# Robotics 2 Project

Robotics & Control 2

Tommaso Borlina, Stefano Lacovara, Sara Petrelli

January 2024

## Introduction - Problem definition I

| | |
|---|---|
| ***Initialization:*** | Consider 3 mobile robots (unicycles) starting from two sets of random initial conditions. |
| ***Consensus:*** | Make them agree on a rendez-vous point. |
| ***Regulation:*** | Move them towards the consensus point. |
| ***Switch:*** | When they are close to it (threshold $\varepsilon = 0.01$), switch to the tracking controller. |
| ***Tracking:*** | Make them follow a circular trajectory centered in the rendez-vous point. |

# Introduction - Problem definition II

**Design choices:**

- Unicycle as point mass, described by the kinematic model

$$\Sigma : \begin{cases} \dot{x}(t) = v(t)\,\cos\theta(t) \\ \dot{y}(t) = v(t)\,\sin\theta(t) \\ \dot{\theta}(t) = \omega(t) \end{cases}$$

- Three robots treated as independent agents and no handle of collision avoidance.
- Average consensus in one step.
- Gains chosen especially by trial and error.

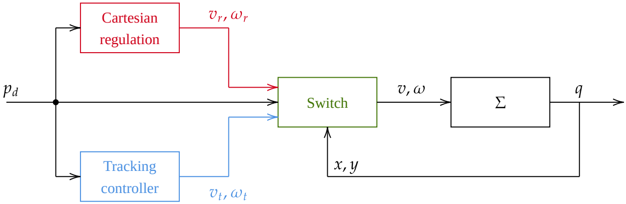
**Implementation choices:**

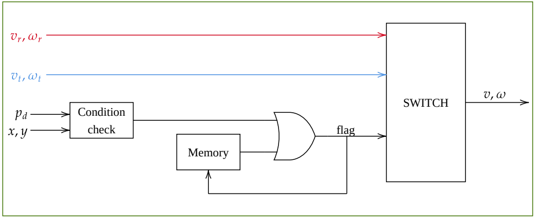
- Derivative implemented as a $1^{st}$ order high-pass filter

$$H(s) = \frac{s}{f_c\, s + 1}, \quad \text{with} \quad f_c = \frac{1}{2\pi\, 20}.$$

- Unwrap block placed after the $\mathrm{atan2}$ function.
- Initial conditions $q_0$ set in the Integrator blocks of the model.

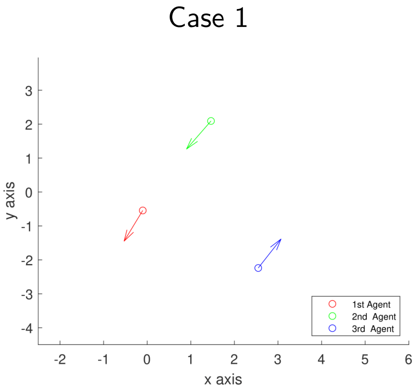
# Introduction - Switch

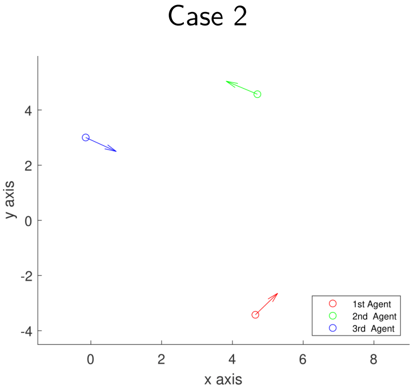



Note: In the first block diagram the output feedback should enter also into the two controllers, though, it has been neglected for clarity.

# Introduction - Initial Conditions




Case 1: $$q_{0,1} = \begin{bmatrix} -0.10 \\ -0.54 \\ 4.27 \end{bmatrix}, \quad q_{0,2} = \begin{bmatrix} 1.46 \\ 2.09 \\ 4.12 \end{bmatrix}, \quad q_{0,3} = \begin{bmatrix} 2.55 \\ -2.24 \\ 1.02 \end{bmatrix}.$$

Case 2: $$q_{0,1} = \begin{bmatrix} 4.65 \\ -3.42 \\ 0.89 \end{bmatrix}, \quad q_{0,2} = \begin{bmatrix} 4.71 \\ 4.57 \\ 2.65 \end{bmatrix}, \quad q_{0,3} = \begin{bmatrix} -0.15 \\ 3.00 \\ 5.75 \end{bmatrix}.$$

# Consensus - Algorithm

Given a matrix $P \in \mathbb{R}^{3x3}$, the consensus dynamics is $x_{cons}(k+1) = P\, x_{cons}(k)$, initialized with the initial positions of the agents, with $x_{cons}(k) = [x_{cons,1} | x_{cons,2}] = \begin{bmatrix} x_1(k) & y_1(k) \\ x_2(k) & y_2(k) \\ x_3(k) & y_3(k) \end{bmatrix}$.

**Algorithm**

i = 0, $\varepsilon = 0.01$
flag = false
set $P$, $x_{cons}(0)$

**while** flag = false **do**

  **if** i $\geq 100$ **then**
    flag = true

  **if** $\left|\max\left\{x_{cons,1}\right\} - \min\left\{x_{cons,1}\right\}\right| \leq \varepsilon/10$ **then**
    **if** $\left|\max\left\{x_{cons,2}\right\} - \min\left\{x_{cons,2}\right\}\right| \leq \varepsilon/10$ **then**
      flag = true

  $x_{cons}(\mathrm{i}+1) = P\, x_{cons}(\mathrm{i})$
  i = i + 1

# Consensus - Theory

**Proposition** [Strongly connected and aperiodic digraphs and primitive adjacency matrices]

A matrix $A \in \mathbb{R}^{nxn}$ is primitive $\iff$ The associated digraph $\mathcal{G}$ is strongly connected and aperiodic.

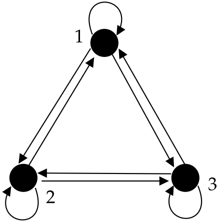
**Proposition** [Sufficient condition for consensus]

Given the update rule $x(k+1) = A\,x(k)$ with $A \in \mathbb{R}^{nxn}$ primitive and row-stochastic, then $x(k) = P^k\,x(0)$ reaches consensus for $k \to \infty$.

**Proposition** [Sufficient condition for average consensus]

Given the update rule $x(k+1) = A\,x(k)$ with $A \in \mathbb{R}^{nxn}$ primitive and doubly-stochastic, then $x(k) = P^k\,x(0)$ reaches average consensus for $k \to \infty$.

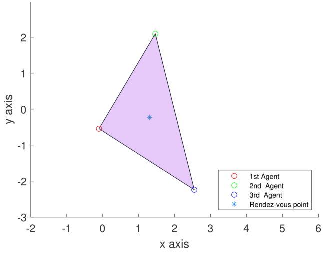
# Consensus - Average Asymptotic Consensus in One Step



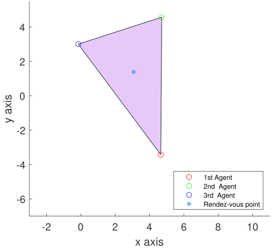

$P$ primitive and doubly stochastic

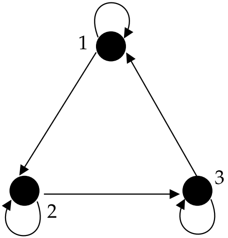
$$P = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

Case 1




Case 2

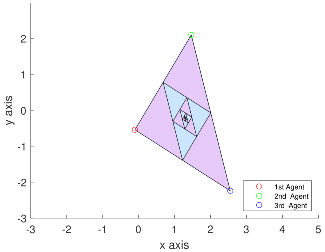




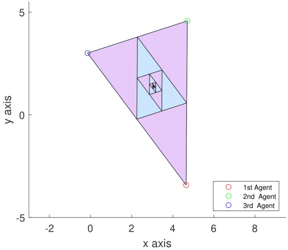

$P$ primitive and doubly stochastic

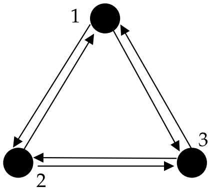
$$P = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \end{bmatrix}$$

Case 1




Case 2

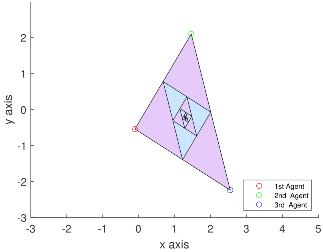




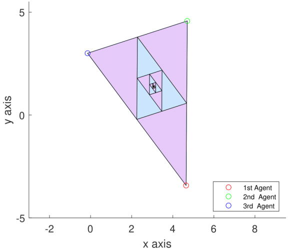

$P$ primitive and doubly stochastic

$$P = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

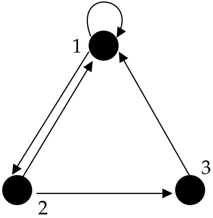
Case 1




Case 2



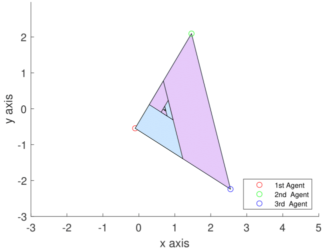
# Consensus - Asymptotic Consensus



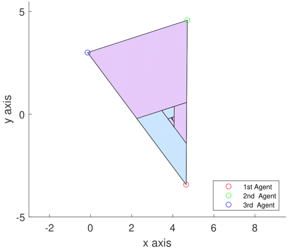

$P$ primitive and row stochastic

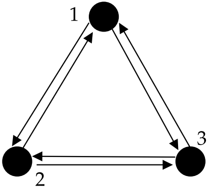
$$P = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1/2 \\ 1 & 0 & 0 \end{bmatrix}$$

Case 1




Case 2

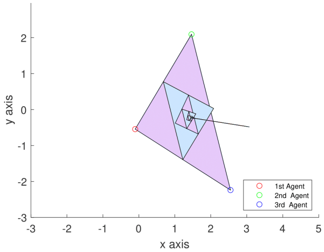




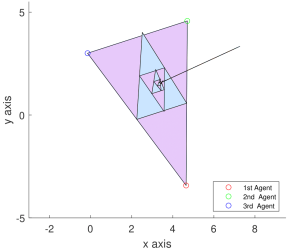

$P$ primitive

$$P = \begin{bmatrix} 0 & 0.55 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

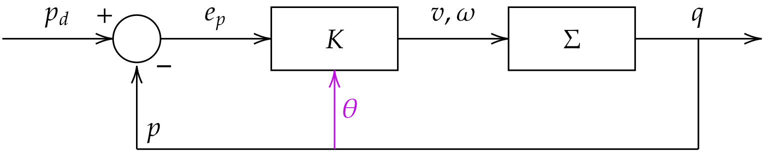
Case 1




Case 2



# Regulation - Cartesian I

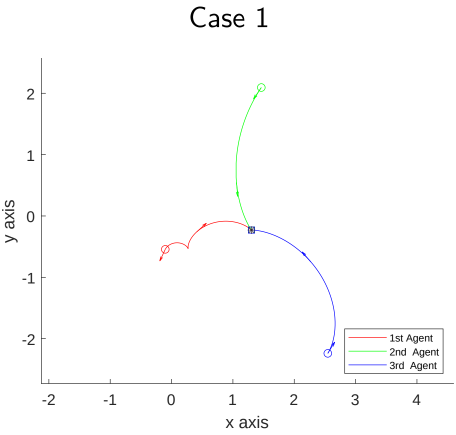


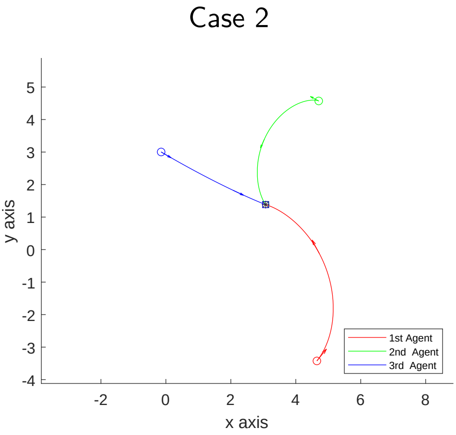

Rendez-vous point: $p_d = \begin{bmatrix} x_d & y_d \end{bmatrix}^T$

Control law: $\begin{cases} v = -K_v \overbrace{(e_{p,x} \cos\theta + e_{p,y} \sin\theta)}^{=\langle e_p, n \rangle} \\ \omega = -K_\omega \underbrace{(\mathrm{atan2}(e_{p,y}, e_{p,x}) + \pi - \theta)}_{=\gamma} \end{cases}$

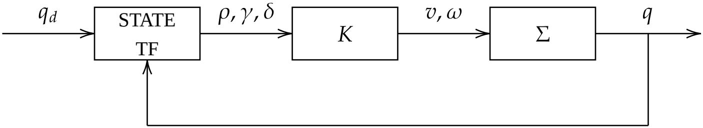
Gains: $K_v = 3$, $K_\omega = 5$.

# Regulation - Cartesian II



# Regulation - Posture with singularity at the rendez-vous point I



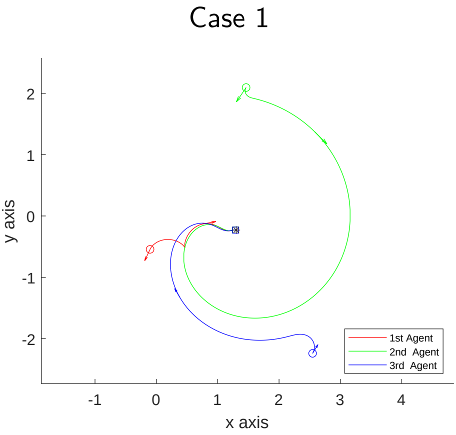

Regulation desired point: $q_d = \begin{bmatrix} x_d & y_d & \theta_d \end{bmatrix}^T$, with $x_d, y_d$ rendez-vous point and $\theta_d$ arbitrary

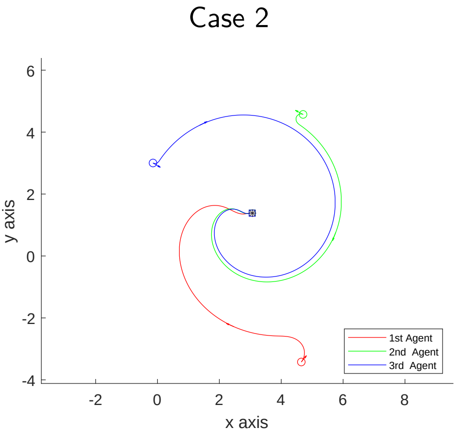
State Transformation: $\begin{cases} \rho = \sqrt{e_{q,x}^2 + e_{q,y}^2} \\ \gamma = \operatorname{atan2}(e_{q,y}, e_{q,x}) + \pi - \theta \\ \delta = \gamma + e_{q,\theta} \end{cases}$

Control law: $\begin{cases} v = K_1 \overbrace{\rho \cos\gamma}^{=\langle e_p, n\rangle} \\ \omega = K_2\,\gamma + K_1 \dfrac{\sin\gamma\,\cos\gamma}{\gamma}\,(\gamma + K_3\,\delta) \end{cases}$

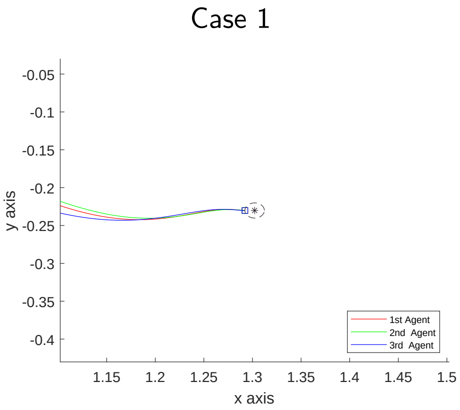
Gains: $K_1 = K_2 = K_3 = 5$.

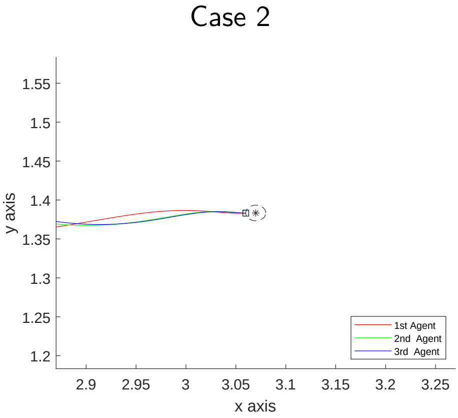
# Regulation - Posture with singularity at the rendez-vous point II



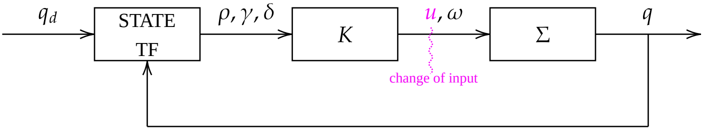



# Regulation - Posture with singularity at the rendez-vous point III



# Regulation - Posture without singularity at the rendez-vous point I




Regulation desired point: $q_d = \begin{bmatrix} x_d & y_d & \theta_d \end{bmatrix}^T$, with $x_d, y_d$ rendez-vous point and $\theta_d$ arbitrary

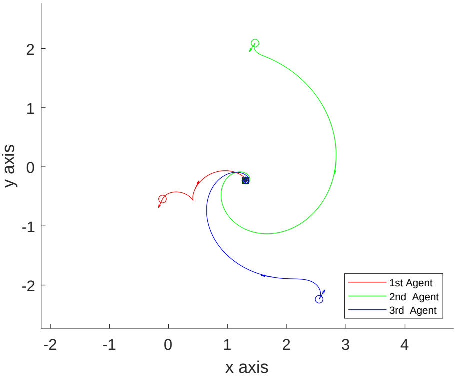
State Transformation: $\begin{cases} \rho = \sqrt{e_{q,x}^2 + e_{q,y}^2} \\ \gamma = \mathrm{atan2}(e_{q,y}, e_{q,x}) + \pi - \theta \\ \delta = \gamma + e_{q,\theta} \end{cases}$

Control law: $\begin{cases} u = \dfrac{v}{\rho} = K_1 \cos\gamma \\ \omega = K_2\,\gamma + K_1 \dfrac{\sin\gamma\,\cos\gamma}{\gamma}\,(\gamma + K_3\,\delta) \end{cases}$

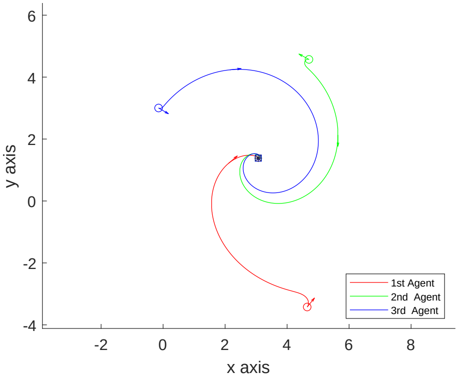
Gains: $K_1 = K_2 = 3$, $K_3 = 2$.

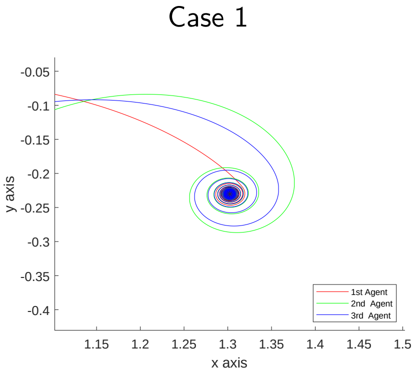
# Regulation - Posture without singularity at the rendez-vous point II

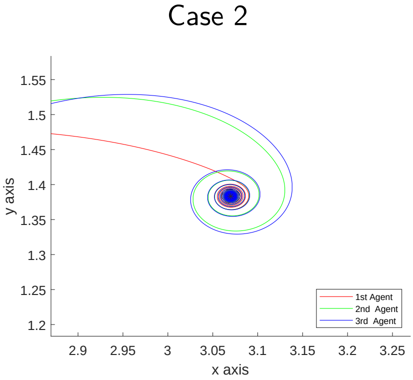
Case 1

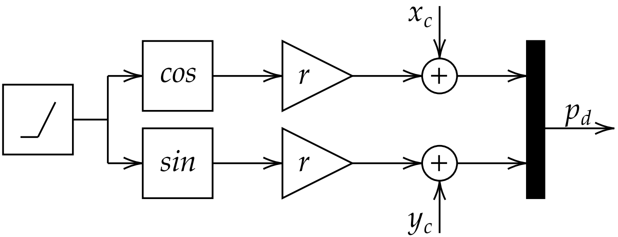



Case 2




Case 1
y axis
x axis
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
-0.4
1.15
1.2
1.25
1.3
1.35
1.4
1.45
1.5
1st Agent
2nd Agent
3rd Agent


Case 2
y axis
x axis
1.55
1.5
1.45
1.4
1.35
1.3
1.25
1.2
2.9
2.95
3
3.05
3.1
3.15
3.2
3.25
1st Agent
2nd Agent
3rd Agent

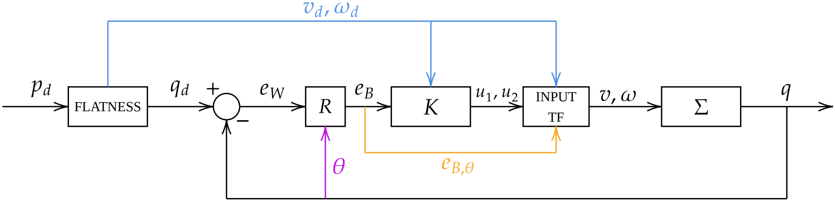



Center: $[x_c \quad y_c]^T$ = rendez-vous point, Radius: $r = 5\ [m]$

Phases: $\varphi_1 = 0,\ \ \varphi_2 = \frac{2}{3}\pi,\ \ \varphi_3 = \frac{4}{3}\pi$

Trajectory: $p_d = \begin{cases} x_d(t) = x_c + r\ \cos\Big(\theta(t) + \varphi_i\Big) \\ y_d(t) = y_c + r\ \sin\Big(\theta(t) + \varphi_i\Big) \end{cases}, \quad \theta(t) = \mathrm{ramp}(t) + \varphi_i,\ \ i = 1, 2, 3.$

# Tracking - Linearization of state error dynamics I



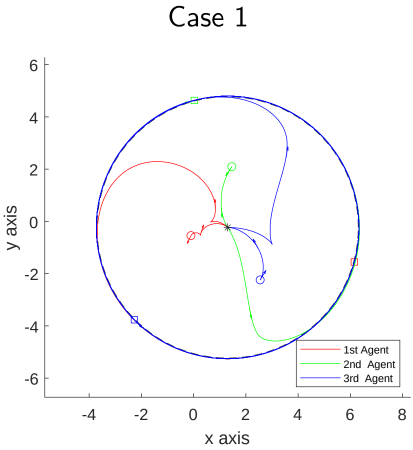

Flatness: $\begin{cases} v_d = \sqrt{\dot{x}_d^2 + \dot{y}_d^2} \\ \omega_d = \dfrac{\dot{x}_d \ddot{y}_d - \ddot{x}_d \dot{y}_d}{\dot{x}_d^2 + \dot{y}_d^2} \end{cases}$, W2B transformation: $\underbrace{\begin{bmatrix} e_{B,x} \\ e_{B,y} \\ e_{B,\theta} \end{bmatrix}}_{= e_B} = \underbrace{\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{= R} \underbrace{\begin{bmatrix} x_d - x \\ y_d - y \\ \theta_d - \theta \end{bmatrix}}_{= e_W}$

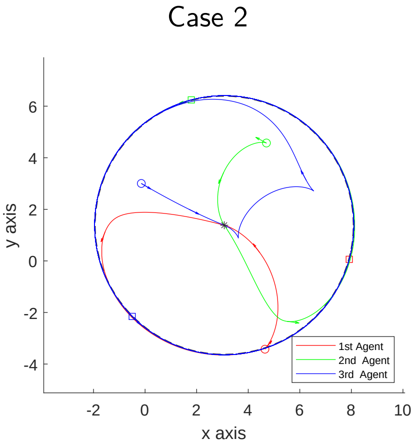
Control law: $\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -K_1 & 0 & 0 \\ 0 & -K_2 & -K_3 \end{bmatrix}}_{= K} \underbrace{\begin{bmatrix} e_{B,x} \\ e_{B,y} \\ e_{B,\theta} \end{bmatrix}}_{= e_B}$

Input Transformation: $\begin{cases} v = v_d \cos e_{B,\theta} - u_1 \\ \omega = \omega_d - u_2 \end{cases}$

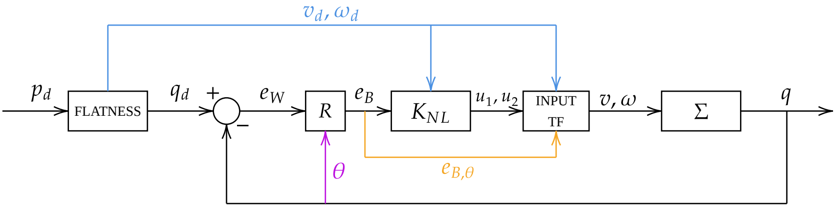
Gains: $K_1 = K_3 = 2\,\xi\,a,\ K_2 = \dfrac{a^2 - \omega_d^2}{v_d}$ and $a = 3,\ \xi = 0.9$.

# Tracking - Linearization of state error dynamics II



# Tracking - Non-Linear controller of state error dynamics I

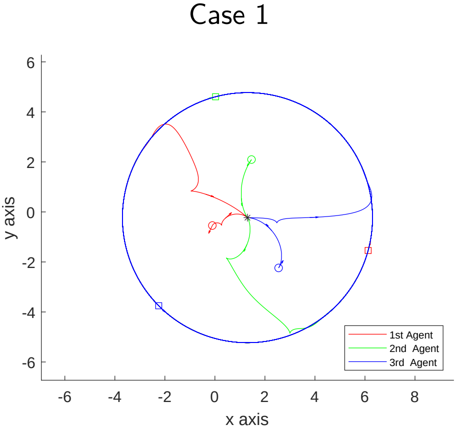


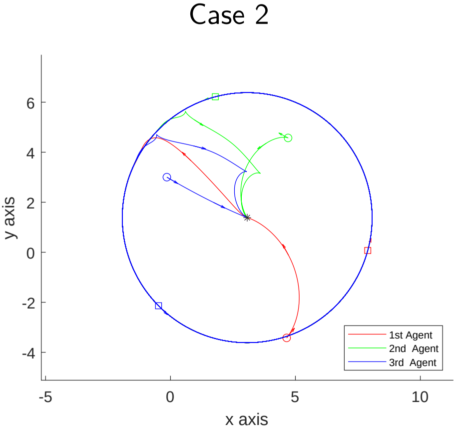

Flatness: $\begin{cases} v_d = \sqrt{\dot{x}_d^2 + \dot{y}_d^2} \\ \omega_d = \dfrac{\dot{x}_d \ddot{y}_d - \ddot{x}_d \dot{y}_d}{\dot{x}_d^2 + \dot{y}_d^2} \end{cases}$ , W2B transformation: $\underbrace{\begin{bmatrix} e_{B,x} \\ e_{B,y} \\ e_{B,\theta} \end{bmatrix}}_{= e_B} = \underbrace{\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{= R} \underbrace{\begin{bmatrix} x_d - x \\ y_d - y \\ \theta_d - \theta \end{bmatrix}}_{= e_W}$

Control law: $\begin{cases} u_1 = -K_1\, e_{B,x} \\ u_2 = -K_2 \left( v_d \dfrac{\sin e_{B,\theta}}{e_{B,\theta}} \right) e_{B,y} - K_3\, e_{B,\theta} \end{cases}$

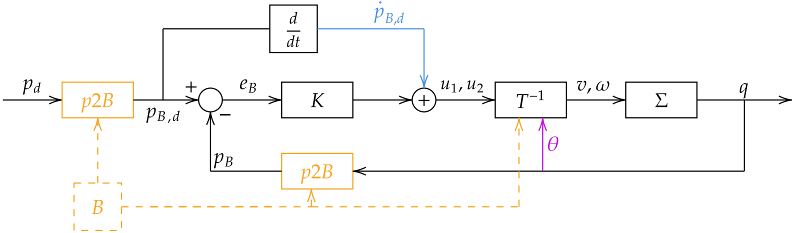
Input Transformation: $\begin{cases} v = v_d \cos e_{B,\theta} - u_1 \\ \omega = \omega_d - u_2 \end{cases}$

Gains: $K_1 = K_3 = 2\,\xi \sqrt{b\, v_d + w_d^2}$, $K_2 = b$, with $b = 20$, $\xi = 0.8$.


Case 1
y axis
x axis
6
4
2
0
-2
-4
-6
-6
-4
-2
0
2
4
6
8
1st Agent
2nd Agent
3rd Agent


Case 2
y axis
x axis
6
4
2
0
-2
-4
-5
0
5
10
1st Agent
2nd Agent
3rd Agent

# Tracking - Output Feedback Linearization based on a reference point on the sagittal axis I

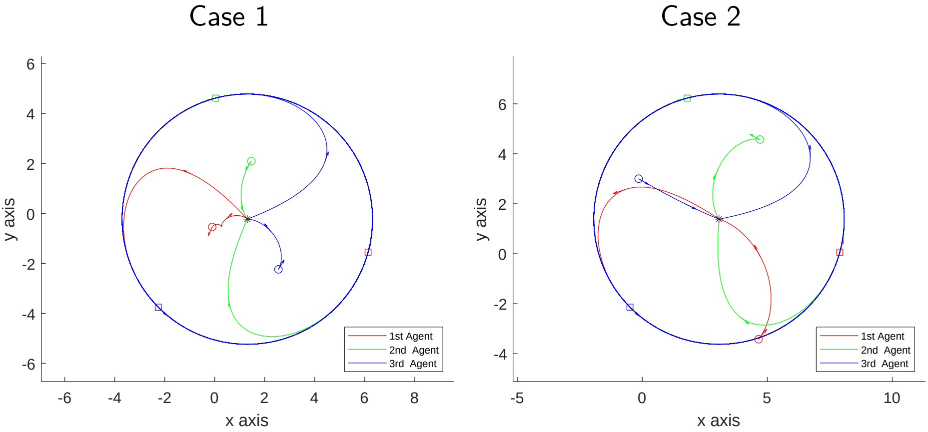



p2B transformation: $\begin{cases} x_B = x + b\,\cos\theta \\ y_B = y + b\,\sin\theta \end{cases}$ , with $b = 0.01\,[\mathrm{m}]$ , New dynamics: $\begin{cases} \dot{x}_B = u_1 \\ \dot{y}_B = u_2 \end{cases}$

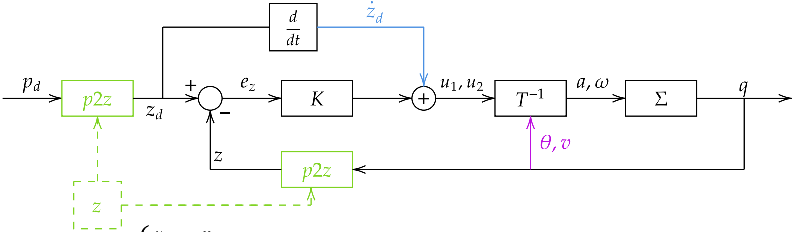
Control law: $\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \underbrace{\begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix}}_{=K} \underbrace{\begin{bmatrix} x_{B,d} - x_B \\ y_{B,d} - y_B \end{bmatrix}}_{=e_B} + \underbrace{\begin{bmatrix} \dot{x}_{B,d} \\ \dot{y}_{B,d} \end{bmatrix}}_{=\dot{p}_{B,d}}$

Input transformation: $\begin{bmatrix} v \\ \omega \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta & \sin\theta \\ -\dfrac{\sin\theta}{b} & \dfrac{\cos\theta}{b} \end{bmatrix}}_{=T^{-1}} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$, Gains: $K_1 = K_2 = 3$.

# Tracking - Output Feedback Linearization based on a reference point on the sagittal axis II



# Tracking - Output Feedback Linearization based on second order derivatives I

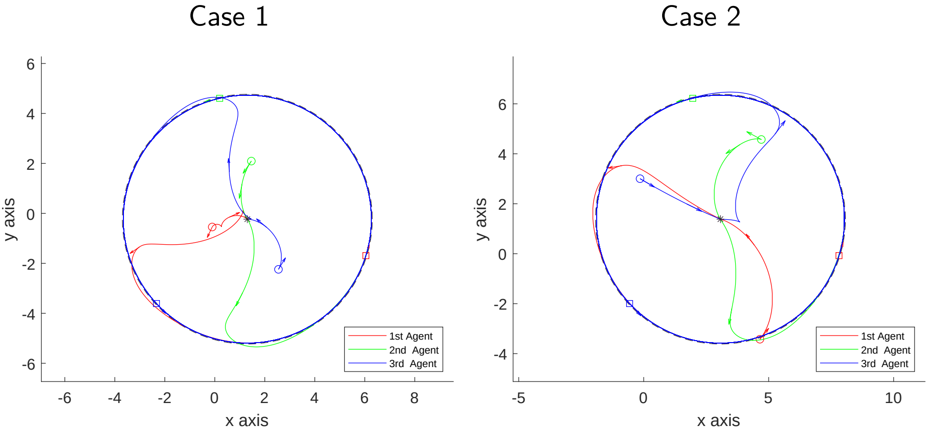



p2z transformation: $\begin{cases} z_1 = x \\ z_2 = y \\ z_3 = \dot{x} \\ z_4 = \dot{y} \end{cases}$ , New dynamics: $\begin{cases} \ddot{z}_1 = u_1 \\ \ddot{z}_2 = u_2 \end{cases}$

Control law: $\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \underbrace{\begin{bmatrix} K_1 & 0 & K_3 & 0 \\ 0 & K_2 & 0 & K_4 \end{bmatrix}}_{=K} \underbrace{\begin{bmatrix} x_d - x \\ y_d - y \\ \dot{x}_d - \dot{x} \\ \dot{y}_d - \dot{y} \end{bmatrix}}_{=e_z} + \begin{bmatrix} \ddot{x}_d \\ \ddot{y}_d \end{bmatrix}, \quad v = \sqrt{\dot{x}^2 + \dot{y}^2}$

Input Transformation: $\begin{bmatrix} a \\ w \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta & \sin\theta \\ -\dfrac{\sin\theta}{v} & \dfrac{\cos\theta}{v} \end{bmatrix}}_{=T^{-1}} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$, Gains: $K_1 = K_2 = 30, \; K_3 = K_4 = 5$

# Tracking - Output Feedback Linearization based on second order derivatives II



# Conclusions I

**Regulation**

| | | |
|---|---|---|
| *Cartesian* | Easy to implement | No orientation control |
| *Posture w/ singularity* | Orientation control | Singularity in $e_p = 0$ |
| *Posture w/out singularity* | Orientation control | Spiraling approach to $q_d$<br>No deceleration approaching $q_d$ |

**Tracking**

| | | |
|---|---|---|
| *Linearization* | Linear dynamics<br>Poles allocation | W2B transformation of $e$<br>Local approximation around $e = 0$<br>Differential flatness |
| *NonLinear* | No approximation around $e = 0$ | W2B transformation of $e$<br>Differential flatness |
| *B-Point* | Simple implementation<br>Linearized and decoupled dynamics<br>$\theta_d$ not required | Translation of $b \neq 0$<br>Smoothness of trajectory<br>$\theta$ not directly controllable |
| $2^{nd}$ *Derivatives* | Linearized and decoupled dynamics<br>$\theta_d$ not required | $2^{nd}$ derivatives computations<br>$\theta$ not directly controllable<br>$v \neq 0$ |

# Conclusions II

In this specific scenario the most suitable controllers are:

- Regulation task: Cartesian controller;
- Tracking task: Either the B-Point or the Non-Linear controller. However, the former requires a perfect knowledge of the model, hence in a non-ideal case the performances could worsen.

Some possible future improvements or further work could be:

- Exploit the posture regulation to position the robots with the appropriate orientation in order to follow the circular trajectory;
- Consider real-world applications, implementing saturation blocks, collision avoidance and gain tuning to satisfy given performance specifications, even in presence of noise.

# Thank you for your attention