

**Figure 2.34.** Current source driving a resistor as load: an *amplifier*!

Figure 2.35. Blocking capacitor  $C$  is chosen so that all frequencies of interest are passed by the highpass filter it forms in combination with the parallel resistance of the base biasing resistors<sup>21</sup>; that is,

$$C \geq \frac{1}{2\pi f(R_1 \parallel R_2)}.$$

The quiescent collector current is 1.0 mA because of the applied base bias and the 1.0 k emitter resistor. That current puts the collector at +10 volts (+20 V, minus 1.0 mA through 10 k). Now imagine an applied wiggle in base voltage  $v_B$ . The emitter follows with  $v_E = v_B$ , which causes a wiggle in emitter current

$$i_E = v_E/R_E = v_B/R_E$$

and nearly the same change in collector current ( $\beta$  is large). So the initial wiggle in base voltage finally causes a collector voltage wiggle

$$v_C = -i_C R_C = -v_B (R_C/R_E)$$

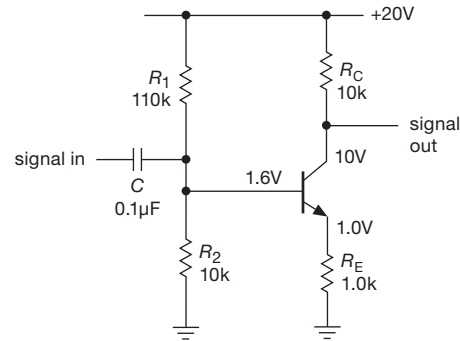
Aha! It's a *voltage amplifier*, with a voltage amplification (or "gain") given by

$$\text{gain} = v_{\text{out}}/v_{\text{in}} = -R_C/R_E \quad (2.6)$$

In this case the gain is  $-10,000/1000$ , or  $-10$ . The minus sign means that a positive wiggle at the input gets turned into a negative wiggle (10 times as large) at the output. This is called a *common-emitter amplifier* with emitter degeneration.

### A. Input and output impedances of the common-emitter amplifier

We can easily determine the input and output impedances of the amplifier. The input signal sees, in parallel, 110 k, 10 k, and the impedance looking into the base. The latter is



**Figure 2.35.** An ac common-emitter amplifier with emitter degeneration. Note that the output terminal is the collector rather than the emitter.

about 100 k ( $\beta$  times  $R_E$ ), so the input impedance (dominated by the 10 k) is about 8 k. The input coupling capacitor thus forms a highpass filter, with the 3 dB point at 200 Hz. The signal driving the amplifier sees 0.1  $\mu$ F in series with 8 k, which to signals of normal frequencies (well above the 3 dB point) just looks like 8 k.

The output impedance is 10 k in parallel with the impedance looking into the collector. What is that? Well, remember that if you snip off the collector resistor, you're simply looking into a current source. The collector impedance is very large (measured in megohms), and so the output impedance is just the value of the collector resistor, 10 k. It is worth remembering that the impedance looking into a transistor's collector is high, whereas the impedance looking into the emitter is low (as in the emitter follower). Although the output impedance of a common-emitter amplifier will be dominated by the collector load resistor, the output impedance of an emitter follower will not be dominated by the emitter load resistor, but rather by the impedance looking into the emitter.

### 2.2.8 Unity-gain phase splitter

Sometimes it is useful to generate a signal and its inverse, i.e., two signals 180° out of phase. That's easy to do – just use an emitter-degenerated amplifier with a gain of  $-1$  (Figure 2.36). The quiescent collector voltage is set to  $0.75V_{CC}$ , rather than the usual  $0.5V_{CC}$ , in order to achieve the same result – maximum symmetrical output swing without clipping at either output. The collector can swing from  $0.5V_{CC}$  to  $V_{CC}$ , whereas the emitter can swing from ground to  $0.5V_{CC}$ .

Note that the phase-splitter outputs must be loaded with equal (or very high) impedances at the two outputs to maintain gain symmetry.

<sup>21</sup> The impedance looking into the base itself will usually be much larger because of the way the base resistors are chosen, and it can generally be ignored.

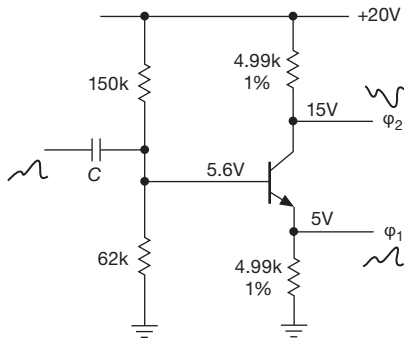


Figure 2.36. Unity-gain phase splitter.

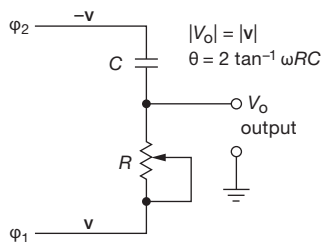


Figure 2.37. Constant-amplitude phase shifter.

### A. Phase shifter

A nice use of the phase splitter is shown in Figure 2.37. This circuit gives (for a sinewave input) an output sinewave of adjustable phase (from zero to  $180^\circ$ ) and with constant amplitude. It can be best understood with a phasor diagram of voltages (§1.7.12); with the input signal represented by a unit vector along the real axis, the signals look as shown in Figure 2.38.

Signal vectors  $\mathbf{v}_R$  and  $\mathbf{v}_C$  must be at right angles, and they must add to form a vector of constant length along the real axis. There is a theorem from geometry that says that the locus of such points is a circle. So the resultant vector (the output voltage) always has unit length, i.e., the same amplitude as the input, and its phase can vary from nearly zero to nearly  $180^\circ$  relative to the input wave as  $R$  is varied from nearly zero to a value much larger than  $X_C$  at the operating frequency. However, note that the phase shift depends on the frequency of the input signal for a given setting of the potentiometer  $R$ . It is worth noting that a simple  $RC$  highpass (or lowpass) network could also be used as an adjustable phase shifter. However, its output amplitude would vary over an enormous range as the phase shift was adjusted.

An additional concern here is the ability of the phase-splitter circuit to drive the  $RC$  phase shifter as a load. Ideally, the load should present an impedance that is large

compared with the collector and emitter resistors. As a result, this circuit is of limited utility where a wide range of phase shifts is required. You will see improved phase-splitter techniques in Chapter 4, where we use op-amps as impedance buffers, and in Chapter 7, where a cascade of several phase-shifter sections generates a set of “quadrature” signals that extends the phase-shifting range to a full  $0^\circ$  to  $360^\circ$ .

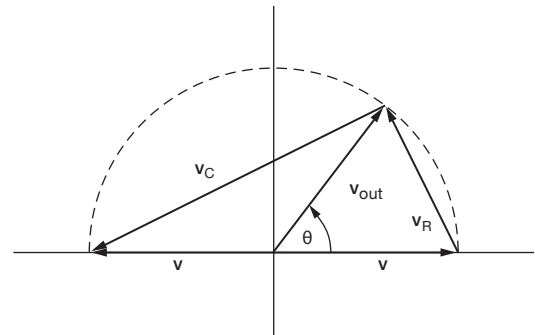
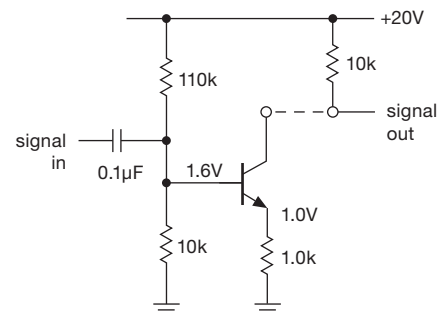
Figure 2.38. Phasor diagram for phase shifter, for which  $\theta = 2 \arctan(\omega RC)$ .

Figure 2.39. The common-emitter amplifier is a transconductance stage driving a (resistive) load.

### 2.2.9 Transconductance

In the preceding section we figured out the operation of the emitter-degenerated amplifier by (a) imagining an applied base voltage swing and seeing that the emitter voltage had the same swing, then (b) calculating the emitter current swing; then, ignoring the small base current contribution, we got the collector current swing and thus (c) the collector voltage swing. The voltage gain was then simply the ratio of collector (output) voltage swing to base (input) voltage swing.

There's another way to think about this kind of amplifier. Imagine breaking it apart, as in Figure 2.39. The first

part is a voltage-controlled current source, with quiescent current of 1.0 mA and gain of  $-1 \text{ mA/V}$ . Gain means the ratio of output to input; in this case the gain has units of current/voltage, or  $1/\text{resistance}$ . The inverse of resistance is called *conductance*.<sup>22</sup> An amplifier whose gain has units of conductance is called a *transconductance* amplifier; the ratio of changes  $\Delta I_{\text{out}}/\Delta V_{\text{in}}$  (usually written with small-signal changes indicated in lowercase:  $i_{\text{out}}/v_{\text{in}}$ ) is called the transconductance,  $g_m$ :

$$g_m = \frac{\Delta I_{\text{out}}}{\Delta V_{\text{in}}} = \frac{i_{\text{out}}}{v_{\text{in}}}. \quad (2.7)$$

Think of the first part of the circuit as a transconductance amplifier, i.e., a voltage-to-current amplifier with transconductance  $g_m$  (gain) of  $-1 \text{ mA/V}$  ( $1000 \mu\text{S}$ , or  $1 \text{ mS}$ , which is just  $1/R_E$ ). The second part of the circuit is the load resistor, an “amplifier” that converts current to voltage. This resistor could be called a *transresistance* converter, and its gain ( $r_m$ ) has units of voltage/current, or resistance. In this case its quiescent voltage is  $V_{CC}$ , and its gain (transresistance) is  $10 \text{ V/mA}$  ( $10 \text{ k}\Omega$ ), which is just  $R_C$ . Connecting the two parts together gives you a voltage amplifier. You get the overall gain by multiplying the two gains. In this case the voltage gain  $G_V = g_m R_C = -R_C/R_E$ , or  $-10$ , a unitless number equal to the ratio (output voltage change)/(input voltage change).

This is a useful way to think about an amplifier, because you can analyze performance of the sections independently. For example, you can analyze the transconductance part of the amplifier by evaluating  $g_m$  for different circuit configurations or even different devices, such as field-effect transistors FETs. Then you can analyze the transresistance (or load) part by considering gain versus voltage swing tradeoffs. If you are interested in the overall voltage gain, it is given by  $G_V = g_m r_m$ , where  $r_m$  is the transresistance of the load. Ultimately the substitution of an active load (current source), with its extremely high transresistance, can yield single-stage voltage gains of 10,000 or more. The *cascode* configuration, which we will discuss later, is another example easily understood with this approach.

In Chapter 4, which deals with operational amplifiers, you will see further examples of amplifiers with voltages or currents as inputs or outputs: voltage amplifiers (voltage to voltage), current amplifiers (current to current), and transresistance amplifiers (current to voltage).

### A. Turning up the gain: limitations of the simple model

The voltage gain of the emitter-degenerated amplifier is  $-R_C/R_E$ , according to our model. What happens as  $R_E$  is reduced toward zero? The equation predicts that the gain will rise without limit. But if we made actual measurements of the preceding circuit, keeping the quiescent current constant at 1 mA, we would find that the gain would level off at about 400 when  $R_E$  is zero, i.e., with the emitter grounded. We would also find that the amplifier would become significantly nonlinear (the output would not be a faithful replica of the input), the input impedance would become small and nonlinear, and the biasing would become critical and unstable with temperature. Clearly our transistor model is incomplete and needs to be modified to handle this circuit situation, as well as others we will talk about presently. Our fixed-up model, which we will call the transconductance model, will be accurate enough for the remainder of the book.

### B. Recap: the “four topologies”

Before jumping into the complexity just ahead, let’s remind ourselves of the four transistor circuits we’ve seen, namely the switch, emitter follower, current source, and common-emitter amplifier. We’ve drawn these very schematically in Figure 2.40, omitting details like biasing, and even the polarity of transistor (i.e., *npn* or *pnp*). For completeness we’ve included also a fifth circuit, the *common-base amplifier*, which we’ll meet soon enough (§2.4.5B).

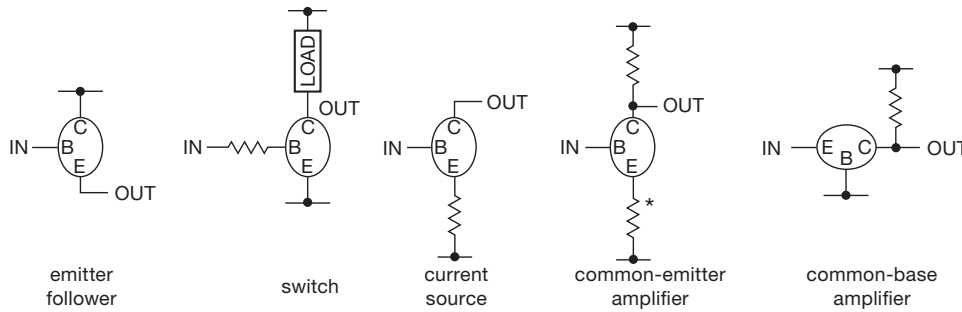
## 2.3 Ebers–Moll model applied to basic transistor circuits

We’ve enjoyed seeing some nice feats that can be accomplished with the simplest BJT model – switch, follower, current source, amplifier – but we’ve run up against some serious limitations (hey, would you believe, *infinite* gain?!). Now it’s time to go a level deeper, to address these limitations. The material that follows will suffice for our purposes. And – good news – for many BJT applications the simple model you’ve already seen is completely adequate.

### 2.3.1 Improved transistor model: transconductance amplifier

The important change is in rule 4 (§2.1.1), where we said earlier that  $I_C = \beta I_B$ . We thought of the transistor as a current amplifier whose input circuit behaved like a diode. That’s roughly correct, and for some applications it’s good enough. But to understand differential amplifiers,

<sup>22</sup> The inverse of reactance is *susceptance* (and the inverse of impedance is *admittance*), and has a special unit, the *siemens* (“S,” not to be confused with lowercase “s,” which means seconds), which used to be called the *mho* (ohm spelled backward, symbol “ $\Omega$ ”).



**Figure 2.40.** Five basic transistor circuits. Fixed voltages (power supplies or ground) are indicated by connections to horizontal line segments. For the switch, the load may be a resistor, to produce a full-swing voltage output; for the common-emitter amplifier, the emitter resistor may be bypassed or omitted altogether.

logarithmic converters, temperature compensation, and other important applications, you must think of the transistor as a *transconductance* device – collector current is determined by base-to-emitter voltage.

Here’s the modified rule 4.

**4. Transconductance amplifier** When rules 1–3 (§2.1.1) are obeyed,  $I_C$  is related to  $V_{BE}$  by<sup>23</sup>

$$I_C = I_S(T) \left( e^{V_{BE}/V_T} - 1 \right), \quad (2.8)$$

or, equivalently,

$$V_{BE} = \frac{kT}{q} \log_e \left( \frac{I_C}{I_S(T)} + 1 \right), \quad (2.9)$$

where

$$V_T = kT/q = 25.3 \text{ mV} \quad (2.10)$$

at room temperature (68°F, 20°C),  $q$  is the electron charge ( $1.60 \times 10^{-19}$  coulombs),  $k$  is Boltzmann’s constant ( $1.38 \times 10^{-23}$  joules/K, sometimes written  $k_B$ ),  $T$  is the absolute temperature in degrees Kelvin ( $K = ^\circ\text{C} + 273.16$ ), and  $I_S(T)$  is the *saturation current* of the particular transistor (which depends strongly on temperature,  $T$ , as we’ll see shortly). Then the base current, which also depends on  $V_{BE}$ , can be approximated by

$$I_B = I_C/\beta,$$

where the “constant”  $\beta$  is typically in the range 20 to 1000, but depends on transistor type,  $I_C$ ,  $V_{CE}$ , and temperature.  $I_S(T)$  approximates the reverse leakage current (roughly  $10^{-15}$  A for a small-signal transistor like the 2N3904). In the active region  $I_C \gg I_S$ , and therefore

the  $-1$  term can be neglected in comparison with the exponential:

$$I_C \approx I_S(T) e^{V_{BE}/V_T}. \quad (2.11)$$

The equation for  $I_C$  is known as the Ebers–Moll equation.<sup>24</sup> It also describes approximately the current versus voltage for a diode, if  $V_T$  is multiplied by a correction factor  $m$  between 1 and 2. For transistors it is important to realize that the collector current is accurately determined by the base–emitter voltage, rather than by the base current (the base current is then roughly determined by  $\beta$ ), and that this exponential law is accurate over an enormous range of currents, typically from nanoamps to milliamps. Figure 2.41 makes the point graphically.<sup>25</sup> If you measure the base current at various collector currents, you will get a graph of  $\beta$  versus  $I_C$  like that in Figure 2.42. Transistor beta versus collector current is discussed further in Chapter 2x.

Although the Ebers–Moll equation tells us that the base–emitter voltage “programs” the collector current, this property is not easy to use in practice (biasing a transistor by applying a base voltage) because of the large temperature coefficient of base–emitter voltage. You will see later how the Ebers–Moll equation provides insight and solutions to this problem.

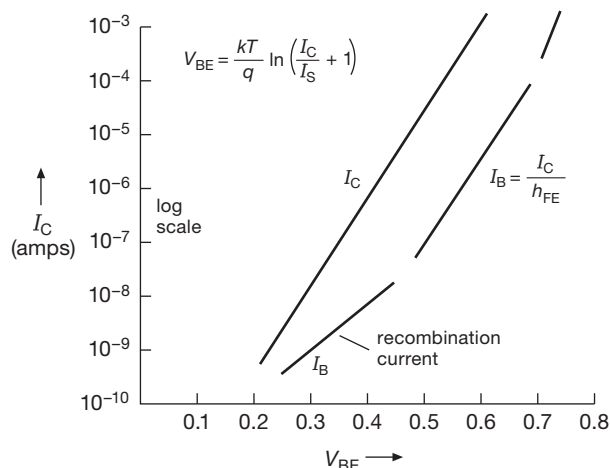
### 2.3.2 Consequences of the Ebers–Moll model: rules of thumb for transistor design

From the Ebers–Moll equation (2.8) we get these simple (but handy) “ratio rules” for collector current:  $I_{C2}/I_{C1} = \exp(\Delta V_{BE}/V_T)$  and  $\Delta V_{BE} = V_T \log_e(I_{C2}/I_{C1})$ . We

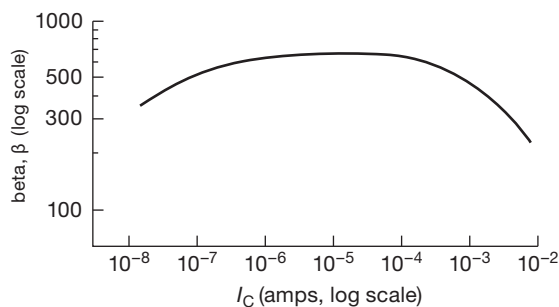
<sup>23</sup> We indicate the important temperature dependence of  $I_S$  by explicitly showing it in functional form – “ $I_S(T)$ ”.

<sup>24</sup> J. J. Ebers & J. L. Moll, “Large-signal behavior of junction transistors,” *Proc. IRE* **42**, 1761 (1954).

<sup>25</sup> This is sometimes called a Gummel plot.



**Figure 2.41.** Transistor base and collector currents as functions of base-to-emitter voltage  $V_{BE}$ .



**Figure 2.42.** Typical transistor current gain ( $\beta$ ) versus collector current.

also get the following important quantities we will be using often in circuit design.

### A. The steepness of the diode curve.

How much do we need to increase  $V_{BE}$  to increase  $I_C$  by a factor of 10? From the Ebers–Moll equation, that’s just  $V_T \log_e 10$ , or 58.2 mV at room temperature. We like to remember this as *base–emitter voltage increases approximately 60 mV per decade of collector current*. (Two other formulations: collector current doubles for each 18 mV increase in base–emitter voltage; collector current increases 4% per millivolt increase in base–emitter voltage.) Equivalently,  $I_C = I_{C0} e^{\Delta V/25}$ , where  $\Delta V$  is in millivolts.<sup>26</sup>

<sup>26</sup> The “25” in this and the following discussion is more precisely 25.3 mV, the value of  $k_B T/q$  at room temperature. It’s proportional to absolute temperature – engineers like to say “PTAT,” pronounced *pee’-tat*. This has interesting (and useful) consequences, for example the

### B. The small-signal impedance looking into the emitter, $r_e$ , for the base held at a fixed voltage.

Taking the derivative of  $V_{BE}$  with respect to  $I_C$ , you get

$$r_e = V_T / I_C = 25 / I_C \text{ ohms}, \quad (2.12)$$

where  $I_C$  is in milliamps.<sup>27</sup> The numerical value  $25/I_C$  is for room temperature. This *intrinsic* emitter resistance,  $r_e$ , acts as if it is in series with the emitter in all transistor circuits. It limits the gain of a grounded-emitter amplifier, causes an emitter follower to have a voltage gain of slightly less than unity, and prevents the output impedance of an emitter follower from reaching zero. Note that the transconductance<sup>28</sup> of a grounded emitter amplifier is just

$$g_m = I_C / V_T = 1 / r_e \quad (= 40 I_C \text{ at room temp}). \quad (2.13)$$

### C. The temperature dependence of $V_{BE}$ .

A glance at the Ebers–Moll equation suggests that  $V_{BE}$  (at constant  $I_C$ ) has a positive temperature coefficient because of the multiplying factor of  $T$  in  $V_T$ . However, the strong temperature dependence of  $I_S(T)$  more than compensates for that term, such that  $V_{BE}$  (at constant  $I_C$ ) *decreases* about 2.1 mV/°C. It is roughly proportional to  $1/T_{\text{abs}}$ , where  $T_{\text{abs}}$  is the absolute temperature. Sometimes it’s useful to cast this instead in terms of the temperature dependence of  $I_C$  (at constant  $V_{BE}$ ):  $I_C$  *increases* about 9%/°C; it doubles for an 8°C rise.

There is one additional quantity we will need on occasion, although it is not derivable from the Ebers–Moll equation. It is known as the Early effect,<sup>29</sup> and it sets important limits on current-source and amplifier performance.

### D. Early effect.

$V_{BE}$  (at constant  $I_C$ ) varies slightly with changing  $V_{CE}$ . This effect is caused by the variation of effective base width as  $V_{CE}$  changes, and it is given, approximately, by

$$\Delta V_{BE} = -\eta \Delta V_{CE}, \quad (2.14)$$

where  $\eta \approx 10^{-4}$ – $10^{-5}$ . (As an example, the npn 2N5088 has  $\eta = 1.3 \times 10^{-4}$ , thus a 1.3 mV change of  $V_{BE}$  to maintain constant collector current when  $V_{CE}$  changes by 10 V.)

opportunity to make a “silicon thermometer.” We’ll see more of this in Chapter 2x, and again in Chapter 9.

<sup>27</sup> We like to remember the fact that  $r_e = 25 \Omega$  at a collector current of 1 mA. Then we just scale inversely for other currents: thus  $r_e = 2.5 \Omega$  at  $I_C = 10$  mA, etc.

<sup>28</sup> At the next level of sophistication we’ll see that, since the quantity  $r_e$  is proportional to absolute temperature, a grounded emitter amplifier whose collector current is PTAT has transconductance (and gain) independent of temperature. More in Chapter 2x.

<sup>29</sup> J. M. Early, “Effects of space-charge layer widening in junction transistors,” *Proc. IRE* **40**, 1401 (1952). James Early died in 2004.

This is often described instead as a linear increase of collector current with increasing collector voltage when  $V_{BE}$  is held constant; you see it expressed as

$$I_C = I_{C0} \left( 1 + \frac{V_{CE}}{V_A} \right), \quad (2.15)$$

where  $V_A$  (typically 50–500 V) is known as the Early voltage.<sup>30</sup> This is shown graphically in Figure 2.59 in §2.3.7A. A low Early voltage indicates a low collector output resistance; *pnp* transistors tend to have low  $V_A$ , see measured values in Table 8.1. We treat the Early effect in more detail in Chapter 2x.<sup>31</sup>

These are the essential quantities we need. With them we will be able to handle most problems of transistor circuit design, and we will have little need to refer to the Ebers–Moll equation itself.<sup>32</sup>

### 2.3.3 The emitter follower revisited

Before looking again at the common-emitter amplifier with the benefit of our new transistor model, let's take a quick look at the humble emitter follower. The Ebers–Moll model predicts that an emitter follower should have nonzero output impedance, even when driven by a voltage source, because of finite  $r_e$  (item B in the above list). The same effect also produces a voltage gain slightly less than unity, because  $r_e$  forms a voltage divider with the load resistor.

These effects are easy to calculate. With fixed base voltage, the impedance looking back into the emitter is just  $R_{out} = dV_{BE}/dI_E$ ; but  $I_E \approx I_C$ , so  $R_{out} \approx r_e$ , the intrinsic emitter resistance [recall  $r_e = 25/I_C(\text{mA})$ ]. For example, in Figure 2.43A, the load sees a driving impedance of  $r_e = 25\ \Omega$ , because  $I_C = 1\text{ mA}$ . (This is paralleled by the emitter resistor  $R_E$ , if used; but in practice  $R_E$  will always be much larger than  $r_e$ .) Figure 2.43B shows a more typical situation, with finite source resistance  $R_S$  (for simplicity we've omitted the obligatory biasing components –

base divider and blocking capacitor – which are shown in Figure 2.43C). In this case the emitter follower's output impedance is just  $r_e$  in series with  $R_S/(\beta + 1)$  (again paralleled by an unimportant  $R_E$ , if present). For example, if  $R_S = 1\text{ k}\Omega$  and  $I_C = 1\text{ mA}$ ,  $R_{out} = 35\ \Omega$  (assuming  $\beta = 100$ ). It is easy to show that the intrinsic emitter  $r_e$  also figures into an emitter follower's *input* impedance, just as if it were in series with the load (actually, parallel combination of load resistor and emitter resistor). In other words, for the emitter follower circuit the effect of the Ebers–Moll model is simply to add a series emitter resistance  $r_e$  to our earlier results.<sup>33</sup>

The voltage gain of an emitter follower is slightly less than unity, owing to the voltage divider produced by  $r_e$  and the load. It is simple to calculate, because the output is at the junction of  $r_e$  and  $R_{load}$ :  $G_V = v_{out}/v_{in} = R_L/(r_e + R_L)$ . Thus, for example, a follower running at 1 mA quiescent current, with 1k load, has a voltage gain of 0.976. Engineers sometimes like to write the gain in terms of the transconductance, to put it in a form that holds for FETs also (see §3.2.3A); in that case (using  $g_m = 1/r_e$ ) you get  $G_V = R_L g_m / (1 + R_L g_m)$ .

### 2.3.4 The common-emitter amplifier revisited

Previously we got wrong answers for the voltage gain of the common-emitter amplifier with emitter resistor (sometimes called emitter degeneration) when we set the emitter resistor equal to zero; recall that our wrong answer was  $G_V = -R_C/R_E = \infty$ !

The problem is that the transistor has  $25/I_C(\text{mA})$  ohms of built-in (intrinsic) emitter resistance  $r_e$  that must be added to the actual external emitter resistor. This resistance is significant only when small emitter resistors (or none at all) are used.<sup>34</sup> So, for instance, the amplifier we considered previously will have a voltage gain of  $-10\text{ k}/r_e$ , or  $-400$ , when the external emitter resistor is zero. The input impedance is not zero, as we would have predicted earlier ( $\beta R_E$ ); it is approximately  $\beta r_e$ , or in this case (1 mA quiescent current) about  $2.5\text{ k}\Omega$ .<sup>35</sup>

<sup>30</sup> The connection between Early voltage and  $\eta$  is  $\eta = V_T/(V_A + V_{CE}) \approx V_T/V_A$ ; see Chapter 2x.

<sup>31</sup> Previewing some of the results there, the Early effect (a) determines a transistor's collector output resistance  $r_o = V_A/I_C$ ; (b) sets a limit on single-stage voltage gain; and (c) limits the output resistance of a current source. Other things being equal, *pnp* transistors tend to have low Early voltages, as do transistors with high beta; high-voltage transistors usually have high Early voltages, along with low beta. These trends can be seen in the measured Early voltages listed in Table 8.1.

<sup>32</sup> The computer circuit-analysis program SPICE includes accurate transistor simulation with the Ebers–Moll formulas and Gummel–Poon charge models. It's a lot of fun to “wire up” circuits on your computer screen and set them running with SPICE. For more detail see the application of SPICE to BJT amplifier distortion in Chapter 2x.

<sup>33</sup> There's more, if you look deeper: at high frequencies (above  $f_T/\beta$ ) the effective current gain drops inversely with frequency; so you get a linearly rising output impedance from an emitter follower that is driven with low  $R_S$ . That is, it looks like an inductance, and a capacitive load can cause ringing or even oscillation; these effects are treated in Chapter 2x.

<sup>34</sup> Or, equivalently, when the emitter resistor is bypassed with a capacitor whose impedance at signal frequencies is comparable with, or less than,  $r_e$ .

<sup>35</sup> These estimates of gain and input impedance are reasonably good, as long as we stay away from operation at very high frequencies or

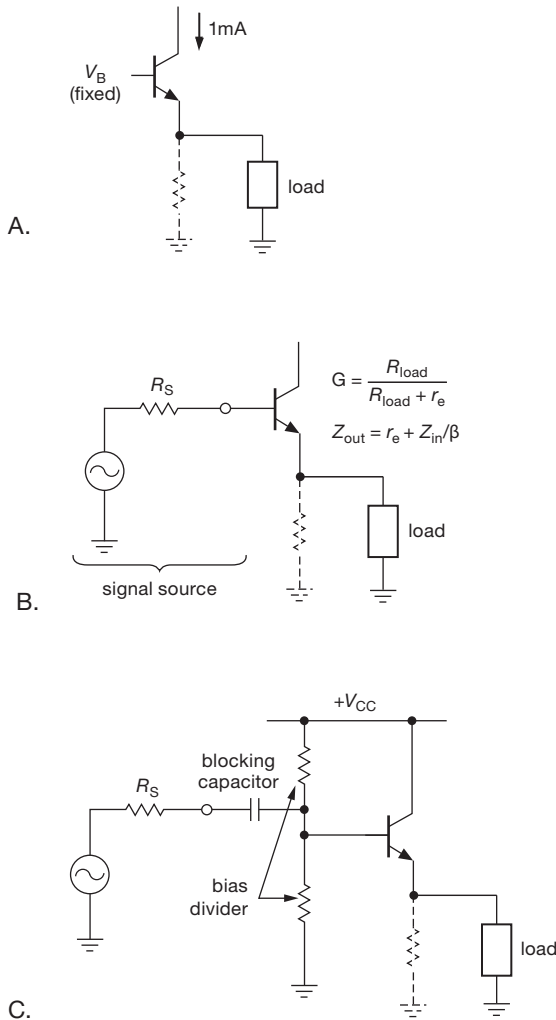


Figure 2.43. Output impedance of emitter followers (see text).

The terms “grounded emitter” and “common emitter” are sometimes used interchangeably, and they can be confusing. We will use the phrase “grounded-emitter amplifier” to mean a common-emitter amplifier with  $R_E = 0$  (or equivalent bypassing). A common-emitter amplifier stage may have an emitter resistor; what matters is that the emitter circuit is common to the input circuit and the output circuit.

from circuits in which the collector load resistor is replaced with a current source “active load” ( $R_C \rightarrow \infty$ ). The ultimate voltage gain of a grounded-emitter amplifier, in the latter situation, is limited by the Early effect; this is discussed in more detail Chapter 2x.

### A. Shortcomings of the single-stage grounded emitter amplifier

The extra voltage gain you get by using  $R_E = 0$  comes at the expense of other properties of the amplifier. In fact, the grounded-emitter amplifier, in spite of its popularity in textbooks, should be avoided except in circuits with overall negative feedback. In order to see why, consider Figure 2.44.

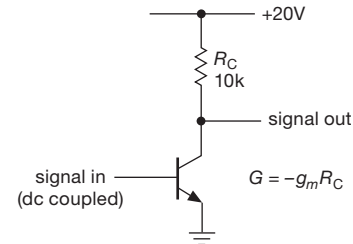


Figure 2.44. Common-emitter amplifier without emitter degeneration.

**1. Nonlinearity.** The voltage gain is  $G = -g_m R_C = -R_C/r_e = -R_C I_C(\text{mA})/25$ , so for a quiescent current of 1 mA, the gain is  $-400$ . But  $I_C$  varies as the output signal varies. For this example, the gain will vary from  $-800$  ( $V_{\text{out}} = 0$ ,  $I_C = 2 \text{ mA}$ ) down to zero ( $V_{\text{out}} = V_{CC}$ ,  $I_C = 0$ ). For a triangle-wave input, the output will look as in Figure 2.45. The amplifier has lots of distortion, or poor linearity. The grounded-emitter amplifier without feedback is useful only for small-signal swings about the quiescent point. By contrast, the emitter-degenerated amplifier has a gain almost entirely independent of collector current, as long as  $R_E \gg r_e$ , and can be used for undistorted amplification even with large-signal swings.

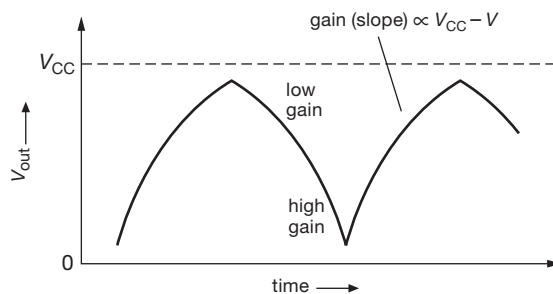
It’s easy to estimate the distortion, both with and without an external emitter resistor. With a *grounded* emitter, the incremental (small-signal) gain is  $G_V = -R_C/r_e = -I_C R_C/V_T = -V_{\text{drop}}/V_T$ , where  $V_{\text{drop}}$  is the instantaneous voltage drop across the collector resistor. Because the gain is proportional to the drop across the collector resistor, the nonlinearity (fractional change of gain with swing) equals the ratio of instantaneous swing to average quiescent drop across the collector resistor:  $\Delta G/G \approx \Delta V_{\text{out}}/V_{\text{drop}}$ , where  $V_{\text{drop}}$  is the average, or quiescent, voltage drop across the collector resistor  $R_C$ . Because this represents the extreme variation of gain (i.e., at the peaks of the swing), the overall waveform “distortion” (usually stated as the amplitude of the residual waveform after subtraction of the strictly linear component) will be smaller by roughly a factor of 3. Note that the distortion depends on only the ratio of swing to quiescent drop, and not directly on the operating current, etc.

As an example, in a grounded emitter amplifier powered from +10 V, biased to half the supply (i.e.,  $V_{\text{drop}} = 5 \text{ V}$ ), we measured a distortion of 0.7% at 0.1 V output sinewave amplitude and 6.6% at 1 V amplitude; these values are in good agreement with the predicted values. Compare this with the situation with an added external emitter resistor  $R_E$ , in which the voltage gain becomes  $G_V = -R_C/(r_e + R_E) = -I_C R_C/(V_T + I_C R_E)$ . Only the first term in the denominator contributes distortion, so the distortion is reduced by the ratio of  $r_e$  to the total effective emitter resistance: the non-linearity becomes  $\Delta G/G \approx (\Delta V_{\text{out}}/V_{\text{drop}})[r_e/(r_e + R_E)] = (\Delta V_{\text{out}}/V_{\text{drop}})[V_T/(V_T + I_C R_E)]$ ; the second term is the factor by which the distortion is reduced. When we added an emitter resistor, chosen to drop 0.25 V at the quiescent current – which by this estimate should reduce the nonlinearity by a factor of 10 – the measured distortion of the previous amplifier dropped to 0.08% and 0.74% for 0.1 V and 1 V output amplitudes, respectively. Once again, these measurement agree well with our prediction.

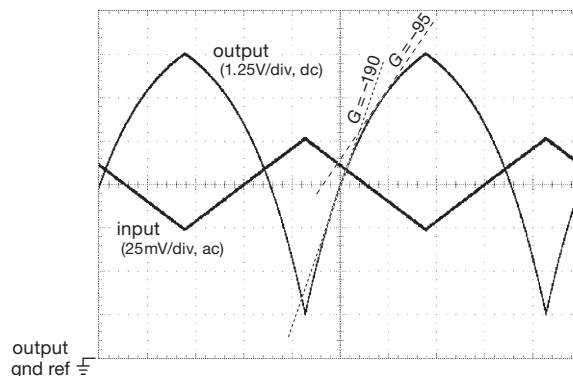
**Exercise 2.12.** Calculate the predicted distortion for these two amplifiers at the two output levels that were measured.

As we remarked, the nonlinearity of a common-emitter amplifier, when driven by a triangle wave, takes the form of the asymmetric “barn roof” distortion sketched in Figure 2.45.<sup>36</sup> For comparison we took a real-life ’scope (oscilloscope) trace of a grounded emitter amplifier (Figure 2.46); we used a 2N3904 with a 5k collector resistor to a +10 V supply, biased (carefully!) to half the supply. With a ruler we estimated the incremental gain at output voltages of +5 V (halfway to  $V_+$ ) and at +7.5 V, as shown, where the collector current is 1 mA and 0.5 mA, respectively. The gain values are in pretty good agreement with the predictions ( $G = R_C/r_e = I_C(\text{mA})R_C/25\Omega$ ) of  $G = -200$  and  $G = -100$ , respectively. By comparison, Figure 2.47 shows what happened when we added a 225  $\Omega$  emitter resistor: the gain is reduced by a factor of 10 at the quiescent point ( $G = R_C/(R_E + r_e) \approx R_C/250\Omega$ ), but with much improved linearity (because changes in  $r_e$  contribute little to the overall resistance in the denominator, which is now dominated by the fixed 225  $\Omega$  external emitter resistor).

For sinusoidal input, the output contains all harmonics of the fundamental wave. Later in the chapter we’ll see how to make differential amplifiers with a pair of transistors; for these the residual distortion is symmetric, and contains only the odd harmonics. And in Chapter 2x we’ll see some very clever methods for cancelling distortion in differential



**Figure 2.45.** Nonlinear output waveform from grounded-emitter amplifier.



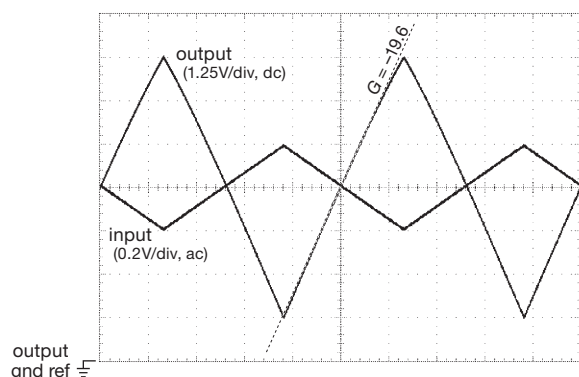
**Figure 2.46.** Real life! The grounded-emitter amplifier of Figure 2.44, with  $R_C = 5\text{k}$ ,  $V_+ = +10 \text{ V}$ , and a 1 kHz triangle wave input. Top and bottom of the screen are +10V and ground for the dc-coupled output trace (note sensitive scale for the ac-coupled input signal). Gain estimates (tangent lines) are at  $V_{\text{out}}$  values of  $0.5V_+$  and  $0.75V_+$ . Horizontal: 0.2 ms/div.

amplifiers, along with the use of SPICE simulation software for rapid analysis and circuit iteration. Finally, to set things in perspective, we should add that any amplifier’s residual distortion can be reduced dramatically by use of *negative feedback*. We’ll introduce feedback later in this chapter (§2.5), after you’ve gained familiarity with common transistor circuits. Feedback will finally take center stage when we get to *operational amplifiers* in Chapter 4.

**2. Input impedance.** The input impedance is roughly  $Z_{\text{in}} = \beta r_e = 25\beta/I_C(\text{mA})$  ohms. Once again,  $I_C$  varies over the signal swing, giving a varying input impedance. Unless the signal source driving the base has low impedance, you will wind up with nonlinearity because of the nonlinear (variable) voltage divider formed from the signal source and the amplifier’s input impedance. By contrast, the input impedance of an emitter-degenerated amplifier is nearly constant, and high.

**3. Biasing.** The grounded emitter amplifier is difficult to

<sup>36</sup> Because the gain (i.e., the slope of  $V_{\text{out}}$  versus  $V_{\text{in}}$ ) is proportional to the distance from the  $V_{\text{CC}}$  line, the shape of the curve is in fact an exponential.



**Figure 2.47.** Adding a  $225\,\Omega$  emitter resistor improves the linearity dramatically at the expense of gain (which drops by a factor of 10 at the quiescent point). Horizontal: 0.2 ms/div.

bias. It might be tempting just to apply a voltage (from a voltage divider) that gives the right quiescent current according to the Ebers–Moll equation. That won't work, because of the temperature dependence of  $V_{BE}$  (at fixed  $I_C$ ), which varies about  $2.1\,\text{mV}/^\circ\text{C}$  [it actually decreases with increasing  $T$  because of the variation of  $I_S(T)$  with temperature; as a result,  $V_{BE}$  is roughly proportional to  $1/T$ , the absolute temperature]. This means that the collector current (for fixed  $V_{BE}$ ) will increase by a factor of 10 for a  $30^\circ\text{C}$  rise in temperature (which corresponds to a 60 mV change in  $V_{BE}$ ), or about  $9\%/^\circ\text{C}$ . Such unstable biasing is useless, because even rather small changes in temperature will cause the amplifier to saturate. For example, a grounded emitter stage biased with the collector at half the supply voltage will go into saturation if the temperature rises by  $8^\circ\text{C}$ .

**Exercise 2.13.** Verify that an  $8^\circ\text{C}$  rise in ambient temperature will cause a base-voltage-biased grounded emitter stage to saturate, assuming that it was initially biased for  $V_C = 0.5V_{CC}$ .

Some solutions to the biasing problem are discussed in the following sections. By contrast, the emitter-degenerated amplifier achieves stable biasing by applying a voltage to the base, most of which appears across the emitter resistor, thus determining the quiescent current.

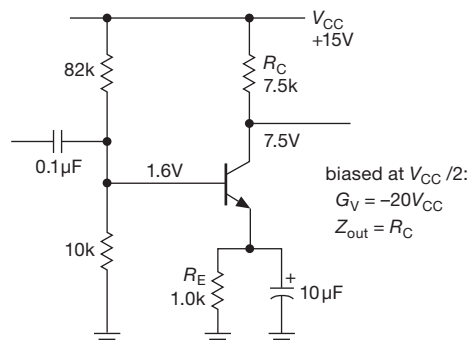
## B. Emitter resistor as feedback

Adding an external series resistor to the intrinsic emitter resistance  $r_e$  (emitter degeneration) improves many properties of the common-emitter amplifier, but at the expense of gain. You will see the same thing happening in Chapters 4 and 5, when we discuss *negative feedback*, an important technique for improving amplifier characteristics by feeding back some of the output signal to reduce the effective input signal. The similarity here is no coincidence –

the emitter-degenerated amplifier itself uses a form of negative feedback. Think of the transistor as a transconductance device, determining collector current (and therefore output voltage) according to the voltage applied between the base and emitter; but the input to the amplifier is the voltage from base to *ground*. So the voltage from base to emitter is the input voltage, *minus a sample of the output* (namely  $I_E R_E$ ). That's negative feedback, and that's why emitter degeneration improves most properties of the amplifier (here improved linearity and stability and increased input impedance.<sup>37</sup>) Later in the chapter, in §2.5, we'll make these statements quantitative when we first look at feedback. And there are great things to look forward to, with the full flowering of feedback in Chapters 4 and 5!

## 2.3.5 Biasing the common-emitter amplifier

If you must have the highest possible gain (or if the amplifier stage is inside a feedback loop), it is possible to arrange successful biasing of a common-emitter amplifier. There are three solutions that can be applied, separately or in combination: bypassed emitter resistor, matched biasing transistor, and dc feedback.



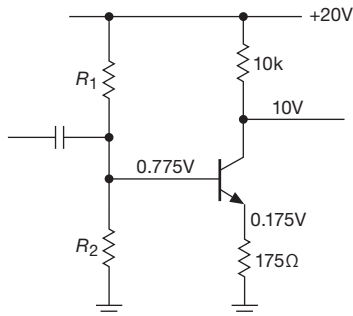
**Figure 2.48.** A bypassed emitter resistor can be used to improve the bias stability of a grounded-emitter amplifier.

## A. Bypassed emitter resistor

You can use a bypassed emitter resistor, biasing as for the degenerated amplifier, as shown in Figure 2.48. In this case  $R_E$  has been chosen about  $0.1R_C$ , for ease of biasing; if  $R_E$  is too small, the emitter voltage will be much smaller than the base–emitter drop, leading to temperature instability of the quiescent point as  $V_{BE}$  varies with temperature. The emitter bypass capacitor is chosen to make its impedance small compared with  $r_e$  (not  $R_E$  – why?) at the

<sup>37</sup> And, as we'll learn, the output impedance would be reduced – a desirable feature in a voltage amplifier – if the feedback were taken directly from the collector.

lowest frequency of interest. In this case its impedance is  $25\ \Omega$  at 650 Hz. At signal frequencies the input coupling capacitor sees an impedance of  $10\text{k}$  in parallel with the base impedance, in this case  $\beta \times 25\ \Omega$ , or roughly  $2.5\text{k}$ . At dc, the impedance looking into the base is much larger ( $\beta$  times the emitter resistor, or about  $100\text{k}$ ).



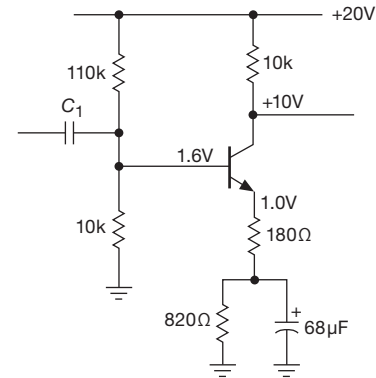
**Figure 2.49.** Gain-of-50 stage presents bias stability problem.

A variation on this circuit consists of using two emitter resistors in series, one of them bypassed. For instance, suppose you want an amplifier with a voltage gain of 50, quiescent current of 1 mA, and  $V_{CC}$  of +20 volts, for signals from 20 Hz to 20 kHz. If you try to use the emitter-degenerated circuit, you will have the circuit shown in Figure 2.49. The collector resistor is chosen to put the quiescent collector voltage at  $0.5V_{CC}$ . Then the emitter resistor is chosen for the required gain, including the effects of the  $r_e$  of  $25/I_C(\text{mA})$ . The problem is that the emitter voltage of only 0.175 V will vary significantly as the  $\sim 0.6\text{ V}$  of base-emitter drop varies with temperature ( $-2.1\text{ mV}/^\circ\text{C}$ , approximately), since the base is held at constant voltage by  $R_1$  and  $R_2$ ; for instance, you can verify that an increase of  $20^\circ\text{C}$  will cause the collector current to increase by nearly 25%.

**Exercise 2.14.** Show that this statement is correct.

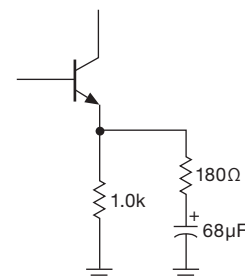
The solution here is to add some bypassed emitter resistance for stable biasing, with no change in gain at signal frequencies (Figure 2.50). As before, the collector resistor is chosen to put the collector at 10 volts ( $0.5V_{CC}$ ). Then the unbypassed emitter resistor is chosen to give a gain of 50, including the intrinsic emitter resistance  $r_e = 25/I_C(\text{mA})$ . Enough bypassed emitter resistance is added to make stable biasing possible (one-tenth of the collector resistance is a good guideline). The base voltage is chosen to give 1 mA of emitter current, with impedance about one-tenth the dc impedance looking into the base (in this case about  $100\text{k}$ ). The emitter bypass capacitor is chosen to have low impedance compared with  $180 + 25\ \Omega$  at the lowest signal frequencies. Finally, the input coupling ca-

pacitor is chosen to have low impedance compared with the *signal-frequency* input impedance of the amplifier, which is equal to the voltage-divider impedance in parallel with  $\beta \times (180 + 25)\ \Omega$  (the  $820\ \Omega$  is bypassed and looks like a short at signal frequencies).



**Figure 2.50.** A common-emitter amplifier combining bias stability, linearity, and large voltage gain.

An alternative circuit splits the signal and dc paths (Figure 2.51). This lets you vary the gain (by changing the  $180\ \Omega$  resistor) without bias change.



**Figure 2.51.** Equivalent emitter circuit for Figure 2.50.

## B. Matched biasing transistor

You can use a matched transistor to generate the correct base voltage for the required collector current; this ensures automatic temperature compensation (Figure 2.52).<sup>38</sup>  $Q_1$ 's collector is drawing 1 mA, since it is guaranteed to be near ground (about one  $V_{BE}$  drop above ground, to be exact); if  $Q_1$  and  $Q_2$  are a matched pair (available as a single device, with the two transistors on one piece of silicon), then  $Q_2$  will also be biased to draw 1 mA, putting its collector at

<sup>38</sup> R. Widlar, "Some circuit design techniques for linear integrated circuits," *IEEE Trans. Circuit Theory* **CT-12**, 586 (1965). See also US Patent 3,364,434.