Resource Management and Scheduling

Multiple Servers and Load Balancing

In the previous section, we established that for a given total capacity, a single pooled **"super-server" offers the best theoretical performance**. However, this conclusion comes with several important caveats.

This section revisits that debate and explores why modern data centers are built with thousands of independent servers and the crucial role that **load balancing** plays in making such systems efficient.

Why Multiple Servers? Revisiting the Debate

There are **three main arguments** against the single-server ideal: a theoretical objection, a scaling argument, and a practical objection.

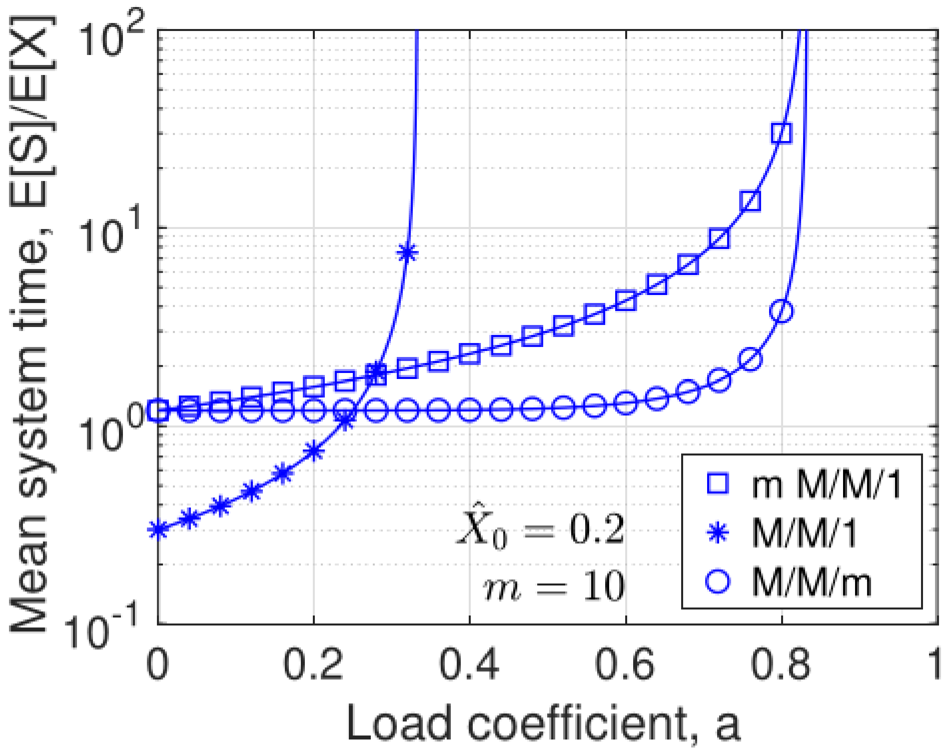
1. The Theoretical Objection: Overhead and Variability

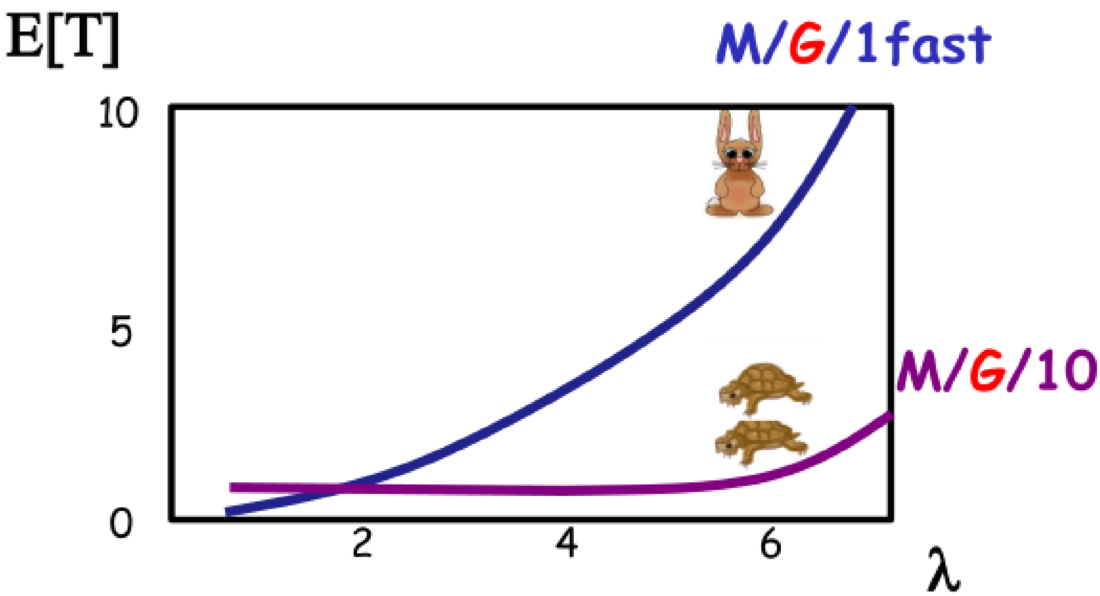
The single-server model assumes that service **capacity scales perfectly**. In reality, many services have a **fixed overhead** component that **does not scale**.

- **Service Time with Overhead:** A more realistic model for service time (X) might be the sum of a **fixed overhead** (X_0) and a **scalable component** (X_1/m):

$$X = X_0 + X_1/m$$

    - The average service time becomes: ☑ = X_0 + \frac{1}{m \mu}$$
- **Performance Impact:** The graphs show that when you factor in this overhead, or when the service time has very high variability, the **conclusion changes**.
    - A system of multiple slower servers (**M/G/k**) can actually provide a lower mean response time than a single, pooled super-server (**M/G/1fast**).





Service time PDF has SCOV = 100.

2. The Scaling Argument: The Halfin-Whitt Regime

Is it possible to design a system that achieves both **near-perfect utilization** ($\rho \to 1$) and **near-zero waiting time** ($E[W] \to 0$)?

- It seems like a paradox, but the answer is **YES**, in a large-scale, many-server system.

This is described by the **Halfin-Whitt** (or Quality-and-Efficiency-Driven) asymptotic regime.

- **The Setup:** We consider a system where the **number of servers**, N, is **only slightly larger** than the **mean offered traffic load**, A.

Specifically, the excess capacity is proportional to the square root of the load:

$$N = A + \beta\sqrt{A}$$

Where β is a fixed parameter that controls how much spare capacity the system has.

- **Deriving the Utilization Coefficient (ρ):** The utilization is defined as the total load divided by the number of servers, $\rho = A/N$.
  - By substituting the Halfin-Whitt condition for N, we get:

    $$\rho = \frac{A}{A + \beta\sqrt{A}}$$

  - Dividing the numerator and denominator by A gives the final expression:

    $$\rho = \frac{1}{1 + \beta/\sqrt{A}}$$

    From this, we can see that as the system scales up (as $A \to \infty$), the term $\beta/\sqrt{A}$ goes to zero, and therefore **utilization ρ approaches 1**.

- **Deriving the Mean Wait (W):** The normalized mean waiting time in an M/M/N system is given by

  $$W = \frac{C(N, A)}{N - A}$$

  where $C(N, A)$ is the ***Erlang C formula*** for the probability of queuing.

  In the Halfin-Whitt regime, we know that $N - A = \beta\sqrt{A}$.

  - Advanced asymptotic analysis (shown on slide 210) provides a simplified expression for the Erlang C formula in this regime.
  - Substituting that result into the waiting time formula gives (from slide 211):

    $$W \sim \frac{\phi(\beta)}{\beta[\beta\Phi(\beta) + \phi(\beta)]}\frac{1}{\sqrt{A}}$$

    Where $\phi(\cdot)$ and $\Phi(\cdot)$ are the **probability density function** and **cumulative distribution function** of a standard normal distribution, respectively.
  - The crucial part of this result is the $1/\sqrt{A}$ term.
  - It shows that as the system scales up (as $A \to \infty$), the **mean waiting time W approaches 0**.

- **The Result:** As the system scales up (as $A \to \infty$), something remarkable happens:
  - The **server utilization**, $\rho = A/N$, approaches **100%**.
  - The **normalized mean waiting time**, W, approaches **0**.

- **Example (Slide 213):**

  - With an offered load of $A = 100$ and $\beta = 1$, we need $N = 110$ servers. This system runs at over **90%** utilization, but the mean wait time is only **2.23%** of the mean service time.
  - With an offered load of $A = 900$ and $\beta = 2$, we need $N = 960$ servers. This system runs at **93.75%** utilization, and the mean wait time is a negligible **0.04%** of the mean service time.

This proves that large-scale, multi-server systems can be both extremely efficient and provide excellent quality of service, a key reason for their use in data centers.

3. The Practical Objection: COTS Hardware and Failures

The most compelling reason for using multiple servers is practical.

- **Feasibility and Cost:** A single "super-server" with the combined power of thousands of machines may be physically impossible or prohibitively expensive to build.
- **Reliability:** A single super-server is also a **single point of failure**.
- **The Data Center Philosophy:** The modern approach is to achieve massive processing capacity by aggregating thousands of inexpensive **Commercial Off-The-Shelf (COTS)** servers.

This approach, however, introduces **two fundamental engineering challenges** that define modern data center design:

1. **Communication:** How do you efficiently interconnect thousands of servers? (This is solved by the **DCN topologies** we studied previously).
2. **Load Balancing:** How do you intelligently distribute incoming tasks across these thousands of servers?

Single vs. Multiple Servers: A Reconciliation

A critical question arises after concluding that large, multi-server systems offer superior scalability and performance: Why did we spend time analyzing scheduling policies for a single server?

The answer lies in the layered nature of modern data centers and the pervasive use of virtualization. The theories are not contradictory; they apply at different levels of the resource management stack.

- **Single-Server Theory is for Intra-Machine Management:** The scheduling policies studied (FCFS, PS, SRPT, etc.) are directly applicable to how the operating system and hypervisor manage competing processes **within a single physical machine**. Virtual Machines (VMs), containers (like Docker), and individual applications all contend for the CPU, memory, and I/O resources of one physical server.

Multiple Servers and Load Balancing

While single-server scheduling theory is crucial for managing resources *within* a single machine, a data center is a massive farm of many servers.

The **key challenge** at this scale is **load balancing**: *the intelligent distribution of incoming tasks across all available servers to optimize performance.*

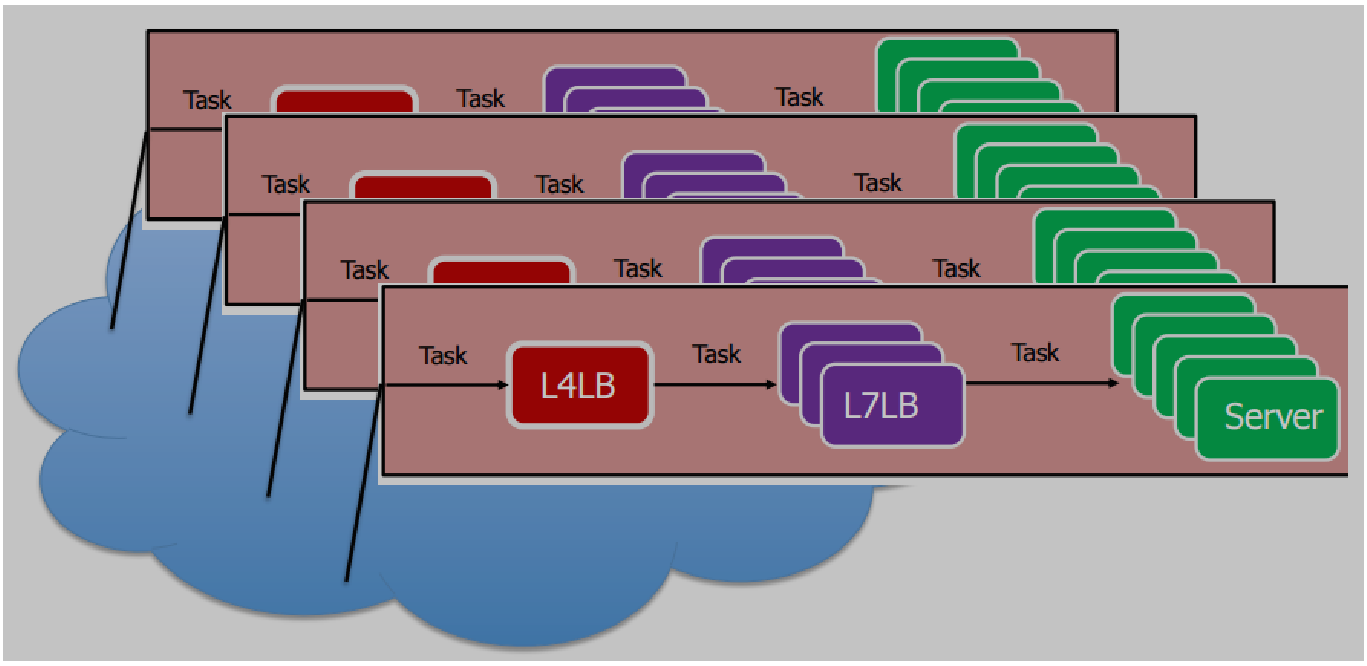
Load Balancing Architecture

The component responsible for distributing work across the server farm is the **Load Balancer (LB)**.

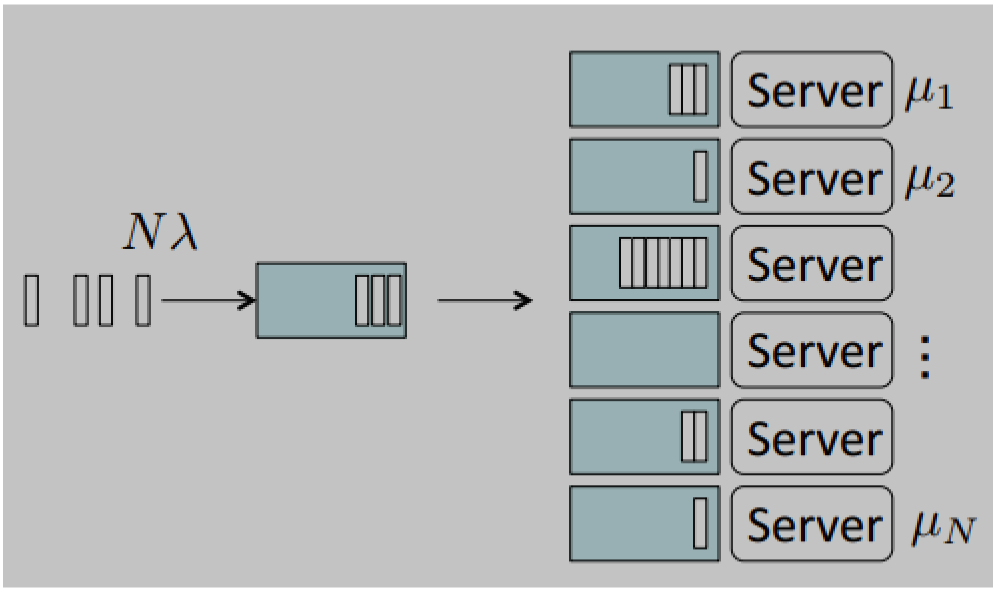
- It is a critical piece of infrastructure for **reducing latency** and **improving overall throughput**.

The architecture often involves **multiple layers of load balancing**:

1. Incoming traffic is first handled by a **Level 4 Load Balancer (L4LB)**, which operates at the **transport layer** and can route traffic based on IP addresses and TCP/UDP ports.
2. This L4LB might direct traffic to a server that is itself running a **Level 7 Load Balancer (L7LB)**, which operates at the **application layer** and can make more sophisticated routing decisions based on application-specific data like HTTP headers or cookies.
3. The L7LB then dispatches individual tasks to be processed.



The System Model



To analyze load balancing policies, we use a **standardized queueing model**. This model consists of a **central dispatcher** (*load balancer*) that routes incoming tasks to one of N parallel servers, each with its own queue.

The **key parameters** of this model are:

- N: The **total number of servers** in the system.
- $\Lambda = N\lambda$: The **mean total arrival rate** of tasks to the dispatcher.
- μ_j: The **task processing rate** of an individual server j.
  - This allows for **heterogeneous** systems where servers can have different speeds.
- $\mu = \sum_{j=1}^{N} \mu_j$: The **total service capacity** of the entire system.

Once a task is dispatched to a server, it enters that server's queue and remains there until its service is complete.

Performance Goal

The primary goal of any load balancing policy is to **intelligently choose the right server** for each incoming task.

- In this context, "*right*" has a very specific meaning: the choice that results in the **lowest possible latency** for the task.

Delay Components

The **total end-to-end latency** experienced by a task can be broken down into **two distinct components**:

1. **Dispatching Delay:** The time elapsed from a task's arrival at the dispatcher to the moment it joins a server's queue.
   - For **"push" policies** that need to query servers, this delay is non-zero.
   - For **"pull" policies** where the dispatcher already knows of an available server, this delay can be effectively zero.
2. **Response Time:** The time a task spends at the server.
   - This is the **sum** of the time spent waiting in the server's queue plus the actual service (processing) time.

- **Key Insight**: A **good load balancing** policy aims to **minimize the sum of these two components**.

Illustrative Example: Load Balancing with Heterogeneous Servers

To understand the **trade-offs** in load balancing, we can analyze a simple but insightful scenario involving **two servers** with **different speeds**.

The Setup

- **Arrivals:** Tasks arrive at a central dispatcher. The arrival process is Poisson, meaning tasks arrive randomly and independently, with a long-term average rate of Λ tasks per second.
- **Servers:** There are two servers available to process these tasks. They are heterogeneous (have different speeds):
  - Server 1 is the faster server, with a service rate of μ_1.
  - Server 2 is the slower server, with a service rate of μ_2.
  - The relationship between their speeds is given by $\mu_1 = \alpha \cdot \mu_2$, where $\alpha > 1$ is the speed ratio. For example, if $\alpha = 2$, Server 1 is twice as fast as Server 2.

- **Dispatching Policy:** The dispatcher uses a very simple probabilistic rule. For each arriving task, it flips a biased coin:
  - With probability p, it sends the task to Server 1.
  - With probability $1-p$, it sends the task to Server 2.
- **The Goal:** Our objective is to determine the best strategy for choosing the probability p, particularly as the performance gap between the servers (α) becomes large.

Mathematical Formulation

To analyze this system, we model each server as an independent **M/M/1 queue**. This model is appropriate because:

- **"M" (Markovian/Poisson Arrivals):** When a Poisson stream of arrivals with rate Λ is split probabilistically, the resulting streams are also Poisson.
  - Arrival rate for Server 1 is $\lambda_1 = p\Lambda$.
  - Arrival rate for Server 2 is $\lambda_2 = (1-p)\Lambda$.
- **"M" (Markovian/Exponential Service):** We assume the service times are exponentially distributed, with mean $1/\mu_1$ and $1/\mu_2$ respectively.
- **"1" (Single Server):** Each queue has a single server.

The **mean system time** (or response time) for a standard M/M/1 queue is given by the formula:

$$E[S] = \frac{1}{\mu - \lambda}$$

The overall average system time for our two-server system, $E[S]$, will be the weighted average of the system times at each server, where the weights are the probabilities of a task being sent to that server.

Step 1: Calculate the Mean System Time for each server.

- For Server 1: $E[S_1] = \frac{1}{\mu_1 - \lambda_1} = \frac{1}{\mu_1 - p\Lambda}$
- For Server 2: $E[S_2] = \frac{1}{\mu_2 - \lambda_2} = \frac{1}{\mu_2 - (1-p)\Lambda}$

Step 2: Calculate the Overall Mean System Time. The overall mean time is p times the average time at Server 1 plus $1-p$ times the average time at Server 2.

$$E[S] = p \cdot E[S_1] + (1-p) \cdot E[S_2]$$

Substituting the formulas from Step 1 gives the final expression:

$$E[S] = \frac{p}{\mu_1 - p\Lambda} + \frac{1-p}{\mu_2 - (1-p)\Lambda}$$

Step 3: Define the Stability Conditions. For a queue to be stable (i.e., not grow infinitely long), its arrival rate must be less than its service rate ($\lambda < \mu$). We must satisfy this for both servers simultaneously.

- For Server 1: $p\Lambda < \mu_1 \implies p < \frac{\mu_1}{\Lambda}$
- For Server 2: $(1-p)\Lambda < \mu_2 \implies 1-p < \frac{\mu_2}{\Lambda} \implies p > 1 - \frac{\mu_2}{\Lambda} \implies p > \frac{\Lambda - \mu_2}{\Lambda}$

Since p is a probability, it must also be between 0 and 1. Combining these conditions gives us the valid range for p:

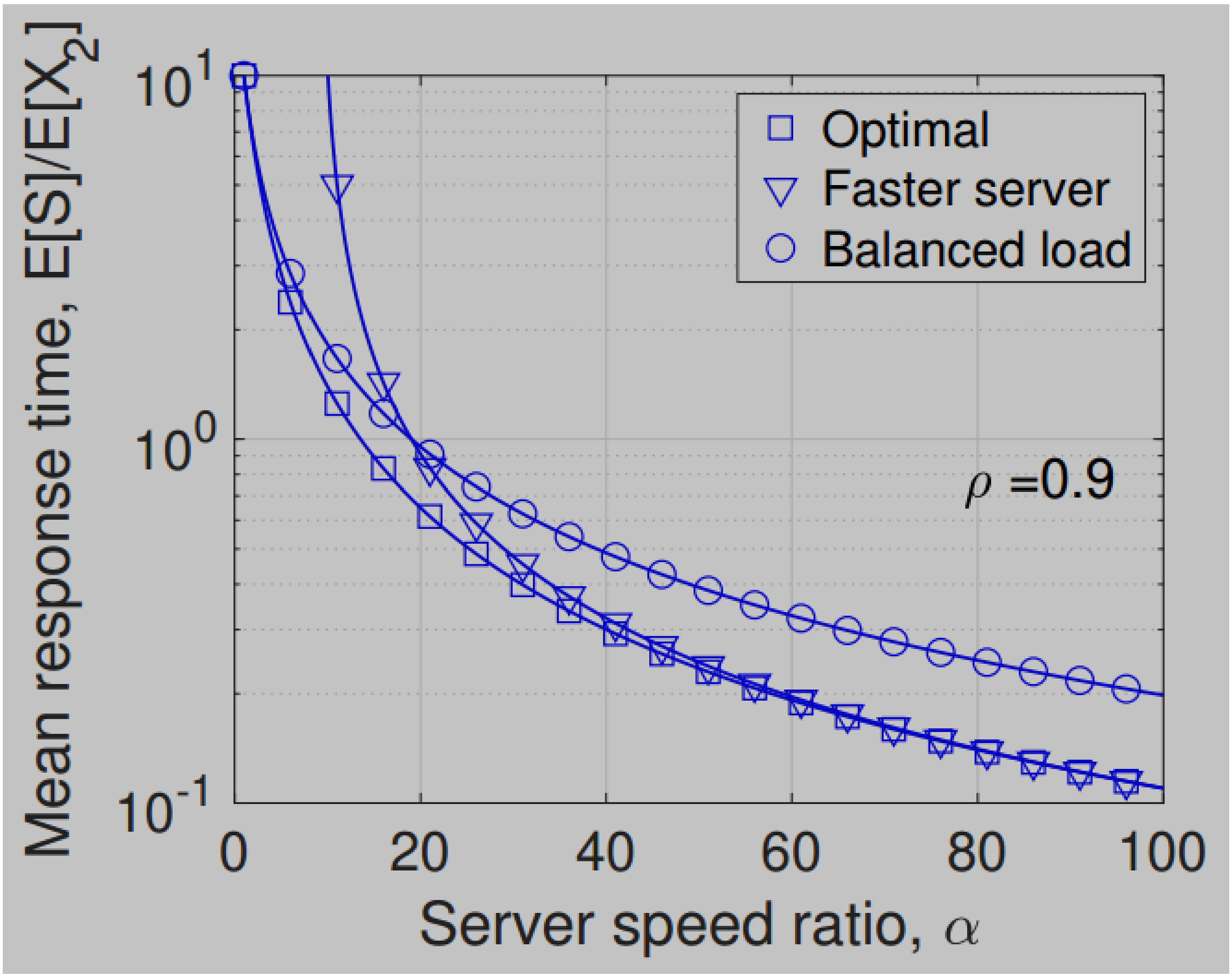
$$p_{min} < p < p_{max}$$

Where:

- $\large p_{min} = \max\left(0, \frac{\Lambda - \mu_2}{\Lambda}\right)$
- $\large p_{max} = \min\left(1, \frac{\mu_1}{\Lambda}\right)$

Numerical Results

We now compare three strategies for choosing p by plotting their performance as the server speed ratio (α) increases.



Analysis of the Policies:

- **Balanced Load:** This policy aims to make the server utilizations equal ($\rho_1 = \rho_2$).
  - It performs very well when servers are of similar speed (α is small).
  - However, as α grows, its performance becomes extremely poor because it continues to send a significant fraction of jobs to the increasingly slow server, creating a **severe bottleneck**.
- **Faster Server:** This policy sends all traffic to the faster server ($p = 1$).

    - It performs poorly when α is small because it ignores the available capacity of the second server.
    - But as α becomes very large, this policy becomes the optimal strategy, as the contribution of the slow server becomes negligible.
- **Optimal:** This policy calculates the value of p that minimizes the $E[S]$ formula at each point.
    - It shows the true best-possible performance, transitioning smoothly from a balanced approach when speeds are similar to an all-or-nothing approach when one server is vastly superior.

Key Takeaway: This simple example demonstrates that there is **no single "best" static load balancing policy**.

- The optimal strategy depends on the specific characteristics of the servers.
- A good load balancing algorithm must be able to adapt to the state of the system.

Formal Performance Goals

To rigorously compare different load balancing policies, we need formal definitions of what makes a policy "good".

- We evaluate policies against two primary goals: **throughput optimality** (stability) and **delay optimality** (performance).

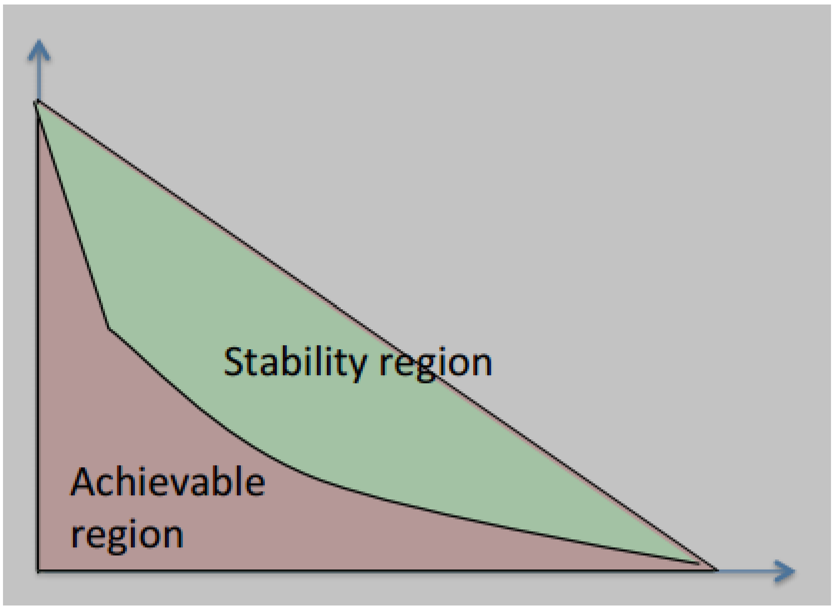
Throughput Optimality

Throughput optimality is the most fundamental property of a load balancing policy. It defines whether a policy can keep the system **stable** under a given workload.

1. **Stability:** A system is considered stable if its **queues do not grow infinitely long** over time.
    - *Formally*: The probability that the total number of tasks in the system, $\sum_{j=1}^{N} Q_j(t)$, exceeds any arbitrarily large number C goes to zero as time goes to infinity.

$$\textbf{Stability:} \quad \limsup_{C\to\infty} \limsup_{t\to\infty} \mathcal{P}\left(\sum_{j=1}^{N} Q_j(t) > C\right) = 0$$

2. **Stability Region vs. Achievable Region:**
    - The **Stability Region** is the set of **all possible total arrival rates**, Λ, for which the system is *theoretically* stable.
        - For **our system**, this condition is simply that the total arrival rate must be less than the total service capacity: $\Lambda < \sum_{j=1}^{N} \mu_j$.
    - The **Achievable Region** is the set of arrival rates for which a *specific load balancing policy* is able to keep the **system stable**.



3. **Definition of Throughput Optimality:** A load balancing policy is said to be **throughput optimal** if its **achievable region is identical to the system's stability region**.
   - In other words, a throughput-optimal policy can successfully manage any workload that doesn't exceed the system's total physical capacity.

Delay Optimality

While throughput optimality ensures a policy doesn't crash the system, it doesn't say anything about performance.

Delay optimality: measures how effective a policy is at minimizing the response time for tasks.

To define this, we must first establish the theoretical best-case performance, which is represented by the **Resource Pooled System**.

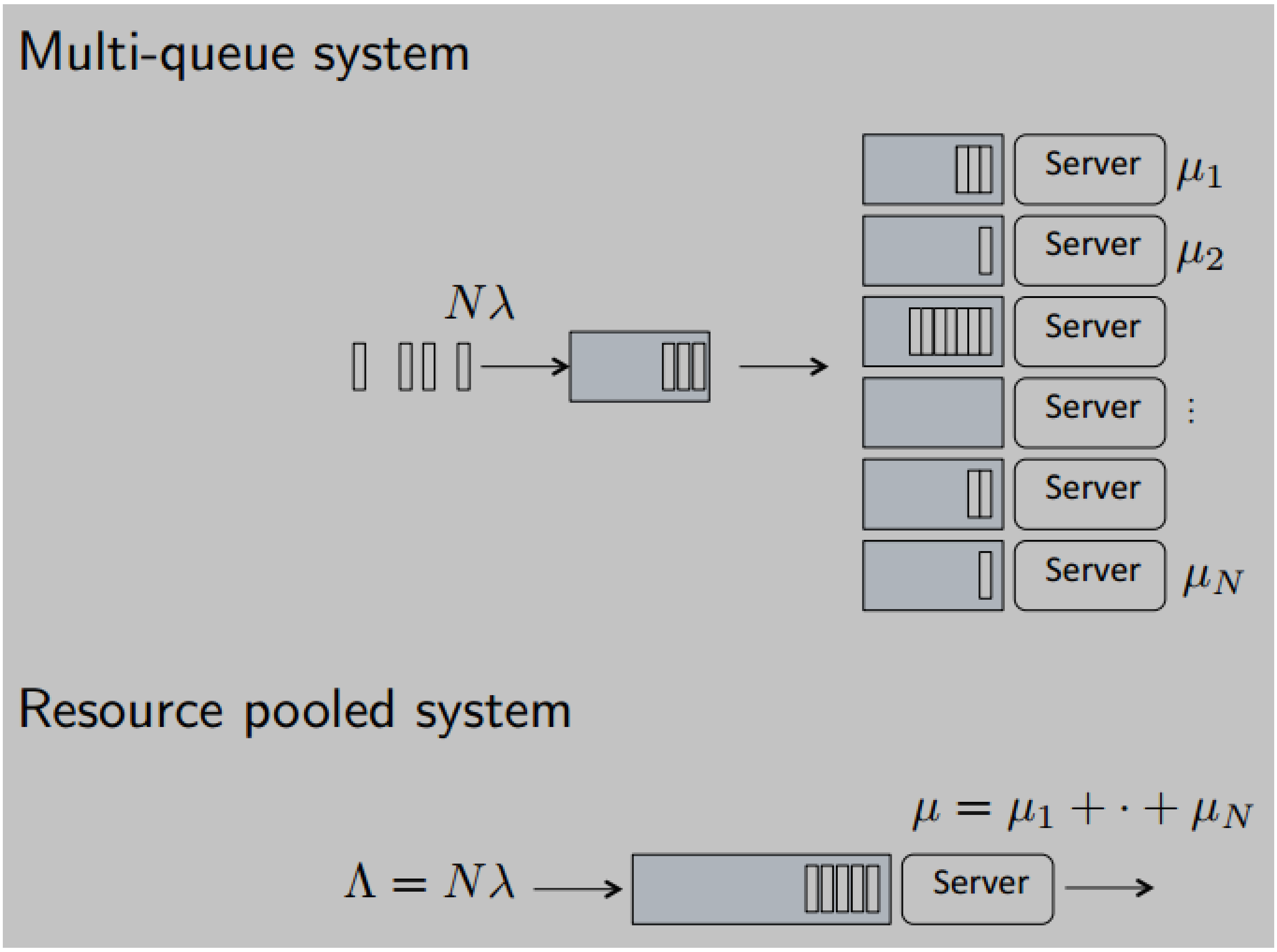
- **Multi-queue system:** This represents the **real-world** scenario we are analyzing.
  - A dispatcher distributes an incoming stream of tasks with a total arrival rate of $N\Lambda$ among N individual servers.
  - Each server ($\mu_1, \mu_2, ..., \mu_N$) has its own independent queue.

2. **Resource pooled system:** This is a **hypothetical, ideal** model.
   - It imagines that all the processing power of the individual servers is combined or "***pooled***" into a **single, powerful "super-server"**.
   - This ideal server has a total service capacity of $\mu = \sum_{k=1}^{N} \mu_k$ and handles the total arrival rate $\Lambda^* = N\Lambda$.

- **The Performance Lower Bound:** A fundamental theorem in queueing theory states that the average number of tasks in a multi-queue system, $E[Q]$, can never be lower than the average number of tasks in its corresponding resource-pooled system, $E[q]$.

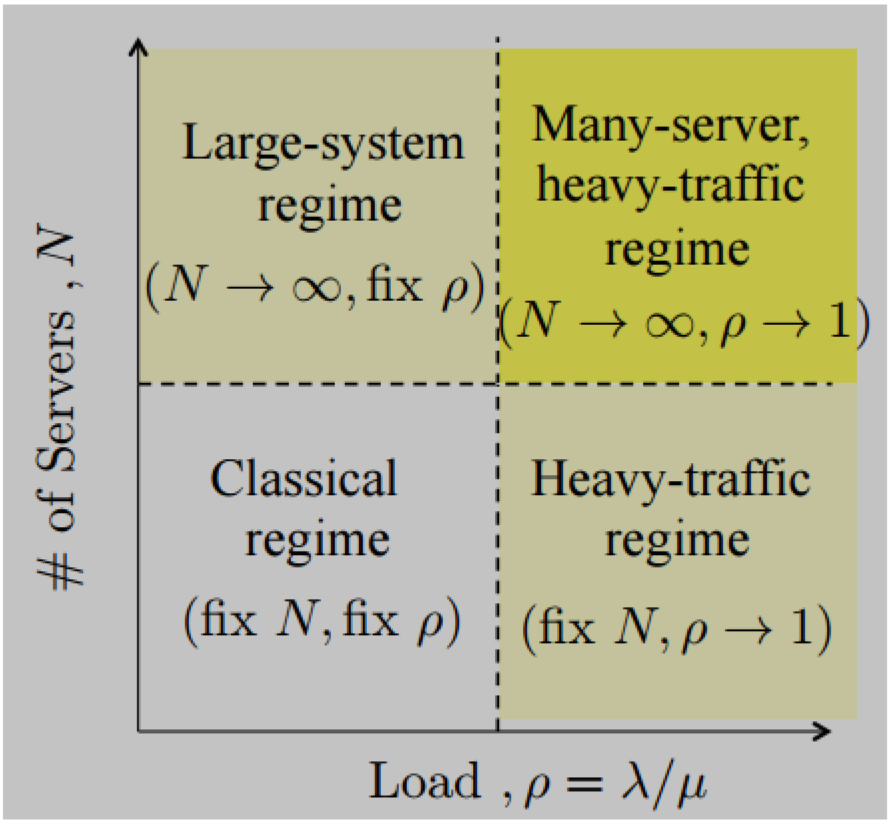
$$E\left[\sum_{n=1}^{N} Q_n(t)\right] \geq E[q(t)]$$

This establishes the resource-pooled system as the gold standard for delay performance.



Heavy-Traffic (HT) Delay Optimality

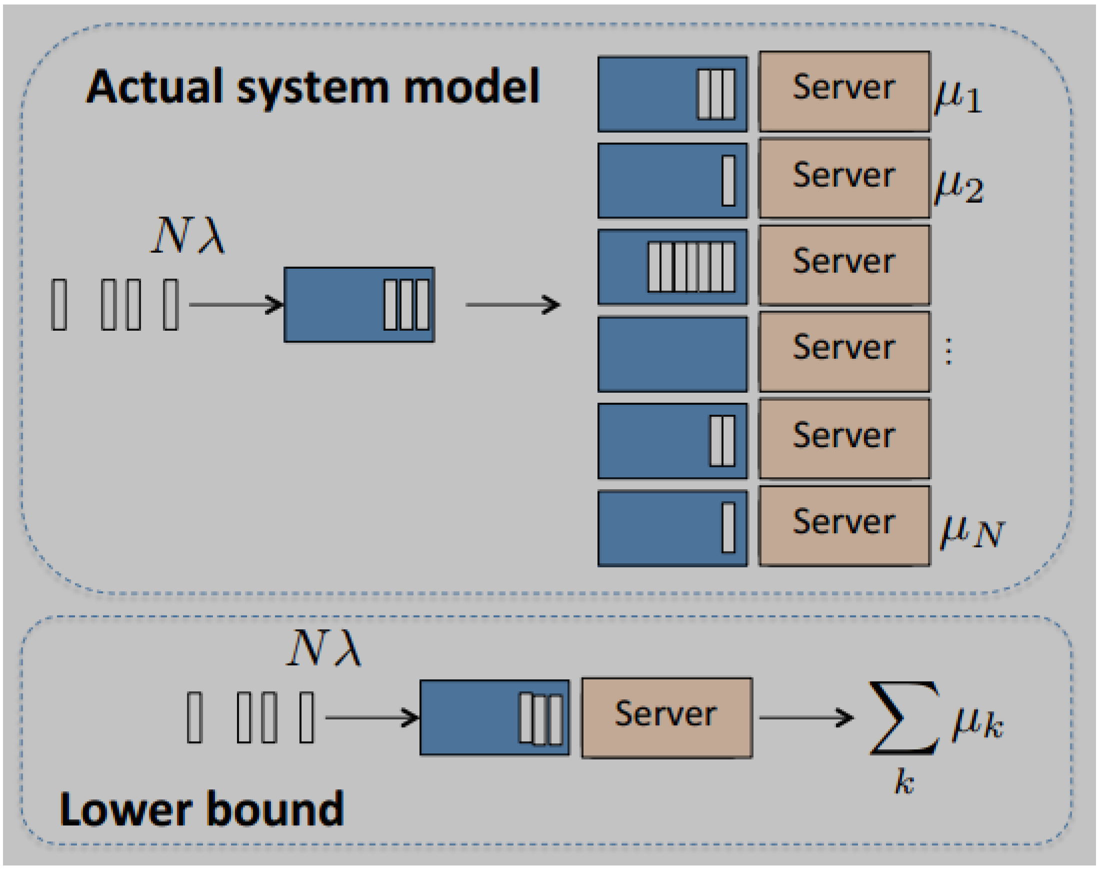
The most important and challenging performance regime for a load balancer is the **heavy-traffic (HT) regime**, where the system is operating very close to its full capacity ($\rho \to 1$).



- **Definition of HT Delay Optimality:** A load balancing policy is considered **HT delay-optimal** if its average delay performance converges to the (best possible) performance of the resource-pooled system as the load approaches 1.
  - *Formally*: we define $\epsilon = \mu - \Lambda$ as the small amount of spare capacity.
    - The HT regime is the limit as $\epsilon \to 0$.
    - A policy is ***HT delay-optimal*** if:

$$\lim_{\epsilon \to 0} \epsilon E[Q] = \lim_{\epsilon \to 0} \epsilon E[q]$$

- **Practical Relevance:** This concept is crucial because almost any load balancing policy will perform well when the system is lightly loaded. The **true differentiator** between policies is their ability to maintain **low latency and stability** when the system is **under stress**, which is precisely the condition described by the heavy-traffic regime.



Classification of Load Balancing Policies

Load balancing schemes can be categorized based on which entity—the dispatcher or the server—initiates the task assignment process.

- **Pull Policies:** The **servers** themselves take the initiative.
  - An idle or under-loaded server sends a message to the dispatcher to "pull" a new task.
  - The dispatcher maintains a state representing which servers are available and assigns new tasks based on these announcements.
- **Push Policies:** The **dispatcher** takes the initiative.
  - It actively polls or queries a subset of servers to gather information about their current load (e.g., queue length).
  - Based on the replies, the dispatcher decides where to "push" the next task.

Overview of Famous Policies

The most well-known load balancing policies can be classified based on whether they use server state information and whether they know the job size in advance (***anticipative vs. non-anticipative***).

- **Non-Anticipative, No State Information:**
  - **RANDOM**
  - **ROUND-ROBIN**
- **Non-Anticipative, With State Information:**

- **JSQ** (Join Shortest Queue)
- **JIQ** (Join Idle Queue)
- **JBT** (Join Below Threshold)

- **Anticipative, With State Information:**
    - **SITA** (Size-Interval Task Assignment)
    - **LWL** (Least Work Left)

The performance ranking of these policies is not **straightforward**; it depends heavily on factors like:

- Variability of **job sizes**
- **Scheduling policy** used at each individual server (e.g., FCFS or PS)
- Whether tasks are **dispatched immediately** or **held in a central queue**.

Simple Algorithms: RANDOM and ROUND-ROBIN

These are the **simplest policies** as they do not require any communication or state information from the servers.

- **RANDOM:** Upon task arrival, the dispatcher selects one of the N servers uniformly at random and sends the task there.
    - **Pros:** Requires **zero state information** and has **no messaging overhead**.
    - **Cons:** Performance is known to be **poor under heavy traffic**, especially when job sizes have **high variability**, as it can easily lead to situations where some servers are overloaded while others are idle.
- **ROUND-ROBIN:** The dispatcher assigns tasks to servers in a fixed, cyclic order ($1, 2, ..., N, 1, 2, ...$).
    - **Pros:** Similar to RANDOM in its simplicity and lack of messaging.
    - **Cons:** Performance is also similar to RANDOM. It only requires minimal state at the dispatcher to remember which server was assigned the last task.

Core "Push" Policies

"*Push*" policies are **proactive**; the dispatcher gathers information about the state of the servers *before* sending a task.

Join Shortest Queue (JSQ)

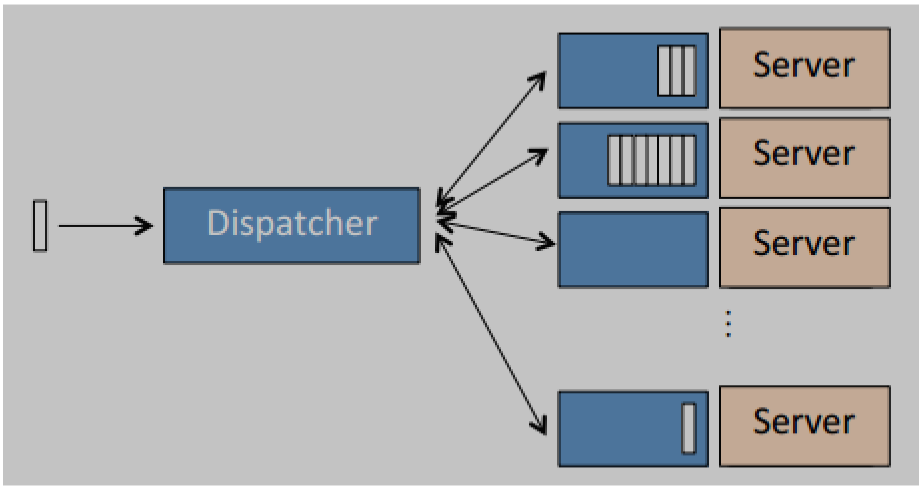
Join Shortest Queue (JSQ) is the most well-known and theoretically optimal "push" policy.

- **Policy:** Upon a task's arrival, the dispatcher polls **all** N servers to determine their current queue length.
    - It then sends the task to the server with the absolute shortest queue.
- **Pros:**
    - JSQ is proven to be **delay-optimal** in the heavy-traffic regime.

    - It provides the best possible average delay performance among policies that do not use job size information.

- **Cons:**

    - **High Message Overhead:** The policy is extremely "chatty."
        - It requires querying every server for every single incoming task, resulting in $2N$ **messages** (a query and a response for each server) **per arrival**.
        - This makes it impractical for systems with a large number of servers.
    - **Dispatching Delay:** There is a **non-zero delay** between when a task arrives and when it is dispatched, as the dispatcher must wait to receive the queue length information from all servers.



Power-of-d (PoD)

The Power-of-d (PoD) policy, often called Power-of-2 when $d = 2$, is a highly effective and practical compromise that captures most of the benefits of JSQ with a **fraction of the overhead**.

- **Policy:** Upon a task's arrival, the dispatcher polls a small, fixed number, d, of **randomly chosen servers** (e.g., $d = 2$).

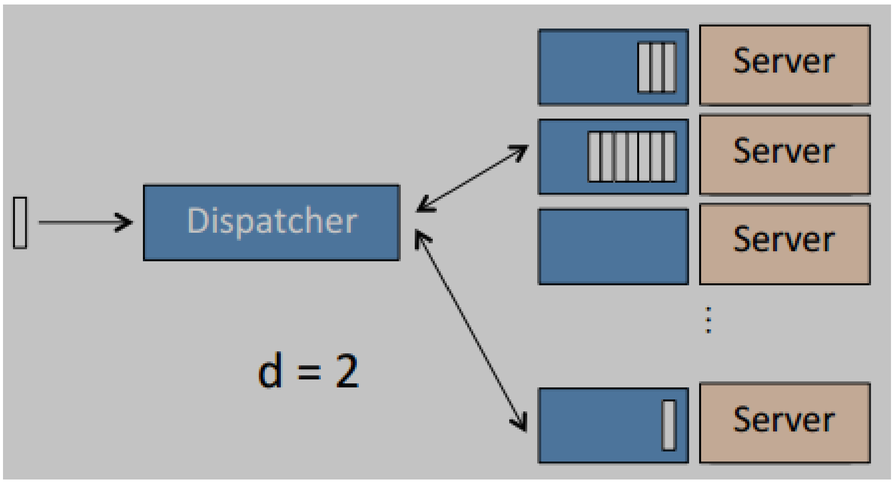
    - It then sends the task to the server with the shortest queue *among that small sample*.

- **Pros:**

    - Achieves a dramatic performance improvement over a simple RANDOM policy.
        - This phenomenon is often called the "power of two choices."
    - Like JSQ, it is **asymptotically delay-optimal** in the heavy-traffic regime.

- **Cons:**

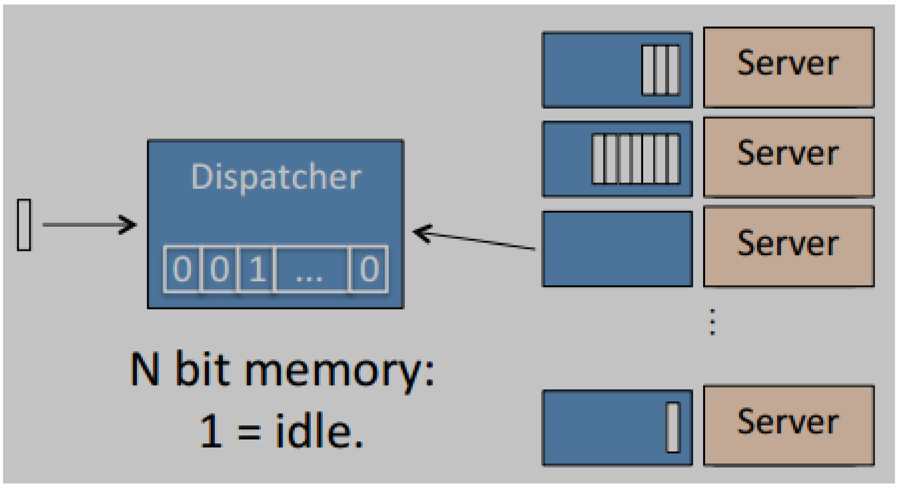
    - It still incurs a **non-zero dispatching delay** while it polls the d servers.



Join Idle Queue (JIQ)

In contrast to "push" policies, "pull" policies are initiated by the servers themselves when they become available for work.

The canonical example is *Join Idle Queue* (**JIQ**).



- **Policy:** The JIQ algorithm works as follows:
    - The dispatcher maintains a simple state, often a bitmap or a list, indicating which of the N servers are currently idle.
    - When a server finishes processing all tasks in its queue and becomes idle, it sends a message to the dispatcher to signal its availability.
    - When a new task arrives at the dispatcher, it checks its list of idle servers.
        - If **one or more servers are idle**, it sends the task to a randomly chosen server from that list.
        - If **all servers are busy**, the dispatcher defaults to a simple policy like **RANDOM**, sending the task to a random server from the entire pool.
- **Pros:**
    - **Low Message Overhead:** JIQ is very efficient in terms of communication.
        - At most, one message is sent per task departure (when a server becomes idle), which is significantly less than polling-based "push" policies.

- **Zero Dispatching Delay:** When an idle server is available, the dispatcher can send a task to it immediately without any polling delay.
- Under **moderate loads**, JIQ often provides **better delay performance** than polling-based policies like Power-of-d.

- **Cons:**

  - The primary drawback is its **poor performance in the heavy-traffic (HT) regime**.
    - Once all servers are busy, the "idle list" at the dispatcher is always empty.
    - In this state, JIQ's behavior degenerates to that of the simple and inefficient **RANDOM** policy, leading to high delays.

The "Big Picture": A Policy Dilemma

The policies discussed so far present a clear **trade-off**:

- **Push Algorithms (JSQ, PoD)**
  - **Pro**: These are **delay-optimal in the heavy-traffic regime**, providing excellent performance when the system is under stress.
  - **Cons**: However, they suffer from **non-zero dispatching delays** and can have **high messaging overhead**.
- **Pull Algorithms (JIQ)**
  - **Pro**: These have **zero dispatching delay** and very **low message overhead**.
  - **Cons**: However, their performance is poor in the heavy-traffic regime, as they devolve into a simple RANDOM policy.

The **goal**, therefore, is to design a policy that combines the best properties of both approaches: **the low overhead of "pull" policies with the high-performance guarantees of "push" policies.**

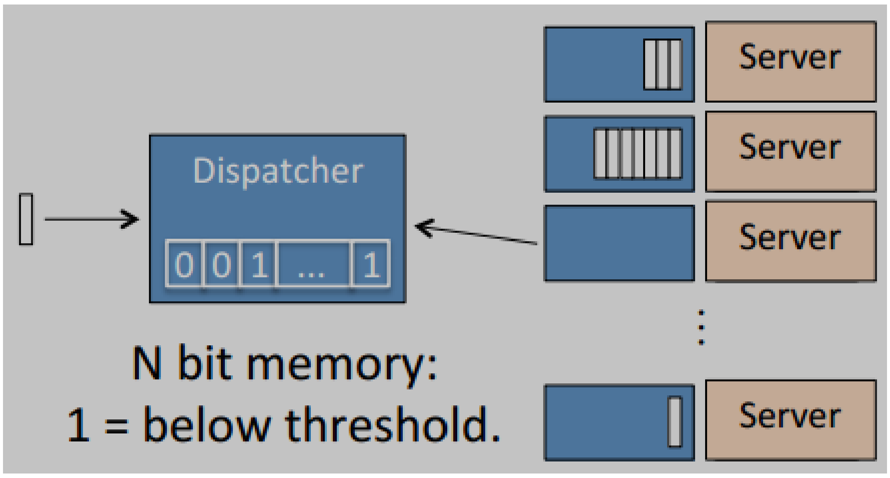
A Hybrid Approach: The Join-Below-Threshold (JBT) Algorithm

The Join-Below-Threshold (JBT) algorithm is a hybrid "pull-based" scheme designed to achieve this goal. It operates with two key components: the dispatcher and the server.

JBT(r) Algorithm: Dispatcher's Role

The **dispatcher's logic** is as follows:

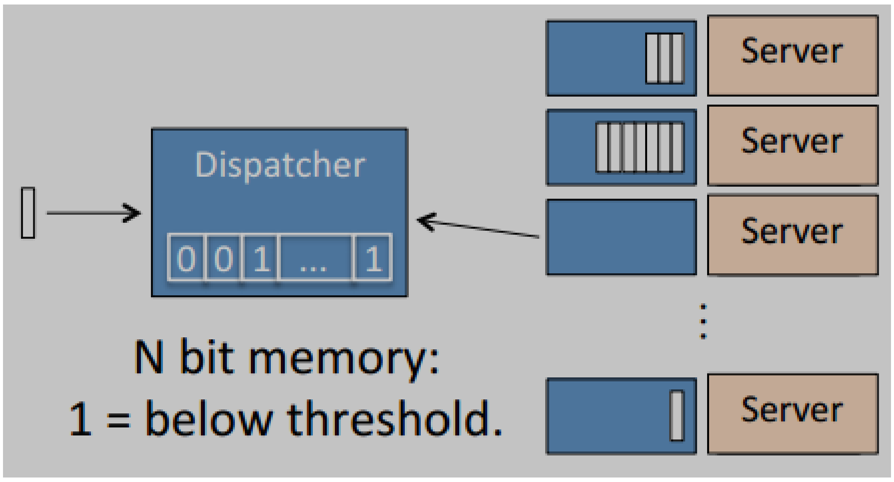
1. It maintains a list of server IDs that are known to have a queue length less than a predefined threshold, r.
2. When a new task arrives, the dispatcher checks its list of available servers.
   - If the list is **not empty**, it picks a server ID from the list at random, sends the task to that server, and **immediately removes that ID from the list**.
     - This is an **optimistic measure**, assuming that sending one task might push the server's queue over the threshold.
   - If the list is **empty**, it defaults to a simple policy like RANDOM.
3. The dispatcher's list is only repopulated when it receives "I'm available" messages from the servers themselves.



JBT(r) Algorithm: Server's Role

The server's role is very simple and communication-efficient:

1. The server continuously monitors its own task queue.
2. A server sends a message to the dispatcher **only** at the specific moment its queue length drops from being equal to the threshold r to $r - 1$.
    - This single message announces that it is now "below threshold" and available to be added back to the dispatcher's list.



Practical Implementation: The Adaptive JBT-d Algorithm

A **fixed threshold** r is **not optimal**, as the ideal queue length threshold changes with the overall system load.

The **JBT-d** algorithm is a practical, **adaptive** version that dynamically adjusts this threshold.

1. **Adaptive Threshold Update:** The threshold is no longer a fixed value.
    - Periodically (e.g., every T time units), the dispatcher polls a small, random sample of d servers.
    - It then sets the **new threshold** to be the **minimum queue length** it observed among that sample.
    - This new threshold is then communicated to all servers.
2. **Server Behavior:** Each server operates as in the basic JBT policy, sending its ID to the dispatcher whenever its queue length falls below the *current, dynamically updated* threshold.

3. **Dispatcher Behavior:** Upon a new task arrival, the dispatcher sends it to a server from its list of available (below-threshold) servers.
   - If the list is empty, it dispatches the task randomly.

JBT-d Properties and Performance Analysis

The adaptive JBT-d algorithm has a powerful **theoretical guarantee of optimality**, making it a **highly robust** and **efficient policy**.

- **Optimality Theorem:** For any finite update period T and any sample size $d \geq 1$, the JBT-d policy is proven to be both **throughput optimal** and **delay optimal in the heavy-traffic regime**.

  - This means it can stabilize any manageable workload and provides the best possible delay performance when the system is under high stress.

- **Generalization to Heterogeneous Servers (JBTG-d):** The algorithm can be extended to handle servers with different processing speeds (μ_j).

  - When a server reports that it is below the threshold, it also includes its service rate, μ_j.
  - The dispatcher then uses this information to perform a **weighted random assignment**.
    - When it picks a server from its available list, it doesn't choose uniformly.
    - Instead, the probability of sending a task to server j is proportional to its speed:

$$\phi_j = \frac{\mu_j}{\sum_{k \in L(t)} \mu_k}$$

  Where $L(t)$ is the **list of available servers**.

  - This ensures that faster servers are assigned proportionally more tasks.
  - If no servers are on the list, it defaults to a weighted random choice among all servers.

Policy Comparison Summary

The following table summarizes the key properties of the load balancing policies discussed. It compares them on messaging overhead per task (Msg/task) and their optimality for both **homogeneous** (**Ho**) and **heterogeneous** (**He**) server environments.

Policy	Msg/task	TO-Ho	TO-He	DO-Ho	DO-He
Random	0	✓	✗	✗	✗
JSQ	$2N$	✓	✓	✓	✓
PoD(d) / SQ(d)	$2d$	✓	✗	✓	✗
JIQ	≤ 1	✓	✗	✗	✗
JBT-d	$\leq 1 + \frac{N+2d}{T}$	✓	✗	✓	✗
JBTG-d	$\leq 1 + \frac{N+2d}{T}$	✓	✓	✓	✓

Key Takeaways:

- **JSQ** is theoretically perfect but has an impractically high message overhead that scales with the number of servers, N.
- Simple policies like **PoD** and **JIQ** fail to achieve optimality in heterogeneous environments.
- The generalized, adaptive **JBTG-d** policy is the **standout**.
  - It is the only policy listed that achieves **both throughput and delay optimality for both homogeneous and heterogeneous systems**, all while maintaining a low and controllable message overhead.

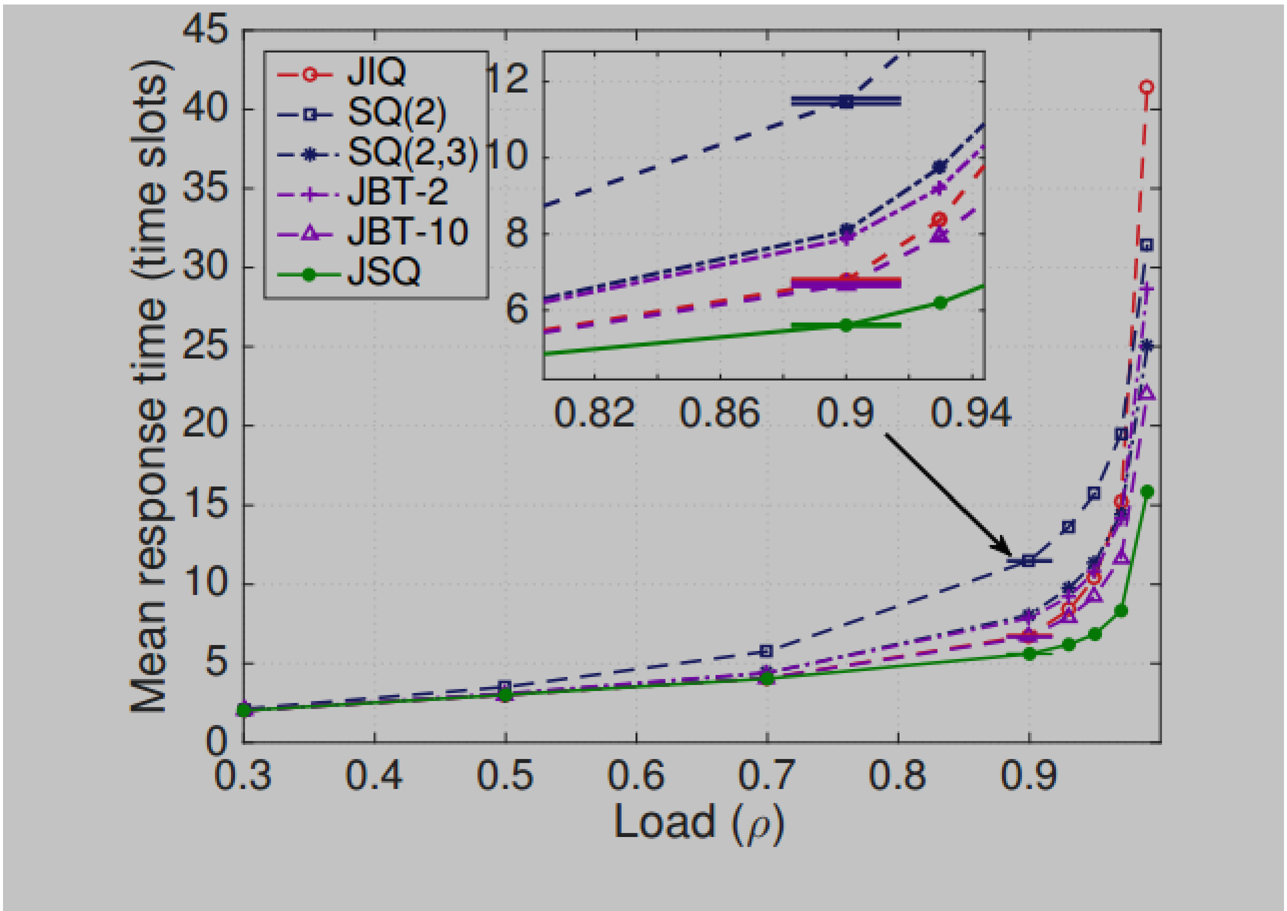
Numerical Performance Results

Simulation results validate the theoretical properties of JBT-d, showing its **superior performance compared to other policies**, especially **under heavy load**.

- **Delay Performance (Homogeneous Servers)**

  This graph plots the **mean response time** versus **system load** (ρ) for a system with 10 identical servers.

  - As predicted by theory, the performance of **JIQ** degrades rapidly as the load increases beyond 90%.
  - In contrast, the **JBT-d** policy's performance remains low and closely tracks that of the theoretically optimal (but impractical) **JSQ** policy, even under very heavy load.

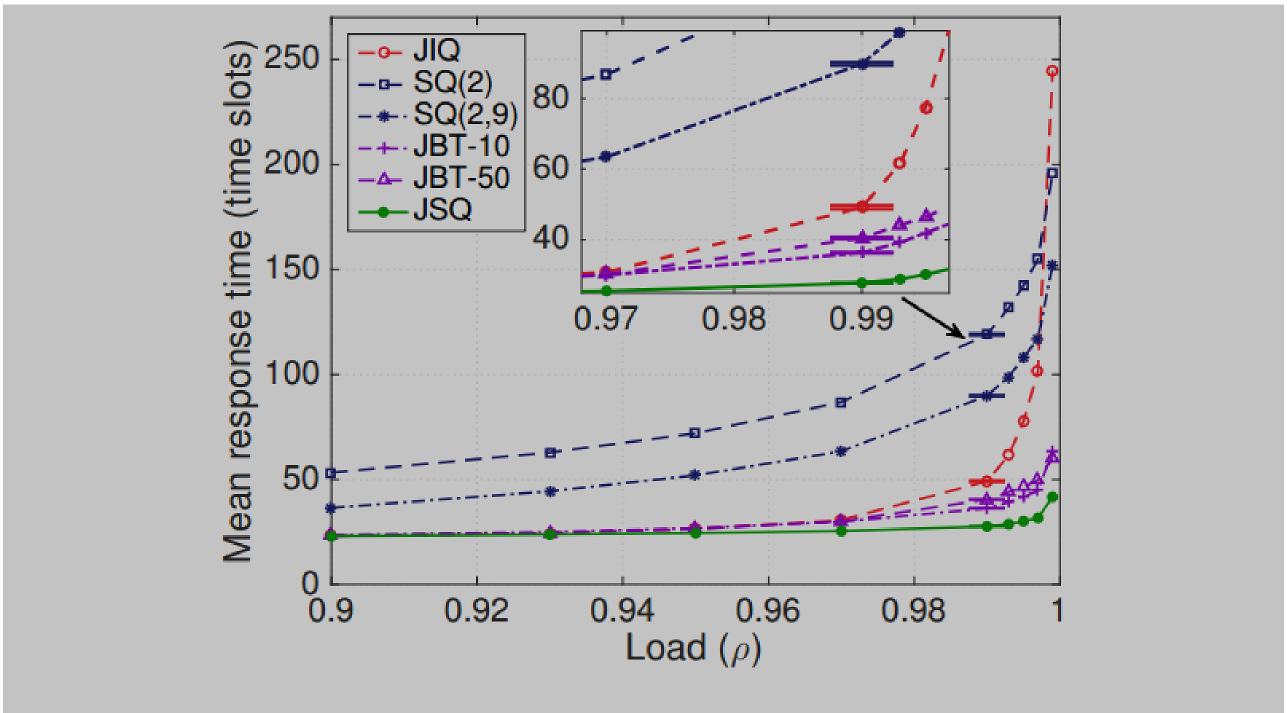


Delay performance under 10 homogeneous servers.

- **Heavy-Traffic Delay Performance (Homogeneous Servers)**

  This graph provides a zoomed-in view of the heavy-traffic regime ($\rho > 0.9$) for systems with 10 and 50 servers.

  - These results clearly demonstrate **JBT-d's heavy-traffic optimality**.
  - While JIQ's delay explodes, JBT-d maintains a **very low and stable mean response time**, confirming its robustness under stress.

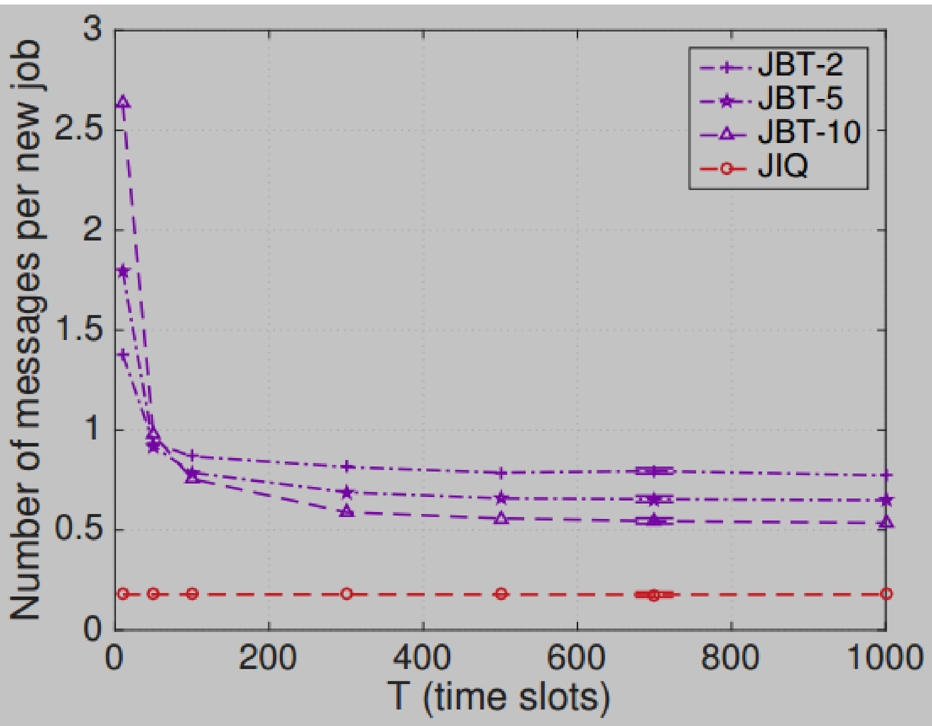


Heavy-traffic delay performance under 50 homogeneous servers.

- **Message Overhead Analysis** (*CHECK CORRECTNESS*)

  This final set of graphs shows the trade-off between message overhead and delay performance for the JBT-d algorithm.

  - **Purple Lines (Message Rate vs. T):** This shows that the number of messages per new job for JBT-d decreases as the threshold update period, T, increases. This means the communication overhead of the policy is **controllable**.
  - **Red Line (Delay vs. T):** This shows that the mean response time of JBT-d is **not very sensitive** to the value of T.



Message per new job arrival under 10 homogeneous servers.

Conclusion

Together, these graphs show the **key advantage of JBT-d**.

- An operator can choose a larger value for T to reduce the messaging overhead to be as low as JIQ's, while still retaining the vast majority of JBT-d's superior delay performance.
- This makes it a highly practical and efficient policy.

Anticipative Algorithms: Using Job Size Information

This class of algorithms assumes the **dispatcher has knowledge of a job's size upon its arrival**.

- This information allows for more intelligent and often more performant load balancing decisions.

Size-Interval Task Assignment (SITA)

- **Policy:** The range of possible job sizes is **partitioned** into N intervals using a set of thresholds, $s_0, s_1, \ldots, s_N$.
  - When a job of size X arrives, it is deterministically assigned to a specific server i based on its size
  - Assign job of size X such that $s_{i-1} < X \leq s_i$, the job goes to server i.
- **Challenge:** The main difficulty in implementing SITA is **finding the optimal threshold** values to balance the load effectively across the servers.

Least Work Left (LWL)

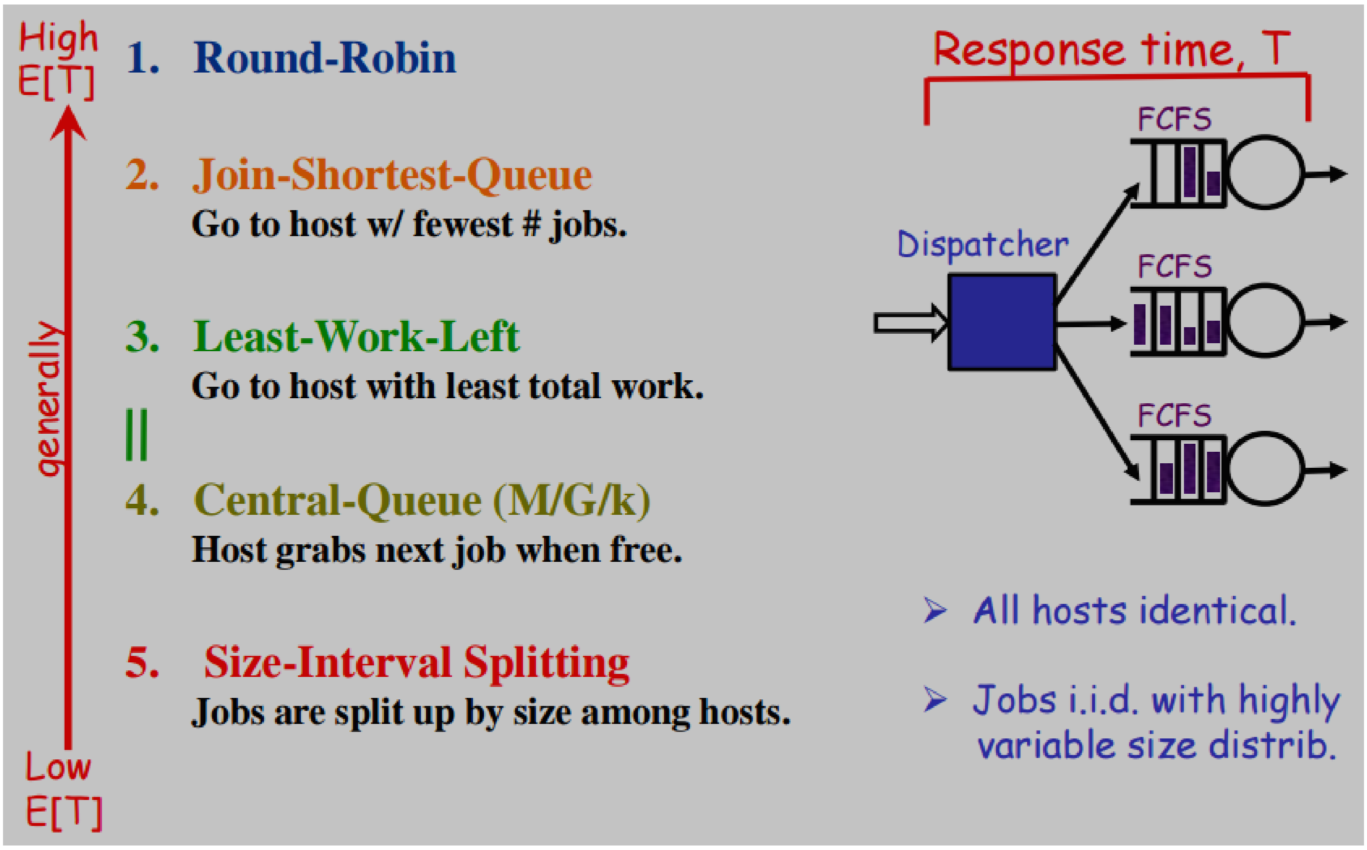
- **Policy:** Upon a task's arrival, the dispatcher polls all servers to determine the total amount of *unfinished work* in each server's queue (i.e., the sum of the remaining processing times of all jobs waiting at that server).
    - It then sends the new task to the server with the minimum unfinished work.

Performance Ranking (with FCFS Servers)

When the individual servers in the farm each use a simple First-Come, First-Served (**FCFS**) scheduling policy, and **job sizes are highly variable**, the performance of different load balancing policies can be ranked as follows.

(**CHECK**)

1. **Central-Queue (M/G/k):** The theoretical ideal (the resource-pooled system), which provides the lowest possible mean response time.
2. **SITA:** Performs very well by segregating jobs by size.
3. **LWL:** Also performs well by trying to balance the total workload.
4. **JSQ:** Performance degrades because knowing only the number of jobs is not enough when sizes are highly variable.
5. **Round-Robin:** Performs the worst in this scenario.



The Deceptive Ranking: LWL vs. SITA

The simple ranking on the previous slide is deceptive because it **doesn't capture the full picture**.

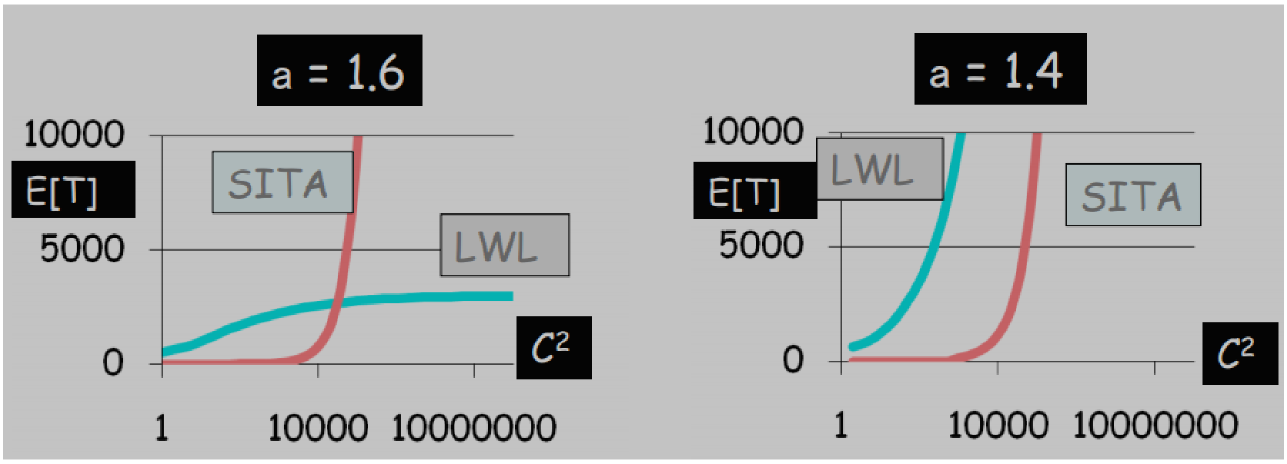
- The relative performance of LWL and SITA is highly dependent on the specific statistical properties of the job size distribution, particularly its "heavy-tailedness" (shape).

A **key question** in performance analysis is: *what happens to the performance of LWL and SITA as service time variability (measured by the **Squared Coefficient of Variation**, or **SCOV**) becomes **extremely high**.*

- **No single answer**; the outcome depends entirely on the specific statistical properties of the job size distribution.
  - Either policy could be superior, or both could perform well or poorly.

This dependency is demonstrated when comparing performance for a **Bounded Pareto distribution**, a common model for heavy-tailed workloads, but with a slightly different shape parameter, a:

- **When the shape parameter $a = 1.6$.**
  - As variability increases, the performance of **LWL improves**, while SITA's degrades.
  - In this scenario, **LWL is the better policy**.
- **When the shape parameter $a = 1.4$**
  - With a small change to the distribution's shape, the situation completely reverses.
  - Now, LWL's performance degrades with variability, while **SITA's improves**.
  - In this scenario, **SITA is the better policy**.



Summary

- For workloads with **high service time variability**, there is **no single winner** between SITA and LWL.
  - The best choice depends on the precise nature of the job sizes.
  - Non-anticipative policies like RANDOM, Round-Robin, and JSQ perform poorly in this environment.
- For workloads with **low service time variability** (e.g., all jobs are roughly the same size), the situation is different.
  - Simple policies like **RANDOM and Round-Robin perform well**, as there is no advantage to be gained from using size information.

Performance with Processor Sharing (PS) Servers

(check if more math explan. is needed)

The analysis changes significantly if the individual servers use a **Processor Sharing (PS)** scheduling discipline instead of FCFS.

SITA with PS Servers

If we use the SITA policy to dispatch jobs to a farm of PS servers, the goal is to set the size thresholds (s_i) to minimize the overall mean response time.

The mean response time is given by:

$$E[R] = \frac{1}{N\lambda} \sum_{i=1}^{N} \frac{\rho_i}{1-\rho_i}$$

This must be minimized subject to the constraint that the sum of the loads on each server, ρ_i, equals the total system load, A.

$$\sum_{i=1}^{N} \rho_i = A, \quad 0 \leq \rho_i < 1$$

The optimal solution is to choose the thresholds such that the load is **perfectly balanced** across all servers, meaning $\rho_i = \rho = A/N$ for all i.

A key result is that for PS servers, an optimally configured SITA policy yields the **exact same performance as the much simpler RANDOM** policy. The fairness of the PS discipline at each server completely negates the advantage of segregating jobs by size at the dispatcher.

Final Ranking with PS Servers

When servers use PS, the performance ranking is as follows:

1. **OPT (Optimal):** The theoretical best performance.
2. **JSQ:** Performance is extremely close to optimal and is insensitive to job size variability.
3. **LWL:** Performs much better than SITA/RANDOM.
4. **SITA / RANDOM:** These have the worst performance in this scenario.