Path Semantical Outer Quality

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In this paper I introduce a relation that represents path semantical outer quality, which relates to path semantical quality that is thought of as an inner quality. The two forms of quality forms a finite lattice with inner quality as the bottom element and outer quality as the top element. Other path semantical operators such as aquality and contravariant quality are also elements of this lattice and come in inner and outer variants. This lattice shows how models of Path Semantics can be axiomatically different while at the same time be intuitively similar and share many theorems.

Path Semantical Outer Quality is defined as following, using the Path Semantical Qubit[1] operator:

psoq(a, b) <=> (a == b) ∧ ¬~¬a ∧ ¬~¬b

Viewed in relation to Path Semantical Quality[2], one can think of various path semantical operators[3][4] as elements of some lattice where normal quality is “inner quality”:

000	(a == b) ∧ ~a ∧ ~b	inner quality
001	(a == b) ∧ ~a ∧ ~¬b	inner reverse contravariant quality
010	(a == b) ∧ ~¬a ∧ ~b	inner contravariant quality
011	(a == b) ∧ ~¬a ∧ ~¬b	inner aquality
100	(a == b) ∧ ¬~a ∧ ¬~b	outer aquality
101	(a == b) ∧ ¬~a ∧ ¬~¬b	outer contravariant quality
110	(a == b) ∧ ¬~¬a ∧ ¬~b	outer reverse contravariant quality
111	(a == b) ∧ ¬~¬a ∧ ¬~¬b	outer quality

The binary code associated with each operator has the property that when performing a uniform involution, e.g. `000 => 111` or `101 => 010`, one turns “inner” into “outer” and vice versa.

This view of path semantical operators is important because in some models of Path Semantics, there are definitions of e.g. aquality that only makes sense in an “inner” sense and not an “outer” sense, or vice versa. One such model is Dit Calculus[5], which only has “outer aquality”.

In the standard model of Path Semantics, the qubit operator `~` has the following invariant[1]:

¬~a == ~¬a

This makes inner aquality and outer aquality equal to each other.

Outer quality and inner quality in the standard model is only equal to each other when `¬¬a => a`. This holds in classical logic with excluded middle[6] (PL), but not in constructive logic[7] (IPL).

An extended lattice can be constructed using `000 – 222`, where `2` annotates use of `¬¬`.
When reading these operators out load, one can use “existential” if the number contains `2`,
while using the code to distinguish ambiguous cases from each other.

For example:

102	(a == b) ∧ ¬~¬a ∧ ¬~¬¬b	existential outer contravariant quality
201	(a == b) ∧ ¬¬~a ∧ ¬¬~¬b	existential inner reverse contravariant quality

References:

[1] "Path Semantical Qubit"
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