Last Order Logic

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In this paper I present a logical language which is kind of like First Order Logic, but funny.

Last Order Logic (LOL) is a language given by the following syntax:

1	term for truth
0	term for falsehood
I	unit interval type
(a, b)	tuple of `a` and `b`
¬a	logical NOT
a ∧ b	logical AND
a ∨ b	logical OR
a ⊻ b	logical XOR
a == b	logical EQ
a => b	logical IMPLY
a ~= b	a path from `a` to `b`
(a ~= b) ~ 0 : a	path start point
(a ~= b) ~ 1 : b	path end point
((a ~= b) ~= (c ~= d)) ~ (1, 0) : c	surface point
f(a)	lambda application
\(i : I) = f(i)	lambda abstraction
∀ i : I { f(i) } : un(T)	where `f` has true type `T` for all `i`
∀ i : I { f(i) } : nu(U)	where `f` has false type `U` for at least one `i`
∃ i : I { f(i) } : nu(T)	where `f` has true type `T` for at least one `i`
∃ i : I { f(i) } : un(U)	where `f` has false type `U` for all `i`
f(a ~= b) == (f(a) ~= f(b))	lambda application for paths
lift(a) : a	`a` is lifted to type level
(0 ~= 1) : (I ~= I)	the path `0 ~= 1` has path type `I ~= I`

Notice that path points and quantifiers `∀` and `∃` do not evaluate, they produce a type instead. The quantifiers `∀` and `∃` are homogenous, which means when it has e.g. a true type, it has the same type for every true case. The reason for this design choice is to introduce multiple senses of truth values. In First Order Logic[1], there is only one sense of truth which is `false` and `true`. In Last Order Logic, one can have e.g. `un(1)` which means "uniformly true". This can be nested, e.g. `nu(un(1))` which means "non-uniformly uniformly true". Furthermore, a path can be a truth value, e.g. `un(a ~= a)` means "uniformly `a` in one dimension".

un	**un**iform	alternatives: objective, "clothes on"
nu	**n**on-**u**niform	alternatives: personal, "nude"

The terminology "uniform" and "non-uniform" comes from Avatar Extensions[2].
These senses of truth are "truthful" when every symbol is concrete, such as `un(1)` or `nu(1 ~= 0)`.
When the symbols are not concrete, the uniformity or non-uniformity is not to be taken seriously.
The intuition is that a uniform truth can turn into non-uniform and vice versa.
This idea borrows from the mathematical universe called "The Joker"[3].

This paper only presents the language of Last Order Logic and does not include inference rules.
Rest of this paper are examples.

Example 1:

$\because \quad f(i : I) = (1 \sim= 0) \sim i$

$\therefore \quad f(0) : 1 \qquad f(1) : 0 \qquad \exists\, i \{ f(i) \} : \mathrm{nu}(1)$

Example 2:

$\because \quad f(i : I) = ((1 \sim= 0) \sim= (1 \sim= 0)) \sim i$

$\therefore \quad f(0) : (1 \sim= 0) \qquad f(1) : (1 \sim= 0) \qquad \forall\, i : I \{ f(i) \} : \mathrm{un}(1 \sim= 0)$

Example 3:

$\because \quad p \sim 0 : 1 \qquad p \sim 1 : 0$

$\therefore \quad p == (1 \sim= 0)$

Example 4:

$\because \quad \exists\, i : I \{ p \sim i \} : \mathrm{nu}(1) \qquad p \sim 1 : 0$

$\therefore \quad p == (1 \sim= 0)$

Example 5:

$\because \quad \exists\, i : I \{ p \sim i \} : \mathrm{nu}(1 \sim= 0) \qquad p \sim 0 : (0 \sim= 0)$

$\therefore \quad p == ((0 \sim= 0) \sim= (1 \sim= 0))$

Example 6:

$\because \quad \forall\, i : I, j : I \{ p \sim (i, j) \} : \mathrm{un}(\mathrm{un}(1))$

$\therefore \quad p = ((1 \sim= 1) \sim= (1 \sim= 1))$

Example 7:

$\because \quad \forall\, i : I \{ p \sim i \} : \mathrm{un}(1)$

$\therefore \quad \forall\, i : I \{ \neg p \sim i \} : \mathrm{un}(0)$

Example 8:

$\because \quad \forall\, i : I \{ p \sim i \} : \mathrm{un}(1)$

$\therefore \quad \neg\exists\, i : I \{ \neg p \sim i \} : \mathrm{un}(1)$

Example 9:

$\because \quad \forall\, i : I \{ p \sim i \} : \mathrm{un}(1)$

$\therefore \quad \exists\, i : I \{ \neg p \sim i \} : \mathrm{un}(0)$

Example 10:

$\because \quad \exists\, i : I \{ p \sim i \} : \mathrm{nu}(1)$

$\therefore \quad \neg\exists\, i : I \{ p \sim i \} : \mathrm{nu}(0)$

Example 11:

$\because \quad x : \mathrm{un}(0)$

$\therefore \quad \neg x : \mathrm{un}(1)$

Example 12:

$\because \quad x : \mathrm{nu}(0)$

$\therefore \quad \neg x : \mathrm{nu}(1)$

Example 13:

$\because \quad \neg\forall\, i : I \{ p \sim i \} : \mathrm{nu}(1)$

$\therefore \quad \exists\, i : I \{ \neg p \sim i \} : \mathrm{nu}(1)$

Example 14:

$\because \quad \exists\, i : I, j : I \{ p \sim (i, j) \} : \mathrm{un}(\mathrm{false})$

$\therefore \quad \exists\, i : I \{ \neg p \sim i \} : \mathrm{nu}(\mathrm{true})$

References:

[1] “First Order Logic”
Wikipedia
https://en.wikipedia.org/wiki/First-order_logic

[2] “Avatar Extensions”
AdvancedResearch – Summary page on Avatar Extensions
https://advancedresearch.github.io/avatar-extensions/summary.html

[3] “The Joker”
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https://github.com/advancedresearch/path_semantics/blob/master/papers-wip2/the-joker.pdf