Module 4 Problem Set Hsuan-You(Shaun) Lin

1. T(n) = 3 T(n/2) + n^2
My Answer:

$a=3,\ b=2,\ k=2,\ p=0 \Rightarrow a<b^k,\ p\geq 0$

$\Rightarrow O(n^2 \overset{1}{\cancel{\log^0}}(n)) = \underline{O(n^2)}$ #

2. T(n) = 4 T(n/2) + n^2
My Answer:

$a=4, b=2, k=2, p=0 \Rightarrow a=b^k, p>-1$

$\Rightarrow O(n^{\log_2(4)} \log^{0+1}(n)) = \underline{O(n^2 \log n)}$ #

3. T(n) = T(n/2) + n^2
My Answer:

$a=1, b=2, k=2, p=0 \Rightarrow a<b^k, p\geq 0$

$\Rightarrow O(n^2 \overset{1}{\cancel{\log^0}}(n)) = \underline{O(n^2)}$ #

4. T(n) = 16 T(n/4) + n
My Answer:

$a=16, b=4, k=1, p=0 \Rightarrow a>b^k$

$\Rightarrow O(n^{\log_4(16)}) = \underline{O(n^2)}$ #

5. T(n) = 2 T(n/2) + n log(n)
My Answer:

$a=2, b=2, k=1, p=1 \Rightarrow a=b^k, p>-1$

$\Rightarrow O(n^{\log_2(2)} \log^{1+1}(n)) = \underline{O(n \log^2 n)}$ #

6. T(n) = 2 T(n/2) + n/log(n)
My Answer:

$a=2, b=2, k=1, p=-1 \Rightarrow a=b^k, p=-1$

$\Rightarrow O(n^{\log_2(2)} \log(\log(n))) = \underline{O(n \log(\log n))}$ #

7. T(n) = 2 T(n/4) + n^(0.51)
My Answer:

$a=2, b=4, k=0.51, p=0 \Rightarrow a<b^k, p\geq 0$

$\Rightarrow O(n^{0.51} \overset{1}{\cancel{\log^0}}(n)) = \underline{O(n^{0.51})}$ #

8. T(n) = 6 T(n/3) + n^2 log(n)
My Answer:

$a=6, b=3, k=2, p=1 \Rightarrow a<b^k, p\geq 0$

$\Rightarrow O(n^2 \log^1(n)) = \underline{O(n^2 \log n)}$ #

9. T(n) = 7 T(n/3) + n^2
My Answer:

$a=7, b=3, k=2, p=0 \Rightarrow a<b^k, p\geq 0$

$\Rightarrow O(n^2 \log^0(n)) = \underline{O(n^2)}$ #

10. T(n) = sqrt(2) T(n/2) + log(n)
My Answer:

$a=\sqrt{2}, b=2, k=0, p=1 \Rightarrow a>b^k$

$\Rightarrow O(n^{\log_2(\sqrt{2})}) = \underline{O(n^{0.5})}$ #

11. T(n) = 3T(n/3) + 1
T(1) = 2
My Answer:

$a=3, b=3, k=0, p=0 \Rightarrow a > b^k$

$\Rightarrow O(n^{\log_3(3)}) = O(n) \rightarrow$ Master Theorem

$\Rightarrow T(n) = C_1 n + C_0$

$T(1) = 2$

$\rightarrow T(1) = C_1(1) + C_0 = 2$

$C_1 + C_0 = 2$

$\therefore C_1 = \frac{5}{2}, C_0 = -\frac{1}{2}$

Solve:

$3T(\frac{n}{3}) + 1 = 3(C_1\frac{n}{3} + C_0) + 1$

$C_1 n + C_0 = C_1 n + 3C_0 + 1$

$-1 = 2C_0, C_0 = -\frac{1}{2}$

Confirm:

$\rightarrow 3T(\frac{n}{3}) + 1 = 3(\frac{5}{2}\frac{n}{3} - \frac{1}{2}) + 1$

$= \frac{5}{2}n - \frac{3}{2} + 1$

$= \frac{5}{2}n - \frac{1}{2} = T(n)$

$\therefore \underline{T(n) = \frac{5}{2}n - \frac{1}{2}}$ #

12. T(n) = nT(n-1)
T(2) = 150
My Answer:

$T(2) = 2T(2-1) = 2T(1) \rightarrow T(1) = \frac{150}{2} = 75$

$T(3) = 3T(2) = 6T(1) = 3!T(1) = 75 \times 3!$

$\therefore \underline{T(n) = 75\,n!}$ #

13. T(n) = T(9n/10) + T(n/10) + n
My Answer:

$$\begin{cases} T(9n/10) = T(81n/100) + T(9n/100) + 9n/10 \\ T(n/10) = T(9n/100) + T(n/100) + n/10 \end{cases}$$

$\rightarrow T(n) = T(9n/10) + T(n/10) + n$

$= T(81n/100) + T(9n/100) + 9n/10 + T(9n/100) + T(n/100) + n/10 + n$

$= T(81n/100) + 2T(9n/100) + T(n/100) + 9n/10 + n/10 + n$

$= T(81n/100) + 2T(9n/100) + T(n/100) + 2n$

$\therefore T(n) = T(81n/100) + 2T(9n/100) + T(n/100) + 2n = O(n\log n)$

$\therefore$ <u>The asymtotic complexity of this recursion is $O(n\log n)$</u> #

14. T(n) = T(an) + T((1-a)n) + n
My Answer:

$$\begin{cases} T(an) = T(a^2n) + T(a\cdot(1-a)n) + an \\ T((1-a)n) = T(a\cdot(1-a)n) + T((1-a)^2) + n \end{cases}$$

$\rightarrow T(n) = T(an) + T((1-a)n) + n$

$= T(a^2n) + 2T(a(1-a)n) + T((1-a)^2) + 2n$

$\therefore T(n) = T(a^2n) + 2T(a(1-a)n) + T((1-a)^2) + 2n = O(n\log n)$

$\therefore$ <u>The asymtotic complexity of this recursion is $O(n\log n)$</u> #