

Listing-Möbius Shifts

by Sven Nilsen, 2021

In this paper I introduce two simple shift functions on lists of objects with involutions, that have some interesting mathematical properties when applied to higher dimensional mathematical objects.

An involution^[1] is a function that is its own inverse:

$$\text{inv} \cdot \text{inv} \Leftrightarrow \text{id}$$

$$\text{inv} : T \rightarrow T$$

Assuming some involution, left and right Listing-Möbius shifts are two functions:

$$\text{lmob} : [T] \rightarrow [T]$$

Left Listing-Möbius shift

$$\text{rmob} : [T] \rightarrow [T]$$

Right Listing-Möbius shift

Left Listing-Möbius shift pops the first item in the list and pushes its inverse:

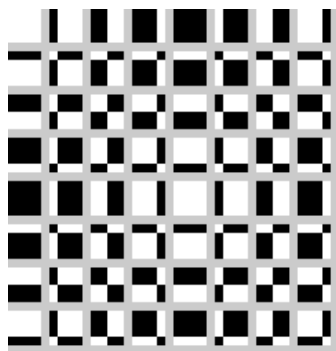
$$\text{lmob}([a, b, c, d]) = [b, c, d, \text{inv}(a)]$$

Right Listing-Möbius shift pops the last item in the list and pushes its inverse to the front:

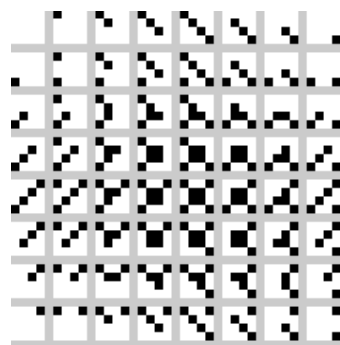
$$\text{rmob}([a, b, c, d]) = [\text{inv}(d), a, b, c]$$

The name “Listing-Möbius” is in honour of the German mathematicians Johann Benedict Listing and August Ferdinand Möbius, who are attributed the independent discovery of the Möbius strip in 1858^[2].

Listing-Möbius shifts can be used to study higher dimensional mathematical structures, because the embedded rules commute in a way that might be related to symmetries:



Row/columns go together



Opposite diagonals go together

*However, e.g. row vs diagonal does not go together!
Open problem: Is this related to symmetries somehow?*

References:

- [1] “Involution (mathematics)”
Wikipedia
[https://en.wikipedia.org/wiki/Involution_\(mathematics\)](https://en.wikipedia.org/wiki/Involution_(mathematics))

- [2] “Möbius strip”
Wikipedia
https://en.wikipedia.org/wiki/M%C3%B6bius_strip