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CORRELATIONS BACKGROUND

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1

Revisiting Wang

A brief set of notes on (Wang, n.d.); a seminal paper on portfolio modeling for insurance.

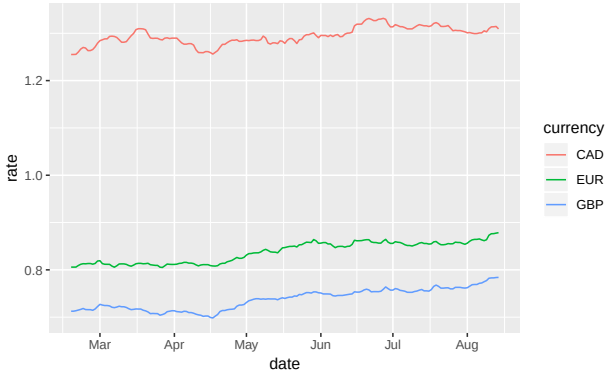
Thoughts, thinkity, thunk.

2

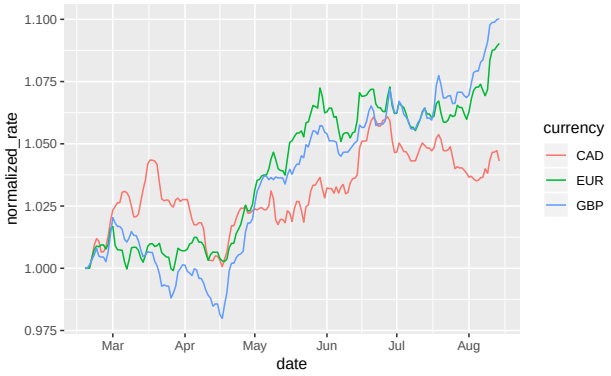
Example data

```
library(tidyverse)
library(quantmod)

get_fx_tibble <- function(which_fx) {

  obj_xts <- getFX(which_fx, auto.assign = FALSE) %>%
    as.data.frame() %>%
    as_tibble(rownames = 'date')

}

tbl_fx <- map_dfc(
    c('USD/GBP', 'USD/EUR', 'USD/CAD'), get_fx_tibble
  ) %>%
  select(-date1, -date2) %>%
  mutate(date = as.Date(date)) %>%
  select_all(gsub, pattern = 'USD.', replacement = '') %>%
  tidyr::gather(currency, rate, -date) %>%
  group_by(currency) %>%
  arrange(date) %>%
  mutate(
      rate_change = rate / dplyr::lag(rate) - 1
    , normalized_rate = rate / first(rate)
  ) %>%
  ungroup()

save(
  tbl_fx
  , file = file.path('data', 'fx.rda')
)

tbl_fx %>%
  ggplot(aes(date, rate, color = currency)) +
```

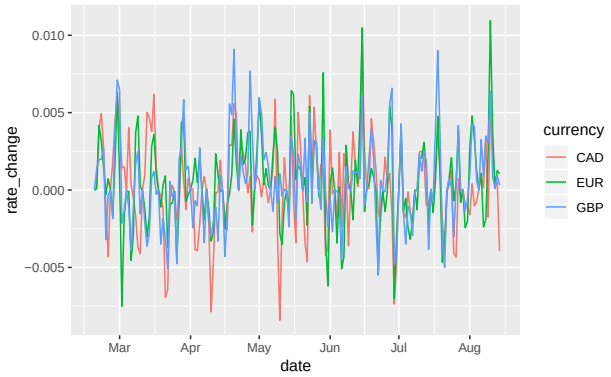
```
  geom_line()
```



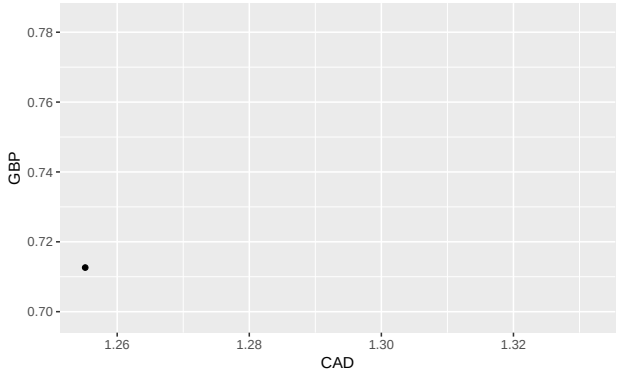

```
tbl_fx %>%
  ggplot(aes(date, normalized_rate, color = currency)) +
  geom_line()
```




```
tbl_fx %>%
  ggplot(aes(date, rate_change, color = currency)) +
  geom_line()
#> Warning: Removed 3 rows containing missing
#> values (geom_path).
```

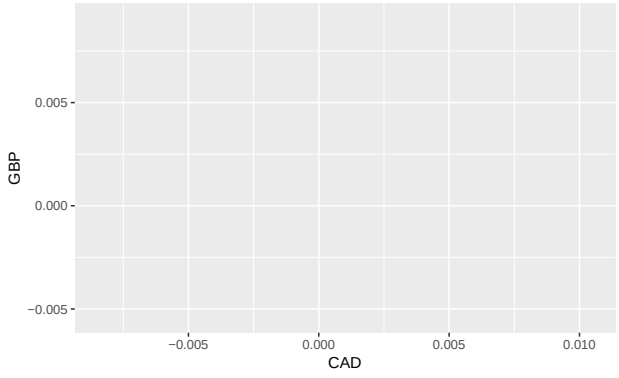


```
tbl_fx %>%
  select(-rate_change) %>%
  tidyr::spread(currency, rate) %>%
  ggplot(aes(CAD, GBP)) +
  geom_point()
#> Warning: Removed 534 rows containing missing values
#> (geom_point).
```



A visual inspection of the plot suggests that *changes* in FX are not as strongly correlated.

```
tbl_fx %>%
  select(-rate) %>%
  tidyr::spread(currency, rate_change) %>%
  ggplot(aes(CAD, GBP)) +
  geom_point()
#> Warning: Removed 535 rows containing missing values
#> (geom_point).
```



3

Copula refresher

What's a copula? It's a multivariate distribution, with support on $\Re^N$. We use them to simulate multivariate losses which need not be independent and identically distributed.

3.1 Support in R packages

Comes primarily from the `copula` package though there are some others.

```
library(copula)
```

3.2 Visualize multivariate data

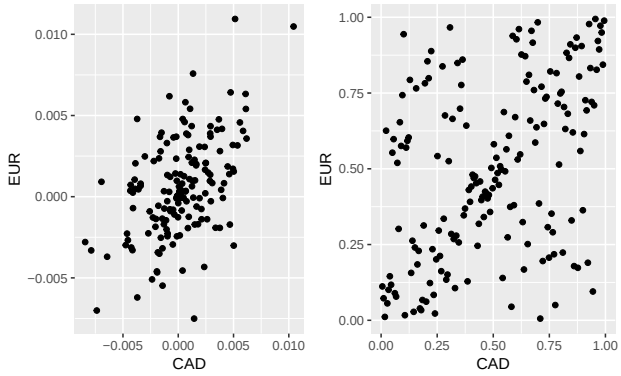
3.2.1 Simulations

3.2.2 Actual data

Load in our sample FX data.

```
load(
  file = file.path('data', 'fx.rda')
)

mat_obs <- tbl_fx %>%
  select(date, currency, rate_change) %>%
  filter(!is.na(rate_change)) %>%
  tidyr::spread(currency, rate_change) %>%
  select(-date) %>%
  as.matrix()

mat_pseudo <- mat_obs %>%
  copula::pobs()

plt_obs <- mat_obs %>%
  as.data.frame() %>%
```

```
  ggplot(aes(CAD, EUR)) +
  geom_point()

plt_pseudo <- mat_pseudo %>%
  as.data.frame() %>%
  ggplot(aes(CAD, EUR)) +
  geom_point()

grid.arrange(plt_obs, plt_pseudo, nrow = 1)
```



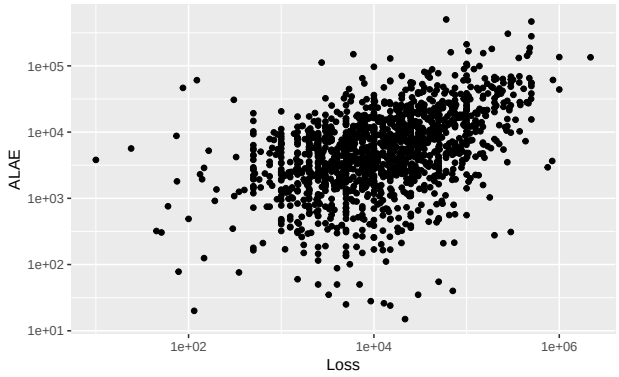
3.3 Fitting a copula

(Charpentier, n.d.) references four methods for fitting a copula. The example below will use maximum likelihood.

```
fit_fx <- fitCopula(
    gumbelCopula(dim = 3)
  , data = mat_pseudo
  , method = 'ml'
)

fit_fx
#> Call: fitCopula(copula, data = data, method = "ml")
#> Fit based on "maximum likelihood" and 178 3-dimensional observations.
#> Copula: gumbelCopula
#> alpha
#>  1.59
#> The maximized loglikelihood is 78.4
#> Optimization converged
confint(fit_fx)
#>       2.5 % 97.5 %
#> alpha  1.46   1.72
```

Another example from (Charpentier, n.d.) using loss and ALAE data.

```r
library(CASdatasets)
data("lossalae")

lossalae %>%
  ggplot(aes(Loss, ALAE)) +
  geom_point() +
  scale_x_log10() +
  scale_y_log10()
```



4

Copula literature review

4.1 Textbooks

There are at least two textbooks which give a foundational presentation of copulas for actuaries. The first is (Charpentier, n.d.). I have a physical copy of this book and it's pretty good. The second is (Parodi, n.d.), which I'd never heard of until I did a search on O'Reilly's Safari service. It's very broad, but the chapter on multiline modeling looks promising.

(Nelsen, n.d.) is a book that I borrowed from a colleague in Munich. Hoo-boy. The first chapter was super mathy and not terribly grounded. I couldn't get past it. Enter at your own risk.

(Hofert et al. 2018) looks promising. Against my better judgment, I might buy a copy. It was written by the same people who wrote the `copula` package, so there you are.

(Joe, n.d.) is another interesting one that I stumbled across. Harry Joe - the author - gets name checked in (Venter et al.).

4.2 Variance

- Multivariate Copulas for Financial Modeling (Venter et al.)
- Dependence Models and the Portfolio Effect (Mango and Sandor)
- Quantifying Correlated Reinsurance Exposures with Copulas (Venter)
- Tails of copulas

Here, Venter (Venter 2002) talks through some issues about dependence in the tail of the copula.

Venter introduces measurements for right and left tail concentration:

$$R(z) = Pr(U > z|V > z)$$

4.2.1 Multivariate Copulas for Financial Modeling

(Venter et al.) looks at the issues associated with copulas higher than bivariate.

We learn about a few new copulas: the IT, and the Joe.

As an example, the authors fit several copulas to currency exchange data.

5

Stochastic Reserving Review

5.1 Mack

5.2 Other stuff

6

Stochastic Reserving Literature

This will mirror the methodology review, once I figure out how that's structured.

7

Solvency Regulation Literature

Solvency I, solvency II Solvency me, solvency you!!

References

Charpentier, Arthur. n.d. *Computational Actuarial Science with R*.

Hofert, Marius, Ivan Kojadinovic, Martin Maechler, and Jun Yan. 2018. *Elements of Copula Modeling with R*. Springer Use R! Series. http://www.springer.com/de/book/9783319896342.

Joe, Harry. n.d. *Dependence Modeling with Copulas*.

Mango, Donald, and James Sandor. "Dependence Models and the Portfolio Effect." https://www.casact.org/research/dare/index.cfm?fa=view&abstrID=5151.

Nelsen, Roger. n.d. *An Introduction to Copulas*. Springer.

Parodi, Pietro. n.d. *Pricing in General Insurance*.

Venter, Gary. 2002. "Tails of Copulas." *Proceedings of the Casualty Actuarial Society*, no. 89: 68—113.

———. "Quantifying Correlated Reinsurance Exposures with Copulas." https://www.casact.org/research/dare/index.cfm?fa=view&abstrID=5290.

Venter, Gary, Jack Barnett, Rodney Kreps, and John Major. "Multivariate Copulas for Financial Modeling." https://www.variancejournal.org/issues/01-01/103.pdf.

Wang, Shaun. n.d. "AGGREGATION of Correlated Risk Portfolios: MODELS and Algorithms." https://www.casact.org/library/wang.pdf.