Index Theorem

by Sven Nilsen, 2020

In this paper I present an index theorem found in Path Semantical Logic.

The Index Theorem is a proof in Path Semantical Logic[1]:

> (tr, fa, i, one) (𝔹, ℕ):
> (𝔹 ∧ ¬i)=fa, (𝔹 ∧ i)=tr, (ℕ ∧ i)=one => tr=one

Where the tuple `(tr, fa, i, one)` has level 1 and the tuple `(𝔹, ℕ)` has level 0.

Notice that `tr` and `fa` are propositions that model booleans (`𝔹`).

The theorem assigns each member of `𝔹` an index, using `¬i` and `i`.
Since `i` is used to assign `one` to `ℕ`, an equality is propagated to `tr=one`.
However, this is not trivial, since the theorem no longer holds when removing `(𝔹 ∧ ¬i)=fa`.

Notice that implications is not expressed directly, but follows from the use of equality.

References:

[1] “Path Semantical Logic”
AdvancedResearch, reading sequence on Path Semantics
https://github.com/advancedresearch/path_semantics/blob/master/sequences.md#path-semantical-logic