# Minimum Sequence
# of Reciprocal Prime Unitary Sums

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*In this paper I introduce a new sequence of ordered lists of natural numbers corresponding to each natural number giving the length of the list. These lists have the property that any number can be removed from the list and be easily restored. The lists contain primes, usually among the lower ones, implying that some primes occur more frequently than others.*

In arithmetic, students learn that `1` can be divided by `3`:

1 / 3 + 1 / 3 + 1 / 3 = 1

However, if one sums `1 / p` where `p` is a prime, for example:

1 / 2 + 1 / 3 = 5 / 6

There is no such sum that adds up to `1` (conjecture). Yet, since there are infinite number of primes, one can get arbitrary close to `1` by selecting `1 / p` where `p` is large.

The question is how close is it possible to get to `1` for `n` arbitrary selected primes?

This question is interesting because in one sense, the sum must progress relatively quickly toward `1` as there is only a finite number of steps that can be taken. On the other hand, the sum must not progress too quickly as it must exploit small steps by selecting large primes at the end. The combination of these two requirements, plus the lack of an efficient formula to compute primes, plus that there are infinite number of primes to choose from, makes it a challenge to find the closest sum for arbitrary `n` selected primes. For some reason, lower primes seems to occur frequently.

I used probabilistic search to suggest the following minimum distance to `1` for range `[2, 10]`:

2: [2, 3] = 0.8333333333333333
3: [2, 3, 7] = 0.976190476190476
4: [2, 3, 7, 43] = 0.9994462901439645
5: [2, 3, 11, 17, 59] = 1.0000151061965619
6: [2, 3, 7, 83, 167, 173] = 1.0000070397344656
7: [2, 3, 11, 23, 71, 107, 113] = 1.0000005440688902
8: [2, 3, 11, 41, 67, 71, 83, 97] = 1.000000019443045
9: [2, 5, 7, 19, 37, 43, 47, 53, 71] = 1.0000005901002629
10: [2, 3, 11, 47, 53, 97, 103, 137, 227, 257] = 1.0000005677302628

So far, there is no proof that these are the closest sums. Notice how the accuracy increases until 9. This could mean that either there exists a closer sum for 9, or that it is not possible to sum 9 primes in a such way that you get closer to `1` than with 8 primes. One might imagine that there is some finite sequence of primes that is closer to `1` than any other, however, would not this contradict the conjecture that no such sum adds up to `1`?

One can restore a removed number with `round(1 / (1 – x))` where `x` is defect sum.