Answered Modal Logic

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In this paper I introduce a modal logic for the answered predicate of questions.

The meta-knowledge of the answer of a question can be modeled using a set of the following symbols:

{!◇, ¬!◇, □}	All possible states of knowledge about question (*unknown* **unanswered**)
□	The question is answered (*known* **answered**)
¬!◇	There exists two cases, one answered and one unanswered (*unknown* **answered**)
!◇	There exists no case where the question is answered (*known* **unanswered**)

Inversion laws:

¬□ = {!◇, ¬!◇, □} ∀ x { !!x = x } ∀ x { ¬¬x = x } ∀ x { ¬!x = !¬x }

The symbols \`◇\` and \`□\` can be interpreted as "possible" and "necessary" as in classical Modal Logic.
The \`!\` operator corresponds to the associated classical Modal Logic inversion operator.

When a law is in the form \`X => {!◇, ¬!◇, □}Y\` one can choose:

◇(X => □Y)
◇(X => ¬!◇Y)
◇(X => !◇Y)

Here, the \`◇\` operator reflects on the semantics of the logic itself.
When used this way, it is not an operator of questions directly, but as a meta-operator.

Notice that this logic deviates from epistemic modal logic, which uses semantics "it is known *that* X".
Here, the logic refers to the knowledge of the answer, without describing what the answer is.

For example:

□"A ∧ B" => ¬!◇"A"

This can be read as "If I know value of \`A ∧ B\`, then there exists two knowledge cases of \`A\`".

In general, the internal semantics of the questions is irrelevant for this logic.

Instead, the questions are treated as black boxes, with partial knowledge described e.g. in the form:

□X => ¬!◇Y

It is the partial knowledge described using this modal logic that can derive other partial knowledge.
The internal semantics of the questions is only relevant for grounding the initial partial knowledge.

I will now prove the following:

∴ (□X => ¬!◇Y) => ◇(¬!◇X => {¬!◇, □}Y)

∵ □X => ¬!◇Y
∴ ¬◇Y => ¬□X
∴ ¬◇Y => {!◇, ¬!◇, □}X
∴ ◇(¬◇Y => !◇X) Choosing `!◇` among possible interpretations
∴ ◇(¬!◇X => ◇Y)
∴ ◇(¬!◇X => ¬!◇Y ∨ □Y)
∴ ◇(¬!◇X => {¬!◇, □}Y)
∴ Q.E.D.

When choosing the other possible interpretation:

∴ ◇(¬◇Y => ¬!◇X)
∴ ◇(!◇X => ◇Y)
∴ ◇(!◇X => ¬!◇Y ∨ □Y)
∴ ◇(!◇X => {¬!◇, □}Y)

∴ (□X => ¬!◇Y) => ◇(!◇X => {¬!◇, □}Y)

These two possible interpretations are not contradictory.
When there exists a case where `¬!◇X` (*unknown* **answered**) implies `{¬!◇, □}Y` (`◇Y`),
it also possible that when `!◇X` (*known* **unanswered**) that `{¬!◇, □}Y`.
Answering `X` or not does not change the answer of `Y`.