Optimal Collective Intelligence

by Sven Nilsen, 2018

An optimal collective intelligence is a set of agents which every agent performs equally or better in some environment than all other agents in the same collective:

```
optimal_collective_intelligent := \(agents : {agent})
    = ∀ a : agents { ∃ e : environment {
        performance(a, e) >= max b : agents { performance(b, e) }
    } }

performance : agent × environment → real
```

In a strictly optimal collective intelligence, `>=` is replaced by `>`. A such set of agents does not contain redundant skills. For any two agents, there is an environment where one is better than the other:

```
strictly_optimal_collective_intelligent := \(agents : [agent])
    = ∀ a : agents { ∃ e : environment {
        performance(a, e) > max b : agents ∧ (¬= a) { performance(b, e) }
    } }
```

What we mean by optimal collective intelligence depends on how we define the environments. In principle, there could be a way to construct an environment such that an arbitrary agent is best.

For example, an environment that identifies the agent and return a score 1, returning 0 otherwise, is guaranteed to make the agent perform better than any other agent.

So, in order to define environments that are useful, they must be “fair” in some sense. Since any agent performing better than others contains some information that makes it perform better, the notion of “fair” is hard to formalize.

Optimal collective intelligence and strictly optimal collective intelligence are uniform set properties:

```
optimal_collective_intelligent : uniform_set_property
strictly_optimal_collective_intelligent : uniform_set_property
```

This means that no matter how we remove agents from the set, the new sub-set has the same property.

Every optimal collective intelligence has a sub-set that is strictly optimal.

“Optimal” does not mean that the set of agents beat every other set of agents on their specified environments. On the other hand, it means that the amount of information needed to keep track of agents is minimized. It is just as easy to pick a champion from the optimal set than any other set. By predicting which champion to select from information about the environment, the collective agent behavior will make its best effort across environments.