UNIVERSITY OF SOUTHERN MINDANAO

MATH121E

Calculus 2

Topic Outline

Applications of Definite Integral

- Plane areas
- Areas Between curves
- Other Applications
    - a. Volume
    - b. Work
    - c. Hydrostatic Pressure

Definite Integral Application-WORK

Work is the scientific term used to describe the action of a force which moves an object. When a constant force F is applied to move an object a distance d, the amount of work performed is

W = F. d

The SI unit of force is the newton, (kg·m/s2), and the SI unit of distance is a meter (m). The fundamental unit of work is one newton-meter, or a joule (J). That is, applying a force of one newton for one meter performs one joule of work. In Imperial units (as used in the United States), force is measured in pounds (lb) and distance is measured in feet (ft), hence work is measured in ft-lb

Definite Integral Application-WORK

- Let F(x) be a continuous function [a , b] on describing the amount of force being applied to an object in the direction of travel from distance x=a to distance x=b. The total work (W) done on [a,b] is

$$W=\int_a^b F(x)dx.$$

Definite Integral Application-WORK

Example 1:

A spring has a natural length of 0.2m. A 40-N force is required to stretch (and to hold the spring) to a a length of 0.3m. How much work is done in stretching the spring from 0.35m to 0.38m?

Definite Integral Application-WORK

SOLUTION:

Given:

$l_o = 0.2m$

$l = 0.3m$

$F = 40N$

$Req'd:$

$W = ?\ if\ l_o = 0.35m\ \&\ l = 0.38m$

$Sol'n:$

$x = l - l_o$

$F = kx = k(l - l_o)$

$$\frac{F}{(l - l_o)} = \frac{k\cancel{(l - l_o)}}{\cancel{(l - l_o)}}$$

$$k = \frac{F}{(l - l_o)} = \frac{40N}{(0.3m - 0.2m)} = \frac{40N}{0.1m}$$

$$k = 400\frac{N}{m}$$

$F(x) = kx = 400x$

$$\boxed{\int u^n dx = \frac{1}{n+1}u^{n+1} + C, n \neq -1}$$

$$W = \int_{0.35}^{0.38} 400x\, dx = 400\int_{0.35}^{0.38} x\, dx$$

$$W = 400\left[\frac{1}{1+1}x^{1+1}\right]_{0.35}^{0.38} = 400\left[\frac{1}{2}x^2\right]_{0.35}^{0.38} = 200\left[x^2\right]_{0.35}^{0.38}$$

$$W = 200\left[0.38^2 - 0.35^2\right] = 200(0.144 - 0.123) = 200(0.021)$$

$$\underline{W = 4.2J}$$

$$W = \int_a^b F(x)\, dx$$

K= spring constant
X= displacement

Definite Integral Application-WORK

Example 2:

We have a cable that weighs 2lb/ft attached to a bucket filled with a coal that weighs 800lbs. The bucket is initially at the bottom of a 500ft mine shaft. Determine the amount of work required to lift the bucket all the way up to the shaft.

Definite Integral Application-WORK

From Newton's Law of Motion

SOLUTION:

Given:

$$cable = \frac{weight}{unit\ length} = 2lb/ft$$

$$coal\ weight = 800lb$$

$$h = 500\ ft$$

Req'd:

$$W = ?$$

Sol'n:

$$F = \frac{w}{d}(h-y) + w_{coal}$$

$$F(y) = 2(500-y) + 800$$

$$F(y) = 1000 - 2y + 800$$

$$F(y) = 1800 - 2y$$

$$\boxed{\int u^n dx = \frac{1}{n+1}u^{n+1} + C, n \neq -1}$$

$$W = \int_a^b F(y)dy$$

$$W = \int_0^{500}(1800-2y)dy = \int_0^{500}1800dy - \int_0^{500}2ydy$$

$$W = 1800\int_0^{500}dy - 2\int_0^{500}ydy$$

$$W = 1800[y]_0^{500} - 2\left[\frac{1}{1+1}y^{1+1}\right]_0^{500} = 1800[y]_0^{500} - 2\left[\frac{1}{2}y^2\right]_0^{500}$$

$$W = 1800[y]_0^{500} - [y^2]_0^{500}$$

$$W = 1800[500-0] - [500^2 - 0^2] = 1800(500) - 250{,}000$$

$$W = 900{,}000 - 250{,}000$$

$$\underline{W = 650{,}000\ lb-ft}$$

Definite Integral Application-WORK

Example 3: A cable weighing 3lb/ft is unwinding from a cylindrical drum. If 50ft is already unwound, find the work done by the force of gravity as an additional 250ft unwound.

Definite Integral Application-WORK

$Given:$

$$cable = \frac{weight}{unit\ length} = 3lb/ft$$

$length\ of\ unwound\ cable = 50\ ft$

$additional\ length\ of\ unwound\ cable = 250\ ft$

$Req'd:$

$W = ?$

$Sol'n:$

$x = length\ of\ cable\ unwound\ at\ any\ time$

$$F = \frac{w}{d}(x) = 3x$$

$$W = \int_a^b F(x)dx$$

$$\boxed{\int u^n dx = \frac{1}{n+1}u^{n+1} + C, n \neq -1}$$

$$W = \int_{50}^{300} 3x dx = 3\int_{50}^{300} x dx$$

$$W = 3\left[\frac{1}{1+1}x^{1+1}\right]_{50}^{300} = 3\left[\frac{1}{2}x^2\right]_{50}^{300} = \frac{3}{2}\left[x^2\right]_{50}^{300}$$

$$W = \frac{3}{2}\left[300^2 - 50^2\right]$$

$$W = \frac{3}{2}\left[90{,}000 - 2{,}500\right] = \frac{3}{2}(87{,}500)$$

$$W = 131{,}250\ lb - ft$$

Definite Integral Application-WORK

Reference:

https://www.youtube.com/watch?v=iIsLdMk1z3I

End of Topic

Thank you

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