Contraction Theorem

by Sven Nilsen, 2020

In this paper I present a contraction theorem found in Path Semantical Logic.

The Contraction Theorem is a proof in Path Semantical Logic[1]:

∴   (a, b) (A, B):
∴ $a(A \land B), b(A \land B) \implies contr(A, B)$

∵ $contr(x, y) = (x \land y) \veebar (\neg x \land \neg y)$

Where the tuple `(a, b)` has level 1 and the tuple `(A, B)` has level 0.
The notation `a(T)` means `a=>T` where `T` is at a lower level.

The `contr` function was derived in the paper "Contractible Types"[2].

The Contraction Theorem says that when two propositions at level 1 are associated with two propositions in level 0, then the two propositions in level 0 are in contractible type family.

This means that the two propositions in level 0 behaves together like a single proposition.
When the first proposition is `true`, the second proposition is `true`.
When the first proposition is `false`, the second proposition is `false`.

References:

[1]   "Path Semantical Logic"
        AdvancedResearch, reading sequence on Path Semantics
        https://github.com/advancedresearch/path_semantics/blob/master/sequences.md#path-semantical-logic

[2]   "Contractible Types"
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        https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/contractible-types.pdf