UNIVERSITY OF SOUTHERN MINDANAO

MATH121E

Calculus 2

Topic Outline

Applications of Definite Integral

- Plane areas
- Areas Between curves
- Other Applications
    - a. Volume
    - b. Work
    - c. Hydrostatic Pressure

Definite Integral Application-VOLUME

Solid of Revolution

A solid of revolution is generated by revolving a plane area about a line, called the axis of rotation

Three Methods in Finding the Volume:

1. Disk Method
2. Washer Method
3. Cylindrical/Shell Method

Definite Integral Application-VOLUME

Disk Method

- This method is useful when the axis of rotation is part of the boundary of the plane area.

Washer Method

- This method is useful when the axis of rotation is not part of the boundary of the plane area.

Cylindrical Method

-Uses a cylinder to compute the volume

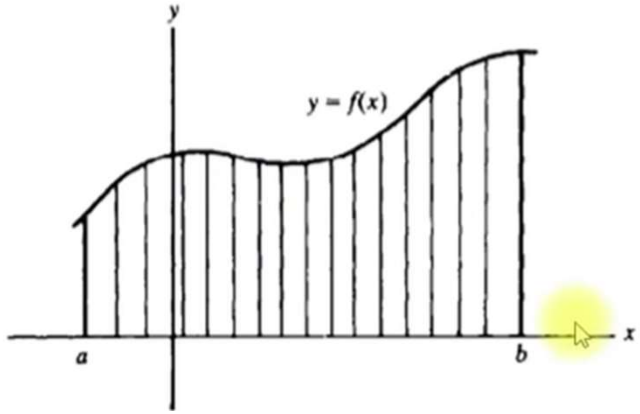
Definite Integral Application-VOLUME

Disk Method

a. When the axis of rotation is the **x-axis** , and the top of the plane area is given by the curve y = f(x), between x=a and x=b , then the volume is given by

$$V = \int_a^b \pi y^2 \, dx = \pi \int_a^b [f(x)]^2 \, dx$$

dx- the Axis of rotation is at x-axis

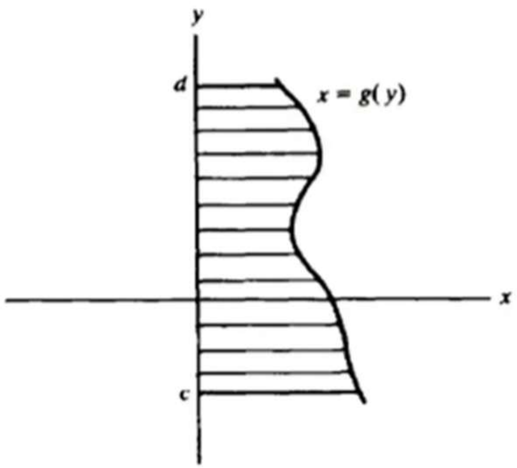


Definite Integral Application-VOLUME

Disk Method

b. When the axis of rotation is the **y-axis** , and one side of the plane area is given by the curve x = g(y), between y=c and y=d , then the volume is given by

$$V = \int_c^d \pi x^2 \, dy = \pi \int_c^d [g(y)]^2 \, dy$$



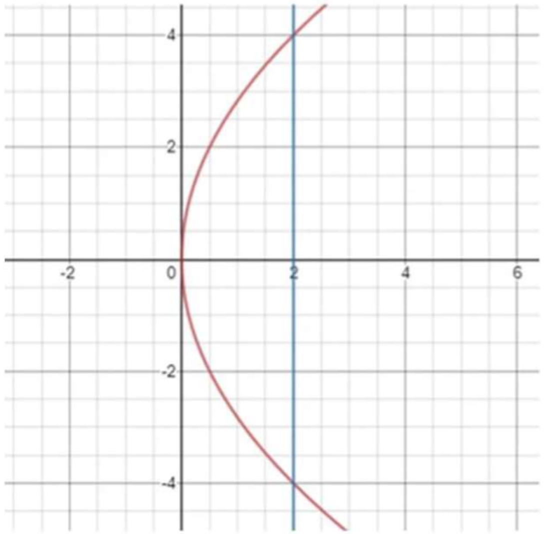
dy- the Axis of rotation is at y-axis

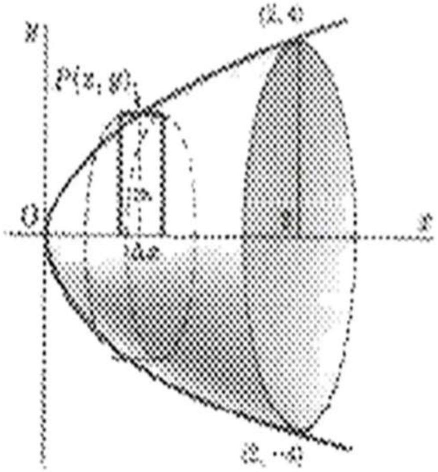
Definite Integral Application-VOLUME

Example 1:

Find the volume generated by revolving the first quadrant area bounded by the parabola $y^2 = 8x$ and its latus rectum $x=2$ about the x-axis

Vertex = (0,0)





Definite Integral Application-VOLUME

$$V = \pi \int_a^b y^2\,dx \rightarrow \text{axis of rotation is } x\text{-axis}$$

$$y^2 = 8x$$

$$V = \pi \int_0^2 y^2\,dx = \pi \int_0^2 8x\,dx$$

$$V = \pi \left[4x^2\right]_0^2 = \pi\left[4(2)^2 - 0\right] = \boxed{16\pi \text{ cubic units}}$$

Definite Integral Application-VOLUME

Example 2:

Determine the volume of the solid rotating the region bounded by

$y = x^2 - 4x + 5$, $x=1$ and $x=4$ and the x-axis

Vertex can be solved using the formula ;

$x = \dfrac{-b}{2a}$

2.) $y = x^2 - 4x + 5$, x-axis

$x = 1$

$x = 4$

$$V = \pi \int_a^b y^2\,dx$$

$$y - 5 = x^2 - 4x$$

$$4 + y - 5 = x^2 - 4x + 4$$

$$y - 1 = (x-2)^2$$

$$V(2,1)$$

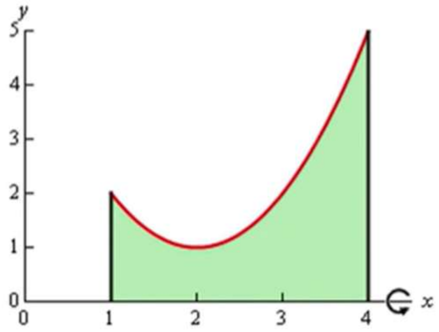
$$V = \pi \int_1^4 (x^2 - 4x + 5)^2\,dx$$

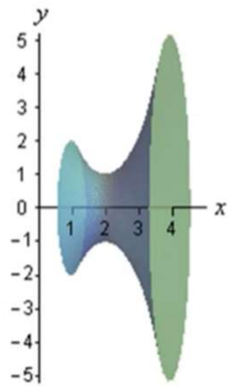
$$V = \pi \int_1^4 (x^4 - 8x^3 + 26x^2 - 40x + 25)\,dx$$

$$V = \pi \left[\frac{1}{5}x^5 - 2x^4 + \frac{26}{3}x^3 - 20x^2 + 25x \right]_1^4$$

Definite Integral Application-VOLUME

$$V = \frac{78}{5}\pi \text{ cubic units}$$



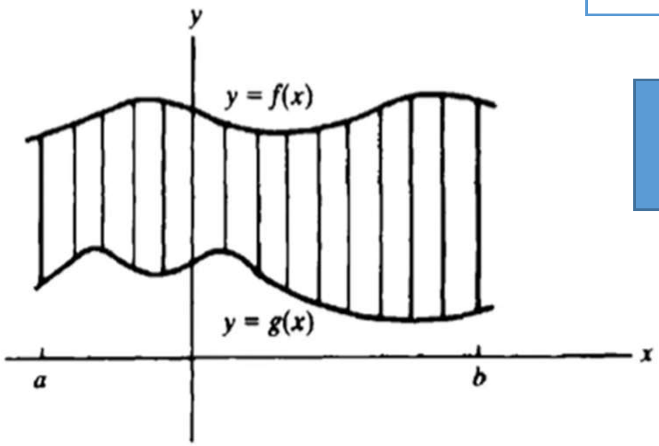


Definite Integral Application-VOLUME

Washer Method

a. If the axis of rotation is the **x-axis**, the upper boundary of the plane is given by y = f(x) and the lower boundary by y=g(x) and the region runs from x=a and x=b , then the volume is given by

$$V = \pi \int_a^b \{[f(x)]\}^2 - \{[g(x)]\}^2 \, dx$$



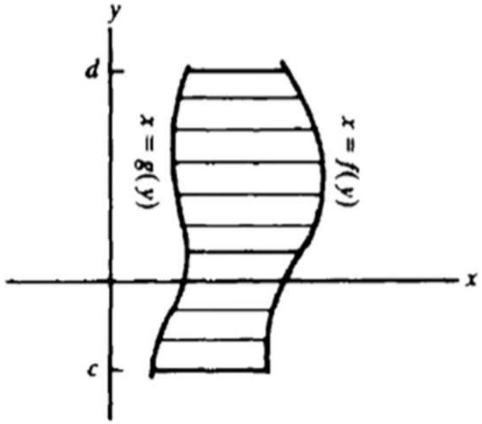
dx- the Axis of rotation is at x-axis

Definite Integral Application-VOLUME

Washer Method

b. Similarly, If the axis of rotation is the **y-axis**, and the plane area is bounded to the right is given by x = f(y) to the left x=g(y) above y=d and below y=c , then the volume is given by

$$V = \pi \int_{c}^{d} \{[f(y)]\}^2 - \{[g(y)]\}^2 \, dy$$



dy- the Axis of rotation is at y-axis

Definite Integral Application-VOLUME

Example 3. Determine the volume of the solid obtained by rotating the portion of the region bounded by $y = \sqrt[3]{x}\ and\ y = \frac{1}{4}x$ that lies in the first quadrant about the y-axis.

Vertex = (0,0)

outer radius   inner radius

$$V = \pi \int_c^d (f(y))^2 - (g(y))^2\, dy$$

$$V = \pi \int_0^2 \left[(4y)^2 - (y^3)^2\, dy \right]$$

$$V = \pi \int_0^2 (16y^2 - y^6)\, dy$$

$$V = \pi \left[\frac{16}{3} y^3 - \frac{1}{7} y^7 \right]_0^2 = \boxed{\frac{512\pi}{21}}$$

$y = \sqrt[3]{x} \Rightarrow x = y^3$

$y = \frac{1}{4}x$

$x = 4y$

$4\cancel{y} = y^{\cancel{3}}$

$4 = y^2$

$y = \pm 2$

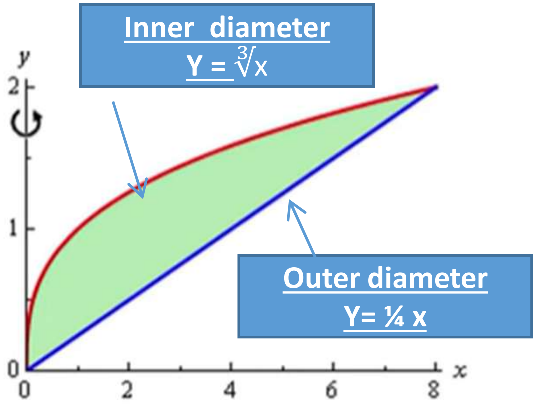
$y = 2$

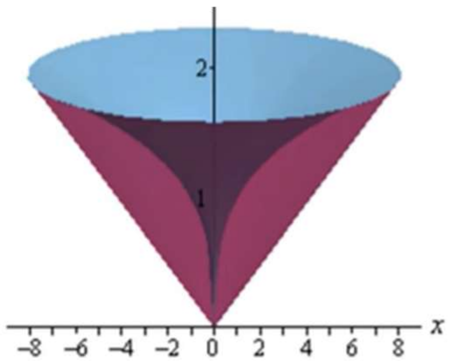
$4y = y^3$

$4y = y^2(y)$

Definite Integral Application-VOLUME

Example 3. Determine the volume of the solid obtained by rotating the portion of the region bounded by $y = \sqrt[3]{x}\ and\ y = \frac{1}{4}x$ that lies in the first quadrant about the y-axis.





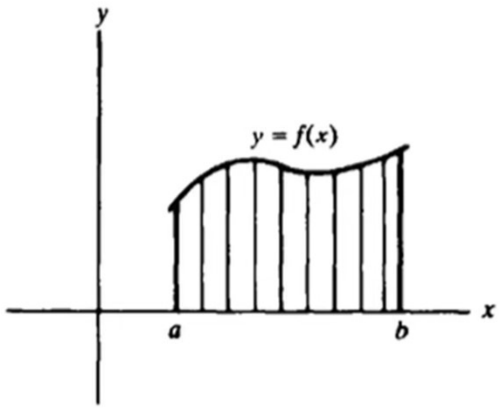
Definite Integral Application-VOLUME

Cylindrical/Shell Method

a. If the axis of rotation is the **y-axis** and the plane area, in the first quadrant, is bounded below by the x-axis above y =f(x) from x=a and x=b then the volume is given by

$$V = 2\pi \int_a^b xy\,dx = 2\pi \int_a^b x\,f(x)\,dx$$

dx- the Axis of rotation is at y-axis

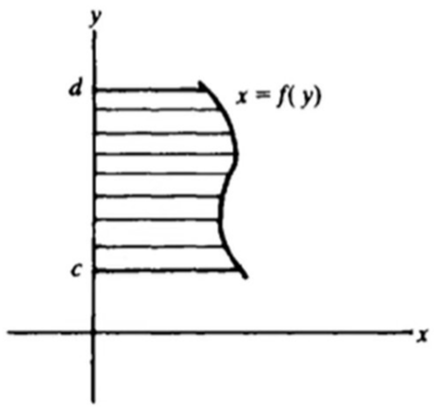


Definite Integral Application-VOLUME

Cylindrical/Shell Method

b. If the axis of rotation is the **x-axis** and the plane area, in the first quadrant, is bound to the left by the y-axis to the right by x =f(y) from y=c and y=d then the volume is given by

$$V = 2\pi \int_c^d xy\, dy = 2\pi \int_c^d y\, f(y)\, dy$$



dy- the Axis of rotation is at x-axis

Definite Integral Application-VOLUME

Example 3:

Determine the volume of the solid obtained by rotating the portion of a region bounded by $y = \sqrt[3]{x}$ and $y = \frac{1}{4}x$ that lies in the first quadrant about the y-axis.

By looking at the equation,
Vertex = (0,0)

$$V = 2\pi \int_a^b xy\,dy \rightarrow x\text{-axis}$$

$$V = 2\pi \int_c^d xy\,dx \rightarrow y\text{-axis}$$

$$y = \sqrt[3]{x} \rightarrow x = y^3$$

$$V = 2\pi \int xy\,dy$$

$$x = 8$$

$$y^3 = 8$$

$$y = 2$$

Definite Integral Application-VOLUME

radius, right

$$V = 2\pi \int_0^2 x\,y\,dy$$

$$y = \sqrt{x} \rightarrow x = y^3$$

$$x = 8$$

$$y^3 = 8$$

$$y = 2$$

$$V = 2\pi \int_0^2 y(8 - y^3)\,dy$$

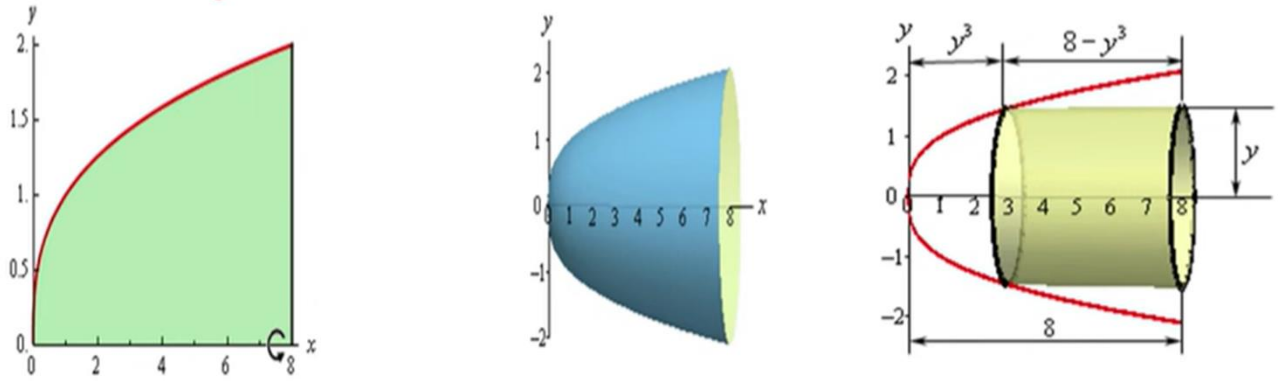
$$V = 2\pi \int_0^2 (8y - y^4)\,dy$$

$$V = 2\pi \left[4y^2 - \frac{1}{5}y^5\right]_0^2 = \boxed{\frac{96\pi}{5}}$$

Definite Integral Application-VOLUME

4. Determine the volume of the solid obtained by rotating the region bounded by $y = \sqrt[3]{x}, x = 8$ and the x-axis about the x-axis.

Ans: $\frac{96\pi}{5}$ cubic units



Definite Integral Application-VOLUME

Reference:

https://www.youtube.com/watch?v=gYjKX124sxo

End of Topic

Thank you

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