Split Adjoint Operators

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In this paper we generalize split-imaginary numbers to split adjoint operators.

An split-imaginary number[1] is defined as following:

$$\mathbf{i}^2 = 1$$

By adding a minus sign to the each side:

$$-\mathbf{i}^2 = -1$$

Using Avatar Covers[2], it is natural to use the avatar cover \`xor\` for this product:

$$\mathbf{i} \cdot (-\mathbf{i}) = (-\mathbf{i}) \cdot \mathbf{i} = -1$$

$$\mathrm{mul}[\mathrm{neg}]_a \Leftrightarrow \mathrm{xor}$$

We use the same process as in the paper “Imaginary Adjoint Operators”[3].

Hence, for any symmetric avatar cover \`xor\`:

$$f[g]_a \Leftrightarrow \mathrm{xor}$$

An Split Adjoint Operator \`g\` is defined as the following relation with \`f\`:

$$\exists\ e\ \{\ \exists\ \mathbf{i}\ \{\ f(\mathbf{i}, g(\mathbf{i})) = f(g(\mathbf{i}), \mathbf{i}) = g(e)\ \}\ \land\ \forall\ y\ \{\ f(y, e) = f(e, y) = y\ \}\ \}$$

Here, \`e\` is some unit element of \`f\`.
The element \`**i**\` is an imaginary element.
Notice that \`-1\` is represented as \`g(e)\`.

References:

[1] “Split-complex number”
Wikipedia
https://en.wikipedia.org/wiki/Split-complex_number

[2] “Avatar Covers”
Sven Nilsen, 2020
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/avatar-covers.pdf

[3] “Imaginary Adjoint Operators”
Adam Nemecek, Sven Nilsen, 2020
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/imaginary-adjoint-operators.pdf