Open Morphisms

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In this paper I introduce the use of Path Semantical Quality in Higher Category Theory. This can be interpreted as a generalization of open sets in topology, but for morphisms instead of sets.

In the field of topology[1], open sets[2] are used to define continuous maps[3]. An open set is a generalization of an open interval in the real line. An open set, over some set \`X\`, can be constructed out of:

- Unions of open sets in \`X\`
- Finite intersections of open sets in \`X\`
- The empty set in \`X\`
- The whole set \`X\` itself

This very loose definition of open sets allows great flexibility, so the notion of "open" is depending on the kind of topological space under consideration. It is worth noticing that since the whole set \`X\` itself is an open set, this requires at least one subjective choice. Therefore, there is no way to make a such definition universal, except for the empty set.

In Path Semantics[4], such subjective choices are common when working with Path Semantical Quality[5]. On one side, there are specific axioms associated with theories using quality. On the other side, it is difficult to understand all the theories combined. This raises the question: What is quality?

This open problem has resisted detailed analysis and research for several years. During the research, new possible theories have been developed and thus further deepened the problem. One suggestion, from Kent Palmer (PhD), was to unify quality with Hegel's philosophy. Palmer believes that Hegel thinks about groupoids. This is historically plausible since groupoids are related to topology and Hegel drew inspiration from Leibniz, one the of pioneers in the development of topology as a field.

One breakthrough came when quality was used to express solutions of the imaginary inverse[6].
A first draft included an axiom:

~f => ~inv(f)

However, it turned out that the first draft of the axioms did not fit properly with Higher Category Theory[7]. By taking inspiration from Higher Category Theory, the above axiom was removed and instead using both \`~f\` and \`~inv(f)\` to express an isomorphism. This means that the solution is interpreted as directional.

Interpreted as open morphisms, composition replaces union and intersection of open sets:

~f ∧ ~g => ~(g · f)

This means, if \`f\` and \`g\` are open, then the composition \`g · f\` is open.
Naturally, the identity morphism \`id\` is open, which corresponds to the "empty set".
The "whole set" as open might be \`~true\`[8] which can be thought of as the whole category.

A natural interpretation of open morphisms in Higher Category Theory is that one can use internal n-morphisms to express weak equivalences[9] and avoid infinite regress[10] by using quality.

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