Catuṣkoṭi Communication

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In this paper we give an intuitive explanation of Cubical Binary Codes in the AML Catuṣkoṭi bridge.

In communication, an error correction message[1] contains some information that can be used to check another message. When applied to the Liar's paradox[2], the sentence "this sentence is false" is both a message and an error correction message. The expected result, due to error correction, is `false`, which also adds the assumption `x : [value_of] false`[3] as a belief that is evaluated internally when interpreting the message:

```
x : [value_of] false        Belief by error correction message
x : [value_of] false        Message
----------------------
true                        Internally consistent, externally inconsistent
```

The reason the message is internally consistent, is because the belief by the error correction message is consistent with what the message says. However, this does not imply that the message has been checked for errors. The reason the message is externally inconsistent, is because the internal consistency conflicts with the expected `false`, which means the message contains an error.

In such systems (which we here call "The Normal"), the message is interpreted only once (in addition to error correction messages) and the Liar's paradox is detected. However, these systems are unable to experience tautological self referential statements[4], but only "see" one of two possibilities:

"this sentence is true"

```
x : [value_of] true         Belief by error correction message
x : [value_of] true         Message
---------------------
true                        Internally consistent, externally consistent
```

The Joker is a mathematical universe where messages are not taken seriously. Now, watch this:

"this sentence is true"

```
x : [value_of] false        Disbelief by error correction message (The Joker)
x : [value_of] true         Message
----------------------
false                       Internally inconsistent, externally consistent
```

The message is internally inconsistent, which is expected of The Joker. However, the message is externally consistent! It has no errors! It is *correct* that "this sentence is true" is false, because if the sentence lies about itself, then it is **indeed false**!

We define Catuṣkoṭi[5] Communication to be the union of The Normal and The Joker.

In The Normal, it is common to reduce the amount of bits required for communication. This is done by assuming the error correction message “the next sentence is true” for all messages, unless specified otherwise. When sending “the next sentence is false”, one can swap the expected value temporarily. Therefore, when sending “the next sentence is true”, it is *obvious*, because the information is true but not relevant for communication.

In Catuṣkoṭi Communication, both “the next sentence is false” and “the next sentence is true” are obvious. This is because all messages are checked for both `true` and `false`. Notice that these error correction messages are obvious in the sense that the information is both true and irrelevant. They are not obvious in The Normal sense!

Catuṣkoṭi Communication has 4 truth values per interpretation:

00	the expected value is `false` and the received value is `false`
01	the expected value is `false` and the received value is `true`
10	the expected value is `true` and the received value is `false`
11	the expected value is `true` and the received value is `true`

For every message, there is two interpretations, so the truth values come in pairs:

(0?, 1?)

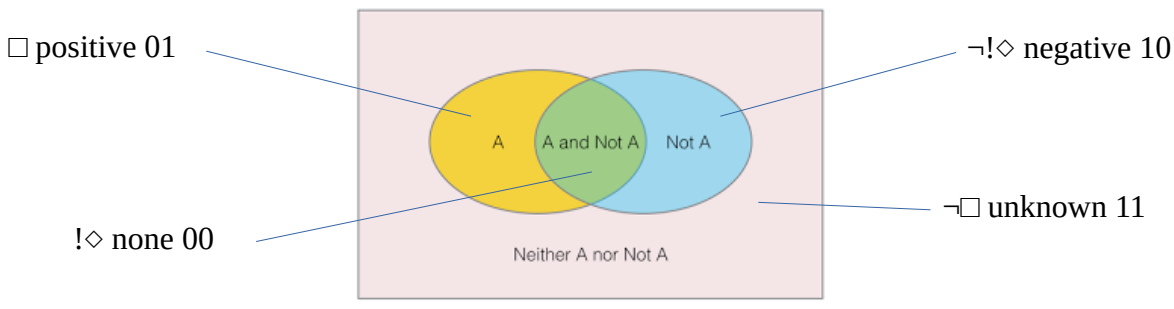
The first element starts always with `0` and the second element starts always with `1`.
This means that these bits can be skipped and we only need the second bit from each element of the pair. Instead of storing this bit directly, we will store whether the second bit from the first element is equal to `0`, and whether the second bit from the second element is equal to `1`:

00	Inconsistent with The Joker, inconsistent with The Normal	none
01	Inconsistent with The Joker, consistent with The Normal	positive
10	Consistent with The Joker, inconsistent with The Normal	negative
11	Consistent with The Joker, consistent with The Normal	unknown

By thinking of `none/positive/negative/unknown` as measurements, one arrives at AML Catuṣkoṭi[6].

The intuition is that in both positive (01) and negative (10) cases, the receiver learns something. The none (00) and unknown (11) states have in common that the receiver is not learning anything, but they are different in the reason **why** one is not learning anything.

Daniel Fischer found an illustration[7] that might help to visualise these states as points in a space:



References:

[1] “Error detection and correction”
Wikipedia
https://en.wikipedia.org/wiki/Error_detection_and_correction

[2] “Liar paradox”
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https://en.wikipedia.org/wiki/Liar_paradox

[3] “Sub-Types as Contextual Notation”
Sven Nilsen, 2018
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/sub-types-as-contextual-notation.pdf

[4] “Paradoxes and Tautologies”
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[6] “Answered Modal Logic Catuṣkoṭi”
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[7] “The four sided negation: A tool for overcoming bias in science, atheism and democracy”
the red pill – concentrated musings on psychohistory, teleportation and AI
http://the-redpill.blogspot.com/2015/11/the-four-sided-negation-tool-for.html