Quantum Knight Functions

by Sven Nilsen, 2020

In this paper I present a family of quantum non-deterministic functions that are always partially observed in two different states and which other observed probabilities follows a "chess knight" rule.

A quantum knight function is a quantum non-deterministic function `f`:

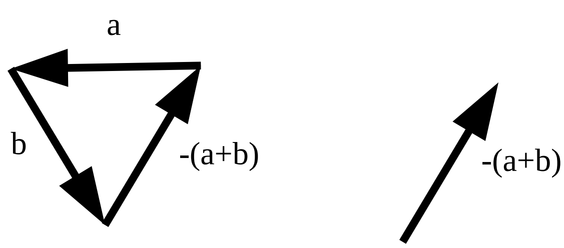
$f : () \to \mathbb{B}^2$ $\qquad$ $\exists_{pc}f : \mathbb{B}^2 \to \mathbb{C}$

With a complex probability distribution generated by two arbitrary basis vectors `a` and `b`:

$a + b \neg= 0$

$a : \mathbb{C}$
$b : \mathbb{C}$

With the intuition that two complex probability amplitudes are non-zero and equal to `-(a+b)`:



Usually, the complex probability amplitudes are assigned the labels:

00	01	10	11
-(a+b)	a	b	-(a+b)

A partial observation `g` is when a deterministic function removes some information about `f`.

$g \cdot f$	Partial observation
$g : \mathbb{B}^2 \to B$	`g` removes some information about `f`

The kind of partial observation used here is to look for any of the states:

g = {(= 00), (= 01), (= 10), (= 11)}	Look for any concrete state

State	**$P([f]\ [g]\ true) = (\exists_p(g \cdot f))(true)$**
00	1
01	$\lvert a\rvert^2 / (5\lvert a\rvert^2 + 4(a \cdot b) + \lvert b\rvert^2)$
10	$\lvert b\rvert^2 / (\lvert a\rvert^2 + 4(a \cdot b) + 5\lvert b\rvert^2)$
11	1

Here are probabilities of states `01` and `10` for some constrained solutions:

Constrained solution	**01**	**10**
$a = b$	$1/10$	$1/10$
$(a \cdot b) = 0 \quad \wedge \quad \lvert a\rvert = n\lvert b\rvert$	$1/(5 + n^2)$	$n^2/(1 + 5n^2)$
$\lvert a\rvert = \lvert b\rvert = \lvert a+b\rvert$	$1/8$	$1/8$
$(a \cdot (a + b)) = 0 \quad \wedge \quad \lvert a\rvert = n\lvert b\rvert$	$1/(1 + n^2)$	$n^2/(5n^2 - 3)$

The name "quantum knight" comes from Chess, where the knight piece can only move 1 forward and 2 sideways, or 2 forward and 1 sideways. Similarly, the complex probability amplitude of `g · f` returning `false` for states `01` and `10`, are the sums:

01	$-(2a + b)$	$\lvert-(2a + b)\rvert^2 = \lvert 2a + b\rvert^2 = 4a^2 + 4(a \cdot b) + b^2$
10	$-(a + 2b)$	$\lvert-(a + 2b)\rvert^2 = \lvert a + 2b\rvert^2 = a^2 + 4(a \cdot b) + 4b^2$

Quantum knight functions are impossible to construct with pure functions extended with random sources. This is because it makes no sense that a function `f` always returns both `00` and `11`.

- If one looks for `00`, then `f` will always return `00` as a classical non-deterministic function
- If one looks for `11`, then `f` will aways return `11` as a classical non-deterministic function

In all pure functions extended with random sources, the probabilities of `f` returning `00` and `11` will add up to 100%, but for quantum knight functions they add up to 200%. Probabilities for quantum non-deterministic functions only adds up to 100% when a choice of partial observation is committed.

Intuitively, a quantum knight function behaves in a such way that they can predict the observer. Semantically, it is easy to be mislead by this counter-intuitive property, e.g. by thinking that this proves the quantum knight functions knows *intention* of the observer.

However, this is not technically the right way of interpreting what is happening:

- When looking for `00` or `11`, one gets an answer `true` or `false`
- The observer's *intention* is not encoded into whether the answer is `true` or `false`
- Both `00` and `11` will be observed to return `true`, independent of the observer's *intention*
- The partial observation function `g` is just a choice of an arbitrary function, e.g. `(= 00)`

With other words, a quantum knight function can only fool an observer expecting to see results from pure functions extended with random sources. Any observer used to quantum non-determinism will not be surprised. It is just a mathematical fact of the nature of quantum non-deterministic functions. Neither is there any hidden semantics carrying over to any underlying physical system. The semantics of quantum knight functions is closed in their self-describing representation. Their mathematical properties would still be true, even if one lived in a classical physical universe. However, how quantum physical systems is to be interpreted, is a complete different question.

How does `f` "know" whether one looks for `00` or `11`? The answer is that quantum destructive interference cancels out the complex probability amplitudes of `g · f` returning `false` to zero, such that the only possible state is returning `true`. In pure functions extended with random sources, this would mean that `f` always returns `true` for states `00` and `11`.