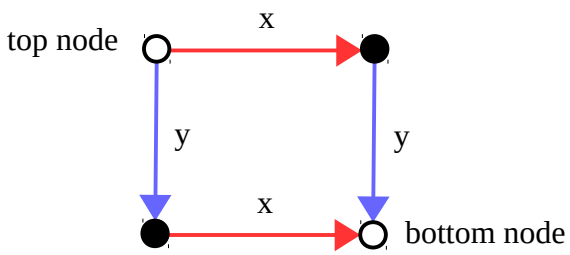
Permutations as Cubical Paths

by Sven Nilsen, 2020

In this paper I show that permutations can be modeled using paths on n-cubes.

Consider the following square where edges are arrows in different colors by parallel dimension:



There are two ways of navigating from the top node to the bottom node:

x, y y, x

Similarly, for a cube:

x, y, z	x, z, y	y, x, z	y, z, x	z, x, y	z, y, x

For a 4-cube:

x, y, z, w	x, y, w, z	x, z, y, w	x, z, w, y	x, w, y, z	x, w, z, y
y, x, z, w	y, x, w, z	y, z, x, w	y, z, w, x	y, w, x, z	y, w, z, x
z, x, y, w	z, x, w, y	z, y, x, w	z, y, w, x	z, w, x, y	z, w, y, x
w, x, y, z	w, x, z, y	w, y, x, z	w, y, z, x	w, z, x, y	w, z, y, x

For an n-cube, the paths are modeled using permutations.

Therefore, permutations of `n` elements can be modeled using paths on an n-cube.