Contraction Theorem

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In this paper I present a contraction theorem found in Path Semantical Logic.

The Contraction Theorem is a proof in Path Semantical Logic[1]:

$\therefore$ (a, b) (A, B):
$\therefore$ a(A ∧ B), b(A ∧ B) => contr(A, B)

$\because$ contr(x, y) = (x ∧ y) ⊻ (¬x ∧ ¬y)

Where the tuple \`(a, b)\` has level 1 and the tuple \`(A, B)\` has level 0.
The notation \`a(T)\` means \`a=>T\` where \`T\` is at a lower level.

The \`contr\` function was derived in the paper "Contractible Types"[2].

The Contraction Theorem says that when two propositions at level 1 are associated with two propositions in level 0, then the two propositions in level 0 are in contractible type family.

This means that the two propositions in level 0 behaves together like a single proposition.
When the first proposition is \`true\`, the second proposition is \`true\`.
When the first proposition is \`false\`, the second proposition is \`false\`.

It is not possible to prove \`contr(A, B)\` from \`a(A ∧ B)=b(A ∧ B)\`.

References:

[1] “Path Semantical Logic”
AdvancedResearch, reading sequence on Path Semantics
https://github.com/advancedresearch/path_semantics/blob/master/sequences.md#path-semantical-logic

[2] “Contractible Types”
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https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/contractible-types.pdf