String Rewriting Rules for Cartesian Combinatorics

by Sven Nilsen, 2020

In this paper I describe two simple rules that generates every pure Cartesian product.

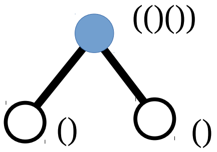
The following two rules for string rewriting generates every pure Cartesian product:

() → (())
(()) → ()()

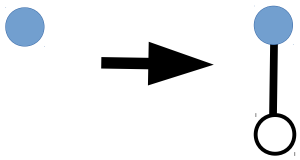
To construct a “proof” of a pure Cartesian product, one can apply these rules in any reverse order. For example, starting with `(()())`, one can trace back the construction in reverse:

(()())		this pure Cartesian product has a unique construction
((()))	(()) → ()()	can only apply the second rule in reverse
(())	() → (())	can only apply the first rule in reverse
()	() → (())	can only apply the first rule in reverse

Pure Cartesian products can be represented as a tree:

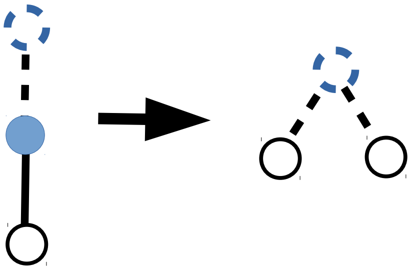


The two string rewriting rules can only be applied to a leaf or singleton branch of the tree:



() → (())

Applied to leaf



(()) → ()()

Applied to
singleton branch

All trees can be constructed using these two rules. Since the string rewriting rules are acyclic, trees form a partial ordered set, or a kind of “power tree” under these two rules. Since there are two rules, the power tree is binary.