## **Permutative Binary Numbers**

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In this paper I represent a number system that extends the binary numbers with permutations of the positions of non-zero bits. Using this system, I show there is a close mathematical connection between sub-groups of permutations and non-zero bits of binary numbers. The combined system can be used to reason about one system in terms of the other, making some proofs in number theory easier.

In Location arithmetic, an representaion of binary numbers, the powers of 2 are assigned a letter each:

a = 1 b = 2 c = 3 d = 4 e = 5

For example, the number `7` can be written "abc", but also "acb" and every permutation. All permutations of letters are equal.

What if the binary numbers were extended such that these permutations were ordered? It would correspond to permuting the powers of 2 of non-zero bits:

```
00000 0
00001 1
             a
00010 2
             b
00011 3
             ab
                   ba
00100 4
             C
00101 5
             ac
                   ca
00110 6
             bc
                   cb
00111 7
                   acb
                          bac
                                bca
                                             cba
             abc
                                       cab
01000 8
             d
01001 9
             ad
                   da
01010 10
             bd
                   db
01011 11
                   adb
                                             dba
             abd
                          bad
                                bda
                                       dab
01100 12
             cd
                   dc
01101 13
                   adc
                                       dac
             acd
                          cad
                                cda
                                             dca
01110 14
             bcd
                   bdc
                          cbd
                                cdb
                                       dbc
                                             dcb
01111 15
             abcd
                   abdc
                         acbd
                                acdb
                                       adbc
                                             adcb
                                                    bacd badc bcad bcda
10000 16
             e
```

Notice that the sub-groups of permutations occur previously defined for some number. This is because when you subtract a bit, you end up with a binary number for which the sub-group must be defined:

```
00111 - 00001 = 00110 "6" is sub-group of "7a" since 7 - 1(a) = 6
```

By grouping sub-permutations by recursive sequences, one gets the following pattern:

000000	0						
000001	1	a					
000011	2	•	ь				
			b				
000011	3	a2	<b>b</b> 1	_			
000100	4			C			
000101	5	a4		c1			
		u-r	L 4	-3			
000110	6	_	b4	c2			
000111	7	a6	<b>b</b> 5	c3	_		
001000	8				d		
001001	9	a8			d1		
001011	10	uo	LO.		d2		
		4.0	b8				
001011	11	a10	<b>b9</b>		d3		
001100	12			c8	d4		
001101	13	a12		c9	d5		
001110	14		<b>b12</b>	c10	d6		
		4.4					
001111	15	a14	<b>b13</b>	<b>c11</b>	<u>d7</u>	_	
010000	16					e	
010001	17	a16				e1	
010010	18		<b>b</b> 16			62	
		-10				e2 e3	
010011	19	a18	<b>b</b> 17	_		es	
010100	20			c16		e4	
010101	21	a20		c17		e5	
010110	22		b20	c18		<mark>e6</mark>	
010111	23	a22	b21	c19		e7	
		dZZ	021	C19	140		
011000	24				d16	e8	
011001	25	a24			<b>d17</b>	e9	
011010	26		b24		d18	e10	
011011	27	a26	b25		d19	e11	
		420	023	-24	d20		
011100	28			c24		e12	
011101	29	a28		c25	d21	e13	
011110	30		b28	c26	<b>d22</b>	e14	
011111	31	a30	b29	c27	<b>d23</b>	e15	
100000	32						f
100001	33	a32					f1
		d32	1.22				
100010	34		b32				f2
100011	35	a34	<b>b33</b>				f3
100100	36			<b>c32</b>			f4
100101	37	a36		c33			f5
100110	38		<b>b</b> 36	c34			f6
		-20					f7
100111	39	a38	<b>b</b> 37	c35			
101000	40				d32		f8
101001	41	a40			<b>d33</b>		f9
101010	42		<b>b40</b>		d34		f10
101011	43	a42	b41		d35		f11
		a42	041	-40			
101100	44	-		c40	d36		f12
101101	45	a44		c41	d37		f13
404440							f14
101110	46		b44	c42	d38		C4 =
		a46					TIS
101111	47	a46	b44 b45	c42 c43	d38 d39	<u>.21</u>	f15
101111 110000	<b>47</b> 48					e32	f16
101111 110000 110001	<b>47</b> 48 49	a46 a48	b45			<mark>e33</mark>	f16 f17
101111 110000	<b>47</b> 48		b45 b48			e33 e34	f16
101111 110000 110001 110010	<b>47</b> 48 49 50	a48	b45 b48			e33 e34	f16 f17 f18
101111 110000 110001 110010 110011	47 48 49 50 51		b45	c43		e33 e34 e35	f16 f17 f18 f19
101111 110000 110001 110010 110011 110100	47 48 49 50 51 52	a48 a50	b45 b48	c43 c48		e33 e34 e35 e36	f16 f17 f18 f19 f20
101111 110000 110001 110010 110011 110100 110101	47 48 49 50 51 52 53	a48	b45 b48 b49	c43 c48 c49		e33 e34 e35 e36 e37	f16 f17 f18 f19 f20 f21
101111 110000 110001 110010 110011 110100 110101 110110	47 48 49 50 51 52 53 54	a48 a50 a52	b48 b49 b52	c43 c48 c49 c50		e33 e34 e35 e36 e37 e38	f16 f17 f18 f19 f20 f21 f22
101111 110000 110001 110010 110011 110100 110101	47 48 49 50 51 52 53	a48 a50	b45 b48 b49	c43 c48 c49		e33 e34 e35 e36 e37	f16 f17 f18 f19 f20 f21
101111 110000 110001 110010 110011 110100 110101 110110	47 48 49 50 51 52 53 54	a48 a50 a52	b48 b49 b52	c43 c48 c49 c50		e33 e34 e35 e36 e37 e38 e39	f16 f17 f18 f19 f20 f21 f22
101111 110000 110001 110010 110011 110100 110101 110110	47 48 49 50 51 52 53 54 55 56	a48 a50 a52 a54	b48 b49 b52	c43 c48 c49 c50	d39 d48	e33 e34 e35 e36 e37 e38 e39 e40	f16 f17 f18 f19 f20 f21 f22 f23 f24
101111 110000 110001 110010 110011 110100 110101 110110	47 48 49 50 51 52 53 54 55 56	a48 a50 a52	b45 b48 b49 b52 b53	c43 c48 c49 c50	d39 d48 d49	e33 e34 e35 e36 e37 e38 e39 e40	f16 f17 f18 f19 f20 f21 f22 f23 f24 f25
101111 110000 110001 110010 110011 110101 110101 110110	47 48 49 50 51 52 53 54 55 56 57	a48 a50 a52 a54 a56	b45 b48 b49 b52 b53	c43 c48 c49 c50	d39 d48 d49 d50	e33 e34 e35 e36 e37 e38 e39 e40 e41	f16 f17 f18 f19 f20 f21 f22 f23 f24 f25 f26
101111 110000 110001 110010 110011 110101 110101 110110	47 48 49 50 51 52 53 54 55 56 57 58 59	a48 a50 a52 a54	b45 b48 b49 b52 b53	c48 c49 c50 c51	d48 d49 d50 d51	e33 e34 e35 e36 e37 e38 e39 e40 e41 e42 e43	f16 f17 f18 f19 f20 f21 f22 f23 f24 f25 f26 f27
101111 110000 110001 110010 110011 110101 110101 110110	47 48 49 50 51 52 53 54 55 56 57	a48 a50 a52 a54 a56 a58	b45 b48 b49 b52 b53	c48 c49 c50 c51	d48 d49 d50 d51 d52	e33 e34 e35 e36 e37 e38 e39 e40 e41	f16 f17 f18 f19 f20 f21 f22 f23 f24 f25 f26
101111 110000 110001 110010 110011 110101 110101 110110	47 48 49 50 51 52 53 54 55 56 57 58 59	a48 a50 a52 a54 a56	b45 b48 b49 b52 b53	c48 c49 c50 c51	d48 d49 d50 d51	e33 e34 e35 e36 e37 e38 e39 e40 e41 e42 e43	f16 f17 f18 f19 f20 f21 f22 f23 f24 f25 f26 f27
101111 110000 110001 110010 110011 110100 110101 110110	47 48 49 50 51 52 53 54 55 56 57 58 59 60	a48 a50 a52 a54 a56 a58	b45 b48 b49 b52 b53	c48 c49 c50 c51	d48 d49 d50 d51 d52 d53	e33 e34 e35 e36 e37 e38 e39 e40 e41 e42 e43	f16 f17 f18 f19 f20 f21 f22 f23 f24 f25 f26 f27 f28
101111 110000 110001 110010 110011 110100 110101 110110	47 48 49 50 51 52 53 54 55 56 57 58 59 60 61	a48 a50 a52 a54 a56 a58	b45 b48 b49 b52 b53 b56 b57	c48 c49 c50 c51	d48 d49 d50 d51 d52	e33 e34 e35 e36 e37 e38 e39 e40 e41 e42 e43 e44	f16 f17 f18 f19 f20 f21 f22 f23 f24 f25 f26 f27 f28 f29

For every non-zero bit, there is a sub-group, which follows the frequency of that bit. When it is turned on, the sub-group appears, relative to the sum. When it is turned off, the sub-group disappears. Also, notice that many primes have a sub-group that is a prime.