Binary Square Matrix Combinatorics

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In this paper I formalize binary square matrix combinatorics using Directional Set Algebra.

For all `n : nat`, there is an associated binary square matrix combinatorics:

0	Empty matrix set
D	Diagonal matrix set
U	Upper strictly triangular matrix set
L	Lower strictly triangular matrix set
1	All matrices

The following law holds with Directional Set Algebra:

$$D + U + L \Rightarrow 1$$

Sizes of sets, since they share the zero matrix, gets subtracted one when added together:

$$|x + y| = |x| + |y| - 1 \qquad \text{if } |x| > 0 \wedge |y| > 0$$

$$|0| = 0$$
$$|D| = 2^n$$
$$|1| = 2^{(n \cdot n)}$$
$$|U| = |L| = 2^{(n \cdot (n - 1) / 2)}$$

Sub-types of binary matrix sets can be constructed using elements `0`, `1` and `?`.
The following laws holds with Directional Set Algebra, where `?` is top and there is no bottom:

$$0 + 1 = ?$$

For example, for `n = 3`:

```
? 0 0       0 ? ?       0 0 0       ? ? ?
0 ? 0   +   0 0 ?   +   ? 0 0   =>  ? ? ?
0 0 ?       0 0 0       ? ? 0       ? ? ?
```