Adversarial Path of Fuel Distance

by Sven Nilsen, 2018

In this paper I represent the adversarial path of fuel distance, which is common in economics.

An adversarial path of fuel distance is defined by the following:

```
x ~ 0 := \(distance : real ∧ (<= x)) = x – distance

x : real
x ~ 0 : real → real
x ~ 1 : real
```

This means that when committing to making a choice of fuel distance, one must travel some direction that is shorter than the fuel distance. It is not possible to travel farther distances than the fuel permits, which eliminates all choices that exist outside the reachable distance.

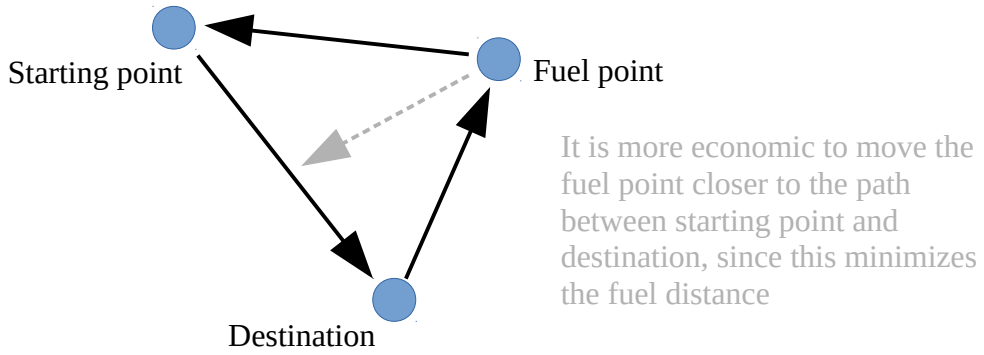
It is also possible to travel a negative distance, which equals “filling the tank”. When this is possible everywhere, there might exist a path with infinitely travelled length (requiring infinite choices):

```
(x ~ 0)(-y) = x + y
```

By traveling `0` distance, one ends up exactly with as much fuel as before:

```
(x ~ 0)(0) = x
```

An “infinite cycle problem” is figuring out how to travel a closed route such that the fuel distance ends up greater or equal than the starting point, which means that one has to travel somewhere to “fill up the tank”. In all such problems, the fuel point is desirable to move closer to the path between starting point and destination:



In problems of multi-starting points and multi-destinations, the optimal fuel points are placed at central locations. One can also consider space-time variants of this problem in anthropic astronomy, where the world-line of a planet containing life is likely located at a fixed orbit relative to a star over time.