

# Dual Adjoint Operators

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*In this paper we generalize dual-imaginary numbers to dual adjoint operators.*

An dual-imaginary number<sup>[1]</sup> is defined as following:

$$\epsilon^2 = 0$$

By adding a minus sign to the each side:

$$-\epsilon^2 = 0$$

Using Avatar Covers<sup>[2]</sup>, it is natural to use the avatar cover `xor` for this product:

$$\epsilon \cdot (-\epsilon) = (-\epsilon) \cdot \epsilon = 0$$

$$\text{mul}[\text{neg}]_a \Leftrightarrow \text{xor}$$

We use the same process as in the paper “Imaginary Adjoint Operators”<sup>[3]</sup>.

Hence, for any symmetric avatar cover `xor`:

$$f[g]_a \Leftrightarrow \text{xor}$$

An Dual Adjoint Operator `g` is defined as the following relation with `f`:

$$\exists z \{ \exists \epsilon \{ f(\epsilon, g(\epsilon)) = f(g(\epsilon), \epsilon) = z \} \wedge \forall y \{ f(y, z) = f(z, y) = z \} \}$$

Here, `z` is some zero element of `f`.

The element `ε` is a dual-imaginary element.

## References:

- [1] “Dual number”  
Wikipedia  
[https://en.wikipedia.org/wiki/Dual\\_number](https://en.wikipedia.org/wiki/Dual_number)
- [2] “Avatar Covers”  
Sven Nilsen, 2020  
[https://github.com/advancedresearch/path\\_semantics/blob/master/papers-wip/avatar-covers.pdf](https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/avatar-covers.pdf)
- [3] “Imaginary Adjoint Operators”  
Adam Nemecek, Sven Nilsen, 2020  
[https://github.com/advancedresearch/path\\_semantics/blob/master/papers-wip/imaginary-adjoint-operators.pdf](https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/imaginary-adjoint-operators.pdf)