Obscure Functions

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In this paper I formalize an obscure function, which identity remains unknown inside a black box.

An obscure function `f` is a function where the probability of it returning some output `x`
is less or equal to an infinitesimal `ε`:

$$\text{obscure}(f : A \to B) := \exists\ x\ \{\ (\exists_p f)(x) <= \varepsilon\ \} \qquad\qquad \varepsilon = 1 / \infty$$

This condition obscures the identity of a black box function. A black box function means that the source code or how the function is constructed is unknown.

Intuitively, one always think of black box functions as obscure, because it is very hard to derive all properties that it has by simply looking at the input and the output. However, given infinite theorem proving capabilities, there are many black box functions which can be identified, under some simplifying assumptions, such as the function being deterministic and having a finite type.

What makes obscure functions special, is that they might be deterministic and have a finite type, and still remain obscure when put inside black boxes for any level of theorem proving capabilities.

When `f` is treated as a black box function, it is impossible to distinguish the following cases:

$$(\exists_p f)(x) = 0 \qquad <=> \qquad x : [\exists_p f]\ 0$$
$$(\exists_p f)(x) = \varepsilon \qquad <=> \qquad x : [\exists_p f]\ \varepsilon$$

These cases can not be distinguished because `$\exists_p f$` can only be approximated through sampling. In practice, any event which is infinitesimally unlikely is expected to never be observed. This makes sampling insufficient as a way of deriving the function `f`'s identity.

By Leibniz' law, when it is unknown whether a property of an object is true, the identity of the object is unknown. For the identity of two objects to be the same, their properties must also be the same. If it is known that the some property remains unknown, there exists at least two objects that satisfy all known properties. Hence, the identity of the object in question is not unique.

However, if `x : ∃f` then the ambiguity can be resolved:

$$x : \exists f \qquad <=> \qquad x : [\exists f]\ \text{true} \qquad <=> \qquad x : [\exists_p f]\ (>= \varepsilon)$$

$$[\exists_p f]\ (>= \varepsilon) \wedge (\ [\exists_p f]\ 0 \vee [\exists_p f]\ \varepsilon\ ) \qquad => \qquad [\exists_p f]\ \varepsilon$$

Therefore, the existential path of obscure functions **can not** be derived experimentally by treating them as black box functions.

This is an example where some epistomological consequence of probabilistic path semantics is derived for discrete path semantics. It can not be derived without the concept of a probabilistic existential path.