Higher Order De Morgans' Laws for Unary Binary Functions

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In this paper I represent a higher order version of De Morgan's laws for unary binary functions.

Given two unary binary functions `f` and `g`, their symmetric paths by `not` have the following laws:

f[not] ∧ g[not] <=> (f ∨ g)[not]

f[not] ∨ g[not] <=> (f ∧ g)[not]

f : bool → bool
g : bool → bool

The laws were checked using an experimental automated theorem prover:

false1[not] ∧ false1[not]	<=>	(false1 ∨ false1)[not]	ok
false1[not] ∧ not[not]	<=>	(false1 ∨ not)[not]	ok
false1[not] ∧ id[not]	<=>	(false1 ∨ id)[not]	ok
false1[not] ∧ true1[not]	<=>	(false1 ∨ true1)[not]	ok
not[not] ∧ false1[not]	<=>	(not ∨ false1)[not]	ok
not[not] ∧ not[not]	<=>	(not ∨ not)[not]	ok
not[not] ∧ id[not]	<=>	(not ∨ id)[not]	ok
not[not] ∧ true1[not]	<=>	(not ∨ true1)[not]	ok
id[not] ∧ false1[not]	<=>	(id ∨ false1)[not]	ok
id[not] ∧ not[not]	<=>	(id ∨ not)[not]	ok
id[not] ∧ id[not]	<=>	(id ∨ id)[not]	ok
id[not] ∧ true1[not]	<=>	(id ∨ true1)[not]	ok
true1[not] ∧ false1[not]	<=>	(true1 ∨ false1)[not]	ok
true1[not] ∧ not[not]	<=>	(true1 ∨ not)[not]	ok
true1[not] ∧ id[not]	<=>	(true1 ∨ id)[not]	ok
true1[not] ∧ true1[not]	<=>	(true1 ∨ true1)[not]	ok
false1[not] ∨ false1[not]	<=>	(false1 ∧ false1)[not]	ok
false1[not] ∨ not[not]	<=>	(false1 ∧ not)[not]	ok
false1[not] ∨ id[not]	<=>	(false1 ∧ id)[not]	ok
false1[not] ∨ true1[not]	<=>	(false1 ∧ true1)[not]	ok
not[not] ∨ false1[not]	<=>	(not ∧ false1)[not]	ok
not[not] ∨ not[not]	<=>	(not ∧ not)[not]	ok
not[not] ∨ id[not]	<=>	(not ∧ id)[not]	ok
not[not] ∨ true1[not]	<=>	(not ∧ true1)[not]	ok
id[not] ∨ false1[not]	<=>	(id ∧ false1)[not]	ok
id[not] ∨ not[not]	<=>	(id ∧ not)[not]	ok
id[not] ∨ id[not]	<=>	(id ∧ id)[not]	ok
id[not] ∨ true1[not]	<=>	(id ∧ true1)[not]	ok
true1[not] ∨ false1[not]	<=>	(true1 ∧ false1)[not]	ok
true1[not] ∨ not[not]	<=>	(true1 ∧ not)[not]	ok
true1[not] ∨ id[not]	<=>	(true1 ∧ id)[not]	ok
true1[not] ∨ true1[not]	<=>	(true1 ∧ true1)[not]	ok