Rewriting and the Core Axiom

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In this paper I interpret Rewriting Systems using the Core Axiom of Path Semantics.

A Rewriting System[1] is a system where terms are rewritten using rules:

F_0	The side of the rule that binds to some expression
X_0	The side of the rule that synthesises some new expression
F_1	The expression that is matched against by \`$F_0$\`
X_1	The synthesised expression generated by \`$X_0$\`

In the notation of the core axiom of Path Semantics[2], there are the following relations:

$F_0(X_0)$	\`$F_0$\` produces \`$X_0$\`
$F_1(X_1)$	\`$F_1$\` produces \`$X_1$\`
$F_0 > X_0$	\`$F_0$\` is related to \`$X_0$\` using some non-circular definition
$F_0 = F_1$	\`$F_0$\` matches somewhere and is bound to \`$F_1$\`
$X_0 = X_1$	\`$X_0$\` matches somewhere and synthesises \`$X_1$\`

The core axiom of Path Semantics is the following:

$$\frac{F_0(X_0), F_1(X_1), F_0 > X_0, F_0 = F_1}{X_0 = X_1}$$

Interpreted in the context of Rewriting Systems, one can see the direct relation between rewriting and mathematical languages. In some sense, the core axiom gives rewriting a semantics as proof systems.

The non-circular definition of \`$F_0 > X_0$\` can be thought of as a way of distinguishing the expression being matched against from the synthesised expression. This behavior is embedded in the system classified as a Rewriting System, usually as some form of “time” or as an internal representation.

References:

[1] “Rewriting”
Wikipedia
https://en.wikipedia.org/wiki/Rewriting

[2] “Path Semantics”
Sven Nilsen, 2016-2019
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/path-semantics.pdf