Trivial Commutative Symmetry

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A function `f` has trivial commutative symmetry for some symmetry operator `g` when:

$$f[g \times g \to id] \iff f$$

For example:

$$mul(neg(a), neg(b)) = mul(a, b)$$

Notice that since the arguments are not swapped,
this also holds when multiplication is anti-commutative.

In exterior algebra, the exterior product is anti-commutative, yet it has trivial commutative symmetry:

$\therefore \quad ((-a)\mathbf{e_1} + (-b)\mathbf{e_2}) \wedge ((-c)\mathbf{e_1} + (-d)\mathbf{e_2})$
$\therefore \quad ((-a)(-d) - (-b)(-c))\ \mathbf{e_1} \wedge \mathbf{e_2}$
$\therefore \quad (ad - bc)\ \mathbf{e_1} \wedge \mathbf{e_2}$

$\because \quad (a\mathbf{e_1} + b\mathbf{e_2}) \wedge (c\mathbf{e_1} + d\mathbf{e_2}) = (ad - bc)\ \mathbf{e_1} \wedge \mathbf{e_2}$