

Normal Paths as Function Sub-Types

by Sven Nilsen, 2017

Here I represent a new insight that unites logical equivalence of paths with sub-types of functions defined using paths. This makes function sub-types similar to variable sub-types. This new notation has strengths and weaknesses different from both paths and their equivalent equations.

In path semantics^[1] one can define sub-types^[2] of variables by using functions. Such sub-types makes sense only when the existential path^[3] returns `true`:

$$\begin{aligned} \because & \quad a : [g] \, b \\ \because & \quad b : [\exists g] \, \text{true} \\ \\ \because & \quad g : A \rightarrow B \\ \because & \quad \exists g : B \rightarrow \text{bool} \end{aligned}$$

The new idea is that sub-types of functions can be defined using paths:

$$\begin{aligned} \because & \quad f : [[g]] \, h \\ \\ \because & \quad f : A \times A \rightarrow A \\ \because & \quad g : A \rightarrow B \\ \because & \quad h : B \times B \rightarrow B \end{aligned}$$

This is the same as writing:

$$\begin{aligned} \because & \quad f[g] \iff h \\ \because & \quad g(f(a, b)) = h(g(a, b)) \end{aligned}$$

In asymmetric notation^{[4][5]}, one can add sub-script indices like this:

$$f : [[g_{i-n}]] \, h$$

Like for normal sub-types, there is an existential path that returns `true` if the path set is non-empty:

$$h : [\exists[g]] \, \text{true}$$
$$\exists[g] \iff \exists[g \times g \rightarrow g]$$

The `∃[g]` notation is used since the brackets makes it easy to use notation for symmetric and asymmetric paths. `∃g` would lead to confusion since it is used for existential path of `g`.

There is still motivation to use the notation of logical equivalence of functions in proofs.

One does not need to write double brackets, e.g. `f[g] ⇔ h`.

Also, path sets^[6], written `f[g] ⇔ {h₀, h₁}`, are useful for describing partial functions in terms of function spaces and might not be easily expressed as a function sub-type.

References:

- [1] “Path Semantics”
Sven Nilsen, 2016-2019
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/path-semantics.pdf
- [2] “Sub-Types as Contextual Notation”
Sven Nilsen, 2018
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/sub-types-as-contextual-notation.pdf
- [3] “Existential Paths”
Sven Nilsen, 2017
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/existential-paths.pdf
- [4] “Algebraic Notation for Asymmetric Paths”
Sven Nilsen, 2017
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/algebraic-notation-for-asymmetric-paths.pdf
- [5] “Path Function Product Notation”
Sven Nilsen, 2017-2019
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/path-function-product-notation.pdf
- [6] “Path Sets”
Sven Nilsen, 2017
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/path-sets.pdf