Adversarial Path of Cartesian Product

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In this paper I represent the adversarial path of Cartesian products.
This is based on ideas from a discussion with Adam Nemecek.

A Cartesian product is just a tuple:

$$a : A$$
$$b : B$$
$$(a, b) : A \times B$$

The paper "Adversarial Paths" introduced choices and adversarial choices.
Making a choice `A` is written `A ~ 0`, which has the type:

$$A \sim 0 : T \rightarrow A \sim 1$$

A Cartesian product of choices is itself a choice.
In the case of a making a choice of a Cartesian product of choices:

$$(a, b) \sim 0 \Leftrightarrow (a \sim 0, b \sim 0)$$

For example, when `a` and `b` are lists:

$$(a, b) : [A] \times [B]$$
$$(a, b) \sim 0 : ([A] \times [B]) \sim 0$$
$$(a \sim 0, b \sim 0) : [A] \sim 0 \times [B] \sim 0$$
$$(a \sim 0, b \sim 0) : (nat \rightarrow A \sim 1) \times (nat \rightarrow B \sim 1)$$
$$(a \sim 0, b \sim 0) : nat \times nat \rightarrow A \sim 1 \times B \sim 1$$
$$(a \sim 0, b \sim 0) : nat \times nat \rightarrow (A \times B) \sim 1$$

Therefore:

$$(a, b) \sim 0 : nat \times nat \rightarrow (A \times B) \sim 1$$

$$a : [A]$$
$$b : [B]$$

A such tuple of list of choices is called "undecided" because it lacks the necessary information to arrive at any concrete new resource `(A × B) ~ 1`. To do this, one has to apply the tuple to arguments:

$$(a, b)(x, y) : (A \times B) \sim 1 \qquad \text{`(a, b)(x, y)` is "decided"}$$

$$x, y : nat$$