Exponential Duplication

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In this paper I give a formula which provides some intuition about exponential growth. It is a formula that tells how long time it takes for an exponential system to reach a state where the growth of the system matches the initial seed per time unit. With other words, if the system does not collapse within this time span, the system becomes self-seeding and even with a 100% destruction of the original seed after this point in time, the system as a whole is robust against destruction.

A lot of systems can be described as exponential growth when observed locally in time.
Yet, exponential growth is hard for human brains to reason about, because of our bias toward linearity.

It is easy to understand that the systems are growing, but when non-linearity kicks in one can be surprised over consequences. Most people who learn about the exponential function, for example to invest money rationally, reasons about returns over time in various ways. A common measure is percentage per year. However, since constant growth does not add up linearly, a lot of people do not realize that e.g. 10% in one year becomes 21% for two years, instead of 20%. Locally, the exponential function is almost linear, but over time, there are non-linear effects.

Exponential duplication is a property that all exponentially growing systems have. It might seem counter-intuitive at first, because it is a measure of growth which meaning does not take into account all the accumulated growth. However, it is useful to train intuition about exponential growth. Like percentages, exponential duplication has no physical unit on its own, but inherits from units of time.

The number of time units it takes to grow a system such that its growth per time unit matches the seed:

exp_dup(x : real) = 1 – ln(x) / ln(1+x)

Here, `x` is the fraction of the growth per time unit.

For example:

- `0.1 = 10%` duplicates exponentially approximately every `25.16` period.
- `0.2 = 20%` duplicates exponentially approximately every `9.83` period.
- `0.3 = 30%` duplicates exponentially approximately every `5.59` period.

If you invest an amount of 100$ now, at a 10% constant growth per year, it will take over 25 years before you receive more than 100$ every year in returns. However, at 20% constant growth per year, it will take only 10 years before you receive 100$ every year in returns.

Here are some more examples:

x	exp_dup(x)
0.00000000000001	3226197758559035.5
0.0000000000001	299575506151924.8
0.000000000001	27628564920432.688
0.00000000001	2532843392738.713
0.0000000001	230258490260.24457
0.000000001	20723264133.657482
0.00000001	1842068095.8006923
0.0000001	161180965.47452262
0.000001	13815518.466854958
0.00001	1151299.302942619
0.0001	92109.00881320896
0.001	6912.208581263756
0.01	463.8157851175218
0.1	25.158857928096783
0.2	9.827469119589406
0.3	5.58893594662972
0.4	3.7232283444107126
0.5	2.709511291351455
0.6	2.086854636815819
0.7	1.672174810189183
0.8	1.3796335722440531
0.9	1.1641503119754075
0.99	1.014605188124168
0.999	1.0014444590872258
0.9999	1.0001442871264212
0.99999	1.000014427126614
0.999999	1.000001442696803
0.9999999	1.0000001442695217
0.99999999	1.0000000144269507
0.999999999	1.000000001442695
0.9999999999	1.0000000001442695
1	1

Exponential duplication can be derived by solving the following equation for `y`:

$$(1+x)^{(y-1)} * x = 1$$

$$(1+x)^{(y-1)} = 1/x$$
$$(y-1) * \ln(1+x) = \ln(1/x)$$
$$(y-1) * \ln(1+x) = \ln(1) - \ln(x)$$
$$(y-1) * \ln(1+x) = 0 - \ln(x)$$
$$y - 1 = -\ln(x) / \ln(1+x)$$

$$y = 1 - \ln(x) / \ln(1+x)$$

In the limit, when `x` goes to infinity, the exponential duplication goes to `0`:

$$\lim_{x \to \infty} \{ 1 - \ln(x) / \ln(1+x) \} = 0$$

The intuition of this is that when you start investing at infinite growth, the returns activate immediately.

Systems that do not collapse before the time period required to reach exponential duplication are extremely robust against destruction. One can destruct the original seed without preventing the system as a whole to grow further. After exponential duplication happens, one can also take out the growth and use it as a seed elsewhere.

Exponential duplication is a kind of discrete sub-growth within the total accumulated growth.
The growth from previous intervals of time is ignored, because it is not taken out of the system.
Previous growth is used to reach a state where the system can produce a copy of itself per unit of time.
It is therefore useful to choose a unit of time which reflects the desired production.