# Constrained Uniform Properties of Sets

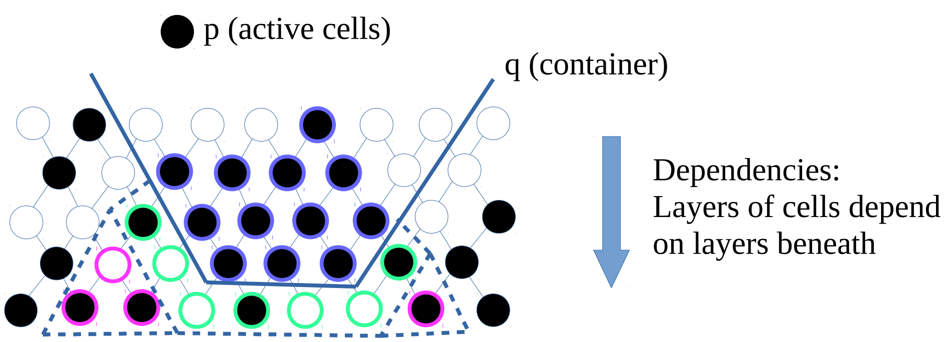
by Sven Nilsen, 2019

*In this paper I represent a general method to prove soundness of constrained uniform solvers and a general method to solve constrained uniform problems with unconstrained uniform solvers.*

Constrained uniform properties of sets can be translated into uniformed properties[1] (unconstrained):

$$\begin{aligned} p'(x) \quad = \quad & p(x) \land q(x) \quad \lor \\ & S( \backslash(y) = p(y) \land q(y) )(x) \land \neg S(S( \backslash(z) = p(z) \land q(z) ))(x) \land \neg q(x) \quad \lor \\ & S( \backslash(v) = S( \backslash(w) = p(w) \land q(w) )(v) \land \neg q(v) )(x) \end{aligned}$$

Using Higher Order Operator Overloading[2] and a visualization, it becomes more understandable:




$$p' \quad = \quad p \land q \quad \lor \quad S(p \land q) \land \neg S(S(p \land q)) \land \neg q \quad \lor \quad S(S(p \land q) \land \neg q)$$

| | |
|---|---|
| $p' : set \rightarrow bool$ | Unconstrained uniform property |
| $p : set \rightarrow bool$ | Constrained uniform property |
| $q : set \rightarrow bool$ | Constraints |

| | |
|---|---|
| $p \land q$ | "Evaluation condition" |
| $S(p \land q) \land \neg S(S(p \land q)) \land \neg q$ | "Termination condition" |
| $S(S(p \land q) \land \neg q)$ | "Ignorance condition" |

The function \`S(x)\` returns "all subsets of each set in a set" and is defined as:

$$S(x : set \rightarrow bool) = \backslash(y : set) = y \in \cup\ z : x \ \{ P(z) \land \neg z \}$$

$$S : (set \rightarrow bool) \rightarrow (set \rightarrow bool)$$

Where \`P\` is the powerset operator[3].

A uniform solver is a problem solver that assumes reductionism[4]. All constrained uniform solvers that satisfy the evaluation condition and the termination condition are sound. Constrained uniform problems can be solved with unconstrained uniform solvers by composing the 3 parts together as specified.

## References:


[1] "Uniform Properties of Sets"
Sven Nilsen, 2018
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/uniform-properties-of-sets.pdf

[2] "Higher Order Operator Overloading"
Sven Nilsen, 2018
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/higher-order-operator-overloading.pdf

[3] "Power set"
Wikipedia
https://en.wikipedia.org/wiki/Power_set

[4] "Reductionism"
Wikipedia
https://en.wikipedia.org/wiki/Reductionism