Existence of Normal Paths

by Sven Nilsen, 2019

In this paper I introduce a non-ambiguous notation for existence of normal paths.

Existence of a normal path[1] is expressed in path semantics by putting `∃?` in front of a normal path:

$$∃?f[g_{i→n}] : bool$$

It is equivalent to asking whether the path set[2] is non-empty:

$$∃?f[g_{i→n}] \quad <=> \quad f[g_{i→n}] <¬=> \{\}$$

It can also be expressed in first-order logic[3]:

$$∃?f[g_{i→n}] <=> ∃\ h\ \{\ ∀\ x : ∀f\ \{\ f(g_0(x_0), g_1(x_1), \ldots, g_{n-1}(x_{n-1})) = g_n(h(x_0, x_1, \ldots, x_{n-1}))\ \}$$

Here, `∀f` means the trivial path (domain) of `f`[4].

A common way to write is a function that can be substituted with another:

$$f[g_{i→n}] <=> h$$

The substitution is valid if and only if the following equation has a solution `h`:

$$∀\ x : ∀f\ \{\ f(g_0(x_0), g_1(x_1), \ldots, g_{n-1}(x_{n-1})) = g_n(h(x_0, x_1, \ldots, x_{n-1}))\ \}$$

For multiple solutions, one can write (where `h` is a function):

$$f[g_{i→n}] => h$$

Multiple solutions can also be expressed as the path set[2] containing at least two functions:

$$f[g_{i→n}] <=> \{h_0, h_1, \ldots\}$$

The notation `∃?` is used to not mix notation with the existential path:

`∃f` Existential path (a function that tells what `f` returns, codomain)

`∃?f` Existence of function (`true` if the function `f` exists, `false` otherwise)

It makes only sense to ask whether a function exists in a language
that can talk about the function indirectly.
The expression `f[$g_{i→n}$]` points to a function “out there” that
predicts property `$g_n$` from properties `$g_i$` of `f`.

References:

[1] "Normal Paths"
Sven Nilsen, 2019
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/normal-paths.pdf

[2] "Path Sets"
Sven Nilsen, 2017
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/path-sets.pdf

[3] "First-order logic"
Wikipedia
https://en.wikipedia.org/wiki/First-order_logic

[4] "Constrained Functions"
Sven Nilsen, 2017
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/constrained-functions.pdf