Complexity of Path Semantical Logic

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In this paper I derive the time complexity of Path Semantical Logic for level 1 and 0.

Path Semantical Logic separates propositions into levels, such that an equality between two propositions in level N+1, propagates into equality between uniquely associated propositions in level N.

In the paper "Faster Brute Force Proofs"[1], I suggested a method to speed up brute force propositional logic separating propositions into levels 1 and 0. To show the potential for improved performance, I provided a table with fractions[2] as a function of `|F|` (level 1) and `|X|` (level 0). However, I did not provide a method to calculate this fraction directly without counting. Since the denominator of the fraction is simply `$2^{|F|+|X|}$`, only a formula for computing the numerator is missing.

The formula for computing the numerator is also a measure of the time complexity for level 1 and 0.

There are two conditions that determines whether a case should be counted:

1. No more than one zero in `F`
2. All `X`s are equal

These two conditions are combined using logical OR.
For simplicity, I will refer to the first condition as `O(F)` and the second as `O(X)`.

It is simpler to analyze the individual conditions and use the following relationship:

$$O(F \vee X) = O(F) + O(X) - O(F \wedge X)$$

The simplest term is `O(X)`, because there are only two solutions `00000…0` and `111111…1`.
For each solution, one enumerates the entire `F`:

$$O(X) = 2 \cdot 2^{|F|} = 2^{1+|F|}$$

To compute `O(F)`, one enumerates the entire `X` when there are zero or one `0`s in `F`:

1111…1 0111…1 1011…1 1101…1 …

$$O(F) = (1 + |F|) \cdot 2^{|X|}$$

The third term `$O(F \wedge X)$` is the intersection, which is simply a product of `2` and `1 + |F|`:

$$O(F \wedge X) = 2 \cdot (1 + |F|)$$

Finally, putting this together and simplifying `$O(F \vee X)$` to `O(|F|, |X|)`:

$$O(|F|, |X|) = 2^{1+|F|} + (1 + |F|) \cdot (2^{|X|} - 2)$$

References:

[1] “Faster Brute Force Proofs”
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https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/faster-brute-force-proofs.pdf

[2] “Fractions”
Wikipedia
https://en.wikipedia.org/wiki/Fraction

[3] “Time complexity”
Wikipedia
https://en.wikipedia.org/wiki/Time_complexity