Problem Set 6

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1. Assume the underlined number is the pivot of current iteration, and boldface ones are numbers being considered currently, and red-colored numbers are ones being swapped given current outer-loop iteration. Then:

   Iteration #1: use 3 as pivot

   <u>3</u> 17 -5 4 13 8 7 6 9

   <u>3</u> -5 17 4 13 8 7 6 9

   -5 <u>3</u> 17 4 13 8 7 6 9

   Iteration #2: use 17 as pivot

   -5 3 **<u>17</u> 4 13 8 7 6 9**

   -5 3 **9 4 13 8 7 6 <u>17</u>**

   Iteration #3: use 9 as pivot

   -5 3 **<u>9</u> 4 13 8 7 6** 17

   -5 3 **<u>9</u> 4 6 8 7 13** 17

   -5 3 **7 4 6 8 <u>9</u> 13** 17

   Since 9 is at the 7th index, we found the 7th smallest element.

2. The median of this 5-length array is equivalent to finding 3rd smallest element. Using the same paradigm as previous question, here we track the step:

   Iteration #1: use 9 as pivot

   <u>9</u> 8 6 4 -100

   -100 8 6 4 <u>9</u>

   Iteration #2: use -100 as pivot

   <u>-100</u> 8 6 4 9

   (Nothing changes since -100 is the smallest given current subarray)

   Iteration #3: use 8 as pivot

   -100 **<u>8</u> 6 4** 9

   -100 **4 6 <u>8</u>** 9

Iteration #4: use 4 as pivot

-100 **4 6** 8 9

(Nothing changes since 4 is the smallest given current subarray)

Iteration #5: use 6 as pivot

-100 4 **6** 8 9

(Nothing changes since there is only one element here)

Since 6 is at the 3rd index, we found the median of this array.

3. For possible pivot elements, all numbers in the left partition should be less than pivot, and vice versa.
   (1) Considering 3, since 1 is smaller but on the right partition, it is impossible.
   (2) Considering 1, since 3 is larger but on the left partition, it is impossible.
   (3) Considering 2, since 3 is larger but on the left partition, it is impossible.
   (4) Considering 4, two partitions are valid, so it is possible.
   (5) Considering 5, two partitions are valid, so it is possible.
   (6) Considering 8, since 6 is smaller but on the right partition, it is impossible.
   (7) Considering 7, since 6 is smaller but on the right partition, it is impossible.
   (8) Considering 6, since 8 is larger but on the left partition, it is impossible.
   (9) Considering 9, two partitions are valid, so it is possible.

   Therefore, possible pivot elements include 4, 5, 9.

4. We first analyze possible sequences from input, intermediate steps, to outputs (note that indices are started with 0):
   Step #1: **1**, 2, 3, 4, 5, where the number at 0th index is swapped with the 0th one.
   Step #2: 1, **2**, 3, 4, 5, where the number at 1st index is swapped with the 1st one.
   Step #3: 1, **3**, **2**, 4, 5, where the number at 2nd index is swapped with the 1st one.
   Step #4: 1, **4**, 2, **3**, 5, where the number at 3rd index is swapped with the 1st one.
   Step #5: 1, 4, 2, **5**, **3**, where the number at 4th index is swapped with the 3rd one.

   Thus, the sequence from the random number generator is [0, 1, 1, 1, 3].

5. The analysis is as follows:
   (a) Each iteration has 3 candidate positions, so total number of permutations is:
   $$3^3 = 27$$

(b) Each iteration has i candidate positions, so total number of permutations is:

$3! = 6$

(c) Shuffling 3 cards should produce $C_2^3 = 6$ outcomes with even probability. KFY generates 6 permutations and is equal to the number of combinations, so it implies an even distribution. However, using the faulty algorithm generates 27 permutations, which implies outcomes are not evenly distributed since 27 is not divisible by 6.