金融工程课程实验:

最优套期保值比的确定

桑梓洲 141292018

选题

期货套利和套期保值交易

了解期货套利方法和策略。理解套期保值的概念、原理及作用；掌握基差及其基本原理；了解套期保值的基本方法；进行套期保值模拟交易实验；利用时间序列分析的方法建立模型以计算**最优套期保值比率系数。**

目录

$$S_t - S_{t-1} = \alpha + \beta(F_t - F_{t-1}) + \epsilon_t.$$

$$s_t = \alpha_{0s} + \alpha_{1s}(S_{t-1} - \delta F_{t-1}) + \epsilon_{st}$$
$$f_t = \alpha_{0f} + \alpha_{1f}(S_{t-1} - \delta F_{t-1}) + \epsilon_{ft},$$

$$y_t = \epsilon_t h_t^{1/2},$$
$$h_t = \alpha_0 + \alpha_1 y_{t-1}^2,$$

$$e = \left(1 - \frac{V(R)^*}{x_t^2 \sigma_t^2}\right) \text{ or } e = \rho^2.$$

EViews

The Theory of hedging and Speculation in Commodity Futures, Johnson(1960)

between market i and market j, a *combination* of positions in i and j has a total variance of return $V(R)$ due to price change given by :

(1) $$V(R) = x_i^2 \sigma_i^2 + x_j^2 \sigma_j^2 + 2x_i x_j \operatorname{cov}_{ij}$$

The combination also has an actual return R and an expected return $E(R)$ due to price change given respectively by :

(2) $$R = x_i B_i + x_j B_j$$

and

(3) $$E(R) = x_i u_i + x_j u_j$$

p. 143

where B_i, B_j denotes the *actual* price changes from t_1 to t_2 in i and j, and u_i and u_j denote the price changes from t_1 to t_2 *expected at* t_1. As such, u_i and u_j are the mean values of the probability distributions of return existing in the i and j markets respectively at time t_1.[1]

Differentiating equation (1) with respect to x_j and setting the derivative equal to 0, we have the value x_j^* minimizing the variance of return for the combination x_i, x_j^*.

(4) $$x_j^* = -\frac{x_i \operatorname{cov}_{ij}}{\sigma_j^2}$$

Substituting the value x_j^* for x_j in equation (1) and letting $V(R)^*$ denote the total variance of return of the combination x_i, x_j^*, we have :

$$V(R)^* = x_i^2 \sigma_i^2 + \frac{x_i^2 \operatorname{cov}_{ij}^2}{\sigma_j^2} - \frac{2x_i^2 \operatorname{cov}_{ij}^2}{\sigma_j^2}$$

or $$V(R)^* = x_i^2 \left(\sigma_i^2 - \frac{\operatorname{cov}_{ij}^2}{\sigma_j^2}\right)$$

Since the coefficient of correlation, ρ, estimated by the trader is equal to $\frac{\operatorname{cov}_{ij}}{\sigma_i \sigma_j}$ then $V(R)^* = x_i^2 \sigma_i^2 (1 - \rho^2)$. Generally speaking the larger the (absolute) value of the coeffi-

The Hedging Performance of the New Futures Markets, Ederington(1979)

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If the expected change in the basis is zero, then clearly the expected gain or loss is reduced as $b \to 1$. It is also obvious that expected changes in the basis may add to or subtract from the gain or loss which would have been expected on an unhedged portfolio $\{E(U) = X_s E(S)\}$.

Holding X_s constant, let us consider the effect of a change in b, the proportion hedged, on the expected return and variance of the portfolio R.

$$\frac{\partial \operatorname{Var}(R)}{\partial b} = X_s^2\{2b\sigma_f^2 - 2\sigma_{sf}\} \tag{8}$$

so the risk minimizing b, b^*, is

$$b^* = \frac{\sigma_{sf}}{\sigma_f^2} \tag{9}$$

$$\frac{\partial E(R)}{\partial b} = -X_s[E(\Delta B) + E(S)] - \frac{\partial K(X_s, b)}{\partial b} \tag{10}$$

opportunity locus in figure 1, and corresponds to the variance of the return on a portfolio where b equals the b^* defined in equation 9. The measure of hedging effectiveness used in this paper is, therefore, the percent reduction in the variance or

$$e = 1 - \frac{\operatorname{Var}(R^*)}{\operatorname{Var}(U)}$$

where varR^* denotes the minimum variance on a portfolio containing security futures.

Substituting equation 9 into equation 5 yields

$$\operatorname{Var}(R^*) = X_s^2\left\{\sigma_s^2 + \frac{\sigma_{sf}{}^2}{\sigma_f^2} - 2\frac{\sigma_{sf}{}^2}{\sigma_f^2}\right\} = X_s^2\left(\sigma_s^2 - \frac{\sigma_{sf}{}^2}{\sigma^2 f}\right)$$

Consequently

$$e = \frac{\sigma_{sf}{}^2}{\sigma_s^2\sigma_f^2} = \rho^2$$

where ρ^2 is the population coefficient of determination between the change in the cash price and the change in the future's price.

In order to judge the market's effectiveness at reducing risk, we estimated e using the sample coefficient of determination, r^2, for hedges of two arbitrary lengths (two and four weeks) and using the sample variances and sample covariance of the two and four week price changes over the observed period to estimate b^* as well as σ_s^2, σ_f^2 and σ_{sf}. As noted above, the GNMA and T-Bill markets were established in October 1975 and January 1976 respectively. Since it seemed prudent to allow the markets to gain some depth before analyzing them, weekly data collection for the GNMA market began in January 1976 and for the T-Bill market in March 1976. Both data sets were continued through December 1977. For comparison purposes we also collected data (January 1976—December 1977) and calculated e for two established and heavily traded futures: corn and wheat.[6]

Time Varying Distributions and Dynamic Hedging with Foreign Currency Futures, Kroner & Sultan(1993)

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Two potential problems are prevalent in these empirical studies. First, if the spot and futures rates are cointegrated, then Regression (1) is misspecified because it involves overdifferencing the data and obscuring the long-run relationship between S_t and F_t (Engle and Granger (1987)). This implies a downward bias in $\hat{\beta}$ (Brenner and Kroner (1993)). Second, these studies implicitly assume that the risk in spot and futures markets is constant over time, implying that the minimum risk hedge ratio will be the same irrespective of when the hedging is undertaken. But this assumption contrasts sharply with reality because as new information is received by the market, the riskiness of each of these assets changes. See Bollerslev (1990) or Kroner and Sultan (1991) for evidence of this. This assumption implies that the risk-minimizing hedge ratio is time varying. Therefore, conventional models like (1) cannot produce risk-minimizing hedge ratios, raising important concerns regarding the risk reduction properties of conventional hedging models.

In this paper, we demonstrate a method of calculating the risk-minimizing futures hedge that addresses both of these issues, and apply the method to several different currencies. We propose and estimate a bivariate error correction model (ECM) in ΔS_t and ΔF_t with a GARCH error structure. The error correction term imposes the long-run relationship between S_t and F_t, and the GARCH error structure permits the second moments of the distribution to change through time. The time-varying hedge ratios can then be calculated from the estimated covariance matrix from the model. Both within-sample tests and out-of-sample tests reveal

(11)
$$\begin{aligned} s_t &= \alpha_{0s} + \alpha_{1s}\left(S_{t-1} - \delta F_{t-1}\right) + \epsilon_{st} \\ f_t &= \alpha_{0f} + \alpha_{1f}\left(S_{t-1} - \delta F_{t-1}\right) + \epsilon_{ft}, \end{aligned}$$

(12)
$$\left[\begin{array}{c} \epsilon_{st} \\ \epsilon_{ft} \end{array}\right] \Big| \Psi_{t-1} \sim N(0, H_t),$$

(13)
$$H_t = \left[\begin{array}{cc} h_{ss,t} & h_{sf,t} \\ h_{sf,t} & h_{ff,t} \end{array}\right] = \left[\begin{array}{cc} h_{s,t} & 0 \\ 0 & h_{f,t} \end{array}\right]\left[\begin{array}{cc} 1 & \rho \\ \rho & 1 \end{array}\right]\left[\begin{array}{cc} h_{s,t} & 0 \\ 0 & h_{f,t} \end{array}\right],$$

(14)
$$\begin{aligned} h^2_{s,t} &= c_s + a_s\epsilon^2_{s,t-1} + b_s h^2_{s,t-1} \\ h^2_{f,t} &= c_f + a_f\epsilon^2_{f,t-1} + b_f h^2_{f,t-1}, \end{aligned}$$

t can be computed as the ratio of conditional covariance between s and f to the conditional variance of f (both measured at time t), i.e., as

(15)
$$\hat{b}^*_t = \frac{\hat{h}_{sf,t}}{\hat{h}_{ff,t}}.$$

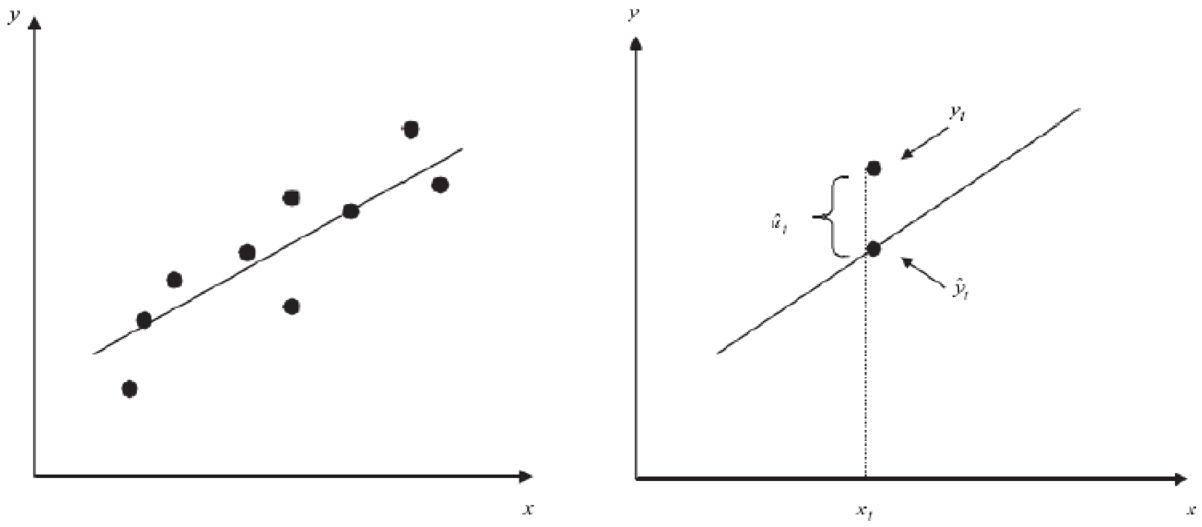
OLS, Intro

$$y_t = \alpha + \beta x_t + u_t$$

普通最小二乘（OLS）法认为，α、β的估计值，分别记为$\hat{\alpha}$、$\hat{\beta}$，应能最小化下面的式子，

*Ordinary Least Square

$$L = \sum_{t=1}^{T} u_t^2 = \sum_{t=1}^{T} (y_t - \alpha - \beta x_t)^2 \qquad (7)$$



OLS, Intro

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对扰动项的假设

④ $E(u_t) = 0$　　⑤ $\mathrm{var}(u_t) = \sigma^2 < \infty$　　⑥ $\mathrm{cov}(u_i, u_j) = 0$

⑦ $\mathrm{cov}(u_t, x_t) = 0$　　⑧ $u_t \sim N(0, \sigma^2)$

符合这 8 个假设下的(6)式称为**经典线性回归模型**（CLRM）。其中，①～

⑥的假设确保了 OLS 估计量的优良性质，被称为**高斯-马尔科夫假设**。

*Classic Linear Regression Model

OLS 估计量是“最优线性无偏估计量（BLUE）”

- 线性估计量：OLS 估计量是随机变量 y_t 的线性组合
- 无偏估计量：$E(\hat{\alpha}) = \alpha$，$E(\hat{\beta}) = \beta$
- 最优线性估计量(高斯-马尔科夫定理)：在所有的线性估计量中，OLS 估计量的方差最小（效率最高）

*Best Linear Unbiased Estimator

OLS, Intro

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对扰动项的假设

④ $E(u_t)=0$ ⑤ $\mathrm{var}(u_t)=\sigma^2<\infty$ ⑥ $\mathrm{cov}(u_i,u_j)=0$

⑦ $\mathrm{cov}(u_t,x_t)=0$ ⑧ $u_t\sim N(0,\sigma^2)$

符合这 8 个假设下的(6)式称为**经典线性回归模型**（CLRM）。其中，①～

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OLS 估计量是“最优线性无偏估计量（BLUE）”

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- 线性估计量：OLS 估计量是随机变量 y_t 的线性组合
- 无偏估计量：$E(\hat{\alpha})=\alpha$，$E(\hat{\beta})=\beta$
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  OLS 估计量的方差最小（效率最高）

Hedge Ratio, Naïve OLS

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从Wind获取**2017/2/20-2017/3/22**日**苏宁云商**等几家上市公司的数据，按心情构建投资组合作为被套期保值资产；再获取同阶段**沪深300股指期货IF1704**数据，作为套保资产，拟分别用实际值、收益率、对数差分进行套保比测算。

PORTFOLIO
portfolio=cenbest+suning+2*boc+njcb
Modified: 2/20/2017 3/22/2017 =>
portfolio=0.3*cenbest+suning+3*boc+njcb

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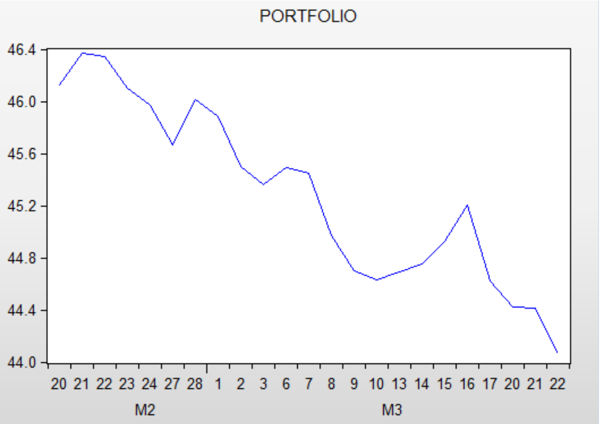
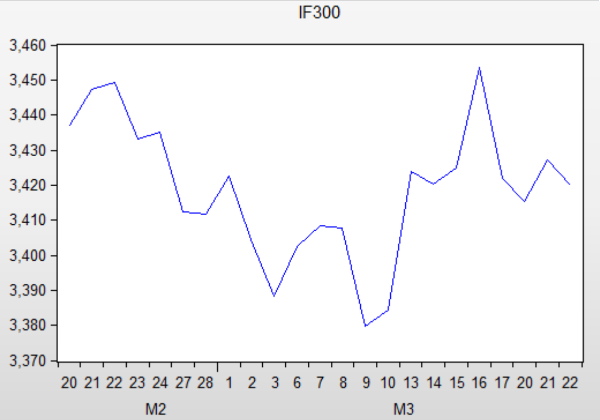
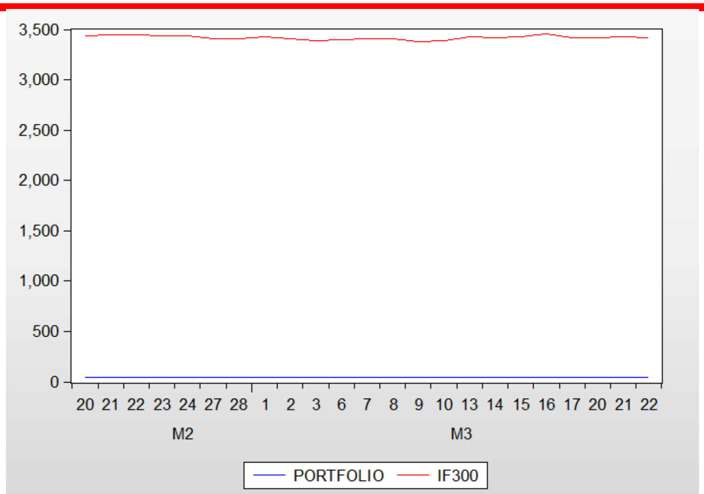
	CENBEST	SUNING	NJCB	BOC
	CENBEST	SUNING	NJCB	BOC
2/20/2017	37.80	11.25	12.47	3.69
2/21/2017	37.99	11.55	12.29	3.71
2/22/2017	38.07	11.40	12.36	3.72
2/23/2017	38.09	11.27	12.21	3.73
2/24/2017	37.66	11.26	12.25	3.72
2/27/2017	37.61	11.20	12.11	3.69
2/28/2017	38.00	11.33	12.24	3.68
3/01/2017	37.88	11.36	12.15	3.67
3/02/2017	37.93	11.20	11.91	3.67
3/03/2017	37.89	11.20	11.84	3.65
3/06/2017	37.76	11.23	12.04	3.63
3/07/2017	37.35	11.16	12.16	3.64
3/08/2017	36.22	11.10	12.08	3.64
3/09/2017	36.52	10.98	11.84	3.64
3/10/2017	36.63	10.99	11.76	3.63
3/13/2017	36.19	11.03	11.79	3.67
3/14/2017	36.40	11.00	11.85	3.66
3/15/2017	36.92	11.01	11.86	3.66
3/16/2017	36.91	11.08	12.01	3.68
3/17/2017				





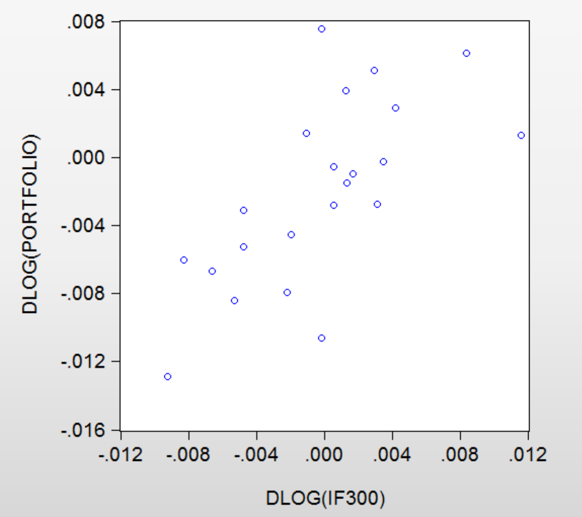
Hedge Ratio, OLS

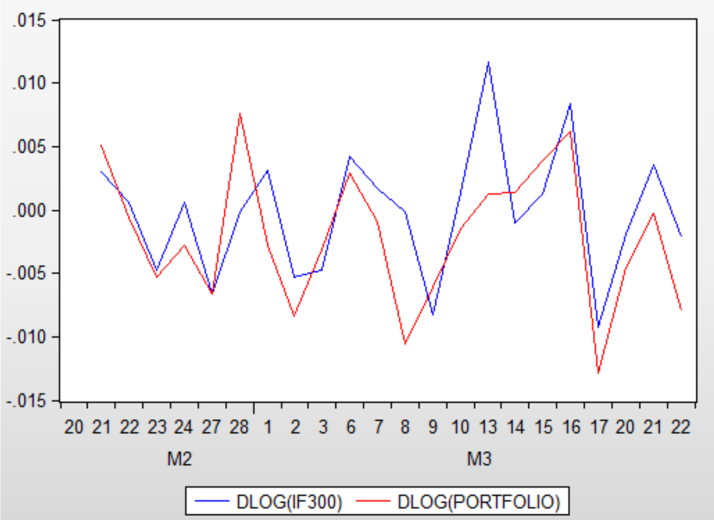


Dependent Variable: PORTFOLIO
Method: Least Squares
Date: 03/27/17 Time: 10:17
Sample: 2/20/2017 3/22/2017
Included observations: 23

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-3.272617	24.03688	-0.136150	0.8930
IF300	0.014204	0.007031	2.020303	0.0563

R-squared	0.162734	Mean dependent var	45.28839
Adjusted R-squared	0.122864	S.D. dependent var	0.686114
S.E. of regression	0.642584	Akaike info criterion	2.036302
Sum squared resid	8.671194	Schwarz criterion	2.135041
Log likelihood	-21.41748	Hannan-Quinn criter.	2.061135
F-statistic	4.081622	Durbin-Watson stat	0.110685
Prob(F-statistic)	0.056290		





Dependent Variable: DLOG(PORTFOLIO)
Method: Least Squares
Date: 03/27/17 Time: 10:18
Sample (adjusted): 2/21/2017 3/22/2017
Included observations: 22 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.001915	0.000871	-2.199535	0.0398
DLOG(IF300)	0.731374	0.175553	4.166106	0.0005

R-squared	0.464617	Mean dependent var	-0.002084
Adjusted R-squared	0.437848	S.D. dependent var	0.005441
S.E. of regression	0.004079	Akaike info criterion	-8.079245
Sum squared resid	0.000333	Schwarz criterion	-7.980060
Log likelihood	90.87170	Hannan-Quinn criter.	-8.055880
F-statistic	17.35644	Durbin-Watson stat	1.999034
Prob(F-statistic)	0.000477		

Dependent Variable: D(PORTFOLIO)/PORTFOLIO(-1)
Method: Least Squares
Date: 03/27/17 Time: 10:18
Sample (adjusted): 2/21/2017 3/22/2017
Included observations: 22 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.001908	0.000870	-2.194280	0.0402
D(IF300)/IF300(-1)	0.728273	0.175312	4.154156	0.0005

R-squared	0.463188	Mean dependent var	-0.002068
Adjusted R-squared	0.436348	S.D. dependent var	0.005428
S.E. of regression	0.004076	Akaike info criterion	-8.081133
Sum squared resid	0.000332	Schwarz criterion	-7.981947
Log likelihood	90.89246	Hannan-Quinn criter.	-8.057767
F-statistic	17.25702	Durbin-Watson stat	1.997592
Prob(F-statistic)	0.000491		

$$\frac{Y_t - Y_{t-1}}{Y_{t-1}} = \frac{Y_t}{Y_{t-1}} - 1 = \ln\frac{Y_t}{Y_{t-1}}$$

ECM, Intro

*Error Correction Model

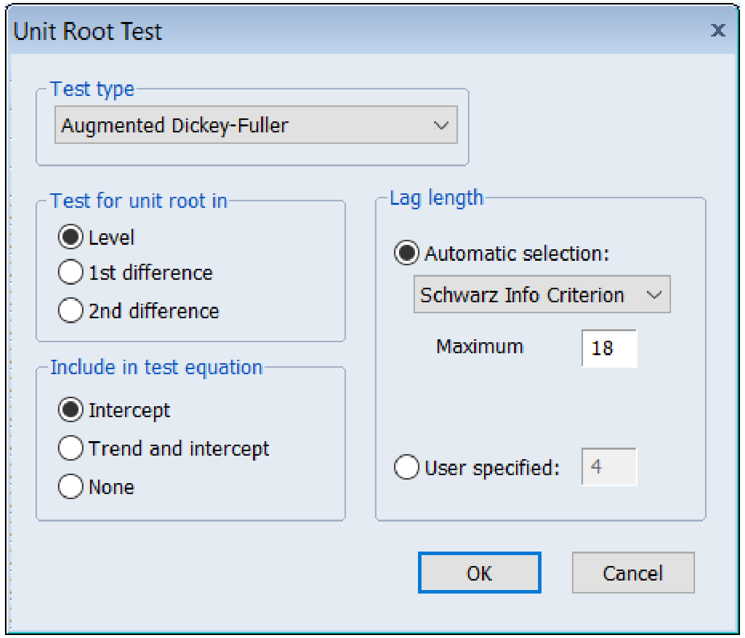
我们不给出严格的定义，十分简要地陈述以下事实：

通常我们更喜欢**平稳**的时间序列，非平稳会导致一些方法的失效。一般金融数据中非平稳的数据经过一次差分后会成为平稳的，记作 $y \sim I(1)$，类似地经过d次差分后平稳则记为 $y \sim I(d)$，称作d阶**单整** (Integration)，平稳的数据则为 $I(0)$ 。

平稳性与单位根检验：**ADF**检验

H_0: series contains a unit root versus H_1: series is stationary.

协整(Co-integration)：若两个 $I(1)$ 变量的线性组合是平稳（$I(0)$）的，则称两个变量是协整的。协整说明随着时间推移两个变量会一起移动，也许短期会偏离他们的关系但从**长期**会恢复到他们原有的联系去。



Hedge Ratio, ECM

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$$\Delta y_t = \beta \Delta x_t + u_t$$

$$\Delta y_t = \beta_1 \Delta x_t + \beta_2 (y_{t-1} - \gamma x_{t-1}) + u_t$$

Augmented Dickey-Fuller Unit Root Test on CSI300

Null Hypothesis: CSI300 has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on SIC, maxlag=4)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.456904	0.3436
Test critical values:	1% level	-4.440739	
	5% level	-3.632896	
	10% level	-3.254671	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Unit Root Test on D(CSI300)

Null Hypothesis: D(CSI300) has a unit root
Exogenous: None
Lag Length: 1 (Automatic - based on SIC, maxlag=4)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-5.154368	0.0000
Test critical values:	1% level	-2.685718	
	5% level	-1.959071	
	10% level	-1.607456	

*MacKinnon (1996) one-sided p-values.

Engle-Granger Cointegration Test

Date: 03/27/17 Time: 16:33
Series: CSI300 IF300
Sample: 2/20/2017 3/22/2017
Included observations: 23
Null hypothesis: Series are not cointegrated
Cointegrating equation deterministics: C @TREND
Automatic lags specification based on Schwarz criterion (maxlag=4)

Dependent	tau-statistic	Prob.*	z-statistic	Prob.*
CSI300	-4.238728	0.0517	-41.77732	0.0000
IF300	-3.486000	0.1715	-16.00686	0.1570

*MacKinnon (1996) p-values.
Warning: p-values may not be accurate for fewer than 25 observations.

$$S = \hat{\alpha} + \hat{\beta} F + \varepsilon$$

$$\{\varepsilon_t\} = \{S_t - (\hat{\alpha} + \hat{\beta} F_t)\}$$

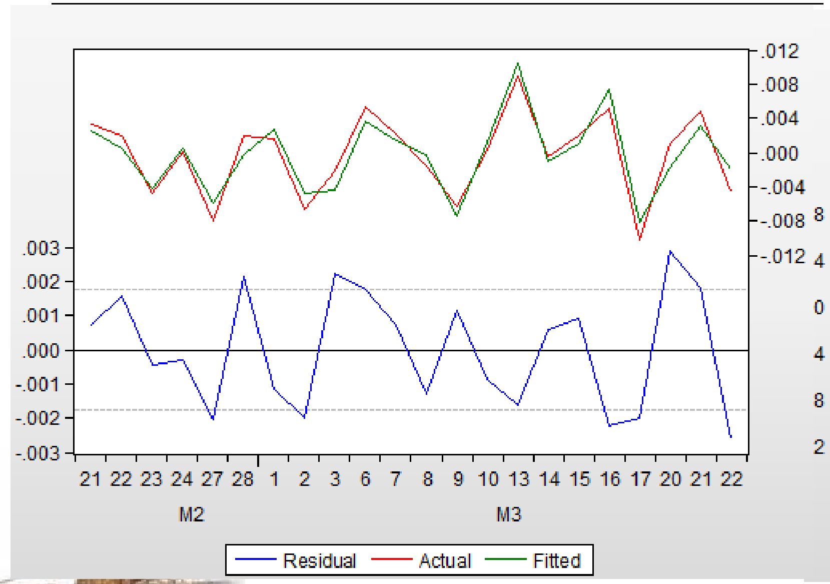
$$\Delta S = c + \hat{\beta}_1 \Delta F + \hat{\beta}_2 \varepsilon_{t-1} + u_t$$

Hedge Ratio, ECM

Dependent Variable: DLOG(CSI300)
Method: Least Squares
Date: 03/27/17 Time: 10:44
Sample (adjusted): 2/21/2017 3/22/2017
Included observations: 22 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-6.89E-05	0.000370	-0.186307	0.8541
DLOG(IF300)	0.894492	0.074606	11.98962	0.0000

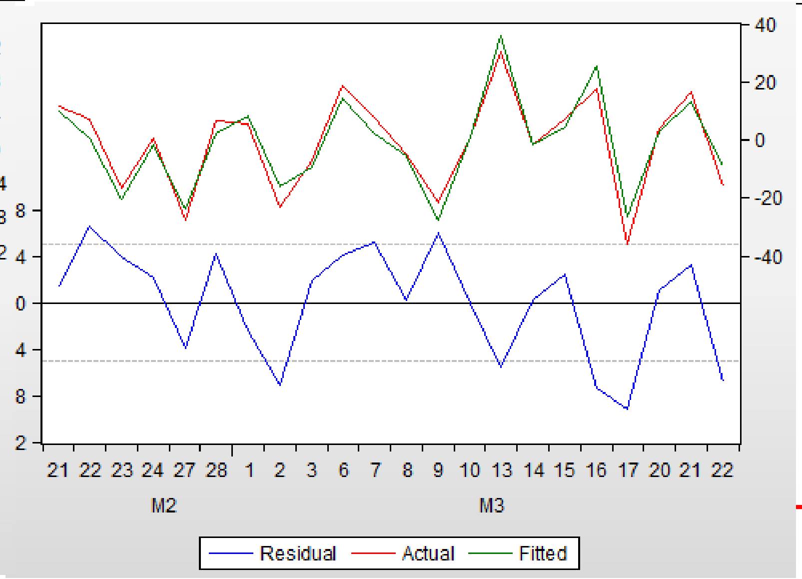
R-squared	0.877863	Mean dependent var	-0.000275
Adjusted R-squared	0.871756	S.D. dependent var	0.004841
S.E. of regression	0.001734	Akaike info criterion	-9.790700
Sum squared resid	6.01E-05	Schwarz criterion	-9.691514
Log likelihood	109.6977	Hannan-Quinn criter.	-9.767335
F-statistic	143.7509	Durbin-Watson stat	2.135376
Prob(F-statistic)	0.000000		



Dependent Variable: D(CSI300)
Method: Least Squares
Date: 03/26/17 Time: 11:57
Sample (adjusted): 2/21/2017 3/22/2017
Included observations: 22 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.009936	1.065792	-0.009322	0.9927
D(IF300)	0.933953	0.063433	14.72346	0.0000
RSD_CSI_IF(-1)	-0.695761	0.221860	-3.136038	0.0054

R-squared	0.919740	Mean dependent var	-0.952727
Adjusted R-squared	0.911291	S.D. dependent var	16.72708
S.E. of regression	4.981996	Akaike info criterion	6.175662
Sum squared resid	471.5854	Schwarz criterion	6.324441
Log likelihood	-64.93228	Hannan-Quinn criter.	6.210710
F-statistic	108.8649	Durbin-Watson stat	1.620844
Prob(F-statistic)	0.000000		



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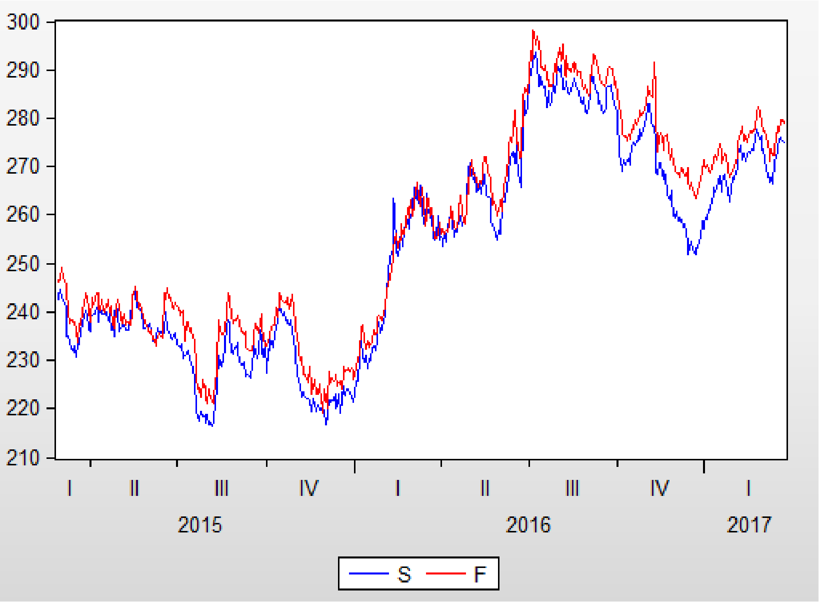

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Dependent Variable: S
Method: Least Squares
Date: 03/26/17 Time: 15:54
Sample: 2/25/2015 3/27/2017
Included observations: 544

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.895235	1.666775	1.137067	0.2560
F	0.976996	0.006472	150.9667	0.0000

R-squared	0.976771	Mean dependent var	252.6386
Adjusted R-squared	0.976728	S.D. dependent var	21.34470
S.E. of regression	3.256155	Akaike info criterion	5.202641
Sum squared resid	5746.580	Schwarz criterion	5.218446
Log likelihood	-1413.118	Hannan-Quinn criter.	5.208820
F-statistic	22790.96	Durbin-Watson stat	0.674328
Prob(F-statistic)	0.000000		

Equation: DLOG_OLS Workfile: AU::Untitled\

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: DLOG(S)
Method: Least Squares
Date: 03/26/17 Time: 15:59
Sample (adjusted): 2/26/2015 3/27/2017
Included observations: 543 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000170	0.000369	0.460167	0.6456
DLOG(F)	0.270785	0.042938	6.306469	0.0000

R-squared	0.068481	Mean dependent var	0.000233
Adjusted R-squared	0.066759	S.D. dependent var	0.008895
S.E. of regression	0.008593	Akaike info criterion	-6.672165
Sum squared resid	0.039943	Schwarz criterion	-6.656337
Log likelihood	1813.493	Hannan-Quinn criter.	-6.665976
F-statistic	39.77155	Durbin-Watson stat	2.362073
Prob(F-statistic)	0.000000		

London Gold(S) & AU1706(F)

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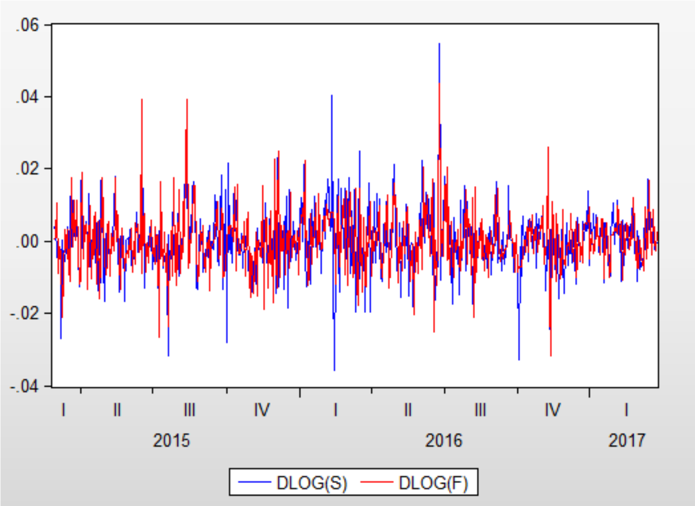

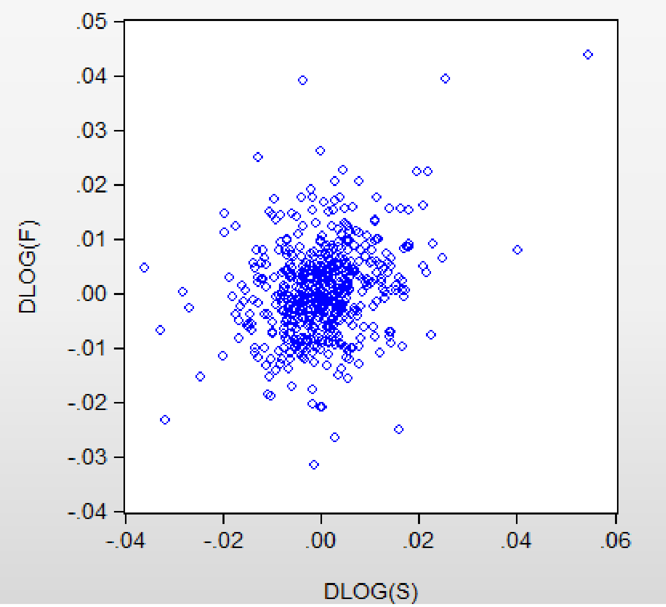

ARMA, Intro

AR(*p*) $y_t = \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + u_t$ *Auto-Regressive

MA(*q*) $y_t = \mu + u_t + \theta_1 u_{t-1} + \theta_2 u_{t-2} + \cdots + \theta_q u_{t-q}$ *Moving Average

$u_t \ (t = 1, 2, 3, \ldots)$为白噪声过程

ARMA ARMA(p, q)模型是 AR(p)与 MA(q)的组合

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.608	0.608	370.01	0.000
		2	0.381	0.019	515.59	0.000
		3	0.221	-0.028	564.49	0.000
		4	0.132	0.006	581.93	0.000
		5	0.070	-0.013	586.80	0.000
		6	0.055	0.029	589.85	0.000
		7	0.053	0.021	592.72	0.000
		8	0.037	-0.014	594.09	0.000
		9	0.017	-0.013	594.39	0.000
		10	-0.005	-0.019	594.41	0.000

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.464	-0.464	215.73	0.000
		2	-0.028	-0.310	216.50	0.000
		3	0.030	-0.184	217.40	0.000
		4	-0.045	-0.178	219.44	0.000
		5	0.045	-0.095	221.44	0.000
		6	-0.026	-0.089	222.12	0.000
		7	-0.019	-0.103	222.48	0.000
		8	-0.015	-0.133	222.71	0.000
		9	0.050	-0.063	225.20	0.000
		10	0.036	0.031	226.51	0.000

ARMA, Hedge Ratio

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Correlogram of Residuals Squared

Date: 03/27/17 Time: 11:20
Sample: 2/25/2015 3/27/2017
Included observations: 543

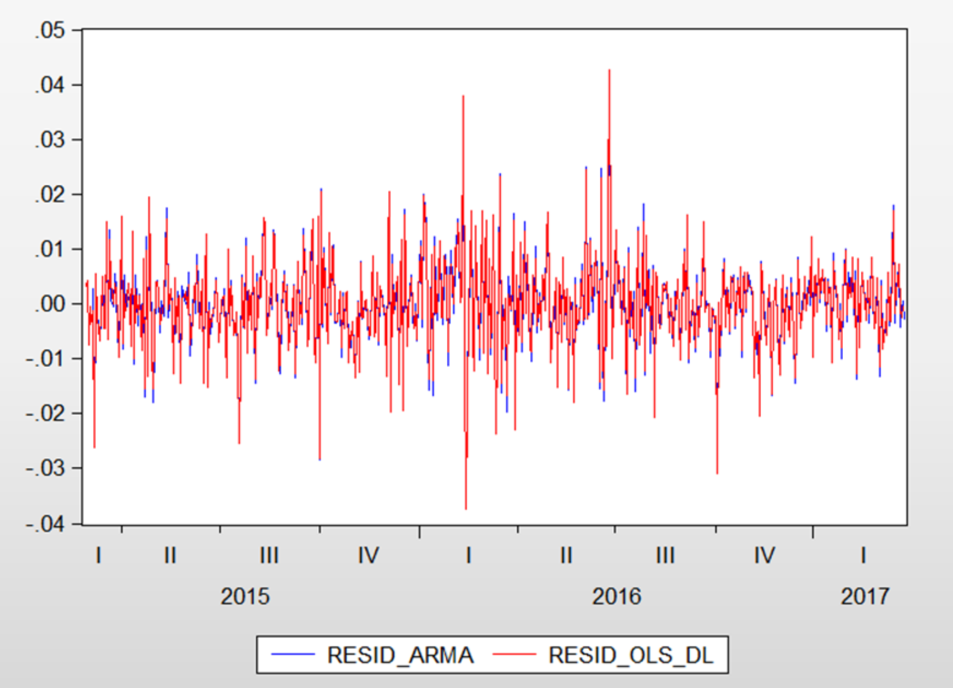
	AC	PAC	Q-Stat	Prob
1	0.049	0.049	1.3121	0.252
2	0.092	0.090	5.9582	0.051
3	0.030	0.021	6.4432	0.092
4	0.007	-0.003	6.4738	0.166
5	0.127	0.123	15.287	0.009
6	0.033	0.022	15.895	0.014
7	-0.036	-0.063	16.631	0.020
8	0.019	0.014	16.839	0.032
9	0.018	0.026	17.027	0.048
10	0.034	0.016	17.686	0.061
11	-0.027	-0.043	18.106	0.079
12	0.058	0.070	20.009	0.067
13	0.007	0.007	20.039	0.094
14	-0.039	-0.062	20.881	0.105

Dependent Variable: DLOG(S)
Method: Least Squares
Date: 03/26/17 Time: 18:39
Sample (adjusted): 3/05/2015 3/27/2017
Included observations: 538 after adjustments
Convergence achieved after 21 iterations
MA Backcast: 2/26/2015 3/04/2015

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000175	0.000379	0.462938	0.6436
DLOG(F)	0.273771	0.041892	6.535110	0.0000
AR(2)	-0.468204	0.027941	-16.75671	0.0000
AR(5)	-0.540410	0.026360	-20.50126	0.0000
MA(2)	0.449606	0.015744	28.55685	0.0000
MA(5)	0.627853	0.013657	45.97421	0.0000

R-squared	0.100113	Mean dependent var	0.000239
Adjusted R-squared	0.091655	S.D. dependent var	0.008928
S.E. of regression	0.008509	Akaike info criterion	-6.684279
Sum squared resid	0.038519	Schwarz criterion	-6.636459
Log likelihood	1804.071	Hannan-Quinn criter.	-6.665574
F-statistic	11.83702	Durbin-Watson stat	2.379410
Prob(F-statistic)	0.000000		

Inverted AR Roots	.63-.57i -.79	.63+.57i	-.23+.95i	-.23-.95i
Inverted MA Roots	.65-.58i -.82	.65+.58i	-.24+.97i	-.24-.97i



GARCH, Intro

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ARCH(*q*) $\sigma_t^2 = \text{var}(u_t \mid u_{t-1}, u_{t-2}, \ldots)$

*Autoregressive Conditional Heteroscedasticity

$$y_t = \beta_1 + \beta_2 x_{2t} + \beta_3 x_{3t} + \beta_4 x_{4t} + u_t \qquad u_t \sim \text{N}(0, \sigma_t^2)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 \qquad \sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \cdots + \alpha_q u_{t-q}^2$$

GARCH(*p*, *q*)

*Generalized ARCH

$$y_t = \beta_1 + \beta_2 x_{2t} + \beta_3 x_{3t} + \beta_4 x_{4t} + u_t$$

$$u_t = v_t \sigma_t \qquad v_t \sim \text{N}(0, 1)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2$$

ARCH term volatility

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

一般来讲GARCH(1,1)已经足够好(Sufficient)。

long-term average value

fitted variance

GARCH term

GARCH vs OLS

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Heteroskedasticity Test: ARCH

F-statistic	1.874429	Prob. F(10,522)	0.0463
Obs*R-squared	18.47584	Prob. Chi-Square(10)	0.0474

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 03/27/17 Time: 17:42
Sample (adjusted): 3/12/2015 3/27/2017
Included observations: 533 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	5.25E-05	1.07E-05	4.888347	0.0000
RESID^2(-1)	0.046021	0.043769	1.051453	0.2935
RESID^2(-2)	0.103243	0.043778	2.358334	0.0187
RESID^2(-3)	0.010412	0.044008	0.236606	0.8131
RESID^2(-4)	-0.014901	0.043225	-0.344727	0.7304
RESID^2(-5)	0.127110	0.043209	2.941762	0.0034
RESID^2(-6)	0.025601	0.043205	0.592537	0.5537
RESID^2(-7)	-0.067705	0.043219	-1.566547	0.1178

Dependent Variable: DLOG(S)
Method: Least Squares
Date: 03/26/17 Time: 15:59
Sample (adjusted): 2/26/2015 3/27/2017
Included observations: 543 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000170	0.000369	0.460167	0.6456
DLOG(F)	0.270785	0.042938	6.306469	0.0000

R-squared	0.068481	Mean dependent var	0.000233
Adjusted R-squared	0.066759	S.D. dependent var	0.008895
S.E. of regression	0.008593	Akaike info criterion	-6.672165
Sum squared resid	0.039943	Schwarz criterion	-6.656337
Log likelihood	1813.493	Hannan-Quinn criter.	-6.665976
F-statistic	39.77155	Durbin-Watson stat	2.362073
Prob(F-statistic)	0.000000		

Heteroskedasticity Test: ARCH

F-statistic	0.550690	Prob. F(10,522)	0.8539
Obs*R-squared	5.564247	Prob. Chi-Square(10)	0.8504

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 03/27/17 Time: 17:48
Sample (adjusted): 3/12/2015 3/27/2017
Included observations: 533 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.950703	0.160206	5.934259	0.0000
WGT_RESID^2(-1)	0.015652	0.043778	0.357532	0.7208
WGT_RESID^2(-2)	0.027571	0.043767	0.629947	0.5290
WGT_RESID^2(-3)	-0.022375	0.043782	-0.511063	0.6095
WGT_RESID^2(-4)	-0.019828	0.042236	-0.469465	0.6389
WGT_RESID^2(-5)	0.065014	0.042235	1.539349	0.1243
WGT_RESID^2(-6)	0.001284	0.042239	0.030409	0.9758
WGT_RESID^2(-7)	-0.063057	0.042236	-1.492985	0.1360

Dependent Variable: DLOG(S)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 03/27/17 Time: 17:45
Sample (adjusted): 2/26/2015 3/27/2017
Included observations: 543 after adjustments
Convergence achieved after 15 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	2.11E-05	0.000376	0.056273	0.9551
DLOG(F)	0.287810	0.028834	9.981478	0.0000
Variance Equation				
C	7.94E-06	4.40E-06	1.805576	0.0710
RESID(-1)^2	0.053483	0.019792	2.702208	0.0069
GARCH(-1)	0.838454	0.075769	11.06585	0.0000

R-squared	0.067945	Mean dependent var	0.000233
Adjusted R-squared	0.066222	S.D. dependent var	0.008895
S.E. of regression	0.008595	Akaike info criterion	-6.686612
Sum squared resid	0.039966	Schwarz criterion	-6.647044
Log likelihood	1820.415	Hannan-Quinn criter.	-6.671141
Durbin-Watson stat	2.384494		

GARCH模型可显著消除ARCH效应。

ECM-GARCH, the Model

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Engle-Granger Cointegration Test

Date: 03/27/17 Time: 19:15
Series: F S
Sample: 2/25/2015 3/27/2017
Included observations: 544
Null hypothesis: Series are not cointegrated
Cointegrating equation deterministics: C
Automatic lags specification based on Schwarz criterion (maxlag=18)

Dependent	tau-statistic	Prob.*	z-statistic	Prob.*
F	-3.954335	0.0089	-33.45841	0.0028
S	-3.976901	0.0083	-33.86799	0.0026

*MacKinnon (1996) p-values.

Workfile: FINAL - (c:\users\pistachio\desktop\final.wf1)

View | Proc | Object | Save | Freeze | Details+/- | Show | Fetch | Store | Delete | Genr | Sample

Range: 2/25/2015 3/27/2017 -- 544 obs Filter: *
Sample: 2/25/2015 3/27/2017 -- 544 obs Order: Name

c
eq1
eq2
f
garch01
garch02
h
ols
resid
resid_f
resid_s
s
z

Equation: EQ1 Workfile: FINAL::Untitled\

View | Proc | Object | Print | Name | Freeze | Estimate | Forecas

Specify/Estimate...
Forecast...
Make Residual Series...
Make Regressor Group
Make GARCH Variance Series...
Make Gradient Group
Make Derivative Group
Update Coefs from Equation
Add-ins

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0.7)
ARCH(
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085
195
Variance Equation

Covariance Analysis: Ordinary
Date: 03/26/17 Time: 17:14
Sample (adjusted): 2/26/2015 3/27/2017
Included observations: 543 after adjustments
Balanced sample (listwise missing value deletion)

Correlation	RESID_F	RESID_S
RESID_F	1.000000	
RESID_S	0.305200	1.000000

$$S = \alpha + \beta F + \varepsilon$$

$$\{ect_t\} = \{S_t - \hat{\alpha} - \hat{\beta} F_t\}$$

$$\Delta S_t = c_1 + \delta_1 ect_{t-1} + \varepsilon_{s,t-1}$$

$$\Delta F_t = c_2 + \delta_2 ect_{t-1} + \varepsilon_{f,t-1}$$

series h= ρ * (GARCH01/GARCH02)^0.5

$$s_t = \alpha_{0s} + \alpha_{1s}\left(S_{t-1} - \delta F_{t-1}\right) + \epsilon_{st} \quad (11)$$

$$f_t = \alpha_{0f} + \alpha_{1f}\left(S_{t-1} - \delta F_{t-1}\right) + \epsilon_{ft},$$

$$\begin{bmatrix} \epsilon_{st} \\ \epsilon_{ft} \end{bmatrix} \Bigg| \Psi_{t-1} \sim N(0, H_t), \quad (12)$$

$$H_t = \begin{bmatrix} h_{ss,t} & h_{sf,t} \\ h_{sf,t} & h_{ff,t} \end{bmatrix} = \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix} \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix}, \quad (13)$$

$$h_{s,t}^2 = c_s + a_s \epsilon_{s,t-1}^2 + b_s h_{s,t-1}^2 \quad (14)$$

$$h_{f,t}^2 = c_f + a_f \epsilon_{f,t-1}^2 + b_f h_{f,t-1}^2,$$

ECM-GARCH, Hedge Ratio

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Dependent Variable: D(F)
Method: ML - ARCH
Date: 03/26/17 Time: 17:05
Sample (adjusted): 2/26/2015 3/27/2017
Included observations: 543 after adjustments
Convergence achieved after 14 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	1.740521	0.111731	15.57776	0.0000
B(-1)	0.426790	0.021420	19.92499	0.0000
Variance Equation				
C	0.314174	0.072556	4.330103	0.0000
RESID(-1)^2	0.167326	0.036957	4.527624	0.0000
GARCH(-1)	0.763672	0.040288	18.95516	0.0000

R-squared	0.219932	Mean dependent var	0.061510
Adjusted R-squared	0.218490	S.D. dependent var	2.193985
S.E. of regression	1.939550	Akaike info criterion	4.077622
Sum squared resid	2035.164	Schwarz criterion	4.117190
Log likelihood	-1102.074	Hannan-Quinn criter.	4.093093
Durbin-Watson stat	1.476420		

Dependent Variable: D(S)
Method: ML - ARCH
Date: 03/26/17 Time: 17:01
Sample (adjusted): 2/26/2015 3/27/2017
Included observations: 543 after adjustments
Convergence achieved after 26 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.027101	0.142541	0.190128	0.8492
B(-1)	0.002738	0.029847	0.091718	0.9269
Variance Equation				
C	0.345556	0.171449	2.015507	0.0439
RESID(-1)^2	0.036261	0.013951	2.599090	0.0093
GARCH(-1)	0.896137	0.045135	19.85446	0.0000

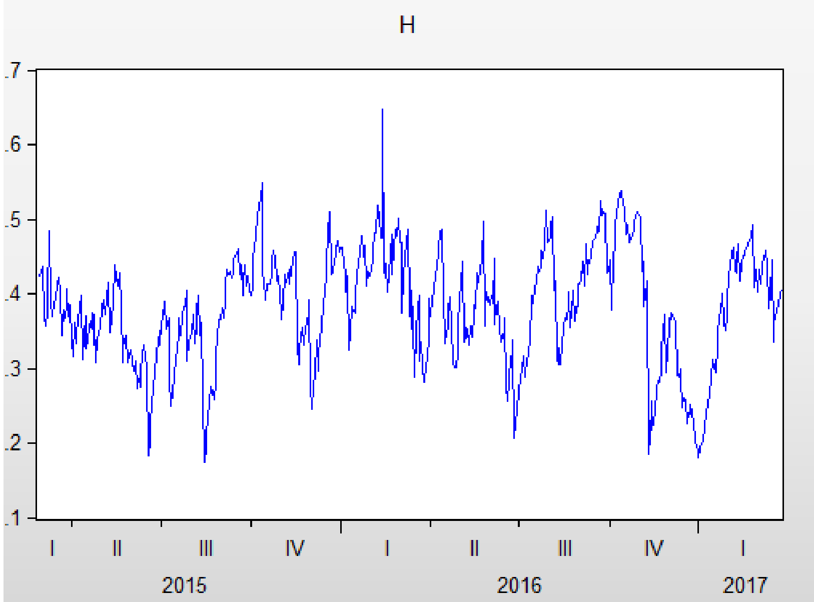
R-squared	-0.000507	Mean dependent var	0.060265
Adjusted R-squared	-0.002356	S.D. dependent var	2.251498
S.E. of regression	2.254149	Akaike info criterion	4.454757
Sum squared resid	2748.921	Schwarz criterion	4.494325
Log likelihood	-1204.466	Hannan-Quinn criter.	4.470228
Durbin-Watson stat	1.999968		

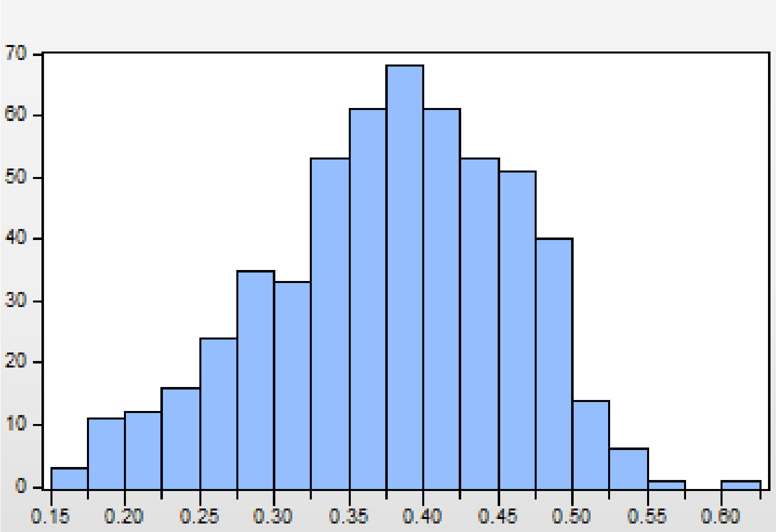
Correlogram of Standardized Residuals Squared

Date: 03/27/17 Time: 13:56
Sample: 2/25/2015 3/27/2017
Included observations: 543

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	-0.009	-0.009	0.0479	0.827
		2	-0.033	-0.033	0.6527	0.722
		3	0.001	0.001	0.6539	0.884
		4	-0.043	-0.045	1.6901	0.793
		5	0.028	0.028	2.1292	0.831
		6	0.038	0.035	2.9049	0.821
		7	-0.028	-0.026	3.3529	0.851
		8	0.027	0.027	3.7658	0.878
		9	-0.030	-0.030	4.2774	0.892
		10	-0.003	0.001	4.2828	0.934
		11	-0.011	-0.017	4.3445	0.959
		12	-0.017	-0.015	4.5036	0.973
		13	0.015	0.012	4.6361	0.982
		14	0.007	0.005	4.6636	0.990

ECM-GARCH, Hedge Ratio

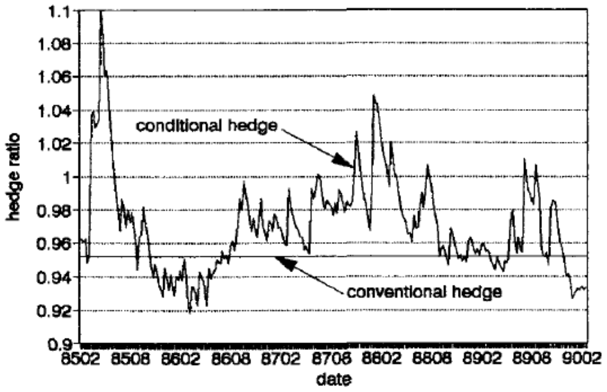




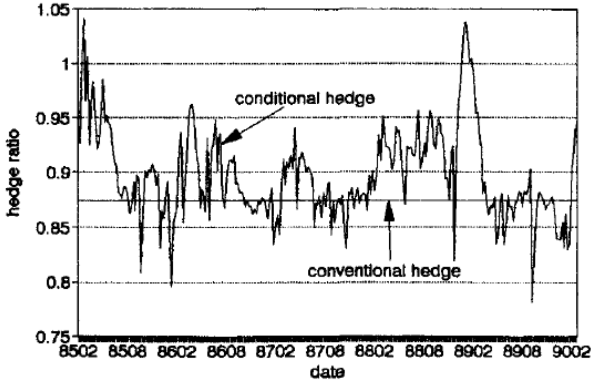
Series: H
Sample 2/25/2015 3/27/2017
Observations 543

Mean	0.377783
Median	0.382918
Maximum	0.624912
Minimum	0.167635
Std. Dev.	0.081612
Skewness	-0.323951
Kurtosis	2.650638
Jarque-Bera	12.25891
Probability	0.002178

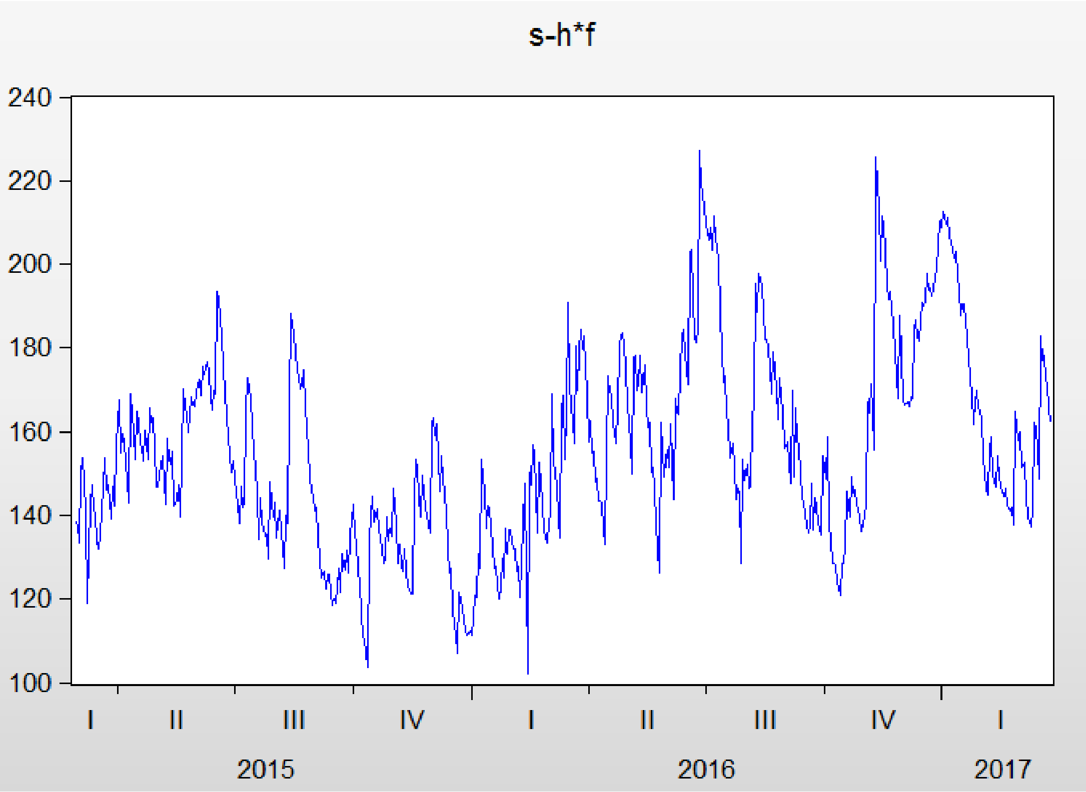
British Pound
Hedge Ratios

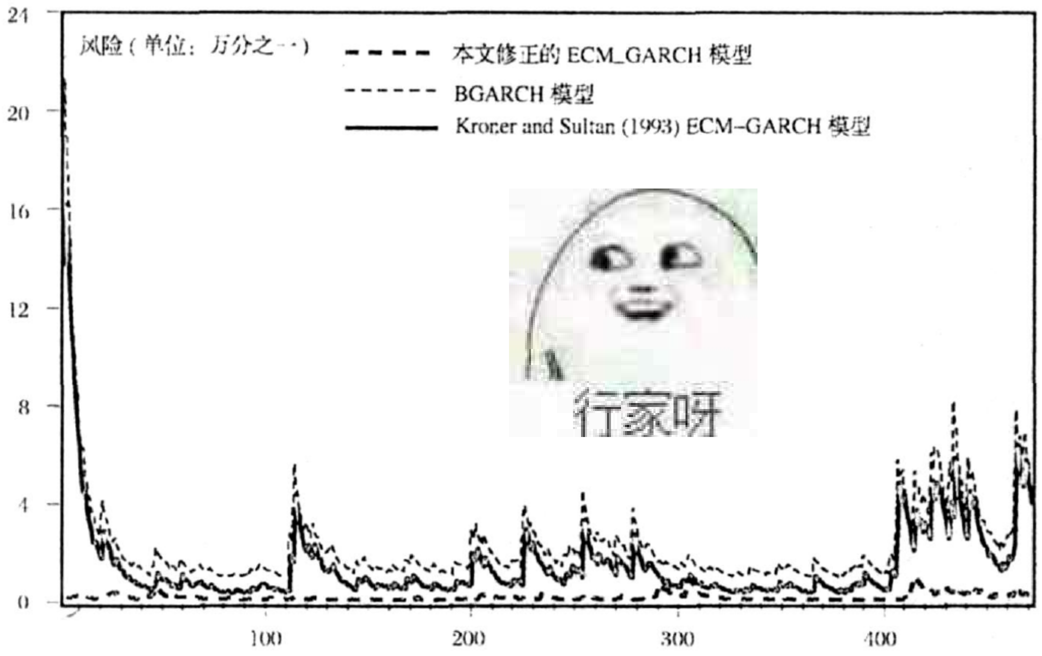


Canadian Dollar
Hedge Ratios



GARCH, Efficiency





基于 ECM-BGARCH 模型对中国黄金期货套期保值比率的研究

表 6. 1 OLS、ECM、ECM-BGARCH 三种模型得出的套期保值绩效

方法	套期保值率 h	套期保值后资产组合方差 Var(Ht)	套期保值绩效 H_e
OLS	0.693559	0.278710	0.906354
ECM	0.721125	0.230759	0.922466
ECM-BGARCH	0.742425	0.196806	0.933874

Conclusion, Room for Improvement

对不同的期货品种应对多种套期保值模型进行尝试，或许不存在绝对最优；
不要盲目相信论文上的数据、结论，有可能是巧合，不一定具备普适性；

缺乏对套期保值效果的检验及定量描述
应采用部分样本内预测，用样本外数据进行检测
对理论没有细致研究，缺乏扎实理论根基及深入理解
部分数据过少，不具有说服力
没有对比，未在一个具体案例上做出对比

Bibliography & Further Links

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Thank you~