Abstract Implication Theorem

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In this paper I present an abstract implication theorem found in Path Semantical Logic.

The Abstract Implication Theorem is a proof in Path Semantical Logic[1]:

(a, b) (T, U):
a=>b, a(T)=b(U) => T=>U

Where the tuple `(a, b)` has level 1 and the tuple `(T, U)` has level 0.
The notation `a(T)` means `a=>T` where `T` is at a lower level.

The Abstract Implication Theorem is similar to the Implication Theorem[2],
except that `a(T)` and `b(U)` can not be proved from the premises.

The Abstract Implication Theorem says that implications carries over to lower levels, over equalities of associations, even if the associations themselves are not asserted formally. It is sufficient that the equality of associations are asserted.

A slightly more generalized version shows that this technique has expressive power:

$(a, b_1, b_2), (T, U_1, U_2)$:
$a=>(b_1 \vee b_2), a(T)=(b_1(U_1) \wedge b_2(U_2)) \Rightarrow T=>(U_1 \vee U_2)$

The premise `a(T)=(b₁(U₁) ∧ b₂(U₂))` is necessary to carry over the relationship `∨`.
This is not provable using the premise `a(T)=(b₁(U₁) ∨ b₂(U₂))` instead.

References:

[1] “Path Semantical Logic”
AdvancedResearch, reading sequence on Path Semantics
https://github.com/advancedresearch/path_semantics/blob/master/sequences.md#path-semantical-logic

[2] “Implication Theorem”
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https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/implication-theorem.pdf