UNIVERSITY OF SOUTHERN MINDANAO

MATH121E

Calculus 2

Topic Outline

Applications of Definite Integral

- Plane areas
- Areas Between curves
- Other Applications
  - a. Volume
  - b. Work
  - c. Hydrostatic Pressure

Plane Areas

$$A = \int_{a}^{b} (Y_{upper} - Y_{lower})\, dx$$ This is for vertical Strip

$$A = \int_{a}^{b} (X_{right} - X_{left})\, dy$$ This is for horizontal strip Strip

Plane Areas

Steps to Solve the problem

1. Find for the vertex
2. Find the intersection point
3. Draw the curves
4. Solve the area

For vertex= we have the standard form

$F(x) = ax^2+bx+C$, for vertex we have the formula

$$x = \frac{-b}{2a}$$

$F(y) = ay^2+by+C$; vertex is $y = \dfrac{-b}{2a}$

Plane Areas

1.) Find the area bounded by the curve $y = x^2$ and the x-axis and the ordinates $x = 1$ and $x = 3$

$y = x^2$ is a parabola facing upward

$$A = \int_a^b (y_{upper} - y_{lower})\,dx$$

$$= \int_1^3 (x^2 - 0)\,dx$$

$$= \int_1^3 x^2\,dx$$

$$= \left.\frac{x^3}{3}\right|_1^3$$

Vertex: $y = x^2$

from ; $x = \frac{-b}{2a}$

$a = 1$, $b = 0$

$x = \frac{-0}{2(1)} = 0$

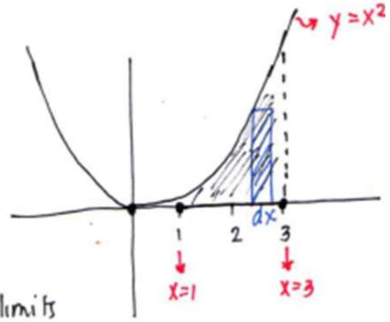
Substitute value of x in

$y = x^2$

$y = 0^2$

$y = 0$

$V(0,0)$



Substituting the limits

$$A = \left.\frac{x^3}{3}\right|_1^3$$

$$= \frac{3^3}{3} - \frac{1^3}{3}$$

$$= \frac{27}{3} - \frac{1}{3}$$

$$\boxed{A = \frac{26}{3} \text{ sq. units}}$$

Plane Areas

2. Find the area lying above the x-axis and under the parabola

$$y = 4x - x^2$$

a.) Vertex ;

to solve for vertex , we have a standard equation

$$f(x) = ax^2 + bx + c$$

using the given equation

$$y = 4x - x^2$$
$$y = -x^2 + 4x$$
$$a = -1 \;,\; b = 4$$

from the formula

$$x = \frac{-b}{2a}$$
$$x = -\frac{4}{2(-1)}$$
$$x = \frac{-4}{-2}$$

$x = 2 \rightarrow$ substitute this in the equation of parabola

$$y = -x^2 + 4x$$
$$y = -(2)^2 + 4(2)$$
$$y = -4 + 8$$
$$y = 4$$

V (2, 4)

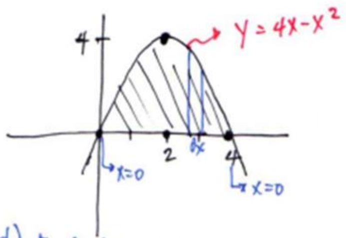
b.) for the limits and point of intersection

$$y = 4x - x^2$$

let $y = 0$;

$$0 = 4x - x^2$$
$$0 = 4x - x^2$$
$$0 = x(4 - x)$$
$$x = 0 \quad x = 4$$

c.) Draw the curve



Note: Y_{lower} is equal to zero because the intersection or the line touches the x axis

d.) find the area

$$A = \int_a^b (y_{upper} - y_{lower})\,dx$$
$$= \int_0^4 \left[4x - x^2 - (0)\right] dx$$
$$= \int_0^4 (4x - x^2)\,dx$$
$$= \frac{4x^2}{2} - \frac{x^3}{3}\Big|_0^4 = 2x^2 - \frac{x^3}{3}\Big|_0^4$$
$$= \left(2(4)^2 - \frac{4^3}{3}\right) - 0$$
$$= 2(16) - \frac{64}{3}$$
$$= 32 - \frac{64}{3}$$
$$= \frac{96 - 64}{3} = \boxed{\frac{32}{3} \text{ sq. units}}$$

Plane Areas

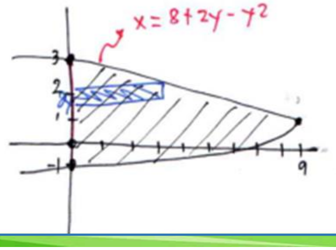
3. Find the area bounded by the parabola $x = 8 + 2y - y^2$ and the y-axis and the lines $y = -1$ and $y = 3$.

a.) Solve for vertex;

$$f(y) = ay^2 + by + c$$
$$x = 8 + 2y - y^2$$
$$a = -1$$
$$b = 2$$
$$y = -\frac{b}{2a}$$
$$y = \frac{-2}{2(-1)}$$
$$y = 1 \rightarrow \text{substitute in}$$
$$x = 8 + 2y - y^2$$
$$x = 8 + 2(1) - 1^2$$
$$x = 9$$
$$V(9, 1)$$

b. limits and point of intersection is already given ; $y = -1$ and $y = 3$

c. Draw the curve



note: $X_{left} = 0$ since the given lines touches the y-axis

d. Solve for area

$$A = \int_a^b (X_{right} - X_{left})\,dy$$
$$A = \int_{-1}^{3} \left[(8 + 2y - y^2) - 0\right] dy$$
$$= \int_{-1}^{3} (8 + 2y - y^2)\,dy$$
$$= 8y + \frac{2y^2}{2} - \frac{y^3}{3} \Big|_{-1}^{3}$$
$$= 8y + y^2 - \frac{y^3}{3} \Big|_{-1}^{3}$$
$$= \left[8(3) + 3^2 - \frac{(3)^3}{3}\right] - \left[8(-1) + 1^2 - \frac{(-1)^3}{3}\right]$$
$$= 24 + 9 - \frac{27}{3} - \left[-8 + 1 + \frac{1}{3}\right]$$
$$= 24 + 9 - 9 - \left[-7 + \frac{1}{3}\right]$$
$$= 24 - \left[\frac{-21 + 1}{3}\right]$$
$$= 24 - \left[-\frac{20}{3}\right]$$
$$= 24 + \frac{20}{3}$$
$$= \frac{72 + 20}{3} = \boxed{\frac{92}{3} \text{ sq. units}}$$

Plane Areas

4. Find the area bounded by the parabola $y^2 = 4x$ and the line $y = 2x - 4$

a.) Solve for Vertex of Parabola

$y^2 = 4x$

$x = \frac{y^2}{4}$

vertex ; $y = -\frac{b}{2a}$

$a = \frac{1}{4}$

$b = 0$

$y = -\frac{0}{2(\frac{1}{4})}$

$y = 0 \rightarrow$ substitute in

$x = \frac{y^2}{4}$

$x = \frac{0^2}{4}$

$x = 0$

$V(0,0)$

b. Solve for the limits and point of intersection

$y^2 = 4x$; $y = 2x - 4$ ↳ substitute this in parabola equation

$y^2 = 4x$

$(2x-4)^2 = 4x$

$4x^2 - 2(2x)(4) + 4^2 - 4x = 0$

$4x^2 - 16x - 4x + 16 = 0$

$[4x^2 - 20x + 16 = 0]\frac{1}{4}$

$x^2 - 5x + 4 = 0$

$(x-1)(x-4) = 0$

$x = 1$ and $x = 4 \rightarrow$ substitute this in the equation of the line to get the value of y.

$x = 1$

$y = 2x - 4$

$y = 2(1) - 4$

$y = 2 - 4$

$y = -2$

$P_1(1,-2)$

$x = 4$

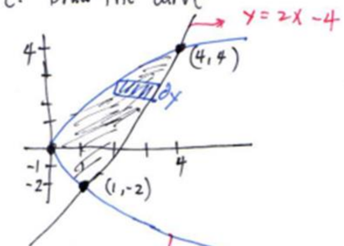
$y = 2(4) - 4$

$y = 8 - 4$

$y = 4$

$P_2(4,4)$

c. Draw the curve



d. Solve for Area

$A = \int_a^b (x_{right} - x_{left})$

$x = \frac{y^2}{4}$

$y = 2x - 4$

$\frac{y+4}{2} = \frac{2x}{2}$

$x = \frac{y}{2} + 2$

$A = \int_{-2}^{4} \left(\frac{y}{2} + 2 - \frac{y^2}{4}\right) dy$

$A = \frac{y^2}{4} + 2y - \frac{y^3}{4(3)} \Big|_{-2}^{4}$

$A = \frac{y^2}{4} + 2y - \frac{y^3}{12} \Big|_{-2}^{4}$

$A = \left[\frac{4^2}{4} + 2(4) - \frac{4^3}{12}\right] - \left[\frac{(-2)^2}{4} + 2(-2) - \frac{(-2)^3}{12}\right]$

$= \left[4 + 8 - \frac{64}{12}\right] - \left[1 - 4 + \frac{8}{12}\right]$

$= \left[\frac{48 + 96 - 64}{12}\right] - \left[\frac{12 - 48 + 8}{12}\right]$

$= \frac{80}{12} + \frac{28}{12}$

$= \frac{108}{12} = \boxed{9 \text{ sq. units}}$

End of Topic

Thank you

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