Avatar Covers

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In this paper I introduce Avatar Covers, which strengthens symmetric self-normal and adjoint paths.

An Avatar Cover is a boolean function that describes the “avatar cover pattern” of normal paths[1] to themselves. This is used e.g. when working with binary operators for interpreting Avatar Graphs[2]:

$$f[g_{i \rightarrow n}]_a := \backslash(a : \mathbb{B}, b : \mathbb{B}) = \forall\ x, y\ \{\ g_n(f(x, y)) = f(\text{if } a\ \{g_0\} \text{ else } \{id\}(x), \text{if } b\ \{g_1\} \text{ else } \{id\}(y))\ \}$$

$f[g_{i \rightarrow n}]_a : \mathbb{B} \times \mathbb{B} \rightarrow \mathbb{B}$	Uses notation for normal paths \`$f[g_{i \rightarrow n}]$\` with an subscript \`a\`
$f : T \times T \rightarrow T$	A binary operator \`f\`
$g_{i \rightarrow n} : [T \rightarrow T]$	A path function product[4] of unary operators \`$g_{i \rightarrow n}$\`

Symmetric Avatar Cover is the avatar cover pattern of a symmetric normal path:

$f[g]_a$ Subscript \`a\` after bracket stands for “avatar cover”

When the Symmetric Avatar Cover is \`and\`, it is the same as a symmetric normal path to itself:

$f[g]_a \Leftrightarrow \text{and} \quad\quad \Rightarrow \quad\quad f[g] \Leftrightarrow f$

This rule is called “Normal Symmetric Avatar Cover Transform” (NS-ACT) in theorem proving.
One can use NS-ACT in reverse when NS-ACT holds, since no information is lost using the transform.
For example, negation of addition has a symmetric avatar cover \`and\`:

∵	$-(a + b) = (-a) + (-b)$	
∴	$neg(add(a, b)) = add(neg(a), neg(b))$	
∴	$add[neg \times neg \rightarrow neg] \Leftrightarrow add$	Using equational form of normal paths
∴	$add[neg] \Leftrightarrow add$	Using notation for symmetric normal paths
∴	$add[neg]_a \Leftrightarrow and$	Using reverse NS-ACT

When the Symmetric Avatar Cover is \`xor\`, it is the same as a self-adjoint path[3] \`f\` of \`g · f\` by \`g\`.

$f[g]_a \Leftrightarrow \text{xor} \quad\quad \Rightarrow \quad\quad (g \cdot f)[g \times id \rightarrow id] \Leftrightarrow (g \cdot f)[id \times g \rightarrow id] \Leftrightarrow f$

This rule is called “Adjoint Symmetric Avatar Cover Transform” (AS-ACT) in theorem proving.
One can use AS-ACT in reverse when AS-ACT holds, since no information is lost using the transform.
For example, negation of multiplication has a symmetric avatar cover \`xor\`:

∵	$-(a \cdot b) = (-a) \cdot b = a \cdot (-b)$	
∴	$neg(mul(a, b)) = mul(neg(a), b) = mul(a, neg(b))$	
∴	$mul[neg \times id \rightarrow neg] \Leftrightarrow mul[id \times neg \rightarrow neg]$	Equational form
∴	$(neg \cdot mul)[neg \times id \rightarrow id] \Leftrightarrow (neg \cdot mul)[id \times neg \rightarrow id]$	Adjoint path form
∴	$mul[neg]_a \Leftrightarrow xor$	Reverse AS-ACT

References:

[1] "Normal Paths"
Sven Nilsen, 2019
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/normal-paths.pdf

[2] "Avatar Graphs"
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[3] "Adjoint Paths"
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[4] "Path Function Product Notation"
Sven Nilsen, 2017-2019
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/path-function-product-notation.pdf