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A mixed C-vine copula model for hedging price and volumetric risk in wind power trading

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When energy trading companies enter into long-term agreements with wind power producers, where a fixed price is paid for the fluctuating production, they are facing a joint price and volumetric risk. Since the pay-off of such agreements is non-linear, a hedging portfolio would ideally consist of not only forwards, but also a basket of e.g. call and put options. Illiquidity and an almost non-existent market for options challenge however the optimal hedging of joint price and volumetric risk in many market places. Here, we consider the case of the Danish power market, and exploit its strong positive correlation with the much more liquid German market to construct a proxy hedge. We propose a three-dimensional mixed vine copula to model the evolution of the Danish and German spot electricity prices and the Danish wind power production. We construct a realistic hedging portfolio by identifying various instruments available in the market, such as real options in the form of the right to transfer electricity across the border and the right to convert electricity to heat. Using the proposed vine copula to determine optimal hedging decisions, we show that significant benefits are to be drawn by extending the hedging portfolio with the proposed instruments.

Keywords: Spot electricity prices; Wind power production; Mathematical finance; Copula vines; Energy derivatives; Hedging

JEL Classification: C22, C51, G13

1. Introduction

In recent years, the share of wind power production has increased significantly in many market places. As a consequence, considering in more detail the joint price and volumetric risk associated with wind power trading has become highly relevant. Not only are both spot electricity prices and wind power production notoriously volatile and unpredictable, but the mechanism of spot price formation implies a negative relation between the two that adds another dimension to the problem. Since spot prices are set by matching supply and demand curves, with supply curves prioritizing the cheapest generation sources (hereby wind turbines), the negative relation intensifies with a growing share of wind power production in the electrical grid. Wind power producers can rarely manage such involved risk, which would require constructing and rebalancing hedging portfolios through trading different suitable derivative instruments on the exchange, and they often seek to transfer their risk to another party. Alternatively in some market places, the price risk of wind power producers might automatically be removed (or partially removed) by support mechanisms initiated to promote the growth in renewable generation.

In this paper, we consider the problem of hedging joint price and volumetric risk in the context of an energy trading company entering into longer term financial agreements with wind power producers, where a pre-determined fixed price is paid for the fluctuating wind power production. Such financial agreements are becoming common in e.g. Denmark, Sweden, and the UK, which all rely to some extent on wind power generation.

The problem of joint price and volumetric risk was first addressed by McKinnon (1967) in relation to the classical farmer's problem, and has since been studied in different contexts. In the case of the energy markets, the main focus has been directed towards the joint price and volumetric risk associated with the customers' electricity demand. Some of the first to address many aspects of this problem, including hedging, were Oum *et al*. (2006) and Oum and Oren (2009, 2010). However, as acknowledged by Oum and Oren (2010), their proposed hedge had limited application in practice, as they allowed for trading in calls and puts written on the spot electricity price with a full spectrum of strikes, which was not possible at the time—and neither is it now. Later, the work of Coulon *et al*. (2013) focused on constructing a structural model for hedging

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the same type of risk, and used hedging instruments that are more available.

Our problem differentiates itself from the existing literature in different aspects. First, we consider volumetric risk on the supply rather than the demand side, which raises different modelling challenges. Second, we choose to analyse data from the Danish power market, since Denmark is among the top wind power countries in the world. This comes however at the cost of considering a market place that is characterized as rather illiquid, and where in addition the availability of options is limited. Nonetheless, this is the situation in the majority of European power markets. Since the hedging possibilities are naturally limited, we extend the modelling framework to a three-dimensional case, where we exploit the strong positive relation between the Danish and the German spot electricity prices to enlarge the set of available hedging instruments.

To model the joint behaviour of electricity prices in Denmark and Germany and wind power production in Denmark, we consider a vine copula, which is essentially a construction formed using pairs of bivariate copulas. Joe (1997) was the first to propose such a construction, and more general settings were later proposed by Bedford and Cooke (2001, 2002), who introduced the regular vines with the subclasses known as D-vines and canonical (C)-vines. Vine copula models allow for very flexible multivariate distributions, which motivates our model choice.

Copula models have found various applications in economics, finance and risk management. The literature is extensive for the bivariate case, with some examples being Cherubini and Luciano (2002) who consider bivariate option pricing, Patton (2006) who provides an application to exchange rates, Benth and Kettler (2011) who consider the spark spread, Avdulaj and Barunikl (2015) who investigate oil-stock diversification, and Elberg and Hagspiel (2015) who study spatial dependencies of wind power and interrelations with the spot price in Germany. Copulas are also a popular choice when it comes to higher dimensional problems, and the range of applications is again broad: Grothe and Schnieders (2011) investigate the optimal allocation of wind farms, Gatfaoui (2016) links the gas, oil and stock markets through trivariate copulas, Brechmann and Czado (2013) consider vines in the context of financial risk management, and Reboredo and Ugolini (2015) model systemic sovereign debt risk using vine copulas.

We offer two main contributions in this paper: First, we construct a flexible empirical model that captures the marginal behaviour of the individual variables accurately, as well as the dependency structure between the variables. Second, we identify instruments that can be used in practice to hedge the multiplicative price and volumetric risk, and study their hedging benefits. Based on empirical examples, we find that the variance of the revenue distribution can be significantly reduced by including these instruments, compared to the common strategy of using solely power forwards in a hedging portfolio. Furthermore, we employ different alternative models to facilitate hedging decisions, with the purpose of highlighting the effects of heavy tails in the margins, tail dependence and time variation in the dependence structure.

The paper is structured as follows: In section 2 we present the data. Section 3 introduces the modelling framework and presents empirical results. In section 4, we study hedging applications and compare different vine copula specifications against each other. Finally we conclude in section 5.

2. Data

Our empirical study relies on data from the Danish and the German power market for the period 1 January 2012 to 12 December 2016, corresponding to 1808 daily observations. Since the Danish power market is divided into two price areas, namely Western Denmark (DK1) and Eastern Denmark (DK2), where the share of wind power is significantly higher in DK1, we restrict our attention to this area. Hence, the first data input consists of the daily spot electricity price in DK1. Next, we consider the DK1 wind power production as a percentage of the installed capacity (wind power production index) on a daily basis, and the last data input consists of the daily spot electricity price in Germany (DE). The three time series are illustrated in figures 1(a)–(c). Regarding the price data, we have truncated a few extreme observations as they are considered outliers. Specifically, four extreme observations were truncated in the DK1 price series, and six extreme observations were truncated in the case of DE.

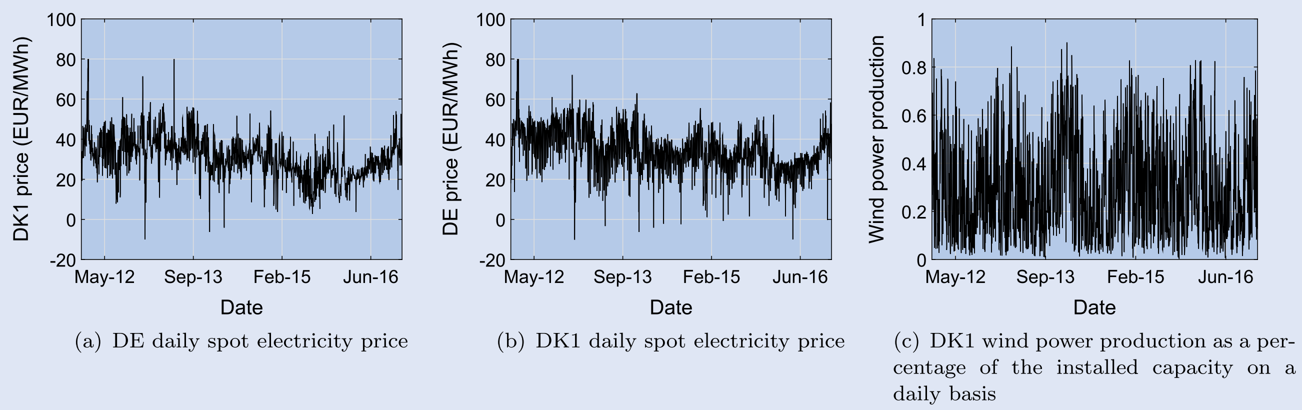
In figures 1(d)–(f), the data are plotted against each other pairwise. The first plot reveals a strong positive relation between the two prices, substantiating the idea of a proxy hedge, that is using instruments from the much more liquid DE market to hedge price and volume risk in DK1. The two remaining plots show a negative relation between wind power production and prices in the two areas we consider. As mentioned previously, this relation is expected due to the mechanism of price formation, and implies an additional correlation risk when dealing with a simultaneous exposure to spot electricity price and wind power production.

3. A vine copula model for spot electricity prices and wind power production

Let us denote by $\boldsymbol{x}_t = (x_{1,t}, x_{2,t}, x_{3,t})$ for $t = 1, \ldots, T$, the three-dimensional time series consisting of the spot electricity price in DK1, the spot electricity price in DE and the wind power production in DK1. We are interested in modelling the joint (conditional) distribution of $\boldsymbol{x}_t$ using a vine copula approach. In the following, we elaborate on this approach in the context of model construction and estimation. Since our problem is three-dimensional, we will concentrate on such a case.

3.1. Decomposing a trivariate distribution function using pair-copulas

In a joint distribution function, information regarding both the marginal behaviour of each random variable and the dependence structure between the random variables is generally embedded. The copula is introduced as a tool to isolate the study of the dependence structure from that of the marginal behaviour of the individual variables. A copula is a multivariate distribution function C defined on the unit cube, with



(a) DE daily spot electricity price

(b) DK1 daily spot electricity price

(c) DK1 wind power production as a percentage of the installed capacity on a daily basis

(d) DK1 price vs. DE price

(e) DK1 price vs. DK1 wind

(f) DE price vs. DK1 wind

Figure 1. Historical data.

standard uniform margins. The central result when working with copulas, Sklar's Theorem [Sklar (1959)], shows how the marginal distributions and the copula are connected to the joint distribution. For the three-dimensional case, the theorem states that if we let F be the joint distribution function of the random vector $\boldsymbol{X} = (X_1, X_2, X_3)$ with marginal distribution functions F_1, F_2, F_3, then there exists a three-dimensional copula C such that

$$F(x_1, x_2, x_3) = C\{F_1(x_1), F_2(x_2), F_3(x_3)\}. \tag{1}$$

If the marginals are continuous, then the copula is unique and defined through

$$C(u_1, u_2, u_3) = F\{F_1^{-1}(u_1), F_2^{-1}(u_2), F_3^{-1}(u_3)\}, \tag{2}$$

where u_i are standard uniform variables and F_i^{-1} are the ordinary inverses of the marginal distribution functions, for $i = 1, 2, 3$. Increasing further the practical applicability of this representation is the fact that the converse of Sklar's Theorem holds: If we start with a copula C and margins F_1, F_2, F_3, then F defined in equation (1) is a joint distribution function with margins F_1, F_2, F_3. This allows us to construct multivariate distributions in a flexible way, using arbitrary margins and copulas as building blocks.

Next, we turn our attention to density functions. Assuming densities exist, the joint probability density function f corresponding to the joint distribution F defined in equation (1) is given by

$$\begin{aligned} &f(x_1, x_2, x_3) \\ &= f_1(x_1) \cdot f_2(x_2) \cdot f_3(x_3) \cdot c\{F_1(x_1), F_2(x_2), F_3(x_3)\}, \end{aligned} \tag{3}$$

where f_i denote the marginal density functions, for $i = 1, 2, 3$, and c denotes a trivariate copula density.

Using that a joint density function $f(x_1, x_2, x_3)$ can be factorized as e.g.

$$f(x_1, x_2, x_3) = f_1(x_1) \cdot f_{2|1}(x_2|x_1) \cdot f_{3|2,1}(x_3|x_2, x_1), \tag{4}$$

and the fact that each conditional density in equation (4) can be written as the appropriate pair-copula multiplied by a (conditional) marginal density, an iterative decomposition can be used to obtain the expression for the three-dimensional vine structure:

$$\begin{aligned} f(x_1, x_2, x_3) = f_1(x_1) \cdot f_2(x_2) \cdot f_3(x_3) \cdot c_{12}\{F_1(x_1), F_2(x_2)\} \\ \cdot c_{13}\{F_1(x_1), F_3(x_3)\} \\ \cdot c_{23|1}\{F_{2|1}(x_2|x_1), F_{3|1}(x_3|x_1)\}. \end{aligned} \tag{5}$$

For more details on the recursive conditioning used to obtain equation (5) and more general constructions, see Aas *et al.* (2009) and Czado *et al.* (2012). We note here that the decomposition given in equation (5) is not unique since different orderings of the variables are possible, but will elaborate on the procedure used to select the vine copula model in section 3.2. Furthermore, the marginal conditional distributions $F_{2|1}(x_2|x_1)$ and $F_{3|1}(x_3|x_1)$ entering the decomposition in equation (5) are often referred to as $h-$functions, and are defined as

$$h(u_i|u_j; \Theta) = F(x_i|x_j) = \frac{\partial C\{F(x_i), F(x_j); \Theta\}}{\partial F(x_j)}, \tag{6}$$

where $u_i = F(x_i)$ and $u_j = F(x_j)$ are standard uniform, and C is a parametric bivariate copula with parameters Θ.

Keeping in mind the decomposition given in equation (5), we describe below the two-step procedure we follow to model the joint behaviour of prices in DK1, prices in DE and wind power production in DK1.

In the first step, we consider each time series individually to filter out seasonality and serial time dependence. To correct each series for deterministic seasonality, we consider appropriate seasonal functions which we fit by simple linear regression. Then, we fit ARMA-GARCH type models to the deseasonalized data in order to filter out the serial dependence in the conditional mean and the conditional variance. To allow for more flexibility, we relax the usual Gaussian assumption for the standardized innovation processes.

In the second step, we consider a vine copula model for the approximately $i.i.d.$ standardized innovations obtained in the first step. That is, we model the dependency of processed data, after seasonality and marginal time dependencies have been removed. Specifically, the type of vine we consider is the so-called mixed C-vine. The C refers to the graphical representation of a sequence of trees where each tree has a central node, and the term *mixed* refers to having no restrictions on the copula family chosen for each individual pair.

3.2. *Inference for the three-dimensional model*

When considering full inference in the context of a vine copula, three issues must be addressed: (1) the selection of a specific decomposition, (2) the selection of bivariate copula families in the vine—given that an estimation procedure is in place, and (3) estimation of the full model.

Regarding the selection of a specific decomposition, we follow the procedure in Czado *et al*. (2012) and order variables according to their importance. To achieve this, we compute Kendall's τ for all possible pairings of the variables. Then, the variable most related to all other variables is set to be the central node of the first tree, and so forth.

Selection of the bivariate copulas in the vine is based on the Akaike information criterion (AIC), which is a commonly used criterion in this context, see e.g. Czado *et al*. (2012) and Elberg and Hagspiel (2015). The goodness-of-fit (GoF) of the chosen copulas is verified by performing the Cramér-von Misses (CvM) test, see Genest *et al*. (2009) and Berg (2009) for a review and power study of many GoF tests available for copulas.

Estimation of the model is based on the numerical maximization of the log-likelihood. In the three-dimensional case, the log-likelihood for the full model is given by

$$\begin{aligned}\log \mathcal{L} &= \sum_{t=1}^{T} \log f(y_{1,t}, y_{2,t}, y_{3,t} | \mathcal{F}_{t-1}; \Theta) \\ &= \sum_{i=1}^{3}\sum_{t=1}^{T} \log f_i(y_{i,t}|\mathcal{F}_{t-1}; \Theta_i) \\ &\quad + \sum_{t=1}^{T} \log c(u_{1,t}, u_{2,t}, u_{3,t}|\mathcal{F}_{t-1}; \Theta^c), \end{aligned} \tag{7}$$

where $\Theta = (\Theta_1, \Theta_2, \Theta_3, \Theta^c)$ denotes the parameters for the full model, and $\mathcal{F}_{t-1}$ is the filtration. Furthermore, the trivariate copula log-likelihood in equation (7) is decomposed as

$$\begin{aligned}&\sum_{t=1}^{T} \log c(u_{1,t}, u_{2,t}, u_{3,t}|\mathcal{F}_{t-1}; \Theta^c) \\ &= \sum_{t=1}^{T} \log c_{12}(u_{1,t}, u_{2,t}|\mathcal{F}_{t-1}; \Theta_1^c) \\ &\quad + \sum_{t=1}^{T} \log c_{13}(u_{1,t}, u_{3,t}|\mathcal{F}_{t-1}; \Theta_2^c) \\ &\quad + \sum_{t=1}^{T} \log c_{23|1}\{h(u_{2,t}|u_{1,t}; \Theta_1^c), h(u_{3,t}|u_{1,t}; \Theta_2^c)|\mathcal{F}_{t-1}; \Theta_3^c\},\end{aligned} \tag{8}$$

where $\Theta^c = (\Theta_1^c, \Theta_2^c, \Theta_3^c)$ denotes the parameters for the entire copula vine. As in most applications, we consider here a stepwise procedure, where the three marginal models are estimated independently of the copula. Estimation of the copula vine is additionally split up into two steps, as in Aas *et al*. (2009). First, we consider a sequential procedure where the parameters of each bivariate copula in the vine are estimated. Second, a joint maximization of the full copula log-likelihood is performed, using as start values the parameters obtained in the sequential estimation.

3.3. *Empirical results*

3.3.1. Marginal models. We start our empirical study by considering the individual behaviour of our three variables. In the case of the wind power production time series, we are dealing with data that is bounded between 0 and 1, cf. figure 1(c). A standard time series model such as the ARMA model is clearly not suitable for this data, since nothing in the model ensures that we remain within the natural bounds when performing simulations. Therefore, we perform the logit transformation defined as $\Lambda(x) = \ln(x/(1-x))$ to the wind data in order to obtain a time series that takes values on the entire real line. Regarding the price data, the log transformation usually used in the literature when modelling electricity spot price data is unfeasible here, since we observe negative prices in both the DK1 and the DE price areas. As a result, we work with the raw electricity prices.

To model the seasonal component of the wind power production in DK1, we consider the function

$$f_t^{\text{Wind}} = a + c_1 \sin(2\pi t/365) + c_2 \cos(2\pi t/365), \tag{9}$$

where a is a constant, and c_1 and c_2 denote the coefficients for a yearly cycle. To model the seasonal component of the electricity prices in DK1 and DE, we consider the slightly more involved function

$$\begin{aligned}f_t^{Price} &= a + bt + c_1 \sin(2\pi t/365) + c_2 \cos(2\pi t/365) \\ &\quad + d_1 \sin(4\pi t/365) + d_2 \cos(4\pi t/365) \\ &\quad + \sum_{j=1}^{6} w_j W_t^j,\end{aligned} \tag{10}$$

where a again denotes a constant, b is the trend coefficient, c_1, c_2, d_1 and d_2 represent coefficients for the yearly and half-yearly cycles, respectively, and lastly w_j for $j = 1, \ldots, 6$

Table 1. OLS estimates for the parameters of the seasonal functions. Reference day-of-week is Monday.

	$\hat{a}$	$\hat{b}$	$\hat{c}_1$	$\hat{c}_2$	$\hat{d}_1$	$\hat{d}_2$	$\hat{w}_1$	$\hat{w}_2$	$\hat{w}_3$	$\hat{w}_4$	$\hat{w}_5$	$\hat{w}_6$
Electricity price DK1	40.980	−0.010	−2.184	0.814	−0.571	−1.400	1.025	1.350	0.955	−0.266	−4.535	−7.364
Electricity price DE	45.347	−0.010	−3.195	2.133	0.222	−1.173	2.098	1.869	1.572	0.437	−6.373	−13.219
Logit wind DK1	−1.110	–	0.068	0.489	–	–	–	–	–	–	–	–

are coefficients corresponding to the day-of-week dummies denoted W^j. Parameter estimates obtained by fitting the proposed seasonal functions to the data are displayed in table 1.

Next, we fit ARMA(p, q), ARMA(p, q)—GARCH(1,1) and ARMA(p, q)—GJR(1,1) models to the deseasonalized data, where we let $p = 0, \ldots, 5$ and $q = 0, \ldots, 5$. Also, we relax the normality assumption for the residuals, and allow for the more flexible skew t distribution instead, as in Patton (2013). The order of the ARMA model and the type of variance model are chosen based on the Bayesian information criterion (BIC). In table 2, the specifications for the marginal models and the corresponding parameter estimates are displayed.

We also report p-values resulting from performing the Ljung-Box (LB) Q-test of serial independence on the standardized residuals, which we denote r, and the squared standardized residuals, r^2. The LB Q-tests all show strong evidence of no serial dependence left in the standardized residuals. Lastly, we report the p-values resulting from simulation-based Kolmogorov–Smirnov (KS) and CvM GoF tests. The tests are based on 999 bootstraps, and we examine the adequacy of two distributions for the standardized residuals, namely the normal distribution and the skew t (or skew normal) distribution.† The results show that the skew t (or skew normal) specification is suitable in all marginal models, whereas the normal distribution is rejected at a 5% significance level. Having verified the adequacy of the three proposed marginal models, we proceed to modelling the dependence structure with copulas.

3.3.2. Copula models. By applying the probability integral transform to the standardized residuals resulting from the marginal models, we obtain the approximately standard uniforms U_{DK1}, U_{DE}, and U_W, which are the input variables in our copula. The variable U_{DK1} refers to the price in DK1, U_{DE} refers to the price in DE, and lastly U_W refers to the wind power production in DK1. In figure 2, we plot the empirical copula densities for the three possible pairings of our variables. Since the variable with *most influence* (following the decomposition procedure in Czado *et al.* (2012)) is the electricity price in DK1, we let this variable be the central node of the first tree. The pair-copula decomposition we obtain is illustrated in figure 3. We note that a C-vine coincides with a D-vine in the three-dimensional case; hence, the C in C-vine is solely used to allude to the decomposition method.

According to figure 3, we must specify two bivariate copulas for the unconditional dependence between the pair U_{DK1}, U_{DE} and the pair U_{DK1}, U_W. Subsequently, we must specify one bivariate copula for the conditional dependence of U_{DE}, U_W given U_{DK1}. This is done in a recursive manner: First, we select the most suitable copulas for the pairs in the first tree, and then conditional on this selection, we move onto choosing a copula family that characterizes the conditional dependence in the second tree. For all pairs in the C-vine, we fit nine different bivariate copulas, and base selection on the AIC. The results are illustrated in table 3, showing that the Student t copula is preferred for the dependence between prices in DK1 and DE, and that the Gaussian copula is preferred for the dependence between wind power production in DK1 and price in DK1. Given this selection and the associated estimated copula parameters, we apply the h-functions, cf. equation (6), to obtain the variables $U_{DE|DK1}$ and $U_{W|DK1}$. We then fit the nine different copulas to this pair and find that given prices in DK1, wind power production in DK1 and prices in DE are likely to be independent, and hence we choose the independence copula for this pair.

We note that since only the Gaussian and the Student t copulas allow for negative dependence, a rotation was performed in order to fit the remaining seven copulas to the pair U_{DK1}, U_W in table 3. Specifically, we fit these copulas to the pair U_{DK1}, $1 - U_W$ instead of the pair U_{DK1}, U_W. Regarding the standard errors for the estimates of the copula parameters stated in table 3, these are simulation-based and take into account the estimation error coming from the ARMA–GARCH models, as in Patton (2013).

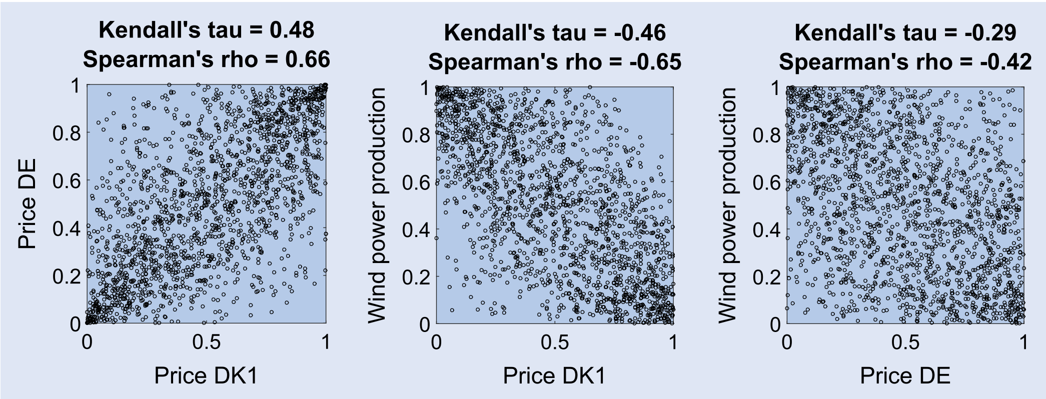
To test for the GoF of the selected copula families, we perform the CvM test, which is based on a comparison with the empirical copula. We obtain p-values of 0.584, 0.157 and 0.348 for the Student t copula, the Gaussian copula, and the independence copula, respectively, meaning that we cannot reject the null that the copula family is well-specified in neither one of the three cases. Having the independence copula describe the conditional dependence in the second tree simplifies our problem somewhat, since a joint estimation would yield the same results as the sequential estimation we have performed, and is therefore redundant.

Since the dimension of our problem is so small, one naturally wonders why not just use a trivariate copula instead of what might seem like an intricate decomposition of the problem. Also, is mixing different copula families really necessary? To provide some evidence for the benefit of the proposed mixed C-vine, we consider three other copula specifications: First, we consider the most standard case, i.e. fitting a trivariate Gaussian copula to our data. Second, we fit a trivariate Student t copula, and third, we fit a C-vine consisting of three Student t bivariate copulas, that is no mixing is allowed. The results are given in table 4, and show that the mixed C-vine is the preferred copula model based on the AIC and the BIC.

†For the wind data, the skew t distribution converges towards the skew normal distribution since we obtain an estimate for the degrees of freedom above 300. Therefore, we replace the skew t with the skew normal distribution in this case.

Table 2. Type and order of marginal models, parameter estimates and p-values.

	Electricity price DK1		Electricity price DE		Logit wind DK1	
Marginal model	ARMA(2,1)—GARCH(1,1)		ARMA(2,1)—GARCH(1,1)		ARMA(1,1)	
			Conditional mean			
AR1 $\hat{\phi}_1$	1.441		1.409		0.303	
AR2 $\hat{\phi}_2$	−0.460		−0.456		–	
MA1 $\hat{\theta}_1$	−0.837		−0.804		0.263	
Variance $\hat{\sigma^2}$	–		–		1.014	
			Conditional variance			
Constant $\hat{\omega}_G$	2.520		3.633		–	
ARCH $\hat{\alpha}_G$	0.142		0.120		–	
GARCH $\hat{\beta}_G$	0.795		0.772		–	
			Marginal distribution			
	Skew t		Skew t		Skew normal	
Shape $\hat{\nu}$	5.663		6.050		–	
Skew $\hat{\eta}$	−0.053		−0.174		−0.218	
			Testing model assumptions (p-values)			
LB Q-test (5 lags of r)	0.467		0.630		0.679	
LB Q-test (10 lags of r)	0.285		0.194		0.256	
LB Q-test (5 lags of r^2)	0.169		0.327		0.238	
LB Q-test (10 lags of r^2)	0.402		0.724		0.585	
	Normal	Skew t	Normal	Skew t	Normal	Skew Normal
KS test	0.004	0.225	0.001	0.828	0.011	0.426
CvM test	0.003	0.546	0.001	0.687	0.014	0.544



(a) U_{DK1} vs. U_{DE} (b) U_{DK1} vs. U_W (c) U_{DE} vs. U_W

Figure 2. Empirical copula densities obtained by applying the probability integral transform to the standardized residuals resulting from the marginal models for DK1 prices, DE prices and wind power production in DK1, respectively.

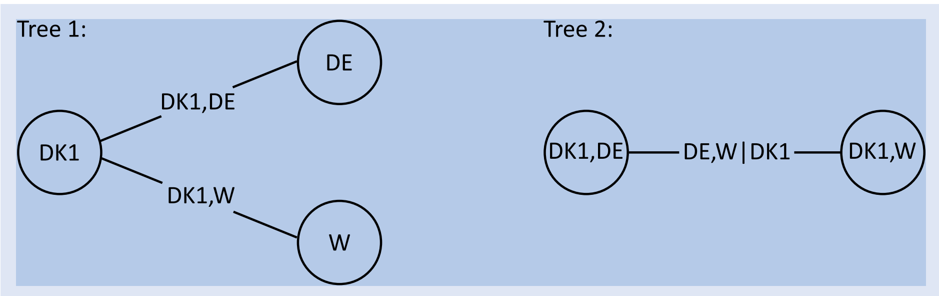


Figure 3. Proposed C-vine structure for the electricity price in DK1, the electricity price in DE, and the wind power production in DK1.

Table 3. Estimation results, where nine copula specifications are considered for each pair-dependence in the vine. The maximized value of the copula log-likelihood is denoted $\log \mathcal{L}_c$. The bold numbers in the first tree correspond to the optimal copulas, where selection is based on AIC. SJC is short for the Symmetrized Joe-Clayton copula introduced in Patton (2006). The Rot. Gumbel and Rot. Clayton refer to the rotated copulas. For the functional forms of the considered copulas and other characteristics, we refer to Joe (1997) and Nelsen (1999). The asterisk refers to copulas fitted to rotated data, that is the pair U_{DK1}, $1 - U_W$ instead of the pair U_{DK1}, U_W. Standard errors are based on 999 simulations.

		Tree 1						Tree 2		
			$c_{DK1,DE}$			$c_{DK1,W}$			$c_{DE,W\|DK1}$	
Copula model		Param. (s.e.)	$\log \mathcal{L}_c$	AIC	Param. (s.e.)	$\log \mathcal{L}_c$	AIC	Param. (s.e.)	$\log \mathcal{L}_c$	AIC
Gaussian	$\hat{\rho}$	0.667 (0.013)	531.72	−1,061.44	−0.640 (0.014)	**476.58**	**−951.16**	0.018 (0.025)	0.29	1.42
Gumbel	$\hat{\theta}$	1.801 (0.040)	503.59	−1,005.18	1.675* (0.036)	407.26	−812.52	1.012 (0.010)	0.67	0.66
Rot. Gumbel	$\hat{\theta}$	1.844 (0.041)	540.58	−1,079.16	1.706* (0.047)	437.02	−872.04	1.016 (0.009)	1.09	−0.18
Clayton	$\hat{\theta}$	1.282 (0.061)	458.82	−915.64	1.065* (0.058)	369.63	−737.26	0.039 (0.022)	1.44	−0.88
Rot. Clayton	$\hat{\theta}$	1.149 (0.068)	398.03	−794.06	0.971* (0.055)	331.25	−660.50	0.003 (0.024)	0.01	1.98
Plackett	$\hat{\theta}$	9.809 (0.670)	532.08	−1,062.16	7.539* (0.442)	456.95	−911.90	1.086 (0.085)	0.66	0.68
Frank	$\hat{\theta}$	5.355 (0.190)	511.43	−1,020.86	4.933* (0.160)	472.05	−942.10	0.162 (0.115)	0.64	0.72
SJC	$\hat{\tau}^U$	0.433 (0.015)	546.57	−1,089.14	0.373* (0.030)	426.14	−848.28	0.000 (0.014)	0.81	2.38
	$\hat{\tau}^L$	0.515 (0.025)			0.448* (0.025)			0.001 (0.016)		
Student *t*	$\hat{\rho}$	0.677 (0.015)	**565.14**	**−1,126.28**	−0.640 (0.017)	476.58	−949.16	0.021 (0.025)	2.83	−1.66
	$\hat{\nu}$	5.671 (1.159)			379.173 (138.202)			20.704 (27.093)		

Although the Student t C-vine has a slightly larger log-likelihood, the number of parameters in this model doubles compared to our proposed mixed C-vine. Also, estimation of this model is more involved, since a simultaneous estimation of the full copula log-likelihood is required. The trivariate Student t copula is less appropriate because it has only one degree of freedom. This is especially a drawback in our situation, where the dependency between the two prices exhibits significant tail dependence, whereas the dependence between each price and the wind power production exhibits no tail dependence, and thus pulling the one common degree of freedom in opposite directions. Consequently, although the difference in log-likelihoods between the proposed mixed C-vine and the trivariate Student t copula model is not striking, the latter would produce misleading tail dependence estimates which is especially problematic from a risk management perspective. Finally, the poorer fit of the trivariate Gaussian copula is due to similar arguments as in the case of the trivariate Student t, since the Gaussian copula has no tail dependence.

To sum up, our proposed mixed C-vine allows for a lot of flexibility while only having three parameters, which coincides with the smallest alternative model. Furthermore, the model is easily estimated since a sequential procedure suffices.

3.3.3. Time-varying copula models. So far in our analysis, we have assumed that the dependence is static. However, since this is rarely the case in practice, a natural extension is to investigate whether or not time variation should be introduced in the copula model. Having established that the mixed C-vine copula is our preferred model cf. table 4, we will restrict our attention to this model, and thus to introducing time variation in the Student t and Gaussian copulas, which are the selected specifications for the unconditional dependence of the pair U_{DK1}, U_{DE} and the pair U_{DK1}, U_W, respectively.

Our approach to capturing time-varying dependence is through the Generalized Autoregressive Score (GAS) model of Creal *et al.* (2013). As the name suggests, the GAS model allows the time-varying copula parameter to evolve as a function of lagged values of the copula parameter and lagged values of the (scaled) score function of the copula log-likelihood. Assuming a copula with parameter ρ, the GAS specification we consider is:

$$h_{t+1} = \omega + \alpha h_t + \beta s_t I_t^{-1/2}, \tag{11}$$

where

$$h_t = g(\rho_t), \tag{12}$$

$$s_t = \frac{\partial}{\partial \rho} \log c((u_{1,t}, u_{2,t}); \rho_t), \tag{13}$$

$$I_t = \mathbb{E}_{t-1}[s_t^2]. \tag{14}$$

In equations (11)–(14), h_t denotes the transformed copula parameter obtained by applying the tranformation $g(\cdot)$ to the copula parameter ρ_t, s_t denotes the score of the copula log-likelihood, and I_t is the Fisher information. For applications of the GAS model in the context of copulas, and also details on estimation and comparison with alternative models such as the

Table 4. Comparison of the mixed C-vine copula with other competing models. Numbers in bold correspond to the preferred model, where selection is based on the AIC and the BIC. The maximized value of the full copula log-likelihood is denoted log $\mathcal{L}_*$. Standard errors are simulation-based (999 simulations), and take into account the estimation error coming from the marginal models.

Mixed C-vine		Student t C-vine			3-dim. Student t		3-dim. Gaussian	
Param.	Final (s.e.)	Param.	Seq. (s.e.)	Final (s.e.)	Param.	Final (s.e.)	Param.	Final (s.e.)
$\rho_{DK1,DE}$	0.677 (0.015)	$\rho_{DK1,DE}$	0.677 (0.015)	0.677 (0.015)	$\rho_{DK1,DE}$	0.680 (0.013)	$\rho_{DK1,DE}$	0.667 (0.013)
$\nu_{DK1,DE}$	5.671 (1.159)	$\nu_{DK1,DE}$	5.671 (1.159)	5.634 (0.995)	$\rho_{DK1,W}$	−0.639 (0.013)	$\rho_{DK1,W}$	−0.640 (0.015)
$\rho_{DK1,W}$	−0.640 (0.014)	$\rho_{DK1,W}$	−0.640 (0.017)	−0.641 (0.014)	$\rho_{DE,W}$	−0.423 (0.020)	$\rho_{DE,W}$	−0.420 (0.019)
		$\nu_{DK1,W}$	379.173 (138.202)	379.173 (47.786)	ν	13.195 (3.735)		
		$\rho_{DE,W\|DK1}$	0.021 (0.025)	0.021 (0.027)				
		$\nu_{DE,W\|DK1}$	20.678 (27.125)	20.694 (24.754)				
log $\mathcal{L}_*$	**1,041.72**		1,044.55	1,044.57		1,028.65		1,008.43
AIC	**−2,077.44**		−2,077.10	−2,077.14		−2,065.30		−2,010.86
BIC	**−2,060.94**		−2,044.10	−2,044.14		−2,027.30		−1,994.36

ARMA-type processes employed in Patton (2006), we refer to e.g. Creal *et al*. (2013), Patton (2013), Avdulaj and Barunikl (2015) and Pircalabu *et al*. (2017).

Given our choice of a Student t and a Gaussian copula as elements of the vine, we model the correlation parameter of each of the two copulas according to the GAS equation. The degrees of freedom parameter of the Student t copula is kept constant. Further, since correlation is restricted to lie in the interval $(-1, 1)$, we let $h_t = \log(\rho_t + 1) - \log(1 - \rho_t)$ to ensure this.

The estimated parameters for the GAS models are reported in table 5, pointing to evidence of time-varying dependence in both models. To provide a visual indication of how the correlations change with the time, we plot in figure 4 the dynamics of the correlation parameter estimates implied by the two GAS models.

Having introduced time-varying copulas in the first tree of the vine, we note that this can affect the choice of copula in the second tree. To check that the independence copula remains a good specification for the conditional dependence in the second tree, we compute new variables $U_{DE|DK1}$ and $U_{W|DK1}$ based on the h-functions that now take as input the time-varying copula parameters ρ_t implied by the fitted GAS models. Performing the CvM test on the new conditional pair, we obtain a p-value of 0.239, confirming the GoF of the independence copula. Basing the model selection on values of the AIC, both time-varying models yield lower values than the corresponding constant models, implying that the time-varying mixed C-vine copula model is preferred to the constant one. Hence, we will keep this time-varying model as our preferred specification and use it in the next section as a simulation tool based on which hedging decisions are made.

4. Hedging joint price and volumetric risk

In this section, we address the hedging of joint price and volumetric risk in the context of an energy trading company entering into longer-term contracts with wind power producers, where the future production is bought at a pre-determined fixed price; we shall refer to such contracts as fixed-for-fluctuating agreements.

First, let us introduce some notation and simplifying assumptions. We will assume that at time t_0, a hypothetical energy trading company enters into fixed-for-fluctuating agreements in DK1, corresponding to an installed capacity of c MW. The company offers a common fixed price $S_{t_0}^{\text{fixed}}$ per MWh to all wind power producers, and we consider a delivery period from T_1 to T_2. Furthermore, we will assume that the total wind power production in DK1 is perfectly correlated to the total production associated with the agreements entered into by the company. Since we model the total daily wind power production in DK1 relative to the total installed capacity, which we denote Q_t^{DK1}, we simply multiply Q_t^{DK1} by $24c$ to get the total daily production measured in MWh that the company has under management. Under these conditions, the total revenue R^f of the energy trading company over the period $[T_1, T_2]$ can be expressed as

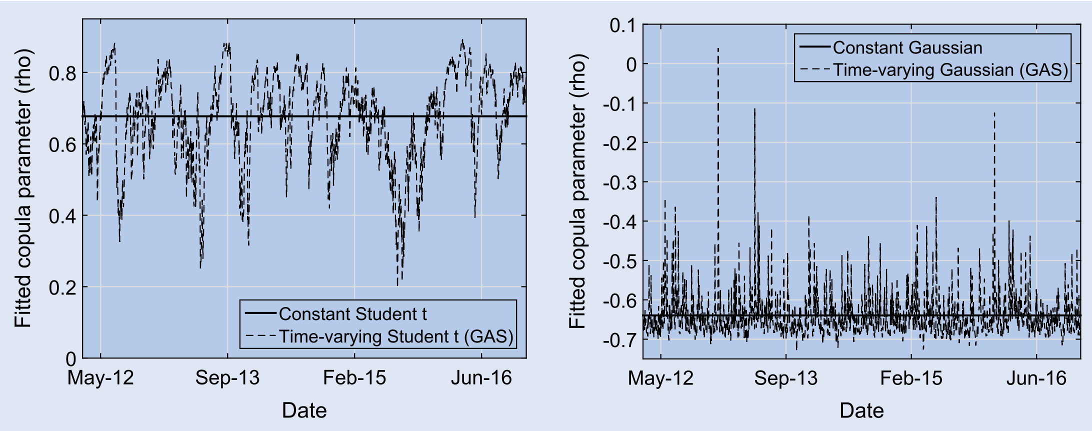
$$R^f = 24c \sum_{t=T_1}^{T_2} Q_t^{DK1}(S_t^{DK1} - S_{t_0}^{\text{fixed}}), \qquad \text{for } t_0 < T_1 < T_2. \tag{15}$$

where S_t^{DK1} denotes the average spot electricity price in DK1 at day t, and we assume that all days have 24 h. Moreover, according to equation (15), we assume that the delivered wind power production is sold on the day-ahead market, and ignore possible balancing costs.

To construct a hedging portfolio, we first need to identify the available financial instruments in the market. Since we are attempting to hedge joint price and volumetric risk in the DK1 market, the selection of hedging instruments to choose from is small, and in addition, the market place is characterized as being rather illiquid. To overcome these limitations, we include hedging instruments with reference to the DE spot electricity price, and use them to construct a proxy hedge. This extension is believed to be advantageous due to the strong

Table 5. Estimation results for time-varying copulas in the mixed C-vine, where the linear correlations evolve according to the GAS specification. Standard errors are simulation-based. The maximized value of the copula log-likelihood is denoted $\log \mathcal{L}_c^{tv}$.

	Tree 1				Tree 2	
	Student *t* GAS		Gaussian GAS		Independence	
	$c_{DK1,DE}$		$c_{DK1,W}$		$c_{DE,W\|DK1}$	
	Param.	s.e.	Param.	s.e.	Param.	s.e.
$\hat{\omega}$	0.057	(0.027)	−0.696	(0.301)	–	–
$\hat{\alpha}$	0.967	(0.016)	0.545	(0.196)	–	–
$\hat{\beta}$	0.120	(0.026)	0.142	(0.044)	–	–
$\hat{\nu}$	6.711	(2.274)	–	–	–	–
$\log \mathcal{L}_c^{tv}$	612.67		485.04		–	
AIC	−1,217.34		−964.08		–	



(a) Pair $\hat{U}_{DK1}$, $\hat{U}_{DE}$ (b) Pair $\hat{U}_{DK1}$, $\hat{U}_W$

Figure 4. Linear correlation implied by the constant copula models and the GAS models.

positive correlation between the spot electricity prices in DK1 and DE, which we have demonstrated earlier.

Here, we shall concentrate on a static hedge that is performed by the energy trading company at time t_0, i.e. the same time as entering the fixed-for-fluctuating agreements. Due to market incompleteness, we are clearly not attempting to replicate the pay-off of fixed-for-fluctuating agreements exactly, and thus to attain a perfect hedge.

The hedging instruments we allow are as follows:

(1) *Power forwards and futures*
Power forwards and futures are the most widely used hedging instruments in electricity markets. Depending on the market place, they can have different characteristics, e.g. physical vs. financial settlement, delivery during the base, peak or off-peak hours, and delivery periods corresponding to a year, season, quarter, month, etc. Here, we will allow trading these types of contracts with reference to both the DE and the DK1 spot electricity price. Also, we will assume a deterministic interest rate, thus equating forwards and futures. When referring to such contracts on the DE market, we will denote them DE futures, and when referring to them in DK1, we will use the term DK1 forwards. DE futures are among the most liquid contracts traded on the European Energy Exchange (EEX). Trading in DK1 forwards is somewhat more cumbersome, since two positions must be entered into: a futures contract on the Nordic spot system price and an electricity price area differential (EPAD), which is a forward contract with reference to the difference between the DK1 area price and the Nordic system price.
For a contracted quantity of 1 MW, a long position in a DE futures contract yields the following revenue, denoted by $R^{F,DE}$:

$$R^{F,DE} = 24 \sum_{t=T_1}^{T_2} S_t^{DE} - F_{t_0}^{DE}, \qquad (16)$$

where $F_{t_0}^{DE}$ denotes the DE futures price at time t_0, with delivery period from T_1 to T_2, and S_t^{DE} denotes the average DE spot electricity price at day t. Since we work on a daily basis, we implicitly assume base load delivery, and the contracted quantity of 1 MW entails delivering 24 MWh at each day $t \in [T_1, T_2]$. To obtain the revenue associated with a DK1 forward contract, which we denote $R^{F,DK1}$, DE is simply replaced by DK1 in equation (16), with the obvious changes in the interpretation of variables.

(2) *Wind index futures*
Wind index futures are at the present time the only standardized exchange-traded instruments with the underlying being the actual wind index. For a contracted quantity of 1 lot, their revenue denoted by R^{WIF} can be expressed as

$$R^{WIF} = 24(T_2 - T_1 + 1)(I^{\text{Actual}} - I_{t_0}^{\text{Locked}}) \cdot 100 \text{ EUR}, \quad \text{for } t_0 < T_1 < T_2, \quad (17)$$

where I^{Actual} denotes the actual average wind index for the contracted period, which is obtained by dividing the total actual wind power production by the total available capacity and the number of hours in the delivery period, i.e. $0 \leq I^{\text{Actual}} \leq 1$. Furthermore, $I_{t_0}^{\text{Locked}}$ is the locked-in level set at time t_0. This level can be thought of as the market's expectation of the future average wind index corresponding to the delivery period.†
Wind index futures were launched late 2015 by NASDAQ OMX Commodities Europe and late 2016 by EEX. They can currently be traded on the German wind power index, and according to EEX (2014), the next step is to expand this product to the Danish and the UK market. Since we model the wind power production in DK1, we will allow for the inclusion of DK1 wind index futures in our hedging portfolio based on this outlook.

(3) *Financial transmission rights (spread options)*
A financial transmission right (FTR) is an option written on the hourly spot electricity price difference in two price areas that are interconnected. It gives the holder the right to 'transfer' electricity from e.g. DK1 to DE (or vice versa) whenever the price difference $S_h^{DE} - S_h^{DK1}$ (or $S_h^{DK1} - S_h^{DE}$) is positive for hour h. In reality, no physical delivery of electricity is made and only the price spread, if positive, is paid out to the holder of the option; hence the name FTR. Since we work with aggregated data on a daily basis, we shall consider the daily price spread instead of the hourly price spreads. If we denote by $P^{DK1\rightarrow DE}$ the pay-off associated with the right to transfer electricity from DK1 to DE each day t in the delivery period from T_1 to T_2, we have a sum of daily spread options represented by the pay-off

$$P^{DK1\rightarrow DE} = 24 \sum_{t=T_1}^{T_2} \left(S_t^{DE} - S_t^{DK1} \right)^+,$$

where we assume a contracted quantity of 1 MW. The revenue resulting from a 1 MW long position in such daily spread options is

$$R^{DK1\rightarrow DE} = P^{DK1\rightarrow DE} - V_{t_0}^{DK1\rightarrow DE}, \quad (18)$$

where $V_{t_0}^{DK1\rightarrow DE}$ denotes the sum of fair option values at time t_0. Similarly, $R^{DE\rightarrow DK1}$ is obtained by considering the opposite spot price spread.

(4) *Electric boilers (out-of-the-money put options)*
In DK1, there are a number of combined heat and power (CHP) plants equipped with electric boilers. These plants can typically produce heat in one of three ways: (1) by the use of gas boilers, (2) by the use of electric boilers, where electricity is converted to heat, and (3) using gas as an input to produce electricity, with heat being a by-product. We do not go into details with the optimization problem that CHP plants face every day in order to minimize their customers' heat expenses. We note however that when profitable, say when the price of electricity is at most K, plant owners will use electricity to produce heat. So, if we could sell a fixed amount of electricity to CHP plants at the price K each time the daily spot goes below K, we would be entering a string of daily put options. Such products can in fact be traded OTC in DK1, at a strike K which is usually significantly below the average electricity spot price in DK1, translating into the fact that the daily puts are out-of-the-money (OTM). The benefit of such instruments is clear, as they reduce the risk associated with high wind and low price scenarios by providing a price floor. Nonetheless, they are mostly available/useful during winter periods.‡ For a contracted quantity of 1 MW, their pay-off P^{Put} is given by

$$P^{\text{Put}} = 24 \sum_{t=T_1}^{T_2} \left(K - S_t^{DK1} \right)^+,$$

where the strike K is kept constant throughout the delivery period. Like in the case of FTRs, we consider here a sum of daily options. The revenue associated with a 1 MW long position is given by

$$R^{\text{Put}} = P^{\text{Put}} - V_{t_0}^{\text{Put}}, \quad (19)$$

where $V_{t_0}^{\text{Put}}$ denotes the sum of fair option values at time t_0.

For a portfolio consisting of fixed-for-fluctuating agreements as well as all hedging instruments described above, the associated total revenue for the delivery period from T_1 to T_2 can be expressed as

$$R^{\text{Total}} = R^f + \sum_{n=1}^{N} \theta_n R^{(n)}, \quad (20)$$

where the $R^{(n)}$'s represent the revenues corresponding to each of the hedging instruments (equations (16)–(19)), and the θ_n's denote the corresponding contracted quantities. Specifically, $N = 6$ in equation (20), with the following correspondence: DK1 forwards ($n = 1$), DE futures ($n = 2$), wind index futures ($n = 3$), FTRs in the direction DE→DK1 ($n = 4$), FTRs in the direction DK1→DE ($n = 5$), and lastly OTM puts ($n = 6$).

†We note that our product description of wind index futures is based on Nasdaq (2015) and EEX (2014).

‡Due to lower demand for heat during summer, CHP plants have a lower capacity. Moreover, high wind / low price scenarios are less likely.

To find the fair value of the financial contracts in the portfolio at t_0, we assume a zero interest rate, and let the risk neutral measure equal the physical measure, i.e. $\mathbb{Q} = \mathbb{P}$, as in Coulon *et al*. (2013). With these simplifying assumptions, all contracts in the portfolio can be priced by performing simulations from our proposed C-vine copula model. Specifically, the fair value of $S_{t_0}^{\text{fixed}}$ can be computed by the usual practice of setting the risk neutral expectation of the revenue R^f given in equation (15) equal to zero, which yields

$$S_{t_0}^{\text{fixed}} = \frac{\mathbb{E}_{t_0}^{\mathbb{Q}}\left[\sum_{t=T_1}^{T_2} Q_t^{DK1} S_t^{DK1}\right]}{\mathbb{E}_{t_0}^{\mathbb{Q}}\left[\sum_{t=T_1}^{T_2} Q_t^{DK1}\right]}. \tag{21}$$

The same approach is used to obtain forward and futures prices, which we assume are unbiased estimators of the future spot electricity prices. The expected wind index needed to obtain the locked in level in wind index futures is obtained by averaging across simulated paths of future wind power production corresponding to the delivery period. Lastly, fair option values are obtained by computing the risk neutral expectations of simulated pay-offs. The assumption of $\mathbb{Q} = \mathbb{P}$ is of course unrealistic, but we do not go into details here and refer instead to Burger *et al*. (2004), Benth *et al*. (2008), and Kolos and Ronn (2008) for comprehensive discussions on the matter. It simplifies nonetheless our analysis a great deal, since the mean of the total revenue distribution is zero per construction, and is unaffected by varying hedge quantities. As a result, we are left with the variance aspect of the problem. To obtain the optimal static hedge, we consider the traditional variance-minimizing criterion

$$\min_{\boldsymbol{\theta}} \mathbb{V}\mathrm{ar}_{t_0}\left[R^f + \sum_{n=1}^{N} \theta_n R^{(n)}\right] \tag{22}$$

$$\text{s.t. } \theta_n \geq 0, \;\; n = 4, \ldots, 6, \tag{23}$$

where short-selling constraints are introduced regarding the options. This restriction is added due to practical reasons, since the hedger, i.e. the energy trading company, does not own the DK1–DE interconnector. Moreover, we assume that the hedger does not own CHP plants.

4.1. *Hedging results*

In order to exemplify the benefits of the proposed hedging instruments, we consider an energy trading company entering into monthly fixed-for-fluctuating agreements at $t_0 = 12$ December 2016, which corresponds to the last date in our sample. As delivery we consider the out-of-sample period from $T_1 = 1$ January 2017 to $T_2 = 31$ January 2017. Hedging is also performed at t_0, and all hedging instruments have the same delivery period as the fixed-for-fluctuating agreements. We let the installed capacity $c = 500$ MW and the daily strike of the OTM put options $K = 12$ EUR/MWh.

Before considering the minimization problem stated in equations (22)–(23), we study the individual effectiveness of the proposed hedging instruments. First, we compute the fair value of the fixed-for-fluctuating agreements and all the hedging instruments at t_0, based on 100 000 simulations from the time-varying mixed C-vine copula model.† Then, we compute revenues for each simulated path, where we vary the hedge quantities θ_n for each instrument, $n = 1, \ldots, N$. The results are displayed in figure 5, where we consider as measures of risk the standard deviation of the revenue distributions in figure 5(a) and the 5% Value-at-Risk (VaR) of the revenue distributions in figure 5(b).

Not surprisingly, we observe a large variation in the hedging benefits of the proposed instruments. The DK1 forwards are the most effective, as they secure the revenue associated with a significant part of the production in advance. Then, only the remaining production—the difference between the actual and the hedged volume—will be exposed to spot price risk. The DE futures are less effective at reducing risk, which is expected since they are settled against the DE and not the DK1 spot prices. Somehow in contrast to DK1 forwards, the wind index futures fix a revenue associated with the difference between actual and expected production. Hence, this fixed revenue is associated with a much smaller volume than in the case of the DK1 forwards, which explains the inferiority of wind index futures at reducing risk.

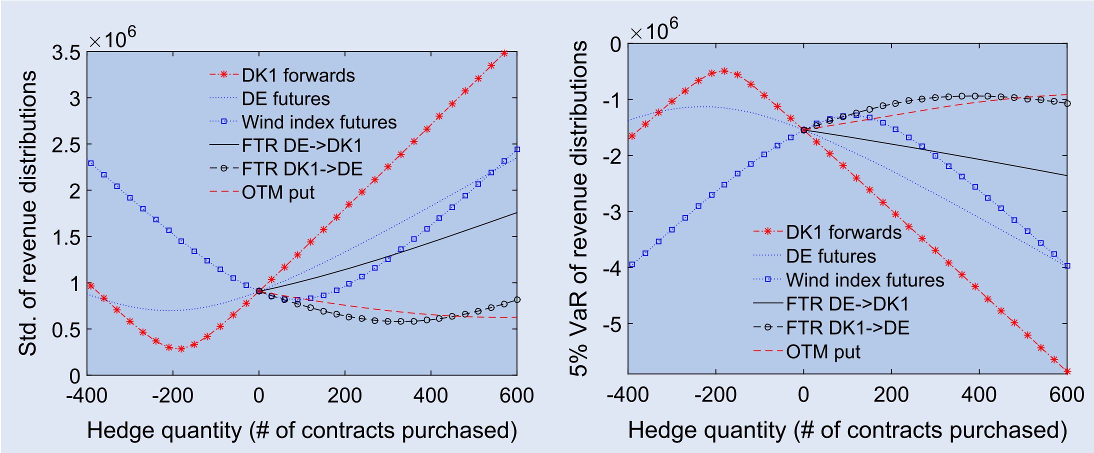
When considering the options, we observe that the OTM puts are rather effective, owing to the fact that they provide a floor to the DK1 spot prices, thus reducing the risk related to a low price/high volume scenario. Lastly, the FTRs in the direction DK1 to DE are also effective, and are preferable to the opposite direction. The difference in hedging benefits between the FTRs relates mostly to the short selling constraint.

Next, we turn our attention to the optimal static hedge when allowing trading in all instruments simultaneously. In this context, we note that the classical parity argument (see Carmona and Durrleman 2003) implies that an FTR in one particular direction can be written as a linear combination of an FTR in the opposite direction, a DK1 forward and a DE futures. Consequently, allowing trading in all these four instruments would result in the variance minimization problem in equations (22)–(23) having many possible optimal solutions. To avoid this, we remove the FTRs in the direction DK1→DE from the hedging portfolio, and present below one of many optimal solutions.

In figure 6(a) we plot three different simulated revenue distributions for comparison purposes. The blue dashed line displays the simulated monthly revenue distribution associated with the fixed-for-fluctuating agreements before performing a hedge. The black solid line represents the distribution after performing an optimal hedge using DK1 forwards only, and lastly the red dashed-dotted line corresponds to the distribution after performing an optimal hedge based on all proposed hedging instruments simultaneously. Figure 6(b) provides a zoom of the left tails of the simulated distributions, and figure 6(c) specifies the optimal hedging strategy associated with the latter revenue distribution, and is obtained by solving equations (22)–(23).

Figures 6(a) and (b) clearly show that significant benefits are to be drawn using all proposed hedging instruments. In table 6, we provide the reduction in the variance and the 5% VaR of the monthly revenue distribution achieved by (1) using DK1 forwards as opposed to no hedge, and (2) using all instru-

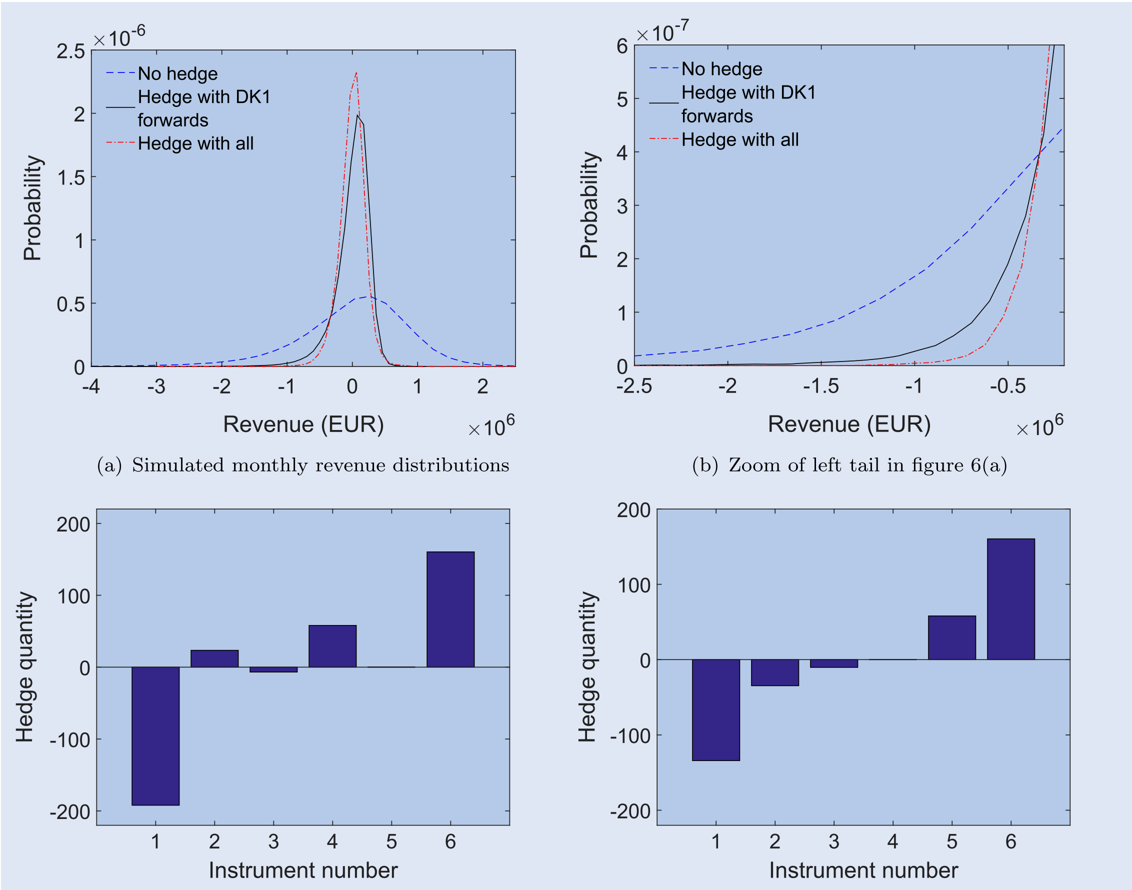
†Sampling from a copula vine is based on a recursive procedure and using the inverse of the h-functions, cf. equation 6. We refer to Aas *et al*. (2009) for a detailed description of the procedure.



(a) Effect on the standard deviation

(b) Effect on the 5% VaR

Figure 5. Isolated effect of different hedging strategies.



(a) Simulated monthly revenue distributions

(b) Zoom of left tail in figure 6(a)

(c) Optimal hedging strategy

(d) Optimal hedging strategy when including costs associated with trading in DK1 forwards

Figure 6. Hedging results under different assumptions. The hedging instruments are numbered as (1) DK1 forwards, (2) DE futures, (3) Wind index futures, (4) FTRs DE → DK1, (5) FTRs DK1 → DE, and (6) OTM put.

ments as opposed to using only DK1 forwards in the hedging portfolio. In both cases, significant reductions are attained. However, when examining the isolated effect of each hedging instrument in combination with the DK1 forwards, we see that it is mostly the OTM puts that reduce risk additionally.

Our results so far indicate that DK1 forwards and OTM puts are the most powerful, and that including additional instruments to the hedging portfolio does not contribute substantially to the variance reduction of the revenue distribution. Since neither of these two instruments depends on the DE price,

Table 6. The upper panel displays the effects of hedging the joint price and volumetric risk implied by fixed-for-fluctuating agreements with DK1 forwards only, as opposed to no hedge at all. The lower panel displays the effects of adding other instruments to a hedging portfolio initially consisting of DK1 forwards.

Initial hedging portfolio No hedge			Isolated effect DK1 forwards			Combined effect
Reduction in variance of revenue dist.			90.36%			–
Reduction in 5% VaR of revenue dist.			68.23%			–

Initial hedging portfolio DK1 forwards only			Isolated effect			Combined effect
	DE futures	Wind index futures	FTR DE→DK1	FTR DK1→DE	OTM put	
Reduction in variance of revenue dist.	0.05%	1.40%	14.15%	11.62%	43.24%	46.72%
Reduction in 5% VaR of revenue dist.	0.20%	0.42%	9.60%	7.22%	30.42%	31.88%

the benefit of including this variable in our copula vine seems minimal. To highlight the importance of hedging instruments written on the DE price, we regard the optimal hedge quantity of DK1 forwards, which is approx. -190 MW cf. figure 6(c). As mentioned earlier, the DK1 market place is characterized by illiquidity, and so being able to enter a short position corresponding to e.g. -190 MW DK1 forwards at t_0 would imply a significant cost due to the rather large difference between the bid price and the mid-point of the bid-ask spread. We consider therefore including such a cost in our revenue equation, and in consequence minimize the total revenue variance and the expected cost of the replicating portfolio subject to an arbitrage free constraint and the same short-selling constraints as before, i.e.

$$\min_{S_{t_0}^{fixed},\boldsymbol{\theta}} \left\{ \mathbb{V}\mathrm{ar}_{t_0}\left[R^f + \sum_{n=1}^{N} \theta_n R^{(n)} - 24\xi \,|\theta_{n=1}| \right] + \mathbb{E}_{t_0}\left[\sum_{n=1}^{N} \theta_n R^{(n)} + 24\xi \,|\theta_{n=1}| \right] \right\} \tag{24}$$

$$\text{s.t. } \mathbb{E}_{t_0}\left[R^f + \sum_{n=1}^{N} \theta_n R^{(n)} - 24\xi \,|\theta_{n=1}| \right] = 0 \tag{25}$$

$$\theta_n \geq 0, \ n = 4, \ldots, 6, \tag{26}$$

where ξ denotes the cost in EUR/MWh associated with a position of $\theta_{n=1}$ MW in DK1 forwards. We fix $\xi = 1$ EUR/MWh, which is a realistic choice considering recent numbers for volumes traded and bid-ask spreads associated with DK1 forwards, and solve the constrained minimization problem posed in equations (24)–(26). We include FTRs in both directions here, since the indifference implied by the parity argument from before no longer holds. The solution is displayed in figure 6(d), and not surprisingly, the hedging strategy shifts towards less DK1 forwards. Still, a significant amount of DK1 forwards is required, which is due to the inability to short FTRs in the direction DE→DK1. The same degree of variance reduction is obtained with this alternative hedging strategy; we note however that in order to satisfy the no-arbitrage constraint and thus keep the expected revenue at zero, the price $S_{t_0}^{\text{fixed}}$ is re-estimated, with the solution yielding a lower price than under the assumption of a frictionless market.

4.2. An example highlighting the benefits of wind index futures

Since the examples in section 4.1 have to a limited extent brought out the usefulness of the wind index futures, one might think of them as inferior products compared to all other proposed hedging instruments. Although this is true in the case where a simultaneous hedge is performed at t_0, when also entering into fixed-for-fluctuating agreements, there are other scenarios that can better highlight their benefits. Let us consider the following example:

- At some time t^* before t_0 = 12 December 2016, an energy trading company enters into fixed-for-fluctuating agreements with wind power producers.
- Delivery period for the agreements is the same as before, i.e. T_1 = 1 January 2017 and T_2 = 31 January 2017, and the installed capacity c remains fixed to 500 MW. We again here assume perfect correlation between the wind power production that the company buys and the total wind power production in the price area.
- Also at time t^*, the company performs a hedge corresponding to a short position in DK1 forwards, where the contracted volume equals the company's expectation of the total future production for January 2017. We assume that the expected wind index is 0.38, and thus the hedge quantity equals $-0.38 \cdot 500$ MW.
- We fix $S_{t^*}^{\text{fixed}} = 33$ EUR/MWh and $F_{t^*}^{DK1} = 37$ EUR/MWh.
- As time goes by and we reach t_0, the price level has dropped. This is indeed reflected in our simulations, where we estimate lower values for S^{fixed} and F^{DK1} at t_0.
- At t_0, the trading company wishes to rebalance their hedging portfolio, and they consider whether to use additional DK1 forwards and/or wind index futures.

By performing similar calculations as in the previous examples, we obtain the standard deviations of revenue distributions under different rebalancing strategies consisting of trading in DK1 forwards and wind index futures, respectively. The results are summarized in figure 7(a), showing that wind index futures are more effective than DK1 forwards. This can be explained by considering e.g. a high wind/low price scenario, which causes the realized production to exceed the company's expectation. The excess production will have to be sold on the day-ahead market, and since prices have fallen substantially since t^*, the spot price will be below the fixed price that the company promised to pay wind power producers. In this situation, a long position in wind index futures will yield a premium corresponding to precisely the excess production, independent of the current price level cf. equation (17).

We complement these results with figures 7(b) and (c), where we illustrate the revenue distribution and a zoom of the left tail before the hedge (blue dashed line), after an optimal hedge in DK1 forwards at t^* (black solid line), and after rebalancing with DK1 forwards and wind index futures at t_0 (red dashed-dotted line). As expected, the revenue distribution is shrunk after the extra hedge, compared to the revenue distribution obtained after the initial hedge in DK1 forwards. According to table 7, the benefit is measured to be a reduction in the variance of the revenue distribution of approx. 29%. When considering the isolated effects, table 7 reveals that an extra hedge based on wind index futures alone achieves almost the same risk reduction. So given wind index futures in the extra hedge, adding DK1 forwards yields a minimal benefit in this example.

4.3. Comparison with alternative models

So far in our hedging analysis, we have concentrated on applications where the copula model remains fixed to our preferred specification, which is the time-varying mixed C-vine copula model. In this section, we wish to compare this model against *suboptimal* alternatives, and highlight the importance of capturing characteristics such as heavy tails in the marginal distributions, tail dependence, and time variation in the copula. To this end, we consider four models, which in the interest of clarity are stated in table 8.

Given the same contractual specifications as in section 4.1, we perform out-of-sample simulations from all four models in table 8. Then, we solve for the optimal hedging strategy when (1) allowing for only DK1 forwards in the hedging portfolio, and (2) allowing for all proposed hedging instruments in the hedging portfolio. Finally, we compute reductions in the variance and 5% VaR of the revenue distributions obtained when hedging with all proposed hedging instruments as opposed to using only DK1 forwards. Also, we compute the values in EUR corresponding to the 5% VaR and the 5% Expected Shortfall (ES) of the revenue distributions obtained by including all hedging instruments. To avoid the effect of simulation errors, the same random seed is used for simulations from the four models. The results are given in table 9.

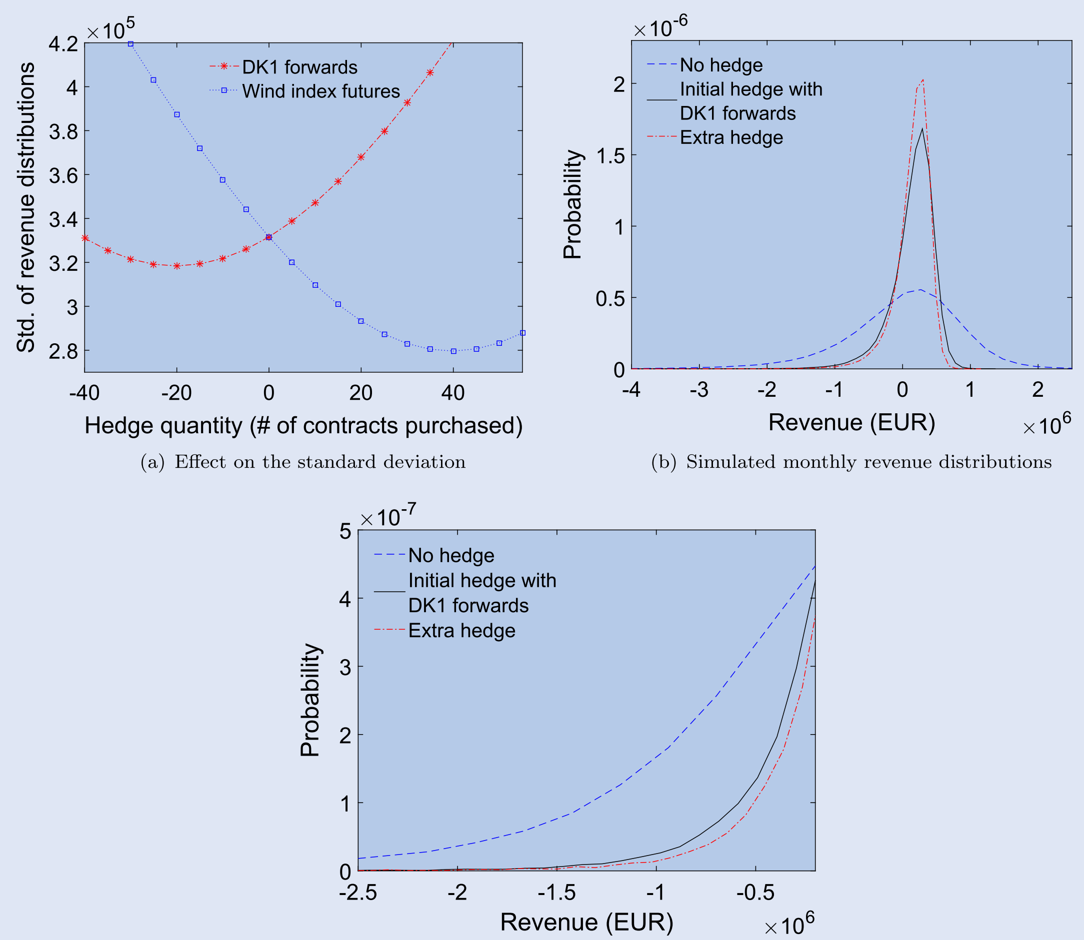
Let us start by considering the first two rows of table 9 and discuss the differences in the reductions. When regarding the difference between Model 1 and 2, we see that the introduction of heavy tails implies an increase in the hedging benefits by yielding higher values for the reduction in variance and 5% VaR. The introduction of tail dependence between the DK1 and DE prices (i.e. the difference between Model 2 and 3) lowers the hedging benefits, which is expected since tail dependence lowers the benefits of the FTRs in the sense that a positive probability of extreme prices occurring simultaneously lowers the overall potential payout. Lastly, the introduction of time-varying dependence (i.e. the difference between Model 3 and 4) lowers further the hedging benefits. While tail dependence is in this case always expected to entail lower hedging benefits, the effects of time-varying vs. static dependence can go both ways, depending on the difference between the levels of dependence implied by the two models. In our example, the lower hedging benefits produced with Model 4 are caused by the stronger dependence between the DK1 and DE prices implied by the GAS model. This stronger dependence weakens once again the usefulness of the FTRs, thereby contributing negatively to the overall benefits.

Moving onto the last two rows of table 9, we notice that the values of the 5% VaR and 5% ES increase when considering the models in decreasing order: Our preferred model (Model 4) implies most risk, followed by Model 3 and so forth.

Since the prices of the fixed-for-fluctuating agreements and all hedging instruments have already been computed to perform our hedging exercise above, we also quantify in table 10 the effects of the different models from a pricing perspective. Cf. table 10, the introduction of heavy tails in the margins (measured by Δ_1) has a mixed but noteworthy effect on most instrument prices. The largest absolute impact is on the price of the OTM puts, which is not surprising, since OTM options are much more affected by tail behaviour than at-the-money or in-the-money options.

Regarding Δ_2 and Δ_3, most interesting to consider are the FTRs and the fixed-for-fluctuating agreements. This is because their pay-offs depend on two underlyings, and hence the choice of copula can influence the prices of these instruments to a large extent, as opposed to all other instruments. For the FTRs, the introduction of tail dependence, followed by time-varying dependence, have both a negative and significant effect on prices. The overall reduction in prices from Model 1 to Model 4 reaches around 11% for the direction DE→DK1, and around 9% for the direction DK1→DE. For the fixed-for-fluctuating agreements, the fact that Δ_2 is zero is not surprising, since tail dependence is introduced only between the DK1 and DE prices, and not the DK1 price and wind power production. As a result, the transition from a trivariate Gaussian to a mixed C-vine copula model has no implications for the price of this instrument. Δ_3 being zero for S^{fixed} can be explained by the fact that the mean value of the time-varying dependence implied by the GAS model for the DK1 price and wind power production is very close to the dependence implied by the static model.

As a last remark, we investigate the effects of using *suboptimal* models for hedging. Specifically, we let pricing and hedging decisions be based on one of Model 1, 2 or 3, while afterward assuming that the 'real' prices and wind power production evolve according to Model 4. That is, based on the prices and the hedging strategies obtained earlier with the more naive models, we construct revenue distributions where the simulated paths of our three variables come from our preferred model. Then, we ask the following question: By how much do we under- or overestimate risk?



(a) Effect on the standard deviation

(b) Simulated monthly revenue distributions

(c) Zoom of left tail in figure 7(b)

Figure 7. Illustrating the usefulness of wind index futures.

Table 7. Effects of rebalancing the hedging portfolio due to decreasing prices.

	Isolated effect		
	Wind index futures	DK1 forwards	Combined effect
Reduction in variance of revenue dist.	28.28%	7.80%	28.92%
Reduction in 5% VaR of revenue dist.	22.25%	8.67%	22.31%

Table 8. Alternative models for comparison study, and their characteristics. TV is short for time-varying.

	Model 1	Model 2	Model 3	Model 4
Marginal distributions	Normal	Skew *t* (skew normal)	Skew *t* (skew normal)	Skew *t* (skew normal)
Copula model	Trivariate Gaussian	Trivariate Gaussian	Mixed C-vine	TV mixed C-vine
Heavy tails (margins)	×	✓	✓	✓
Tail dependence	×	×	✓	✓
TV dependence	×	×	×	✓

The results are given in table 11, and we report the 5% VaR, the 5% ES and the expected value of the revenue distribution in two cases: (a) using the same model to compute instrument prices and an optimal hedging strategy, and also to construct a revenue distribution (as in table 9); (b) using Models 1–3 to compute instrument prices and an optimal hedging strategy, while constructing the revenue distribution using simulations from Model 4. The columns denoted Δ state the relative differences between results obtained in (a) and (b), showing that basing decisions on the *wrong* model leads to an underestimation of risk in all three cases. Furthermore, we note that the expected value of the revenue distribution is no longer zero in the case of columns (b). If we based pricing and hedging decisions on Model 1, the expected revenue would actually be

Table 9. Combined effects of adding the other proposed instruments to a hedging portfolio initially consisting of DK1 forwards. The results are obtained with four different models. The values in EUR of the 5% VaR and 5% ES correspond to the revenue distributions obtained by hedging with all proposed instruments.

Initial hedging portfolio DK1 forwards only	Model 1	Model 2	Model 3	Model 4
Reduction in variance of revenue dist.	40.49%	49.25%	48.12%	46.72%
Reduction in 5% VaR of revenue dist.	29.03%	33.15%	32.70%	31.88%
Value of the 5% VaR (EUR)	−321,440	−326,840	−328,410	−334,890
Value of the 5% ES (EUR)	−440,690	−462,200	−465,720	−476,940

Table 10. Prices computed with different models. All prices are given in EUR/MWh with the exception of I^{Locked}, which represents an index. Δ_1, Δ_2 and Δ_3 refer to the relative differences between Models 1 and 2, Models 2 and 3, and Models 3 and 4, respectively, and are highlighted if different from zero.

Estimated prices Pricing date (t_0): 12 December 2016 Delivery period: January 2017	Model 1	Model 2	Model 3	Model 4	Δ_1	Δ_2	Δ_3
S^{fixed}	23.661	23.736	23.736	23.732	**0.32%**	0.00%	0.00%
F^{DK1}	26.322	26.324	26.324	26.324	0.00%	0.00%	0.00%
F^{DE}	26.931	26.935	26.936	26.937	0.00%	0.00%	0.00%
I^{Locked}	0.378	0.382	0.382	0.382	**1.06%**	0.00%	0.00%
$V^{DE\to DK1}$	2.919	2.859	2.775	2.585	**−2.06%**	**−2.94%**	**−6.85%**
$V^{DK1\to DE}$	3.529	3.471	3.386	3.198	**−1.64%**	**−2.45%**	**−5.88%**
V^{Put}	0.519	0.548	0.548	0.548	**5.59%**	0.00%	0.00%

Table 11. Values in EUR of the mean, 5% VaR and 5% ES of revenue distributions obtained under different assumptions: (a) pricing and hedging decisions are performed with a suboptimal model while the simulated data used to construct revenue distributions also evolves according to the suboptimal model, and (b) pricing and hedging decisions are performed with a suboptimal model while the simulated data used to construct revenue distributions evolves according to Model 4, the time-varying mixed C-vine copula.

	Model 1			Model 2			Model 3		
	(a)	(b)	Δ	(a)	(b)	Δ	(a)	(b)	Δ
5% VaR	-321,440	−325,150	−1.14%	−326,840	−351,040	−6.89%	−328,410	−345,910	−5.06%
5% ES	−440,690	−458,890	−3.97%	−462,200	−494,050	−6.45%	−465,720	−488,200	−4.60%
Mean	0	586	–	0	−14,508	–	0	−10,221	–

positive, implying a gain on average. Despite the suboptimal hedging strategy and the overestimated FTR prices, which all push the revenue distribution to the left, the decisive factor that ends up shifting the revenue distribution to the right is the price of fixed-for-fluctuating agreements, which is too low under Model 1. If we based pricing and hedging decisions on Model 2 or 3, we would expect a loss on average. This is due to the hedging strategy implied by Model 2 and 3, respectively, which pulls the revenue distribution to the left. Adding to this effect are the FTR prices, which continue to be overestimated in both Model 2 and 3.

5. Conclusion

In this paper we have considered the joint price and volumetric risk in wind power trading, and proposed a flexible model that has practical applicability and can be used to determine optimal hedging decisions. The problem has been addressed from an energy company's perspective, who pays wind power producers a fixed price in return for their fluctuating wind power production. Our study concentrates on the joint price and volumetric risk in Denmark, a market place characterized by illiquidity and a restricted range of available hedging instruments. As a consequence, we have extended the modelling framework to include the German spot electricity price, thereby exploiting the strong positive relation between the two price series to increase the hedging possibilities.

We have proposed and fitted a three-dimensional time-varying mixed C-vine copula model to our variables of interest, capturing rather well the marginal behaviour of each variable and also the dependence between the variables. Based on simulations from the proposed model, we have shown through different examples that significant benefits are to be drawn from including other than the standard and usually employed power forwards in a hedging portfolio. We find that derivative instruments associated with the right to convert electricity to heat are especially effective at reducing risk. Moreover, instruments with reference to the German spot electricity price are especially beneficial when accounting for illiquidity on the Danish power market, in the sense that they contribute to lowering the cost of the hedging portfolio. Lastly, wind index futures have risk reducing benefits when rebalancing the hedging portfolio under certain market conditions.

To highlight the importance of capturing heavy tails in the marginal distributions, tail dependence and time variation in the copula model—all characteristics that are captured by the proposed empirical model—similar calculations were performed with alternative and more naive models. We find that the inability to capture these characteristics leads to an underestimation of risk. Also, some instrument prices are affected to a large extent.

Power markets can differ greatly from region to region, and our framework is *tailored* to the case of Denmark. Depending on the availability of hedging instruments in other power markets, one might wish to include additional variables to the model. In e.g. Coulon *et al*. (2013), the authors propose gas-fired plants to hedge joint price and volumetric risk associated with load-serving obligations. We acknowledge that gas-fired (or coal-fired) plants could potentially be beneficial in our application as well. In this connection, the evolution of gas (or coal) prices would have to be taken into account. What is particularly useful about our model framework is that other variables can easily be incorporated in the vine copula, and hence different situations can be accommodated without much difficulty.

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