Radiation balance of the Earth

Modelling of Physical Systems

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1 Mean Earth temperature assuming no atmosphere

The average temperature of the Earth was calculated assuming no atmosphere. The calculation was done using equation number 4, which was obtained by transforming equations number 1, 2, 3.

$$P_S = S \cdot \frac{Pow_Z}{4} \cdot (1 - A) \tag{1}$$

$$P_z = \sigma \cdot T^4 \cdot Pow_z \tag{2}$$

$$P_Z = P_S \tag{3}$$

$$T = \sqrt[4]{\frac{S \cdot (1 - A)}{4 \cdot \sigma}} \tag{4}$$

where:

P_S - Power of solar radiation arriving to the Earth (short wave radiation),
P_z - Power of radiation emitted from Earth (long wave radiation),
A - mean albedo of the Earth surface (0.3),
S - solar constant ($1366W/m^2$),
Pow_z - area of the Earth,
σ - Stefan-Boltzmann constant ($5.67 \cdot 10^{-8}W/m^2K^4$),
T_m - mean Earth temperature.

To compute given equation and mean Earth temperature the following code was used:

```
A = 0.3;
S = 1366;
sigma = 5.67*(10^(-8));

Tm = nthroot(S*(1-A)/(sigma*4), 4);
disp(Tm);
Tm_celc = Tm - 273.15;
disp(Tm_celc);
```

Mean Earth temperature equaled 254.82 K, which is −18.33 °C. It's definitely too low value, because of not including atmosphere in calculation. The atmosphere is an important factor affecting global heat transfer and the mean temperature of the Earth.

2 Relationship between mean temperature and solar constant

The next step was to add the atmosphere factor to the calculations. The equations became more complicated and had forms like equation number 5 and number 6.

$$(-t_a)(1-a_s)\frac{S}{4} + c(T_s - T_a) + \sigma T_s^4 (1 - a_a') - \sigma T_a^4 = 0 \tag{5}$$

$$-(1 - a_a - t_a + a_s t_a)\frac{S}{4} - c(T_s - T_a) - \sigma T_s^4 (1 - t_a' - a_a') + 2\sigma T_a^4 = 0 \tag{6}$$

where:

t_a - transmission of the atmosphere for short wave radiation (0.53),
a_a - albedo of the atmosphere for short wave radiation (0.30),
a_s - surface albedo for short wave radiation (0.19),
$t_a{}'$ - transmission of the atmosphere for long wave radiation (0.06),
$a_a{}'$ - albedo of the atmosphere for long wave radiation (0.31),
c - heat transfer coefficient ($2.7\ Wm^{-2}K^{-1}$),
T_a - mean temperature of the atmosphere,
T_s - mean surface temperature.

To compute given equations and mean Earth temperature with atmosphere factor the following code was used:

```
A = 0.3;
S = 1366;
sigma = 5.67*(10^(-8));
a_s = 0.19;
t_a = 0.53;
a_a = 0.30;
t_ap = 0.06;
a_ap = 0.31;
c = 2.7;

fun = @f;
x0 = [300, 300];
Tm2 = fsolve(@(T)fun(T, a_s, t_a, a_a, S, sigma, c, a_ap, t_ap), x0);
disp(Tm2);
Tm2_celc = Tm2 - [273.15, 273.15];
disp(Tm2_celc);

function result = f(T, a_s, t_a, a_a, S, sigma, c, a_ap, t_ap)
    T_a = T(1);
    T_s = T(2);

    result = (((-t_a) * (1-a_s) * S/4 + c*(T_s-T_a) + (sigma*(T_s^(4))*(1-a_ap)) ...
        - sigma*T_a^4) - (1-a_a - t_a + a_s*t_a)*S/4 - c*(T_s-T_a) - ...
        (sigma*(T_s^(4))*(1-t_ap-a_ap)) + 2*sigma*T_a^4);
end
```

Mean Earth's atmosphere temperature equaled 248.52 K, which is −24.63 °C, and mean Earth's surface temperature equaled 285.99 K, which is 12.84 °C. As expected, adding the atmosphere factor significantly improved the results and brought them closer to the actual, real values.

Then, average temperatures were then calculated for S ranging from 80% to 120% of the value using the following code:

```
Ts = [];
Ta = [];
S_ar = [];

for S1 = 0.8*S:0.01*S:1.2*S
    S_ar = [S_ar S1];
    S = S1;
    T = fsolve(@(T)fun(T, a_s, t_a, a_a, S, sigma, c, a_ap, t_ap), x0);
    Ts = [Ts T(2) -273.15];
    Ta = [Ta T(1) -273.15];
end

plot(S_ar, [Ts', Ta'], '.', 'MarkerSize', 20);
legend('atmospere', 'surface');
xlabel('Solar constant [W/m^2]');
ylabel('Temperature [C]');
```

It generated figure number 1.

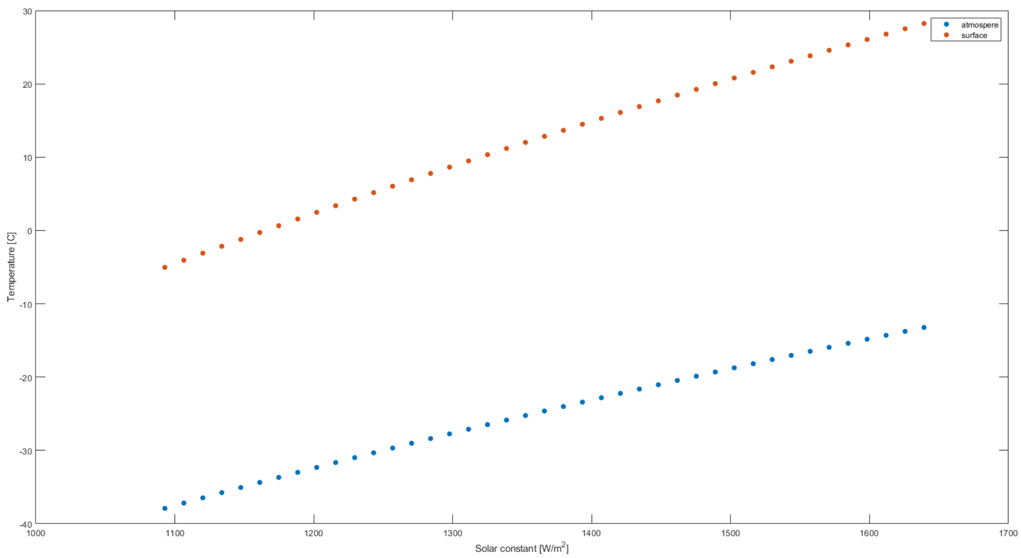

Figure 1: Relationship between mean temperature and solar constant

It shows that when the solar constant grows, both surface and atmosphere temperatures are rising, though surface temperature is mounting faster.

3 Glaciation mechanism

Next factor introduced to the model was glaciations mechanism. It assumed changing albedo as a function of ground temperature. When the temperature drops below -10 degrees celcius, the Earth's surface begins to be covered by ice (simulating ice age), which increases the Earth's albedo.

```
T2s = [];
T2s2 = [];
Ts = [];
Ta = [];
S_ar = [];
a_s = 0.11;
S = 1366;

for S1 = 1.4*S:(-0.01*S):0.6*S
    S = S1;
    temp = fsolve(@(T)fun(T, a_s, t_a, a_a, S, sigma, c, a_ap, t_ap), x0);
    T2s = [T2s temp(2)-273.15];

    if(temp(2) - 273.15 < -10 )
        a_s = 0.65;
```

```
    else
        a_s = 0.11;
    end
end

S = 1366;

for S1 = 0.6*S:0.01*S:1.4*S
    S_ar = [S_ar S1];
    S = S1;
    temp = fsolve(@(T)fun(T, a_s, t_a, a_a, S, sigma, c, a_ap, t_ap), x0);
    T2s2 = [T2s2 temp(2)-273.15];

    if(temp(2) - 273.15 < -10)
        a_s = 0.65;
    else
        a_s = 0.11;
    end
end

plot(S_ar, [flip(T2s'), T2s2'], 'LineWidth', 3);
legend('descrease', 'increase');
xlabel('Solar constant [W/m^2]');
ylabel('Temperature [C]');
```

It generated figure number 2.

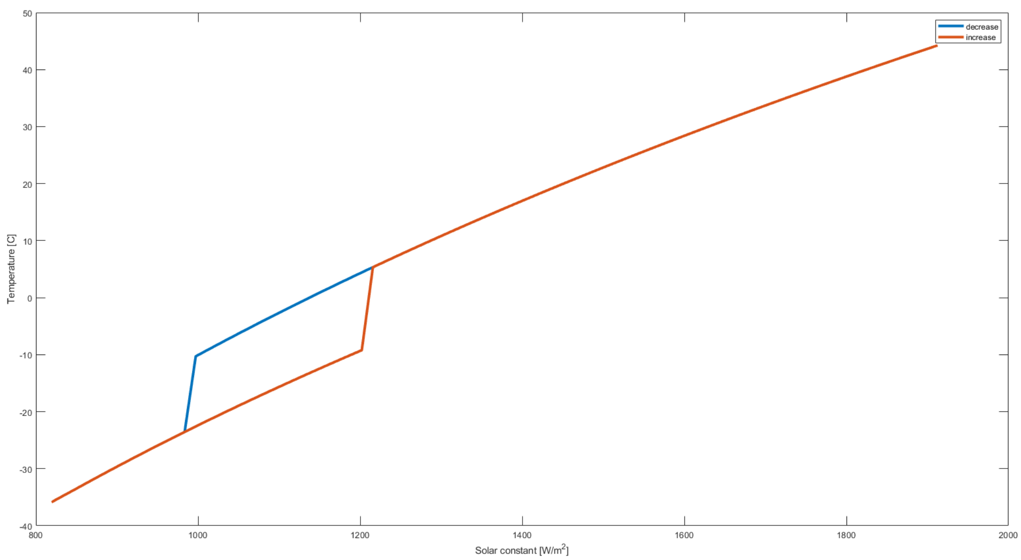

Figure 2: Relationship between mean temperature and solar constant with changing albedo

As figure number 2 shows, when solar constant is increasing and decreasing with added mechanism of changing Earth's albedo, it gives different mean temperature values. Albedo changes at -10 °C, which is simulating appearance of ice and snow on the Earth's surface. It causes different solar constant value for which mean temperature changes rapidly (when ice and snow is melting), thus resulting in a different appearance of two curves.

4 Results from provided 1D model

In last step code provided by the tutor was used to simulate 1D model of Earth's energy balance, the results of which are shown in figure number 3.

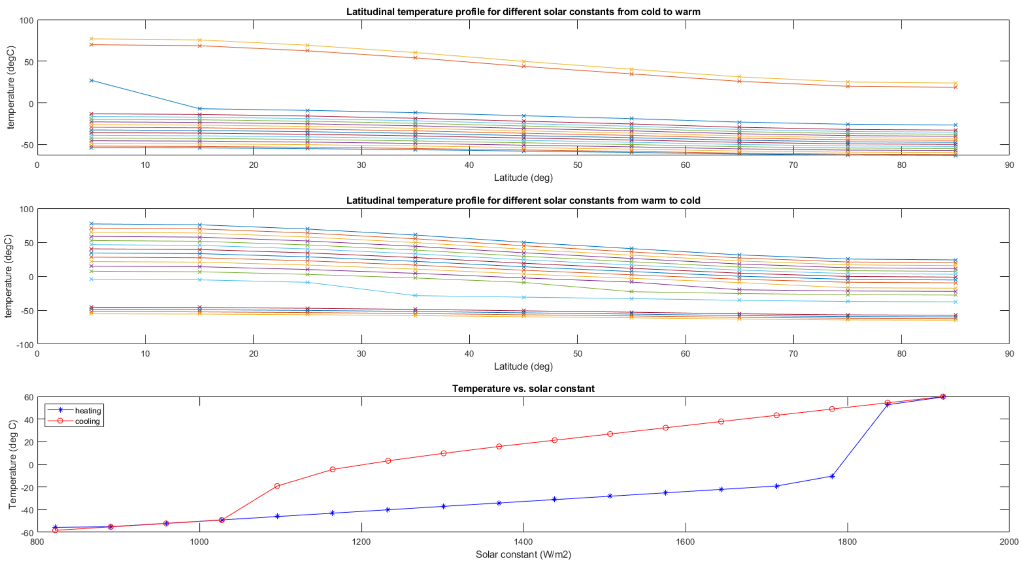

Figure 3: Relationship between mean temperature and solar constant with changing albedo

When comparing results from computing set of energy balance equations with atmosphere mechanism (figure 2) with results from 1D model (figure 3), one can clearly see that in the first the temperature change is more rapid then in the second. Additionaly the hysteresis is wider in the second model and whole temperature change is more non linear.

5 Conclusions

The first and most important conclusion is that the atmosphere has a significant and crucial influence on the Earth's temperature. Without it, life on earth would not be possible due to excessively cold temperatures.

Secondly, as shown in the examples, the solar constant and the average temperature of the Earth are directly correlated and proportional - the greater the solar constant the greater the average temperature, and vice versa.

Thirdly, albedo also significantly affects the average temperature of the Earth, independent of solar constant. The greater the albedo, the lower the temperature, due to the greater reflection of sunlight by the Earth's surface.