ba, cba, dcba \}$$

$$L = \{ a^n b^n \mid n \geq 1 \} \Rightarrow L^R = \{ b^n a^n \mid n \geq 1 \}$$

$$L = \{ a^n b^n \mid n \geq 1 \}, K = \{ 0^n 1^n \mid n \geq 1 \}$$

$$LK = \{ a^n b^n 0^n 1^n \mid n \geq 1 \}$$



$$LK = \{ a^n b^n 0^m 1^m \mid n \geq 1, m \geq 1 \}$$

$$L^2 = ?$$

## \* / Star Operation on Languages

$$\Sigma = \{0, 1\}$$

$$\Sigma^* = \Sigma^0 \cup \Sigma \cup \Sigma^2 \cup \Sigma^3 \cup \dots \cup \Sigma^n \cup \dots$$

$$\Sigma^0 = \{\varepsilon\}, \quad \Sigma^n = \underbrace{\Sigma \Sigma \dots \Sigma}_n$$

$$\{0,1\}^* = \{\varepsilon\} \cup \{0,1\} \cup \{0,1\}^2 \cup \dots \cup \{0,1\}^n \cup \dots$$

$$= \{ \varepsilon, \underline{0}, \underline{1}, \underline{00}, \underline{01}, \underline{10}, \underline{11}, 000, 001, 010, 011, 111, \dots \}$$

## Empty string / language

Denote  $\varepsilon$  as empty string

$$|\varepsilon| = 0, \quad w\varepsilon = \varepsilon w = w$$

Denote  $\phi$  as empty language

$$\phi = \{\}, \quad \phi L = L\phi = \phi$$

Denote  $\Sigma^+ = \Sigma \cup \Sigma^2 \cup \Sigma^3 \cup \dots \cup \Sigma^n \cup \dots$

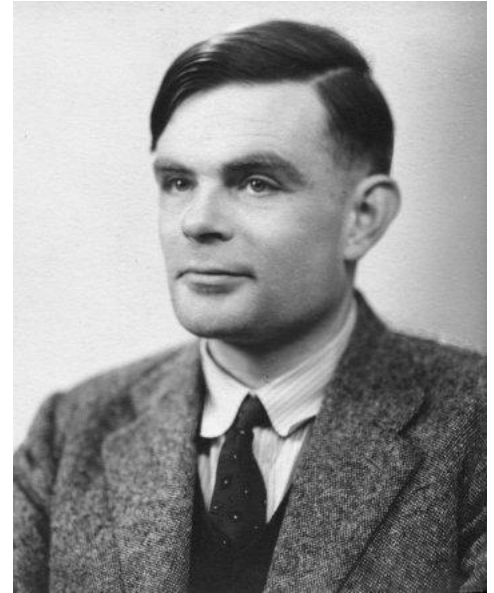
$$= \Sigma^* - \Sigma^0$$

$$= \Sigma^* - \{\varepsilon\}$$

# Automata

## Alan Marthison Turing

- On Computable Numbers  
With an Application to  
the Entscheidungs Problem
- Turing Machine



# Automata

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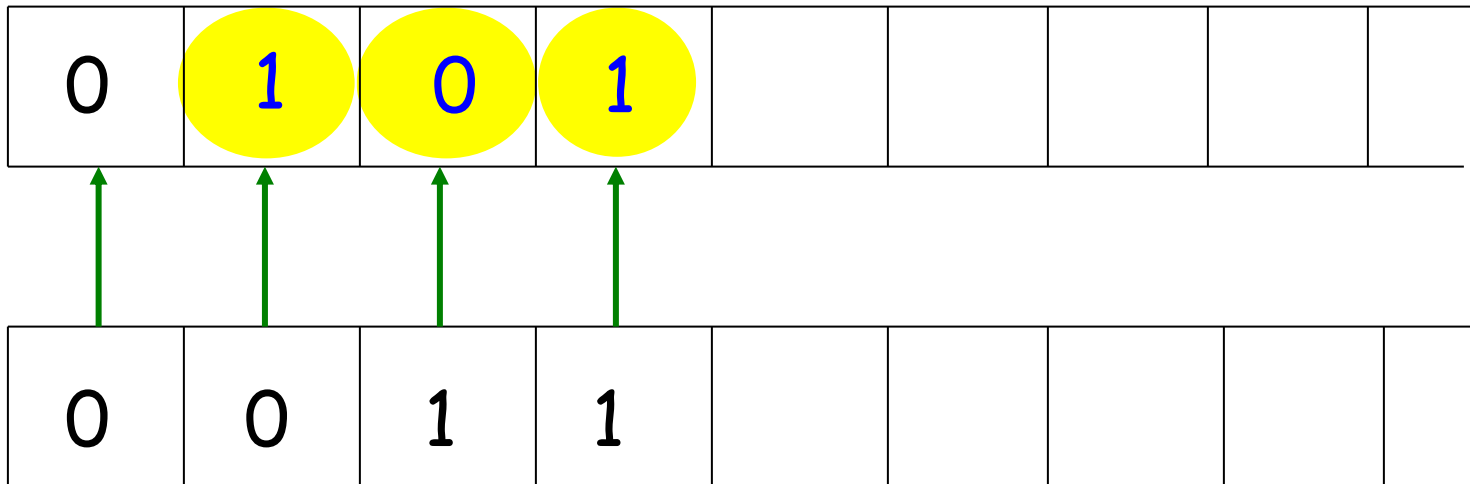
- ◆ Finite Automata
  - Deterministic Finite Automata
  - Non-deterministic Finite Automata
- ◆ Push Down Automata
- ◆ Turing Machine



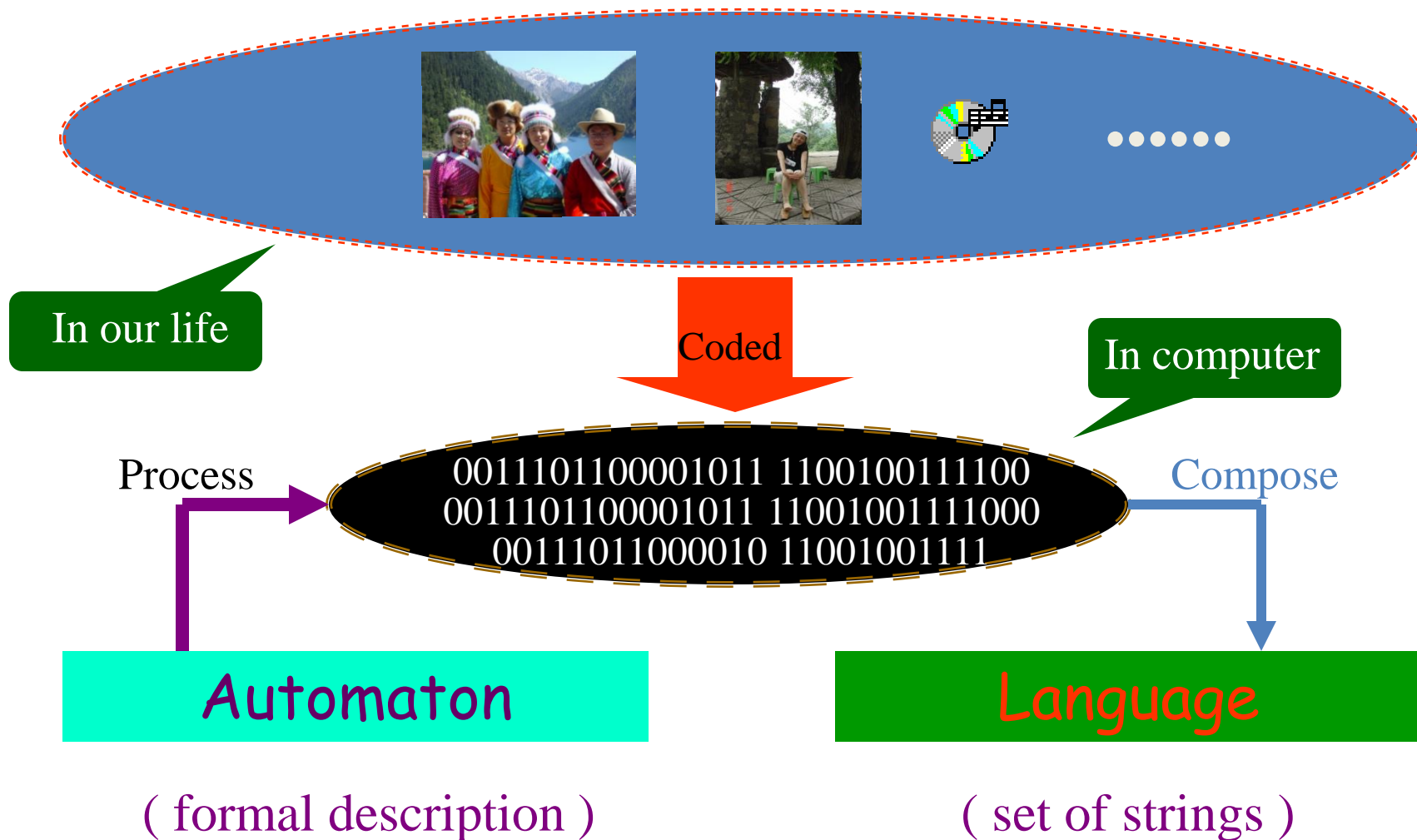
# Computation

$$\begin{array}{r} 2 \\ + 3 \\ \hline 5 \end{array}$$

$$\begin{array}{r} 0010 \\ + 0011 \\ \hline 0101 \end{array}$$



# Computation



# Computation

- ◆ Computable Problems
  - write a program to solve
- ◆ Intractable Problems
  - find someway to work around

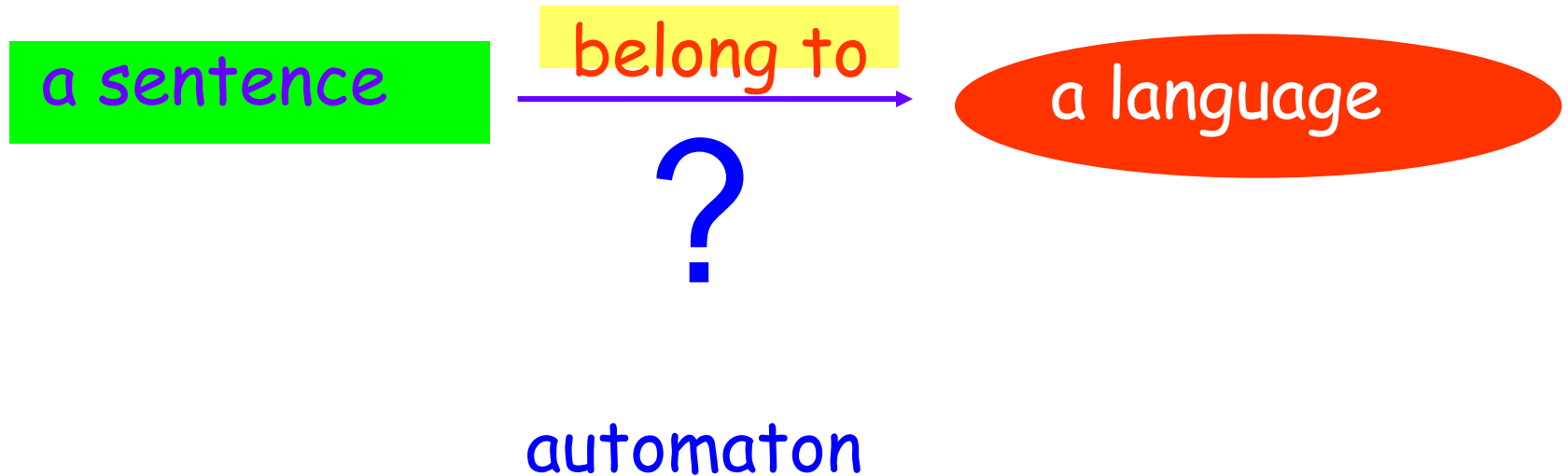
# Undecidable Problem

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```
main ( )
{
    Int n, total, x, y, z ;
    scanf("%d", &n);
    total=3;
    while(1){
        for(x=1;x<=total-2;x++)
            for(y=1;y<=total-x-1;y++){
                z=total-x-y;
                if(exp(x,n)+exp(y,n)==exp(z,n))
                    printf("hello,world\n");
            }
        total++;
    }
}
```


$$x^n + y^n = z^n$$

# Undecidable Problem



# Content

Automata

Languages

Grammars

Construction

Properties

Design

Finite  
Automaton

Regular  
Language

Regular  
Expression

Recognize

Generate

Push Down  
Automaton

Context Free  
Language

Context Free  
Grammar

Turing  
Machine

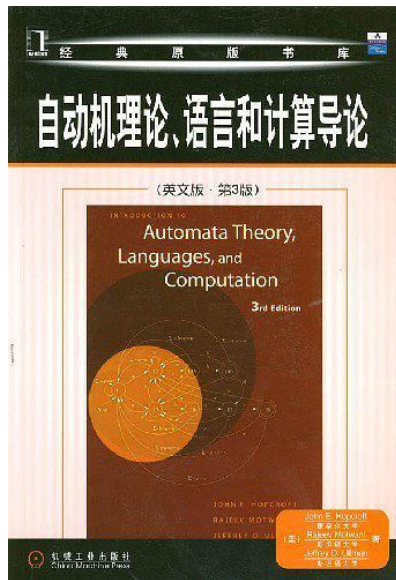
Recursively  
Enumerable

( Phrase  
Grammar)

# Text book

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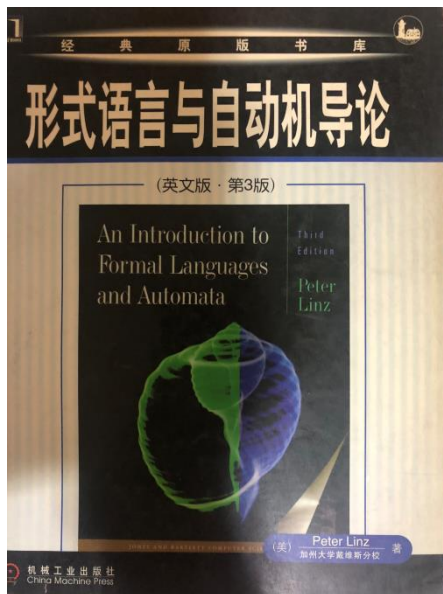
## 1. Introduction to Automata Theory , Languages , and Computation (Third Edition )



—— John E. Hopcroft  
Rajeev Motwani  
Jeffrey D. Ullman

## 2. An Introduction to Formal Languages and Automata (Third Edition )

—— Peter Linz





# Goal

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1. Understanding "theoretical" concepts  
----- method of formal description
2. To realize mathematics
3. Improving reading ability in English

# Homework

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- ♦ All exercises showed on qq-group
- ♦ Write on A4 papers
- ♦ Discussions - 4 times

# Honor and Collaboration

- ◆ Collaboration is strongly encouraged
- ◆ Solutions must be written independently
- ◆ Responsible for Understanding and explaining

# Examination

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- ◆ Only final exam
- ◆ Closed exam

Nothing allowed except one pen



# Grading Policy

- ◆ Homework : 20% //including Class Performance
- ◆ Final exam : 80%



# Information

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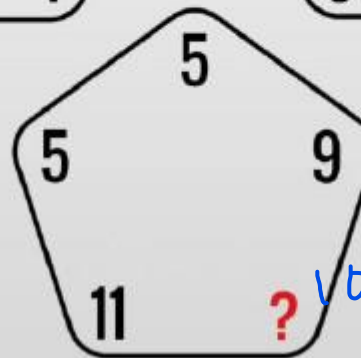
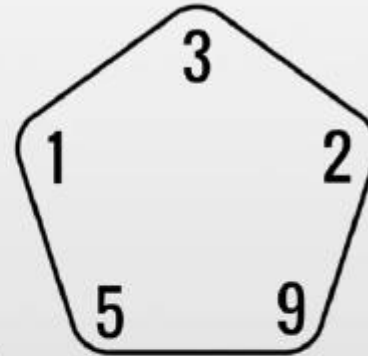
- ◆ Tutor : 孙大烈
- ◆ Office : 综合楼 520
- ◆ E-mail : [sdl@hit.edu.cn](mailto:sdl@hit.edu.cn)
- ◆ 课程群 : 自动机/104804608 (qq)
- ◆ MOOC :



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- <https://www.icourse163.org/learn/HIT-1206319802>
- ◆ 画状态图: <http://madebyevan.com/fsm/>

CAN YOU FIND THE MISSING NUMBER?



Good good study  
day day up!