The Inquiry for Semantics of Choices

by Sven Nilsen, 2019

The world around us is described with modern technology using bits of information, yet theoretical physics uses quantum logic to more accurately describe how nature behaves. The Turing machine is fundamental for theoretical understanding of computers, yet mathematical foundations require semantics going way beyond the computable. What are those gaps between practical models and theoretical understanding? It turns out that this boundary is defined by choices.

What are choices? How do we understand choices mathematically versus physically?

In one way a choice is very simple, in another way it is very deep and complex.

A choice can be a state, for example:

x : bool

Here, `x` can have values `false` or `true`. Which concrete value `x` has is a choice.

In any physical system, a state has a cause which can be examined by possible histories. Therefore, there is no need to prove anything beyond observing previous states of the physical system and predict future states using a scientific theory.

It is implicitly understood from what we know about the physical world that choices always are caused by something, even if the cause might be a random quantum fluctuation. We predict that the future will not stop to "think" about choices at the particle level of reality. A human person might think about choices, yet the thinking is caused by physical particles and not by some magical abstract substance representing the act of making choices.

However, even if physics happen in a predictable way, what do we mean when we talk about this process going on over and over from a mathematical perspective?

In Adversarial Path Semantics, an Adversarial Path decouples the commitment of a choice from the information required to finish the choice:

A ~ 0 : T → A ~ 1

This is interpreted the following way:

(A ~ 0) : T → (A ~ 1)

An adversarial path consumes a resource, then "feed in" new information to produce a new resource.

If `A ~ 0` means "I am going on a vacation", then `T` could mean the vacation destination, e.g. New York and `A ~ 1` could mean what activities I can do in New York. With other words, one can make a choice without knowing the final outcome of that choice, yet knowing some choice has been made.

By formalizaing choices as adversarial paths, it is possible to reason about their meaning without a full definition. Adversarial paths can be combined to create more complex adversarial paths.

An important property of how Turing machines differ from informal theorem proving is the following:

Turing machines eliminate choices, but informal theorem proving do not eliminate all choices

When a program executes on a Turing machine, there is no “loose end” which makes the behavior undefined. The machine simply continues to run, executing one instruction after another.

Under informal theorem proving, there is no way to tell how the whole proof should be written. A person doing informal theorem proving might use a bag of tricks, some more efficient than others.

The difference between these two modes of execution, is that informal theorem proving contains some state where it is undefined how the information arrives such that the choice was finished. Informal theorem proving commits to choices, just as Turing machines do, but the way it finishes is not fully specified.

Semantics of choices is very important for motivation of mathematical and programming language design. While Turing machines are able to perform any computation desirable, the meaning of languages in general depends heavily on choices.

For example, when somebody asks for your name, you could think of it as a program taking two strings and e.g. storing it in a file. However, it could also be audio records of you introducing yourself. These records might be sent to a different computer etc. The meaning of “somebody asks for your name” has a lot of implicit and complex choices built into it, even when talking about it in the context of possible computer programs.

Without the ability to use and build abstractions based on choices, there would be no human general intelligence. It is a central part of how languages work which makes it possible to communicate and collaborate efficiently.

From a mathematical perspective, there seems to be no reason that choices should be “special”. One can construct arbitrary choices and study them like any other mathematical object. Yet, we depend on them heavily in practical applications of mathematics.

In psychology, the semantics of choices could play a vital role in understanding how consciousness work. People make choices and this become part of their identity over time. One choice leads to another, but there is no well defined boundary between the person making the choice and the rest of the world. How is the experience of making a choice related to the physical world? Can consciousness be simulated by a Turing machine?

For example, if choices in physics is defined in a way such that certain sub-types of bounded Turing machines can not simulate them, it might lead to a new worldview. This could be e.g. definition of quantum randomness, expanding universe etc.

Figuring out semantics of choices grounded in the physical world is even harder than figuring out semantics of choices from a mathematical perspective. For example, an extreme anthropic observer selection effect might change the semantics of an underlying physical theory.