Left and Right Inverse

by Sven Nilsen, 2022

In this paper I explain the underlying model of left and right inverse.

The left and right inverse have the following definitions[1]:

$$\text{left\_inv}_{X,Y} := \backslash(f : X \to Y) = \exists\, g : Y \to X \;\{\; g \cdot f <=> id_X \;\}$$
$$\text{right\_inv}_{X,Y} := \backslash(f : X \to Y) = \exists\, g : Y \to X \;\{\; f \cdot g <=> id_Y \;\}$$

Notice that left and right inverse are two perspectives of the same underlying model:

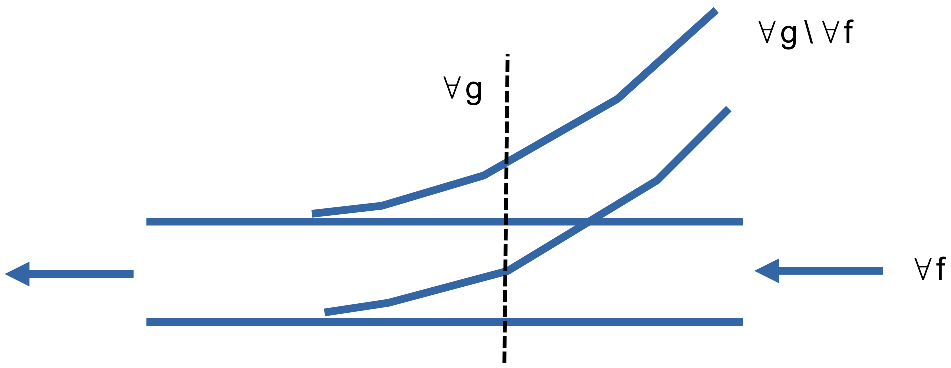
$$g \cdot f <=> id$$

The function \`g\` is the left inverse of \`f\`.
The function \`f\` is the right inverse of \`g\`.

The domain of \`g\` is greater or equal than the domain of \`f\`[2]:

$$|\forall g| >= |\forall f|$$

One intution one can use, is that \`id\` might be thought of as a looping railroad of domain \`∀f\`:



The can be an entrance railroad for trains to get into the loop of domain \`∀g \ ∀f\`, such that:

$$|\forall g| = |\forall f| + |\forall g \setminus \forall f|$$

Where \`\\` is the relative complement operator[3].

When \`f\` has both a left and right inverse, \`|∀g \ ∀f| == |∀f \ ∀g| == 0\`.

The entrance railroad does not have to map one-to-one, but can map arbitrarily.
For example, when \`|∀f| == 2\` and \`|∀g| == 3\`, \`|∀g \ ∀f| == 1\`.

The looping railroad might be thought of as “straight” from a topological perspective.
This is because any swap operation that \`f\` performs, must be cancelled by \`g\`.

References:

[1] “Terminology for Morphisms”
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https://github.com/advancedresearch/path_semantics/blob/master/papers-wip2/terminology-for-morphisms.pdf

[2] “Constrained Functions”
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https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/constrained-functions.pdf

[3] “Relative complement”
Wikipedia
https://en.wikipedia.org/wiki/Complement_(set_theory)#Relative_complement