Constrained Implication Theorem

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In this paper I present a constrained implication theorem found in Path Semantical Logic.

The Constrained Implication Theorem is a proof in Path Semantical Logic[1]:

(a, b) (T, U):
a=>b, a(T)=>b(U) => T=>U

Where the tuple `(a, b)` has level 1 and the tuple `(T, U)` has level 0.
The notation `a(T)` means `a=>T` where `T` is at a lower level.

The Constrained Implication Theorem is similar to the Abstract Implication Theorem[2], except that the assumption `a(T)=b(U)` is weakened to `a(T)=>b(U)`.

The Constrained Implication Theorem says that implications carries over to lower levels, over implications of associations, even if the associations themselves, not even their equalities, are asserted formally. It is sufficient that the implications of associations are asserted.

One can think about the 3 implication theorems as a hierarcy:

1. The Constrained Implication Theorem
2. The Abstract Implication Theorem[2]
3. The (Normal) Implication Theorem[3]

The 1) theorem implies 2), which both in turn implies 3).

The Constrained Implication Theorem has similar expressive power to the Abstract Implication Theorem, which is shown by a slightly more generalized version:

$(a, b_1, b_2)\ (T, U_1, U_2)$:
$a \Rightarrow (b_1 \vee b_2),\ a(T) \Rightarrow (b_1(U_1) \wedge b_2(U_2)) \;\;\Rightarrow\;\; T \Rightarrow (U_1 \vee U_2)$

The premise `$a(T) \Rightarrow (b_1(U_1) \wedge b_2(U_2))$` is necessary to carry over the relationship `∨`.
This is not provable using `$a(T) \Rightarrow (b_1(U_1) \vee b_2(U_2))$` instead.

References:

[1] "Path Semantical Logic"
AdvancedResearch, reading sequence on Path Semantics
https://github.com/advancedresearch/path_semantics/blob/master/sequences.md#path-semantical-logic

[2] "Abstract Implication Theorem"
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https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/abstract-implication-theorem.pdf

[3] "Implication Theorem"
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https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/implication-theorem.pdf