Monotonic Integral Transform

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In this paper I introduce an integral transform based on monotonic functions.

A monotonic integral transform `∫'f` of `f` is defined as following:

$$\int' f = \int 0\ x\ \{\ \mathrm{sign}(f'(x)) \cdot f'(x) \cdot dx\ \} + f(0)$$

When `f` is monotonic in the interval `(a, b)`, where `a >= 0` and `b >= 0`:

$$\forall\ x : (a, b)\ \{\ \mathrm{sign}(f'(x)) == \mathrm{sign}(f'((a + b) / 2))\ \}$$

The sign of the derivative determines the following relationships to the integral `∫f`:

$$(\int f)(a, b) = (b - a)f(a) \qquad \text{if the sign is `0`}$$
$$(\int f)(a, b) = (b - a)(f(a) \mp (\int' f)(a)) \pm (\int\int' f)(a, b) \qquad \text{if the sign is `1` or `-1`, respectively}$$

One can think about the monotonic integral transform as way of “unfolding” a function.

For example, if there is a function from time to space:

$$f : \mathrm{time} \rightarrow \mathrm{space}$$

The monotonic integral transform `∫'f` moves in one direction only.

This can be combined with the constant speed transform, such that space-time events can be placed along a path to give the illusion of moving forward and backward in space, while the observer is actually moving at constant speed in one direction.

For continuous functions, the observer will experience going over a hill top as seemlessly integrated from the incline into the decline, without noticing the swap of direction, since events near the top will be separated by time intervals large enough to perform the swap continuously.

The new function has the same shape as the original function, so it can be used to compute the integral of the original function. However, some information from the original function is required:

- The sign of the derivative `sign(f'((a + b) / 2))`
- The value at the lower bound `f(a)`