Transitive Mirror Theorems

by Sven Nilsen, 2021

In this paper I present three transitive mirror theorems found in Path Semantical Logic.

Similar to Normal[1] and Abstract[2] Implication Theorems, there are two Transitive Mirror Theorems, which are proofs in Path Semantical Logic[3]:

(a, b) (C, D): Normal Transitive Mirror Theorem
(a => b), b(C), b(D) => (C = D)

(a, b) (C, D): Abstract Transitive Mirror Theorem
(a => b), b(C)=b(D) => (C = D)

(a, b) (C, D): Transitive One-Way Mirror Theorem
(a => b), b(C)=>b(D) => (C => D)

Here, the tuple `(a, b)` has level 0 and the tuple `(C, D)` has level 1.
The notation `b(C)` means `b=>C` where `C` is at a higher level.
Notice that these levels follow the new standard order[4].

There is no corresponding analogue of the Constrained Implication Theorem[5].
However, there is a related "one-way mirror" version that has `C => D` instead.

The Normal Transitive Mirror Theorem written in an alternative notation:

(a, b) (c, d):
a => b
b => c
b => d
---
c = d

The "mirror" happens by `a => b` being transported into `c = d`, despite `a => b` being directional.
Since `b` implies both `c` and `d`, by transitivity `a` also implies both `c` and `d`.

This can be interpreted as `c` and `d` being mirrored or swapped at each end-point of `a => b`:

(a => c) ∧ (b => d) (a => d) ∧ (b => c)

References:

[1] “Implication Theorem”
Sven Nilsen, 2020
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/implication-theorem.pdf

[2] “Abstract Implication Theorem”
Sven Nilsen, 2020
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/abstract-implication-theorem.pdf

[3] “Path Semantical Logic”
AdvancedResearch – Reading sequence on Path Semantical Logic
https://github.com/advancedresearch/path_semantics/blob/master/sequences.md#path-semantical-logic

[4] “New Standard Order for Levels”
Sven Nilsen, 2021
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip2/new-standard-order-for-levels.pdf

[5] “Constrained Implication Theorem”
Sven Nilsen, 2020
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/constrained-implication-theorem.pdf