Introduction to Digital Twins - Portfolio

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March 21, 2023

1. Write the following systems of differential equations as systems of first order equations [10 points].

   (a) $\ddot{y}(t) = y(t)^2 - x(t) + e^t$
   $\ddot{x}(t) = y(t) - x(t)^2 - e^t$

   (b) $\ddot{x}(t) = 2y(t) - 4t^2x(t)$
   $\ddot{y}(t) = -2x(t) - 2t\dot{x}(t)$

   (c) $\ddot{x}(t) = \ddot{y}(t)x(t) + \dot{y}(t)^2x(t)$
   $\ddot{y}(t) = -y(t)$

   (d) $\frac{d^3y}{dt^3}(t) = \ddot{y}(t)x(t)^2 - 3\dot{y}(t)^2x(t)$
   $\ddot{y}(t) = t + \dot{x}(t)$

2. Write the following differential equations as systems of first order equations [10 points].

   (a) $\ddot{x}(t) + t^2\dot{x}(t) + 3x(t) = 0$

   (b) $\ddot{x}(t) = -2x(t) - 10\dot{x}(t)$

   (c) $\ddot{y}(t) = 2\sqrt{e^{2t} - y(t)^2}$

   (d) $2\ddot{x}(t) - 5\dot{x}(t) + x(t) = 0$

3. Solve equations (b) and (d) in 2) numerically using the Euler's and Runge-Kutta's method [10 points].

4. Solve the following system on the interval [0,2] using Euler and Runge-Kutta method [10 points].
   $\ddot{x}(t) = 2y(t) - \sin(4t^2x(t)), \;\; x(0) = 1, \, \dot{x}(0) = 2$
   $\ddot{y}(t) = -2x(t) - \frac{1}{2t^2\dot{x}(t)^2+3}, \;\; y(0) = 1, \, \dot{y}(0) = 0$

5. Make a simulation of a digital twin for autonomous ship that travel from A to B in Unity 3D. The simulation will be evaluated based on the following criteria:

   (a) The presentation of mathematical modelling of the ship [10 points]. The state generated by the model will serve as the state of the physical ship.

   (b) The presentation of the simulation and visualization in the Unity 3D [10 points].

   (c) The presentation of state and parameter estimation of the digital model in Unity 3D. The estimation algorithm should be based on the following paper: A. Hasan, M. Tahavori, and H. Midtiby, Model-based fault diagnosis algorithms for robotic systems, IEEE Access, vol. 11, pp. 2250-2258, 2023 [30 points].

   (d) A video demonstration of the autonomous ship that goes from A to B [10 points].