

Non-Constructibility of Quantum Non-Determinism

by Sven Nilsen, 2020

In this paper I show that quantum non-deterministic functions are not constructible in general.

For every constructible function f , the output has some probability distribution $\exists_p f$.
Knowing the probability distribution is not the same as knowing how to construct the function.

For example, by permuting the order of the outputs one gets another function f' such that:

$$\exists_p f \Leftrightarrow \exists_p f'$$

This means that for functions with more than one output, the probability distribution does not give a unique description of the function. Therefore, one can not determine f from $\exists_p f$. However, one can construct f by enumerating every possible function having the same probability distribution.

For pure functions plus extensions of non-deterministic using random sources:

$$\exists_p f : T \rightarrow \mathbb{R}$$

It takes some type T and returns a real number.

In a quantum non-deterministic function, the probability distribution has the type:

$$\exists_{pc} f : T \rightarrow \mathbb{C}$$

Complex numbers \mathbb{C} is a generalization of real numbers \mathbb{R} .
Therefore, quantum non-deterministic functions are generalizations of pure functions.

The problem is that one can not enumerate every possible function having the same $\exists_{pc} f$.
Why? Because we do not know how to construct a *single* quantum non-deterministic function which the normal path $f[id \rightarrow re]$ does not have a probability distribution with zero imaginary components:

$$\exists_{pc} f[id \rightarrow re] \Leftrightarrow \exists_{pc} f \quad \text{The quantum non-deterministic functions that are constructible}$$

Where re is a function returning the real component of a complex number.

There are quantum non-deterministic functions that are not constructible, the constructibility of quantum non-deterministic does not hold in general, but only for a subset. Although, one can model the probability distributions of quantum non-deterministic functions, the functions themselves are not accessible.