

Path Semantical Quality

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In this paper I introduce a partial equivalence operator for use in Path Semantics.

A path semantical^[1] quality between a and b is written:

$$a \sim b \quad \text{psq}(a, b)$$

Where \sim is infix notation and psq is the explicit function name.

When $=$ is used instead of \sim , one might use \sim .

Path semantical quality is logically close to biconditional^[2], but is also a partial equivalence relation^[3]. The “quality” word is used to distinguish it from “equality” and mere “partial equivalence”.

Being a partial equivalence relation, it is symmetric and transitive, but not necessarily reflexive:

$$\forall a, b \{ (a \sim b) == (b \sim a) \} \quad \text{Symmetry}$$

$$\forall a, b, c \{ (a \sim b) \wedge (b \sim c) \Rightarrow (a \sim c) \} \quad \text{Transitivity}$$

What makes psq different from eqb is how biconditions are lifted, requiring symbolic distinction^[4]:

$$(a \Rightarrow b) \wedge (b \Rightarrow a) \quad \Leftrightarrow \quad a == b$$

$$(a \Rightarrow b) \wedge (b \Rightarrow a) \wedge \text{sd}(a, b) \quad \Rightarrow \quad a \sim b$$

Where sd is the special operator for symbolic distinction.

Notice that \Rightarrow is used instead of \Leftrightarrow because the converse^[5] does not hold.

The converse drops the the requirement of symbolic distinction:

$$a \sim b \quad \Rightarrow \quad a == b$$

So, all path semantical qualities are equalities, but not vice versa.

This means path semantical quality, when assumed, is a stronger notion of equality.

There are two ways of arriving at $a \sim a$:

$$(a == b) \wedge (a \neq b) \quad \Rightarrow \quad a \sim a \quad \text{Product Witness}$$

$$(a \sim a) \wedge \neg \exists b : \arg \{ \text{sd}(a, b) \Rightarrow (a == b) \} \quad \text{Loop Witness}$$

The Product Witness arrives at $a \sim a$ through $a \sim b$ using transitivity and symmetry.

The Loop Witness arrives at $a \sim a$ through directly assuming it.

Product Witness and Loop Witness are part of the theory of Avatar Extensions^[6].

References:

- [1] “Path Semantics”
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