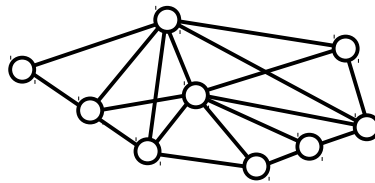


# Homotopy Physics and Simplex Approximations

by Sven Nilsen, 2021

*In this paper I show that simplex approximation is useful to understand the distribution of avatars.*

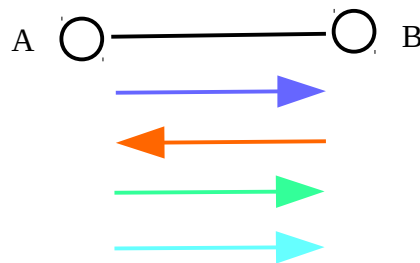
Assume the entire universe is like an undirected graph where each edge is a possible path for a particle:



The entire universe is assumed to be like an undirected graph where edges are possible paths

A “possible path” does not mean that a particle travels in the sense of classical mechanics, but in the sense of the Path Integral Formulation of quantum mechanics. In Homotopy Physics, a particle does not travel along a single path, but is made up of many homotopy maps between the “possible paths”.

However, since the word “travel” is not used in this context, one can use it to mean something else. When a particle “travels” from A to B, the path is assigned some continuous map of type  $I \rightarrow \mathbb{C}^M$  where  $M$  is a natural number:



The undirected edge is a potential for possible paths

Directed edges are assignments of continuous maps of type  $I \rightarrow \mathbb{C}^M$  to the undirected edge

There can be multiple assignments, so directed edges are pictured having different colors. The “travel” from B to A corresponds to the red arrow shown in the illustration.

These assignments result from the possible path being intersecting with the homotopy maps. The homotopy maps have type  $I^N \rightarrow \mathbb{C}^M$ , where  $N$  and  $M$  are natural numbers. An edge is a special case where  $N = 1$ .

A simplex approximation means that for every pair of nodes in the graph, there is an undirected edge. Translated into possible paths, it means that particles can “travel” from any point to any other point.

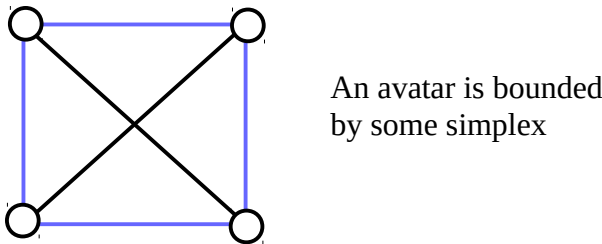
While this approximation is not physically realistic, it is very useful to understand the distribution of avatars by complexity throughout the universe under different assumptions.

A simplex has the mathematical property that each “face” on the simplex is also a simplex:

Simplex	Faces	Number of faces
0-simplex (point)	0	1
1-simplex (line)	0 1	2 1
2-simplex (triangle)	0 1 2	3 3 1
3-simplex (tetrahedron)	0 1 2 3	4 6 4 1
4-simplex (5-cell)	0 1 2 3 4	5 10 10 5 1

The number of faces in a simplex corresponds to numbers found in Pascal’s triangle.

An avatar is a homotopy map of type  $I^N \rightarrow \mathbb{C}^M$ , which is like a hypercube. However, it is always possible to embed a hypercube onto some n-simplex. This means that the pattern of a hypercube, e.g. a square, can be drawn like this:



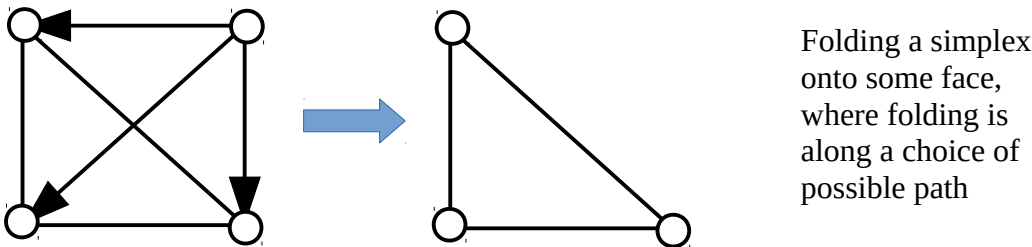
A hypercube has  $2^N$  vertices, which can be embedded onto a  $2^N-1$ -simplex.

There are often many ways to embed avatars onto a simplex, but by grouping avatars using the simplex as boundary, it simplifies reasoning about the distribution of avatars.

It is impossible to measure the distribution of avatars directly, since measurements are restricted. One can only measure the outcomes where possible paths contribute to some probability of an event. This means an observer can only know a tiny fraction of the total information in the universe.

With a such high level of ignorance, due to only knowing a tiny fraction, it makes no sense to develop highly detailed models of the distribution of avatars. Any such model is likely to be wrong.

However, Avatar Extensions hints at how avatars might be created, by using an introduction operator. Starting with the universal wavefunction, one can imagine creating avatars from it. This introduction operator is analogue to the *folding* of a simplex into some face. The avatar is bounded, in an analogue sense, to the folded simplex, where the folding operation is unknown, but along some possible path:



With other words, every avatar is the universe wavefunction folded onto itself.

This might not make much sense from an objective physical perspective, because all probabilities measured by experiments can be explained using the universe wavefunction. Why should one think about avatars, entities hidden from the available information?

The motivation of folding the universe wavefunction, is due to *observers*.

Each observer experience the universe from a unique perspective, which is *different* from any other observer. The universe wavefunction can only explain probabilities on events that observers agree on.

The universe wavefunction is some sort of language which observers use to describe reality independent of their own perspectives. To explain observers themselves, one must restore the subjectivity of observers in the universe, which corresponds the unknown folding operation.

Very little is known, or assumed, about this folding operation:

- The folding operation is assumed to be symmetric for any possible path
- The end result of the folding operation is associated with the distribution of avatars

With other words, if there are multiple ways to create an avatar from the universe wavefunction by performing a sequence of folding operations, then the order of these operations or the number of ways does not matter for reasoning about the avatar distribution. The only thing that matters is the final result of the folding, which corresponds to some simplex.

These two assumptions are made to cover as many possible theories that are consistent with the probabilities that are measured by experiments. If the number of ways mattered by folding, then there will always be more ways to fold into simpler avatars than complex ones and as a consequence the probability of an observer having complexity higher than a fundamental particle would be zero. Likewise, experiments only show that probabilities depend on possible paths and not on higher homotopy maps themselves. If homotopy maps contributed significantly to probabilities, then one can imagine the number of ways to fold, which might be homotopy maps, having some influence.

The folding operation can also have uncountable choices, e.g. over meeting points, but due to symmetry such uncountable choices are factored out. This means one can reason as if the folding operation is approximately discrete and in this case corresponds to faces of a simplex.

In the simplex approximation, these two assumptions makes it possible to reason about the distribution of avatars by counting faces. An avatar can have any complexity, from a single point to the whole universe. Since normalized Pascal's triangle has a binomial distribution, it follows that the distribution of avatars might, as a simple approximation, have a binomial distribution in complexity.

As a consequence, an observer is likely to have a subjective experience that is somewhere in between the two extremes of existing as a single point and existing as the whole universe.