Terminology for Morphisms

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$1_{XX} = 1_X$ $0_{XX} = 0_X$ $1_0 = 0_0$

$1_X : X \to X$	=>	$\forall g : X \to Y \{ g \cdot 1_X = g \}$
	=>	$\forall g : Y \to X \{ 1_X \cdot g = g \}$

$inv(1_X) = 1_X$
$inv(0_{XY}) = 0_{YX}$

$0_X : X \to X$	=>	$\forall g : X \to Y \{ g \cdot 0_X = 0_{XY} \}$
	=>	$\forall g : Y \to X \{ 0_X \cdot g = 0_{YX} \}$
f : mono	=>	$\forall g_0, g_1 \{ (f \cdot g_0 = f \cdot g_1) \Rightarrow (g_0 = g_1) \}$
	=>	inv(f) : epi
$f : X \to Y \land$ left_inv	=>	$\exists g : Y \to X \{ g \cdot f = 1_X \}$
	=>	f : mono
	=>	inv(f) : right_inv
f : epi	=>	$\forall g_0, g_1 \{ (g_0 \cdot f = g_1 \cdot f) \Rightarrow (g_0 = g_1) \}$
	=>	inv(f) : mono
$f : X \to Y \land$ right_inv	=>	$\exists g : Y \to X \{ f \cdot g = 1_Y \}$
	=>	f : epi
	=>	inv(f) : left_inv
f : iso	=>	f : left_inv
	=>	f : right_inv
	=>	f : mono
	=>	f : epi

1_X : iso	1_0 : iso	0_0 : iso
1_X : left_inv	1_0 : left_inv	0_0 : left_inv
1_X : right_inv	1_0 : right_inv	0_0 : right_inv
1_X : mono	1_0 : mono	0_0 : mono
1_X : epi	1_0 : epi	0_0 : epi

comp : iso × iso → iso
comp : left_inv × left_inv → left_inv
comp : right_inv × right_inv → right_inv
comp : mono × mono → mono
comp : epi × epi → epi

$comp(0_{XY}, g : Y \to Z) = 0_{XZ}$
$comp(f : X \to Y, 0_{YZ}) = 0_{XZ}$