Imaginary Adjoint Operators

by Adam Nemecek, Sven Nilsen, 2020

In this paper we generalize imaginary numbers to imaginary adjoint operators.

An imaginary number[1] is defined as following:

$$\mathbf{i}^2 = -1$$

By moving the minus sign to the other side:

$$-\mathbf{i}^2 = 1$$

Using Avatar Covers[2], it is natural to use the avatar cover `xor` for this product:

$$\mathbf{i} \cdot (-\mathbf{i}) = (-\mathbf{i}) \cdot \mathbf{i} = 1$$

$$\mathrm{mul}[\mathrm{neg}]_a \iff \mathrm{xor}$$

We will use the assumption that symmetry forcing[3] is vital for interpretation of Avatar Extensions[4]. Basically, this means it is sufficient to introduce a $1 \rightarrow 1$ avatar map for 1-avatars and let products fill out the rest of the category[4], using the avatar cover to construct isomorphisms[5] to "old" products.

Normally, one thinks of an imaginary number as having special properties. Under the interpretation that symmetry forcing is vital for Avatar Extensions, it is actually the "minus sign" that is extended with imaginary adjoint semantics. The imaginary number is just some symbol that satisfies the algebra of an imaginary adjoint operator.

Hence, for any symmetric avatar cover `xor`:

$$f[g]_a \iff \mathrm{xor}$$

This is necessary to make a strong enough statement that the operator `g` is non-collapsing.

An Imaginary Adjoint Operator `g` is defined as the following relation with `f`:

$$\exists\ e\ \{\ \exists\ \mathbf{i}\ \{\ f(\mathbf{i}, g(\mathbf{i})) = f(g(\mathbf{i}), \mathbf{i}) = e\ \}\ \land\ \forall\ y\ \{\ f(y, e) = f(e, y) = y\ \}\ \}$$

Here, `e` is some unit element of `f`. The element `**i**` is an imaginary element.

References:

[1] “Imaginary number”
Wikipedia
https://en.wikipedia.org/wiki/Imaginary_number

[2] “Avatar Covers”
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https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/avatar-covers.pdf

[3] “Symmetry Forcing”
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https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/symmetry-forcing.pdf

[4] “Avatar Extensions”
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https://github.com/advancedresearch/path_semantics/blob/master/sequences.md#avatar-extensions

[4] “Category theory”
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https://en.wikipedia.org/wiki/Category_theory

[5] “Isomorphism”
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