Quantum Propagation

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In this paper I present an algorithm for simulating quantum functions, by quantum propagation.

A quantum binary function `f` is non-deterministic with a complex probabilistic existential path:

$f : () \to \mathbb{B}^n$ $\qquad\qquad$ $\exists_{pc} f : \mathbb{B}^n \to \mathbb{C}$

A partial observation `g` of `f` is a deterministic function that removes some information:

$g \cdot f$ $\qquad\qquad$ $g : \mathbb{B}^n \to \mathbb{B}^m$ $\qquad\qquad$ $m < n$

The probabilistic existential path of `$g \cdot f$` is computing by summing over complex probability amplitudes and taking the norm squared. The norm squared can be written as a product:

$|x|^2 <=> x \cdot x^*$

Now, since `x` is a sum of complex probability amplitudes, one can expand the product:

$(\sum i \{ x_i \})(\sum j \{ x_j \})^* = \sum i, j \{ x_i x_j^* \}$

From `n` amplitudes, this forms `$n^2$` basis vectors, which are symmetric since `$x_i x_j^* = x_j x_i^*$`.
These are still complex numbers, only their sum has a zero imaginary component.

Instead of summing over outcomes, one can pick a random basis vector for each outcome.
This is the basic principle for simulating quantum functions using this technique.

- Probabilities can be computed directly by summing over propagated basis vectors
- In the limit, this sum converges toward a real probability for each outcome
- At any given instant, every outcome is equally probable if `f` is semi quantum

For example, if `$f : () \to \mathbb{B}^2$` and `$g : \mathbb{B}^2 \to B$`, there are two outcomes:

[f] [g] true $\qquad$ [f] [g] false

Each of these outcomes has a set of basis vectors. One basis vector is picked at random from each set.

$(a, b) : \mathbb{C} \times \mathbb{C}$ $\qquad$ Two basis vectors `a` and `b`, one for each outcome

$|a| / (|a| + |b|)$ $\qquad$ The probability of `[g] true` happening at an instant (50% for semi q.f.)

If `g <=> and`, then there is a set for `00, 01, 10` with size `$3^2 = 9$`, and a set for `11` with size `$1^2 = 1$`.

The norm of the sum over such basis vectors is a statistical sample with a built-in uncertainty principle.
When the sample size is short, there can be non-zero outcomes that converges to zero probability.