Natural Propositions

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In this paper I introduce Natural Propositions as an explanatory device for Path Semantical Logic.

Natural numbers[1] are often thought of as "positive", although they have no internal structure to describe positivity or negativity. This is because integers[2] are thought of as an extension of natural numbers with positivity and negativity.

Imagine that integers were invented before natural numbers, but we had no language to express positivity or negativity. This would make it difficult to think of natural numbers, because we would not be able to express the idea "like integers, but positive".

Propositions are the fundamental objects in Propositional Logic[3]. Path Semantical Logic[4] extends Propositional Logic with layers of propositions. The layers are connected with the core axiom of Path Semantics[5].

One strange thing about Path Semantical Logic is that in order to prove the following:

```
(F, G) (X, Y):
F(!X), G(Y)  =>  !X=Y
```

A third proposition `Z` must be added:

```
(F, G) (X, Y, Z):
F(!X), G(Y), Z=!X  =>  !X=Y
```

This problem leads to the idea of *Natural Propositions*.

The proposition `Z` is "natural" because it is an argument to the proof.

This way of thinking is a bit similar to the following reasoning about integers:

```
-x = y          If `y` is positive, then `-x` is positive.
```

Since propositions migth be thought of as "positive" in Path Semantical Logic, it can be confusing since "positive" could mean "true". In order to avoid misunderstanding, I suggest the phrase "natural".

Natural Propositions form a dominating/dominated logic, where naturality is propagated and dominates unnaturality. As notation I suggest Symmetric Avatar Paths[6], in order to derive rules and semantics:

```
? <=> ?[natural => natural_alignment]

?not            The behavior of `not` for Natural Propositions
?eq             The behavior of `eq` for Natural Propositions
```

References:

[1] "Natural number"
Wikipedia
https://en.wikipedia.org/wiki/Natural_number

[2] "Integer"
Wikipedia
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[3] "Propositional calculus"
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[4] "Path Semantical Logic"
AdvancedResearch – Reading sequence on Path Semantics
https://github.com/advancedresearch/path_semantics/blob/master/sequences.md#path-semantical-logic

[5] "Path Semantics"
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[6] "Symmetric Avatar Paths"
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