Afternoon

Regular Expression

- Definition
- Design
- Equivalence with FA

Arithmetical Expression

$$0,\ 1+2,\ 3\times(5-2),\ (56-7)^2,\ \ldots\ldots$$

- Traditional definition

0 1 2

- Inductive definition
  - Any number is a arithmetical expression
  - If **a** and **b** are arithmetical expressions , then

    so is $a+b, a-b, a\div b, a\times b, a^n, (a)$.

Building Regular Expressions

BASIS

1. ε is a RegExp, denoting the language $\{\varepsilon\}$.

2. ϕ is a RegExp, denoting the language ϕ.

3. For each a in Σ, a is a RegExp and denotes the language $\{a\}$.

Notice : language is a set of strings.

Building Regular Expressions

INDUCTION

1. If E and F are RegExp, denoting the language $L(E)$ and $L(F)$, then $E+F$, EF and E^* are RegExp that denote the languages $L(E) \cup L(F)$, $L(E)L(F)$ and $(L(E))^*$.

2. If E is a RegExp, then so is (E) .

Example 1 Language for

$$r = (a+b)^*(a+bb)$$

$a \rightarrow \{ \text{a} \}$, $b \rightarrow \{ \text{b} \}$

$a + b \rightarrow \{ \text{a} \} \cup \{ \text{b} \} = \{\text{a, b}\}$

$bb \rightarrow \{ \text{b} \} \{ \text{b} \} = \{ \text{bb} \}$

$a + bb \rightarrow \{ \text{a} \} \cup \{ \text{bb} \} = \{ \text{a, bb} \}$

$(a + b)^* \rightarrow \{ \text{a ,b} \}^*$

$(a + b)^* (a + bb) \rightarrow \{ \text{a ,b} \}^* \{ \text{a, bb} \}$

$L(r)$ = { a,bb,aa,abb,ba,bbb, …… }

Example 2 Language for

$$r = (aa)^*(bb)^*b$$

$L(r) = (\{a\}\{a\})^*(\{b\}\{b\})^*\{b\}$

$= (\{aa\})^*(\{bb\})^*\{b\}$

$= \{aa\}^*\{bb\}^*\{b\}$

$= \left\{ a^{2n}b^{2m+1} \mid n \geq 0, m \geq 0 \right\}$

Example 3 RegExp for

$\{ w \mid w$ consists of alternating 0's and 1's $\}$

Partition

010101...0101	$\longrightarrow$	$(01)^*$
101010...1010	$\longrightarrow$	$(10)^*$
0101010...1010	$\longrightarrow$	$0(10)^*$ or $(01)^*0$
101010...10101	$\longrightarrow$	$(10)^*1$ or $1(01)^*$

The regular expression

$$(01)^*+(10)^*+0(10)^*+(10)^*1$$

Example 3 RegExp for

{ w | w consists of alternating 0's and 1's }

$$\{\varepsilon , 1 \}\{ 010101\ldots0101 \} \{\varepsilon , 0 \}$$

$$\{\varepsilon , 1 \} \{ 0\ 1\}^* \{\varepsilon , 0 \}$$

$$(\varepsilon + 1) (0\ 1)^* (\varepsilon + 0)$$

$$(01)^*+(10)^*+0(10)^*+(10)^*1 \Leftrightarrow (\varepsilon + 1)(0\ 1)^* (\varepsilon + 0)$$

$$\Leftrightarrow (\varepsilon + 0)(1\ 0)^* (\varepsilon + 1)$$

Example 4 RegExp for

$L = \{ w \mid w \in \{0, 1\}^*$ and w has no pair of consecutive 0's $\}$

Partition

no 0	$\{1\}^*$	$\longrightarrow$	1^*
one 0	$\{1\}^*\{0\}\{1\}^*$	$\longrightarrow$	1^*01^*
more 0's	$\{1\}^*\{\{0\}\{1\}\{1\}^*\}^*\{0, \varepsilon\}$		$\{0110111\ldots\ldots\}$

$\Downarrow$

$$1^* (\; 0\,1\,1^*)^* (0 + \varepsilon)$$

Example 4 RegExp for

$L = \{ w \mid w \in \{0, 1\}^*$ and w has no pair of consecutive 0's $\}$

$\{01\}^*$ $\{\varepsilon, 01, 0101, 010101, 01010101, \ldots\ldots\}$

$\{1, 01\}^*$ $\{\varepsilon, 1, 01, 11, 101, 011, 0101, 111, 1101, 1011, \ldots\ldots\}$

$\Downarrow$

$(1+01)^* \Rightarrow (1+01)^*(0 + \varepsilon)$

Exercises RegExp for (Σ={0,1})

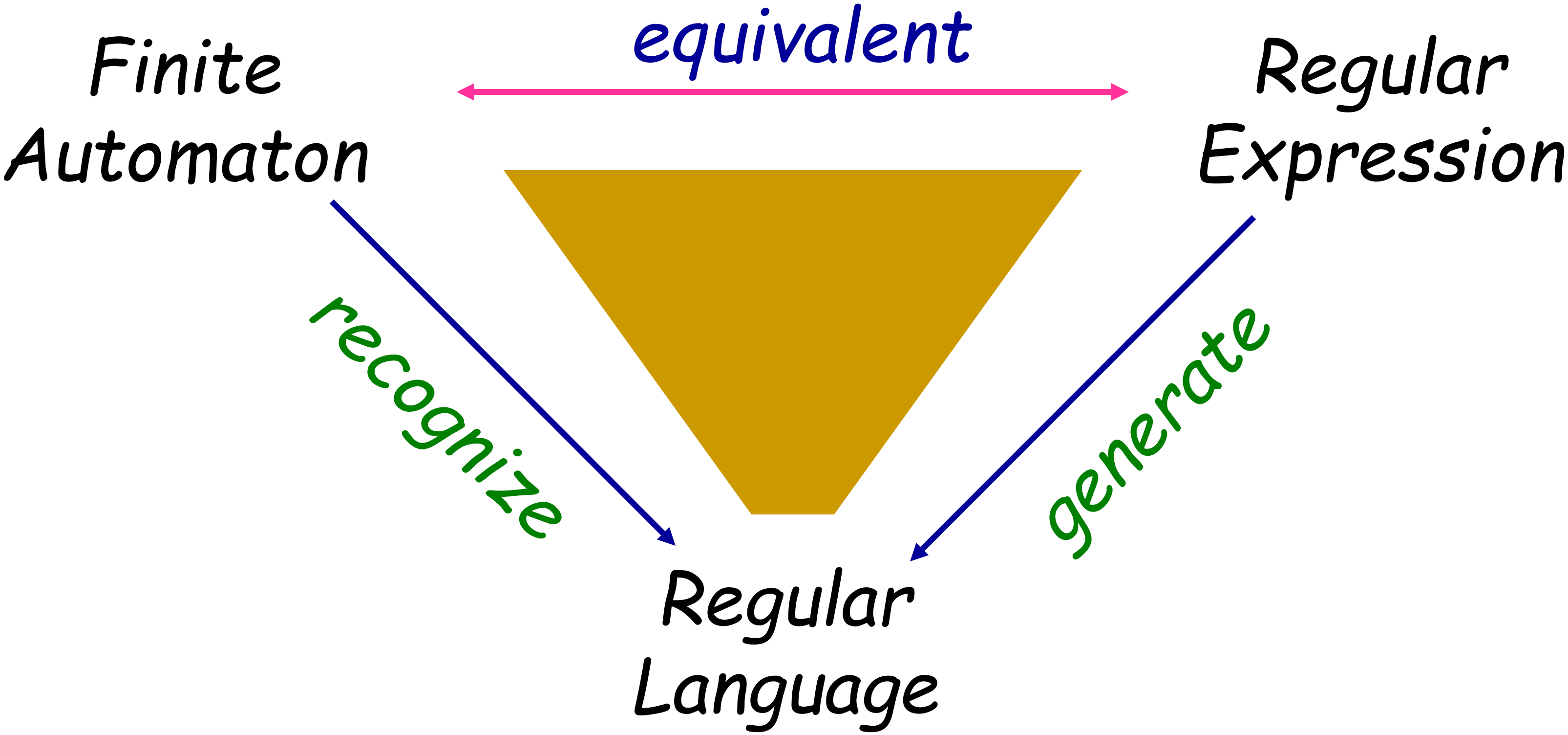
1. $\{w \mid w$ has exactly a single 1 $\}$

2. $\{w \mid w$ contains 001 $\}$

3. $\{w \mid$ length(w) $\geq$ 3 and the third symbol is 0 $\}$

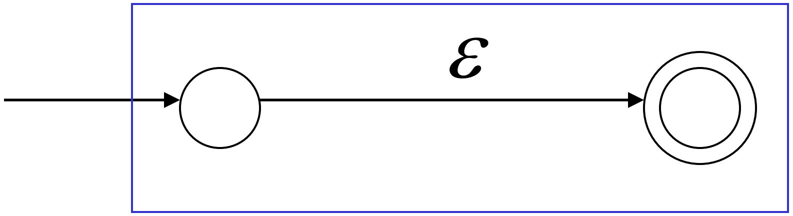
4. What language does the RegExp ϕ^* represent ?

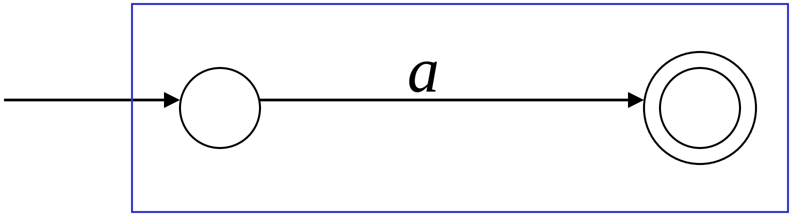
FA & RegExp


Finite
Automaton
equivalent
Regular
Expression
recognize
generate
Regular
Language

RegExp $\Rightarrow$ FA

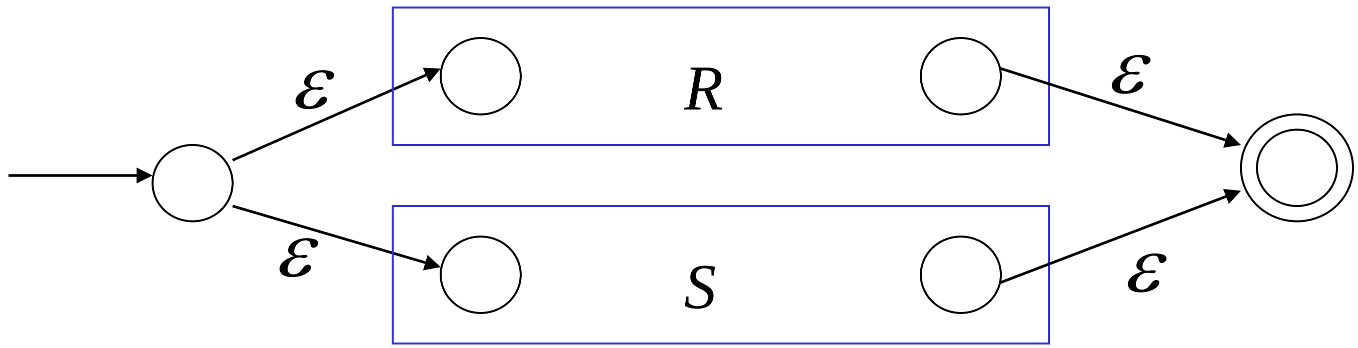
Basis :

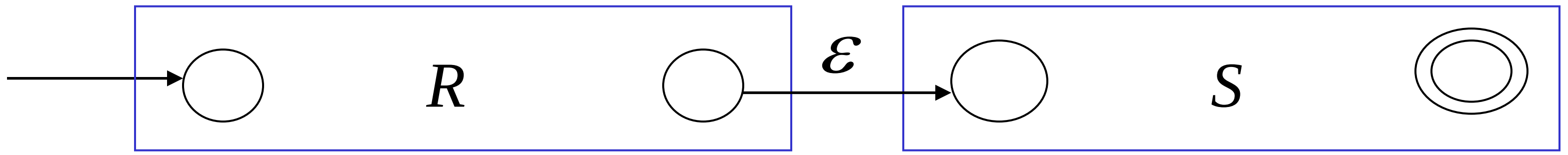


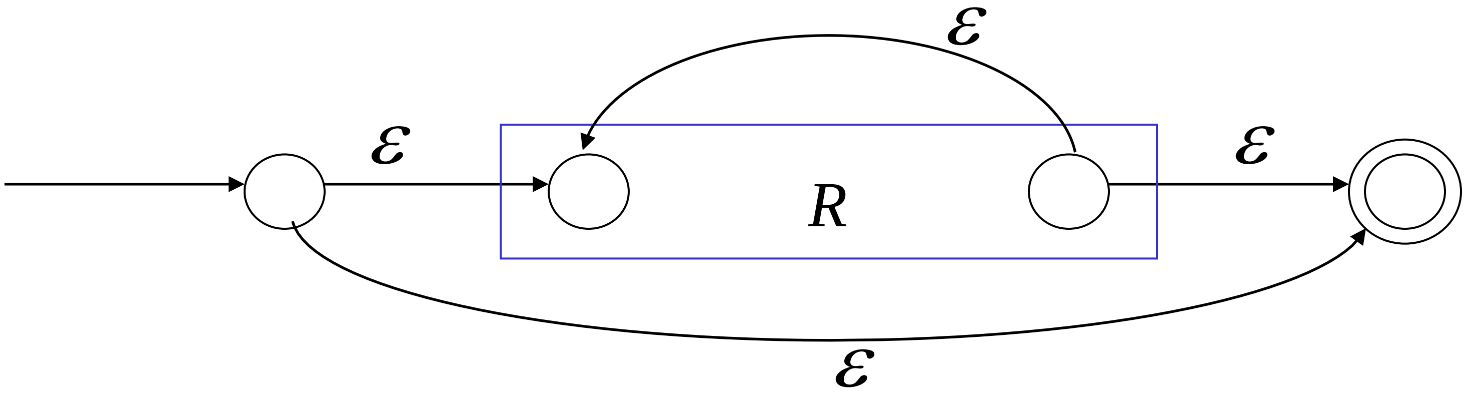


RegExp ⇒ FA

Induction :



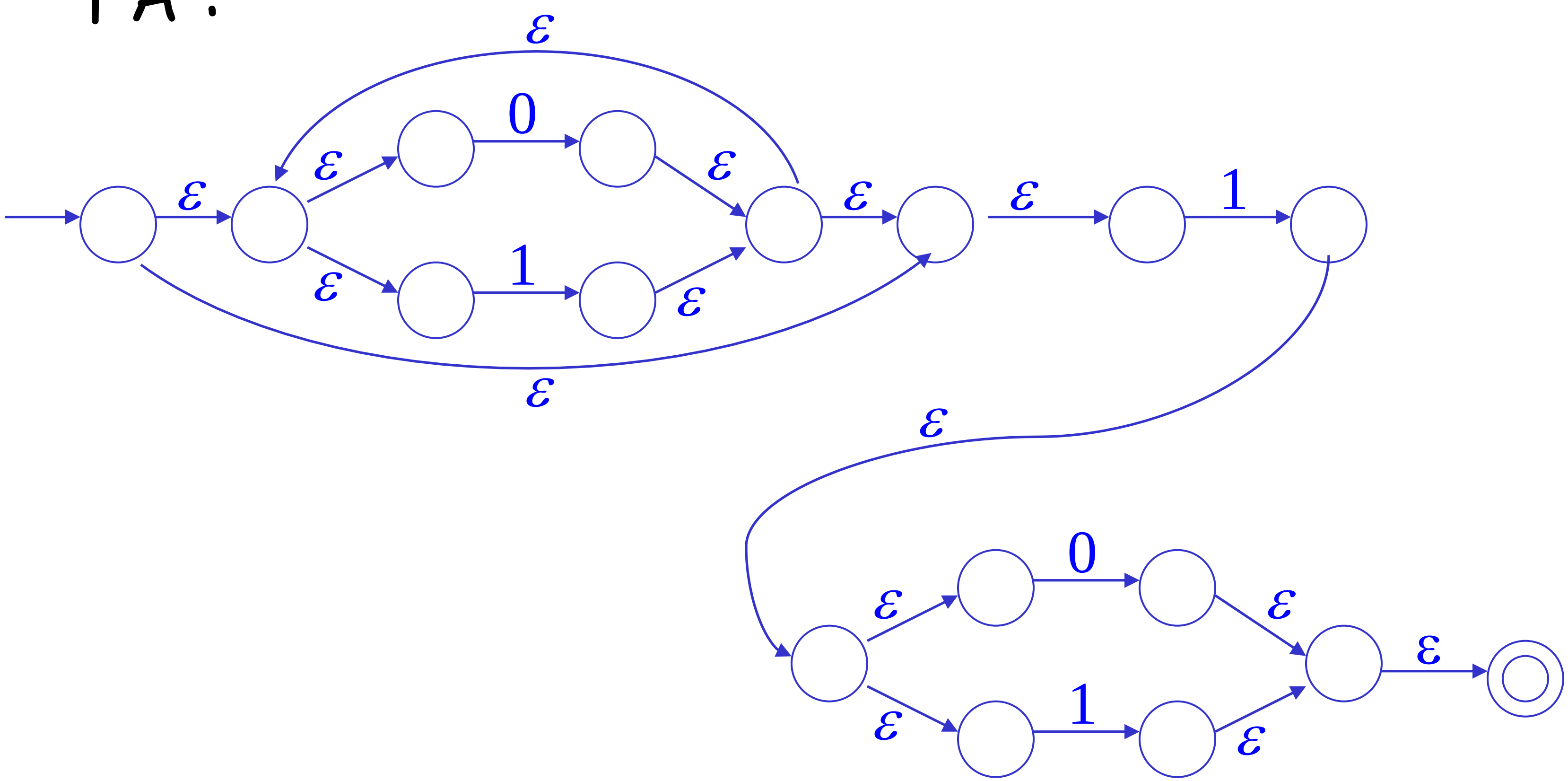




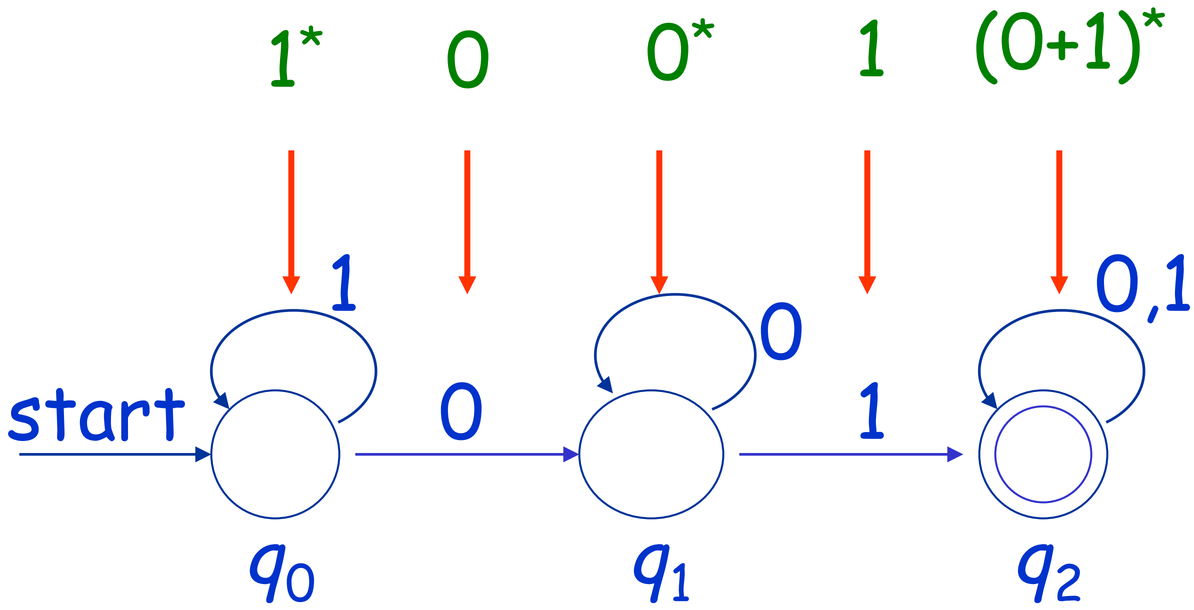
Example 5 RegExp $\Rightarrow$ FA

RegExp : $(0+1)^*1(0+1)$

FA :



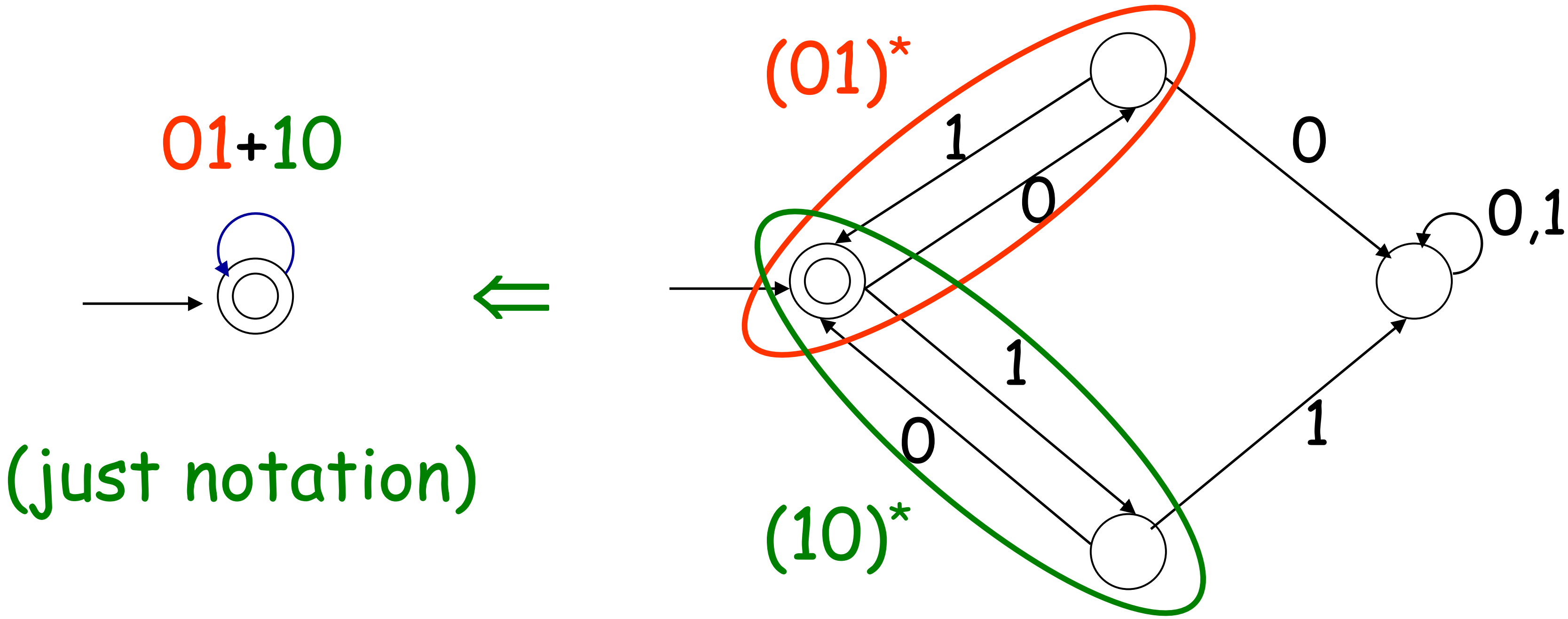
Example 6 FA $\Rightarrow$ RegExp



$L = \{ w \mid w \in \{0, 1\}^* \text{ and } w \text{ contains } 01 \}$

RE : $(0+1)^*01(0+1)^* \Rightarrow 1^*00^*1(0+1)^*$

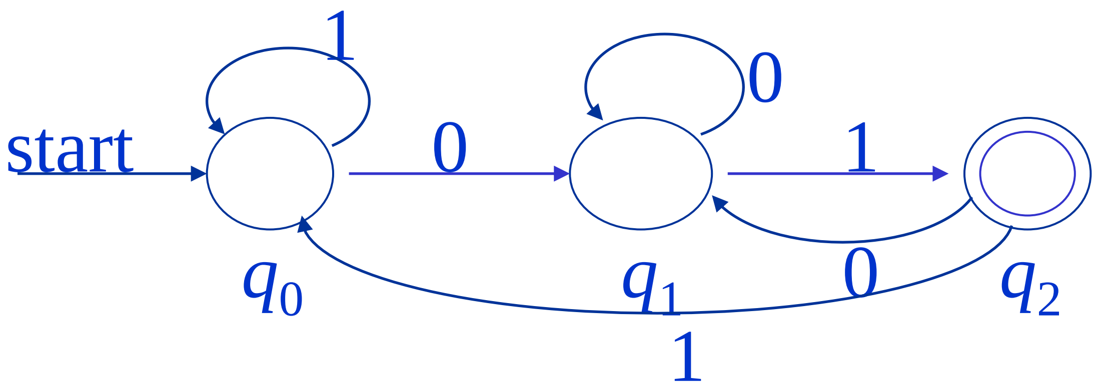
Example 7 FA ⇒ RegExp



$(01+10)^*$

Example 8 FA $\Rightarrow$ RegExp

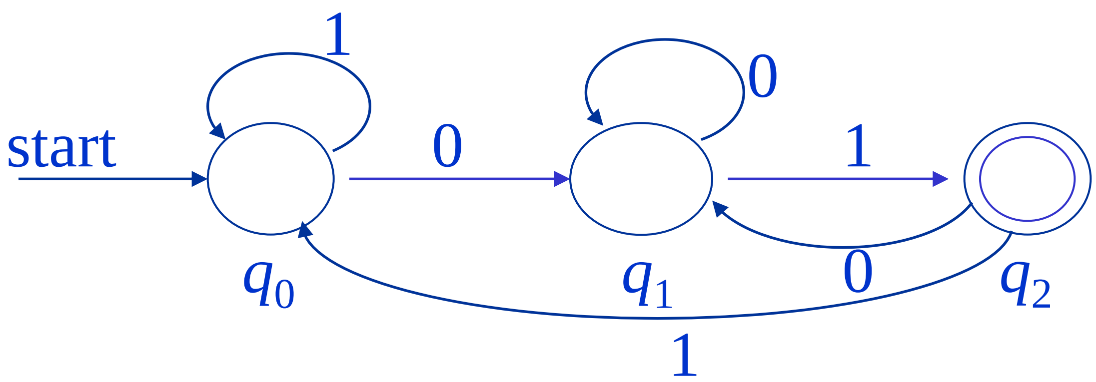
$1^* \quad 0 \quad 0^* \quad 1 \quad (00^*1)^*$



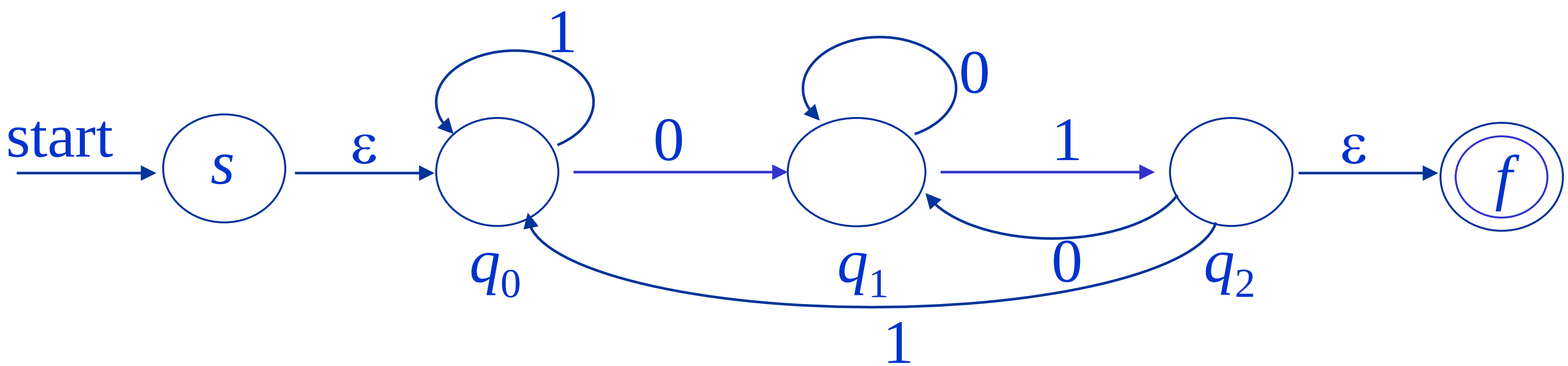
$1^* \quad 0 \quad 0^* \quad 1 \quad (11^*00^*1)^*$

$1^*00^*1\ (00^*1 + 11^*00^*1)^*$

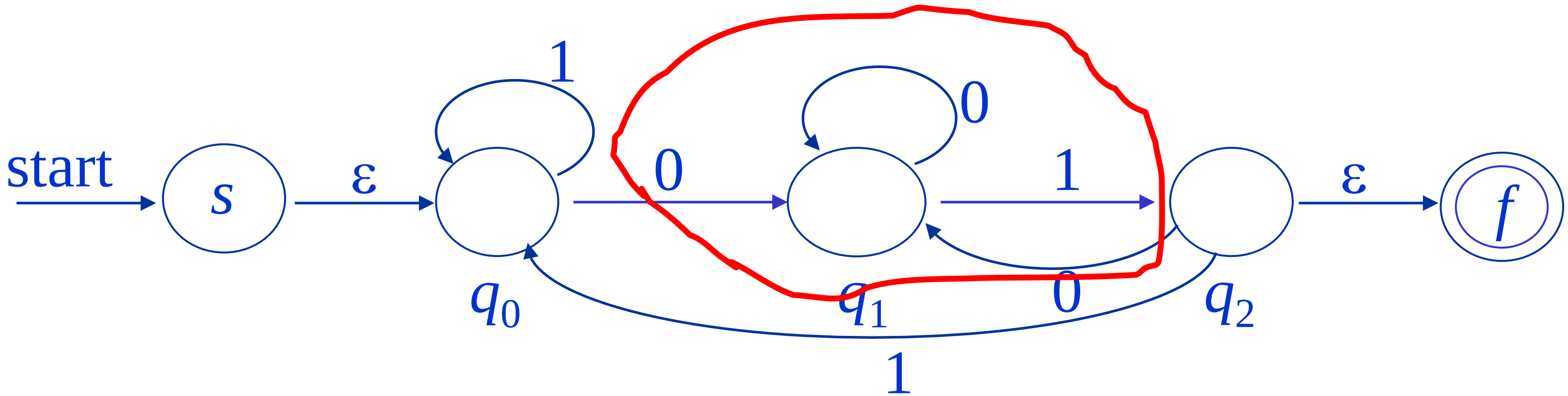
FA ⇒ RegExp - Delete states



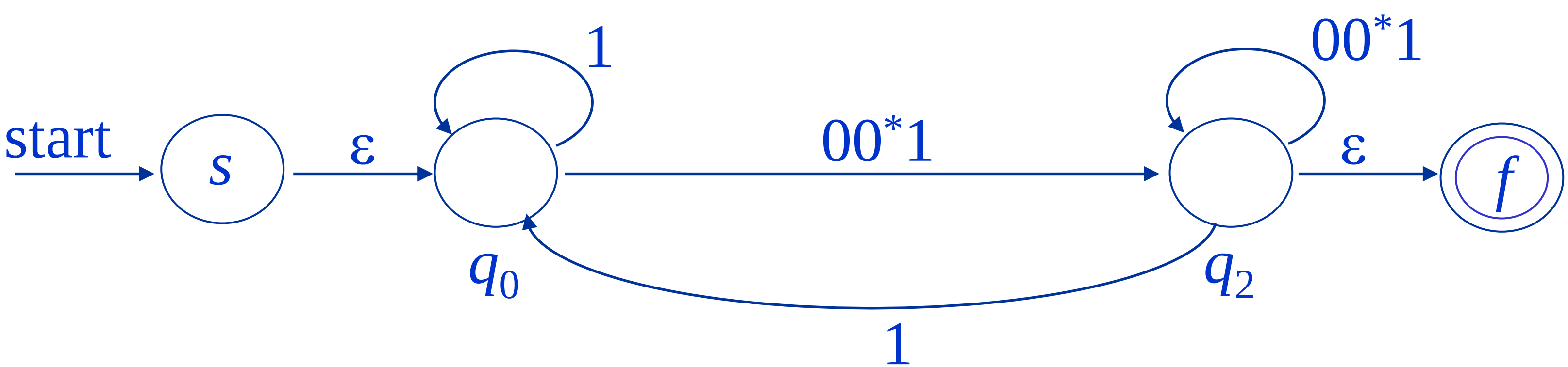
add two states, s and f :



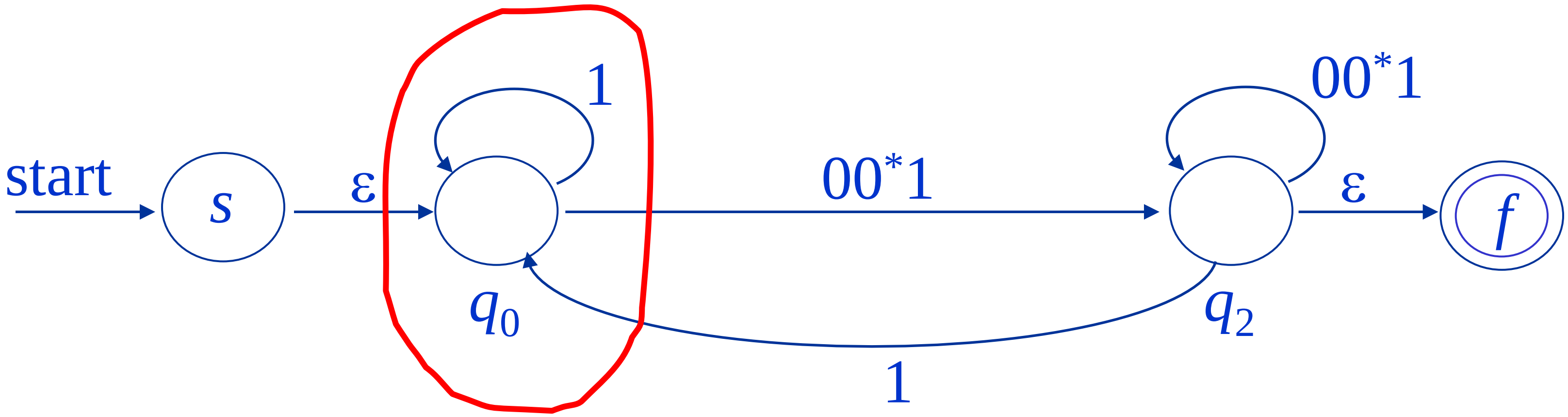
FA ⇒ RegExp - Delete states



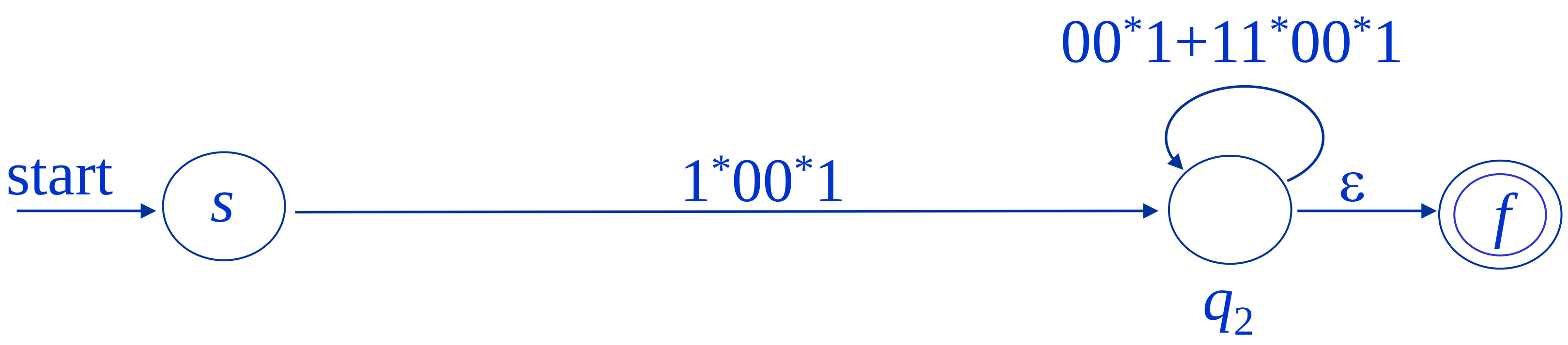
delete q_1 :



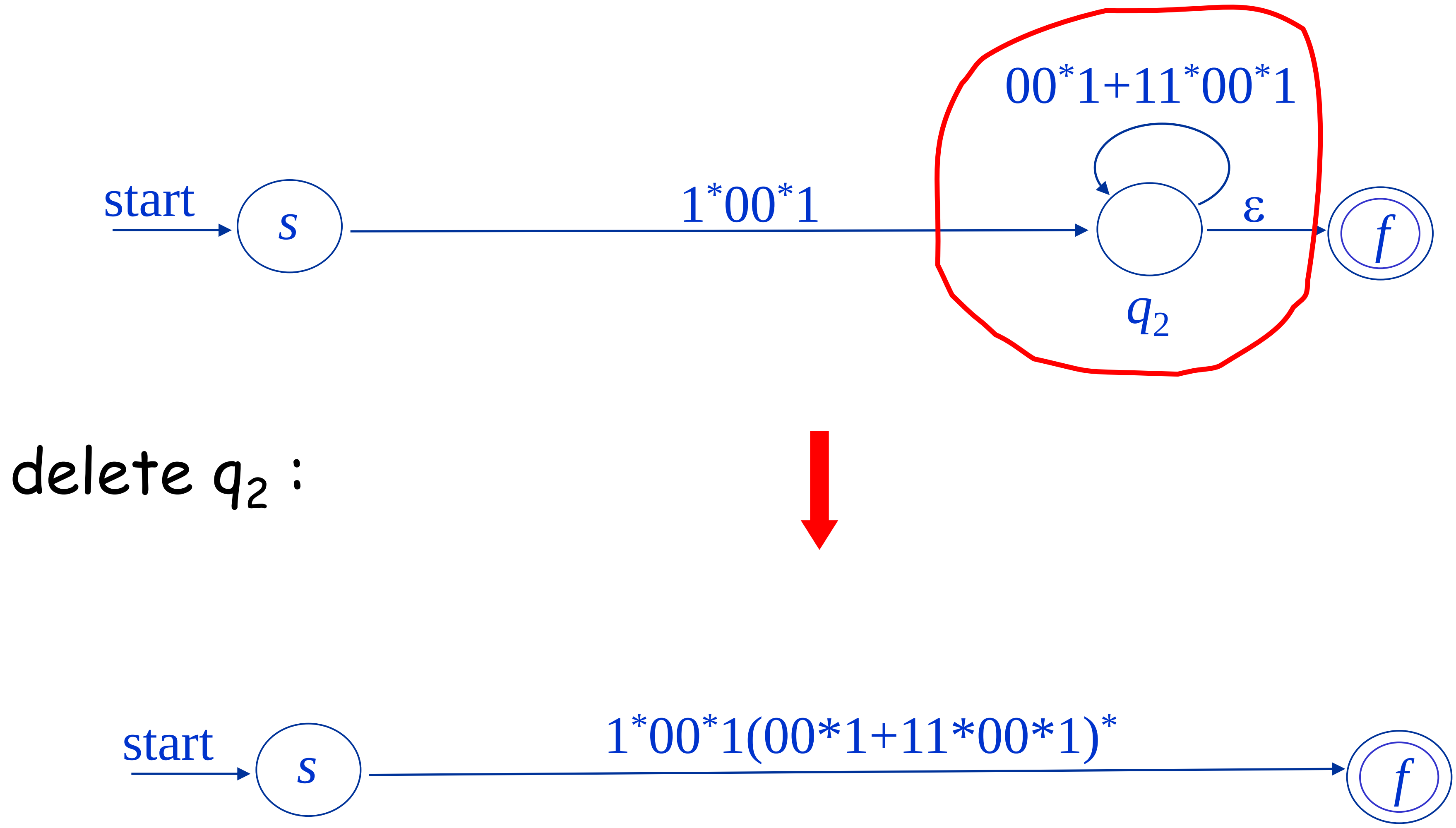
FA $\Rightarrow$ RegExp - Delete states



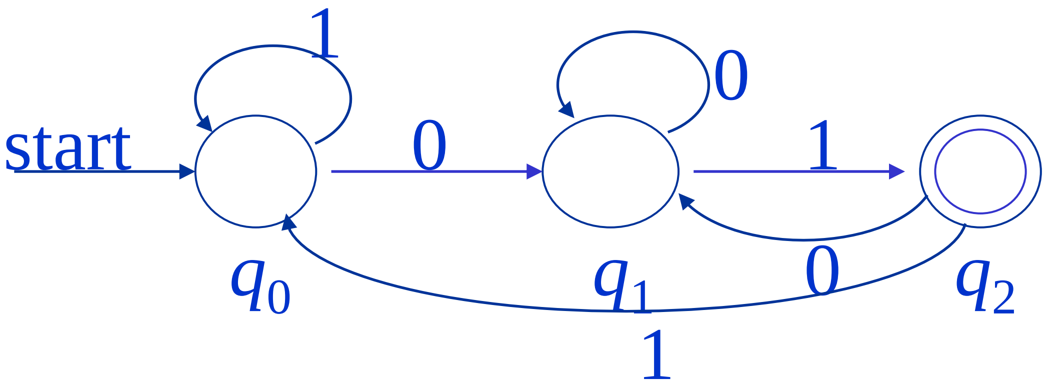
delete q_0 :



FA ⇒ RegExp - Delete states



FA $\Rightarrow$ RegExp - Induction



- Pick every label on the path from q_0 to q_2

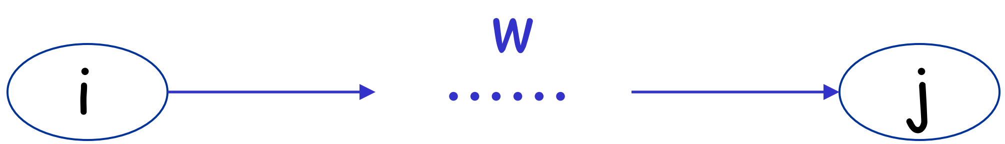
---- one by one

- Form every regexp on the path from q_0 to q_2

---- one by one

FA $\Rightarrow$ RegExp - Induction

- Q={1,2,3,……,n}
- $R_{ij}^{(k)} : 0 \leq k \leq n$
  - regular expression of path from *i* to *j*
  - no inner node is greater than *k*   不超过 k

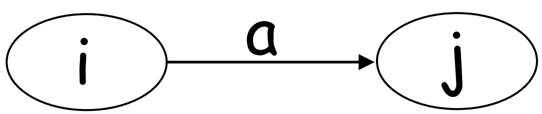


$$R_{ij}^{(k)} \Rightarrow w$$

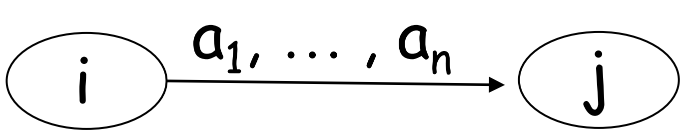
FA $\Rightarrow$ RegExp - Induction

Basis $k = 0, \; i \neq j$

$$\Rightarrow \quad R_{ij}^{(0)} = \phi$$



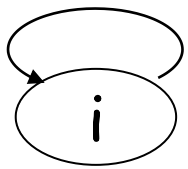
$$\Rightarrow \quad R_{ij}^{(0)} = a$$



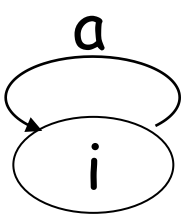
$$\Rightarrow \quad R_{ij}^{(0)} = a_1 + a_2 + \Lambda + a_n$$

FA $\Rightarrow$ RegExp - Induction

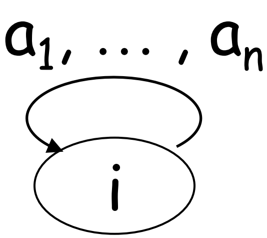
Basis $k = 0,\ i = j$



$$\Rightarrow \quad R_{ij}^{(0)} = \varepsilon + \phi$$



$$\Rightarrow \quad R_{ij}^{(0)} = \varepsilon + a$$



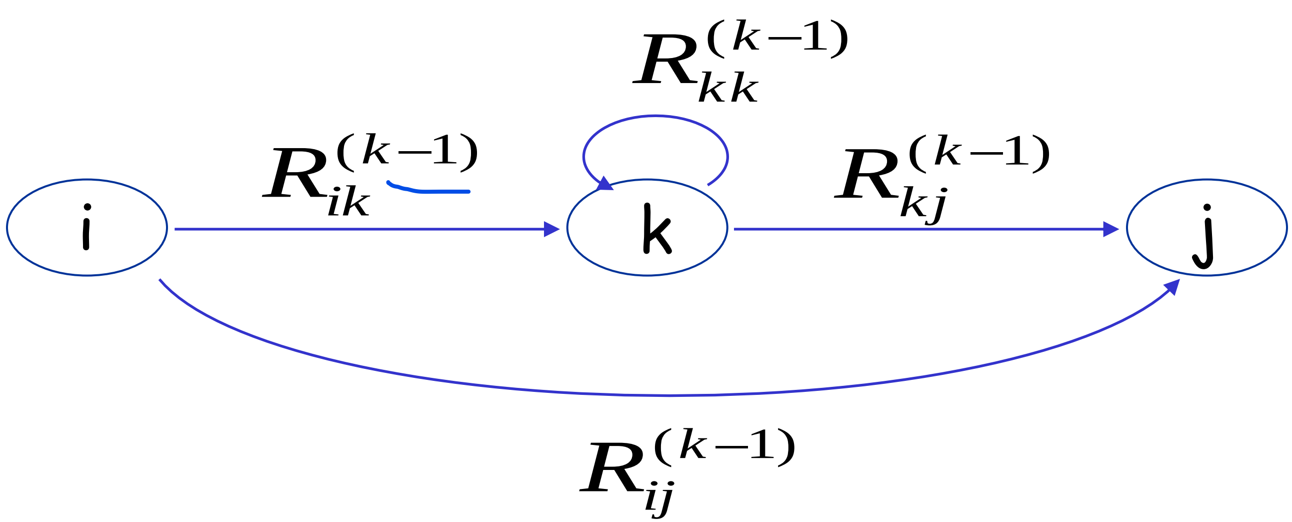
$$\Rightarrow \quad R_{ij}^{(0)} = \varepsilon + a_1 + a_2 + \Lambda + a_n$$

FA $\Rightarrow$ RegExp - Induction

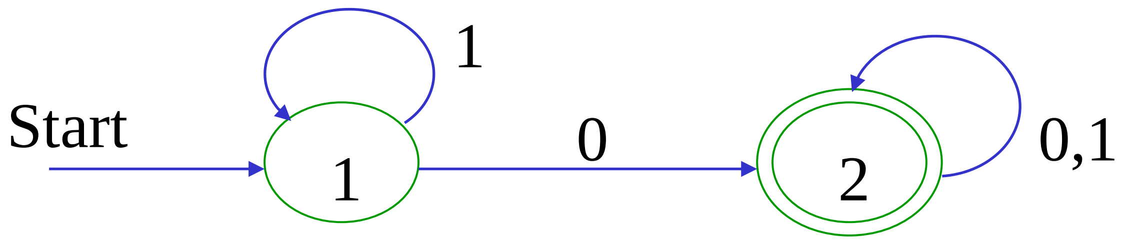
只考虑 k之前

Induction $k \geq 1$

$$R_{ij}^{(k)} = R_{ij}^{(k-1)} + R_{ik}^{(k-1)}(R_{kk}^{(k-1)})^* R_{kj}^{(k-1)}$$



Example 9 FA $\Rightarrow$ RegExp



$$R_{11}^{(0)} = \varepsilon + 1, \quad R_{12}^{(0)} = 0, \quad R_{21}^{(0)} = \phi, \quad R_{22}^{(0)} = \varepsilon + 0 + 1$$

Example 9 FA $\Rightarrow$ RegExp

$$R_{ij}^{(k)} = R_{ij}^{(k-1)} + R_{ik}^{(k-1)} (R_{kk}^{(k-1)})^* R_{kj}^{(k-1)}$$

$$R_{11}^{(0)} = \varepsilon + 1, \quad R_{12}^{(0)} = 0, \quad R_{21}^{(0)} = \phi, \quad R_{22}^{(0)} = \varepsilon + 0 + 1$$

$$R_{11}^{(1)} = \varepsilon + 1 + (\varepsilon + 1)(\varepsilon + 1)^* (\varepsilon + 1) = 1^*$$

$$R_{12}^{(1)} = 0 + (\varepsilon + 1)(\varepsilon + 1)^* 0 = 1^* 0$$

$$R_{21}^{(1)} = \phi + \phi = \phi$$

$$R_{22}^{(1)} = \varepsilon + 0 + 1 + \phi = \varepsilon + 0 + 1$$

Example 9 FA $\Rightarrow$ RegExp

$$R_{ij}^{(k)} = R_{ij}^{(k-1)} + R_{ik}^{(k-1)}\left(R_{kk}^{(k-1)}\right)^* R_{kj}^{(k-1)}$$

$$R_{11}^{(1)} = 1^*,\ R_{12}^{(1)} = 1^*0,\ R_{21}^{(1)} = \phi,\ R_{22}^{(1)} = \varepsilon + 0 + 1$$

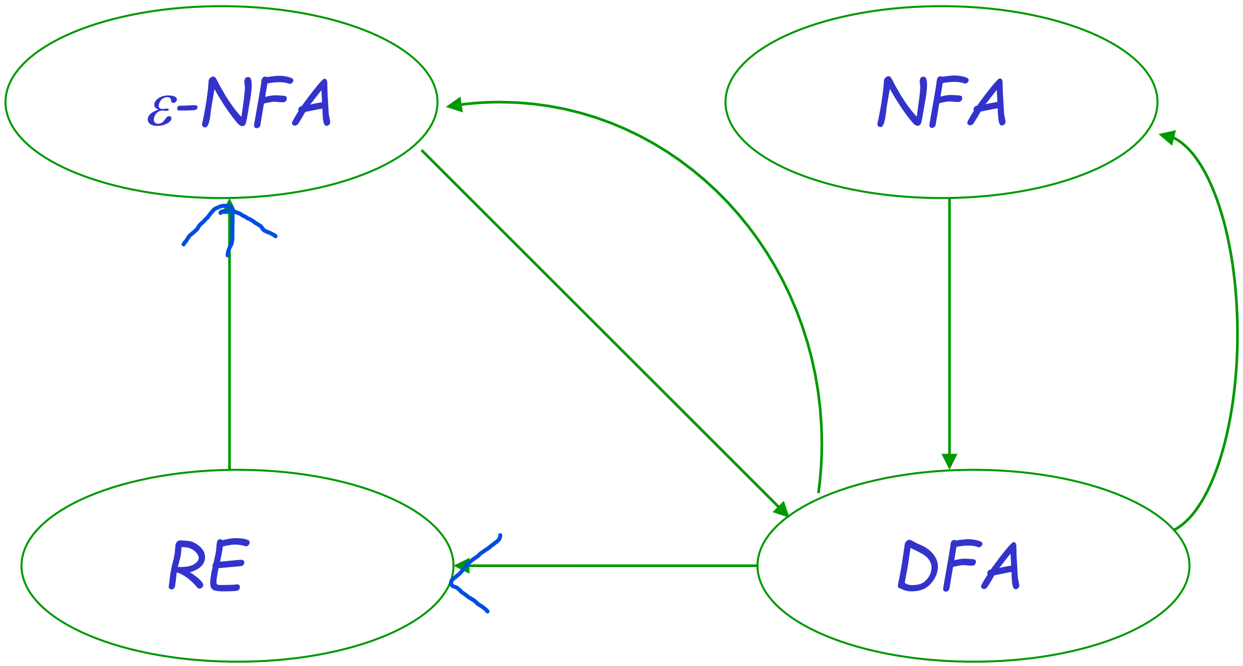
$$R_{11}^{(2)} = 1^* + 1^*0(0 + 1)^*\phi = 1^*$$

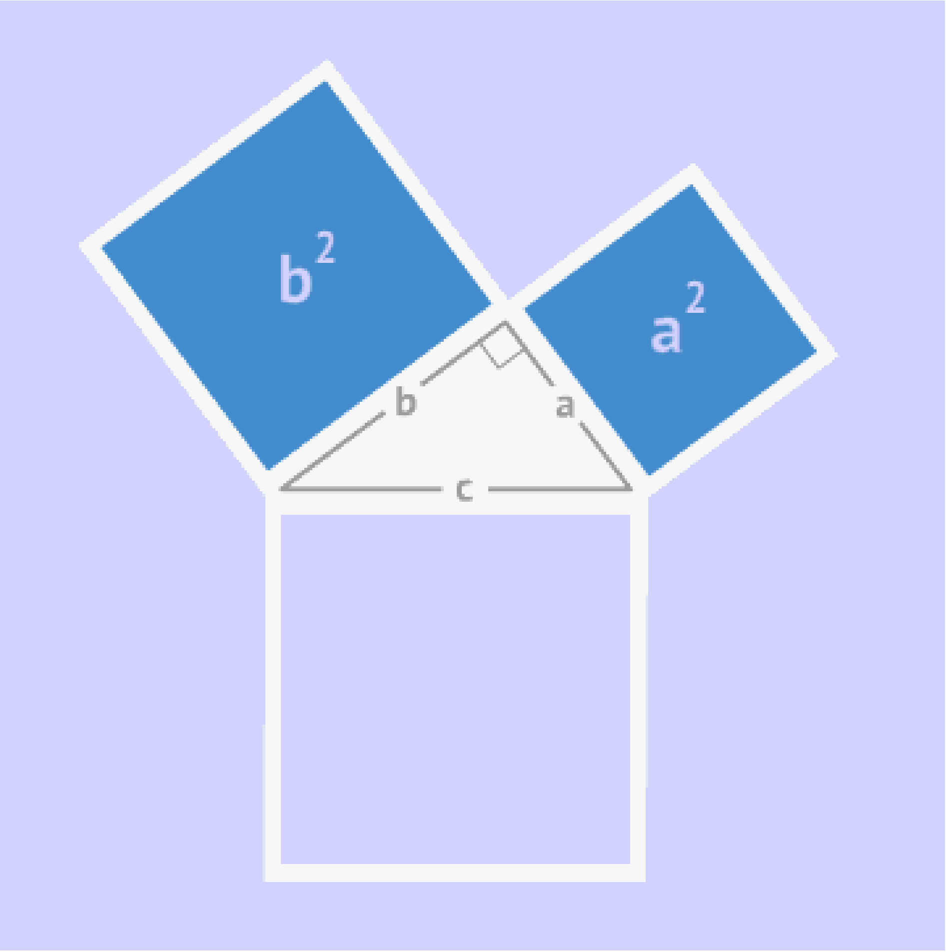
$$R_{12}^{(2)} = 1^*0 + 1^*0(\varepsilon + 0 + 1)^*(\varepsilon + 0 + 1) = 1^*0(0 + 1)^*$$

$$R_{21}^{(2)} = \phi + \phi = \phi$$

$$R_{22}^{(2)} = \varepsilon + 0 + 1 + (\varepsilon + 0 + 1)^* = (0 + 1)^*$$

FA & RE




b2
a2
b
a
c

Good good study

day day up !