What Accumulator is and How to Price them

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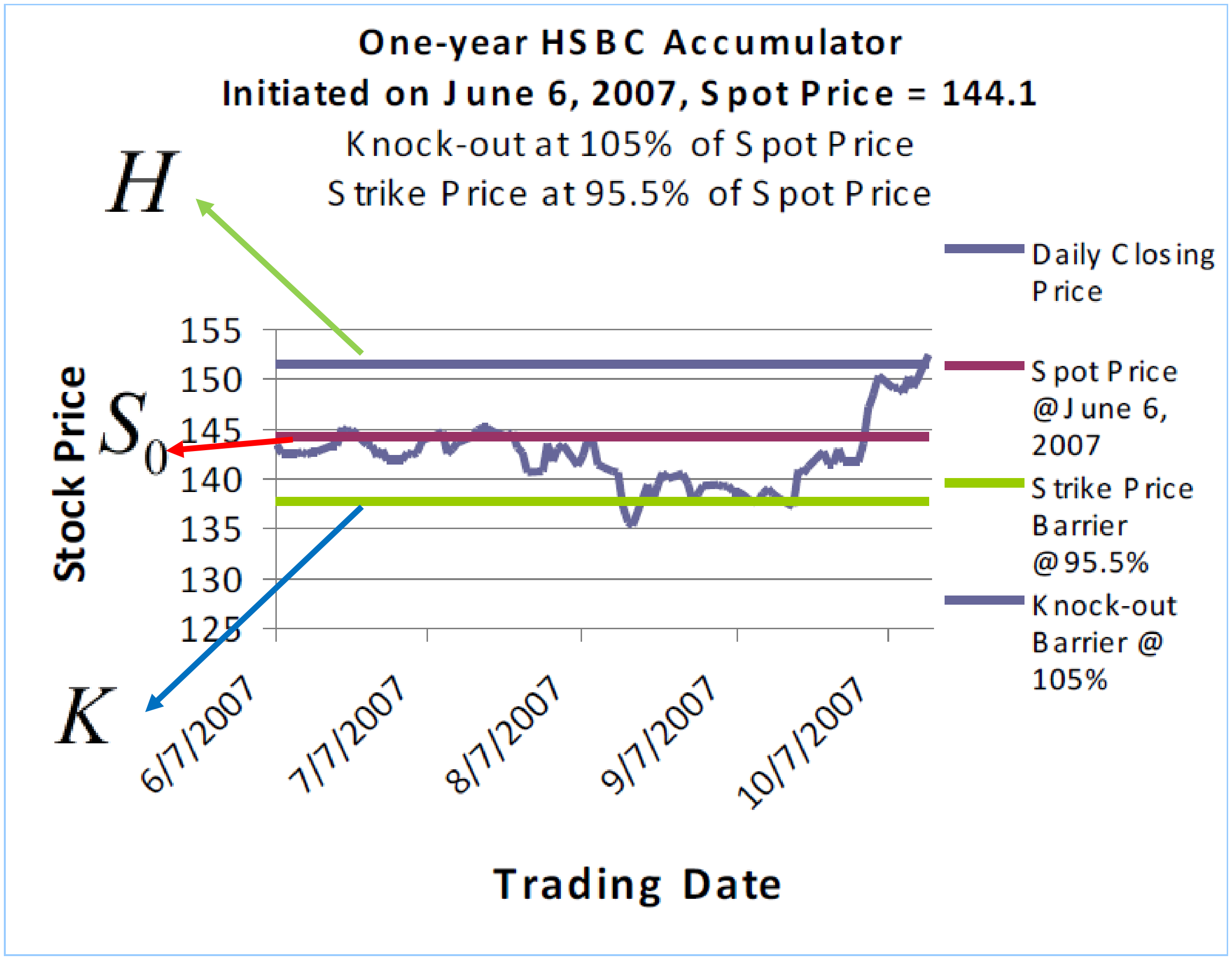
累计期权

- 场外合约：高级客户和投行签署，未来一年每个交易日中按约定价格购买约定数量的标的资产
- **Knock-Out Discount Accumulator**, KODA
  - Discount：合约价K是签时现货资产S_0的折扣价
  - Knock-out: 日后价格S_t若高于H则合约终止
  - Accumulate: 若$S_t > K$且未敲出每日按K累积Q单位资产
  - Step-up：若S_t低于K，则必须按K购买$2Q$数量资产
  - Zero-Cost: 签约只需付购买价，合约本身无费用
  - 高额解约费，亏损时无法退出
  - 因具有赌博性质，在美国被禁止销售

Accumulator



I kill'u later

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- 以一份汇丰(00005.HK)累计期权为例，假设你于2008年3月初签订，当时股价为$122港元，合约列明需要于12个月内(约250个交易日)，每日以折让价$105买入400股汇控，终止价则订为$138。
- 2008年5月初，汇控一度触及$136.8的高位，但仍未被knock out。然后股价于2008年10月尾跌穿$105，你必须按合约以$105高价接货。其后，汇控股价一蹶不振，直至合约到期时，更跌至$37。
- 粗略估算，当合约到期，你合共已买入100,000股只值$37的汇控，账面亏损高达680万！即使市况畅旺，你这张Accumulator也最多只能为你带来330万的利润。

Review of Accumulator Pricing Methods

- Analytical:
  - Reduced to Barrier Options Pricing
  - BS framework and solutions by Reiner and Rubinstein(1991)
  - Lévy process and Fourier cosine
  - Limitations: assume constant r, σ and lognormal distribution
- Numerical:
  - PV of Expected Return under risk-neutral random walk
  - Good for Path Dependent Options
  - Monte Carlo Simulation
  - Binomial Tree(lattice)

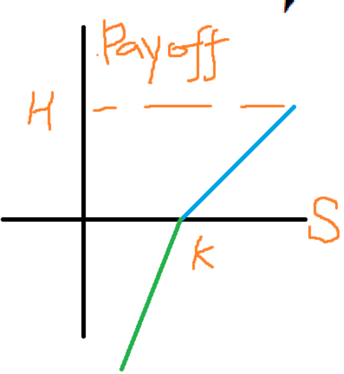
Analytical Method

- 考虑交易日t_i的Payoff (K. Lam, *et al.*, 2009):

$$
\begin{cases}
0 & \textit{if } \max_{0\le\tau\le t_i} S_\tau \ge H \\
S_{t_i} - K & \textit{if } \max_{0\le\tau\le t_i} S_\tau < H, \quad S_{t_i} \ge K \\
2(S_{t_i} - K) & \textit{if } \max_{0\le\tau\le t_i} S_\tau < H, \quad S_{t_i} < K
\end{cases}
$$

- 等价于做多一份向上敲出看涨期权与做空两份向上敲出看跌期权的组合 (K. Lam, *et al.*, 2009):

$$V = \sum_{i=1}^{n} \{C_{uo}(t_i, K, H) - 2 \cdot P_{uo}(t_i, K, H)\}$$



Closed-form Solutions for Barrier Options

- According to K. Lam, *et al.*(2009):
- Up-Out-Call:

$$C_{uo}(K,H,t_i) = S_0 e^{-qt_i}\left\{N(x(t_i)) - N(x_1(t_i)) + \left(\frac{H}{S}\right)^{2\lambda}\left[N(-y(t_i)) - N(-y_1(t_i))\right]\right\}$$

$$-e^{-rt_i}K\left\{N\left(x(t_i)-\sigma\sqrt{t_i}\right) - N\left(x_1(t_i)-\sigma\sqrt{t_i}\right) + \left(\frac{H}{S}\right)^{2\lambda-2}\left[N\left(-y(t_i)+\sigma\sqrt{t_i}\right) - N\left(-y_1(t_i)+\sigma\sqrt{t_i}\right)\right]\right\}$$

- Up-Out-Put:

$$P_{uo}(K,H,t_i) = e^{-rt_i}K\left\{N\left(-x(t_i)+\sigma\sqrt{t_i}\right) - \left(\frac{H}{S}\right)^{2\lambda-2} N\left(-y(t_i)+\sigma\sqrt{t_i}\right)\right\}$$

$$-S_0 e^{-qt_i}\left\{N(-x(t_i)) - \left(\frac{H}{S}\right)^{2\lambda} N(-y(t_i))\right\}$$

- Where

$$x(t_i) = \frac{\log(S/K) + (\mu+\sigma^2)t_i}{\sigma\sqrt{t_i}},\ x_1(t_i) = \frac{\log(S/H) + (\mu+\sigma^2)t_i}{\sigma\sqrt{t_i}},\quad y(t_i) = \frac{\log(H^2/SK) + (\mu+\sigma^2)t_i}{\sigma\sqrt{t_i}},\ y_1(t_i) = \frac{\log(H/S) + (\mu+\sigma^2)t_i}{\sigma\sqrt{t_i}},$$

$$\mu = r - q - \sigma^2/2,\qquad \lambda = 1 + \frac{\mu}{\sigma^2}.$$

Further Extension-Analytical

- Delayed Settlement (K. Lam, *et al.*, 2009):
  - 考虑不必每天结算(现实中常按月)
  - 设对于每一交易日t_i存在到交割日T_i的映射
  - $V^{Delay}=\sum_{i=1}^{n}\{C_{uo}^{F}(t_i,K,H,T_i)-2\cdot P_{uo}^{F}(t_i,K,H,T_i)\}$
- Discrete Barrier (K. Lam, *et al.*, 2009):
  - 考虑不必实时监控是否敲出(而只监控日收盘价)
  - 修正项$\widetilde{H}=He^{\beta\sigma\sqrt{T/m}}$, $V_{Discrete}(H)=V(\widetilde{H})$
  - 其中$\beta=-\zeta(\frac{1}{2})/\sqrt{2\pi}\approx 0.5826$, m是监控点个数
  - 效果拔群👍

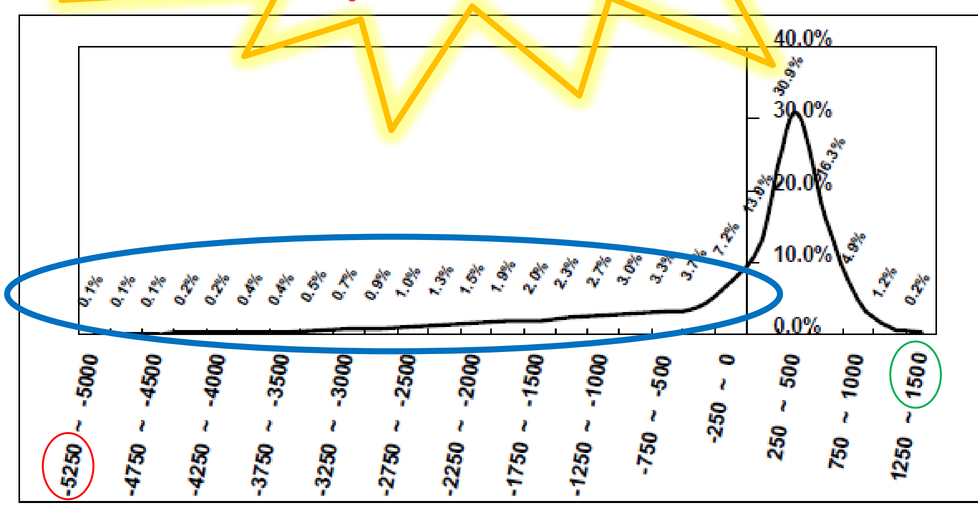
Pricing a Sample Accumulator

- $S_0 = 100, K = 90, H = 105$
- $r = 0.03, \sigma = 0.2, T = 1$
- 每日购买1股，杠杆率为2(跌破购买2股)
- 每月按21天(交易日)计
- 股息支付率为0

FAIR VALUES OF ACCUMULATOR CONTRACTS

Volatility (σ)	Discounted Purchase Price K				Zero-structure discounted price
	78	84	90	96	
10%	2639.5	1821.5	978.4	24.2	96.14
15%	1785.8	1108.4	369.8	-499.5	92.70
20%	1217.4	604.0	-82.2	-883.4	89.32
25%	790.0	211.6	-437.1	-1180.8	86.04
30%	445.2	-109.3	-727.2	-1423.3	82.86
35%	155.2	-380.6	-972.4	-1629.2	79.80
40%	-95.5	-615.9	-1185.4	-1809.6	76.84

5% chance of -$2424.50



Pricing by Monte Carlo

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```
%AccumulatorMC.m
function[P,CI]=AccumulatorMC(S0,K,r,sigma,H,NSteps,NRepl)
Payoff=zeros(NRepl,1);
for i=1:NRepl
    Path=AssetPaths(S0,r,sigma,1,NSteps,1);
    Payoff(i)=AccCalc(Path,K,r,NSteps,H);
end;
[P,aux,CI]=normfit(Payoff)
```

```
function SPaths=AssetPaths(S0,mu,sigma,T,NSteps,NRepl)
%NRepl represents # of paths to be generated
%NSteps represents # of steps per path
SPaths = zeros(NRepl,1+NSteps);
SPaths(:,1)=S0;% Every path beg

dt=T/NSteps;
nudt=(mu-0.5*sigma^2)*dt;
sidt=sigma*sqrt(dt);
for i=1:NRepl
    for j=1:NSteps
        SPaths(i,j+1)=SPaths(i,j)*exp(nudt+sidt*randn);
    end;
end;
```

```
%AccCalc.m
function payoff=AccCalc(Path,K,r,NSteps,H)
mpayoff=zeros(12,1);
for i=1:NSteps
    j=floor((i-1)/21)+1;
    if Path(i)>=H
        break;
    elseif  Path(i)>=K
        mpayoff(j)=mpayoff(j)+Path(i)-K;
    else
        mpayoff(j)=mpayoff(j)+2*(Path(i)-K);
    end;
end;
a=1:12;
d=exp(-1/12*a*r);
payoff=d*mpayoff;
```

```
>> [P,CI]=AccumulatorMC(100,90,0.03,0.2,105,252,5000000)
```

TABLE I
MONTE CARLO VS CLOSED FORM VALUE OF ACCUMULATOR CONTRACT

Monthly settlement accumulator	Closed form value	MC value	Difference in percentage
$V^{Delay}_{discrete}$	-84.845	-82.96	2.27%

```
P =

  -89.0332

CI =

  -90.2532
  -87.8132
```

Pricing by Formulae

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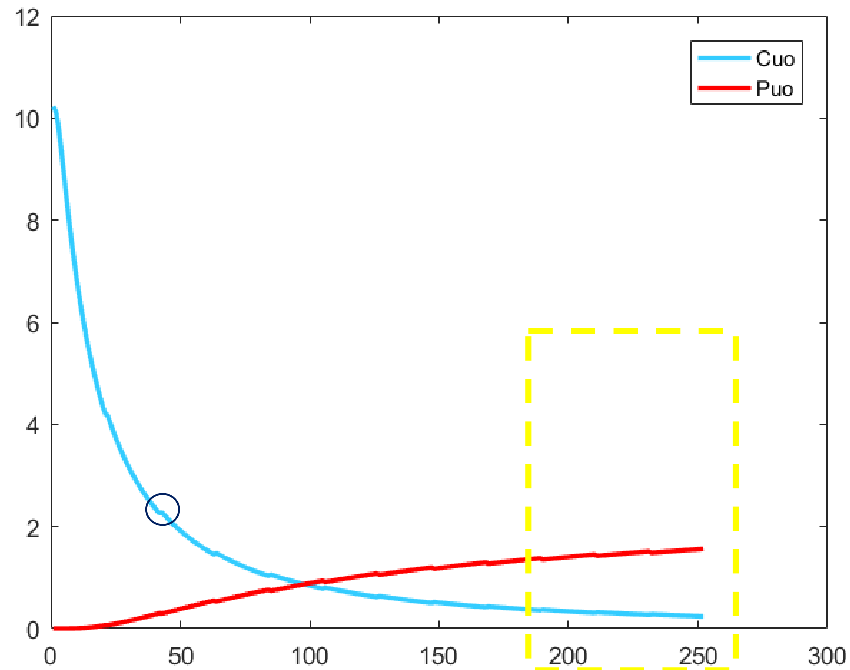
```
%UpOut.m
function [C,P]=UpOut(S0,K,r,T,sigma,H)
a=(H/S0)^(-1+2*r/sigma^2);
b=(H/S0)^(1+2*r/sigma^2);
d1=(log(S0/K)+(r+sigma^2/2)*T)/(sigma*sqrt(T));
d2=(log(S0/K)+(r-sigma^2/2)*T)/(sigma*sqrt(T));
d3=(log(S0/H)+(r+sigma^2/2)*T)/(sigma*sqrt(T));
d4=(log(S0/H)+(r-sigma^2/2)*T)/(sigma*sqrt(T));
d5=(log(S0/H)-(r-sigma^2/2)*T)/(sigma*sqrt(T));
d6=(log(S0/H)-(r+sigma^2/2)*T)/(sigma*sqrt(T));
d7=(log(S0*K/H^2)-(r-sigma^2/2)*T)/(sigma*sqrt(T));
d8=(log(S0*K/H^2)-(r+sigma^2/2)*T)/(sigma*sqrt(T));

Ts=(floor((T*252-1)/21)+1)/12;
C=S0*(normcdf(d1)-normcdf(d3)...
    -b*(normcdf(d6)-normcdf(d8)))...
    -K*exp(-r*Ts)*(normcdf(d2)-normcdf(d4)...
    -a*(normcdf(d5)-normcdf(d7)))
P=-S0*(1-normcdf(d1)-b*normcdf(d8))...
    +K*exp(-r*Ts)*(1-normcdf(d2)-a*normcdf(d7))
```

```
for i=1:252
    [c,p]=UpOut(100,90,0.03,i/252,0.2,105);
    sum=sum+c-2*p;
end;
```

```
>> sum

sum =

-114.0580
```

```
for i=1:252
    [c,p]=UpOut(100,90,0.03,i/252,0.2,105*1.0074);
    sum=sum+c-2*p;
end;
```

```
>> sum

sum =

-84.6954
```



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The END

Thanks !