Problem Set 12

Daniel Wang (S01435533)

1. The Python code is as follows. The complexity is O(n^2).

```python
def Lx_solver(L, b):
    n = len(L)

    for i in range(n):
        b[i] /= L[i][i]
        L[i][i] = 1

        for j in range(i+1, n):
            b[j] -= L[j][i] * b[i]
            L[j][i] = 0

    return b
```

2. The Python code is as follows. The complexity is O(n^2).

```python
def Ux_solver(U, b):
    n = len(U)

    for i in range(n-1, -1, -1):
        b[i] /= U[i][i]
        U[i][i] = 1

        for j in range(i):
            b[j] -= U[j][i] * b[i]
            U[j][i] = 0

    return b
```

3. The answer is [3, 2, 15], and the trace for each iteration i is as follows:

[Iteration 0]

L = [[1, 0, 0], [0, 1, 0], [0, 5, 1]]

b = [3.0, 2.0, 25.0]

[Iteration 1]

L = [[1, 0, 0], [0, 1, 0], [0, 0, 1]]

b = [3.0, 2.0, 15.0]

[Iteration 2]

L = [[1, 0, 0], [0, 1, 0], [0, 0, 1]]

b = [3.0, 2.0, 15.0]

4. The Python code is as follows:

```python
def LU_decomposition(A):
    n = len(A)

    for k in range(n):
        for i in range(k+1, n):
            A[i][k] = A[i][k] / A[k][k]
        for i in range(k+1, n):
            for j in range(k+1, n):
                A[i][j] = A[i][j] - A[i][k] * A[k][j]

        print(f"Iteration {k}")
        print(f"Current compact A matrix is {A}")

    L = [[0 for _ in range(n)] for _ in range(n)]
    U = [[0 for _ in range(n)] for _ in range(n)]

    for i in range(n):
        L[i][i] = 1

    for i in range(n):
        for j in range(i):
            L[i][j] = A[i][j]
        for j in range(i, n):
            U[i][j] = A[i][j]

    return L, U
```

The output trace of compact A matrix is:

```
Iteration 0
Current compact A matrix is [[4, -5, 6], [2.0, 4.0, -5.0], [3.0, 8.0, -6.0]]
Iteration 1
Current compact A matrix is [[4, -5, 6], [2.0, 4.0, -5.0], [3.0, 2.0, 4.0]]
Iteration 2
Current compact A matrix is [[4, -5, 6], [2.0, 4.0, -5.0], [3.0, 2.0, 4.0]]
```

5. The Python code is as follows:

```
def karatsuba_multiplication(x, y):
    a, b = x // 100, x % 100
    c, d = y // 100, y % 100

    # Karatsuba's algorithm
    t1 = a * c
    t2 = b * d
    t3 = (a+b) * (c+d)
    t4 = t3 - t1 - t2

    print(t1, t2, t3, t4)
    return t1 * 10000 + t4 * 100 + t2
```

For 5822 x 4104, the output is 23893488. The values of t1 to t4 are 2378, 88, 3600, 1134, respectively.