Chapter 4

正则语言的性质

1. Prove that the language $L = \{a^n b^m c^n \mid m, n \geq 0\}$ is not regular with pumping lemma.

2. [Exercise 4.1.2] Prove that the following are not regular languages.

   a) $\{0^n \mid 0$ is a perfect square$\}$ n n^2

   b) $\{0^n \mid 0$ is a perfect cube$\}$ n n^3

   c) The set of strings of 0's and 1's whose length is a perfect square.

   d) The set of strings of 0's and 1's that are of the form ww, that is some string repeated.

3. [Exercise 4.2.2] If L is a language, and a is a symbol, then L/a, the quotient 商 of L and a, is the set of strings w such that wa is in L. For example, if $L = \{a, aab, baa\}$, then $L/a = \{\varepsilon, ba\}$. Prove that if L is regular, so is L/a. Hint: Start with a DFA for L and consider the set of accepting states.

4. [Exercise 4.2.6] Show that the regular languages are closed under the following operations:

   (a) $\min(L) = \{w|w$ is in L, but no proper prefix of w is in $L\}$.

   suffix 后缀

   (b) $\max(L) = \{ w \mid w$ is in L and for no x other than ε is wx in $L\}$

   (c) $\text{init}(L) = \{ w \mid$ for some x, wx is in $L \}$

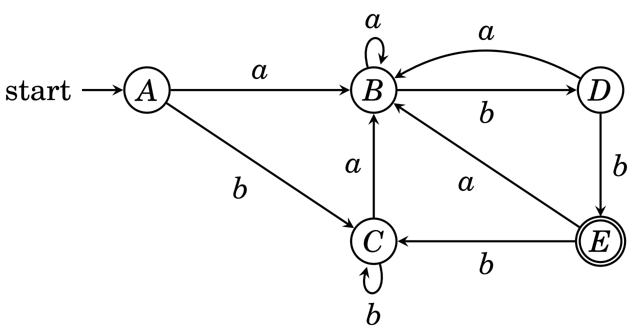
5. [Exercise 4.2.8] Let L be a language. Define $half(L)$ to be the set of first halves of strings in L, that is, $\{w \mid$ for some x such that $|x| = |w|$, we have wx in $L\}$. For example, if L = $\{\varepsilon, 0010, 011, 010110\}$ then $half(L) = \{\varepsilon, 00, 010\}$. Notice that odd-length strings do not contribute to $half(L)$. Prove that if L is a regular language, so is $half(L)$.

6. Let L be a language. Define $\sqrt{L}$ to be the set $\{w \mid ww$ is in $L\}$. Prove that if L is regular, then so is $\sqrt{L}$. *Hint*: Given a DFA for L with n many states, one can build a DFA for $\sqrt{L}$ with n^n many states.

   More generally, for $k \geq 1$, define $\sqrt[k]{L}$ to be the set $\{w \mid w^k$ is in $L\}$. Also define $\sqrt[*]{L}$ to be the set $\{w \mid w \text{ is in } \sqrt[k]{L} \text{ for some } k \geq 1\}$. Similar constructions prove that if L is regular, then all these other languages are regular.

7. [Exercise 4.2.11] Show that regular languages are closed under the following operations: $cycle(L) = \{w \mid \text{we can write } w \text{ as } w = xy, \text{ such that } yx \text{ is in } L\}$. For example, if $L = \{01, 011\}$, then $cycle(L) = \{01, 10, 011, 110, 101\}$. *Hint*: Start with a DFA for L and construct an ε−NFA for $cycle(L)$.

8. Minimize the given DFA.



9. Prove or disprove: if L_1 is regular and $L_1 \cup L_2$ is also regular, then L_2 must be regular.

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