

Quantum Andor Functions

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In this paper I present a quantum function that is “almost” constructible using pure functions plus extensions of non-determinism using random sources.

A quantum andor function is a semi quantum non-deterministic function f :

$$f : () \rightarrow \mathbb{B}^2 \quad \exists_{pc} f : \mathbb{B}^2 \rightarrow \mathbb{C}$$
$$f : [\text{semi_quantum}] \text{ true}$$
$$\text{and} \cdot f : [\text{semi_quantum}] \text{ true}$$
$$\text{or} \cdot f : [\text{semi_quantum}] \text{ true} \quad \text{“and”} + \text{“or”} = \text{“andor”}$$

For more information about semi quantum functions, see paper “Semi Quantum Non-Determinism”.

The complex probability distribution can be generated with two arbitrary basis vectors a and b :

$$\begin{array}{cccc} 00 & 01 & 10 & 11 \\ a & b & -b & a \end{array}$$

$$|a| = |b|$$

$$a : \mathbb{C}$$
$$b : \mathbb{C}$$

The only partial observations that can distinguish a quantum andor function from some function constructed with pure functions plus extensions of non-determinism using random sources:

$$\begin{array}{ll} (= 01) \cdot f & \text{Measure for `01`} \\ (= 10) \cdot f & \text{Measure for `10`} \end{array}$$

These two partial observations give the same real probability distribution:

$$\begin{array}{cc} 0 & 1 \\ \frac{3}{4} & \frac{1}{4} \end{array}$$

With other words, `01` and `10` is given 25% chance of being observed, respectively.

All other functions of type $\mathbb{B}^2 \rightarrow \mathbb{B}$ corresponds to the intuition that f is constructible using:

$$f() = \text{if random}() < 0.5 \{ 00 \} \text{ else } \{ 11 \}$$

This intuition gives `01` and `10` a 0% chance of being observed, which is a contradiction. Therefore, quantum andor functions are not constructible using pure functions plus extensions of non-determinism using random sources. However, they are “almost” constructible.