

Binary Square Matrix Combinatorics

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In this paper I formalize binary square matrix combinatorics using Directional Set Algebra.

For all $n : \text{nat}$, there is an associated binary square matrix combinatorics:

0	Empty matrix set
D	Diagonal matrix set
U	Upper triangle matrix set
L	Lower triangle matrix set
1	All matrices

The following law holds with Directional Set Algebra:

$$D + U + L = 1$$

Sizes of sets:

$$\begin{aligned} |0| &= 0 \\ |D| &= 2^n \\ |1| &= 2^{(n \cdot n)} \\ |U| &= |L| = 2^{(n \cdot (n-1) / 2)} \end{aligned}$$

Sub-types of binary matrix sets can be constructed using elements 0 , 1 and $?$.

The following laws holds with Directional Set Algebra, where $?$ is top and there is no bottom:

$$0 + 1 = ?$$

For example, for $n = 3$:

$$\begin{array}{ccccc} ?00 & & 0?? & & 000 & & ??? \\ 0?0 & + & 00? & + & ?00 & => & ??? \\ 00? & & 000 & & ??0 & & ??? \end{array}$$