Stair Pairs

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In this paper I present a way to enumerate pairs that resembles a stair pattern.

A pair is a tuple `(a, b) : (nat, nat)` where `a < b`.
Pairs can be mapped to and from natural numbers.
By enumerating natural numbers, one can also enumerate pairs.

However, mapping to and from natural numbers is not the only way to enumerate pairs.
One can select a pair, decrease `a` and increase `b` until `a = 0`.

$$(3, 4) \quad (2, 5) \quad (1, 6) \quad (0, 7)$$

Notice that the sum `a + b` is the same for all such pairs.
The problem is to find a way to select initial pairs such that this method covers every possible pair.

A stair pair is a pair corresponding to each natural number `n`:

$$\text{stair\_pair}(n : nat) = \text{if even}(n) \; \{((n + 2) / 2 - 1, (n + 2) / 2)\}$$
$$\text{else} \; \{((n + 3) / 2 - 2, (n + 3) / 2)\}$$

$$\text{stair\_pair\_number}((a, b) : (nat, nat)) = a + b - 1$$

For example:

$$\text{stair\_pair}(0) == (0, 1)$$
$$\text{stair\_pair}(1) == (0, 2)$$
$$\text{stair\_pair}(2) == (1, 2)$$
$$\text{stair\_pair}(3) == (1, 3)$$

One can see the stair pattern when connecting dots in the matrix representation of pairs: