

# A presentaion on A\* search algorithm

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# Introduction



Figure: Using Navigation Assistants

# Definition

## What is Graph Traversal?

Graph traversal (also known as graph search) refers to the process of visiting (checking and/or updating) each vertex in a graph.

# Definition

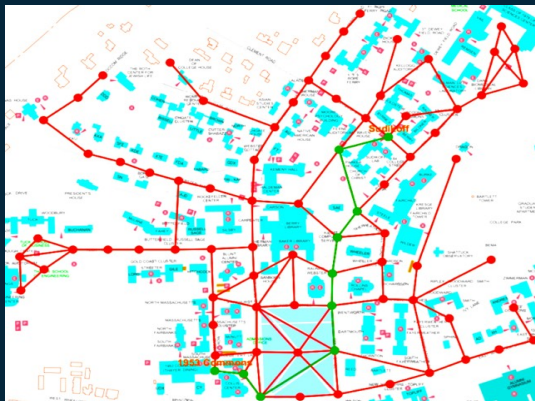


Figure: Using Graph Traversal Algorithms

# Definition

## What is A\* search?

A\* search is a path-finding and graph traversal algorithm which is basically an evolution of Dijkstra's algorithm

# Why A\* search?

As we have already mentioned, A\* search is basically a modification of Dijkstra's algorithm.

- But why do we need A\* search?

# Why A\* search?

As we have already mentioned, A\* search is basically a modification of Dijkstra's algorithm.

- But why do we need A\* search?
- What are the benefits of A\* over Dijkstra's algorithm or other such algorithms which validates the modification?



# A\* search Overview

To answer that question we first need to know how the A\* search algorithm works

- You see, Dijkstra's algorithm is a greedy algorithm and so it always immediately chooses the best possible path at every step

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To answer that question we first need to know how the A\* search algorithm works

- You see, Dijkstra's algorithm is a greedy algorithm and so it always immediately chooses the best possible path at every step
- Even if the chosen path strays away from the destination, it isn't smart enough to detect that
- That's why it needs to visit a lot more nodes to come to a conclusive decision

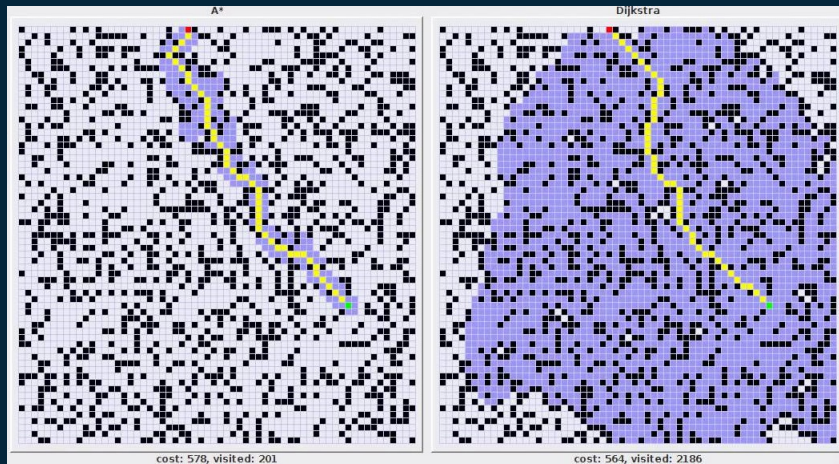


Figure: A\* search vs Dijkstra

How does A\* manage to be this efficient?

## HEURISTICS

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- It employs a practical method not guaranteed to be optimal or perfect
- But it is sufficient for the immediate goals

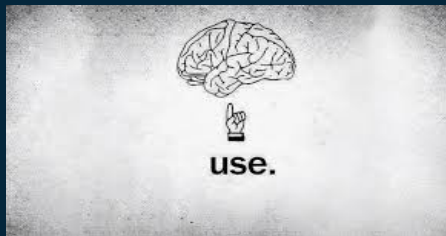


Figure: Using Heuristics



# Let's come back to this diagram again

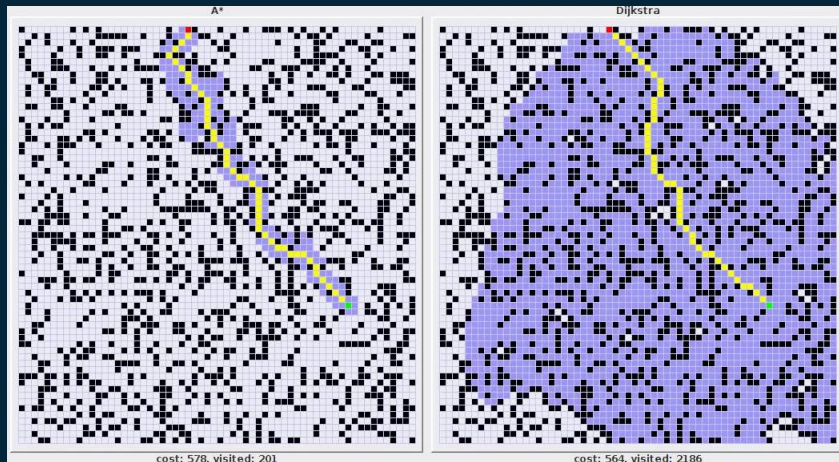


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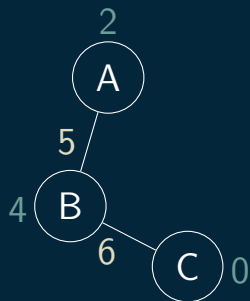
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- For example, the Euclidean distance can be an obvious one
- The shorter the Euclidean distance of a node from the destination the more likely it is to be included in the optimal path



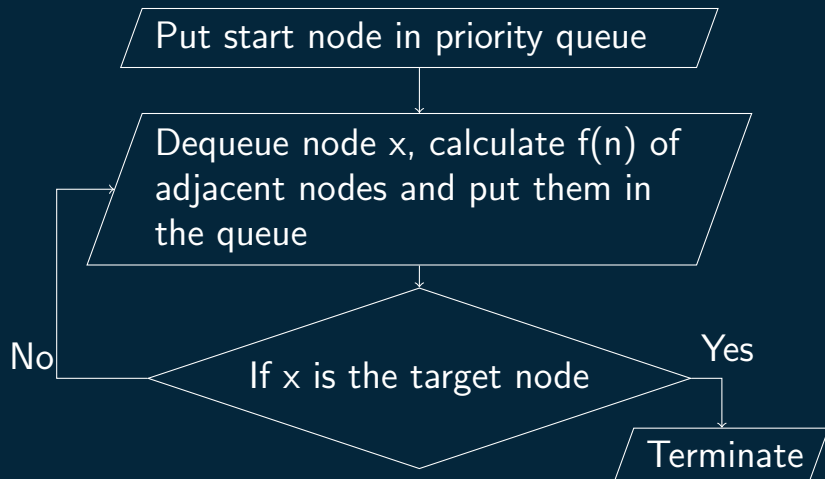
Figure: Using Euclidean Distance as a Heuristic measure

# A\* Search Algorithm



$$f(n) = d(n) + h(n)$$

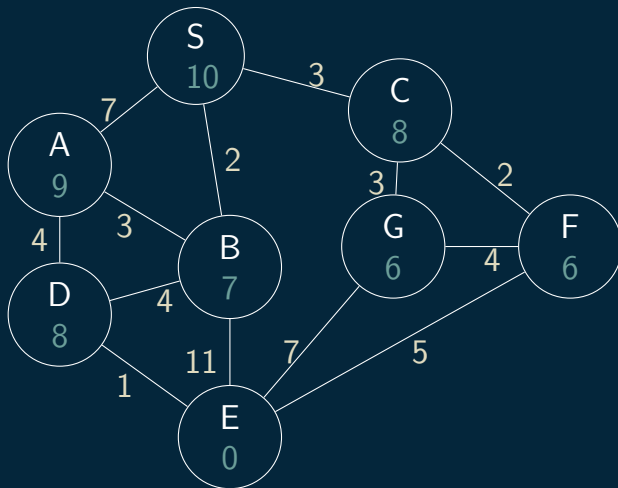
# Flow Chart



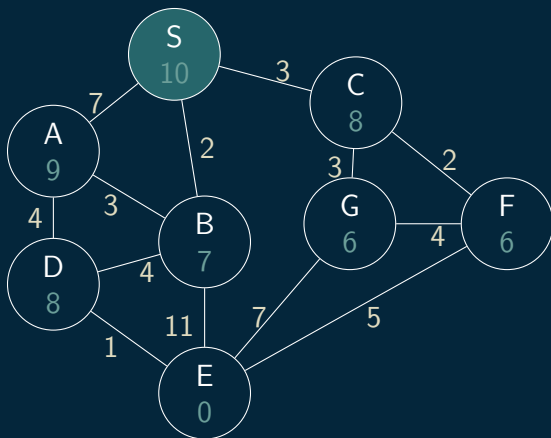


# Example

## Graph



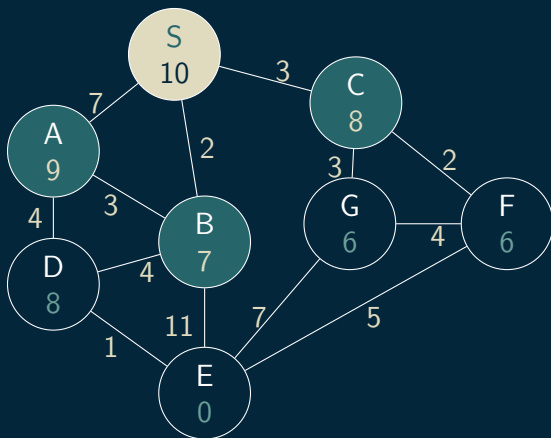
# Example



Priority Queue

S(NULL) -> 0 -> 10

# Example



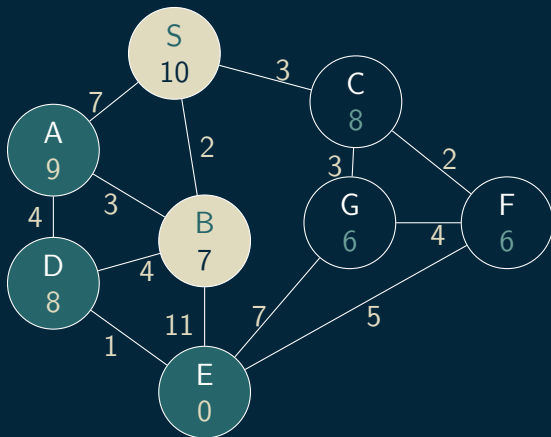
Priority Queue

$B(S) \rightarrow 2 \rightarrow 9$

$C(S) \rightarrow 3 \rightarrow 11$

$A(S) \rightarrow 7 \rightarrow 16$

# Example



Priority Queue

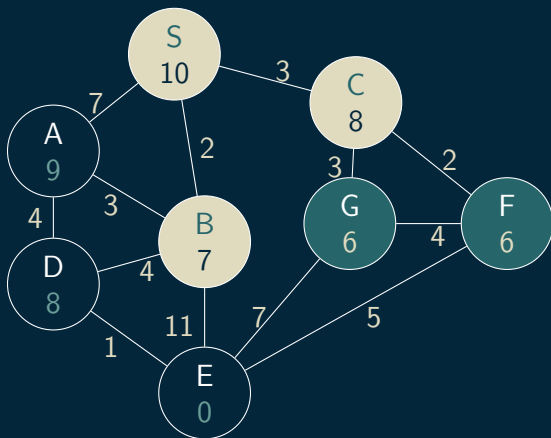
$C(S) \rightarrow 3 \rightarrow 11$

$E(B) \rightarrow 13 \rightarrow 13$

$A(B) \rightarrow 5 \rightarrow 14$

$D(B) \rightarrow 6 \rightarrow 14$

# Example



## Priority Queue

F(C)  $\rightarrow$  5  $\rightarrow$  11

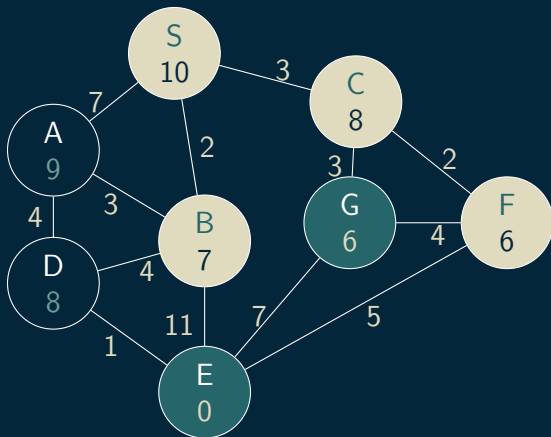
G(C)  $\rightarrow$  6  $\rightarrow$  12

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Priority Queue

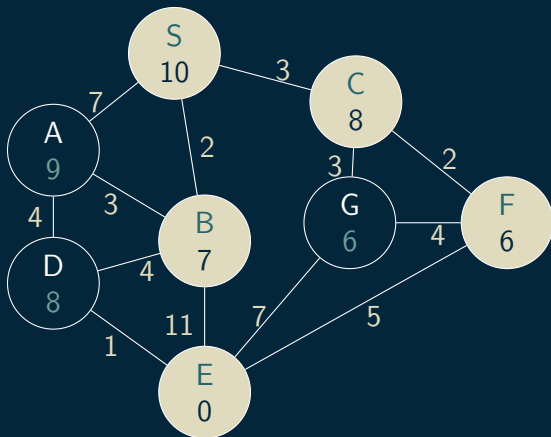
$E(F) \rightarrow 10 \rightarrow 10$

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Priority Queue

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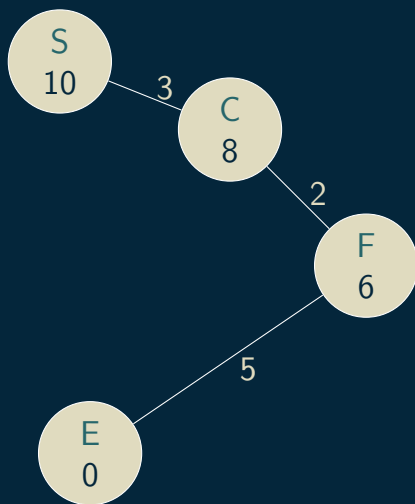


Figure: Expected shortest path



# Exact Heuristics

Using exact distance  
from any node  $n$  to  
goal node as  $h(n)$

- Precompute distances  
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- Precompute distances between each pair of nodes
- Time-consuming
- Equal to Euclidean distance if there is no other node on the direct path

# Manhattan Distance

$$h(n) = |n.x - g.x| + |n.y - g.y|$$

- Suitable for 2D grids
- Movement is allowed only in  $x$  or  $y$  axis (4 directions)

# Diagonal Distance

$$h(n) = \max\{|n.x - g.x|, |n.y - g.y|\}$$

- Suitable for games like chess
- Both axial and diagonal movements are allowed (8 directions)

# Euclidean Distance

$$h(n) = \sqrt{(n.x - g.x)^2 + (n.y - g.y)^2}$$

- Suitable for graphs
- Movement in any direction is allowed

# Implementation

Implementation details that can  
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- Breaking ties in deciding minimum element of priority queue

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Implementation details that can affect the performance of  $A^*$

- Breaking ties in deciding minimum element of priority queue
- Alternative heap implementations of priority queue



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- **cross product of s-g and n-g vectors**
  - Prefer paths on the start-goal straight line
- **LIFO like DFS**
  - Prefer new insertions to old insertions

# Which heap to use?

Binary  
Heap

vs

Fibonacci  
Heap

# Which heap to use?

Operation	Delete-min	Insert	Decrease-key
Binary Heap	$\Theta(\log n)$	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$
Fibonacci Heap	Amortized $\mathcal{O}(\log n)$	$\Theta(1)$	Amortized $\Theta(1)$

Table: Time-complexity comparisons

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Time Complexity :  $\mathcal{O}(b^d)$  or  $\mathcal{O}(|E|)$

# Where do we use A\*?



Figure: Tower Defense

# Where do we use $A^*$ ?

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- **Artificial Intelligence**
  - informed search algorithm for agents
- **Navigation**
  - Google Maps & other digital maps

# THE END

Thank You !  
Feel free to ask any questions