Avatar Binary Relations

by Sven Nilsen, 2020

In this paper I introduce avatar binary relations, that satisfies avatar extensions.

In the paper “Unique Universal Binary Relations”[1], I introduced binary relations with the axioms:

p(a, b) role_of(b) = p p(a) = b

By using roles, this makes it possible to use `(a, b)` to completely specify the relation.
This simplifies modeling of theories with many constraints, compared to using unconstrained relations.

The problem is that this logic is sometimes too constrained, which led to the idea of using avatars.

An 1-avatar `q’` in this logic is way of lifting up a symbol `b` such that it can be assigned a new role:

p_2(a, q’(b)) role_of(q’(b)) = p_2 p_2(a) = q’(b)

This solves the problem of relaxing constraints for universality, but it does not relax uniqueness.
Yet, Avatar Graphs[2] were used to make a guess of how this pattern generalizes into higher dimensions.
It was understood that 2-avatars and higher avatars need to “integrate information” of lower avatars.

During a discussion with Cristian Urlea about Avatar Graphs[3], he realized an important insight about the interpretation: Avatar Graphs represents the role totality of the core. With other words, a kind of space covering representation. I connected this with permutations as paths along an n-cube[4]. This idea lets one simplify an n-avatar to a theory that creates different contexts over permutations:

x = a, y = b Permutations of assignments over a set `{a, b}` is a 2-avatar (2 elements)
x = b, y = a

p(x, q’(y)) role_of(q’(y)) = p p(x) = q’(y) Uniqueness interpreted in different contexts

It turns out that this interpretation can be reduced to the following axioms:

p(a, q’(b)) role_of(q’(b)) = p p(a) = {q’(_)} : ∋ q’(b)

The last axiom means that uniqueness is relaxed to a set containing only elements of the same 1-avatar.
This set contains the right argument of the relation.

References:

[1] "Unique Universal Binary Relations"
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https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/unique-universal-binary-relations.pdf

[2] "Avatar Graphs"
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https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/avatar-graphs.pdf

[3] "Applied Category Theory Server"
Discord
https://discord.gg/vzukJWC

[4] "Permutations as Cubical Paths"
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https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/permutations-as-cubical-paths.pdf