Natural Frequency

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In this paper I clarify what it means for energy to be quantized.

The Planck-Einstein relation is the following:

$$E = hf$$

E	energy
h	Planck's constant
f	frequency

By quantized energy, what one actually means is that `f` is a natural number:

$$f : \mathbb{N}$$

If one takes a look at the time-dependent Schrödinger equation:

$$i\hbar\psi'(t) = H\psi(t)$$

The Hamiltonian `H` can be thought of as the total energy:

$$i\hbar\psi'(t) = E\psi(t)$$

Using Planck-Einstein relation, the Schrödinger equation can be simplified:

$$i\hbar\psi'(t) = hf\psi(t)$$
$$(ih / 2\pi)\psi'(t) = hf\psi(t)$$
$$(i / 2\pi)\psi'(t) = f\psi(t)$$

When setting `τ = 2*π*` *and `n = f`, one can write this in an elegant form:*

$$i\psi'(t) = n\tau\psi(t)$$

Or, written using the `d` function as the derivative:

$$i \cdot d(\psi) = n \cdot \tau \cdot \psi$$

With other words, energy and Planck's constant disappear from the equation, because they are not intrinsic to the wavefunction. After all, the wavefunction only contains complex probability amplitudes. Only the natural frequency `n` is used to relate the wavefunction to its complex conjugated derivative. The constant `τ` is a mathematical consequence of the derivative of Phi functions (for more information about Phi functions, see the paper "Quantum Schrödinger Functions").