Rule-Free Logic

by Sven Nilsen, 2025

In this paper, I introduce an entire new sub-field of logic which studies rule-free logic. This is a logic that is without rules and it is not possible to prove anything. However, while seemingly trivial, it is not trivial to study rule-free logic. This is because any study of rule-free logic requires assumptions about methodology or at least some philosophical position. Since rule-free logic has not been studied before, it is not certain that people have the same views on rule-free logic and what it means. Some people might believe it would be useless or impossible to study, but at the very minimum, such positions should be looked into more closely, to better understand what people mean by a useful logic or some logic that is possible to study.

People have through millenia since the invention of writing thought about spirituality and existence, which has resulted in a lot of negative answers bringing about a world where most people do no longer believe in e.g. ghosts. However, the essence about spirituality and existence has remained illusive and difficult to grasp, specially when treating these topics on an abstract level.

The problem might be that people have been aiming too high. If we already assume something about spirituality and existence, even without believing that there are some objects out there that have the properties we assume, then we might have missed something basic on the way that could lead to problems later on when attempting to achieve better understanding.

One tool that came out of the process of people trying to understand spirituality and existence, is mathematics. Mathematics can be reduced in many cases to logic. Yet, logic itself is not very well understood. There are many languages in logic and some are weaker or stronger than others. Weaker or stronger, in some sense, means here that one can add axioms to one language to get similar properties to another language.

The most important logical language is Intuitionistic Propositional Logic (IPL), that forms the basis for checking computer software. By adding excluded middle axiom to IPL, one gets classical logic. It is possible to construct brute force theorem provers that prove every theorem in classical logic, without such systems being easily modifiable to support constructive logic. Therefore, the system design is very important for how practical or limited the language is while being used. Logicians often use multiple languages, e.g. to prove theorems at the meta-level about another logic. A recent breakthrough in IPL was introducing a new operator that allows meta-theorem proving from within IPL, which uses HOOO EP axioms. These axioms, found by the author, are even more fundamental than Modal Logic, which is a large sub-field of the logic discipline. This means, it was not until recently that people could understand IPL in some complete sense that included both object- and meta-level. Consider that IPL has been studied for over a century and underlies many advances in technology and proved itself as reliable tool, the state of the field itself is still at its very beginning.

Logicians are studying weaker logical languages than IPL, due to e.g. quantum behavior that is not distributive, or for optimization using e.g. linear logic. Yet, IPL is considered the central logic for the discipline, because one can use it to reason about other logical languages. There is a sense that weaker logics are less useful, but also, they have applications in many domains and purposes. The trade-offs between system design and generality are not easily quantifiable. It is from this perspective that I suggest to start a new sub-field of logic that studies rule-free logic, in order to provide more tools for studying relative weakness properties of weaker logical languages. Rule-free logic is an ideal logic which is weaker than any other logical language. Yet, there is no current agreement of whether rule-free logic is unique or even its existence. It has not been studied before.

For example, one person might think about rule-free logic as some language where one can only say e.g. `a` or `b` or give any name for propositions, but is not able to prove anything using the propositions. Here, there is an immediately disagreement among people whether one can prove `a` by assuming `a` in rule-free logic, or whether it is even possible to assume `a` in the first place. It is not trivial that one can say `a` and make it work as an assumption. Most logicians have these expectations from working with IPL or perhaps weaker logical languages, but when confronted with what should be possible to do in rule-free logic, one starts questioning the expectations themselves.

Obviously, rule-free logic can not be used for anything practical. However, it does not follow from this fact that the study of rule-free logic will not result in any practical applications as a side effect of investigations. It happens many times before in history when people started studying something, most people did not see the applications, but they turned up at a later point in time. We should not just dismiss studying something because it seems useless at the outset. When it comes to rule-free logic, one can immediately predict, using human intuition, that a rule-free logic, if it exists, then it is not useful at all. Hence, this bias of looking at the usefulness of the result as an aim, might overlook the insights by the process or social activity itself.

One advantage of studying rule-free logic, is that to beginners might seem like a field with low entry requirements. Since this field has not been well developed before and there are no expectations to use the results for applications, since they are by definition impossible to predict in advance, people who do not feel comfortable contributing to other sub-fields of logic might find a place and by luck or hard work, discover something new. Basically, everything we discover about rule-free logic is a bonus, so the lowered expectations and possible rewards might attract people.

If it turns out that rule-free logic is easily exhaustible as a field of knowledge, then in the very least we would have completed this sub-field of logic as a discipline. However, at the current moment, we know next to nothing about rule-free logic. This means, we can not just assume that this sub-field will be exhaustible. At least, we have to be able to show why this would be the case.

For example, one can apply the scientific method to the study of rule-free logic. If two people A and B have hypotheses about rule-free logic that are in contradiction with each other, then perhaps there is some experiment that can be performed to rule out at least one of the hypotheses. There has been none such experiment performed up to this moment, despite that most logicians would probably have some intuitions about which hypotheses to make. Just assuming that everyone would agree with one’s own hypothesis, or assuming that there are no contradictions, is premature reasoning.

One idea I have is to let A and B try two different systems for theorem proving and try to modify the source code to get closer to a rule-free logic. When they no longer can make further modifications easily, they can ask an impartial judge to evaluate the two systems and make a comparison. In some sense, if A and B get stuck at different or the same points toward rule-free logic, then this is itself of scientific interest for the study of rule-free logic.

This experiment requires no formal definition of rule-free logic itself. It is simply basic science.

There are many research questions that can be immediately asked about potential results of such experiments. Personally, I believe I have some intuition of what might happen, but also, I am curious about the results of an actual experiment. For now, I believe that preparations and predictions of a such experiment should be made in dialogue with people that are interested in the study of rule-free logic. My intention with this paper was just to introduce the sub-field and let other people be able to contribute. I hope that this field can touch on some philosophical questions related to other fields, without carrying the baggage of these disciplines. Perhaps rule-free logic can clarify some concepts and ideas that were originally phrased somewhere else.