Answered Modal Logic

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In this paper I introduce a modal logic for the answered predicate of questions.

The meta-knowledge of the answer of a question can be modeled using a set of the following symbols:

{□, ◇, ¬◇}	All possible states of knowledge about question (*unknown* **unanswered**)
□	The question is answered (*known* **answered**)
◇	There exists a case where the question is answered (*unknown* **answered**)
¬◇	There exists no case where the question is answered (*known* **unanswered**)

Inversion laws:

¬□ = {◇, ¬◇}
¬¬□ = {□}
¬◇ = {¬◇}
¬¬◇ = {◇}

When a law is in the form `X => {□, ◇, ¬◇}Y` one can choose:

◇(X => □Y)
◇(X => ◇Y)
◇(X => ¬◇Y)

Here, the `◇` operator reflects on the semantics of the logic itself.
When used this way, it is not an operator of questions directly, but as a meta-operator.

Notice that this logic deviates from epistemic modal logic, which uses semantics “it is known *that* X”.
Here, the logic refers to the knowledge of the answer, without describing what the answer is.

For example:

□”A ∧ B” => ◇”A”

This can be read as “If I know value of `A ∧ B`, then there exists a case where I know value of A”.

In general, the internal semantics of the questions is irrelevant for this logic.

Instead, the questions are treated as black boxes, with partial knowledge described e.g. in the form:

□X => ◇Y

It is the partial knowledge described using this modal logic that can derive other partial knowledge.
The internal semantics of the questions is only relevant for grounding the initial partial knowledge.

I will now prove the following:

$\therefore \quad (\Box X => \Diamond Y) => \Diamond(\Diamond X => \Diamond Y)$

$\because \quad \Box X => \Diamond Y$

$\therefore \quad \neg\Diamond Y => \neg\Box X$

$\therefore \quad \neg\Diamond Y => \{\Diamond, \neg\Diamond\}X$

$\therefore \quad \Diamond(\neg\Diamond Y => \neg\Diamond X)$ Choosing `¬◇` among possible interpretations

$\therefore \quad \Diamond(\Diamond X => \Diamond Y)$

$\therefore$ Q.E.D.

When choosing the other possible interpretation:

$\therefore \quad \Diamond(\neg\Diamond Y => \Diamond X)$

$\therefore \quad \Diamond(\Diamond X => \neg\Diamond Y)$

$\therefore \quad (\Box X => \Diamond Y) => \Diamond(\Diamond X => \neg\Diamond Y)$

This might seem like a contradiction to the following:

$\therefore \quad (\Box X => \Diamond Y) => \Diamond(\Diamond X => \Diamond Y)$

However, when you think about this intuitively, it actually works.

When there exists a case where `X` is answered means `Y` is *unknown* **answered**,
there also might exist a case where `X` is answered but `Y` is *known* **unanswered**.
This is a possibility that can not be left out, without knowing more.

The reason it seems like a contradiction at first, is because of the following in propositional logic.

$\because \quad (P => Q) \wedge (P => \neg Q)$

$\therefore \quad P => (Q \wedge \neg Q)$

$\therefore \quad P =>$ false

$\therefore \quad \neg P \vee$ false

$\therefore \quad \neg P$

So, the only case where this statement is true is when `P` is false.

However, when using the meta-operator:

$\because \quad \Diamond(P => Q) \wedge \Diamond(P => \neg Q)$

Those statements are not contradictory, because they can be valid in different contexts.
It would only be a contradiction if the meta-operator was not used.