

# Symmetry Forcing

by Sven Nilsen, 2020

*In this paper I describe a property of Avatar Graphs which forces equal objects as graph symmetry.*

In Linear Logic<sup>[1]</sup>, the notion of a resource means that it can only be used once.

Another way to phrase the same constraint is that there is no sharing of the resource.

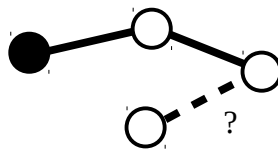
I had a discussion with Cristian Urlea about Avatar Graphs<sup>[2]</sup> having a similar, but weaker, property which can be thought of as a constraint for some Cartesian product<sup>[3]</sup> of two elements  $a$  and  $b$ :

$(a, b)$  if and only if  $a = b$  is not provable by “trivial” reflection

By “trivial” reflection, I mean that  $a$  and  $b$  can not refer to the same node in the graph.

This property is enforced by the graph not being a multigraph, but the property is non-trivial because Avatar Graphs might be thought of as an encoded category<sup>[4]</sup> with initial objects and products. The original category might be a multigraph, by adding new nodes and edges. For simplicity, I will refer to “trivial reflection” as “reflection” and all non-trivial reflections/isomorphisms as “isomorphisms”.

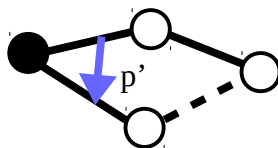
For example, you have a core, a 1-avatar  $a$  and you want to create a 2-avatar using a Cartesian product. However, a 2-avatar requires two 1-avatars. Where do you get the second 1-avatar?



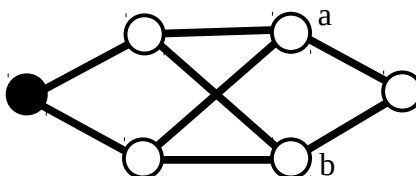
One way to solve this, is to introduce a map from all existing 1-avatars which creates new 1-avatars.

$p'(a)$        $p'(a) \simeq a$       Isomorphisms are used because they generalize naturally

Here,  $p'$  is a map such that an isomorphism to the inner object  $a$  can be proved, but not by reflection.



However, it is possible in a product  $(a, b)$  to reuse 1-avatars in both  $a$  and  $b$ :



With other words, the notion of a resource here means at least one copy of 1-avatars when reused.

An open problem is whether symmetry forcing is vital for the interpretation of Avatar Extensions<sup>[5]</sup>.

## References:

- [1] “Linear logic”  
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[https://en.wikipedia.org/wiki/Linear\\_logic](https://en.wikipedia.org/wiki/Linear_logic)
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[https://github.com/advancedresearch/path\\_semantics/blob/master/papers-wip/avatar-graphs.pdf](https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/avatar-graphs.pdf)
- [3] “Cartesian product”  
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- [4] “Category theory”  
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- [5] “Avatar Extensions”  
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