Uncovered Permutation Grammars

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In this paper I show that some permutation grammars are not covered by single permutation groups.

A single permutation group is the permutations generated by a single permutation.

For example:

> 10423

Generates:

> 10423 + 01342 + 10234 + 01423 + 10342 + 01234

A permutation grammar is “uncovered” if it can not be generated by any single permutation group.

For example, the permutation grammar:

> (ab)(cd) = abcd + bacd + abdc + badc

Is not covered by any single permutation group.

Proof: Check every permutation of size 4 and see whether any matches.

> abcd
> abdc + abcd
> acbd + abcd
> acdb + adbc + abcd
> adbc + acdb + abcd
> adcb + abcd
> bacd + abcd
> badc + abcd
> bcad + cabd + abcd
> bcda + cdab + dabc + abcd
> bdac + dcba + cadb + abcd
> bdca + dacb + abcd
> cabd + bcad + abcd
> cadb + dcba + bdac + abcd
> cbad + abcd
> cbda + dbac + abcd
> cdab + abcd
> cdba + badc + dcab + abcd
> dabc + cdab + bcda + abcd
> dacb + bdca + abcd
> dbac + cbda + abcd
> dbca + abcd
> dcab + badc + cdba + abcd
> dcba + abcd
> *No single permutation group covers the permutation grammar of size 4.*

Qed.