2010 Fourth International Conference on Network and System Security

Credit Spread Option Pricing by Dynamic Copulas

Ping Li
School of Economics and Management
Beihang University
Beijing 100083, China
Email: lipingbuaa@yahoo.cn

Guangdong Huang
School Information and Engineering
China Geosciences University (Beijing)
Beijing 100191, China
Email: gdhuang@cugb.edu.cn

***Abstract*—This paper extends the dynamic copula model for bivariate option pricing in Goorbergh et al (2004) to price credit spread options. We use GARCH-t model to describe the marginal distributions for corporate bonds and treasury, and combine them with dynamic Gaussian copula to obtain the joint distribution. As an application we use this model to price credit spread options written on American corporate bonds. Unlike other approaches for credit spread option pricing, this model is based on the two components of the spread rather than the spread itself, and the dependence structure is time-varying.**

***Index Terms*—Credit Spread Option; Dynamic Copula; GARCH-t Model**

I. INTRODUCTION

In recent years the market of credit derivatives grows rapidly. These instruments are broadly used not only for hedging purposes but also as a way to improving the return on capital.

Credit Spread Options (CSO) are options whose payment depend on a particular credit spread of a risky bond, which is the excess return offered by the risky bond beyond a risk-free bond, both having the same characteristics such as maturity and payment. The spread represents the premium which the market demands for holding the risky bonds. In this paper the risky bonds we will use are corporate bonds.

There are two main approaches to CSO pricing: structural model and reduced form model. In structural approach one assumes that the firm value follows a stochastic process and the default occurs when the firm value reaches a certain boundary, which is pre-specified. Das (1995) used the structural approach to price credit risk derivatives, Longstaff-Schwartz (1995) assumed the riskless interest rate as well as the spread follow correlated mean reversion diffusion processes and developed a valuation model to price credit spread derivatives; Tahani (2000) modeled the process of the logarithm of credit spread using the GARCH model and obtained closed-form pricing formulas for credit spread options; Chacko-Das (2002) derived pricing formulas for credit spread options based on the uncorrelated Cox-Ingeresoll-Ross processes for the spot rate and spot spread, and the parameters in the stochastic processes were assumed to be constant. Chu-Kwok(2003) extended the Longstaff-Schwartz model by assuming time-dependent drift functions in the mean reversion processes of the riskless interest rate and credit spread and obtain the pricing formulas for credit spread options.

Reduced form approach is based on the arbitrage-free evolution of spreads term structures, describing the default event separately from the firm value process. Jarrow-Lando-turnbull (1997) constructed a Markov chain model for valuation of risky debt and credit derivatives that incorporates the credit ratings of a firm as an indicator of the likelihood of default, Kijima-Komoribayashi (1998) proposed a technique of risk premium adjust-

978-0-7695-4159-4/10 $26.00 © 2010 IEEE
DOI 10.1109/NSS.2010.93

ment to overcome the problem in JLT model that bonds with high credit ratings may experience no default within the sample period, so the risk premium adjustments become ill-defined as the estimated default probabilities are zero; Das-Sundaram(2000) proposed a discrete-time Heath-Jarro-Merton model for valuation of credit derivatives.

A main feature of the methods mentioned above is that they model credit spread directly. This will lead to the loss of information implied in the dependence structure between the underlying assets, which is important in valuing multivariate options. Mougeot (2000) analyzed the importance of modeling the two underlying components of spread rather than spread itself in pricing credit spread option. Under the assumption that the underlying assets are independent, he compared the prices of CSO derived form different approaches and found the mispricing is significant. In his paper, the marginal distributions were characterized by Ornstein-Uhlenbeck processes, and the dependence parameter by linear correlation. In application he simply assumed that the two margins are independent.

Goorbergh (2004) used a dynamic copula model to price better-of-two-markets and worse-of-two-markets options. In this paper, we will use the similar dynamic copula model to price CSO. The marginal distributions are modeled by GARCH(1, 1) process, and the dependence structure between corporate bonds and treasury is modeled dynamically through a copula function. After comparing many types of GARCH, we choose a GARCH(1,1) model different from Goorbergh. Our model is then applied to price credit spread options on corporate bonds with Moody's rating.

The remainder of this paper is organized as follows. Section 2 describes the characteristics of CSO; Section 3 explains in detail the proposed dynamic CSO pricing scheme; Section 4 presents the empirical results, and conclusions are given in Section 5.

II. Credit Spread Options Specification

Credit Spread Options enable investors to separate credit risk from market risk. When the credit rating of a particular bond downgrades, the credit spread widens and the bond's price falls. An investor can protect against this risk by buying a credit spread call option written on this bond. On the contrary if the investor believes that the credit rating of a particular bond will upgrade, in which case the credit spread would decrease, a credit spread put option can capitalize on that development without actually buying the bond.

Theoretically credit spread is calculated as the spread between the yields on the reference risky bond and the LIBOR with the same maturity, whereas in US credit spread is the difference between the yields of a corporate bond and the treasury interest rate. The price of a CSO is calculated by multiplying an amount pre-specified. Here we assume this amount to be 1000, which is usually the principal of bonds.

Credit Spread Options can be written either on the spread level or on the price level of an underlying corporate bond. Here we use V_c to denote the value of a credit spread call option written on the *spread level*, while V_p denotes the value of a credit spread put option written on the *price level*. Then the payoff of a call option written on *credit spread* can be expressed as

$$\begin{aligned} & V_c(\tau, R_1, R_2, T, K_r) \\ = & \ 1000 \times \max((R_2 - R_1) - K_r, 0) \end{aligned}$$

where τ is the maturity of the option, T is the maturity of the underlying corporate bond, K_r is the strike spread pre-specified, R_1 and R_2 are the rates of return of treasury bond and corporate bond, respectively.

Similarly the payoff of a put option written on *price level* is

$$\begin{aligned} & V_p(\tau, P_1, P_2, T, K_p) \\ = & \ 1000 \times \max(K_p - (P_1 - P_2), 0) \end{aligned}$$

where P_1 , P_2 are the prices of the treasury bond and corporate bond, respectively, and K_p is the price spread pre-specified.

The two types of option mentioned above are identical in use, so it is possible to derive the price of a credit spread put/call option written on *price level* when we know the price of the credit spread call/put option written on the *spread level* with the same underlying, and vice versa. See Giacometti (2004) for details. We will concentrate on the pricing of CSO written on *spread level*.

III. Pricing Model for Credit Spread Options

In this section we present the dynamic copula model for CSO pricing. Contrary to earlier works, we model the returns of underlying assets rather than the spread itself, and use dynamic copulas to describe the dependence structure.

The proposed valuing scheme is structured as follows: firstly, each objective marginal distribution is modeled by GARCH (1, 1) process, and then we transform them to risk-neutral distributions; the dependence structure is modeled by dynamic copulas, with the assumption that the objective and the risk-neutral copulas are the same. Time variation in the copula is modeled by allowing Kendall's tau to evolve with time forced by the conditional volatilities of the underlying bonds. Model is implemented by Monte Carlo simulations. The fair value of the option is the expected payoff discounted at risk-free rate.

A. Marginal distribution

The purpose of GARCH (1, 1) specification is that while the return of financial assets is always leptokurtic and fat-tailed, GARCH (1, 1) can capture volatility clustering and allows for an easy change of measure. After comparing different types of GARCH model, we employ the following model for marginal distributions. For $i = 1, 2$,

$$\begin{cases} R_{i,t} = \mu_i + c_i R_{i,t-1} + \xi_{i,t} \\ \xi_{i,t} = \sqrt{h_{i,t}}\eta_{i,t}, \quad \eta_{i,t}|\psi_{t-1} \sim t_v \\ h_{i,t} = \omega_i + \alpha_i \xi_{i,t-1}^2 + \beta_i h_{i,t-1} \end{cases} \tag{1}$$

where $R_{i,t}$ denotes the return of corporate bond or treasury bond; μ_i is the mean of $R_{i,t}$; $\xi_{i,t}$ is the residual of $R_{i,t}$; $h_{i,t}$ is the innovation; t_v is a student distribution with degree of freedom v; ω_i, α_i, β_i and c_i are parameters to be estimated. The information set ψ_{t-1} includes all realized returns on both bonds. The marginal distributions are specified conditional on this common information set, so that copula theory can be used to construct a joint conditional distribution. See Patton (2003) for details.

Duan (1995) showed that, under certain conditions, the change of measure comes down to a change of drift. The law of the returns under a risk-neutral probability measure Q is given by:

$$\begin{cases} R_{i,t} = R_f - \frac{1}{2}h_{i,t-1} + c_i R_{i,t-1} + \xi_{i,t}^* \\ \xi_{i,t}^* = \sqrt{h_{i,t}}\eta_{i,t}, \quad \eta_{i,t}|\psi_{t-1} \sim t_v \\ h_{i,t} = \omega_i + \alpha_i (R_{i,t-1} - \mu_i)^2 + \beta_i h_{i,t-1} \end{cases} \tag{2}$$

where R_f is the constant risk-free rate. Under the risk-neutral measure, actualized prices are martingales.

It's important to notice that though marginal distributions are conditional on the common information set, each conditional margin is assumed to be only dependent on its own past, that is

$$\xi_{i,t-1}|\psi_{t-1} = \xi_{i,t-1}|\psi_{i,t-1}$$

This means that return spillovers or volatility spillovers from one bond to the other are excluded. This restriction is necessary for the application of Duan's change of measure.

B. Dynamic Copula

A copula is a function that joins a multivariate distribution function to its univariate marginal distributions. Readers interested in details can refer to Nelsen (1999) or Cherubini (2004). We only give some useful concepts relevant to our paper. We use C to denote a copula function and all of our investigations are based on, but not limited in bivariate cases.

Theorem III.1: (Sklar, 1959) Let $H(x, y)$ be a joint distribution function with continuous margins $F(x)$ and $G(y)$, then there exists a unique copula function C such that for all x, y in $\bar{R}$ ($\bar{R}$ denotes the extended real line $[-\infty, +\infty]$)

$$H(x, y) = C(F(x), G(y)). \tag{3}$$

If $F(x)$ and $G(y)$ are continuous, then C is unique; otherwise, C is uniquely determined on $RanF \times RanG$. Conversely, if C is a copula function, and $F(x)$, $G(y)$ are distribution functions, then the function defined by equation (3) is a joint distribution function with margins $F(x)$ and $G(y)$.

Sklar's theorem is very important since it provides a direct way to link the marginal and joint distribution by a copula function.

Definition III.1: (Kruskal,1958) The sample version of the measure of association known as Kendall's tau is defined in terms of concordance as follows: Let $\{(x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n)\}$ be the random sample of n observations from a vector (X, Y) of continuous random variables. There are C_n^2 distinct pairs (x_i, y_i) and (x_j, y_j) of observations in the sample. They are said to be concordant if $(x_i - x_j)(y_i - y_j) > 0$, and discordant if $(x_i - x_j)(y_i - y_j) < 0$. Let c denote the number of concordant pairs, and d the number of discordant pairs. Then Kendall's tau for the sample is defined as

$$\tau = \frac{c - d}{c + d} \tag{4}$$

Kendalll's tau is one of the most widely used scale-invariant measures of association. We will use it to capture the time-varying property of dynamic copula.

In this paper we assume the objective and risk-neutral copulas are identical, so that the risk-neutral joint returns can be obtained by combining the risk-neutral marginal distribution with objective copula. Rosenberg (2003) makes this assumption as well.

The dynamic copula considered here is bivariate Gaussian copula, since it is widely used in dynamic copula study and easy to manipulate. Alternative copula families such as t-copula, Archimedean copula or Joe-Clayton copula can also be considered. The bivariate Gaussian copula function is

$$\begin{aligned} C_G(\mu, \nu|\rho) &= \int_{-\infty}^{\Phi^{-1}(\mu)} \int_{-\infty}^{\Phi^{-1}(\nu)} \frac{1}{2\pi\sqrt{1-\rho^2}} \\ &\times \exp\{\frac{-(s^2 - 2\rho st + t^2)}{2(1-\rho^2)}\} ds dt \end{aligned}$$

where $\Phi^{-1}(\dot{})$ is the inverse of univariate standard normal distribution function, and ρ is the correlation coefficient, $-1 < \rho < 1$. For Gaussian copula, the following relationship holds

$$\tau(\rho) = \frac{2}{\pi} \arcsin \rho \tag{5}$$

Dynamic property of the copula is reflected in the evolving of the dependence parameter forced by the conditional volatilities of the underlying return. The time variation in the copula is governed by

$$\tau_t = \gamma_0 + \gamma_1 \log \max(h_{1,t}, h_{2,t}) \tag{6}$$

where τ_t is Kendall's tau at time t, $h_{1,t}$ and $h_{2,t}$ are variances in GARCH processes. Patton (2003) proposes an ARMA-type process linking the dependence parameter to absolute differences in return innovations, which is another way to capture the time-varying property.

The time-varying Kendall's tau τ_t for the standardized return innovations is rolling-window estimated with window sizes of two month, that is, Kendall's tau at time t is computed using the 20 trading days prior to day t, day t itself, and 20 trading days after day t. The parameters γ_0 and γ_1 is estimated by regressing the Kendall's tau on the log maximum conditional volatility.

IV. Empirical Results

In this section we apply our model to price two credit spread options written on American Aaa and Baa corporate bonds. The data comes from yahoo finance and the set includes daily series of interest rates for one-year treasury bonds and one-year Aaa and Baa corporate bonds. The interest rates run from January 2, 1998 to June 30, 2006. Both of the underlying bonds mature in one year and the options mature in one month.

We use maximum likelihood method to estimate the GARCH(1,1) parameters for the marginal bond returns. The results are shown in Table 1.

TABLE I
ESTIMATION OF THE GARCH PARAMETERS

Parameters	Aaa	Baa	1 year Treasury
μ	0.0087 (0.0069)	0.014 (0.008)	0.0016 (0.0018)
c	0.9984 (0.0011)	0.9979 (0.0011)	1.0 (0.0004)
ω	8.58E-05 (3.69E-05)	5.76E-05 (2.48E-05)	4.33E-05 (1.25E-05)
α	0.0406 (0.0109)	0.038046 (0.0094)	0.0587 (0.0102)
β	0.9202 (0.0247)	0.9344 (0.0185)	0.9192 (0.0135)
ν	10.388 (1.9134)	10.203 (1.8761)	5.3418 (0.5532)

*Figures in brackets are robust quasi-maximum likelihood standard errors.

The sample version of Kendall's tau for the standardized return innovations is rolling-window estimated. The parameters γ_0 and γ_1 are estimated by regression of Kendall's tau on the logarithm of the maximum return volatility. Table 2 shows the results. The slope coefficients γ_1 for two bonds are positive, which means that dependence increases when the markets fluctuate acutely.

TABLE II
ESTIMATION OF PARAMETERS γ_0 AND γ_1 OF KENDALL'S TAU

Parameters	Aaa	Baa
γ_0	0.3981 (0.0097*)	0.3904 (0.0099)
γ_1	0.0607 (0.0078)	0.0423 (0.00851)

*Figures in brackets are robust quasi-maximum likelihood standard errors.

Some parameters in the pricing model for credit spread options need to be pre-specified. The initial levels of volatility of the underlying bonds are defined as the estimated unconditional variance $\omega/(1-\alpha-\beta)$. The risk-free rate is assumed to be 5.2% annually, which is the mean in recent half year. The strike spreads of the options are taken as the average spread in June 2006, that is, 110% and 90%, respectively. The prices of options are obtained by 10,000 times of Monte Carlo simulation. Table 3 shows the prices derived from our model.

TABLE III
CSO PRICES DERIVED FROM THE MODEL

	Aaa		Baa	
Strike (%)	K=0.73	K=0.82	K=1.52	K=1.69
Put ($)	0.19	1.93	0.37	2.34
Call ($)	29.64	1.72	24.04	2.56

V. CONCLUSIONS

This paper studies the pricing of credit spread options by a dynamic copula method. By investigating the return series of the underlying bonds as well as the relationship between the price of credit spread option and the dependence structure of the underlying bonds, we use GARCH-t model to describe the marginal distributions, and apply time-varying bivariate Gaussian copula to capture the dependence structure between two underlying bonds. A remarkable feature of the proposed model is that, contrary to earlier works on credit spread option pricing, we model the two components of credit spread instead of credit spread itself. As an application, we use this model to price two types of credit spread options written on corporate bonds. Further work is still needed, such as taking into account other models to describe margins and other types of copula functions. Comparisons of this model with earlier pricing models are still under consideration.

ACKNOWLEDGMENT

This work is supported by the National Natural Science Foundation of China (No.

70971006, 70831001), and the National Basic Research Program of China (973 Program, Grant No. 2007CB814906).

REFERENCES

[1] IEEEhowto: Das Chacko, G. and Das, S., Pricing Interest Rate Derivatives: A General Approach, Review of Financial Studies, 2002, 15:195-241.
[2] IEEEhowto: CLV Cherubini, U. Luciano,E. and Vecchiato, W., Copula Methods in Finance, John Wiley & Sons, 2004.
[3] IEEEhowto: Chu Chu, C. C. and Kowk, Y. K., No-Arbitrage Approach to Pricing Credit Spread Derivatives, The Journal of Derivatives, Spring, 2003, 10: 51-64.
[4] IEEEhowto: DAS1 Das, S.R., Credit Risk Derivatives, The Journal of Derivatives, Spring 1995, 2:385-407.
[5] IEEEhowto: DAS2 Das, S.R. and Sundaram, R.K., A Discrete-Time Approach to Arbitrage-Free Pricing of Credit Derivatives, Management Science, 2000, 46: 46-62.
[6] IEEEhowto: Duan Duan, J.C., The GARCH Pricing Model, Mathematical Finance, 1995, 5:13-32.
[7] IEEEhowto: GT Giacometti, R. and Teocchi, M., On Pricing of Credit Spread Options, European Journal of Operational Reserch, 163 (2005):52-64.
[8] IEEEhowto: GGW Goorbergh R., Genest C. and Werker B., Bivariate option pricing using dynamic copula models, Insurance Math. Economic, 37 (2005): 101-114.
[9] IEEEhowto: Jarrow Jarrow, R., Lando, D., and Turnbull, S. M., A Markov Model for the Term Structure of Credit Risk Spreads, Review of Financial Studies, 1997, 10:481-523.
[10] IEEEhowto: KK Kijima, M. and Komoribayashi, K., A Markov Chain Model for Valuing Credit Risk Derivatives, The Journal of Derivatives, 1998, 5:97-108.
[11] IEEEhowto: Mou Mougeot, N., Credit Spread Specification and the Pricing of Spread Options, Working paper, Ecole des HEC, 2000.
[12] IEEEhowto: Nelson Nelsen, R.B., An Introduction to Copulas, New York: Springer-Verlag, 1999.
[13] IEEEhowto: Patton Patton, A.J., Modeling Asymmetric Exchange Rate Dependence, International Economic Review, Volume 47, Number 2, May 2006, pp. 527-556.
[14] IEEEhowto: Rosen Rosenberg, J.V., Nonparametric Pricing of Multivariate Contingent Claims, Federal Reserve Bank of New York. 2003, 162(3).