Linear Algebra

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0 Introduction

These Linear Algebra notes are based on the book Friedberg Linear Algebra, a rigorous linear algebra book and the lecture notes of Terence Tao. Linear algebra is the study of linear transformations and their algebraic properties.

1 Vector Spaces

Definition (Vector Spaces)**.** A *vector space* (or *linear space*) V over a field $\mathbb{F}$ consists of a set on whicb two operations (called addition and scalar multiplication , respectively) are defined so that for each pair of elements $\mathbf{x}$, $\mathbf{y}$. in V there is a unique element $\mathbf{x}+\mathbf{y}$ in V (closed under addition), and for each element a in $\mathbb{F}$ and each element $\mathbf{x}$ in V there is a unique element $a\mathbf{x}$ in V (closed under scalar multiplication), such that the following conditions hold.

(VS 1) For all $\mathbf{v}, \mathbf{w} \in V$, $\mathbf{v}+\mathbf{w}=\mathbf{w}+\mathbf{v}$ (Addition is commutative)

(VS 2) For all $\mathbf{u}, \mathbf{v}, \mathbf{w} \in V$, $(\mathbf{u}+\mathbf{v})+\mathbf{w}=\mathbf{u}+(\mathbf{v}+\mathbf{w})$ (Addition is associative)

(VS 3) There is a vector $\mathbf{0}$ such that $\mathbf{0}+\mathbf{v}=\mathbf{v}$. (Additive identity)

(VS 4) For all vectors $\mathbf{v} \in V$, there is a vector $(-\mathbf{v})$ such that $-\mathbf{v}+(\mathbf{v})=\mathbf{0}$ (Additive inverse)

(VS 5) The scalar 1 has the property that $1\mathbf{v}=\mathbf{v}$ for all $v \in V$ (Multiplicative identity)

(VS 6) For any scalars $\lambda, \mu \in \mathbb{F}$ and any vector $\mathbf{v} \in V$, we have $\lambda(\mu\mathbf{v})=(\lambda\mu)\mathbf{v}$

(VS 7) For any scalar $\lambda \in \mathbb{F}$ and any vectors $v, w \in V$, $\lambda(\mathbf{v}+\mathbf{w})=\lambda\mathbf{v}+\lambda\mathbf{w}$.

(VS 8) For any scalars $\lambda, \mu \in \mathbb{F}$ and any vector $v \in V$, $(\lambda+\mu)\mathbf{v}=\lambda\mathbf{v}+\mu\mathbf{v}$.