Binary Square Matrix Combinatorics

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In this paper I formalize binary square matrix combinatorics using Directional Set Algebra.

For all `n : nat`, there is an associated binary square matrix combinatorics:

0	Empty matrix set
D	Diagonal matrix set
U	Upper strictly triangular matrix set
L	Lower strictly triangular matrix set
1	All matrices

The following law holds with Directional Set Algebra:

D + U + L => 1

Sizes of sets, since they share the zero matrix, gets subtracted one when added together:

|x + y| = |x| + if x == y { 0 } else { |y| – if |x| > 0 ∧ |y| > 0 { 1 } else { 0 } } }

|0| = 0
|D| = 2^n
|1| = 2^(n · n)
|U| = |L| = 2^(n · (n – 1) / 2)

Notice that `|x + y|` operates on the symbolic level.

Sub-types of binary matrix sets can be constructed using elements `0`, `1` and `?`.
The following laws holds with Directional Set Algebra, where `?` is top and there is no bottom:

0 + 1 = ?

For example, for `n = 3`:

```
? 0 0         0 ? ?         0 0 0         ? ? ?
0 ? 0    +    0 0 ?    +    ? 0 0    =>   ? ? ?
0 0 ?         0 0 0         ? ? 0         ? ? ?
```