Dual Adjoint Operators

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In this paper we generalize dual-imaginary numbers to dual adjoint operators.

An dual-imaginary number[1] is defined as following:

$$\boldsymbol{\varepsilon}^2 = 0$$

By adding a minus sign to the each side:

$$-\boldsymbol{\varepsilon}^2 = 0$$

Using Avatar Covers[2], it is natural to use the avatar cover `xor` for this product:

$$\boldsymbol{\varepsilon} \cdot (-\boldsymbol{\varepsilon}) = (-\boldsymbol{\varepsilon}) \cdot \boldsymbol{\varepsilon} = 0$$

$$\text{mul}[\text{neg}]_a \Leftarrow\!\!\Rightarrow \text{xor}$$

We use the same process as in the paper “Imaginary Adjoint Operators”[3].

Hence, for any symmetric avatar cover `xor`:

$$f[g]_a \Leftarrow\!\!\Rightarrow \text{xor}$$

A Dual Adjoint Operator `g` is defined as the following relation with `f`:

$$\exists\ z\ \{\ \exists\ \boldsymbol{\varepsilon}\ \{\ f(\boldsymbol{\varepsilon}, g(\boldsymbol{\varepsilon})) = f(g(\boldsymbol{\varepsilon}), \boldsymbol{\varepsilon}) = z\ \}\ \wedge\ \forall\ y\ \{\ f(y, z) = f(z, y) = z\ \}\ \}$$

Here, `z` is some zero element of `f`.
The element `**ε**` is a dual-imaginary element.

References:

[1] "Dual number"
Wikipedia
https://en.wikipedia.org/wiki/Dual_number

[2] "Avatar Covers"
Sven Nilsen, 2020
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/avatar-covers.pdf

[3] "Imaginary Adjoint Operators"
Adam Nemecek, Sven Nilsen, 2020
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/imaginary-adjoint-operators.pdf