Contractible Types

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In this paper I show that there is a way of modeling contractible types in Path Semantical Logic.

In the paper "Noncontractible Types"[1], I showed that noncontractible type families can be modeled in Path Semantical Logic[2]. Since a type family is contractible when it is not noncontractible, one can use the following law:

$$\neg(\neg(X_0 \wedge X_1 \wedge \ldots \wedge X_{n-1}) \wedge (X_0 \vee X_1 \vee \ldots \vee X_{n-1}))$$
$$=$$
$$(X_0 \wedge X_1 \wedge \ldots \wedge X_{n-1}) \vee (\neg X_0 \wedge \neg X_1 \wedge \ldots \wedge \neg X_{n-1})$$

Which can be reduced to two cases:

$$(X_0 \wedge X_1 \wedge \ldots \wedge X_{n-1}) \veebar (\neg X_0 \wedge \neg X_1 \wedge \ldots \wedge \neg X_{n-1})$$

To check that a type family is contractible, modeled in Path Semantical Logic, it is sufficient to check whether all propositions are `true` or all propositions are `false`.

This fits the intuition in Path Semantics that the functions `\true` and `\false` contracts to a single point.

References:

[1]   "Noncontractible Types"
       Sven Nilsen, 2020
       https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/noncontractible-types.pdf

[2]   "Path Semantical Logic"
       AdvancedResearch, reading sequence on Path Semantics
       https://github.com/advancedresearch/path_semantics/blob/master/sequences.md#path-semantical-logic