Eleven Algexenic Types

by Sven Nilsen, 2022

In this paper I present eleven natural types of Algexenic numbers under self-reference.

Algexenic numbers are natural numbers written in Algexenotation^[1].

Assume some function `f` that maps ordinary natural numbers to Algexenotation:

 $f : nat \rightarrow algexeno$

To store `f` efficiently in computer memory, one can use self-reference.

This means that `f` does not translate completely into Algexenotation, but uses original natural numbers when branching. If one branch corresponds to a hyperprime, it is inlined, because a hyperprime does not take up more memory than an original natural number. Factorization is assumed to be in ascending order. The resulting structure is a forest^[2] that consists of 11 types:

Type	Description	Example
0	Intrinsic original	0' => 0'
1	Hyperprime	2' => 0
2	Hyperprime power	4' => 0^0
3	Hyperprime product	6' => 0*1
4	Left hyperprime sum	7' => 1+4'
5	Right hyperprime product	12' => 4'*1
6	Left hyperprime product	14' => 0*7'
7	Left hyperprime power	16' => 0^4'
8	Original product	28' => 4'*7'
9	Right hyperprime power	49' => 7'^0
10	Original power	2401' => 7'^4'

```
0
       0, 1
1
       2, 3, 5, 11, 31, 127, 709, 5381, 52711, 648391, 9737333, 174440041, 3657500101, ...
2
       4, 8, 9, 25, 27, 32, 121, 125, 243, 961, 1331, 2048, 3125, 16129, 29791, 161051, 177147, ...
3
       6, 10, 15, 22, 33, 55, 62, 93, 155, 254, 341, 381, 635, 1397, 1418, 2127, 3545, 3937, ...
       7, 13, 17, 19, 23, 29, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, ...
4
5
       12, 20, 24, 30, 40, 44, 45, 48, 60, 66, 77, 80, 88, 90, 96, 99, 110, 120, 124, 132, 135, 154, ...
6
       14, 18, 21, 26, 34, 35, 38, 39, 46, 50, 51, 54, 57, 58, 65, 69, 74, 75, 82, 85, 86, 87, 94, 95, ...
       16, 64, 81, 128, 256, 512, 625, 729, 1024, 2187, 4096, 6561, 8192, 14641, 15625, 19683, ...
7
8
       28, 36, 42, 52, 56, 63, 68, 70, 72, 76, 78, 84, 91, 92, 100, 102, 104, 105, 108, 112, 114, ...
       49, 169, 289, 343, 361, 529, 841, 1369, 1681, 1849, 2197, 2209, 2809, 3481, 3721, 4489, ...
9
       2401, 28561, 83521, 117649, 130321, 279841, 707281, ...
10
```

References:

- [1] "Algexenotation"
 Daniel Fischer, Sven Nilsen, 2022
 https://github.com/advancedresearch/path-semantics/blob/master/papers-wip2/algexenotation.pdf
- [2] "Tree (data structure)"
 Wikipedia
 https://en.wikipedia.org/wiki/Tree (data structure)