

Equivalences Between Equivalences as Swaps of Swaps

by Sven Nilsen, 2020

In this paper I show that equivalences between equivalences means the same as swaps of swaps.

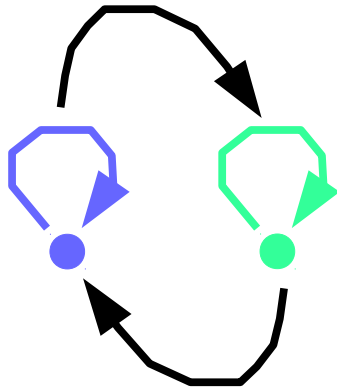
When a function is applied to an equivalence of equivalences:

$$\begin{aligned} f((a \sim b) \sim (c \sim d)) \\ f(a \sim b) \sim f(c \sim d) \\ (f(a) \sim f(b)) \sim (f(c) \sim f(d)) \end{aligned}$$

To find out what this means, one can use a simpler example:

$$f((x \sim x) \sim (y \sim y))$$

One can visualize this as the usual swap, but instead of just drawing objects, I include the self-arrows:



Now, replace $x \sim x$ with $a \sim b$ and $y \sim y$ with $c \sim d$ (think about swaps as objects):

