Restricted Dual Composition

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In this paper I introduce an inference rule for Avatar Logic that restricts composition to dual targets.

A monotonic solver is like nuclear physics: It is easy to create explosions, but hard to harnest the energy for useful purposes. Similarly, in Category Theory composition "explodes" combinatorically, but it is hard to find compositions that are relevant for specific proofs. In Avatar Logic, the rule for composition is:

$\quad$ (X, Z) :- (X, Y), (Y, Z).

Restricted Dual Composition solves this problem by modifying the rule for composition:

$\quad$ (X, Z) :- (X, Y), (Y, Z), (X*, X).

Here, `X$^*$` means that there is some dual of `X`, which for a single 1-avatar `c` corresponds to 2 rules:

$\quad$ (X, Z) :- (X, Y), (Y, Z), (c'(X), X).
$\quad$ (c'(X), Z) :- (c'(X), Y), (Y, Z), (X, c'(X)).

It means that when `(X$^*$, X)` is specified, it behaves like a goal that "targets" `(X, X$^*$)` and vice versa.

Composition in this logic is restricted to outgoing arrows of `X` or `X$^*$`.

The idea is that composition propagates in a controllable fashion, such that isomorphism between the duals is proved if there exists a directional loop back to `X`: