Normal Path Diagrams

by Sven Nilsen, 2020

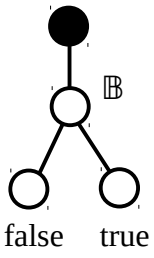
In this paper I present a diagram notation for normal paths based on Avatar Graphs.

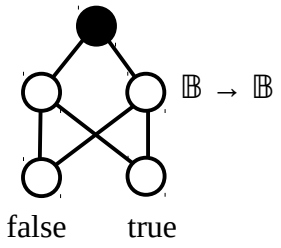
A normal path[1] diagram is an abstract representation of what a normal path means.
It is based on the same diagram notation as for Avatar Graphs[2], but using the following rules instead:

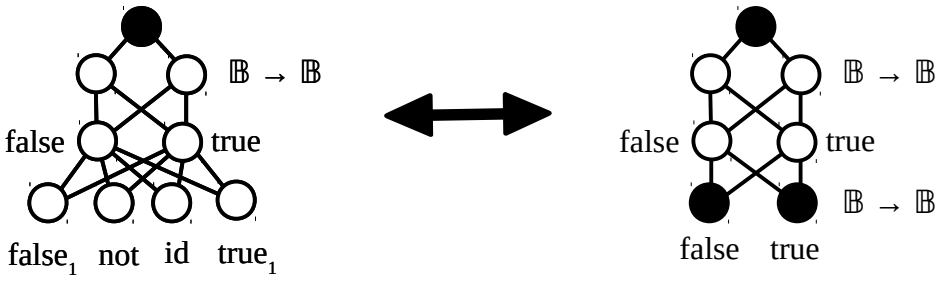
- Nodes are order from left to right, top to bottom
- There is a black node at the top
- Edges connecting white nodes to a black node above form a function type[3]
- Edges connecting white nodes to a white node above form a sum type[4]
- Edges connecting black nodes to a white nodes above form a function family
- All edges are connecting two nodes at levels `n` and `n+1`, no other edges are valid

The set of valid paths from the top to the bottom is an instance of a normal path.
In the case the paths crosses black nodes (a function family), one can expand into white nodes.

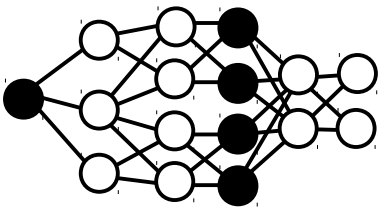
Some examples:







A function family is a compact way of representing all possible functions of a type. Instead of listing every function in the family, one simply uses a black node for every possible output.



One can also draw the diagram sideways. Here there is a binary function where the second argument is a union type of the first argument and the output. There are two function families isomorphic to booleans which elements are isomorphic to the family `𝔹 → 𝔹`.

References:

[1] “Normal Paths”
Sven Nilsen, 2019
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/normal-paths.pdf

[2] “Avatar Graphs”
Sven Nilsen, 2020
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/avatar-graphs.pdf

[3] “Function type”
Wikipedia
https://en.wikipedia.org/wiki/Function_type

[4] “Tagged union”
Wikipedia
https://en.wikipedia.org/wiki/Tagged_union