Fun Tings

1. Intermediate Value Theorem (IVT)

If f is continuous on $[a, b]$ and $f(a) < 0 < f(b)$ (or vice versa),
then there exists $c \in (a, b)$ such that $f(c) = 0$.

> Used when you want to prove a function hits a specific value (often 0) **without solving** the function.

---

2. Mean Value Theorem (MVT)

If f is continuous on $[a, b]$ and differentiable on (a, b),
then there exists $c \in (a, b)$ such that:

$f\prime(c) = \frac{f(b)-f(a)}{b-a}$

> Use this when a function's **rate of change** needs to be related to its **average rate of change**.

---

3. Rolle's Theorem

If f is continuous on $[a, b]$, differentiable on (a, b), and $f(a) = f(b)$,
then $\exists c \in (a, b))$ such that $f\prime(c) = 0$.

> It's basically the MVT with flat endpoints. Often used to prove a derivative **is zero somewhere**.

---

4. Fundamental Theorem of Calculus (FTC)

Part 1:

If f is continuous on $[a, b]$, define:

$F(x) = \int_a^x f(t)dt$

Then F is continuous on $[a, b]$, differentiable on (a, b), and:

$F'(x) = f(x)$

Part 2:

$\int_a^b f(x)dx = F(x) - F(a)$

where F is **any** antiderivative of f.

> These are your bread and butter for integrating and differentiating **inversely**.

---

5. First Derivative Test

If $f\prime(x)$ changes sign around a point c, then you can determine whether $f(c)$ is a local max or min.

If $f''(c) > 0$, then local **minimum** at c;
If $f''(c) < 0$, then local **maximum** at c.

> Proofs about **increasing/decreasing behavior** or extremums usually involve this.

---

6. Continuity and Differentiability Rules

- Every differentiable function is continuous, but not vice versa.
- Continuous functions over closed intervals **attain** maxima and minima (Extreme Value Theorem).
- If a continuous function’s integral is zero over an interval, it must take on both positive and negative values or be identically zero somewhere.