Non-Standard Symmetric Binary

by Sven Nilsen, 2022

In this paper I discuss a non-standard binary system related to the balanced ternary system.

A balanced ternary system[1] has bases `D = {-1, 0, 1}` and has the following evaluation:

$$v(d : [D]) = \sum i \{ d_i \cdot 3^i \}$$

A non-standard[2] symmetric binary system has bases `D = {0, ±1}` and has a non-trivial evaluation:

$$v(d : [D], c : [C]) = \sum i \{ |d_i| \cdot c_i \cdot 2^i \}$$

Where `|0| = 0` and `|±1| = 1`.
`C = {-1, 1}` and picks a sign choice for each symbol.

When a symbol in `d` is `0`, the corresponding sign choice is irrelevant.
This means that there are many choices that correspond to the same evaluations.

To represent choices uniquely and efficiently, one can exploit the property that for any `d` there is at most one value `x = v(d, c)` up to the relevance of `c`.

Borrowing the notation from Avatar Logic[3], this is written `x'(d)`.
The sign is factored out, e.g. `+x'(d)` or `-x'(d)`

One reads this as "`x` as `d`".

For example, `2'(6)` is read "2 as 6", where `6` is binary of `D` (`±1, ±1, 0`).
A way to clarify is by reading "2 as 1-avatar of ±1, ±1, 0" which gives `+1, -1, 0`:

$$(+1) \cdot 2^2 + (-1) \cdot 2^1 + 0 \cdot 2^0 = 2$$

d	**avatars**
0	0'(0)
1	±1'(1)
2	±2'(2)
3	±1'(3), ±3'(3)
4	±4'(4)
5	±3'(5), ±5'(5)
6	±2'(6), ±6'(6)
7	±1'(7), ±3'(7), ±5'(7), ±7'(7)
8	±8'(8)
9	±7'(9), ±9'(9)
10	±6'(10), ±10'(10)
11	±5'(11), ±7'(11), ±9'(11), ±11'(11)
12	±4'(12), ±12'(12)
13	±3'(13), ±5'(13), ±11'(13), ±13'(13)
14	±2'(14), ±6'(14), ±10'(14), ±14'(14)

d	**avatars**
15	±1'(15), ±3'(15), ±5'(15), ±7'(15), ±9'(15), ±11'(15), ±13'(15), ±15'(15)
16	±16'(16)
17	±15'(17), ±17'(17)
18	±14'(18), ±18'(18)
19	±13'(19), ±15'(19), ±17'(19), ±19'(19)
20	±12'(20), ±20'(20)
21	±11'(21), ±13'(21), ±19'(21), ±21'(21)
22	±10'(22), ±14'(22), ±18'(22), ±22'(22)
23	±17'(23), ±19'(23), ±21'(23), ±23'(23)
24	±8'(24), ±24'(24)
25	±7'(25), ±9'(25), ±23'(25), ±25'(25)
26	±6'(26), ±10'(26), ±22'(26), ±26'(26)
27	±5'(27), ±7'(27), ±9'(27), ±11'(27), ±21'(27), ±23'(27), ±25'(27), ±27'(27)
28	±4'(28), ±12'(28), ±20'(28), ±28'(28)
29	±3'(29), ±5'(29), ±11'(29), ±13'(29), ±19'(29), ±21'(29), ±27'(29), ±29'(29)

References:

[1] “Balanced ternary”
Wikipedia
https://en.wikipedia.org/wiki/Balanced_ternary

[2] “Non-standard positional numeral systems”
Wikipedia
https://en.wikipedia.org/wiki/Non-standard_positional_numeral_systems

[3] “Avatar Logic”
AdvancedResearch – Summary page on Avatar Extensions
https://advancedresearch.github.io/avatar-extensions/summary.html#avatar-logic