Path Semantical Quality as Path Connected Spaces

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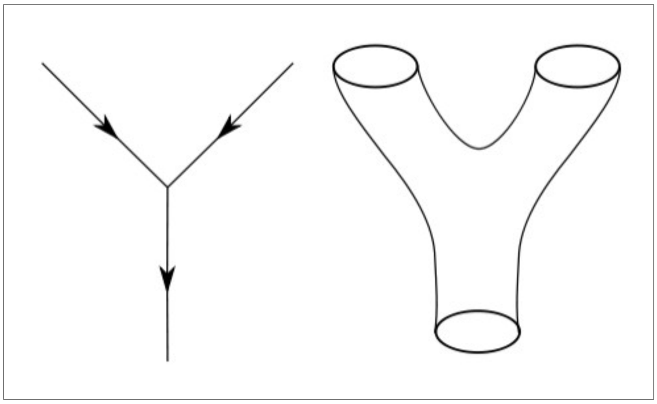
In this paper I discuss the interpretation of path semantical quality as path connected spaces.

There is a notion of ambiguity when considering path connected spaces[1], that depends on the choice of perspective. It means that it is not trivial to talk about what is actually meant by a path connected space, as the notion of a path connected space puts constraints on possible interpretations in a such way that it resembles circular reasoning. Basically, a path connected space is path connected, simply because we say it is and there is no further elaboration of what is meant by being path connected. As soon as we start to talk about why a space is path connected, we run into problems of communicating properly the properties of the space.

For example, in String Theory[2], two closed strings can merge into one:



In space-time, the two closed strings that merge into one closed string are path connected through the world sheet. Yet, at one moment in time, the two closed strings are separated. At another moment in time, the two closed strings are joined together. A dynamic interpretation of a such interaction produces an ambiguity of path connectedness since it is possible to glue together moments, without changing the internal structure of moments.

The ambiguity is not due to the dynamics of two closed strings merging together, as intuitive it might seem at first, that two strings are either path connected or not. On the contrary, there is no reason why two strings should not have this kind of dynamics. The actual ambiguity arises from the statement that the interaction is a higher dimensional path connected space than moments. Is there any more reason to believe this statement, than there is reason to disbelieve this statement? Do we consider that statement true only because we say it is true? Whether we believe or disbelieve this statement, does not affect the predictions given, through per-moment observation of the interaction.

In mathematics, it is common to overlook such statements because they are implicitly assumed from the context of some problem or theorem being discussed. Furthermore, it is also evident that once you start questioning and doubting such statements, the mathematics of a particular situation becomes more complex and full of ambiguity. People feel a sense of being overwhelmed and reach for methods to provide clarity and reach some sort of closure[3]. They prefer to not think about it, out of fear of getting stuck on their way, to reach some goal or some imagined reward in the future. In these situations, it is easier to pretend that this is solved than to admit the assumptions being made.

However, why do we tend believe that mathematics demands of us a such insight and clarity? I see no reason for not just being honest about it and say “I assume this space is path connected” and move on. The anxiety of people displayed when facing ambiguity of this sort comes from the false idea that we ought to be ultimately certain about what we are talking about in mathematics. Instead, we make assumptions implicitly all the time and this is not something to be ashamed of.

The attempt to make mathematics clearer than it is, often results in sloppy reasoning. This makes it hard for people who come later to keep track of what kind of assumptions that underlies reason in a particular field. This can result in foundational problems of mathematics.

For example, when mathematicians talk about connectedness of a set, there is no such formal notion actual in place, but merely a sloppy interpretation of what a set means. There is no reason that a set can not be represented as disconnected, e.g. using colors to distinguish different sets. The statement “the set of blue pixels on the screen” is just as well founded interpretation, as a set in a Venn diagram. However, when people are taught Venn diagrams, they are represented with examples where sets are usually connected. This leads to the false idea that sets have a formal notion of connectedness, when in fact there is no such thing in the actual Set Theory foundation[4].

The major foundational problem of Set Theory as a foundation for mathematics, is that path connectedness is completely orthogonal to the interpretations and representations of sets.

The basic primitive notion of a set comes from propositional logic[5], where the statement:

$$a \wedge b$$

Is taken to mean \`a\` and \`b\` are both true. In an interpretation of path connectedness between \`a\` and \`b\`, one can not express a such statement without also implying path connectedness:

$$a \wedge b \quad\quad => \quad a == b$$

However, from the perspective of path disconnectiveness (by modus tollens):

$$\neg(a == b) \quad\quad => \quad \neg(a \wedge b)$$

It means, if \`a\` then \`¬b\` and vice versa, creating the property that all members of a set are different.

The ambiguity of path connectedness in sets is also the characteristic property of sets.

Therefore, it is not only the case that path connectedness is orthogonal to the interpretations and representations of sets, but it is also necessary of sets to have this ambiguous property. By overlooking such statements that are usually implicitly assumed, e.g. by trying to explain them away in the sense of sets “ought to” be interpreted in this or that way, one also misses the very insight of sets and this undermines reasoning about mathematics as a whole. The paradox lies in taking sets to be primitive notions of mathematics yet at the same time pretend that they have additional properties, arbitrarily chosen, while in fact sets rely on allowing such arbitrary choices.

After taking these steps of seemingly radical honesty about ambiguity, where sets are projections into tautological propositional statements, that allow sets to be interpreted as foundational objects:

$a \wedge b$	=>	$a == b$	Path connectedness perspective of sets
$\neg(a == b)$	=>	$\neg(a \wedge b)$	Path disconnectedness perspective of sets

One can “lift” \`a ∧ b\` into \`~a ∧ ~b\` using the path semantical qubit operator \`~\`[6], to represent this choice of path connectedness perspective of sets, at the same time as path connectedness is made explicit in the form of \`a == b\`, one obtains the definition of path semantical quality \`~~\`[7]:

$$(a \sim\sim b) := (a == b) \wedge (\sim a \wedge \sim b)$$

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