Answered Modal Logic in Cubical Binary Codes

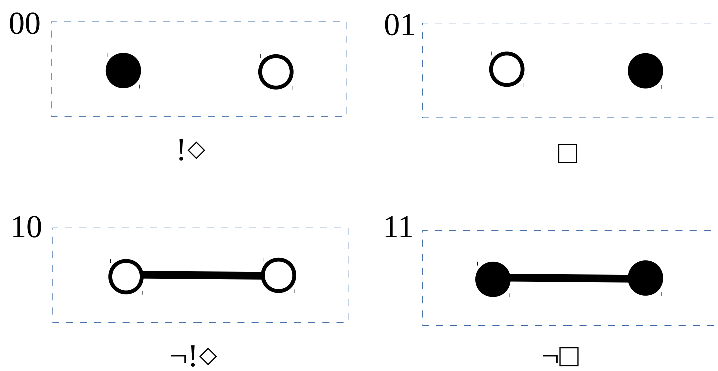
by Sven Nilsen, 2020

In this paper I describe the simplest case of encoding Answered Modal Logic in Cubical Binary Codes.

Answered Modal Logic has a modal set consisting of 3 elements:

¬□ = {!◇, ¬!◇, □}

The smallest Cubical Binary Code that fits this set is the following:



The reason for this assignment is because `□` and `!◇` are natural opposites:

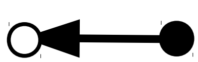
01	□	For all cases, the predicate returns `true`
00	!◇	There exists no case for which the predicate returns `true`

To clarify, the cases here refers to context, not inputs of the predicate.
The logic is for reasoning about the predicate across possible worlds, hence the modality.

The natural way of assigning `¬!◇` is 10:

10	¬!◇	There exists two cases, where the predicate returns `true` and `false`

If one models `◇ = {¬!◇, □}` as an interval, then one can say it includes `□` at one end:

◇ = (!◇, □]  One can think about `◇` as an arrow

The semantics of the code `11` (`¬□`) is naturally referring to the whole modal set:

 ¬□ = {!◇, ¬!◇, □}

This encoding satisfies the requirement that the building block of the logic can not fully model sets.
There is no way to model the empty modal set `{}`, hence a full model of a set is impossible to express.