Quantum Schrödinger Functions

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In this paper I formalize Quantum Schrödinger Functions and introduce the Phi quantum function.

The time-dependent Schrödinger equation for a `$\psi$` wavefunction over a Hilbert space:

$i\hbar\psi'(t) = H\psi(t)$ time-dependent Schrödinger equation

$\psi : \text{time} \to \mathbb{H}$

The type of complex numbers is a Hilbert space with the inner product:

∴ $\langle a, b\rangle \iff ab^*$

∵ $(\mathbb{C}, \text{mul}_{\mathbb{C}} \cdot (\text{id}, \text{conj})) : [\text{hilbert\_space}]\ \text{true}$
∵ $\text{conj}(x) = \text{re}(x) - \text{im}(x)i$

A quantum Schrödinger function `f` is a higher order quantum function depending on time:

$f : \text{time} \to T$ where `T` is usually `$\mathbb{B}^n$` or `$\mathbb{R}^n$` and `$T^0 \iff ()$`
$\exists_{cp}f(t) \iff \psi(t)$

$\psi : \text{time} \to T \to \mathbb{C}$ `$\psi$` satisfies time-dependent Schrödinger for all `T`

The Phi quantum function is a basic building block for quantum Schrödinger functions:

$\text{phi} : \text{frequency} \to \text{time} \to ()$
$\exists_{cp}\text{phi}(f)(t) \iff \varphi(f)(t)$

$\varphi(f)(t) = \sin(2\pi ft) + i\cos(2\pi ft)$
$\varphi(f)'(t) = (2\pi f)(\cos(2\pi ft) - i\sin(2\pi ft))$

Observed alone, `phi` always returns, but nevertheless it is has a complex probability amplitude.
The function `$\varphi(f)(t)$` simply rotates around in a unit circle at frequency `f`.

One can derive the analogue of the Schrödinger equation for `$\varphi(t)$`:

∴ $\hbar\varphi(f)'(t) = (hf)(\cos(2\pi ft) - i\sin(2\pi ft))$ multiply with `$\hbar$` (`$\hbar 2\pi = \hbar h / \hbar = h$`)
∴ $i\hbar\varphi(f)'(t) = E((-i \cdot i)\sin(2\pi ft) + i\cos(2\pi ft))$ multiply with `$i$`
∴ $i\hbar\varphi(f)'(t) = E\varphi(f)(t)$

∵ $\hbar = h / 2\pi$ reduced Planck's constant
∵ $E = hf$ Planck-Einstein relation

The Hamiltonian `E` is a measure of the total energy, so Phi is a quantum Schrödinger function.