Invertible Adjoint Normal Paths

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*In this paper I prove that when the normal path of an adjoint path is invertible,
the adjoint operators are logically equivalent to the identity function.*

An adjoint path is defined as following:

$$f[g_0 \times id \to id] <=> f[id \times g_1 \to id]$$

When this normal path is invertible, one can prove the following:

$$g_0 <=> g_1 <=> id$$

As a consequence, the adjoint normal path of \`f\` is \`f\`:

$$f[id] <=> f$$

Proof:

$$f[g_0 \times id \to id] <=> h$$
$$h^{-1} \cdot f[g_0 \times id \to id] <=> h^{-1} \cdot h$$
$$f[g_0 \times id \to h^{-1} \cdot id] <=> id$$
$$f[g_0 \times id \to h^{-1}] <=> id$$
$$g_0 \times id <=> id_{T \times T}$$ One can see this from \`$f(x, y) = h(g_0(x), y)$\`
$$g_0 <=> id$$

$$f[g_0 \times id \to id] <=> f[id \times g_1 \to id]$$
$$f[id \times id \to id] <=> f[id \times g_1 \to id]$$
$$f[id] <=> f[id \times g_1 \to id]$$
$$f <=> f[id \times g_1 \to id]$$
$$g_1 <=> id$$

Q.E.D.