Unary Embedding Termination

by Sven Nilsen, 2020

In this paper I discuss a termination property of unary numbers embedded in binary numbers.

A unary number can be embedded in binary as following:

0	0
1	1
2	11
3	111
4	1111
…	
n	$2^n - 1$ in binary

Imagine that the number of bits available are infinite, such that a unary number has the pattern:

…0000001111111…

This pattern is a kind of termination property (called “unary embedding termination”).
For any unary number \`n\`, at the n-th bit the pattern changes from ones to zeroes.
It is known that there are only zeroes on the left side of the change and only ones at the right side.
Any binary number has this termination property if and only if it is an encoding of a unary number.

However, how can one prove that there exists no such binary number that is not embedded?

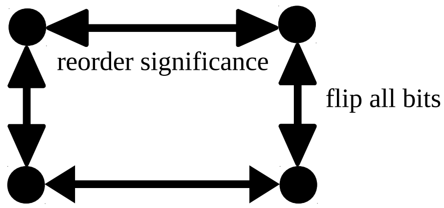
The problem is that the semantics of this property depends on a choice of representing binary numbers.

Among all the possible permutations of ordering the bits, there exists only two representations where all natural numbers, encoded as unary numbers embedded inside binary numbers, have this property:

1. Ordering significance from right to left (standard)
2. Ordering significance from left ro right (e.g. \`10\` is written \`01\`)

Since the semantics is depending on the representation, unary embedding termination is not provable within any axiomatic system that models only natural numbers and nothing else.

Also, if you flip a single bit, turning \`0\` into \`1\` and vice versa, then the representation looses this unary embedding termination property. However, if you flip all bits, then the unary embedding termination property is preserved.



There are 4 different embeddings of unary numbers in binary numbers which preserves the unary termination property, generated by two operations: 1) Reorder significance and 2) flip all bits.