

Quantum Schrödinger Functions

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In this paper I formalize Quantum Schrödinger Functions and introduce the Phi quantum function.

The time-dependent Schrödinger equation for a ψ wavefunction over a Hilbert space:

$$i\hbar\psi'(t) = H\psi(t) \quad \text{time-dependent Schrödinger equation}$$

$$\psi : \text{time} \rightarrow \mathbb{H}$$

The type of complex numbers is a Hilbert space with the inner product:

$$\therefore \langle a, b \rangle \Leftrightarrow ab^*$$

$$\therefore (\mathbb{C}, \text{mul}_{\mathbb{C}} \cdot (\text{id}, \text{conj})) : [\text{hilbert_space}] \text{ true}$$

$$\therefore \text{conj}(x) = \text{re}(x) - \text{im}(x)i$$

A quantum Schrödinger function f is a higher order quantum function depending on time:

$$f : \text{time} \rightarrow T \quad \text{where } T \text{ is usually } \mathbb{B}^n \text{ or } \mathbb{R}^n \text{ and } T^0 \Leftrightarrow ()$$
$$\exists_{\text{cp}} f(t) \Leftrightarrow \psi(t)$$

$$\psi : \text{time} \rightarrow T \rightarrow \mathbb{C} \quad \psi \text{ satisfies time-dependent Schrödinger for all } T$$

The Phi quantum function is a basic building block for quantum Schrödinger functions:

$$\text{phi} : \text{frequency} \rightarrow \text{time} \rightarrow ()$$
$$\exists_{\text{cp}} \text{phi}(f)(t) \Leftrightarrow \varphi(f)(t)$$

$$\varphi(f)(t) = \sin(2\pi ft) + i\cos(2\pi ft)$$
$$\varphi(f)'(t) = (2\pi f)(\cos(2\pi ft) - i\sin(2\pi ft))$$

Observed alone, phi always returns, but nevertheless it has a complex probability amplitude. The function $\varphi(f)(t)$ simply rotates around in a unit circle at frequency f .

One can derive the analogue of the Schrödinger equation for $\varphi(t)$:

$$\therefore \hbar\varphi(f)'(t) = (hf)(\cos(2\pi ft) - i\sin(2\pi ft))$$
$$\therefore i\hbar\varphi(f)'(t) = (hf)(i\sin(2\pi ft) - i^2\cos(2\pi ft))$$
$$\therefore i\hbar\varphi(f)'(t) = E\varphi(f)(t)$$

$$\therefore \hbar = h / 2\pi \quad \text{reduced Planck's constant}$$

$$\therefore E = hf \quad \text{Planck-Einstein relation}$$

The Hamiltonian H is a measure of the total energy, so Phi is a quantum Schrödinger function.