

Invertible Domain Constraints

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In this paper I show that for all functions of finite types, there exist a trivial path such that the function maps uniquely, yet the existential path of the function remains unchanged.

When the trivial path^[1] $\forall f$ of a function f changes, the identity of f changes as well^[2]. Usually, the problem is that the existential path^[3] $\exists f\{\forall f\}$ changes, which can result in unsoundness.

$\exists f\{\forall f\}$ The meaning of the existential path of f depends on the trivial path of f

However, sometimes the trivial path can change without the existential path changing. The most extreme of this scenarios is when the trivial path is constrained such that $\{f\forall f\}$ maps uniquely, but without changing the existential path. This is called an “invertible domain constraint”.

For example, addition is a binary operator on the natural numbers^[4]:

$\text{add} : \text{nat} \times \text{nat} \rightarrow \text{nat}$

For every natural number n , there exists $n+1$ possible inputs to add that constructs the number:

0	=	0+0
1	=	1+0, 0+1
2	=	2+0, 1+1, 0+2
3	=	3+0, 2+1, 1+2, 0+3

When an invertible domain constraint happens for add , the $n+1$ possible inputs are reduced to 1 :

0	=	0+0
1	=	1+0, 0+1
2	=	2+0, 1+1, 0+2
3	=	3+0, 2+1, 1+2, 0+3

Every invertible domain constraint for add must contain $0+0$, since there is only one possibility.

The total number of invertible domain constraints for add is:

$$\prod i : \text{nat} \{ i+1 \}$$

For the intersection with $(< n)$, the number of invertible domain constraints is the factorial of n :

$$\prod i : (< n) \{ i+1 \} = n!$$

For example, for addition that constructs all natural numbers less than 4 , the number of invertible domain constraints is $4 \cdot 3 \cdot 2 \cdot 1 = 24$.

References:

- [1] “Constrained Functions”
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- [4] “Natural number”
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