Answered Modal Logic in Cubical Binary Codes

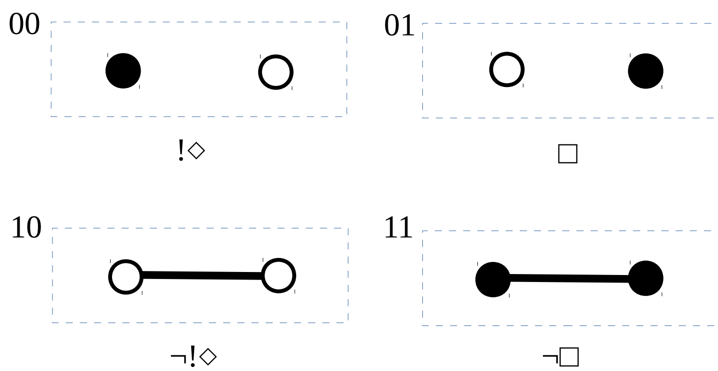
by Sven Nilsen, 2020

In this paper I describe the simplest case of encoding Answered Modal Logic in Cubical Binary Codes.

Answered Modal Logic has a modal set consisting of 3 elements:

¬□ = {!◇, ¬!◇, □}

The smallest Cubical Binary Code that fits this set is the following:



The reason for this assignment is because \`□\` and \`!◇\` are natural opposites:

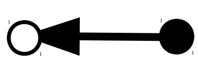
00	□	For all cases, the predicate returns \`true\`
01	!◇	There exists no case for which the predicate returns \`true\`

To clarify, the cases here refers to context, not inputs of the predicate.
The logic is for reasoning about the predicate across possible worlds, hence the modality.

The natural way of assigning \`¬!◇\` is 10:

10	¬!◇	There exists two cases, where the predicate returns \`true\` and \`false\`

If one models \`◇ = {¬!◇, □}\` as an interval, then one can say it includes \`□\` at one end:

◇ = (!◇, □]  One can think about \`◇\` as an arrow

The semantics of the code \`11\` (\`¬□\`) is naturally referring to the whole modal set:

 ¬□ = {!◇, ¬!◇, □}

This encoding satisfies the requirement that the building block of the logic can not fully model sets.
There is no way to model the empty modal set \`{}\`, hence a full model of a set is impossible to express.