

Tautological Self-Referential Identity

by Sven Nilsen, 2018

In this paper I prove that a language consisting of only self-referential identity is tautological.

This paper builds on ideas from the paper “Comments on Leibniz’s law” (Sven Nilsen, 2018).

Assume there are two objects, `a` and `b`, and a language:

$$L = \{ [\text{identical}(L, a)] \text{ true}, [\text{identical}(L, b)] \text{ true} \}$$

Both `a` and `b` are identical to themselves.

Either `a` is identical to `b` or not. If `a` is identical to `b`, then the evaluation looks as following:

$$L = \{ \text{true}, \text{true} \}$$

If `a` is not identical to `b`, then the evaluation looks as following:

$$\begin{array}{ll} L = \{ \text{true}, \text{false} \} & \text{for `a`} \\ L = \{ \text{false}, \text{true} \} & \text{for `b`} \end{array}$$

So, whether one assumes that `a` and `b` are identical or not, does not lead to a contradiction in the self-referential language. This means that the language, or this family of languages, is tautological.

In general:

$$L := \set{(\text{objects} : [\text{any}]) = \cup x : \text{objects} \{ [\text{identical}(L, x)] \text{ true} \}}$$

The meaning of a such language is precisely the same as the assumption of identity between objects. There exists one language for every way of assigning identities. `L` is not a function since it refers to itself. Instead, it is a family of functions where each member satisfies equality of identities.

This is interesting because any identities, assigned for any language, corresponds to a similar assignment of a self-referential language that contains only identity. The construction of a such language is a representation of the identity assignment itself, as something self-contained and separate concept from the underlying meaning of the language.

When constructing some language, the objects and the assignment of identities are not encoded. They are not given as a part of the description of the language. However, through this tautological self-referential family of languages, it is possible to talk about what it means to assign consistent identities among objects. Any consistent assignment is encoded in the description of this family of languages. So, for whatever assignment of identities between objects that is used, it is consistent if and only if it has a language in this family.