Binary Square Matrix Combinatorics

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In this paper I formalize binary square matrix combinatorics using Directional Set Algebra.

For all `n : nat`, there is an associated binary square matrix combinatorics:

0	Empty matrix set
D	Diagonal matrix set
U	Upper triangle matrix set
L	Lower triangle matrix set
1	All matrices

The following law holds with Directional Set Algebra:

D + U + L => 1

Sizes of sets, since they share the zero matrix, gets subtracted one when added together:

$|x + y| = |x| + |y| - 1$ if $|x| > 0 \land |y| > 0$

$|0| = 0$
$|D| = 2^n$
$|1| = 2^{(n \cdot n)}$
$|U| = |L| = 2^{(n \cdot (n - 1) / 2)}$

Sub-types of binary matrix sets can be constructed using elements `0`, `1` and `?`.
The following laws holds with Directional Set Algebra, where `?` is top and there is no bottom:

0 + 1 = ?

For example, for `n = 3`:

```
? 0 0        0 ? ?        0 0 0        ? ? ?
0 ? 0   +    0 0 ?   +    ? 0 0   =>   ? ? ?
0 0 ?        0 0 0        ? ? 0        ? ? ?
```