Invertible Domain Constraints

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In this paper I show that for all discrete functions, there exist a trivial path such that the function maps uniquely, yet the existential path of the function remains unchanged.

When the trivial path[1] `∀f` of a function `f` changes, the identity of `f` changes as well[2].
Usually, the problem is that the existential path[3] `∃f{∀f}` changes, which can result in unsoundness.

∃f{∀f} The meaning of the existential path of `f` depends on the trivial path of `f`

However, sometimes the trivial path can change without the existential path changing. The most extreme of this scenarios is when the trivial path is constrained such that `f{∀f}` maps uniquely, but without changing the existential path. This is called an "invertible domain constraint".

For example, addition is a binary operator on the natural numbers[4]:

$$\text{add} : \text{nat} \times \text{nat} \rightarrow \text{nat}$$

For every natural number `n`, there exists `n+1` possible inputs to `add` that constructs the number:

0	=	0+0			
1	=	1+0,	0+1		
2	=	2+0,	1+1,	0+2	
3	=	3+0,	2+1,	1+2,	0+3

When an invertible domain constraint happens for `add`, the `n+1` possible inputs are reduced to `1`:

0	=	0+0			
1	=	1+0,	0+1		
2	=	2+0,	1+1,	0+2	
3	=	3+0,	2+1,	1+2,	0+3

Every invertible domain constraint for `add` must contain `0+0`, since there is only one possibility.

The total number of invertible domain constraints for `add` is:

$$\prod i : \text{nat} \{ i+1 \}$$

For the intersection with `(< n)`, the number of invertible domain constraints is the factorial of `n`:

$$\prod i : (< n) \{ i+1 \} = n!$$

For example, for addition that constructs all natural numbers less than `4`, the number of invertible domain constraints is `4 · 3 · 2 · 1 = 24`.

References:

[1] "Constrained Functions"
Sven Nilsen, 2017
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/constrained-functions.pdf

[2] "Function Identity"
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https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/function-identity.pdf

[3] "Existential Paths"
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https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/existential-paths.pdf

[4] "Natural number"
Wikipedia
https://en.wikipedia.org/wiki/Natural_number