Reals Defined Using Satisfied Natural Numbers

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In this paper I define real numbers using Satisfied Natural Numbers.

A real number is a potentially orthogonally infinite object which can be broken down into projective finite pieces. When the real number is orthogonally infinite, the breaking down process never stops.

One can think about a real number as some kind of iterator that keeps track of the state such that when it outputs some piece of information, one does not have to relearn that piece of information later to know how the iterator works. However, it is not necessary that perfect knowledge is possible about the iterator itself. E.g. most irrational numbers are not knowable.

Using Satisfied Natural Numbers, I can define a real number `x` as the following:

$$\text{real}(x) := \quad x == \sum i \{ y_i \} \quad \text{where} \quad \exists i \{ y_i > -\text{aleph}_0 \land y_i < \text{aleph}_0 \}$$

And:

$$\text{real}(x - y[\text{why}(\exists i \{ y_i > -\text{aleph}_0 \land y_i < \text{aleph}_0 \})])$$

Notice that this requires the satisfied term to be between plus and minus countable infinity `$\text{aleph}_0$`.

However, this does not require the satisfied terms themselves to be of countable cardinality.
For example, `$\tau$` (`$2 \cdot \pi$`) is an irrational number and hence not of countable cardinality.
Yet, ` $\tau > -\text{aleph}_0 \land \tau < \text{aleph}_0$` is provable and therefore a satisfied term.

The idea that one must prove satisfied terms to be on the real line might seem strange. The real line and the real numbers seems to be one and the same. Yet, have anyone actually observed the whole real line? What if there are real numbers that behave extensionally as if they are on the real line, but they have a "secret" that they are not actually on the real line under some hypertask?

One such kind of real numbers are limits. They can be located at infinity, yet can be approached arbitrarily close. E.g. `f(x) = 1 / x`. Here, `f(0)` is not a real number. Yet, I can input any `x ¬= 0`. In order to express that one is approaching some limit, the remaining distance needs to become "smaller" in some sense, such that the remaining distance can be proved to be on the real line. However, the limit itself is not located on the real line, because, as one tries to evaluate at the limit, one gets an answer that is not at the real line.

Some limits can not be proven to not be on the real line extensionally, because there is no real input one can use to determine that the answer is not at the real line. On the other hand, the distances to these kind of limits are infinite (and hence not real numbers) or non-analytic (and hence can not be revealed to be non-reals). These limits can be transformed soundly as if they were higher order real numbers. All operations that are valid on real numbers are also valid for such limits (in lifted form).

Limits are not real numbers themselves, but they are used to calculate real numbers. One can build objects of intrinsic higher order which project down to lower orders, such as reals.