Algebraic Notation for Asymmetric Paths

by Sven Nilsen, 2017

This is a notation you can use to write asymmetric paths with same algebraic rules as for symmetric paths. The symmetric notation[1] is the following:

$$f[g] <=> h$$

Which means:

$$\forall\ x_0, x_1, \ldots x_{n-1}\ \{\ h(g(x_0), g(x_1), \ldots g(x_{n-1})) = g(f(x_0, x_1, \ldots, x_{n-1}))\ \}$$

An asymmetric path can use different functions on the arguments and return value:

$$\forall\ x_0, x_1, \ldots x_{n-1}\ \{\ h(g_0(x_0), g_1(x_1), \ldots g_{n-1}(x_{n-1})) = g_n(f(x_0, x_1, \ldots, x_{n-1}))\ \}$$

This can be written as:

$$f[g_0 \times g_1 \times \ldots \times g_{n-1} \rightarrow g_n] <=> h$$

For example, the following function predicts whether multiplying two numbers results in a prime[2]:

$$mul([is\_one]\ a, [is\_prime]\ b) \rightarrow [is\_prime]\ a \wedge b$$

This can also be written this way:

$$mul[is\_one \times is\_prime \rightarrow is\_prime] <=> and$$

References:

[1] “Normal Paths”
Sven Nilsen, 2019
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/normal-paths.pdf

[2] “Prime number”
Wikipedia
https://en.wikipedia.org/wiki/Prime_number