Algexenotation

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In this paper we suggest a version of Tic Xenotation that is algebraic compressible.
This also provides an algebraic compressible form of ordered Cartesian combinatorics.

Tic Xenotation[1] maps ordered Cartesian combinatorics[2] (multisets[3]) uniquely to natural numbers:

() = 2’
(()) = 3‘ 3 is the 2^{nd} prime
()() = 4’
((())) = 5’ 5 is the 3^{rd} prime
…

We use `x’` to express that `x` is some expression in an ordinary number system.

The trick is that `(X)` interprets `X` as the X-th prime in its own representation. Since multisets are ordered Cartesian products and any natural number greater than 1 is a product of primes[4], one can represent any multiset in Cartesian form as a natural number.

The problem with Tic Xenotation is that the format is not very compressible.
For example, `31’ = ((((()))))` and `16’ = ()()()()`.
For small numbers, this is not a problem, but we would like to represent large numbers as well.

The trick we use is that we introduce `+, *, ^` as operators and use numbers as hyperprimes.

31’ = ((((())))) = 4 + () = 4 + 0 = 4
16’ = ()()()() = 0^(0^0)

We call this notation for “Algexenotation”.

In Algexenotation, multiplication is trivial, but addition is expensive.
Hyperprimes are trivial, since they are encoded as ordinary natural numbers.
The sum of two hyperprimes is a hyperprime using normal addition, for example:

3 + 2 = 5 11’ (3) hyperprimed with 5’ (2)

For multiplication and exponents, one can use standard algebraic rules:

(a*b)^c = a^c*b^c
(a^b)^c = a^(b*c)

For example:

100’ = (0*2)^0 = 0^0*2^0
1 000’ = (0*2)^1 = 0^1*2^1
1 000 000’ = (0*2)^(0*1) = 0^(0*1)*2^(0*1)
((2^10)^2)’ = (0^(0*2))^0 = 0^(0^0*2)

On the next page you will find a table of numbers from 0-99.

0’-9’	0’	1’	0	1	0^0
	2	0*1	1+0^0	0^1	1^0
10’-19’	0*2	3	0^0*1	1+0*1	0*(1+0^0)
	1*5	0^(0^0)	2+0^0	0*1^0	1+0^1
20’-29	0^2*2	1*(1+0^0)	0*3	1+1^0	0^1*1
	2^0	0*(1+0*1)	1^1	0^0*(1+0^0)	1+0*2
30’-39’	0*1*2	4	0^2	1*3	0*(2+0^0)
	2*(1+0^0)	0^0*1^0	1+0^0*1	0*(1+0^1)	1*(1+0*1)
40’-49’	0^1*2	2+0*1	0*1*(1+0^0)	1+0*(1+0^0)	0^0*3
	1^0*2	0*(1+1^0)	1+1*2	0^(0^0)*1	(1+0^0)^0
50’-59’	0*2^0	1*(2+0^0)	0^0*(1+0*1)	1+0^(0^0)	0*1^1
	2*3	0^1*(1+0^0)	1*(1+0^1)	0*(1+0*2)	3+0^0
60’-69’	0^0*1*2	1+0*1^0	0*4	1^0*(1+0^0)	0^(0*1)
	2*(1+0*1)	0*1*3	2+0^1	0^0*(2+0^0)	1*(1+1^0)
70’-79’	0*2*(1+0^0)	1+0^2*2	0^1*1^0	1+1*(1+0^0)	0*(1+0^0*1)
	1*2^0	0^0*(1+0^1)	(1+0^0)*3	0*1*(1+0*1)	1+0*3
80’-89’	0^(0^0)*2	1^(0^0)	0*(2+0*1)	2+1^0	0^0*1*(1+0^0)
	2*(2+0^0)	0*(1+0*(1+0^0))	1*(1+0*2)	0^1*3	1+0^1*1
90’-99’	0*1^0*2	(1+0^0)*(1+0*1)	0^0*(1+1^0)	1*4	0*(1+1*2)
	2*(1+0^1)	0^2*1	1+2^0	0*(1+0^0)^0	1^0*3

Proof that distributivity of addition and multiplication does not hold in Algexenotation:

Assume that:

∵ a * (b + c) = a * b + a * c

∵ 0*(1+1*2) = 94’
∴ (0*1+0*1*2)
∴ (1*0+1*0*2)
∴ 1*(0+0*2)
∴ 1*0*2
∴ 0*1*2 = 30’

Since `30’ != 94’` this is a proof that distributivitiy does not hold, using proof by contradiction.

Otherwise, addition is associative and commutative.
Multiplication is associative and commutative.

This means that Algexenotation is an example of an algebra of natural numbers which has addition and multiplication, but no distributivity of addition and multiplication.

References:

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https://en.wikipedia.org/wiki/Cartesian_product

[3] "Multiset"
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[4] "Fundamental theorem of arithmetic"
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https://en.wikipedia.org/wiki/Fundamental_theorem_of_arithmetic