Split Adjoint Operators

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In this paper we generalize split-imaginary numbers to split adjoint operators.

An split-imaginary number[1] is defined as following:

$$\mathbf{j}^2 = 1$$

By adding a minus sign to the each side:

$$-\mathbf{j}^2 = -1$$

Using Avatar Covers[2], it is natural to use the avatar cover `xor` for this product:

$$\mathbf{j} \cdot (-\mathbf{j}) = (-\mathbf{j}) \cdot \mathbf{j} = -1$$

$$\mathrm{mul}[\mathrm{neg}]_a \Leftrightarrow \mathrm{xor}$$

We use the same process as in the paper “Imaginary Adjoint Operators”[3].

Hence, for any symmetric avatar cover `xor`:

$$f[g]_a \Leftrightarrow \mathrm{xor}$$

An Split Adjoint Operator `g` is defined as the following relation with `f`:

$$\exists\, e \;\{\; \exists\, \mathbf{j} \;\{\; f(\mathbf{j}, g(\mathbf{j})) = f(g(\mathbf{j}), \mathbf{j}) = g(e) \;\} \;\wedge\; \forall\, y \;\{\; f(y, e) = f(e, y) = y \;\} \;\}$$

Here, `e` is some unit element of `f`.
The element **`j`** is a split-imaginary element.
Notice that `-1` is represented as `g(e)`.

References:

[1] "Split-complex number"
Wikipedia
https://en.wikipedia.org/wiki/Split-complex_number

[2] "Avatar Covers"
Sven Nilsen, 2020
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/avatar-covers.pdf

[3] "Imaginary Adjoint Operators"
Adam Nemecek, Sven Nilsen, 2020
https://github.com/advancedresearch/path_semantics/blob/master/papers-wip/imaginary-adjoint-operators.pdf