Answered Modal Logic

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In this paper I introduce a modal logic for the answered predicate of questions.

The meta-knowledge of the answer of a question can be modeled using a set of the following symbols:

{□, ◇, ¬◇}	All possible states of knowledge about question (*unknown* **unanswered**)
□	The question is answered (*known* **answered**)
◇	There exists a case where the question is answered (*unknown* **answered**)
¬◇	There exists no case where the question is answered (*known* **unanswered**)

Inversion laws:

¬□ = {◇, ¬◇}
¬¬□ = {□}
¬◇ = {¬◇}
¬¬◇ = {◇}

When a law is in the form `X => {□, ◇, ¬◇}Y` one can choose:

◇(X => □Y)
◇(X => ◇Y)
◇(X => ¬◇Y)

Here, the `◇` operator reflects on the semantics of the logic itself.
When used this way, it is not an operator of questions directly, but as a meta-operator.

Notice that this logic deviates from epistemic modal logic, which uses semantics "it is known *that* X".
Here, the logic refers to the knowledge of the answer, without describing what the answer is.

For example:

□"A ∧ B" => ◇"A"

This can be read as "If I know value of `A ∧ B`, then there exists a case where I know value of `A`".

In general, the internal semantics of the questions is irrelevant for this logic.

Instead, the questions are treated as black boxes, with partial knowledge described e.g. in the form:

□X => ◇Y

It is the partial knowledge described using this modal logic that can derive other partial knowledge.
The internal semantics of the questions is only relevant for grounding the initial partial knowledge.

I will now prove the following:

$\therefore \quad (\Box X => \Diamond Y) => \Diamond(\Diamond X => \Diamond Y)$

$\because \quad \Box X => \Diamond Y$
$\therefore \quad \neg\Diamond Y => \neg\Box X$
$\therefore \quad \neg\Diamond Y => \{\Diamond, \neg\Diamond\}X$
$\therefore \quad \Diamond(\neg\Diamond Y => \neg\Diamond X)$ Choosing `¬◇` among possible interpretations
$\therefore \quad \Diamond(\Diamond X => \Diamond Y)$
$\therefore$ Q.E.D.

When choosing the other possible interpretation:

$\therefore \quad \Diamond(\neg\Diamond Y => \Diamond X)$
$\therefore \quad \Diamond(\neg\Diamond X => \Diamond Y)$

$\therefore \quad (\Box X => \Diamond Y) => \Diamond(\neg\Diamond X => \Diamond Y)$

These two possible interpretations are not contradictory.
When there exists a case where `X` is answered means `Y` is *unknown* **answered**,
it also possible that when `X` is *known* **unanswered** that `Y` is *unknown* **answered**.
Answering `X` or not does not change the answer of `Y`.