Universal Existential Paths

by Sven Nilsen, 2017

An existential path[1] is a total function[2] that determines the truth value of whether there exists some input to a function that returns some output:

∴ $\exists f := \backslash(x') = \exists x \{ f(x) = x' \}$

∵ $x' = f(x)$
∵ $f : X \to X$

This can be generalized using domain constraint notation[3]:

∵ $\exists f\{T_X\} := \backslash(x') = \exists x : T_X \{ f(x) = x' \}$

In domain constraint notation you can create the universal `f{}`[3],
so one can also create a universal existential path:

∴ $\exists f\{\} := \backslash(T_x) = \backslash(x') = \exists x : T_X \{ f(x) = x' \}$

All universal existential paths are reducible to normal existential paths using `$true_1$`:

∴ $\exists f\{[true_1]\ true\} <=> \exists f\{true_1\} <=> \exists f$

∵ $true_1 := \backslash(\_) = true$

A sub-existential path is when you pass any other function than `$true_1$` to a universal existential path:

∴ $(\exists f\{\})([g(c)]\ true) <=> \exists f\{[g(c)]\ true\} <=> \exists f\{g(c)\}$

∴ $\exists f\{g(c)\} : B \to bool$
∵ $f : A \to B$
∵ $g : C \to A \to bool$

When one passes a function of more than one argument to a universal existential path,
one creates a higher order existential path:

$\exists f\{g\} : C \to B \to bool$

Name	**Notation**	**Type**
Sub-existential path	$\exists f\{g(c)\}$	$B \to bool$
Higher order existential path	$\exists f\{g\}$	$C \to B \to bool$
Universal existential path	$\exists f\{\}$	$T_C \to B \to bool$

References:

[1] “Existential Paths”
Sven Nilsen, 2017
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/existential-paths.pdf

[2] “Partial function”
Wikipedia
https://en.wikipedia.org/wiki/Partial_function

[3] “Domain Constraint Notation”
Sven Nilsen, 2017
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/domain-constraint-notation.pdf