Higher Order De Morgan's Laws for Unary Binary Functions

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In this paper I represent a higher order version of De Morgan's laws for unary binary functions.

Given two unary binary functions `f` and `g`, their symmetric paths by `not` have the following laws:

 $f[not] \land g[not]$ <=> $(f \lor g)[not]$ $f[not] \lor g[not]$ <=> $(f \land g)[not]$ $f: bool \rightarrow bool$ $g: bool \rightarrow bool$

The laws were checked using an experimental automated theorem prover:

```
false1[not] \( \) false1[not]
                                    <=>
                                                (false1 v false1)[not]
                                                                                    ok
false1[not] \( \text{not[not]} \)
                                                (false1 v not)[not]
                                    <=>
                                                                                    ok
false1[not] \land id[not]
                                    <=>
                                                (false1 v id)[not]
                                                                                    ok
false1[not] \( \text{true1[not]} \)
                                    <=>
                                                (false1 v true1)[not]
                                                                                    οk
not[not] ∧ false1[not]
                                    <=>
                                                (not v false1)[not]
                                                                                    ok
not[not] \land not[not]
                                    <=>
                                                (not v not)[not]
                                                                                    ok
not[not] ∧ id[not]
                                    <=>
                                                (not v id)[not]
                                                                                    ok
not[not] ∧ true1[not]
                                    <=>
                                                (not v true1)[not]
                                                                                    ok
id[not] ∧ false1[not]
                                                (id v false1)[not]
                                    <=>
                                                                                    ok
id[not] ∧ not[not]
                                    <=>
                                                (id v not)[not]
                                                                                    ok
id[not] \land id[not]
                                    <=>
                                                (id v id)[not]
                                                                                    ok
id[not] ∧ true1[not]
                                                (id v true1)[not]
                                                                                    ok
true1[not] \( \text{false1[not]} \)
                                                (true1 v false1)[not]
                                                                                    ok
                                                (true1 v not)[not]
true1[not] \land not[not]
                                    <=>
                                                                                    ok
true1[not] \( \text{id[not]} \)
                                                (true1 v id)[not]
                                                                                    ok
true1[not] \( \text{true1[not]} \)
                                    <=>
                                                (true1 v true1)[not]
                                                                                    ok
false1[not] v false1[not]
                                    <=>
                                                (false1 ∧ false1)[not]
                                                                                    ok
false1[not] v not[not]
                                    <=>
                                                (false1 \land not)[not]
                                                                                    ok
false1[not] v id[not]
                                    <=>
                                                (false1 \(\lambda\) id)[not]
                                                                                    οk
false1[not] v true1[not]
                                                (false1 ∧ true1)[not]
                                                                                    ok
not[not] v false1[not]
                                    <=>
                                                (not \land false1)[not]
                                                                                    ok
                                                (not \land not)[not]
not[not] v not[not]
                                    <=>
                                                                                    ok
not[not] v id[not]
                                    <=>
                                                (not \land id)[not]
                                                                                    ok
not[not] v true1[not]
                                    <=>
                                                (not ∧ true1)[not]
                                                                                    ok
id[not] v false1[not]
                                                (id ∧ false1)[not]
                                    <=>
                                                                                    οk
id[not] v not[not]
                                    <=>
                                                (id \land not)[not]
                                                                                    ok
id[not] v id[not]
                                                (id \land id)[not]
                                    <=>
                                                                                    ok
id[not] v true1[not]
                                                (id ∧ true1)[not]
                                    <=>
                                                                                    ok
true1[not] v false1[not]
                                    <=>
                                                (true1 \land false1)[not]
                                                                                    ok
true1[not] v not[not]
                                    <=>
                                                (true1 \land not)[not]
                                                                                    ok
true1[not] v id[not]
                                    <=>
                                                (true1 \land id)[not]
                                                                                    ok
true1[not] v true1[not]
                                    <=>
                                                (true1 \( \true1 \)[not]
                                                                                    ok
```