Dual Identity Lift

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In this paper I introduce a technique for lifting identity morphisms to Restricted Dual Composition.

The following two rules lifts identity morphisms in Avatar Logic[1] to Restricted Dual Composition[2]:

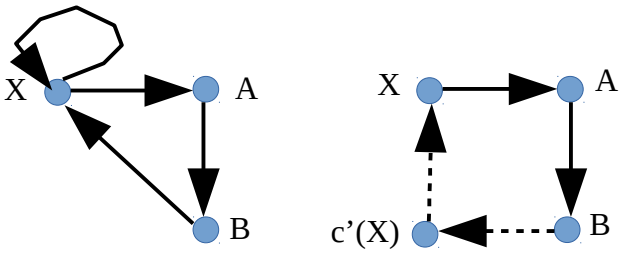
```
(c'(X), X) :- (X, Y), (X, X), X != Y, X != c'(T), Y != c'(U).
(Y, c'(X)) :- (Y, X), (X, X), X != Y, X != c'(T), Y != c'(U).
```

The first rule says that when there is an outgoing arrow from `X` to `Y`	(X, Y)
and an identity morphism on `X`,	(X, X)
where `X` is not equal to `Y`	X != Y
and `X` is not some 1-avatar	X != c'(T)
and `Y` is not some 1-avatar	Y != c'(U)
there is a morphism from the 1-avatar `c'(X)` to `X`	(c'(X), X)

The second rule says that when there is an incoming arrow from `Y` to `X`	(Y, X)
and an identity morphism on `X`,	(X, X)
where `X` is not equal to `Y`	X != Y
and `X` is not some 1-avatar	X != c'(T)
and `Y` is not some 1-avatar	Y != c'(U)
there is a morphism from `Y` to the 1-avatar `c'(X)`	(Y, c'(x))

The first rule makes sure that when there is an identity morphism on `X`, a goal is specified that activates restricted dual composition for `X`. The second rule makes sure that the incoming arrow is connected to the dual such that the loop can be lifted.

The loop is lifted because the goal `(c'(X), X)` completes the loop when the incoming `(Y, X)` is lifted to `(Y, c'(X))`. One can also think about this technique as a kind of identity morphism transformation in a linear solver. Notice that in a monotonic solver, the unlifted morphisms are preserved.



Unlifted *Lifted*

The 1-avatar `c'(X)` is called the "dual" of `X` and written in dual notation as `X*`. However, `X` is also the "dual" of `c'(X)`. This means that rules with dual notation can be thought of as generating multiple rules over 1-avatars.

In the case of dual identity lift, dual notation is not needed because Restricted Dual Composition is used merely as a technique to detect loops locally from `X`. There is only need for one direction.

References:

[1] "Avatar Logic"
AdvancedResearch – Summary Page on Avatar Extensions
https://advancedresearch.github.io/avatar-extensions/summary.html

[2] "Restricted Dual Composition"
Sven Nilsen, 2021
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip2/restricted-dual-composition.pdf