Chapter 3

正则表达式

1. Give regular expressions for each of the following languages over the alphabet $\{0,1\}$.

   (1) Strings that end with the suffix $0^5 = 00000$

   

   (2) All strings containing the substring 000.

   

   (3) All strings *not* containing the substring 000.

   

   (4) All strings *not* containing the substring 110.

   

   (5) Every string except 000.

(6) All strings w such that in every prefix of w, the number of 0s and 1s differ by at most 1.

(7) All strings w such that in every prefix of w, the number of 0s and 1s differ by at most 2.

(8) All strings that start with 00 and contain at least one 1.

(9) All strings that start with 01 and contain at least two 0s.

(10) All strings that start with 1 and contain at least two 0s.

(11) All strings containing at least two 0s and at least one 1.

(12) All strings in which the substring 000 appears an even number of times. (For example, 0001000 and 10000 are in this language, but 000110000 and 00100000 are not.)

(13) Strings in which every occurrence of the substring 00 appears before every occurrence of the substring 11.

(14) Strings in which the number of 0s and the number of 1s differ by a multiple of 3.

(15) Strings that contain an even number of 1s and an odd number of 0s.

(16) Strings that represent a number divisible by 5 in binary.

(17) All strings in which every run of 0s has length at least 3.

(18) All strings containing at least three 0s.

(19) All strings containing at least two 0s and at least one 1.

2. [Execise 3.1.1] Write regular expressions for the following languages:

   (a) The set of strings over alphabet $\{a, b, c\}$ containing at least one a and at least one b.

(b) The set of strings of 0's and 1's whose tenth symbol from the right end is 1.

(c) The set of strings of 0's and 1's with at most one pair of consecutive 1's.

3. [!Execise 3.1.2] Write regular expressions for the following languages:

(a) The set of all strings of 0's and 1's such that every pair of adjacent 0's appears before any pair of adjacent 1's.

(b) The set of strings of 0's and 1's whose number of 0's is divisible by five.

4. [!!Execise 3.1.3] Write regular expressions for the following languages:

(a) The set of all strings of 0's and 1's not containing 101 as a substring.

(b) The set of all strings with an equal number of 0's and 1's, such that no prefix has

two more 0's than 1's, nor two more 1's than 0's.

(c) The set of all strings of 0's and 1's whose number of 0's is divisible by five and whose number of 1's is even.

(d) The set of all strings of 0's and 1's whose number of 0's is even and whose number of 1's is even.

5. [!Execise 3.1.4] Give English descriptions of the languages of the following regular expressions:

(a) $(\mathbf{1}+\boldsymbol{\varepsilon})(\mathbf{00}^*\mathbf{1})^*\mathbf{0}^*$

(b) $(\mathbf{0}^*\mathbf{1}^*)^*\mathbf{000}(\mathbf{0}+\mathbf{1})^*$

(c) $(\mathbf{0}+\mathbf{10})^*\mathbf{1}^*$

6. [!Execise 3.1.4] $\emptyset$ and $\{\varepsilon\}$ are only two languages whose closure is finite.

7. [Exercise 3.2.1]: Here is a transition table for DFA:

	0	1
$\rightarrow q_1$	q_2	q_1
q_2	q_3	q_1
$*q_3$	q_3	q_2

   (a) Give all the regular expressions $R_{ij}^{(0)}$. *Note*: Think of state q_i as if it were the state with integer number i.

   (b) Give all the regular expressions $R_{ij}^{(1)}$. Try to simplify the expressions as much as possible.

   (c) Give all the regular expressions $R_{ij}^{(2)}$. Try to simplify the expressions as much as possible.

   (d) Give a regular expression for the language of the automaton.

   (e) Construct the transition diagram for the DFA and give a regular expressions for its language by eliminating state q_2.

8. [Exercise 3.2.2]: Repeat Exercise 3.2.1 for the following DFA:

   Note that solutions to parts (a), (b) and (e) are *not* available for this exercise.

	0	1
$\rightarrow q_1$	q_2	q_3
q_2	q_1	q_3
$*q_3$	q_2	q_1

9. [Exercise 3.2.3]: Convert the following DFA to a regular expression, using the state-elimination technique of Section 3.2.2.

	0	1
$\rightarrow *p$	s	p
q	p	s
r	r	q
s	q	r

10. [Exercise 3.2.4]: Convert the following regular expressions to NFA's with ε-transitions.

    (a) 01^*.

    (b) $(0+1)01$.

    (c) $00(0+1)^*$.

11. [Exercise 3.2.5]: Eliminate ε-transitions from your ε-NFA's of Exercise 3.2.4. A solution to part (a) appears in the book's Web pages.

12. Design regular expressions for languages over $\Sigma = \{0, 1\}$.

    (1) All strings contain the substring 010.

    (2) $L = \{w \mid w$ does not contain 00 as a substring. $\}$

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