

Last Order Logic

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In this paper I present a logical language which is kind of like First Order Logic, but funny.

Last Order Logic (LOL) is a language given by the following syntax:

1	term for truth
0	term for falsehood
I	unit interval type
(a, b)	tuple of `a` and `b`
$\neg a$	logical NOT
$a \wedge b$	logical AND
$a \vee b$	logical OR
$a \underline{\vee} b$	logical XOR
$a == b$	logical EQ
$a => b$	logical IMPLY
$a \leadsto b$	a path from `a` to `b`
$(a \leadsto b) \sim 0 : a$	path start point
$(a \leadsto b) \sim 1 : b$	path end point
$((a \leadsto b) \leadsto (c \leadsto d)) \sim (1, 0) : c$	surface point
$f(a)$	lambda application
$\backslash i : I = f(i)$	lambda abstraction
$\forall i : I \{ f(i) \} : \text{un}(T)$	where `f` has true type `T` for all `i`
$\forall i : I \{ f(i) \} : \text{nu}(U)$	where `f` has false type `U` for at least one `i`
$\exists i : I \{ f(i) \} : \text{nu}(T)$	where `f` has true type `T` for at least one `i`
$\exists i : I \{ f(i) \} : \text{un}(U)$	where `f` has false type `U` for all `i`
$f(a \leadsto b) == (f(a) \leadsto f(b))$	lambda application for paths
$\text{lift}(a) : a$	`a` is lifted to type level
$(0 \leadsto 1) : (I \leadsto I)$	the path `0 ~ 1` has path type `I ~ I`

Notice that path points and quantifiers \forall and \exists do not evaluate, they produce a type instead. The quantifiers \forall and \exists are homogenous, which means when it has e.g. a true type, it has the same type for every true case. The reason for this design choice is to introduce multiple senses of truth values. In First Order Logic^[1], there is only one sense of truth which is `false` and `true`. In Last Order Logic, one can have e.g. $\text{un}(1)$ which means “uniformly true”. This can be nested, e.g. $\text{nu}(\text{un}(1))$ which means “non-uniformly uniformly true”. Furthermore, a path can be a truth value, e.g. $\text{un}(a \leadsto a)$ means “uniformly `a` in one dimension”.

un	uniform	alternatives: objective, “clothes on”
nu	non-uniform	alternatives: personal, “nude”

The terminology “uniform” and “non-uniform” comes from Avatar Extensions^[2]. These senses of truth are “truthful” when every symbol is concrete, such as $\text{un}(1)$ or $\text{nu}(1 \leadsto 0)$. When the symbols are not concrete, the uniformity or non-uniformity is not to be taken seriously. The intuition is that a uniform truth can turn into non-uniform and vice versa. This idea borrows from the mathematical universe called “The Joker”^[3].

This paper only presents the language of Last Order Logic and does not include inference rules. Rest of this paper are examples.

Example 1:

$\because f(i : I) = (1 \sim= 0) \sim i$
 $\therefore f(0) : 1 \quad f(1) : 0 \quad \exists i \{ f(i) \} : \text{nu}(1)$

Example 2:

$\because f(i : I) = ((1 \sim= 0) \sim= (1 \sim= 0)) \sim i$
 $\therefore f(0) : (1 \sim= 0) \quad f(1) : (1 \sim= 0) \quad \forall i : I \{ f(i) \} : \text{un}(1 \sim= 0)$

Example 3:

$\because p \sim 0 : 1 \quad p \sim 1 : 0$
 $\therefore p == (1 \sim= 0)$

Example 4:

$\because \exists i : I \{ p \sim i \} : \text{nu}(1) \quad p \sim 0 : 0$
 $\therefore p == (1 \sim= 0)$

Example 5:

$\because \exists i : I \{ p \sim i \} : \text{nu}(1 \sim= 0) \quad p \sim 0 : (0 \sim= 0)$
 $\therefore p == ((0 \sim= 0) \sim= (1 \sim= 0))$

Example 6:

$\because \forall i : I, j : I \{ p \sim (i, j) \} : \text{un}(\text{un}(1))$
 $\therefore p = ((1 \sim= 1) \sim= (1 \sim= 1))$

Example 7:

$\because \forall i : I \{ p \sim i \} : \text{un}(1)$
 $\therefore \forall i : I \{ \neg p \sim i \} : \text{un}(0)$

Example 8:

$\because \forall i : I \{ p \sim i \} : \text{un}(1)$
 $\therefore \neg \exists i : I \{ \neg p \sim i \} : \text{un}(1)$

Example 9:

$\because \forall i : I \{ p \sim i \} : \text{un}(1)$
 $\therefore \exists i : I \{ \neg p \sim i \} : \text{un}(0)$

Example 10:

$\because \exists i : I \{ p \sim i \} : \text{nu}(1)$
 $\therefore \neg \exists i : I \{ p \sim i \} : \text{nu}(0)$

Example 11:

$\because x : \text{un}(0)$
 $\therefore \neg x : \text{un}(1)$

Example 12:

$\because x : \text{nu}(0)$
 $\therefore \neg x : \text{nu}(1)$

Example 13:

$\because \neg \forall i : I \{ p \sim i \} : \text{nu}(1)$
 $\therefore \exists i : I \{ \neg p \sim i \} : \text{nu}(1)$

Example 14:

$\because \exists i : I, j : I \{ p \sim (i, j) \} : \text{un}(\text{false})$
 $\therefore \exists i : I \{ \neg p \sim i \} : \text{nu}(\text{true})$

References:

- [1] “First Order Logic”
Wikipedia
https://en.wikipedia.org/wiki/First-order_logic

- [2] “Avatar Extensions”
AdvancedResearch – Summary page on Avatar Extensions
<https://advancedresearch.github.io/avatar-extensions/summary.html>

- [3] “The Joker”
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https://github.com/advancedresearch/path_semantics/blob/master/papers-wip2/the-joker.pdf