Wildcard Sets

by Sven Nilsen, 2020

In this paper I represent sets that match any set under set equality of undecidable infinitesimals.

An undecidable infinitesimal `x` has the property:

$$f(x) \neg= f(x + \varepsilon) \qquad \vee \qquad f(x) \neg= f(x - \varepsilon)$$

$f : \text{real} \rightarrow \text{bool}$
$\varepsilon : \text{real}$
$\varepsilon^2 = 0$

There are infinite number of sets that have this property everywhere, called “wildcard sets”.

In order to describe wildcard sets, one can define how these sets are constructed locally at infinitesimal level. Each position is separated from the previous one with an infinitesimal `ε`.

The two most obvious wildcard sets are:

…**0**10101010101… f(0) = 0
…**1**01010101010… f(0) = 1

The **bold digit** is which value the set returns for `0`.

The two sets above are the only sets who undecidable infinitesimals are bidirectional everywhere.
This means that at every position, the value varies both to the left and to the right.
Notice that one can construct one set from the other by either flipping all bits or shifting one position.

There are four wildcard sets that are unidirectional everywhere.
This means that at every position, the value varies to the left or to the right, but not both:

…**0**11001100110…
…**1**00110011001…
…0**1**1001100110…
…1**0**0110011001…

Notice that these sets can be constructed from any one of them by shifting up to 3 times left or right.

To make it easier to reason about wildcard sets, one can use four letters `U, D, B, T`:

U = 01 “up”
D = 10 “down”
B = 00 “bottom”
T = 11 “top”

There are 10 valid ways to put these four letters together:

UU	0101
DD	1010
UD	0110
DU	1001
UB	0100
TU	1101
DT	1011
BD	0010
BT	0011
TB	1100

To the right, one can make these choices:

U	U, D, B
D	U, D, T
B	D, T
T	U, B

To the left, one can make these choices:

U, D, T	U
U, D, B	D
U, T	B
D, B	T

Any wildcard set can be constructed using this method.

Since the two letters `U` and `D` are isomorphic to bits, it means that there are at least as many wildcard sets as there are ways to define sets of real numbers. While the set of wildcard sets is a subset of the powerset of sets of real numbers, this subset is not smaller than the entire powerset. With other words, the powerset of sets of real numbers can be thought of as containing itself.

In any wildcard set, the number of zeroes is equal to the number of ones.

All wildcard sets have the same properties with respect to set equality with undecidable infinitesimals. Here, the `<=>` operator is changed from functional equality to describe this kind of set equality. Using any wildcard set, one can prove that `\false` equals `\true`:

∵ ∀ w : wildcard { \false <=> w }
∵ ∀ w : wildcard { \true <=> w }

∴ ∀ _ : wildcard { \false <=> \true }

wildcard : (real → bool) → bool

The same holds for any set of real numbers. A wildcard set collapses the consistency of proofs.

This might seem like a huge problem. However, I claim wildcard sets are not a problem, because:

- Wildcard sets are impossible to construct using a decidable algorithm (conjecture)
- There exists no decidable algorithm for determining all wildcard sets (conjecture)
- Proofs that prove inconsistency without wildcard sets are uncountable infinite (conjecture)

So, although it is true that generalizing propositional logic to undecidable infinitesimals is inconsistent from a classical point of view, this does not lead to inconsistent proofs constructively.