Avatar Hypergraph Rewriting

by Sven Nilsen, 2021

In this paper I present an extension of hypergraph rewriting with symbolic distinction.

An Avatar Hypergraph Rewriting system (AHR), is given by the following grammar:

```
expr ::= <nat> | {expr…} | <avatar>'(expr)
rule ::= {expr...} → {expr...}
```

This is similar to models used in the Wolfram Physics Project[1], but extended with avatars[2]:

p'(0)	the 1-avatar `p` of `0`

An avatar is only used on the left side of a rule, never on the right side or in a state.

When two nodes share the same avatar, they are allowed to identical:

{p'(0), p'(1)} → …	the nodes `0` and `1` are allowed to be identical

However, when two nodes have different avatars, they are not allowed to be identical:

{p'(0), q'(1)} → …	the nodes `0` and `1` are not allowed to be identical

Avatars can be used on any hypersurface:

{p'({0, 1}), q'({2, 3})}	the directed edge `{0, 1}` is not allowed to be same as `{2, 3}`

Avatars can also be nested:

```
{p'({r'(0), s'(1)}), q'({r'(2), s'(3)})}
```

Here, `{0, 1}` is a different edge than `{2, 1}`, but `0 == 2` and `1 == 3` is allowed.
So, `{0, 1}` might share a node with `{2, 1}`, but not both at the same time.

Any node which is not wrapped into an avatar can match with any avatars.

{p'(0), 1}	`0` and `1` are allowed to be identical
{p'(0), 1, q'(2)}	`1` can be identical to `0` or `2`, but not both

Avatars in AHR are used to express symbolic distiction[3].
Symbolic distinction might be expressed in other ways, e.g. using "where" clauses to rules.
I chose avatars because they fit better with the syntax and are very expressive.

There are some limitations of using avatars. It is impossible to control how symbolic distinction works beyond a binary relation between any two nodes. This binary relation is either `0` or `1` and has no dependency on the symbolic distinction of other nodes. If this is too limiting, then one can extend the grammar further.

References:

[1] "The Wolfram Physics Project"
A project to find the fundamental theory of physics
https://wolframphysics.org/

[2] "Avatar Extensions"
AdvancedResearch – Summary page on Avatar Extensions
https://advancedresearch.github.io/avatar-extensions/summary.html

[3] "Symbolic Distinction"
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https://github.com/advancedresearch/path_semantics/blob/master/papers-wip2/symbolic-distinction.pdf