AND with Cubical Binary Codes

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In this paper I show how to encode the Propositional Logic interpretation of generalized AND for Answered Modal Logic, using the simplest Cubical Binary Codes that fits the modal set.

The generalized AND gate using the Propositional Logic interpretation of Answered Modal Logic can be encoded with Cubical Binary Codes as following:

and(00, 00) = 00	and(!◇, !◇) = !◇
and(00, 01) = 00	and(!◇, □) = !◇
and(00, 10) = 00	and(!◇, ¬!◇) = !◇
and(00, 11) = 00	and(!◇, ¬□) = !◇
and(01, 00) = 00	and(□, !◇) = !◇
and(01, 01) = 01	and(□, □) = □
and(01, 10) = 10	and(□, ¬!◇) = ¬!◇
and(01, 11) = 11	and(□, ¬□) = ¬□
and(10, 00) = 00	and(¬!◇, !◇) = !◇
and(10, 01) = 10	and(¬!◇, □) = ¬!◇
and(10, 10) = 10	and(¬!◇, ¬!◇) = ¬!◇
and(10, 11) = 10	and(¬!◇, ¬□) = ¬!◇
and(11, 00) = 00	and(¬□, !◇) = !◇
and(11, 01) = 11	and(¬□, □) = ¬□
and(11, 10) = 10	and(¬□, ¬!◇) = ¬!◇
and(11, 11) = 11	and(¬□, ¬□) = ¬□

For the right-most bit, one can use logical AND:

and(?0, ?0) = ?0
and(?0, ?1) = ?0
and(?1, ?0) = ?0
and(?1, ?1) = ?1

For the left-most bit, one can use \`and($a_l$, $b_l$) ∨ and($a_l$, $b_r$) ∨ and($a_r$, $b_l$)\`:

and(0?, 0?) = 0?
and(00, 10) = 0?
and(00, 11) = 0?
and(01, 10) = 1?
and(01, 11) = 1?
and(10, 00) = 0?
and(10, 01) = 1?
and(11, 00) = 0?
and(11, 01) = 1?
and(1?, 1?) = 1?