Probabilistic Paths as Sub-Types

by Sven Nilsen, 2019

In this paper I introduce an interpretation of probabilistic paths as sub-types.

The major result of this paper is the following statement:

$$([g_i]_p\ b_i = p_{ib}) \wedge [f]_p\ [g_n]\ a \qquad = \qquad f[g_{i \to n}]_p(b_i \mathbin{++} a,\ p_{ib})$$

With other words, a sub-set of truth values of a probabilistic sub-type is defined by a probabilistic path. Here, equality is defined as the truth value, ignoring the input variable (See “Truth Values of Sub-types” for more detail). The operator`++` is `concat`. The statement uses Nilsen-Cartesian product notation which is common notation in asymmetric path semantics.

Starting from simple rules, one can generalize step by step:

$$[f]_p\ a \qquad = \qquad (\exists_p f)(a)$$

A probabilistic sub-type, no domain constraints.

$$[g]\ b \wedge [f]_p\ a \qquad = \qquad (\exists_p f\{[g]\ b\})(a)$$

A probabilistic sub-type with a domain constraint.

$$([g]_p\ b = p_b) \wedge [f]_p\ a \qquad = \qquad f[g \to id]_p([b, a], [p_b])$$

A probabilistic sub-type with a probabilistic domain constraint.

$$([g_i]_p\ b_i = p_{ib}) \wedge [f]_p\ a \qquad = \qquad f[g_i \to id]_p(b_i \mathbin{++} a,\ p_{ib})$$

A probabilistic sub-type with multiple probabilistic domain constraints.

$$([g_i]_p\ b_i = p_{ib}) \wedge [f]_p\ [g_n]\ a \qquad = \qquad f[g_{i \to n}]_p(b_i \mathbin{++} a,\ p_{ib})$$

A probabilistic path expressed as a sub-type.

This shows that a probabilistic path defines a sub-set of truth values for probabilistic sub-types.

The expression `$[g]_p\ b = p_b$` is interpreted as `$([g]_p\ b) = p_b$`. When there exists some `$p_b$` such that:

$$[g]_p\ b = p_b$$

It includes `0`, therefore it is a generalization of boolean sub-types. However, the semantics is different, because no sub-type is defined for `$p_b$`, while in boolean sub-types the sub-type is greater than zero:

$$[g]\ b \qquad \Leftrightarrow \qquad [g]_p\ b > 0$$