# Contracting Havox Diagrams

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A Havox diagram is an undirected graph where nodes are judged to be identical (solid lines) or different (dotted lines), and where edges can be judged to be identical or different. In this paper I show a proof technique by contracting Havox diagrams.

Assume a closed circuit that has lines connected to black boxes. All the black boxes has a secret switch that chooses between `not` or `id` (bool → bool). By pushing a button, all the boxes switches with 50% probability each, such that you never know which function they choose. You push the button.
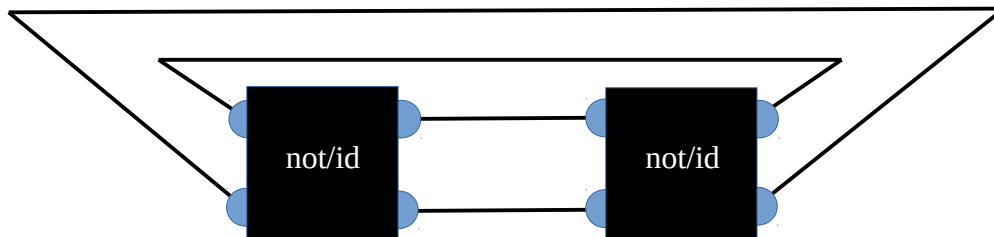
In the circuit above, the following property:

$$x = f_0(f_1(x)) \qquad <=> \qquad x : [f_1] \, [f_0] \, x$$
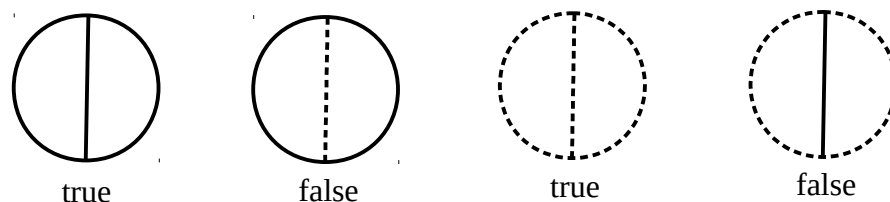
Holds only when the internally connected circuit can be extracted into two separate circles:
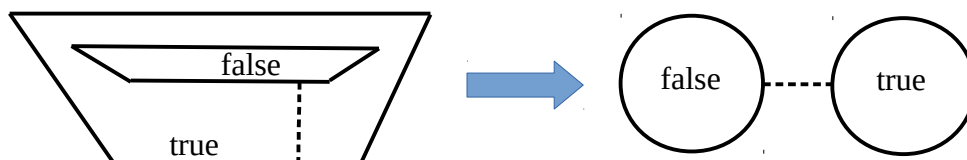
These two circles represent some measurement on a pair of inputs or outputs. However, instead of measuring, one can just attach a line between the pair to say "these lines are supposed to be separate". When one box chooses `not`, instead of two circles, one gets a single circle with a dotted line inside:

Rules (dotted edges have nodes where they intersect another line, for solid edges it does not matter):

true   false   true   false

Have fun. :)