

Linear Algebra

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March 18, 2021

Vector Spaces

Vector spaces, vectore definition, linear transformations, spans, linear combinations,
bases,... [1]

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0 Introduction

These Linear Algebra notes are based on the book Friedberg Linear Algebra, a rigorous linear algebra book and the lecture notes of Terence Tao. Linear algebra is the study of linear transformations and their algebraic properties.

1 Vector Spaces

1.1 Vector Spaces

Definition (Vector Spaces). A *vector space* (or *linear space*) V over a field \mathbb{F} consists of a set on which two operations (called addition and scalar multiplication, respectively) are defined so that for each pair of elements \mathbf{x}, \mathbf{y} in V there is a unique element $\mathbf{x} + \mathbf{y}$ in V (closed under addition), and for each element a in \mathbb{F} and each element \mathbf{x} in V there is a unique element $a\mathbf{x}$ in V (closed under scalar multiplication), such that the following conditions hold.

- (VS 1) For all $\mathbf{v}, \mathbf{w} \in V$, $\mathbf{v} + \mathbf{w} = \mathbf{w} + \mathbf{v}$ (Addition is commutative)
- (VS 2) For all $\mathbf{u}, \mathbf{v}, \mathbf{w} \in V$, $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$ (Addition is associative)
- (VS 3) There is a vector $\mathbf{0}$ such that $\mathbf{0} + \mathbf{v} = \mathbf{v}$. (Additive identity)
- (VS 4) For all vectors $\mathbf{v} \in V$, there is a vector $(-\mathbf{v})$ such that $-\mathbf{v} + (\mathbf{v}) = \mathbf{0}$ (Additive inverse)
- (VS 5) The scalar 1 has the property that $1\mathbf{v} = \mathbf{v}$ for all $v \in V$ (Multiplicative identity)
- (VS 6) For any scalars $\lambda, \mu \in \mathbb{F}$ and any vector $\mathbf{v} \in V$, we have $\lambda(\mu\mathbf{v}) = (\lambda\mu)\mathbf{v}$
- (VS 7) For any scalar $\lambda \in \mathbb{F}$ and any vectors $\mathbf{v}, \mathbf{w} \in V$, $\lambda(\mathbf{v} + \mathbf{w}) = \lambda\mathbf{v} + \lambda\mathbf{w}$.
- (VS 8) For any scalars $\lambda, \mu \in \mathbb{F}$ and any vector $\mathbf{v} \in V$, $(\lambda + \mu)\mathbf{v} = \lambda\mathbf{v} + \mu\mathbf{v}$.

1.2 Exercises

Exercise.

1.3 Subspaces

A subset W of a vector space V over a field F is called a *subspace* of V if W is a vector space over F with the operations of addition and scalar multiplication defined on V .