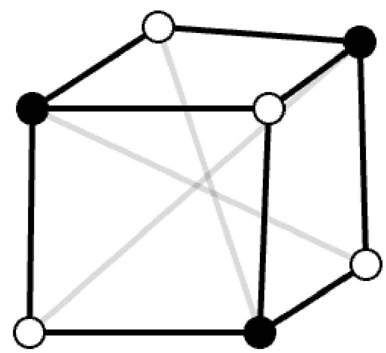
# Introduction to Avatar Graphs

by Sven Nilsen, 2020

*In this paper I give an introduction to Avatar Graphs, based on a recent discussion with Christian Urlea.*

When I talk to people about Path Semantics[1], the topic of Avatar Graphs[2] seems to be something that people are most interested in. This is likely because Avatar Graphs are intended to guess how mathematical theories might be extended at higher dimensions. This is closely related to Higher Category Theory[3] and people working in this area study these abstract mathematical structures.



*An avatar graph with 3 core candidates*

The basic idea of an Avatar Graph is that there is a "core" and a graph/network of avatars that have a particular relationship to the core. Informally, one says that an avatar is like the core, but playing a different role. Avatars "integrate information" with the core. The core is usually the original mathematical theory and the extended theory is represented as the graph/network.

The rules of Avatar Graphs are designed to isolate the patterns which are interesting in this paradigm:

- Non-contractibility – All n-avatars where n>1 have more than one child closer to the core
- Connectivity – The whole graph is connected
- Unique highest avatar – Each core has a unique highest avatar in the graph
- Reachability – All nodes are reachable when walking from highest avatar to the core
- Avatar connectivity – An n-avatar can only pass information to any partition of lower avatars

These rules are repeated for each node, such that core candidates are separated from n-avatars where the smallest n is greater than one. This operation is too complex for the human brain, so to explore Avatar Graphs there is a tool where the user creates the graph and computer algorithms analyzes it[4].

In Path Semantics, there is a technique that lets one replace binary relations with a version that is more efficient for theorem proving and abstract generalizations. A binary relation is an ordered pair `(a, b)`. Normally, there are no constraints, which makes it hard to model theories with many constraints using logical relations. However, Unique Universal Binary Relations[5] allows one to write `(a, b)` and this contains all the information needed. For a predicate `p(a, b)`, the right argument `b` is assigned the role `p`, such that `role_of(b) = p` and `p(a) = b`. Hence, unconstrained binary relations can be replaced by this new and more efficient way of modeling theories.

1-avatars can be used when Unique Universal Binary Relations are too constrained for a theory. This is done by introducing a symbol which "lifts" an existing symbol such that it can be assigned a new role.

For example, a person named Bob says: When he is working, his collegue Alice is his most trusted partner, but when he is not working, he considers Alice his friend. Here, Bob plays two different avatar roles, `working_bob` and `not_working_bob`, where `role_of(working_bob) = trusted_partner` and `role_of(not_working_bob) = friend`. So, `(alice, working_bob)` and `(alice, not_working_bob)`.

The relations were specified from the perspective of Bob, meaning that the theory is subjective. However, from the perspective of Alice, this does not seem to work:

trusted_partner(alice) = working_bob
friend(alice) = not_working_bob

Alice might have more trusted partners than Bob when he is working, and she might have more friends than Bob when he is not working. However, one can swap Bob and Alice, just fine:

trusted_partner(bob) = working_alice
friend(bob) = not_working_alice

Notice that the same 1-avatar lifting is done for Alice and this would be consistent if one could separate the contexts using the roles, because of the imposed constraints of Unique Universal Binary Relations.

The problem here, is that the separate theories for Alice or Bob are sufficient seen from the subjective perspective of a person, but is too constrained for an objective model of social relations. One would like a mechanism that allows transitioning from a good subjective model to a good objective model.

A 2-avatar is a method of extending a “subjective” theory such that the information in one subjective theory can be integrated with the information in another subjective theory.

A trivial way to do this, is to reintroduce normal unconstrained binary relations.
However, in some circumstances, it is easier or more efficient to use a subjective theory as template.

To reuse the subjective theory as a template, one can permute a set `{alice, bob}`:

x = alice, y = bob
x = bob, y = alice

trusted_partner(x) = working_y
friend(x) = not_working_y

For a 3-avatar, one can permute a set `{alice, bob, carl}`, creating `3! = 6` different contexts:

e.g.  x = alice, y = bob, z = carl

∀ a ∈ {y, z} { trusted_partner(a) = working_x }
∀ a ∈ {y, z} { friend(a) = not_working_x }

This idea of permuting a set, is based on a discussion I had with Christian Urlea[6] about Avatar Graphs. He realized that Avatar Graphs represents the role totality of the core. With other words, a kind of space covering representation. I connected this with permutations as paths along an n-cube[7].

Basically, a subjective theory is some path when walking from the highest avatar to the core. The objective theory is what you get from taking all paths and integrating the information in all of them.

This technique is very powerful, because it allows arbitrary individual differences between subjective theories to vary using conditional constraints. For example, `if a == alice { … }`.

## References: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[3] “Higher category theory”
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[4] “Avatar-Graph – A library for Avatar Graphs”
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[5] “Unique Universal Binary Relations”
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https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/unique-universal-binary-relations.pdf

[6] “Applied Category Theory Server”
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[7] “Permutations as Cubical Paths”
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