Afternoon

手机关机或静音放包/兜里

Deterministic Finite Automata

- Definition
- Notation
- Construction
- Regular Language

Formal Definition

Deterministic finite automaton is a five-tuple ,

such as $M = (Q, \Sigma, \delta, q_0, F)$

Where Q is a finite set of *states* ,

Σ is a finite set of *input symbols* ,

q_0 is a *start state* ,

F is a set of *final state* ,

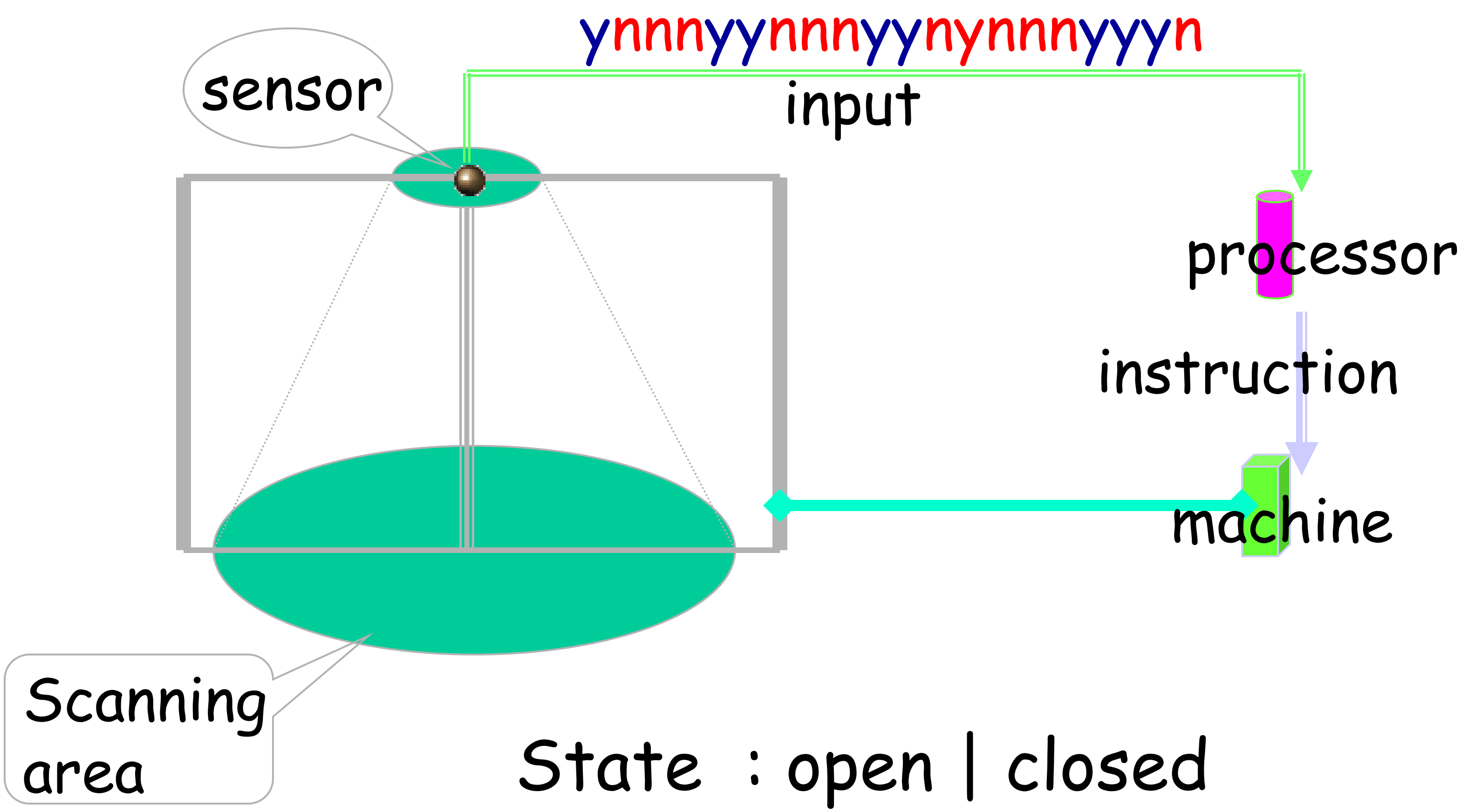
δ is *transition function* , which is a mapping

from $Q \times \Sigma$ to Q .

Auto-Door

Auto-Door

Auto-Door



State : open | closed

Action : open | close

input : y | n

Auto-Door

Σ

Input symbols : { 0 , 1 } State : { Closed , Open }

State transition :

δ

(Closed, 0) $\Rightarrow$ Closed

(Closed, 1) $\Rightarrow$ Open

(Open , 1) $\Rightarrow$ Open

(Open , 0) $\Rightarrow$ Closed

Start state : Closed

Final state : Closed

和 或

和

Auto-Door

字符集

形式化

Input symbols : { 0 , 1 } State : { q , p }

State transition function:

$$\delta(q, 0) = q$$

$$\delta(q, 1) = p$$

$$\delta(p, 1) = p$$

$$\delta(p, 0) = q$$

Start state : q

Final state : q

Automaton

DFA for Auto-Door

$M = (Q, \Sigma, \delta, q_0, F)$

$Q = \{closed, open\}, \quad \Sigma = \{n, y\}$

$q_0 = closed, \quad F = \{closed\}$ 接受

$\delta:$

$\delta(closed, n) = closed$

$\delta(closed, y) = open$

$\delta(open, n) = closed$

$\delta(open, y) = open$

DFA for Auto-Door

formal expression

$$M = (Q, \Sigma, \delta, q_0, F)$$

$Q = \{p, q\}$, $\Sigma = \{0, 1\}$

$q_0 = q$, $F = \{q\}$

δ :

Can you see the Door ?

$\delta(q, 0) = q$

$\delta(q, 1) = p$

$\delta(p, 0) = q$

$\delta(p, 1) = p$

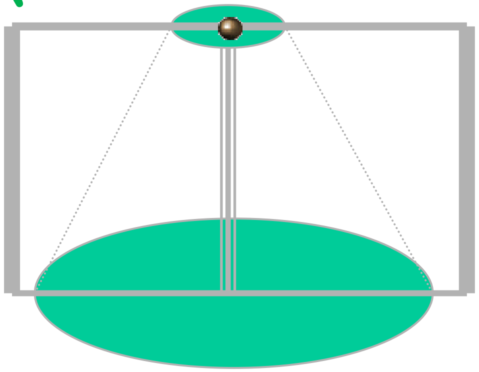
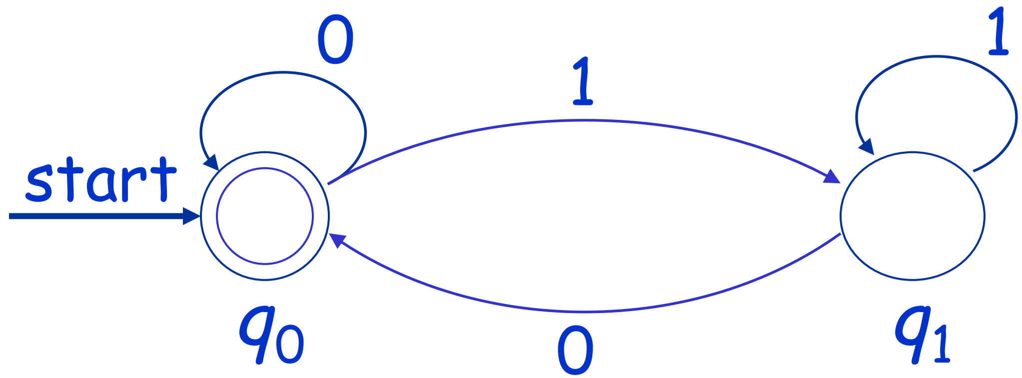


Diagram Notation

图示法



$$M = (\{q_0, q_1\}, \{0,1\}, \delta, q_0, \{q_0\})$$

δ :

$$\delta(q_0, 0) = q_0, \quad \delta(q_0, 1) = q_1$$

$$\delta(q_1, 0) = q_0, \quad \delta(q_1, 1) = q_1$$

Table Notation

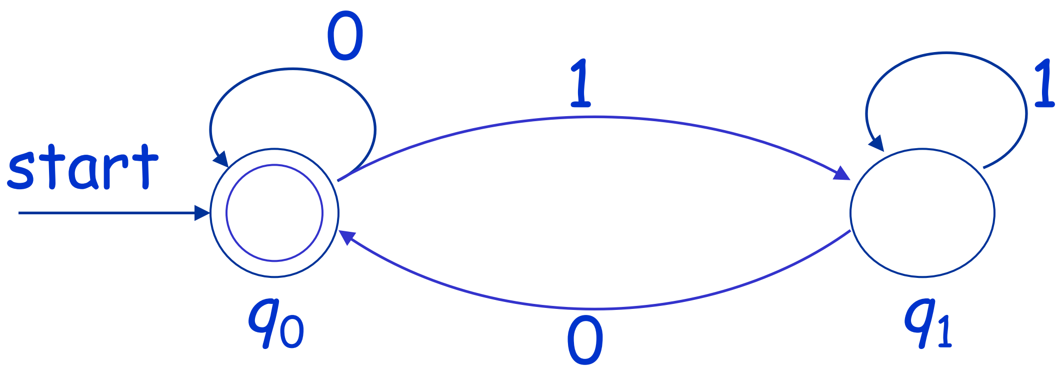
	0	1
$\rightarrow * q_0$	q_0	q_1
q_1	q_0	q_1

$$M = (\{q_0, q_1\}, \{ 0,1 \}, \delta , q_0, \{ q_0 \})$$

δ :

$$\delta(q_0 , 0) = q_0 , \quad \delta(q_0 , 1) = q_1$$
$$\delta(q_1 , 0) = q_0 , \quad \delta(q_1 , 1) = q_1$$

Partition Strings



理解

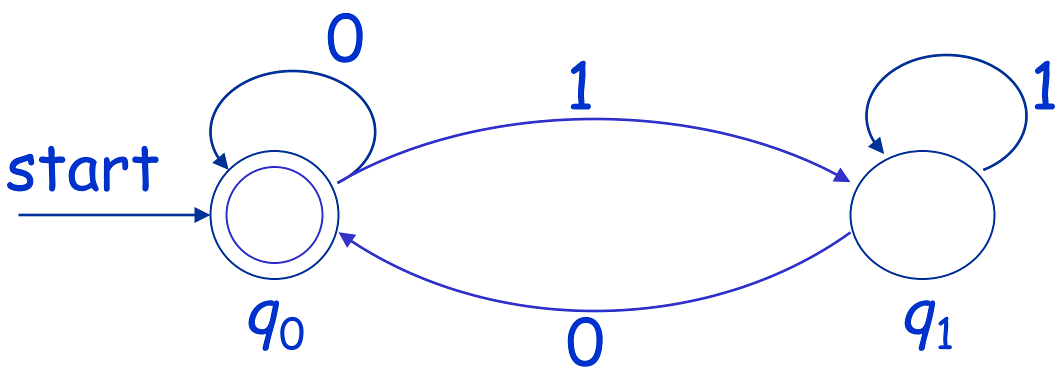
M partitions all strings into two groups:

$$L_1 = \{\, w \in \{0,1\}^* \mid w \text{ end with } 0 \,\} \cup \{\, \varepsilon \,\}$$

$$L_2 = \{\, w \in \{0,1\}^* \mid w \text{ end with } 1 \,\}$$

DFA as a recognizer of language

识别器



M "recognize" the following language :

$$L = \{ w \in \{0,1\}^* \mid w \text{ end with } 0 \} \cup \{ \varepsilon \}$$

With the language L, and a string $w \in \{0,1\}^*$

M tell us whether w belongs to L , or not

Decision problem

Given a language L, and a string w

Is w belong to L ?

Example 1

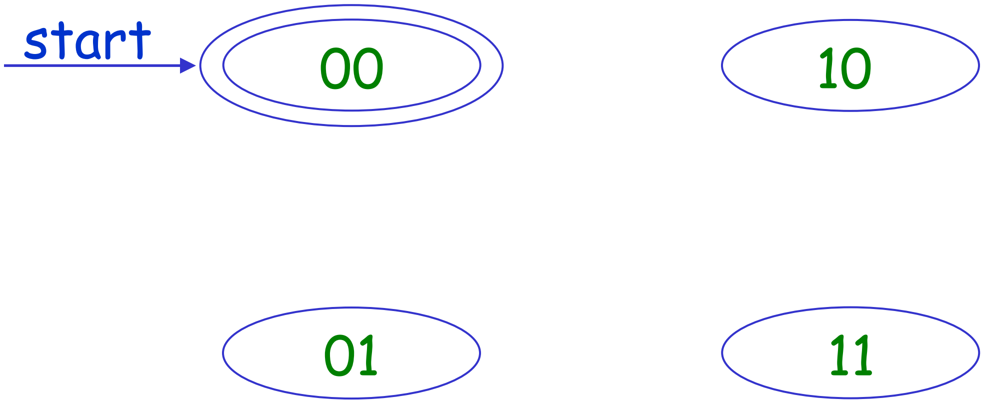
$L = \{w \in \{0,1\}^* \mid w$ has both an even number of 0's and an even number of 1's $\}$

- Partition strings into four groups
  - 00 : even 0 and even 1
  - 01 : even 0 and odd 1
  - 10 : odd 0 and even 1
  - 11 : odd 0 and odd 1

Example 1

$$L = \{w \in \{0,1\}^* \mid w \text{ has both an even number of 0's and an even number of 1's}\}$$

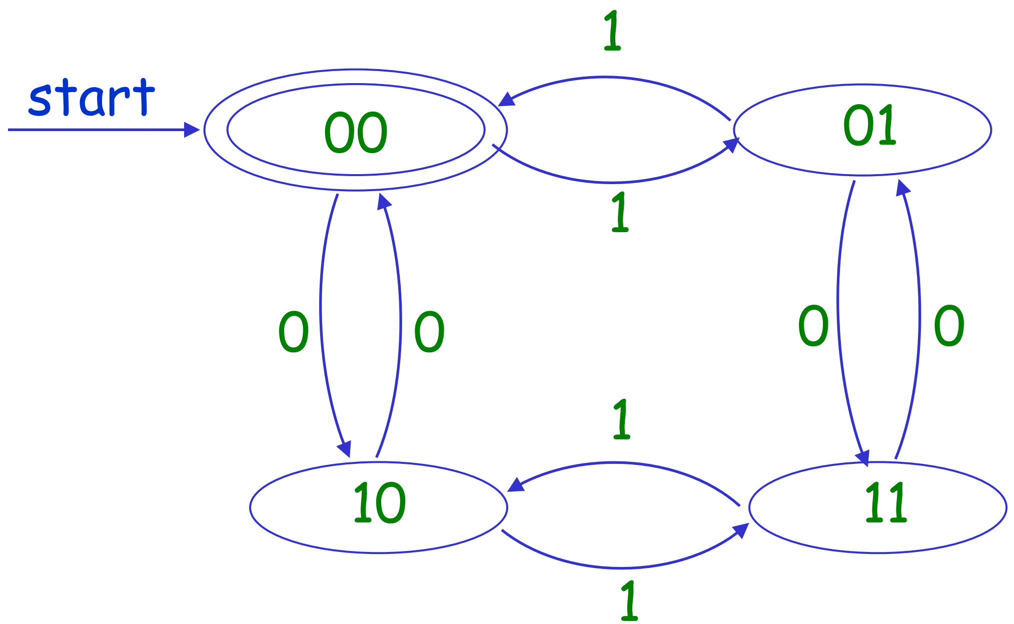
- Set states corresponding to partitions



Example 1

$L = \{w \mid w$ has both an even number of 0's and an even number of 1's $\}$

- Put transition arcs between states



Example 2

序列识别器

L = {w | w consists of 0's and 1's , and contains sub-string 01}

or {x01y | x and y are consists of any number of 0's and 1's }

Problem :

How to decide whether a given string w belongs to L ?

Example 2

L = {x01y | x and y are consists of any number of 0's and 1's }

How to start our work ?

- What is the meaning " w belongs to L "
- Partition strings by properties of L
- Set states which correspond to the partitions
- Put transition arcs between states

Example 2

L = {x01y | x and y are consists of any number of 0's and 1's }

- Partition strings by properties of L

{ ε,0,1,00,01,10,11,000,001,010,100,011,101,110,111 }

{ 0000,0001,0010,0100,1000,0011,0101,1001,...... }

$\delta(q, 1) = p$

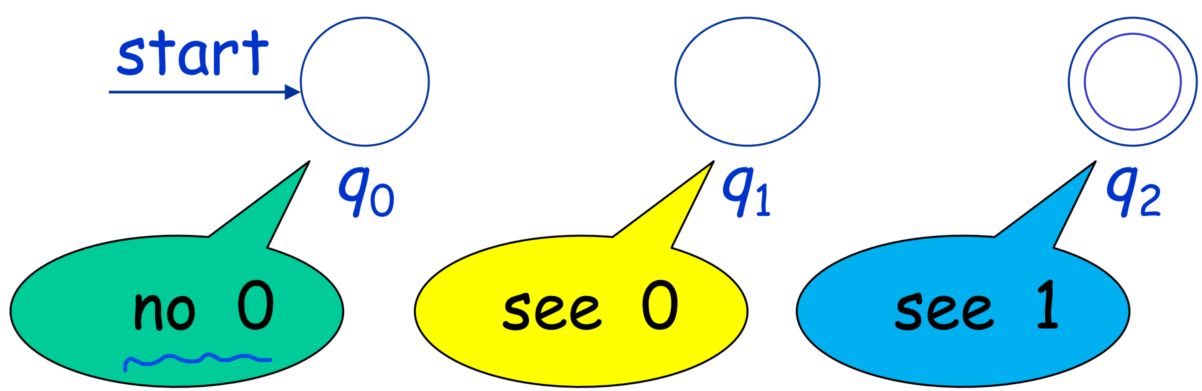
1 1 1 0 0 0 1 1 0 0 1

1 1 1 0 0 0 1 1 0 0 1

Example 2

$L = \{x01y \mid x$ and y are consists of any number of 0's and 1's $\}$

- Set states which correspond to the partitions



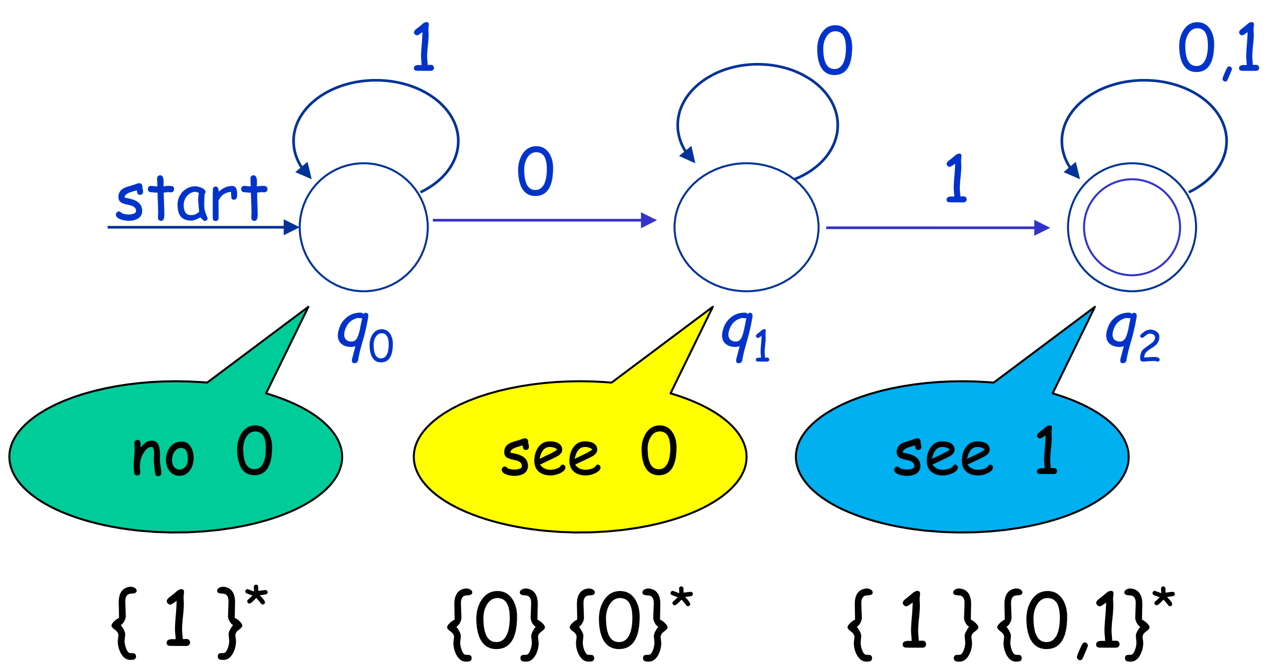
认知

形式化： $\{1\}^*$ $\{0\}\{0\}^*$ $\{1\}\{0,1\}^*$

Example 2

$L = \{x01y \mid x \text{ and } y \text{ are consists of any number of 0's and 1's}\}$

- Put transition arcs between states



Extending δ to string

递归定义

$\delta(q, a) = p$

BASIS

$$\hat{\delta}(q, \varepsilon) = q.$$

hat

INDUCTION

归纳

Suppose w is a string of the form xa, that is, a is the last symbol of w, and x is the string consisting of all but the last symbol. Then

$$\hat{\delta}(q, w) = \delta(\hat{\delta}(q, x), a)$$

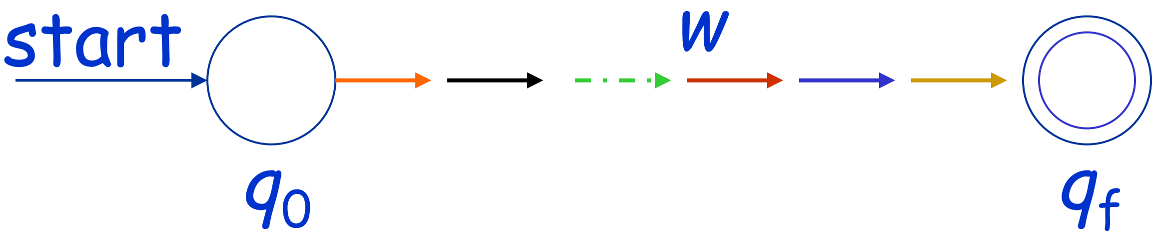
先递 再归 (计算)

Language of a DFA

dfa (识别、接受) 的 L

Definition The *language of a DFA* A is denoted $L(A)$, and defined as

$$L(A) = \{w \mid \hat{\delta}(q_0, w) \ is \ in \ F\}$$



Regular language

Definition

正则语言

If L is $L(A)$ for some DFA A, then we say L is a *regular language*.

RegL = { L | There is a DFA accepting L }

Note : a kind of languages accepted by DFA's

L(A)是一类语言

Exersizes

Construct DFA for following languages :

a) $\{ 0 \}^*$

b) $\{w \mid w \in \{0,1\}^*$ and begin with 0 $\}$

c) $\{w \mid w$ consists of any number of 0's followed by any number of 1's $\}$

d) $\{ \varepsilon \}$

e) ϕ

2

1010
1111

11001
1010

100011

$5 = 4+1$

101

$10 = 8+2$

1010

1111

$50 = 32+16+2$

110010

$122 = 64+32+16$
$+8+2$

1111010

1010
+ 1010

10100
10100

101000

Good good study

day day up !