Incomplete Proof Search

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In this paper I outline an idea of incomplete proof search, based the idea behind Zeno's paradoxes. This idea can be applied to systems of reasoning that reasons about smarter versions of themselves.

Zeno's paradoxes is perhaps some of the most famous philosophical thought experiments in history. For thousands of years, they still engage philosophers and computer scientists.

Instead of using Zeno-like paradoxes to argue against some idea of e.g. motion or completion of a super-task, I would like to flip it inside out and argue from the assumption that:

1. Either the super-task is successful, or…
2. … a proof that the super-task fails is found in a finite number of steps

With other words, if a negative proof is not found within a finite number of steps, then one has a proof that the super-task is successful.

For example, in the Dichotomy paradox, Atlanta wishes to walk to the end of a path. Before she gets there, she must get halfway there. Before she can get halfway there, she must get a quarter of the way there, and so on. This generates a list of infinite tasks that must be completed.

Zeno thinks that completing an infinite list of tasks is impossible.

I on the other hand, using my assumptions, believe Atlanta can walk to the end of a path. There is no obstacle on the way, no matter how small, so no proof that Atlanta can't walk is found. This holds no matter how many times the path is divided into sub-paths.

Of course, my assumptions only works on the problem when there are no infinitely small obstacles.

Apparently, this solves the paradox. Yet, it is not the reason I use a Zeno's paradox as an example.

The problem is, one can not be absolutely sure that the super-task is successful. In order to be absolutely sure, one must have completed an infinite number of steps looking for the negative proof, hence performing a super-task on its own of theorem proving.

However, under logical uncertainty, the confidence that the super-task is possible, increases with the number of steps I checked. No matter what the super-task is, under my assumptions, the confidence is ordered, such that the confidence after `n+1` steps is a bit higher than the confidence after `n` steps. Why? Because it means that no negative proof was found when taking that step.

So, by choosing any finite number of steps, I can get arbitrarily confident in my conclusion.

Assuming that my maximum ability is `x` steps, I can reason about somebody who's maximum ability is `y` where `y > x`, but otherwise is identical to myself. This means that I can reason about somebody who are "smarter than" myself.