Directional Set Products

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In this paper I introduce product rules for Directional Set Algebra.

In Directional Set Algebra, the product rules are the following:

$x \cdot (y \cdot z) = (x \cdot y) \cdot z$	Associative
$x \cdot z + y \cdot z = (x + y) \cdot z$	Distributive
$x \cdot y <\neg=> y \cdot x$	Non-commutative

The semantics of this product is similar to a Cartesian product.

The complement follows from these rules:

$$x \cdot y + x \cdot \neg y = x \cdot 1$$

Proof:

$$\begin{array}{l} x \cdot y + x \cdot \neg y \\ x \cdot (y + \neg y) \\ x \cdot (y + 1 - y) \\ x \cdot (y - y + 1) \\ x \cdot (0 + 1) \\ x \cdot 1 \end{array}$$

For example:

$\because$ $0 + 1 = ?$ Top is \`?\`, no bottom

$\therefore$ $0 \cdot 0 + 0 \cdot 1 + 1 \cdot 0 + 1 \cdot 1 = 0 \cdot ? + 1 \cdot ? = ? \cdot ?$

One can use this in a short hand notation to create grammars for binary numbers:

000 + 001 + 010 + 011 + 100 + 101 + 110 + 111
00? + 01? + 10? + 11?
0?? + 1??
???