The Invisible Third Player in Partial Reversible Games

by Sven Nilsen, 2018

Partial reversible game theory is an extension of traditional game theory where the assumption of irreversibility is weakened. What makes these kind of games special is that the model of the world, required to behave rationally, might be arbitrary complex. In this paper I formalize partial reversible games in the framework of traditional game theory as reasoning about the strategy of an invisible third player who's payoffs are partially unknown and decision made after the first two players. I also show that any complete decision theory of partial reversible games requires a finite type for rewards.

A partial reversible game can be thought of as a reward matrix with two components. The first component is an irreversible reward. The second component is a reward kept only if the game is NOT reversed. A such reward matrix might look like the following for two players A and B:

A/B (irreversible, reversible)	**A1**	**A2**
B1	1, 3 / 1, 3	-1, 3 / -2, 3
B2	0, 4 / 0, 5	5, 0 / 4, 0

One imporant assumption often studied in traditional game theory is when both players behave rationally. They both want to maximize their rewards given the available information before making their decisions.

To formalize partial reversible games in traditional game theory, I split the reward matrix into two parts, where the third player makes decision 1 and 2 (C1 and C2) after A and B made their decisions:

A/B//C1	**A1**	**A2**
B1	1 / 1 // ?	-1 / -2 // ?
B2	0 / 0 // ?	5 / 4 // ?

A/B//C2	**A1**	**A2**
B1	4 / 4 // ?	2 / 1 // ?
B2	4 / 5 // ?	5 / 4 // ?

C1 corresponds to a reversed game and C2 corresponds to an irreversed game. Notice that the rewards in the second matrix is the sum of the rewards in the partial reversible game. The `//` operator has lower presedence than `/` and expresses the group order of making decisions. A and B first make their decisions, then C makes its decision.

Another way of expressing C's information is the reward matrix of A and B plus knowledge of the decisions made by A and B. This is an advantage C has over A and B when playing rationally.

There are 3 columns per cell in the matrix, with `?` inserted for the third player. This is because I will exploit a mathematical trick to embed partial reversible games in traditional game theory. The trick is to model any condition for reversing the game as a rational player, which knowledge about rewards (the `?`s) can be thought of as a vector `$R_c$` (for player C) of a path semantical sub-type:

R_c : [f] t

f : [real] → T

`[real]` stands for a vector with real components. The `T` above stands for some arbitrary type.

The meaning of what is said is grounded in the mathical language framework of path semantics. This is called a “sub-type”, by specifying some function `f` which takes the reward vector and computes some value `t` of type `T`.

With other words, something is said about the third player’s reward vector. The players A and B need to understand what is said, which sets requirement on complexity of their world model in order to behave rationally.

A logician might prefer to replace `f` with a boolean function `g`:

[f] t <=> [g] true

g := \(x : [real]) = f(x) == t
g : [real] → bool

Semantically it might be beneficial to use the `[f] t` notation. One can think of the function `f` as a class of games where `t` is a learned rational behavior within that class. When `f` is known, one can restrict games to a specific domain where *rational behavior* is well defined. It avoids the problem of defining rational behavior for all models of the world. Yet, one means to say that rational behavior is defined for *some* function, without necessary revealing the function’s identity in the generic case.

This is also a proof that the model required to behave rationally can be *arbitrary complex*. The number of functions of type `[real] → T` is uncountable. However, by replacing `[real]` with some finite type, e.g. `[nat ∧ (< 100)]` the set of functions becomes finite and the model required might be finite.

So, when dealing with partial reversible games, in order to speak of rational behavior one is required to have some match between the A and B players’ model of the world and some language that expresses the third player’s reward vector. Since the set of functions of type `[real] → T` is uncountable, any complete decision theory of finding a such match in general is *incomputable*. Therefore, if a decision theory can be formalized, it is only defined for *some* `f`. However, by replacing `[real]` with some finite type, e.g. `[nat ∧ (< 100)]` the set of functions becomes finite and the decision theory decidable.

The assumption that the third player is rational can be weakened. This happens when the strategy of the third player C is indeterministic. Since this kind of weakened assumptions also holds for players A and B, there is a one-to-one relationship with a sub-set of traditional game theory. Whatever mathematical tools that are developed under traditional game theory, might be used to prove theorems of partial reversible games, as long the translation from the first form to the second form is found.