Quantum Non-Determinism

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In this paper, I show that quantum non-determinism corresponds to partial observations where the sum over probabilities are over complex numbers, with an extra step that obfuscates learning distributions.

In the paper "Partial Observations", I showed that probability distributions can be learned through partial observations. For example, a non-deterministic function `f : () → bool^n`:

00	01	10	11
0.2	0.3	0.1	0.4

A partial observation `g : bool^n → bool^m` where `m < n` can learn parts of `f` when:

$|[g]\ b| == 1$ for some `b`

$g \cdot f \quad : () \to bool^m$

Quantum non-deterministic functions uses complex numbers for probabilities instead of real numbers:

00	01	10	11
0.2 + 0.4i	0.3 – 0.1i	0.1 + 0.3i	-0.4 – 0.2i

Like before, the partial observation "sums over" the complex probability amplitudes:

0	1		
0.5 + 0.3i	-0.3 + 0.1i	first bit	
0.3 + 0.7i	-0.1 + -0.3i	second bit	
0.6 + 0.6i	**-0.4 – 0.2i**	logical AND	learns `f() == 11`
0.2 + 0.4i	0 + 0i	logical OR	learns `f() == 00`

If one could measure the complex probability amplitude directly, then one could use partial observations to learn the probability distribution of `f`. However, to observe a quantum system requires an extra step. To measure e.g. `0` one takes the norm squared divided by the sum of norms squared:

$|p_0|^2 / (|p_0|^2 + |p_1|^2) \quad \Longleftrightarrow \quad p_0 p_0^* / (p_0 p_0^* + p_1 p_1^*)$ `p*` replaces `i` with `-i`.

Which produces the following probabilities:

0	1		
0.773	0.227	first bit	
0.853	0.147	second bit	
0.783	**0.217**	logical AND	no longer learns `f() == 11`
1	0	logical OR	only observes `f() == 00`

The destructive interference of complex amplitudes obfuscates the learning through partial observation.