Binary Square Matrix Combinatorics

by Sven Nilsen, 2020

In this paper I formalize binary square matrix combinatorics using Directional Set Algebra.

For all `n : nat`, there is an associated binary square matrix combinatorics:

0	Empty matrix set
D	Diagonal matrix set
U	Upper strictly triangular matrix set
L	Lower strictly triangular matrix set
1	All matrices

The following law holds with Directional Set Algebra:

    D + U + L => 1

Sizes of sets, since they share the zero matrix, gets subtracted one when added together:

```
|x + y| = if x == 1 ∨ y == 1 { |1| }
          else {
                |x| +
                if x == y { 0 }
                else {
                        |y| – if |x| > 0 ∧ |y| > 0 { 1 } else { 0 }
                }
          }
```

```
|0| = 0
|D| = 2^n
|1| = 2^(n · n)
|U| = |L| = 2^(n · (n – 1) / 2)
```

Notice that `|x + y|` operates on inputs `0, D, U, L, 1` and normalized compositions.
Composed inputs, such as `(D + U) + D`, must be normalized to `D + U`.

Sub-types of binary matrix sets can be constructed using elements `0`, `1` and `?`.
The following laws holds with Directional Set Algebra, where `?` is top and there is no bottom:

    0 + 1 = ?

For example, for `n = 3`:

```
? 0 0       0 ? ?       0 0 0       ? ? ?
0 ? 0   +   0 0 ?   +   ? 0 0   =>  ? ? ?
0 0 ?       0 0 0       ? ? 0       ? ? ?
```