# Twin Implication Core Theorem

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*In this paper I present a proof in Propositional Logic that the core axiom of Path Semantics is equivalent to the combination of two mirrored Constrained/Abstract Implication Theorems.*

The following is a proof in Propositional Logic[1] that the core axiom of Path Semantics[2] can be broken up into two parts, one the mirror twin of the other, which are both Constrained Implication Theorems[3]:

$$
\begin{array}{ll}
a,\ b,\ A,\ B: & \\
 & \\
((a=>A \land b=>B \land a=b) \ \ => \ \ A=B) & \text{Core axiom of Path Semantics} \\
= & \\
( & \\
\quad (a=>b \land (a=>A)=>(b=>B) \ \ => \ \ A=>B) \land & \text{Constrained Implication Theorem 1} \\
\quad (b=>a \land (b=>B)=>(a=>A) \ \ => \ \ B=>A) & \text{Constrained Implication Theorem 2} \\
) &
\end{array}
$$

Notice that the Constrained Implication Theorem only holds in Path Semantical Logic. It does not hold in normal Propositional Logic. However, Propositional Logic can prove that these two forms are equivalent.

Similarly, one can show that two twin Abstract Implication Theorems[4] are equal to the core axiom:

$$
\begin{array}{ll}
(a=>A \land b=>B \land a=b) \ \ => \ \ A=B) & \text{Core axiom of Path Semantics} \\
= & \\
( & \\
\quad (a=>b \land (a=>A)=(b=>B) \ \ => \ \ A=>B) \land & \text{Abstract Implication Theorem 1} \\
\quad (b=>a \land (b=>B)=(a=>A) \ \ => \ \ B=>A) & \text{Abstract Implication Theorem 2} \\
) &
\end{array}
$$

This is because the Constrained Implication Theorem equals the Abstract Implication Theorem:

$$
\begin{array}{ll}
(a=>b \land (a=>A)=>(b=>B) \ \ => \ \ A=>B) & \text{Constrained Implication Theorem} \\
= & \\
(a=>b \land (a=>A)=(b=>B) \ \ => \ \ A=>B) & \text{Abstract Implication Theorem}
\end{array}
$$

Although, the Constrained Implication Theorem is more general, because the following **does not hold**:

$$
\begin{array}{ll}
(a=>b \land (a=>A)=>(b=>B) & \text{Assumption of Constrained Implication Theorem} \\
= & \\
(a=>b \land (a=>A)=(b=>B) & \text{Assumption of Abstract Implication Theorem}
\end{array}
$$

The proofs of this paper was checked by the automated theorem prover library Pocket-Prover[5].

Q.E.D.

## References:


[1] “Propositional calculus”
Wikipedia
https://en.wikipedia.org/wiki/Propositional_calculus

[2] “Path Semantics”
Sven Nilsen, 2016-2019
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/path-semantics.pdf

[3] “Constrained Implication Theorem”
Sven Nilsen, 2020
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/constrained-implication-theorem.pdf

[4] “Abstract Implication Theorem”
Sven Nilsen, 2020
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/abstract-implication-theorem.pdf

[5] “Pocket-Prover – a fast, brute force, automatic theorem prover for first order logic”
AdvancedResearch
https://github.com/advancedresearch/pocket_prover