Afternoon

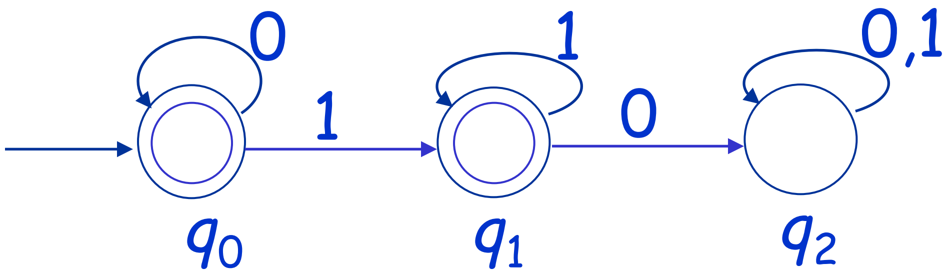
Pushdown Automata

- Definition
- Construction
- Configuration
- Deterministic PDA

The limit of FA

$L=\{\, 0^n1^n \mid n \geq 0 \,\}$

$M=\{0^n1^m \mid n \geq 0,\, m \geq 0 \,\}$



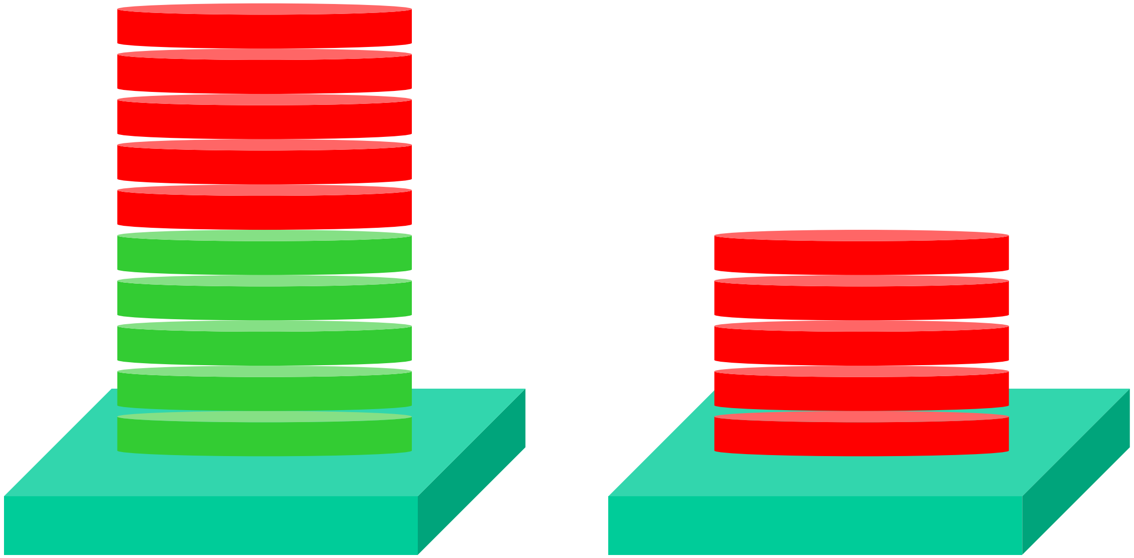
Why is there no any FA to recognize L ?

$L=\{\, 01, 0011, 000111, 00001111, 0000011111 , \ldots\ldots \}$

---- Remember the same number of 0's and 1's

Red/green discs

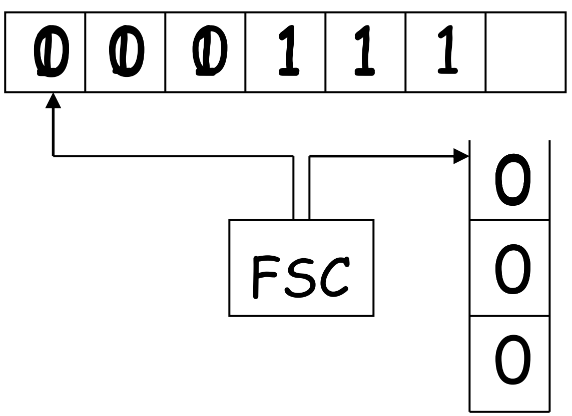
Is same the number of red and green discs ?



- Take red off , and put it on right table, one by one
- Take green off with red corresponding to it, one by one

Modify FA

$L = \{ 0^n 1^n \mid n \geq 1 \}$



read : 1 1 1

pop : 0 0 0

- read one 0, push one 0
- read one 1, pop one 0

Push-down automaton/PDA

PDA is a seven-tuple $P = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

- Q is finite set of states
- Σ *is finite set of input symbols*
- Γ *is finite set of stack symbols*
- q_0 *is start state*
- z_0 *is initial stack symbol*
- F *is finite set of accepting state*
- δ is transition function : $Q \times (\Sigma \cup \{\varepsilon\}) \times \Gamma \Rightarrow 2^{Q \times \Gamma^*}$

$$\delta(q, a, X) = \{(p, \alpha) \mid p \in Q, \alpha \in \Gamma^*\}$$

PDA for $L=\{0^n1^n \mid n \geq 1\}$

$$P(L) = (\{q, p, r\}, \{0,1\}, \{0, z\}, \delta, q, z, \{r\})$$

δ is defined as follows :

$$\delta(q, 0, z) = (q, 0z)$$

$$\delta(q, 0, 0) = (q, 00)$$

$$\delta(q, 1, 0) = (p, \varepsilon)$$

$$\delta(p, 1, 0) = (p, \varepsilon)$$

$$\delta(p, \varepsilon, z) = (r, z)$$

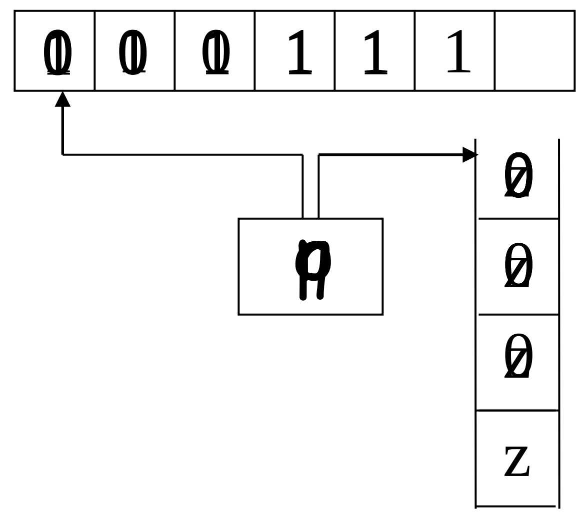
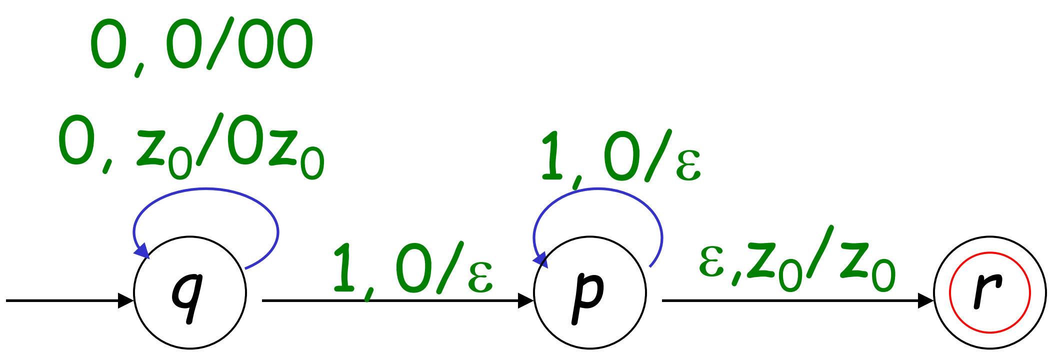


Diagram notation

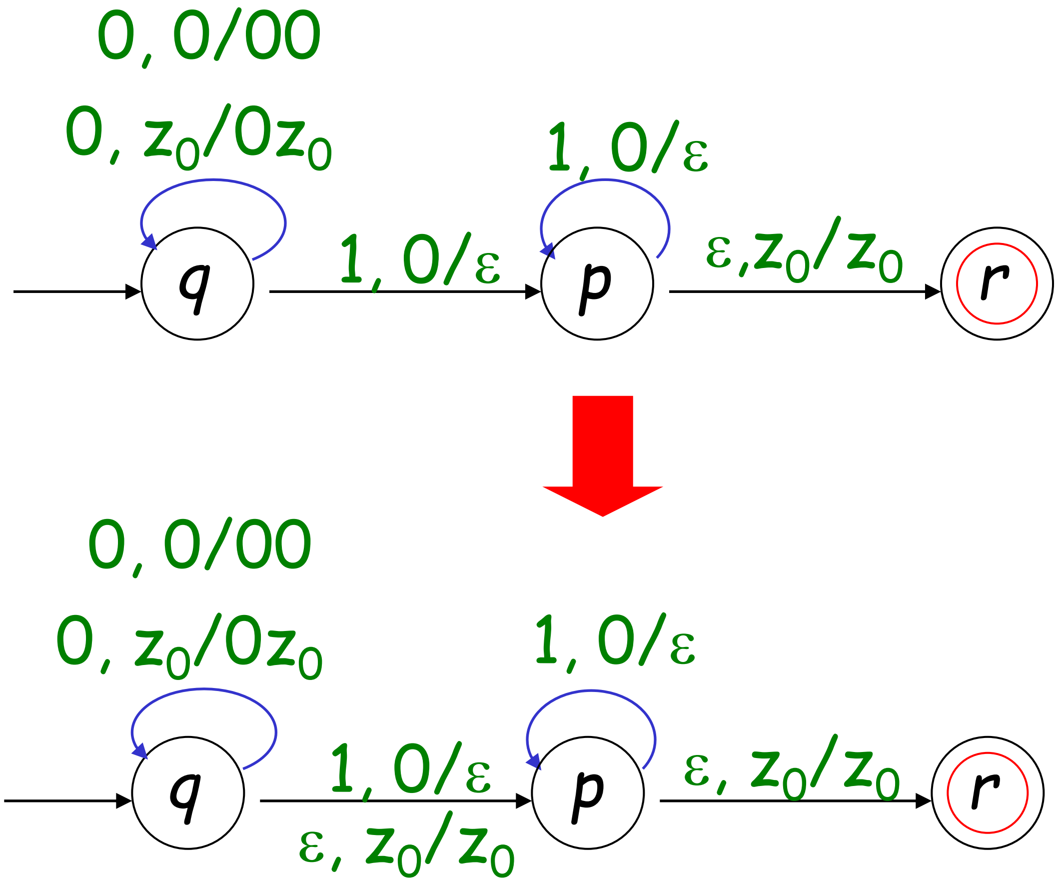
- adding stack symbol to arc
- diagram of PDA for $L=\{\ 0^n1^n \mid n\geq 1\ \}$



$$\delta(q, 0, z_0) = (q, 0z_0)$$
$$\delta(q, 0, 0) = (q, 00)$$
$$\delta(q, 1, 0) = (p, \varepsilon)$$
$$\delta(p, 1, 0) = (p, \varepsilon)$$
$$\delta(p, \varepsilon, z_0) = (r, z_0)$$

- What is the PDA for $L=\{0^n1^n \mid n\geq 0\}$?

Example 1 PDA for $L=\{0^n1^n \mid n \geq 0\}$

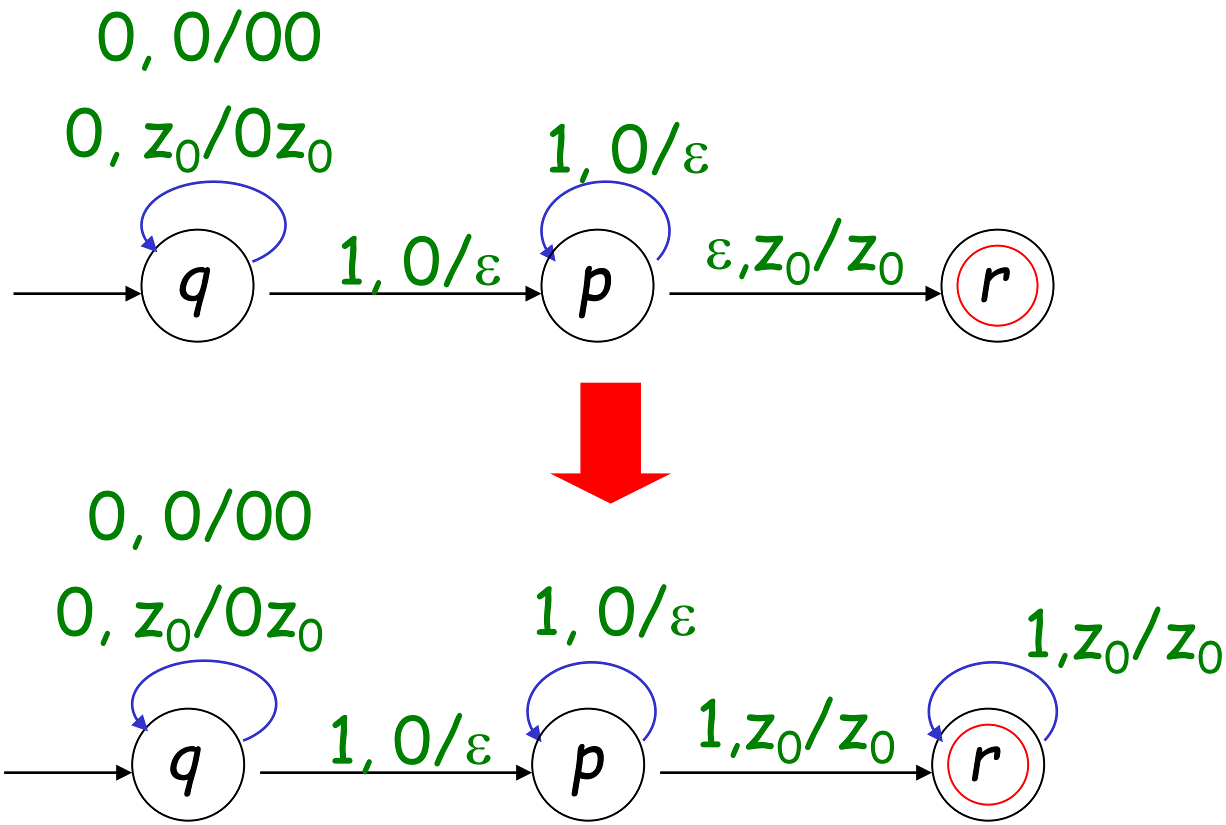


$n \geq 1$

$n \geq 0$

Example 2 PDA for $L=\{ 0^n1^m \mid n < m \}$

$$w = 0^n1^m = 0^n1^n1^{m-n}, \; m-n>0$$



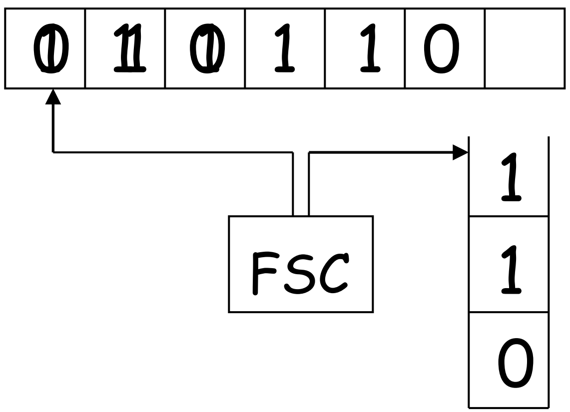
m=n

m>n

Example 3 PDA for

$L=\{ww^R \mid w \in \{0,1\}^*\}$

$w = 0\ 1\ 1\ 1\ 1\ 0$



read : 1 0

pop : 1 0

- read w, push w
- read w^R, pop w

Example 3 PDA for $L=\{ww^R \mid w \in \{0,1\}^*\}$

- step 1. Push w into stack one by one

$$\delta(q, 0, z_0) = (q, 0z_0),\ \delta(q, 1, z_0) = (q, 1z_0)$$
$$\delta(q, 0, 0) = (q, 00),\quad \delta(q, 1,0) = (q, 10)$$
$$\delta(q, 0, 1) = (q, 01),\quad \delta(q, 1,1) = (q, 11)$$

- step 2. Pop wR out of stack one by one

$$\delta(q, 1, 1) = (p, \varepsilon),\quad \delta(q, 0, 0) = (p, \varepsilon)$$
$$\delta(p, 1, 1) = (p, \varepsilon),\quad \delta(p, 0, 0) = (p, \varepsilon)$$

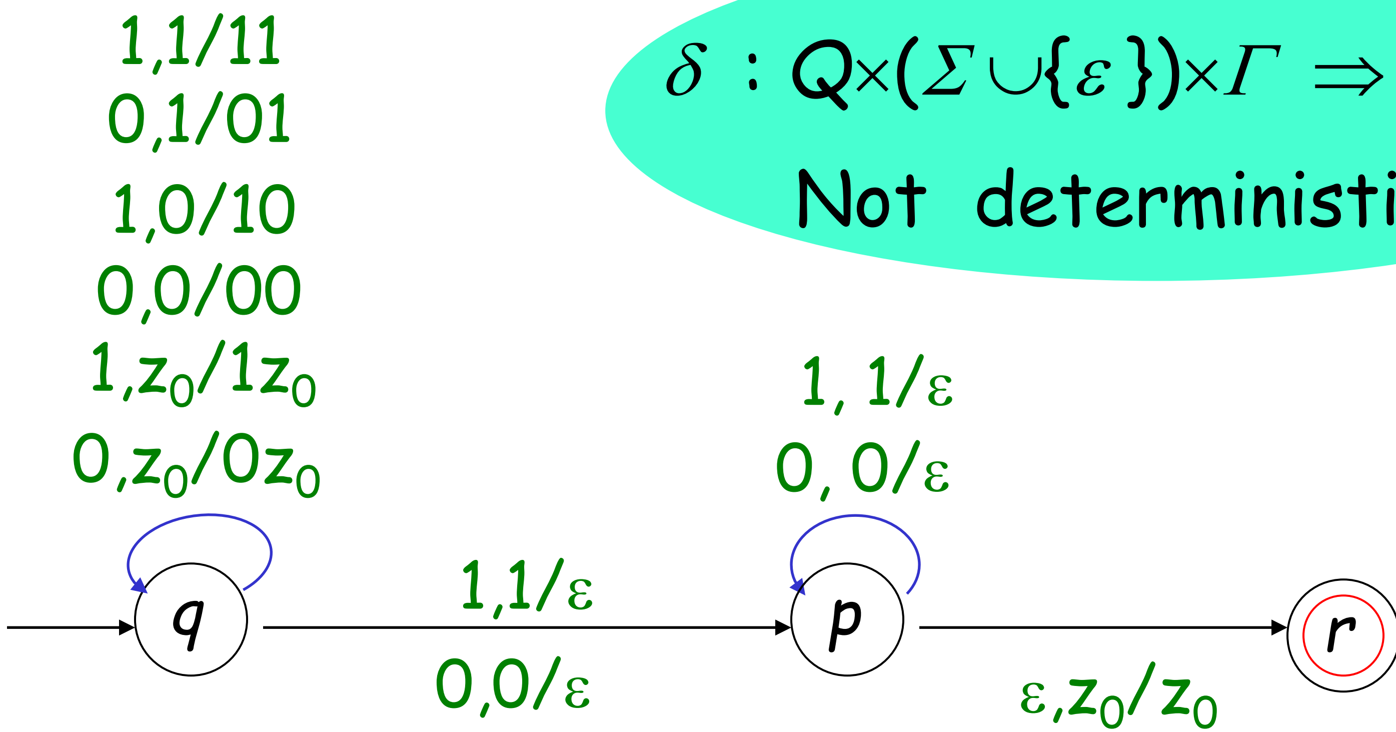
- finally $\delta(p,\varepsilon,z_0) = (r, z_0)$

Example 3 PDA for

$L = \{ ww^R \mid w \in \{0,1\}^* \}$

$\delta : Q \times (\Sigma \cup \{\varepsilon\}) \times \Gamma \Rightarrow 2^{Q \times \Gamma^*}$

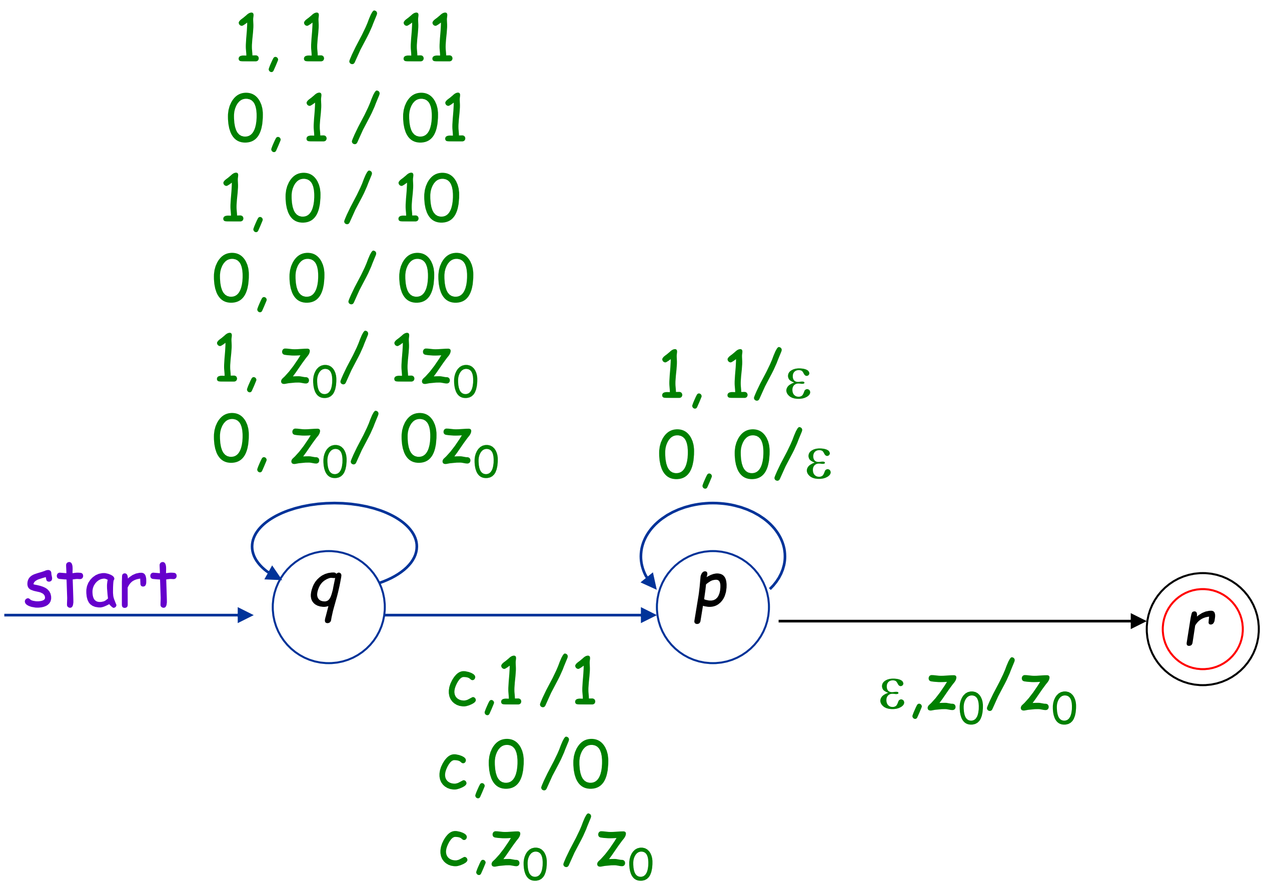
Not deterministic !



w = 0 1 1 1 1 0

Example 4 PDA for

$L=\{\, wcw^R \mid w \in \{0,1\}^* \,\}$



Deterministic push-down automaton

A PDA $P = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$ is said to be deterministic , when

- $\delta(q, a, X)$ has at most one member for any q in Q, a in Σ or a=ε , and X in Γ
- *If $\delta(q, a, X)$ is nonempty for some a in Σ , then $\delta(q, \varepsilon, X)$ must be empty .*

Deterministic push-down automaton

- If $\delta(q, a, X)$ is nonempty for some a in Σ, then $\delta(q, \varepsilon, X)$ must be empty .

$$\delta(q, a, X)$$

$$\delta(q, \varepsilon, X)$$

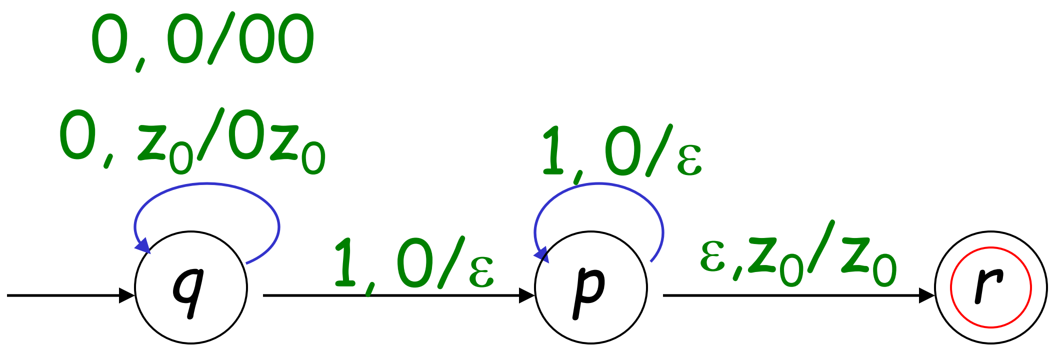
read a or ε

read a or not read a

Non deterministic

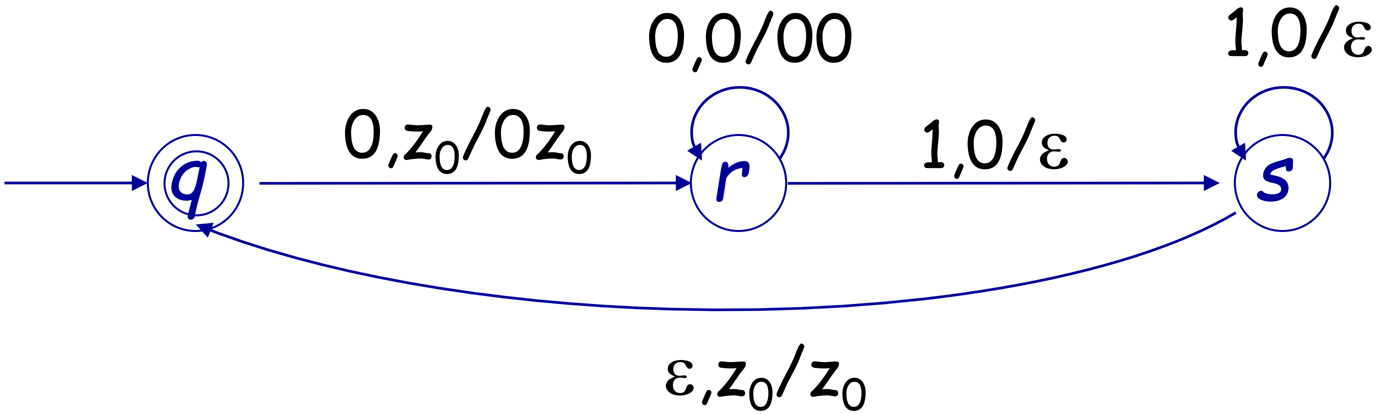
Example 5 DPDA for

$$L = \{ 0^n 1^n \mid n > 0 \}$$



Example 6 DPDA for

$$L = \{ 0^n 1^n \mid n \geq 0 \}$$



Is it right ?

Configuration

configuration--> (q, w, α)

q : state in which the PDA is

w : left symbols that PDA is going to read

α : string within stack

In PDA for $\{ 0^n1^n \mid n \geq 1 \}$, Let w=0011

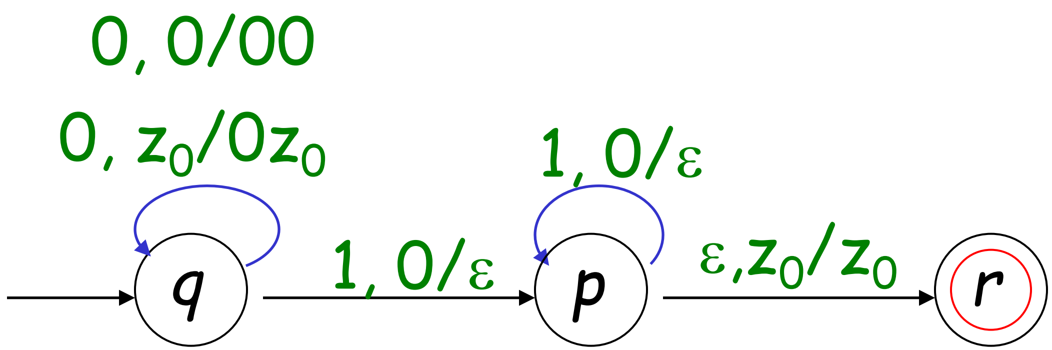
Initial configuration : (q, 0011, z)

Inner configuration : (q, 011, 0z), (q, 11, 00z), ...

Final configuration : (r, ε, z)

Instantaneous Description

- PDA for $L=\{0^n1^n \mid n\geq 1\}$



Let w=0011,

$$(q,0011,z_0) \vdash (q,011,0z_0) \vdash (q,11,00z_0) \vdash (p,1,0z_0)$$

$$\vdash (p,\varepsilon,z_0) \vdash (r,\varepsilon,z_0)$$

Compact : $(q,0011,z_0) \vdash^* (r,\varepsilon,z_0)$

Language of PDA

- Acceptance by final state

$$L(P) = \{w \mid (q_0, w, z_0) \vdash^* (q, \varepsilon, \alpha), q \in F\}$$

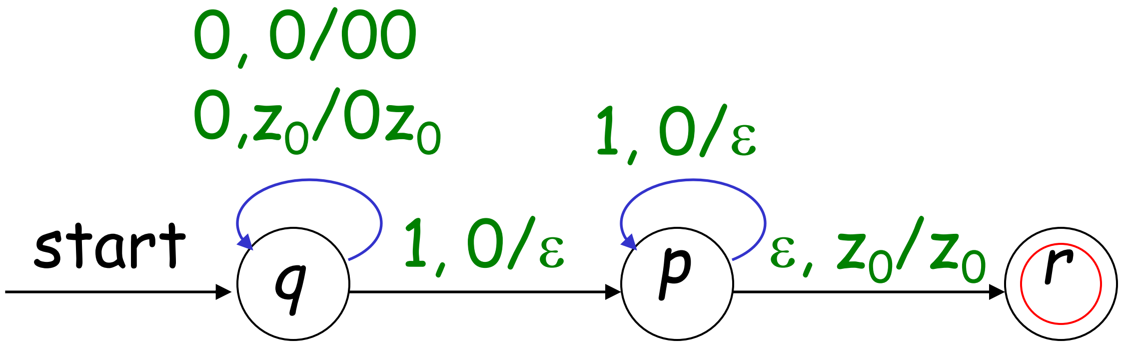
- Acceptance by empty stack

$$N(P) = \{w \mid (q_0, w, z_0) \vdash^* (q, \varepsilon, \varepsilon)\}$$

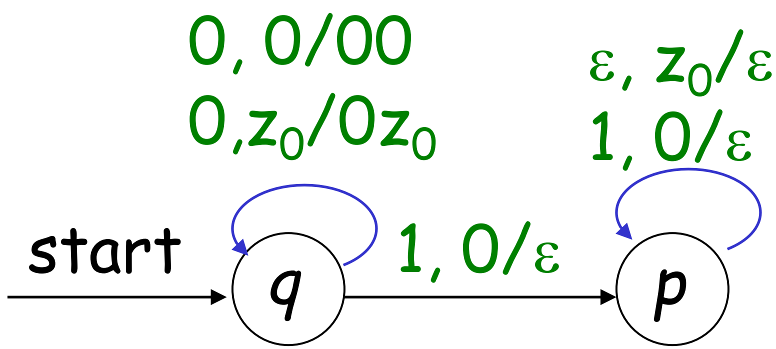
- Equivalence of two acceptance

$$L(P) \Leftrightarrow N(P)$$

Equivalence of two acceptance

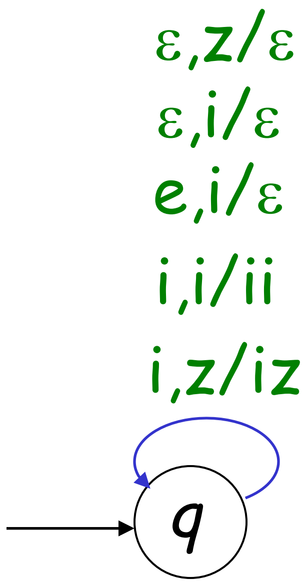


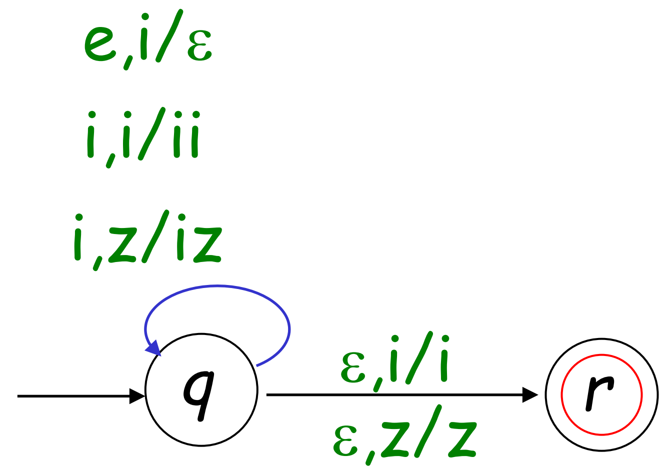
Accept by final state



Accept by empty stack

Example 7 PDA for if-else

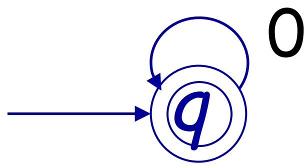




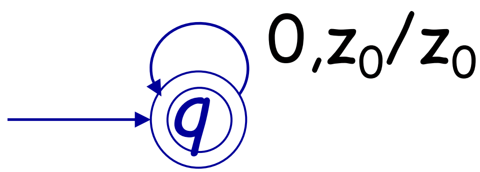
Two acceptance of DPDA

$$L = \{ 0^n \mid n \geq 0 \} = \{ 0 \}^*$$

FA :



DPDA :



---- by final state

by empty stack ?

Two acceptance of DPDA

- prefix property of language

  There are no two distinct string x and y in the language such that x is a prefix of y.

- yes : wcw^R . no : 0^*

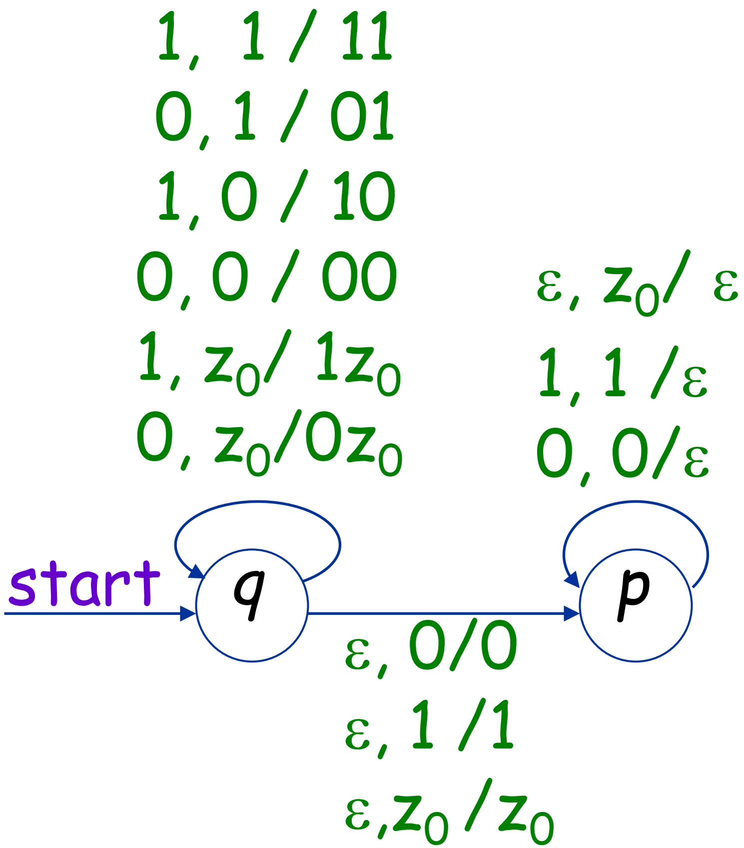
- L is accepted by DPDA P by empty stack $\Leftrightarrow$ L is accepted by DPDA P' by final state and L has prefix property.

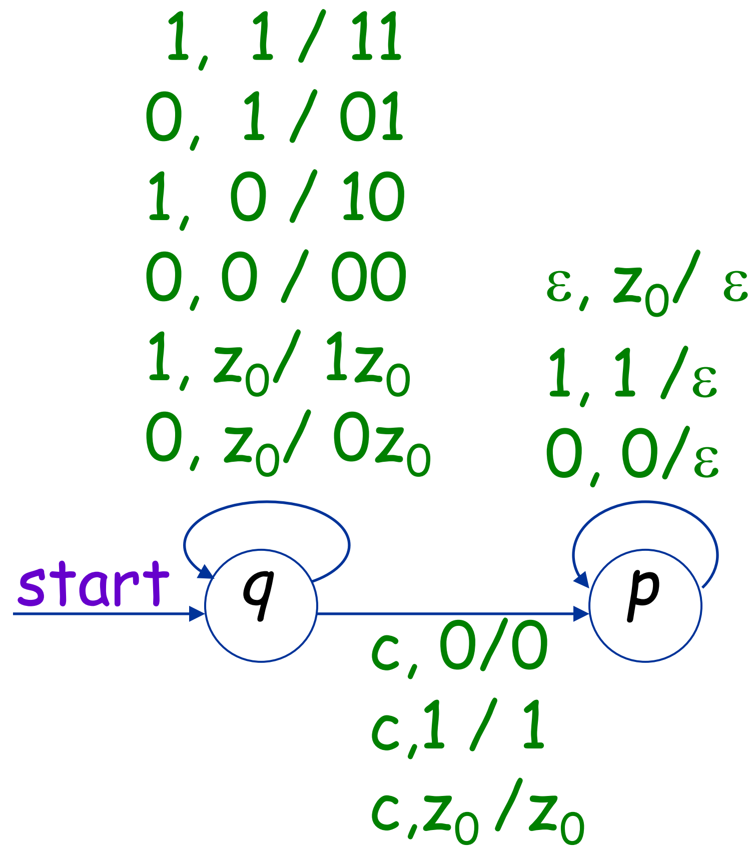
DPDA & PDA

Equivalent ?

$$L(FA) \subset L(DPDA) \subset L(PDA)$$

DPDA & PDA





$L_{wwr}=\{ww^R \mid w \in \{0,1\}^*\}$

$L_{wcwr}=\{wcw^R \mid w \in \{0,1\}^*\}$

FA & DPDA

FA $A = (Q, \Sigma, \delta, q_0, F)$

DPDA $P = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

If L is accepted by a FA, then it must be accepted by a DPDA .

$$\delta_A(q, a) = p \Rightarrow \delta(q, a, z_0) = (p, z_0)$$

The stack is never used .

Good good study

day day up !

$$2592 = 2^5 \cdot 9^2$$

Notable Properties of Specific Numbers (page 14) at MROB