Avatar Algebra Superposition

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In this paper I discuss the principle of superposition for Avatar Algebra.

Avatar Algebra is the idea that theorems in algebra are related to theorems about Avatar Extensions[1].

In the paper "Natural One-Avatar"[2], I showed that the natural choice of a 1-avatar is the identity of a product. For Avatar Algebra, this means that a product is a kind of introduction operator for avatars.

Using the natural one-avatar, a new 1-avatar `x` might be trivially introduced as following:

$$1 \cdot x = x \cdot 1 = x$$

At first sight, this might not make much sense. Why not just introduce `x` directly?

The product as introduction operator becomes more important when talking about superposition[3]:

$$(a + b) \cdot f(x) = a \cdot f(x) + b \cdot f(x)$$

Here, superposition of mathematical objects is introduced using the distributive property[4] in algebra.
The `+` operator can be thought of as a symmetry operator.
The symmetry between two 1-avatars `a + b` is lifted into a symmetry between mathematical objects.

Normally, superposition is formulated as:

$$F(y_1 + y_2) = F(y_1) + F(y_2) \qquad \text{Additivity}$$
$$F(a \cdot y) = a \cdot F(y) \qquad \text{Homogeneity}$$

By choosing `F = (mul f(x))`, one one gets Additivity from Distribution:

$$F(y_1 + y_2) = (\text{mul } f(x))(y_1 + y_2) = (y_1 + y_2) \cdot f(x) = y_1 \cdot f(x) + y_2 \cdot f(x) = F(y_1) + F(y_2)$$

Homogeneity follows from Distribution and Associativity[5]:

$$F(a \cdot y) = (\text{mul } f(x))(a \cdot y) = (a \cdot y) \cdot f(x) = a \cdot (y \cdot f(x)) = a \cdot F(y)$$

Associativity is a tautology in Avatar Algebra since the product is quantified over all 1-avatars.
Introduction does not mean anything deeper than introducing objects (the operation is always allowed).
The output of the introduction operator must always refer to the inputs, being parametric dependent.

Therefore, the distributive property in Avatar Algebra implies superposition.

However, when you look at the distributive law, it does not require linearity over the argument `x`.
This means that although Avatar Algebra is linear, the introduced mathematical objects does not need to be linear in themselves.

References:

[1] “Avatar Extensions”
AdvancedResearch – Reading Sequence on Path Semantics
https://github.com/advancedresearch/path_semantics/blob/master/sequences.md#avatar-extensions

[2] “Natural One-Avatar”
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https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/natural-one-avatar.pdf

[3] “Superposition principle”
Wikipedia
https://en.wikipedia.org/wiki/Superposition_principle

[4] “Distributive property”
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https://en.wikipedia.org/wiki/Distributive_property

[5] “Associative property”
Wikipedia
https://en.wikipedia.org/wiki/Associative_property