Natural Implication

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In logic, material implication might produce non-intuitive results: When the assumption is false, anything follows. In this paper I show that when we say "A implies B" in natural language, we mean that "A", by how it is used in argumentation, implies that it is sound or true, but not necessarily both.

The problem starts when we say that `A` means something as an argument:

meaning_of("A") = (A => B) ∧ (A => ¬B)

For example:

It looks like the Sun is orbiting the Earth (`A => B`).
It looks like the Earth is rotating around its own axis (`A => ¬B`).

We think of "It looks like" as an argument "A", which can be used to explain why some people believe either the Sun is orbiting the Earth or why some people are believing the Earth is rotating around its own axis.

Using the notation from Naive Zen Logic:

(B ? some people) => ((A => B) ? some people)
(¬B ? some people) => ((A => ¬B) ? some people)

Whenever something is believed by some people, it must be because it looked that way at first:

∀ X { (X ? some people) => ((A => X) ? some people) }

So, our "A" is an explanation for whatever is believed by some people.

The problem is that in logic interpreted as each proposition representing truth:

((A => B) ∧ (A => ¬B)) = (A = false)

What we mean by "It looks like" has a different interpretation than truth. If we try to substitute "A" with "It looks like", we must also ask how we choose to think about `"It looks like" = false`.

The way we resolve this problem is by substituting "true" and "false" with "sound" and "unsound":

((A => B) ∧ (A => ¬B)) = (A = unsound)

Since we substitute all truth values, the Logic is given a different interpretation than truth.

Soundness is isomorphic to Logic and therefore occupy the whole space of possible logical constructions. This means one can not mix truth and soundness within the same Logic.

However, one can create a combinatorial Logic that combines truth and soundness. While not being isomorphic to Logic, it is possible to encode it into Logic, which makes theorem proving possible.

A sound argument can be true, but it can also can be false.
A true argument can be sound, but it can also be unsound.

This constraint is what underlies the semantics of natural implication.

	A is true	**A is false**
A is sound	coherent	anti-coherent
A is unsound	anti-coherent	coherent

When the two logics of truth and soundness are the same, they are coherent:

$$\text{coherent} \Rightarrow \forall P \{ ((P = \text{true}) \Rightarrow (P = \text{sound})) \wedge ((P = \text{false}) \Rightarrow (P = \text{unsound})) \}$$

When the two logics of truth of soundness are opposites, they are anti-coherent:

$$\text{anti-coherent} \Rightarrow \forall P \{ ((P = \text{true}) \Rightarrow (P = \text{unsound})) \wedge ((P = \text{false}) \Rightarrow (P = \text{sound})) \}$$

Other states corresponds to a constraint of one or more propositions in one of the logics:

	A is true	**A is false**
A is sound	true-decoherent	false-decoherent
A is unsound	true-decoherent	false-decoherent

	A is true	**A is false**
A is sound	sound-decoherent	sound-decoherent
A is unsound	unsound-decoherent	unsound-decoherent

Natural implication imposes a such constraint as a relationship between two propositions of truth:

$$\begin{aligned} &\text{natural\_implication}(A, B) = \\ &\quad ((A = \text{false}) \Rightarrow (((A \Rightarrow B) = \text{false}) \wedge ((A \Rightarrow \neg B) = \text{false}))) \vee \\ &\quad ((A = \text{unsound}) \Rightarrow (((A \Rightarrow B) = \text{unsound}) \wedge ((A \Rightarrow \neg B) = \text{unsound}))) \end{aligned}$$

This is logically equivalent to:

$$\text{natural\_implication}(A, B) = (A = \text{true}) \vee (A = \text{sound})$$

Now, since we proved that `( (A => B) ∧ (A => ¬B) ) = ( A = unsound )`, then “A” must be true:

$$\frac{(A = \text{unsound}) \wedge ((A = \text{true}) \vee (A = \text{sound}))}{A \Rightarrow \text{true}}$$