Proof of Equivalence

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In this paper I show the intuition behind proofs of equivalence.

A proof of equivalence is searching for a rule of the form `A ~> B` (called the "goal") using an axiomatic system for rules. An equivalence does not necessary mean that the two objects are syntactically equal, but that there exists an isomorphism between them.

Consider a simple system consisting of two axioms:

(X ~> Y) ∧ (Y ~> Z) → (X ~> Z)	Transport axiom for rules
(X ~> Y) → (_(X)_ ~> _(Y)_)	Context axiom for rules

Starting with a simple rule:

AB ~> BA

One can prove the following:

AABB ~> BBAA	Goal
A(AB)B ~> A(BA)B (AB)(AB) ~> (BA)(BA) B(AB)A ~> B(BA)A	Proof

Now, consider a larger example (reusing existing proofs):

AAABBB ~> BBBAAA	Goal
A(AABB)B ~> A(BBAA)B (AB)BA(AB) ~> (BA)BA(BA) B(ABAB)A ~> B(BBAA)A	Proof

The intution of proofs of equivalence is to show that a goal is true by constructing it, using the rules and axioms for producing new rules.

From the simple system above, one can create a more powerful rule of the kind:

A^nB^m ~> B^mA^n

This rule follows from the existing rules, but it is not *provable* within the existing system. However, one can show that it holds for any finite `n` and `m` by enumerating all rules of this kind and then show that there exists a proof for every case. Also, even though this rule is more powerful, it only handles goals of a particular form which is a special case of the whole system.