

Higher Order De Morgan's Laws for Unary Binary Functions

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In this paper I represent a higher order version of De Morgan's laws for unary binary functions.

Given two unary binary functions `f` and `g`, their symmetric paths by `not` have the following laws:

$$f[\text{not}] \wedge g[\text{not}] \quad \Leftrightarrow \quad (f \vee g)[\text{not}]$$

$$f[\text{not}] \vee g[\text{not}] \quad \Leftrightarrow \quad (f \wedge g)[\text{not}]$$

$f : \text{bool} \rightarrow \text{bool}$

$g : \text{bool} \rightarrow \text{bool}$

The laws were checked using an experimental automated theorem prover:

false1[not] \wedge false1[not]	\Leftrightarrow	(false1 \vee false1)[not]	ok
false1[not] \wedge not[not]	\Leftrightarrow	(false1 \vee not)[not]	ok
false1[not] \wedge id[not]	\Leftrightarrow	(false1 \vee id)[not]	ok
false1[not] \wedge true1[not]	\Leftrightarrow	(false1 \vee true1)[not]	ok
not[not] \wedge false1[not]	\Leftrightarrow	(not \vee false1)[not]	ok
not[not] \wedge not[not]	\Leftrightarrow	(not \vee not)[not]	ok
not[not] \wedge id[not]	\Leftrightarrow	(not \vee id)[not]	ok
not[not] \wedge true1[not]	\Leftrightarrow	(not \vee true1)[not]	ok
id[not] \wedge false1[not]	\Leftrightarrow	(id \vee false1)[not]	ok
id[not] \wedge not[not]	\Leftrightarrow	(id \vee not)[not]	ok
id[not] \wedge id[not]	\Leftrightarrow	(id \vee id)[not]	ok
id[not] \wedge true1[not]	\Leftrightarrow	(id \vee true1)[not]	ok
true1[not] \wedge false1[not]	\Leftrightarrow	(true1 \vee false1)[not]	ok
true1[not] \wedge not[not]	\Leftrightarrow	(true1 \vee not)[not]	ok
true1[not] \wedge id[not]	\Leftrightarrow	(true1 \vee id)[not]	ok
true1[not] \wedge true1[not]	\Leftrightarrow	(true1 \vee true1)[not]	ok
false1[not] \vee false1[not]	\Leftrightarrow	(false1 \wedge false1)[not]	ok
false1[not] \vee not[not]	\Leftrightarrow	(false1 \wedge not)[not]	ok
false1[not] \vee id[not]	\Leftrightarrow	(false1 \wedge id)[not]	ok
false1[not] \vee true1[not]	\Leftrightarrow	(false1 \wedge true1)[not]	ok
not[not] \vee false1[not]	\Leftrightarrow	(not \wedge false1)[not]	ok
not[not] \vee not[not]	\Leftrightarrow	(not \wedge not)[not]	ok
not[not] \vee id[not]	\Leftrightarrow	(not \wedge id)[not]	ok
not[not] \vee true1[not]	\Leftrightarrow	(not \wedge true1)[not]	ok
id[not] \vee false1[not]	\Leftrightarrow	(id \wedge false1)[not]	ok
id[not] \vee not[not]	\Leftrightarrow	(id \wedge not)[not]	ok
id[not] \vee id[not]	\Leftrightarrow	(id \wedge id)[not]	ok
id[not] \vee true1[not]	\Leftrightarrow	(id \wedge true1)[not]	ok
true1[not] \vee false1[not]	\Leftrightarrow	(true1 \wedge false1)[not]	ok
true1[not] \vee not[not]	\Leftrightarrow	(true1 \wedge not)[not]	ok
true1[not] \vee id[not]	\Leftrightarrow	(true1 \wedge id)[not]	ok
true1[not] \vee true1[not]	\Leftrightarrow	(true1 \wedge true1)[not]	ok