# Algebraic Properties of Addition

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*In this paper I formalize the algebraic properties of addition using natural numbers.*

(nat, add : nat × nat → nat) : associative ∧ commutative ∧
[\(nat, add) = distributive(nat, add, mul)] true ∧
[identity_element] 0 ∧ inverse_element

associative := \(t : type, op : t × t → t) =
∀ x, y, z : t { op(op(x, y), z) == op(x, op(y, z)) }

commutative := \(t : type, op : t × t → t) =
∀ x, y : t { op(x, y) == op(y, x) }

distributive := \(t : type, f : t × t → t, g : t × t → t) =
∀ x, y, z : t { g(x, f(y, z)) == f(g(x, y), g(x, `)) }

identity_element :=
\(t : type, op : t × t → t ∧ ∃ e : t { ∀ x : t { (op(x, e) == x) ∧ (op(e, x) == x) }) = e

inverse_element := \(t : type, op : t × t → t ∧ [identity_element(t)] e) =
∀ x { ∃ y { op(x, y) == e } }