Non-Constructibility of Quantum Non-Determinism

by Sven Nilsen, 2020

In this paper I show that quantum non-deterministic functions are not constructible in general.

For every constructible function `f`, the output has some probability distribution `$\exists_p$f`.
Knowing the probability distribution is not the same as knowing how to construct the function.

For example, by permuting the order of the outputs one gets another function `f’` such that:

$$\exists_p f <=> \exists_p f'$$

This means that for functions with more than one output, the probability distribution does not give a unique description of the function. Therefore, one can not determine `f` from `$\exists_p$f`. However, one can construct `f` by enumerating every possible function having the same probability distribution.

For pure functions plus extensions of non-deterministism using random sources:

$$\exists_p f : T \to \mathbb{R}$$

It takes some type `T` and returns a real number.

In a quantum non-deterministic function, the probability distribution has the type:

$$\exists_{pc} f : T \to \mathbb{C}$$

Complex numbers `ℂ` is a generalization of real numbers `ℝ`.
Therefore, quantum non-deterministic functions are generalizations of pure functions.

The problem is that one can not enumerate every possible function having the same `$\exists_{pc}$f`.
Why? Because we do not know how to construct a *single* quantum non-deterministic function which the normal path `f[id → re]` does not have a probability distribution with zero imaginary components:

$$\exists_{pc} f[id \to re] <=> \exists_{pc} f$$ The quantum non-deterministic functions that are constructible

Where `re` is a function returning the real component of a complex number.

There are quantum non-deterministic functions that are not constructible, the constructibility of quantum non-deterministic does not hold in general, but only for a subset. Although, one can model the probability distributions of quantum non-deterministic functions, the functions themselves are not accessible.