Subset Sentences

by Sven Nilsen, 2018

In this paper I formalize the semantics of subset sentences in path semantics and discuss some ideas.

A subset sentence is a constructive symbol that takes zero, one or more arguments, with the property that all valid sentences starting with the same symbol has a subset relationship to itself, if a subset relationship is defined for the sub-sentences starting with same symbol. Zero arguments means either the union of all subsets or the intersection of all subsets. If two sub-sentences starting with same symbol have no internal subset relationship, the subset relationship for the symbol can be overloaded.

```
inclusive := \(s : subset_sentence) = ∀ x { s x ⊆ s }
exclusive := \(s : subset_sentence) = ∀ x { s ⊆ s x }
positive := \(s : subset_sentence) = ∀ x { ∀ y : (⊆ x) { s y ⊆ s x } }
negative := \(s : subset_sentence) = ∀ x { ∀ y : (⊆ x) { s x ⊆ s y } }
```

For example:

My : inclusive ∧ positive	“My” is a subset sentence
My := {person family group}	Grammar
person ⊆ group	
family ⊆ group	
My person ⊆ My family	Overloading for “person” and “family”
My person ⊆ My group	Overloading for “person” and “group”
me := My person	Alias for “me”
mine := My	Alias for “mine”

Here is a proof using the subset sentence “My”:

```
me ⊆ My group
My person ⊆ My group
```

Here is a proof that “me” is a subset of “mine”:

```
me ⊆ mine
My person ⊆ My
```

Notice that the overloaded meaning might be associated with other ideas, such as personal freedom. One reason that natural language is ambiguous, is because of different associations people might have.

The purpose of subset sentences in communication is to zoom in or out in a context.

For example, instead of saying that “x got a new home” for every person, one says things like “My family got a new home”, because this implies all the other things one wants to say:

```
got_new_home(My family) => got_new_home(My person)
```

Formally, when a predicate `p` uses a subset sentence `p(s)`, one means:

    p(s) <=> ∀ x : (⊆ s) { p(x) }

Notice that `⊆` is natural here while `⊂` (strict subset that does not contain itself) is unnatural. If one says “my family moved”, one does not only mean that every person in the family moved, but one also means that the family, as a whole, moved. This is important, because in discrete combinatorical spaces, there is more information in systems that are put together than in the individual parts.

A subset sentence can also be an alias:

    My := Belonging _ me

Notice that `me` is an alias of “My”, making “My” a recursive alias:

    My X
    Belonging X me
    Belonging X (My person)
    Belonging X (Belonging _ me)
    Belonging X (Belonging _ (My person))
    Belonging X (Belonging _ (Belonging _ (My person)))
    ...

However, at some point the expanded alias recurses on itself, such that:

    Belonging _ Y := Belonging _ (Belonging _ Y)

In formal languages, it is sometimes necessary to transform expressions into a unique representation for comparison. Instead of expanding forever, the unique form of the expression uses the shortest version. One could either use “My person” or “me” here, but since they are both aliases, the shortest version is:

    My := Belonging _ me

Therefore, a formal language subset sentences might support recursive aliases and deduce the shortest form using introspection on the definition.

Since `My` is inclusive and positive, it implies that `Belonging` is inclusive and positive too, at least when the second argument is “me”:

> My : inclusive ∧ positive <=> Belonging _ me : inclusive ∧ positive

If this holds for all arguments, then it implies that “Your” is also inclusive and positive:

> you := Your person
> Your := Belonging _ you
>
> ∀ x { Belonging _ x : inclusive ∧ positive }
> Belonging _ you : inclusive ∧ positive
> Your : inclusive ∧ positive

Notice that this possible ambiguity rise to interesting interpretations of natural language. A person might be thinking of “My” as inclusive, that means it contains everything that he or she owns or relates to, while at the same time thinking of “Your” as exclusive, that means it contains nothing.

Mathematically, one can speak of beauty associated with symmetry of such sentences, so if “My” is inclusive, then “Your” is inclusive, and so on.

- My : i+ Inclusive and positive, “My” contains everything, “My person ⊆ My group”
- My : i- Inclusive and negative, “My” contains everything, “My group ⊆ My person”
- My : e+ Exclusive and positive, “My” contains nothing, “My person ⊆ My group”
- My : e- Exclusive and negative, “My” contains nothing, “My group ⊆ My person”

Instead of using subset sentences as a formal language, one can also introspect on natural language by writing down words that come to mind for various symmetries/asymmetries:

	My : i+	**My : i-**	**My : e+**	**My : e-**
Your : i+	trade	lead	parent	integrate
Your : i-	learn	compete	worship	revolt
Your : e+	demand	exploit	share	master
Your : e-	mob	rob	follow	cultivize

In the table above, I thought of `+` as a person feeling part of a society, while `-` meant feeling above or outside. For “My” the person feels about itself, but for “Your” the person believes the other person feels or want to feel that way. Likewise `i` means gathering to oneself or feeling right to own ,while `e` means giving away or feeling poor. The ideas for associations came from imagining situations where people might be thinking these kind of thoughts frequently. I wrote down the first word that came to mind and changed them only when it did not match the correct interpretation.

The table has a symmetry in the way that you can flip “My” and “Your” by taking the transpose of it as a matrix, and see the perspective of a situation from the other person. I also colored a “neighborhod” of states where `i` or `e` is the fixed behavior, in a way that corresponds to colors interpreted as frequently occuring emotions.

Similarly, the table can be rearranged to show a different neighborhood where `+` and `-` are fixed:

	My : i+	**My : e+**	**My : i-**	**My : e-**
Your : i+	trade	parent	lead	integrate
Your : e+	demand	share	exploit	master
Your : i-	learn	worship	compete	revolt
Your : e-	mob	follow	rob	cultivize

One of the reasons I started thinking about these sentences was to be able to organize and reason about the aspects of behavior of agents who think about themselves and others. It seems to me that it is not only the ability to reflect on thoughts that matters, but associations with the combinatorial structure of language. These associations could be developed by bias from what one finds useful, where the bias is visible in the ambiguity that rises from generalizing some subset sentence for the person.

In a language that allows multiple interpretations of `inclusive/exclusive` and `positive/negative`, one could develop a syntax for resolving the ambiguity.

For example:

> I want to trade my jacket against your scarf

Could be disambiguied by assinging labels:

> I want to trade my (i+) jacket against your (i+) scarf

Another example:

> I demand to have my (i+) umbrella delivered by your (e+) servant

The key insight here is that words describing the intention occurs in same sentence or context.

A language could distinguish between subset sentences that are interpreted differently, also by the sub-sentence that implies the default interpretation when taking out of context:

> my (i+) umbrella your (i+) umbrella
> my (e+) gift your (e+) gift
> my (i-) duty your (i-) duty
> my (e+) help your (e+) help

Sometimes, a sentence can be confusing because it contains conflicting clues to its interpretation, not necessary on the part of the receiver, but because of the intentions from the sender are ambiguous:

> I demand your (?+) help demand => my (i+) help => my (e+)

An example of how confusing it becomes when this is mixed terribly (implies correlation of meaning):

> I want to share my (??) duty with you share => my (e+) duty => my (i-)