Avatar Algebra Symmetry

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In this paper I discuss symmetry in Avatar Algebra.

Avatar Algebra is the idea that theorems in algebra are related to theorems about Avatar Extensions[1].

In the paper "Avatar Algebra Superposition"[2], I discussed the principle of superposition for Avatar Algebra, which follows from the distribution property in normal algebra. I also showed that the introduction operator is naturally associative, since it does not mean anything deeper than introducing new objects. Another way to think about it, is like a concatenated list or a Cartesian product[3].
However, the distributivity property still says something deep:

$$(a + b) \cdot f(x) = a \cdot f(x) + b \cdot f(x)$$

Here, `a + b` means `a` and `b` are related by some symmetry[4].
This symmetry is lifted into a symmetry between two mathematical objects.
In some way, the distributivity property can be used to create copies of mathematical objects.

The question is: What does a symmetry mean?

The simplest kind of symmetry is commutativity[5] of addition:

$$a + b = b + a$$

Therefore, from superposition one gets the following:

$$a \cdot f(x) + b \cdot f(x) = b \cdot f(x) + a \cdot f(x)$$

It means, when `a` and `b` can be swapped, one can also swap the mathematical objects.

This swapping is not the kind that takes two objects and swaps them *physically*.
Instead, it might be thought of as *reconstructing* the two objects such that they look swapped.
However, in many situations, one can exchange one kind of swap with another.

Hence, usually this kind of symmetry is *sufficient*.

References:

[1] “Avatar Extensions”
AdvancedResearch – Reading Sequence on Path Semantics
https://github.com/advancedresearch/path_semantics/blob/master/sequences.md#avatar-extensions

[2] “Avatar Algebra Superposition”
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https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/avatar-algebra-superposition.pdf

[3] “Cartesian product”
Wikipedia
https://en.wikipedia.org/wiki/Cartesian_product

[4] “Symmetry in mathematics”
Wikipedia
https://en.wikipedia.org/wiki/Symmetry_in_mathematics

[5] “Commutative property”
Wikipedia
https://en.wikipedia.org/wiki/Commutative_property