

Adjoint Paths in Boolean Algebra

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In this paper we derive adjoint paths in Boolean Algebra.

Here are the adjoint paths of type $\text{bool}^2 \rightarrow \text{bool}$ which have `not` as self-adjoint operator:

```
\false
eq
xor
\true
```

The normal paths of these adjoint paths are:

```
\false[not x id → id] <=> \false[id x not → id] <=> \false
eq[not x id → id] <=> eq[id x not → id] <=> xor
xor[not x id → id] <=> eq[id x not → id] <=> eq
\true[not x id → id] <=> \true[id x not → id] <=> \true
```

Proof:

00	10	01
01	11	00
10	00	11
11	01	10

This gives two constraints:

```
10 == 01
11 == 00
```

Enumerating all functions of type $\text{bool}^2 \rightarrow \text{bool}$ and remove all that violate the constraints:

```
0000 \false
0110 xor
1001 eq
1111 \true
```

Q.E.D.