# Canonical Form of Answered Modal Logic

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*In this paper I introduce the canonical form of Answered Modal Logic.*

The canonical form of Answered Modal Logic is the following syntax:

$$(a_0 \land a_1 \land \ldots a_n) \lor (b_0 \land b_1 \land \ldots b_n) \lor \ldots$$

For brevity, the parantheses can be omitted.
Each term is prefixed with one of members of the modal set `{!◇, ¬!◇, □}`.

The inversion rule `¬□ = {!◇, ¬!◇, □}` can be used with `{!◇, ¬!◇, □}X = !◇X ∨ ¬!◇X ∨ □X`.

This form is used to reduce an expression into one that can be compared with other expressions.

For example:

∵ □A ¬= □B — Notice that `¬=` uses `not`
∴ (not . eq)(□A, □B)
∴ (eq[not] . (not . fst, not . snd))(□A, □B)
∴ eq[not](not(□A), not(□B))
∴ xor(!◇A, !◇B)
∴ (!◇A ∧ not(!◇B)) ∨ (not(!◇A) ∧ !◇B)
∴ (!◇A ∧ □B) ∨ (□A ∧ !◇B)

After normalizing to the canonical form, the expressions can be extracted to a table:

| | !◇A | ¬!◇A | □A |
|---|---|---|---|
| !◇B | 0 | 0 | 1 |
| ¬!◇B | 0 | 0 | 0 |
| □B | 1 | 0 | 0 |

Another example:

∵ □A => □B
∴ not(□A) ∨ □B
∴ !◇A ∨ □B

| | !◇A | ¬!◇A | □A |
|---|---|---|---|
| !◇B | 1 | 0 | 0 |
| ¬!◇B | 1 | 0 | 0 |
| □B | 1 | 1 | 1 |

When a variable is unmentioned, e.g. `B` is not mentioned in `!◇A`, one can fill out the row/column.