Primbix

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In this paper I introduce primbix, a natural number associated with every natural number, in order to motivate further research on prime numbers. Primbix generalizes primes and has several desirable properties seen from a game design perspective. I also present a new sequence of natural numbers that contains the primes of increasing minimum primbixes. Some game design discussion.

Prime numbers have fascinated mathematicians for millenia. The reason is that every natural number can be thought of as a multiple of `0`, `1` (obviously) and primes. Another reason is that the sequence of primes makes it easier to find new primes. So, the knowledge about primes generates future utility. While the sequence of primes has no known efficient formula, there are good approximations. In the era of abundant computational capacity, this makes use of primes practical.

At the same time, the interest in prime numbers dwindles slowly as the low hanging fruit is picked up. Researchers have moved on to focus on harder properties about prime number, such as prime gaps in arithmetic progressions. In spirit of this development, I suggest moving in a direction of using creative ways to formulate problems about primes. Such problems should be judged based on aesthetic measures of game design. A problem is beautiful if it relates to our current knowledge by certain desirable properties, that motivates researchers and hobbyists, such as:

- Computationally hard (finding new numbers is challenging)
- Superexponential (low density as the number of digits grows)
- Prime coverage (the results encode knowledge about primes)
- Few known numbers (finding a new number is breaking news)

In order to achieve these properties above, I suggest the following generalization of primes:

> A **primbix** is an associated natural number for every natural number.
>
> Every composite number has **primbix** `0`.
>
> Every prime has **primbix** greater or equal to `1`,
> which is `1` unless it has the formula `1 + 2 · r · s` where `r` and `s` are primes,
> in which case the **primbix** is `primbix(r) + primbix(s)`.

With other words, if you have two primes `r` and `s`, multiply them with `2` and add `1`, then the answer has a predictable primbix which is either `0` or the primbixes sum of `r` and `s`. Therefore, primbixes are kind of like building a house of cards, where you either get the full reward or nothing. One can think of the primbix as a “value” of a number which is hard to obtain, but which also has potential to produce even higher primbixes. This is similar to how primes make it easier to find new primes. However, high primbixes are even harder to find and relies on good methods to find primes! This property is intentional, in order to motivate further research on primes. An additional benefit is that for hobbyists, collecting high primbixes is fun and exciting with new discoveries.

With this introduction, here is the most precious sequence of all, the minimum primbix sequence:

> 0, 2, 13, 53, 317, 3407, 8243, 49459, 197837, 1187023, 18314419, 112006043,
> 491325707, 2047857083, 9869296583, …

The minimum primbix sequence contains the lowest numbers of increasing primbixes. This means, any two primes, used to get a new higher primbix, which primbix sum outperforms this sequence is guaranteed to be a composite. So, for a certain class of operations, one can predict primality in a negative sense, relative to knowledge about the mimimum primbix sequence.

Notice how this turns primbix knowledge on its head to talk about primality. One of the reasons that interest in primes dwindles slowly over time, is because there is only a little efficiency possible to gain from knowledge of existing primes. Each known prime contributes a tiny amount which is negligable when there exist an efficient approximation algorithm. By leveraging primbix, one can construct new classes of operations that relate primality of higher numbers to primbix of lower numbers. Since it is much more efficient to calculate using lower numbers than higher numbers, this might boost incentives for improving prime algorithms.

The evidence so far suggests that the minimum primbix sequence is super-exponential. This means that as new numbers in the sequence are found, the efficiency of using primbix to determine compositeness goes up. This is the basic idea: One defines a super-exponential sequence which is computationally hard to derive, but once found, it can be used to improve other algorithms by reducing a problem into smaller recursive problems. This might not work efficiently to find solutions, but it might produce more efficient filters against non-solutions.

I chose the primbix definition because it was the simplest one of this kind I could think of. There might be other definitions that are more practical for use in various algorithms. When thinking along these lines, one can argue that the overall problem of primes is that it returns just a single bit of information about the number:

prime : nat → bool primbix : nat → nat

More information means more utility. A natural number can contain any finite amount of information. The type of the `primbix` function, `nat → nat`, is like a general principle when there is a hard problem of this kind. Most research on prime numbers has focused on the composite side of the prime filter, such as whether two numbers are coprime. However, this increases the amount of information about composite numbers. What I try here, is to increase the amount of information on the prime side. Primbix provides more information about primes, such that there are some primes that are more rare than others, aka Mersienne primes or arithmetic progressions.

The problem I think of Mersienne primes and arithmetic progressions when it comes to motivating research on primes, is that they have a too simple structure that does not allow a lot of creative freedom in the choice of algorithms. I believe that binary trees are simple yet also flexible. I would like to see more discrete topologies applied in this way, which combines combinatorics with number theory, usually in the form of discrete spaces. People can ask many questions about the binary trees that are interesting and generate tables for fast lookup. They do not have to agree on a single common practice that is only available to a small group of experts. At the same time of allowing greater diversity, there must be some utility in sharing knowledge with each other. I think primbix satisfies this property, since one can easily (relatively) derive missing gaps of knowledge from knowing a single number. It is not my intention here to criticize the research on Mersienne primes and arithmetic progressions. I want to point out that there is room for some better game design that can motivate people to get into prime research.

In Academia, people often treat research as a matter of prestige and to make a name for oneself. As a person who design stuff, including mathematical notations, game engines and scripting languages, I think that most of the value of research is to leverage technology to empower the ability of other people, so they can do something interesting and have fun. It is a different goal, but taken seriously.

For example, when learning about Euclid's theorem of infinite primes, people might take Euclid's proof as "serious" because Euclid is a great name within the mathematical community. However, most likely, Euclid came up with this theorem because it was fun to do so.

Moreover, the name "Euclid" might have been introduced by European mistranslations from Arabic (who used Euclid as a reference to "greek man"). This might have been intentional or not, as a way to present the author of Element as a white male. In reality, it might very well happen that the author was a black female.

The point is that very little is known about the historical Euclid, so when one gets the impression today that prime research ought to be "serious", it is not clear to me in which footsteps these people follow. Maybe original prime research was considered a "fun" activity to engage people? It is kind of obvious from the limited computational capacity at the time that primes present a sort of balanced game design. Something that is neither too complex or too simple for the mind.

When stuff starts to get boring in a game, the game designer introduces new ideas to increase the complexity. This makes the game starting to be fun again, hopefully without just adding more frustration. The today's world consists of people being experts on making each other miserable, due to social pressure of everything must happen at small scales of time. For example, one is not expected to work on something that two millenia afterwards, still engages people. Like Euclid.

So, are you following in Euclid's footsteps or not?

I think every person needs to answer this question privately and not excuse themselves, as when giving into social pressure, to be able to claim of following Euclid's footsteps. Because, honestly, I think this is precisely what it is like to head in the wrong direction. The "fun" research is so much more demanding and more difficult, because you are not evaluating the result based on something you can easily predict in advance. You have to take risks and look for new opportunities all the time.

This mindset, which I could call "the Euclid mindset", has gotten a bad reputation due to some people, who often do not know anything or very little, thinks that one ought to work on stuff that is prestige. It just happens that the things people do for prestige often exclude other people from its activity. It is like an entertaining monkey, doing an impressive trick in front of a paying audience. In Academia, research is payed for by people who expect to see something. I do not think most people aiming for prestige are thinking about their own research as a form of game design. You have to create problems for people to solve, not just solve problems. Otherwise, it gets boring and people lose the ability to understand your research. I hope this makes it easier for you to understand the genius of Euclid. If Euclid did not invent stuff that was intriguing but had no social utility at the time, then the other stuff that Euclid wrote would not be very interesting for people.

My attempt here is to make prime research even more "prime"-like, kind of how a game designer that takes a previous version of a game and changes it a bit to make it more appealing for users who advanced to a higher level of mastering complexity. At the same time, this new version still requires basic prime research to advance. The difference is that for normal people, progress is more easily noticeable and they have a vested interest in it. You still get the same kind of progress, but you get it faster, perhaps not in a single direction, but along multiple directions and with more diversity, since you have more people engaged. The progress is "fun" for more people and attracts the sort of brilliant minds who got a distaste for prestige. This means you are engaging people who never would be interested in academic research. Maybe you can convince them that not all of it is bad and something of it is worth looking into?

One thing that many people working in Academia tends to forget, is that the world, in general, is in a state of suffering. There are millions of people who suffer from deseases, mental illness, malnutrition and marginalization. If you do not understand how these people think, then you are not able of engaging them. You might be successful by every standard that people around you uses to judge your performance. However, the value is not in the hundreds of students that all memorizes every little detail of theory to pass an exam or how many papers you get published to journals. The value comes from a few individuals that are given an extraordinary opportunity to contribute to society. Ironically, society celebrates such successful individuals as “geniuses” instead of thinking of the rare moments where these individuals are allowed to participate in research. It is the social connection that empowers people to build each other up. Behind every “genius” there is a hidden “village” of people providing all sorts of support roles. When you are benchmarking people to exclude most of them, you are not allowing the sort of community that flourishes in support of few gifted individuals that are needed in order to cause major contributions. If exams were designed such that a good result also provided support for friends and family of the person who passed the exam, then the competition would be more productive. This is the sort of improvements that people miss because they are out to judge a single person, which is a very destructive mindset and causes most of the misery in the world.

I am not aiming for people to think that e.g. Academia should go away. On the contrary, I think that people who work in Academia ought to look at what things Academia is good at and try to find ways to make them even better. This means not being afraid of asking some though questions to yourself. The world does not end only because you one day figured out that you were heading in a wrong direction. Perhaps you have to take some risks when it comes to how other people see you in society in order to make a change in your life. This has happened thousands of times before to many people before you. Think of this as a natural process.

The world is full of people who asks themselves the question “how can software make me more money?”. However, the a less frequently asked question is “how can software make life better for people?”. Depending on which question you ask yourself, the result of any success you make might look very different. If you get a positive answer to the first question and a negative to the second, then overall you contribute to increasing misery in the world. On the other hand, if you have a positive answer to the second question, but unfortunately it was negative to the first, then you still have improved life in general. No harm was done. It is the second question that should be asked first, because it is more important. Just think about all the things that make yourself miserable. One likely reason you are miserable is because out of complex chains of causality, people are making each other miserable in society.

Prime research is not important just because of its own research goals. It is important because it is the gateway for many people to start thinking more deeply about mathematics. This was something that Euclid probably understood. Who would be interested in geometry derived from axioms otherwise? Perhaps people like Kurt Gödel, but very few else. You have to keep the gateway open to mathematics, or mathematics as a whole will become irrelevant for people. I have seen this happen in the past century and I try to help turning this around. At the same time, programming has exploded in popularity and is dwarfing mathematics. The regress in mathematics is not due to lack of opportunity, but of the mindset of people in the community. When people learn a programming language, they have tutorials and software to help them get started. Where are the tutorials and software in mathematics? It does not mean that there are lack of tools, perhaps they already exists, but they are just not used and advocated among researchers as a way to reach out to people. Thinking in this way can slowly change how mathematics as a field interacts with society. The basic idea is to start thinking about research from a perspective of game design. A game needs a lot of work to be built, but once built, it engages people more efficiently than personal outreach. The primbix is like a way to upgrade the gateway of mathematics to be challenging with our technology.

It is not sufficient to build a community around a research topic, because there are plenty of such dreams that never succeeded in the long term. The mathematical community is one of the most successful communities through history. It has continued without interruption for over 2 millenia. However, despite teaching mathematics to all students in school, the mathematical community has shrunk in relation to the overall population size, across the world. If you only think in short terms of time, then you might not consider this a problem. However, a few centures ahead, humanity could be almost wiped out from pollution. If the population size is drastically reduced, then the mathematical community will be a bottleneck for humanity to recover. The irony is that there never has been so many people who were given an opportunity to get interested in mathematics, at the same time it fails so spectacularly. This is because, despite efforts to build communities, the research topic itself is just not interesting enough to keep people engaged.

This is because people still think that mathematics ought to be about solving problems!

By leveraging existing knowledge from game design, there is no difficulty in changing how people engage with mathematics. If people from the mathematical community could stop discriminating programmers for a second, then they might see that they could actually learn something (for once, since the exam :P). However, doing game design is hard and you can not expect to master it in a day. Actually, changing how you think is hard because people around you are not trying to change how they think. Still, one does not need to look outside mathematics to find examples of good game design. Just think about the ideas that inspire and intrigue you the most. I bet that these ideas are produced by people with game design skills.

When people in the mathematical community discriminates programmers, they behave like people who programmers in general consider as “total losers”. This is because such behaviour is unusual within the programming community. In programming, it is natural to be curious and open to new ideas, because you might know something that can push a good idea even further. If you are going around calling other people names, then this is considered a trait of insanity or sleep deprivation. I am sure that people in the mathematical community do not realize that some behaviour they normalized is considered very rude and inappropriate outside the community. No, it is not a sign of confidence with a hint of arrogance. It is just bullying and showing how much of a loser you are.

So, why are people in the mathematical community behaving like losers? This could be simply because the mathematical community is losing overall. The initial elitist positions turned out into toxicity, which alienated people from the community, further driving up toxicity and thus creating an aging, increasingly socially irrelevant movement out of the longest intellectual tradition of humanity. I do not take any pleasure in telling people this, but somebody has to tell them.

Over and over, history shows that rational thinking often comes surprisingly early on the stage, but gets replaced or out-competed by mythical thinking. For example, the natural philosophers could have kick started science at an earlier point, but was replaced by an anthropomorphizing philosophy which continues to this day. This is usually attributed to Socrates, Plato and Aristoteles. However, people actually forgot how Aristoteles was blamed for his failures. Most people do not know that Aristoteles was the first zoologist! He was actually contributing to science before he became the teacher of Alexander the Great. It is through the role as a zoologist that Aristoteles got a bad reputation, when this work should be celebrated precisely since it was an important demonstration of how to be wrong. People do not even have to look for examples of philosophy outside the usual academic circle of texts. They just have to look closer and learn more about the very great names they admire (or despise)! It is simply: Just do philosophy better at what it is already good at!

Similarly, I am not asking to change mathematics much. Philosophy went through the same phase of immaturity. Start designing “fun” problems. There is plenty of room for “serious” research later.