Non-Deterministic Existential Paths

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The deterministic existential path is defined by the following:

∃f{∀f} <=> \(y) = ∃ x { (∀f)(x) ∧ (y == f(x)) }

However, this is not sufficient for non-deterministic function, because `f(x)` is not unique.

Instead, a recursive definition is required, using a more primitive version of a non-deterministic path:

∃f{∀f} <=> \(y) = ∃ x { (∀f)(x) ∧ (∃f{(= x)})(y) }

Substituting `∀f` with `(= z)` to show that this works:

∃f{(= z)} <=> \(y) = ∃ x { (= z)(x) ∧ (∃f{(= x)})(y) }
∃f{(= z)} <=> \(y) = ∃ x { (= z)(z) ∧ (∃f{(= z)})(y) }
∃f{(= z)} <=> \(y) = ∃ x { true ∧ (∃f{(= z)})(y) }
∃f{(= z)} <=> \(y) = ∃ x { (∃f{(= z)})(y) }
∃f{(= z)} <=> \(y) = (∃f{(= z)})(y)
∃f{(= z)} <=> ∃f{(= z)}
true

This means that `∃f{(= _)}` must be defined first.