Randomary Nth Contractibility

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In this paper I show that randomary numbers are Nth contractible if they have the same sign for `N`.

Assume the following equation:

$$-1 + \mathbf{r_i} + \mathbf{r_j} \quad = \quad \mathbf{r_i} - \mathbf{r_j}$$

It is not allowed to add or subtract randomary numbers to both sides of the equation.

For example, this is invalid:

$$\begin{array}{lll} -1 + \mathbf{r_i} + \mathbf{r_j} & = & \mathbf{r_i} - \mathbf{r_j} \qquad (+\ \mathbf{r_j}) \\ -1 + \mathbf{r_i} + 2\mathbf{r_j} & = & \mathbf{r_i} \end{array}$$

However, the following is valid for any `n`:

$$\begin{array}{lll} -1 + \mathbf{r_i} + \mathbf{r_j} & = & \mathbf{r_i} - \mathbf{r_j} \qquad (+\ n\mathbf{r_k}) \\ -1 + \mathbf{r_i} + \mathbf{r_j} + n\mathbf{r_k} & = & \mathbf{r_i} - \mathbf{r_j} + n\mathbf{r_k} \end{array}$$

If it is valid for any `n`, then it is valid for any `n + 1`.
This means that `$\mathbf{r_k}$` can be added or subtracted to both sides later.

Why is not the same possible for `$\mathbf{r_j}$`?

This is because some randomary numbers are equal but non-contractible along a dimension.

Two randomary numbers are Nth contractible if they have the same sign for `N`.

For example, the following is valid:

$$\begin{array}{lll} -1 + \mathbf{r_i} + \mathbf{r_j} & = & \mathbf{r_i} - \mathbf{r_j} \qquad (-\ \mathbf{r_i}) \\ -1 + \mathbf{r_j} & = & \mathbf{-r_j} \end{array}$$

Now, one can use the commutation of indices to solve the equation correctly:

$$\begin{array}{lll} -1 + \mathbf{r_i} + \mathbf{r_j} & = & \mathbf{r_i} - \mathbf{r_j} \qquad (\mathbf{r_{i-j}} = \mathbf{r_{j-i}}) \\ -1 + \mathbf{r_i} + \mathbf{r_j} & = & \mathbf{r_j} - \mathbf{r_i} \\ -1 + \mathbf{r_i} + \mathbf{r_j} & = & \mathbf{r_j} - \mathbf{r_i} \qquad (+\ \mathbf{r_j}) \\ -1 + \mathbf{r_i} + 2\mathbf{r_j} & = & 2\mathbf{r_j} - \mathbf{r_i} \end{array}$$