Countable Infinity
from Existential Paths

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In this paper I show how to prove when a set does not exceed countable infinity using existential paths.

The function `f` takes a set `a` of size up to countable infinity, but no more, when:

$\exists$f <=> a \ b
|b| >= 1

f : a → T
a : T → bool
b : T → bool

Notation:

`$\exists$f`	existential path of `f`
`a \ b`	`a` except `b`
`|b|`	size of `b`

All members of `a` is mapped by `f` to `a \ b`. If this set is finite, then at least two members of `a` is mapped to the same output. In order to map every member of `a` to some unique member of `a \ b`, the set of `a` must be infinite.

Assume that `a` is infinite. By constraining `f` with `a \ b`:

$\exists$f{a \ b} <=> a \ b \ f(b)

This works because when the `b` is not allowed as input and `f` maps to unique outputs, there must be one map that is missing: The output `f(b)`.

By constraining `f` with `a \ b \ f(b)`:

$\exists$f{a \ b \ f(b)} <=> a \ b \ f(b) \ f(f(b))

Repeating this process:

$\exists$f{a \ b \ f(b) \ \ f^2(b) … \ f^n(b)} <=> a \ b \ f(b) \ f^2(b) \ f^3(b) \ … \ f^{n+1}(b)

Since `f` maps to unique outputs, there is no case where `$f^n$(b) = $f^m$(b)` when `n ¬= m`.
The entire set `a` can be counted by repeating this process.
It is kind of like counting with numbers, except you count like this: `(>= 0), (>= 1), (>= 2) …`.
Therefore, since `a` is either finite or countable infinite, it can only be up to countable infinite in size.