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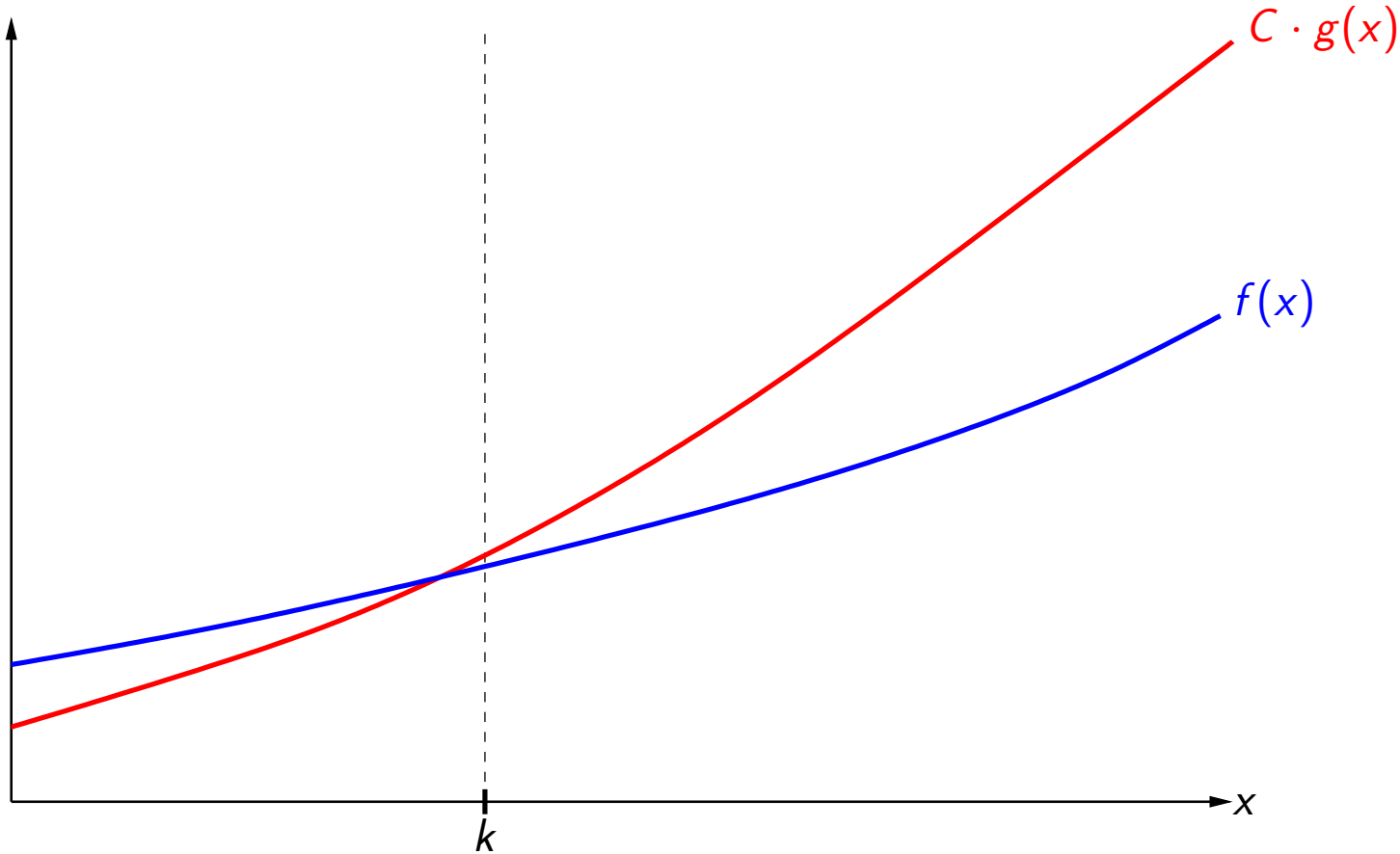

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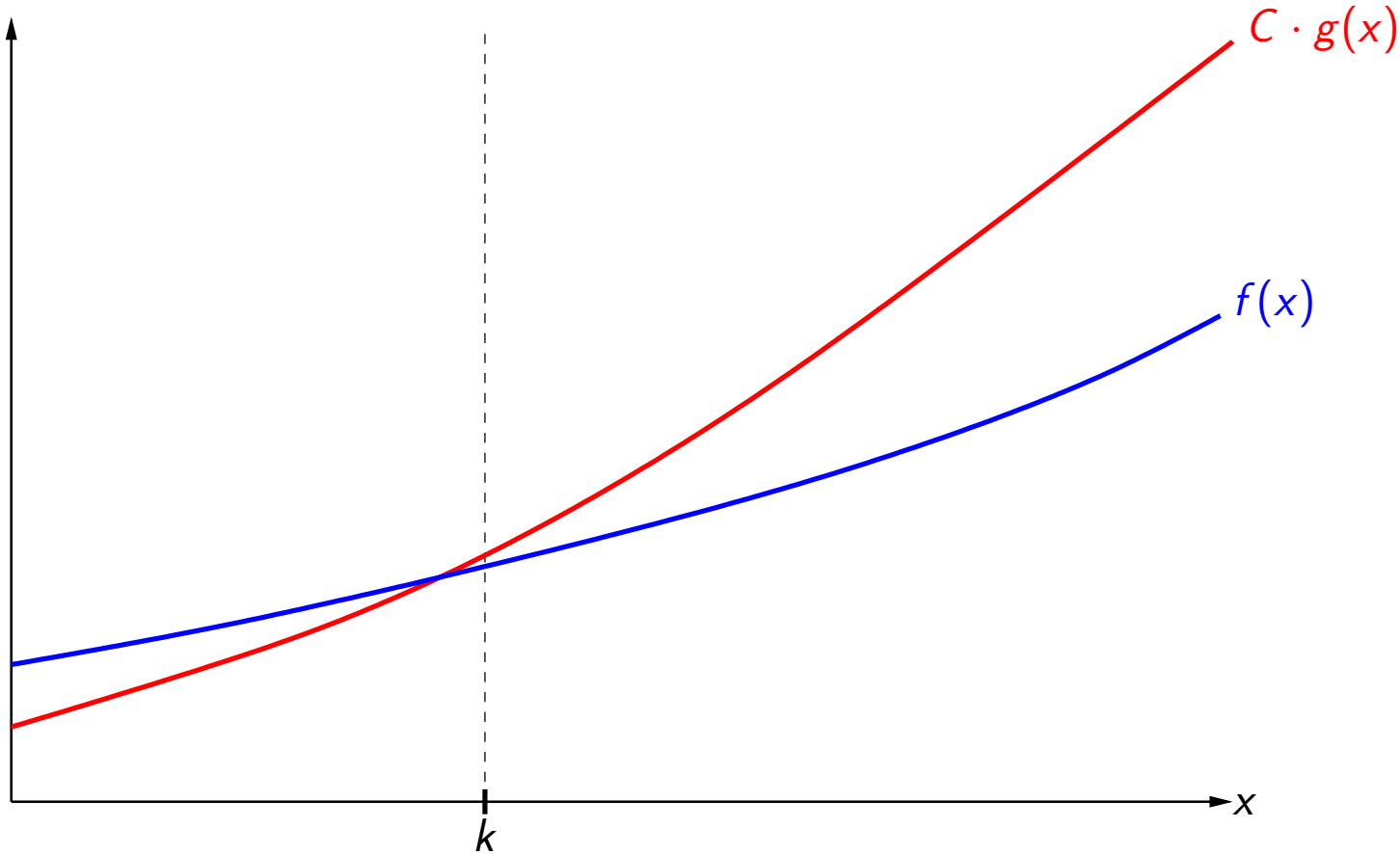

- From $x = k$ onward, $f(x)$ never exceeds $C \cdot g(x)$.

Illustration of witnesses that work

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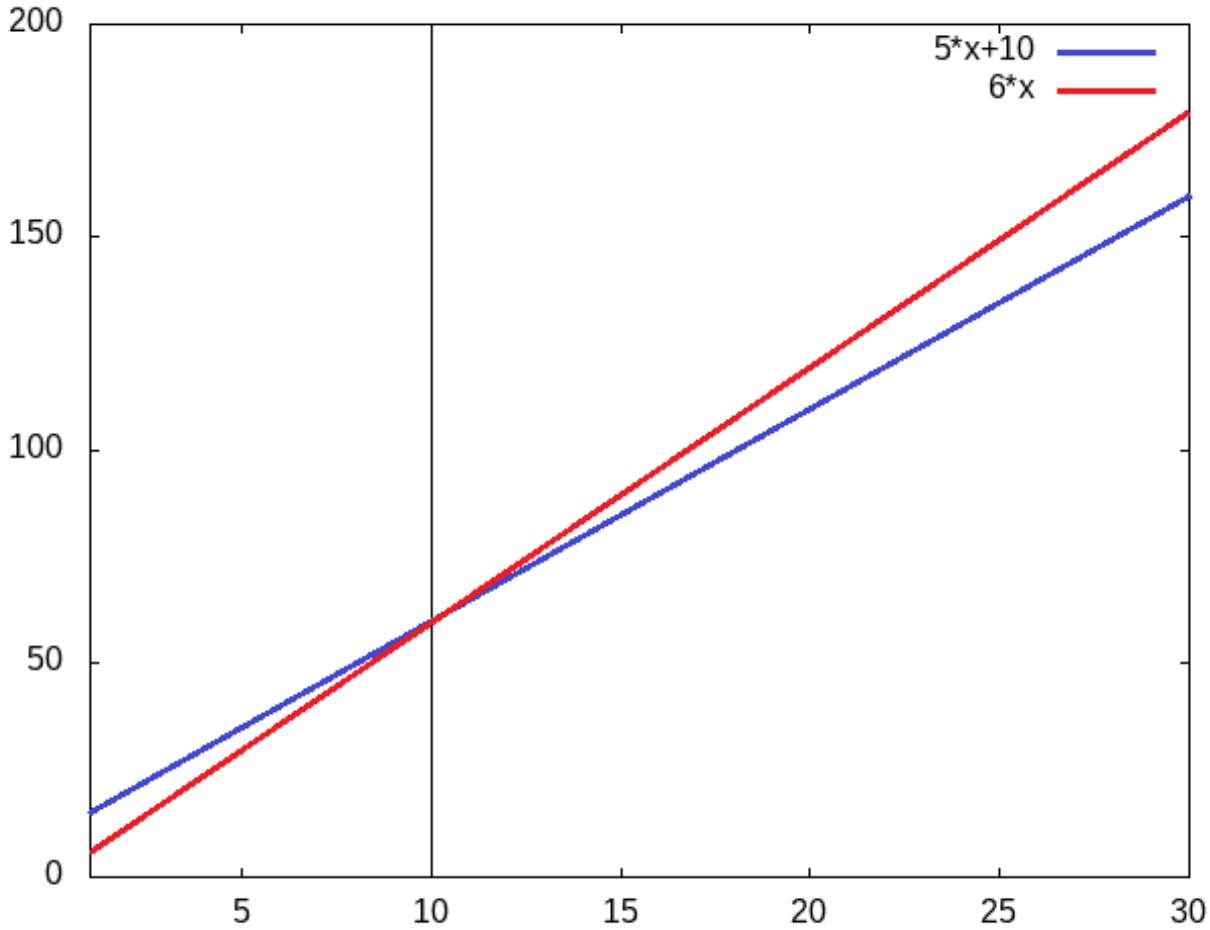

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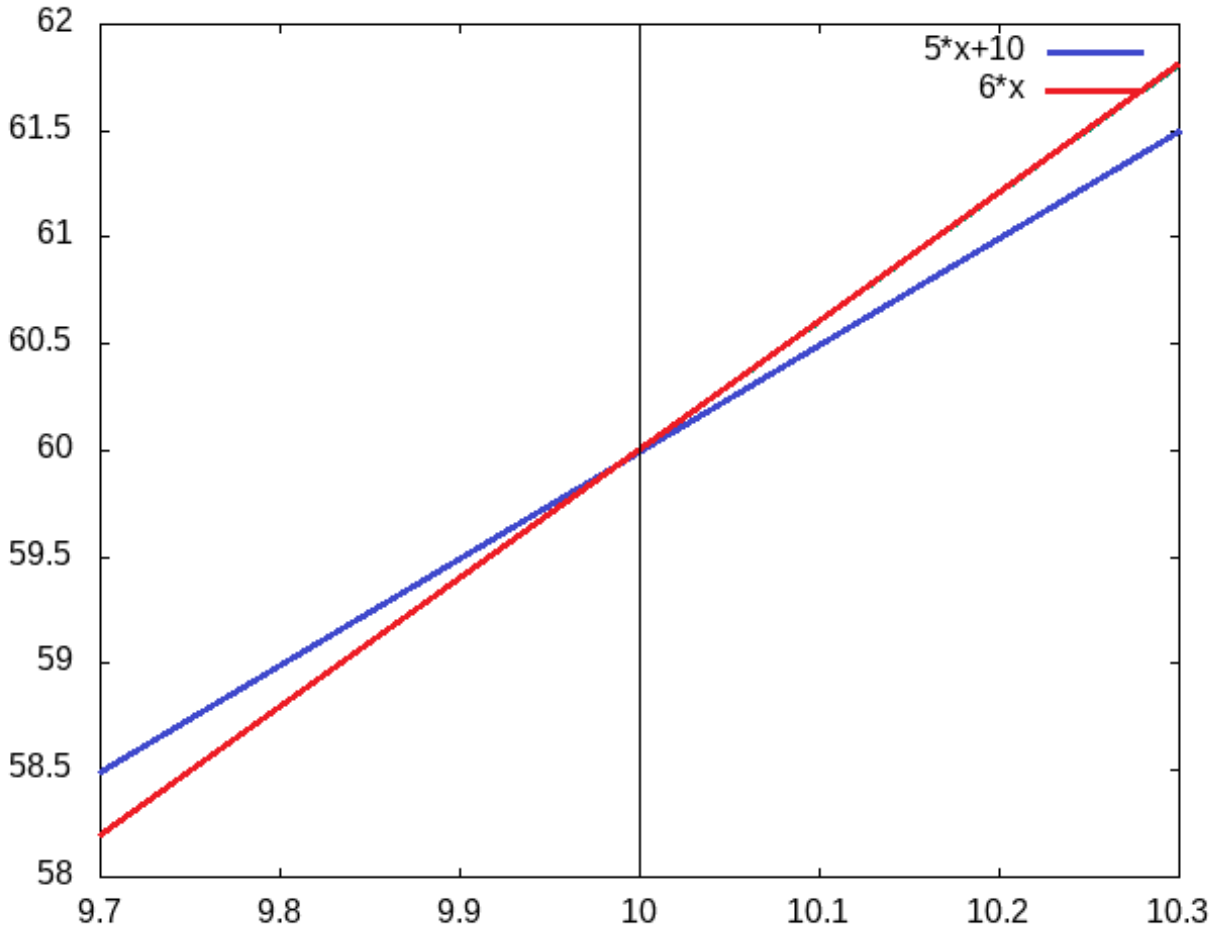

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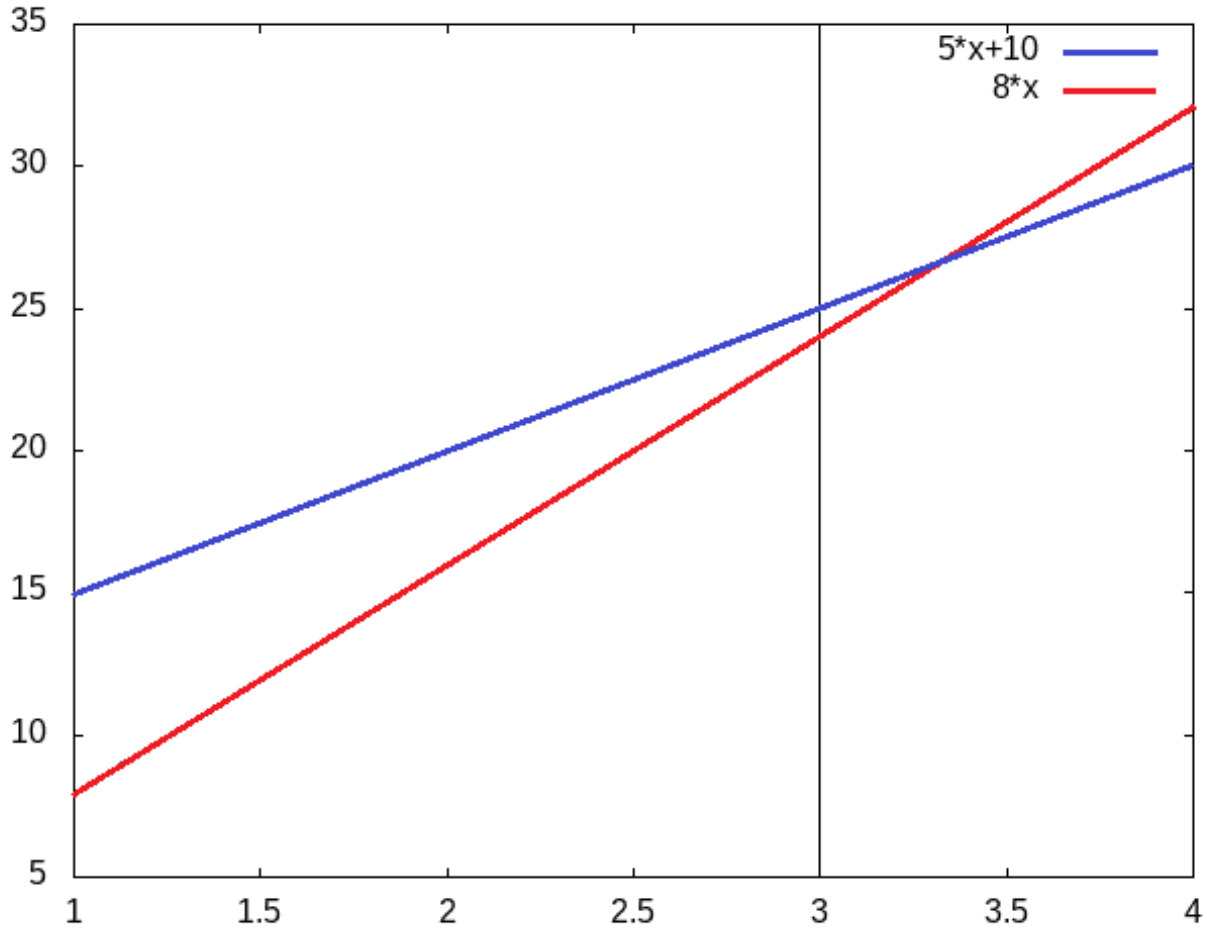
Illustration of witnesses that don't work

- $C = 8$, $k = 3$ are **NOT** witnesses for $5x + 10 = O(x)$



- At $x = 3$, $5x + 10 = 25$ exceeds $8 \cdot x = 24$.

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- We do say: The time complexity of our algorithm is $O(n^2)$.

Using big-O notation in a natural way

If the time complexity of our algorithm is:

$$12 + 17\sqrt{n} + 5n + 17n \log n$$

How should we express this with big-O notation? Should we say that the time complexity is in:

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- $O(n^2)$: correct but not natural: why give a larger bound than necessary?