Non-Constructibility of Quantum Non-Determinism

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In this paper I show that quantum non-deterministic functions are not constructible in general.

For every constructible function `f`, the output has some probability distribution `∃ₚf`.
Knowing the probability distribution is not the same as knowing how to construct the function.

For example, by permuting the order of the outputs one gets another function `f'` such that:

$$∃_pf <=> ∃_pf'$$

This means that for functions with more than one output, the probability distribution does not give a unique description of the function. Therefore, one can not determine `f` from `∃ₚf`. However, one can construct `f` by enumerating every possible function having the same probability distribution.

For pure functions plus extensions of non-deterministism using random sources:

$$∃_pf : T → ℝ$$

It takes some type `T` and returns a real number.

In a quantum non-deterministic function, the probability distribution has the type:

$$∃_{pc}f : T → ℂ$$

Complex numbers `ℂ` is a generalization of real numbers `ℝ`.
Therefore, quantum non-deterministic functions are generalizations of pure functions.

The problem is that one can not enumerate every possible function having the same `∃_{pc}f`.
Why? Because we do not know how to construct a *single* quantum non-deterministic function which has a different probability distribution than the normal path `f[id → re]`:

$$∃_{pc}f <=> ∃_{pc}f[id → re]$$ The quantum non-deterministic functions that are constructible

These functions represents groups of normal paths that are rotated in the complex plane.
The function `re` returns the real component of a complex number:

$$re(x : ℂ) = (x + x^*) / 2$$ Adding the conjugate zeroes the imaginary component

All functions that are constructible have aligned complex probabilities. Since there are quantum non-deterministic functions which complex probabilities are not aligned, only a subset of quantum non-deterministic functions can be constructed. Although, one can model the probability distributions of quantum non-deterministic functions, the functions themselves are not accessible.