Emph Notation

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In this paper I derive notation for subsets of an abstract path, called "emphs".

Assume the following two pairs that are unique universal binary relations:

(a, b)   (b, c)

role_of(b) = p
role_of(c) = q

Using function composition:

$q(p(a)) = (q \cdot p)(a) = c$

This composition can not be turned into a unique universal binary relation, because `c` is already assigned the role `q` and can not have `q · p` as an additional role. Neither does it make sense when `p` and `q` are different roles:

(a, c)   is not valid because it would imply the role of `b` and `c` are the same

However, using the semantics of an Avatar Graph, one can construct an "avatar" of `c`. This avatar behaves like `c` except that it is assigned the role `q · p`:

$(a, c_{q \cdot p})$

Since `q` is already known from `c`, one can simplify this notation further:

(a, c_p)

Ideally, one would like to avoid mentioning `p`, since it makes abstract generalizations harder. Instead of `p`, one could use:

(a, c_{ab})

However, `a` is already known from the pair, so this can be reduced to:

(a, c_b)

Now, instead of using subscript `$c_b$`, it is easier to compose using an arrow notation:

$(a, b \rightarrow c)$

Likewise, it is possible to construct an "avatar" `a → b` of `b`, such that `(a → b, c)`.
These two descriptions emphasize different aspects of the same underlying abstract path using avatars.

There are two different choices of how to interpret the emphasis in a readable way:

1. `(a, b→c)` emphasizes `b→c`
2. `(a, b→c)` emphasizes `(a, b)`

The first version only refers to the avatar `b→c` of `c`.
The second version refers to a subset of the path `(a, b)`.

I choose the second version, because it refers to a subset of the path.

In general, this notation can be used with n-tuples, where `,` and `→` are separators:

(a→b→c, d)	emphasizes `(c, d)`
(a, b→c, d)	emphasizes `(a, b)` and `(c, d)`
(a, b, c, d)	emphasizes the entire path
(a→b→c→d)	emphasizes no part of the path

Because of the emphasis of a subset of the path, it is called "Emph Notation".

For example, in standard path semantics one can write a subtype:

false : [not] true

This is path where the emphasis is:

(not(false), true→bool)	emphasizes `(not(false), true)`

The role of `true` is `value_of`, so the full unique universal binary relation is:

value_of(not(false), true)

That `true` has the type `bool` is not important, although it proves that the subtype makes sense.
If the type of `true` was emphasized, then one would "leave" the input to the function `not`:

(not(false)→true, bool)	emphasizes `(true, bool)`
type_of(true, bool)	full emphasized unique universal binary relation

Emph notation is used to signify which part of a path that is important to focus on.
It describes a subset of the path, which contains at least one relation when the subset is non-empty.

While `(a, b→c)` emphasizes `(a, b)`, it also describes a relation `(a, $c_b$)` which is the abstract path from `a` to `c`. With other words, an "emph" describes both a proof and a part of the proof that is considered more important than the rest of the proof.