

Answered Modal Logic in Cubical Binary Codes

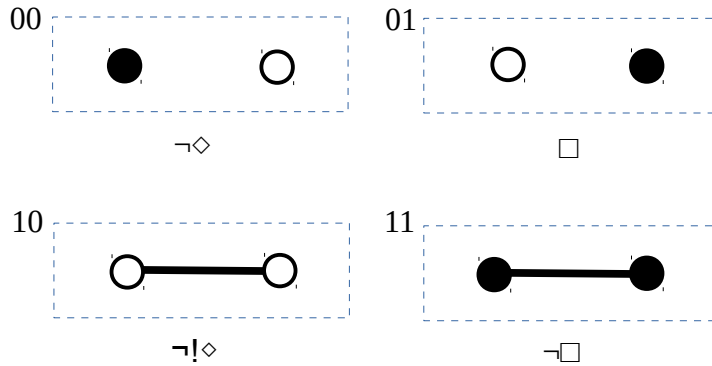
by Sven Nilsen, 2020

In this paper I describe the simplest case of encoding Answered Modal Logic in Cubical Binary Codes.

Answered Modal Logic has a modal set consisting of 3 elements:

$$\neg\Box = \{!\Diamond, \neg!\Diamond, \Box\}$$

The smallest Cubical Binary Code that fits this set is the following:



The reason for this assignment is because $\neg\Box$ and $!\Diamond$ are natural opposites:

00	\Box	For all cases, the predicate returns `true`
01	$!\Diamond$	There exists no case for which the predicate returns `true`

To clarify, the cases here refers to context, not inputs of the predicate.

The logic is for reasoning about the predicate across possible worlds, hence the modality.

The natural way of assigning $\neg!\Diamond$ is 10:

10	$\neg!\Diamond$	There exists two cases, where the predicate returns `true` and `false`
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If one models $\Diamond = \{\neg!\Diamond, \Box\}$ as an interval, then one can say it includes $\neg\Box$ at one end:

$$\Diamond = (\neg\Diamond, \Box] \quad \text{○} \leftarrow \text{●} \quad \text{One can think about } \Diamond \text{ as an arrow}$$

The semantics of the code `11` ($\neg\Box$) is naturally referring to the whole modal set:

$$\text{●} \text{---} \text{●} \quad \neg\Box = \{!\Diamond, \neg!\Diamond, \Box\}$$

This encoding satisfies the requirement that the building block of the logic can not fully model sets.

There is no way to model the empty modal set $\{\}$, hence a full model of a set is impossible to express.