

# Heat transfer simulation

## Modelling of Physical Systems

Piotr Cenda  
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## 1 Physical object

To simulate heat transfer we used physical object model presented on Fig. 1. The red area represents the heater, the blue edge represents the border, which is thermally isolated from the environment, and the white space between is metal object, which transferred heat.

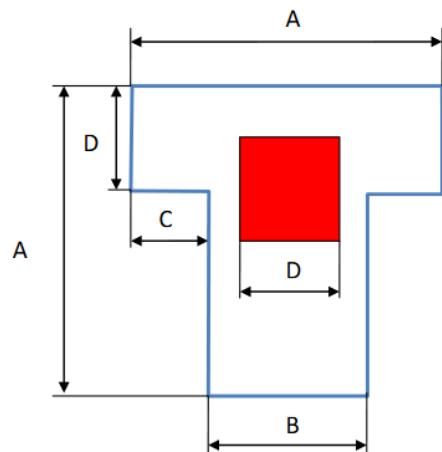


Figure 1: Physical object dimensions

where:

A = 200mm,  
B = 100mm,  
C = 50mm,  
D = 50mm.

## 2 Heat transfer programme

To simulate heat transfer the following equation was used:

$$\frac{T_{i,j}^{n+1} - T_{i,j}^n}{\Delta t} = \frac{K}{c_w \rho} \left( \frac{T_{i+1,j}^n - 2T_{i,j}^n + T_{i-1,j}^n}{(\Delta x)^2} + \frac{T_{i,j+1}^n - 2T_{i,j}^n + T_{i,j-1}^n}{(\Delta y)^2} \right) \quad (1)$$

which was, in order to begin numerical computation, discretized to:

$$T_{i,j}^{n+1} = T_{i,j}^n + \frac{K \Delta t}{c_w \rho (\Delta x)^2} [T_{i+1,j}^n - 2T_{i,j}^n + T_{i-1,j}^n] + \frac{K \Delta t}{c_w \rho (\Delta y)^2} [T_{i,j+1}^n - 2T_{i,j}^n + T_{i,j-1}^n] \quad (2)$$

where:

$dx$  - distance between computational nodes in x direction,

$dy$  - distance between computational nodes in y direction,

$dt$  - time step,

$K$  - thermal conductivity coefficient,

$Nt$  - number of time steps,

$NX$  - number of computational nodes in  $x$  direction,

$NY$  - number of computational nodes in  $y$  direction.

That equation, model of physical object and both boundary conditions were implemented as in the following code:

```

1 clear ;
2 clc ;
3
4 % dimensions of the model
5 A = 0.2; %m
6 B = 0.1; %m
7 C = 0.05; %m
8 D = 0.05; %m
9 heater_thickness = 0.002; %m
10
11 % initialization temperatures
12 init_temp_heater = 80; %celcius
13 init_temp_obj = 20; %celcius
14 init_temp_border = 10; %celcius
15
16 material = 1;
17 % 1 - alumina , 2 - cooper , 3 - stainless steel
18 density = [2700, 8920, 7860]; %kg/m^3
19 sp_heat = [900, 380, 450]; %J/kgK
20 conduct = [237, 401, 58]; %W/mK
21
```

```

22 % numerical parameters
23 dt = 0.001; %s
24 dx = 0.001; %m
25 dy = 0.001; %m
26 nax = round(A/dx);
27 nay = round(A/dy);
28 nbx = round(B/dx);
29 nby = round(B/dy);
30 ncx = round(C/dx);
31 ncy = round(C/dy);
32 ndx = round(D/dx);
33 ndy = round(D/dy);

34
35 % thermal equation parameters for specified material
36 efx = (conduct(material)*dt)/(density(material)*sp_heat(material)
   *(dx^2));
37 efy = (conduct(material)*dt)/(density(material)*sp_heat(material)
   *(dy^2));

38
39 % boundary condition
40 bc = 2;

41
42 % heater heat transfer in boundary condition 2
43 P = 100; %W - power of heater
44 dT = (P*dt)/(sp_heat(material)*(D^2)*heater_thickness*density(
   material));

45
46 % plane of model declaration
47 % 0 - nothing , 1 - border , 2 - object , 3 - heater
48 plane = zeros(nay+2, nax+2);
49 plane(2:(ndy-1), 2:nax) = 2;
50 plane(2:(end-1), ncx:(end-ncx-1)) = 2;
51 plane(2, 2:(end-1)) = 1;
52 plane(2:ndy, 2) = 1;
53 plane(2:ndy, (end-1)) = 1;
54 plane(ndy, 2:ncx) = 1;
55 plane(ndy, (end-ncx-1):(end-1)) = 1;
56 plane(ndy:(end-1), ncx) = 1;
57 plane(ndy:(end-1), (end-ncx-1)) = 1;
58 plane((end-1), ncx:(end-ncx-1)) = 1;
59 plane((ndy/2):(ndy*3/2), (ncx+(nbx-ndx)/2):(ncx+(nbx+ndx)/2)) = 3;

60
61 % plane of temperature declaration
62 plane_temp = zeros(nax+2, nay+2);
63 plane_temp(plane == 1) = init_temp_border;

```

```

64 plane_temp(plane == 2) = init_temp_obj;
65
66 if bc == 2
67     plane_temp(plane == 3) = init_temp_obj;
68 else
69     plane_temp(plane == 3) = init_temp_heater;
70 end
71
72 n_steps = 200000;
73
74 f1 = figure('units', 'normalized', 'outerposition', [0 0 1 1]);
75
76 for n=1:n_steps
77     surf(plane_temp, 'EdgeColor', '#8e8e8e', 'FaceColor', 'interp',
78          , 'LineWidth', 0.1);
79     view(-135, 30);
80     xlim([0 nax+2]);
81     ylim([0 nay+2]);
82     colorbar;
83
84     if (n == 10000 && bc == 2)
85         plane(plane==3) = 2;
86     end
87
88     new_plane = plane_temp;
89
90     for i=1:nax
91         for j=1:nay
92             % if it is the object near heater
93             if (plane(i, j) == 3 && bc == 2)
94                 new_plane(i, j) = plane_temp(i, j) + dT;
95             % if it is the object
96             elseif plane(i, j) == 2
97                 new_plane(i, j) = temp_next(i, j, plane_temp, efx,
98                                              efy);
99             end
100        end
101    end
102
103    max_dt = max(abs(new_plane-plane_temp), [], 'all');
104    disp(max_dt);
105
106    % if max temperature is close to zero, then equilibrium is
107    % established
108    % and simulation can be stopped

```

```

106 if max_dt < 0.001
107     break
108 end
109
110 plane_temp = new_plane;
111 pause(0.001);
112 end
113
114 function temp_new = temp_next(i, j, temp, efx, efy)
115     temp_l = temp(i, j-1);
116     temp_r = temp(i, j+1);
117     temp_down = temp(i+1, j);
118     temp_up = temp(i-1, j);
119
120     temp_new = temp(i, j) + efx*((temp_down+273.15) - 2*(temp(i, j)
121         +273.15) + (temp_up+273.15)) ...
122         + efy*((temp_r+273.15) - 2*(temp(i, j)+273.15) + (temp_l
123             +273.15));
124 end

```

It computed for chosen material, initial temperatures and boundary conditions gradient of temperature for each point on object. For boundary condition nr 2 it also computed additional heat added to heater through power of heating itself.

### 3 Initial conditions

Initial conditions for simulations of first boundary condition were: for heater was 80°C, for isolated border 10°C and for rest of the object it was 20°C (Fig. 2 and (Fig. 2)).

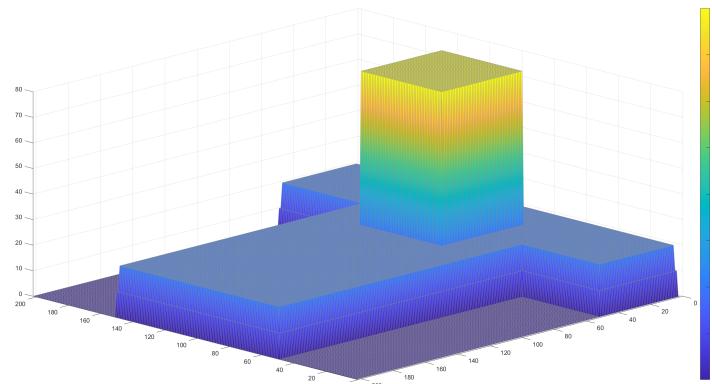


Figure 2: Initial conditions for first boundary condition

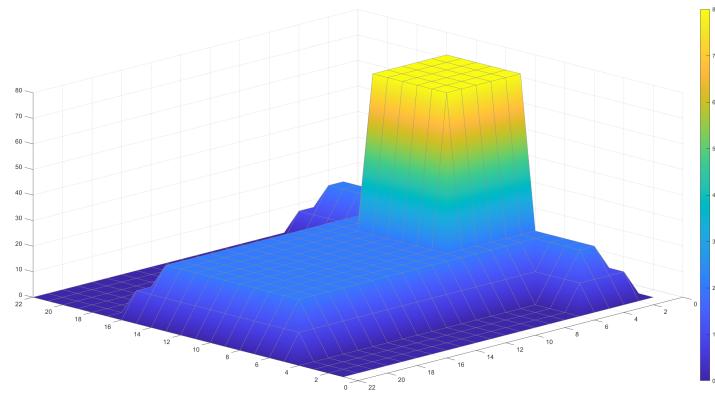


Figure 3: Initial conditions for first boundary condition with smaller dx and dy

Initial conditions for simulations of second boundary condition were: for isolated border  $10^{\circ}\text{C}$  and for rest of the object it was  $20^{\circ}\text{C}$  (Fig. 4).

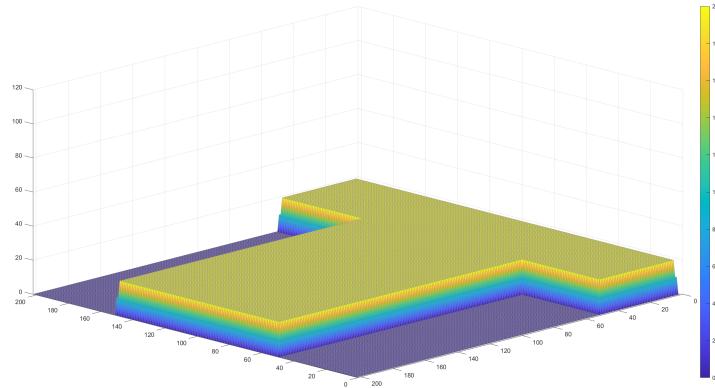


Figure 4: Initial conditions for second boundary condition

## 4 Boundary condition nr 1 for three materials

Using shown code, visualisations were made for three different materials such as alumina, copper and stainless steel. They differed in three physical properties: density, specific heat and thermal conductivity (Fig. 1).

	Alumina	Copper	Stainless steel
Density ( $kg/m^3$ )	2700	8920	7860
Specific heat ( $J/kgK$ )	900	380	450
Thermal conductivity ( $W/mK$ )	237	401	58

Table 1: Properties of used three different materials

For each material, using its unique properties, simulation with boundary condition nr 1 was made. Initial temperature for heater was 80°C, for isolated border 10°C and for rest of the object it was 20°C. Each simulation ended when equilibrium was reached. It was determined that the simulation reached equilibrium when the largest temperature change for a single point between steps was less than 0.001.

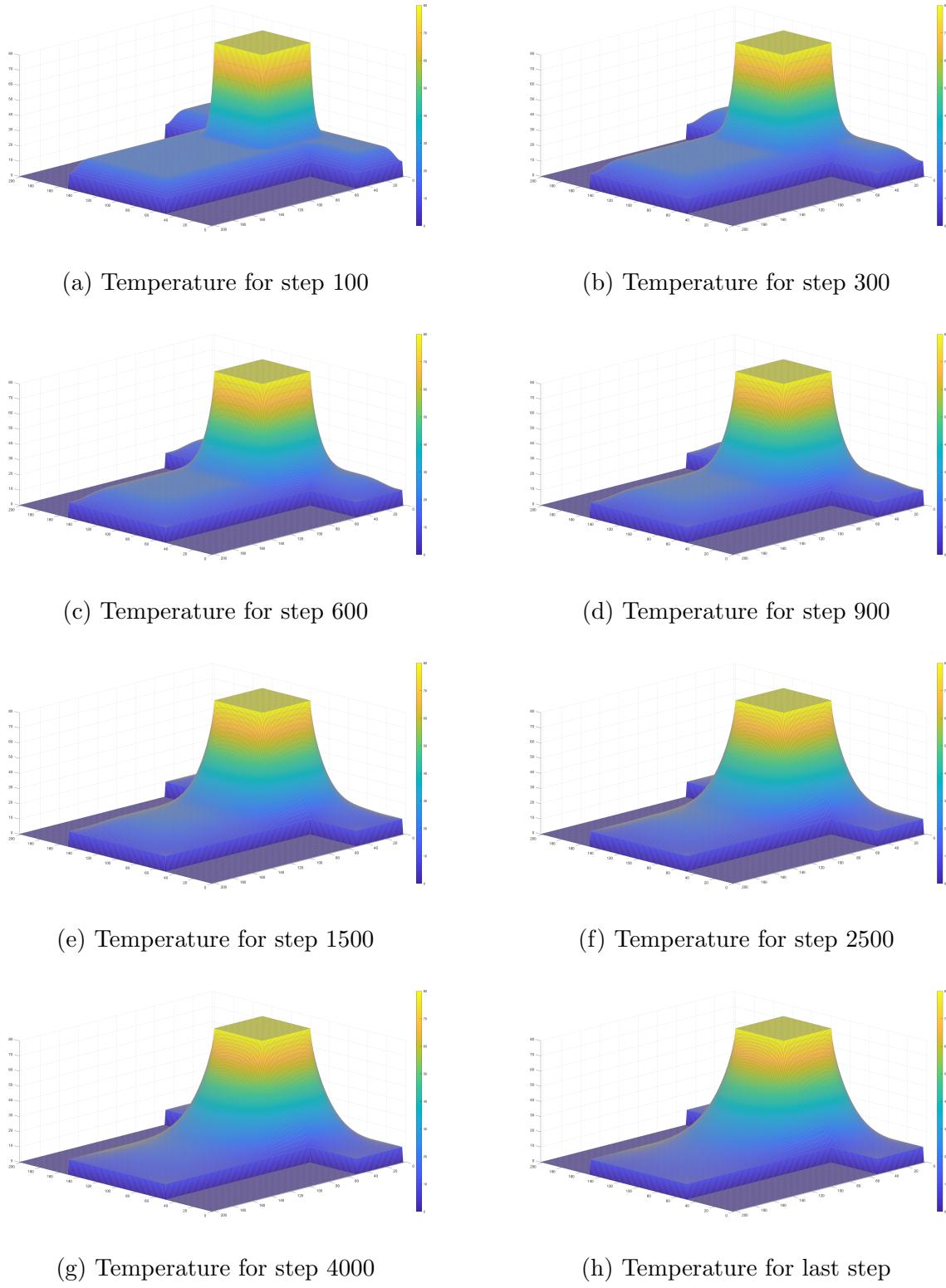


Figure 5: Boundary condition nr 1 simulation for alumina

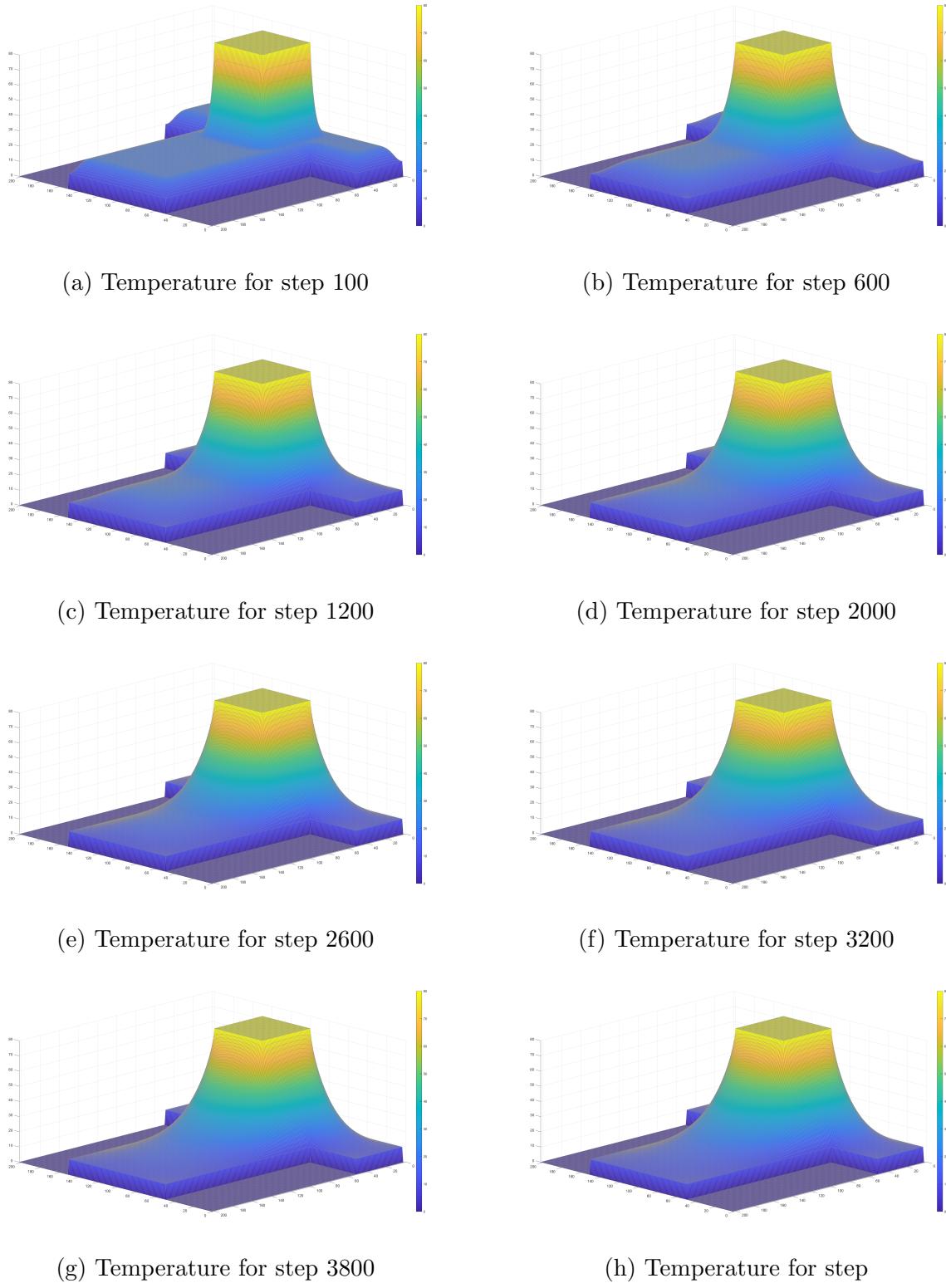
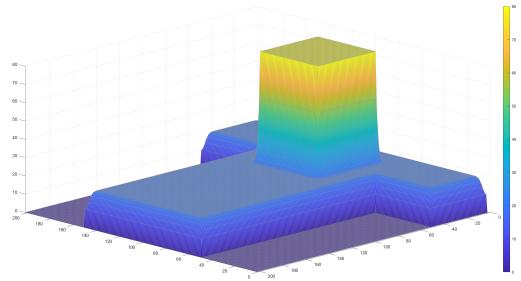
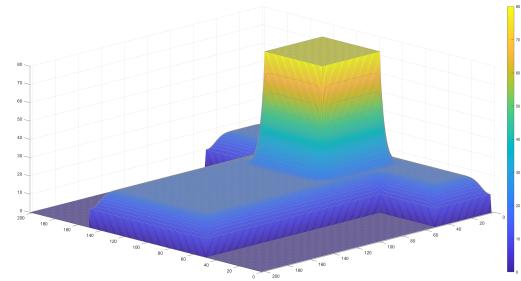


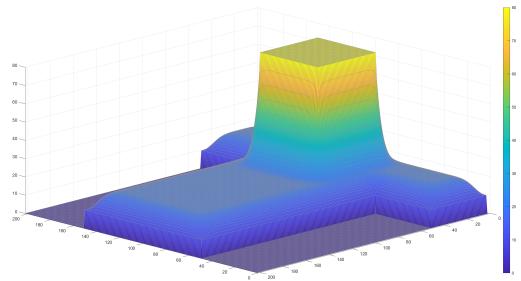
Figure 6: Boundary condition nr 1 simulation for copper



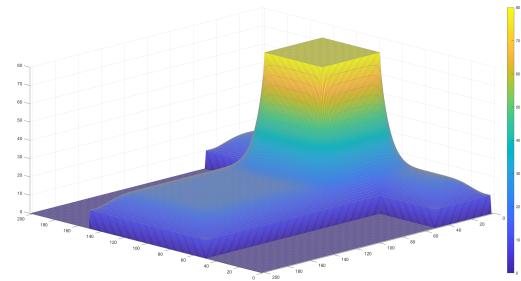
(a) Temperature for step 100



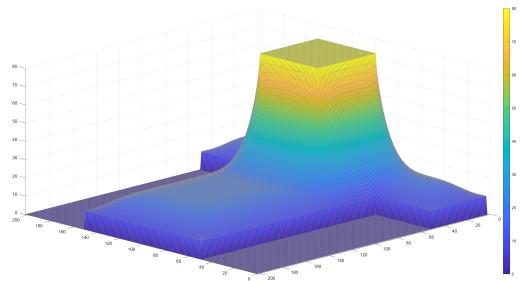
(b) Temperature for step 600



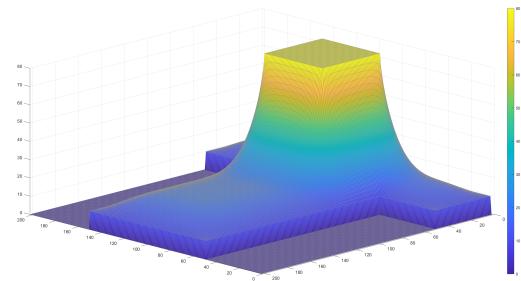
(c) Temperature for step 1000



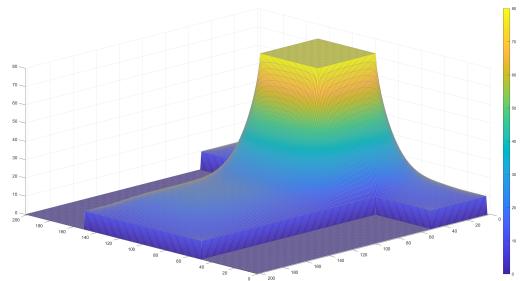
(d) Temperature for step 2500



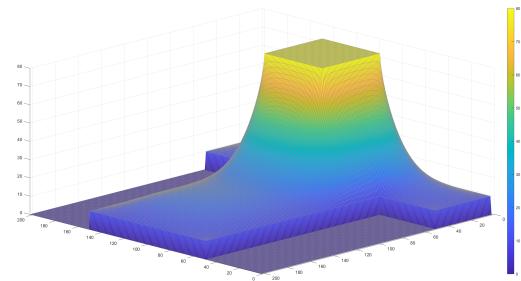
(e) Temperature for step 4500



(f) Temperature for step 7000



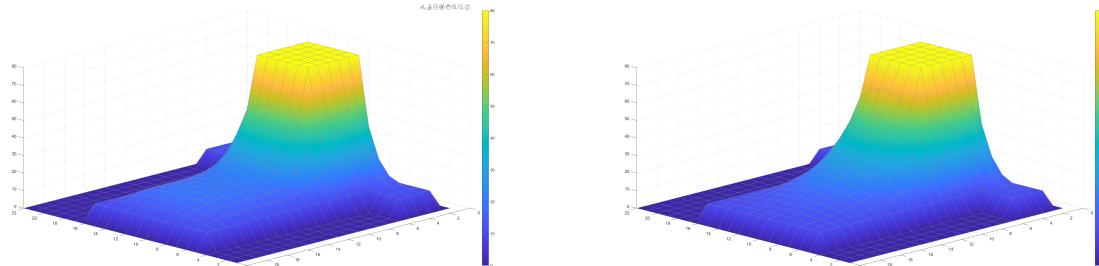
(g) Temperature for step 9000



(h) Temperature for last step

Figure 7: Boundary condition nr 1 simulation for steel

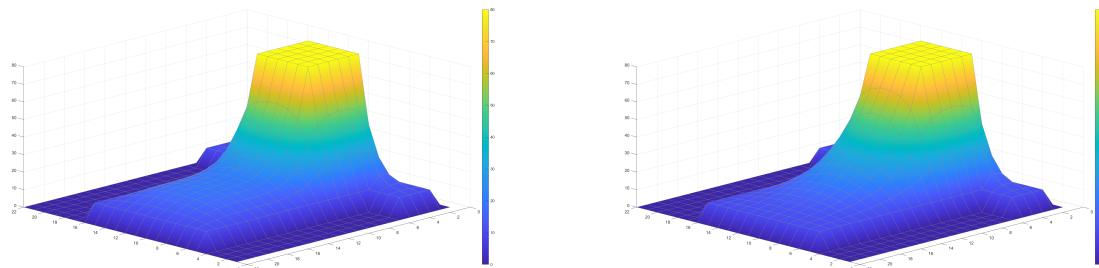
Then the same simulation for each material was made again with smaller dx and dy, which were equal to 0.01.



(a) Temperature for step 200

(b) Temperature for last step

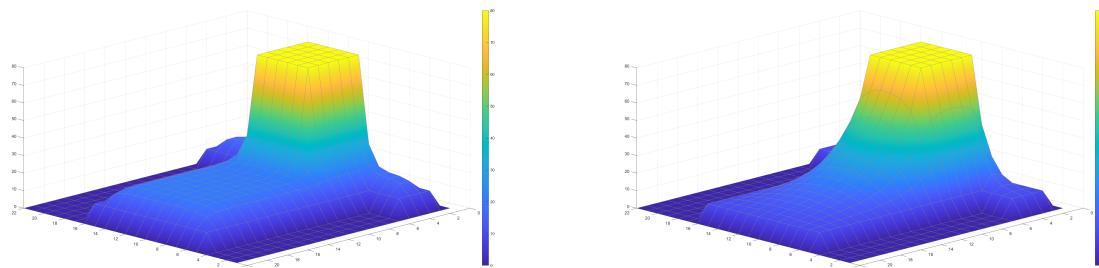
Figure 8: Boundary condition nr 1 simulation for alumina with smaller dx and dy



(a) Temperature for step 200

(b) Temperature for last step

Figure 9: Boundary condition nr 1 simulation for copper with smaller dx and dy



(a) Temperature for step 200

(b) Temperature for last step

Figure 10: Boundary condition nr 1 simulation for steel with smaller dx and dy

## 5 Boundary condition nr 2 for alumina and copper

During boundary condition nr 2 for first 10000 steps (10s in simulation time) heater was turned on and red area on modeled object was given additional heat every  $dt$  according to the equation 3. Power of heater was set to 100W.

$$\Delta T = \frac{P \cdot dt}{c_w \cdot D^2 \cdot h \cdot \rho} \quad (3)$$

where:

$P$  - power of the heater,

$dt$  - time,

$D^2$  - heater area,

$c_w$  - specific heat of the object material,

$\rho$  - density,

$h$  - object thickness.

Using known parameters theoretical  $\Delta T$  was calculated for alumina (equation 4) and for copper (equation 5).

$$\Delta T_{alumina} = \frac{100W \cdot 10s}{900 \frac{J}{kgK} \cdot (0.05m)^2 \cdot 0.002m \cdot 2700 \frac{kg}{m^3}} = 82.305K \quad (4)$$

$$\Delta T_{copper} = \frac{100W \cdot 10s}{380 \frac{J}{kgK} \cdot (0.05m)^2 \cdot 0.002m \cdot 8920 \frac{kg}{m^3}} = 59.004K \quad (5)$$

For each material, using its unique properties, simulation with boundary condition nr 2 was made. Initial temperature for isolated border 10°C and for rest of the object it was 20°C. Each simulation ended after 20000th (20s in simulation) step was reached. After 10000th step (10s in simulation) heater was turned off. It was performed only for alumina and copper.

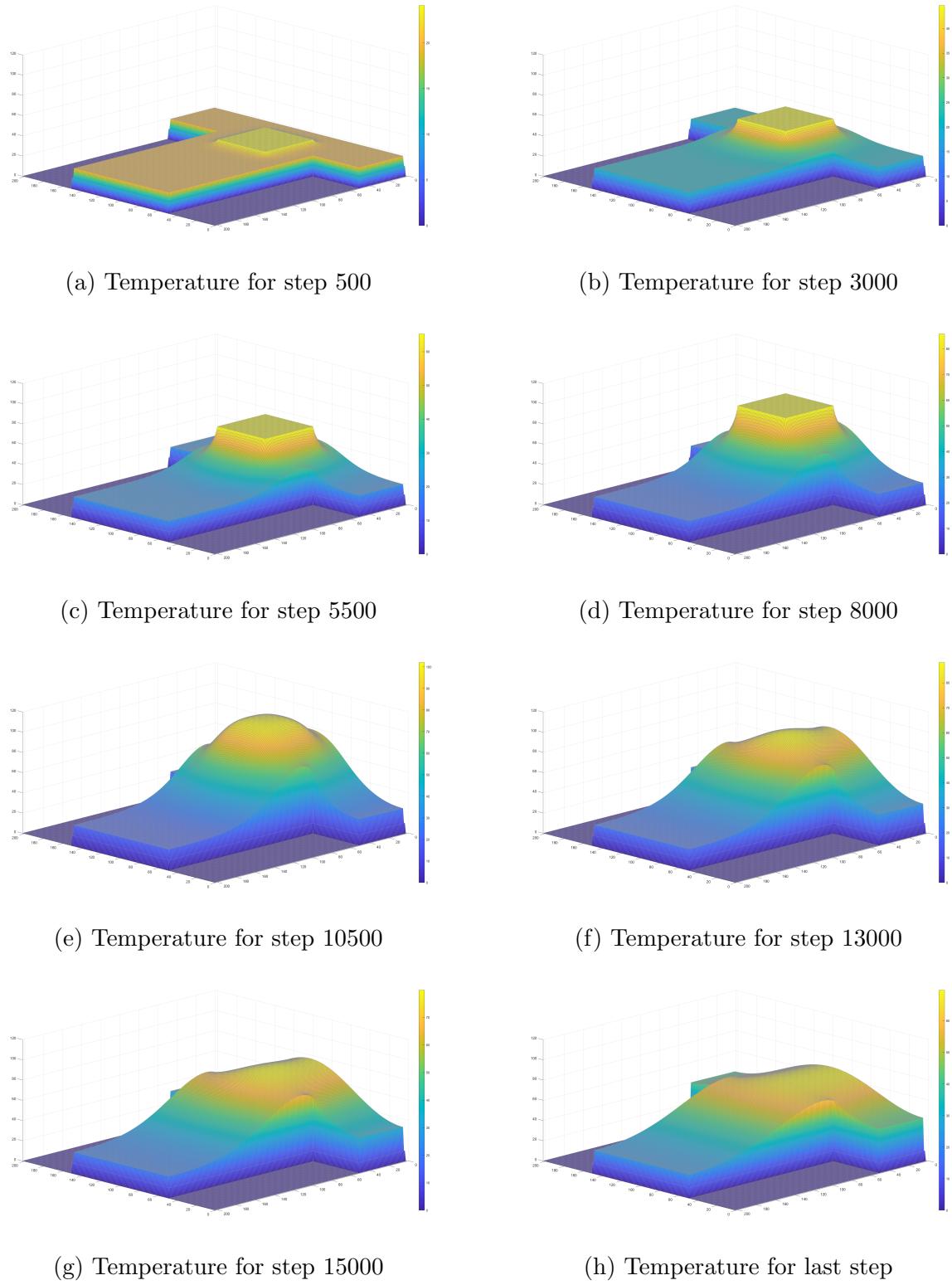


Figure 11: Boundary condition nr 2 simulation for alumina

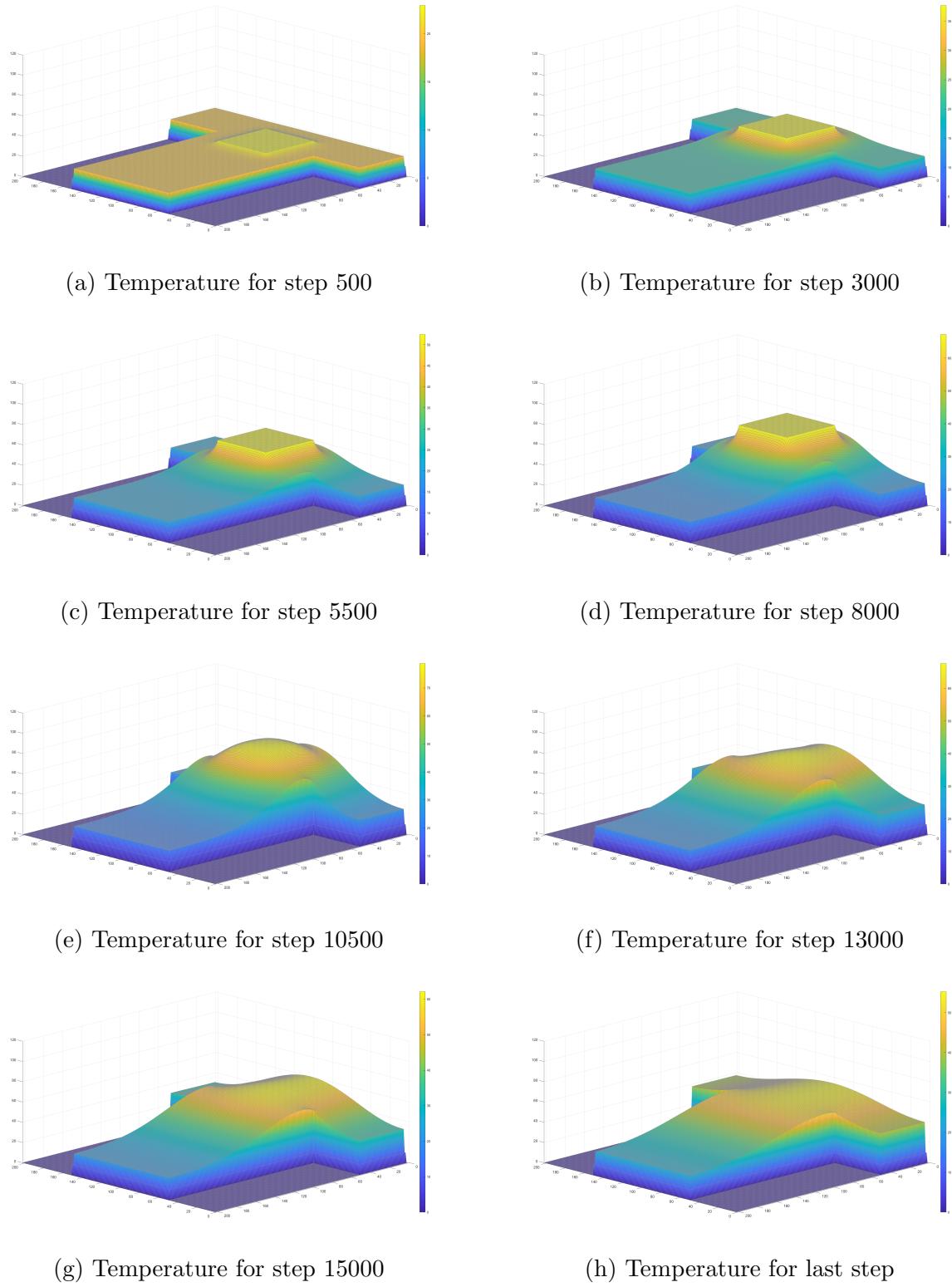


Figure 12: Boundary condition nr 2 simulation for copper

## 6 Theoretical heat balance calculation

Previously calculated theoretical  $\Delta T$  for alumina (59.004K) and for copper (82.305K), as well as  $\Delta T$  obtained from the simulation as mean temperature of whole object (26.788K for alumina and 20.506K for copper) was used to calculate theoretical heat balance with given equation 6.

$$Q = c_w \cdot m \cdot \Delta T \quad (6)$$

where:

$Q$  - quantity of heat,

$\Delta T$  - temperature difference,

$c_w$  - specific heat of the object material,

$m$  - object mass.

$$Q_{aluminatheor} = 900 \frac{J}{kgK} \cdot (0.000005m^3 \cdot 2700 \frac{kg}{m^3}) \cdot 59.004K = 716.899J \quad (7)$$

$$Q_{aluminasim} = 900 \frac{J}{kgK} \cdot (0.000005m^3 \cdot 2700 \frac{kg}{m^3}) \cdot 26.788K = 3254.74J \quad (8)$$

$$Q_{coppertheor} = 380 \frac{J}{kgK} \cdot (0.000005m^3 \cdot 8920 \frac{kg}{m^3}) \cdot 82.305K = 1394.91J \quad (9)$$

$$Q_{coppersim} = 380 \frac{J}{kgK} \cdot (0.000005m^3 \cdot 8920 \frac{kg}{m^3}) \cdot 20.506K = 3475.36J \quad (10)$$

## 7 Conclusions

Presented simulations of first boundary condition showed that material of which object was made significantly changed heat transfer and equilibrium state seen on simulation. Best performance of heat transfer, which was equal to the fastest establishment of equilibrium, was obtained for copper as it only needed little more than 3800 steps (3.8s in simulation time). Next was alumina and then stainless steel, which needed twice as many steps as copper and alumina.

Simulations for smaller dx and dy parameters obtained equilibrium faster than simulations with higher spatial resolution, because gradient bigger and was changing more rapidly between neighbouring points.

Simulations of second boundary conditions, where after 10000 steps heater was turned off, again showed, that copper is better heat conductor than alumina. It distributed the heat from heater on whole object more effectively and faster. Calculated heat quantities derived from theoretical data and from simulations deviated from each other due to numerous approximations present in used numerical and modelling methods used in the simulation.