

Answered Modal Logic

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In this paper I introduce a modal logic for the answered predicate of questions.

The meta-knowledge of the answer of a question can be modeled using a set of the following symbols:

$\{!\diamond, \neg!\diamond, \Box\}$ All possible states of knowledge about question (*unknown **unanswered***)

\Box The question is answered (*known **answered***)

$\neg!\diamond$ There exists two cases, one answered and one unanswered (*unknown **answered***)

$!\diamond$ There exists no case where the question is answered (*known **unanswered***)

Inversion laws:

$$\neg\Box = \{!\diamond, \neg!\diamond, \Box\} \quad \forall x \{ !!x = x \} \quad \forall x \{ \neg\neg x = x \} \quad \forall x \{ \neg!x = !\neg x \}$$

The symbols \diamond and \Box can be interpreted as “possible” and “necessary” as in classical Modal Logic. The $!\neg$ operator corresponds to the associated classical Modal Logic inversion operator.

When a law is in the form $X \Rightarrow \{!\diamond, \neg!\diamond, \Box\}Y$ one can choose:

$$\diamond(X \Rightarrow \Box Y)$$

$$\diamond(X \Rightarrow \neg!\diamond Y)$$

$$\diamond(X \Rightarrow !\diamond Y)$$

Here, the \diamond operator reflects on the semantics of the logic itself.

When used this way, it is not an operator of questions directly, but as a meta-operator.

Notice that this logic deviates from epistemic modal logic, which uses semantics “it is known *that* X”. Here, the logic refers to the knowledge of the answer, without describing what the answer is.

For example:

$$\Box“A \wedge B” \Rightarrow \neg!\diamond“A”$$

This can be read as “If I know value of $A \wedge B$, then there exists two knowledge cases of A ”.

In general, the internal semantics of the questions is irrelevant for this logic.

Instead, the questions are treated as black boxes, with partial knowledge described e.g. in the form:

$$\Box X \Rightarrow \neg!\diamond Y$$

It is the partial knowledge described using this modal logic that can derive other partial knowledge. The internal semantics of the questions is only relevant for grounding the initial partial knowledge.

I will now prove the following:

$$\therefore (\Box X \Rightarrow \neg !\Diamond Y) \Rightarrow \Diamond(\neg !\Diamond X \Rightarrow \Diamond Y)$$

$$\because \Box X \Rightarrow \neg !\Diamond Y$$

$$\therefore \neg \Diamond Y \Rightarrow \neg \Box X$$

$$\therefore \neg \Diamond Y \Rightarrow \{!\Diamond, \neg !\Diamond, \Box\} X$$

$$\therefore \Diamond(\neg \Diamond Y \Rightarrow !\Diamond X) \quad \text{Choosing `!\Diamond` among possible interpretations}$$

$$\therefore \Diamond(\neg !\Diamond X \Rightarrow \Diamond Y)$$

$$\therefore \text{Q.E.D.}$$

When choosing the other possible interpretation:

$$\therefore \Diamond(\neg \Diamond Y \Rightarrow \neg !\Diamond X)$$

$$\therefore \Diamond(!\Diamond X \Rightarrow \Diamond Y)$$

$$\therefore (\Box X \Rightarrow \neg !\Diamond Y) \Rightarrow \Diamond(!\Diamond X \Rightarrow \Diamond Y)$$

These two possible interpretations are not contradictory.

When there exists a case where `X` is answered means `Y` is *unknown answered*,
it also possible that when `X` is *known unanswered* that `Y` is *unknown answered*.
Answering `X` or not does not change the answer of `Y`.