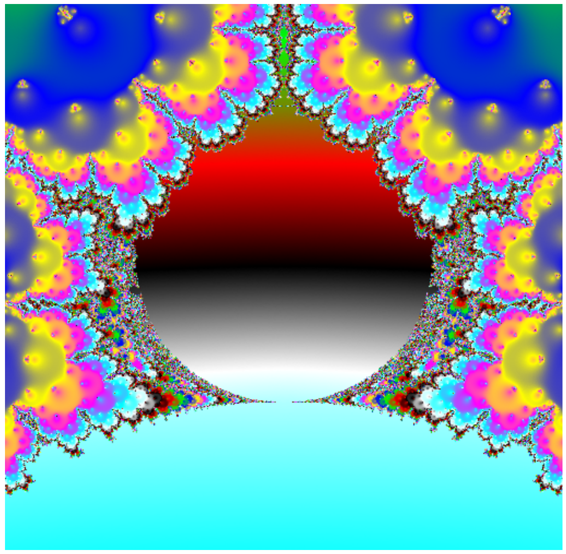
Golden Measure

by Daniel Fischer, Sven Nilsen, 2021

In this paper we derive a measure for approximity of Golden ratio solutions and introduce an associated fractal based on the same technique as for the Mandelbrot set.



The solutions to the Golden ratio equation:

$$x^2 = x + 1$$

are the Golden ratio solutions:

$$½ \cdot (1 \pm \mathrm{sqrt}(5))$$

These solutions are related to each other using two involutions:

$$inv_0(x) = 1 – x$$
$$inv_1(x) = -1 / x$$

By squaring the difference, one obtains a measurement:

$$golden\_measure(x) = (1 – x + 1/x)^2 = x^2 + 1/x^2 – 2 \cdot x + 2/x – 1$$

The solutions to `x : [golden_measure] 0` are the solutions of the Golden ratio equation.

The fixpoints are given by:

$$x = x^2 + 1/x^2 – 2 \cdot x + 2/x – 1$$

This has has 4 solutions in the complex plane. Most noticeable is the fixpoint `1`.

There are 4 solutions `x : [golden_measure] 1`:

-1 $\quad$ 1 $\quad$ $1 + \mathrm{sqrt}(2)$ $\quad$ $1 – \mathrm{sqrt}(2)$

There is an associated fractal using the same technique as for the Mandelbrot set:

$$z_0 = 1$$
$$z_{n+1} = golden\_measure(z_n) + c$$

This fractal is centered around the fixpoint `1`.