The Classical Model of Path Semantical Qubit

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In this paper I explain how the classical model of path semantical qubit works.

In the paper "Path Semantical Qubit"[1], I introduced the qubit operator for use in Path Semantics. The qubit operator might be thought of as the most central operator in entire Path Semantics, due to path semantical quality being possible to define using normal logic in combination with the qubit operator. In turn, path semantical quality is necessary for the core axiom of Path Semantics[3].

Since the qubit operator is central to Path Semantics, it is important for path semanticists to have a working understanding of how the qubit operator works in various models.

Perhaps the most important model is in constructive logic[4], as it is a superset of classical logic. However, constructive logic and classical logic comes with very different notions of what kind of models one is talking about:

- In constructive logic, the underlying notion is about proof models
- In classical logic, the underlying notion is about parallelism of Boolean algebra[5]

It might come as a surprise to some people that classical logic is usually about parallelism of Boolean algebra. This is because most people think about classical logic from the perspective of First Order Logic[6]. In First Order Logic, one thinks about relations and predicates. However, the underlying language for First Order Logic is Zeroth Order Logic, or classical propositional logic. One can obtain this language by adding the axiom of excluded middle to constructive propositional logic, also known as intuitionistic propositional logic[4].

In a brute force theorem proving model of classical propositional logic, there are no branches in the computational processing. This means, one can generate bit vectors for every proposition and check a proof by a single pass.

Consider the following truth table of logical AND:

A	**B**	**A ∧ B**
0	0	0
0	1	0
1	0	0
1	1	1

One way of processing is top to bottom, where one sets every possible state of A and B.
Another way of processing is left to right, where A is a bit vector `0011` and B is `0101`.
The AND operator might be thought of as a parallel instruction that reads all bits of A and B and outputs new bits as a bit vector. The output depends on the input, but in a such way that the individual bits within a bit vector do not interfere with each other. This is possible only when there are no conditional branches that depend on the order of bits.

A parallelism perspective of classical propositional logic is the key to understand the qubit operator.

Path semanticists consider the qubit operator to be “natural” in mathematics and logic. This is because it has very important mathematical properties and Path Semantics as a field builds the entire theory on this single operator. In a way, one can easily distinguish path semanticists from other mathematicians and logicians by whether they consider the path semantical qubit operator important or not. The fact that the qubit operator is a non-trivial mathematical object, has multiple models, e.g. depending on whether one works with Type Theory for constructive logic or a brute force theorem prover in classical logic, or other languages, shows how deep and rich the theory of Path Semantics has been developed from its humble beginnings. However, to outsiders who never heard about the qubit operator and struggle to form a big-picture perspective of Path Semantics, it might seem very strange that there is so much focus on the qubit operator, quality and the core axiom. In heated discussions about Path Semantics, the problem is usually not that path semanticists are wrong about something. Most frequently, the outsiders are simply not aware about certain aspects of logic that path semanticists take for granted. This means that in order to understand the basics of Path Semantics, people have to study these aspects of logic that they did not know about.

The qubit operator is different from every other operator in normal logic. This is an indisputible fact and can be demonstrated easily due to its property of tautological congruence. However, this part of approaching the basics of Path Semantics, when being confronted with the qubit operator as a new idea, e.g. breaking the intuition one develops from First Order Logic, is the major obstacle that logicians and mathematicians face when learning Path Semantics. Think about it like this: How would you feel if, in your entire life, you thought you understood something essential about mathematics and one day you discover that not only were you ignorant about how it worked, you also discover that there is an entire world of math right around the corner that you did not see? It is not even like discovering an elephant in your living room one day, but more like an entire zoo, and yet at the same day one discovers a portal to a multiverse in your closet? Furthermore, the personal identity that people develop as part of having a lifelong experience in math and logic, how they see themselves and how they think about people seeing them, contributes to the trauma of experiencing a surprising perspective of mathematics that puts everything they know into question.

Still, as shocking this surprise is to some people, this psychological breakthrough of Path Semantics as a theory, is not something that path semanticists worry about on a daily basis. They go with the flow, having learned that there is much more to mathematics, even at fundamental level, perhaps more than humans can possibly know or understand. Every person who learns Path Semantics goes through this psychological process. Once one gets used to the qubit operator, there is no going back. What is seen can not be unseen. When you get to the other side, where the qubit operator is taken for granted, you settle down and one day see a path semanticist in the mirror: A person who thought normal logic was the terminal station for a foundation of mathematics, when it turned out as just a speck of dust, or a grain of sand, in the infinite desert of Path Semantics. It is not just about discovering the incredible level of ignorance about logic, but also accepting and embracing it.

There are many models where Boolean algebra does not require parallelism, e.g. most mainstream programming languages. The particular form of parallelism that is possible in classical propositional logic is a peculiarity that is not easily perceived. So, when learning about the qubit operator for the first time, there is a kind of pre-shock which preceds the later major shock of Path Semantics. At this pre-shock stage of understanding logic, one perceives classical propositional logic as more universal than it actually is. The parallelism property of classical propositional logic removes this illusion. It demonstrates that classical propositional logic is not as general and universal as once thought, despite having incredible power as a logical language. Classical propositional logic is not even very universal with respect to Boolean algebra! However, one has no time to dwell on this property for too long or its philosophical consequences. To even understand the qubit operator in the classical model, one must embrace this perspective of parallelism, despite knowing it is not the full story and there is so much more to explore.

Perhaps you are one of the few people who knew about this parallelism property of classical propositional logic. However, it is more likely that you learned about this here for the first time. You see, the way most people are taught classical logic is by a set of axioms and rules of inference. This is a language bias that hides the axiom-free underlying theory. If you knew about the parallelism property before the axioms and inference rules, then you might also understand that every axiom and inference rule can be deduced directly from this property. It is not only that most people's level of ignorance excludes some peculiar property of the very language they are studying, but they also do not understand the power and simplicity that comes from it, that overshadows their own knowledge. Not only is everything that people think they know not enough, but everything they already know can be replaced by something they did not know. Try to emphasize with the psychological process these people have to go through: Their knowledge was completely redundant and insignificant, after putting a lot of work into it to get there. Yet, discovering this level of ignorance is not even about making significant advancements in the understanding of mathematical foundations. It is more like trying to take the first steps, to learn to walk, as a path semanticist.

Now, I assume you got built up an impression that not only does the parallelism property of classical propositional logic show that the language is limited, but also that it is the very defining property out of which everything else in this language is a mere consequence. You have changed your perspective of logic completely, like a meditating munk, or nun, on top of a mountain, ready to reach the new stage of enlightenment by having your mind emptied of all non-sensical thoughts about axioms and inference rules. The paralllelism property has crystallized in your mind, as the single object of focus, to which an entirely new insight will occur:

What if: You introduce a unary operator that depends on the entire input bit vector?

Now, you might not have thought this through. I asked you to embrace the perspective of parallelism and what you are asked to do now, is breaking this property completely. Smash it, destroy it, take the straw of hope that you have grasped, giving up your old intuition of classical propositional logic, and burn it! Are you really willing to pay this price? A Platonic everything?

This is where people pass through the singularity of their perspective of mathematics and become path semanticists. Not only does this new insight break the deep secret of logic that was hidden beneath the shallow surface that people perceived as perfect knowledge, but that this hidden secret is sacrificed in a such radical manner that completely obliterates what you thought you knew about mathematics, twice. Yet, the reason people become path semanticists, is not due to loss of hope of understanding mathematics, after understanding their own position. The reason is what comes next.

One obstacle to understanding the qubit operator in the classical model, is the intuition one develops after a lifelong experience of programming. Most people do not benefit from this because they have not been programming for that long. Even most people with a lifelong experience in programming, never get this insight. To expect that people who just were taught some axioms and inference rules in a university during a logic class, to get this level of intuition, is unreasonable. To these people, it is not understandable why lifelong programmers see what follows as a thrill: They never developed the intuition that depending on the entire input bit vector, is basically impossible.

From an axiomatic perspective, depending on the entire input bit vector seems not particular difficult. Similar stuff is done in other areas of mathematics all the time. What is the big deal? However, from a brute force theorem perspective, or the perspective of classical propositional logic as parallelism, this is a major undertaking. It involves questions about finite number of arguments in proofs, versus infinite number of arguments. It seems to present a such enormous challenge to conceptualize, that one can hardly grasp how any such axioms would make sense. It is an intellectual challenge of proportions that seems to defy the gravity, the natural laws of logic.

Yet, for path semanticists, this is a child's play. The idea of using parallelism to leverage a useful perspective and later sacrifice it completely to obtain something new, is not a particular uncommon experience when studying Path Semantics. You have not seen very much of what path semanticists do, nor understood what it all is about, before such techniques become common in your toolbox. The Platonists viewing mathematics through axioms and inference rules, on the other hand, might feel a bit pride in not getting their hands "dirty" through taking such perspectives. They have not understood the value of thinking like a Seshatist[7] yet. Poor Platonists, who do not get along Seshatists, can never get out of their comfort zone. This is why path semanticists embrace their own level of ignorance: Challenging mathematical comfort zones is just something you need to do.

It turns out, that if this operator, depending on the entire input bit vector, outputs random bits which are order free, that means one can not detect any imperfections in the noise that can reveal how the dependence on the input bits work, then there exists a choice of dependence on the input bits such that all the problems with the intuition, from a lifelong programming experience, goes away.

If your perspective is only through axioms and inference rules, then you can just forget ever appreciating this. What is needed is hands-on experience. You need to consult somebody with a lot of programming experience to explain you why this is an incredible mathematical property. No amount of pride can teach you the lessons of humility. Theoretically, the output depends on the entire input bit vector, but in practical implementations, the programmer writing the code makes an arbitrary choice that seemingly is at hand, no matter how complex the model of the qubit operator is needed under various circumstances. You have to appreciate this from both the impossible theoretical perspective and the incredible lightness and simplicity of the practical perspective. The leverage of both perspectives is optimized, forging a single coherent mathematical object.

It is as if the munk, or nun, who meditates at the top of the mountain, after finding the focus of all perfect knowledge, reaches out the hand and bends a single grass straw. A sort of mysterious act, performed and seen in relation to the focused mind. This act would look different in different parallel universes, yet has some kind of deeper meaning to it, a unity beyond each individual universe. These munks and nuns in the parallel universes behave as one, they become one in relation to their focus.

A such act is done to model the axiom `¬~a == ~¬a`, also called "Sesh". This Sesh property makes outputs of the qubit operator behave like classical propositions. In the constructive model, Sesh is not enough to get decidability, but it is a step in this direction. Here, in the classical model, the programmer checks an arbitrary bit of the input, for example the first bit, and flips all bits if e.g. this bit is `1`. After generating the random bits, the bits are flipped back, such that there are two inputs and two outputs that align with each other, like a splicing DNA helix creating a copy of itself from each part. From the outside, there is no way to detect which bit was checked before flipping the bits when processing all bits in parallel. Furthermore, even when there is only a single bit processed at a time, this is enough. The single bending of a grass straw, once in every parallel universe, is enough.

You might now have an impression of how deep one must go in order to appreciate the qubit operator both from a theoretical and practical perspective. Yet, it does not stop there: Just ask yourself this question "What does the output mean?". It does not mean anything. Or, it can take on any meaning. In fact, there is no proposition that is symbolic indistinct from it, nor entirely symbolic distinct from it, except from the aligned ones by Sesh. Actually, the qubit operator introduces an entirely new proposition, like a virtual particle, excited into a real particle by forming relations to other particles. The notions of observation and of measurement, become consequences of this idea. The pure classical picture of logic as a perfect world, shattered into a mixture of perfect worlds and probabilistic worlds. In a such way that "probably true" is both problematic and sensible at the same time. A new multiverse has been born from a single operator. The qubit.

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