Exclusive Theorem

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In this paper I present an exclusive theorem found in Path Semantical Logic.

The Exclusive Theorem is a proof in Path Semantical Logic[1]:

$$
\begin{aligned}
&(tr, fa)\ (B, C): \\
&tr(B),\ B \Rightarrow (tr \veebar fa),\ B \neg= C \ \ \Rightarrow \ \ tr \veebar fa
\end{aligned}
$$

Where the tuple `(tr, fa)` has level 1 and the tuple `(B, C)` has level 0.
The notation `tr(B)` means `tr=>B` where `B` is at a lower level.

The propositions `tr` and `fa` are used with Boolean intuition, where `true` and `false` are exclusive.

Thing that are not provable:

- fa(B)
- B
- ¬(tr=>C)
- ¬(fa=>C)

One can prove `C=>fa`.

If `fa(B)` is added as assumption, then one can prove `C=>tr` and therefore `¬C` and `B`.

One can prove `fa(B) ∨ fa(C)`, but not `fa(B) ⊻ fa(C)`.
Similarly, one can prove `tr(B) ∨ tr(C)` (trivial, since `tr(B)`), but not `tr(B) ⊻ tr(C)`.

One can prove:

$$fa(B) \land tr(B) \ \ \neg= \ \ fa(C) \land tr(C)$$

Here is a table of the possible cases:

B	**C**
tr	tr, fa
tr, fa	tr
tr, fa	fa

Notice that there is no case where `C` is uninhabited.

References:

[1] “Path Semantical Logic”
AdvancedResearch, reading sequence on Path Semantics
https://github.com/advancedresearch/path_semantics/blob/master/sequences.md#path-semantical-logic