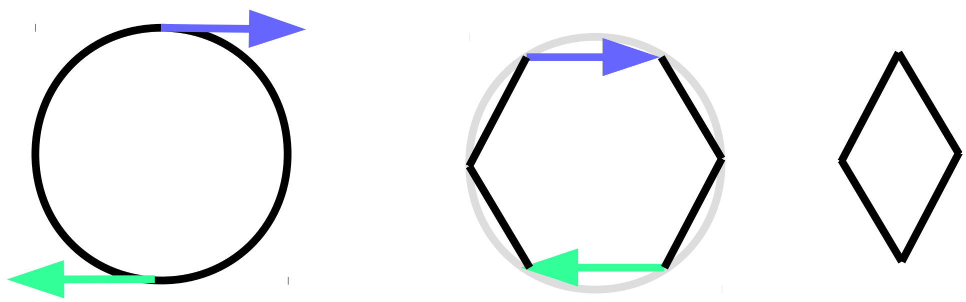
Cocyclic Graphs

by Sven Nilsen, 2020

In many branches of mathematics, such as topology, homotopy, group and proof theory, the idea of a loop or cycle plays a central role. In this paper I introduce a constructive axiom for graphs to model a generalization of loops, called "cocyclic graphs".

The tightening of loops on hyperspheres is essential in topology, e.g. the Poincaré conjecture.
However, what is the geometric intuition that makes tightening a loop possible?

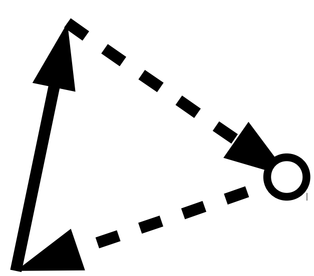
For example, at the parameterized circle, for every point and tangent,
there exists an opposite point with the opposite tangent.
Intuitively, one can remove these two points and glue the new pieces together:



This process can be repeated until a single point remains.
If the loop contains any irregularities, then those parts can be fixed first so it becomes symmetric.

Now, instead of doing this by choice from unlabeled edges, one can do this by construction in reverse, by introducing a stronger axiom that generalizes loops.

This axiom is as following: For every vector and point, there exists a canonical directional triangle:



A graph is called "cocyclic" if it is constructed from such canonical directional triangles recursively:

- Each triangle generates two new vectors
- New points can be chosen arbitrarily and old edges can be erased
- Each vector sum of a cocyclic graph has a characteristic interpretation