One-Dimensional Real Wave

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In this paper I describe a physics model for a one-dimensional real-valued wave.

Fundamentally, a real-valued wave behaves just like a particle at any position:

$$s' = s + v\ dt + ½\ a\ dt^2$$

The difference is that the topology of the model is invariant over time.
Each particle has two neighbors, which remains neighbors at all times.
A particle only interacts with its neighbors, which is done by acceleration.

Mathematicians developed notation to simplify modeling physical systems in arbitrary dimensions.

For example, in one dimension each particle has two neighbors. In two dimensions they have four. In three dimensions they have six and so on. The number of neighbors in `n` dimensions is `2n`. To abstract over this relationship they invented a symbol `∇`. This is called "nabla" or "del". It represents the partial derivatives in a vector field. In this case it means roughly that by choosing a direction of a string, the neighbor value in the positive direction minus the neighbor value in the negative direction, divided by 2 to get the change per unit.

The operator of interest here is `∇ · ∇`, or `$\nabla^2$`. This corresponds to directional invariant measure which intuitively describes the displacement from equilibrium. An equilibrium is when the particle has the average value of its neighbors. One can think about it as a spring in a state where there are no structural forces making it constract or expand. A spring can still have momentum, but the general idea is that the spring force can be generalized to waves.

In equations, the `$\nabla^2$` operator is used to reason abstractly about a physical system. When programming, one must turn this into something computable:

$$\nabla^2\psi = (2n / \delta^2)(\psi_{avg} - \psi)$$

Here, `$\psi_{avg}$` is the average value of neighbors, `n` is number of dimensions and `δ` is usually `1`.
The acceleration of a wave propagating at a speed `c` is the following:

$$a = c^2\nabla^2$$

When `$c^2 > 2$`, the wave will diverge into two "parts" because acceleration cancels out velocity. It will look like two waves because every point alternates across the center. One must decrease the time step and iterate more times to simulate higher velocities in real time.

The rest of the equation `s + v dt` can be thought of as a basic skeleton of the physical system. One way to make optimize the simulation is to precompute constants for fixed time steps:

$$s' = k_0 s - s_{prev} + k_1(s_{+1} + s_{-1}) \qquad k_0 = 2 - c^2dt^2 \qquad k_1 = c^2dt^2 / 2$$