Spatial Sum Functions

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A spatial sum function `f` takes some set, such that the union of disjoint sub-sets maps to their sum:

$$\text{spatial\_sum}(f : \text{set} \rightarrow T) = \forall s_0 : \text{set}, s_1 : \text{set} \wedge [\cap\ s_0]\ \varnothing\ \{\ f(s_0 \cup s_1) = f(s_0) + f(s_1)\ \}$$

$$`+` :\ T \times T \rightarrow T$$

A spatial sum is a similar to a uniform set property. If a function is a spatial sum, then all its partial functions constrained by any set property is also a spatial sum:

$$(\ f : \text{spatial\_sum}\ ) \Rightarrow \forall\ p : \text{set} \rightarrow \text{bool}\ \{\ (\ f\{p\} : \text{spatial\_sum}\ )\ \}$$

Spatial sum functions are used to analyze optimization problems where some energy or utility function can be evaluated everywhere over some set. Under such circumstances, greedy algorithms might be counter-productive using actions that have global effects, if there are no other actions with local effects that contribute by composition with previous global actions. This requires splitting the set into smaller parts and only use actions with local effects. This higher order optimization problem is relevant for all spatial sum functions.