Permutative Binary Numbers

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In this paper I represent a number system that extends the binary numbers with permutations of the positions of non-zero bits. Using this system, I show there is a close mathematical connection between sub-groups of permutations and non-zero bits of binary numbers. The combined system can be used to reason about one system in terms of the other, making some proofs in number theory easier.

In Location arithmetic, an represenation of binary numbers, the powers of 2 are assigned a letter each:

a = 1
b = 2
c = 3
d = 4
e = 5
…

For example, the number \`7\` can be written "abc", but also "acb" and every permutation. All permutations of letters are equal.

What if the binary numbers were extended such that these permutations were ordered? It would correspond to permuting the powers of 2 of non-zero bits:

00000	0	.										
00001	1	a										
00010	2	b										
00011	3	ab	ba									
00100	4	c										
00101	5	ac	ca									
00110	6	bc	cb									
00111	7	abc	acb	bac	bca	cab	cba					
01000	8	d										
01001	9	ad	da									
01010	10	bd	db									
01011	11	abd	adb	bad	bda	dab	dba					
01100	12	cd	dc									
01101	13	acd	adc	cad	cda	dac	dca					
01110	14	bcd	bdc	cbd	cdb	dbc	dcb					
01111	15	abcd	abdc	acbd	acdb	adbc	adcb	bacd	badc	bcad	bcda	…
10000	16	e										
…												

Notice that the sub-groups of permutations occur previously defined for some number. This is because when you subtract a bit, you end up with a binary number for which the sub-group must be defined:

00111 – 00001 = 00110 "6" is sub-group of "7a" since \`7 – 1(a) = 6\`

By grouping sub-permutations by recursive sequences, one gets the following pattern:

000000	0	.					
000001	1	a					
000010	**2**		b				
000011	**3**	a2	b1				
000100	4			c			
000101	**5**	a4		c1			
000110	6		b4	c2			
000111	**7**	a6	b5	c3			
001000	8				d		
001001	9	a8			d1		
001010	10		b8		d2		
001011	**11**	a10	b9		d3		
001100	12			c8	d4		
001101	**13**	a12		c9	d5		
001110	14		b12	c10	d6		
001111	15	a14	b13	c11	d7		
010000	16					e	
010001	**17**	a16				e1	
010010	18		b16			e2	
010011	**19**	a18	b17			e3	
010100	20			c16		e4	
010101	21	a20		c17		e5	
010110	22		b20	c18		e6	
010111	**23**	a22	b21	c19		e7	
011000	24				d16	e8	
011001	25	a24			d17	e9	
011010	26		b24		d18	e10	
011011	27	a26	b25		d19	e11	
011100	28			c24	d20	e12	
011101	**29**	a28		c25	d21	e13	
011110	30		b28	c26	d22	e14	
011111	**31**	a30	b29	c27	d23	e15	
100000	32						f
100001	33	a32					f1
100010	34		b32				f2
100011	35	a34	b33				f3
100100	36			c32			f4
100101	**37**	a36		c33			f5
100110	38		b36	c34			f6
100111	**39**	a38	b37	c35			f7
101000	40				d32		f8
101001	**41**	a40			d33		f9
101010	42		b40		d34		f10
101011	**43**	a42	b41		d35		f11
101100	44			c40	d36		f12
101101	45	a44		c41	d37		f13
101110	46		b44	c42	d38		f14
101111	**47**	a46	b45	c43	d39		f15
110000	48					e32	f16
110001	49	a48				e33	f17
110010	50		b48			e34	f18
110011	**51**	a50	b49			e35	f19
110100	52			c48		e36	f20
110101	**53**	a52		c49		e37	f21
110110	54		b52	c50		e38	f22
110111	55	a54	b53	c51		e39	f23
111000	56				d48	e40	f24
111001	**57**	a56			d49	e41	f25
111010	58		b56		d50	e42	f26
111011	**59**	a58	b57		d51	e43	f27
111100	60			c56	d52	e44	f28
111101	**61**	a60		c57	d53	e45	f29
111110	62		b60	c58	d54	e46	f30
111111	**63**	a62	b61	c59	d55	e47	f31

For every non-zero bit, there is a sub-group, which follows the frequency of that bit. When it is turned on, the sub-group appears, relative to the sum. When it is turned off, the sub-group disappears. Also, notice that many primes have a sub-group that is a prime.