

# Constrained Uniform Properties of Sets

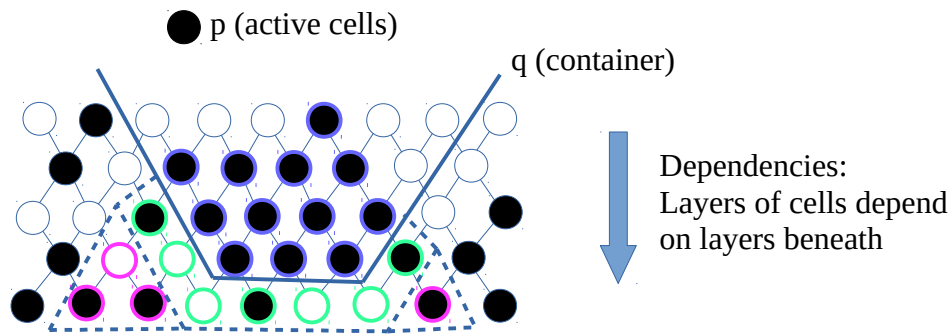
by Sven Nilsen, 2019

*In this paper I represent a general method to prove soundness of constrained uniform solvers and a general method to solve constrained uniform problems with unconstrained uniform solvers.*

Constrained uniform properties of sets can be translated into uniformed properties<sup>[1]</sup> (unconstrained):

$$p'(x) = p(x) \wedge q(x) \vee S(\setminus(y) = p(y) \wedge q(y))(x) \wedge \neg S(S(\setminus(z) = p(z) \wedge q(z)))(x) \wedge \neg q(x) \vee S(\setminus(v) = S(\setminus(w) = p(w) \wedge q(w))(v) \wedge \neg q(v))(x)$$

Using Higher Order Operator Overloading<sup>[2]</sup> and a visualization, it becomes more understandable:



$$p' = p \wedge q \vee S(p \wedge q) \wedge \neg S(S(p \wedge q)) \wedge \neg q \vee S(S(p \wedge q) \wedge \neg q)$$

$p' : \text{set} \rightarrow \text{bool}$	Unconstrained uniform property
$p : \text{set} \rightarrow \text{bool}$	Constrained uniform property
$q : \text{set} \rightarrow \text{bool}$	Constraints

$p \wedge q$	“Evaluation condition”
$S(p \wedge q) \wedge \neg S(S(p \wedge q)) \wedge \neg q$	“Termination condition”
$S(S(p \wedge q) \wedge \neg q)$	“Ignorance condition”

The function  $\setminus S(x)$  returns “all subsets of each set in a set of sets” and is defined as:

$$S(x : \text{set} \rightarrow \text{bool}) = \setminus(y : \text{set}) = y \in \cup z : x \{ P(z) \wedge \neg z \}$$

$$S : (\text{set} \rightarrow \text{bool}) \rightarrow (\text{set} \rightarrow \text{bool})$$

Where  $\setminus P$  is the powerset operator<sup>[3]</sup>.

A uniform solver is a problem solver that assumes reductionism<sup>[4]</sup>. All constrained uniform solvers that satisfy the evaluation condition and the termination condition are sound. Constrained uniform problems can be solved with unconstrained uniform solvers by composing the 3 parts together as specified.

## References:

- [1] “Uniform Properties of Sets”  
Sven Nilsen, 2018  
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- [2] “Higher Order Operator Overloading”  
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[https://github.com/advancedresearch/path\\_semantics/blob/master/papers-wip/higher-order-operator-overloading.pdf](https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/higher-order-operator-overloading.pdf)
- [3] “Power set”  
Wikipedia  
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- [4] “Reductionism”  
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