Universal Non-deterministic Sampler

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In this paper I represent a universal higher order function for sampling discrete probability distributions with non-deterministic path semantics. It is called a "non-deterministic sampler" and grounds probabilistic non-deterministic path semantics in probabilistic path semantics.

A non-deterministic sampler is a higher order function `ɣ` such that:

$$\exists_p f \iff \exists_p ɣ(\exists_p f)$$

$$f : A \to B$$
$$ɣ : (B \to prob) \to () \to B$$

A probabilistic path `$\exists_p f$` returns the probability of a function returning some output, which is used by `ɣ` to weight bias of random decisions. It takes the probabilistic path of a function and returns a function which probabilistic path is non-deterministic logically equivalent to the argument. The function returned by the non-deterministic sampler takes no arguments and can be used to sample the probability distribution which the probabilistic path represents. Since it takes no argument, it means that the function's decisions relies **only** on a source of randomness and from a non-deterministic logical perspective is unique (instances can not be distinguished from each other). It follows that:

The non-deterministic sampler is universal because every function it returns is unique.

The non-deterministic sampler is nilpotent over probabilistic-existential-path-argument-composition:

$$\exists_p f \iff \exists_p ɣ(\exists_p f) \iff \exists_p ɣ(\exists_p ɣ(\exists_p f)) \iff \exists_p ɣ(\exists_p ɣ(\exists_p ɣ(\exists_p f))) \iff \exists_p ɣ(\exists_p ɣ(\exists_p ɣ(\ldots)))$$

Notice that this is **not** function composition, since `$\exists_p$` has lower operator precedence than application.

For example, the non-deterministic sampler of `$id_{bool}$` is same as a random coin flip with `false/true`:

$$ɣ(\exists_p id_{bool}) \iff \backslash() = \text{if random}() < 0.5 \; \{ \text{ false } \} \text{ else } \{ \text{ true } \}$$

$$id_{bool} := \backslash(x : bool) = x$$

The probabilistic existential path of `$id_{bool}$` returns `0.5` for both `true` and `false`:

$$\exists_p id_{bool} := \backslash(x : bool) = 0.5$$

We can derive the probabilistic existential path of the non-deterministic sampler of `$id_{bool}$`:

$$\exists_p id_{bool} \iff \exists_p ɣ(\exists_p id_{bool})$$
$$\exists_p ɣ(\exists_p id_{bool}) := \backslash(x : bool) = 0.5$$

Therefore, when you flip a `false/true` coin, you get `false` half the time and `true` half the time.