Answered Modal Logic in Cubical Binary Codes

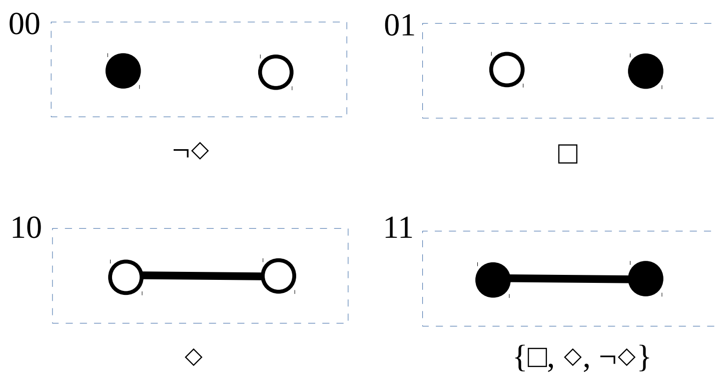
by Sven Nilsen, 2020

In this paper I describe the simplest case of encoding Answered Modal Logic in Cubical Binary Codes.

Answered Modal Logic has a modal set consisting of 3 elements:

{□, ◇, ¬◇}

The smallest Cubical Binary Code that fits this set is the following:



The reason for this assignment is because \`□\` and \`¬◇\` are natural opposites:

00	□	Means that for all cases, the predicate returns \`true\`
01	¬◇	Means that there exists no case for which the predicate returns \`true\`

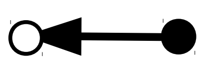
To clarify, the cases here refers to context, not inputs of the predicate.
The logic is for reasoning about the predicate across possible worlds, hence the modality.

The closest one gets to assigning \`◇\` naturally is 10:

10	◇	Means that there exists a case for which the predicate returns \`true\`

Technically, if one modeled this as an interval, then one can say it includes \`□\` at one end:

◇ = (¬◇, □]



One can think about \`◇\` as an arrow

The semantics of the code \`11\` is naturally refering to the whole modal set:

{□, ◇, ¬◇}

This encoding satisfies the requirement that the building block of the logic can not fully model sets.
There is no way to model the empty modal set \`{}\`, hence a full model of a set is impossible to express.