Associativity Avatar Graph

by Sven Nilsen, 2020

In this paper I present an Avatar Graph that models associativity.

The Associativity Avatar Graph[1] is the following, as illustrated:

Reading from the bottom to the top:

1. The bottom node is the unit `()`
2. Introduction of 3 elements `a, b, c`,
    which can be thought of as `() → a, () → b, () → c`
3. Two products `ab` and `bc`
4. Two new products `a(bc)` and `(ab)c`
5. A witness of the equality `a(bc) = (ab)c`

This avatar graph was discovered by the author during a discussion in the Bivector Discord Server[3] when investigating axioms of the geometric product by comparing them with theorems in Avatar Algebra[4].
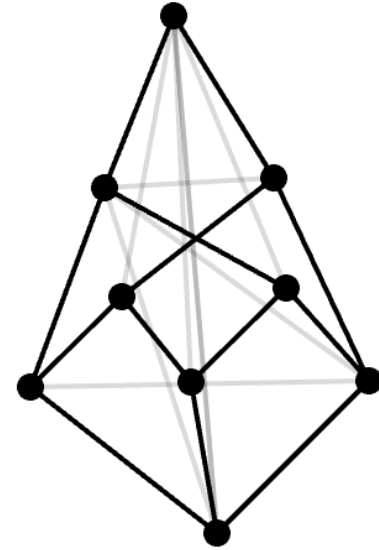
The Associative Avatar Graph has 9 vertices, which is uncommon for filled avatar graphs (usually 2^n).

This graph was not listed previously in the House of Graphs, a database of interesting graphs. Graph properties calculated by the House of Graphs[2]:

Acyclic:	No	Edge Connectivity:	2	Minimum Degree:	2
Algebraic connectivity:	1.268	Eulerian:	No	Min. Dominating Set:	3
Average degree:	2.889	Genus:	1	Number of Components:	1
Bipartite:	No	Girth:	4	Number of Edges:	13
Chromatic Index:	4	Hamiltonian:	Yes	Number of Triangles:	0
Chromatic Number:	3	Independence Number:	4	Number of Vertices:	9
Circumference:	9	Index:	2.928	Planar:	No
Claw-Free:	No	LLE*:	5.414	Radius:	2
Clique Number:	2	Longest Induced Cycle:	6	Regular:	No
Connected:	Yes	Longest Induced Path:	5	SLE*:	1.277
Density:	0.361	Matching Number:	4	Smallest Eigenvalue:	-2.414
Diameter:	3	Maximum Degree:	3	Vertex Connectivity:	4

* LLE = Laplacian Largest Eigenvalue
* SLE = Second Largest Eigenvalue

References:

[1]   "Avatar Graphs"
        Sven Nilsen, 2020
        https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/avatar-graphs.pdf

[2]   "Associativity, Filled Avatar Graph"
        The House of Graphs
        https://hog.grinvin.org/ViewGraphInfo.action?id=44059

[3]   "Bivector"
        Discord
        https://discord.gg/vGY6pPk

[4]   "Avatar Algebra"
        Summary of Avatar Extensions
        https://advancedresearch.github.io/avatar-extensions/summary.html#avatar-algebra