Probabilistic Liar’s Paradox

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A Liar’s paradox is a paradox like the sentence “this sentence is false”. However, in systems that reasons about uncertainty, it is desirable to use probability instead of booleans as truth values. In order to extend Liar’s paradox from booleans to probability, it is necessary to do the following:

1. Assume that a sentence that contains a reference to a sentence is to be interpreted as if it refers to a member of the powerset of all sentences. It refers to every sentence which is a solution of the sentence containing the reference. This approach was used in the paper “Liar’s Paradox and Complete Functions”.
2. Provide a prior probability distribution over the set of all sentences. Every sentence is given a weight which is normalized over the member of the powerset such that its probability can be calculated.
3. Probabilities listed in the sentence that contains a reference to a sentence is then calculated from the set of probabilities which the member of the powerset of all sentences contains.

With other words, probabilities are considered free variables that are determined from the normalized weights of the underlying member of the powerset of all sentences.

For example:

> This sentence is 50% likely to be true.

Assume that this sentence talks about a language where 50% of sentences are true and 50% of sentences are false. Each sentence is assigned a weight 1.

The sentence is sound for all sets which contains 50% true sentences and 50% false sentences. So, one could pick a sentence that is true and a sentence that is false, and say this about these sentences. Or, one could pick two sentences that are true and two sentences that are false, etc.

In the set of all sentences that are either true or false, there is no sentence that is 50% likely to be true. So, it can not refer to any sentence in particular, but only speaking about a set of sentences.

However, by extending the set of all sentences to contain sentences of probabilistic truth value, it can contain the sentence “This sentence is 50% likely to be true”. This sentence still has weight 1, but only 0.5 of it is true. So, picking out this sentence as the only member of the set makes it possible to talk about a particular sentence. The sentence becomes self-referential.

Now, when probabilistic truth values become self-referential, one gets a probabilistic Liar’s paradox about the set of sentences which the sentence refers to.

If it is 100% true that a sentence is 50% true, then the set which the sentence refers to must contain another sentence that is 0% true. However, if the set contains two sentences, it is 0% true that it is 100% true that a sentence is 50% true. Therefore, by assigning a weight 1 to each case, it is only 50% true that the sentence is 50% true. Which again, is 100% true and the Liar’s paradox follows.