Countable Infinity from Existential Paths

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In this paper I show how to prove when a function's decidability does not exceed countable infinity.

The function `f` decides on set `a` of size up to countable infinity, but no more, when:

$$\exists f <=> a \setminus b$$
$$|b| >= 1$$

$$f : a \to T$$
$$a : T \to bool$$
$$b : T \to bool$$

Notation:

`∃f`	existential path of `f`
`a \ b`	`a` except `b`
`|b|`	size of `b`

All members of `a` is mapped by `f` to `a \ b`. If this set is finite, then at least two members of `a` is mapped to the same output. Maximum decidability of `f` is when all members are mapped uniquely. In order to map every member of `a` to some unique member of `a \ b`, the set of `a` must be infinite.

Assume that `a` is infinite. By constraining `f` with `a \ b`:

$$\exists f\{a \setminus b\} <=> a \setminus b \setminus f(b)$$

This works because when the `b` is not allowed as input and `f` maps to unique outputs, there must be one map that is missing: The output `f(b)`.

By constraining `f` with `a \ b \ f(b)`:

$$\exists f\{a \setminus b \setminus f(b)\} <=> a \setminus b \setminus f(b) \setminus f(f(b))$$

Repeating this process:

$$\exists f\{a \setminus b \setminus f(b) \setminus \setminus f^2(b) \dots \setminus f^n(b)\} <=> a \setminus b \setminus f(b) \setminus f^2(b) \setminus f^3(b) \setminus \dots \setminus f^{n+1}(b)$$

Since `f` maps to unique outputs, there is no case where `$f^n(b) = f^m(b)$` when `n ¬= m`.
The entire segmented sub-sets of `a` by `f` can be counted by repeating this process.
It is kind of like counting with numbers, except you count like this: `(>= 0), (>= 1), (>= 2) ...`.
Since `f` decides either finite or countable infinite, it can only decide up to countable infinite in size.
However, there can be chunks of `a`'s interpretation as a set, which exceed countable infinity.