

Binary Square Matrix Combinatorics

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In this paper I formalize binary square matrix combinatorics using Directional Set Algebra.

For all $n : \text{nat}$, there is an associated binary square matrix combinatorics:

0	Empty matrix set
D	Diagonal matrix set
U	Upper strictly triangular matrix set
L	Lower strictly triangular matrix set
1	All matrices

The following law holds with Directional Set Algebra:

$$D + U + L \Rightarrow 1$$

Sizes of sets, since they share the zero matrix, gets subtracted one when added together:

$$|x + y| = |x| + |y| - \text{if } |x| > 0 \wedge |y| > 0 \{ 1 \} \text{ else } \{ 0 \}$$

$$|0| = 0$$

$$|D| = 2^n$$

$$|1| = 2^{(n \cdot n)}$$

$$|U| = |L| = 2^{(n \cdot (n - 1) / 2)}$$

Sub-types of binary matrix sets can be constructed using elements 0 , 1 and $?$.

The following laws holds with Directional Set Algebra, where $?$ is top and there is no bottom:

$$0 + 1 = ?$$

For example, for $n = 3$:

$$\begin{array}{ccc} ?00 & 0?? & 000 \\ 0?0 & 00? & ?00 \\ 00? & 000 & ??0 \end{array} \Rightarrow \begin{array}{c} ??? \\ ??? \\ ??? \end{array}$$