Path Semantical Qubit

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In this paper I introduce a qubit operator for use in Path Semantics.

In the paper “Path Semantical Quality”[1], I introduced a partial equivalence operator for Path Semantics[2]. After doing a lot of research in Prop[3] and Pocket-Prover[4], I have come to the conclusion that the special operator for symbolic distinction[5] `sd` can be replaced, by using a `qubit` operator, which is written using unary `~`:

sd(a, b) => ~a ∧ ~b

Such that the definition of path semantical quality is the following:

a ~~ b <=> (a == b) ∧ ~a ∧ ~b

It follows that `qubit` might be thought of as self-quality:

a ~~ a <=> ~a

Hence, `~` and `~~` are equally fundamental for Path Semantics.

The `qubit` operator has the following invariant:

¬~a == ~¬a

This corresponds to the normal paths[6]:

qubit[not] <=> qubit
not[qubit] <=> not

In classical logic[7], the `qubit` operator generates a new random proposition that depends on the argument for a single case. This means that some proofs become probabilistic. To preserve the normal paths, the implementation checks one bit position and flips all bits when the bit is zero, before using it as a seed to a pseudo-random generator which output bits is flipped back if the checked bit was zero.

In constructive logic[8], the `qubit` operator is simply a 1-avatar[9] that wraps the core proposition into a newtype. A `qubit` can only be assumed and never created directly. This is similar to how `false` behaves in constructive logic as an empty type.

The name “qubit” comes from the intuition that each argument proposition in classical logic might be thought of as having a singular frequency, which is projected into a quantum superposition[10] of all frequencies when using `qubit`.

References:

[1] "Path Semantical Quality"
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[4] "Pocket-Prover"
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https://github.com/advancedresearch/pocket_prover

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https://github.com/advancedresearch/path_semantics/blob/master/papers-wip2/symbolic-distinction.pdf

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