Adjoint Paths

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In this paper I formalize adjoint paths using path semantics.

An adjoint path is a relationship between two asymmetric normal paths of `f`:

$$f[g_0 \times id \rightarrow id] <=> f[id \times g_1 \rightarrow id]$$

$$f : T \rightarrow U$$

Since `id` maps to same type `T → T`, it follows that `$g_0$` and `$g_1$` also maps to same type:

$$g_0 : T \rightarrow T$$
$$g_1 : T \rightarrow T$$

Since these two normal paths are the same, it means that they both use the same function:

$\because$ $f[g_0 \times id \rightarrow id] <=> h$ $\quad f(x, y) = h(g_0(x), y)$
$\because$ $f[id \times g_1 \rightarrow id] <=> h$ $\quad f(x, y) = h(x, g_1(y))$

$\therefore$ $h(g(x), y) = h(x, g(y))$

The `$g_0$` is called the “left side” and `$g_1$` is called the “right side”.

When the left side is equal to the right side, `f` is a self-adjoint.

The `id` function is a self-adjoint for every function.

$$f[id] <=> f[id]$$