UNIVERSITY OF SOUTHERN MINDANAO

MATH121E

Calculus 2

Topic Outline

INTEGRATION TECNIQUES

- Improper Integral

Improper Integral

- A definite integral whose region of integration is unbounded or includes a point at which the integrand is undefined or tends to infinity.

Improper Integral

There are two types of Improper Integrals:

1. Definition of an Improper Integral of Type 1 – when the limits of integration are infinite.

2. Definition of an Improper Integral of Type 2 – when the integrand becomes infinite within the interval of integration.

Improper Integral

Example a

Evaluate the following integral.

$$\int_1^{\infty} \frac{1}{x}\, dx$$

We will replace the infinity with a variable ***(usually t),*** do the integral and then take the limit of the result as **t goes to infinity**.

We will call these integrals **convergent** if the associated **limit exists** and is a finite number (*i.e.* **it's not plus** or minus infinity) and **divergent** if the associated limit either doesn't exist or is (plus or minus) **infinity**.

When we say infinity, it is either in the lower or upper limit of the integral sign

Improper Integral

$$\int_1^{\infty} \frac{1}{x}\, dx$$

$$= \lim_{t \to \infty} \int_1^{t} \frac{1}{x}\, dx$$

$$= \lim_{t \to \infty} \ln x \,]_1^t$$

$$= \lim_{t \to \infty} \ln |t| - \ln |1|$$

$$= \lim_{t \to \infty} \ln |t| - \lim_{t \to \infty} \ln |1|$$

$$= \lim_{t \to \infty} \ln |t| - 0 = \infty$$

So, this particular integral is divergent. It doesn't converge

Improper Integral

Example b $\int_1^{\infty} \frac{1}{x^2}\,dx$

$= \lim_{t\to\infty} \int_1^{t} \frac{1}{x^2}\,dx = \lim_{t\to\infty} -\frac{1}{x}\Big|_1^{t} = \lim_{t\to\infty} -\frac{1}{t} - \lim_{t\to\infty} -\frac{1}{1}$

Take the integral of $\int \frac{1}{x^2}\,dx$

$= \int x^{-2}\,dx$

$= x^{-1} + C$

$= -\frac{1}{x} + C$

$= \lim_{t\to\infty} \left| -\frac{1}{t} + 1 \right|$

Replacing t with ∞ and $-\frac{1}{\infty} = 0$

$= \lim_{t\to\infty} \left| -\frac{1}{0} + 1 \right|$

= 1

Since we got a finite number, this integral is convergent

Improper Integral

P- series $\int_1^{\infty} \frac{1}{x^{\,p}}\,dx$

If p is >1 , integral is convergent
If p is ≤ 1 , integral is divergent

Example c $\int_1^{\infty} \frac{1}{x^{1}}\,dx = \infty$

From above condition, since p is equal to 1 , integral is divergent

Improper Integral

From our Example b

$$\int_1^{\infty} \frac{1}{x^2}\, dx = 1$$

Applying the given conditions since p = 2, and 2 is greater than 1.The integral is convergent

Improper Integral

Find the integral of below

1. $\int_1^{\infty} \frac{1\ dx}{(3x+1)^2}$

2. $\int_{-\infty}^{1} \frac{1\ dx}{(2x-5)}$

3. $\int_0^{\infty} \frac{x\ dx}{(x^2+2)^2}$

4. $\int_{-\infty}^{\infty} (2-v^4)\ dv$

References

1. Differential and Integral Calculus 6th Edition by Clyde E. Love and Earl D. Rainville
2. YouTube: The Organic Chemistry Tutor

End of Topic

Thank you

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