Un- and Re-Sesh

by Sven Nilsen, 2024

In this paper, I introduce two new operators used to get rid of the Sesh property and later restore it from path semantical qubit. I describe how this is implemented in Pocket-Prover. This means that standard Path Semantics can cover theories that do not use the Sesh axiom. While practical, one has to be careful since these operators do not work property in single-bit brute force solvers.

In standard Path Semantics[1], it is common to assume the Sesh axiom by default:

(!~a == ~!a)^true   for all `a`

The problem is that some theories do not use the Sesh axiom[2]. It is still convenient to use the symbol `~` for such qubit operators, as long one makes it clear that the Sesh axiom does not apply.

The challenge is: How can one check theorems of such theories using e.g. brute force solvers like Pocket-Prover[3] that are based on standard Path Semantics?

Amazingly, there is a trick that works as long the brute force solver operates on bit-vectors with more than one bit at the time:

1. Use a seed for a random generator that changes each time the proof function is called
2. Generate two bit patterns using the random generator
3. Pick a random bit location using the seed and check if the two patterns differ at that bit
4. Swap one of the bit patterns if the selected bit differs
5. Use the bit in the input-vector to determine which pattern to XOR with the input

This function is `un_sesh` and is invertible, using a corresponding `re_sesh` function:

re_sesh . un_sesh <=> id

For a reference implementation, see the Pocket-Prover project[3].

For example, to get a qubit operator that do not have the Sesh axiom, one can use the following:

platonic_qubit   :=   un_sesh . qubit

The name `platonic_qubit` is due to the Sesh axiom being incompatible with excluded middle for agnostic language bias[2], so since the Sesh axiom is Seshatic biased[4] by forcing constructivity in agnostic reasoning, the suppression of the Sesh axiom corresponds to a Platonic language bias which can be used with excluded middle (e.g. in brute force theorem proving).

One should be careful when using this technique, because the operators do not work properly with single bit brute-force solvers. When a single bit brute-force solver is used, the Sesh property is conserved with `un_sesh`. In most cases this is not a problem. The work around is to drop the swap of one of the bit patterns in the `un_sesh` implementation, which makes `un_sesh` no longer invertible. If one needs `re_sesh` one can fall back to modifying `qubit` directly.

The motivation is to use standard Path Semantics to deal with theories that do not use the Sesh axiom, without needing to rebuild the standard qubit operator on top of a custom defined qubit.

References:

[1] “Overview of Path Semantical Logics”
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[3] “Pocket-Prover”
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[4] “Seshatism vs Platonism”
AdvancedResearch – Summary page on Avatar Extensions
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