Sovereign Debt Spreads in a Markov Switching Regime

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Abstract

In a sovereign debt model with endogenous default, I add uncertainty about the long-run prospects of the emerging country to account for the high spreads and relatively frequent default episodes observed in these countries. The existing literature cannot account for the business cycle moments of the spreads: the spreads are generally too low and when they are increased through some adjustments to the model, volatilities of spreads exceed the data.

In the model of this paper, agents (the country and international investors) do not have direct information about the long-run output trend, but can infer a long run trend through period output realizations. If the country is already stagnating and there is pessimism about the future then default is less costly for the country and it might choose to default. In addition, because of this uncertainty, the country is willing to pay high interest rates in the short run as it tries to determine whether a bad output realization is just temporary or whether the long-run prospects have indeed deteriorated.

The model is able to account for business cycle moments of spreads, and also generate other emerging market business cycle facts such as countercyclical interest rates, a countercyclical current account, and more volatile consumption relative to output.

JEL Classification: E44, F32, F34

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1 Introduction

In the last thirty years emerging markets have been subject to relatively frequent default episodes, some of which include the recent defaults of Argentina, Ecuador and Russia. In addition, emerging markets have certain business cycle features that they do not share with developed countries. Neumeyer and Perri (2005) shows that emerging market interest rates are countercyclical, with

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spreads increasing as the country enters a recession, while developed country interest rates are roughly acyclical. In addition, although for both developed countries and emerging markets, the current account is countercyclical, it is much more so for emerging markets. In these markets, a reduction in output is accompanied by a strong outflow of capital, reducing further what is available for consumption and investment, so that consumption volatility exceeds output volatility. The opposite is true for developed countries.

There has been an expanding literature of sovereign debt models with endogenous default which aims to generate all of these facts in a unified framework. In these models, the small open economy can trade one-period discount bonds in the international markets, and it can choose to default on its debt if it finds it optimal to do so. This structure generates endogenous fluctuations in spreads depending on the incentives of the country to default. Although the model has been successful with qualitative results, it has been largely unsuccessful in accounting for the business cycle moments in spreads; mainly the spreads in the model are too low, and when they are increased through some adjustments to the model, volatilities of spreads exceed the data.

I show in this paper that an enrichment to the model that incorporates uncertainty about the long-run output trend of the emerging country can address this problem. The uncertainty about the long run can be legitimized by the observation that emerging markets are subject to major and frequent policy changes which affect their long run output prospects. Whether the policies are to stabilize inflation, increase investment, or integrate the country into financial markets, the outcome of these policies are highly uncertain. In addition, we would expect a country to be more reluctant to default if it believes itself to be on a high output path because default implies a break from the current policies and it is followed by a financial crash impeding the output at least temporarily. On the other hand, if the economy is already stagnating and is expected to continue that way, then the low output that follows default would not be a further punishment to the country, and would imply a relatively low cost of default for the country.

To incorporate all of these features into the model, I assume that the country can potentially follow two different output regimes: one with a high expected output and the other with a low expected output. There is an exogenously given probability of switching between these regimes. Agents (the country and financial investors) do not know which output regime the country is in, but the period output realizations are immediately observable, and agents can infer information about the long-run trend from actualized outputs. The aftermath of default is modeled as a switch to a new regime, which is with high probability the low output regime. As a result, default is a big loss in times of high confidence (that is, if the country is with a high probability in the high output regime), and is nearly costless at times of low confidence.

The model is able to generate high default probabilities and a relatively high debt level simultaneously. The difficulty of generating these two outcomes simultaneously is that, if default is likely at all times then a significantly positive debt level is unsustainable as it prevents trading at reasonable prices. In this model, this difficulty is overcome by the asymmetry in the cost of default. Low confidence periods result in default because there is nothing further to lose by the country. At high confidence periods, the country is willing to pay back high debts because the possibility of switching to a low-output regime is very costly, making relatively high debt levels sustainable. In addition, the uncertainty about the long run gives the country the incentive to undertake the cost of paying high interest rates for some period of time to determine whether a low output realization is just temporary or whether the long run prospects for the country are bleak.

In summary, due to low patience (a standard assumption used to induce the emerging country to borrow), the country borrows strongly during high confidence periods because it faces low cost to borrowing (low spreads). When nature switches, the country is stuck with a high debt level and high interest rates, but delays defaulting until it is reasonably sure that the state of nature has indeed switched.

In addition to high and countercyclical spreads, this mechanism also generates a countercyclical current account. As explained above, the country does more borrowing when it is cheap to do so, which is during high confidence/high output periods, resulting in a countercyclical current account. In the endowment economy used in the paper, this results in consumption that is more volatile relative to output as the country borrows even more to consume during high output shocks and pays back the debt at high interest rates during low output shocks.

I perform the quantitative exercise for Argentina and examine the model's fit with the data. The model can fit the average spreads and also account for the volatility of spreads in Argentina. In addition, the model generates the countercyclical spreads, the countercyclical current account, and more volatile consumption relative to output observed for Argentina, the reasons for which are explained above. I also show that the model can account for the Argentinian spread series for the period 1994-2001.

The paper is related to the quantitative models of sovereign debt with endogenous default. The sovereign default model builds on the approach of Eaton and Gersovitz (1981) where assets traded between countries are limited to one-period bonds and repayments are enforced by the threat of financial autarky. Arellano (2006) extends it to study the quantitative implications of the model for Argentina. Aguiar and Gopinath (2005) adds trend shocks to the model and examines its potential to generate higher default rates and spreads. Yue (2006) models renegotiation of debt that follows default and examines its implications for other business cycle statistics for Argentina. My model is closer to Arellano (2006); it has transitory shocks and debt level is reduced to zero following default. I enrich the model by adding another realistic feature about debtor countries, the uncertainty they face about their long run trends, and show that it improves the model significantly in accounting for the business cycle moments of the spreads.

The paper proceeds as follows. Section 2 introduces the model, Section 3 characterizes the equilibrium, Section 4-5 provides quantitative analysis and Section 6 concludes.

2 The Model Environment:

I study sovereign default in a dynamic small open economy. The asset markets are incomplete and the country can only trade one-period discount bonds in international financial markets. In addition, debt contracts are not enforceable and the country can default on its debt if it finds it optimal to do so.

There is a single consumption good that is nonstorable but tradable in international markets. The country is risk-averse and the utility of the country for the single consumption good is given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t)$$

where $0 < \beta < 1$ is the discount factor,and c is the consumption good and $U(.)$ is the period utility that is strictly concave and increasing.

Financial intermediaries are risk-neutral and behave competitively. They are ready to borrow or lend any amount for an expected return of r, the constant risk-free world interest rate.

2.1 The endowment process

The country receives a stochastic endowment y_t in each period. There are two possible output regimes (or states of nature) for the country, denoted by $s_t \in \{H, L\}$. The country output follows a different process depending on which regime the country is in. I assume that the endowment process is a first order Markov process in each regime, denoted by probability distribution functions $G_H(y_{t+1}|y_t)$ and $G_L(y_{t+1}|y_t)$. More specifically, I am going to assume that in each regime, the endowment process is $AR(1)$, with same autocorrelation and variance, but different means. H denotes the process with the higher conditional/unconditional mean:

$$\text{Process H:} \quad \ln(y_t) = \mu_H + \rho \ln(y_{t-1}) + \varepsilon_t, \qquad \varepsilon_t \tilde{} N(0, \sigma^2) \tag{1}$$

$$\text{Process L:} \quad \ln(y_t) = \mu_L + \rho \ln(y_{t-1}) + \varepsilon_t, \qquad \varepsilon_t \tilde{} N(0, \sigma^2)$$

with $\mu_H > \mu_L$

Except for the periods of default, the state of nature $s_t \in \{H, L\}$ also follows a first-order Markov chain. The transition probability π_{ij} gives the probability that state i will be followed by state j:

$$\pi_{ij} = \Pr(s_t = j | s_{t-1} = i), \quad i, j \in \{H, L\}$$

2.2 Information structure

The information set is the same for all agents. The output of the country is observable in the period that it is actualized. The state of nature s_t is not directly observable, but an inference can be made about its value based on actualizations of y_t. The sum of the probabilities of being in states H and L is by construction equal to 1. I will express the equilibrium in terms of θ_t, the probability of the country being in process H.

From the perspective of agents in the economy; the next period endowment y_{t+1}, given θ_t and y_t, is distributed according to the probability distribution function $F(y_{t+1}|y_t, \theta_t)$:

$$F(y_{t+1}|y_t, \theta_t) = \theta_t G_H(y_{t+1}|y_t) + (1 - \theta_t) G_L(y_{t+1}|y_t)$$

2.3 Asset Structure and Default

The country has access to financial markets where it can buy or sell one period discount bonds. Financial intermediaries commit to paying back their debt, but the country can choose to default on its debt if it finds it optimal to do so. B_t is the debt level of the country and q_t is the price of the one-period bond. One unit of the bond with price q_t therefore delivers one unit of the consumption good next period in the states it does not default.

The country might choose to default on its debt when debt level is positive. Default is attractive to the country because after default, debt level is reduced to zero.[1] There are two costs of default. One is that, the history of the state of nature $s_t \in \{H, L\}$ is erased and nature draws state of nature H with probability θ_P. θ_P is low so that default is a punishment in most cases.

The second punishment concerns access to financial markets. Default is followed by temporary financial autarky. [2] In autarky, the sovereign government is not able to save or borrow in financial markets. As the consumption good is nonstorable, the country simply consumes its endowment stream in autarky. [3] The binary variable $h_t \in \{0, 1\}$ denotes the credit score of the country: $h_t = 1$

[1] Yue (2006) has renegotiation of debt following default and the debt level after default is potentially positive depending on the bargaining weight of the country. A previous version of the model had renegotiation without significant effect on the results. I chose this formulation for ease of exposition of the model.

[2] This approach follows the framework of Eaton and Gersovitz (1981) in which repayment is enforced by the threat of financial autarky.

[3] The assumption of not being able to save in financial autarchy can be relaxed without any change in the quantitative results of the paper. In the quantitative analysis, the discount factor β is fairly low, and the low patience dominates the precautionary saving motive of the country. Even if savings are permitted during autarky, it would not result in positive asset holdings in equilibrium.

indicates that the country has a good credit score and that it has access to financial markets; $h_t = 0$ is a bad credit score and denotes that the country is in financial autarky. During autarky, the country's credit score upgrades to 1 again with an exogenously given probability λ. There are no further defaults in autarky as debt level is reduced to zero following default.

3 Characterization of Recursive Equilibrium

In this section, I give the equations that characterize the recursive equilibrium The exposition is divided into four sections. The first part provides the transition schedule of beliefs about which regime the country is in. This is the same for all agents, the country and financial intermediaries, as each observes the same information set. It is followed by the sovereign government's and the financial intermediary's problems. Finally, the definition of the recursive equilibrium is given.

3.1 Updating of beliefs

Beliefs about which regime the country is in is updated according to Bayes' rule. The transition probability between states of nature (regimes) $s_t \in \{H, L\}$ depends on whether or not the country defaults. If the country does not default, the transition is governed by the first order Markov process specified in Section 2.1. If it defaults then the last realization of s_t does not matter and nature draws state H with probability θ_P.

Accordingly, the transition schedule of beliefs depends on whether or not the country has defaulted in the current period. Let $\theta'_N(\theta, y, y')$ denote next period's updated belief after observing next period's shock y' in the case the country has not defaulted today. $\theta'_D(y, y')$ denotes the same for the case where the country defaults in the current period. θ'_D does not depend on the current belief θ as the history about the state of nature is erased in the case of default.

If the country has not defaulted:

$$\theta'_N(\theta, y, y') = \frac{G_G(y'|y)\left[\theta\pi_{HH} + (1-\theta)\pi_{LH}\right]}{G_G(y'|y)\left[\theta\pi_{HH} + (1-\theta)\pi_{LH}\right] + G_B(y'|y)\left[\theta\pi_{HL} + (1-\theta)\pi_{LL}\right]} \tag{2}$$

If the country has defaulted:

$$\theta'_D(y, y') = \frac{G_G(y'|y)\left[\theta_P\pi_{HH} + (1-\theta_P)\pi_{LH}\right]}{G_G(y'|y)\left[\theta_P\pi_{HH} + (1-\theta_P)\pi_{LH}\right] + G_G(y'|y)\left[\theta_P\pi_{HL} + (1-\theta_P)\pi_{LL}\right]} \tag{3}$$

The only thing that has changed for the transition of beliefs following default is that θ_P is used instead of θ as nature draws the good state with probability θ_P.

3.2 Sovereign Government's Problem:

The sovereign government maximizes the lifetime utility of the representative agent. The government faces different constraints that depend on the country's credit score $h_t \in \{0, 1\}$. Let, $V(\theta, y, B)$ and $V_A(\theta_D, y)$ denote the value function of the country under credit score 1 and 0 respectively. I will also refer to V_A as the autarky value.

When $h = 1$, the sovereign government has the option to default. $V(\theta, y, B)$ satisfies:

$$
\begin{aligned}
V(\theta, y, B) &= \max\{V_N(\theta, y, B), V_D(y)\} \quad \text{if } B > 0 \\
&= V_N(\theta, y) \qquad\qquad\qquad\qquad \text{else}
\end{aligned}
$$

where V_N and V_D refer to the value function of the country if it does not default and defaults respectively. The value of default V_D is not contingent on either the market belief or the debt level at the start of the period. If there is a default, current beliefs become irrelevant and the debt level is reduced to zero. The country that has a debt level $B > 0$ will default only when $V_D > V_N$.

Let $\Delta(\theta, y, B)$ denote the default function that takes on the value D or N:

$$
\begin{aligned}
\Delta(\theta, y, B) &= D \text{ if } V_N(\theta, y, B) < V_D(\theta, y) \text{ and } B > 0 \\
& \quad N \quad \text{else}
\end{aligned}
$$

The value function when the sovereign government does not default is:

$$
V_N(\theta, y, B) = \max_{B'} \left\{ U(y - B + q(\theta, y, B')B') + \beta \int V(\theta'_N(\theta, y, y'), y', B', 1) F(y'|y, \theta) dy' \right\}
$$

If the sovereign government does not default, it must pay back the current debt B and choose a level of borrowing B'. The borrowing allows the country to increase current consumption by $q(\theta, y, B')B'$. In addition to (θ, y), the bond price $q(\theta, y, B')$ depends on how much the country must pay back next period B'. In equilibrium, the country is more probable to default at high levels of debt, and the price schedule q is decreasing in debt level B', implying that the spreads it has to pay is increasing B'. The country internalizes the effect a higher debt level has on the spreads it has pay. For next period's updated belief I use the transition schedule θ'_N as the country does not default in the current period.

The value of default V_D satisfies:

$$V_D(\theta) = U(y)+\beta\left((1-\lambda)\int V_A(\theta'_D\left(y,y'\right),y')F(y'|y,\theta_P)dy' + \lambda\int V(\theta'_D\left(y,y'\right),y',0)F(y'|y,\theta_P)dy'\right)$$

When the country defaults, its debt level is reduced to zero and it is excluded from financial markets, so it consumes only its endowment. In the next period, it is either redeemed (i.e. readmitted) back into the financial markets with probability λ and realizes the value $V(\theta'_D, y', 0)$; else it stays at credit rating $h = 0$ and receives its autarky value $V_A(\theta'_D, y')$. For next period's updated belief the transition schedule θ'_D is used as the country defaults in the current period.

The autarky value of the country is:

$$V_A(\theta, y) = \left\{U(y) + \beta\left((1-\lambda)\int V_A(\theta'_N, y')F(y'|y,\theta)dy' + \lambda\int V(\theta'_N, y', 0)F(y'|y,\theta)dy'\right)\right\}$$

There is no decision made by the country in autarky. In each exclusion period, the country consumes its endowment. In the next period, its credit score is upgraded to $h = 1$ with probability λ or it stays at $h = 0$ and continues to receive its autarky value.

3.3 Financial Intermediaries' Problem

Intermediaries are risk-neutral and behave competitively. They are willing to lend or borrow any amount given that their expected return is equal to the risk free-interest rate r. $q(\theta, y, B')$ is the price schedule the country faces given that it has access to financial markets today (i.e. $h = 1$ and it does not default in the current period):

$$q(\theta, y, B') = \frac{1}{1+r}\int \left(I\left(\Delta(\theta'_N, y', B') = D\right)\right) F(y'|y,\theta)dy' \tag{4}$$

where $I(.)$ is the indicator function which takes the value one if the statement inside is true, and 0 otherwise. As seen, the price schedule depends on the default probability of the country in the following period. θ'_N is used to update beliefs as the country cannot default in the current period if it is to trade in financial markets and borrow B'. At a future node (y'), financial intermediaries receive one unit of the consumption good if the country does not default , else they get zero. The expected payment per unit of debt is discounted by $1/(1 + r)$ to obtain q.

3.4 Definition: Recursive Equilibrium

Definition: The recursive equilibrium is (i) the transition schedules of beliefs under default and no default $\theta'_D(y, y')$, $\theta'_N(\theta, y, y')$, (ii) the borrowing function $B'(\theta, y, B)$ and the default function $\Delta(\theta, y, B)$ for the government, and (iii) the price schedule for bonds $q(\theta, y, B)$ such that:

1. The transition schedules of beliefs $\theta'_D(y, y')$, $\theta'_N(\theta, y, y')$, satisfy Bayes' rule; equations(2) , (3).

2. Given the transition schedules of beliefs, $\theta'_D(y, y')$, $\theta'_N(\theta, y, y')$, and the bond price function $q(\theta, y, B)$; the country's asset holding $B'(\theta, y, B)$ and default $\Delta(\theta, y, B)$ functions satisfy the government's optimization problem.

3. Given the transition schedules of beliefs $\theta'_D(y, y')$, $\theta'_N(\theta, y, y')$, and default function $\Delta(\theta, y, B)$; the price schedule $q(\theta, y, B)$ satisfies the lender's zero profit condition; equation (4).

4 Parameter Choices

The quantitative exercise is done for the Argentinian economy. I define one period as a quarter. For the utility function I use CRRA preferences:

$$U = \frac{c^{1-\sigma}}{1-\sigma}$$

The coefficient of relative risk aversion 2 is standard. The risk-free interest rate is taken as 1% to fit the quarterly risk-free interest rate of the US. The probability of redemption λ is chosen as 0.1 which implies financial autarky of an average duration of 2.5 years. The discount factor chosen is 0.93 which, although low, is higher than what is used in most papers in the literature. The low discount factor is needed so that the country is on average a net borrower and might choose to default on its debt.

For the Argentinian output process I use an autocorrelation coefficient of $\rho = 0.95$ with a standard deviation of $\sigma = 0.034$. For the transition probability between the two processes I use $\pi_{GG} = \pi_{LL} = \pi = 0.9$, and the difference between the mean of the two processes is chosen as $\mu_H - \mu_L = 0.017$ which is half of the standard deviation of the output process. I choose the default punishment θ_D to fit the mean interest rates [4] As for the future, I intend to calibrate these parameters by examining a panel of emerging markets to isolate their long run trend. It is common

[4] In most papers the model is calibrated so that it fits a yearly 3% default probability which is the average for Argentina in the last 200 years; instead in this paper I fit the average spreads. The obvious drawback of considering the last 200 years is the changing structure of the economy. In particular, it ignores the fact that financial markets were closed to Argentina for a significant portion of the 19th century, and that during the second half of the 20th century most of the international lending was through governments and international institutions rather than private investors. If the time frame after financial liberalization of 1977 is considered and periods of financial exclusion is left out (after default in 1982, Argentina could not borrow until the Brady agreement in 1992), default probability jumps up to 11%. (Damill et al.(2005) gives a thorough account of Argentina's debt for the second half of the 20th century.) As default is a low probability event and the relevant time period is short, fitting average spreads is a more reasonable exercise than fitting default probability. With risk-neutral investors, there is one-to-one correspondence between spreads and default probability. Klingen et al.(2004) demonstrate that the expost returns to investors during the time frame 1970-2000 have been around the same the same as returns on a ten-year U.S. treasury bond (most of foreign currency borrowing in emerging markets is long term also), which is consistent with risk-neutral financial intermediaries assumption.

that emerging markets are subject to long-lasting periods of stagnation and expansions. The transition probabilities between the two processes, and the difference in the mean of the processes can be calibrated using the relative frequency of expansions and recessions, and the expected output differences between these processes.

Table 1. Parameter Values

Parameter	Definition	Value
β	discount factor	0.93
γ	risk aversion	2
λ	redemption probability	0.1
θ_P	default punishment	0.068
ρ	autocorrelation	0.95
σ	standard deviation	0.034
π	transition probability	0.9
$\mu_H - \mu_L$	difference between the processes	0.017

5 Results:

5.1 Data

Selected business cycle moments of Argentina are given in Table 2. Argentina displays many of the peculiar features of emerging market economy business cycles. Average spreads are high at 5.16% and the volatility of spreads is even higher at 5.55 %. Spreads are strongly countercyclical with the correlation between output and spreads at -0.44. In addition, the trade balance is countercyclical at -0.67. Countercyclical trade balance is also observed for developed countries, but the relationship is generally much weaker.[5] Another feature that separates Argentina and most emerging markets from developed countries is that the volatility of consumption is higher than the volatility of output. For Argentina, volatility of consumption is 1.12 times the volatility of output. The inability to smooth consumption might be a result of incompleteness of asset markets, or because the borrowing/saving is not done for consumption smoothing purposes, or as a combination of both as in this paper.

[5] For a thorough analysis of the difference between developed and emerging market business cycles, see Aguiar and Gopinath(2004).

		Y	R_S	C	TB
Standard Deviation		4.73	5.55	5.33	1.35
Quarterly Autocorrelation		0.81	-	-	-
	Y	1	-0.44	0.97	-0.67
Correlation Matrix	R_S	-	1	-0.39	0.64
	C	-	-	1	-0.75
	TB	-	-	-	1
Average spread			5.16		

Table 1: Business Cycle Statistics for Argentina

Notes: All series are quarterly data. The output (Y), consumption (C) and trade balance (TB) are real series taken from Ministry of Finance (MECON) and seasonally adjusted. They are in logs as deviations from HP trend with smoothing parameter 1600, and their standard deviations are reported in percentage terms. The annualized short-term spread series (R_S) for the period 1994:02-2001:04 are from Broner, F., G. Lorenzoni, and S. Schmukler (2005).

5.2 Mechanism of the model

I use a discrete state space method to solve the model. The detailed description is given in the Appendix.

Before I examine the key business cycle moments of the model, it is of use to explain how different confidence levels about the economy θ affect the interest rates and borrowing levels in the model. In Figure 1. I choose two levels of θ, one high and the other low, and show the spreads for different levels of borrowing (fixing the current output shock and debt level). It is seen that the spread is lower for any borrowing level with a high θ and in addition the price schedule is flatter, meaning an increase in the debt level results in a smaller increase in spreads when θ is high. The reason for the very low spreads at high confidence periods is related to the gradual nature of learning. When a high confidence period is followed by a low output realization, the country delays default to make sure that the regime has indeed switched (i.e. that it was not just a temporary shock). As default probability in the next period is nearly zero in high confidence periods, spreads are almost zero.

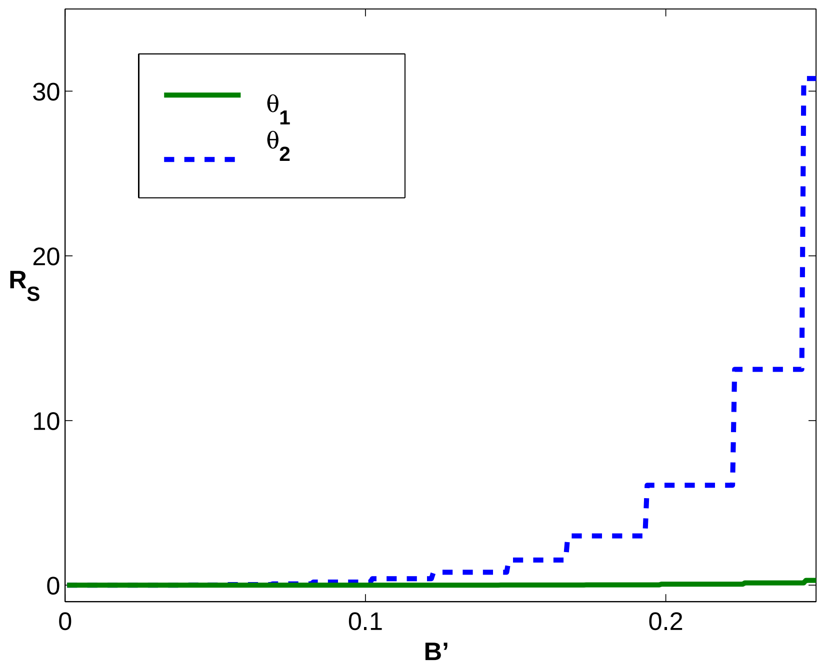

Figure1: Spreads conditional on borrowing

Notes: The figure gives the spread the country faces for each level of borrowing. The correspondence to the model is: $R_S = 1/q(\theta_i, \overline{y}, B') - (1 + r)$, where $i \in \{1, 2\}$ and $\theta_1 > \theta_2$. The current output shock $\overline{y}$ is the same for both lines.

Figure 2 displays the borrowing choice of the country, conditional on not defaulting, as a function of today's debt level for two different levels of θ (holding today's output fixed). For any debt level, choice of borrowing is higher when θ is high. Because of low patience, the country would like to borrow whenever it is cheap to do so and when more borrowing does not increase by too much the interest rate it must pay on its existing debt. As seen in Figure 1, both of these conditions are satisfied in times of high confidence, inducing more borrowing by the country during those periods.

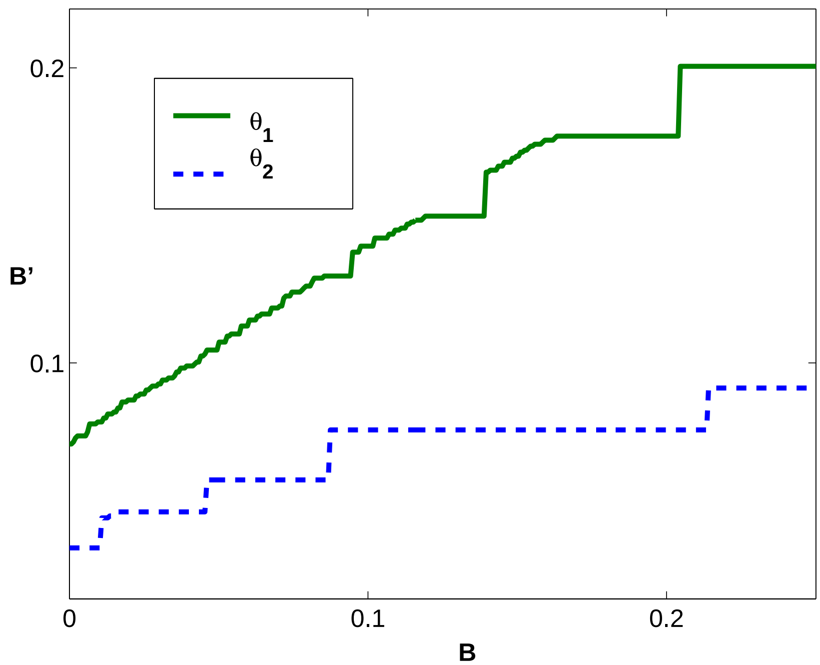

Figure 2: Borrowing conditional on period debt level

Notes: The figure gives the equilibrium borrowing conditional on current debt level; $B'(\theta_i, \overline{y}, B)$ where $i \in \{1, 2\}$ and $\theta_1 > \theta_2$. Current output is fixed at $\overline{y}$.

As seen from Figure 1 and Figure 2, there are two potential effects of θ on equilibrium spreads. On the one hand, for any choice of borrowing level, spreads are higher when confidence is low; on the other hand, the borrowing level is lower in low confidence periods which might decrease the equilibrium interest rates in those periods.

Figure 3 gives the equilibrium spreads the country faces at the optimal choice of borrowing as a function of today's debt level. The exercise is again done for a high and low θ, holding current output constant. The equilibrium spread is higher when the assessment about the economy is more pessimistic. Although debt level is reduced in low confidences period, equilibrium interest rates are still higher.

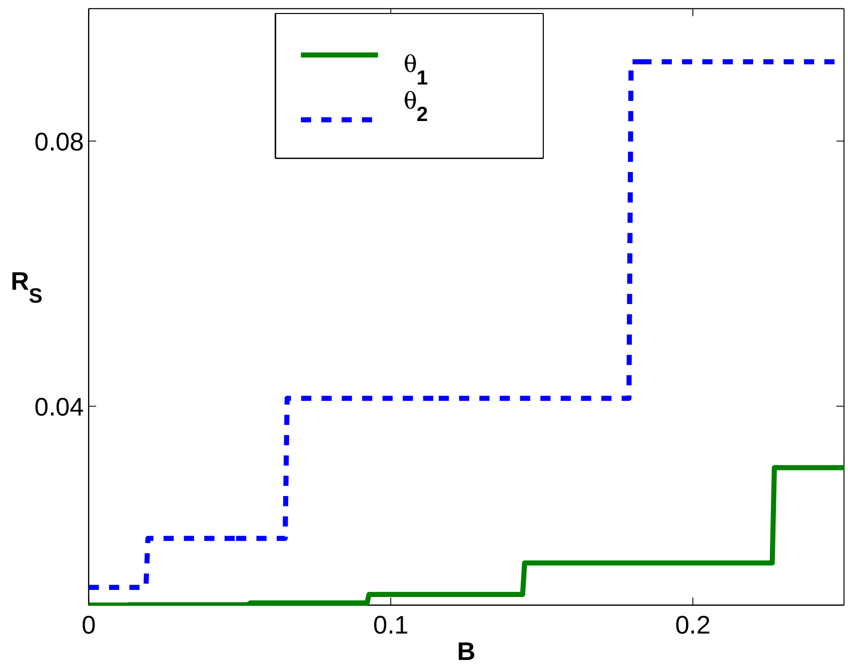

Figure 3: Equilibrium spread conditional on current debt level

Notes: The figure gives the equilibrium spreads at the equilibrium borrowing conditional on current debt level B. In terms of the notation in the paper; $R_S = 1/q(\theta_i, \overline{y}, B'(\theta_i, \overline{y}, B)) - (1+r)$ where $i \in \{1,2\}$ and $\theta_1 > \theta_2$.

5.3 Business cycle results of the model

Table 3 reports the correlation of θ with spreads and the trade balance as well as other key business cycle moments for Argentina using the benchmark parameters. The very strong negative correlation between the spreads and θ (implied by Figure 3); and the strong negative correlation between trade balance and θ (implied by Figure 2) indicate that the driving force of fluctuations in these variables is the assessment about the country θ. Spreads and output are also negatively correlated, but the correlation is weaker than the one between spreads and θ. The reason is that default risk is more sensitive to the assessment about the country (θ) than it is to its output. The countercyclicality of spreads comes mostly through the indirect effect that output and beliefs are strongly positively correlated at 0.74. Likewise output and current account is negatively correlated, but more weakly than the correlation between θ and current account.

Compared with the data, the model does a reasonable job with respect to these correlations. The negative correlation between spreads and output is very close to the data. The negative correlation between current account and output is weaker than in the data, but it is expected as this is an endowment economy. In the model, volatility of consumption is higher than volatility of output as in the data. In an endowment economy such as the one examined here, countercyclical trade balance leads to this result as the country borrows further to consume during high output shocks and pays back during low output shocks. The volatility of current account is higher than in the data.

		Y	R_S	C	TB	θ
Standard Deviation		4.54	4.81	5.37	1.94	-
Quarterly Autocorrelation		0.77	-	-	-	-
	Y	1	-0.47	0.94	-0.25	0.74
Correlation Matrix	R_S	-	1	-0.61	0.60	-0.69
	C	-	-	1	-0.57	0.77
	TB	-	-	-	1	-0.39
	θ	-	-	-	-	1
Average spread			5.15%			
Mean Asset Holding			-8.3%			

Table 3: Simulation Results for the Model

Notes: The simulation results reported are averages over 50 simulations each of length 500. Apart from deseasonalization, simulated data is treated in an identical manner to empirical data.

5.3.1 Spreads

The average spread was a target statistic. The volatility of spreads is fairly close to the data. The model also generates an average debt level of 8.3% which is between 6 % of Arellano (2006) and 9.7 % of Yue (2006).

A few words are in order to compare the exercise done in this paper with prior quantitative papers on sovereign debt with endogenous default. In the previous literature, it has been a challenge to generate the high spreads. Aguiar and Gopinath (2004) increases the default probability through IMF bail-outs; if the country defaults then financial intermediaries are partially compensated for their loss. With this mechanism, higher default rates do not result in an increase in spreads because of the implicit guarantee by the IMF. Yue (2006) generates a 3% default probability with a quarterly discount factor of 0.73 (yearly, 0.3) which seems too low even for an emerging market.

The default punishment assumed in these papers is a proportionate output loss in the case where the country defaults. The problem is that the loss needs to be high enough so that a reasonable debt level is sustainable; however with high losses, default occurs too rarely. In this paper, this is overcome because of the asymmetry in the default punishment. In low confidence periods, default is costless resulting in frequent defaults, while in high confidence periods default is very costly and the country is willing to pay back high levels of debt. The default punishment θ_P gives a degree of freedom to fit the average spread.

Arellano (2006) adds a similar asymmetry to the default punishment to generate high interest rates. In her model when the country defaults, there is no output loss if the output is below some threshold level, and if it is above this threshold there is a proportionate output loss. At low outputs, the country defaults and at high outputs, default is costly and a high debt level is sustainable. Spreads can be increased by changing this threshold level. The problem with this setting is that it leads to very volatile interest rates. The simulation results generates a 6.4% volatility in spreads for an average spread of 3.6%. This problem does not exist in the model of this paper because of the gradual nature of learning. The country unloads some of its debt before spreads increase by too much.

I further look at the ability of the model to replicate the bond spreads of Argentina for the period 1994:01-2001:04. For this purpose, I first obtain the series for θ using the output series for Argentina. The initial belief does not matter because I begin Bayesian updating at date 1983, while the spreads series used is after period 1992. Later, I feed the series for output and θ into the model to obtain the spread series for the relevant period. The initial asset level does not have a significant effect as the country hits its endogenous "borrowing limit" very soon because of the low patience level. The spreads from the data and the model are displayed in Figure 4. The model does a fairly good job to account for the spreads.

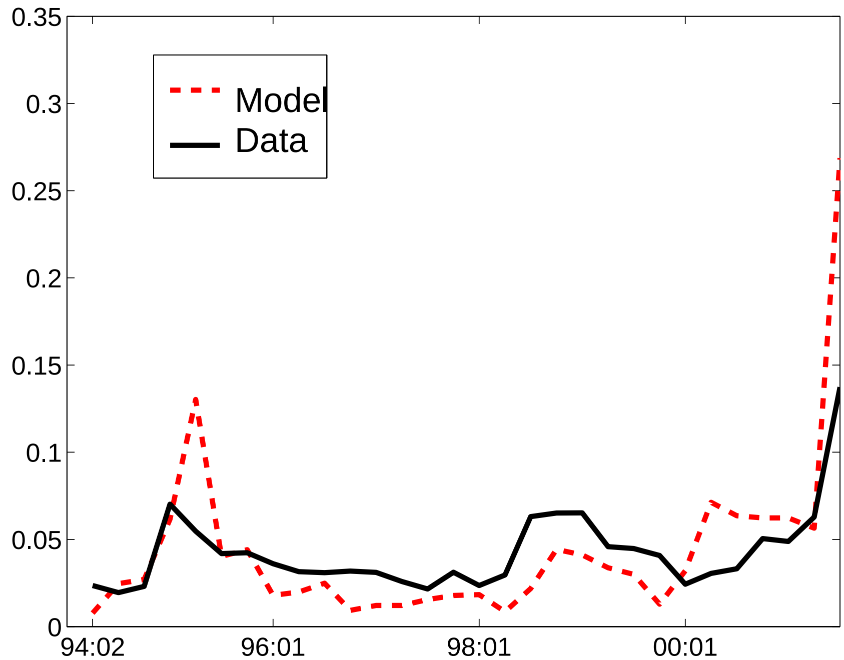

Figure 4: Bond Spreads in the Data and the Model (1994:02-2004:04)

6 Conclusion

In this paper, I enrich a standard defaultable debt model by adding uncertainty about the long run trend and show that it improves the results of the model in terms of its fit with emerging economy defaults, spreads and other business cycle dynamics. The country might be either in a high or low output path and the country and international investors lack information about which path the country is in. If the country believes itself to be in a low output path, then default is almost costless and it defaults. In addition the uncertainty increases the willingness of the country to pay high interest rates in the short run to distinguish which process it is in.

The model is applied to Argentina. The model can fit the high spreads observed in Argentina without resorting to unreasonable parameters and it can account for the cyclicality of spreads. In addition, the model is able to generate the countercyclical trade balance and more volatile consumption relative to output that is observed for Argentina and many other emerging countries.

7 Appendix

To solve the model numerically, I use the discrete state space method. I discretize the output shock y into 50 grids, the belief θ into 90 grids and the debt level B into 350 grids. I approximate the two continuous $AR(1)$ processes $G_i(y'|y)$, $i \in (H, L)$ with a discrete Markov chain. For that purpose, I integrate the normal distribution over each interval to compute the Markov transition matrix. The discrete Markov process for $F(y'|y, \theta)$ is constructed by putting weight θ on the Markov chain for G_H and weight $(1 - \theta)$ on the Markov chain for G_L. For the the transition schedule of beliefs $\theta'_D(y, y')$ and $\theta'_N(\theta, y, y')$, Bayesian updating is done for each possible state, and θ' is set at the grid that is nearest to the updated belief.

The rest of the solution follows as:

(i) Assume an initial pricing function $q^0(\theta, y, B')$. I start at risk-free prices $1/(1 + r)$.

(ii) Use $q^0(\theta, y, B')$ and an initial guess for $V^0(\theta, y, B)$ to iterate over the value function till it converges. The comparison of $V(\theta, y, B)$ and $V_D(y)$ gives the default function $\Delta(\theta, y, B)$.

(iii) Using the default function $\Delta(\theta, y, B)$, update $q^1(\theta, y, B')$ using equation (4). Using $q^1(\theta, y, B')$, repeat steps (i) and (ii) until $q(\theta, y, B')$ converges.

This procedure converges to a pricing function. One thing to be aware of is the possibility of multiple equilibria. One 'trivial' equilibria is where the bond price q is equal to zero in every state and the country defaults on its debt in every state, so there is no borrowing or lending in equilibrium. I initialize the price at risk-free rate $1/(1 + r)$, to get the equilibrium with highest prices and the least probability of default.

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