Theoretical vs Existential Possibility

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In this paper I introduce stronger and weaker notions of intuitionistic modal possibility that corresponds to theoretical and existential propositional possibility, correspondingly.

In most theories of Modal Logic[1], it is common to use the following definition of possibility:

$$\Diamond p \quad <=> \quad \neg\Box\neg p$$

With introduction of Higher Order Operator Overloading Exponential Propositions[2] (HOOO EP) in Path Semantics[3], a corresponding Intuitionistic Modal Logic[4] can be derived and there is a natural way to define a stronger notion of possibility (as implemented in the Prop[5] library):

$$\Diamond p \quad <=> \quad p^{true} \vee \neg(p^{true} \vee false^{p})$$
$$\Box p \quad <=> \quad p^{true}$$

Here, the statement `¬(p^true ∨ false^p)` is often written `theory(p)`, since this corresponds naturally to an assumption that can be used to prove something that is not tautological nor absurd.

In Intuitionistic Propositional Logic (IPL), the weaker notion of possibility is the following:

$$\neg\neg\Diamond p \quad <=> \quad \neg\Box\neg p$$

In HOOO EP, there is another way to write this as `∃ true { p }` which equals `¬((¬p)^true)`. The `∃ a { b }` notation might be thought of as a binary analogue of the unary modality `◇`. Therefore, one can think about the weaker notion `¬¬◇p` as existential possibility. Correspondingly, the stronger notion `◇p` might be thought of as theoretical possibility.

Existential possibility is the most common form, due to the definition derived using necessity. However, this notion of possibility is not strong enough in constructive logic to arrive at theoretical possibility. While theoretical possibility does not contain enough information to tell what kind of assumption might be made in a theory, it does require properties which points to something deeper about this stronger notion of possibility.

Theoretical possibility is more affirmative than existential possibility. In Path Semantics, propositional theories are important because they can be used to lift equality into quality[6].

For example, in fundamental physics, IPL is a model of hypergraph rewriting, popularized as Wolfram models. Moments of time in these models are emergent and can be thought of as ways to slice up the evolution of space-time. Different relativistic frames of reference correspond to different ways to make such slices without intersections. However, IPL in this sense has no mechanism for explicit transition of consciousness. There is a hypothesis that consciousness in Wolfram models are related to symbolic distinction of rule matching under hypergraph rewriting, which explains why we humans observe a complex universe. From within a coherent mind in the emergent space time, moments seem to pass like local propositional spaces of thoughts, which can be modeled using the core axiom of Path Semantics. So, when two symbols are associated, one can think about it as quality lifted from equality (state), hence theoretical possibility of consciousness.

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