Adjoint Trivial Paths

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In this paper we introduce adjoint trivial paths for path semantics.

An adjoint trivial path is one of the two following:

$\forall_0(g_0, h)$ such that $\forall$ a : [$\forall_0(g_0, h)$] true, b : T { $h(a, h(g_0(a), b)) == b$ }
$\forall_1(g_1, h)$ such that $\forall$ a : T, b : [$\forall_1(g_1, h)$] true { $h(h(a, g_1(b)), b) == a$ }

$\forall_0 \iff \forall_{adj}(0)$ $\forall_1 \iff \forall_{adj}(1)$
$\forall_{adj} : \mathbb{B} \to (T \to T) \to (T \times T \to T) \to (T \to \mathbb{B})$
$g : T \to T$ $h : T \times T \to T$

Where `$\forall_0(g_0, h)$` is called "left adjoint trivial path",
and `$\forall_1(g_1, h)$` is called "right adjoint trivial path".

An adjoint trivial path refers to the largest set which has this property.
When an adjoint trivial path is said to exist, one means that the adjoint trivial path is not `\false`.
For self-adjoints operators `$g_0 \iff g_1$`, one can use `$\forall_{adj}(g, h)$`.

It follows that when `h` is associative, it has a left and right adjoint norm (distinct from vector norm):

∵ $h(a, h(b, c)) == h(h(a, b), c)$ `h` is associative

∴ $h(a, h(g_0(a), b)) == b$
∴ $h(h(a, g_0(a)), b) == b$
left_adjoint_norm(g_0, h)(a) := h(a, g_0(a))

∴ $h(h(a, g_1(b)), b) == a$
∴ $h(a, h(g_1(b), b)) == a$
right_adjoint_norm(g_1, h)(b) := h(g_1(b), b)

When `left_adjoint_norm($g_0$, h) <=> right_adjoint_norm($g_1$, h)`, one can use `adjoint_norm($g_1$, h)`.

Negation and addition has adjoint trivial paths that covers all values:

$\forall_{adj}$(neg, add) <=> \true adjoint_norm(neg, add) <=> \0

Complex conjugate and complex multiplication:

∵ \(a : $\mathbb{C}$, b : $\mathbb{C}$) = adjoint_norm(conj, $mul_{\mathbb{C}}$)(a) · b == b Start with ` $h(a, h(g_0(a), b)) == b$`
∴ \(a : $\mathbb{C}$, b : $\mathbb{C}$) = $(re(a)^2 + im(a)^2) \cdot b == b$
∴ \(a : $\mathbb{C}$, b : $\mathbb{C}$) = $re(a)^2 + im(a)^2 == 1$
∴ \(a : $\mathbb{C}$) = $re(a)^2 + im(a)^2 == 1$ Remove parameter `b`
∴ $\forall_{adj}$(conj, $mul_{\mathbb{C}}$)