Normal Re-paths

by Sven Nilsen, 2020

A normal path has the following form (using constrained functions):

$$f[g_{i \to n}] \iff h$$

$$h : \exists g_i\{\forall f\} \to \exists g_n\{\exists f\}$$

This solution might exist or not, hence a "path" or a "proof" semantics.

A normal re-path replaces the path function with a relation:

$$h' : \exists g_i\{\forall f\} \times \exists g_n\{\exists f\} \to \text{bool}$$

This is a boolean function that returns `true` for these sub-types and `false` otherwise.

Since relations are generalized functions, normal re-paths are generalized normal paths.

In order to express the form of normal re-paths, one must redefine `f` as a relation:

$$f' : \forall f \times \exists f \to \text{bool}$$

The normal re-path is then defined as following:

$$f'[g_{in} \to \text{id}] \iff h'$$

This is just a lifted normal path on the lifted functions describing the relations.

However, unlike normal paths, a normal re-path always has a solution.

For example, the normal path `mul{(< 3), (< 3)}[prime]` does not exist, but its re-path does:

$f := \text{mul}\{(< 3), (< 3)\}$

$f : \text{nat} \times \text{nat} \to \text{nat}$
$\text{prime} : \text{nat} \to \text{bool}$

$f'[\text{prime} \to \text{id}] \iff h'$

$f' : (\text{nat} \times \text{nat}) \times \text{nat} \to \text{bool}$
$h' : (\text{bool} \times \text{bool}) \times \text{bool} \to \text{bool}$

Where `h'` has the following relation matrix:

h'	**(false, false)**	**(false, true)**	**(true, false)**	**(true, true)**
false	true	true	true	true
true	false	true	true	false

Written as a function: $h'((a, b) : \text{bool} \times \text{bool}, c : \text{bool}) = \neg(a == b \land c)$