Binary Square Matrix Combinatorics

by Sven Nilsen, 2020

In this paper I formalize binary square matrix combinatorics using Directional Set Algebra.

For all `n : nat`, there is an associated binary square matrix combinatorics:

0	Empty matrix set
I	Identity matrix set
U	Upper triangle matrix
L	Lower triangle matrix
1	All matrices

The following law holds with Directional Set Algebra:

$$I + U + L = 1$$

Sizes of sets:

$$|0| = 0$$
$$|I| = 1$$
$$|1| = 2^{(n \cdot n)}$$
$$|U| = |L| = 2^{(n \cdot (n - 1) / 2)}$$

Sub-types of binary matrix sets can be constructed using elements `0`, `1` and `?`.
The following laws holds with Directional Set Algebra, where `?` is top and there is no bottom:

$$0 + 1 = ?$$

Upper triangle matrix:

```
0 ? ?
0 0 ?
0 0 0
```

Lower triangle matrix:

```
0 0 0
? 0 0
? ? 0
```

All matrices:

```
? ? ?
? ? ?
? ? ?
```