An invitation to quantum computing

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Why the Q-word?

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Motivation from Moore's law:

Classical mechanics breaks down at sub-atomic level.
With computer parts shrinking ever more, quantum effects are inevitable.

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Ultimate goal: build a quantum computer!

Principles of quantum mechanics in a nutshell

Think of quantum states as linear superpositions of classical states.
A qubit is the simplest non-trivial quantum state.

$$\psi = \alpha\chi_0 + \beta\chi_1$$

where $\chi_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\chi_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $\alpha, \beta \in \mathbb{C}$ satisfy

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Born's rule: Given a state $|\psi\rangle$ and an orthomormal basis $\{|\chi\rangle, |\xi\rangle\}$, the probability of measuring χ is $|\langle\chi||\psi\rangle|^2$ and the probability of measuring ξ is $|\langle\xi||\psi\rangle|^2$.

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Intuition: Choosing an orthonormal basis in a 2D Hilbert space can be likened to choosing two half planes to measure the probability of finding a particle in either.

How does a quantum state evolve?

Measure it in an orthonormal basis! If we perform said measurement in the orthonormal basis $\{|\chi\rangle, |\xi\rangle\}$ we have the transformation

$$|\psi\rangle \mapsto |\phi\rangle$$

where $\phi \in \{\chi, \xi\}$ is the outcome of the measurement according to Born's rule. This is known as the *collapse* of the wavefunction.

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Note that the states $\sqrt{\frac{1}{2}}|0\rangle + \sqrt{\frac{1}{2}}|1\rangle$ and $\sqrt{\frac{1}{2}}|0\rangle - \sqrt{\frac{1}{2}}|1\rangle$ are different whereas $\sqrt{\frac{1}{2}}|0\rangle + \sqrt{\frac{1}{2}}|1\rangle$ and $e^{i\theta}\left(\sqrt{\frac{1}{2}}|0\rangle + \sqrt{\frac{1}{2}}|1\rangle\right)$ are indistinguishable. (See later in Deutsch's algorithm).

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Note that by definition all quantum gates are invertible. This is not necessarily true for good old logical gates.

How to understand measurement?

Einstein to Bohr: ‘God doesn’t throw dice.’
Bohr to Einstein: ‘Don’t tell God what not to do.’

For those in need of procrastination:

Bohr–Einstein debates

14 languages

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From Wikipedia, the free encyclopedia

The **Bohr–Einstein debates** were a series of public disputes about quantum mechanics between Albert Einstein and Niels Bohr. Their debates are remembered because of their importance to the philosophy of science, insofar as the disagreements—and the outcome of Bohr's version of quantum mechanics becoming the prevalent view—form the root of the modern understanding of physics.[1] Most of Bohr's version of the events held in the Solvay Conference in 1927 and other places was first written by Bohr decades later in an article titled, "Discussions with Einstein on Epistemological Problems in Atomic Physics".[2][3] Based on the

Niels Bohr (left) with Albert Einstein (right) at Paul Ehrenfest's home in Leiden (December 1925)

How to understand measurement?

Schrödinger's thought experiment:

A cat is placed in a box with a vial of poison. Before the box is opened, the cat is in a superposition of the states 'DEAD' and 'ALIVE'. It is only 'DEAD' OR 'ALIVE' once the box is opened.

A life-size cat figure in the garden of Huttenstrasse 9, Zurich, where Erwin Schrödinger lived from 1921 to 1926. Depending on the light conditions, the figure appears to be either a live cat or a dead one.

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All possible outcomes of measurements happen in different physical worlds.

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Perhaps quantum systems are just sensitive to interaction with the outside environment. This phenomenon is called decoherence and is a big obstacle to building quantum computers.

Multi-particle systems

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$$(\lambda v_1 + \mu w_1) \otimes v_2 = \lambda v_1 \otimes v_2 + \mu w_1 \otimes v_2$$

and

$$v_1 \otimes (\lambda v_2 + \mu w_2) = \lambda v_1 \otimes v_2 + \mu v_1 \otimes w_2$$

for all $v_1, w_1 \in \mathcal{H}_1$, $v_2, w_2 \in \mathcal{H}_2$ and $\lambda, \mu \in \mathbb{C}$.

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Example. Consider the 2-particle state

$$|\psi\rangle = \frac{1}{2}\left(|00\rangle + |10\rangle - |01\rangle - |11\rangle\right).$$

We can write

$$|\psi\rangle = \left(\sqrt{\frac{1}{2}}|0\rangle + \sqrt{\frac{1}{2}}|1\rangle\right) \otimes \left(\sqrt{\frac{1}{2}}|0\rangle - \sqrt{\frac{1}{2}}|1\rangle\right).$$

$|\psi\rangle$ is said to be a *separable state*.

The Bell state

Not all states are separable!
The *Bell state* or *Einstein-Podolski-Rosen (EPR)* state is an example of a 2-qubit state that cannot be written as the tensor product of two single qubit states.

$$|\psi\rangle = \sqrt{\frac{1}{2}} \left(|00\rangle + |11\rangle \right).$$

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Most of the power of quantum information over classical comes from harnessing the Bell state. It encodes a correlation that cannot be simulated by classical information.

Some more quantum gates

Example. The following is known as the *Hadamard gate*

$$H = \sqrt{\frac{1}{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

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The Hadamard operator satisfies $H^\dagger = H^{-1} = H$.
Example. The control-NOT (CNOT) gate acts on a 2 qubit system on the boolean basis as follows:

$$|00\rangle \mapsto |00\rangle$$

$$|01\rangle \mapsto |01\rangle$$

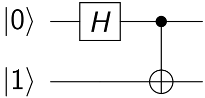
$$|10\rangle \mapsto |11\rangle$$

$$|11\rangle \mapsto |10\rangle.$$

How to write it in matrix form?

Example of a circuit

We can use the following quantum circuit to build the Bell state.



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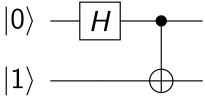
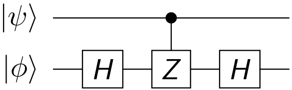


Example. What does this circuit do?



N.B. In the above $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

Deutsch-Jozsa Algorithm

Let $f : \{0,1\} \to \{0,1\}$ be one of the following Boolean functions.

	f_{00}	f_{01}	f_{10}	f_{11}
$f(0)$	0	0	1	1
$f(1)$	0	1	0	1

That is, f is either *constant* or *balanced*.

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Classically, we need two calls to f. Quantumly, we need only one!

Theorem (Deutsch-Jozsa)

Given a boolean function on n bits that is promised to be constant or balanced, there is a quantum algorithm to tell whether f is constant or balanced using only one call to f.

Deutsch's Algorithm (the case $n = 1$)

Let U_f be the following quantum gate: for $x, y \in \{0, 1\}$

$$U_f |x\rangle |y\rangle = |x\rangle |y \oplus f(x)\rangle .$$

Note that

$$U_f |x\rangle \otimes \sqrt{\frac{1}{2}} (|0\rangle - |1\rangle) = (-1)^{f(x)} |x\rangle \otimes \sqrt{\frac{1}{2}} (|0\rangle - |1\rangle) .$$

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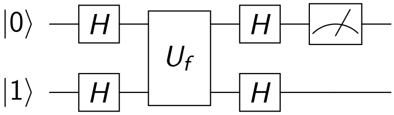
$$U_f \left|x\right\rangle \otimes \sqrt{\frac{1}{2}} \left(\left|0\right\rangle - \left|1\right\rangle\right) = \left(-1\right)^{f(x)} \left|x\right\rangle \otimes \sqrt{\frac{1}{2}} \left(\left|0\right\rangle - \left|1\right\rangle\right) .$$

The following circuit determines whether f is constant or balanced.



A love story to motivate quantum teleportation

Romeo and Juliet are a sweet couple in XVth century Verona but their parents want them to break up before they head to University. Juliet will be going to Oxbridge but Romeo will head to Durham. Their bond, however, has been fortified to a point that they share a Bell state:

$$|\psi\rangle = \sqrt{\frac{1}{2}}\left(|00\rangle + |11\rangle\right)\ldots$$

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Their parents hire an investigator, Eavesdropping Eve, who is rather old fashioned and knows nothing about quantum communication, so she has access only to Romeo and Juliet's classical communications.

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where 1 is 'I love you' and 0 is 'I HATE YOU!'.

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> *QUANTUM TELEPORTATION: a protocol that transfers a quantum state to any geographical location requiring the parties to exchange only classical information, so that Eve is unable to observe ξ.*

Quantum teleportation: the protocol

STEP 1: Juliet entangles $|\xi\rangle$ with the Bell state she shares with Romeo:

$$|\psi_1\rangle = |\xi\rangle \otimes |\psi\rangle = \sqrt{\frac{1}{2}}\alpha\,|0\rangle\left(|00\rangle + |11\rangle\right) + \sqrt{\frac{1}{2}}\beta\,|1\rangle\left(|00\rangle + |11\rangle\right).$$

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STEP 2: Juliet applies the CNOT gate to her particle in the Bell pair conditional on ξ. Namely,

$$CNOT\,|\psi_1\rangle = \sqrt{\frac{1}{2}}\alpha\,|0\rangle\,(|00\rangle + |11\rangle) + \sqrt{\frac{1}{2}}\beta\,|1\rangle\,(|10\rangle + |01\rangle)\,.$$

Quantum teleportation: the protocol

STEP 3: Finally, Juliet acts on her feelings state with the Hadamard operator to obtain

$$H\hat{C}\,|\psi_1\rangle = \frac{1}{2}\alpha\,(|0\rangle + |1\rangle)\,(|00\rangle + |11\rangle) + \frac{1}{2}\beta\,(|0\rangle - |1\rangle)\,(|10\rangle + |01\rangle)\,.$$

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Or, by regrouping we get the state

$$|\psi_2\rangle = \begin{array}{c} \frac{1}{2}\,|00\rangle\,(\alpha\,|0\rangle + \beta\,|1\rangle) + \\ \frac{1}{2}\,|10\rangle\,(\alpha\,|0\rangle - \beta\,|1\rangle) + \\ \frac{1}{2}\,|01\rangle\,(\alpha\,|1\rangle + \beta\,|0\rangle) + \\ \frac{1}{2}\,|11\rangle\,(\alpha\,|1\rangle - \beta\,|0\rangle)\,. \end{array}$$

Quantum teleportation: the protocol

The communication via the classical channel (that Eve has access to) is as follows:

Alice's measurement	Instructions to Bob
00	Keep your state as it is
10	Act on it with Z
01	Act on it with NOT
11	Act on it with NOT and then Z

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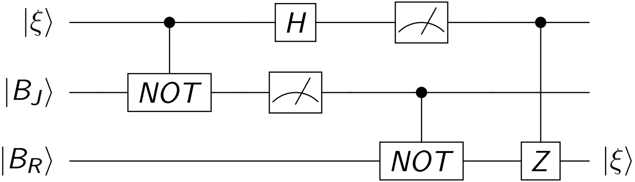
So Eve has no idea what Juliet's mixed feelings are!

> *'My only love sprung from my only hate!'*
> *(Romeo and Juliet, Act I)*

Quantum teleportation: a circuit



Quantum teleportation: the state of the art

1993: Bennet, Brassard, Crepeau, Jozsa, Peres and Wooters first discover quantum teleportation.

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2025: Teleportation of quantum gates achieved by Oxford group!

No cats were harmed when writing these slides