Answered Modal Logic Catuṣkoṭi

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In this paper we introduce a bridge from Catuṣkoṭi to Answered Modal Logic using quantum systems.

Catuṣkoṭi[1] has multiple applications in the Dharmic traditions of Indian Logic.

Answered Modal Logic[2] is a modal logic for the answered predicate of questions.

Since both Answered Modal Logic and Catuṣkoṭi are 4-value logic, Daniel Fischer suggested that there might be a bridge between them. The problem was to figure out how, if possible, because Answered Modal Logic has multiple involution operators (the basics \`not\` and \`¬\`, with the extension \`!\`).

The solution was to use measurement kinds of quantum systems[3]:

Quantum Systems	**Catuṣkoṭi**	**Answered Modal Logic**	**Cubical Binary Code[4]**
Positive measurement	P; That is being	□	01
Negative measurement	not P; That is not being	¬!◇	10
None measurement	P and not P; That is being and that is not being	!◇	00
Unknown measurement	not(P or not P); That is neither being nor that is not being	¬□	11

The intuition is that a quantum system might be in a definite state, or in superposition. When measured, one gets an answer. The superposition collapses to a definite state after measurement. The only way to not collapse the superposition is by not performing any measurement.

On the other hand, when measuring where a quantum particle is *not*, one might collapse the superposition if sufficient information is received by the negative measurement. Or, the superposition might be at least partially preserved.

This means that the involution on quantum systems is not like the \`not\` in Catuṣkoṭi:

not(positive measurement) = none measurement   not(□) = !◇
not(negative measurement) = unknown measurement   not(¬!◇) = ¬□

Futhermore, the \`not\` in the 4th Catuṣkoṭi is different from the others, resulting in these relations:

not(P) = P and not P   because not measuring a definite state is a none measurement
not P = P or not P   because \`not(P or not P) = not(not P)\`

References:

[1] "*Catuṣkoṭi*"
Wikipedia
https://en.wikipedia.org/wiki/Catu%E1%B9%A3ko%E1%B9%ADi

[2] "Answered Modal Logic"
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https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/answered-modal-logic.pdf

[3] "Measurement in Quantum Mechanics"
Wikipedia
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[4] "Answered Modal Logic in Cubical Binary Codes"
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https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/answered-modal-logic-in-cubical-binary-codes.pdf