# Derivative

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*In this paper I describe the notation for derivative of functions in standard path semantics.*

The notation for the derivative in traditional mathemantics has always been messy, because of the intuition e.g. “with respect to some variable `x`” when other variables are held constant.

To make this precise, one can look at the `line` function:

$\because \quad line_A := \backslash(a : A, b : A) = \backslash(x : real) = x * (b - a) + a$

The variables `a` and `b` are captured by the closure, such that they are constants in the function:

$\therefore \quad line_A(a, b) : real \rightarrow A$

The derivative is a higher order function defined as following:

$\because \quad d_A(f : real \rightarrow A) = \backslash(x : real) = \lim h \rightarrow 0 \; \{ \; (f(x + h) - f(x)) \; / \; h \; \}$

$\therefore \quad d_A : (real \rightarrow A) \rightarrow (real \rightarrow A)$

The `A` is a type and should not be confused with a variable.

For example, in Euler’s notation for the derivative operator:

D <=> d One can exchange `D` with `d` when `D` is not annotated

However, when annotating `D` with some variable for clarification, e.g. `$D_x$`:

$D_x \; f \iff d_T(f)$

$f : real \rightarrow T$

This notation is invariant with respect to the name of the argument.

Also, notice that `$d_T$` is applied to `f`. If `$d_T$` was composed with `f`, then `f` would be higher order:

$\because \quad f : U \rightarrow (real \rightarrow T)$
$\therefore \quad d_T \cdot f : U \rightarrow (real \rightarrow T)$

To find the n-th derivative with respect to `x` of `f(a, b, x)` one can use function currying:

$\therefore \quad d^n/dx^n \; f \iff d_T{}^n((\; \backslash(a, b) = \backslash(x) = f(a, b, x) \;)(a, b)) \iff d_T{}^n(f(a, b))$

$\because \quad f : A \times B \times real \rightarrow T \qquad a : A \qquad b : B$