Countable Infinity from Existential Paths

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In this paper I show how to prove when a set does not exceed countable infinity using existential paths.

The function \`f\` takes a set \`a\` of size up to countable infinity, but no more, when:

∃f <=> a \ b
|b| > 0

f : a → T
a : T → bool
b : T → bool

Notation:

\`∃f\`	existential path of \`f\`
\`a \ b\`	\`a\` except \`b\`
\`|b|\`	size of \`b\`

All members of \`a\` is mapped by \`f\` to \`a \ b\`. If this set is finite, then at least two members of \`a\` is mapped to the same output. In order to map every member of \`a\` to some unique member of \`a \ b\`, the set of \`a\` must be infinite.

Assume that \`a\` is infinite. By constraining \`f\` with \`a \ b\`:

∃f{a \ b} <=> a \ b \ f(b)

This works because when the \`b\` is not allowed as input and \`f\` maps to unique outputs, there must be one map that is missing: The output \`f(b)\`.

By constraining \`f\` with \`a \ b \ f(b)\`:

∃f{a \ b \ f(b)} <=> a \ b \ f(b) \ f(f(b))

Repeating this process:

∃f{a \ b \ f(b) \ \ f^2(b) … \ f^n(b)} <=> a \ b \ f(b) \ f^2(b) \ f^3(b) \ … \ f^{n+1}(b)

Since \`f\` maps to unique outputs, there is no case where \`$f^n$(b) = $f^m$(b)\` when \`n ¬= m\`.
The entire set \`a\` can be counted by repeating this process.
It is kind of like counting with numbers, except you count like this: \`(>= 0), (>= 1), (>= 2) ...\`.
Therefore, since \`a\` is either finite or countable infinite, it can only be up to countable infinite in size.