

The Id Unit Function

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In this paper I define a function that constructs a function tuple erasing a single argument.

The `id_unit` function is defined as following^[1]:

$$\text{id_unit} := \lambda(n : \text{nat}) = \lambda(i : \text{nat}) = \prod j [0, i) \{ \text{id} \} \times \text{unit} \times \prod j [i+1, n) \{ \text{id} \}$$

Here, the `∏` loop means taking the Cartesian product^[2] of the loop body.

The motivation for this function is to erase a single argument in normal paths^[3].

For example, a function `f` taking 4 arguments can erase the 3rd argument with `id_unit(4)(2) → id`:

$$f[\text{id_unit}(4)(2) \rightarrow \text{id}] : T \times T \times () \times T \rightarrow U$$

$$f : T^4 \rightarrow U$$

This normal path does not exist if `f` depends on the 3rd argument for some input.

When a subscript index is used with the id unit function, the size of the tuple is inferred:

$$f[\text{id_unit}_2 \rightarrow \text{id}] \Leftrightarrow f[\text{id_unit}(4)(2) \rightarrow \text{id}]$$

When multiple parameters are erased, a subscript interval can be used as a shorthand:

$$\text{id_unit}_{[0, n)} \Leftrightarrow \text{id_unit}_0 \cdot \text{id_unit}_1 \cdot \dots \cdot \text{id_unit}_{n-1} \Leftrightarrow \text{unit}^n$$

One can also use the interval notation with the last argument:

$$\text{id_unit}(4)[0, 3) = (\text{unit}, \text{unit}, \text{unit}, \text{id})$$

References:

- [1] “Alphabetic List of Functions – Standard Dictionary for Path Semantics”
Sven Nilsen, 2017
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/alphabetic-list-of-functions.pdf

- [2] “Cartesian product”
Wikipedia
https://en.wikipedia.org/wiki/Cartesian_product

- [3] “Normal Paths”
Sven Nilsen, 2019
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/normal-paths.pdf