Practice Final Exam: Modules 8~13 Hsuan-You(Shaun) Lin

<Short Answers>

1. What is the **worst-case** complexity for each of these algorithms? (Use Big Oh notation):

1.1. Selection Sort of N items
Ans: O(N^2)
1.2. Insertion Sort of N items
Ans: O(N^2)
1.3. Shell sort of N items
Ans: O(N^3/2)
1.4. A sequence of M union-find operations on N disjoint sets
Ans: O(N log*M)
1.5. The copying cost when using the doubling algorithm for a sequence of N push operations.
Ans: O(N)
1.6. Quicksort of N items
Ans: O(N^2)
1.7. Merge sort of N items
Ans: O(N*log(N))
1.8. A LSB radix sort over an alphabet of size A, for fixed-length strings of length L. there are N such strings.
Ans: O(NL)
1.9. Given a sample string of length N, and a fixed pattern (NOT a regular expression) of length M, what is the complexity of searching the sample string for the pattern?
Ans: O(M+N)
1.10. Given a sample string of length N, and a (fully parenthesized) regular expression of length M, what is the complexity of searching the sample string for the pattern?
Ans: O(MN)
1.11. Find the maximum item in a max heap with N items
Ans: O(1)
1.12. Finding an item in a random binary search tree of size N.
Ans: O(N)
1.13. Finding an item in a left-leaning red-black tree of size N.
Ans: O(log N)
1.14. Inserting an item into a random binary tree of size N
Ans: O(N)
1.15. Inserting an item into a left-leaning red-black tree of size N
Ans: O(log N)
1.16. Insert 1 element into a hash-table that has N elements.
Ans: O(N)
1.17. Find an item in a hash-table that has N items.
Ans: O(N)
1.18. For a graph G = (V,E) with no negative edge weights, how long does Dijkstra's algorithm take to find the shortest path between distinguished vertices s and t.
Ans: O(E logV)
1.19. How long does the LU factorization take for a matrix of size NxN?
Ans: O(N^3)
1.20. How long does Strassen's algorithm take to multiply 2 square matrices of size N x N?
What is the average case complexity for each of these algorithms
Ans: $O(N^{\log_2 7})$

2. What is the **average case** complexity for each of these algorithms
2.1. Selection Sort of N items ?
Ans: O(N^2)
2.2. Insertion Sort of N items ?
Ans: O(N^2)
2.3. Shell sort of N items?
Ans: O(N*log N)
2.4. A sequence of M union-find operations on N disjoint sets
Ans: O(M log*N)
2.5. The copying cost when using the doubling algorithm for a sequence of N push operations
Ans: O(1)
2.6. Quicksort of N items
Ans: O(N*log N)
2.7. Merge sort of N items
Ans: O(N*log N)
2.8. Insert 1 item into a priority queue of size N
Ans:
2.9. A LSB radix sort over an alphabet of size A, for fixed-length strings of length L. there are N such strings
Ans:
2.10. Finding an item in a random binary search tree of size N
Ans: O(log N)
2.11. Finding an item in a left-leaning red-black tree of size N
Ans:
2.12. Inserting an item into a random binary tree of size N
Ans:
2.13. Inserting an item into a left-leaning red-black tree of size N
Ans:
2.14. Insert 1 element into a hash-table that has N elements.
Ans: O(1)
2.15. Find an item in a hash-table that has N items
Ans:
2.16. For a graph G = (V,E) with no negative edge weights, how long does Dijkstra's algorithm take to find the shortest path between distinguished vertices s and t.
Ans:
2.17. Given a graph G with no negative cycles, but might have negative edge weights, what is the complexity of Dijkstra's algorithm to find the shortest path between 2 distinguished vertices?
Ans:
2.18. Given a graph G = (V,E) with negative edge weights, but no negative cycles, what is the best known asymptotic complexity for finding the shortest path between 2 distinguished vertices s and t?
Ans: O(VE)
2.19. Given a graph G = (V,E) with negative edge weights, but no negative cycles, what is the best known asymptotic complexity for finding the shortest path from 1 distinguished vertex s to all other vertices in the graph?
Ans: O(VE)
2.20. Given a graph G = (V,E), with weighted edges given by the function w(e) for e $\in$E, what is the best known asymptotic complexity to find the minimum cost spanning tree?
Ans: O(E logV)
2.21. How long does the LU factorization take for a matrix of size NxN?
Ans: O(N^3)
2.22. How long does Strassen's algorithm take to multiply 2 square matrices of size N x N ?
Ans: $O(N^{\log_2 7})$

<Not-so-short Answers>

1. What is the computational complexity of the following program? (Big Oh notation is sufficient). Assume that X[i,j] references the (i,j) component of matrix X. Also, assume that N parameter describing the size of X as X[N,N].

```
X[N,N].
for (i = 1; i <= N; i++){
        v[i] = 0;
        for (j = i + 1; j <= N; j++) {
                v[i] += A[i,j] * b[j];
        }
 }
```

   Be sure to give your reasons for your complexity calculation

Ans: O(N^2)

2. For the Union-Find f array below:

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
f[i]	2	2	4	2	6	7	2	4	5	6	11	13	11	16	11	11

2.1. Draw the forest corresponding to the f-array.

Ans: Same as Module 2

2.2. Can the above be a result of running weighted quick union? Explain why this is impossible OR ELSE give a sequence of union-find operations that produce the above table.

Ans: Same as Module 2

3. Using any method you like, find the Big Oh complexity of the recurrence

   T(n) = 5T(n/4) + n^3 log(n)

Ans:

4. Using any method you like, find the Big Oh complexity of the recurrence

   T(n) = T(2n/3) + T(n/3) + n

Ans:

5. You are building a web crawler and storing each full HTML URL in an array. You notice this taking up a lot of room on your laptop. You also notice that the HTML URLs have a lot of common prefixes.

What data structure do you recommend to avoid storing more that 1 copy of each prefix?
Ans: Trie
How would you find the longest common prefix (LCP) of 2 URLs in your data structure?
Ans:
What is the complexity of your LCP algorithm?
Ans: O(Max(L1, L2))

6. Show a trace of quickselect to find the median of the following sequence (use 1st element pivots):

3, 17, -5, 4, 13, 8, 7, 6, 9, 15,-15

Ans: Loop up in Module 6

7. A variation of the rod-cutting problem is the following:
Given a rod of length n inches, a table of prices pi for i = 1, 2 ..., and a cost C for cutting a rod, determine the maximum revenue R obtainable by cutting up the rod and selling the pieces.
Example:
Consider the case when n = 6, with the price (in $) breakdown given below, and the cost of a cut is $1.

length i	1	2	3	4	5	6	7	8	9	10
price pi	1	5	8	9	10	17	17	20	24	30

One possible way choice is not to cut the rod at all, but to sell it intact for $9
Another alternative would be to cut it into 6 1 inch pieces for $6 in hardware sales - $5 in cutting cost, giving a net profit of $1 — Not as good as our first option.

7.1. Give an algorithm that solves the rod cutting problem for any length rod and price breakdown list.
Ans: Same as Module 13

7.2. Show a trace of your algorithm running to solve the initial example shown above
Ans: Same as Module 13

8. You are given a list of meeting times as pairs of numbers. For example:
   #1 (7, 9) = from 7AM to 9AM
   #2 (15, 16) = from 3 PM to 4 PM
   #3 (8, 11) = from 8AM to 11AM

   In the list, some meetings will overlap. For example meeting #1 and meeting #3 overlap.
   For this problem, you should find an algorithm to merge all overlapping meetings.
   Your output should be a list of new meeting times that do not overlap.

   In the given example, your final output would be:

   #1 (7, 11) = 7AM to 11AM, merge #1 & #3 above
   #2 (15, 16). = 3 PM to 4 PM (nothing to merge)

Note that consecutive meetings should be merged. (2,3) and (3,4) should be merged into (2,4)
Similarly, meetings that are subsumed should be merged into the inclusive meeting. (1, 5) and (2,4) should be merged into (1,5).
The input to your merging algorithm will be an array of N pairs. Give the complexity of your algorithm in terms of N.
Ans:

9. Given the following set of strings:
   abe
   baa
   eee
   abc
   bad
   bab
   abc

Every string has length 3 (ignore leading spaces). Furthermore, every string is drawn from only 5 characters: a,b,c,d,e
What algorithm has the best asymptotic complexity to sort this set?
Write pseudocode for that algorithm, and show a trace of the sorting procedure.
Ans:

10. Solve the following linear system using the LU factorization and triangular solves

Be sure to show your steps, including factorization Answers can be given in decimals if you want.

$$\begin{pmatrix} 1.0 & 2.0 & 3.0 \\ 2.0 & 3.0 & 4.0 \\ 3.0 & 4.0 & 5.0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ 12 \\ 15 \end{pmatrix}$$

Ans: Same as Module 12

11. A cycle in a directed graph. G = (V,E) is defined as a path <v1,v2,...v1>, where the last vertex is the same as the first vertex.Given a directed graph G=(V,E), determine if G has a directed cycle.
Hint: Use Depth First Search
Ans:

12. You are given 2 strings X=x1x2x3x4...xm and Y=y1y2y3...yn . The weighted edit distance is defined as the minimum cost of an edit instruction sequence to convert string X to string Y.
The edit operations are:

1. d: delete a character, cost = cd
2. i: insert a character, cost = ci
3. r: replace a character, cost = cr

Note that these operations, however, do not necessarily have the same cost. For example, cost(d) = cost(i) = 1, and cost(r) = 4. then sequences X = a and Y = b, then edit distance = 2 —> delete(a),insert(b). The alternative replace(a,b) has cost(r) = 4. For cost(r) = cost(i) = cost(d) = 1 then weighted edit distance for X, Y = replace(a,b), so edit distance = 1.

Devise an efficient algorithm to solve the weighted edit distance problem.
Ans:

13. Let G = (V,E) be a graph with edge weights given by w(E). Suppose we also know that that

$$1 \leq w(E) \leq 10.$$

Give a modification to Kruskal's algorithm for the minimum spanning tree that exploits the bounds on the edge weights. What is asymptotic complexity of your modified algorithm?
Ans: