Stair Pairs

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In this paper I present a way to enumerate pairs that resembles a stair pattern.

A pair is a tuple `(a, b) : (nat, nat)` where `a < b`[1].
Pairs can be mapped to and from natural numbers[2].
By enumerating natural numbers, one can also enumerate pairs.

However, mapping to and from natural numbers is not the only way to enumerate pairs.
One can select a pair, decrease `a` and increase `b` until `a = 0`.

$$(3, 4) \quad (2, 5) \quad (1, 6) \quad (0, 7)$$

Notice that the sum `a + b` is the same for all such pairs.
The problem is to find a way to select initial pairs such that this method covers every possible pair.

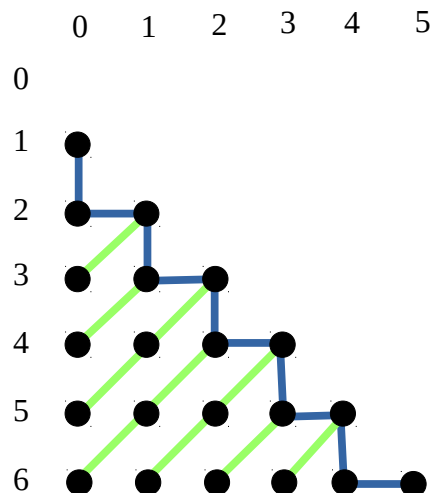A stair pair is a pair corresponding to each natural number `n`:

$$\text{stair\_pair}(n : nat) = \text{if even}(n) \ \{((n + 2) / 2 - 1, (n + 2) / 2)\}$$
$$\text{else} \ \{((n + 3) / 2 - 2, (n + 3) / 2)\}$$

$$\text{stair\_pair\_number}((a, b) : (nat, nat)) = a + b - 1$$

For example:

$$\text{stair\_pair}(0) == (0, 1)$$
$$\text{stair\_pair}(1) == (0, 2)$$
$$\text{stair\_pair}(2) == (1, 2)$$
$$\text{stair\_pair}(3) == (1, 3)$$

One can see the stair pattern when connecting dots in the matrix[3] representation of pairs:

References:

[1]   "Ordered pair"
        Wikipedia
        https://en.wikipedia.org/wiki/Ordered_pair

[2]   "Natural number"
        Wikipedia
        https://en.wikipedia.org/wiki/Natural_number

[3]   "Matrix (mathematics)"
        Wikipedia
        https://en.wikipedia.org/wiki/Matrix_(mathematics)