Skolem and Herbrand Normal Form

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A sentence in first-order logic can be transformed to Skolem normal form through a process called "skolemization". The Skolem normal form is satisfiable if and only if the sentence is satisfiable:

$$\forall\ y, x\ \{\ p(x, y) \rightarrow p(s(y), y)\ \}$$

Which implies:

$$\forall\ y\ \{\ \exists\ x\ \{\ p(x, y)\ \} = p(s(y), y)\ \}$$

The trick is that if \`x\` exists such that \`p\` returns \`true\` when second argument is \`y\`, then one can produce an example of \`x\`. However, if there exists no \`x\` such that \`p\` treturns \`true\` when second argument is \`y\`, then it does not matter which \`x\` one uses.

Similarly, a sentence in first-order logic can be transformed to Herbrand normal form through a processed called "herbrandization". The Herbrand normal form is satisfiable if and only if the sentence is satisfiable:

$$\forall\ y, x\ \{\ \neg p(x, y) \rightarrow \neg p(h(y), y)\ \}$$

Which implies:

$$\forall\ y\ \{\ \forall\ x\ \{\ p(x, y)\ \} = p(h(y), y)\ \}$$

The trick is that if \`p\` returns \`true\` for all \`x\` when second argument is \`y\`, then it is not possible to produce a counter-example of \`x\`, so it does not matter which \`x\` one uses. However, if \`p\` returns \`false\` for some \`x\` when second argument is \`y\`, then one can produce a counter-example of \`x\`.