Ourobra Tables

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In this paper I introduce a natural way of constructing algebraic tables with involutions.

An Ourobra Table is an algebraic table (e.g. addition or multiplication) constructed using the pattern:

00	01
10	11

Where \`0\` and \`1\` describes the two sides of an involution.
The name "Ourobra" comes from "ouroboros", an ancient symbol of a serpent eating its own tail[1].

For example, given an involution between \`a\` and \`b\` gives a standard Cayley table[2]:

	a	**b**
a	aa	ab
b	ba	bb

This table can now be used to construct a larger table with negation:

	a	**b**	**-a**	**-b**
a	aa	ab	a(-a)	a(-b)
b	ba	bb	b(-a)	b(-b)
-a	(-a)a	(-a)b	(-a)(-a)	(-a)(-b)
-b	(-b)a	(-b)b	(-b)(-a)	(-b)(-b)

For addition, assuming \`∀ a, b { -(a + b) = (-a) + (-b) }\`:

	a	**b**	**-a**	**-b**
a	a+a	a+b	a+(-a)	a+(-b)
b	b+a	b+b	b+(-a)	b+(-b)
-a	-(a+(-a))	-(a+(-b))	-(a+a)	-(a+b)
-b	-(b+(-a))	-(b+(-b))	-(b+a)	-(b+b)

For multiplication, assuming \`∀ a, b { -a*b = (-a)*b = a*(-b) }\`:

	a	**b**	**-a**	**-b**
a	a*a	a*b	-a*a	-a*b
b	b*a	b*b	-b*a	-b*b
-a	-a*a	-a*b	a*a	a*b
-b	-b*a	-b*b	b*a	b*b

Notice that this gives a nice pattern which makes it easy to visualise how the upper 2x2 matrix is copied and modified to construct the rest of the matrix. It is not suitable for encoding the full information of a specific algebra, but works for reasoning about abstract algebras with involutions.

References:

[1] "Ouroboros"
Wikipedia
https://en.wikipedia.org/wiki/Ouroboros

[2] "Cayley table"
Wikipedia
https://en.wikipedia.org/wiki/Cayley_table