Symbolic Indistinction as Propositional Infinity

by Sven Nilsen, 2023

In this paper I present the idea that symbolic indistinction might be thought of as the analogue of infinity in number theory, but for propositions.

In the paper “Symbolic Distinction”[1] I introduced an operator for symbolic distinction and explained how symbolic distinction is a valid way of generalizing logic, while symbolic indistinction can lead to unsoundness problems.

In logic, when one says `a == b` where `a` and `b` are propositions, there is a notion of soundness that the logic should treat `a` and `b` as equal and kind of “forget” the differences between them. This is the common view, but it is not necessarily so for all perspectives of logic.

For example, when using propositional exponentials[2] one can still say how `a` and `b` differ, like e.g. `(a == b)^c` means that `a == b` is provable under the assumption `c`, with no futher assumptions necessary. This is different from saying `c => (a == b)`, that allows capturing additional assumptions from the environment during theorem proving.

Yet, when saying `(a == b)^true`, which means `a == b` under none assumptions, the differences between `a` and `b` are completely erased. However, nothing prevents one from assuming `(a == b)^true` is the case. As a result, logic can not “reach” the ultimate level of knowledge that is gained by proving `(a == a)^true`, that is, how `a` is symbolic indistinct from `a`. There is no way to distinguish a proof of reflexivity from a proof of tautological equality from within the internal language of some logic that is sound. What is provable without making any assumptions, depends on what information is known or assumed without making any assumptions. Since this knowledge is unknown as a whole, there is no way one can be absolutely sure from within logic that `a == a` is not like `a == b` in some sense, despite how intuitive this knowledge is by looking at the representation of these expressions in the meta-language of logic.

The way this is solved is by using symbolic distinction to “approach” symbolic indistinction. While `a == a` is an impossible level of knowledge within logic, there are many cases where various symbolic distinctions can be ruled out.

There are many cases in mathematics where symbolic distinction is used, e.g. in number theory where `1 = 0.9999999...`. From the inner perspective of number theory, these two representations are indistinguishable. Yet, when encoding numbers as bits of information, the difference in representation can become significant. One can think about the knowledge of `1 = 1` vs `1 = 0.9999999...` as some kind of “infinite” knowledge. The knowledge can be approached gradually, in step by step, yet never reach the goal. This is kind of like a Zeno’s paradox[3].

Therefore, one can reason about symbolic indistinction as a kind of “infinite” logical proposition.

Reasoning about infinity in number theory works symbolically using the symbol `∞`. Yet when one talks about what `∞` means, it becomes a complex topic in mathematics. Similarly, one can reason about reflexivity of propositional equality as something trivial, in the sense of analyzing a text string “a == a”, yet when one talks about what it means from within logic, it becomes non-trivial.

References:

[1] “Symbolic Distinction”
Sven Nilsen, 2021
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip2/symbolic-distinction.pdf

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[3] “Zero’s paradoxes”
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https://en.wikipedia.org/wiki/Zeno's_paradoxes