Absurdity of Binary Relations

by Sven Nilsen, 2022

In this paper I introduce a definition of absurdity of binary relations.

The absurdity of some binary relation[1] \`f\` is defined as following:

$$\text{absurdity} := \backslash(f : T \times T \to T) := \exists\, x \, \{ \forall\, y \, \{ f(x, y) \neg= x \} \land \exists\, y \, \{ f(f(x, y), y) \neg= x \} \}$$

Here, \`T\` is some type with equality[2] (strictly speaking it suffices to use partial equivalence[3]).

$$\text{eq} : T \times T \to \text{bool}$$

The absurdity can be used to distinguish \`imply\` from \`rimply\`[4]:

$$\begin{aligned}
&\text{imply} : [\text{absurdity}]\ \text{true} \\
&\text{rimply} : [\text{absurdity}]\ \text{false}
\end{aligned}$$

This can be used without knowing the concrete type of \`T\` and without knowing about truth or false values, nor even knowing about other operators within the same calculus. The algorithm works by picking out \`false\` that can be used to prove anything, but which is distinguished from \`true\`:

$$\exists\, x \, \{ \forall\, y \, \{ (x \Rightarrow y) \neg= x \} \land \exists\, y \, \{ ((x \Rightarrow y) \Rightarrow y) \neg= x \} \} \qquad f \Leftrightarrow \text{imply}$$

$$\begin{aligned}
&\forall\, y \, \{ (\text{false} \Rightarrow y) \neg= \text{false} \} \land \exists\, y \, \{ ((\text{false} \Rightarrow y) \Rightarrow y) \neg= \text{false} \} \qquad x == \text{false} \\
&\forall\, y \, \{ \text{true} \neg= \text{false} \} \land \exists\, y \, \{ (\text{true} \Rightarrow y) \neg= \text{false} \} \\
&\forall\, y \, \{ \text{true} \} \land \exists\, y \, \{ y \neg= \text{false} \} \\
&\text{true} \land \exists\, y : (= \text{true}) \, \{ y \neg= \text{false} \} \\
&\exists\, y : (= \text{true}) \, \{ \text{true} \} \\
&\text{true}
\end{aligned}$$

$$\begin{aligned}
&\forall\, y \, \{ (\text{true} \Rightarrow y) \neg= \text{true} \} \land \exists\, y \, \{ ((\text{true} \Rightarrow y) \Rightarrow y) \neg= \text{true} \} \qquad x == \text{true} \\
&\forall\, y \, \{ y \neg= \text{true} \} \land \exists\, y \, \{ (y \Rightarrow y) \neg= \text{true} \} \\
&\forall\, y \, \{ y \neg= \text{true} \} \land \exists\, y \, \{ \text{true} \neg= \text{true} \} \\
&\forall\, y \, \{ y \neg= \text{true} \} \land \exists\, y \, \{ \text{false} \} \\
&\forall\, y \, \{ y \neg= \text{true} \} \land \text{false} \\
&\text{false}
\end{aligned}$$

$$\exists\, x \, \{ \forall\, y \, \{ (x \Leftarrow y) \neg= x \} \land \exists\, y \, \{ ((x \Leftarrow y) \Leftarrow y) \neg= x \} \} \qquad f \Leftrightarrow \text{rimply}$$

$$\begin{aligned}
&\forall\, y \, \{ (\text{false} \Leftarrow y) \neg= \text{false} \} \land \exists\, y \, \{ ((\text{false} \Leftarrow y) \Leftarrow y) \neg= \text{false} \} \qquad x = \text{false} \\
&\forall\, y : (= \text{true}) \, \{ (\text{false} \Leftarrow \text{true}) \neg= \text{false} \} \land \exists\, y : (= \text{false}) \, \{ ((\text{false} \Leftarrow \text{false}) \Leftarrow \text{false}) \neg= \text{false} \} \\
&\forall\, y : (= \text{true}) \, \{ \text{false} \neg= \text{false} \} \land \exists\, y : (= \text{false}) \, \{ (\text{true} \Leftarrow \text{false}) \neg= \text{false} \} \\
&\forall\, y : (= \text{true}) \, \{ \text{false} \} \land \exists\, y : (= \text{false}) \, \{ \text{true} \neg= \text{false} \} \\
&\text{false} \land \exists\, y : (= \text{false}) \, \{ \text{true} \} \\
&\text{false}
\end{aligned}$$

$$\begin{aligned}
&\forall\, y \, \{ (\text{true} \Leftarrow y) \neg= \text{true} \} \land \exists\, y \, \{ ((\text{true} \Leftarrow y) \Leftarrow y) \neg= \text{true} \} \qquad x == \text{true} \\
&\forall\, y \, \{ \text{true} \neg= \text{true} \} \land \exists\, y \, \{ (\text{true} \Leftarrow y) \neg= \text{true} \} \\
&\forall\, y \, \{ \text{false} \} \land \exists\, y \, \{ \text{true} \neg= \text{true} \} \\
&\forall\, y \, \{ \text{false} \} \land \exists\, y \, \{ \text{false} \} \\
&\forall\, y \, \{ \text{false} \} \land \text{false} \\
&\text{false}
\end{aligned}$$

References:

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