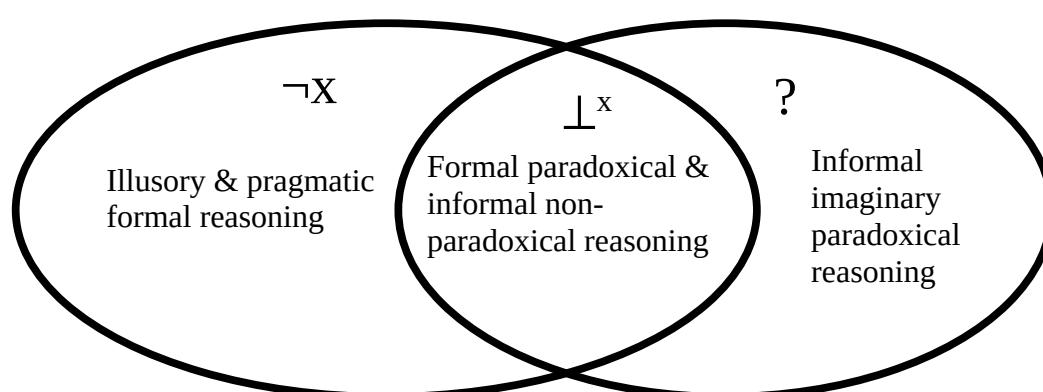


Semantics of Falsehood

by Sven Nilsen, 2022

In this paper I present a theory about semantics of falsehood in formal and informal sense. This theory can be thought of as how humans lie both formally and informally and that these lies fall into different categories of falsehood. The theory is derived from HOOO Exponential Propositions and therefore should be interpreted as from a perspective “What if everything humans think naturally about falsehood is more or less wrong?”, in attempt to either uncover weaknesses in the foundation, or to reveal flaws in human intuition about logic.



Venn diagram over 3 categories of falsehood, of which two contains semantic contradictory drifts. The universe of \perp^x is a subset of larger formal universe of $\neg x$, but also a subset of an informal imaginary paradoxical universe.

“What if everything humans think naturally about falsehood is more or less wrong?” is a sentence that can be interpreted as a joke. However, this joke also has more than one meaning. In one sense it is comical, since per definition, whatever humans think about falsehood is wrong (falsehood and wrong means the same thing). In another sense it is tragic, since humans might have misunderstood what they mean precisely about falsehood, both formally and informally. Yet, in another sense, the joke is about despair, because if humans are unable to think precisely about what is falsehood, then they might not be able to correct themselves, as their attempt to address where they are wrong also requires the ability to reason accurately about falsehood, which leads to a potentially infinite loop of confusion.

The solution to this comical & tragical situation of despair is to ground one’s reasoning in a formal language that can be studied to explain what is happening formally without doubt, plus guide our intuition forward to a possible future where humans might have a solid sense of what falsehood means, given enough time. I suggest using HOOO Exponential Propositions^{[1][2]} for this direction, although this theory is a very recent development and perhaps not yet complete, nor has it high confidence in the consistency of the axioms. Regardless of the solidity of the foundation, the optimal utility of research is when the theory being tested has a 50% chance of survival. Either we might discover a flaw in the foundation, or we might reveal flaws in human intuition about logic.

One famous thought experiment about imaginary paradoxical reasoning, was popularized in the 1986 movie “Labyrinth” directed by Jim Henson^[3]. In this movie, Sarah approaches two gates guarded by one who always says the truth and one who always lie. Behind one door is the path toward the goal and behind the other is the path to certain death. One question only is answered. I will not spoil the movie for those who have not seen it yet, but Sarah finds a clever solution.

Since humans live in a social world of similar minds where nobody has access to the ultimate truth of reality, it is not strange that we have developed this habit of thinking of paradoxes as something mysterious and fascinating. In some sense, a paradox is thought of as neither true nor false and therefore outside the domain of logic. However, this imaginary way of thinking about paradoxes comes from reasoning about partial knowledge and not about partial truth or falsehood. The contradiction lies in the fact that humans are capable of recognizing a paradox, which should in principle also be recognized as an absurd statement. In logic, all absurd statements are false in some way or another. There is no such thing as absurd reasoning without falsehood, hence this intuitive way of reasoning about paradoxes, based on partial knowledge, is imaginary.

The problem here is not just that imaginary paradoxical reasoning does not exist in a world where all knowledge is total and there is perfect information only^[4]. In order, just to explain how people think, we have to make up this category that exists outside logic, since there is nothing in the foundation that can express this precisely. Internally consistent, but also imaginary and externally inconsistent, is the idea that imaginary paradoxical reasoning is outside logic.

To explain the notation of internal vs external consistency used here: One can think about internal vs external consistency as a language barrier, where something that makes sense within the language as internal consistency, while something that makes sense outside the language as external consistency. For example, circular reasoning is internal consistent but not external consistent. Distinguishing between internal and external consistency is useful since not all languages share the same notions of truths.

In one sense it is true, because there is no way to express this mysterious dimension of paradoxes within logic. In another sense it is false, because there is a way to express the same ideas as formal paradoxical reasoning in which this mysterious dimension attached to paradoxes does not exist.

For example, in the Liar's paradox^[5], one uses imaginary paradoxical reasoning to be fascinated over the statement "this sentence is false". Instead of being interpreted as a simply false statement (since it is absurd), one attributes it an additional dimension of "paradoxical truth". Formally, the proof of Liar's paradox using HOOO Exponential Propositions is far from trivial^[6], but it basically says:

$$\perp \wedge ((\perp \wedge x) \wedge (x \wedge \top) \wedge (x \wedge \top) \wedge (\perp \wedge x))$$

It means, the Liar's paradox is indeed a paradox and hence absurd (false). However, if it is false, then what do we mean when we say that it is a paradox?

To make things worse: The precise logical language about formal paradoxical reasoning, overlaps with the informal non-paradoxical reasoning than humans have. When we simply state that something is obviously false, we are actually using the same kind of reasoning that goes into formal paradoxical expressions in logic. It is not enough that humans have created an entirely new category of imaginary paradoxical reasoning on its own, but also replaced the original category of reasoning with ordinary intuitive sense of falsehood. This means, there is no room to put back the imaginary paradoxical reasoning into language where it belongs formally.

HOOO Exponential Propositions is not a theory that one can learn to master without some help from computers. At the very least, I do not yet know any humans that can. However, humans seem to have some intuition about the falsehood aspects of this theory. The notion of $\perp \wedge x$ as falsehood is stronger than the notion of $\neg x$ and not expressible in normal logic. If anything, then it is surprising how weak $\neg x$ becomes once one has learned to use the theory properly. Yet, normal logic has been around for many years, so how do humans trained in the field reason about $\neg x$?

The way logicians think about $\neg x$ falls into two categories: Illusory and pragmatic reasoning.

- Under illusory reasoning, $\neg x$ might come out of absurdity due to making 'false' provable within the theory
- Under pragmatic reasoning, $\neg x$ becomes a consequence where there is a proper theory, neither tautological nor paradoxical, which is accepted due to $\neg x$ being sensible in a context that also makes sense

Illusory reasoning is a kind of reasoning where one makes statements, but they do not really mean anything, since one is reasoning from a perspective of absurdity. There is nothing that "causes" true statements to be true, since they are not distinguished from statements that are false. Think about a system that produces random characters: Sooner or later it will produce whole sentences that seem coherent, but that does not imply that the sentences are true nor that they refer to anything real except in the eyes of the interpreter who looks for meaning in random patterns. This form of reasoning can be thought of as making 'false' provable within the theory.

Pragmatic reasoning is where logicians apply logic to problems. This is not possible without assuming some theory that at least has some false edge cases that are not provable in normal logic. It is impossible to prove anything new from a pure tautological theorem, except theorems that are already true in normal logic. In this context, one says $\neg x$ but this might not be interpreted literally. Instead, it can be said to hold within some theory. When we change the theory, the truth of $\neg x$ might also change.

However, none of these two categories of illusory and pragmatic formal reasoning is provable within normal logic. It requires HOOO Exponential Propositions or some kind of Modal Logic^[7] just to express how illusory vs pragmatic formal reasoning works. As result, normal logic can not distinguish illusory or pragmatic reasoning internally.

It is not surprising, considering humans being generally intelligent and able to recognize paradoxes, that they have a stronger sense of falsehood than normal logic. Normal logic does not only got problems expressing how its own pragmatic usage works formally, but it also can not express precisely what humans mean when they say something is false \perp^x .

For example, when we say $a == b$ for two arbitrary propositions a, b , one can not prove in normal logic that this is true, neither can one prove that this is false. The truth value of $a == b$ lives inside an exponential complex space of meaning which is formed by the relations of all propositions. One possible way to solve this problem is to contract the space such that every statement that can not be proven tautologically (all cases are true), is by definition false. However, we have not fixed normal logic in any way by doing this, only applied a convention from the outside of what a proof means. Putting other problems aside, such as loss of constructivity, this does not express anything about falsehood from within the logic. The result is that normal logic is not strong enough to reason about falsehood properly.

On the other hand, what humans do with this universe of informal imaginary paradoxical reasoning, is such out of scope for normal logic that it seemingly does not belong to that mode of thinking that logicians prefer. It is only at the boundary between formal and informal reasoning about falsehood where one can understand, somewhat, what is going on. Unfortunately, this requires a complex theory like HOOO Exponential Propositions to be handled properly. The actual paradox here is that the closer one attempts to reach human intuitive reasoning by formal means, the less intuitive the logic itself seems to become.

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