Linear Models. List 1

Facts:

- For a random vector $X = (X_1, \ldots, X_p)^T$, we define:
  - a vector of expected values by $\mu^X = (EX_1, \ldots, EX_p)^T$
  - a variance–covariance matrix by $\Sigma^X_{p \times p}$, where $\Sigma^X(i,j) = Cov(X_i, X_j) = E(X_i X_j) - EX_i EX_j$.
- Let us denote by A a matrix with k–rows and p–columns, by B a k-dimensional vector, and by $Y = AX + B$ a random vector. It holds: $\mu^Y = A\mu^X + B$ and $\Sigma^Y = A\Sigma^X A^T$.
- Let X be a multivariate normal vector $N(\mu, \Sigma)$, then its density has a following form

$$f(x) = det(2\pi\Sigma)^{-1/2} exp\left((x-\mu)^T \Sigma^{-1}(x-\mu)/2\right) \quad .$$

- If Y = AX+B is an affine transformation of X (a multivariate normal vector), then Y has a multivariate normal distribution.

Exercises:

1) Use a *rnorm* function (in *R*) to generate 100 random vectors from a two–dimensional normal distribution $N(0, I)$ and plot them.

2) Find an affine transformation, which transforms above cloud of points into a cloud of points from a normal distribution $N(\mu, \Sigma)$, where:
   - $\mu = (4,2), \quad \Sigma = \begin{pmatrix} 1 & 0.9 \\ 0.9 & 1 \end{pmatrix} \quad .$
   - $\mu = (4,2), \quad \Sigma = \begin{pmatrix} 1 & -0.9 \\ -0.9 & 1 \end{pmatrix} \quad .$
   - $\mu = (4,2), \quad \Sigma = \begin{pmatrix} 9 & 0 \\ 0 & 1 \end{pmatrix} \quad .$

   Plot a cloud of points for each case.

3) Use a *rnorm* function (in *R*) to generate 200 random vectors from a multivariate normal distribution $N(0, I_{100\times100})$. Insert them into a matrix $X_{200\times100}$ (each row is a separate generation of a one vector). Construct a matrix A, such that rows of a matrix $\tilde{X} = XA$ contains 200 vectors from a multivariate normal distribution $N(0, \Sigma_{100\times100})$, where $\Sigma(i,i) = 1$ and $\Sigma(i,j) = 0.9$ for $i \neq j$. In order to verify your solution, calculate a sample variance–covariance matrix. Next, on its basis, plot a histogram of obtained sample variances and calculate their mean. Do the same for sample covariances. Summarize results.