Symmetric Paths of Matrices

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In this paper I explain how to present symmetric paths as linear algebra equations.

A matrix is a table of numbers that also serves as a single argument function for vectors.

$$\text{matrix} : \text{vector} \rightarrow \text{vector}$$

A matrix can also be thought of as a transformation of other matrices,
where column vectors in the input matrix gets transformed into row vectors in the output matrix.

$$\text{matrix} : \text{matrix} \rightarrow \text{matrix}$$

Therefore, a matrix can be thought of as a function in more than one way.
However, a matrix is a single argument function in both cases of vectors and matrices.

In mathematics, it is common to use a big letter for matrices, but small letters for functions.
Since this can be a bit confusing, I will use small letters for easier translation from path semantics.

$f : \text{matrix}$	
$g : \text{matrix}$	
$h : \text{matrix}$	
$id = I$	The identity matrix is named `id`

A symmetric path of a single argument function is the following two laws:

$$f[g] <=> h \qquad g \cdot f <=> h \cdot g$$

However, for matrices in particular, one can switch `<=>` with an equal sign:

$$f[g] = h \qquad g \cdot f = h \cdot g$$

Where `·` is matrix multiplication.

If `g` is invertible, one can compute the symmetric path of `f` by `g` directly:

$$g \cdot f \cdot g^{-1} = h \cdot g \cdot g^{-1}$$
$$g \cdot f \cdot g^{-1} = h \cdot id$$
$$g \cdot f \cdot g^{-1} = h$$
$$f[g] = g \cdot f \cdot g^{-1} = h$$