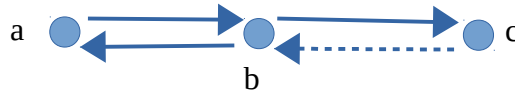


Naive Isomorphism Propagation

by Sven Nilsen, 2021

In this paper I explain how the naive core axiom in Avatar Logic propagates isomorphisms through outgoing arrows of any endpoint in the isomorphism.

Naive Isomorphism Propagation means that an isomorphism^[1] from `a` to `b`, propagates into an isomorphism `b` to `c` if there is an outgoing arrow from `b` to `c`.



$$(C=c, D=b) :- (A=b, B=a), (B=a, A=b), (A=b, C=c), (B=a, D=b).$$

It might be hard to spot how Naive Isomorphism Propagation works, so here is the rule filled in.

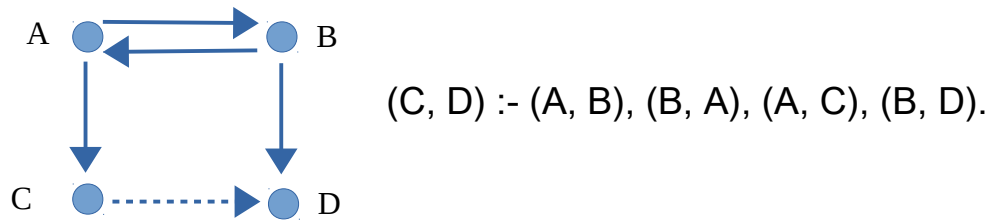
An isomorphism, in the sense of Category Theory^[2], is a set of two objects A and B with two morphisms, one from A to B and another from B to A:



Usually for isomorphisms of sets, the morphisms are thought of as maps between sets, converting every element of A into an element of B and vice versa such that every element can be mapped back to itself.

However, it is important to recognize that the interpretation of a such isomorphisms is left out in pure Category Theory: An isomorphism is merely the fact that there are two arrows, one from A to B and another from B to A. One can also think of this as a bidirectional edge between nodes in a graph.

An Avatar Logic^[3], the naive core axiom can be modelled as following:



$$(C, D) :- (A, B), (B, A), (A, C), (B, D).$$

The word “naive” means that the order assumption is left out of the core axiom of path semantics^[4].

The rule `(C, D) :- (A, B), (B, A), (A, C), (B, D).` is a Horn clause^[5], but for pairs instead of predicates. Avatar Logic uses pairs (binary relations) with some additional axioms. Here, only pairs are relevant.

The naive core axiom is the only rule. There are no axioms of Category Theory. Interpreting arrows as maps between sets does not make sense. By “isomorphism in the sense of Category Theory”, it is meant how objects and arrows are used in Category Theory, but without using the same axioms.

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