Listing-Möbius Shift Symmetry on Closed Time Loops

by Sven Nilsen, 2023

In this paper I present a symmetry on closed time loops that involves a Listing-Möbius Shift. This symmetry can be used to detect the topological difference between a path that is contractible to a point and a path that goes around a hole, without needing to attempt contraction.

A symmetry[1] is when a property of some system is invariant over some action, such as translation, rotation, mirror flip etc. Now, it can be difficult to come up with novel categories of symmetries, because they generalise to many kinds of systems. This means that novel categories of symmetries require some explanation in order to build an intuition of what they mean, since most common sense symmetries have already been exhausted.

Imagine that you are walking around a lake. The walking route has a topology of a large circle that ends up where you started. When you have completed one round, you do another, but this time walking backwards. A friend captures this on a camera and later edits the clip such that the second round is played backwards in time. What will you see watching the clip?

You will see yourself walking in the reverse direction around the lake, retracing the route that you took the first time. Notice that if you take any path in an open space and retrace the route that you took, walking in normal direction, it is not always possible to record a movie of that path and edit it such that it looks like you are doing two loops in the same direction. So, when you have a closed loop, you can edit the movie such that it looks like you are retracing your steps and vice versa, by walking backwards the second round, but this is not possible in general for any path.

In fact, if you record a movie of yourself walking in any path, then ignoring the direction you are walking, this is all you need to create a movie where you retrace your path. However, in order to do this, the time must be reversed in half of the movie, so you have a sum time zero for the entire path. This means, the path is a closed time loop. However, if you repeat a closed path in a circle twice, then it is possible to create a similar movie that looks like a closed time loop with sum time zero, by editing the movie.

This symmetry between looping twice and retracing a path, only exists if there is a topological hole.

As a consequence, there is a way to detect topological holes without needing to attempt contraction of a path to a point. Instead of contracting the path, one uses an equivalence class of paths containing this symmetry. Since every composition of paths that are reversible are contractible to a point and every path in the equivalence class containing the symmetry has this property, the equivalence class containing this symmetry is a subset of the equivalence class of contractible paths.

This structure on paths is nice, because it does not require any higher equivalence class than the class of contractible paths. Every non-trivial equivalence class is a subset of the trivial one.

Furthermore, there is a built-in ambiguity of path connectedness, such that all paths allow an interpretation of being composed out of smaller paths that might not be connected to each other. This ambiguity appears in the interpretation of path semantical quality as path connected spaces[2].

The ambiguity of path connectedness comes from the symmetry between a path that loops twice and a path that retraces its route. If a such symmetry exists, then it means that one must allow cuts, inverses and glue operations on the path as long no new gaps are introduced. These operations extend the normal operations allowed in topology[3].

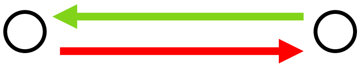
One can also think about these extended operations as invariants that preserve path connectedness before and after the transformation, but allow intermediate exceptions that violate path connectedness. In some sense, a kind of quantum tunnelling effect or movie editing operation.

However, if it is possible to distinguish direction of time in such paths by some action, then the action requires a Listing-Möbius shift[4] in the second round when moving in closed time loops.

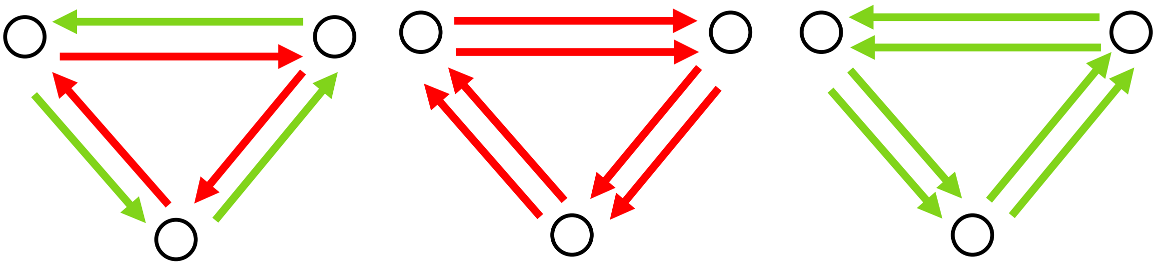
A Listing-Möbius shift is an involution on some space such that the space contains both positive and negative values. There is a slice of the space that contains all the information about the space, but the slice can be transformed gradually in a way that it gets the opposite polarity, while preserving the total information about the space.

When applying Listing-Möbius shift symmetry on closed time loops, there is a corresponding symmetry on actions that erases information about time direction. This means, one can not know from within the space whether time is moving forwards or backwards in other ways than holding information about the topology of the space in some memory. Without memory, time might as well move backwards, in the opposite direction, as forward.

Explained visually:



For every red arrow, there is a green arrow going in the opposite direction. No matter which path is taken, there exists a reversal of the path. However, the green arrow might be thought of as being ambiguous. In one sense, it represents cancelling of the red arrow. In another sense, it represents a kind of doubling of the red arrow, but only if there exists a path such that one can go back to the starting point without visiting the same position twice.



In a triangle, there are three possible states that are invariant under this symmetry. When there is a red arrow and a green arrow between two nodes, one calls it an “unbiased” state. The corresponding biased state of an edge, either towards red or green, is only valid when the exact opposite bias is possible. Therefore, the two biases are mirror images of each other.

Now, it is not trivial how to think about such spaces, because there is no trivial way to define exactly where the Listing-Möbius shift will take place. It does not depend on local information about the space, but how paths are traversed in loops. We know it is possible to construct a such shift in principle, but since it is a side effect of turning an unbiased state into a biased state, there is no information in the space itself that distinguishes two biased states from each other. This is why the symmetry is kind of non-trivial, in particular for complex cases of spaces.

References:

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[3] “Topology”
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