

# Non-Constructibility of Quantum Non-Determinism

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*In this paper I show that quantum non-deterministic functions are not constructible in general.*

For every constructible function  $f$ , the output has some probability distribution  $\exists_p f$ .  
Knowing the probability distribution is not the same as knowing how to construct the function.

For example, by permuting the order of the outputs one gets another function  $f'$  such that:

$$\exists_p f \Leftrightarrow \exists_p f'$$

This means that for functions with more than one output, the probability distribution does not give a unique description of the function. Therefore, one can not determine  $f$  from  $\exists_p f$ . However, one can construct  $f$  by enumerating every possible function having the same probability distribution.

For pure functions plus extensions of non-determinism using random sources:

$$\exists_p f : T \rightarrow \mathbb{R}$$

It takes some type  $T$  and returns a real number.

In a quantum non-deterministic function, the probability distribution has the type:

$$\exists_{pc} f : T \rightarrow \mathbb{C}$$

Complex numbers  $\mathbb{C}$  is a generalization of real numbers  $\mathbb{R}$ .  
Therefore, quantum non-deterministic functions are generalizations of pure functions.

The problem is that one can not enumerate every possible function having the same  $\exists_{pc} f$ .  
Why? Because we do not know how to construct a *single* quantum non-deterministic function which the normal path  $f[id \rightarrow re]$  does not have a probability distribution with zero imaginary components:

$$\exists_{pc} f[id \rightarrow re] \Leftrightarrow \exists_{pc} f \quad \text{The quantum non-deterministic functions that are constructible}$$

Where  $re$  is a function returning the real component of a complex number.

Since there are quantum non-deterministic functions that are not constructible, the constructibility of quantum non-deterministic functions does not hold in general, but only for a strict subset. Although, one can model the probability distributions of quantum non-deterministic functions, the functions themselves are not accessible.