Normal Path Diagram of AND

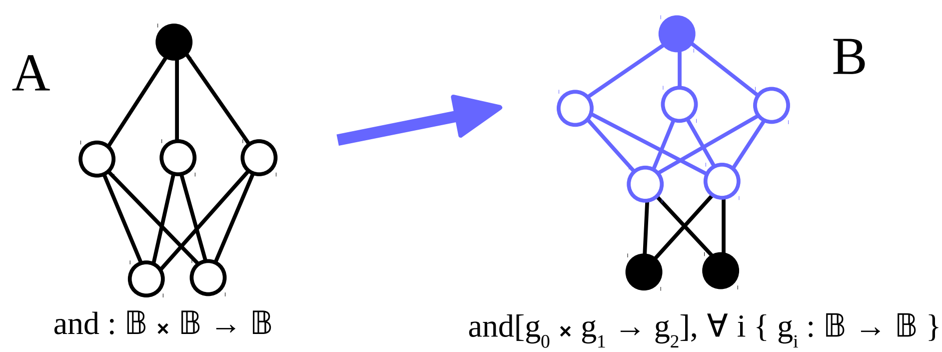
by Sven Nilsen, 2020

In this paper I explain how AND can be visualized using a normal path diagram.

What do I mean when I say “and” (the function `and : $\mathbb{B} \times \mathbb{B} \rightarrow \mathbb{B}$`)? The meaning behind words is often much deeper and richer in complexity than it seems at first.

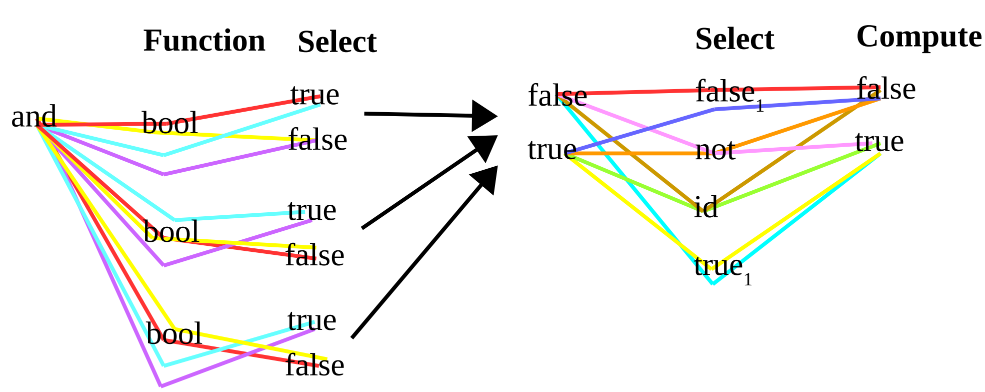
One way to illustrate this is to imagine that the meaning of “and” is a set of paths, where each path is different from all the others in the same set. There can be multiple sets, because the word “and” can have multiple meanings, depending on which context one uses it.

For example, for the function `and` as illustrated below, using Normal Path Diagrams[1], there are two sets of paths `A` and `B`, but they do not have the same number of paths. The set `B` has 4 times as many paths (because there are 4 paths of type `$\mathbb{B} \rightarrow \mathbb{B}$`)!



Although the set of paths `A` and `B` are different, one is talking about the same object `and`. For every path in `B`, there is a way of assigning it a path in `A`, such that each path in `A` got 4 paths.

One can also visualize the paths using colors. The function `and` consists of a set of paths such that all paths satisfy the rules for defining a function and no more paths can be added. On the other hand, `and[$g_0 \times g_1 \rightarrow g_2$]` represents some choices one can make, e.g. `and[not]`:



One can replace `and` with any function `f : $\mathbb{B} \times \mathbb{B} \rightarrow \mathbb{B}$`, without changing the number of paths in `B` relative to `A`.

References:

[1] “Normal Path Diagrams”
Sven Nilsen, 2020
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/normal-path-diagrams.pdf