Closed Natural Numbers and The Fundamental Identity

by Sven Nilsen, 2020

In this paper I show that closed natural numbers is consistent with the fundamental identity.

Closed natural numbers is a stronger axiomatic system built on Peano axioms, where:

$1 + 1 + 1 + \ldots = 0$	$0 \neg= \infty$
$\|closed\_nat\| = \infty$	$(\infty : closed\_nat) = false$

In combinatorics, the fundamental identity is a generating function:

$$\sum n \infty \{ x^n \} = 1 / (1 - x)$$

Setting `x = 1`, the left hand side becomes:

$$\sum n \infty \{ x^n \} = 1 + 1 + 1 + \ldots$$

Setting `x = ∞ + 1`, the right hand side becomes:

$$1 / (1 - (\infty + 1)) = 1 / \infty \sim= 0$$

If one thinks about `∞ + a` as "looping around" for every `a`, then the fundamental identity shows:

$$1 + 1 + 1 + \ldots = 0$$

This means that closed natural numbers is consistent with the fundamental identity.

One can use this result to interpret the fundamental identity more accurately.

For diverging limits, the fundamental identity can be thought as "looping around" infinity.
With other words, the equality symbol `=` is interpreted a bit differently than normal.
For every `x = a` on the left side, the right side is interpreted as `x = ∞ + a`.
However, as everybody knows, `1 / (1 - (∞ + a)) ~= 0` for all `a >= 1`. How can this be true?

The answer is that for diverging limits, the fundamental identity encodes information at the infinitesimal level, which is close to `0`, such that when scaling up this information:

$$\lim y \to \infty \{ y \cdot (1 / (1 - (y + a)) \} \sim= 1 / (1 - a)$$

When setting `a = 1`, the information `(1 / (1 – (y + a))` becomes undecidable when scaled up.
However, undecidable means that one can decide this instead using the "looping around" trick:

The left side is interpreted with `x = 1`, the right side `x = ∞ + 1`, therefore `1 + 1 + 1 + … = 0`.