Synchronizability and Cosynchronizability

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*In this paper I introduce **synchronizability** and **cosynchronizability**, which are related to equivalence classes in Outside theories of Avatar Extensions, imposing weaker conditions than normalizability.*

In Avatar Extensions[1], specifically Avatar Schema Theory[2], theories are categorised into two groups:

- Inside: Models external objects as unknowns
- Outside: At least one symbol does not refer to the theory

Formal languages are Inside theories, or at least can be interpreted as Inside theories, while Outside theories are used to talk about meaning that goes beyond the mathematical languages developed.

The normalization property[3] is important for equivalence classes[4] in Inside theories. Philosophically, Inside theories interpreted in Outside theories require *cosynchronizability* to be normalizable. To develop the analogue of normalization property for Outside theories, I introduce two new functions:

synchronize : T × Time → opt[T]
time : T → Time

The `Time` type is an associated type of `T`, which might be thought of as some kind of abstract reference frame that allows operations to be well-defined on objects of `T`.

From the `synchronize` function, I define a new helper function:

can_synchronize : T × Time → bool := \(x, t) = is_some(synchronize(x, t))

Synchronizability and cosynchronizability are defined as following:

synchronizability(T) = ∃ a : T { ∀ b : T { can_synchronize(a, time(b)) } }
cosynchronizability(T) = ∃ a : T { ∀ b : T { can_synchronize(b, time(a)) } }

Equivalence is defined as (with lifted reflexivity, symmetricity and transitivity[5]):

equiv : T × T → opt[bool] := \(a, b) = synchronize(a, time(b)) = synchronize(b, time(b)) ⋁
synchronize(a, time(a)) = synchronize(b, time(a))

Here, operations are lifted into `opt[T]` since `none()` is used when some object is not well-defined.

`=` : opt[T] × opt[T] → opt[bool]	Equality
`⋁` : opt[bool] × opt[bool] → opt[bool]	Logical OR

References:

[1] "Avatar Extensions"
AdvancedResearch – Summary page on Avatar Extensions
https://advancedresearch.github.io/avatar-extensions/summary.html

[2] "Avatar Schema Theory"
AdvancedResearch – Summary page on Avatar Extensions
https://advancedresearch.github.io/avatar-extensions/summary.html#avatar-schema-theory

[3] "Normalization property (abstract rewriting)"
Wikipedia
https://en.wikipedia.org/wiki/Normalization_property_(abstract_rewriting)

[4] "Equivalence class"
Wikipedia
https://en.wikipedia.org/wiki/Equivalence_class

[5] "Equivalence relation"
Wikipedia
https://en.wikipedia.org/wiki/Equivalence_relation