Directional Set Algebra

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In Set Theory, one is reasoning about sets as some kind of universal structure. The operators on sets are total. However, when formalizing more rigid structures it can be convenient to use partial operators. Directional Set Algebra is designed for formalization with sets using partial operators.

A Directional Set Algebra has the following operators, elements and laws:

+	Addition (associative, commutative)
-	Subtraction (associative)
=>	Directional equality
=	Bidirectional equality

0	Bottom (optional, not included in every algebra)
1	Top (sometimes taking another symbol, e.g. \`?\`)

A + 0 = A   A + A => A   A + 1 => 1   ¬A => 1 – A

The left side of directional equality \`=>\` is uniquely associated with the right side.
For example, \`A => B\` and \`A => C\` is not allowed.

Bidirectional equality is used when both sides contain sets that are exclusive.
One can replace bidirectional equality with directional equalities, by expanding using algebra.

For example:

A + B = C

This equals the following laws:

A + B => C
C – A => B
C – B => A

On the other hand, the following does not imply anything else:

A + B => C

A property of Directional Set Algebra is that one does not need to worry about what the symbols mean. In Set Theory, a set \`A\` means that it is a set, but in Directional Set Algebra, \`A\` just means \`A\`, unless specified otherwise. The fact that \`A\` behaves as a set is a consequence of the rules, not because \`A\` contains any objects.