

Unitary Symmetric Paths

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In this paper I describe unitary symmetric paths.

A unitary symmetric path of f by g is the inverse of f :

$$f[g] \Leftrightarrow f^{-1}$$

It follows that g is unique, because the inverse of f is unique.

Also, it follows that $g^2 \Leftrightarrow \text{id}$ and this can be used to find the inverse unitary symmetric path:

$$\begin{aligned} f &\Leftrightarrow f \\ f[\text{id}] &\Leftrightarrow f \\ f[g^2] &\Leftrightarrow f \\ f[g][g] &\Leftrightarrow f \\ f^{-1}[g] &\Leftrightarrow f \end{aligned}$$

Some examples:

$$\begin{aligned} \text{id}[\text{id}] &\Leftrightarrow \text{id} \\ \text{not}[\text{not}] &\Leftrightarrow \text{not} \\ \text{inc}_{\mathbb{Z}}[\text{neg}] &\Leftrightarrow \text{dec}_{\mathbb{Z}} \\ (\cdot 2)\{(\neg = 0)\}[\text{inv}] &\Leftrightarrow (/ 2)\{(\neg = 0)\} \end{aligned}$$

The two last examples have inverse unitary symmetric paths:

$$\begin{aligned} \text{dec}_{\mathbb{Z}}[\text{neg}] &\Leftrightarrow \text{inc}_{\mathbb{Z}} \\ (/ 2)\{(\neg = 0)\}[\text{inv}] &\Leftrightarrow (\cdot 2)\{(\neg = 0)\} \end{aligned}$$