Canonical Form of Answered Modal Logic

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In this paper I introduce the canonical form of Answered Modal Logic.

The canonical form of Answered Modal Logic is the following syntax:

$$(a_0 \wedge a_1 \wedge \ldots a_n) \;\vee\; (b_0 \wedge b_1 \wedge \ldots b_n) \;\vee\; \ldots$$

For brevity, the parantheses can be omitted.
Each term is prefixed with one of members of the modal set `{□, ◇, ¬◇}`.

The inversion rule `¬□ = {◇, ¬◇}` can be used with `{◇, ¬◇}X = ◇X ∨ ¬◇X` to normalize.

This form is used to reduce an expression into one that can be compared with other expressions.

For example:

∵ □A ¬= □B
∴ (not . eq)(□A, □B)
∴ (eq[not] . (not . fst, not . snd))(□A, □B)
∴ eq[not](¬□A, ¬□B)
∴ xor(¬□A, ¬□B)
∴ (¬□A ∧ ¬¬□B) ∨ (¬¬□A ∨ ¬□B)
∴ (¬□A ∧ □B) ∨ (□A ∨ ¬□B)
∴ ({◇, ¬◇}A ∧ □B) ∨ (□A ∨ {◇, ¬◇}B)
∴ ((◇A ∨ ¬◇A) ∧ □B) ∨ (□A ∨ (◇B, ¬◇B))
∴ ((◇A ∧ □B) ∨ (¬◇A ∧ □B)) ∨ ((□A ∨ ◇B) ∨ (□A ∨ ¬◇B))
∴ (◇A ∧ □B) ∨ (¬◇A ∧ □B) ∨ (□A ∨ ◇B) ∨ (□A ∨ ¬◇B)

After normalizing to the canonical form, the expressions can be extracted to a table:

	¬◇A	◇A	□A
¬◇B	0	1	1
◇B	1	0	0
□B	1	0	0

Another example:

∵ □A => □B
∴ ¬□A ∨ □B
∴ {◇, ¬◇}A ∨ □B
∴ (◇A ∨ ¬◇A) ∨ □B
∴ ◇A ∨ ¬◇A ∨ □B

	¬◇A	◇A	□A
¬◇B	1	1	0
◇B	1	1	0
□B	1	1	1

When a variable is unmentioned, e.g. `B` is not mentioned in `◇A`, one can fill out the row/column.