Permutative Functions

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A permutative function is a function where any permutation of arguments commutes.
This property generalizes the commutative property[1] of functions to any number of arguments.

For example, a commutative function of 2 arguments has the following property:

f(a, b) = f(b, a)

Since `f` is commutative, it is also permutative.

A permutative function of 3 arguments has the following property:

f(a, b, c) = f(a, c, b) = f(b, a, c) = f(b, c, a) = f(c, a, b) = f(c, b, a)

All functions returning a single value for all arguments are permutative,
for example a proof in propositional logic must return `true` for all inputs:

f <=> \true

This means that the arguments can be applied in any order while preserving the proof.

For example, modus ponens[2] can be constructed in 2 ways using 2 arguments:

(a ∧ (a => b)) => b
(b ∧ (b => a)) => a

Although proofs are interesting, they are trivial permutative functions, because arguments are ignored.

Here are the truth tables of two non-trivial permutative functions of 3 arguments:

	$\mathbf{f_0}$	$\mathbf{f_1}$
000	1	1
001	1	0
010	1	0
011	0	1
100	1	0
101	0	1
110	0	0
111	1	1

All non-trivial permutative functions depend on all arguments.

References:

[1] "Commutative property"
Wikipedia
https://en.wikipedia.org/wiki/Commutative_property

[2] “Modus ponens”
Wikipedia
https://en.wikipedia.org/wiki/Modus_ponens