Randomary Numbers

by Sven Nilsen, 2020

In this paper I present the algebraic rules of real random numbers in the unit interval.

A real random number in the unit interval is generated by a function:

For brevity, values generated by this function is annotated `r`, often with one or more index `i` or `j`.

One can then construct an algebra:

$$a + b \textbf{r}_i + c \textbf{r}_j + d \textbf{r}^{ij} + \dots \\ \hspace{1cm} : rnd(real)$$

a:real

b: real

 $rnd: type \rightarrow type$

With the following rules:

$$\mathbf{r}_{ij} = \left(\mathbf{r}_i + \mathbf{r}_j\right) / 2 \qquad \qquad \mathbf{r}_{i - j} = \mathbf{r}_i - \mathbf{r}_j \qquad \qquad \mathbf{r}^{ij} = \mathbf{r}^i \cdot \mathbf{r}^j$$

$$\mathbf{r}_{i-i} = \mathbf{r}_i - \mathbf{r}_i$$

$$\mathbf{r}^{ij} = \mathbf{r}^i \cdot \mathbf{r}^j$$

$$a\mathbf{r}_i + b\mathbf{r}_i = (a + b)\mathbf{r}_{ij}$$
 $\mathbf{r}_{i-j} = 2\mathbf{r}_{ij} - 1$

$$\mathbf{r}_{i-j} = 2\mathbf{r}_{ij} - 1$$

The indices commute everywhere:

$$\label{eq:rij} \mathbf{r}_{ij} = \mathbf{r}_{ji} \hspace{1cm} \mathbf{r}_{i\text{-}j} = \mathbf{r}_{j\text{-}i} \hspace{1cm} \mathbf{r}^{ij} = \mathbf{r}^{ji}$$

A randomary number is invariant when replacing every occurrence of a source with its inverse.

$$\forall x : rnd(real), i : nat \{ x[r_i := 1 - r_i] \le x \}$$