Symmetric Avatar Paths

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In this paper I present a model in logic of knowledge, using "answered" and "truth" predicates.
I also derive efficient notation based on the intuition of Avatar Graphs, called symmetric avatar paths.

A symmetric avatar path is a 2-avatar over 1-avatar "almost" symmetric paths:

∴	$f_0 \leftarrow [g_0]$	=>	$f_1 \leftarrow [g_1]$	$?[g_0 => g_1]f_0 <=> f_1$	2-avatar
∴	$f_0 \leftarrow [g_0] : T^n \to A$		$f_1 \leftarrow [g_1] : T^n \to B$		1-avatars

∵ $f_0 : A^n \to A$ $f_1 : B^n \to B$ $g_0 : T \to A$ $g_1 : T \to B$

This notation requires some explanation to be understood,
so I will use a model in logic of knowledge as an example.

Assume there is a hidden state, a bit, which value is unknown.
One can ask two questions about the state: "Is the bit `0`?" or "Is the bit `1`?".

The answer to one of these questions also answers the other. This is true no matter what the answer is.
In first-order logic, one can use `answered` and `truth` predicates that take questions as arguments:

> answered("Is the bit `0`?") = answered("Is the bit `1`?")
> truth("Is the bit `0`?") = not(truth("Is the bit `1`?"))
>
> answered : question → bool
> truth : question → bool

Notice the similarity to the symmetric path equation `g(f(a)) = h(g(a))` for `f[g] <=> h` (one argument).
However, instead of one equation, there are two, and the symmetric path equation is not satisfied.
Instead of symmetric paths, it would be nice to express the relationship between these two equations.

One can think about the `not` function of truths as having the `id` function of answered values:

$not \leftarrow [truth]$ => $id \leftarrow [answered]$	One function "implies" the other, but not same type
?[truth => answered]not <=> id	Symmetric avatar path expresses this correctly

This notation is a 2-avatar of symmetric path, by automatically creating 1-avatars of `not` and `id` for questions, both of type `question → bool`, but for different roles of questions. The role of a 2-avatar is to integrate information processed by 1-avatars. For more information, see the paper "Avatar Graphs".

Some symmetric avatar paths, using a simplified notation `?` meaning `?[truth => answered]`:

?\false <=> \false	?and <=> and	Notice that the answered functions model
?not <=> id	?or <=> and	which arguments truth functions depends on.
?id <=> id	?fst <=> fst	This means that knowledge basically tracks
?\true <=> \false	?snd <=> snd	flow of information from one place to another.