

Adjoint Paths

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In this paper I formalize adjoint paths using path semantics.

An adjoint path is a relationship between two asymmetric normal paths of f :

$$f[g_0 \times \text{id} \rightarrow \text{id}] \Leftrightarrow f[\text{id} \times g_1 \rightarrow \text{id}]$$

$$f : T \rightarrow U$$

Since id maps to same type $T \rightarrow T$, it follows that g_0 and g_1 also maps to same type:

$$g_0 : T \rightarrow T$$

$$g_1 : T \rightarrow T$$

Since these two normal paths are the same, it means that they both use the same function:

$$\therefore f[g_0 \times \text{id} \rightarrow \text{id}] \Leftrightarrow h \quad f(x, y) = h(g_0(x), y)$$

$$\therefore f[\text{id} \times g_1 \rightarrow \text{id}] \Leftrightarrow h \quad f(x, y) = h(x, g_1(y))$$

$$\therefore h(g(x), y) = h(x, g(y))$$