Semi Quantum Non-Determinism

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In this paper I introduce semi quantum non-deterministic functions, that share some properties with pure functions plus extensions of non-determinism using random sources.

A semi quantum non-deterministic function has the property that all norms of complex probability amplitudes are equal to each other. With other words, a semi quantum function behaves almost like a random source, except that the orientation of complex probability amplitudes allows interference.

A semi quantum function of type `() → 𝔹` is indistinguishable from a coin flip, since:

$|0| = |1|$

$|0|^2 / (|0|^2 + |1|^2) = |0|^2 / (|0|^2 + |1|^2) = |0|^2 / 2|0|^2 = 0.5$

There is no way to measure this function using a partial observation, because it only returns one output.

A semi quantum function of type `() → 𝔹²` has the following constraint:

$|00| = |01| = |10| = |11|$

In general, for binary functions, the definition of a semi quantum function is:

$\text{semi\_quantum}(f) = \forall\ x, y\ \{\ |(\exists_{pc}f)(x)| = |(\exists_{pc}f)(x)|\ \}$

$f : () \to \mathbb{B}^n \qquad \exists_{pc}f : \mathbb{B}^n \to \mathbb{C}$ Quantum binary function

When constructing quantum functions for various purposes, it is common to add the requirement of semi quantumness to either the quantum source `f`, the partial observation `g · f`, or both:

f : [semi_quantum] true	Requires the quantum source to be semi quantum
g · f : [semi_quantum] true	Requires the partial observation to be semi quantum

The property of semi quantumness comes from the constraint that the complex probability distribution behaves like a real probability distribution.

$|x|^2 / (|x|^2 + |y|^2) = |x| / (|x| + |y|) \quad \Rightarrow \quad |x| = |y|$
$|x|^2 / (|x|^2 + |y|^2 + |z|^2) = |x| / (|x| + |y| + |z|) \quad \Rightarrow \quad |x|^2 / (|y|^2 + |z|^2) = |x| / (|y| + |z|)$
$|x|^2 / (|y|^2 + |z|^2) = |x| / (|y| + |z|) \quad \land \quad |y| = |z| \quad \Rightarrow \quad |x| = |y|$

Normalized, `$|\exists_{pc}f|^2 \Leftrightarrow \exists_p f$` holds for all quantum functions. `$|\exists_{pc}f| \Leftrightarrow \exists_p f$` holds for semi quantum.

With other words, a semi quantum function behaves like a real probability distribution even before the measuring step where complex probability amplitudes are converted into real probabilities. This makes it easier to construct quantum functions that are "almost" like other non-deterministic functions.