Important Formulas from `1_signals.pdf`

1. Continuous-time vs. Discrete-time Signals

- **Sampling (Conversion from Continuous to Discrete):**

$$x[n] = x(nT)$$

  (where $x(t)$ is continuous, $x[n]$ is discrete, T is sampling period, n is integer index)
- **Euler's Formula (Fundamental for complex exponentials):**

$$e^{j\theta} = \cos\theta + j\sin\theta$$

2. Basic Discrete-time Signals

- **Unit Impulse $\delta[n]$:**

$$\delta[n] = \left\{ 1, \quad n = 0 \; 0, \;\; n \neq 0 \right.$$

- **Unit Step $u[n]$:**

$$u[n] = \left\{ 1, \quad n \geq 0 \; 0, \;\; n < 0 \right.$$

- **Exponential Signal (e.g., decay/growth):**

$$x[n] = a^n u[n]$$

- **Discrete-time Sinusoid (example form):**

$$x[n] = A\cos(\omega n + \phi) \quad \text{or} \quad x[n] = A\cos(2\pi k n + \phi)$$

3. Elementary Operations on Discrete-time Signals

- **Time Shift (Delay if $k > 0$, Advance if $k < 0$):**

$$y[n] = x[n-k]$$

- **Scaling:**

$$y[n] = \alpha x[n]$$

- **Sum:**

$$y[n] = x[n] + w[n]$$

- **Product:**

$$y[n] = x[n]w[n]$$

- **Integration (Discrete-time Running Sum):**

$$y[n] = \sum_{m=-\infty}^{n} x[m]$$

- **Differentiation (Discrete-time First-Order Difference):**

$$y[n] = x[n] - x[n-1]$$

4. Energy and Power of Discrete-time Signals

- **Energy of $x[n]$:**

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

- **Average Power of $x[n]$:**

$$P_x = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

5. Classes of Discrete-time Signals

- **Periodic Signal (with period N):**

$$\tilde{x}[n] = \tilde{x}[n+kN] \quad (\text{for any integer } k)$$

- **Periodization of a finite-support signal $\overline{x}[n]$ (to create $\tilde{x}[n]$ with period N):**

$$\tilde{x}[n] = \sum_{k=-\infty}^{\infty} \overline{x}[n-kN]$$

6. Linear Algebra for Signals

- **Inner Product of $x, y \in \mathbb{C}^N$:**

$$(x, y) = x^H y = \sum_{n=0}^{N-1} x^*[n] y[n]$$

- **Norm (Squared) and Energy from Inner Product:**

$$||x||^2 = (x, x) = x^H x = \sum_{n=0}^{N-1} |x[n]|^2 = E_x$$

- **Squared Euclidean Distance:**

$$||x-y||^2 = ||x||^2 + ||y||^2 - 2\mathrm{Re}(x, y)$$

- **Orthogonality Condition:**

$$(x, y) = 0$$

- **Cauchy-Schwarz Inequality:**

$$|(x, y)| \leq ||x|| \cdot ||y||$$

7. Signal Expansion over a Basis

- **General Signal Expansion (x in terms of basis u_k):**

$$x = \sum_{k=0}^{N-1} X_k u_k$$

- **Coefficients for Orthogonal Basis u_k:**

$$X_k = \frac{(u_k, x)}{||u_k||^2} = \frac{u_k^H x}{u_k^H u_k}$$

- **Orthonormal Basis Condition:**

$$(u_k, u_l) = \delta[k-l]$$

- **Coefficients for Orthonormal Basis u_k:**

$$X_k = (u_k, x) = u_k^H x$$

- **Parseval's Relation (for Orthonormal Basis):**

$$E_x = ||x||^2 = \sum_{k=0}^{N-1} |x[n]|^2 = \sum_{k=0}^{N-1} |X_k|^2$$

- **Unitary Matrix Transform (U with orthonormal columns):** Analysis: $X = U^H x$ Synthesis: $x = UX$

8. Best Approximations (Projection Theorem)

- **Best K-term Approximation $\tilde{x}$ (using orthogonal basis u_k):**

$$\tilde{x} = \sum_{k=0}^{K-1} X_k u_k = \sum_{k=0}^{K-1} \frac{(u_k, x)}{||u_k||^2} u_k$$

- **Energy Decomposition (Error $e = x - \tilde{x}$):**

$$||x||^2 = ||\tilde{x}||^2 + ||e||^2 \quad (\text{since } e \perp \tilde{x})$$

9. Analysis and Synthesis Formulas (General)

- **Analysis (Coefficients X_k from signal x using analysis vectors a_k):**

$$X_k = a_k^H x \quad (\text{or matrix form } X = A^H x)$$

- **Synthesis (Signal x from coefficients X_k using synthesis vectors s_k):**

$$x = \sum_k X_k s_k \quad (\text{or matrix form } x = SX)$$

- **Biorthogonality Condition (for perfect reconstruction with a_k, s_k):**

$$a_k^H s_l = \delta[k-l]$$