Symmetric Path of Function Composition

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In this paper I show that the symmetric path of function composition is the function composition of symmetric paths.

Given two functions `$f_0$` and `$f_1$` and two symmetric paths `$f_0[g]$` and `$f_1[g]$`:

$$(f_1 \cdot f_0)[g] \Leftrightarrow f_1[g] \cdot f_0[g]$$

Proof:

$g \cdot f \Leftrightarrow h \cdot g$	Symmetric path equation for `$f[g] \Leftrightarrow h$`
$g \cdot f \cdot g^{-1} \Leftrightarrow h \cdot g \cdot g^{-1}$	Compose `$g^{-1}$` on both sides
$g \cdot f \cdot g^{-1} \Leftrightarrow h$	Using `$g \cdot g^{-1} \Leftrightarrow id$`
$g \cdot f \cdot g^{-1} \Leftrightarrow f[g]$	

$f_1[g] \cdot f_0[g]$	
$(g \cdot f_1 \cdot g^{-1}) \cdot (g \cdot f_0 \cdot g^{-1})$	
$g \cdot f_1 \cdot g^{-1} \cdot g \cdot f_0 \cdot g^{-1}$	
$g \cdot f_1 \cdot f_0 \cdot g^{-1}$	Using `$g^{-1} \cdot g \Leftrightarrow id$`
$g \cdot (f_1 \cdot f_0) \cdot g^{-1}$	
$(f_1 \cdot f_0)[g]$	

If `g` has an inverse `$g^{-1}$`, the symmetric path of function composition can be simplified:

$$(f_1 \cdot f_0)[g] \Leftrightarrow g \cdot f_1 \cdot f_0 \cdot g^{-1}$$