Order-Free Quantum Non-Determinism

by Sven Nilsen, 2020

In this paper I suggest that when quantum non-deterministic functions are order-free, their probability distribution identifies the function, although the function itself can not be constructed. In a sense, this is like constructing an imaginary function, similar to how complex numbers are constructed.

An order-free non-deterministic function `f` can be determined by its probabilistic distribution `$\exists_p f$`:

$$f : () \to T$$
$$\exists_p f : T \to \mathbb{R}$$

Order-free non-deterministic functions requires certain restrictions,
such as sources of perfect randomness, described in the paper “Order-Free Non-Determinism”.

A quantum non-deterministic function replaces the probability distribution with complex numbers:

$$f : () \to T$$
$$\exists_{pc} f : T \to \mathbb{C}$$

Quantum non-deterministic functions are not constructible in general.
The reason for this is that although the complex probability distribution is known,
it does not imply that the identity of the function can be determined.
For non-deterministic functions it is possible to enumerate all functions to construct `f`,
which is not the case for quantum non-deterministic functions, hence non-constructible.

However, by asserting that a quantum non-deterministic function is order-free, all of a sudden the complex probability distribution determines the identity of the function.

This is a way of cheating, by constructing the function “indirectly”,
although the function itself can not be written down.

The analogue of this in complex analysis is the invention of an imaginary number `i = sqrt(-1)`.
Although this number can not be expressed as a real number, there is nothing ambiguous about its meaning in mathematics, since all its properties can be determined.

One can say that imaginary numbers do not hide any abstraction.

Similarly, one can say that order-free quantum non-deterministic functions do not hide an abstraction.

Order-free quantum non-deterministic functions can be used in mathematics to reason fully about some quantum systems, through their properties in mathematical logic. In practice the restrictions, that are described in the paper “Order-Free Non-Determinism”, are consistent with physical observations so far:

- Time traveling is not possible, including traveling faster than the speed of light
- Quantum randomness is as far anyone can tell, perfectly random sources of information