Quantum Propagation

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In this paper I present an algorithm for simulating quantum functions, by quantum propagation.

A quantum binary function `f` is non-deterministic with a complex probabilistic existential path:

$f : () \rightarrow \mathbb{B}^n \qquad \exists_{pc}f : \mathbb{B}^n \rightarrow \mathbb{C}$

A partial observation `g` of `f` is a deterministic function that removes some information:

$g \cdot f \qquad g : \mathbb{B}^n \rightarrow \mathbb{B}^m \qquad m < n$

The probabilistic existential path of `g · f` is computing by summing over complex probability amplitudes and taking the norm squared. The norm squared can be written as a product:

$|x|^2 \Leftrightarrow x \cdot x^*$

Now, since `x` is a sum of complex probability amplitudes, one can expand the product:

$(\sum i \{ x_i \})(\sum j \{ x_j \})^* = \sum i, j \{ x_i x_j^* \}$

From `n` amplitudes, this forms `$n^2$` basis vectors, which are symmetric since `$x_i x_j^* = x_j x_i^*$`.
These are still complex numbers, only their sum has a zero imaginary component.

Instead of summing over outcomes, one can pick a random basis vector `$x_i x_j^*$` such that:

g(i) = g(j) <=> (i, j) : [g] [eq] true

The random basis vector is added to the corresponding outcome.
This is the basic principle for simulating quantum functions using this technique.

- Probabilities can be computed directly by summing over propagated basis vectors
- In the limit, this sum converges toward a real probability for each outcome
- At any given instant, every outcome is equally probable if `f` is semi quantum

For example, if `$f : () \rightarrow \mathbb{B}^2$` and `$and : \mathbb{B}^2 \rightarrow B$`, there are two outcomes:

[f] [and] false	00	01	10	$3^2 = 9$
[f] [and] true	11			$1^2 = 1$

Even `[f] [and] false` seems to happen 9 times as often as `[f] [and] true`,
it is correct to just sum this up, because the equation `$|x|^2 = x \cdot x^*$` also holds for sums.
In the limit, this converges toward a real probability when normalized `$|x|^2 / (|x|^2 + |y|^2)$`.
When the sample size is short, there can be non-zero outcomes that converges to zero probability.
Also, the deviation of `P(x)` for small sample sizes affects the corresponding `P(y)`.