Quantum Schrödinger Functions

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In this paper I formalize Quantum Schrödinger Functions and introduce the Phi quantum function.

The time-dependent Schrödinger equation for a `$\psi$` wavefunction over a Hilbert space:

$i\hbar\psi$'(t) = Hψ(t)   time-dependent Schrödinger equation

ψ : time → $\mathbb{H}$

The type of complex numbers is a Hilbert space with the inner product:

∴   ⟨a, b⟩ <=> ab*

∵   ($\mathbb{C}$, mul$_\mathbb{C}$ · (id, conj)) : [hilbert_space] true
∵   conj(x) = re(x) - im(x)i

A quantum Schrödinger function `f` is a higher order quantum function depending on time:

f : time → T   where `T` is usually `$\mathbb{B}^n$` and `$\mathbb{B}^0$ <=> ()`
∃$_{cp}$f(t) <=> ψ(t)

ψ : time → $\mathbb{C}$

The Phi quantum function is a basic building block for quantum Schrödinger functions:

phi : frequency → time → ()
∃$_{cp}$phi(f)(t) <=> φ(f)(t)

φ(f)(t) = sin(2πft) + $i$cos(2πft)
φ(f)'(t) = (2πf)(cos(2πft) – $i$sin(2πft))

The `phi` function always returns, but nevertheless it is has a complex probability amplitude.
The function `$\varphi$(f)(t)` simply rotates around in a unit circle at frequency `f`.

One can derive the analogue of the Schrödinger equation for `$\varphi$(t)`:

∴ $\hbar\varphi$(f)'(t) = (hf)(cos(2πft) – $i$sin(2πft))
∴ $i\hbar\varphi$(f)'(t) = (hf)($i$sin(2πft) – i^2cos(2πft))
∴ $i\hbar\varphi$(f)'(t) = Eφ(f)(t)

∵ $\hbar$ = h / 2π   reduced Planck's constant
∵   E = hf   Planck-Einstein relation

The Hamiltonian `H` is a measure of the total energy, so Phi is a quantum Schrödinger function.