Project (2): -

Marmara University - Faulty of Engineering

Department of Computer Engineering

Data Structures (Autumn 2022)

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AVL-Tree Vs Splay Tree

Sections Of the Report: -

- Section (1): Problem Definition.
- Section (2): Implementation Details.
- Section (3): Test Cases.

To compare the performance of AVL and Splay trees, you can follow these steps:

1. Read the input text file and store the characters in a list or array.
2. Create two empty AVL and Splay trees.
3. Iterate over the list of characters and perform the following operations for each character:
   - Insert the character in both the AVL and Splay trees if it does not exist. If it already exists, update its frequency in the text.
   - For the AVL tree, if there is an AVL condition violation after inserting the new node, perform the necessary rotations to restore the balance of the tree. Keep track of the total number of rotations performed.
   - For the Splay tree, perform the necessary splay(s) after reading each character. Keep track of the total number of splays performed.
4. At the end of the iteration, calculate the total cost for both the AVL and Splay trees as follows:
   - For the AVL tree, the total cost is the sum of the number of comparisons (successful and unsuccessful) and the number of rotations performed.
   - For the Splay tree, the total cost is the sum of the number of comparisons (successful and unsuccessful) and the number of splays performed.

5. Compare the total costs of the AVL and Splay trees to determine which tree performed better.

It is important to note that the performance of AVL and Splay trees may vary depending on the input data. You may want to test your program with different input datasets to see how the trees perform under different conditions.

The recurrence formulas for AVL and Splay trees can be used to calculate the asymptotic complexity (i.e., the time and space complexity) of these data structures.

AVL Tree:

The time complexity of an AVL tree is determined by the height of the tree, which is O(log n) in the average case and O(n) in the worst case.

Insertion:

- Time complexity: O(log n)
- Space complexity: O(1)

Deletion:

- Time complexity: O(log n)
- Space complexity: O(1)

Search:

- Time complexity: O(log n)
- Space complexity: O(1)

Splay Tree:

The time complexity of a Splay tree is also determined by the height of the tree, which is O(log n) in the average case and O(n) in the worst case.

Insertion:

- Time complexity: O(log n)
- Space complexity: O(1)

Deletion:

- Time complexity: O(log n)
- Space complexity: O(1)

Search:

- Time complexity: O(log n)
- Space complexity: O(1)

It is important to note that the above time and space complexities are for the average and worst cases, and the actual complexity may vary depending on the specific implementation and the input data.

AVL Tree:

An AVL tree is a self-balancing binary search tree, which means that the height of the tree is kept balanced to ensure that the time complexity of the basic operations (insertion, deletion, and search) is O(log n). An AVL tree is named after its inventors, G.M. Adelson-Velsky and E.M. Landis, who published it in their 1962 paper "An algorithm for the organization of information".

The balance factor of an AVL tree is defined as the difference in the heights of the left and right subtrees of a node. An AVL tree is considered balanced if the balance factor of every node is in the range [-1, 1]. If the balance factor of a node is outside this range, then the tree is considered unbalanced and rotations are performed to restore the balance.

Insertion:

To insert a new node in an AVL tree, follow the same steps as in a binary search tree, but also check the balance factor of the nodes after each insertion. If the balance factor of a node is outside the range [-1, 1], perform the necessary rotations to restore the balance of the tree.

The time complexity of insertion in an AVL tree is O(log n) in the average case and O(n) in the worst case.

Deletion:

To delete a node from an AVL tree, follow the same steps as in a binary search tree, but also check the balance factor of the nodes after each deletion. If the balance factor of a node is outside the range [-1, 1], perform the necessary rotations to restore the balance of the tree.

The time complexity of deletion in an AVL tree is O(log n) in the average case and O(n) in the worst case.

Search:

To search for a node in an AVL tree, follow the same steps as in a binary search tree. The time complexity of search in an AVL tree is O(log n) in the average case and O(n) in the worst case.

Splay Tree:

A Splay tree is a self-balancing binary search tree, similar to an AVL tree. The main difference between the two is that in a Splay tree, the recently accessed elements are moved to the top of the tree, while in an AVL tree, the balance factor of the nodes is maintained to ensure a balanced tree.

Insertion:

To insert a new node in a Splay tree, follow the same steps as in a binary search tree, but also perform splay operations to bring the recently inserted node to the top of the tree.

The time complexity of insertion in a Splay tree is O(log n) in the average case and O(n) in the worst case.

Deletion:

To delete a node from a Splay tree, follow the same steps as in a binary search tree, but also perform splay operations to bring the parent of the deleted node to the top of the tree.

The time complexity of deletion in a Splay tree is O(log n) in the average case and O(n) in the worst case.

Search:

To search for a node in a Splay tree, follow the same steps as in a binary search tree, but also perform splay operations to bring the searched node to the top of the tree. The time complexity of search in a Splay tree is O(log n) in the average case and O(n) in the worst case.

Input File (1): - Calculations:-

AVL Tree (1): -

Num. of Nodes	Num. of Comparisons	Num. of Rotations	AVL Total Cost	AVL Performance
1	0	0	0	
2	0	0	0	
3	1	0	1	1
4	3	2	5	0.2
5	5	2	7	0.142857
6	8	4	12	0.083333
7	11	4	15	0.066667
8	13	4	17	0.058824
9	16	6	22	0.045455
10	19	6	25	0.04
11	22	6	28	0.035714
12	25	6	31	0.032258
13	29	7	36	0.027778
14	33	7	40	0.025
15	38	9	47	0.021277
16	41	9	50	0.02
17	44	9	53	0.018868
18	49	10	59	0.016949
19	54	11	65	0.015385
20	58	11	69	0.014493
21	62	11	73	0.013699
22	66	11	77	0.012987
23	70	11	81	0.012346
24	73	11	84	0.011905
25	76	11	87	0.011494
26	81	13	94	0.010638
27	85	13	98	0.010204
28	89	13	102	0.009804
29	94	13	107	0.009346

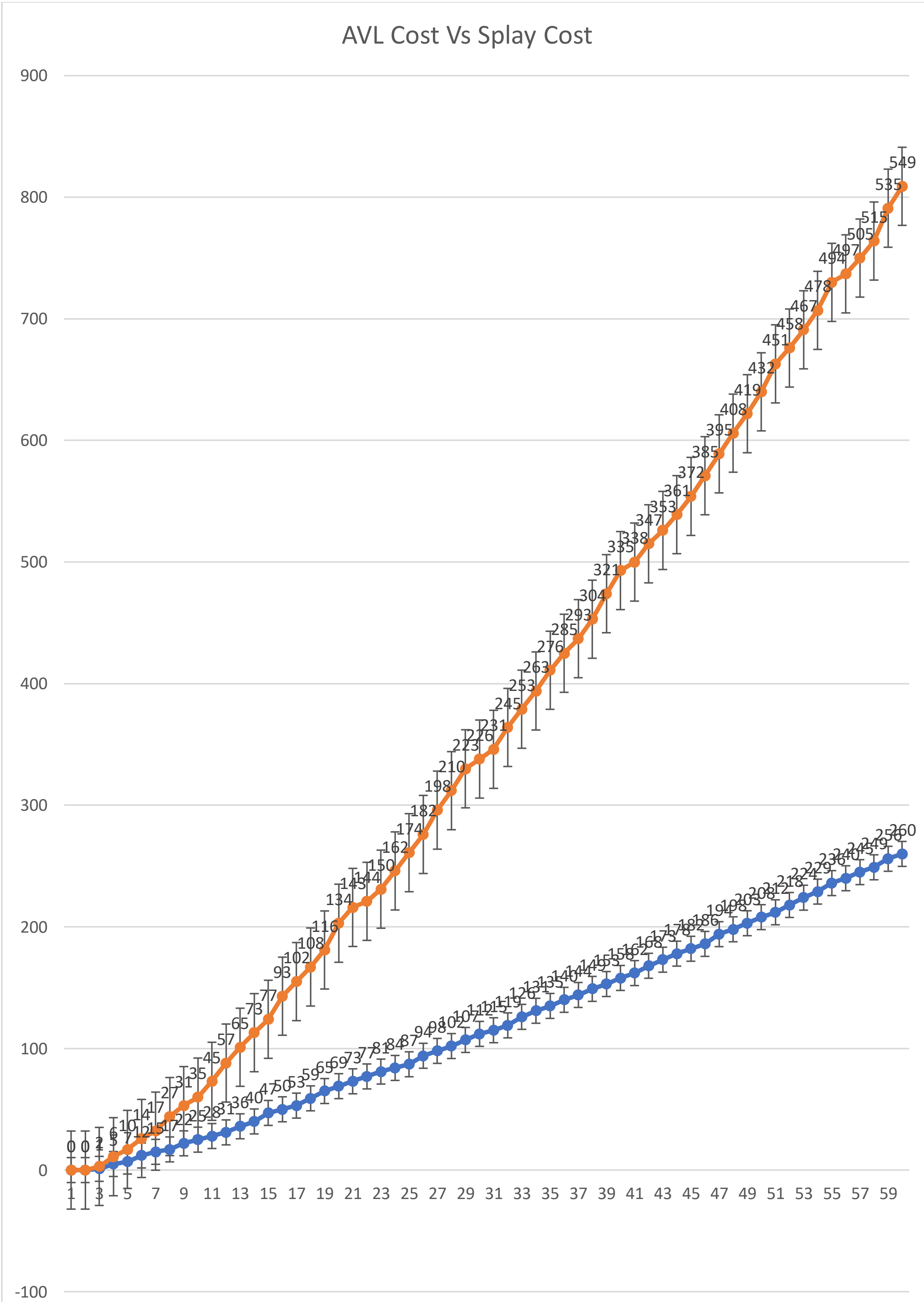
30	99	13	112	0.008929
31	102	13	115	0.008696
32	106	13	119	0.008403
33	111	15	126	0.007937
34	116	15	131	0.007634
35	120	15	135	0.007407
36	125	15	140	0.007143
37	129	15	144	0.006944
38	134	15	149	0.006711
39	138	15	153	0.006536
40	143	15	158	0.006329
41	147	15	162	0.006173
42	153	15	168	0.005952
43	158	15	173	0.00578
44	163	15	178	0.005618
45	167	15	182	0.005495
46	171	15	186	0.005376
47	177	17	194	0.005155
48	181	17	198	0.005051
49	186	17	203	0.004926
50	191	17	208	0.004808
51	195	17	212	0.004717
52	201	17	218	0.004587
53	207	17	224	0.004464
54	212	17	229	0.004367
55	217	19	236	0.004237
56	221	19	240	0.004167
57	226	19	245	0.004082
58	230	19	249	0.004016
59	236	20	256	0.003906
60	240	20	260	0.003846

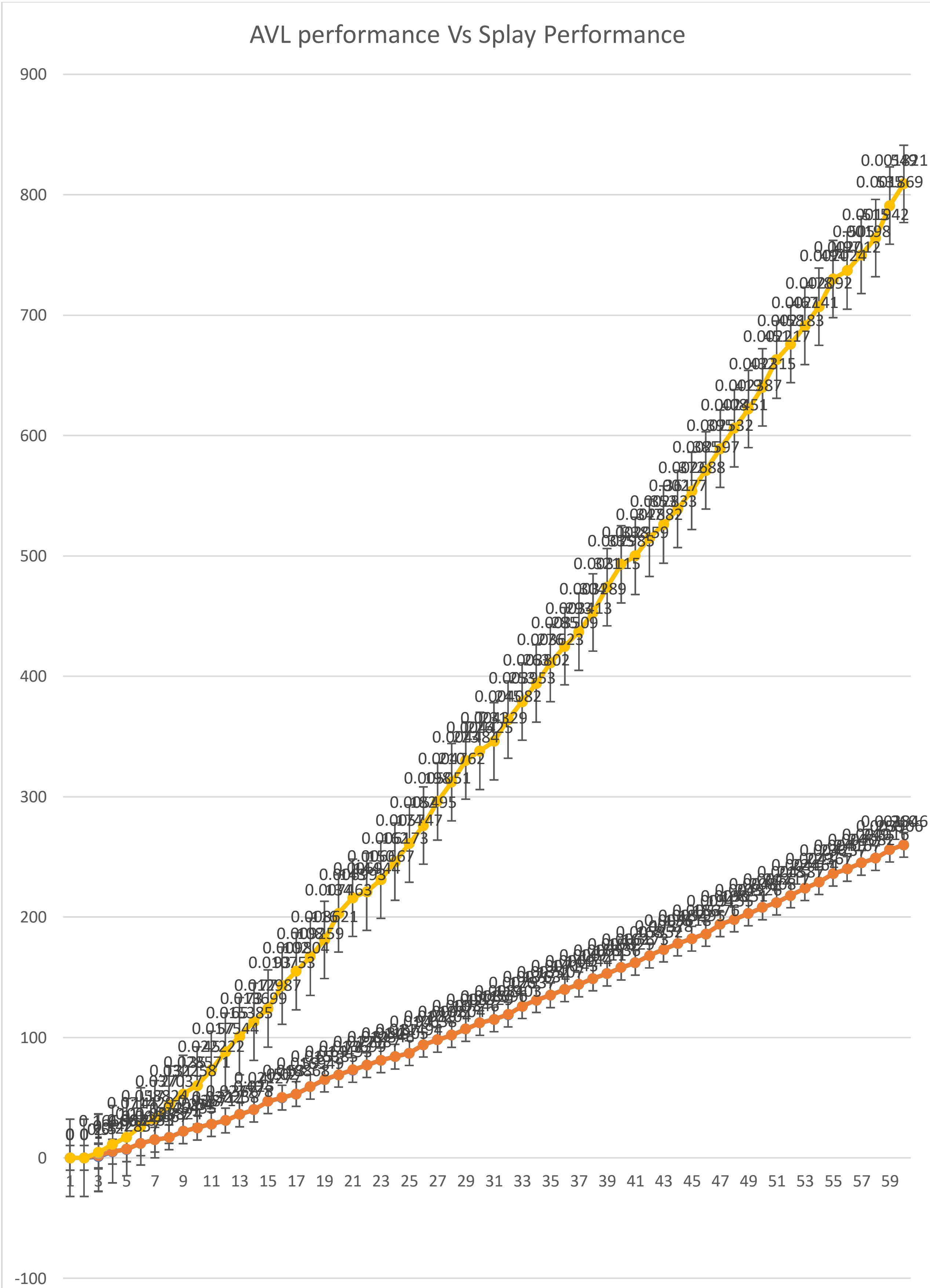
Splay Tree (1): -

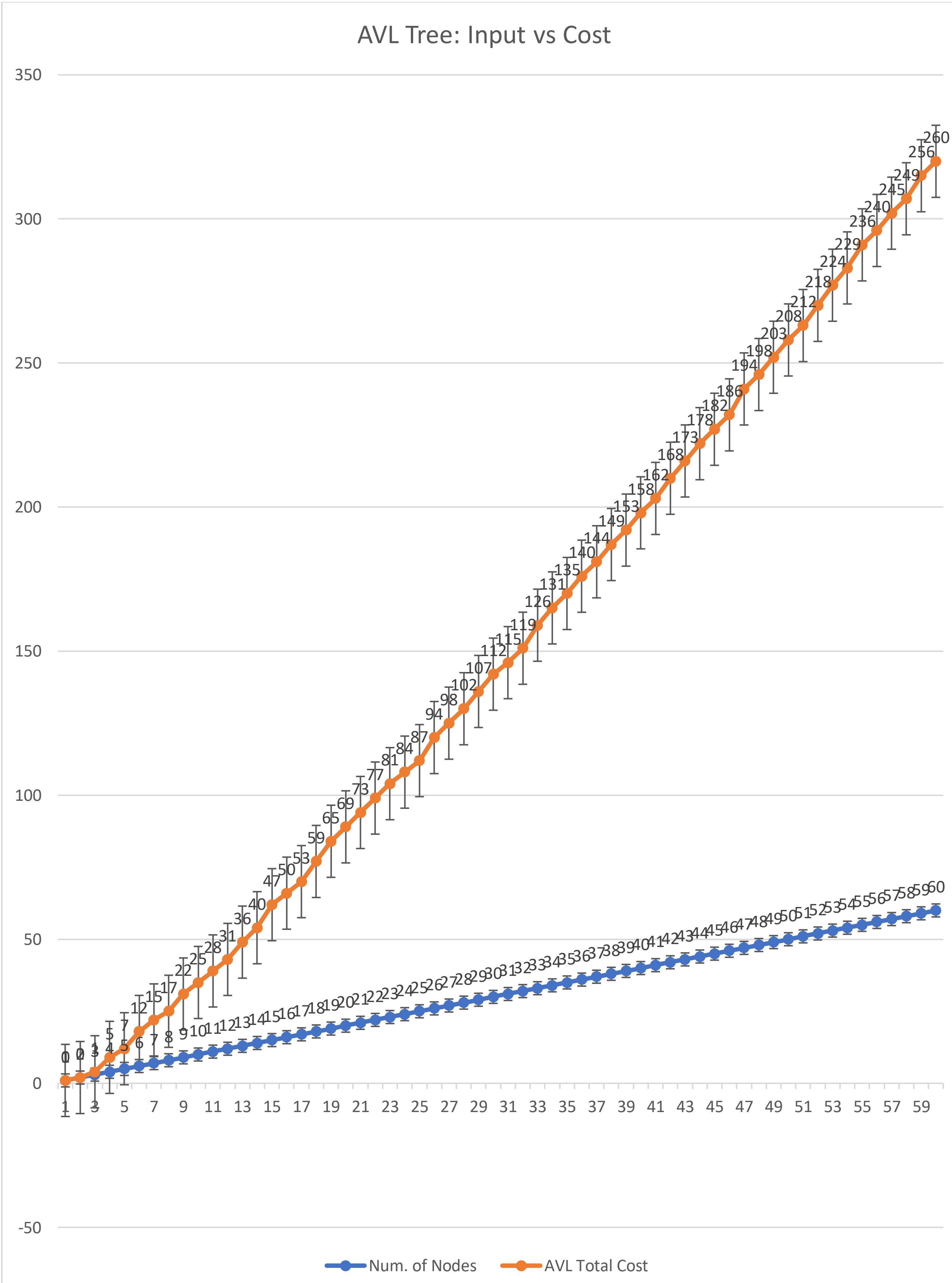
Num. of Nodes	Num. of Comparisons	Num. of Rotations	Splay Total Cost	Splay Performance
1	0	0	0	0
2	0	0	0	0
3	1	1	2	0.5
4	3	3	6	0.166667
5	5	5	10	0.1

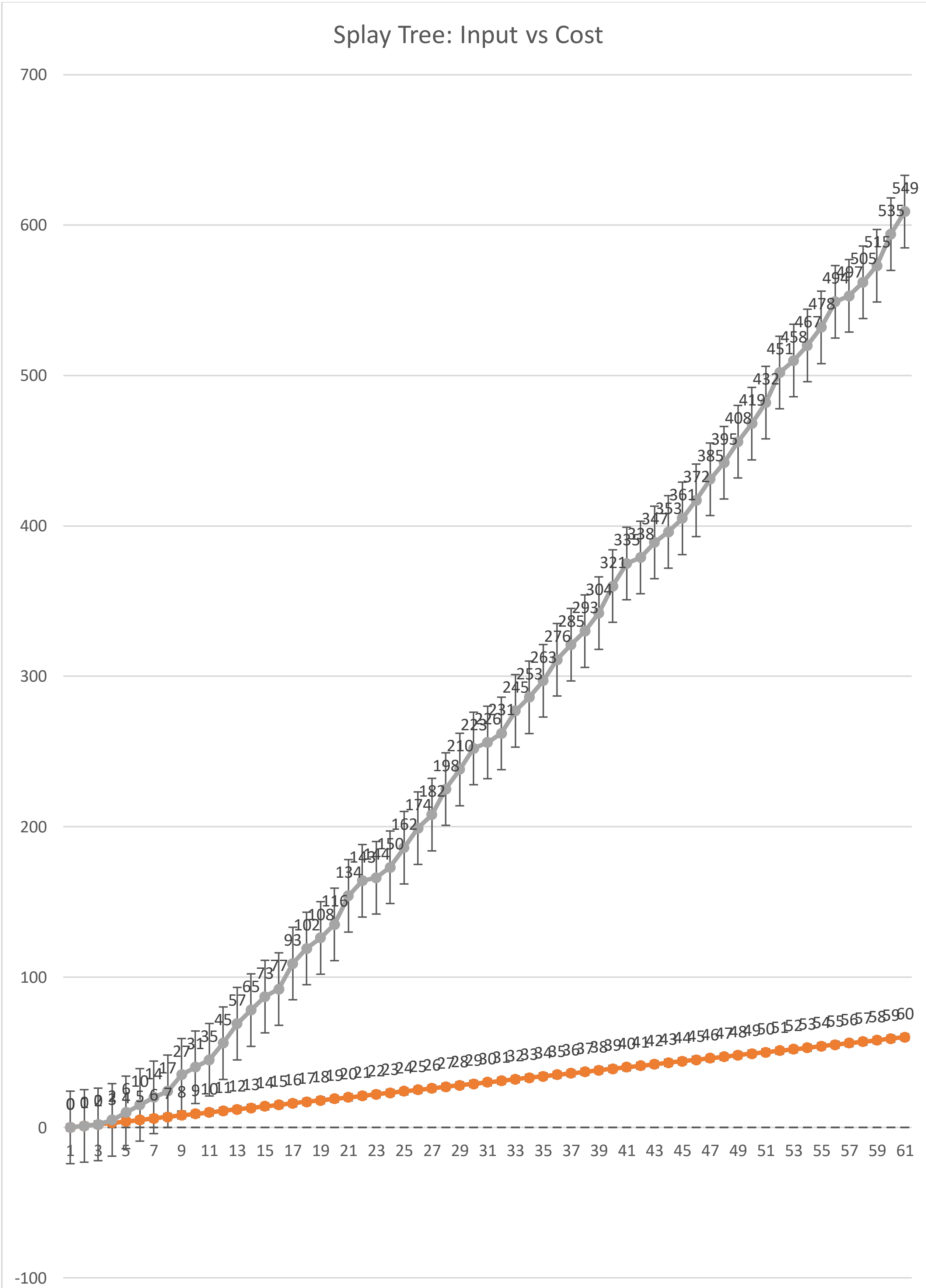
6	7	7	14	0.071429
7	9	8	17	0.058824
8	14	13	27	0.037037
9	16	15	31	0.032258
10	18	17	35	0.028571
11	23	22	45	0.022222
12	29	28	57	0.017544
13	33	32	65	0.015385
14	37	36	73	0.013699
15	39	38	77	0.012987
16	47	46	93	0.010753
17	52	50	102	0.009804
18	55	53	108	0.009259
19	59	57	116	0.008621
20	68	66	134	0.007463
21	73	70	143	0.006993
22	74	70	144	0.006944
23	77	73	150	0.006667
24	83	79	162	0.006173
25	89	85	174	0.005747
26	93	89	182	0.005495
27	101	97	198	0.005051
28	107	103	210	0.004762
29	114	109	223	0.004484
30	116	110	226	0.004425
31	119	112	231	0.004329
32	126	119	245	0.004082
33	130	123	253	0.003953
34	135	128	263	0.003802
35	142	134	276	0.003623
36	147	138	285	0.003509
37	151	142	293	0.003413
38	157	147	304	0.003289
39	166	155	321	0.003115
40	173	162	335	0.002985
41	175	163	338	0.002959
42	180	167	347	0.002882
43	183	170	353	0.002833
44	187	174	361	0.00277
45	193	179	372	0.002688
46	200	185	385	0.002597
47	205	190	395	0.002532
48	212	196	408	0.002451

49	218	201	419	0.002387
50	225	207	432	0.002315
51	235	216	451	0.002217
52	239	219	458	0.002183
53	244	223	467	0.002141
54	250	228	478	0.002092
55	258	236	494	0.002024
56	260	237	497	0.002012
57	264	241	505	0.00198
58	269	246	515	0.001942
59	279	256	535	0.001869
60			549	0.001821


AVL Cost Vs Splay Cost


AVL performance Vs Splay Performance
900
800
700
600
500
400
300
200
100
0
-100
1 3 5 7 9 11 13 15 17 19 21 23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59


AVL Tree: Input vs Cost
350
300
250
200
150
100
50
0
-50
1 3 5 7 9 11 13 15 17 19 21 23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59
Num. of Nodes
AVL Total Cost



Splay Tree: Input vs Cost
700
600
500
400
300
200
100
0
-100

Input File (2): - Calculations:-

AVL-Tree (2): -

Num. of Nodes	Num. of Comparisons	Num. of Rotations	AVL Total Cost	AVL Performance
1	0	0	0	
2	0	0	0	
3	1	0	1	1
4	3	2	5	0.2
5	5	2	7	0.142857
6	8	4	12	0.083333
7	11	6	17	0.058824
8	14	8	22	0.045455
9	17	8	25	0.04
10	20	8	28	0.035714
11	23	8	31	0.032258
12	26	8	34	0.029412
13	30	10	40	0.025
14	34	10	44	0.022727
15	37	10	47	0.021277
16	41	10	51	0.019608
17	45	11	56	0.017857
18	49	12	61	0.016393
19	53	12	65	0.015385
20	57	12	69	0.014493
21	62	13	75	0.013333
22	66	13	79	0.012658
23	70	13	83	0.012048
24	75	14	89	0.011236
25	79	14	93	0.010753
26	83	14	97	0.010309
27	88	15	103	0.009709
28	93	17	110	0.009091
29	97	17	114	0.008772
30	102	17	119	0.008403
31	106	17	123	0.00813
32	110	17	127	0.007874
33	115	19	134	0.007463
34	119	19	138	0.007246
35	124	20	144	0.006944
36	128	20	148	0.006757
37	132	20	152	0.006579

38	136	20	156	0.00641
39	141	20	161	0.006211
40	146	20	166	0.006024
41	151	22	173	0.00578
42	156	23	179	0.005587
43	162	24	186	0.005376
44	168	25	193	0.005181
45	173	25	198	0.005051
46	178	25	203	0.004926
47	181	25	206	0.004854
48	186	25	211	0.004739
49	191	25	216	0.00463
50	197	27	224	0.004464
51	202	27	229	0.004367
52	208	28	236	0.004237
53	213	28	241	0.004149
54	218	28	246	0.004065
55	223	28	251	0.003984
56	229	28	257	0.003891
57	235	30	265	0.003774
58	241	30	271	0.00369
59	246	30	276	0.003623
60	252	31	283	0.003534
61	258	31	289	0.00346
62	263	31	294	0.003401
63	269	31	300	0.003333
64	276	33	309	0.003236
65	281	33	314	0.003185
66	285	33	318	0.003145
67	290	33	323	0.003096
68	297	33	330	0.00303
69	303	33	336	0.002976
70	309	33	342	0.002924
71	316	35	351	0.002849
72	321	35	356	0.002809
73	325	35	360	0.002778
74	330	35	365	0.00274
75	331	35	366	0.002732
76	335	35	370	0.002703
77	341	35	376	0.00266
78	346	35	381	0.002625
79	352	35	387	0.002584
80	357	35	392	0.002551

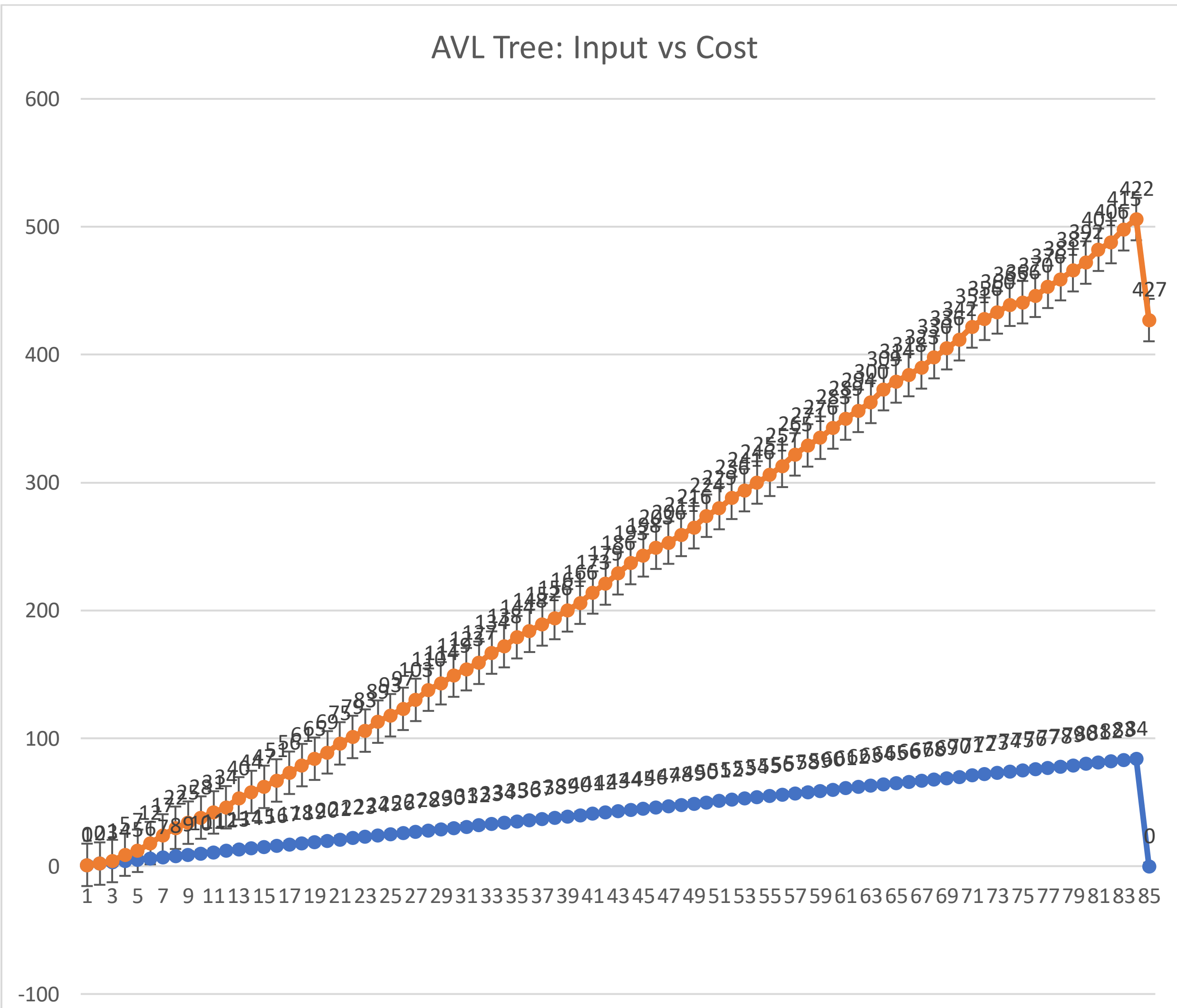
81	364	37	401	0.002494
82	369	37	406	0.002463
83	376	39	415	0.00241
84	383	39	422	0.00237
			427	0.002342

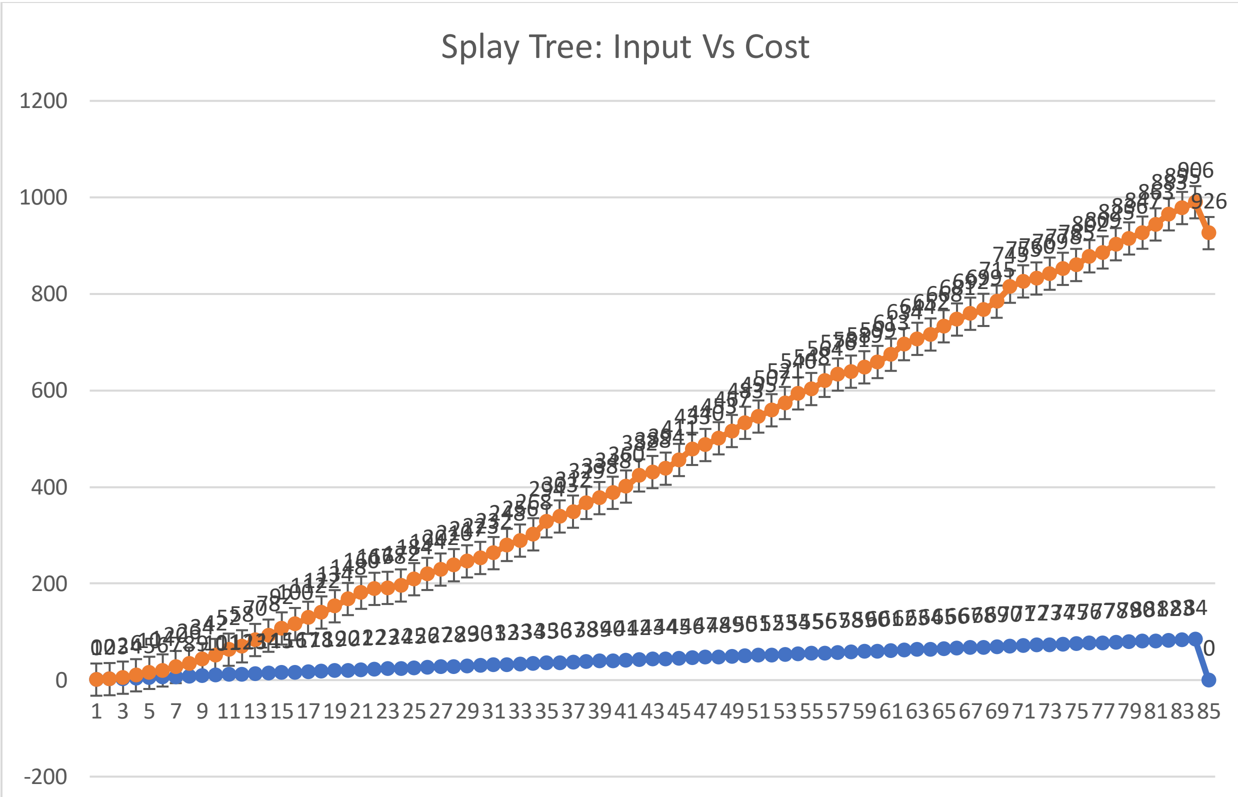
Splay Tree (2): -

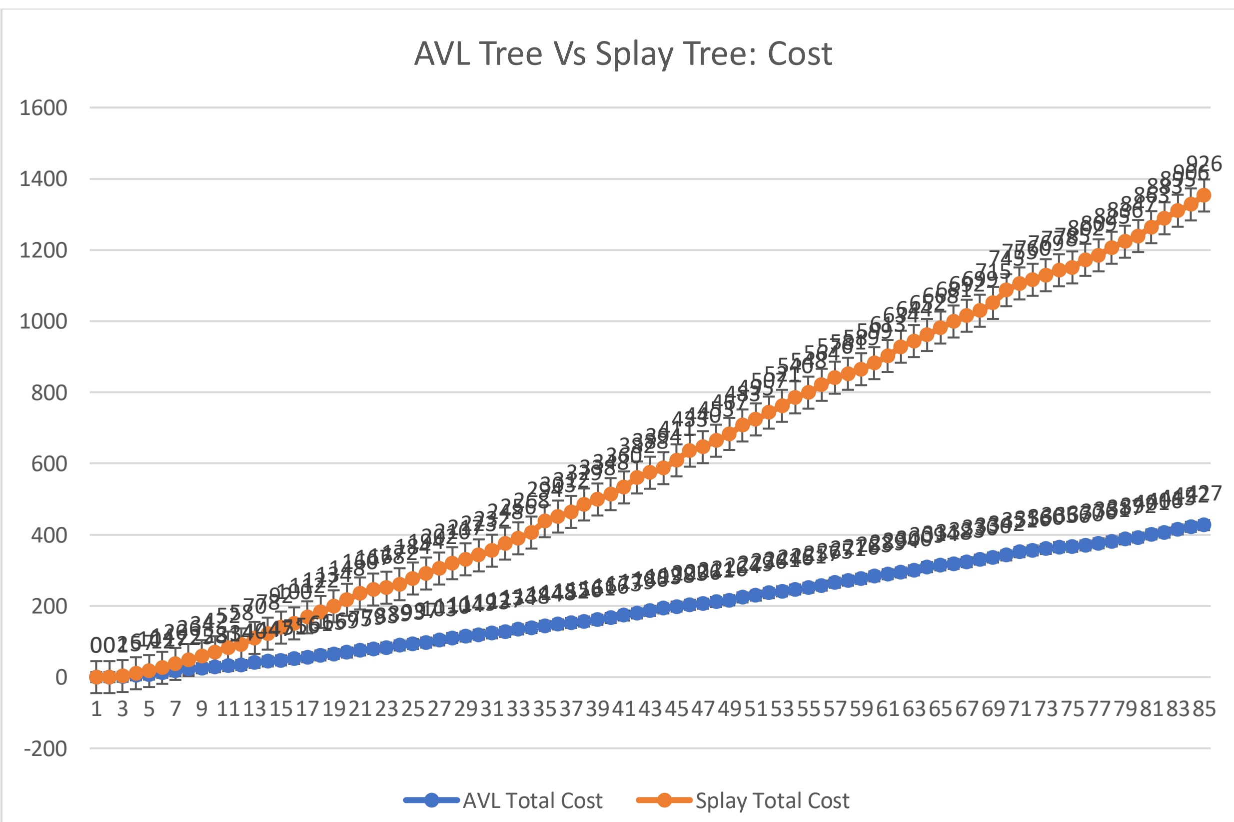
Num. of Nodes	Num. of Comparisons	Num. of Rotations	Splay Total Cost	Splay Performance
1	0	0	0	
2	0	0	0	
3	1	1	2	0.5
4	3	3	6	0.166667
5	5	5	10	0.1
6	7	7	14	0.071429
7	10	10	20	0.05
8	13	13	26	0.038462
9	17	17	34	0.029412
10	21	21	42	0.02381
11	26	26	52	0.019231
12	29	29	58	0.017241
13	35	35	70	0.014286
14	39	39	78	0.012821
15	46	46	92	0.01087
16	50	50	100	0.01
17	56	56	112	0.008929
18	61	61	122	0.008197
19	67	67	134	0.007463
20	74	74	148	0.006757
21	80	80	160	0.00625
22	84	83	167	0.005988
23	85	83	168	0.005952
24	87	85	172	0.005814
25	93	91	184	0.005435
26	98	96	194	0.005155
27	102	100	202	0.00495
28	106	104	210	0.004762
29	110	107	217	0.004608
30	113	110	223	0.004484
31	118	114	232	0.00431

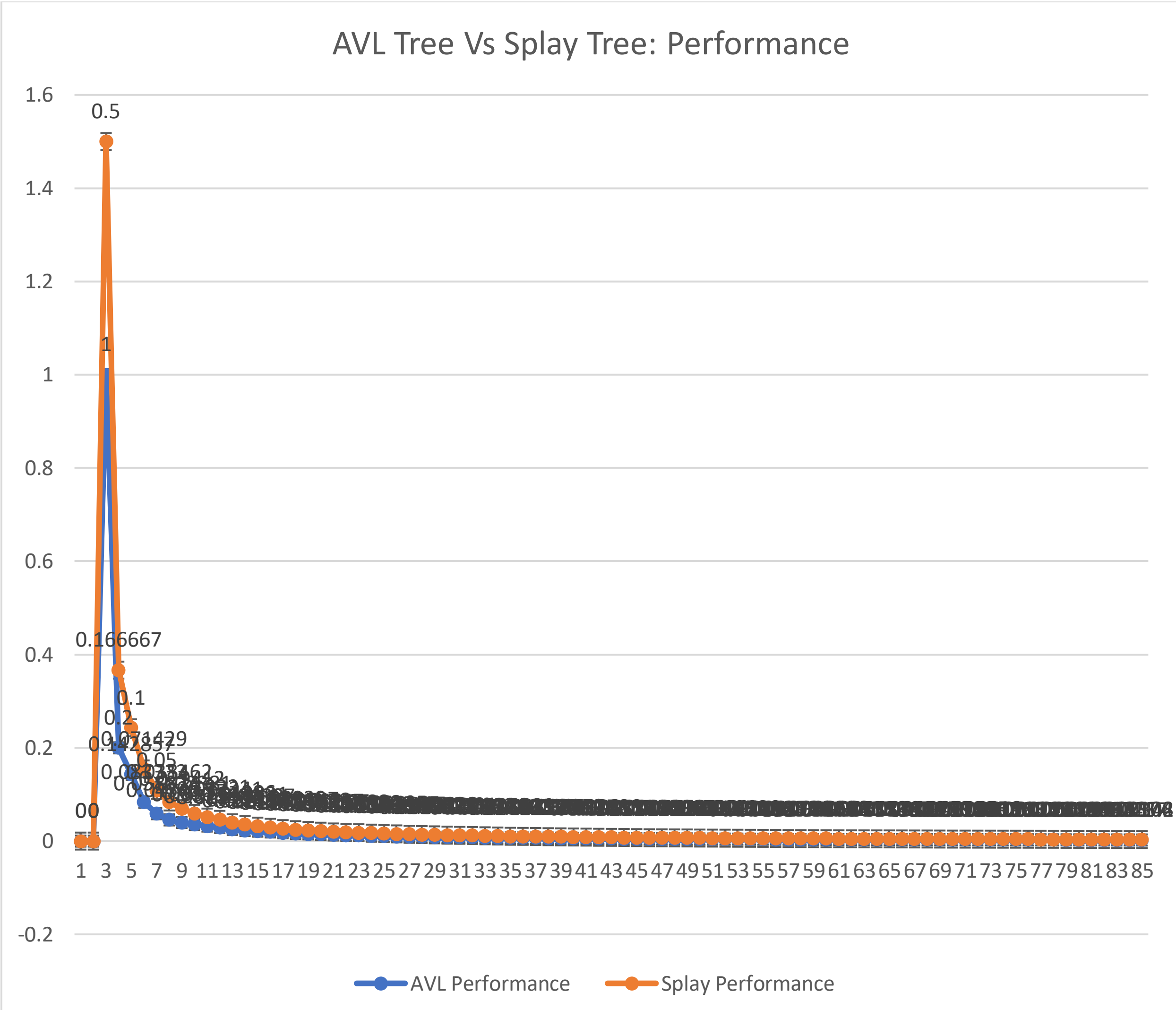
32	126	122	248	0.004032
33	130	126	256	0.003906
34	136	132	268	0.003731
35	149	145	294	0.003401
36	154	149	303	0.0033
37	159	153	312	0.003205
38	168	161	329	0.00304
39	173	165	338	0.002959
40	178	170	348	0.002874
41	184	176	360	0.002778
42	195	187	382	0.002618
43	198	190	388	0.002577
44	201	193	394	0.002538
45	210	201	411	0.002433
46	221	212	433	0.002309
47	225	215	440	0.002273
48	232	221	453	0.002208
49	239	228	467	0.002141
50	247	236	483	0.00207
51	253	242	495	0.00202
52	259	248	507	0.001972
53	266	255	521	0.001919
54	276	264	540	0.001852
55	280	268	548	0.001825
56	288	276	564	0.001773
57	294	282	576	0.001736
58	297	284	581	0.001721
59	301	288	589	0.001698
60	306	293	599	0.001669
61	313	300	613	0.001631
62	324	310	634	0.001577
63	329	315	644	0.001553
64	333	319	652	0.001534
65	341	327	668	0.001497
66	348	333	681	0.001468
67	354	338	692	0.001445
68	358	341	699	0.001431
69	366	349	715	0.001399
70	381	364	745	0.001342
71	386	369	755	0.001325
72	389	371	760	0.001316
73	394	375	769	0.0013
74	399	379	778	0.001285

75	403	382	785	0.001274
76	412	390	802	0.001247
77	416	393	809	0.001236
78	424	401	825	0.001212
79	430	406	836	0.001196
80	436	411	847	0.001181
81	444	419	863	0.001159
82	454	429	883	0.001133
83	460	435	895	0.001117
84	466	440	906	0.001104
			926	0.00108




Splay Tree: Input Vs Cost
1200
1000
800
600
400
200
0
-200


AVL Tree Vs Splay Tree: Cost
1600
1400
1200
1000
800
600
400
200
0
-200
AVL Total Cost
Splay Total Cost



From Previous Calculations: We conclude some points: -

Difference in terms of Cost of operations and Performance:-

AVL and Splay trees are both self-balancing binary search trees, which means that they both have a time complexity of O(log n) for the basic operations (insertion, deletion, and search). However, there are some differences between the two in terms of the cost of operations and performance:

1. Balance maintenance: In an AVL tree, the balance of the tree is maintained by checking the balance factor of the nodes after each insertion or deletion and performing rotations if necessary. In a Splay tree, the balance of the tree is maintained by moving the recently accessed elements to the top of the tree.
2. Rotation cost: In an AVL tree, rotations are performed to maintain the balance of the tree. A single rotation in an AVL tree costs one time unit (tu), while a double rotation costs two tus. In a Splay tree, splay operations are performed to move the recently accessed elements to the top

of the tree. The cost of a splay operation is equal to the number of depth levels that the node has moved through.

3. Search performance: In an AVL tree, the search performance is not affected by the recent access patterns, as the tree is balanced based on the balance factor of the nodes. In a Splay tree, the search performance may be affected by the recent access patterns, as the tree is balanced based on the access history of the elements.

Overall, AVL and Splay trees are both efficient data structures for storing and accessing data, but the specific performance and cost of operations may vary depending on the implementation and the input data.

For Small Data Input: -

For small input sizes, the performance of AVL and Splay trees may not be significantly different, as the time complexity of both data structures is O(log n) for the basic operations (insertion, deletion, and search). However, there are some differences between the two data structures that may affect their performance for small input sizes:

1. Rotation cost: In an AVL tree, rotations are performed to maintain the balance of the tree. A single rotation in an AVL tree costs one time unit (tu), while a double rotation cost two tus. In a Splay tree, splay operations are performed to move the recently accessed elements to the top of the tree. The cost of a splay operation is equal to the number of depth levels that the node has moved through. For small input sizes, the cost of rotations or splays may not be significant compared to the overall cost of the operation.
2. Search performance: In an AVL tree, the search performance is not affected by the recent access patterns, as the tree is balanced based on the balance factor of the nodes. In a Splay tree, the search performance may be affected by the recent access patterns, as the tree is balanced based on the access history of the elements. For small input sizes, the difference in search performance between AVL and Splay trees may not be significant.

Overall, for small input sizes, the performance and cost of operations for AVL and Splay trees may not be significantly different. It is important to test the performance of both data structures with different input sizes and access patterns to determine which one performs better in a particular scenario.

For Large Data Input: -

For large input sizes, the performance of AVL and Splay trees may differ, as the time complexity of both data structures is O(log n) for the basic operations (insertion, deletion, and search). However, there are some differences between the two data structures that may affect their performance for large input sizes:

1. Rotation cost: In an AVL tree, rotations are performed to maintain the balance of the tree. A single rotation in an AVL tree costs one time unit (tu), while a double rotation costs two tus. In a Splay tree, splay operations are performed to move the recently accessed elements to the top of the tree. The cost of a splay operation is equal to the number of depth levels that the node

has moved through. For large input sizes, the cost of rotations or splays may be significant compared to the overall cost of the operation.

2. Search performance: In an AVL tree, the search performance is not affected by the recent access patterns, as the tree is balanced based on the balance factor of the nodes. In a Splay tree, the search performance may be affected by the recent access patterns, as the tree is balanced based on the access history of the elements. For large input sizes, the difference in search performance between AVL and Splay trees may be significant, depending on the access patterns.

Overall, for large input sizes, the performance and cost of operations for AVL and Splay trees may differ. It is important to test the performance of both data structures with different input sizes and access patterns to determine which one performs better in a particular scenario.

For Inputs with Duplicates: -

For input with duplicates, both AVL and Splay trees will perform similarly, as the time complexity of both data structures is O(log n) for the basic operations (insertion, deletion, and search). However, there are some differences between the two data structures that may affect their performance for input with duplicates:

1. Rotation cost: In an AVL tree, rotations are performed to maintain the balance of the tree. A single rotation in an AVL tree costs one time unit (tu), while a double rotation costs two tus. In a Splay tree, splay operations are performed to move the recently accessed elements to the top of the tree. The cost of a splay operation is equal to the number of depth levels that the node has moved through. The cost of rotations or splays may be significant for input with many duplicates, as the number of operations may increase.

2. Search performance: In an AVL tree, the search performance is not affected by the recent access patterns, as the tree is balanced based on the balance factor of the nodes. In a Splay tree, the search performance may be affected by the recent access patterns, as the tree is balanced based on the access history of the elements. The search performance of both data structures may be affected by the presence of many duplicates, as the number of comparisons may increase.

Overall, for input with duplicates, the performance and cost of operations for AVL and Splay trees may not be significantly different. It is important to test the performance of both data structures with different input sizes and access patterns to determine which one performs better in a particular scenario.

The cost of operations in AVL and Splay trees is measured in time units (tus), which represent the number of basic operations (comparisons or rotations/splays) required to perform a particular task. The specific cost of operations in AVL and Splay trees may vary depending on the implementation and the input data.

Here is a comparison of the cost of some basic operations in AVL and Splay trees in terms of tus:

Insertion:

- In an AVL tree, the cost of insertion is determined by the number of comparisons required to find the appropriate position for the new node and the number of rotations required to maintain the balance of the tree. The total cost of insertion in an AVL tree is O(log n) in the average case and O(n) in the worst case.
- In a Splay tree, the cost of insertion is determined by the number of comparisons required to find the appropriate position for the new node and the number of splays required to move the recently inserted node to the top of the tree. The total cost of insertion in a Splay tree is O(log n) in the average case and O(n) in the worst case.

Deletion:

- In an AVL tree, the cost of deletion is determined by the number of comparisons required to find the node to be deleted and the number of rotations required to maintain the balance of the tree after the deletion. The total cost of deletion in an AVL tree is O(log n) in the average case and O(n) in the worst case.
- In a Splay tree, the cost of deletion is determined by the number of comparisons required to find the node to be deleted and the number of splays required to move the parent of the deleted node to the top of the tree. The total cost of deletion in a Splay tree is O(log n) in the average case and O(n) in the worst case.

Search:

- In an AVL tree, the cost of search is determined by the number of comparisons required to find the node. The time complexity of search in an AVL tree is O(log n) in the average case and O(n) in the worst case.
- In a Splay tree, the cost of search is determined by the number of comparisons required to find the node and the number of splays required to move the searched node to the top of the tree. The time complexity of search in a Splay tree is O(log n) in the average case and O(n) in the worst case.

It is important to note that the above cost estimates are for the average and worst cases, and the actual cost may vary depending on the specific implementation and the input data.
